Discussion Paper Series – CRC TR 224

Discussion Paper No. 331
Project B 05

Collusion Between Non-differentiated Two-Sided Platforms

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April 2022
(*First version: January 2022*)

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Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Collusion between non-differentiated two-sided platforms*

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April 7, 2022

Abstract

Platform competition can be intense when offering non-differentiated services. However, competition is somewhat relaxed if platforms cannot set negative prices. If platforms collude they may be able to implement the outcome that maximizes industry profits. In an infinitely repeated game with perfect monitoring, this is feasible if the discount factor is sufficiently large. When this is not possible, under some condition, a collusive outcome with one-sided rent extraction along the equilibrium path can be sustained that leads to higher profits than the non-cooperative outcome.

Keywords: Two-sided markets, tacit collusion, cartelization, price structure, platform competition

JEL classifiaction: L41, L13, D43

*Martin Peitz gratefully acknowledges financial support by Deutsche Forschungsgemeinschaft (DFG) through CRC TR 224 (Project B05).

1 Introduction

Markets with two-sided platforms featuring positive cross-group network effects and non-differentiated services have the tendency that one platform attracts all users on both sides. However, the presence of the non-active platform constrains the behavior of the focal platform in a static non-cooperative setting. If two-sided platforms repeatedly compete with each other, they may have the ability to obtain higher profits by using supergame strategies that contain punishments for deviations from a collusive outcome. In this paper we investigate the scope for tacit collusion in such platform markets.

We analyze tacit collusion using grim trigger strategies in markets with homogeneous, single-homing users on both sides of the platforms. We determine the critical discount factor above which the monopoly outcome can be sustained by platforms which set participation fees on each side of the market in each period to maximize profits. Below this critical discount factor the non-cooperative solution of the static game or some partially collusive outcome can be sustained. As we show, for some range of discount factors a partially collusive outcome can be supported that features the monopoly price on the side of the market which exerts a lower cross-group network effect and the non-cooperative price of the single-period game on the other side. This outcome can be sustained for lower discount factors than the monopoly outcome. Thus, for an intermediate range of discount factors, the equilibrium with one-sided collusion exists, while the equilibrium with full collusion does not. One-sided collusion features more asymmetric prices across the two sides than the fully collusive equilibrium, which can be supported for larger discount factors, and the non-cooperative equilibrium, which emerges for lower discount factors.

Related literature: It has been recognized for some time that collusion between two-sided platforms is a relevant real-world topic; for an informal discussion; for instance, see Evans and Schmalensee (2008, 2013). Formal analyses are provided by Ruhmer (2011), Dewenter, Haucap, and Wenzel (2011), Boffa and Filistrucchi (2014), and Lefouili and Pinho (2020). While the specifics of the models differ, all have in common that they feature differentiated two-sided platforms such that both under one-shot Nash and collusive behavior both platforms are active.[1] This is in line with the strand of literature in two-sided markets initiated by Rochet and Tirole (2003) and Armstrong (2006). In particular, Ruhmer (2011) and Lefouili and Pinho (2020) consider the infinitely repeated version of the Armstrong (2006) model with differentiated platforms and two-sided single-homing – the latter also consider

[1] In static settings, positive market shares of more than one platform can also arise with non-differentiated platforms, when participants on one side of the market are engaged in imperfect competition, as shown by Karle, Peitz, and Reisinger (2020). They can also arise when platforms can commit to quantities on each side (see Correia-da-Silva et al., 2019).

the competitive bottleneck model. By contrast, our setting features non-differentiated platforms as in Caillaud and Jullien (2001, 2003). They show that non-differentiated platforms that can charge any participation fees (i.e., a positive or negative price) and can also charge per transaction, set prices such that all users join the same platform and platform profits are zero. In this paper, we consider the infinitely repeated game version of the related model in which platforms cannot charge transaction fees and participation fees are non-negative.[2] We provide conditions that a less competitive outcome than the one in the one-shot game can be sustained through tacit collusion and establish conditions under which the fully collusive or a partially collusive outcome can be sustained.

2 Model

We consider a model with two non-differentiated platforms that interact repeatedly over an infinite horizon, $t \in \{0, 1, ...\}$. We assume that the platforms have a common discount factor $\delta \in (0, 1)$. There are two consumer sides labelled as A and B with consumer mass n_A and n_B, respectively. Consumers choose to subscribe to one of the platforms in each period. We assume that consumers single-home in each period and that there are no intertemporal demand linkages. Dropping the time index, the per-period net surplus for a side-A and side-B consumer from subscribing to platform i is

$$u_A + \alpha n_B^i - f_A^i \text{ and}$$
$$u_B + \beta n_A^i - f_B^i, \text{ respectively,}$$

where u_A is the intrinsic value of subscribing to platform i for a side-A consumer and u_B is the intrinsic value for a side-B consumer; n_A^i is the number of side-A consumers on platform i and n_B^i is the number of side-B consumers on platform i; $f_A^i \geq 0$ is the non-negative participation fee for a side-A consumer on platform i and $f_B^i \geq 0$ is the non-negative participation fee for a side-B consumer on platform i.[3] The parameter $\alpha > 0$ measures the external benefit that a side-A consumer enjoys from the presence of each side-B consumer on the same platform, and $\beta > 0$ measures the external benefit that a side-B consumer obtains from an additional side-A consumer. We assume, without loss of generality, that $\alpha \geq \beta$. In other words, at the margin, side B exerts the weakly larger cross-group network effect on the other side.

[2] Several papers in the two-sided market literature look at markets in which prices cannot be negative. For a recent contribution in which non-negative prices feature prominently, see Choi and Jeon (2021) in the context of bundling. In their extension of competition between non-differentiated platforms to two-sided pricing, Karle, Peitz, and Reisinger (2020) also assume that prices have to be non-negative.

[3] Allowing for negative participation fees would gives rise to the problem of non-existence of a pure-strategy equilibrium in the one-shot game.

Platform i's action in the stage game is to set non-negative prices on the two sides of the market: f_A^i and f_B^i. We assume that platforms incur zero cost of serving consumers on both sides and cannot make side-payments to each other. The profit for platform i in each period is

$$\pi^i(f_A^i, f_B^i, f_A^j, f_B^j) = f_A^i n_A^i(f_A^i, f_B^i, f_A^j, f_B^j) + f_B^i n_B^i(f_A^i, f_B^i, f_A^j, f_B^j).$$

We specify the numbers of consumers who participate as a function of their utilities. In particular, we assume that a side-A consumer participates if

$$\max\left\{u_A + \alpha n_B^1 - f_A^1;\ u_A + \alpha n_B^2 - f_A^2\ \right\} \geq 0$$

where 0 is the outside option for consumer k on side A. Similarly, a side-B consumer participates if

$$\max\left\{u_B + \beta n_A^1 - f_B^1;\ u_B + \beta n_A^2 - f_B^2\ \right\} \geq 0.$$

Since consumers have the same value for the outside option, we can treat the total number of participants on each side, n_A and n_B, as exogenous parameters in the sense that if there is any participation on one side, everybody on this side participates and $n_A^1 + n_A^2 = n_A$ and $n_B^1 + n_B^2 = n_B$.

The timing within each period is as follows:

1. a public randomization device determines which of the two platforms is focal: in each period there is an independent drawn according to which each platform is focal with probability 1/2, with realization $\phi \in \{1, 2\}$;

2. platforms simultaneously set non-negative participation fees (f_A^i, f_B^i);

3. consumers on sides A and B decide simultaneously which, if any, platform to join;

4. platforms observe the resulting allocation.

We call platform i focal if market participants on both sides coordinate their actions to join this platform, whenever this can be supported as equilibrium at stage 3. The public randomization device is drawn independently in each period. Thus, there is no persistence of focality – for a discussion, see the end of the next section. According to the timing, platforms set prices after learning which one is focal; this means that the non-focal platform can decide to deviate from collusive prices after it has learnt that it is non-focal.[4] In a

[4] For simplicity, we assume that the non-focal platform j sets the same prices as the focal one, i, along the equilibrium path. There are other outcome-equivalent equilibria in which the non-focal patform j sets prices on side A with $f_A^j > f_A^i - \alpha n_B$ and $f_B^j > f_B^i - \beta n_A$.

collusive equilibrium, the non-focal platform will make zero profit in the current period. At stage 3 of the game there may exist an equilibrium in the static game in which there is zero participation. However, whenever there is an equilibrium with positive participation, we discard the zero participation equilibrium. In particular, we assume that market participants coordinate on a (stable) equilibrium with maximal participation.

In each period t, after observing which platform is focal, platforms maximize present-discounted profits. A strategy of platform i in period t conditional on the realization of the plublic random device, ϕ, is thus a mapping from all possible histories of prices and realizations of the public randomization device into $\mathbb{R}^2_+$. We consider subgame-perfect Nash equilibria of this infinitely repeated game. As stated above, we use the equilibrium selection criterion proposed by Caillaud and Jullien (2003) according to which consumers coordinate on the equilibrium most favorable to one of the platforms, which thus becomes the focal platform in a given period.

3 Analysis

We analyze tacit collusion between platforms in which subsidization of consumers is not possible. We first characterize the focal subgame-perfect Nash equilibrium of the single-period game. We then characterize the minimum discount factor for which the monopoly profit is sustainable in the industry using the Nash-reversion grim-trigger strategies (sustainability of full collusion). Finally, we consider collusive strategies with one-sided rent extraction leading to lower than monopoly profits.

Noncooperative outcome We start by characterizing the "noncooperative" outcome; i.e., the equilibrium that obtains in the single-period game under our selection rule. We denote the equilibrium prices of the focal platform in the stage game by f_A^n and f_B^n and the associated competitive payoff by π^n. We summarize properties of the noncooperative equilibrium in Lemma 1.

Lemma 1 *The following is an equilibrium of the non-cooperative stage game. Platforms set* $f_A^n = \alpha n_B$ *and* $f_B^n = \beta n_A$ *and all consumers on both sides subscribe to the focal platform.*

To see that the above is an equilibrium, note that because consumers are homogeneous, it must be true that all consumers on a given side subscribe to the same platform. Given the prices charged by the platforms, the consumers on side A receive a net surplus equal to u_A and the consumers on side B receive a net surplus of u_B. The non-focal platform could deviate to prices on both sides equal to zero. Because of focality, consumers weakly prefer to subscribe to the focal platform as it is not possible to earn a payoff higher than u_A (or u_B) by switching to the non-focal platform. It is a best response for each platform to set the prices as in Lemma 1, because consumers prefer to subscribe to the non-focal platform for all prices of the focal platform greater than αn_B and βn_A after the non-focal platform deviated to sufficiently small but positive prices. Thus, the focal platform's noncooperative profit is $\pi^n = (\alpha + \beta) n_A n_B$ and the non-focal platform has a profit of zero.

Collusion on the monopoly outcome Next, we examine the tacitly collusive equilibria. In this subsection we first focus on the equilibrium in which the monopoly outcome is implemented as a collusive market outcome. Here, the focal firm attracts all consumers from both sides, and each firm will be focal with probability 1/2.[5]

With full consumer participation, the profit-maximizing monopoly prices should be such that consumers on both sides get zero payoff. The fully collusive prices along the equilibrium path solve the monopoly problem and are given by $f_A^c = u_A + \alpha n_B$ and $f_B^c = u_B + \beta n_A$. In each period, platform i charges the monopoly prices f_A^c and f_B^c and, in expectations, receives half of the monopoly profit $\pi^c = (u_A + \alpha n_B) n_A + (u_B + \beta n_A) n_B$. In expectations, platforms share equally the stakes of collusion in the next period by conditioning on the absence of deviations from the collusive mechanism in every previous period. We assume that deviations are publicly observable, and a deviation is followed by choosing the equilibrium of the single-period game. Thus collusion is supported by a grim trigger strategies.

We analyze whether a platform can profitably deviate from the monopoly strategy. Potentially profitable deviations in this setting are to charge sufficiently low prices on one side only or on both sides to affect the subscription decisions of consumers. It is easy to show that the deviant platform is always better off by lowering only one price as it is enough to induce consumers on one side to switch and the other consumer side would follow because of the positive cross-group network effects. Clearly, lowering the price is only profitable in the instance that this platform is non-focal. In this case, it has to lower the price on one side to make sure that this side will always participate.

If the deviator targets side A, then the deviation price has to be less than u_A to ensure that the side-A consumers subscribe to the deviator even if no side-B consumers were to join. Since prices are publicly observable, consumers on side B know that all consumers on side A will subscribe to the deviating platform. The optimal deviation price on side B is $f_B^d = u_B + \beta n_A$.

Depending on whether it is more profitable to target side A or side B, the optimal deviation prices (f_A^d, f_B^d) are either $f_A^d = u_A$ and $f_B^d = u_B + \beta n_A$ or $f_A^d = u_A + \alpha n_B$ and

[5] We return to this point in the conclusion.

$f_B^d = u_B$. The deviation payoff is, therefore, given by

$$\begin{aligned}\pi^d &= \max\{u_A n_A + (u_B + \beta n_A) n_B, (u_A + \alpha n_B) n_A + u_B n_B\} \\ &= u_A n_A + u_B n_B + \max\{\alpha, \beta\} n_A n_B \\ &= u_A n_A + u_B n_B + \alpha n_A n_B,\end{aligned}$$

where the last equality follows from the fact that we have assumed $\alpha \geq \beta$. This shows that the best deviation is to target side B.

With the usual arguments, the monopoly outcome is sustainable in this setting for sufficiently high δ. We label the critical discount factor below which the monopoly outcome is not sustainable as δ^c. As stated above, if reversion to the competitive play occurs as a result of deviation, then, in subsequent periods, the focal platform is drawn randomly with equal probabilities. Thus, δ^c is given when the incentive constraint of the non-focal platform

$$\frac{\delta}{1-\delta}\frac{1}{2}\pi^c \geq \pi^d + \frac{\delta}{1-\delta}\frac{1}{2}\pi^n$$

binds. Here, $\pi^n = (\alpha + \beta) n_A n_B$ is the noncooperative payoff for the focal platform. The expression for the expected discounted value along the collusive path reflects that, in the current period, the non-focal platform gets zero profit. Thus, a deviation generates a short-term gain of π^d and a long-term loss of $\pi^c - \pi^n$ in each future period in which the platform turns out to be focal.

After rearranging terms, the critical discount factors above which π^c is sustainable, δ^c, is expressed as

$$\begin{aligned}\delta^c &= \frac{2\pi^d}{2\pi^d + (\pi^c - \pi^n)} \\ &= \frac{2(u_A n_A + u_B n_B + \alpha n_A n_B)}{3(u_A n_A + u_B n_B) + 2\alpha n_A n_B} < 1.\end{aligned}$$

Lemma 2 *The fully collusive outcome with prices $f_A^c = u_A + \alpha n_B$ and $f_B^c = u_B + \beta n_A$ in each period can be sustained in equilibrium of the infinitely repeated game for $\delta \geq \delta^c$.*

We note that the critical discount factor δ^c is increasing in α, n_A, and n_B, while decreasing in u_A and u_B.

Collusion on non-monopoly outcome For a discount factor $\delta < \delta^c$, the monopoly outcome can not be supported as a subgame-perfect Nash equilibrium of the game. However, tacitly collusive equilibria may exist in which platforms charge lower than the monopoly prices. In what follows, we focus on analyzing such equilibria. A particular collusive strategy consists of charging the monopoly price on one side and the noncooperative price on the other. We refer to this strategy as the one-sided rent-extraction strategy and show for which discount factors such prices are chosen along the equilibrium path of a subgame-perfect equilibrium.

Definition 1 *The strategy of one-sided rent extraction on side A is a partially collusive price strategy $(f_A^o,\ f_B^o)$ with $f_A^o = f_A^c$ and $f_B^o = f_B^n$; i.e., platforms set the monopoly price on side A and the competitive price on side B.*

We denote the payoff associated with the one-sided rent-extraction strategy by π^o. Thus, $\pi^o = \pi^c - u_B n_B$. We denote the critical discount factor below which the one-sided rent extraction outcome is not sustainable as δ^o.

Lemma 3 *The one-sided collusive outcome with prices $f_A^o = f_A^c$ and $f_B^o = \beta n_A$ in each period can be sustained in equilibrium of the infinitely repeated game for $\delta \geq \delta^o$ with*

$$\delta^o = \begin{cases} \frac{2(u_A n_A + u_B n_B + \beta n_A n_B)}{2(u_A n_A + u_B n_B + \beta n_A n_B) + u_A n_A}, & \text{if } u_B \leq \beta n_A, \\ \frac{2(u_A + 2\beta n_B)}{2(u_A + 2\beta n_B) + u_A}, & \text{if } u_B > \beta n_A. \end{cases}$$

Proof. Consider the one-sided rent-extraction strategy $(f_A^o,\ f_B^o)$ with $f_A^o = f_A^c$ and $f_B^o = \beta n_A$. Given $(f_A^o,\ f_B^o)$, it is impossible for the deviator to target side B for any $\hat{f}_B^d \geq 0$. If the deviator targets side A, it optimally sets $\hat{f}_A^d = u_A$.

Consider first the case $u_B \leq \beta n_A$. Then, $\hat{f}_B^d = u_B + \beta n_A$. This gives deviation profit $\hat{\pi}^d = u_A n_A + u_B n_B + \beta n_A n_B$.

The per-period industry payoff under the one-sided rent-extraction strategy is sustainable if the following incentive constraint is satisfied:

$$\frac{\delta}{1-\delta}\frac{1}{2}\pi^o \geq \hat{\pi}^d + \frac{\delta}{1-\delta}\frac{1}{2}\pi^n.$$

After rearranging terms, the critical discount factor δ^o above

$$\delta^o \equiv \frac{2\hat{\pi}^d}{2\hat{\pi}^d + \pi^o - \pi^n},$$

where $\pi^n = (\alpha+\beta) n_A n_B$ is the competitive payoff, $\pi^o = (u_A + \alpha n_B)\, n_A + \beta n_A n_B$ is the payoff under one-sided collusion, and $\hat{\pi}^d$ is the deviation payoff when platforms use the one-sided rent-extraction strategy, as defined above. We note that $\pi^o - \pi^n = u_A n_A$. Thus,

$$\delta^o = \frac{2[u_A n_A + u_B n_B + \beta n_A n_B]}{2[u_A n_A + u_B n_B + \beta n_A n_B] + u_A n_A}.$$

Consider second the case $u_B > \beta n_A$. Then, a consumer on side B obtains net benefit $u_B - \beta n_A$ on the focal platform when no consumer from side A joins. The deviating non-focal platform has to leave this net surplus to side-B consumers. Since it attracts all side-A consumers, a side-B consumer receives a net surplus of $u_B + \beta n_A - \hat{f}_B^d$. Thus, the deviating platfrom can set at most $\hat{f}_B^d = 2\beta n_A$. This gives deviation profit $\hat{\pi}^d = u_A n_A + 2\beta n_A n_B$. Hence, the critical discount factor becomes

$$\delta^o = \frac{2(u_A n_A + 2\beta n_A n_B)}{2(u_A n_A + 2\beta n_A n_B) + u_A n_A}.$$

■

For $u_B \leq \beta n_A$, the critical discount factor δ^o is increasing in β, n_A, n_B, and u_B, while decreasing in u_A. For $u_B > \beta n_A$, the critical discount factor δ^o is increasing in β and n_B, while decreasing in u_A.

We are now in the position to compare the sustainability of one-sided collusion to full collusion.

Proposition 1 *There exist parameter constellations such that one-sided collusion is sustainable, whereas full collusion is not. In particular, for any given parameter values β, u_A, u_B, n_A, n_B, the critical discount factor δ^c is strictly greater than δ^o if α is sufficiently large – that is,*

$$\alpha > \left(\beta + \min\left\{\beta, \frac{u_B}{n_A}\right\}\right) \frac{u_A n_A + u_B n_B}{u_A n_A}. \tag{1}$$

Proof. The monopoly payoff π^c is sustainable if the following incentive constraint is satisfied:

$$\frac{\delta}{1-\delta}\frac{1}{2}\pi^c \geq \pi^d + \frac{\delta}{1-\delta}\frac{1}{2}\pi^n.$$

The payoff π^o is sustainable if the following incentive constraint is satisfied:

$$\frac{\delta}{1-\delta}\frac{1}{2}\pi^o \geq \hat{\pi}^d + \frac{\delta}{1-\delta}\frac{1}{2}\pi^n$$

There exists a non-empty range of parameter δ for which π^o is sustainable, but π^c is not; i.e., $0 < \delta^o < \delta^c$, if and only if the following inequality is satisfied:

$$\begin{aligned} \frac{\pi^c - \pi^n}{\pi^d} &< \frac{\pi^o - \pi^n}{\hat{\pi}^d} \\ \iff (\pi^c - \pi^n)\hat{\pi}^d &< (\pi^o - \pi^n)\pi^d, \end{aligned}$$

which is equivalent to $\pi^o \pi^d - \pi^c \hat{\pi}^d > (\pi^d - \hat{\pi}^d)\pi^n$. Since $\hat{\pi}^d$ is defined piece-wise, we have to consider the two cases $u_B \leq \beta n_A$ and $u_B > \beta n_A$.

If $u_B \leq \beta n_A$, then $\hat{\pi}^d = \pi^d - (\alpha - \beta) n_A n_B$ and $\pi^o = \pi^c - u_B n_B$. By plugging in the expressions for π^o and $\hat{\pi}^d$, we can rewrite the inequality as

$$(\alpha - \beta)\frac{(\pi^c - \pi^n)}{\pi^d} > \frac{u_B}{n_A}.$$

By further substituting for π^c, π^d and π^n and rearranging terms, the above inequality reduces to

$$\begin{aligned} \alpha - \beta &> \frac{u_B(u_A n_A + u_B n_B + \alpha n_A n_B)}{n_A(u_A n_A + u_B n_B)} \\ \Longleftrightarrow \alpha - \beta &> \frac{u_B}{n_A} + \alpha \frac{u_B n_B}{u_A n_A + u_B n_B} \\ \Longleftrightarrow \alpha &> \left(\beta + \frac{u_B}{n_A}\right) \frac{u_A n_A + u_B n_B}{u_A n_A}. \end{aligned} \tag{2}$$

If $u_B > \beta n_A$, then $\hat{\pi}^d = \pi^d - (\alpha - \beta) n_A n_B - (u_B - \beta n_A)\, n_B$. The condition $(\pi^c - \pi^n)\hat{\pi}^d < (\pi^o - \pi^n)\pi^d$ can be written as $(u_A n_A + u_B n_B)\hat{\pi}^d < u_A n_A \pi^d$, which, after substituting and rearranging, becomes

$$\alpha > 2\beta \frac{u_A n_A + u_B n_B}{u_A n_A}. \tag{3}$$

Combining inequalities (2) and (3) gives (1). ∎

The intuition is as follows. Since α is greater than β, the deviant platform tends to prefer to target side B than side A since greater rents can be extracted from the latter due to the larger network benefit. Therefore, to limit the profitability of deviations, it is useful for the focal platform to ensure consumer participation on side B (by sacrificing all supra-competitive rents on that side) and extract the monopoly rents from the consumers on side A.

In the Appendix we show that when $\delta < \min\{\delta^o, \delta^c\}$ the cartel cannot obtain higher profits than in the non-cooperative outcome with per-period profit π^n. For $\delta > \delta^c$ the monopoly outcome can be achieved. When $\delta^o < \delta^c$ and $\delta \in [\delta^o, \delta^c)$, the maximal cartel profit is achieved with the one-sided rent-extraction strategy given in Lemma 3. In particular, the partially collusive outcome in which all rents are extracted on side B never maximizes cartel profits.

Discussion Our main result shows that, by adopting the one-sided rent-extraction strategy, platforms can enhance the sustainability of collusion beyond the sustainability of the monopoly outcome: When network effects are sufficiently large on side A, there exists a range of discount factor δ for which the monopoly outcome is not sustainable but the one-sided collusive outcome is. Rent extraction takes place on the side that is subject to the larger cross-group network effect.

Our finding relates to the two-sided single-homing model with differentiated platforms investigated by Lefouili and Pinho (2020). They find for an intermediate range of discount factors that platforms set the monopoly price on one side and a price between the competitive and the monopoly price on the other side. Since platforms are non-differentiated the market outcome is markedly different from the one in Ruhmer (2011) and Lefouili and Pinho (2020): in our model, one platform carries all the trade in each period, whereas in Ruhmer (2011) and Lefouili and Pinho (2020), where platforms are sufficiently differentiated relative to the strength of the network effects, each platform carries half the trade in equilibrium.

In our analysis, we assumed that there is no persistence. One way to model persistence is to assume that the platform that attracted all participants in the previous period is more likely to be focal in the next period. Then, the non-focal platform has a stronger incentive to deviate from the collusive price and it becomes more difficult to sustain collusion. This implies higher critical discount factors (δ^c and δ^o) and, thus, shows that more persistence (which may be interpreted to be an "incumbency" advantage) is detrimental to collusion.

We also assumed that platforms set prices after the realization of the public randomization device. If platforms were to set prices before they know which one is focal, both platforms obtain the same expected profit when following the collusive strategy. This reduces the deviation incentive of any platforms because each platform knows that a deviation is costly in the present period if it turns out to be focal. Thus, collusion would become easier to sustain.

4 Conclusion

In this note, we provide a simple analysis of tacit collusion between homogeneous two-sided platforms which can not set negative prices. In the infinitely repeated game with discounting, we analyze equilibria when platforms use grim trigger strategies. Due to the constraints that prices can not be negative, it is more difficult to challenge the focal platform and, therefore, platforms set strictly positive prices on both sides in the equilibrium of the single-period game. This is the unique outcome in the infinitely repeated game for sufficiently low discount factors. For sufficiently high discount factors, an equilibrium can be sustained in which platforms set monopoly prices on both sides along the equilibrium path. For intermediate discount factors, platforms can improve on the non-cooperative outcome by setting the monopoly price on one side and offering their service at the competitive price on the other along the equilibrium path.

As mentioned in Section 2, removing the non-negative price constraint gives rise to the problem of non-existence of a pure-strategy equilibrium in the one-shot game. One remedy would be to consider sequential price setting as in Jullien (2011). Future work may want to look at collusive strategies in the infinitely repeated game in which platforms set prices sequentially in each period.

Future work may also want to allow for richer network effects. In particular, it may be interesting to allow for negative cross-group network effects experienced on one side.

Our setting with non-differentiated platforms has the feature that consumers coordinate on the same platform and one platform carries all trade also with collusion. From an applied perspective, this may seem unappealing. An extension would be to include some loyal users on each side for each platform and allow for price discrimination between loyals and illoyals. Such a straightforward extension would lead the non-focal platform to extract rents from its loyal consumers and thus to enjoy some profit.

Alternatively, if platforms are present in multiple markets, both platforms can make strictly positive profits in each period (in some markets one platform is not active and in others the other platform). Future work may explicitly take such multi-market contact into consideration.

5 Appendix

For given parameters $(\delta, \alpha, \beta, u_A, u_B, n_A, n_B)$, a platform cartel maximizes the total collusive payoff subject to the incentive compatibility and no subsidization constraints:

$$\max_{f_A, f_B} \pi^{pc}(f_A, f_B)$$
$$\text{s.t. } \hat{\pi}^d(f_A, f_B) \leq \frac{\delta(\pi^{pc}(f_A, f_B) - \pi^n)}{2(1-\delta)}$$
$$f_A \geq 0, \;\; f_B \geq 0$$

If parameters are such that the monopoly payoff is sustainable, then, obviously, the optimal cartel prices are equal to the fully collusive prices f_A^c and f_B^c. Otherwise, the optimal cartel strategy is a partially collusive strategy involving at least one price that is lower than the monopoly price. We adopt the following notation: $f_A^{pc} \equiv f_A^c - \varepsilon_A$ and $f_B^{pc} \equiv f_B^c - \varepsilon_B$, where $\varepsilon_A \geq 0$ and $\varepsilon_B \geq 0$ are nonnegative numbers. We denote the optimal ε_A and ε_B as $(\varepsilon_A^*, \varepsilon_B^*)$. It is easy to see that $\varepsilon_A^* \leq u_A$ and $\varepsilon_B^* \leq u_B$ must hold; i.e the optimal cartel price on each side must be greater than or equal to the competitive price.[6] To determine ε_A^* and ε_B^*, we characterize the highest sustainable collusive payoff.

It is useful to spell out how a deviating platform sets its prices, as this explains why we distinguish between different "scenarios." It can set its prices such that users on one side will definitely join the deviating platform; it can then extract rents on the other side. If the deviating platform targets side B, it will charge the price below $\hat{f}_B^d = u_B - \varepsilon_B$ that makes uses on side B indifferent between joining the (empty) platform B and joining the (full) focal platform. Then, to attract consumers on side A, the deviating platform must give users on that side a utility that is the maximum between the net surplus they get on the other platform (which, with the deviation, no longer has any side-B users), $\varepsilon_A - \alpha n_B$, and the outside option, 0:

$$\hat{f}_A^d = \begin{cases} u_A + 2\alpha n_B - \varepsilon_A & \text{for } \varepsilon_A \geq \alpha n_B \\ u_A + \alpha n_B & \text{else.} \end{cases}$$

Correspondingly, if the deviating platform targets side A, it sets (just below) $\hat{f}_A^d = u_A - \varepsilon_A$ and

$$\hat{f}_B^d = \begin{cases} u_B + 2\beta n_A - \varepsilon_B & \text{for } \varepsilon_B \geq \beta n_A \\ u_B + \beta n_A & \text{else.} \end{cases}$$

[6] Charging a competitive price on a given side ensures that the consumers on that side do not switch for any nonnegative price offered by the deviant platform.

Scenario 1.a. $\{\varepsilon_A \leq \alpha n_B$ and $\varepsilon_B \leq \beta n_A\}$ and deviator targets side B.

The deviator prefers to target side B rather than A, if $\varepsilon_B n_B \leq \varepsilon_A n_A + (\alpha - \beta) n_A n_B$. The relevant deviation payoff is $\hat{\pi}^d = \pi^d - \varepsilon_B n_B$ and the IC is written as

$$\pi^d - \varepsilon_B n_B \leq \frac{\delta(\pi^c - \varepsilon_A n_A - \varepsilon_B n_B - \pi^n)}{2(1-\delta)}$$

which is equivalent to

$$2(1-\delta)\pi^d \leq \delta\left(\pi^c - \pi^n\right) + (2-3\delta)\varepsilon_B n_B - \delta\varepsilon_A n_A. \tag{4}$$

Thus, when $\delta \geq \frac{2}{3}$, π^c is not sustainable implies that any other collusive payoff π^{pc} is also not sustainable. If $\delta < \frac{2}{3}$, it is possible that π^c is not sustainable, while π^{pc} is sustainable. Assuming $\delta < \frac{2}{3}$, we rewrite (4) as

$$\varepsilon_B n_B \geq \frac{\delta}{2-3\delta}\varepsilon_A n_A + \underbrace{\frac{2(1-\delta)\pi^d - \delta\left(\pi^c - \pi^n\right)}{2-3\delta}}_{\equiv\Delta}$$

which is equivalent to

$$\varepsilon_B n_B \geq \frac{\delta}{2-3\delta}\varepsilon_A n_A + \Delta \tag{5}$$

where

$$\begin{aligned}\Delta &= \frac{2(1-\delta)\left(u_A n_A + u_B n_B + \alpha n_A n_B\right) - \delta\left(u_A n_A + u_B n_B\right)}{2-3\delta} \\ &= u_A n_A + u_B n_B + \frac{2\alpha(1-\delta)}{2-3\delta} n_A n_B.\end{aligned}$$

As $\frac{2(1-\delta)}{2-3\delta} > 1$ is satisfied for all $\delta > 0$, it follows from (5) that $\varepsilon_B n_B \geq \alpha n_A n_B$ should hold for the IC to be satisfied. Since in this scenario we consider $\varepsilon_B \leq \beta n_A$ and given $\alpha > \beta$, we find a contradiction. Thus, there does not exist $\{\varepsilon_A,\ \varepsilon_B\}$ such that $\varepsilon_A \leq \alpha n_B$ and $\varepsilon_B \leq \beta n_A$ and the deviator targets side B, which ensures that π^{pc} is sustainable when π^c is not sustainable.

Scenario 1.b. $\{\varepsilon_A \leq \alpha n_B$ and $\varepsilon_B \leq \beta n_A\}$ and deviator targets side A.

The deviator prefers to target side A rather than B, if the following holds:

$$\varepsilon_B n_B > \varepsilon_A n_A + (\alpha - \beta) n_A n_B.$$

Because $\varepsilon_B \leq \beta n_A$ in this scenario, the following should hold

$$\varepsilon_A n_A + (\alpha - \beta) n_A n_B < \beta n_A n_B. \tag{6}$$

The relevant deviation payoff is $\hat{\pi}^d = \pi^d - \varepsilon_A n_A - (\alpha - \beta) n_A n_B$ and the IC is

$$\pi^d - \varepsilon_A n_A - (\alpha - \beta) n_A n_B \leq \frac{\delta(\pi^c - \varepsilon_A n_A - \varepsilon_B n_B - \pi^n)}{2(1-\delta)}$$

which is equivalent to

$$2(1-\delta)\pi^d \leq \delta\left(\pi^c - \pi^n\right) + (2-3\delta)\varepsilon_A n_A - \delta\varepsilon_B n_B + 2(1-\delta)(\alpha-\beta) n_A n_B \tag{7}$$

If $\delta \geq \frac{2}{3}$, then, as in *Scenario 1.a*, we demonstrate that π^c is not sustainable implies that any other collusive payoff π^{pc} is also not sustainable. It is sufficient to show that

$$(3\delta - 2)\varepsilon_A n_A + \delta\varepsilon_B n_B > 2(1-\delta)(\alpha-\beta) n_A n_B$$

Since in *Scenario 1.b,* $(\alpha - \beta) n_A n_B < \varepsilon_B n_B - \varepsilon_A n_A$, the following holds:

$$\frac{2}{3}(\varepsilon_B n_B - \varepsilon_A n_A) > \frac{2}{3}(\alpha-\beta) n_A n_B > 2(1-\delta)(\alpha-\beta) n_A n_B$$

It is then easy to see that

$$(3\delta - 2)\varepsilon_A n_A + \delta\varepsilon_B n_B > \frac{2}{3}(\varepsilon_B n_B - \varepsilon_A n_A) > 2(1-\delta)(\alpha-\beta) n_A n_B$$

which is what we wanted to show.

Next, we consider only the case $\delta < \frac{2}{3}$ and rewrite (7) as

$$\varepsilon_A n_A \geq \frac{\delta}{2-3\delta}\varepsilon_B n_B + \Delta - \frac{2(1-\delta)(\alpha-\beta)}{2-3\delta} n_A n_B, \tag{8}$$

where Δ is defined as in *Scenario 1.a* Since $\varepsilon_B n_B > \varepsilon_A n_A + (\alpha - \beta) n_A n_B$, it follows from (8) that

$$\varepsilon_A n_A \left(1 - \frac{\delta}{2-3\delta}\right) > \Delta + \frac{\alpha-\beta}{2-3\delta}\left(\delta - 2(1-\delta)\right) n_A n_B$$

which is equivalent to

$$\varepsilon_A n_A \frac{2-4\delta}{2-3\delta} > u_A n_A + u_B n_B + \left(\frac{2\alpha(1-\delta)}{2-3\delta} - (\alpha-\beta)\right) n_A n_B. \tag{9}$$

Since for all $\delta > 0$, $\frac{2\alpha(1-\delta)}{2-3\delta} > \alpha$ holds, it follows from (9) that

$$\varepsilon_A n_A \frac{2-4\delta}{2-3\delta} > \beta n_A n_B. \tag{10}$$

Since $\frac{2-4\delta}{2-3\delta} < 1$ and $2\beta - \alpha < \beta$ (as $\alpha > \beta$ by assumption), inequalities (6) and (10) cannot be satisfied simultaneously. Thus, there does not exist a pair $(\varepsilon_A,\ \varepsilon_B)$ such that $\varepsilon_A \leq \alpha n_B$ and $\varepsilon_B \leq \beta n_A$ and the deviator targets side A, ensuring that π^{pc} is sustainable when π^c is not sustainable.

Scenario 2.a. $\{\varepsilon_A \leq \alpha n_B$ and $\varepsilon_B > \beta n_A\}$ and deviator targets side B.

The deviator prefers to target side B if the following holds:

$$\varepsilon_B n_B \leq \varepsilon_A n_A + (\varepsilon_B - \beta n_A)\, n_B + (\alpha - \beta) n_A n_B$$

The relevant deviation payoff is $\hat{\pi}^d = \pi^d - \varepsilon_B n_B$ and the IC is given by (5) as in *Scenario 1.a*

$$\varepsilon_B n_B \geq \frac{\delta}{2-3\delta} \varepsilon_A n_A + \Delta$$

If such $(\varepsilon_A, \varepsilon_B)$ exists, then it must be true that $\varepsilon_B > u_B$ (follows from the definition of Δ) implying that $\varepsilon_B n_B + \varepsilon_A n_A > u_B n_B$. But then the one-sided rent-extraction strategy yields a strictly higher payoff (with $\varepsilon_B = u_B$ and $\varepsilon_A = 0$). Situations (in this and subsequent scenarios) in which the one-sided rent extraction strategy is not sustainable are dealt with towards the end of the proof.

Scenario 2.b. $\{\varepsilon_A \leq \alpha n_B$ and $\varepsilon_B > \beta n_A\}$ and deviator targets side A.

The deviator prefers to target side A if the following holds:

$$\begin{aligned} \varepsilon_B n_B \quad &> \quad \varepsilon_A n_A + (\varepsilon_B - \beta n_A)\, n_B + (\alpha - \beta) n_A n_B \\ \Longleftrightarrow \varepsilon_A n_A \quad &< \quad (2\beta - \alpha)\, n_A n_B. \end{aligned} \tag{11}$$

The relevant deviation payoff is $\hat{\pi}^d = \pi^d - \varepsilon_A n_A - (\varepsilon_B - \beta n_A)\, n_B - (\alpha - \beta) n_A n_B$ and the IC is given by

$$\begin{aligned} \pi^d - \varepsilon_A n_A - (\varepsilon_B - \beta n_A)\, n_B - (\alpha - \beta) n_A n_B \quad &\leq \quad \frac{\delta(\pi^c - \varepsilon_A n_A - \varepsilon_B n_B - \pi^n)}{2(1-\delta)} \\ \Longleftrightarrow \quad \varepsilon_A n_A + \varepsilon_B n_B \quad &\geq \quad \Delta + \frac{2(1-\delta)}{2-3\delta} (2\beta - \alpha)\, n_A n_B. \end{aligned} \tag{12}$$

It follows from (11) and (12) that $\varepsilon_B n_B \geq \Delta$.

If such $(\varepsilon_B; \varepsilon_A)$ exists, then it must be true that $\varepsilon_B > u_B$ which yields strictly lower payoff than the one-sided rent-extraction strategy with $\varepsilon_B = u_B$ and $\varepsilon_A = 0$.

Scenario 3.a. $\{\varepsilon_A > \alpha n_B$ and $\varepsilon_B \leq \beta n_A\}$ and deviator targets side B.

The deviator prefers to target side B if the following holds:

$$\begin{aligned} \varepsilon_B n_B + (\varepsilon_A - \alpha n_B) n_A &\leq \varepsilon_A n_A + (\alpha - \beta) n_A n_B \\ \iff \varepsilon_B n_B &\leq (2\alpha - \beta) n_A n_B. \end{aligned}$$

The relevant deviation payoff is $\hat{\pi}^d = \pi^d - \varepsilon_B n_B - (\varepsilon_A - \alpha n_B) n_A$ and the IC is:

$$\pi^d - \varepsilon_B n_B - (\varepsilon_A - \alpha n_B) n_A \leq \frac{\delta(\pi^c - \varepsilon_A n_A - \varepsilon_B n_B - \pi^n)}{2(1-\delta)}$$

which is equivalent to

$$\varepsilon_A n_A + \varepsilon_B n_B \geq \Delta + \frac{2(1-\delta)\alpha}{2-3\delta} n_A n_B. \tag{13}$$

If such $(\varepsilon_A, \varepsilon_B)$ exists, then it must be true that $\varepsilon_B n_B + \varepsilon_A n_A \geq u_B n_B$. But then the one-sided rent-extraction strategy with $\varepsilon_B = u_B$ and $\varepsilon_A = 0$ yields a strictly higher payoff than any other partially collusive strategy within this scenario.

Scenario 3.b. $\{\varepsilon_A > \alpha n_B$ and $\varepsilon_B \leq \beta n_A\}$ and deviator targets side A.

The deviator prefers to target side A if $\varepsilon_B n_B > (2\alpha - \beta) n_A n_B$ or, equivalently, $\varepsilon_B > (2\alpha - \beta) n_A$ holds. Since $\alpha > \beta$, we have a contradiction to $\varepsilon_B \leq \beta n_A$. Thus, this scenario is not possible.

Scenario 4.a. $\{\varepsilon_A > \alpha n_B$ and $\varepsilon_B > \beta n_A\}$ and deviator targets side B.

The deviator prefers to target side B if the following holds:

$$\varepsilon_B n_B + (\varepsilon_A - \alpha n_B) n_A \leq \varepsilon_A n_A + (\varepsilon_B - \beta n_A) n_B + (\alpha - \beta) n_A n_B.$$

This is equivalent to $\beta n_A n_B \leq \alpha n_A n_B$, which always holds.

The relevant deviation payoff is $\hat{\pi}^d = \pi^d - \varepsilon_B n_B - (\varepsilon_A - \alpha n_B) n_A$ and the IC is given by (13) as in *Scenario 3.a.* If such $(\varepsilon_B, \varepsilon_A)$ exists, then it must be true that $\varepsilon_B n_B + \varepsilon_A n_A > u_B n_B$. But then the one-sided rent-extraction strategy yields a strictly higher payoff as in this case $\varepsilon_B = u_B$ and $\varepsilon_A = 0$.

Scenario 4.b. $\{\varepsilon_A > \alpha n_B$ and $\varepsilon_B > \beta n_A\}$ and deviator targets side A.

The deviator prefers to target side B if $\beta n_A n_B > \alpha n_A n_B$ which can not hold since $\alpha \geq \beta$. So, this scenario is not possible.

Hence, we as showed above, collusion may be sustainable only in Scenarios 2.a, 2.b, 3.a and 4.a. If the one-sided rent extraction strategy is not sustainable and some other partially collusive strategy is sustaniable, then it follows from the incentive compatibility constraints in these scenarios that $\varepsilon_A n_A + \varepsilon_B n_B > u_A n_A + u_B n_B$ must hold. But then the partially collusive payoff should be less than the competitive payoff which is not possible.

We note that for very high discount factors there also exist collusive equilibria in which the non-focal platform extracts all surplus and the focal platform is idle. The non-focal platform then sets prices $f_A = u_A + \alpha n_B$ and $f_B = u_B + \beta n_A$. The focal platform does not attract any consumer for its prices sufficiently high (i.e., $f_A > u_A + \alpha n_B, f_B > u_B$ or $f_A > u_A, f_B > u_B + \beta n_A$) and all consumers on both sides will join the non-focal platform. The incentive constraint of the focal platform is

$$\pi^c \leq \frac{\delta}{2(1-\delta)}(\pi^c - \pi^n)$$

The critical discount factor that satisfies the constraint with equality is $\frac{2\pi^c}{3\pi^c - \pi^n}$, which is larger than δ^c.

We conclude that when the monopoly payoff is not sustainable, the one-sided rent-extraction strategy yields a strictly higher payoff than any other sustainable partially collusive strategy. When neither the monopoly payoff nor the partially collusive outcome with one-sided rent extraction is sustainable, the non-cooperative outcome will prevail.

References

[1] Armstrong, M. (2006). Competition in Two-Sided Markets. Rand Journal of Economics 37, 668-691.

[2] Boffa, F., and L. Filistrucchi (2014). Optimal Cartel Prices in Two-Sided Markets. NET Institute Working Paper 14-19.

[3] Caillaud, B. and B. Jullien (2001). Competing Cybermediaries. European Economic Review 45, 797-808.

[4] Caillaud, B. and B. Jullien (2003). Chicken & Egg: Competition among Intermediation Service Providers. Rand Journal of Economics 34, 309-328.

[5] Choi, J. P. and D.-S. Jeon (2021). A Leverage Theory of Tying in Two-Sided Markets with Non-Negative Price Constraints. American Economic Journal: Microeconomics 13, 283-337.

[6] Correia-da-Silva, J., Jullien, B., Lefouili, Y., and J. Pinho (2019). Horizontal Mergers between Multisided Platforms: Insights from Cournot Competition. Journal of Economics & Management Strategy 28, 109-124.

[7] Dewenter, R., Haucap, J., and T. Wenzel (2011). Semi-collusion in Media Markets. International Review of Law and Economics 31, 92-98.

[8] Evans, D. S. and R. Schmalensee (2008). Markets with Two-sided Platforms. Issues in Competition Law and Policy (ABA Section of Antitrust Law), 1.

[9] Evans, D. S. and R. Schmalensee (2013). The Antitrust Analysis of Multi-sided Platform Businesses. NBER Working Paper No. 18783, February.

[10] Jullien, B. (2011). Competition in Multi-sided Markets: Divide and Conquer. American Economic Journal: Microeconomics 3, 186-220.

[11] Karle, H., Peitz, M., and M. Reisinger (2020). Segmentation versus Agglomeration: Competition Between Platforms with Competitive Sellers. Journal of Political Economy 128, 2329-2374.

[12] Lefouili, Y. and J. Pinho (2021). Collusion Between Two-Sided Platforms. International Journal of Industrial Organization 72, 102656.

[13] Rochet, J.-C. and J. Tirole (2003). Platform Competition in Two-sided Markets. Journal of the European Economic Association 1, 990-1029.

[14] Ruhmer, I. (2010). Platform Collusion in Two-sided Markets. Unpublished manuscript, University of Mannheim.