Discussion Paper Series – CRC TR 224

Discussion Paper No. 303
Project B 04, C 03

Strategic Pricing and Ratings

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July 2021

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Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Strategic Pricing and Ratings*

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July 2, 2021

Abstract:

A seller serving two generations of short lived heterogeneous consumers sells a product under uncertain demand. We characterize the seller's optimal pricing, taking into account that the current period's price affects the information transmission to the next period consumers via consumer ratings. While the seller always prefers to generate more information, it is not necessarily in the consumers' interest. We characterize situations in which consumer surplus and welfare are decreasing in additional information. We provide conditions under which aggregate consumer surplus and welfare are lower with than without a rating system.

JEL-classification: D83, L12, L13, L81.
Key words: Online Markets, Rating, Reputation

1. Introduction

Information asymmetries between buyers and sellers about the quality of goods and services or of the associated transaction are a common phenomenon, and take on critical importance in anonymous markets, in particular online markets, where buyers and sellers do not meet in person. Apart from the costly service of independent certifiers that are employed by sellers or buyers for only a few goods, the most credible source of information which allows to mitigate these asymmetries are arguably past buyers. While the information provided by them is conditional on their typically unobserved tastes, it reflects a common interest in purchasing the good. Most importantly, the information is credible because its provision tends not to be biased by self-interest. Yet sellers influence this information channel by their strategic choices, and in particular their pricing decisions. It is of natural importance to understand the implications on the generation of market information, and on seller profits and consumer surplus, as well as on welfare.

Consider a long lived seller who enters a market with a new product of exogenously determined binary quality. This quality is unknown *ex ante* to both the seller and consumers.[1] Consumers

*Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 (Project B04 and C03) is gratefully acknowledged.

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[1]The common uncertainty reflects the seller's noisy perception of the demand function, which is directly induced by consumers' uncertainty.

are short-lived, i.e. active in only one of two periods, demand at most one unit, and differ in their willingness to pay. Consumers that purchase, and only those, leave reviews that are available either individually or in some aggregate to future consumers.[2] In this setting, the early period's consumers' priors are exogenous (but assumed to contain all information available in the market), while the late period's consumers' priors depend on the information contained in the ratings.

A key observation is that individual consumers' reviews are heterogeneous in terms of their informational content. While setting a low price will always increase the quantity sold by the seller and with it the number of reviews, heterogeneity in consumers' expertise implies that the information available to future consumers does not necessarily increase. In particular, if expertise is negatively correlated with willingness to pay, a lower price leads to the inclusion of less informative individual reviews, especially when reported in aggregate form.[3] This tends to lower the informativeness, in particular of an aggregate rating statistic.[4] When analyzing the strategic choices of the seller, it is therefore important to allow for both possibilities.

We capture these considerations by allowing the generated information to be positively or negatively associated with increased sales in a given period. We analyze the strategic decisions of the seller, taking into account the current period's decisions' impact on information transmission and future market outcomes. This allows us to address questions such as whether it is in the seller's interest to enhance or restrict information generation, how the seller achieves this, and whether consumers benefit or are harmed by the intertemporal information transmission.

This analysis is nontrivial for several reasons. Both the consumers' perception of the good's quality and the seller's perception of demand change with the revealed information. From the consumers' perspective, more information allows them to make better informed decisions, which increases their surplus. However, the firm's strategic actions also reflect the revealed information, which may harm consumers. From the seller's perspective, it is unclear *a priori* whether the benefits from taking strategic choices contingent on the revealed information outweigh potential costs from consumers' choices reflecting the same information, and whether the benefits from learning that the good is of high quality outweigh the costs of learning that it is not. The relative importance of these considerations in turn is affected by multiple factors, such as the difference in buyers' willingness to pay for high and low quality goods and associated market demand, and the cost structure of the firm when supplying different quantities to the market.

Within a static equilibrium analysis involving just one period, we show that it is *always* beneficial for the seller when the information available to the market improves prior to his strategic in-period choices. This holds in spite of the possibility that the outcome, by revealing that the good is of low quality, is unfavorable.

By contrast, this is *not necessarily* the case for the consumers. Consumers incorporate the revealed information into their decision whether to purchase, which increases consumer welfare. At the same time, the resulting strategic adjustment by the seller in terms of the induced

[2]The communication of ratings via aggregates opens the possibility to manipulate the aggregation in the interest of either party. We ignore this strategic aspect in the present analysis.

[3]See, e.g., Moe and Schweidel (2012) and Li and Hitt (2008) in their empirical analysis.

[4]The problem is relevant even when the reviews are reported individually, unless the reviews are dated and the typical consumer is able to account for this within a fully Bayesian formation of her prior.

equilibrium price-quantity-combination may more than offset these gains. Notably, this may not only lead consumer surplus to be adversely affected, but also total welfare. We provide conditions under which increases, and losses in consumer surplus and welfare materialize.

These observations have direct implications on the effects involving the intertemporal transmission of information. By increasing the information provision, the seller always sacrifices early-period profits to enhance late-period profits. It does so by distorting the equilibrium price-quantity combination away from the optimum, with the direction of the distortion depending on the relationship between the quantity sold (consumers served) and the degree of information transmission. When information transmission is enhanced by increased sales, early consumers benefit as the first period price decreases. Conversely, when information transmission is promoted by restricting initial sales to limit the smutching of information by non-expert buyers, the first period consumers are harmed. In the second period, as information provision is always enhanced by the firm, consumers benefit from the additional information but are detrimentally affected by the seller's strategic reaction when the quality of the good turns out to be high. We derive conditions under which the second effect outweighs the first one.

When comparing the situations with, and without the introduction of a rating system, our analysis gives rise to several interesting intertemporal trade-offs. First, it is possible that early consumers are detrimentally affected by its introduction, while late consumers benefit. However, the converse can also materialize.

Second, it is possible that consumers in both periods suffer. This materializes whenever the seller strategically reduces the quantity in the first period to enhance information provision, and the enhanced information provision detrimentally affects late consumers. Overall, this can worsen even the aggregate surplus in both periods relative to the setting with no such improvement—in spite of an improvement in market information via across-period information transmission. Even if aggregate surplus not always worsened, the differential impact on the market participants over the life cycle of a product warrants further investigation and careful analysis of the role of rating systems in modern marketplaces.

In the remainder of this section we report on the literature we immediately relate to. In the ensuing Section 2 we specify our model. In Section 3 we derive our results. Section 4 contains illustrative examples. We conclude with Section 5. All proofs are relegated to the Appendix.

1.1. Related Literature

The literature on reputation systems and in particular rating systems in online markets is vast. We refer to the excellent surveys by Bar-Isaac and Tadelis (2008); Hendrikx et al. (2015); Tadelis (2016) and only highlight the most closely related papers below.

We model how sellers' strategic actions influence both current and future market outcomes when information is transmitted across periods via a rating system. Our setup is motivated by empirical observations that ratings have substantial impact on demand, see, e.g., Luca and Zervas (2016). The possibility that increased sales are associated with noisier average induced reviews is documented by Sorokin (2021) for video games on the platform Steam.

In theoretical work, Kovbasyuk and Spagnolo (2018) and Hoerner and Lambert (2021) study the interaction between alternative rating systems and incentives for strategic seller action. They argue that reporting recent ratings rather than the full history best resolves the trade-off between mitigating moral hazard for sellers while enhancing information transmission. Vellodi (2020) addresses the natural asymmetries in ratings records between incumbents and entrants, and argues in favor of policies to reduce the resulting entry barriers.

In many online markets, ratings are selective in that only purchasing consumers are allowed to leave reviews.How this selection impacts the information transmission process is the focus of, e.g., Ifrach et al. (2019) and Acemoglu et al. (2019). More closely related to our work are papers which explicitly consider the role of prices as a means to select purchasing, and thus reviewing, consumers. Li and Hitt (2008) show that the self-selection into purchasing affects the information transmission across periods as ratings reflect heterogeneous consumer tastes—it follows that higher prices induce consumers with intrinsically higher motivation to purchase and thus positively impacts induced reviews. At the same time, there is a direct price effect whereby prices affect the value for money which impacts product reviews, see Li and Hitt (2010). Recent work focuses on balancing these opposing effects in the dynamic strategic pricing decision by sellers, see Crapis et al. (2017); Feng et al. (2019); Stenzel et al. (2020). Empirically, the relationship between prices and induced reviews depends on product characteristics. For books, the selection effect dominates (Zegners, 2019), while the price effect dominates for USB sticks (Cabral and Li, 2015) and AirBnB ratings (Schaefer et al., 2021).

Our focus on how pricing affects information transmission differs from these contributions. Their focus is on the impact of prices on the review *scores* left by individual consumers, where individual reviews reflect both the value-for-money consideration and the idiosyncratic tastes. We complement these contributions by considering the relationship between selection based on willingness-to-pay and the expertise of consumers in assessing the product in a setting of common uncertainty.

With the idea that the seller includes information provision effects in his pricing decisions, we relate to the literature on experimentation in monopoly and duopoly markets. Mirman et al. (1993) provide conditions under which the early period's quantity will be distorted upwards or downwards in a setting where the monopolist is uncertain about the mean demand curve. In line with their results and work by Bose et al. (2006), information is desirable to the seller in our context. Belleflamme and Bloch (2001) compare the value of information to firms across monopoly and duopoly in price and quantity setting games. They show that this value and the associated early period distortions vary with the market structure: information is always beneficial to the monopolist, but may be harmful for Bertrand duopolists. While we restrict attention to a monopolistic seller, we explicitly account for the effects of the firm's strategic choices on consumers and aggregate surplus in both periods.

It is important to note that there is no distinction between the monopolist setting prices or quantities in our setup—this is because of the commonality of information between the monopolist and consumers. In, e.g., Belleflamme and Bloch (2001) this commonality is violated because the firms learn about demand, which implicitly means that consumers have an informational advantage.

Gill and Sgroi (2012) study a signaling model in which a monopolist can choose whether and to publicly run a test of its product before launching it. They show that the monopolist always benefits from launching the test as profits are convex in the consumers' beliefs. Our finding on profits is analogous but we depart from Gill and Sgroi (2012) by deriving conditions for when consumers are harmed by information provision, and by endogenizing the cost of the test: the cost of running a test with a certain informativeness derives from the departure from the monopoly quantity in the first period.

2. Model

We consider a firm that produces a single good. The cost of production depends on the quantity produced, q, and is denoted $c(q)$ with marginal cost $c'(q) \geq 0$ and $c''(q) \geq 0$. The good is of quality $\theta \in \{\overline{\theta}, \underline{\theta}\}$, with a common prior $\mu = \Pr(\theta = \overline{\theta})$ that the good is of high quality. The firm is active for two periods, $t \in \{1, 2\}$, and has a discount factor $\delta \in (0, 1]$. A mass 1 of short-lived consumers in each period is active for one period only. Consumers maximize expected utility with utility functions $\overline{u}(q)$ when the state is $\overline{\theta}$ and $\underline{u}(q)$ when the state is $\underline{\theta}$. Our specification of consumers' preferences encompasses consumer heterogeneity in various forms (see, e.g., Anderson et al., 1988, for one of several alternatives).

For our analysis, it is convenient to operate with the implied inverse demand $P(q, \mu)$ in each period, that depends on the quantity q supplied and the current belief μ that the product is of high quality. From the typical consumer's utility maximization problem we get that $P(q, 1) = \overline{u}'(q)$ and $P(q, 0) = \underline{u}'(q)$ and hence that inverse demand is linear in μ and given by[5]

$$P(q, \mu) = \mu P(q, 1) + (1 - \mu) P(q, 0). \tag{1}$$

We impose the standard assumptions that $P(q, \mu)$ is twice continuously differentiable and decreasing in quantity, $P_q(q, \mu) < 0$, and that profits are concave in quantity for every belief μ, $q\,(P_{qq}(q, \mu) - c''(q)) + 2\,(P_q(q, \mu) - c'(q)) < 0$.[6] Moreover, we assume that the willingness to pay is non-decreasing in the quality, implying that $P_\mu(q, \mu) \geq 0$. Finally, we impose that $q = P^{-1}(0, \mu) \geq 0$ such that demand is well-defined for all prices.

Timing and Information Structure In the first period, the firm decides on the quantity q_1 which induces a price $P(q_1, \mu_1)$. Prior to the second period, the firm as well as the second-period consumers observe a public signal about the true quality. The informational content of this signal depends on the quantity sold in the first period and captures the learning about the quality from ratings provided by previous consumers.

Specifically, the true quality of the product is revealed with probability $\psi(q_1)$ where $\psi : \mathbb{R}_+ \to [0, 1]$ is assumed to be continuously differentiable and monotone.[7] Hence, the common belief

[5]Given belief μ, the consumer's first-order condition given price p is $\mu \underbrace{\overline{u}'(q)}_{P(q,1)} + (1 - \mu) \underbrace{\underline{u}'(q)}_{P(q,0)} - p = 0$.

[6]Subscripts denote partial derivatives, e.g., P_q denotes the partial derivative of inverse demand with respect to q. Throughout, we simplify the notation where possible by omitting the dependent variables.

[7]While we impose monotonicity of $\psi(q)$ for expositional purposes we do not require this assumption. An extension to, e.g., a single-peaked $\psi(q)$ is conceptually straightforward. However, it comes at the cost of

μ_2 in the second period is $\mu_2 = 1$ if the good is of high quality and the state is revealed. This outcome occurs with probability $\psi(q_1) \cdot \mu_1$. Alternatively, when the quality is revealed and the product is of low quality, the belief is $\mu_2 = 0$. This occurs with probability $\psi(q_1) \cdot (1 - \mu_1)$. Finally, with probability $1 - \psi(q_1)$ the quality is not revealed and, independent of the true quality, the belief in period 2 is equal to the prior, $\mu_2 = \mu_1$. Note that the state-revelation probability ψ is independent of the state itself so that whether or not the state is revealed is *not* itself informative.

In our analysis, we distinguish between cases where a higher quantity leads to more information, $\psi'(q) > 0$, and where a higher quantity leads to less information, $\psi'(q) < 0$. This distinction is important. There is ample evidence that consumers are heterogeneous with respect to their reviewing behavior, both in terms of whether they provide reviews in the first place, and the content of reviews conditional on the decision to do so. But heterogeneity in individual reviews' informativeness implies that a larger quantity and hence more consumers leaving a review can lower the informativeness of aggregate statistics (such as the average star rating on amazon.com or AirBnB), which in turn may have a sizeable impact on future consumers' inference and demand.[8]

This issue is particularly relevant whenever there is a negative correlation between consumers' willingness-to-pay and the informativeness of their individual reviews. We view this as a natural assumption for many products, for which consumers who are intrinsically most interested in such a product that is expressed by a high willingness to pay. These consumers tend to devote ample time to learning about the product's specific features, allowing them to more accurately judge its true quality. While this is in itself not sufficient to ensure that increased sales adversely affect the informational content of ratings, there is empirical evidence in this regard. Specifically, Sorokin (2021) documents that reviews on the video game platform Steam exhibit an increased variance following a price reduction and associated increase in sales for a given product.

3. Analysis

We proceed as follows. We begin by analyzing the role of information provision on profits, consumer surplus, and total surplus within a static model. The properties derived in this static model serve as the building block for the analysis of the dynamic incentives of the firm taking the effect of current actions on information transmission into account, as well as for the welfare analysis of rating systems. Towards this, we derive results for two benchmark cases. First, we consider the two-period model without information transmission, i.e., where the firm and consumers base their decisions exclusively on the exogenous prior in both periods. Second, we allow for information transmission, but consider a myopic firm maximizing flow profits period by period. We then characterize the optimal behavior of the strategic, forward-looking firm,

first-order conditions that are not always sufficient in the firm's dynamic optimization problem, which makes the analysis more cumbersome.

[8]To illustrate this with a simple example, suppose that the true quality of a good is μ and that there are two consumers $i = 1, 2$ who leave in expectation unbiased reviews $r_i \sim N(\mu, \sigma_i^2)$. If we compare the precision $\frac{1}{\sigma_1^2}$ of the review left by consumer 1 with the average review left by both reviewers, $r_s = \frac{r_1+r_2}{2}$, which is given by $\frac{4}{\sigma_1^2+\sigma_2^2}$, we can immediately see that this average review is less precise provided that the second consumer produces a review which is sufficiently more noisy than the first consumers' review, $\sigma_2^2 > 3\sigma_1^2$.

and compare the outcomes to the two benchmarks. This allows us to speak to the impact of rating systems and information transmission in general (first benchmark), and what part of this impact is attributed to the strategic actions of the firm (second benchmark).

3.1. Information in a Static Model

To evaluate whether providing information to consumers is beneficial to the firm, the consumers, and total surplus, it is sufficient to study the curvature of profits and consumer surplus in their belief. The curvature determines whether additional information is desirable as beliefs are a martingale and, thus, the expected posterior is equal to the prior. By Jensen's inequality, the expected value of a convex function is greater than the value of the function at the expected value. This implies that information is desirable (undesirable) for firms, total surplus or consumers if profits, total surplus or consumer surplus are convex (concave) in the belief. We first consider the curvature of profits, then of total surplus and finally of consumer surplus which by definition is the difference between total surplus and profits.

The effect of information on profits. We define the profit function of a seller who sets quantity q and whose information is identical to that of consumers and given by the prior μ that the good is of high quality $\theta = \overline{\theta}$ as

$$\Pi(q, \mu) = P(q, \mu)q - c(q). \tag{2}$$

Information revelation has two effects on the profit of the seller. First, more precise information of consumers about the quality might reduce profits when the quality is low and improve them when the quality is high. Due to the linearity of $P(q, \mu)$ in μ, it is straightforward to see that the ex ante profits of the seller that sets the same quantity in both states is the same irrespective of whether consumers learn the true quality or not and equals to $\mu P(q, 1)q+(1-\mu)P(q, 0)q-c(q) = P(q, \mu)q - c(q)$. Second, the seller can increase its profit by conditioning the quantity on the revealed information and earn higher profits in each state. Define $\overline{q}$ and $\underline{q}$ as the maximizers of $P(q, \mu)q - c(q)$ for $\mu = 1$ and $\mu = 0$, respectively. It immediately follows that

$$\mu P(q, 1) + (1 - \mu)P(q, 0) - c(q) < \mu(P(\overline{q}, 1)\overline{q} - c(\overline{q})) + (1 - \mu)(P(\underline{q}, 0)\underline{q} - c(\underline{q})), \tag{3}$$

which implies that the firm reaches higher profits when information about the true quality is publicly revealed. We sum up this discussion in the following proposition.[9]

Proposition 1 *The seller makes higher profits when information of the true quality is revealed.*

More generally, it is useful to explore the curvature of the seller's equilibrium profit in the common belief μ. By assumption, $\Pi(q, \mu)$ is strictly concave in μ and there exists a unique maximizer of $\Pi(q, \mu)$ which we denote $q(\mu)$. Consequently, we can define the maximal profit given μ as $\Pi(\mu) = \Pi(q(\mu), \mu)$. We can establish that this equilibrium profit is convex irrespective of the demand and the cost functions.

Proposition 2 $\Pi(\mu)$ *is convex in* μ.

[9]In Appendix A we show that the the result extends to general information structures.

This result is in line with Mirman et al. (1993) and Bose et al. (2006) who establish that a monopolistic seller prefers to have more information about uncertain demand. To understand the intuition, consider a marginal increase in μ and the impact on the seller's profit $\Pi(q(\mu), \mu)$. Recall that for a fixed quantity the profit is linear in μ. Therefore, the curvature of the equilibrium profit cannot be strictly concave as that would contradict the optimally of the seller's quantity adjustment.

The effect of information on consumer surplus. While the firm always desires information, we establish here that this is not necessarily the case for consumers. Although more information allows consumers make better purchasing decisions, the seller's strategic adjustment may affect the induced equilibrium price-quantity-combinations to the consumers' detriment, in particular when the good is revealed to be of high quality.

Consumer surplus as a function of the (equilibrium) quantity sold q and the common prior μ can be expressed as

$$CS(q, \mu) = \int_0^q [P(x, \mu) - P(q, \mu)]dx. \tag{4}$$

Define consumer surplus in the case of no information revelation as

$$\begin{aligned} CS(\mu) &= \int_0^{q(\mu)} [P(x, \mu) - P(q(\mu), \mu)]dx & (5)\\ &= \mu \int_0^{q(\mu)} [P(x, 1) - P(q(\mu), 1)]dx + (1 - \mu) \int_0^{q(\mu)} [P(x, 0) - P(q(\mu), 0)]dx & (6)\\ &= \mu CS(q(\mu), 1) + (1 - \mu)CS(q(\mu), 0), & (7) \end{aligned}$$

where $q(\mu)$ maximizes (2). Note that if the true state θ is revealed, then consumer surplus is equal to $\mu CS(1)+(1-\mu)CS(0)$. Thus consumers holding priors μ strictly prefer full information (no information) if and only if

$$\Delta CS = \mu CS(1) + (1 - \mu)CS(0) - CS(\mu) > (<)0. \tag{8}$$

In general, it is possible that consumers benefit from information for some priors, while they are worse off for others. However, we focus on exploring the sufficient conditions under which consumers strictly prefer full information (no information) for all priors $\mu \in (0, 1)$. This boils down to exploring the curvature of consumer surplus as consumer are better off from full information (no information) for all priors if and only if $CS(\cdot)$ is a strictly convex (strictly concave) function.

Curvature of consumer surplus. It is useful to define the pass-through rate of beliefs μ into the price and its behavior with respect to μ as

$$\alpha = \frac{d}{d\mu} P(q(\mu), \mu) = P_q q_\mu + P_\mu \tag{9}$$

$$\beta = \frac{d\alpha}{d\mu} \tag{10}$$

Both α and β play key roles for our results. Note that the sign of β generally depends on the sign of the third order derivatives of the demand and cost functions with respect to q which contains information about the behavior of the curvature of demand. We can use α and β to write the first and the second derivatives of CS with respect to μ (omitting the arguments) as

$$CS'(\mu) = \int_0^q (P_\mu(x,\mu) - \alpha)dx \tag{11}$$

$$CS''(\mu) = (P_\mu - \alpha)q_\mu - \beta q \tag{12}$$

The marginal change of $CS(\mu)$ when μ increases is represented by the change in surplus of infra-marginal consumers. The curvature of $CS(\mu)$ reflects how the benefit/losses of infra-marginal consumers differ in μ. The first term of $CS''(\mu)$ represents the change in surplus of the marginal consumer (the extensive margin) and is always positive as $(P_\mu - \alpha)q_\mu = -P_q q_\mu^2 > 0$. The second term of $CS''(\mu)$ represents the behavior of the surplus of all infra-marginal consumers (the intensive margin) and is negative if the equilibrium price increases with significantly increasing rate. The following proposition summarizes this discussion.

Proposition 3 *Consumer surplus is globally convex (concave) if and only if* $(P_\mu - \alpha)q_\mu - \beta q$ *for all* $\mu \in (0,1)$.

Note that if the pass-through rate of beliefs into price decreases in μ (so that the equilibrium price increases in μ at a decreasing rate), i.e., if $\beta < 0$, then $(P_\mu - \alpha)q_\mu - \beta q > 0$. In this case, $CS(\mu)$ is convex which implies that consumers are better off under full information.

Corollary 1 *If* $\beta \leq 0$*, then consumers are better off under full information.*

This implies that the necessary condition for consumer surplus and total welfare to be concave is $\beta > 0$. In general, there exist demand and costs functions for which consumer surplus can be convex and concave, and we provide detailed examples which illustrate their genericity in Section 4. Specifically, we show that both concavity and convexity obtain both with zero and non-zero production costs, and when the belief acts as a pure demand shifter or pure demand rotator.

The effect of information on total surplus. To explore whether or not total surplus given by $S(\mu) = \Pi(\mu) + CS(\mu)$ is higher when information is revealed, it is necessary and sufficient to determine the sign of

$$\Delta S = \mu S(1) + (1-\mu)S(0) - S(\mu). \tag{13}$$

Generally, the sign of ΔS depends on the prior belief μ. We focus again on exploring the sufficient condition under which total surplus is higher (lower) when information is revealed for *all* priors $\mu \in (0,1)$. Write the curvature of total surplus $S''(\mu)$ as

$$S''(\mu) = \Pi''(\mu) + CS''(\mu) = \Pi_{q\mu}q_\mu - P_q q_\mu^2 - \beta q \tag{14}$$

$$= \Pi_{q\mu}q_\mu \left(1 + \frac{1}{2-\sigma}\right) - \beta q \tag{15}$$

$$= \Pi''(\mu) \left(1 + \frac{1}{2-\sigma}\right) - \beta q, \tag{16}$$

where $\sigma < 2$ is the curvature of inverse demand. In (16), the first term represents the additional surplus from marginal consumers (the extensive margin), which is always positive, while the second term reflects the change in the surplus of inframarginal consumers (the intensive margin). Given Proposition 2 and Proposition 3, we immediately obtain the following result:

Proposition 4 *Let β denote the pass-through rate of beliefs into price. The following holds:*

(i) If $\beta \leq 0$, then $S''(\mu) > 0$.

(ii) If $\beta > 0$ and sufficiently high so that

$$\beta > \frac{1}{\Pi''(\mu)} \frac{q}{\left(1 + \frac{1}{2-\sigma}\right)} \text{ for all } q, \tag{17}$$

then $S''(\mu) < 0$.

The intuition behind Proposition 4 is straightforward. Recall that the firm's equilibrium profits are convex. As such, total surplus as the sum of the firm's profits and consumer surplus, can only be concave provided that consumer surplus is sufficiently concave. Specifically, it is necessary that consumer surplus is adversely affected by information (consumer surplus is sufficiently concave), and that this overcomes the convexity of the seller's profits in the belief. Towards this, we require in turn a sufficiently strongly increasing rate to which beliefs are reflected in the equilibrium price.

3.2. Intertemporal Analysis

Having characterized the impact of information on the outcomes in the static model, we turn now to the development and characterization of the results in the two-period model with intertemporal information transmission. We first characterize the two benchmark results where information transmission does not take place, and where the firm does not internalize the impact of its first-period behavior on second-period outcomes via the induced information transmission.

Benchmark 1: No Information Transmission By not allowing for information transmission between periods, we capture for example a platform before a rating system is introduced. This benchmark allows us to study the welfare effects of the introduction of a rating system, and in particular to respond to a central question: Under which conditions are consumers harmed by the introduction of a rating system?

Absent information transmission across periods, the common belief is the same in both periods, $\mu_2 = \mu_1$. In this case, the firm maximizes flow profits in both periods, i.e. sets $q_1^* = q_2^* = \hat{q}$ where $\hat{q}$ is uniquely characterized by $\hat{q} = q(\mu_1) = q(\mu_2) = \arg\max_q \Pi(q, \mu_1)$.

Benchmark 2: Myopic Seller Here, we allow for information transmission, but consider a myopic seller who maximizes flow profits period by period. This benchmark serves to isolate the effect of information transmission on consumers without interference by the seller's dynamic incentives.

It is immediate that the first-period quantity will still be $q_1^* = \hat{q} = q(\mu_1)$. Information transmission is characterized by the probability $\psi(\hat{q})$ with which the true quality is revealed prior to the second period. This implies three possible cases.

With probability $(1-\psi(\hat{q}))$, no information is revealed and the firm sells $q_2^* = \hat{q}$ units again. With probability $\psi(\hat{q})$, the state is revealed and the consumers learn that quality is high with conditional probability μ_1 or that quality is low with conditional probability $1-\mu_1$. Recall that $\overline{q}$ and $\underline{q}$ denote the optimal quantity choices in the second period when information is revealed and note that our assumptions are *not* sufficient to ensure $\overline{q} > \hat{q} > \underline{q}$.[10]

A myopic seller will choose the optimal quantity $\hat{q}$ in the second period if no information is revealed, and $\overline{q}$ or $\underline{q}$, otherwise, depending on whether the good is actually of high or low quality. First-period profits, consumer surplus and aggregate surplus remain unchanged compared to the absence of information transmission. However, a myopic seller clearly benefits from information in the second period due to the convexity of profits established in Proposition 2. Whether consumer surplus and aggregate surplus increase due to the rating system when the seller is myopic depends on the conditions on the demand function derived in Proposition 3 and Proposition 4.

Sophisticated Seller Finally, consider the sophisticated seller who internalizes the informational spillover in her first period quantity choice. In the second period, the seller will earn $\Pi(\mu)$ where μ is the realized posterior. Thus, the seller's profit in both periods is given by

$$\Pi(q_1,\mu_1) + \delta\left(\Pi(\mu_1) + \psi(q_1)\Delta\Pi\right), \tag{18}$$

where $\Delta\Pi = \mu_1\Pi(1) + (1-\mu_1)\Pi(0) - \Pi(\mu_1)$. The relevance of the profit curvature is immediate. Convexity of Π implies that information is desirable to the seller and hence that $A \equiv \mu_1\Pi(1) + (1-\mu_1)\Pi(0) - \Pi(\mu_1)$ is positive. Assessing the firm's first-order condition with respect to q_1, we obtain

$$\underbrace{q_1 P_q(q_1,\mu_1) + P(q_1,\mu_1) - c'(q_1)}_{\text{effect on flow profits}} + \underbrace{\psi'(q_1)\cdot\left[\mu_1\Pi(1) + (1-\mu_1)\Pi(0) - \Pi(\mu_1)\right]}_{\text{effect on continuation profits}} = 0. \tag{19}$$

Hence the relationship between the optimal quantity q_1^* and the myopically optimal quantity $\hat{q}$ depends on the sign of $\psi'(\cdot)$. From Proposition 2, the firm benefits from information and therefore wants to increase the probability that information is revealed, $\psi(q_1)$. As $\hat{q}$ maximizes $q_1 P(q_1,\mu_1)$, we know that $q_1 > \hat{q}$ if $\psi'(q) > 0$ and $q_h < \hat{q}$ if $\psi'(q) < 0$.

This is a natural result. The firm desires information and thus distorts the quantity choice in the first period away from the myopically optimal quantity $\hat{q}$—the direction of this distortion is shaped by whether additional information is generated via supplying additional quantity $(\psi'(q) > 0)$ or via rationing $(\psi'(q) < 0)$.

Corollary 2 (First period quantity and price) *The equilibrium quantity in the first period*

[10] It is possible that $\underline{q} > \hat{q} > \overline{q}$. Specifically, this may materialize when inverse demand in the high quality state $P(q,1)$ is sufficiently more responsive to price changes. A simple example satisfying our assumptions is given by $P(q,1) = 1-2q$ and $P(q,0) = \frac{2}{3} - 3q^2$, which imply optimal quantities $\overline{q} = \frac{1}{4}$ and $\underline{q} = \frac{\sqrt{2}}{3\sqrt{3}} \approx 0.27 > \overline{q}$.

q_1^* lies above the myopic optimum $\hat{q}$ if and only if informativeness increases in quantity, $\psi'(q) > 0$. Conversely, it lies below the myopic optimum $\hat{q}$ if and only if $\psi'(q) < 0$.

The distortion in the equilibrium quantity q_1^* relative to $\hat{q}$ is important, as it determines whether first period consumer and total surplus increase or decrease compared to the no information transmission benchmark. Specifically, consumer surplus is strictly increasing in the quantity supplied by the firm because a larger quantity is associated with a lower price, while beliefs are unaffected. As such, consumer surplus is larger (smaller) in the first period compared to the no information benchmark whenever q_1^* is larger (smaller) than $\hat{q}$. The effect on total surplus is unambiguously negative when the quantity supplied by the firm decreases as both the firm and consumers suffer. When the quantity increases, it is in principle possible that total surplus falls. However, this would require the intertemporal incentives to be sufficiently strong to induce the firm to supply sufficiently in excess of the static perfect competition quantity (which in turn exceeds $\hat{q}$). In principle, total surplus will hence increase when $q_1^* > \hat{q}$; absent production costs, an increased equilibrium quantity always increases total surplus.

Intertemporal Consumer Surplus The above characterizations allow us to explore the necessary and sufficient condition for consumer surplus to be higher or lower when a rating system is introduced. The aggregate consumer surplus is given by

$$CS(q_1^*, \mu) + \delta(CS(\mu) + \psi(q_1^*)\Delta CS), \tag{20}$$

where q_1^* maximizes (18). The following proposition establishes conditions for consumers to benefit and be adversely affected, respectively.

Proposition 5 *Consumers are better off (worse off) from the introduction a rating system if and only if*

$$\delta \Delta CS \psi(q_1^*) > (<) CS(\mu) - CS(q_1^*, \mu). \tag{21}$$

From Corollary 2, $\psi' \geq 0 (< 0)$ implies $q \geq (<) q(\mu)$. Consider both cases separately:

Case 1: $\psi' > 0$. In this case the seller will increase the quantity in period 1, $q_1^* > q(\mu)$. Since

$$CS_q(q, \mu) = -P_q(q, \mu)q > 0, \tag{22}$$

there exists $\xi \in [q(\mu), q_1^*]$ such that $CS(\mu) - CS(q_1^*, \mu) = CS'_q(\xi, \mu)(q(\mu) - q_1^*) < 0$. Therefore if $\Delta CS > 0$, then consumers benefit from information transmission, i.e., the introduction of a rating system.

Case 2: $\psi' < 0$. In this case the seller will decrease the quantity in period 1, $q_1^* < q(\mu)$. Therefore $CS(\mu) - CS(q_1^*, \mu) > 0$. It is straightforward to see that if $\Delta CS < 0$, then consumers are worse off with a rating system.

Corollary 3 *If $\psi' > 0$ and $\Delta CS > 0$, then consumers are better off with a rating system. If $\psi' < 0$ and $\Delta CS < 0$, then consumers are worse off with a rating system.*

3.3. Summary of Results

Based on our analysis, we are able to consider the marginal and overall impact of allowing for information transmission between consumers.

	Consumers					
	information desired			information not desired		
$\psi'(q) > 0 \implies q_1^* > \hat{q}$	$\pi_1 \downarrow$ $\pi_2 \uparrow$	$CS_1 \uparrow$ $CS_2 \uparrow$	$TS_1 \uparrow$ $TS_2 \uparrow$	$\pi_1 \downarrow$ $\pi_2 \uparrow$	$CS_1 \uparrow$ $CS_2 \downarrow$	$TS_1 \uparrow$ $TS_2 \uparrow\downarrow$
$\psi'(q) < 0 \implies q_1^* < \hat{q}$	$\pi_1 \downarrow$ $\pi_2 \uparrow$	$CS_1 \downarrow$ $CS_2 \uparrow$	$TS_1 \downarrow$ $TS_2 \uparrow$	$\pi_1 \downarrow$ $\pi_2 \uparrow$	$CS_1 \downarrow$ $CS_2 \downarrow$	$TS_1 \downarrow$ $TS_2 \uparrow\downarrow$

Table 1: Impact of information on profits (π), consumers surplus (CS) and total surplus (TS)

Table 1 summarizes the results.[11] It characterizes the impact on realized equilibrium profits π_t, consumer surplus CS_t and total surplus TS_t in the two periods $t = 1, 2$ by evaluating whether these measures have increased or decreased relative to the benchmark scenario with no information transmission in which the firm supplies $\hat{q}$ in both periods. The top row captures the case where information provision increases in the supplied quantity, $\psi'(q) > 0$, while the bottom row captures the reverse scenario in which information provision is enhanced by reducing sales. In a similar vein, we distinguish between the left column in which consumers like information provision (in the sense that $CS(\mu)$ is convex in μ), and the right column in which consumers dislike information provision (concave $CS(\mu)$).

The firm always desires to enhance information provision. It will hence distort the quantity in the first period from the myopically optimal $\hat{q}$ upwards (top row) or downwards (bottom row), respectively, to increase information transmission. In doing so, it sacrifices flow profits in the first period ($\pi_1 \downarrow$) to increase expected continuation profits in the second period ($\pi_2 \uparrow$). Consumers and total surplus benefit from this distortion in the first period, whenever it leads to an increase in the first period quantity ($CS_1 \uparrow$ & $TS_1 \uparrow$, top row), and are detrimentally affected when it leads to a decrease ($CS_1 \downarrow, TS_1 \downarrow$, bottom row). In the second period, the increased information provision benefits consumers when $CS(\mu)$ is convex (left column) and is to their detriment otherwise (right column). The overall effect on total surplus in the second period is unambiguously positive when both consumers and the firm benefit (left column) and ambiguous when the firm benefits to the detriment of consumers (right column).

Overall, the main takeaways are as follows. First, it is possible that second period consumers are adversely affected by information transmission between consumers, while the firm benefits. Second, it is in fact possible that allowing for information transmission between consumers is universally detrimental for total surplus—it may be adversely affected in the first period if the firm strategically reduces the supplied quantity to enhance information provision in the case

[11] The table and subsequent discussion implicitly assume that $TS \uparrow$ in the case where $q_1^* > \hat{q}$. We do this for a more streamlined exposition. As discussed previously, it is theoretically possible that total surplus decreases in this case, which implies additional ambiguity as to the total impact of the introduction of a rating system on total surplus across the two periods.

where $\psi' < 0$, and adversely affected in the second period because the strategic response by the firm due to improved information reduces consumer surplus by more than the firm's profits increase. Necessary for this is that consumers are adversely affected by enhanced information, i.e. that $CS(\mu)$ is concave.

Finally, even when total surplus is not adversely affected in both periods, there is scope for an intertemporal trade-off in terms of consumer surplus and/or total surplus. This always materializes when the first period quantity lies below the myopic optimum ($\psi'(q) < 0$) and consumers benefit from information transmission ($CS(\mu)$ convex), in which case first period total surplus is reduced but second period total surplus increased by allowing for information transmission. In addition, it may materialize when information is not desired by consumers.

4. Examples

The previous analysis, in particular Proposition 3 and Proposition 4 provide sharp conditions under which consumer and total surplus are increased and detrimentally affected by enhanced information provision. This in turn has implications for the intertemporal trade-offs and overall assessment of the impact of rating systems on market outcomes as illustrated by Table 1.

In this section, we provide examples to further the understanding of which types of demand and cost structures satisfy these conditions. The main goal is to show that all types of behavior are generic in the sense that we do not need to rely on knife-edge cases to, e.g., have total surplus being adversely affected by information. Towards this, we proceed in two steps. We first abstract from production costs, i.e., set $c(q) = 0$, and investigate the implications when the belief acts as a demand shifter and demand rotator, respectively. Turning then to positive production costs, we focus on a simple case in which demand is linear and productions costs are quadratic. This allows us to analyze the role of cost structure parameters in determining the overall value of information to different market participants.

4.1. No Production Costs

We first characterize the necessary and sufficient condition for convexity (concavity) of consumer surplus when production costs are zero, i.e., $c(q) = 0$ for all $q \geq 0$. We separately analyze the cases of demand shift, where only the intercept of the demand curves for $\overline{\theta}$ and $\underline{\theta}$ differ and their slope stays the same at all quantity levels; and demand rotation.

For the analysis, it us useful to define the curvature of the inverse demand function $P(q, \mu)$ with respect to quantity as $\sigma = -qP_{qq}/P_q$.[12] The next proposition characterizes the necessary and sufficient conditions for convexity (concavity) of consumer surplus for pure demand shift and pure demand rotation.[13]

Proposition 6 *Suppose that $c(q) = 0$. If $P(q, \mu) = a(1 + \phi\mu) - t(q)$, $a, \phi > 0, t' > 0$ (pure demand shift), then the sign of $CS''(\mu)$ is determined by the sign of $a\phi + \sigma' P(q(\mu), \mu)$. If*

[12]Note that the second order condition of the profit maximization problem directly implies that $\sigma < 2$.
[13]The general conditions for the demand shift and demand rotation can be found in Appendix D.

$P = a - (1 - b\mu)t(q)$, $a, b > 0, t' > 0$ *(pure demand rotation), then the sign of* $CS''(\mu)$ *is determined by the sign of* $\frac{ab}{1-b\mu} + \sigma' P$.

For the case of a pure demand shift in which μ affects only the demand intercept, the pass-through rate of beliefs into the price α is negatively and linearly related to the pass-through rate of costs evaluated at zero cost, denoted ρ, so that $\alpha = a\phi(1-\rho)$. Therefore, the behavior of pass-through rate of beliefs β is determined by the behavior of the pass-trough rate of costs ρ'. As ρ in turn can be expressed as a function of the curvature of the inverse demand function, σ, $\rho = 1/(2-\sigma)$, we obtain that the sign of CS'' depends on the behavior of the demand curvature σ'. If the pass-through rate becomes higher for larger costs, then $CS''(\mu) > 0$ and consumer surplus is convex in the belief μ. Otherwise, if the pass-through rate becomes sufficiently low for larger costs, then $CS''(\mu) < 0$ and consumer surplus is concave. The argument for the pure demand rotation follows the same line of reasoning.

We can refine the conditions in Proposition 6 by considering demand functions with constant curvature. In this case, consumer surplus is convex and consumers prefer more information in both the case of the belief acting as a demand shifter, and as a demand rotator.

Corollary 4 *Suppose that* $c(q) = 0$. *If* $P = a(1 + \phi\mu) - bq^k$, *where* $a, \phi, k > 0$ *(pure demand shift) or* $P = a - (1 - b\mu)cq^k$, *where* $a, b, k > 0$ *(pure demand rotation), then* $CS''(\mu) > 0$ *and consumer surplus is convex.*

Corollary 4 is established by noting that we have $\sigma = 1 - k$ both when considering μ as a pure demand shifter and as a pure demand rotator. As such, the behavior of the curvature which determines β is equal to zero and we obtain $\beta = 0$. Proposition 6 immediately establishes that consumer surplus is convex and therefore that consumers are better off from enhanced information provision.

Bear in mind that Corollary 4 applies only to a particular class of demand functions with constant curvature of inverse demand. The resulting desirability of information to consumers does not extend generically to other demand structures; in fact, we next establish that information can be detrimental to consumer surplus even when μ is a pure demand shifter and pure demand rotator, respectively.

First, consider the following example of a pure demand shift, $P = a(1 + \phi\mu) - t(q)$, where $a, \phi, t' > 0$. Note that if ϕ is close enough to 0 and $\frac{1}{|\sigma'|}$ is bounded, then the sign of CS'' coincides with the sign of σ' for all $\mu \in [0, 1]$. Suppose that $\sigma' < 0$ in a small neighborhood of $\phi = 0$, then CS is concave for all μ in a neighborhood of $\phi = 0$. In this case, linear curvature may lead to concavity of consumer surplus. Consider $P = a(1 + \phi\mu) - be^{kq}$, $k > 0$. The curvature of the demand function is $\sigma = -\frac{qt''}{t'} = -\frac{qk^2e^{kq}}{ke^{kq}} = -kq$ and $\sigma' = -k$. The FOC implies $a(1 + \phi\mu) = be^{kq}(1 + kq)$, then $P = b(kq)e^{kq}$. The sign of CS'' coincides with the sign of $a\phi - kP = a\left(1 + \phi\left(1 - \frac{1}{\phi}\right)\right) - k(a(1 + \phi\mu) - be^{kq})$.

Second, concavity of CS is similarly possible under pure demand rotation, in which $P = a - (1 - b\mu)t(q)$. If b is close enough to 0 and $\frac{1}{|\sigma'|}$ is bounded, then the sign of CS'' coincides with the sign of σ' for all $\mu \in [0, 1]$. Supposing that $\sigma' < 0$ in a small neighborhood of $b = 0$, consumer surplus is concave for all μ in a neighborhood of $b = 0$.

4.2. Linear Demand and Quadratic Production Costs

Linear-quadratic example. To illustrate our results in an example with positive production costs, assume that demand is linear with $P(q,0) = \max\{a_l - b_l q, 0\}$ and $P(q,1) = \max\{a_h - b_h q, P(q,0)\}$ with $a_h > a_l$.[14] The firm's cost is quadratic $c(q) = kq^2$ with $k \geq 0$. We let the intertemporal information provision be increasing in the quantity sold and consider $\psi(q) = \varphi q^\eta$, where $\varphi \geq 0$ is a scaling parameter and varying η allows for increasing and decreasing marginal informativeness of q.

We can analytically solve the model for this class of demand and cost functions. It is straightforward to derive the general parameter conditions such that consumer surplus and aggregate surplus are globally concave. We provide the derivations in Appendix E.

For ease of exposition, we focus on a particular example that nests all relevant cases in the following:

$$\begin{aligned} P(q,1) &= \max\{2-4q, 1-q\} \\ P(q,0) &= 1-q \\ c(q) &= kq^2 \end{aligned} \tag{23}$$

with $k \geq 0$. In this example, a sufficiently optimistic belief $\mu \geq \underline{\mu}(k) := \max\{0, \frac{3}{1+k} - 2\}$ ensures that $\pi(q(\mu)) \geq \pi(q = \frac{1}{2(1+k)})$.[15] For all $\mu < \underline{\mu}(k)$, $q(\mu) = \frac{1}{2(1+k)}$.

Economically, this example illustrates a case in which some consumers have a substantially higher willingness to pay for the high quality good. However, the willingness to pay of these *geeks* decreases relatively steeply. This implies that for sufficiently low production costs, the monopolist sells to fewer customers if the quality is known to be high than if it were known to be low, $\overline{q} < \underline{q}$. If production costs instead are relatively high, the optimal quantities revert: the costs push the monopolist to sell to only very few customers in either case and the marginal revenue under high quality exceeds the one under low quality which leads to $\overline{q} > \underline{q}$.

In this example, it is straightforward that consumer surplus is constant for all $\mu \in [0, \underline{\mu}(k))$ for any given k. Moreover, we can use the derivations in Appendix E to show that consumer surplus is

- convex for all $\mu \in (\underline{\mu}, 1)$ whenever $k \in [0, \frac{2}{25}(\sqrt{34} - 3))$,
- concave for $\mu \in (\underline{\mu}, \frac{7k-2}{6-9k})$ and convex for $\mu \in (\frac{7k-2}{6-9k}, 1)$ whenever $k \in (\frac{2}{25}(\sqrt{34} - 3), \frac{1}{2})$,
- concave for $\mu \in (\underline{\mu}, 1)$ whenever $k \in (\frac{1}{2}, 2)$, and
- convex for $\mu \in (\underline{\mu}, 1)$ whenever $k > 2$.

Similarly, aggregate surplus is also constant for all $\mu \in [0, \underline{\mu}(k))$ and

- convex for $\mu \in (\underline{\mu}, 1)$ whenever $k \in [0, \underline{k})$ with $\underline{k} := \{k : \frac{3}{1+k} - 2 = \frac{6-k(5+2k)}{3(5k-6)}\}$,

[14] The max operator is required to ensure that $P(q,1) \geq P(q,0)$ when $b_h > b_l$.

[15] This condition binds only for $k < 1/2$ and derives from the fact that for pessimistic beliefs μ and low k, the optimal price-quantity combination is on the branch of the demand functions for which $P(q,0) = P(q,1) = 1-q$.

- concave for $\mu \in (\underline{\mu}, \frac{6-k(5+2k)}{3(5k-6)})$ and convex for $\mu \in (\frac{6-k(5+2k)}{3(5k-6)}, 1)$ whenever $k \in (\underline{k}, \sqrt{37}-5)$,
- concave for $\mu \in (\underline{\mu}, 1)$ whenever $k \in (\sqrt{37}-5, 2)$, and
- convex for $\mu \in (\underline{\mu}, 1)$ whenever $k > 2$.

To build intuition for the results in this example, note first that the optimal quantity for $\mu > \underline{\mu}$ is given by $q(\mu) = \frac{1+\mu}{2(1+3\mu+k))}$, which is decreasing in the belief for all $k \in [0, 2)$.[16] This derives from the existence of *geeks* with a high willingness to pay as discussed above. However, once the cost parameter k increases above 2—inducing the quantities to be low in both underlying states—a belief increase leads to a quantity expansion.

Next, consider the effect that a belief increase has on the demand function which is $P_\mu(q, \mu) = a'(\mu) - b'(\mu)q$. $a'(\mu) = 1$ corresponds to a demand shift outwards while $b'(\mu) = 3$ is a rotation that induces the demand function to be steeper. The change in consumer surplus is then determined by the difference between each purchasing consumer's increase in their willingness to pay and the pass-through rate of the belief increase into the price which is given by

$$\begin{aligned}
\alpha &= \underbrace{P_q q'(\mu)}_{\text{price-change due to } q'(\mu)} + \underbrace{a'(\mu) - b'(\mu)q(\mu)}_{\text{WTP change of marginal consumer}} \\
&= -(1-3\mu)q'(\mu) + 1 - 3q(\mu). \qquad (24)
\end{aligned}$$

This pass-through consists of two components. First, holding the quantity fixed, the monopolist can raise the price by $a'(\mu) - b'(\mu)q(\mu)$ which corresponds to the increase in the marginal consumer's willingness to pay. However, the firm also responds with a change in the quantity, $q'(\mu)$, which requires a price adjustment of $P_q q'(\mu)$. The effect of the belief change on consumer surplus is thus given by[17]

$$\begin{aligned}
CS'(\mu) &= \int_0^{q(\mu)} \big((1-3x) - (-(1-3\mu)q'(\mu) + 1 - 3q(\mu))\big)\, dx \\
&= \int_0^{q(\mu)} \big(3(q(\mu) - x) + (1-3\mu)q'(\mu)\big)\, dx \\
&= \left(3\frac{q(\mu)}{2} + (1-3\mu)q'(\mu)\right) q(\mu). \qquad (25)
\end{aligned}$$

Finally, recall that the curvature of consumer surplus is determined by

$$CS''(\mu) = -P_q \left(q'(\mu)\right)^2 - \beta q \qquad (26)$$

where the first term corresponds to the price change due to a quantity adjustment in response to a belief change multiplied by how the amount of inframarginals that are affected by it vary with the belief—an unambiguously positive effect for any demand and cost function. However, how pass-through varies with the belief, β, depends on both the demand and the cost function.

[16]Throughout the subsequent discussion, we focus only on the cases in which $\mu > \underline{\mu}$.

[17]Recall that the only the inframarginal effects matter here as the marginal consumer's surplus is extracted entirely.

In our particular example, we obtain

$$\begin{aligned}\beta &= P_q q''(\mu) + 2P_{q\mu} q'(\mu) \\ &= -(1+3\mu)q''(\mu) - 6q'(\mu). \end{aligned} \tag{27}$$

The intuition for how pass-through behaves with the belief results from the linearity of the demand in the belief and the simplicity of the linear demand and quadratic cost structure. As $P_{\mu\mu} = P_{qq} = 0$ there is no direct second-order effect of the quantity or of the belief on the marginal revenue. Moreover, due to $c'''(q) = 0$, there also is no second-order effect on the marginal cost. Thus, the only second-order effect a belief change has on the firm's choice comes through the cross-effect of a quantity change on the marginal revenue change: $\frac{d^2}{dqd\mu}(P + P_q q) = 2P_{\mu q}$. Thus, we obtain

$$\begin{aligned} q'(\mu) &= -\frac{a'(\mu) - 2b'(\mu)q(\mu)}{2(P_q - k)} \\ &= \frac{1 - 3q(\mu)}{2(1 + 3\mu + k)} \end{aligned} \tag{28}$$

$$\begin{aligned} q''(\mu) &= -2q'(\mu)\frac{P_{\mu q}}{2(P_q - k)} \\ &= -2q'(\mu)\frac{b'(\mu)}{b_\mu + k} \\ &= -2q'(\mu)\frac{3}{1 + 3\mu + k} \end{aligned} \tag{29}$$

We can immediately see that as the belief increases, $|q'(\mu)|$ decreases, i.e., the rate of change declines. Importantly, the relative decline is given by

$$\begin{aligned} \frac{q''(\mu)}{q'(\mu)} &= -2\frac{P_{\mu q}}{P_q - k} \\ &= -\frac{6}{1 + 3\mu + k}. \end{aligned} \tag{30}$$

Thus, we can conclude the sign of β from

$$\beta \geq 0 \iff P_q q'(\mu)\left(\frac{q''(\mu)}{q'(\mu)} + 2\frac{P_{q\mu}}{P_q}\right) \geq 0 \iff 2P_q q'(\mu)\left(-\frac{P_{\mu q}}{P_q - k} + \frac{P_{q\mu}}{P_q}\right) \geq 0. \tag{31}$$

It is immediate that whenever the production cost is zero, $k = 0$, the pass-through rate is constant. However, if the cost is positive and $P_{q\mu}$ negative as in our example, the sign of the pass-through rate's behavior is exclusively determined by the sign of $q'(\mu)$: whenever $q'(\mu) < 0$ the pass-through of beliefs into prices is increasing in μ.

In this example, consumer surplus is globally concave if the cost parameter k is sufficiently high while preserving $q'(\mu) < 0$. If k is too high, $q'(\mu) > 0$ and the pass-through rate is decreasing in the belief. If k is too low, $q'(\mu) < 0$ and the pass-through rate is increasing but the positive effect on consumer surplus, i.e., $P_q q'(\mu)^2$ dominates. Only once the pass-through increases sufficiently in the belief, consumers are harmed by information provision.

Intertemporal Example. We next embed the linear-quadratic example in the intertemporal model to illustrate the findings further. For this, we parametrize $\delta = 1$ and $\mu = 2/5$.

First, consider the case in which the probability that a consumer learns the true quality is increasing in the quantity with the simple parametrization $\psi(q) = q$.

Second, recall from the previous analysis that by varying the cost parameter k from 1/3 to 4 we obtain regions in which consumer surplus as well as aggregate may increase and decrease in information provision.

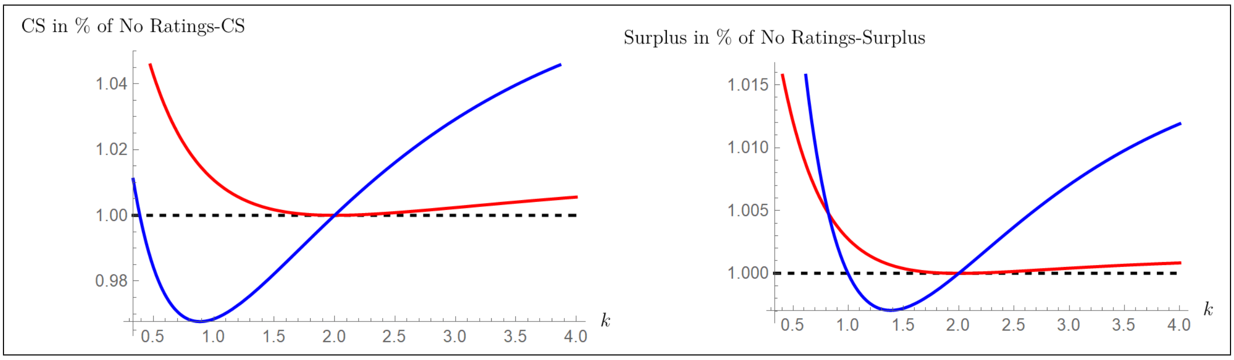

Figure 1: The per-period effect of a rating system on consumer surplus and aggregate surplus when higher quantities increase the informativeness of a rating.
Left panel: The solid red line depicts the consumer surplus in period one with a rating system in % of the consumer surplus without a rating system. These consumers are always better off with the rating system as the firm expands its quantity in period one to increase information in period two. The solid blue line depicts consumer surplus for period two consumers who are harmed by the additional information for intermediate k levels.
Right panel: The solid red line depicts the aggregate surplus in period one with a rating system in % of the aggregate surplus without a rating system. The solid blue line depicts the same for period two aggregate surplus. For intermediate k, consumer surplus is lower with than without a rating system. This effect is so strong that even though the firm benefits from more information, aggregate surplus in period two is lower with the rating system.

Figure 1 shows the effect of the introduction of a rating system on consumer surplus and aggregate surplus in each period. It is immediate from the preceding discussion that for intermediate cost levels period-2 consumer surplus is lower than without the rating system. However, period 1 customers always benefit from the introduction of the rating system as the firm has an incentive to expand its quantity to increase information provision.

The right panel of Figure 1 illustrates that aggregate surplus may also decline due to the introduction of the rating system. The loss in consumer surplus is not compensated by sufficient increases in profits. The losses derive from the second-period consumers who are harmed by the rating system.

It is evident from the left panel in Figure 2 that the loss in consumer surplus due to additional information may outweigh the benefits in this specification that derive from the firm increasing sales in period 1. This occurs when the cost parameter is at an intermediate level—in the region where consumer surplus is concave in the belief.

The right panel of Figure 2 shows that for intermediate cost levels the negative impact that information has on second-period consumers may outweigh all benefits of the rating system,

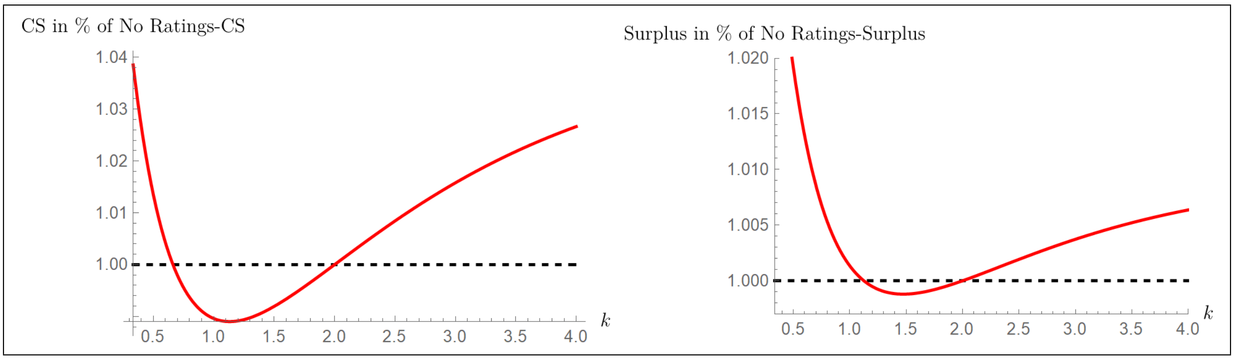

Figure 2: The effect of a rating system on consumer surplus and aggregate surplus when higher quantities increase the informativeness of a rating.
Left panel: The solid red line depicts the sum of consumer surplus in both periods with a rating system in % of the consumer surplus without a rating system. For intermediate k, consumers are worse off with more information. This effect is so strong that even though period one consumers benefit, aggregate consumer surplus is lower than without the rating system.
Right panel: The solid red line depicts the sum of aggregate surplus in both periods with a rating system in % of the aggregate surplus without a rating system. For intermediate k, aggregate consumer surplus is lower than without a rating system. This effect is so strong that even though the firm always benefits from more information, aggregate surplus is lower with the rating system.

specifically higher firm profits overall and higher quantities and thus total surplus in the first period.

5. Conclusion

Motivated by the omnipresence of rating systems allowing for between-consumer information transmission in modern online markets, we analyze a seller's strategic behavior in a framework where current-period outcomes affect the degree to which information is generated and available to market participants in the future.

We show that while the seller always benefits from information transmission via rating systems, this does not extend to consumer and total surplus. From a static perspective, consumers may be harmed by the seller's strategic response to positive information. The loss in consumer surplus resulting from an increase in the equilibrium price can more than offset the gain in surplus from making more informed decisions. The detrimental impact on consumer surplus can render better information undesirable even from a welfare perspective.

From a dynamic perspective, we show that the seller sacrifices flow profits to enhance information transmission and thereby future profits. In view of the induced current-period outcome, the direction of the distortion depends on whether information transmission is enhanced by an increase or a decrease in the number of ratings. An increase in that number is motivated by more information contributed by additional buyers. A decrease is motivated by the possibility that reviews contributed by additional non-expert buyers with a lower willingness to pay smutch the information generation. This gives rise to potential intertemporal trade-offs where late consumers benefit at the expense of early consumers, or vice versa. We provide conditions under which these effects materialize. Our analysis thus sheds light on an important aspect of the role of rating systems in modern marketplaces.

Appendix

A. Proof of Proposition 1 for a general information mechanism.

Proof. Consider a general information mechanism $\lambda(m|s)$, where $m \in \mathcal{M}$ is a message and $s \in \{h, l\}$. If message m is realized, then the profit function is given by $\pi(q, \lambda(h|m))$. Define q_m as the maximizer of $\pi(q, \lambda(h|m))$. We assume that λ is informative and for some q^m is different for at least two messages. Note that the seller prefers λ to no information if and only if

$$\int_{m \in \mathcal{M}} \mathbb{P}(m)[\lambda(h|m)\Pi(q^m, 1) + (1 - \lambda(h|m))\Pi(q^m, 0)]dm - \Pi(q, \mu) > 0.$$

Using the Bayes rule $\int_{m \in \mathcal{M}} \mathbb{P}(m)\lambda(h|m)dm = \mu$, we obtain that

$$\begin{aligned}
&\int_{m \in \mathcal{M}} \mathbb{P}(m)[\lambda(h|m)\Pi(q^m, 1) + (1 - \lambda(h|m))\Pi(q^m, 0)]dm - (\mu\Pi(q, 1) + (1 - \mu)\pi(q, 0)) \\
&= \int_{m \in \mathcal{M}} \mathbb{P}(m)[\lambda(h|m)\Pi(q^m, 1) + (1 - \lambda(h|m))\Pi(q^m, 0)]dm \\
&- \int_{m \in \mathcal{M}} \mathbb{P}(m)[\lambda(h|m)\Pi(q, 1) + (1 - \lambda(h|m))\Pi(q, 0)]dm \\
&= \int_{m \in \mathcal{M}} \mathbb{P}(m)(\Pi(q^m, \lambda(h|m)) - \Pi(q, \lambda(h|m)))dm > 0,
\end{aligned}$$

which implies that the seller strictly prefers any informative signal λ to no information. ■

B. Proof of Proposition 2.

Proof. The FOC of the seller's problem is given by

$$\Pi_q = P(q, \mu) + qP_q(q, \mu) - c'(q) = 0.$$

By applying the envelope theorem we can note that

$$\begin{aligned}
\frac{d}{d\mu}\Pi(\mu) &= \Pi_\mu(q(\mu), \mu) \\
\frac{d^2}{d\mu^2}\Pi(\mu) &= \Pi_{q\mu}q_\mu + \underbrace{\Pi_{\mu\mu}}_{=0}.
\end{aligned}$$

From the implicit function theorem we have that $q_\mu = -\Pi_{q\mu}/\Pi_{qq}$, so

$$\frac{d^2}{d\mu^2}\Pi(\mu) = -\frac{\Pi_{q\mu}^2}{\Pi_{qq}} > 0,$$

as $\Pi_{qq} < 0$ by assumption.

■

C. Proof of Proposition 6.

Proof. It is useful to define the pass-through rate of costs to the price dP/dc evaluated at zero costs is equal to $\rho = \frac{P_q}{\Pi_{qq}} = \frac{1}{2-\sigma}$. Therefore $q_\mu = -\Pi_{q\mu}/\Pi_{pp} = \rho\Pi_{q\mu}/P_q$ and so $q_\mu P_q = \rho\Pi_{q\mu}$. By plugging the last identity back to the expression for $CS''(\mu)$ we obtain

$$\begin{aligned} CS''(\mu) &= \rho\Pi_{q\mu}q_\mu - \beta q \\ &= \rho\Pi''(\mu) - \beta q. \end{aligned} \tag{32}$$

We explore the sign of (32) for the case of pure demand shift and pure demand rotation separately.

Case 1: pure demand shift. Suppose that $P = a(1+\phi\mu) - t(q)$, $a, \phi > 0, t' > 0$, then the associated profits are given by $\Pi(q,\mu) = a(1+\phi\mu)q - qt(q)$. The FOC of the seller's maximization problem implies that $0 = a(1+\phi\mu) - t(q) - qt'(q)$ for $q = q(\mu)$. Consequently, the curvature of the demand function with respect to quantity is given by $\sigma = -\frac{qt''}{t'}$. The SOC implies that $\sigma < 2$ and the pass-through rate of cost evaluated at zero cost is $\rho = \frac{1}{2-\sigma}$.

Then $\frac{d^2}{d\mu^2}(\Pi) = -\frac{\Pi_{q\mu}^2}{\Pi_{qq}} = -\frac{a^2\phi^2}{\Pi_{qq}} = -a^2\phi^2\frac{\rho}{P_q} = a^2\phi^2\frac{\rho}{t'}$. Next we compute α and β. Since $q_\mu = a\phi\frac{\rho}{t'}$

$$\begin{aligned} \alpha &= a\phi(1-\rho) \\ \beta &= -a\phi\rho' \end{aligned}$$

By plugging these expression back to (32) we obtain

$$CS''(\mu) = a^2\phi^2\frac{\rho^2}{t'} + a\phi\rho' q$$

Therefore the sign of CS'' is determined by the sign of

$$\begin{aligned} a\phi\rho^2 + \rho' qt' &= a\phi\rho^2 + \rho' P \\ &= \frac{1}{(2-\sigma)^2}(a\phi + \sigma' P). \end{aligned}$$

Case 2: pure demand rotation. Suppose that $P = a - (1-b\mu)t(q)$, $a, b > 0, t' > 0$. Then the seller's profit is given by $\Pi = aq - (1-b\mu)qt(q)$. The FOC of the profit maximization problem is $0 = a - (1-b\mu)(qt(q))'$. The curvature of the demand function is given by $\sigma = -\frac{qt''}{t'}$. The pass-through rate of cost evaluated at zero cost is $\rho = \frac{1}{2-\sigma}$. Then using the fact that $\pi_{q\mu} = b(qt(q))' = ab/(1-b\mu)$ we obtain

$$(\Pi^*)'' = -\frac{\Pi_{q\mu}^2}{\Pi_{qq}} = -\frac{\rho}{P_q}\frac{a^2b^2}{(1-b\mu)^2} = \frac{\rho a^2 b^2}{(1-b\mu)^3 t'}.$$

Since $q_\mu = \frac{ab}{(1-b\mu)^2 t'}\rho$ we can compute α and β as

$$\begin{aligned}
\alpha &= -\frac{ab}{1-b\mu}\rho + bt \\
\beta &= -\frac{ab^2}{(1-b\mu)^2}\rho - \frac{ab}{1-b\mu}\rho' + bt' q_\mu \\
&= -\frac{ab}{1-b\mu}\rho'
\end{aligned}$$

Using the FOC we have that $q = -P/P_q$. By plugging all these expressions into (32) we find that

$$\begin{aligned}
CS'' &= \frac{\rho^2 a^2 b^2}{(1-b\mu)^3 t'} + \frac{ab}{1-b\mu}\frac{P}{(1-b\mu)t'}\rho' \\
&= \frac{ab}{(1-b\mu)^2 t'}\left(\frac{ab}{1-b\mu}\rho^2 + \rho' P\right) \\
&= \frac{ab}{(1-b\mu)^2 t'}\left(\frac{ab}{1-b\mu} + \sigma' P\right)\rho^2,
\end{aligned}$$

which implies that the sign of $CS''(\mu)$ is determined by the sign of $\frac{ab}{1-b\mu} + \sigma' P$. ■

D. Necessary and sufficient conditions for convexity (concavity) of CS for a general demand function when $c(q) = 0$.

Proposition 7 *Suppose that $c(q) = 0$. If $P(q,\mu) = a(1+\phi\mu) - (w(q) - b\mu)t(q)$, $a, b, \phi > 0$, $w' > 0, t' > 0$ (demand shift and demand rotation), then the sign of $CS''(\mu)$ is determined by the sign of $\rho[\Pi_{q\mu} + \sigma' P] - b(t' - \rho(tq)'')q$.*

Proof. To find the sign of (32) it is useful to calculate the following expressions:

$$\begin{aligned}
\Pi_{q\mu} &= a\phi + b(tq)', \\
P_q &= \mu b t' - (wt)', \\
P_{qq} &= \mu b t'' - (wt)'', \\
P_\mu &= a\phi + bt.
\end{aligned}$$

The pass-though rate of quality and its behavior is given by

$$\begin{aligned}
\alpha &= -\rho\pi_{q\mu} + P_\mu, \\
\beta &= b(t' - \rho(tq)'')q_\mu - \rho'\pi_{q\mu} \\
&= b(t' - \rho(tq)'')q_\mu - \rho^2\pi_{q\mu}\sigma' \\
&= \left(b(t' - \rho(tq)'') + \rho P_q \sigma'\right) q_\mu \\
&= \left(b(t' - \rho(tq)'') - \rho P/q\sigma'\right) q_\mu.
\end{aligned}$$

By plugging this back to the formula for CS'' we find that

$$\begin{aligned} CS''(\mu) &= \rho(\pi^*)'' - \beta q \\ &= q_\mu(\rho\pi_{q\mu} - \beta q/q_\mu) \\ &= q_\mu(\rho[\pi_{q\mu} + \sigma' P] - b(t' - \rho(tq)'')q) \end{aligned}$$

Note that if $b = 0$, then we get that the sign is determined by $a\phi + \sigma' P$. If $\phi = 0$ and $w(q) = 1$, then the sign of CS'' is determined by the sign of $b(tq)' + \sigma' P$.

■

Proof of Corollary 4. Proof. *Case 1: pure demand shift.* Note that the FOC implies that $0 = a(1 + \phi\mu) - (1 + k)bq^k$ and $\sigma = 1 - k$, so if $k > 1$ then $\sigma < 0$ so the curve is concave (convex if $0 < k < 1$). Consequently, $\rho = \frac{1}{1+k}$ and $\rho' = 0$ which implies that $CS''(\mu) = \rho(\pi^*)'' + \rho' q = \frac{1}{1+k}(\pi^*)'' > 0$. This implies that total welfare is also convex.

Case 2: pure demand rotation. Note that the FOC implies that $0 = a - (1 - b\mu)c(1 + k)q^k$ and $\sigma = 1 - k$, so if $k > 1$ then $\sigma < 0$ so the curve is concave (convex if $0 < k < 1$). Consequently, $\rho = \frac{1}{1+k}$ and $\rho' = 0$ which implies that $CS''(\mu) = \frac{\rho^2 a^2 b^2}{(1-b\mu)^3 k q^{k-1}} > 0$. This implies that total welfare is also convex. ■

E. Linear-quadratic case—general conditions.

To simplify notation, denote $a_\mu := \mu a_h + (1 - \mu)a_l$ and $b_\mu := \mu b_h + (1 - \mu)b_l$. The optimal quantity, price, profits, consumer surplus and aggregate surplus given belief μ are

$$\begin{aligned} q(\mu) &= \frac{a_\mu}{2(b_\mu + k)} \\ p(\mu) &= \frac{a_\mu}{2(b_\mu + k)}(b_\mu + 2k) \\ \pi(\mu) &= \frac{a_\mu}{2}\frac{a_\mu}{2(b_\mu + k)} \\ CS(\mu) &= \frac{b_\mu}{2}\left(\frac{a_\mu}{2(b_\mu + k)}\right)^2 \\ S(\mu) &= \frac{3b_\mu + 2k}{2}\left(\frac{a_\mu}{2(b_\mu + k)}\right)^2. \end{aligned}$$

Observation 1 *In the linear-quadratic example, consumer surplus is globally concave if the parameters satisfy one of the following three sets of conditions*

1. $0 < b_h < b_l < 2k, 0 < a_l < a_h < a_l\left(\frac{b_h+k}{b_l+k} + \frac{b_l-b_k}{b_l+k}\frac{2k}{b_l}\right)$
2. $0 < b_l < b_h < 2k, 0 < a_l < a_h < a_l\frac{b_h+k}{b_l+k}$
3. $0 < b_l < b_h, b_h > 2k, 0 < a_l\frac{b_h+k}{b_l+k+2k\frac{(b_h-b_l)}{b_h}} < a_h < a_l\frac{b_h+k}{b_l+k}$.

In the linear-quadratic example, aggregate surplus is globally concave if the parameters satisfy

4. $0 < b_l < b_h, 0 < a_l\frac{(b_h+k)(3b_h+2k)}{(b_l+k)(3b_h+2k)+2k(b_h-b_l)} < a_h < a_l\frac{b_h+k}{b_l+k}$.

In the linear-quadratic example, consumer surplus is locally concave if the parameters satisfy one of the following six sets of conditions

5. $0 < b_h < b_l \frac{2}{b_l/k+3} < b_l < 2k, 0 < a_l < a_l \frac{a_l}{b_l+k}\left(b_h + k - 2k\frac{b_h-b_l}{b_l}\right) < a_h$
   $\mu \in \left(\frac{b_l}{b_h-b_l}\frac{a_h(b_l+k)-a_l(b_h+k)-a_l\frac{2k}{b_l}}{a_l(b_h+k)-a_h(b_l+k)-(a_h-a_l)2k}, 1\right)$

6. $0 < b_l \frac{2}{b_l/k+3} < b_h < b_l < 2k, 0 < a_l < a_l \frac{a_l}{b_l+k}\left(b_h + k - 2k\frac{b_h-b_l}{b_l}\right) < a_h < a_l \frac{b_h+k}{b_l+k+2k(1-\frac{b_h}{b_l}})$
   $\mu \in \left(\frac{b_l}{b_h-b_l}\frac{a_h(b_l+k)-a_l(b_h+k)-a_l\frac{2k}{b_l}}{a_l(b_h+k)-a_h(b_l+k)-(a_h-a_l)2k}, 1\right)$

7. $0 < b_l < 2k < b_h, 0 < a_l < a_h < a_l \frac{b_h+k}{b_l+k+2k\frac{(b_h-b_l)}{b_h}}, \mu \in \left(0, \frac{b_l}{b_h-b_l}\frac{a_h(b_l+k)-a_l(b_h+k)-a_l\frac{2k}{b_l}}{a_l(b_h+k)-a_h(b_l+k)-(a_h-a_l)2k}\right)$

8. $0 < b_h < b_l \frac{2}{b_l/k+3} < 2k < b_l, \mu \in \left(\frac{b_l}{b_h-b_l}\frac{a_h(b_l+k)-a_l(b_h+k)-a_l\frac{2k}{b_l}}{a_l(b_h+k)-a_h(b_l+k)-(a_h-a_l)2k}, 1\right)$

9. $0 < b_l \frac{2}{b_l/k+3} < b_h < 2k < b_l, 0 < a_l < a_h < a_l \frac{b_h+k}{b_l+k+2k\frac{(b_h-b_l)}{b_h}},$
   $\mu \in \left(\frac{b_l}{b_h-b_l}\frac{a_h(b_l+k)-a_l(b_h+k)-a_l\frac{2k}{b_l}}{a_l(b_h+k)-a_h(b_l+k)-(a_h-a_l)2k}, 1\right)$

10. $0 < 2k < b_l < b_h, 0 < al < a_l \frac{b_h+k+2k(1-\frac{b_h}{b_l})}{b_l+k} < a_h < a_l \frac{b_h+k}{b_l+k+2k\frac{(b_h-b_l)}{b_h}},$
    $\mu \in \left(0, \frac{b_l}{b_h-b_l}\frac{a_h(b_l+k)-a_l(b_h+k)-a_l\frac{2k}{b_l}}{a_l(b_h+k)-a_h(b_l+k)-(a_h-a_l)2k}\right).$

In the linear-quadratic example, aggregate surplus is locally concave if the parameters satisfy

11. $0 < b_l < b_h, 0 < a_l < \frac{a_l}{b_l+k}\left(b_l\frac{3b_h+2k}{3b_l+2k} + k\right) < a_h < \frac{a_l(b_h+k)}{b_h\frac{3b_l+2k}{3b_h+2k}+k},$
    $\mu \in \left(0, \frac{b_l}{b_h-b_l}\frac{a_h(b_l+k)-a_l(b_h+k)-a_l\frac{2k}{b_l}}{a_l(b_h+k)-a_h(b_l+k)-(a_h-a_l)2k}\right).$

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