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The Effect of Horizontal Mergers, When Firms Compete in Prices and Investments

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November 2018

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Abstract

We study the effects of mergers when firms offer differentiated products and compete in prices and investments. Since it is in principle ambiguous, we use aggregative game theory to sign the net effect of the merger: We find that only if it entailed sufficient efficiency gains, could the merger raise total investments and consumer surplus. We also prove there exist classes of models for which the results obtained with cost-reducing investments are equivalent to those with quality-enhancing investments. Finally, we show that, from the consumer welfare point of view, a R&D cooperative agreement is superior to any consumer-welfare reducing merger.

JEL classification: K22, D43, L13, L41.

Keywords: horizontal mergers; innovation; investments; research joint ventures; competition.

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1 Introduction

In a series of recent high-profile mergers in the mobile telephony industry in the EU,[1] the telecom industry has urged the European Commission (which had jurisdiction on these mergers) to take into account that the mergers would have led to higher investments.[2] Mobile Network Operators (MNOs) have made two main arguments in support of this claim. The first is related to existence of scale economies of various nature, and as such it is not conceptually controversial (but it would need to be empirically verified). The second argues that a merger favors investments because industry consolidation would give firms stronger incentives to invest. This argument in particular has resonated with politicians and heads of government, and has been widely discussed in the press.[3] Whether mergers encourage or not investment and innovation is an issue which goes well beyond the telecom industry. Antitrust agencies all over the world, for instance, recognize the importance of assessing the dynamic effects of a merger and the possibility that it may reduce innovation and product variety.[4]

This paper studies the competitive effects of horizontal mergers in a context where firms compete both in prices and in investment levels. To our knowledge, and quite surprisingly, despite the intense policy debate, there exists very little work (that we shall refer to below) on this issue. Of course, there exists a wide literature on the related issue of the effects of competition in general on investments and on innovations.[5] However, this literature analyses what happens to investments when some proxy variable for competition intensifies or relaxes symmetrically for all firms, whereas we explicitly study the effect of a merger, which is an inherently asymmetric change: two firms combine their assets whereas the competitive environment (for instance the toughness of competition or the extent of product differentiation) is otherwise the same. Apart from capturing the nature of a merger, our model also allows to uncover the different effects that a merger has on insiders and outsiders, as well as its overall competitive impact (what is

[1] See the European Commission decisions on the Hutchison/Orange (Austria), Hutchison/Telefonica Ireland, Telefonica Deutschland/EPlus, TeliaSonera/Telenor, Hutchison 3G/Telefonica UK, and H3G Italy/Wind cases.

[2] This debate is not confined to the EU. At the time of writing, T-Mobile and Sprint are planning to merge in the US, and their main claim is that the merger will allow them to invest more in the next-generation wireless technology. See e.g. "Sprint and T-Mobile to Merge, in Bid to Remake Wireless Market", by Michael J. de la Merced and Cecilia Kangapril, The New York Times, April 29, 2018.

[3] See for instance Daniel Thomas and Alex Barker, "Telecoms: Europeans scrambled signal", Financial Times, 30 June 2014; "Together we stand", The Economist, 22 August 2015; "Britain must not go from four to three in mobile", Financial Times, 2 February 2016. See also the discussion in Faccio and Zingales (2017).

[4] See the "Horizontal Merger Guidelines" by the U.S. Department of Justice and FTC, August 2010. In the recent DowChemical/DuPont case, for instance, the European Commission found that the merger would have significantly reduced the incentives to invest in R&D in the pesticide market, and hence imposed a major divestiture by DuPont as a condition for clearance. See European Commission Press Release of 27 March 2017 at http://europa.eu/rapid/press-release_IP-17-772_en.htm.

[5] In a seminal paper, Dasgupta and Stiglitz (1980) consider a model in which homogeneous-good Cournot competitors simultaneously set quantity and cost-reducing investments. Vives (2008) extends this framework to models with price competition and differentiated products. For more recent contributions in this line of research, see also Schmutzler (2013) and Marshall and Parra (2016).

the effect on consumers?), a question which is less relevant in a literature that focuses on how investment and R&D effort react to a symmetric shock to competition (consumers typically benefit/suffer as competition intensifies/softens).

As is standard, in the benchmark case - that is, absent the merger -, firms sell one product. The merger will create a new multi-product firm which owns two product varieties, thereby breaking the symmetry in the industry. To model investments, in our base model we follow the literature on process innovation (among others, Dasgupta and Stiglitz, 1980; Spencer, 1984; Bester and Petrakis, 1993; Qiu, 1997; López and Vives, 2016), and assume that (i) firms compete simultaneously on investments and in the product market,[6] and (ii) investments reduce the cost of production. We then extend this framework to separately consider (i) a sequential model in which firms first set investments and then prices, and (ii) product innovation, i.e., a setup where investments increase product quality.

As we shall see, our analysis suggests that, under no (or weak enough) efficiency savings,[7] a merger will reduce aggregate investments and harm consumers. Interestingly, this net effect will be the result of the decrease in investment and rise in prices on the side of the merging parties (the insiders), and the increase in investments, with prices which may either increase or decrease on the side of the outsiders to the merger. These outcomes confirm the result that a merger harms consumers unless there are sufficiently strong efficiency gains in a multi-dimensional setting where firms decide on investments and prices.

Let us now be more specific about what we do in the paper, starting with the model with simultaneous price and cost-reducing investment decisions. For weak or no efficiency gains, the merger results in the insiders raising their prices and reducing their investments. This is ultimately due to the margin-expansion effect of the merger: the merged entity internalizes that a price decrease in one of its products will reduce the demand of the other product it sells, and this determines an upward pressure in prices relative to the benchmark where all firms are independent. In turn, higher prices will lead to a lower quantity sold by the insiders, and a lower marginal revenue from investing for the insiders, whose investments will therefore decrease.

In standard models of mergers with price-setting firms, constant marginal costs, and where investments are not considered, outsiders' prices also increase due to strategic complementarity. In our model with dynamic efficiencies, however, this strategic property does not necessarily hold, which makes the analysis of the effects of the merger far less straightforward. Specifically,

[6] This approach is equivalent to assuming that investment decisions are not observed when firms set prices.

[7] Although, throughout the paper, we refer to efficiency gains, in our model such efficiencies are equivalent to any internal spillover generated by the merger (e.g., related to the sharing of R&D outcomes). In a previous version of this paper, moreover, we show that if firms were operating in an industry characterised by involuntary spillovers among rivals, the merger would allow the internalisation of spillovers, to a similar effect as efficiency gains.

when the insiders increase their prices, this will tend to increase outsiders' prices as well. But since outsiders' prices increase less, their demand tends to increase (their market share will rise for sure), and this will increase their investment levels. That is, two different effects are at work: one which tends to increase outsiders' prices, and the other, through lower costs, which tends to decrease them. At equilibrium, outsiders' prices may either increase or decrease, and indeed we shall show that either outcome may arise according to the assumptions made on consumer demand.

The fact that one cannot be sure any longer of the effect of the merger on outsiders' prices also implies that in general one is not able to establish the effect of the merger on consumers. Not only insiders' prices go up and outsiders' prices may go down, but also, given that products are differentiated, one cannot readily sum up the effects of possibly different signs to find the aggregate 'net effect'. Then we ask: under which conditions can we establish the impact of the merger? To answer this question, we first show that one can reduce the dimensionality of the problem by restricting attention to one decision variable rather than two. In turn, this allows us to make use of recent developments in oligopoly theory, and reformulate the model using aggregative game theory - which is possible whenever the payoff of a player depends on its own action and an additively separable aggregate of all players' actions (Selten, 1970).[8]

By doing this, we establish that - absent efficiency gains - the merger has a negative impact on consumer surplus.[9] Specifically, this holds for the functions that satisfy the Independence of Irrelevant Alternatives, or IIA, property, like the CES and the logit demand models, which are commonly used for merger simulation. We show it also holds in standard parametric product differentiation models - such as the Shubik-Levitan, and the Salop circle models - which do not satisfy the IIA property. We then find a sufficient condition for which the merger decreases total investments.

Another important advantage of using aggregative game theory is that it allows us to establish clearcut comparative statics. Specifically, we prove that the investment-neutral merger, i.e., the merger under which the value of the investments is the same as in the benchmark, is anticompetitive. That is, for the merger to be consumer-welfare neutral, it requires larger levels of efficiency gains. Other than for policy formulation, this result is useful to guide the empirical investigation of mergers in settings with investments. Specifically, it shows that empirically

[8] We use the oligopolistic aggregative game toolkit developed in Anderson et al. (2016) (see Anderson and Peitz, 2015, for an application to two-sided markets). Nocke and Schutz (2018) develop the aggregative game approach to study oligopolistic competition with multi-product firms, but their assumption that there are no fixed costs makes it difficult to apply their setting to our problem.

[9] We call a merger "anticompetitive" if it reduces consumer surplus. This is the standard adopted by the US and European competition agencies when they screen mergers. For completeness we shall also indicate - when we are able to identify them - the effects of the merger on total surplus. For instance, we shall show that in the Salop model total surplus may increase with the merger.

detecting that, post-merger, investments do not fall, or slightly increase, is not a sufficient statistics to establish that the merger is consumer-welfare neutral.

We then extend our analysis in two directions. While in the main model we follow the literature and look at simultaneous (or unobservable) investments, in our first extension we consider a model of sequential investments in which firms first invest, their choice is observed by all, and then choose prices. The (well-known) presence of strategic effects inherent to sequential settings makes it difficult to establish propositions of general validity about the effects of the merger, which explains why the literature has typically considered simultaneous games. Moreover, an aggregative game theory formulation is not generally possible for the sequential game. Nonetheless, the analysis of parametric models confirms the qualitative results found for the simultaneous moves case: the merger harms consumers; it increases prices and decreases investments of the insiders; it increases investments of the outsiders; and it may either decrease or increase outsiders' prices.

Next, we study the effects of a merger in a product-innovation model where firms undertake quality-enhancing investments. Within the general setting, the merger gives rise to a trade-off. On top of the margin-expansion effect that arises with cost-reducing investments, product innovation also gives rise to a demand-expansion effect (see Jullien and Lefouili, 2018). More specifically, on the one hand, the merged entity will internalize the fact that increasing the quality of one product will reduce attractiveness (and profits) of its other product, and this reduces its incentive to invest; on the other, by raising prices the merger will increase the marginal profitability of investments. Again, the question is whether we can find natural conditions such that the results of the analysis are unambiguous. To this end, we prove that there exist two broad classes of models where quality-enhancing investments are equivalent to cost-reducing ones, and which therefore give rise to the same results as discussed above. Importantly, these models can accommodate standard demand functions like logit, CES and Shubik-Levitan, on top of popular vertical product differentiation models.

To guide policy formulation, we also consider the impact on welfare of a Research Joint Venture (RJV) in which firms decide investments to maximise joint profits, but they behave non-cooperatively when setting prices (see, e.g., d'Aspremont and Jacquemin, 1988).[10] We first prove that, compared to the benchmark, the RJV (weakly) lowers prices and (weakly) increases investments for any value of efficiency gain. Intuitively, the RJV does not distort price choices, but allows its members to benefit from cost savings in the investment function. We then compare the RJV with the merger, which is more challenging, because one cannot exclude a priori that responses by outsiders more than offset the effects on the insiders. However, making

[10] Coming back to the telecom industry, several national regulatory authorities have often allowed MNOs to engage in Network Sharing Agreements (NSAs), whereby they share different elements of the network infrastructure and possibly of the spectrum while continuing to behave independently at the retail level.

use of the results that the RJV is preferable to the benchmark and that it exists a consumer-welfare neutral merger, we know that the RJV will dominate any merger that reduces consumer welfare as compared to the benchmark.[11]

Let us now mention the relationship between our paper and related branches of the literature. After writing this paper, others have studied the relationship between mergers and investments. Federico et al. (2017) study stochastic innovations, but focus on the *initial impetus* of the merger (i.e., they do not determine the total effect of the merger in equilibrium) under a symmetric investment strategy.[12] Federico et al. (2018) compute numerically the total effect of a four-to-three merger in an industry characterised by linear demand and stochatsic innovations. Jullien and Bourreau (2018) propose a product-innovation model in which, by expanding consumer demand, a merger to monopoly can be procompetitive. Finally, Loertscher and Marx (forthcoming) consider mergers and investment incentives in markets with buyer power. Unlike our paper, none of these study a general demand framework allowing for differentiated products with both cost-reducing and quality-increasing investments (and possibly asymmetric demand).

A complementary perspective to our analysis is offered by the literature studying dynamic oligopoly games.[13] Specifically, Mermelstein et al. (forthcoming) study the impact of mergers on the evolution of an industry, and derive the optimal dynamic merger policy in a model with capital accumulation and economies of scale. Differently from our model, in their setting two firms bargain over a merger to monopolize the industry. These firms invest to accumulate capital and exploit scale economies. Post-merger, an entrant appears in the market with zero capital. (Apart from having a different aim, their assumptions of homogenous goods and free entry clearly differentiate their environment from ours.) They find that the antitrust authority should depart from the myopic policy suggested by Nocke and Whinston (2010), and instead undertake a more restrictive policy.[14]

As for the empirical literature on the effects of mergers on investments, it is also quite scant, and does not offer clear insights on what are the likely effects of the merger.[15] Of course, there

[11] For higher values of efficiency gains, we rely on parametric simulations and show that the RJV dominates the merger.

[12] Denicolò and Polo (2018) extend the analysis in Federico et al. (2017) to show that, when asymmetric investment strategies are optimal, a merger to monopoly can raise the probability of innovation.

[13] See, among others, Ericson and Pakes (1995), Gowrisankaran (1999), Fershtman and Pakes (2000).

[14] After reducing the dimensionality of the problem, our base model might also be interpreted as one where differentiated firms compete in prices and have decreasing marginal costs. The standard reference for models of mergers under price competition is Deneckere and Davidson (1985). However, they assume constant marginal costs, so when (absent efficiency gains) insiders raise prices, outsiders will increase prices too, unambiguously resulting in lower consumer surplus. As explained above, with decreasing marginal costs the overall effect of an increase in the merging firms' prices is of ambiguous sign in principle. By relying on the aggregative game theory we can show that the net effect of the merger is anticompetitive, even though buyers of outsiders' products may be better off. We are not aware of other models of mergers under decreasing marginal cost functions. Farrell and Shapiro (1990) propose a Cournot model with homogeneous goods and *increasing* marginal costs, and also establish that, absent efficiencies, mergers are anticompetitive.

[15] Ornaghi (2009) finds that mergers in the pharmaceutical industry decreases innovation. Focarelli and

is also a large empirical literature on how competition impacts upon innovations, investments and productivity,[16] but again a merger is not tantamount to a general shift in the competitive pressure in a sector.

The paper continues thus. Section 2 studies the effects of the merger within a simultaneous moves model with cost-reducing investments. In Section 3, we extend the analysis by considering a sequential moves game and quality-enhancing investments. We then consider in Section 4 a Research Joint Venture as alternative to the merger. Section 5 concludes.

2 A model of price competition and cost-reducing investments

We use a model of Bertrand oligopoly with differentiated goods and $n \geq 2$ firms. Demand for the good produced by firm i is given by $q_i(p_i, \bar{p}_{-i})$, where p_i, which is assumed to take values in a compact interval,[17] is the price of firm i and $\bar{p}_{-i}$ is the $(n-1) \times 1$ vector of prices set by firms $-i \neq i$. The number of independent firms, n, is exogenous, reflecting barriers to entry, although it changes with the merger. Function $q_i(p_i, \bar{p}_{-i})$ is symmetric,[18] and twice continuously differentiable whenever $q_i > 0$. As is standard, we also assume that demand of firm i decreases in p_i ($\partial_{p_i} q_i(p_i, \bar{p}_{-i}) < 0$), goods are substitutes ($\partial_{p_j} q_i(p_i, \bar{p}_{-i}) \geq 0$), and own price effects are larger than cross price effects ($|\partial_{p_i} q_i(p_i, \bar{p}_{-i})| > \partial_{p_j} q_i(p_i, \bar{p}_{-i})$) - where ∂_{p_i} and $\partial^2_{p_i p_i}$ denote, respectively, the first- and the second-order partial derivative with respect to p_i, for all $i = 1, ..., n$.

Each firm i simultaneously sets its price p_i and its cost-reducing investment x_i to maximize profits, given rivals' choices. In the model, $c(x_i)$ denotes firm i's marginal cost as function of x_i. We assume that $c' < 0$, $c'' \geq 0$, $c''' \geq 0$ and $c(0) = c \geq 0$. We denote by $F(x_i)$ the fixed cost borne by firm i to invest x_i, with $F(0) = 0$, $F' \geq 0$, $F'' \geq 0$ and $F''' \geq 0$.

Panetta (2003) find that mergers in the Italian retail banking industry have raised prices in the short-run but decreased them in the long-run due to enhanced efficiency. Genakos et al. (2018) estimate an empirical model and use it to predict the impact of (hypothetical and symmetric) four-to-three mergers. They find that prices would increase, per-firm capital expenditures (a proxy for investments) would also increase, but no evidence of effects on total capital expenditures.

[16] See for instance the work by Aghion et al. (2005) which identifies an inverted-U shape relationship between competition and innovation, and Shapiro (2013) for a critique of their analysis; and the surveys by Bartelsman and Doms (2000) and Syverson (2011).

[17] Specifically, we bound prices by ruling out outcomes with negative payoffs.

[18] That is, the demand of firm i when it sets a price equal to p and all the other firms set a price equal to z in vector $\bar{z}$ is the same as the quantity of a firm j setting p given that all other firms set a price equal to z in vector $\bar{z}$ (i.e., $q_i(p, \bar{z}) = q_j(p, \bar{z})$) for all i, j. If firms i and j merge, the condition for symmetry requires that firm i's quantity is the same as firm j's when i and j set p and all other firms set z. In Section 2.5 we show our main results still hold when the assumption of symmetry is relaxed.

A merger between two firms i and k may give rise to cost savings in R&D, which we will refer to as efficiency gains. The parameter $\lambda \in [0, 1)$ captures the importance of these efficiency gains enjoyed by a merged entity, whose total cost is given by $F(x_i) + F(x_k) - \lambda G(x_i, x_k) \geq 0$, with $\partial_{x_i} G(x_i, x_k), \partial_{x_k} G(x_i, x_k) \geq 0$ and $\partial_{x_i} G(x, x) = \partial_{x_k} G(x, x)$.[19] As we shall see, the following conditions are necessary for the uniqueness of firms' investment value given q_i:

$$F'(x_i) - \lambda \partial_{x_i} G(x_i, x_k) \geq 0, \;\; F''(x_i) - \lambda \partial_{x_i x_i} G(x_i, x_k) \geq 0.$$

Roadmap of this section For the remaining part of this section, we proceed in the following way. In Section 2.1 we write the firms' maximization problem at the benchmark (i.e., if the merger does not take place), show that the firms' bi-dimensional (price and investment) variable problem can be written as a one-dimension (price) problem (we resort to this transformation throughout most of the paper), and that the benchmark equilibrium is unique for a general demand function under standard regularity assumptions. We then write the maximization problem and the FOCs in case of a merger, and explain why the characterization of the merger equilibrium is not a straightforward problem. Since part of the complexity is due to the interaction between insiders and outsiders to the merger, we start by abstracting from outsiders' reactions: Section 2.2 fully characterizes the effect of a merger to monopoly under a general demand function (to do so, we rely on the existence and uniqueness of the benchmark equilibrium previously established). Section 2.3 is the main section of the paper. It focuses on classes of demand functions such that a firm's payoff only depends on its own action and the sum of the actions of all the firms in the industry, which allows us to resort to an aggregative game theory formulation of the problem and to establish the main effects of the merger, notably on consumer surplus and on total investments. Section 2.4 analyses specific functional form examples, both to consider models which do not satisfy the sufficient conditions under which some results hold, and to gain further insights on the effects of the merger, for instance on insiders' and outsiders' prices and investments, and on total surplus. Finally, in Section 2.5 we relax the assumption that firms' demand is symmetric.

With the exception of Lemma 1, all the proofs are in the Appendix A.

2.1 Equilibrium analysis

In what follows, we first analyse the benchmark (or status-quo) case, where there are n independent firms. Then, we study the effects of the merger, where two out of the n firms merge.

[19] In previous versions of the paper we modeled efficiency gains as affecting marginal costs of production: $c(x_i, x_k, \lambda) = c(x_i + I_s \lambda x_k)$, with $I_s = 0$ when firms are independent and $I_s = 1$ if they are merged. The results were qualitatively the same as those reported here. Note that in this section we assume that a firm is able to fully appropriate its own investments (for instance because they are fully protected by IPRs or property rights laws).

2.1.1 Benchmark with independent firms

In the benchmark, each firm i solves the following maximization problem:

$$\max_{p_i, x_i} \tilde{\pi}_i(p_i, \bar{p}_{-i}, x_i) = (p_i - c(x_i))q_i(p_i, \bar{p}_{-i}) - F(x_i), \; i = 1, .., n.$$

The associated first-order conditions (FOCs) are:

$$\partial_{p_i}\tilde{\pi}_i = q_i(p_i, \bar{p}_{-i}) + \partial_{p_i} q_i(p_i, \bar{p}_{-i})(p_i - c(x_i)) = 0, \tag{1}$$
$$\partial_{x_i}\tilde{\pi}_i = -c'(x_i)q_i(p_i, \bar{p}_{-i}) - F'(x_i) = 0. \tag{2}$$

Since the FOC with respect to x_i in (2) is independent of rivals' investment x_{-i}, we can express x_i as a function of the firm's quantity $q_i(\cdot, \cdot)$. This allows us to reduce the bi-dimensional (price and investment) problem into a game in which each firm chooses only its price. As we will see, we need to make this transformation into a one-dimensional problem also to write the n-firm aggregative formulation of the game that we use in Section 2.3. We prove all this in the next lemma, invoking the assumptions made thus far and in particular those on functions $F(\cdot)$ and $c(\cdot)$.

Lemma 1. *For any given value of $(p_i, \bar{p}_{-i})$, there is a unique corresponding value of x_i, and each firm i's bi-dimensional (investment and price) maximization problem can be rewritten as one in a single choice variable (price).*

Proof. Since there are no efficiency gains in the benchmark, the investment FOCs in equation (2) can be rewritten as:

$$\partial_{x_i}\tilde{\pi}_i = -c'(x_i)q_i(p_i, \bar{p}_{-i}) = F'(x_i) \iff -\frac{F'(x_i)}{c'(x_i)} = q_i(p_i, \bar{p}_{-i}). \tag{3}$$

We use $-F'(x_i)/c'(x_i) \equiv \phi(x_i)$. Since $F'(\cdot) \geq 0$, $c'(\cdot) < 0$, $F''(\cdot) \geq 0$ and $c''(\cdot) \geq 0$, it follows that $\phi'(\cdot) \geq 0$:

$$\phi'(x_i) = \frac{\partial}{\partial x_i}\left(-\frac{F'(x_i)}{c'(x_i)}\right) = -\frac{(F''(x_i)c'(x_i) - c''(x_i)F'(x_i))}{(c'(x_i))^2} \geq 0. \tag{4}$$

Hence, equation (3), together with the properties of $\phi(\cdot)$, implies that $\phi(\cdot)$ is invertible, with $x_i = \phi^{-1}(q_i(p_i, \bar{p}_{-i})) \equiv \chi(q_i(p_i, \bar{p}_{-i}))$, and, by the properties of the inverse functions, $\chi'(\cdot) \geq 0$:[20] for any given value of $(p_i . \bar{p}_{-i})$, the equilibrium value of firm i's investment is uniquely determined by that firm's quantity.

[20]Specifically, since ϕ is differentiable and χ is its inverse, $\phi' = 1/\chi'$ and $\text{sign}\{\phi'\} = \text{sign}\{\chi'\}$.

We can then write firm i's profits as a function in a single choice variable (p_i):

$$(p_i - c(\chi(q_i(p_i, \bar{p}_{-i}))))q_i(p_i, \bar{p}_{-i}) - F(\chi(q_i(p_i, \bar{p}_{-i}))).$$

Q.E.D.

The lemma shows that x_i is a function of the firm's quantity q_i according to the injective relationship $x_i = \chi(q_i)$, with $\chi'(\cdot) \geq 0$ and $\chi(0) = 0$. Hence, the problem of firm i can be rewritten as

$$\max_{p_i} \ \pi_i(p_i, \bar{p}_{-i}) \equiv (p_i - c(\chi(q_i(p_i, \bar{p}_{-i}))))q_i(p_i, \bar{p}_{-i}) - F(\chi(q_i(p_i, \bar{p}_{-i}))), \tag{5}$$

with

$$x_i = \chi(q_i(p_i, \bar{p}_{-i})). \tag{6}$$

In what follows, we assume that function π_i satisfies the standard assumptions ensuring that a unique regular symmetric interior equilibrium exists (see, e.g., Vives, 1999). Specifically, these assumptions require that the FOCs are downward sloping and have a unique solution.[21] Dropping functional notation for q_i, firm i's FOC with respect to p_i is:

$$\partial_{p_i}\pi_i = (p_i - c(\chi(q_i)))\partial_{p_i}q_i + q_i - \frac{dc(\chi(q_i))}{dp_i}q_i - \frac{dF(\chi(q_i))}{dp_i} = 0, \tag{7}$$

where, invoking the envelope theorem,

$$\begin{aligned} \frac{dc(\chi(q_i))}{dp_i}q_i + \frac{dF(\chi(q_i))}{dp_i} &= q_i c'(\chi(q_i))\chi'(q_i)\partial_{p_i}q_i + F'(\chi(q_i))\chi'(q_i)\partial_{p_i}q_i \\ &= [c'(\chi(q_i))q_i + F'(\chi(q_i))]\chi'(q_i)\partial_{p_i}q_i \\ &= 0 \end{aligned}$$

by the equilibrium condition in equation (6), and $q_i = -F'/c'$ (see equation (3) in the proof of Lemma 1). Then, the FOC in equation (7) can be written as

$$\partial_{p_i}\pi_i = (p_i - c(\chi(q_i)))\partial_{p_i}q_i + q_i = 0. \tag{8}$$

Under the assumptions above, these FOCs are sufficient for optimality. After imposing symmetry, the solution of equation (8) gives the equilibrium value of the price in the benchmark, p^b. Plugging this p^b into equation (6) gives us the unique symmetric equilibrium of a firm's investment in the benchmark, x^b. Therefore, under the (mild) regularity conditions given above,

[21] We then check that these conditions are satisfied in the parametric models we use to illustrate our results.

we obtain the following lemma (the proof follows from the discussion above and is therefore omitted).[22]

Lemma 2. *In the benchmark with n independent firms and simultaneous moves, there exists a unique symmetric equilibrium that features each firm setting a price $p_i = p^b$ and investing $x_i = x^b$, with $i = 1, ..., n$.*

Before proceeding with the analysis of the merger, we note that, after accounting for the dynamic efficiencies generated by investments, our pricing model may exhibit strategic complementarity or substitutability depending on the primitives of the game. Indeed, the cross derivative of π_i with respect to p_i and p_j is given by

$$\partial^2_{p_i p_j}\pi_i = (p_i - c(\chi(q_i)))\partial^2_{p_i p_j} q_i + \partial_{p_j} q_i - c'(\chi(q_i))\chi'(q_i)\partial_{p_j} q_i \partial_{p_i} q_i, \forall j \neq i, \tag{9}$$

with $c' < 0$ and $\chi' \geq 0$ - while it would be $(p_i - c)\partial^2_{p_i p_j} q_i + \partial_{p_j} q_i$ absent investments. The sign of equation (9) depends on the shape of demand and cost functions. Thus, fixing $c = c(\chi(q_i))$, the "dynamic" reaction function solving equation (8) may slope downward or upward.

2.1.2 Merger between firm i and firm k

Next, consider the merging firms' problem. Recall that the merger may generate efficiency gains (measured by λ) at the investment stage. Merging firms i and k choose prices and investments to maximize $\tilde{\pi}_{i,k} \equiv \tilde{\pi}_i(p_i, \bar{p}_{-i}, x_i) + \tilde{\pi}_k(p_k, \bar{p}_{-k}, x_k) + \lambda G(x_i, x_k)$:

$$\begin{aligned}\max_{p_i, p_k, x_i, x_k} \tilde{\pi}_{i,k} &= (p_i - c(x_i))q_i(p_i, \bar{p}_{-i}) + (p_k - c(x_k))q_k(p_k, \bar{p}_{-k}) \\ &\quad - F(x_i) - F(x_k) + \lambda G(x_i, x_k), \quad i, k = 1, ..., n, \ i \neq k.\end{aligned}$$

The FOCs with respect to p_i and x_i are (we omit those for p_k and x_k, which are symmetric):

$$\begin{aligned}&\partial_{p_i}\tilde{\pi}_{i,k} = q_i(p_i, \bar{p}_{-i}) + \partial_{p_i} q_i(p_i, \bar{p}_{-i})(p_i - c(x_i)) + \partial_{p_i} q_k(p_k, \bar{p}_{-k})(p_k - c(x_k)) = 0, \\ &\partial_{x_i}\tilde{\pi}_{i,k} = -\partial_{x_i} c(x_i) q_i(p_i, \bar{p}_{-i}) - F'(x_i) + \lambda \partial_{x_i} G(x_i, x_k) = 0.\end{aligned}$$

Moreover, an outsider firm $j \neq i, k$ solves the following problem:

$$\max_{p_j, x_j} \tilde{\pi}_j(p_j, \bar{p}_{-j}, x_j) = (p_j - c(x_j))q_j(p_j, \bar{p}_{-j}) - F(x_j),$$

[22] In Section 2.3, we shall state a similar lemma which will hold only for demand functions which are consistent with an aggregative game formulation.

so that its FOCs are isomorphic to those of a firm in the benchmark, independently of the value of the efficiency gains λ, which affect the merging firms:

$$\begin{aligned}\partial_{p_j}\tilde{\pi}_j &= q_j(p_j,\bar{p}_{-j}) + \partial_{p_j}q_j(p_j,\bar{p}_{-j})(p_j - c(x_j)) = 0,\\ \partial_{x_j}\tilde{\pi}_j &= -c'(x_j)q_j(p_j,\bar{p}_{-j}) - \partial_{x_j}F(x_j) = 0.\end{aligned}$$

Let us make some preliminary observations following simple inspection of the FOCs in the benchmark and merger configurations (in the following subsections we shall dwell more upon the effects mentioned here). First of all, both the price and investment FOCs of the outsiders do not change with the merger, so it will affect the outsiders' choices only through their best responses to the insiders' post-merger optimal choices.

Second, absent efficiency gains ($\lambda = 0$), the investment FOCs of the insiders are the same too, with and without the merger. Investments will be affected by changes in the quantities: in particular, the lower $q_i(p_i,\bar{p}_{-i})$ the lower the marginal revenue from investing and hence the lower investment levels at equilibrium. With efficiency gains ($\lambda > 0$), instead, the merger will decrease investment costs relative to the benchmark, and stimulate insiders' investments, which in turn will decrease marginal costs of production $c(x_i)$ and will tend to decrease prices.

Third, absent efficiency gains the merger effects will be mainly driven by the new term which appears in the insiders' price FOCs ($\partial_{p_i}q_k(p_k,\bar{p}_{-k})(p_k - c(x_k)) > 0$): when setting the price of product i, the merged entity will internalize the impact of p_i on the quantity demanded of good k. This is the standard market power effect of mergers that we all know, and that will tend to raise prices after the merger. With efficiency gains, this effect may be outweighed by the decrease in marginal costs identified above.

While these forces are clear at first sight, the identification of the net effect of the merger on all the relevant variables at equilibrium is not straightforward. Consider for instance the simpler case of no efficiency gains. The insiders' prices will increase with the merger. With strategic complementarity on prices, we know that this will tend to *increase* outsiders' prices as well, but (under regularity conditions) to a lower extent. Hence, outsiders' quantities will tend to rise because outsiders set a better relative price. But, since the higher $q_j(p_j,\bar{p}_{-j})$ the higher the investments x_j, this will tend to decrease the production costs of outsiders, which feeds back into the outsiders' price FOCs and will tend to *decrease* their prices. A priori, we are not able to say whether the ultimate effect will be to increase or decrease the outsiders' prices.

Furthermore, although intuitively the main effects of the merger will come from the direct price increase of the insiders and the following effects will be indirect and of a lower order of magnitude, we need to find a rigorous framework to assess the net effects of the merger, also taking into account that with differentiated products - unlike with homogenous goods - we need to aggregate the effects deriving from the quantities sold of different products. This is where the aggregative game formulation we adopt in Section 2.3 will be of help. Before that, though, we analyse the effects of a merger to monopoly.

2.2 A merger to monopoly

Let us analyse the effect of a merger in an industry where there are only two firms, but without making restrictive assumptions on the demand function. Focusing on two firms allows us to disregard the indirect effects which may take place through the outsiders to the merger.

We first establish that, absent merger-induced efficiency gains ($\lambda = 0$), the merger to monopoly raises prices and reduces investment. Then, we consider efficiency gains ($\lambda > 0$) and identify the level of λ such that the merger will expand the firms' investments and benefit consumers. Proceeding as in Lemma 1, the merging firms' problem in prices and investments can be rewritten as a problem in p_i and p_k only; thus, the corresponding (standard) second order conditions are:

$$(A0)\text{: } \partial^2_{p_i p_i}\pi_{i,k} < 0,\ \partial^2_{p_k p_k}\pi_{i,k} < 0,\ (\partial^2_{p_i p_i}\pi_{i,k})(\partial^2_{p_k p_k}\pi_{i,k}) > (\partial^2_{p_i p_k}\pi_{i,k})^2,$$

where $\pi_{i,k}$ is the merged entity's profit function. In what follows, we assume that these conditions hold true both with and without efficiency gains at any interior maximum. Due to the absence of outsiders, they will guarantee that the FOCs are sufficient for optimality.

2.2.1 No efficiency gains: $\lambda = 0$.

Consider first what happens when *efficiency gains are absent*: $\lambda = 0$. The proposition below establishes that the merger will decrease investments and increase prices, thereby unambiguously harming consumers.

We set up the merging firms' maximization problem:[23]

$$\max_{p_i, p_k} \quad \pi_{i,k} \equiv \pi_i(p_i, p_k) + \pi_k(p_k, p_i) \tag{10}$$

with

$$x_i = \chi(q_i(p_i, p_k)) \tag{11}$$

$$x_k = \chi(q_k(p_k, p_i)), \tag{12}$$

where π_i is defined in equation (5). Solving this problem under Assumption (*A0*), and considering the results in the benchmark above with $n = 2$, yields the following proposition:

[23]Note that, if $\lambda = 0$, the merging firm's FOCs for investments is isomorphic to the one in the benchmark in (2).

Proposition 1. *With simultaneous moves and absent merger-induced efficiency gains, a merger to monopoly will raise the equilibrium price and decrease investments.*

Intuitively, this result is the consequence of the effects discussed above: given the internalization of the price effect of one product on the demand of the other, at the price and investment levels of the benchmark equilibrium (given by Lemma 2) the merged firms will want to raise prices. This will reduce quantities sold which, in turn, will make them want to decrease investments. But since lower investments will imply higher production cost, this effect will reinforce the increasing effect on the price.

2.2.2 Efficiency gains: $\lambda > 0$.

Next, consider the case where $\lambda > 0$. The following lemma establishes a monotonic relationship between the level of efficiency gains λ and the equilibrium values of the merger: as λ increases, investments increase and prices fall. Note that this lemma holds for any λ, and therefore also describes what happens when efficiency gains increase from $\lambda = 0$.

Lemma 3. *Consider the unique equilibrium solution $(p^m(\lambda), x^m(\lambda))$ of the merged entity problem corresponding to a given level λ of efficiency gains. As λ increases, the merger equilibrium price will decrease and the merger equilibrium investment will increase: $\partial_\lambda p^m(\lambda) < 0, \partial_\lambda x^m(\lambda) > 0$.*

To understand the lemma, suppose that there are no efficiency gains and that the merged entity is at its optimal price and investment choice. Now, if λ became positive (no matter how small), the merged entity will find it optimal to increase investments, because they are cheaper. As a result, production costs will be lower, which will push the firm to lower prices (which, through higher quantities demanded, will push it to further adjust investments upwards, and so on). So, the higher the efficiency gains the lower prices and the higher investment levels.

Next, we identify a level of efficiency gains such that investment levels are the same with and without merger.

Lemma 4. *With simultaneous moves, if the level λ^b of efficiency gains is such that a merger to monopoly results in the same level of investments as in the benchmark, $x^m(\lambda^b) = x^b$, then at λ^b, the merger will result in higher prices than at the benchmark: $p^m(\lambda^b) > p^b$.*

2.2.3 Comparisons between benchmark and merger equilibrium

Armed with the two previous lemmas we can now characterize the benchmark and the merger equilibrium solutions as a function of the efficiency gains. In particular, we know that for $\lambda = 0$ the merger leads to lower investments and higher prices; that as λ increases, the benchmark solutions do not change (when firms are independent they do not benefit from efficiency gains), and the merger performs better (prices decrease and investments increase with λ); that there exists a level of efficiency gains λ^b such that the investment at the merger equilibrium equals the investment at the benchmark equilibrium, but that at that level the merger price is still higher than at the benchmark. Hence, by the monotonicity of the solutions with respect to λ, if there exists a $\widetilde{\lambda} > \lambda^b$ such that $p^m(\widetilde{\lambda}) = p^b$ then it must be that $x^m(\widetilde{\lambda}) > x^b$. This also implies that even if the merger was showed to entail sufficient efficiency gains to increase investments, this may not be sufficient to infer that the merger is competitively neutral (not to say beneficial): there will exist an interval $\lambda \in (\lambda^b, \widetilde{\lambda})$ for which the merger raises investments but also the prices, thereby affecting negatively consumer welfare.

The following summarizes this discussion.[24]

Proposition 2. *In a merger to monopoly, for low efficiency gains, $0 \leq \lambda < \lambda^b$, the merger will lead to lower investments and higher prices; for intermediate levels, $\lambda^b \leq \lambda < \widetilde{\lambda}$, the merger (weakly) increases investments but increases prices; only for high efficiency gains ($\lambda \geq \widetilde{\lambda}$) will the merger (weakly) reduce prices and be pro-competitive.*

2.3 The effects of a merger in a n-firm industry

In this section, we analyse the consequences of the merger in an industry with $n \geq 3$ firms. We will study how prices and investments change after the merger by exploiting methodologies borrowed from aggregative game theory (e.g., Selten, 1970; Jensen, 2010; Corchon and Marini, 2018). To ease exposition, we maintain the assumption that demand is symmetric. As shown in Section 2.5, however, this restriction is not crucial for most of the results obtained in this section.

By Lemma 1, we can recast our price-and-investment simultaneous game into a single-variable problem (p_i), under the condition that $x_i = \chi(q_i(p_i, \bar{p}_{-i}))$ for all i. This allows us to write the firm's profit maximization problem as an aggregative game, that is, a game in which a firm's payoff π_i is a function of its own action (a_i) and the sum of the actions of all the n firms in the industry, the aggregate, $A = \sum_{i=1}^{n} a_i$.

Specifically, we focus on the classes of quasi-linear indirect utility functions of the following type:

$$V(\bar{p}) = \sum_i h(p_i) + \Psi\left(\sum_i \psi(p_i)\right),$$

[24] The proof follows from the discussion preceding the proposition, and is therefore omitted.

with ψ, Ψ and $h(p_i)$ continuous and thrice continuously differentiable, $\psi'(p_i) < 0$ and $\Psi'(\cdot) > 0$. By Roy's identity, the ensuing demand function for product i is given by

$$q_i(p_i, \bar{p}_{-i}) = -h'(p_i) - \psi'(p_i)\Psi'\left(\sum_j \psi(p_j)\right), \text{with } i, j = 1, ..., n. \tag{13}$$

Nocke and Schutz (2018) show that, with $n \geq 3$ differentiated products, this demand function has an aggregative formulation - *a fortiori*, this result extends to our framework, too. In particular, since $\psi(\cdot)$ is strictly decreasing, we can rewrite the demand in equation (13) as a function of $a_i \equiv \psi(p_i)$ and its summation $A \equiv \sum_i \psi(p_i)$ only, i.e., $q_i(a_i, A) = -h'(\psi^{-1}(a_i)) - \psi'(\psi^{-1}(a_i))\Psi'(A)$. Accordingly, we will rewrite $\pi_i(p_i, \bar{p}_{-i})$ as $\pi_i(A, a_i)$.

Since the interval of values of p_i is compact, so is the one of a_i. We further assume that $dq_i/da_i > 0$, with $dq_i/da_i = \partial_{a_i} q_i + \partial_A q_i$, $\partial_{a_j} q_i < 0$, $i \neq j$, and $dq_i/da_i + \partial_{a_j} q_i > 0$.[25]

We now discuss the properties of our demand system:

1. If $h'(p_i) = 0$, the demand in equation (13) satisfies IIA, because $q_i/q_j = \psi'(p_i)/\psi'(p_j)$. Prominent examples of demand functions that fall into this category are the logit and CES demand models. As far as the logit is concerned, recall that

$$q_i = \frac{\exp\{(s - p_i)/\mu\}}{\exp\{(s_0 - p_0)/\mu\} + \sum_{j=1}^n \exp\{(s - p_j)/\mu\}}, \tag{14}$$

   where $s_0, s \in \mathbb{R}$ are quality parameters, μ the degree of preference heterogeneity, and the outside good $j = 0$ has a price $p_0 = 0$. It can be written in aggregative terms by setting $a_i = \exp\{(s - p_i)/\mu\}$. The CES function features $q_i = p_i^{-r-1}/\sum_{j=1}^n p_j^{-r}$, where $r = \rho/(1 + \rho)$ and ρ measure products' substitutability. Its aggregative formulation requires $a_i = p_i^{-r}$.

2. If $h'(p_i) \neq 0$, the demand system fails to satisfy the IIA property. A function that falls into this category is the Shubik-Levitan linear demand system,

$$q_i = \frac{(\alpha - p_i)[1 + (n-1)\gamma] - \gamma \sum_{j=1}^n (\alpha - p_j)}{(1 - \gamma)[1 + (n-1)\gamma]}, \tag{15}$$

   where α is the intercept and $\gamma \in (0, 1)$ measures product substitutability. In this case, $a_i = (\alpha - p_i)$.

[25] These assumptions are satisfied by the three specific demand functions we consider as main examples - CES, logit and linear demand.

As we shall see, whether the IIA property holds has consequences for our welfare analysis. The reason is that under the IIA consumer surplus only depends on A; thus, proving that the aggregate falls with the merger will imply a fall in consumer surplus.

Outline of this sub-section We shall analyse this aggregative game formulation by applying the toolkit developed in Anderson et al. (2016). First, we state the assumptions behind the aggregative game analysis; second, we construct firm i's inclusive reaction function $\tilde{r}_i(A)$ to the aggregator A (Selten, 1970).[26] We shall then write the aggregate inclusive reaction $\sum_{i=1}^{n} \tilde{r}_i(A)$. The equilibrium will then be determined as the fixed point of the problem $\sum_{i=1}^{n} \tilde{r}_i(A) = A$. The same procedure will be applied for both the benchmark and the merger, and we shall then proceed to the analysis of the effects by comparing the two equilibria. After carrying out the analysis in general, we shall develop the full analysis with a particular demand function, as an illustration of the methodology, and then show the robustness of the results when considering an asymmetric demand.

2.3.1 Assumptions on payoffs

In the aggregative formulation of the game, the profit function of firm i is

$$\pi_i(A, a_i) = \left(\psi^{-1}(a_i) - c\left(\chi\left(q_i(A, a_i)\right)\right)\right) q_i(A, a_i) - F\left(\chi(q_i(A, a_i))\right), \tag{16}$$

with $x_i = \chi(q_i(A, a_i))$ for all i.

Let $A_{-i} = A - a_i$ denote the sum of all firms' actions but firm i's (so that $A_{-i} = \sum_{j \neq i} a_i$). Then, a firm's profit function in the aggregative game can be written as $\pi_i(A_{-i} + a_i, a_i)$. Moreover, we set $\pi_i(A_{-i} + 0, 0) = 0$ and denote $r_i(A_{-i})$ as the standard best reaction function - so that $r_i(A_{-i}) = \arg\max_{a_i} \pi_i(A, a_i)$. We assume that π_i satisfies

$$(A1)\text{: } \partial_{A_{-i}} \pi_i(A_{-i} + a_i, a_i) < 0 \;\forall a_i > 0.$$

Assumption (*A1*) means that an increase in the actions of the rivals reduces firm i's profits. Recall that a firm's action a_i varies inversely with its price (as $\psi' < 0$); thus, when other firms increase their action, this amounts to a fall in their prices. Assumption (*A1*) implies that, by raising their own action a_i, firms impose a negative externality on each other.

The reaction function of firm i to other firms' actions, $r_i(A_{-i})$, solves

$$\frac{d\pi_i(A_{-i} + a_i, a_i)}{da_i} = \partial_A \pi_i(A_{-i} + a_i, a_i) + \partial_{a_i} \pi_i(A_{-i} + a_i, a_i) = 0, \tag{17}$$

[26] When doing it, we will review the results in Anderson et al. (2016), to show that they directly hold in our setting.

where $d\pi_i(A_{-i}+a_i, a_i)/da_i$ is the total derivative of $\pi(\cdot,\cdot)$ with respect to a_i. To guarantee that $r_i(A_{-i})$ exists, is continuous and solves equation (17) for interior solutions, we also assume

$$(A2)\colon \frac{d^2\pi_i(A_{-i}+a_i, a_i)}{da_i^2} < 0$$

at any interior maximum, which is the equivalent of the standard assumption made in Section 2 requiring profit function's concavity.

Finally, we assume that

$$(A3)\colon \frac{d^2\pi_i(A_{-i}+a_i, a_i)}{da_i^2} < \frac{d^2\pi_i(A_{-i}+a_i, a_i)}{da_i dA_{-i}},$$

which guarantees that reaction functions are well-behaved.[27]

One can verify that these assumptions are satisfied, for instance, for logit, Shubik-Levitan and CES demands if the cost structure exhibits constant returns of scale (e.g., $c(x) = c - x$) and the investment cost function is quadratic (e.g., $F(x) = x^2/2$), so that $\chi(q) = q$.

Although we lay out our game as one of price competition, the reformulation of a firm's profit function in equation (16) implies that, as already discussed above, the firms' choice variables are not necessarily in a relationship of strategic complementarity. Thus, in the aggregative game formulation of our analysis we might have either strategic complementarity, so that $d^2\pi_i(A_{-i}+a_i, a_i)/da_i dA_{-i} > 0$ or strategic substitutability, so that $d^2\pi_i(A_{-i}+a_i, a_i)/da_i dA_{-i} < 0$. In the first case $r_i(A_{-i})$ is upward sloping. In the second, $r_i(A_{-i})$ is downward sloping.[28] We then say that $r_i(A_{-i})$ takes positive values for all $A_i \leq \overline{A}_{-i}$. Instead, $r_i(A_{-i}) = 0$ for all $A \geq \overline{A}_{-i}$.[29]

2.3.2 Construction of the inclusive reaction function

So far, we have derived the standard reaction function r_i, as a function of A_{-i}. Next, we construct the inclusive reaction function of firm i to the value of the aggregator A, which includes its own action a_i. We will denote it by $\tilde{r}_i(A)$. To begin with, we remark some useful properties of $r_i(A_{-i})$:

Lemma 5. *Assumptions (A2) and (A3) imply that $r'(A_{-i}) > -1$. Then, $A_{-i} + r_i(A_{-i})$ is strictly increasing in A_{-i}.*

[27] Equivalently, in alternative to Assumption $(A3)$ one could invoke Corchon (1994) conditions in games with strategic substitutability and strategic complementarity.

[28] For instance, in the benchmark, if $c(x) = c - x$ and $F(x) = x^2/2$, then actions will be strategic complement under the logit demand function, strategic substitutes under the Shubik-Levitan demand, and could be either - depending on the value of r, under the CES demand function.

[29] Intuitively, since the higher a_i the more aggressive the action, when rivals are very aggressive, A_{-i} is so large that firm i's best reply is $a_i = 0$.

The result is known from Selten (1970), and is important to establish that, as a consequence of the monotonicity of $A_{-i} + r_i(A_{-i})$, the aggregate A defined at the value of the best response of firm i ($A_{-i} + r_i(A_{-i})$) is increasing in A_{-i}. We are now in the position to derive the inclusive reaction function $\tilde{r}(A)$. We can invert $A_{-i} + r_i(A_{-i}) \equiv h_i(A_{-i}) = A$, to obtain $A_{-i} = h_i^{-1}(A) \equiv f_i(A)$. Given this, we can write $\tilde{r}_i(A) \equiv r_i(f_i(A))$. Lemma 6 follows:

Lemma 6. *Assumption (A3) implies that $d\tilde{r}_i/dA = r_i'/(1 + r_i')$ is strictly lower than 1. Thus, with strategic complementarity, $r_i' > 0$ implies that the inclusive reaction function is strictly increasing in the aggregate A. With strategic substitutability, $r_i' \in (-1, 0)$ means that the inclusive reaction function is strictly decreasing for all $A < \overline{A}_{-i}$.*

To conclude this section, we provide a result that will be useful to establish the profitability of the merger. Before doing so, we find it useful to denote by $\bar{\pi}_i(A) \equiv \pi_i(A, \tilde{r}_i(A))$ the value of firm i's profit when it maximizes its profit given the actions of the others and doing so results in an aggregate of value A.

Lemma 7. *Under Assumptions (A1)–(A3), $\bar{\pi}(A)$ is strictly decreasing in $A < \overline{A}_{-i}$ and is zero otherwise.*

We proceed by characterizing the equilibrium of the aggregative game in the benchmark and with the merger.

2.3.3 Benchmark with independent firms

Given the derivation of the inclusive reaction function, we proceed to establish the conditions for the existence of the equilibrium with independent firms. Specifically, an equilibrium exists if it exists a fixed point of the following problem:

$$\sum_{i=1}^{n} \tilde{r}_i(A) = A. \tag{18}$$

Lemma 8. *In the benchmark with n independent firms, an equilibrium A^b of the aggregative game always exists. Moreover, the value of A^b is unique if, at any fixed point, it holds true that*

$$\sum_{i=1}^{n} \tilde{r}_i'(A^b) < 1. \tag{19}$$

Condition (19) is the equivalent of the standard stability conditions that we invoke in the benchmark, and it is well known that it is required for the equilibrium value of the aggregate to be unique in well-behaved aggregative games (Cornes and Hartley, 2005). It implies that the value of $\sum_{i=1}^{n} \tilde{r}_i(A)$ intersects A from above and, given the properties of $\tilde{r}_i$, it means that the equilibrium value of A is unique.[30]

2.3.4 Merger between firm i and firm k

Let firms i and k merge. We maintain Assumptions (*A1*)–(*A3*). Merged firms solve

$$\max_{a_i, a_k} \; \pi_i(A, a_i) + \pi_k(A, a_k). \tag{20}$$

The ensuing FOCs with respect to a_i and a_k are sufficient for optimality:[31]

$$\begin{aligned} \partial_A \pi_i(A, a_i) + \partial_{a_i} \pi_i(A, a_i) + \partial_A \pi_k(A, a_k) &= 0 \qquad (21) \\ \partial_A \pi_k(A, a_k) + \partial_{a_k} \pi_k(A, a_k) + \partial_A \pi_i(A, a_i) &= 0. \qquad (22) \end{aligned}$$

In line with the analysis of the main model equilibrium conditions in Section 2.1, the FOCs in (21) and (22) differ from the benchmark because the merged entity takes into account the impact of changing a_i on the profit of firm k.

Solving for the FOCs of the insiders, and constructing the respective inclusive best reaction functions, yields $\tilde{r}_i^m(A)$ and $\tilde{r}_k^m(A)$, with $\tilde{r}_i^m(A) + \tilde{r}_k^m(A) \equiv \tilde{R}^m(A)$.

Lemma 9. *Assume firms i and k merge. Then, for any A, $\tilde{r}_i^m \leq \tilde{r}_i(A)$ and $\tilde{r}_k^m \leq \tilde{r}_k(A)$; thus, $\tilde{r}_i(A) + \tilde{r}_k(A) > \tilde{R}^m(A)$.*

For given value of the aggregate A, merged firms choose less aggressive actions (i.e., higher prices in our Bertrand game with differentiated products), thus commanding a reduction of respective actions a_i and a_k. Since the merger only affects the inclusive best response functions of the insiders, the equilibrium value of the aggregate under the merger solves the following fixed point problem:

$$\tilde{\Sigma}^m(A) \equiv \sum_{j \neq i,k}^{n} \tilde{r}_j(A) + \tilde{R}^m(A) = A. \tag{23}$$

[30] Note also that Lemma 8 is the equivalent of Lemma 2 for the aggregative formulation of the game.

[31] The FOC with respect to a_k is analogous, thus omitted. Nocke and Schutz (2018) analyse existence and uniqueness of the equilibrium in an aggregative formulation of the oligopolistic pricing game with multi-product firms. They show that these properties apply to the class of demands that we take as leading examples (and in particular, the logit, linear and CES demand systems) under constant returns to scale and no fixed costs; thus, their results do not directly extend to our analysis. We then checked that our assumptions are satisfied by these three demand functions.

By Lemma 9, the value of A that solves equation (23), A^m, is strictly lower than the corresponding value in the benchmark: $A^m < A^b$.[32] Moreover, if $d\tilde{\Sigma}^m(A)/dA < 1$, then A^m is unique. All this yields the following result:[33]

Proposition 3. *Assume firms i and k merge, and that the relevant conditions for uniqueness of the aggregate hold with and without the merger. The aggregate falls from A^b to A^m. Hence, the sum of the profits of all firms in the industry go up.*

With strategic complementarity, the merger raises the profits of insiders and outsiders (see Deneckere and Davidson, 1985). With strategic substitutability, instead, the merger is not necessarily profitable (see Salant et al., 1983).[34] All this has implications when we look at specific models: when actions are strategic complements in the aggregative formulation, merger profitability will always be guaranteed, whereas when they are strategic substitutes, we had to check that the merger is profitable.

2.3.5 Implications for consumer welfare

We proceed by determining the consequences of the merger for consumer welfare.

Proposition 4. *If the demand function satisfies the IIA property, a merger between two firms i and k reduces consumer surplus.*

This proposition follows from the next two considerations.[35] First, Lemma 9 establishes that the equilibrium value of the aggregate under the merger is lower than in the benchmark. Moreover, as shown by Anderson et al. (2016) for Bertrand (pricing) games with differentiated products, if a demand system satisfies the IIA property, consumer welfare only depends on the aggregate. More specifically, it increases in A. Thus, if the aggregate falls the industry becomes less competitive and consumer surplus becomes smaller.[36]

The class of demand functions that satisfy the IIA property include the logit and CES demand systems. It does not include linear differentiated products demand systems, like the Shubik and Levitan demand function in equation (15). This does not necessarily mean that the merger will increase consumer surplus but simply that the sufficient condition in the proposition cannot be applied. In fact, in the parametric analysis developed below, we find that the merger does reduce consumer surplus also in the Shubik-Levitan model whenever the merger turns out to be profitable.

[32] Existence is guaranteed by the same arguments as in the proof of Lemma 8.

[33] The proof of the next proposition is omitted, because it follows from the arguments given above.

[34] Recall that in our game even if prices are strategic complements the existence of investments may turn actions in the aggregative formulation of the game into strategic substitutes.

[35] The proof is therefore not given.

[36] Recall that the IIA property holds true if the ratio of any two demands depends only on their own prices (and is independent of the prices of other options in the choice set).

2.3.6 Implications for investments

Proposition 3 shows that, as a consequence of the fall in insiders' actions established in Lemma 9, the aggregate falls with the merger. Thus, if the industry quantity $Q(A, \overline{a}) = \sum_i q_i(A, a_i)$, where $\overline{a} = (a_1, ..., a_n)$, is such that $Q(A, \overline{a}) = Q(A)$ and $Q'(A) \geq 0$, the merger reduces the total industry quantity, too. Among others, this property is satisfied by the logit and Shubik-Levitan demand functions. For example, for the logit demand system in equation (14), total quantity Q is given by $A/(\exp\{s_0\} + A)$, and thus is strictly increasing in the aggregate for any finite value of s_0.[37] Similarly, in the linear products demands à la Shubik and Levitan in equation (15) one has that $Q = A/(1 + 2\gamma)$.

Consider now the simple case featuring a constant returns to scale technology ($c(x) = c - x$) and quadratic investment cost ($F(x) = x^2/2$). Then, Lemma 1 implies that $x_i = q_i$, so that $Q = \sum_i x_i$, i.e., total investments are equal to Q. As a consequence, if $Q'(A) \geq 0$, a merger that reduces the aggregate also reduces industry quantity and investments. Next, we prove that this result holds for any admissible function $\chi(\cdot)$.

Proposition 5. *If $Q(A, \overline{a}) = Q(A)$ and $Q'(A) \geq 0$, then, in an industry with n symmetric firms, the merger between firms i and k reduces total investments.*

This result establishes that, for the merger to reduce total investments, and given that the merger reduces A in equilibrium, it is sufficient that $Q(A, \overline{a}) = Q(A)$ and that $Q'(A) > 0$ - as is the case for demand functions as logit and Shubik Levitan demand functions. For the CES, these sufficient conditions cannot be used as Q depends on A and (non-linearly) on the elements in $\overline{a}$. Nonetheless, we note two things: first, the merger will reduce total quantity also with the CES demand function when actions are strategic complements. We will then illustrate by means of specific parametric examples that investments fall with the CES also when actions are strategic substitutes.[38]

2.3.7 Efficiency gains: $\lambda > 0$

We now consider the case of efficiency gains from the merger.[39] Since the benchmark is the same, we turn directly to the analysis of the merger.

Assume firms i and k merge. In the presence of efficiency gains, the merged firm solves

$$\max_{a_i, a_k} \; \pi_i(A, a_i|\lambda) + \pi_k(A, a_k|\lambda), \tag{24}$$

[37] This demand function is instead constant in the aggregate in the absence of the outside good (i.e., for all $s_0 \to -\infty$).

[38] Notably, under linear variable cost and quadratic fixed cost assumptions, for all values of r which make the merger profitable.

[39] Recall that this formalisation of efficiency gains makes them equivalent to internal spillovers generated by the merger. Under this interpretation, λ captures how efficient the merged firm is in sharing R&D outcomes.

where

$$\begin{aligned}\pi_i(A, a_i|\lambda) + \pi_k(A, a_k|\lambda) &= \left(\psi^{-1}(a_i) - c\left(\chi\left(q_i(A, a_i)|\lambda\right)\right)\right) q_i(A, a_i) - F\left(\chi(q_i(A, a_i)|\lambda)\right) \\ &\quad + \left(\psi^{-1}(a_k) - c\left(\chi\left(q_k(A, a_k)|\lambda\right)\right)\right) q_k(A, a_k) - F\left(\chi(q_k(A, a_k)|\lambda)\right) \\ &\quad + G\left(\chi(q_i(A, a_i)|\lambda), \chi(q_k(A, a_k)|\lambda)\right),\end{aligned}$$

and $x_i = \chi(q_i(\cdot)|\lambda)$ and $x_k = \chi(q_k(\cdot)|\lambda)$ are constructed as in the proof of Lemma 3 for the case of $n = 2$.[40]

To establish how λ affects the equilibrium value of prices and investments, we take the inclusive best response function $\tilde{r}_i(\cdot)$ constructed in Lemmas 5–7 above. Then, we consider an exogenous shift in λ:

Lemma 10. *The inclusive reaction function of the insiders, $\tilde{r}_i(A|\lambda)$, moves upwards with λ: $d\tilde{r}_i(A|\lambda)/d\lambda > 0$.*

We now establish the impact of efficiency gains on the aggregate. Specifically, since merged firms solve the problem in equation (24), the ensuing FOC with respect to a_i is:

$$\partial_A \pi_i(A, a_i|\lambda) + \partial_{a_i} \pi_i(A, a_i|\lambda) + \partial_A \pi_k(A, a_k|\lambda) = 0.$$

Computing the corresponding inclusive best reaction functions yields $\tilde{r}_i^m(A|\lambda)$ and $\tilde{r}_k^m(A|\lambda)$, with $\tilde{r}_i^m(A|\lambda) + \tilde{r}_k^m(A|\lambda) \equiv \tilde{R}^m(A|\lambda) > \tilde{R}^m(A)$ - where the last inequality relies on Lemma 9 and 10. The higher the value of the efficiency gain λ, the more aggressive the actions of the merged entity. In other words, actions a_i and a_k monotonically increase with λ.

Given $\tilde{R}^m(A|\lambda) > \tilde{R}^m(A)$, the ensuing equilibrium value of the aggregate with efficiency gains is larger than in the case with $\lambda = 0$. Specifically, since firms solve:

$$\Sigma^m(A|\lambda) \equiv \sum_{j \neq i,k}^{n} \tilde{r}_j(A) + \tilde{R}^m(A|\lambda) = A, \tag{25}$$

the value of A that solves equation (25), A_λ^m, is strictly larger than the corresponding value without efficiency gains: $A^m < A_\lambda^m$. Existence of A_λ^m follows from the same arguments as in the proof of Lemma 8 and, if $d\tilde{\Sigma}^m(A|\lambda)/dA < 1$, A_λ^m is unique.

We conclude by establishing the uniqueness of the value of λ that renders the merger welfare neutral with respect to the merger.[41]

[40]The profit function of an outsider firm j is as in equation (16).

[41]It is useful to remark that, if Assumption (*A3*) were replaced by Corchon (1994) conditions, we would again obtain that a positive shock to payoffs is expected to increase the smallest and the largest equilibrium value of the aggregate. Thus, under the ensuing uniqueness conditions, we would find that the equilibrium value of the aggregate increases.

Lemma 11. *Assume firms i and k merge. Then, there exists a unique value of λ such that $A^m(\tilde{\lambda}) = A^b$, with $A^m(\tilde{\lambda}) < A^b$ for all $\lambda < \tilde{\lambda}$ and $A^m(\tilde{\lambda}) \geq A^b$ otherwise.*

We can now draw on the analysis made for the case where $\lambda = 0$ to conclude the analysis of the merger with efficiency gains. In particular, we know that - under the assumption that the IIA property holds - the effects on the consumer surplus depend on whether the aggregate A increases or not with the merger. The next proposition summarizes:

Proposition 6. *Assume firms i and k merge, and that the IIA property is satisfied. If efficiency gains are small enough ($\lambda < \tilde{\lambda}$), the merger reduces consumer surplus. Otherwise ($\lambda \geq \tilde{\lambda}$), it will be (weakly) pro-competitive.*

2.4 Specific functional forms models

We have seen above that, absent efficiency gains, the merger leads to lower consumer surplus for a class of models that we can write as aggregative games and satisfy the IIA property. However, some models which are commonly used in industrial organization do not belong to that class. In this section, therefore, we report parametric results for the study of the merger effects for a model that does not satisfy the aggregative games properties, the Salop circle model, as well as for models which can be written as aggregative games - namely the CES, logit and Shubik-Levitan demand functions. Dealing with closed-form solutions will allow us to illustrate the impact of the merger on all variables, thereby gaining further insight on merger effects.[42]

To begin with, we develop the aggregative game analysis for the Shubik-Levitan demand function and we show that - absent efficiency gains - the loss in consumer surplus caused by the merger occurs for any number of firms n in the industry, but decreases with it. This confirms in a setting where firms choose both investments and prices the standard insight that - *ceteris paribus* - antitrust authorities should be more concerned with mergers in concentrated industries.

We then restrict attention to $n = 3$ symmetric firms (the minimum number which allows us to analyse the effects of the merger on insiders and outsiders), and solve our game for the Salop model, the CES, logit and Shubik-Levitan demand functions.[43] It turns out that the merger always harm consumers. This is mainly due to the insiders' lower investments and higher prices.

[42]Below, we summarise the results we obtain, for the full analysis see the Online Appendix B.

[43]The results are in Table B-1 in Online Appendix B. While we could obtain analytical solutions for the Shubik-Levitan and the Salop models, we could not find closed-form solutions with the CES and logit demand functions in the merger case (which entails asymmetries). Thus, the table reports results for representative values of the parameters.

In some cases, outsiders' prices may decrease with the merger (due to their higher investments), but in none of the cases analysed to such an extent as to lead to a pro-competitive effect.[44]

Note that the merger always increases outsiders' profits (they benefit from the insiders' higher prices and lower investments) and that we make assumptions aimed at guaranteeing that the merger is profitable for the insiders (as otherwise it would not be proposed). In principle the merger may raise total surplus, and we do find that this may happen in the Salop model. Before making too much of this result, though, consider that in the Salop model demand is completely inelastic (all the market is covered and each consumer buys just one unit), hence there will be no dead-weight loss from the merger's higher prices.

2.5 Asymmetric demand

The model has been solved under the assumption that demand is symmetric (Section 2, recalled in Section 2.3). In this section, we relax this restriction to consider the case of asymmetric demand, and discuss the robustness of the results in Propositions 3, 4 and 5. By doing so, we allow for the possibility that firms decide to take asymmetric strategies, like discontinuing the production of a variety after the merger.[45]

First of all, it is useful to remark that nothing in the aggregative formulation of the model requires demand symmetry. Indeed, the advantage of this approach is that it relies on the aggregate of the actions, rather than its composition.[46] Hence the derivation of the benchmark equilibrium does not rely on an assumption of symmetry.

Next, consider the merger. The outsiders' inclusive reaction functions are not affected by the merger, and are the same as in the benchmark. As for the insiders, when we showed that the merger reduces the value of their inclusive reaction functions (Lemma 9), symmetry allowed us to rule out the case in which the new firm treats the two products differently. Consider now the case in which the insiders' products may be treated asymmetrically. If the merged entity keeps both goods active, then the proof proceeds as in Lemma 9, which means that both inclusive best reply functions fall with respect to the benchmark. If instead the merged entity closes down firm i's good, then $\tilde{r}_i^m(A) = 0 < \tilde{r}_i(A)$. Hence, $\tilde{r}_k^m(A) = \tilde{r}_k(A)$. This means that

[44] In the aggregative formulation of our game, strategic complementarity holds under the logit demand function and under the CES function for low enough values of r, e.g. for $r = 1$. With the Shubik-Levitan demand, or CES demand with, e.g., $r = 1.6$, the firms' actions are strategic substitutes, implying that the merger increases insiders' prices but lowers outsiders'. However, the fall in the actions of the insiders is never outweighed by the increase in the actions of the outsiders.

[45] Alternatively, one might consider a setting in which firms offer asymmetric product portfolios. While we do not expect the results of the analysis to differ from a qualitative point of view, the challenging feature of such an asymmetric model is that, based on the scant available literature on multi-product firms, it would be complicated to establish existence and uniqueness of the equilibrium with and without the merger.

[46] Moreover, the demand system that we consider in (13) has an aggregative formulation also if goods are not symmetric.

$\tilde{R}^m < \tilde{r}_i(A) + \tilde{r}_k(A)$ and the merger reduces the aggregate as it was the case with symmetric products (i.e., $A^b < A^m$).

What are the implications for consumer surplus? As argued after Proposition 4, a sufficient condition for the merger to reduce consumer welfare is that the demand satisfies the IIA property (or, equivalently, that the consumer surplus only depends on the aggregate, not on its composition). Thus, with and without symmetry, the same condition is sufficient to show that the merger is anticompetitive.

As far as total industry investment is concerned, the analysis in Proposition 5 proves the fall in investments under the condition that total demand depends on the aggregate. Symmetry allows us to streamline the comparison between the value of investments before and after the merger, and then show that investments fall for any (weakly) concave investment function χ.

When allowing the firm to treat each product differently, the comparison is complicated by the fact that outsiders' rise in investments might more than compensate the fall in insiders', even if total demand falls. However, we obtain the same result as in the proposition if the investment function is linear in x, e.g., $q = \zeta x$, with $\zeta > 0$ – which holds true whenever $-F'(x_i)/c'(x_i) = \zeta x_i$ (see the proof of Lemma 1 for details). Then, $\zeta \sum_i x_i = \sum_i q_i$ and a merger that reduces industry quantity decreases industry investments, too.

Finally, in the model with efficiency gains (Section 2.3.7), we use symmetry to simplify the derivation of the investment function $x_i = \chi(q_i|\lambda)$. However, it is possible to operate our transformation and solve the "price-only" model (and the aggregative version of it) even with asymmetric demand. In that case, the investment function is obtained solving the FOCs for x_i and x_k as function of q_i and q_k. Hence, symmetry is not necessary to derive the results, although admittedly it greatly simplifies the analysis.

3 Extensions

In this section, we study two extensions of our main model. First, we consider a sequential first-investments-then-price game. Second, we consider quality-enhancing investments rather than cost-reducing investments.

3.1 Sequential choices

The game where firms simultaneously choose both cost-reducing investments and prices can be interpreted as one where investments cannot be observed by rivals when firms take pricing decisions. In this section, we look at sequential moves, i.e. the case where firms know all investments made at the time they set prices.

As in the previous subsection, we begin by looking at the benchmark (status-quo) case with n symmetric independent firms, and then at the merger between two out of these n firms. Unfortunately though, in the sequential moves case we cannot rely on either the transformation from a bi-dimensional into a one-dimensional variable maximization problem or the aggregate game formulation, so we shall limit ourselves to compare the FOCs at the benchmark and at the merger, and resort to parametric models to gain some insight on the net effects of the merger.

3.1.1 Benchmark with independent firms

If firms act independently, in the second stage each firm i solves the following problem:

$$\max_{p_i} \; \tilde{\pi}_i(p_i, \bar{p}_{-i}, x_i) = (p_i - c(x_i))q_i(p_i, \bar{p}_{-i}) - F(x_i), \;\; i = 1, ..., n.$$

The FOCs are:

$$\partial_{p_i} \tilde{\pi}_i(\cdot) = q_i(p_i, \bar{p}_{-i}) + \partial_{p_i} q_i(p_i, \bar{p}_{-i})(p_i - c(x_i)) = 0. \tag{26}$$

Assume the system of n FOCs is uniquely solved by the vector of equilibrium prices, $\bar{p}^b(\bar{x}) = \left(p_1^b(\bar{x}), p_2^b(\bar{x}), ..., p_n^b(\bar{x})\right)$, which is function of the vector $\bar{x} = (x_1, x_2, ..., x_n)$ of the n cost-reducing investments.

In the first stage, firms maximize $\tilde{\pi}_i\left(\bar{p}^b(\bar{x}), x_i\right)$ with respect to x_i, which, invoking the envelope theorem, yields the following system of FOCs:

$$\partial_{x_i} \tilde{\pi}_i(\cdot) = -c'(x_i)q_i(\bar{p}^b(\bar{x})) - F'(x_i) + (n-1)\frac{dp_j^b}{dx_i}\partial_{p_j} q_i(\bar{p}^b(\bar{x}))(p_i^b(\bar{x}) - c(x_i)) = 0, \tag{27}$$

for all $i = 1, ..., n$ and $j \neq i$. The conditions in (27) define the equilibrium level of investment in the sequential choice game with independent firms.[47] The difference between (27) and (2) is that, with sequential moves, each firm i takes into account that raising its investment reduces the prices set by its rivals, and this will impact negatively own profits since it makes price competition more fierce. This effect is reflected by the last term in (27). As a consequence, the equilibrium investment values in $\bar{x}^b(\bar{p})$, as set by each firm i solving conditions (27) in the benchmark model with sequential moves, will be lower than in the simultaneous moves case, *ceteris paribus*.

[47] For the stability and the uniqueness of the equilibrium at the investment stage in the benchmark and with the merger, we invoke the conditions derived in Kolstad and Mathiesen (1987). We will check that they are satisfied within the parametric models that we refer to below.

3.1.2 Merger between firm i and firm k

In the second stage, the two merging firms i and k solve:

$$\max_{p_i,p_k} \tilde{\pi}_{i,k} = (p_i - c(x_i))q_i(p_i, \bar{p}_{-i}) + (p_k - c(x_k))q_k(p_k, \bar{p}_{-k}) - F(x_i) - F(x_k) + \lambda G(x_i, x_k),$$

with $i \neq k$. The FOC with respect to p_i (we omit that for p_k which is symmetric) is:

$$\partial_{p_i}\tilde{\pi}_{i,k} = q_i(p_i, \bar{p}_{-i}) + \partial_{p_i}q_i(p_i, \bar{p}_{-i})(p_i - c(x_i)) + \partial_{p_i}q_k(p_k, \bar{p}_{-k})(p_k - c(x_k)) = 0. \quad (28)$$

These FOCs are the same as in the simultaneous moves case: thus, since $\partial_{p_i}q_k(p_k, \bar{p}_{-k}) \geq 0$ and $p_k > c(x_k)$ at the equilibrium, and for given investments, the merger increases the price set by each insider with respect to the benchmark.

As for the outsiders, their FOCs will be the same as in the benchmark.

Let us call $\bar{p}^m(\bar{x})$ the vector of the prices which solves the system of the n FOCs above, and note that it will be composed of the two insiders' prices, $p_i^m(\bar{x}) = p_k^m(\bar{x})$ and $(n-2)$ symmetric outsiders' prices $p_j^m(\bar{x})$ with $j \neq i, k$.

In the first stage, insiders maximize joint profits $\tilde{\pi}_{i,k} = \tilde{\pi}_i(\bar{p}^m(\bar{x}), x_i) + \tilde{\pi}_k(\bar{p}^m(\bar{x}), x_k)$ with respect to x_i and x_k. Using the envelope theorem, the associated FOCs are as follows:

$$\begin{aligned} \partial_{x_i}\tilde{\pi}_{i,k} &= -c'(x_i)q_i(\bar{p}^m(\bar{x})) - F'(x_i) + \lambda\partial_{x_i}G(x_i, x_k) \\ &+ (n-2)\frac{dp_j}{dx_i}\left[\partial_{p_j}q_i(\bar{p}^m(\bar{x}))(p_i^m(\bar{x}) - c(x_i)) + \partial_{p_j}q_k(\bar{p}^m(\bar{x}))(p_k^m(\bar{x}) - c(x_k))\right] = 0 \quad (29) \end{aligned}$$

for all $j \neq i, k$.[48]

As for the outsiders, their FOCs will have the same terms as those in the benchmark, except that different prices are anticipated as solution of the last stage of the game:

$$\partial_{x_j}\tilde{\pi}_j(\cdot) = -c'(x_j)q_j(\bar{p}^m(\bar{x})) - F'(x_j) + \sum_{l \neq j}\frac{dp_l^m}{dx_j}\partial_{p_l}q_j(\bar{p}^m(\bar{x}))(p_j^m(\bar{x}) - c(x_j)) = 0. \quad (30)$$

Let us now compare the FOCs with respect to investments in the merger, (29) and (30), with those of the benchmark, (27). As for the insiders', there are three different effects at work. Two are of the same nature as in the simultaneous case: (i) since the insiders anticipate that they will sell lower quantities than at the benchmark ($q_i(\bar{p}^m(\bar{x})) < q_i(\bar{p}^b(\bar{x}))$ because insiders' prices increase more than outsiders'), this will reduce the marginal revenue from investment, consisting of the term $-c'(x_i)q_i(\bar{p}^m(\bar{x}))$. This will tend to lower investments by insiders. (ii)

[48] The FOC of an outsider is isomorphic to the one of a firm in the benchmark (27), and therefore not reported.

To the extent that efficiency gains exist ($\lambda > 0$), the term $\lambda \partial_{x_i} G(x_i, x_k)$ will decrease marginal costs from investing, and hence will tend to increase insiders' investments.

However, (*iii*) a new effect of the merger exists, and can be seen by comparing the last "strategic" term on the left-hand side (LHS) of expressions (27) and (29). In the benchmark configuration of the sequential game, each firm takes into account that investing an additional dollar will lower its own costs and prices, but also (by strategic complementarity) the prices of all the $(n-1)$ rivals - and this will impact negatively on own profits. A merged entity, though, will take into account that an additional dollar invested in product i will lower the prices of the $(n-2)$ outsiders, and that this will impact negatively on *both* the profits from product i and from product k, and therefore tends to further reduce the incentive to invest by the merged firms with respect to the benchmark.[49]

As for the outsiders' FOCs, there will be a similar effect as in the simultaneous game: since outsiders anticipate they will sell more than in the benchmark at the last stage of the game, they will have a higher marginal revenue from investing, and this will raise their investments at the merger equilibrium.

Beyond these considerations on the different effects at work, it is difficult to establish general results on the net effect that a merger may have - even in the case where no efficiency gains exist - due to the impossibility of resorting to an aggregative game formulation, which is not available for sequential games.[50] In order to get some insights into the effects of a merger in a sequential game, we therefore turn to some specific functional form oligopoly models.

3.1.3 The merger in parametric models with sequential choices

We have studied the sequential game for two different standard differentiated product models, the non-address model characterized by the Shubik-Levitan demand function, and the (address) Salop circle model, under the same assumptions made in the simultaneous model. To illustrate the results we report the solutions for particular parameter values in Table 1.

The results for the sequential model are of the same qualitative nature as for the simultaneous model. The merger always raises the insiders' prices, and it may increase or decrease the outsiders' prices; it always lowers insiders' investments and increases outsiders', but with

[49] The comparison between the two last terms in the FOCs (27) and (29), however, is somehow limited by two other elements: (a) there are $(n-1)$ terms to be summed up in (27), but only $(n-2)$ in (29); and (b) the marginal effect on own profits caused by a reduction in the rivals' prices may differ because the marginal profits are evaluated at two different price equilibria.

[50] Interestingly, in the sequential case even dealing with the merger to monopoly is not straightforward. Consider for simplicity the case $\lambda = 0$. When $n = 2$, the strategic term $\partial_{p_j} q_i(\bar{p}^b(\bar{x}))(p_i^b(\bar{x}) - c(x_i))dp_j^b/dx_i$ disappears from the investment FOCs of the merged entity: while in the benchmark a firm will tend to reduce its investment because it anticipates that this will make competition in the product market fiercer, the insiders' will internalize this effect, so this effect would tend to increase investment relative to the benchmark: a priori, we cannot establish whether this effect may or may not outweigh the effect (see (i) above) due to the lower sales expected in the monopoly equilibrium.

Table 1: Equilibrium outcomes with sequential moves

	Shubik-Levitan	**Salop**	
	$a = 2, \gamma = 0.3$	$t = 0.9$	$t = 1.8$
p^b	0.93	0.97	1.33
p_I^m	1.08	1.17	1.78
p_O^m	0.92	1.02	1.53
q^b	0.67	0.33	0.33
q_I^m	0.53	0.23	0.26
q_I^m	0.76	0.54	0.47
x^b	0.65	0.27	0.27
x_I^m	0.51	0.15	0.18
x_O^m	0.73	0.36	0.31
π^b	0.18	0.04	0.16
$\pi_I^m + \pi_I^m$	0.37	0.12	0.47
π_O^m	0.23	0.14	0.35
$CS^m - CS^b$	-0.18	-0.14	-0.35
$W^m - W^b$	-0.11	-0.002	-0.02

Note: we denote an insider firm by I and an outsider by O. Moreover, CS denotes consumer surplus, and W denotes total surplus. The Shubik-Levitan demand function is defined in (15). For the Salop location model, we assume a linear transportation cost t and a circle of unit length. The number of firms is three.

the former effect dominating so that at equilibrium total investments are always lower than at the benchmark; profits of insiders and outsiders alike increase with the merger; consumers are always harmed by the merger, but total surplus may increase with the merger in particular circumstances. To be more precise, it never increases in the Shubik-Levitan model, but in the Salop model it rises for $t \in [0.6, 0.746)$ - namely, for values where substitutability among the goods is very high - and decreases for $t \geq 0.746$ (we impose $t \geq 0.6$ to ensure profitability of the merger). Recall, though, that the Salop model is very special because aggregate demand is completely inelastic, and the higher prices caused by the merger do not entail any deadweight loss.

3.2 Quality-increasing investments

In this section, we discuss the implications of a model in which the investments carried out by firms increase the quality of their good, rather than decreasing their cost of production. Specifically, we let the quantity of a firm depend on its own and rivals' prices (p) and quality (x) level: $q_i = q_i(p_i, \bar{p}_{-i}, x_i, \bar{x}_{-i})$, with $\partial_{x_i} q_i \geq 0$ and $\partial_{x_i} q_k \leq 0$ (that is, an increase in the quality of firm i implies that q_i rises and q_k reduces, with $i \neq k$, as standard in models of quality differentiation). Consider further the case where the price- and quality-setting stages take place simultaneously, the investment in quality does not generate any efficiency gains and each firm bears a marginal cost of production equal to c.

If firms act independently, each solves the following maximization problem:

$$\max_{p_i,x_i} \hat{\pi}_i(p_i,\bar{p}_{-i},x_i,\bar{x}_{-i}) = q_i(p_i,\bar{p}_{-i},x_i,\bar{x}_{-i})(p_i - c) - F(x_i),\ i = 1,...,n.$$

The associated FOCs are:

$$\partial_{p_i}\hat{\pi}_i = q_i(p_i,\bar{p}_{-i},x_i,\bar{x}_{-i}) + \partial_{p_i}q_i(p_i,\bar{p}_{-i},x_i,\bar{x}_{-i})(p_i - c) = 0, \quad (31)$$

$$\partial_{x_i}\hat{\pi}_i = \partial_{x_i}q_i(p_i,\bar{p}_{-i},x_i,\bar{x}_{-i})(p_i - c) - F'(x_i) = 0. \quad (32)$$

If firms i and k merge, they maximize $\hat{\pi}_{i,k} \equiv \hat{\pi}_i(p_i,\bar{p}_{-i},x_i,\bar{x}_{-i}) + \hat{\pi}_k(p_k,\bar{p}_{-k},x_k,\bar{x}_{-k})$ with respect to p_i, p_k, x_i and x_k, with $i,k = 1,...,n$, and $i \neq k$. The FOCs with respect to p_i and x_i follow (we omit those for p_k and x_k, which are symmetric, and those of the outsiders, which are the same as in the benchmark):

$$\partial_{p_i}\hat{\pi}_{i,k} = q_i(p_i,\bar{p}_{-i},x_i,\bar{x}_{-i}) + \partial_{p_i}q_i(p_i,\bar{p}_{-i},x_i,\bar{x}_{-i})(p_i - c) + \partial_{p_i}q_k(p_k,\bar{p}_{-k},x_k,\bar{x}_{-k})(p_k - c) = 0, \quad (33)$$

$$\partial_{x_i}\hat{\pi}_{i,k} = \partial_{x_i}q_i(p_i,\bar{p}_{-i},x_i,\bar{x}_{-i})(p_i - c) + \partial_{x_i}q_k(p_k,\bar{p}_{-k},x_k,\bar{x}_{-k})(p_k - c) - F'(x_i) = 0. \quad (34)$$

When investments increase a firm's quality, the impact of the merger is a priori ambiguous. Compare first the insiders' FOCs with respect to prices, (31) and (33): under the merger there is the usual internalization of price effects which - *ceteris paribus* - leads the merged entity to raise prices. This will increase the marginal revenue from investing and will therefore tend to increase the insiders' incentive to invest. On the other hand, by comparing the investment FOCs of the insiders, (32) and (34), it turns out that the merged entity will have a lower incentive to invest: this is because it internalizes the fact that any extra dollar of investments on product i will lower the demand of product k, which also belongs to the same firm. It is therefore difficult to say whether insiders' investments will increase or decrease with the merger.

As for the outsiders, their FOCs are not affected by the merger, but because of strategic complementarity, outsiders' prices would tend to rise, which in turn raises their incentives to invest. But of course the change in the insiders' investments will also affect their FOCs, so it is difficult to reach an unambiguous conclusion about the effect of the merger on the outsiders too.

3.2.1 Cost-reducing and quality-increasing: indifference results

Given the ambiguous conclusions of the general model with quality-increasing investment, we show two formulations of the demand model under which the equilibrium results of the game with cost-reducing investments are equivalent to those with quality-increasing investments.

Quality-adjusted model In our main model, the utility of the representative consumer takes the following form $U(q_1, ..., q_n)$. Assume now that the consumer's utility depends on $x_i q_i$. That is, $U(x_1 q_1, ..., x_n q_n)$. In this alternative model, the solution of the utility maximization problem leads to a demand system as in $x_i q_i = D_i(z_i, \bar{z}_{-i})$, with $z_i = p_i/x_i$ and $i = 1, .., n$.

In this context, the gross profits of a firm i is equivalent to the gross profits in our baseline model when considering a quality adjusted value of marginal costs of production (c/x_i):

$$\begin{aligned} \hat{\pi}_i + F(x_i) &= (p_i - c_i)q_i \\ &= (z_i - c/x_i)D_i(z_i, \bar{z}_{-i}). \end{aligned}$$

This equivalence means that all the conclusions derived in the model with cost-reducing investment extend to this model with quality-adjusted prices and investments.[51] It also extends to any model where demand can be written as a function of the price-investment ratio, as is the case, for instance, of the vertical product differentiation version of the CES $(q_i = (p_i/x_i)^{(r-1)} / \sum_{j=1}^{n} (p_j/x_j)^r)$.

Hedonic price transformation We now show that there exists another class of demand models for which we can establish an equivalence between the results with cost-reducing and quality-increasing investments.

Assume that $q_i(p_i, \bar{p}_{-i}, x_i, \overline{x}_{-i})$ can be written as $q_i(p_i - \upsilon(x_i), \bar{p}_{-i} - \upsilon(\overline{x}_{-i}))$, where q_i is decreasing in $(p_i - \upsilon(x_i))$ and increasing in the elements of the vector $\bar{p}_{-i} - \upsilon(\overline{x}_{-i})$, and where $\upsilon(x_j)$ increases in the investment x_j for all j. This describes a model with quality-enhancing investments (the higher x_i the higher the perceived quality, and hence the demand for product i) where each firm has profit

$$\pi_i(p_i, \bar{p}_{-i}, x_i, \overline{x}_{-i}) = (p_i - c)q_i(p_i - \upsilon(x_i), \bar{p}_{-i} - \upsilon(\overline{x}_{-i})) - F(x_i).$$

If one defines $h_i \equiv p_i - \upsilon(x_i)$ as the hedonic price of the quality determined by x_i, then the

[51] A model which presents this feature is the one used by Sutton (1998:58ff) and later used by, e.g., Symeonidis (2003).

profit function above is equivalent to:

$$\pi_i(h_i, \overline{h}_{-i}, x_i, \overline{x}_{-i}) = (h_i - (c - \upsilon(x_i)))q_i(h_i, \overline{h}_{-i}) - F(x_i),$$

which is nothing else than the profit function of a firm i whose investment reduces its marginal cost according to $c(x_i) = c - \upsilon(x_i)$. This is like in the cost-reducing model we dealt with in the previous section.

For instance, let us study a quality-enhancing version of the model by using a logit demand function. In its vertical product differentiation version, this demand can be written as:

$$q_i = \frac{\exp\{(s + \upsilon(x_i) - p_i)\}}{\exp\{(s_0 - p_0)\} + \sum_{j=1}^{n} \exp\{(s + \upsilon(x_j) - p_j)\}},$$

where 0 is the outside good, and each firm i has profit $\pi_i = (p_i - c)q_i(\cdot, \cdot) - F(x_i)$. By defining the hedonic price $h_i = p_i - \upsilon(x_i)$, the profit can be rewritten as: $\pi_i = (h_i - c + \upsilon(x_i))(\exp\{(s - h_i)\})/(\exp\{(s_0 - p_0)\} + \sum_{j=1}^{n} \exp\{(s - h_j)\}) - F(x_i)$, which is a version of the cost-reducing model we analysed above. Not only the logit demand falls within this class of functions, but also the Shubik-Levitan demand system (see Häckner, 2000) of the type $q_i = (\alpha_i - p_i)\beta - \gamma \sum_j (\alpha_j - p_j)$, with $\alpha_i = \alpha + \alpha(x_i)$ measuring quality, $\beta > 0$ and $\gamma \in (0, 1)$. Specifically, this would be equivalent to studying a model of cost-reducing investments $c_i = c - c(x_i)$ with a demand $q_i = (\alpha - p_i)\beta - \gamma \sum_j (\alpha - p_j)$.

In sum, some standard models where firms invest to enhance their quality can be reinterpreted as the cost-reducing models we have studied in the previous sections. Hence, the same conclusions would apply: a merger is anti-competitive unless it entails sufficient efficiency gains.[52]

4 Research Joint Ventures

When assessing a merger proposal, the merger has to be compared with the likely counterfactuals. The status quo (what we call benchmark) is an obvious counterfactual, but in the case of efficiency gains from investment, another natural counterfactual candidate is a situation where some firms agree upon investment decisions, while continuing to behave independently in the product market. Examples of cooperative agreements at the investment stage include Research Joint Ventures (RJVs), where firms set joint R&D programs but then independently market their innovation, and Network Sharing Agreements (NSAs), where Mobile Network Operators

[52]Perhaps the most popular model of vertical product differentiation is the Shaked and Sutton model. In the previous version of the paper we showed that even in that model (in which firms set sequentially qualities and then prices) the merger decreases investments and consumer surplus.

(MNOs) share infrastructure (such as sites, antennas, and other equipment) and/or spectrum, but compete in the retail markets where they sell mobile services independently.

In this section, we consider this type of agreements. We assume that the members of a RJV, firms i and k, choose investment levels x_i and x_k to maximize joint profits, but they choose prices to maximize individual profits. We consider the case of simultaneous investments and price decisions.

Since we assume that RJV members i and k maximize joint profits when setting investment, and behave non-cooperatively when setting the price,[53] the FOCs of a RJV-member firm i are (we omit those for k, which are symmetric, and those for the outsiders, which are unchanged):

$$\partial_{p_i}\tilde{\pi}_{i,k} = q_i(p_i, \bar{p}_{-i}) + \partial_{p_i} q_i(p_i, \bar{p}_{-i})(p_i - c(x_i)) = 0, \tag{35}$$

$$\partial_{x_i}\tilde{\pi}_{i,k} = -\partial_{x_i} c(x_i) q_i(p_i, \bar{p}_{-i}) - \partial_{x_i} F(x_i) + \frac{\lambda}{2}\partial_{x_i} G(x_i, x_k) = 0. \tag{36}$$

The FOC for the price is as in the benchmark (no merger), while the FOC for the investment of an insider firm i is the same as in the merger. As we will show, this implies that, in the simultaneous moves case, the RJV will (weakly) dominate the benchmark in terms of consumer welfare, since it allows the members to the agreement to benefit from efficiency gains.

4.1 Comparing the RJV with the benchmark

We now provide a formal comparison between the RJV and the benchmark equilibrium outcomes.

Let us begin with the case with $n = 2$ - we make use of the transformation of the model in one-action only. Since investments are taken cooperatively by i and k, we can write the maximization problem of a RJV member as $\max_{p_i} \pi_i(p_i, p_k|\lambda)$, subject to $x_i = \chi(q_i|\lambda)$, $q_i = q_i(p_i, p_k)$, and

$$\pi_i(p_i, p_k|\lambda) = (p_i - c(\chi(q_i|\lambda)))q_i - F(\chi(q_i|\lambda)) + \frac{\lambda}{2} G(\chi(q_i|\lambda), \chi(q_k|\lambda)). \tag{37}$$

In (37), we assume that RJV firms, being symmetric, equally share the efficiencies generated by the deal. When comparing the equilibrium outcomes in the benchmark and in the RJV, we then find the following.

[53] While this appears natural in a sequential move game, it may appear less so in a simultaneous move game. However, RJVs are often structured in such a way that investment decisions are fully delegated to a separate joint venture whose managers are to behave independently from the managers of the parent companies. Independence between investment decisions and price decisions is also often a requirement of competition agencies to authorise the RJV. Note that if the RJV were to maximize joint profits with respect to both investments and prices, it would simply be identical to a merger.

Proposition 7. *For $n = 2$, the RJV (weakly) lowers prices and (weakly) increases investments with respect to the benchmark for any value of $\lambda \geq 0$. This holds strictly for $\lambda > 0$. For $n \geq 3$, the RJV (weakly) raises the aggregate with respect to the benchmark for any value of $\lambda \geq 0$.*

The intuition behind this result comes from the fact that the RJV does not distort price choices while at the same time allowing its members to benefit from cost savings in the investment function.

For the case of $n \geq 3$ firms, we rely on the aggregative formulation and obtain an analogous result: the RJV raises the aggregate with respect to the benchmark. It then increases consumer surplus and investments under the sufficient conditions given in Section 2.3.

4.2 Comparing the RJV with the merger

What is more difficult to prove is that the RJV is superior to the merger from the point of view of consumers: one cannot exclude a priori that responses by outsiders may more than offset the effects on the insiders.[54]

We nevertheless can make use of the result in Propositions 2 (for $n = 2$) and Lemma 11 (for $n \geq 3$) that it exists a threshold value of efficiency gains $\tilde{\lambda}$ such that, compared to the benchmark, the merger lowers welfare for all $\lambda < \tilde{\lambda}$.[55] Since the RJV clearly dominates the benchmark for any value of λ, we then know that the RJV improves welfare with respect to the merger for all $\lambda < \tilde{\lambda}$. For $\lambda \geq \tilde{\lambda}$, both the merger and the RJV increase consumer welfare when compared to the benchmark. The following corollary recapitulates:

Proposition 8. *The RJV raises consumer welfare with respect to the merger for all $\lambda < \tilde{\lambda}$. If $\lambda \geq \tilde{\lambda}$, both the merger and the RJV increase consumer welfare when compared to the benchmark.*

4.3 Parametric analysis of the RJV

We now analyse the RJV within two specific functional forms models (the Shubik-Levitan and Salop model we have seen above), and the comparison among the merger, benchmark and RJV equilibrium outcomes suggest that the RJV is (weakly) better for consumers than both the merger and the benchmark.

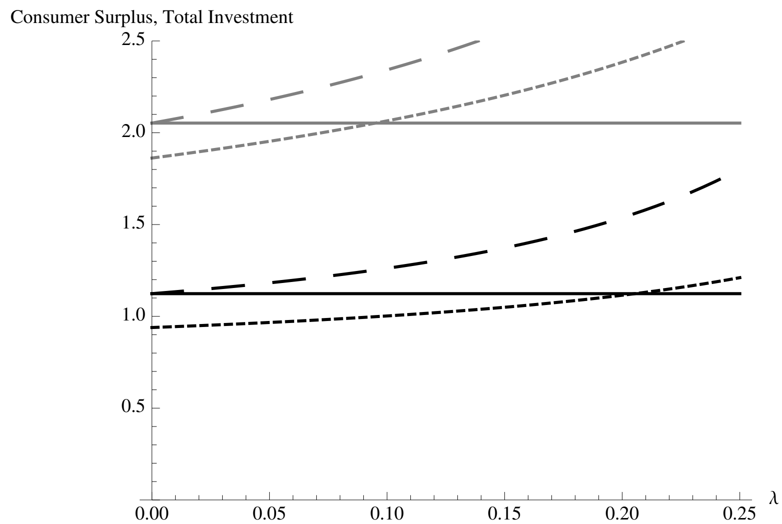
Figure 1, obtained for the Shubik-Levitan model, illustrates that for whatever level of efficiency gains a RJV performs (weakly) better than both the merger and the benchmark: total

[54] For instance, when comparing the merger and the RJV equilibria, the merger leads to higher prices by the insiders and (by strategic complementarity) by the outsiders, a decrease in the quantities sold by the former and an increase for the latter: in principle, it may be that the investments by the outsiders raise more than the decrease in investments by the insiders.

[55] More specifically, when $n \geq 3$, $\lambda \geq \tilde{\lambda}$ directly implies a larger surplus only when the IIA property holds.

investments and consumer surplus are always strictly higher under the RJV (the same would hold for total surplus) for any $\lambda > 0$, and they coincide with the benchmark equilibria for $\lambda = 0$. In line with the analysis of the merger in Section 2.2, there exists a level λ of efficiency gains at which the merger leads to higher investments than the benchmark (but never than the RJV) but that a still higher level is necessary to offset the increase in prices: $CS^m = CS^b$ at a higher level of λ.[56]

Figure 1: Merger and RJV with efficiency gains and simultaneous moves



Note: the black lines refer to consumer surplus and the grey lines to total investments. Moreover, the solid lines correspond to the benchmark, the dotted lines to the merger and the dashed lines to the RJV. The parametric values we use are $n = 3$, $\alpha = 2$, $c = 1$ and $\gamma = 0.3$. The range for λ is chosen so that all parametric restrictions are satisfied.

5 Summary, and some policy implications

We study a model with simultaneous cost-reducing investments and price choices and found that - absent efficiency gains - the merger is anti-competitive: it lowers both investment and consumer surplus. We have also showed that the same results arise in several classes of models where firms invest to enhance the quality of their products, rather than to reduce their costs. This does not mean, of course, that a merger will always be anti-competitive in practice. Indeed, it is possible that by combining their assets two firms will be able to reduce the costs of their investment: if these efficiency gains were large enough, they might increase investments so much as to outweigh the usual detrimental effect of the merger on prices.

However, a remark is in order. We have showed that to the extent that the same efficiency gains can be achieved by a R&D cooperative agreement, such an agreement is likely to be superior to the merger from the welfare point of view. This implies that the merging parties should

[56] We obtain similar results with the Salop model (see the previous version of the paper).

prove not only that the merger will lead to dynamic efficiencies, but also that such efficiencies are merger-specific (that is, they cannot be reached by a less anti-competitive agreement).

One could think of environments where mergers may lead to higher investments even absent efficiency gains, due to the fact that higher profits may relax financial constraints. It would be straightforward to write a model where insufficient profits would limit access to external funding, and consolidation would bring higher earnings that the merged entity could use to finance valuable projects that otherwise would not carry out. However, this positive effect would in general be in conflict with the mechanisms highlighted in this paper, and an analysis of the ensuing trade-off between ability and incentive to invest would be interesting and not obvious.

A Appendix

Proof of Proposition 1. Omitting functional notation for q_i and q_k, the FOC with respect to p_i is given by:[57]

$$\begin{aligned}\partial_{p_i}\pi_{i,k} &= (p_i - c(\chi(q_i)))\partial_{p_i}q_i + q_i - \frac{dc(\chi(q_i))}{dp_i}q_i - \frac{dF(\chi(q_i))}{dp_i} \\ &\quad + (p_k - c(\chi(q_k)))\partial_{p_i}q_k - \frac{dc(\chi(q_k))}{dp_i}q_k - \frac{dF(\chi(q_k))}{dp_i} = 0 \\ &= (p_i - c(\chi(q_i)))\partial_{p_i}q_i + q_i + (p_k - c(\chi(q_k)))\partial_{p_i}q_k = 0. \end{aligned} \tag{A-1}$$

The second equality follows from (11), (12) and $q = -F'/c'$, with $q = \{q_i, q_k\}$. After imposing symmetry, $p_i = p_k = p^m$, we find that the equilibrium price set by the new entity is implicitly defined by

$$p^m = c(\chi(q_i^m)) - \frac{q_i^m}{\partial_{p_i}q_i^m + \partial_{p_i}q_k^m} > c(\chi(q_i^m)), \tag{A-2}$$

where $q_i^m \equiv q_i(p^m, p^m)$. Existence and uniqueness of p^m are guaranteed by *(A0)*. That $p^m > c(\cdot)$ is a consequence of $\partial_{p_i}q_i^m + \partial_{p_i}q_k^m > 0$, which holds by symmetry and the assumption that own price effects are larger than cross price effects.

To show that the price level rises with the merger, $p^m > p^b$, we evaluate (A-1) at $p_i = p^b$. Let $q_i^b \equiv q_i(p^b, p^b)$, by (8), we have

$$(p^b - c(\chi(q_i^b)))\partial_{p_i}q_i^b + q_i^b = 0,$$

so the first two terms in (A-1) equal zero, but the remaining term $(p^b - c(\chi(q_k^b)))\partial_{p_i}q_k^b > 0$ since $\partial_{p_i}q_k^b > 0$. As a result, $\partial_{p_i}\pi_{i,k} > 0$ at $p_i = p_k = p^b$ and the prices after the merger must increase to a price p^m above p^b in order to maximize profits. But since prices increase, the quantity of each brand sold by the merged entity must fall, $q_i^m < q_i^b$. All this yields also a fall in equilibrium investments in (11) and (12), because $\chi' \geq 0$. Q.E.D.

Proof of Lemma 3. First, we show that a similar transformation to the one operated in Lemma 1 applies to the merger problem in the presence of efficiency gains.

The investment FOC of firm i can be rewritten as:

$$\begin{aligned}\partial_{x_i}\tilde{\pi}_{i,k} &= -c'(x_i)q_i(p_i, p_k) = F'(x_i) - \lambda\partial_{x_i}G(x_i, x_k) \\ \iff &\quad -\frac{F'(x_i) - \lambda\partial_{x_i}G(x_i, x_k)}{c'(x_i)} = q_i(p_i, p_k). \end{aligned} \tag{A-3}$$

[57] The FOC with respect to p_k is analogous, thus omitted.

At the symmetric equilibrium, $x_i = x_k = x$. Then,

$$-\frac{F'(x) - \lambda \partial_{x_i} G(x,x)}{c'(x)} = q_i(p_i, p_k).$$

Let $-\left(F'(x) - \lambda \partial_{x_i} G(x,x)\right)/c'(x) \equiv \phi(x|\lambda)$. Since $F'(\cdot) - \lambda \partial_{x_i} G(\cdot,\cdot) \geq 0$, $c'(\cdot) < 0$, $F''(\cdot) - \lambda \partial^2_{x_i x_i} G(\cdot,\cdot) \geq 0$ and $c''(\cdot) \geq 0$, it follows that $\partial_x \phi(\cdot|\lambda) \geq 0$.[58] Hence, $\phi(\cdot|\lambda)$ is invertible and

$$x = \phi^{-1}(q_i(p_i,p_k)|\lambda) \equiv \chi(q_i(p_i,p_k)|\lambda), \tag{A-4}$$

with $\partial_q \chi(\cdot|\lambda) \geq 0$. Moreover, since $\partial_\lambda \phi(\cdot|\lambda) = \partial_{x_i} G(x,x)/c'(x) < 0$, then an increase in λ raises x_i, i.e., $\partial_\lambda \chi(\cdot|\lambda) > 0$.

Therefore, we can rewrite the merging firms' maximization problem as function of p_i and p_k only, in the presence of efficiency gains:

$$\begin{aligned}\max_{p_i,p_k} \pi_{i,k} &= (p_i - c(\chi(q_i(p_i,p_k)|\lambda)))q_i(p_i,p_k) + (p_k - c(\chi(q_k(p_i,p_k)|\lambda)))q_i(p_i,p_k) \\ &\quad - F(\chi(q_i(p_i,p_k)|\lambda)) - F(\chi(q_k(p_k,p_i)|\lambda)) + \lambda G(\chi(q_i(p_i,p_k)|\lambda), \chi(q_k(p_k,p_i)|\lambda)).\end{aligned}$$

Omitting functional notation for q_i and q_k, the FOC with respect to p_i is

$$\begin{aligned}\partial_{p_i}\pi_{i,k} &= (p_i - c(\chi(q_i|\lambda)))\partial_{p_i} q_i + q_i + (p_k - c(\chi(q_k|\lambda)))\partial_{p_i} q_k \\ &\quad - \frac{dc(\chi(q_i|\lambda))}{dp_i} q_i - \frac{dF(\chi(q_i|\lambda))}{dp_i} \\ &\quad - \frac{dc(\chi(q_k|\lambda))}{dp_i} q_k - \frac{dF(\chi(q_k|\lambda))}{dp_i} \\ &\quad + \lambda \frac{dG(\chi(q_i|\lambda), \chi(q_k|\lambda))}{dp_i} = 0.\end{aligned} \tag{A-5}$$

The analysis above implies that

$$\begin{aligned}\frac{dc(\chi(q_i|\lambda))}{dp_i} &= c'(\chi(q_i|\lambda))\partial_{q_i}\chi(q_i|\lambda)\partial_{p_i} q_i, \quad \frac{dc(\chi(q_k|\lambda))}{dp_i} = c'(\chi(q_k|\lambda))\partial_{q_k}\chi(q_k|\lambda)\partial_{p_i} q_k \\ \frac{dF(\chi(q_i|\lambda))}{dp_i} &= F'(\chi(q_i|\lambda))\partial_{q_i}\chi(q_i|\lambda)\partial_{p_i} q_i, \quad \frac{dF(\chi(q_k|\lambda))}{dp_i} = F'(\chi(q_k|\lambda))\partial_{q_k}\chi(q_k|\lambda)\partial_{p_i} q_k\end{aligned}$$

and

$$\begin{aligned}\frac{dG(\chi(q_i|\lambda), \chi(q_k|\lambda))}{dp_i} &= \partial_\chi G(\chi(q_i|\lambda), \chi(q_k|\lambda))\partial_{q_i}\chi(q_i|\lambda)\partial_{p_i} q_i \\ &\quad + \partial_\chi G(\chi(q_i|\lambda), \chi(q_k|\lambda))\partial_{q_k}\chi(q_k|\lambda)\partial_{p_i} q_k.\end{aligned}$$

[58] The calculations are analogous to those in the proof of Lemma 1.

Then, by the envelope theorem,

$$\frac{dc(\chi(q_i|\lambda))}{dp_i}q_i + \frac{dF(\chi(q_i|\lambda))}{dp_i} + \frac{dc(\chi(q_k|\lambda))}{dp_i}q_k + \frac{dF(\chi(q_k|\lambda))}{dp_i} + \lambda\frac{dG(\chi(q_i|\lambda), \chi(q_k|\lambda))}{dp_i} = 0.$$

As a consequence, the FOC in (A-5) reduces to

$$(p_i - c(\chi(q_i|\lambda)))\partial_{p_i}q_i + q_i + (p_k - c(\chi(q_k|\lambda)))\partial_{p_i}q_k = 0.$$

Under our assumptions, this FOC is sufficient for optimality; thus, in the unique symmetric equilibrium the merging firm sets

$$p^m(\lambda) = c(\chi(q_i^m|\lambda)) - \frac{q_i^m}{\partial_{p_i}q_i^m + \partial_{p_i}q_k^m}, \; x^m(\lambda) = \chi(q_i^m|\lambda). \tag{A-6}$$

Next, suppose there is a higher level of efficiency gains, $\lambda' > \lambda$. Since $\partial_\lambda c(\chi(q_i^m|\lambda)) = c'\partial_\lambda\chi \le 0$ and $\partial_\lambda\chi \ge 0$, it follows by the implicit function theorem that $p^m(\lambda') < p^m(\lambda)$ and $x^m(\lambda') > x^m(\lambda)$. Q.E.D.

Proof of Lemma 4. If $p^m(\lambda^b) = p^m$ and $x^m(\lambda^b) = x^b$ denote the price and investment levels which solve the merged entity's problem when $\lambda = \lambda^b$, the merging firm FOC with respect to investments in (A-3) can be written as:

$$\partial_{x_i}\pi_{i,k} = -\partial_{x_i}c(x^b)q_i(p^m, p^m) - \partial_{x_i}F(x^b) + \lambda\partial_{x_i}G(x^b, x^b) = 0.$$

We also know that (p_b, x_b) solve the FOC in the benchmark with $n = 2$ (3). Hence, it must be true that:

$$\partial_{x_i}\pi_i = -\partial_{x_i}c(x^b)q_i(p^b, p^b) - \partial_{x_i}F(x^b) = 0.$$

The last two equations imply:

$$\frac{\partial_{x_i}F(x^b) - \lambda\partial_{x_i}G(x^b, x^b)}{q_i(p^m, p^m)} = \frac{\partial_{x_i}F(x^b)}{q_i(p^b, p^b)}. \tag{A-7}$$

Since $\partial_{x_i}F(x^b) - \lambda\partial_{x_i}G(x^b, x^b) < \partial_{x_i}F(x^b)$, for the above equality to hold it must be $q_i(p^m, p^m) < q_i(p^b, p^b)$. In turn, this is consistent only with $p^m > p^b$. Q.E.D.

Proof of Lemma 5. Writing (17) as

$$\frac{d\pi_i(A_{-i} + r_i(A_{-i}), r_i(A_{-i}))}{da_i} = 0,$$

and taking its total derivative, yields

$$\frac{dr_i(A_{-i})}{dA_{-i}} \equiv r_i'(A_{-i}) = -\frac{\frac{d^2\pi_i(A_{-i}+a_i,a_i)}{da_i dA_{-i}}}{\frac{d^2\pi_i(A_{-i}+a_i,a_i)}{da_i^2}}.$$

The denominator of this expression is negative by Assumption (*A2*) while Assumption (*A3*) implies that the ratio is strictly larger than -1. The monotonicity of $A_{-i}+r_i(A_{-i})$ immediately follows. Q.E.D.

Proof of Lemma 6. The total derivative of $\tilde{r}_i$ results from the observation that, since $\tilde{r}_i(A) \equiv r_i(f_i(A))$,

$$\frac{d\tilde{r}_i(A)}{dA} = \frac{dr_i(A_{-i})}{dA_{-i}}\frac{df_i(A)}{dA} = r'(A_{-i})\frac{df_i(A)}{dA}.$$

Moreover, since $A_{-i} = f_i(A)$ and $A_{-i} + r_i(A_{-i}) = A$, applying the implicit function theorem to $A_{-i} = f_i(A_{-i} + r_i(A_{-i}))$ yields $df_i(A)/dA = 1/(1 + r_i')$. Hence,

$$\frac{d\tilde{r}_i(A)}{dA} = \tilde{r}_i' = \frac{r_i'}{1 + r_i'},$$

with $\tilde{r}_i \in (0, 1)$ for all $r_i' > 0$ (as it holds true with strategic complementarity). Instead, with strategic substitutability, $r_i' \in (-1, 0)$ implies that $\tilde{r}_i' < 0$ for all the values of A in which r_i takes positive values (i.e., for all $A < \overline{A}_{-i}$). Q.E.D.

Proof of Lemma 7. By the definition of $\overline{A}_{-i}$, we have that $\tilde{r}_i(A) = 0$ for all $A \geq \overline{A}_{-i}$. Hence, $\bar{\pi}_i(A) = 0$ in this interval of values of A. For $A < \overline{A}_{-i}$,

$$\frac{d\bar{\pi}_i(A)}{dA} = \frac{d\bar{\pi}_i(A, \tilde{r}_i(A))}{dA} = \partial_A \pi_i(A, \tilde{r}_i(A)) + \partial_{a_i}\pi_i(A, \tilde{r}_i(A))\frac{d\tilde{r}_i(A)}{dA}.$$

From (17),

$$\partial_A \pi_i(A, a_i) = -\partial_{a_i}\pi_i(A, a_i)$$

implies that

$$\partial_A \pi_i(A, \tilde{r}_i(A)) + \partial_{a_i}\pi_i(A, \tilde{r}_i(A))\frac{d\tilde{r}_i(A)}{dA} = \partial_A \pi_i(A, \tilde{r}_i(A))\left(1 - \frac{d\tilde{r}_i(A)}{dA}\right). \qquad \text{(A-8)}$$

Finally, (*A1*) together with Lemma 6 imply that (A-8) is negative, as (*A1*) implies that π_i falls with A for given a_i. Q.E.D.

Proof of Lemma 8. For the existence of the fixed point, we check that the intermediate value theorem assumptions are satisfied. Denote by $\mathcal{A}$ the interval of values of A:

1. The continuity of each $\tilde{r}_i$, $i = 1, ..., n$, implies that also their sum is continuous.
2. Since individual strategies spaces are compact, also $\mathcal{A}$ must be compact.
3. $\sum_{i=1}^{n} \tilde{r}_i(A)$ takes values in $\mathcal{A}$.

Thus, it always exists a fixed point of the problem in (18).

For the uniqueness, a sufficient condition requires that $\sum_{i=1}^{n} \tilde{r}'_i(A) < 1$, as in (19). Q.E.D.

Proof of Lemma 9. By (*A1*), $\pi_k(A, a_k)$ falls in A and the third terms in (21) and (22) are negative. It follows that $\tilde{r}_i^m(A) < \tilde{r}_i(A)$ for any value of a_k. Similarly, $\tilde{r}_k^m(A) < \tilde{r}_k(A)$ for all a_i.[59] Thus, $\tilde{R}^m(A) < \tilde{r}_i(A) + \tilde{r}_k(A)$. Q.E.D.

Proof of Proposition 5. First, due to firms' symmetry, we can define by q^b the equilibrium quantity set by the firms in the benchmark, and by q_I^m and q_O^m the equilibrium quantities set by the insiders (I) and outsiders (O) with the merger, respectively. Given the results in Lemma 9, the goal is to show the conditions under which $n\chi(q^b) \geq 2\chi(q_I^m) + (n-2)\chi(q_O^m)$.

If the industry quantity is proportional to the aggregate A, then $nq^b \geq 2q_I^m + (n-2)q_O^m$. While, by Lemma 9, it is clear that $q_I^m < q^b$,[60] two cases must be considered with respect to the change in the quantity of the outsiders:

1. If $q_O^m < q_b$, then $\chi' \geq 0$ implies that total and each firm's investment fall with the merger.
2. If $q^b \leq q_O^m$, then insiders' investment falls while outsiders' increases. In this case, a sufficient condition for total investments to decrease is that $\chi''(\cdot) \leq 0$. To prove this statement, we first denote $\bar{q} \equiv 2q_I^m/n + (n-2)q_O^m/n$. Then, Jensen's inequality implies that

   $$\chi(\bar{q}) \geq \frac{n-2}{n}\chi(q_I^m) + \frac{2}{n}\chi(q_O^m).$$

   If $q_b = \bar{q}$ and $\chi'' < 0$, the result immediately follows. It follows a fortiori if $q_b > \bar{q}$, as, in this case, $\chi'(\cdot) \geq 0$ implies that $\chi(q_b) > \chi(\bar{q})$.

   Finally, since Jensen's inequality holds only for concave functions, we need to prove that $\chi''(\cdot) \leq 0$ under our assumptions. By the properties of inverse functions, $\chi'' \leq 0$ if and only if $\phi'' \geq 0$, or

   $$\phi''(x_i) = \frac{\partial^2}{\partial x_i^2}\left(-\frac{F'(x_i)}{c'(x_i)}\right) \geq 0. \tag{A-9}$$

[59] Due to products' symmetry, both insiders must be active after the merger takes place.

[60] The fall in a_i and a_k means that the price of the merging parties rises with the merger.

Taking the derivative in (A-9), and omitting functional notation, we find that

$$\text{sign}\{\phi''\} = \text{sign}\left\{-\{(F'''c' - c'''F')(c')^2 - 2c'c''(F''c' - c''F')\}\right\}.$$

Thus, $\phi''(\cdot) \geq 0$ under our restrictions on the functional forms of $c(\cdot)$ and $F(\cdot)$ (specifically, these assumptions require that $c' < 0$, $c'' \geq 0$, $c''' \geq 0$, $F' \geq 0$, $F'' \geq 0$ and $F''' \geq 0$).

Q.E.D.

Proof of Lemma 10. The proof works in two steps. (1) We show that $dr_i(A|\lambda)/d\lambda > 0$ if and only if $d^2\pi_i(A, a_i|\lambda)/(da_i d\lambda) > 0$. (2) We show that, under our assumptions, $dr_i(A|\lambda)/d\lambda > 0$.

1. As observed in Lemma 6, $\tilde{r}_i(A|\lambda) = r_i(f(A|\lambda)|\lambda)$, with $A_{-i} = f_i(A)$ and $A_{-i} + r_i(A_{-i}|\lambda) = A$. Consequently,

$$\frac{d\tilde{r}_i(A|\lambda)}{d\lambda} = \frac{dr_i(A_{-i}|\lambda)}{dA_{-i}}\frac{df(A|\lambda)}{d\lambda} + \frac{dr_i(A_{-i}|\lambda)}{d\lambda}.$$

By the implicit function theorem, $df(A|\lambda)/d\lambda = -\partial_\lambda r_i/(1 + \partial_{A_{-i}} r_i)$; thus,

$$\frac{d\tilde{r}_i(A|\lambda)}{d\lambda} = \frac{\partial_\lambda r_i}{1 + \partial_{A_{-i}} r_i}. \tag{A-10}$$

The denominator of (A-10) is positive by Lemma 5, then the expression is positive if and only if $d^2\pi_i(A, a_i|\lambda)/(da_i d\lambda) > 0$, as $\text{sign}\{\partial_\lambda r_i\} = \text{sign}\{d^2(\pi_i(A, a_i|\lambda) + \pi_k(A, a_k|\lambda))/(da_i d\lambda)\}$.

2. We now prove that, in our aggregative formulation of the game, it holds that $d^2(\pi_i(A, a_i|\lambda) + \pi_k(A, a_k|\lambda))/(da_i d\lambda) > 0$. First take the derivative of $\pi_i(A, a_i|\lambda) + \pi_k(A, a_k|\lambda)$ with respect to a_i. Using the envelope theorem (note that $q_i = q_i(A, a_i)$, but in what follows we omit the functional notation), this expression can be written as

$$(\psi^{-1})' q_i + \left(\psi^{-1} - c(\chi(q_i|\lambda))\right)\frac{dq_i}{da_i} + \left(\psi^{-1} - c(\chi(q_k|\lambda))\right)\partial_{a_i} q_k.$$

In turn, the derivative of the last expression with respect to λ yields

$$-c'(\chi(q_i|\lambda))\partial_\lambda \chi(q_i|\lambda)\frac{dq_i}{da_i} - c'(\chi(q_k|\lambda))\partial_\lambda \chi(q_k|\lambda)\partial_{a_i} q_k.$$

Using symmetry, $q_i = q_k$, and the assumption that own action effects are larger than cross action effects, $dq_i/da_i + \partial_{a_i} q_k > 0$, we obtain that

$$-c'(\chi(q_i|\lambda))\partial_\lambda \chi(q_i|\lambda)\left(\frac{dq_i}{da_i} + \partial_{a_i} q_k\right) > 0.$$

Q.E.D.

Proof of Lemma 11. First note that A^b is independent of λ. Then, by Lemma 10, the FOC with respect to a_i is strictly increasing in λ. Finally, invoking the intermediate value theorem as in the proof of Lemma 8 yields the uniqueness of $\tilde{\lambda}$. Q.E.D.

Proof of Proposition 7. The proof develops in two parts, the first with $n = 2$ and the second with $n \geq 3$.

Let $n = 2$. After dropping the functional notation for q_i, the FOC of (37) with respect to p_i is

$$(p_i - c(\chi(q_i|\lambda)))\partial_{p_i} q_i + q_i + \frac{\lambda}{2}\frac{dG(\chi(q_i|\lambda), \chi(q_k|\lambda))}{dp_i} \\ -(c'(\chi(q_i|\lambda))q_i + F'(\chi(q_i|\lambda)))\chi'(q_i|\lambda)\partial_{p_i} q_i = 0, \quad \text{(A-11)}$$

where

$$\frac{dG(\chi(q_i|\lambda), \chi(q_k|\lambda))}{dp_i} = \partial_\chi G(\chi(q_i|\lambda), \chi(q_k|\lambda))\partial_{q_i}\chi(q_i|\lambda)\partial_{p_i} q_i \\ +\partial_\chi G(\chi(q_i|\lambda), \chi(q_k|\lambda))\partial_{q_k}\chi(q_k|\lambda)\partial_{p_i} q_k.$$

To compare the value of p^b with the price set by a firm in the RJV, we evaluate the LHS of (A-11) at $p = p^b$ and $x_i = \chi(q^b)$. The resulting expression is

$$(p_i - c(\chi(q^b)))\partial_{p_i} q^b + q^b + \frac{\lambda}{2}\frac{dG(\chi(q^b), \chi(q^b))}{dp_i} - (c'(\chi(q^b))q^b + F'(\chi(q^b)))\chi'(q^b)\partial_{p_i} q^b \\ = \frac{\lambda}{2}\frac{dG(\chi(q^b), \chi(q^b))}{dp_i} \leq 0 \quad \forall \lambda \geq 0,$$

where the equality holds by the envelope theorem, and the final inequality follows from

$$\frac{dG(\chi(q^b), \chi(q^b))}{dp_i} = \partial_\chi G(\chi(q^b|\lambda), \chi(q^b|\lambda))\partial_{q_i}\chi(q^b|\lambda)\left(\partial_{p_i} q^b + \partial_{p_k} q^b\right) < 0,$$

which holds true in a symmetric equilibrium by the property that own price effects dominate cross price effects.

This shows that the price of a firm in the RJV is lower than in the benchmark. Since, by symmetry, this is true for both firms, the final quantities produced under the RJV are larger, and so are the values of firms' investments.

Let $n \geq 3$. We then resort to the aggregative formulation of the game. After dropping the functional notation for q_i, the maximization problem of a RJV insider is

$$\max_{a_i} \, (\psi^{-1}(a_i) - c(\chi(q_i|\lambda)))q_i - F(\chi(q_i|\lambda)) + \frac{\lambda}{2} G(\chi(q_i|\lambda), \chi(q_k|\lambda)).$$

The ensuing FOC with respect to a_i is

$$(\psi^{-1})'(a_i)q_i + (\psi^{-1}(a_i) - c(\chi(q_i|\lambda)))\frac{dq_i}{da_i} \\ - (c'(\chi(q_i|\lambda))q_i + F'(\chi(q_i|\lambda)))\, \chi'(q_i|\lambda)\frac{dq_i}{da_i} + \frac{\lambda}{2}\frac{dG(\chi(q_i|\lambda), \chi(q_k|\lambda))}{da_i} = 0. \tag{A-12}$$

Evaluating (A-12) at a_i^b, and invoking symmetry, yields

$$\frac{\lambda}{2}\frac{dG(\chi(q^b|\lambda), \chi(q^b|\lambda))}{da_i} = \frac{\lambda}{2}\left(\frac{dq_i}{da_i} + \frac{dq_k}{da_i}\right) \partial_\chi G(\chi(q^b|\lambda), \chi(q^b|\lambda))\chi'(q^b|\lambda) \geq 0 \tag{A-13}$$

for all $\lambda \geq 0$, where q^b is the quantity of an insider firm at a^b and A^b.[61] Hence, RJV insiders will have an incentive to increase a_i relative to the benchmark equilibrium, which will also result in an increase in the aggregate A. Q.E.D.

[61]The FOC in the benchmark under the aggregative formulation is given by

$$(\psi^{-1})'(a_i)q_i + (\psi^{-1}(a_i) - c(\chi(q_i|\lambda)))\frac{dq_i}{da_i} = 0.$$

B Online Appendix

In this section, we first use the Shubik and Levitan model to illustrate the functioning of the aggregative formulation. We then resort to parametric models to get some further insights on the effects of the merger on some variables of interest.

B.1 Aggregative analysis with linear demand

In this section, we solve the linear demand example to illustrate the construction of the inclusive best response function and the derivation of the equilibrium value of the aggregate. We will do this for the benchmark, and provide a graphical illustration of how the merger changes the aggregate in equilibrium.

Consider the Shubik-Levitan linear demand function. Its aggregative formulation is given by

$$q_i(A, a_i) = \frac{a_i[1 + (n-1)\gamma] - \gamma(a_i + A_{-i})}{(1-\gamma)[1 + (n-1)\gamma]}.$$

Assume also that $c(x) = c - x$ and $F(x) = x^2/2$, so that $x_i = \chi(q_i(A, a_i)) = q_i(A, a_i)$.[62]

First, we find $r_i(A_{-i})$ by solving $d\pi(r(A_{-i}) + A_{-i}, r(A_{-i}))/da_i = 0$ for $r(A_{-i})$. Let us define $B \equiv [1 + (n-1)\gamma]$. We obtain the following expression:

$$r_i(A_{-i}) = \frac{\gamma^2 A_{-i}(1-B) + B(\alpha-1)(1-\gamma)(B-\gamma)}{2B\gamma^2 - \gamma^2 + B^2(1-2\gamma)}.$$

We then invert $A_{-i} + r_i(A_{-i}) = A$, to get $A_{-i} = f_i(A)$. Specifically,

$$f_i(A) = \frac{A\gamma - B[(\alpha-1)(1-\gamma) - A(1-2\gamma)]}{B(B - 2B\gamma + \gamma^2)}(B-\gamma).$$

Inserting this $f_i(A)$ into $r_i(A_{-i})$, we obtain the inclusive best reaction function: $\tilde{r}_i(A) \equiv r_i(f_i(A))$. In our symmetric Shubik-Levitan linear demand system, $\tilde{r}_i(A) = \tilde{r}(A)$ for all i:

$$\tilde{r}(A) = \frac{\gamma^2 A(1-B) + B(\alpha-1)(1-\gamma)(B-\gamma)}{B(B - 2B\gamma + \gamma^2)}. \tag{B-1}$$

To find the equilibrium value of the aggregate A in the benchmark, we then solve $n\tilde{r}_i(A) = A$ and find that

$$A = A^b \equiv \frac{(\alpha-1)B(B-\gamma)(1-\gamma)n}{B^2(1-2\gamma) - \gamma^2 n + B\gamma^2(1+n)}.$$

This A^b is unique by *(A2)*–*(A3)* and $n\tilde{r}_i'(A) < 1$. Specifically, in the benchmark,

$$\begin{aligned} (A2):&\quad B^2(\gamma - B)[B(1-2\gamma) + \gamma] < 0,\\ (A3):&\quad B(B - 2\gamma B + \gamma^2) > 0, \end{aligned}$$

[62]Recall that $a_i = \alpha - p_i$ and $A_{-i} = \sum_{j\neq i} a_j$.

and are both satisfied for all $\gamma \in (0, \bar{\gamma})$, with $\bar{\gamma} \equiv (n - 3 + \sqrt{n^2 + 2n - 3})/2(2n - 3)$.

With the merger, the insiders' inclusive reaction function is[63]

$$\tilde{r}^m(A) = \frac{B\gamma[2 - 2\alpha(1-\gamma) - (2+A)\gamma] + (\alpha - 1)B^2(1-\gamma)}{B(B - 2B\gamma + 2\gamma^2)}.$$

To find the equilibrium value of the aggregate, we then solve $2\tilde{r}^m(A) + (n-2)\tilde{r}(A) = A$, where $\tilde{r}(A)$, the inclusive best reaction function of the outsiders, is as in (B-1). The unique solution of this fixed point problem is

$$A = A^m \equiv \frac{(\alpha - 1)B(1-\gamma)\{2\gamma^3 n - B^2(1-2\gamma)n + B\gamma[2 + n - 2\gamma(1+2n)]\}}{B(1-2\gamma)[2\gamma^2 - 3B\gamma^2 - B^2(1-2\gamma)] - (B-1)\gamma^2(B - 2B\gamma + 2\gamma^2)n}.$$

With the merger, assumptions $(A2)$ and $(A3)$ hold true if

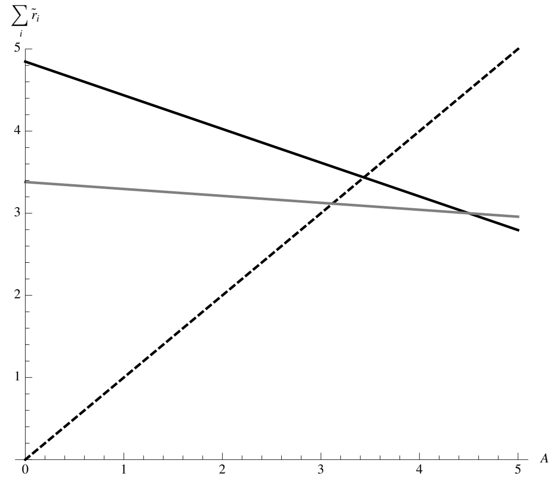
$$\begin{aligned} (A2)&: \quad (B - 2\gamma)[2(B-1)\gamma - B] < 0, \\ (A3)&: \quad \gamma^2(2 - 3B) - B^2(1-2\gamma) < 0. \end{aligned}$$

These conditions are again satisfied for all the positive values of γ below $\bar{\gamma}$. Moreover, confirming the result in Proposition 3, $A^m < A^b$ for all $\gamma \in (0, \bar{\gamma})$.

In line with these findings, Figure B-1 shows that the aggregate is smaller with the merger for $\gamma = 0.4$ and $n = 3$. While the reduction in the aggregate is a sufficient condition for consumer welfare to fall with demand functions like CES or logit, it is not with the Shubik-Levitan demand model. The reason is that the latter does not satisfy the IIA. Hence, in what follows, we look at how the merger changes consumer surplus with respect to the benchmark. Before going there, though, we provide a graphical illustration of the conditions under which assumptions $(A2)$ and $(A3)$ are satisfied in the model with linear demand, and the merger's profitability condition.

[63] We do not report the calculations for the derivation of $\tilde{r}^m$ because we followed the same procedure as in the benchmark.

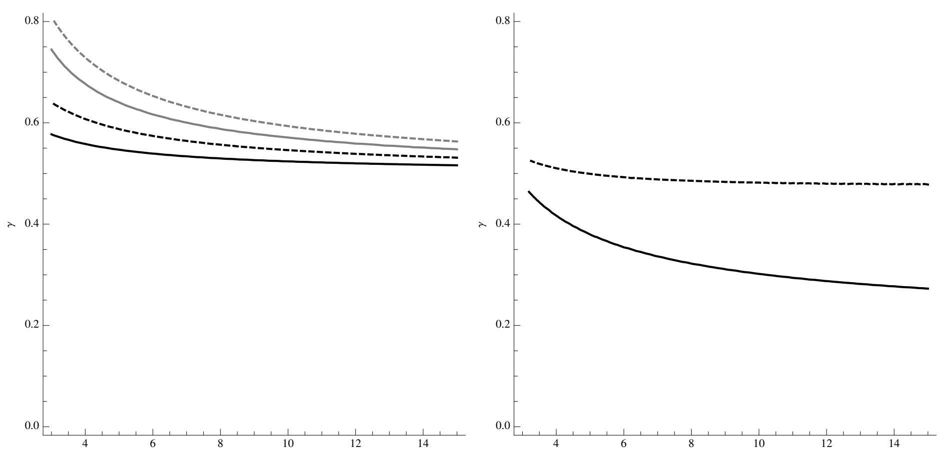
Figure B-1: Benchmark and merger – Aggregative analysis with linear demand



Note: the dashed line corresponds to the 45° line, the black line is the sum of inclusive reaction functions in the benchmark, the grey line is the sum of inclusive reaction functions with the merger. The parametric values we use are $\alpha = 2$, $c = 1$ and $\gamma = 0.4$. As we show in Figure B-2, right panel, the merger is profitable and anticompetitive when $n = 3$ and $\gamma = 0.4$ for any $\alpha > c$.

In Figure B-2, left panel, we illustrate two things: first, the condition implied by (*A3*) is more binding than the one coming from (*A2*) in both the benchmark and the merger. Second, the condition for (*A3*) in the benchmark implies the other three. In particular, the solid black line corresponds to the maximum values below this assumption is satisfied, $\bar{\gamma}$.

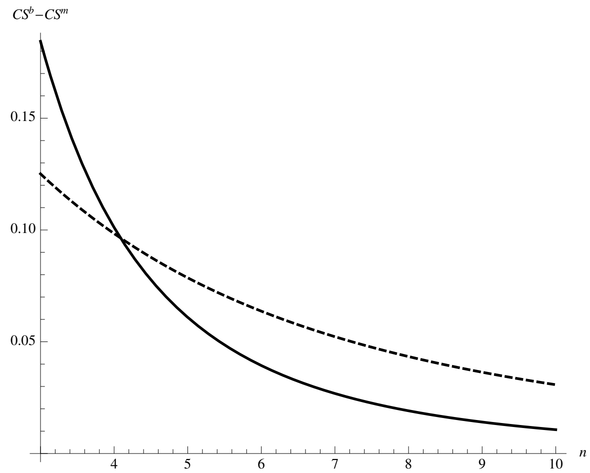
Figure B-2: Assumptions, consumer welfare, and profitability



Note: in the left panel, the solid black (respectively, grey) line corresponds to the maximum value of γ such that assumption (*A3*) holds in the benchmark (respectively, merger). The black (respectively, grey) dashed line gives the maximum value of γ such that assumption (*A2*) is satisfied in the benchmark (respectively, merger). In the right panel, the solid line gives the maximum value of γ below which the merger is profitable, while the dashed line gives the maximum value of γ below which the merger implies a fall in consumer surplus.

We now discuss the conditions for the merger to be profitable and its impact on welfare. In the right panel of Figure B-2, we plot two curves: the solid one gives the maximum values of γ such that the merger is profitable, the dashed ones those below which it reduces consumer welfare. This figure prompts two considerations. First, the profitability condition is more binding than all the parametric assumptions (plotted in the left panel). Second, it shows that the merger is anticompetitive whenever it is profitable.

Figure B-3: Consumer surplus difference



Note: the solid line plots the consumer surplus difference when $\gamma = 0.3$, the dashed line when $\gamma = 0.1$. The merger is profitable in both cases for all $n \leq 10$.

To conclude, we look at how the loss in consumer surplus caused by the merger ($CS^b - CS^m$) evolves with the number of firms, n. It can be showed that as n grows, consumer loss shrinks. (Figure B-3 illustrates this result for different values of γ.) This confirms in a setting where firms choose both investment and price what we know from standard merger theory, namely that the harm caused by a merger is - *ceteris paribus* - the more sizable the more concentrated the industry. If antitrust authorities could prohibit only mergers which create "*significant* lessening of competition" then they might limit their attention to mergers taking place in more concentrated markets.

B.2 Parametric analysis

We have seen above that, absent efficiency gains, the merger leads to lower consumer surplus for a class of models that we can write as aggregative games and satisfy the IIA property. However, some models which are commonly used in industrial organization do not belong to that class. Furthermore, dealing with closed-form solutions will also allow us to illustrate the impact of the merger on all variables, thereby gaining further insight on merger effects.

In this section, therefore, we report parametric results for the study of the merger effects for a model that does not satisfy the aggregative games properties, the Salop circle model, as well as for models which can be written as aggregative games - namely the CES, logit and Shubik-Levitan demand functions.

We restrict attention to $n = 3$ symmetric firms in the industry, the minimum number which allows us to analyse the effects of the merger on insiders and outsiders (by looking at more than three firms would complicate calculations without adding any additional insight). We assume that marginal costs of production are linear, $c(x_i) = 1 - x_i$, that fixed costs are quadratic, $F(x_i) = x_i^2/2$, and (for the moment) that efficiency gains are absent, $\lambda = 0$.[64] Note that given these assumptions, the FOCs with respect to investments, $\partial_{x_i} c(x_i) q_i(p_i, p_{-i}) - \partial_{x_i} F(x_i) = 0$

[64]See Section 4 below for a numerical computation of this model with efficiency gains, where we compare the merger with the benchmark and a Research Joint Venture.

simplify to $q_i(p_i, p_{-i}) = x_i$, entailing the equivalence between outputs and investment levels at all the equilibria.

Table B-1 illustrates the results with particular parameter values. While we could obtain analytical solutions for the Shubik-Levitan and the Salop models, we could not find closed-form solutions with the CES and logit demand functions in the merger case. Thus, we report results for representative values of the parameters.

Table B-1: Equilibrium outcomes with simultaneous moves

	Shubik-Levitan	**Salop**		**CES**		**Logit**	
	$a = 2, \gamma = 0.3$	$t = 0.9$	$t = 1.8$	$r = 1$	$r = 1.6$	$s_0 \to -\infty$	$s_0 = 0$
p^b	0.91	0.97	1.27	2.11	1.51	2.17	2.01
p_I^m	1.06	1.17	1.67	3.10	2.06	2.91	2.12
p_O^m	0.89	0.94	1.39	2.19	1.49	2.39	2.01
$x^b = q^b$	0.68	1/3	1/3	0.16	0.22	0.33	0.096
$x_I^m = q_I^m$	0.54	0.21	0.26	0.09	0.13	0.27	0.087
$x_O^m = q_O^m$	0.79	0.58	0.49	0.19	0.31	0.46	0.097
π^b	0.17	0.04	0.14	0.19	0.137	0.44	0.101
$\pi_I^m + \pi_I^m$	0.36	0.11	0.41	0.41	0.298	1.11	0.2
π_O^m	0.22	0.14	0.31	0.24	0.197	0.74	0.103
$CS^m - CS^b$	-0.18	-0.10	-0.387	-0.24	-0.17	-0.54	-0.021
$W^m - W^b$	-0.09	0.02	-0.004	-0.15	-0.08	-0.24	-0.017

Note: with the merger, we denote an insider firm by I and an outsider firm by O. Moreover, CS denotes consumer surplus, and W denotes total surplus. The Shubik-Levitan demand function is defined in (15). For the Salop location model, we assume a linear transportation cost t and a circle of unit length. The CES demand function is given by $q_i = p_i^{-1-r} / \sum p_i^{-r}$. For values $r > 1.6$ the merger is not profitable. Finally, the logit demand model is defined in (14). We use $\mu = 1$, and distinguish between $s_0 \to -\infty \iff \exp\{s_0\} = 0$, which corresponds to the case without outside good, and $s_0 = 0$, which corresponds to the widely employed case in which $\exp\{s_0\} = 1$.

Description and interpretation of the results. In all the models analysed it turns out that the merger will harm consumers. This is mainly due to the insiders' lower investments and higher prices. In some cases, outsiders' prices may decrease with the merger (due to their higher investments) but in none of the cases analysed to such an extent as to lead to a pro-competitive effect.

To understand these results, we can refer to the mechanisms we have already stressed in this section. When two firms merge, we know from the analysis of their price FOCs that they will raise prices relative to the benchmark. Given investments, the outsider will also tend to raise prices, but by less than the insiders. As a result, the quantity of the insiders fall and that of the outsider increases. From the investment FOCs we know that firms' investments increase with the quantity sold: hence, insiders' investments fall (their costs will then rise) and the outsider's investment rises (its production cost will fall), but total investments decrease. While the investment effect reinforces the rise in the price of the insiders, it moves in the opposite direction for the outsider, as the larger investment lowers its production costs and tends to decrease its price. At the merger equilibrium, the price of the outsider may increase or decrease relative to the benchmark. Indeed, the table above reports cases where the merger decreases the outsider's price.[65]

[65]These results also show that, in the aggregative formulation of our game, strategic complementarity holds

It is also worth stressing that in all models we have studied the merger always decreases consumer surplus. Note that the merger always increases outsiders' profits (they benefit from the insiders' higher prices and lower investments) and that we make assumptions aimed at guaranteeing that the merger is profitable for the insiders.[66] In principle the merger may raise total surplus, and we do find that this may happen in the Salop model. Before making too much of this result, though, consider that in the Salop model demand is completely inelastic (all the market is covered and each consumer buys just one unit), hence there will be no dead-weight loss from the merger's higher prices.

Efficiency gains ($\lambda > 0$) We have carried out an analysis of the Shubik-Levitan and Salop circle model (which do not satisfy the IIA property) under the assumption of efficiency gains, and it confirms the results obtained above in Subsection 2.3.7: while at the benchmark the equilibrium variables are not affected by the level of efficiency gains λ, as λ increases the 'performance' of the merger becomes better and better (total investments increase and consumer surplus increases) until the merger becomes beneficial to consumers. In Section 4 below, we shall report a graphical analysis which illustrates these findings.

under the logit demand function and under the CES function for low enough values of r, e.g. for $r = 1$. With the Shubik-Levitan demand, or CES demand with, e.g., $r = 1.6$, the firms' actions are strategic substitutes, implying that the merger increases insiders' prices but lowers outsiders'. However, the fall in the actions of the insiders is never outweighed by the increase in the actions of the outsiders.

[66] Notably, as substitutability among the products increases, competition becomes fiercer and the insiders will lose more from being less efficient than outsiders (due to lower investments under the merger). Therefore, a common restriction in the models is that products are sufficiently differentiated: this translates into assuming a low enough γ in the Shubik-Levitan model, a large enough t in the Salop model, and a low enough r in the CES model.

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