Internal versus External Growth in Industries with Scale Economies: A Computational Model of Optimal Merger Policy

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Abstract

We study optimal merger policy in a dynamic model in which the presence of scale economies implies that firms can reduce costs through either internal investment in building capital or through mergers. The model, which we solve computationally, allows firms to invest or propose mergers according to the relative profitability of these strategies. An antitrust authority is able to block mergers at some cost. We examine the optimal policy for an antitrust authority who cannot commit to its future policy rule and approves or rejects mergers as they are proposed, considering both consumer value and aggregate value as its possible objectives. We find that the optimal policy can differ substantially from what would be best considering only welfare in the period the merger is proposed. In general, antitrust policy can greatly affect firms' optimal investment behavior, and firms' investment behavior can in turn greatly affect the antitrust authority's optimal policy. Moreover, externalities imposed by mergers on rivals can have significant effects on firms' investment incentives and thereby shape the optimal policy.

1 Introduction

Most analyses of optimal horizontal merger policy in the economics literature are static and focus on the short-run price effects of mergers.[1] But many real-world mergers occur in markets in which dynamic issues are a central feature of competition among firms. As a result, antitrust authorities are regularly confronted with the need to consider likely future effects of a merger on an industry's evolution when deciding whether to approve the merger.[2]

In this paper, we study optimal merger policy in a dynamic setting in which investment plays a central role, as the presence of economies of scale presents firms with the opportunity to lower their average and marginal costs through capital accumulation. These scale economies are also the source of merger-related efficiencies, as a combination of firms' capital through merger lowers average and marginal costs. In such a setting, an antitrust authority's merger approval decisions must weigh any increases in market power against the changes in productive efficiency caused by a merger. Approval of the merger will lower production costs immediately by increasing the scale of the merged firm ("external growth"), which may mean that there is an immediate increase in welfare. However, if the merger is rejected, the firms that wished to merge might instead invest individually to gain scale and lower their costs over time ("internal growth"). Moreover, rivals' investments may change as a result of the merger, altering their efficiency and pricing. Finally, while approval or disapproval of a *particular* merger may affect welfare, merger *policy* can alter firms' *pre-merger* investment behaviors, since those behaviors may be affected by the likelihood that mergers will be approved in the future.

As one example, consider the 2011 attempted merger between AT&T and T-Mobile USA.[3] The merger would have combined the network infrastructure of the two firms. Proponents of the merger argued that this combination would greatly improve both firms' service, creating a more potent rival to Verizon. Opponents countered that the merger would increase market power, and that absent the merger the two firms would each have incentives to independently increase their networks. Thus, the Federal Communications Commission and Department of Justice faced the question of whether the merger would result in a sufficient efficiency improvement (which in this case would be realized on the demand side through enhanced service quality) to offset the increase in market power, taking into account not only any immediate service improvement but also any induced change in the merging firms' future investments. Moreover, the merger would also likely change the investments of the merging firms' rivals, Verizon and Sprint, and possibly potential entrants. Lastly, the investments of firms like T-Mobile could in the future be affected by their expectations of whether mergers such as this would be approved. Similar issues are present in the currently proposed Sprint/T-Mobile USA merger, where the central question is whether the merger would enhance competiton by

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[1] For example, see the classic papers by Williamson (1968) and Farrell and Shapiro (1990).

[2] The U.S. *Horizontal Merger Guidelines*, for example, devote considerable attention to discussions of entry, investment, and innovation.

[3] See Pittman and Li (2013) and DeGraba and Rosston (2014), and the references therein.

creating a stronger third firm.[4]

Our model builds on the computational literature on industry dynamics, pioneered by Pakes and McGuire (1994) and Ericson and Pakes (1995), but with some important differences that make the model more attractive for studying mergers. In that literature, each firm can add one unit of capital in each period, so a merger reduces the investment opportunities both for the merging firms and for the economy. We modify the investment technology to make it *merger neutral*, so that mergers do not change the investment opportunities that are available in the market. Our investment technology also allows for significantly richer investment dynamics, as firms can increase their capital stocks by multiple units, and new entrants can choose endogenously how many units of capital to build when entering.

In addition, we introduce the possibility for firms to merge, as well as an antitrust authority who can block proposed mergers. The decision to propose a merger is endogenous and determined through a bargaining process. We model the authority as a player who cannot commit to its future policy.[5,6] Perhaps surprisingly, issues of policymakers' time consistency have received scant attention in the antitrust literature. We consider both maximization of discounted expected consumer surplus ("consumer value") and discounted expected aggregate surplus ("aggregate value") as possible objectives of the authority, and refer to the policy that emerges as a *Markov perfect policy.*

We begin in Section 2 by describing our model. In each period, firms first bargain over merger proposals. If a merger is proposed, the authority decides whether to allow it and, if so, a new entrant arrives with no capital. Then, the incumbent firms compete in a Cournot fashion. Finally, firms – including any new entrants – decide on capital investment.

In Section 3 we study duopoly markets. A significant challenge in studying optimal merger policy is the lack of a well-accepted canonical model of bargaining in the presence of externalities. While a relatively small share of markets are duopoly markets, and mergers to monopoly are rarely proposed and approved, a significant advantage of examining the behavior of our model in such settings is that the merger bargaining process we adopt for these settings – bilateral Nash bargaining – is well-accepted and easily understood. Throughout most of the section we focus on a single market parameterization so that we can describe equilibrium firm behavior and its interaction with antitrust policy in detail; we discuss afterwards how

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[4]See, for example, "T-Mobile and Sprint: How Fewer Competitors Could Increase Competition," *New York Times*, July 30, 2018. In the EU, similar examples include the Hutchison and Orange Austria, the Hutchison and Telefonica Ireland, the Telefonica Germany and EPlus, the TeliaSonera and Telenor, the Hutchison 3G and Telefonica UK, and the H3G Italy and Wind merger cases.

[5]Despite the existence of merger guidelines in many jurisdictions, antitrust authorities may choose not to follow them when confronted with particular mergers. This was widely viewed to be the case in the years following the release of the 1992 DOJ/FTC Merger Guidelines in the United States. For example, in announcing the release of the 2010 DOJ/FTC Horizontal Merger Guidelines, then Assistant Attorney General Christine Varney commented that "The revised guidelines better reflect the agencies' actual practices." (August 19, 2010 press release)

[6]In the Online Appendix, we also study the optimal commmitment policy.

outcomes vary across a wide range of parameters. When no mergers are allowed, this market spends most of the time in duopoly states and a merger would often increase current-period aggregate surplus.

Our analysis first examines how firm behavior responds when all mergers are allowed or when the antitrust authority implements a static policy that considers only welfare effects in the current period. Not surprisingly, the steady state when all mergers are allowed involves a monopoly or near-monopoly market structure much more often than when mergers are prohibited. It also involves a lower average level of capital. This arises because total investment is lower in monopoly and near-monopoly states. Investment behavior also changes when mergers are allowed. Particularly striking is significantly greater investment by small firms in states in which one firm is very dominant, a form of *"entry for buyout"* [Rasmusen (1988)]. Their investments, made in anticipation of being acquired, are done at high cost and substitute for lower cost investment by larger incumbents, dissipating a great deal of both industry profit and aggregate surplus. Because in this market a merger would increase current-period aggregate surplus in many states, firm behavior with a static aggregate surplus-based policy is essentially equivalent to when all mergers are allowed. In contrast, a static consumer surplus-based policy allows almost no mergers.

We then endogenize merger policy by identifying the Markov perfect policy. With a consumer value objective, the Markov perfect policy basically allows no mergers, just as with the static consumer surplus criterion. With an aggregate value objective, however, the Markov perfect policy allows many fewer mergers than the optimal aggregate surplus-based static policy. The reason is that the inefficient entry for buyout behavior greatly reduces the antitrust authority's desire to approve mergers. The resulting policy significantly reduces the frequency of monopoly and near-monopoly states, and increases both consumer and aggregate value compared to allowing all mergers or following the static aggregate surplus-based policy. Strikingly, it nevertheless results in a lower steady state aggregate value than prohibiting all mergers, or equivalently, having an antitrust authority who seeks to maximize either consumer value or current-period consumer surplus.

In Section 4, we turn our attention to triopoly markets using a variant of the bargaining model of Burguet and Caminal (2015), a model of merger bargaining in the presence of externalities with a number of desirable features. We first confirm that our earlier duopoly results in Section 3 are robust to the possibility of entry of a third firm. We then consider two ways of increasing demand from that considered in Section 3 that lead to markets that spend much of the time as a triopoly when mergers are not allowed. Interestingly, the effects of allowing mergers differ markedly between these two markets. In one market this results in a merger to duopoly, followed by a stable duopoly that almost never attracts entry. In the other, entry of a third firm occurs with regularity, followed by a merger of the entrant with the smaller of the two incumbents, and a repeat of this cycle. Because mergers confer large positive externalities on non-merging firms, allowing mergers creates strong investment incen-

tives in this second market for the duopolist incumbents as they seek to position themselves to be the beneficiary of these externalities. The strong investment by incumbents also reduces the entry for buyout incentives of potential entrants. With the harm arising from entry for buyout either not present or reduced, the aggregate value-based Markov perfect policy is quite permissive in both of these markets; for example, it always allows a merger by symmetric firms who are smaller than their non-merging rival. Overall, compared to not allowing mergers this Markov perfect policy lowers steady state aggregate value in the first market, but leads to little change in aggregate value (despite a strong reduction in consumer value) in the second.

Section 5 concludes and summarizes our insights.

The paper is related to several strands of literature. The first is theoretical work on dynamic merger policy, most notably Nocke and Whinston (2010).[7] In that paper, the dynamics arise from merger opportunities occurring stochastically over time; there is no investment. Another relevant theoretical literature studies the welfare effects of mergers in static models with investment [Motta and Tarantino (2018), Bourreau, Jullien, and Lefoulli (2018), Federico, Langus, and Valletti (2018)].

A second related strand of literature examines mergers in computational dynamic models of industry equilibrium with investment.[8] The closest paper to ours is Gowrisankaran (1999) who introduces an endogenous merger bargaining game into the Pakes-McGuire/Ericson-Pakes framework and examines industry evolution when firms can choose whether, when, and with whom to merge. The adopted investment technology implies that a merger significantly reduces the merging firms' abilities to make future investments, making it unattractive for modeling mergers. There are no scale economies; instead, merger-related efficiencies are assumed to be one-time random benefits. Finally, the model includes a complicated bargaining process whose general properties are unknown; when specialized to the case of two firms, however, it gives the smaller firm the right to make a take-it-or-leave-it offer to the larger firm.[9] Hollenbeck (2017) builds on the approach in our paper, but examines instead settings with investment in quality in an industry with differentiated product price competition. Unlike our paper, he simply compares the outcomes arising if all mergers are allowed to those if a static consumer surplus-based policy is instead followed. Finally, Jerziorski (2015) studies the radio broadcasting industry. He specifies a dynamic model of endogenous mergers with a particular random proposer bargaining process and endogenous station format repositioning investments, and conducts an empirical exercise to estimate his model's parameters. He then simulates the

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[7]Nilssen and Sorgard (1998), Motta and Vasconcelos (2005), and Matushima (2001) analyze static models of competition where two mergers between two non-overlapping pairs of firms can take place sequentially.

[8]Berry and Pakes (1993), Cheong and Judd (2000), and Benkard, Bodoh-Creed, and Lazarev (2010) examine the effects of one-time mergers on industry evolution.

[9]In unpublished work, Gowrisankaran (1997) introduces antitrust policy into the Gowrisankaran (1999) model. Specifically, he examines the effect of commitments to Herfindahl-based policies that block mergers if they result in a Herfindahl index above some maximum threshold and finds little effect of varying the threshold on welfare.

effects of commitments to four specific counterfactual merger policies. He does not examine optimal policy.

Given our focus on only duopoly and triopoly markets, motivated by the plethora of possible approaches to bargaining with externalities with more than two firms, we regard the paper as only a first step in studying optimal merger policy in industries where investment is a central concern. Our results show how optimal policy in dynamic settings with investment can differ in significant ways from what would be statically optimal and provide insights into the factors that affect optimal merger policy in such environments.

2   The Model

We study a dynamic industry model in which a set of $n \geq 2$ firms, $\mathcal{I} \equiv \{1, ..., n\}$, may invest in capacity, or alternatively merge, to increase their capital stocks and harness scale economies. The model follows in broad outline Pakes and McGuire (1994) and Ericson and Pakes (1995), but with some important differences in its investment technology, as well as in the introduction of mergers and merger policy. We focus on symmetric Markov perfect equilibria of our model.

Within each period, the sequence of events is as shown in Figure 1: The firms begin each period observing each others' capital stocks $\mathbf{K} \equiv (K_1, ..., K_n)$, the model's state variable, which affect the firms' production costs. The firms then bargain over which merger, if any, to propose to the antitrust authority. If no merger agreement is reached, the firms proceed to the Cournot competition phase with their current capital levels. If a merger agreement is reached, the merger partners propose their merger to the antitrust authority, who may then decide to block it. If the merger is allowed, the firms combine their capital, and a new entrant appears with no initial capital stock. Following these merger bargaining, merger decision, and entry phases, the active firms engage in Cournot competition given their capital stocks, and earn profits on their sales. Following this Cournot competition stage, the firms choose their capital investments. Finally, depreciation may make obsolete some of a firm's capital. The resulting capital levels after depreciation become the starting values in the next period.

We begin by describing the demand and production costs the firms face (the latter as a function of their capital stocks), and the static Cournot competition that occurs in each period. We then detail how merger bargaining works, the merger policy of the antitrust authority, and the investment, entry, and depreciation processes. The Online Appendix contains a more formal description of our model and computational methods.

2.1   Static Demand, Costs, and Competition

In each period, active firms produce a homogeneous good in a market in which the demand function is $Q(p) = B(A - p)$. The production technology, which requires capital K and labor L, is described by the production function $\left(K^{\beta} L^{(1-\beta)}\right)^{\theta}$, where $\beta \in (0, 1)$ is the capital share

Period begins:	Merger Bargaining:	Merger approval:	Entry:	Cournot competition:	Investment:	Depreciation:
Firms observe capital stocks $\boldsymbol{K}$; value of firm i is $V(K_i, \boldsymbol{K}_{-i})$	Firms bargain after observing proposal costs $\{\Phi_{jk}\}$; a merger combines firms' capital stocks; absent a merger agreement, the firms proceed to the Cournot competition stage	Antitrust authority may block merger M_{jk} after observing blocking cost b_{jk}	If merger occurred between firms j and k, an entrant arrives with no capital, resulting in capital stocks $\boldsymbol{K}' \equiv \left(K_j + K_k, \boldsymbol{K}_{-jk}, 0\right)$	Firms choose outputs given current capital stocks; firm i's interim continuation value at the start of this stage is $\bar{V}(K_i', \boldsymbol{K}_{-i}')$	Firms privately observe investment costs and make capital investment decisions resulting in new capital stocks $\boldsymbol{K}''$	Each unit of capital may become obsolete; resulting capital levels are starting levels in next period

Figure 1: Sequence of events in a single period

and $\theta > 1$ the scale economy parameter. Normalizing the price of labor to be 1, for a fixed level of capital, this production function gives rise to the short-run cost function

$$C(Q|K) = \frac{Q^{1/[(1-\beta)\theta]}}{K^{\beta/(1-\beta)}}$$

with marginal cost

$$C_Q(Q|K) = \left(\frac{1}{(1-\beta)\theta}\right) \frac{Q^{(1/[(1-\beta)\theta])-1}}{K^{\beta/(1-\beta)}}.$$

With this technology, a merger that combines the capital of two identical firms reduces both average and marginal cost if their joint output remains unchanged. This effect will be the source of merger-related efficiencies in our model. Letting R measure the extent of this cost reduction, we have

$$R \equiv \frac{C_Q(2Q|2K)}{C_Q(Q|K)} = \frac{C(2Q|2K)/2Q}{C(Q|K)/Q} = 2^{\left(\frac{1}{1-\beta}\right)\left(\frac{1-\theta}{\theta}\right)}.$$

Note in particular that the marginal cost reduction depends on the scale economy parameter θ and capital share β, but is independent of the output level (and hence demand). In our computations we will focus on a case in which $\beta = 1/3$ and $\theta = 1.1$.[10] Given these values, R is 0.91; that is, a merger of two equal-sized firms results in a 9% efficiency gain.

In each period, active firms engage in Cournot competition given their capital stocks (a firm with no capital produces nothing), resulting in profit $\pi(K_i, \boldsymbol{K}_{-i})$ for a firm with capital stock K_i when the vector of its rivals' capital stocks is $\boldsymbol{K}_{-i} \equiv (K_1, ..., K_{i-1}, K_{i+1}, ..., K_n)$.[11]

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[10] A capital coefficient of 1/3 is routinely assumed in the macroeconomic literature; see, for example, Jones (2005). The scale economy parameter of 1.1 leads to plausible efficiency gains and is selected so that mergers are statically aggregate surplus increasing in a substantial proportion of industry states.

[11] A firm's short-run cost function is strictly convex if $(1-\beta)\theta < 1$, in which case there is a unique Cournot equilibrium if the demand function is weakly concave. In our analysis, these conditions are satisfied.

2.2 Mergers and Bargaining

A merger M_{ij}, which involves the combining of the merging firms' capital, is feasible between any pair $ij \in \mathcal{J} \equiv \{ij | i, j \in \mathcal{I}, i \neq j\}$ of firms. Proposing merger M_{ij} for approval to the antitrust authority involves a cost ϕ_{ij}, which is drawn i.i.d. (both across pairs of firms as well as over time) from a continuous distribution function Φ with support $[\underline{\phi}, \overline{\phi}]$. We introduce these proposal costs primarily for technical reasons to ensure existence of (pure strategy) equilibrium; in the real world, they may represent legal costs.[12] As shown in Figure 1, the antitrust authority may block a merger proposal, while if the firms' merger is allowed a new entrant appears in the market with zero capital.[13]

Let $\overline{V}(K_i, \mathbf{K}_{-i})$ denote the interim expected net present value ("continuation value") of firm i when it has K_i units of capital and the vector of its rivals' capital levels is $\mathbf{K}_{-i}$ at the start of the Cournot competition stage (see Figure 1). If the capital stocks prior to the merger stage are $\mathbf{K}$, then the bilateral value gain from merging, gross of any proposal cost, is

$$\Delta_{ij}(\mathbf{K}) \equiv \overline{V}(K_i + K_j, \mathbf{K}_{-ij}, 0) - (\overline{V}(K_i, \mathbf{K}_{-i}) + \overline{V}(K_j, \mathbf{K}_{-j})), \tag{1}$$

where $\mathbf{K}_{-ij}$ is the vector of capital levels of firms other than i and j. The first term in (1) is the joint interim value if the merger takes place; the second term is the sum of the disagreement payoffs.

The probability that merger M_{ij} gets approved when proposed is denoted $a_{ij}(\mathbf{K})$. The expected change in the merger partners' joint value from proposing merger M_{ij} can thus be written as

$$S_{ij}(\mathbf{K}, \phi_{ij}) \equiv a_{ij}(\mathbf{K})\Delta_{ij}(\mathbf{K}) - \phi_{ij}.$$

The expected externality of the proposal of merger M_{ij} on an outsider $k \neq i, j$ is given by

$$X_k^{ij}(\mathbf{K}) \equiv a_{ij}(\mathbf{K}) \left[\overline{V}(K_k, K_i + K_j, \mathbf{K}_{-ijk}, 0) - \overline{V}(K_k, \mathbf{K}_{-k}) \right],$$

where $\mathbf{K}_{-ijk}$ is the vector of capital levels of firms other than i, j and k.

In each period, at most one merger can be proposed for approval to the antitrust authority. Firms bargain under complete information about which merger to propose (if any) and how to split the surplus, given the vector of merger approval probabilities, $(a_{ij}(\mathbf{K}))_{ij \in \mathcal{J}}$, the vector

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[12] See Doraszelski and Satterthwaite (2010) for a discussion of introducing random private payoffs as a means of ensuring existence.

[13] The immediate arrival of a new entrant following a merger can also be thought of as being the result of a structural remedy imposed by the antitrust authority, which involves the transfer of know how to a firm outside the industry, permitting this firm to become a new entrant. The 2011 U.S. Department of Justice Antitrust Division Policy Guide to Merger Remedies states: "Structural remedies generally will involve [...] requiring that the merged firm create new competitors through the sale or licensing of intellectual property rights." For the case $n = 2$, we have also analyzed in the Online Appendix the case in which the probability of entry is less than one. We also analyze there a case in which only the incumbent manager-owners possess the knowledge of how to operate a firm in this industry so that the new entrant is one of these owners and obtain similar results.

of continuation values in the absence of a merger, $(\overline{V}(K_i, \mathbf{K}_{-i}))_{i \in \mathcal{I}}$, the vector of surpluses, $\left(S_{ij}(\mathbf{K}, \phi_{ij})\right)_{ij \in \mathcal{J}}$, and the matrix of externalities, $\left(X_k^{ij}(\mathbf{K})\right)_{ij \in \mathcal{J}, k \in \mathcal{I}, k \neq i, j}$. For our purposes, the outcome in state $\mathbf{K}$ of a generic bargaining process can be summarized by the vectors $(\vartheta_{ij}(\mathbf{K}))_{ij \in \mathcal{J}}$ and $(V(K_i, \mathbf{K}_{-i}))_{i \in \mathcal{I}}$, where $\vartheta_{ij}(\mathbf{K})$ is the probability that merger M_{ij} gets proposed and approved in state $\mathbf{K}$ and $V(K_i, \mathbf{K}_{-i})$ is the beginning-of-period value of firm i (prior to the realization of proposal costs).

In Section 3, we focus on the case of two firms ($n = 2$) and assume Nash bargaining. In that case, merger M_{12} gets proposed if and only if the bilateral surplus $S_{12}(\mathbf{K}, \phi_{12})$ is positive. The probability that the merger occurs in state $\mathbf{K}$ is therefore given by

$$\vartheta_{12}(\mathbf{K}) = a_{12}(\mathbf{K}) \psi_{12}(\mathbf{K}),$$

where $\psi_{12}(\mathbf{K}) \equiv \Phi(a_{12}(\mathbf{K}) \Delta_{12}(\mathbf{K}))$ is the probability of the merger being proposed. Firm i's beginning-of-period value in state $\mathbf{K}$ includes its possible share of any merger surplus, and equals

$$V(K_i, K_{-i}) = \overline{V}(K_i; K_{-i}) + \frac{1}{2} \int_{\underline{\phi}}^{\overline{\phi}} S_{12}^+(\mathbf{K}, \phi_{12}) d\Phi(\phi_{12}),$$

where $S_{12}^+(\mathbf{K}, \phi_{12}) \equiv \max\{0, S_{12}(\mathbf{K}, \phi_{12})\}$. In Section 4, we explore situations with three firms using an adaptation of the bargaining process of Burguet and Caminal (2015), which we describe there.

2.3 Merger Policy

The antitrust authority has the ability to block mergers. Blocking a proposed merger M_{ij} involves a cost $b_{ij} \in [\underline{b}, \overline{b}]$ drawn each period in an i.i.d. fashion from a distribution H. We introduce these blocking costs primarily for technical reasons to ensure existence of (pure strategy) equilibrium; in the real world, they may represent the opportunity costs of an in-depth merger investigation (which is required for blocking a merger but not for approving a merger) or possible litigation costs.

In our analysis, we focus on a situation in which the antitrust authority cannot commit to its policy.[14] In that case, in any state $\mathbf{K}$, it will decide whether to block a merger by comparing the increase in its welfare criterion from blocking to its blocking cost realization b_{ij}. As welfare criteria, we will consider both consumer value (CV) and aggregate value (AV), the expected net present values of consumer surplus and aggregate surplus, respectively. A Markovian strategy for the antitrust authority is a state-contingent and history-independent threshold $\widehat{b}_{ij}(\mathbf{K})$ describing the highest blocking cost at which it will block merger M_{ij} in a given state $\mathbf{K}$. Equivalently, this can be translated into a merger acceptance probability

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[14]In the Online Appendix, we also consider the case of an antitrust authority who can commit to a deterministic policy $(a_{ij}(\cdot))_{ij \in \mathcal{J}}$ that specifies whether a proposed merger would be approved ($a_{ij}(\mathbf{K}) = 1$) or not ($a_{ij}(\mathbf{K}) = 0$) in each state $\mathbf{K}$.

$a_{ij}(\mathbf{K}) \in [0, 1]$. As we previously noted, we call the equilibrium policy that emerges a Markov perfect policy (MPP). In practice, an antitrust authority may well lack an ability to commit to its future approval policy. While the Department of Justice and Federal Trade Commission in the U.S. periodically issue *Horizontal Merger Guidelines*, which may partially commit these agencies, over time their actual policy often comes to deviate substantially from the *Guidelines'* prescriptions.

2.4 Investment, Entry, and Depreciation

In Pakes and McGuire (1994) and Ericson and Pakes (1995) a firm chooses in each period how much money to invest, with the probability of successfully adding one unit of capital increasing in the investment level. We depart from this technology because in a model of mergers it would impose a significant inefficiency on mergers. In particular, it would restrict the merged firm to adding one unit of capital each period while, if they had not merged, the firms could have *each* added one unit of capital for a total addition of two units.[15] Instead, we specify an investment technology that is *merger neutral* at a market level. By that we mean that a planner who controlled the firms and wanted to achieve at least cost any fixed increase in the market's aggregate capital stock would be indifferent about whether the firms merge. With this assumption we isolate the market-level technological effects of mergers fully in the scale economies of the production function. These technological effects on production costs, combined with firms' behavioral responses in investment, will determine the efficiency benefits of mergers in our model.

We imagine that there are two ways that a firm can invest. The first is *capital augmentation*: each unit j of capital that a firm owns can be doubled at some cost $c_j \in [\underline{c}, \overline{c}]$ drawn from a distribution F. The draws for different units of capital are independent and identically distributed. Thus, for a firm that has K units of capital, there are K cost draws. Given these draws, if the firm decides to augment $m \leq K$ units of capital it will do so for the capital units with the cheapest cost draws. Note that capital augmentation is completely merger neutral: when two firms merge, collective investment possibilities do not change.

The second is *greenfield investment*: a firm can build as many capital units as it wants at a cost $c_g \in [\overline{c}, \overline{c}_g]$ drawn from a distribution G. Greenfield investment allows a firm whose capital stock is zero to invest, albeit at a cost that exceeds that of capital augmentation. We also choose the range of greenfield costs $[\overline{c}, \overline{c}_g]$ to be small so that this investment technology is approximately merger neutral. (It would be fully merger neutral if $\overline{c}_g = \overline{c}$; in our computations we introduce uncertain greenfield investment costs to ensure existence of equilibrium.)

As we noted earlier, our model allows for entry. In contrast to Pakes and McGuire (1994) and Ericson and Pakes (1995), we endow an entrant with the same investment technology as

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[15] Alternatively, if the merged firm kept both investment processes we would need to keep track, as a separate state variable, of how many investment processes a firm possesses, which has no natural bound.

incumbents. The entrant, however, starts with no capital, so it must use greenfield investment.

Note that with our assumptions investment opportunities will be (approximately) merger neutral at the market level. The assumptions also imply the following two properties of investment costs at the firm level:

1. Holding the firm's current capital stock K fixed, the expected per unit cost of adding ΔK units of capital is increasing in the investment size ΔK.[16]

2. Holding the firm's investment size ΔK fixed, the expected investment cost is decreasing in the size of its current capital stock K.[17]

Both properties are consistent with the large literature on capital adjustment costs that Abel (1979) and Hayashi (1982) initiated.[18] The second property is also in line with the large (theoretical and empirical) literature on entry in industrial organization, where it is commonly assumed that potential entrants have to incur a setup cost before entering, implying that new entrants have to incur higher costs than incumbents if they want to add the same amount of productive capital.[19,20]

Put together, the capital augmentation and greenfield investment processes allow for significantly richer investment dynamics than in the typical dynamic industry model. Firms can expand their capital by multiple units at a time through either investment method, and new entrants can decide endogenously how far to jump up in their capital stock.

Capital also depreciates: in each period each unit of capital has a probability $d > 0$ of becoming worthless (including for any future capital augmentation). Depreciation realizations are independent across units of capital. This depreciation process is also merger neutral.[21] Finally, the firms discount the future according to discount factor $\delta < 1$.

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[16] For a firm with no capital, the unit cost of adding ΔK units of capital is constant as such a firm has only access to the greenfield technology.

[17] This implies that investment opportunities – while merger neutral at the market level – are not merger neutral at the firm level.

[18] Most of the literature on capital adjustment costs assumes that adjustment costs are a convex function of the proportional change in the firm's capital stock. With that formulation, the cost of a given sized increase in capital is strictly decreasing in firm size. More recent work, such as Cooper and Haltiwanger (2006), has introduced non-convex components of adjustment costs. But even in those models small firms have very large investment costs.

[19] In the Online Appendix, we also examine the effects of requiring a minimum scale of greenfield investment.

[20] New entrants may also face higher financing costs than established firms. Indeed, there are many empirical studies finding a positive effect of cash flow on investment, pointing to credit constraints; see the influential paper by Fazzari, Hubbard and Petersen (1988) and the survey by Bond and van Reenen (2007). However, these findings cannot easily be mapped into our model as cash flow is likely to be related to retained earnings – which in turn depend on own past capital stocks as well as the rivals' past capital stocks.

[21] This is in contrast to Ericson and Pakes (1995), Gowrisankaran (1997, 1999) and many other papers in the computational IO literature. There, depreciation is modeled as a perfectly correlated industry-wide shock, following which each firm loses one unit of capital, independently of its size. That is, these papers assume that the expected depreciation rate is decreasing with firm size.

In our computations firms will be restricted to an integer number of possible capital levels, with the maximal capital level $\overline{K}$ chosen to be non-binding. We define $\mathcal{S} \equiv \{0, 1, 2, ..., \overline{K}\}$ to be the admissible values of K_i and $\mathcal{S}^n = \mathcal{S} \times ... \times \mathcal{S}$ to be the state space.

3   Merger Policy in Duopoly Markets

In this section we study duopoly markets. While a relatively small share of markets are duopoly markets, and mergers to monopoly are rarely proposed and approved, a significant advantage of examining the behavior of our model in such settings is that the merger bargaining process we adopt for these settings – bilateral Nash bargaining – is well-accepted and easily understood. Throughout most of this section we focus on a single market parameterization so that we can describe equilibrium firm behavior and its interaction with antitrust policy in detail; we discuss afterwards how outcomes vary across a wide range of parameters. We begin in Section 3.1 by describing the parameters of the market we focus on.   In Section 3.2 we examine how firms' behaviors and market performance depends on merger policy, and in Section 3.3 we study the Markov perfect antitrust policy and its positive and normative features. In Section 3.4 we turn to outcomes for other parameters.

3.1   Parameterization

In most of this section, we describe the results for a market in which demand is $Q\left(p\right) = B(A-p)$ with $(A = 3, B = 26)$, while firms' production functions are Cobb-Douglas with capital share parameter $\beta = 1/3$ and scale parameter $\theta = 1.1$ (recall that a merger between two equal-sized firms then lowers marginal and average costs by 9 percent at fixed outputs).

Table 1 gives a sense of this market's static properties with its strong economies of scale and linear demand. It shows the static Cournot equilibrium outcomes for three different states: $(1,0), (10,0)$, and $(5,5)$. The comparison between the $(1,0)$ and $(10,0)$ monopoly states shows the effects of the scale economies on marginal cost. It also shows for state $(1,0)$ the effect of linear demand when price is high and quantity small: demand is quite elastic causing a small price-cost markup. Aggregate surplus in the monopoly $(10,0)$ state is almost identical to that in the duopoly $(5,5)$ state because the strong scale economies almost exactly offset the inefficient monopoly pricing. The distribution of the surplus, however, tilts strongly away from consumers and towards producers.

Table 1: $(A = 3, B = 26)$ **Market Static Equilibria**

State	$(1,0)$	$(10,0)$	$(5,5)$
Marginal Cost (MC)	2.56	1.32	1.54
Price (P)	2.78	2.16	2.02
$P \div MC$	1.09	1.63	1.32
Total Quantity	5.67	21.80	25.40
Total Profit	5.12	26.00	22.80
Consumer Surplus	0.62	9.14	12.40
Aggregate Surplus	5.74	35.12	35.16

Turning to investment costs, the capital augmentation cost for a given unit of capital is independently drawn from a uniform distribution on $[3, 6]$, while the greenfield investment cost c_g is drawn from a uniform distribution on $[6, 7]$.[22] Firms' discount factor is $\delta = 0.8$, corresponding to a period length of about 5 years. We chose this to reflect the time to build new capital. Each unit of capital depreciates independently with probability $d = 0.2$ per period. We take the state space to be $\{0, 1, \ldots, 20\}^2$, so each active firm can accumulate up to 20 units of capital. In this market (and the ones considered in Section 3.4) firms almost never end up outside of the quadrant $\{0, 1, \ldots, 10\}^2$; we allow for capital levels up to 20 so that we can calculate values for mergers and avoid boundary effects. We assume that proposal and blocking costs are uniformly distributed on $[0,1]$.[23]

We focus on these parameter values to highlight the tension between the goals of achieving cost reductions immediately through a merger, preventing increased exercise of market power, and maintaining desirable investment behavior.

Finally, as noted at the end of Section 2, we assume that merger bargaining, which occurs between the two active firms, is described by the bilateral Nash bargaining solution.

3.2 Investment and Merger Incentives under Fixed Merger Policies

In this section we examine the Markov perfect equilibrium for three types of fixed merger policies: (i) the case in which mergers are prohibited—the "no-mergers-allowed" case, (ii) the case in which firms are permitted to merge in any state in which it is profitable for them to do so—the "all-mergers-allowed" case, and (iii) the case of "static" merger policy in which mergers are blocked if and only if they would result in lower current period welfare. In the third case, we consider both current period consumer surplus and aggregate surplus as possible welfare measures. For each policy, we report its long-run steady state distribution over the state space $\mathcal{S}^2$, the consumer, incumbent, entrant, and aggregate values it generates (the discounted

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[22] The large spread of the capital augmentation cost distribution reflects empirical results showing large variation in firms' costs within an industry. See, for example, Bernard, Eaton, Jensen, and Kortum (2003) and Syverson (2004). The Online Appendix includes an extension with a smaller variation in firms' costs.

[23] We know of no empirical literature on proposal and blocking costs. We chose these wide spreads to help ensure convergence of the numerical algorithm. See Doraszelski and Satterthwaite (2010).

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expected value of consumer, incumbent, entrant, and aggregate surpluses, respectively), the investment incentives it creates, and the frequency of mergers it induces.

3.2.1 Equilibria with No-Mergers-Allowed

We begin by examining the equilibrium when no mergers are allowed.[24] Figure 2(a) shows the beginning-of-period steady state equilibrium distribution under a no-mergers-allowed policy. (The other panels of Figure 2 show the steady state distributions for other cases discussed below.) The first column of Table 2 lists some measures of the no-mergers equilibrium.[25] As can be seen, under the no-mergers policy the industry spends most of its time in duopoly states in which both firms are active, but also spends roughly 18 percent of the time in monopoly states. If the industry finds itself in a monopoly state, it can stay there a long time. For example, Figure 3(a) shows the one-period transition probabilities starting from state $(5,0)$; it illustrates the weak entry behavior that allows this monopoly persistence – in fact, starting in state $(5,0)$, the probability that the industry is a monopoly five periods later is 0.84.

There are two cost-based reasons why it is so hard for an entrant starting in state $(5,0)$ to catch up. First, the entrant pays much more per unit of capital purchased: the large firm can add a unit of capital using the lowest of its five cost draws from the uniform distribution on $[3,6]$, whereas the entrant draws from the uniform distribution on $[6,7]$. Second, the large firm's scale economies gives it a marginal cost of 1.70 when setting a monopoly price of 2.35. If the potential entrant should enter with two units of capital, then at state $(5,2)$ the dominant firm sells quantity 14.6 at price of 2.18 with marginal cost 1.62. The entering firm sells 6.7 units with marginal cost 1.92. Profits are 14.5 and 5.1, respectively.

3.2.2 Equilibria with All-Mergers-Allowed

Under an all-mergers-allowed policy equilibrium is quite different. Figure 2(b) shows both the beginning-of-period steady state equilibrium distribution under an all-mergers-allowed policy, as well as the probability that a merger actually happens in each state. Shading shows states in which mergers occur with a darker shade representing a higher probability of a merger happening; cells in which mergers never occur are unshaded: for example, a merger happens with probability 1 in state $(3,3)$, with probability zero in state $(2,2)$, and with probability 0.59 in state $(2,3)$. Observe that firms do not always merge in non-monopoly states. The

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[24]We have assembled the data that we have generated into large Excel workbooks that each contain for each equilibrium, first, a detailed description of the equilibrium strategies of the firms and, for Markov perfect merger policies, of the antitrust authority and, second, a full set of performance statistics. These workbooks are posted on the web as part of our Online Appendix. They enable the reader to explore our results much as we have explored them.

[25]For comparison, in the first-best solution (with price equal to marginal cost and aggregate value-maximizing investment), the aggregate value is 164.7 and the average total capital level is 10.6.
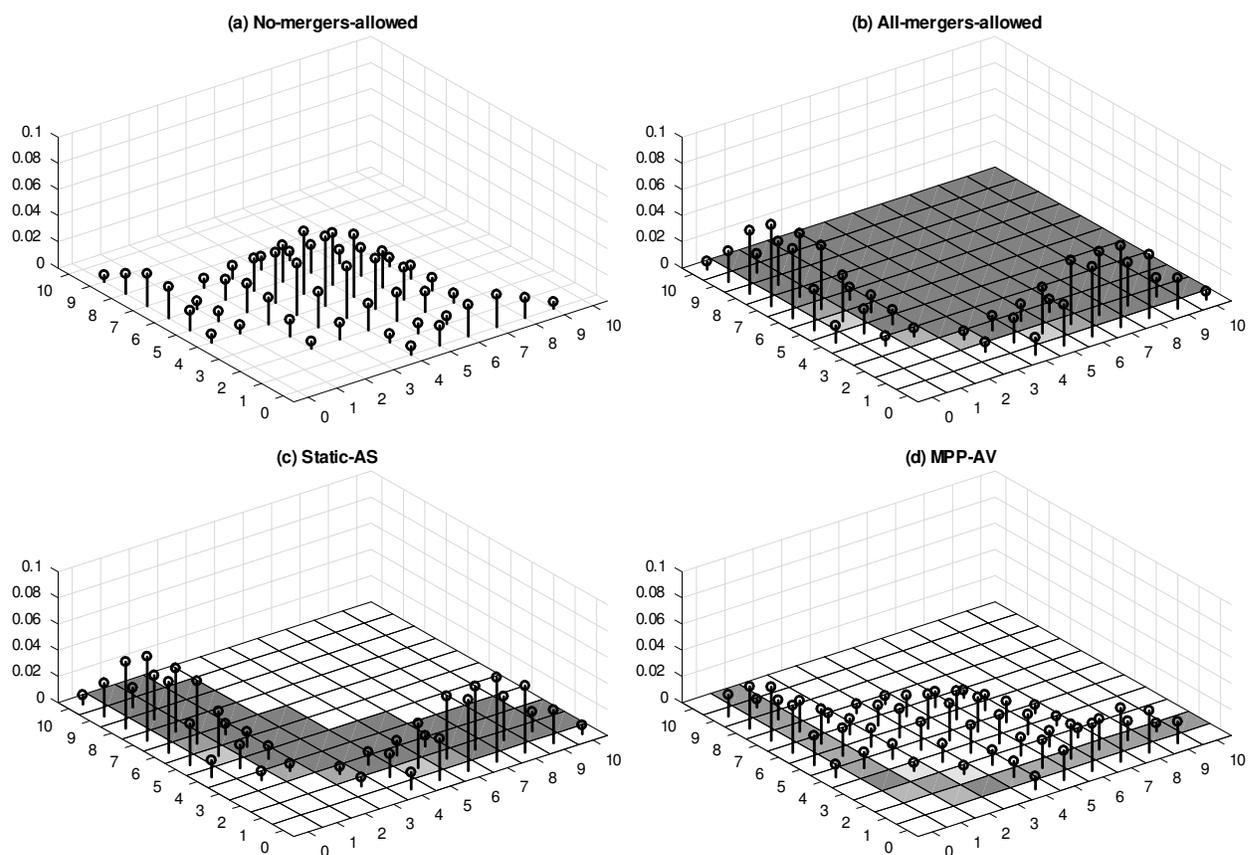
Figure 2: Beginning-of-period steady state distribution of various equilibria. The height of each pin indicates the steady state probability of that state. The shading of each cell reflects the probability of a merger happening in that state (darker grey represents a higher probability).
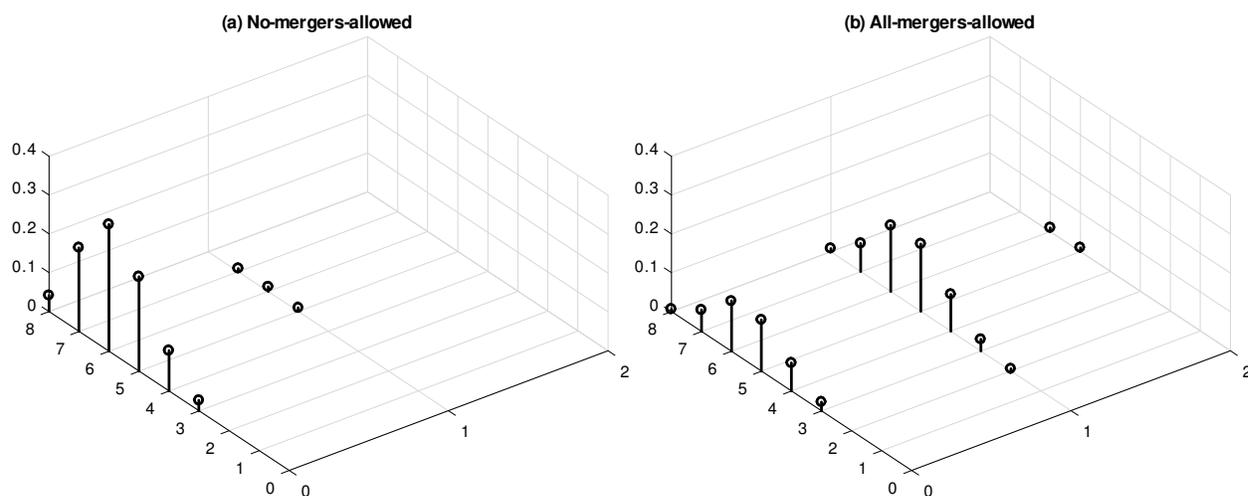
Figure 3: One-period transition probabilities from state (5,0).

reason is that if both firms' capital stocks are low, then merging attracts a new entrant who dissipates the merger's gains.

Table 2: Performance Measures for the $(A = 3, B = 26)$ **Market under Various Policies**

Performance Measure[26]	No-Mergers/ Static-CS/ MPP-CV	All-Mergers	Static-AS	MPP-AV
Avg. Consumer Value	48.1	35.8	35.9	43.3
Avg. Incumbent Value	69.4	68.1	68.5	69.9
Avg. Entrant Value	0.0	1.9	1.8	0.5
Avg. Blocking Cost	0.0	0.0	0.0	-0.1
Avg. Aggregate Value	117.5	105.8	106.2	113.6
Avg. Price	2.15	2.26	2.26	2.19
Avg. Quantity	22.2	19.2	19.2	21.0
Avg. Total Capital	8.0	7.0	7.0	7.7
Merger Frequency	0.0%	37.7%	37.9%	16.1%
% in Monopoly	18.6%	86.0%	88.0%	49.4%
% $\min\{K_1, K_2\} \geq 2$	75.7%	0.9%	0.7%	44.2%
State (0,0) CV	30.3	23.9	24.1	25.6
State (0,0) AV	36.7	34.0	34.1	35.5

The second column of Table 2 shows the properties of the all-mergers-allowed equilibrium. Mergers happen 37.7% of the time, which results in the market being in a monopoly state (at the time of Cournot competition) 86.0% of the time, and in a near-monopoly 99.1% of the time. As a result of allowing mergers, average output falls from 22.2 to 19.2, while the average price rises from 2.15 to 2.26. Average total capital falls from 8.0 to 7.0. Not surprisingly, the change in policy leads to substantial negative changes in consumer value, which falls from 48.1 to 35.8. More surprisingly, average incumbent value falls even though the firms are now allowed to merge whenever they want. This is despite firms' success in raising price, reducing quantity, and limiting total capital. Even once one accounts for future entrants' value, producer value (the sum of incumbent and entrant values) barely rises. Combined with the reduction in CV, aggregate value falls from 117.5 to 105.8.

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[26] All values are ex ante (beginning-of-period) values except % in Monopoly and % $\min\{K_1, K_2\} \geq 2$ (showing the percentages of the time that industry capital is in each type of state) which are at the Cournot competition stage. "No mergers" and "All Mergers" refer to the no-mergers-allowed and all-mergers-allowed policies, respectively. "Static CS" and "Static AS" refer, respectively, to the equilibria under the optimal static consumer surplus-based and aggregate surplus-based merger policies. "MPP CV" and "MPP AV" refer, respectively, to the equilibria when the antitrust authority cannot commit (resulting in a Markov perfect policy) under consumer value and aggregate value welfare criteria. "State (0,0) CV" and "State (0,0) AV" are the values of CV and AV, respectively, for a new industry that starts with no capital.

To explore the reasons behind these results, consider first the reduction in total capital. Allowing mergers does two things. First, it changes the states in which investments are taking place by moving the market to monopoly and near-monopoly states. Second, firms' investment policies change. Table 3 summarizes these effects. Holding investment behavior fixed, average capital addition decreases when weighted by the all-mergers-allowed steady state rather than the no-mergers steady state. However, holding the steady state weighting fixed, average capital addition increases when investment behavior is that of the all-mergers-allowed equilibrium rather than the no-mergers equilibrium. Together, these opposite effects reduce the average capital addition moving from the no-mergers-allowed policy to all-mergers-allowed.

Table 3: Average Capital Addition in the $(A = 3, B = 26)$ Market

		Steady State Distribution	
		No-Mergers	All-Mergers-Allowed .
Investment	No-Mergers	2.0	1.5
Behavior	All-Mergers-Allowed	2.2	1.8

What drives the increased investment incentive? If a merger is certain to occur next period, a firm i's marginal return to investment is $\partial \overline{V}(K_i, K_j)/\partial K_i + (1/2)\partial \Delta_{ij}(K_i, K_j)/\partial K_i$ where $\partial \Delta_{ij}(K_i, K_j)/\partial K_i$ is the marginal effect of K_i on the gain from merger as defined in (1).[27] Each firm is in a state where $\partial \Delta_{ij}(K_i, K_j)/\partial K_i$ is positive 97.5% of the time in the no-mergers steady state and 100% of the time in the all-mergers-allowed steady state; the fact that a firm's gains from a merger are increasing in its capital stock tends to make allowing mergers increase investment incentives.[28]

In the all-mergers-allowed equilibrium the steady state distribution is concentrated in monopoly and near-monopoly states. The increased investment incentive is particularly large and detrimental to producer value in such states. An entrant with zero capital frequently invests in the hope of being bought out: there is a great deal of "*entry for buyout*" behavior [Rasmusen (1988)].[29] For example, Figure 3(b) shows the one-period transition probabilities in state $(5, 0)$ when all mergers are allowed, which can be compared to Figure 3(a) where no-mergers are allowed. The probability that the entrant invests and has non-zero capital after depreciation is 0.57 in the former case, versus 0.04 in the latter. Further, the probability

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[27] This abstracts away from the discrete nature of capital additions.

[28] The change from the no-mergers-allowed to the all-mergers-allowed policy also changes the interim value function $\overline{V}(\cdot)$.

[29] While we are unaware of any formal empirical studies that document the frequency of entry for buyout behavior, Rasmusen (1988) gives a number of examples of entry for buyout in homogeneous goods industries. In the literature on start-ups, acquisition is considered to be one of the primary ways of capturing a start-up's value [see, for example, Gans and Stern (2003)]. Although start-ups frequently introduce product innovations and do not literally fit our homogeneous-goods model, we can reinterpret the capital in our model to be "knowledge capital" and the resulting cost reductions enabled to be consumer value enhancements that increase the firm's profit.

	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	1.1	1.1	1.6	1.7	1.4	1.3	1.1	1.0	1.0	0.9	0.8
k1=1	1.2	1.1	1.5	1.7	1.8	2.0	2.2	2.3	2.5	2.6	2.8
k1=2	(0.6)	0.8	1.5	1.9	2.4	2.7	3.0	3.2	3.4	3.6	3.8
k1=3	(5.4)	0.8	1.6	2.3	2.8	3.1	3.4	3.7	3.9	4.1	4.4
k1=4	(8.3)	0.4	1.8	2.6	3.1	3.4	3.7	4.0	4.2	4.5	4.7
k1=5	(10.1)	0.1	1.9	2.8	3.3	3.6	4.0	4.2	4.4	4.7	4.9
k1=6	(10.5)	(0.1)	2.0	2.9	3.5	3.8	4.1	4.4	4.5	4.8	5.0
k1=7	(10.4)	(0.1)	2.1	3.1	3.6	4.0	4.3	4.5	4.7	4.9	5.1
k1=8	(10.2)	(0.2)	2.2	3.2	3.8	4.1	4.4	4.5	4.7	5.0	5.2
k1=9	(10.1)	(0.2)	2.3	3.3	3.9	4.2	4.5	4.6	4.9	5.1	5.4
k1=10	(9.9)	(0.2)	2.3	3.4	4.0	4.4	4.6	4.7	5.1	5.3	5.5

Figure 4: Beginning-of-period value of the row firm (firm 1) in the all-mergers-allowed equilibrium minus its value in the no-mergers equilibrium. Negative numbers are in parentheses.

of a merger is 0.49 in the first period after the entrant invests when all mergers are allowed, and 0.85 within two periods. Figure 3(b) also shows that the entrant's increased investment lowers the incentive of the incumbent to invest in state $(5, 0)$.

This entry for buyout behavior reduces producer value as the entrants' investments are made at high cost and displace lower cost investments by the incumbent monopolist. Figure 4 illustrates the destructiveness of this behavior for producer value. It shows for each state the change in the row firm's (firm 1) beginning-of-period value that a switch from a no-mergers-allowed to an all-mergers-allowed policy induces. In most states the row firm's value is enhanced but in monopoly states in which the monopolist has at least three units of capital, the monopolist's value falls dramatically. This behavior is also highly detrimental for aggregate value: In both the no-mergers and all-mergers-allowed equilibria, dominant firms generally have insufficient incentives, while entrants have excessive incentives.[30] The entry for buyout phenomenon therefore causes a shift in investment away from the dominant firm, whose incentives are already insufficient, toward the entrant, whose incentives are excessive.

3.2.3 Equilibria with Static Policies

We next consider optimal static merger policy, as in Williamson (1968) and Farrell and Shapiro (1990). These policies block a merger if and only if it decreases welfare (either consumer surplus or aggregate surplus, depending on the criterion) due to production and consumption in the period the merger occurs.[31]

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[30] The Online Appendix contains tables showing the difference between firms' investment incentives and the benefits to social welfare from investment.

[31] Another possible benchmark is the second-best dynamic problem where the planner controls firms' merger decisions as well as their investment decisions, but not their output decisions. This benchmark is analyzed in the Online Appendix. It turns out this second-best merger policy is very similar to the optimal static aggregate surplus-based policy.

	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	-	-	-	-	-	-	-	-	-	-	-
k1=1	-	1.7	1.6	1.4	1.2	1.0	0.9	0.8	0.7	0.6	0.5
k1=2	-	1.6	1.6	1.4	1.2	0.9	0.8	0.6	0.5	0.4	0.3
k1=3	-	1.4	1.4	1.1	0.9	0.6	0.4	0.2	0.1	0.0	(0.1)
k1=4	-	1.2	1.2	0.9	0.5	0.3	0.0	(0.1)	(0.3)	(0.4)	(0.5)
k1=5	-	1.0	0.9	0.6	0.3	(0.0)	(0.3)	(0.5)	(0.7)	(0.8)	(0.9)
k1=6	-	0.9	0.8	0.4	0.0	(0.3)	(0.6)	(0.8)	(1.0)	(1.2)	(1.3)
k1=7	-	0.8	0.6	0.2	(0.1)	(0.5)	(0.8)	(1.1)	(1.3)	(1.5)	(1.6)
k1=8	-	0.7	0.5	0.1	(0.3)	(0.7)	(1.0)	(1.3)	(1.5)	(1.7)	(1.9)
k1=9	-	0.6	0.4	0.0	(0.4)	(0.8)	(1.2)	(1.5)	(1.7)	(1.9)	(2.1)
k1=10	-	0.5	0.3	(0.1)	(0.5)	(0.9)	(1.3)	(1.6)	(1.9)	(2.1)	(2.3)

Figure 5: Static change in aggregate surplus from a merger in the $(A = 3, B = 26)$ market. Negative numbers are in parentheses.

Mergers lower consumer surplus in all but state $(1, 1)$, so the static consumer surplus-based policy is essentially equivalent to allowing no mergers.

In contrast, Figure 5 shows that many mergers increase aggregate surplus. In general, these tend to be states in which the total capital in the industry is not more than 10, though in some asymmetric states with total capital above 10 there is also a gain.[32] The gains in aggregate surplus are generally smaller the larger is the total capital in the industry.[33] An increase in the asymmetry of capital positions, holding total capital fixed, has varying effects on the static gains in aggregate surplus from a merger. This gain gets smaller with increased asymmetry at low levels of total capital, but grows larger with increased asymmetry at greater levels of total capital.

Figure 2(c) shows the beginning-of-period steady state equilibrium distribution under the aggregate surplus-based static merger policy and Table 2 shows equilibrium performance statistics under this policy. As can be seen in the figure and table, the outcome with the aggregate surplus-based static policy is very close to the all-mergers-allowed outcome.

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[32] The only exception is state $(5, 5)$ where the static gain in aggregate surplus is approximately zero.

[33] To understand this result, observe that the change in aggregate surplus from a merger in a symmetric state is approximately

$$Q\left[\left(\frac{\Delta Q}{Q}\right)(P - MC) - \left(1 + \frac{\Delta Q}{Q}\right)\left(\frac{\Delta AC_M}{AC_M}\right)AC_M\right],$$

where $(P - MC)$ is the premerger price-cost margin, AC_M is the average cost if no merger occurs but the output level changes to its post-merger level, and ΔAC_M is the change in average cost at the post-merger output level due to the combination of capital. At larger capital levels, $(P - MC)$ and $|\Delta Q/Q|$ are both greater, $(\Delta AC_M/AC_M)$ is unchanged, and AC_M is smaller, making the sign of the effect on aggregate surplus more likely to be negative for an output-reducing merger. For example, $(P - MC)$ is 0.32 at state $(2, 2)$ and 0.45 at state $(4, 4)$, $(\Delta Q/Q)$ is -0.065 at $(2, 2)$ and -0.126 at $(4, 4)$, and AC_M is 21% lower at $(4, 4)$ than at $(2, 2)$.

3.3 Equilibria with Markov Perfect Merger Policy

We now introduce an optimizing antitrust authority who cannot commit to its future policy, determine its Markov perfect policy, and examine the outcome it induces. In this setting the antitrust authority, like each of the firms, is a player in a dynamic stochastic game; Markov perfection requires that in each state the policy survives the one-stage-deviation test.[34]

As with the static consumer surplus-based policy, the Markov perfect policy outcome when the antitrust authority seeks to maximize consumer value ("CV") is essentially equivalent to the no-mergers-allowed outcome (see Table 2). For the rest of this section, we therefore focus on an authority who seeks to maximize aggregate value ("AV") .

For an antitrust authority following the AV criterion, neither the no-mergers-allowed nor the all-mergers-allowed policy survive the one-stage-deviation test given the firm behavior they induce: assuming future behavior following the no-mergers equilibrium, the antitrust authority would allow many mergers in a one-stage-deviation; assuming future behavior following the all-mergers-allowed equilibrium, the antitrust authority would allow very few mergers.

Figure 2(d)'s shading shows the probability a merger occurs in various states under the Markov perfect policy. The policy differs markedly from all of the policies we have previously considered. The authority approves a proposed merger with positive probability in near-monopoly states in which $\min\{K_1, K_2\} = 1$, as well as in states $(2,2)$, $(3,2)$, and $(2,3)$. Given this policy, mergers are proposed with probability one in all of these states, except in state $(1,1)$, where a merger is never proposed, and in states $(2,1)$, and $(1,2)$, where a merger is proposed with less than full probability. This policy induces an even higher merger probability following entry than the all-mergers-allowed policy: For example, the probability of a merger is 0.69 in the first period after entry in state (5,0), compared to 0.49 in the all-mergers-allowed equilibrium. Firms are more likely to merge in the first period under the Markov perfect policy because if the entrant grows further they are unlikely to be allowed to merge in the second period.

Figure 2(d) also shows the steady state distribution arising under the Markov perfect policy, while Table 2 shows its performance statistics. The industry is in a monopoly state at the Cournot competition stage 49.4% of the time, and in near-monopoly states 55.8% of the time. Compared to the steady state induced when no mergers are allowed, the economy spends much more time in such states. In addition, the average aggregate capital level is lower (7.7 vs. 8.0). The reason is the shift in the steady state distribution toward more asymmetric states, in which investments are lower. However, because a new entrant and the incumbent are not always allowed to merge, monopoly states are less frequent and average capital is greater than under the all-mergers-allowed and static aggregate surplus-based policies.

The Markov perfect policy with the AV criterion is much better for consumers and ag-

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[34]In the Online Appendix we discuss as well the case in which the antitrust authority can commit to its future policy.

gregate value than allowing all mergers or following the static aggregate surplus-based policy. However, it results in a level of steady state AV that is about 3% lower than with the no-mergers policy: AV is 113.6 compared to 117.5 when no mergers are allowed.[35] Firms are only slightly better off – harmed again by the entry for buyout behavior the merger policy induces – while consumers are much worse off: CV is 43.3 (vs. 48.1) and producer value is 70.4 (vs. 69.4). Consumers are harmed both from the monopoly pricing and the reduction in capital. Strikingly, observe that a commitment to maximizing CV or to the static consumer surplus-based policy would be better here for aggregate value than the policy that results when the antitrust authority seeks to maximize AV but cannot commit.[36]

3.4 Results for Other Demand Parameters

Up to this point we have limited our discussion to a single market parameterization. While this focus allowed us to discuss in detail the outcomes and strategies that arise in this case, it naturally leaves open the question of how our results extend to other market conditions. Here we examine the extent to which several of the features of the equilibria discussed extend across a wider range of demand parameters.[37]

We first examine how the no-mergers-allowed and all-mergers-allowed equilibria differ. Figure 6(a) reports on the difference in aggregate value between these two policies for linear demand functions $Q(p) = B(A - p)$, where A is the choke price and B is the market size parameter (e.g., number of consumers). The figure depicts contour lines showing the demand parameters at which the aggregate value difference, $(AV_{No} - AV_{All})/AV_{No}$, achieves a given percentage value. Also shown in the figure are three dots. The middle one is the ($A = 3, B = 26$) market that our discussion above focused on. The other two dots represent a "smaller" and a "larger" market whose equilibria we discuss in greater detail in the Online Appendix, paralleling our discussion above of the ($A = 3, B = 26$) market. In the figure, dashed lines show markets that spend 5%, 20%, and 60% of the time in monopoly states when no mergers

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[35]The finding that the Markov perfect policy with the AV criterion performs worse than the no-mergers policy but better than the all-mergers-allowed policy holds not only for the steady state averages of AV and CV but also for a "new" industry: as shown in Table 2, at state (0,0) the AV (resp. CV) value of the Markov perfect policy is 35.5 (25.6), that of the no-mergers policy 36.7 (30.3), while that of the all-mergers-allowed policy is only 34.0 (23.9).

[36]This conclusion is reminiscent of Lyons (2002), but arises for different reasons.

[37]In the Online Appendix we also consider the effect of varying the production scale parameter θ. The results show similar patterns to those we discuss here, with outcomes closely related to the percentage of time spent in monopoly states when no mergers are allowed.

We also examine there the following modeling extensions: allowing the probability of entry following a merger to be less than one; modifying our greenfield investment technology (used primarily by entrants) to require a minimum scale of investment greater than one unit of capital; reducing the gap of investment costs faced by incumbents and entrants; having bargaining power proportional to capital stocks; allowing a planner to control investment behavior and merger decisions taking as given only Cournot competition; and assuming new entrants are the owners of the firms purchased in mergers.
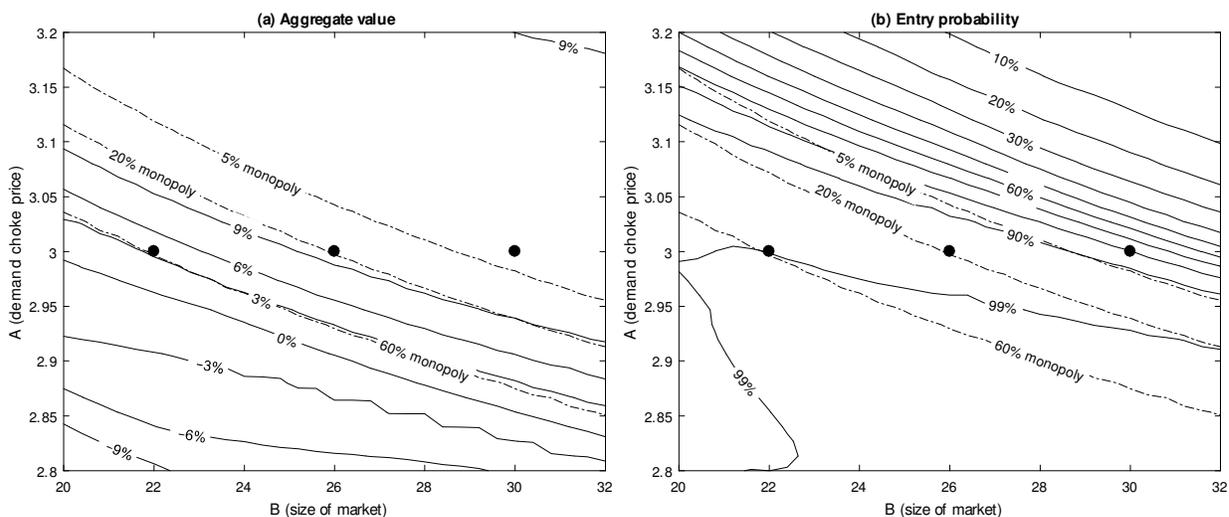
Figure 6: (a) Contour lines of the percentage difference between the steady state aggregate value of the no-mergers and all-mergers-allowed equilibria, $(AV_{No} - AV_{All})/AV_{No}$. (b) Contour lines of the percentage difference between the steady state aggregate value of the no-mergers and all-mergers-allowed equilibria, $(AV_{No} - AV_{All})/AV_{No}$.

are allowed. Market parameters to the Northeast in the figure are large markets with low levels of monopoly, while markets to the Southwest are small markets with high monopoly levels. As can be seen in the figure, aggregate value with no mergers allowed is greater than with all mergers allowed provided that the market is large enough, with aggregate value approximately equal for these two merger policies for markets in which the no-mergers-allowed equilibrium spends about 70% of the time in monopoly states.

For the same range of demand parameters, Figure 6(b) shows the percentage difference in entry probabilities in the no-mergers-allowed and all-mergers-allowed equilibria, $[\Pr(Entry)_{All} - \Pr(Entry)_{No}]/\Pr(Entry)_{All}$.[38] Consistent with the entry for buyout we observed earlier, the level of entry is always weakly greater in the all-mergers-allowed equilibrium, although the difference declines to zero in very large markets where the probability of entry rises to one under either merger policy.

Figure 7 focuses on the Markov perfect policy. Figure 7(a) shows the percentage difference in aggregate value between the Markov perfect policy and the no-mergers-allowed equilibrium, $(AV_{MPP} - AV_{No})/AV_{MPP}$. In small markets, the Markov perfect policy leads to higher aggregate value than when no mergers are allowed. Similar to the comparison between the no-mergers and all-mergers-allowed policies, the no-mergers policy outperforms the

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[38]$\Pr(Entry)_x$ is calculated by weighting the probability of entry in each monopoly state under merger policy x by the probability of that state in the all-mergers-allowed equilibrium. In the Southeast region of the figure, the no-mergers equilibrium has no entry in states that arise with positive probability in the all-mergers-allowed equilibrium, leading the percentage difference in entry probabilities to be 100%.
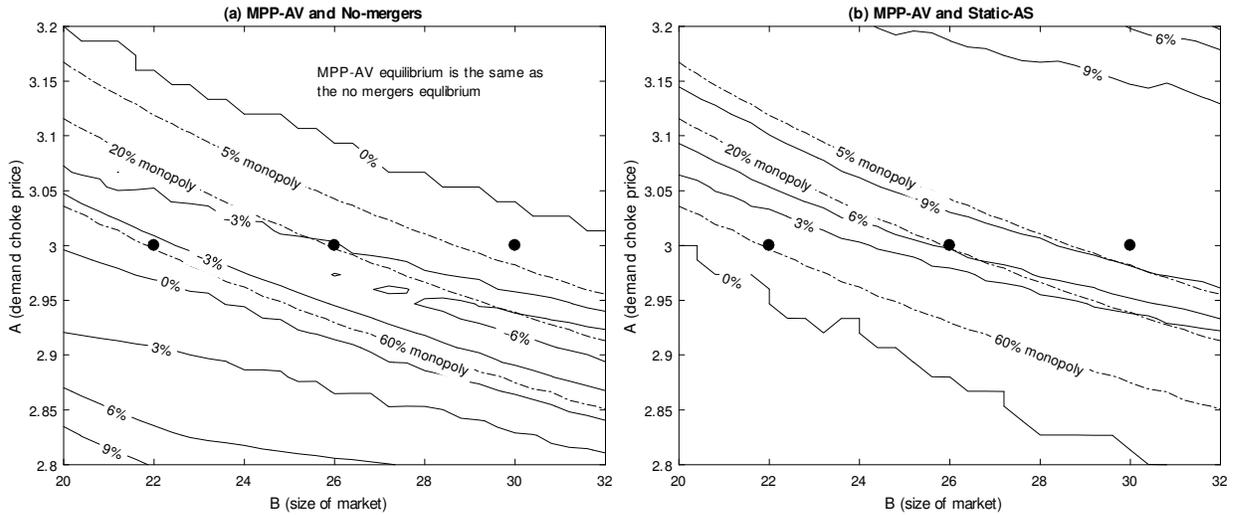
Figure 7: (a) Contour lines of the percentage difference between the steady state aggregate value of the MPP-AV and no-mergers equilibria, $(AV_{MPP} - AV_{No})/AV_{MPP}$. (b) Contour lines of the percentage difference between the steady state aggregate value of the MPP-AV and static aggregate surplus-based policy equilibria, $(AV_{MPP} - AV_{Static})/AV_{MPP}$.

Markov perfect policy provided the market is large enough. However, for the largest markets in the Northeast corner, the Markov perfect policy leads to the same equilibrium as the no-mergers policy because mergers are never consummated. Figure 7(b) shows the same AV comparison but relative to the outcome with the static aggregate surplus-based policy, $(AV_{MPP} - AV_{Static})/AV_{MPP}$. The figure shows that the Markov perfect policy outperforms the static aggregate surplus-based policy provided the market is large enough.

4 Merger Policy in Triopoly Markets

In this section, we extend our framework by introducing a third firm. The key novelty in the triopoly case is that a bilateral merger may now induce an externality on the non-merging firm, which in turn introduces some new investment incentives not present in our earlier duopoly markets. Our analysis here should be viewed as giving a glimpse of the new effects this can introduce, as we do this for one particular three-party bargaining process among many possible ones. Triopoly markets also allow us to study optimal policy toward mergers that combine two weaker (i.e., lower capital stock) firms who face a stronger rival, an issue that arises frequently in merger cases (such as the AT&T/T-Mobile USA and Sprint/T-Mobile USA mergers).

We first examine the robustness of our previous two-firm results to the possibility of a third firm. We show that the $(A = 3, B = 26)$ market that we studied in Section 3 is a "natural duopoly" in the sense that a third firm does not wish to enter when no mergers

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are allowed, although when mergers are allowed the entry for buyout motive sometimes leads a third firm to enter temporarily. Nonetheless, our previous conclusions continue to hold. We then examine merger policy in two "natural triopoly" markets, where when no mergers are allowed the market usually has three firms with positive levels of capital. The presence of externalities on non-merging firms introduces a new effect on incumbent investment that impacts optimal merger policy significantly in one of these markets.

To proceed, we consider a three-firm version of the general model of Section 2. The bargaining stage in each period is a static version of the bargaining protocol in Burguet and Caminal (2015): One firm, say i, is randomly selected as the proposer, with each firm equally likely to be selected. The proposer chooses which of its two rivals to invite for merger negotiations. Suppose firm i invites $j \neq i$. If firm j accepts the invitation, then these two firms enter merger negotiations. Otherwise, firm j invites firm $k \neq i, j$. If firm k accepts, then j and k enter bilateral merger negotiations. If it rejects the invitation, then no merger takes place this period. Bilateral merger negotiations are such that each party is equally likely to be selected to make the other a take-it-or-leave offer. If the offer is accepted, then the merger is proposed to the authority; if it is rejected, then no merger occurs in that period. So, conditional on two firms entering bilateral merger negotiations, the expected payoffs coincide with those in the Nash bargaining solution between those two firms. An attractive feature of this bargaining process is that, no matter which firm is selected as the proposer, each of the three mergers is feasible.[39] As we will see below, another attractive feature is that, generically, the bargaining process has a unique equilibrium for given continuation values.[40]

Recall that (for the case of three firms)

$$\Delta_{ij}(\mathbf{K}) \equiv \overline{V}(K_i + K_j, K_k, 0) - (\overline{V}(K_i, \mathbf{K}_{-i}) + \overline{V}(K_j, \mathbf{K}_{-j}))$$

denotes the joint gain firms i and j get from merging, gross of proposal costs, relative to if no merger occurs, and that

$$S_{ij}(\mathbf{K}, \phi_{ij}) \equiv a_{ij}(\mathbf{K})\Delta_{ij}(\mathbf{K}) - \phi_{ij}$$

is the expected bilateral surplus of firms i and j from entering merger negotiations in state $\mathbf{K}$ (after the realization of the proposal cost ϕ_{ij}), and that $S_{ij}^+(\mathbf{K}, \phi_{ij}) \equiv \max\{0, S_{ij}(\mathbf{K}, \phi_{ij})\}$.

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[39] For example, a simpler random proposer bargaining process in which a proposer is chosen in each period who can make a take-it-or-leave-it merger offer to either of the other firms would have the disadvantage that one of the three mergers would end up being impossible in each period. If there is a clearly most profitable merger, with probability 1/3 the only way for it to happen would be for no merger to occur today in the hope that (with a 2/3 probability) it can happen in the next period. One might think that it is possible to avoid this problem by allowing multiple rounds in each period, with a new proposer chosen randomly in each round should a deal not yet be reached. However, when we experimented with such a procedure we found that cycles could arise in which the equilibrium outcome depended drastically on how many rounds were allowed.

[40] At the same time, there are also features that one might view as less attractive. For example, once an invitation to negotiate is accepted, a firm that is negotiating cannot use the possibility of striking a deal with the excluded firm to improve its deal.

In the following, we will sometimes say that merger M_{ij} is more profitable than merger M_{ik} if $S_{ij}^+(\mathbf{K}, \phi_{ij}) > S_{ik}^+(\mathbf{K}, \phi_{ik})$. Note, however, that this notion of profitability ignores the externality that i and j impose on firm k when entering merger negotiations, which equals $I_{\{S_{ij}^+(\mathbf{K}, \phi_{ij}) > 0\}} X_k^{ij}(\mathbf{K})$.

Note also that the "profitability" of a merger between two firms i and j, $S_{ij}^+(\mathbf{K}, \phi_{ij})$, depends on continuation values. Thus, a merger can be "unprofitable" because it is better for one or both of the firms not to merge in the hopes of benefiting should its rivals merge in the next period.

The following proposition completely characterizes the equilibrium outcome of the merger process:

Proposition 1 *Suppose firm i is selected as the proposer in state $\mathbf{K}$. Then:*

(i) *If $S_{jk}^+(\mathbf{K}, \phi_{jk}) > \max\{S_{ij}^+(\mathbf{K}, \phi_{ij}), S_{ik}^+(\mathbf{K}, \phi_{ik})\}$, then firm i invites either firm j or firm k, and merger M_{jk} gets proposed.*

(ii) *If $S_{ij}^+(\mathbf{K}, \phi_{ij}) > S_{ik}^+(\mathbf{K}, \phi_{ik}) \geq S_{jk}^+(\mathbf{K}, \phi_{jk})$, then firm i invites firm j, and merger M_{ij} gets proposed.*

(iii) *If $S_{ij}^+(\mathbf{K}, \phi_{ij}) > S_{jk}^+(\mathbf{K}, \phi_{jk}) > S_{ik}^+(\mathbf{K}, \phi_{ik})$ and $S_{ij}^+(K, \phi_{ij})/2 > X_i^{jk}(\mathbf{K})$, then firm i invites firm j, and merger M_{ij} gets proposed.*

(iv) *If $S_{ij}^+(\mathbf{K}, \phi_{ij}) > S_{jk}^+(\mathbf{K}, \phi_{jk}) > S_{ik}^+(\mathbf{K}, \phi_{ik})$ and $S_{ij}^+(\mathbf{K}, \phi_{ij})/2 < X_i^{jk}(\mathbf{K})$, then firm i invites firm k, and merger M_{jk} gets proposed.*

(v) *If $S_{ij}^+(\mathbf{K}, \phi_{ij}) = S_{jk}^+(\mathbf{K}, \phi_{jk}) = S_{ik}^+(\mathbf{K}, \phi_{ik}) = 0$, then no merger occurs.*

In case (i), even though firm i is the proposer, merger M_{jk} must be the outcome as both firms j and k prefer their half of the surplus from merger M_{jk}, $S_{jk}^+(\mathbf{K}, \phi_{jk})$, to what they can get bargaining with firm i. In contrast, in case (ii), both j and k prefer to split the bargaining surplus available in a deal with firm i to bargaining with each other. So firm i can get either merger M_{ij} or merger M_{ik}, and prefers the former. In cases (iii) and (iv), firm i will get merger M_{ij} if it proposes bargaining with firm j (j prefers to split surplus with firm i rather than with firm k), but can induce the second-most profitable merger M_{jk} by proposing to bargain with firm k (k prefers to bargain with j); which of these options firm i prefers depends on comparing its split of the bargaining surplus with firm j to the externality it experiences when merger M_{jk} happens. In case (v), no merger has a positive surplus, so no merger occurs.[41] A

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[41] We assume that firms i and j do not merge if $a_{ij}(K)\Delta_{ij}(K) - \phi_{ij} = 0$; however, in case (v) this is a measure zero event — generically we have $a_{ij}(K)\Delta_{ij}(K) - \phi_{ij} < 0$ for all i and j. Note as well that since the surplus measures the bilateral gain from a merger relative to no merger occuring, one reason that a merger may have negative surplus is that one or both firms anticipate the possibility of experiencing a positive externality should rivals merge in a subsequent period.

formal proof is found in the Online Appendix. Observe that merger M_{jk} will happen in two circumstances when firm i is the proposer: when M_{jk} is the most profitable merger, and when M_{jk} is more profitable than M_{ik} and firm i gains more when merger M_{jk} occurs than its half of merger M_{ij}'s surplus.

4.1 Allowing a Third Firm in the $(A = 3, B = 26)$ Market

When no mergers are allowed in the $(A = 3, B = 26)$ market, introducing a third firm has almost no impact as triopoly states are very rare: states with three active firms are visited only about 0.5 percent of the time. That is, the two-firm equilibrium outcome we studied in Section 3 when no mergers are allowed approximates well the outcome of a "free entry" equilibrium. (Equilibrium statistics when we allow a third firm are displayed in the Online Appendix.) In this sense, it is a "natural duopoly" market.[42]

When mergers are allowed, however, the prospect of entry for buyout can lead both a second and a third firm to enter for the prospect of being acquired.[43] When this happens a subsequent merger induces a duopoly rather than a monopoly outcome. As a result, introducing the possibility of a third active firm reduces the steady state frequency of a monopoly state from 86.0% to 85.4%, but increases the probability of a merger from 37.7% to 50.8%.[44]

Despite these quantitative changes in the equilibrium outcome, our previous insights carry over to the three-firm case. Allowing all mergers induces entry for buyout and implies that the industry spends much more time in a monopoly state: the steady state frequency of monopoly increases from 18.0% under the no-mergers policy to 85.4% under the all-mergers-allowed policy. Since firms invest on average less in such monopoly states than in more symmetric states, the average total capital level is lower under the all-mergers-allowed policy.[45] Because the industry spends more time in monopoly and firms invest less, consumers are much worse off when all mergers are allowed: consumer value decreases from 48.3 to 38.0. Despite the large increase in the frequency of monopoly, firms collectively do not gain much from allowing all mergers because of the distortions in firms' investment behavior associated with entry for buyout. As a result, average AV falls from 117.6 to 107.7 when all mergers are allowed.

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[42] The same is true in the "small" $(A = 3, B = 22)$ and "large" $(A = 3, B = 30)$ markets discussed in the Online Appendix.

[43] For example, with two firms the probability that the entrant invests in state (5,0) was 58%. With three firms, in state (5,0,0), the probability that *each* entrant invests is only slightly smaller, namely 51%. This implies that, starting from state (5,0,0), the probability that there are three firms with capital at next period's merger stage is equal to 16.7% (the probability that both entrants invest times the probability that neither entrant's capital depreciates). At the same time, the probability that no entry (and therefore no merger) occurs in state (5,0,0) is considerably lower than in state (5,0), namely 35.0% rather than 53.6%.

Note, however, that our restriction to symmetric strategies implies that in state $(5, 0, 0)$ either both firms invest or neither do. There could be an asymmetric equilibrium in which the entrants invest asymmetrically, perhaps even with one entrant not investing at all.

[44] With three firms, the industry does not spend any time in a triopoly state at the output competition stage.

[45] The average total capital level decreases from 8.0 to 7.6 when all mergers are allowed.

Turning to the Markov perfect policy, when the authority uses the AV criterion the steady state probability that the industry finds itself in a duopoly state pre-merger is high (77.6%), and the likelihood of triopoly is low (2.2%). In duopoly states, the antitrust authority approves a merger only when at least one of the incumbents is very small, and is more restrictive than would be statically optimal; a merger happens 13.3% of the time versus 50.2% if a static aggregate surplus-based policy were instead followed. The performance measures (such as CV, AV, merger frequency, probability of monopoly) under the Markov perfect policy with AV criterion all lie between those of the no-mergers policy and the all-mergers-allowed policy.[46] In particular, the simple commitment policy of never allowing a merger induces again a higher average AV than the Markov perfect policy with the AV criterion.

4.2 Two Natural Triopoly Markets

We now examine merger policy in two "natural triopoly" markets. In the first, we proportionally increase market size by examining a market with $(A = 3, B = 70)$, while in the second we increase the choke price by setting $(A = 4, B = 20)$; all other parameters remain the same as in Section 3. In these markets the industry spends, respectively, 75.5% and 99.4% of the time in a triopoly state when no mergers are allowed. The $(A = 3, B = 70)$ market spends long periods of time in roughly symmetric triopoly states with each firm having about 9 units of capital, but in the unlikely occurrence that one firm depreciates to zero the industry stays in duopoly for a long time, only generating entry if one of the duopolist's capital depreciates to a very low level.[47] In the $(A = 4, B = 20)$ market, on the other hand, the depreciation of one firm to zero capital is soon followed by entry and a return to symmetric triopoly with each firm having roughly 5 units of capital. Indeed, in the $(A = 4, B = 20)$ market there is a positive probability of entry against symmetric duopolists (i.e., a third firm with zero capital investing a positive amount) as long as the incumbents each have less than 11 units of capital; in contrast, in the $(A = 3, B = 70)$ market an entrant will not enter unless the incumbents each have 3 units of capital or less.

4.2.1 Merger Bargaining with Three Firms

To understand some of the effects of merger policy that we observe in these markets it is useful to first examine some features of the merger bargaining process. To do so, we look at the case in which all mergers are allowed.

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[46]The only exception is that the average total capital level under the Markov perfect policy is larger than in the no-mergers policy. As we have seen in the two-firm case, allowing mergers tends to lead to more investment state-by-state but increases the relative frequency of monopoly states in which investment is lower. With only two firms, the second effect outweighs the first. Here, the first effect outweighs the second because introducing a third firm reduces the frequency of monopoly states under the Markov perfect policy (for the same reason as it does under the all-mergers-allowed policy).

[47]We increase the state space to allow up to 30 units of capital for each firm for this market.

Tables 4 and 5 show the probability that a merger occurs, the bargaining surplus, and the merger externality in symmetric states that are around the typical triopoly states in these markets.[48] As in our duopoly markets, a merger occurs with certainty in these symmetric triopoly states unless the capital stocks are low. Notably, however, the gain for the firm not involved in the merger far exceeds the gain for the firms that merge, especially in the $(A = 4, B = 20)$ market.

Table 4: Merger Bargaining Outcomes in Symmetric States
when All Mergers are Allowed [$(A = 3, B = 70)$ Triopoly Market]

Capital Stock for Each Firm	3	4	5	6	7	8	9	10
Pr(Merger)	0%	61%	100%	100%	100%	100%	100%	100%
Bargaining Surplus $S_{ij}(\mathbf{K}, 0)$	-2.64	0.27	5.73	7.86	8.71	9.19	9.59	9.98
Externality $X_k^{ij}(\mathbf{K})$	-1.13	5.12	10.04	12.47	14.06	15.41	16.66	17.85

Table 5: Merger Bargaining Outcomes in Symmetric States
when All Mergers are Allowed [$(A = 4, B = 20)$ Triopoly Market]

Capital Stock for Each Firm	1	2	3	4	5	6	7	8
Pr(Merger)	0%	12%	100%	100%	100%	100%	100%	100%
Bargaining Surplus $S_{ij}(\mathbf{K}, 0)$	-1.75	0.04	2.03	3.52	4.03	4.64	5.20	5.42
Externality $X_k^{ij}(\mathbf{K})$	0.02	1.47	6.91	10.42	12.67	14.56	15.93	17.06

Next, consider asymmetric states. In the all-mergers-allowed steady states of these markets, mergers tend to happen when there are two large incumbents and a small recent entrant. Generally, a merger between the two largest firms generates a negative surplus for the merger partners, as it leads to further entry. But a merger between one of the incumbents and the entrant is worthwhile. Given the large positive externalities on the non-merging firm in these markets, when each large firm is the proposer the resulting merger is between the small firm and the other incumbent.[49] Thus, the relative likelihoods of the two possible mergers is determined by the preference of the smaller firm, who prefers to split surplus with the incumbent that generates the largest merger surplus (net of the proposal costs).

In the $(A = 4, B = 20)$ market, the firms that merge when all mergers are allowed are highly likely to be the two smallest ones. For example, if instead of being at state (5,5,5) the firms are at (5,5,6), then the likelihood that the smaller firms merge is 67%, while 33% of the time one of the smaller firms merge with the larger firm. At state (5,5,7) the likelihood of the

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[48] In these symmetric states each firm has a two-thirds chance of being involved in a merger should one occur. In line with Proposition 1, in any of these symmetric states, a merger will be proposed if $S_{ij}(\mathbf{K}, 0)$ is larger than ϕ_{ij} for some merger M_{ij}. Recall that $S_{ij}(\mathbf{K}, 0)$ incorporates any change in continuation payoff, including expected future externalities, to the merging firms due to their merger.

[49] The chosen incumbent proposer makes an offer to the other incumbent, who then invites the small firm to bargain.

two smallest firms merging rises to 97%, and it is 100% at state (5,5,8). Similarly, at (1,7,8), the likelihood that the two smallest firms merge is 89%, and this increases to 100% at (1,7,9). Overall, conditional on a merger occurring in a state with a unique largest firm, the steady state likelihood that the two smallest firms merge is 97%. The average merger surplus when the smallest two firms merge is 1.26, while the average externality on the largest firm is 3.06. As we will see, the desire to be the firm capturing this externality is an important driver of investment by incumbents in this market.

The effect of asymmetry is much less pronounced, however, in the $(A = 3, B = 70)$ market. For example, at state (2,12,13) the likelihood that the two smallest firms merge is 51%, and only increases to 52% at state (2,12,15). Moreover, in some cases in this market, it is the largest firm that is most likely to merge with the smallest firm: for example, at state (2,7,12) a merger involving the smallest firm is certain to occur, but it is with the largest firm 56% of the time. Overall, conditional on a merger occurring in a state with a unique largest firm, the steady state likelihood that the two smallest firms merge in this market is 49%.

The different effects of asymmetry in these two markets appears to be related to their very different likelihoods of entry and subsequent mergers, which we discuss in the next subsection. In the $(A = 4, B = 20)$ market, the states in which entry occurs lead to situations in which a merger in the current period is fairly likely to lead to further entry and mergers. Hence, a merger today that changes the identity of the largest firm is fairly likely to change who benefits from merger externalities in the following periods, making the smaller incumbent value a merger highly when the incumbents' capital stocks are close. In contrast, in the $(A = 3, B = 70)$ market, the states following entry are highly likely to be ones in which the probability of near-term future mergers is low, making the relative surpluses created by mergers with each of the incumbents fairly unaffected by any effects on future merger bargaining.

The presence of these externalities can also lead to implications for stock price responses to merger announcements, as firms involved in a merger can experience negative returns because the market has learned that the firm will not be benefitting from a merger externality. For example, in the all-mergers-allowed equilibrium of the $(A = 4, B = 20)$ market, the average percentage change in joint value for the two merging firms upon announcement is -0.5%.[50] Conditional on the two merging firms being unequal in size, the average percentage change in value for the larger merging firm is -0.6%, while it is 0.1% for the smaller merging firm.[51] The small value change for the small firm occurs because in most states in which mergers occur

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[50] This calculation assumes that the market knows the firms' capital stocks but not the realization of proposal costs – so value changes occur only when mergers occur, due to proposal cost realizations that were not fully anticipated given the capital state **K**.

[51] In such cases, the large firm has on average 6.96 units of capital, while the small firm has 2.07 units of capital. Positive value changes occur for the large firm only 1.5% of the time and only when the observed merger was the only merger with positive probability but that probability was less than one. Negative returns occur for the small firm only 2.4% of the time and only when there are positive probabilities of it merging with each of two differently-sized larger firms.

a merger is certain to occur and involve the small firm. In such cases, the only surprise can involve which firm it merges with, and often this is fully anticipated as well.

4.2.2 Effects of Fixed Merger Policies

Table 6 reports the outcomes in these two markets when all mergers are allowed. As in our previous two-firm analysis, in both markets switching from the no-mergers-allowed policy to the all-mergers-allowed policy has a significant negative impact on steady state consumer value, here due to the industry spending much less time in triopoly and more time in duopoly states when all mergers are allowed. However, in other respects, when all mergers are allowed these two markets display some important differences from each other and from the outcomes we discussed in Section 3.

In the $(A = 3, B = 70)$ market, mergers almost never happen when all mergers are allowed. The reason is that the industry settles into a roughly symmetric duopoly in which each firm has roughly 12 units of capital, and in which entry is very unlikely.[52] For entry to happen with positive probability one or both incumbents needs to experience a great deal of depreciation. For example, with symmetric incumbents entry only starts to have a positive probability once both have depreciated to 8 units of capital or less. As in Section 3, the prospect of entry for buyout incents entry (when no mergers are allowed entry would not happen unless both symmetric incumbents had less than 3 units of capital), but still not enough for entry to be more than a rare occurrence. Compared to the no-mergers-allowed steady state, the shift from triopoly to duopoly states reduces investment, causing the level of capital and AV to fall, although in contrast to the duopoly situation in Section 3 producer value does increase.

Allowing all mergers in the $(A = 4, B = 20)$ market instead leads to a 27.5% likelihood of entry. This happens because the industry converges to a duopoly outcome in which two active firms each have roughly 8 units of capital and in each period there is approximately a 25% chance of an entrant building one unit of capital.[53] When this happens, the entrant is acquired by one of the two incumbents, investments again lead to a situation in which the two firms each have roughly 8 units of capital, and the process repeats itself. In contrast to what we have seen in the duopoly markets of Section 3 and the $(A = 3, B = 70)$ market, producer value increases enough to make steady state AV almost identical to that when no mergers are allowed, and the average total capital level is higher (14.3 rather than 14.0). The greatly improved relative AV performance of the all-mergers-allowed policy is largely driven by the incumbents' investment responses to the bargaining externality. When there are two incumbent firms with capital

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[52] For example, starting from a symmetric state in which three firms each have 5 units of capital, a merger is certain to happen; after that, entry is very unlikely and the two remaining firms converge over a number of periods to having roughly 12 units of capital each.

[53] For example, starting from a symmetric state in which three firms each have 5 units of capital, a merger is certain to happen; after that the two remaining firms converge over a number of periods toward having roughly 8 units of capital each, until another entry event occurs.

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levels that are not too far apart and an entrant with no capital, this externality incents these incumbents to invest more in an attempt to become the largest firm in the industry, and then benefit from its rival acquiring the entrant. Moreover, this enhanced incumbent investment incentive also curbs the amount of entry for buyout behavior. For example, in state $(0, 7, 7)$, the vector of expected investments is $(0.6, 1.2, 1.2)$ and $(0.3, 1.7, 1.7)$ under the no-mergers and all-mergers-allowed policies, respectively.[54]

Table 6: Performance Measures for the Triopoly Markets

	$(A = 3, B = 70)$			$(A = 4, B = 20)$		
Performance Measure[55]	No-Mergers/ MPP-CV/ Static-CS	All-Mergers/ MPP-AV	Static-AS	No-Mergers/ MPP-CV/ Static-CS	All-Mergers/ MPP-AV	Static-AS
Avg. Consumer Value	209.9	180.0	180.2	161.9	145.1	146.4
Avg. Incumbent Value	199.3	221.7	221.7	126.7	143.0	142.2
Avg. Entrant Value	0.0	0.0	0.0	0.0	0.5	0.7
Avg. Blocking Cost	-	-	-	-	-	-
Avg. Aggregate Value	409.2	401.7	401.7	288.7	288.6	289.3
Avg. Price	1.91	1.99	1.99	2.20	2.30	2.29
Avg. Quantity	76.6	71.0	70.9	36.0	34.0	34.2
Avg. Total Capital	26.1	23.2	23.2	14.0	14.3	14.6
Merger Frequency	-	0.4%	0.4%	-	27.5%	29.4%
% in Monopoly	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
% in Duopoly	24.5%	99.6%	99.6%	0.6%	97.9%	96.7%
% with 1 AV/AS Enhancing Merger	0.0%	0.0%	46.0%	0.0%	5.8%	1.0%
% with 2 AV/AS Enhancing Mergers	0.7%	0.4%	0.0%	0.0%	23.8%	31.7%
% with 3 AV/AS Enhancing Mergers	74.8%	0.0%	0.4%	0.0%	0.0%	0.3%
State (0,0,0) CV	136.4	132.3	132.5	111.0	111.5	111.8
State (0,0,0) AV	143.4	147.6	147.8	134.4	124.3	125.7

In both markets a static aggregate surplus-based policy allows many mergers. For example, in the $(A = 3, B = 70)$ market, when a merger is proposed between symmetric firms the merger is always approved if the firms are each smaller than the non-merging rival, and is often approved even if they are larger than the rival. The static policy is not quite so lenient in the $(A = 4, B = 20)$ market, but is still very permissive: for example, when a merger is proposed between symmetric firms facing a rival with 10 units of capital, the merger will be

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[54] Still, much of the entrant's investment incentive comes from the prospect of being acquired: for example, in state $(0, 7, 7)$ it would not invest at all if the incumbents were following their all-mergers-allowed investment policies but no mergers were allowed.

[55] All values are ex ante (beginning-of-period) values, except the % in Monopoly, % in Duopoly, and State (0,0,0) CV and AV which are at the Cournot competition stage.

approved if and only if the merging firms each have no more than 5 units of capital. As a result, the outcomes in these markets when the authority follows a static aggregate-surplus based policy is very close to that when all mergers are allowed. In contrast, a static consumer surplus-based policy essentially allows no mergers in these markets.

4.2.3 Markov Perfect Merger Policy

Because entry against duopolists is very unlikely in the $(A = 3, B = 70)$ market, with an AV criterion the Markov perfect policy's treatment of a proposed merger of two of three active firms is largely unaffected by any entry for buyout concerns. Without such concerns, this Markov perfect policy is quite permissive.[56] For example, like the static aggregate surplus-based policy it always allows two symmetric firms to merge when each is no larger than their non-merging rival, and often allows them to merge even when they are larger. Given this leniency, the steady state outcome under the Markov perfect policy with an AV criterion is essentially identical to that when all mergers are allowed. As with allowing all mergers, the Markov perfect policy therefore lowers steady state AV compared to a policy of allowing no mergers.[57],[58]

In the $(A = 4, B = 20)$ market, entry happens but the investment incentives noted above curtail the extent of the investment inefficiency following a merger. Overall, the Markov perfect policy with an AV criterion is again fairly lenient and allows two symmetric firms to merge whenever they are each smaller than their non-merging rival. Indeed, in this market it is much more lenient than the static aggregate surplus-based policy. As a result, the steady state outcome under the Markov perfect policy with the AV criterion is again very similar to that under the all-mergers-allowed policy.[59]

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[56]The remaining factors tend to favor a permissive policy here: First, combining two firms' capital stocks increases aggregate surplus in many states. Second, the combination reduces investment costs since investment costs are decreasing in firm size. Third, since investment by small firms in triopoly is often socially excessive (much as in duopoly), mergers of small firms that reduce their investments can be beneficial for aggregate value. The Markov perfect policy is, however, somewhat more stringent than the static policy when the non-merging firm is not large; for example, when the nonmerging rival has 4 units of capital, the Markov perfect policy allows a merger of symmetric firms as long as they have no more than 6 units of capital, while the static policy would allow the merger even if they each have 10 units of capital. The two policies become quite similar when the non-merging firm has more than 8 units of capital.

[57]Observe in Table 6, however, that starting at state (0,0,0) the Markov perfect policy yields a higher AV than allowing no mergers. The Markov perfect policy outcome has all three firms' capital quickly reach a point where a merger does not result in entry, then a merger occurs, followed by a future with a very low likelihood of entry. Note that the antitrust authority's lack of commitment in this market is not very important because once the first merger occurs, future mergers are rare.

[58]As in the $(A = 3, B = 26)$ duopoly market, in both of these triopoly markets the Markov perfect policy with a CV criterion yields a steady state equilibrium equivalent to the no-mergers-allowed policy.

[59]In contrast to the results for the $(A = 3, B = 70)$ market and the $(A = 3, B = 26)$ duopoly market of Section 3, here a commitment to the static aggregate surplus-based policy induces a slightly higher steady state aggregate value than the no-mergers-allowed policy, the all-mergers-allowed policy, and the Markov perfect

In summary, firms respond very differently in these two triopoly markets to policies allowing mergers, driven by the differing likelihoods of entry in these markets. In both markets, however, the negative effects of entry for buyout are either not present or limited, resulting in an AV-based Markov perfect policy that is fairly lenient, a contrast to our finding for duopoly in Section 3. Like the duopoly case, the resulting steady state outcome is not only bad for consumers but fails to raise AV relative to allowing no mergers, although the difference is minimal in the $(A = 4, B = 20)$ market.

5   Conclusion

We have studied optimal merger policy in a dynamic industry model in which scale economies can be achieved through either investment ("internal growth") or merger ("external growth"). In such a setting, an antitrust authority's merger approval decisions must weigh any increases in market power against the changes in productive efficiency caused by a merger, which are affected not only by the immediate cost reductions of the merging parties due to their increased scale, but also the investments of both the merging parties and rivals following the merger. Moreover, an antitrust authority that is able to commit to its policy must also consider how firms' investment behavior is affected by the prospect of future merger approvals.

To shed light on this complicated problem we have developed and computationally solved a dynamic model in which forward-looking Cournot firms invest in capital to produce a homogeneous product. Our model has three significant innovations relative to previous computational dynamic industry models. First each firm in each period can flexibly decide how many additional units of capital it wishes to purchase. Second, this investment technology is (approximately) merger neutral in the sense that the investment opportunities available in the market are unchanged following a merger, offering a much more attractive setting for studying merger policy than the original Ericson and Pakes (1995)/Pakes and McGuire (1994) model. Third, we introduce an antitrust authority as an active, maximizing player that cannot commit to its future merger approval policy. Because of the time inconsistency difficulties that arise in dynamic games the authority is unable to achieve as high a level of welfare as an authority that can commit would be able to achieve.[60]

In much of our main analysis, we have focused on markets with two firms so as to be able to use the familiar and well-accepted Nash bargaining solution. In addition, we have studied markets with three firms, using one particular model of multi-firm bargaining with externalities. Our analysis of these markets provides insights into the factors affecting optimal merger policy when investment behavior and firm scale is a critical determinant of welfare, and shows how optimal policy in dynamic settings with investment can differ in significant

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policy.

[60] In the Online Appendix, we have also analyzed the case of an antitrust authority that can commit to its future merger approval rule.

ways from what would be statically optimal. Specifically, we make five key observations:

First, the desirability of approving a merger can depend importantly on the investment behavior that will follow if it is or is not approved. However, this involves more than just the behavior of the merging firms, as the investment behavior of outsiders to the merger (here, new entrants) can have significant welfare effects. In particular, when entrants (or, more generally, small firms) have higher investment costs than large established incumbents, entry for buyout behavior can impose significant welfare losses and make merger approvals much less attractive for an antitrust authority.

Second, in the other direction, investment behaviors can be greatly influenced by firms' beliefs about future merger policy. Importantly, when the antitrust authority adopts a less restrictive policy, this may spur entry for buyout behavior by firms seeking to be acquired.

Third, the inability to commit may be costly for an antitrust authority. In fact, in cases in which aggregate value is the true social objective, it can often be better to endow the antitrust authority with a consumer value objective (which roughly corresponds to the objective of most antitrust authorities, including the U.S. and EU).

Fourth, the optimal antitrust policy for maximizing aggregate value in our model can differ significantly from the optimal static policy that considers a merger's effects only at the time it would be approved, although it may be either more or less permissive than the static policy.

Finally, externalities on rivals arising from mergers in markets with more than two firms can have significant effects on firms' investment incentives and thereby shape the antitrust authority's optimal policy.

At a more general level, the existing literature on antitrust policy has largely neglected issues relating to investment or firm entry and exit that are inherently dynamic. In a world in which the antitrust authority cannot commit fully to its future actions, analyzing such issues requires modeling the authority as a player who acts dynamically. The present paper is a first step in doing so in a truly dynamic setting. By proposing a merger-neutral investment technology that allows for complex multi-unit investment choices, and yet is tractable, it also contributes to the computational industrial organization literature more generally.

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