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Manipulated Electorates and Information Aggregation

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Abstract

We study the aggregation of dispersed information in elections in which turnout may depend on the state. State-dependent turnout may arise from the actions of a biased and informed "election organizer." Voters are symmetric ex ante and prefer policy a in state α and policy b in state β, but the organizer prefers policy a regardless of the state. Each recruited voter observes a private signal about the unknown state but does not learn the turnout.

First, we characterize how the outcomes of large elections depend on the turnout pattern across states. In contrast to existing results for large elections, there are equilibria in which information aggregation fails whenever there is an asymmetry in turnout; information aggregation is only guaranteed in all equilibria if turnout is state independent. Second, when the turnout is the result of costly voter recruitment by a biased organizer, the organizer can ensure that its favorite policy a is implemented with high probability independent of the state as the voter recruitment cost vanishes. Moreover, information aggregation will fail in all equilibria. The critical observation is that a vote is more likely to be pivotal for the decision if turnout is smaller, leading to a systematic bias of the decision toward the low-turnout state.

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1 Introduction

Voting is considered an effective mechanism for aggregating information about the available policies that is dispersed among voters to determine which policy is best. For instance, consider an election in which voters have to decide between two policies: policy a and policy b. Voters have common interests and prefer policy a in state α and policy b in state β; that is, voters prefer that the implemented policy matches the state of the world. However, no individual voter knows the state, and thus the voters are uncertain about the correct policy. Instead, each voter has a small piece of information about the state in the form of a noisy signal. Feddersen and Pesendorfer (1997, 1998) showed that in large elections, the majority decision will be *as if* there were no uncertainty, in all[1] equilibria. Thus, simple majority rules allow society to aggregate noisy information in order to correctly choose among available options.

In this paper, we uncover an essential feature of this and related models of elections that is necessary for majority rules to reliably aggregate information and implement the voters' preferred outcome in all equilibria: The number of voters has to be independent of the state. We show that if the number of participating voters across the states is not identical, then large elections admit a new type of equilibrium in which information aggregation fails. In both states of the world, the same policy receives the majority of the votes with probability close to 1. The policy that wins in this equilibrium is the policy that is preferred by the voters in the state in which the participation is smaller. For example, whenever voters expect lower participation in state α, policy a wins with probability close to 1 in both states.

This finding raises two concerns. First, a simple majority rule does not aggregate information as reliably as may have been thought previously.[2] Second, and this is the focus of the paper, the failure of information aggregation opens up the possibility of "manipulation." Consider an interested agent ("election organizer") who prefers policy a independently of the state. He may manipulate the outcome in his favor by creating an expectation that the turnout is lower in state α. As discussed, this expectation induces a voting equilibrium that implements his privately preferred outcome a with high probability in both states.

There are numerous examples of activities that may allow an interested party to affect voter turnout. Consider a shareholder vote on a compensation package for the management. Shareholders have a common interest in choosing the package that

[1] When referring to equilibrium without further qualification, we mean symmetric and "responsive" ("non-trivial") equilibria. This is the class of equilibria typically considered in this setting.

[2] Section 7 discusses other mechanisms by which information aggregation may fail, for example, Feddersen and Pesendorfer (1997, Section 6), Mandler (2012), and Bhattacharya (2013).

maximizes the company's value. The management may reasonably be expected to be better informed about the appropriate compensation package and to be biased in the direction of larger compensation. Finally, the management has many tools available to manipulate the shareholder voting turnout; see Yermack (2010). Other examples of activities by which biased agents affect turnout are the bussing of voters to polls in elections or referenda, the strategic choice of the timing and location of elections, and the prodding of colleagues by a department chair. In this paper, we study the effectiveness of such tools to affect election outcomes.

We start our analysis by characterizing voting equilibria for elections in which the participation depends exogenously on the state. We verify the initial claim that whenever there is an imbalance in the number of voters across states—any ratio different from 1—information aggregation fails: there are equilibria in which the majority vote is almost independent of the state as the election becomes large. Information is aggregated in all (symmetric and interior) equilibria only if the number of voters in the two states is the same.

Equilibria in which information aggregation fails are sustained because of a *participation curse* that appears when the number of voters is not identical across states. Holding everything else equal, a vote is more likely to be pivotal in the state with fewer voters. Thus, the election outcome can be systematically biased toward the state with lower participation.

The analysis for exogenously state-dependent numbers has several applications. First, it corresponds to the case in which participation is the result of the organizer's recruitment and the organizer can publicly commit to a recruitment strategy before learning the state. Our results imply that in this case, there is an equilibrium in which the organizer gets his favorite outcome, a: if the organizer commits to any recruitment strategy that implies the participation of fewer voters in state α than in state β, then there will be a continuation equilibrium among the voters in which a majority supports a. Note that, by our general result, any imbalance across states is sufficient when the total number of voters is large, and so there is a sense in which the required commitment power may be small. Second, there are many reasons that the size of the electorate may depend on the state, the strategic manipulation by an organizer is only one of them. Finally, the analysis for a given state-dependent participation is an input into the analysis of a larger game in which the participation is chosen by the organizer without commitment.

Specifically, for the second part of the paper, we introduce an election organizer as an additional player in the election: the organizer privately learns the state of the world, α or β, and then recruits an odd number of voters. Recruitment is a costly activity, and the total recruitment cost is linear in the number of recruited voters. Each voter then has an equal chance of being selected to participate in the election, after which the recruited voters observe noisy signals and cast votes simultaneously. As noted before, the election organizer is biased, in the sense that he prefers policy a independently of the state, and the voters are aware of this conflict of interest. Also as before, the number of recruited voters is not observed by the voters. However, the voters make Bayesian inferences about the state from being recruited. Since the organizer chooses the recruitment privately and after observing the state, this is a scenario in which the organizer cannot commit.

We show that the ability to manipulate turnout still affects the performance of elections significantly: for sufficiently small recruitment costs, there are equilibria in which the election organizer chooses to recruit many voters in each state, with more voters in state β than state α, and the voters, who correctly anticipate the chosen numbers in each state, support a with a strict majority in expectation; see Theorem 2.

Two observations provide intuition. First, as a consequence of our first set of results for general state-dependent participation, if the voters expect the organizer to recruit fewer voters in state α, then there is a voting equilibrium given this expectation in which each individual voter is more likely to support policy a than policy b in both states. Second, we show that if each individual voter is more likely to support policy a than policy b in both states, then it is indeed optimal for the organizer to recruit fewer voters in state α. Finally, we use a fixed-point argument to show that this loop of best replies can be closed. Importantly, in these equilibria, the number of recruited voters is large in both states and the ratio of these numbers is interior in the limit.

We discuss extensions and conduct several robustness checks (abstention, costly voting, competition between multiple organizers, etc.).[3] In particular, we consider the case in which, even conditional on the state, there is residual aggregate uncertainty about the vote totals. As argued by Evren (2012) and others, aggregate uncertainty is natural in many settings, and it implies that the magnitude of the probability of being pivotal is significant and can rationalize realistic turnout levels. Our results extend if the aggregate uncertainty is small. In addition, we argue that when voters receive a noisy public signal about the size of the election, information aggregation continues to fail when there is an imbalance in the number of voters. Moreover, we complete the analysis of the full equilibria from the sec-

[3] We conduct a more extensive robustness check in an older version of this paper, Ekmekci and Lauermann (2014).

ond part and characterize the equilibrium behavior across *all equilibria* when the recruitment cost is small and when the population is large. There are no equilibria in which information is fully aggregated in the limit. We also show how the extent of the manipulation can be diminished when certain policy tools are used (election design) or there is competition between organizers with opposing biases.

Finally, we discuss the paper's contribution to the existing literature and compare our results especially to those from previous work on elections with an uncertain number of voters. Myerson (1998a) shows that there always exists some equilibria that aggregate information. Compared to this work, we show that additional (symmetric and responsive) equilibria exist in which information fails to aggregate unless the population size is essentially state independent.[4] In addition, we endogenize the relation between the number of voters and the state through the activity of an election organizer. Given this endogenous relationship, no equilibrium with full information aggregation exists. We also discuss the relation to a failure of information aggregation in auctions with a state-dependent number of bidders in Lauermann and Wolinsky (2017).

2 Model

A finite number of potential voters, N, has to choose between two policies, $\{a, b\}$, and there are two states of the world, $\omega \in \{\alpha, \beta\}$. Voters share the following utility function:

$$\begin{aligned} u(a, \alpha) &= u(b, \beta) = 1, \\ u(a, \beta) &= u(b, \alpha) = -1, \end{aligned}$$

where $u(x, \omega)$ denotes the utility if policy x is chosen in state ω. In other words, the voters have common interests but are uncertain which policy serves their interest better because this depends on an unknown state of the world.

Information Structure. There is a common prior belief $\rho \in (0, 1)$ that the state is α. Each voter receives a private signal, $s \in S := [0, 1]$. Conditional on the state, the signals are independent across voters and distributed according to a c.d.f. $F(s|\omega)$.

[4] In a separate paper, Ekmekci and Lauermann (2018), we show the existence of equilibria that fail to aggregate information with a state-dependent participation rate in a model where the number of voters is Poisson distributed, both with compulsory voting and with voluntary voting (allowing abstention). As mentioned, we also discuss the case with exogenous aggregate noise leading to significantly higher pivotality probabilities. We discuss that paper in Sections 5.1 and 7 in more detail.

The distribution F admits a continuous density function, denoted by $f(s|\omega)$. We assume the strict Monotone Likelihood Ratio Property (MLRP):[5]

$$\frac{f(s|\alpha)}{f(s|\beta)} \qquad \text{is strictly decreasing in } s. \tag{1}$$

Assumption (1) implies that voters who receive higher signals attach a strictly larger probability to the state of the world being state β.

Our second assumption puts a bound on the informativeness of the signals: There exists a number $\eta > 0$ such that

$$\eta < f(s|\omega) < \frac{1}{\eta} \qquad \text{for all } \omega \in \{\alpha, \beta\} \text{ and } s \in S. \tag{2}$$

The Organizer's Actions and Preferences. There is a single election organizer who observes the realization of the state of the world ω and recruits the voters who participate in the election. If the organizer recruits n pairs of voters, then the number of participants in the electorate is equal to

$$2n + 1 \in \{1, 3, 5, ..., N\}.$$

If the organizer recruits no one, $n = 0$, then one randomly chosen voter becomes the unique voter. Only the recruited voters participate in the election, and so the organizer chooses the turnout. Note that the number of voters is always odd, and a tie in the vote count cannot occur.

The organizer prefers that policy a be implemented, irrespective of the state of the world. Recruitment is costly, and each additional pair of voters costs the organizer $c > 0$. Therefore, the organizer's payoff is

$$u_O(a, n) = 1 - cn,$$
$$u_O(b, n) = -cn,$$

where the first argument is the policy that the majority of the electorate chooses to implement, and the second argument is the number of pairs of voters the organizer recruits.

We assume that the number of potential voters, N, is not too small relative to

[5] Continuity of the densities $f(\cdot|\omega)$ and the strict version of the MLRP are for expositional simplicity. All of our results continue to hold without continuity of the density functions and with the weak version of MLRP, together with a condition that states "$f(\cdot|\alpha)$ is not everywhere identical to $f(\cdot|\beta)$."

c,

$$N \geq \frac{2}{c}. \tag{3}$$

This assumption ensures that the size of the population is never a binding constraint for the organizer.[6] Finally, the choice n is not observed by the voters.[7]

The Timing of the Voting Game.

1. The organizer learns the state.
2. The organizer chooses n.
3. Nature chooses (recruits) $2n+1$ voters, each equally likely, from the population.
4. Each recruited voter observes her private signal, s, but does not observe the number of recruited voters, n.
5. Only the recruited voters participate in the election. Each recruited voter casts a vote for policy a or policy b.
6. The policy that receives the most votes is implemented.

Strategies and Equilibrium. A strategy for the organizer is a pair of distributions over integers,

$$\tilde{\mathbf{n}} = (\tilde{n}_\alpha, \tilde{n}_\beta) \in \Delta(\{0, 1, ..., (N-1)/2\})^2,$$

which denotes the organizer's recruitment choice in states α and β, respectively. We denote as $\mathbf{n} = (n_\alpha, n_\beta)$ a pure strategy.

A pure strategy[8] for a voter is a function

$$d : S \to \{a, b\},$$

that prescribes which policy the voter supports as a function of her signal if she is recruited.[9]

[6]The assumption is a lower bound on the size of the population. Our analysis remains unchanged when the number of potential voters is infinite. The advantage of a finite population is that being recruited is a positive probability event, which facilitates the application of Bayes' formula.

[7]This assumption captures the idea that the voters cannot exactly infer the organizer's recruitment effort (the number of busses, the phone calls made to others, etc.). Note that voters nevertheless make an inference about n from being recruited, as discussed below. In Section 6, we discuss an extension in which voters receive an imperfect public signal about n.

[8]As will become clear, voters' best replies will have a cutoff structure, and therefore, focusing on pure strategies for the voters is without loss of generality.

[9]Recall that when a voter is not recruited, she does not have a ballot to cast.

In the first part of the paper, in Section 3, we consider the voting game for an exogenous participation pattern given by a fixed recruitment strategy $\tilde{\mathbf{n}}$. A symmetric strategy d is a *voting equilibrium* given $\tilde{\mathbf{n}}$ if d is a best response to the strategy profile in which the organizer's strategy is $\tilde{\mathbf{n}}$ and all other voters use strategy d.

In the second part of the paper, in Section 4, we endogenize $\tilde{\mathbf{n}}$. Then, a symmetric Nash equilibrium is a pair $(\tilde{\mathbf{n}}, d)$ in which (i) the organizer's strategy $\tilde{\mathbf{n}}$ is a best response to the voters' strategy d, and (ii) the strategy d is a voting equilibrium given $\tilde{\mathbf{n}}$. We generally refer to a symmetric Nash equilibrium with an endogenous $\tilde{\mathbf{n}}$ simply as an *equilibrium*.

3 Voting Equilibria with Exogenous Participation Pattern

In this section, we study the voting equilibria of large elections in which the number of voters depends on the state in an arbitrary way. Except for an initial example, we will consider pure participation patterns (n_α, n_β) here.

The analysis of elections with an exogenously state-dependent number of voters is interesting in its own right, as discussed in the introduction. There are many scenarios in which the number of voters may be state-dependent. By studying all voting equilibria that arise when the number of voters is state dependent, we gain insights across such scenarios without being distracted by the details. Instead, we can focus on the basic and, we believe, robust mechanism that is driving the failure of information aggregation: Voters are less likely to be pivotal if the electorate is larger.

One immediate scenario to which the analysis applies—in addition to the one considered later—is an organizer who can *commit* to a recruitment policy that creates a state-dependent participation rate. Such a policy may be implemented in the form of rules (say, rules affecting the timing and location of elections, information disclosure requirements, voter ID laws, etc.) that affect participation across elections and that affect participation differentially depending on some state.

Moreover, our analysis for an exogenously state-dependent number of voters lets us connect our work to prior work on such settings by Myerson (1998a), which we discuss later in more detail.

3.1 Inference of Voters and Cutoff Strategies

We first discuss the basic voters' problem for a given participation pattern to prepare the analysis. In our model, voters are consequential, that is, they care only about the implemented policy and not directly about how they vote. A single vote changes the implemented policy only when the number of the other votes that are cast for either alternative is equal. In this event, a single vote is *pivotal*: If she votes for a, then a has a majority and if she votes for b, then b has a majority. In any other event, no single vote can affect the outcome. Thus, when deciding how to vote, it is optimal to condition on the pivotal event and to behave as if her vote were known to be pivotal, as is typical in voting models with incomplete information.

For a given symmetric voter strategy d, the expected vote share for policy a in state ω is

$$q_\omega(d) := \Pr\left(d\left(s\right)=a|\omega\right)=\int_{s\in[0,1]}\mathbf{1}_{d(s)=a}f\left(s|\omega\right)ds.$$

The probability of being pivotal in state ω if the expected vote share is q_ω and the number of recruited voter pairs is n_ω is

$$\binom{2n_\omega}{n_\omega}(q_\omega)^{n_\omega}(1-q_\omega)^{n_\omega}.$$

In our model, there is additional information contained in the event that a voter is recruited. This is because the probability of being recruited depends on the state of the world: The probability of being recruited in state ω if the number of recruited voter pairs is n_ω is

$$\frac{2n_\omega+1}{N}.$$

Taken together, when all other voters are using strategy d and the organizer is using a pure strategy $\mathbf{n}=(n_\alpha,n_\beta)$, then the posterior likelihood ratio that the state is α, conditional on (i) receiving a signal s, (ii) being recruited, and (iii) being pivotal, is given by[10]

$$\Phi(s,\mathrm{piv},\mathrm{rec};\mathbf{n},d):=\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}}\underbrace{\frac{f(s|\alpha)}{f(s|\beta)}}_{\text{signal}}\underbrace{\frac{\frac{2n_\alpha+1}{N}}{\frac{2n_\beta+1}{N}}}_{\text{recruited}}\underbrace{\frac{\binom{2n_\alpha}{n_\alpha}(q_\alpha)^{n_\alpha}(1-q_\alpha)^{n_\alpha}}{\binom{2n_\beta}{n_\beta}(q_\beta)^{n_\beta}(1-q_\beta)^{n_\beta}}}_{\text{pivotal}},\tag{4}$$

where we omit the dependence of q_ω on the voter strategy d for ease of reading.[11]

[10] The extension of the expression to the case in which the organizer uses a mixed strategy $\tilde{\mathbf{n}}$ is straightforward. For completeness, we write the critical likelihood ratio when $\tilde{\mathbf{n}}$ is a mixed strategy in Equation (20) in the Appendix.

[11] We also assume here and for the following discussion that either $0<q_\beta<1$ or $n_\beta=0$, so

This likelihood ratio, which we refer to as *the critical likelihood ratio*, guides a voter's decision. In particular, a voter having a signal s supports policy a if the critical likelihood ratio $\Phi(s,\text{piv},\text{rec};\mathbf{n},d)$ is above 1, and supports policy b otherwise. From the MLRP condition from Assumption (1), Φ is strictly decreasing in s. Therefore, voters use cutoff strategies in all equilibria.[12]

Lemma 1. *Any equilibrium voting strategy has a cutoff structure. There is a signal $\hat{s}$ such that a recruited voter casts a vote for policy b if $s > \hat{s}$ and for policy a if $s < \hat{s}$.*

From here on, we use $\hat{s} \in S$ to denote a generic cutoff strategy and $q_\omega(\hat{s})$ to denote the expected vote share for policy a in state ω when voters use a cutoff strategy $\hat{s}$. Given a cutoff $\hat{s}$, the expected vote share for policy a is simply $q_\omega(\hat{s}) = F(\hat{s}|\omega)$.

By the continuity of Φ, an *interior* cutoff, $0 < \hat{s} < 1$, is an equilibrium cutoff given $\tilde{\mathbf{n}}$ if and only if the cutoff type is indifferent between voting for a and voting for b, that is, if and only if

$$\Phi(\hat{s}, \text{piv}, \text{rec}; \tilde{\mathbf{n}}, \hat{s}) = 1.$$

In the following, we often omit $(\tilde{\mathbf{n}}, \hat{s})$ from the argument of Φ.

3.2 Small Electorates: An Example

As an example, we consider the following participation pattern $\tilde{\mathbf{n}}$: In state α, there is 1 participant; in state β, there is 1 participant with probability λ and there are 3 participants with probability $1-\lambda$. We want to show that when λ is small, then $\hat{s} = 1$ is a (strict) voting equilibrium, that is, all participating citizens vote for a independent of their signal.

For this participation pattern and voting behavior, a citizen is pivotal if and only if she is the only participant; so, the probability to be pivotal in state α is 1 and the probability to be pivotal in state β is λ. The critical likelihood ratio is, therefore,[13]

$$\Phi(s, \text{piv}, \text{rec}) = \frac{\rho}{1-\rho}\frac{f(s|\alpha)}{f(s|\beta)}\frac{1}{\lambda}. \tag{5}$$

For λ small enough, Φ is arbitrarily large for all s. At the extreme, at $\lambda = 0$,

that the probability of being pivotal is strictly positive in state β, and hence Φ is well-defined. We follow the convention that if $n = 0$, then $\binom{2n}{n} = 1$.

[12] If d is such that $0 < q_\omega(d) < 1$, the lemma follows from the previous discussion. If d is such that $q_\omega(d) = 0$ or $= 1$, then d is equivalent to a cutoff strategy with cutoff $\hat{s} = 0$ or $\hat{s} = 1$. This proves the lemma.

[13] With a mixed $\tilde{\mathbf{n}}$, the critical likelihood ratio is formally stated in (20).

a voter can affect the outcome only in state α, and hence should condition on the state being α. Thus, when λ is small, it is a voting equilibrium that all citizens vote for a, that is, $\hat{s} = 1$. Note that this is a strict, nontrivial equilibrium because there is a positive probability that a voter is pivotal.

The example shows that if a biased organizer could commit to a participation pattern in which there is only one voter in state α and sufficiently many voters in the state β, then there is a natural equilibrium in which voters support the biased organizer's preferred policy a with probability 1.

Moreover, there is an interior equilibrium with similar properties for some λ whenever $\frac{\rho}{1-\rho}\frac{f(1|\alpha)}{f(1|\beta)} < 1$. (This condition rules out the situation in which voters believe α to be more likely for all signals, even if $s = 1$.) Given this condition, we show that there exists a voting equilibrium with a cutoff $\hat{s}$ that is close to 1. To see this, note that the condition implies that, for $\lambda = 1$ and for any fixed s' close enough to 1, we have $\Phi(s'$,piv,rec$) < 1$.[14] Moreover, by the same arguments as before, for s' and $\lambda \to 0$, we have $\Phi(s'$,piv,rec$) \to \infty$. The intermediate value theorem, therefore, implies that for any such s' there is a $\lambda(s')$ for which $\Phi(s'$,piv,rec$) = 1$; meaning, $\hat{s} = s'$ is a voting equilibrium given the participation pattern associated with $\lambda(s')$.

Note that, for $s' \in (0,1)$, the participation pattern $\tilde{\mathbf{n}}$ enters in two ways; first, by affecting the inference from participating in the election, and, second, by affecting the inference from being pivotal. These inferences point to opposite states because participation is more likely in state β and being pivotal is more likely in state α; however, for s' close to 1, the inference from being pivotal dominates.

The general result below shows that, for large elections, this construction can be turned around: Starting from any given participation pattern that is asymmetric across states, an interior voting equilibrium with a large cutoff can be found.

3.3 Voting Equilibria with Large, State-Dependent Participation

We now study the voting equilibria of a large election with state-dependent participation. The large number of voters simplifies the characterization of the set of equilibrium outcomes and facilitates the comparison to the existing work on information aggregation in elections. The large election aggregates information if, in each state, the majority chooses the correct policy with probability 1 in the limit.

[14] From (20), the critical likelihood ratio for $s' \in (0,1)$ is

$$\Phi(s', \text{piv}, \text{rec}; \tilde{\mathbf{n}}, s') = \frac{\rho}{1-\rho}\frac{f(s'|\alpha)}{f(s'|\beta)}\frac{\frac{1}{N}1}{\lambda\frac{1}{N}1 + (1-\lambda)\frac{3}{N}\binom{2}{1}F(s'|\beta)(1-F(s'|\beta))}.$$

For s' close to 1, it follows that $\Phi(s'$,piv,rec$;\tilde{\mathbf{n}}, s')$ is close to $\Phi(1$,piv,rec$;\tilde{\mathbf{n}}, 1)$, as expected.

So, we consider a sequence of elections indexed by k, in which the number of voters in each state is given by n_α^k and n_β^k, with $n_\alpha^k = \theta n_\beta^k$ for some $\theta > 0$ and all k. We study the outcomes of the voting equilibria, $\hat{s}^k$, given (n_α^k, n_β^k) in the limit with $n_\alpha^k \to \infty$ and $n_\beta^k \to \infty$.

We show that, as the number of voters becomes large, the election reliably aggregates information in all equilibria if and only if $\theta = 1$. Otherwise, if there is an asymmetry in the number of voters across states, that is, $\theta \neq 1$, there exists an additional equilibrium in which the policy that is best for the voters in the state with a smaller participation rate wins independent of the state.

For the statement of the theorem, let s_ω be the median signal in state ω,

$$q_\omega\left(s_\omega\right) = F(s_\omega|\omega) = 1/2.$$

The limit cutoff is denoted by $s^* = \lim \hat{s}^k$.

Theorem 1. *Fix $\theta > 0$. Take a sequence of elections in which the number of voters is $\{n_\alpha^k, n_\beta^k\}_{k=1}^\infty$, with $n_\alpha^k = \theta n_\beta^k$ for all k, $n_\alpha^k \to \infty$, and $n_\beta^k \to \infty$.*

1. *For all $\theta > 0$, there exists a sequence of voting equilibria with limit cutoff $s^* \in (s_\alpha, s_\beta)$ that aggregates information.*

   For $\theta = 1$, there are no other limit outcomes of interior voting equilibria: Information is always aggregated.

2. *For all $\theta < 1$, there are additional sequences of voting equilibria with limit cutoff $s^* \in (s_\beta, 1)$ in which policy a wins in the limit in both states.*

   There are no other limit outcomes of interior voting equilibria: Either information is aggregated or a wins in both states.

3. *For all $\theta > 1$, there are additional sequences of voting equilibria with limit cutoff $s^* \in (0, s_\alpha)$ in which policy b wins in the limit in both states.*

   There are no other limit outcomes of interior voting equilibria: Either information is aggregated or b wins in both states.

The basic intuition for the existence of additional equilibria is as described in the introduction and in the previous example: If there are more voters in one state than in the other, a voter may be arbitrarily less likely to be pivotal in the state in which the election is larger and hence exclude the possibility of that state when voting. Let us now discuss the argument behind each claim in more detail.

Sketch of the Argument. First, if $n_\alpha^k = n_\beta^k$ for all k, meaning, $\theta = 1$, then information is aggregated in all equilibria. This is simply the "modern Condorcet jury theorem" by Feddersen and Pesendorfer (1998) and others[15] for elections in which the number of voters is independent of the state. The main argument here and in the following is that, in an interior equilibrium, the inference from being pivotal must remain bounded in order for $\Phi(\hat{s}^k,\text{piv},\text{rec}) = 1$. As will be discussed, this requires that the limit cutoff s^* satisfies $q_\alpha(s^*) - 1/2 = 1/2 - q_\beta(s^*)$: In both states, the election must be equally close to being tied in expectation. This limit cutoff implies that all equilibria aggregate information since then $q_\alpha(s^*) > 1/2 > q_\beta(s^*)$. Therefore, as n_α^k and n_β^k diverge to infinity, the weak law of large numbers implies that the majority supports a in state α and b in state β with probability converging to 1.

To see why it cannot be that, in expectation, the election is closer to being tied in one state, note that the ratio of the pivot probabilities is

$$\frac{\binom{2n}{n}(q_\alpha(s))^n(1-q_\alpha(s))^n}{\binom{2n}{n}(q_\beta(s))^n(1-q_\beta(s))^n}, \tag{6}$$

where $n_\alpha = n_\beta = n$. Now, the state in which the election is closer to being tied becomes arbitrarily more likely: Inspection shows that the likelihood ratio goes to ∞ as $n \to \infty$ if $|q_\alpha(s) - 1/2| < |1/2 - q_\beta(s)|$ and to 0 if $|q_\alpha(s) - 1/2| > |1/2 - q_\beta(s)|$.[16] Thus, we have that $\lim \Phi(\hat{s}^k,\text{piv},\text{rec}) \in \{0, \infty\}$ if $q_\alpha(s^*) - 1/2 \neq 1/2 - q_\beta(s^*)$, in contradiction to $\hat{s}^k$ being an interior equilibrium which requires $\Phi(\hat{s}^k,\text{piv},\text{rec}) = 1$.

Now, consider the case in which the number of voters differs, and suppose there are more voters in state β, meaning, $\theta < 1$. To gain intuition, consider first a situation in which the expected vote share is the same in both states, meaning, $q_\alpha = q_\beta \in (0,1)$ (e.g., the vote shares are the same if the signals are pure noise). Then, the asymmetric number of voters affects the critical likelihood ratio in two ways. First, because there are more voters in state β than in state α, a voter is more likely to be recruited in state β, and her posterior belief that the state is β increases when she is recruited. This is the *recruitment effect*. The other effect that works in the opposite direction is the *pivotality effect*. Because there are more voters in state β than in state α, the pivotality probability in state α is larger than the pivotality

[15] Large elections with pure common values are also analyzed in Wit (1998) and Duggan and Martinelli (2001). Feddersen and Pesendorfer (1997) consider a setting with private and common values.

[16] The expression $x(1-x)$ for $x \in [0,1]$ is maximized at 1/2 and symmetric around 1/2. Thus, $q_\alpha > 1 - q_\beta$ and $q_\alpha > q_\beta$ implies $q_\alpha(1-q_\alpha) < q_\beta(1-q_\beta)$. Similarly, $q_\alpha < 1 - q_\beta$ and $q_\alpha > q_\beta$ implies $q_\alpha(1-q_\alpha) > q_\beta(1-q_\beta)$. The claims now follow from $n \to \infty$. The extreme points, $s = 0$ and $s = 1$ can be excluded as limit points by a similar argument using l'Hospital's rule.

probability in state β. Among the two effects, the pivotality effect is dominant, and the net effect supports voting in favor of policy a (provided $q_\alpha = q_\beta \neq \frac{1}{2}$). To see why the pivotality effect dominates, note that the probability of being recruited is increasing *linearly* in the number of voters, but the probability of being pivotal is decreasing *exponentially*.[17] Thus, if the expected vote shares were the same but the turnout was lower in state α, then a voter's posterior probability of state α conditional on being recruited and pivotal would be close to 1.

However, since signals are informative and voters use cutoff strategies, the expected vote share of policy a is necessarily larger in state α than in state β. Given a cutoff $\hat{s}^k$ for which $1 > q_\beta\left(\hat{s}^k\right) > 1/2$, it must be that $1 > q_\alpha\left(\hat{s}^k\right) > q_\beta\left(\hat{s}^k\right) > 1/2$. Since the election is closer in state β, for any given and large number of voters, the election is much more likely to be tied in state β than in state α. Thus, if the expected vote share is larger in state α but the number of voters was the same, then a voter's posterior probability of state α conditional on being pivotal would be close to 0, as we just argued for the case $\theta = 1$. In a voting equilibrium, this effect favoring state β can be shown to exactly balance with the previous effect via the difference in the number of voters that favored state α, establishing the existence of interior cutoffs $\hat{s}^k$ for which $1 > q_\beta\left(\hat{s}^k\right) > 1/2$.

Finally, there always exists an equilibrium that aggregates information as the election becomes large, for any θ. This can be seen using the common interest structure of the game. As observed by McLennan (1998), in a game with common interests, any symmetric strategy profile that maximizes the social surplus is also a Nash equilibrium.[18] Now, if there exists some strategy profile in which the correct action is taken with probability 1 in the limit (meaning, information is aggregated), then this is in particular true for the social surplus maximizing strategy profile; hence, there is a Nash equilibrium sequence that aggregates information. To see that there is such a strategy profile, notice that for any cutoff s' between the median signals, $s' \in (s_\alpha, s_\beta)$, we have $q_\alpha(s') > \frac{1}{2} > q_\beta(s')$. Hence, if all voters follow cutoff s', then as n_α^k and n_β^k diverge to infinity, by the weak law of large numbers, the majority supports a in state α and b in state β with probability 1.

The limit analysis in the proofs of this and later results in the appendix is greatly simplified by the use of Stirling's approximation, which allows us to approximate

[17] See the end of the section for a derivation of the exact rate of convergence.

[18] In our companion paper, Ekmekci and Lauermann (2018), we clarify how the result by McLennan (1998) for a standard game with a fixed number of players extends to a game with a state-dependent and potentially uncertain number of players. With a state-dependent population, this requires some care to determine the correct social surplus criterion.

the probability of being pivotal as follows (dropping the subscript ω):[19]

$$\binom{2n}{n}(q)^n(1-q)^n \approx \frac{(4q(1-q))^n}{\sqrt{\pi n}}. \tag{7}$$

In particular, inspection of the formula verifies the claim that the probability of being pivotal is exponentially declining in n. This is because, for all interior $q \neq 1/2$, the base $4q(1-q)$ is strictly smaller than 1, and so the probability of being pivotal is asymptotically equivalent to an expression of the form $\frac{x^n}{\sqrt{\pi n}}$ for some $x \in (0,1)$, which vanishes exponentially fast to 0 as $n \to \infty$.

Preview of Discussion and Extensions. In Section 5, we provide additional results and extensions.

Population Uncertainty. We discuss the relation to prior work on elections with an uncertain number of voters by Myerson (1998a) and Evren (2012). In particular, allowing for "aggregate uncertainty" as in Evren (2012) implies that the probability of being pivotal is no longer exponentially small. Instead, the probability of being pivotal is linear in this case. This is one response in the literature to address a common concern that the benefits of voting are too small with an exponential pivotality probability.

Election Design and Unanimity. With asymmetric participation, the simple majority rule leads to the existence of bad equilibria. Is there a rule that is potentially more "robust"? We show that unanimity has good properties when participation can be state dependent, in contrast with prior work that has demonstrated that the inferiority of the unanimity rule; see Feddersen and Pesendorfer (1998).

4 Endogenous Participation with a Biased Organizer

We now discuss the extension in which participation in the two states, that is, (n_α, n_β), is determined endogenously by the actions of a biased organizer. This sheds light on what ratios $\theta = \frac{n_\alpha}{n_\beta}$ one may expect. In particular, the model shows that an asymmetry across the two states may arise endogenously if the number of voters can be affected by the actions of an informed party, even if the informed party cannot commit, meaning, its actions must be sequentially rational.[20] An extension shows a stronger result: With endogenous participation, all equilibria fail to aggregate information.

[19] For two functions, we write $f \approx g$ if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1$.

[20] The simple example in Section 3.2 with a small number of voters demonstrated that the organizer can achieve his favorite outcome if he can commit to some $\mathbf{n}$.

4.1 Organizer's Best Response

Recall that the organizer chooses n in order to maximize the probability with which policy a is implemented, less the recruitment cost. The voters do not observe the chosen n of the organizer, and so the organizer's choice of n does not affect voter behavior directly. Thus, the organizer takes the voters' cutoff strategy $\hat{s}$ as given when choosing n in each state. (The essential strategic interaction is as in a simultaneous-move game.)

The organizer's (pure) best-reply correspondence in state ω to a given cutoff strategy $\hat{s}$ of the voters is

$$\underset{n\in\left\{0,1,\ldots,\frac{1}{2}(N-1)\right\}}{\arg\max} \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} \left(q_\omega\left(\hat{s}\right)\right)^i \left(1-q_\omega\left(\hat{s}\right)\right)^{2n+1-i} - nc. \tag{8}$$

The first term in the organizer's objective function is the probability that policy a is implemented when the probability that a randomly selected voter supports policy a is $q_\omega(\hat{s})$, and the turnout is $2n+1$. The second term is the cost of choosing a turnout of $2n+1$.

To get more insight into the organizer's best reply, we calculate the increase in the probability that policy a gets selected when the organizer recruits an additional pair of voters, that is, the *marginal benefit* of increasing n, which is

$$\begin{aligned}\Delta\left(n-1,\omega,\hat{s}\right) := & \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} \left(q_\omega\left(\hat{s}\right)\right)^i \left(1-q_\omega\left(\hat{s}\right)\right)^{2n+1-i} \\ & - \sum_{i=n}^{2n-1} \binom{2n-1}{i} \left(q_\omega\left(\hat{s}\right)\right)^i \left(1-q_\omega\left(\hat{s}\right)\right)^{2n-1-i}.\end{aligned}$$

This expression can be rewritten as[21]

$$\Delta\left(n-1,\omega,\hat{s}\right) = \frac{1}{2}\binom{2n}{n} \left(q_\omega\right)^n \left(1-q_\omega\right)^n \left(2q_\omega-1\right). \tag{9}$$

The increase in the probability that policy a is implemented when the number of

[21] Adding an additional voter pair when there are $2n-1$ voters changes the outcome only if either $n-1$ voters already support a and both of the additional voters support a, or if n voters already support a, and neither of the additional voters support a. Hence, dropping subscript ω,

$$\begin{aligned}\Delta\left(n-1,\omega,\hat{s}\right) &= \binom{2n-1}{n-1} \left(q\right)^{n-1} \left(1-q\right)^n \left(q\right)^2 - \binom{2n-1}{n} \left(q\right)^n \left(1-q\right)^{n-1} \left(1-q\right)^2 \\ &= \frac{1}{2}\binom{2n}{n} \left(q\right)^n \left(1-q\right)^n \left(q-\left(1-q\right)\right).\end{aligned}$$

recruited voters increases from $2n-1$ to $2n+1$ is equal to the probability of a tie in the vote counts for policies a and b, multiplied by the term $\frac{1}{2}(2q-1)$. It is intuitive that the marginal benefit is proportional to the probability that the election is tied since additional voters matter only if the election is close. The term $(2q-1)$ enters because the additional pair may either both vote for a (good) or both vote for b (bad).

If $q_\omega(\hat{s}) \leq 1/2$, then $\Delta(n-1,\omega,\hat{s}) \leq 0$ for every n (the additional pair is more likely to vote for b than a). Therefore, the organizer recruits no additional voter, since recruitment is costly: When the odds are against him, the organizer recruits as few people as possible in order to maximize the variance in the outcome of the election and to save on recruitment cost.

If, however, $1 > q_\omega(\hat{s}) > 1/2$, then $\Delta(n-1,\omega,\hat{s}) > 0$ and $\Delta(n-1,\omega,\hat{s}) > \Delta(n,\omega,\hat{s})$. Therefore, the objective function is strictly concave. There is a unique n such that $\Delta(n-1,\omega,\hat{s}) \geq c$ and $\Delta(n,\omega,\hat{s}) < c$. Notice that when $q > 1/2$, the odds are with the organizer, so he wants to minimize the variance of the election outcome by recruiting many people. For instance, if the organizer recruits an infinite number of voters, then by the law of large numbers, policy a is implemented (but, given positive recruitment costs, the number of voters remains finite for any $c > 0$, of course).

By the strict concavity of the objective function, in both cases, the organizer's best reply is either unique (meaning one integer for each state), or a mixed strategy with support on two adjacent integers for one or for both states. (So, the number of voters is almost deterministic conditional on the state. This justifies also our focus on this case in Section 3.)

Note that $q_\omega(\hat{s}) \lesseqgtr 1/2$ if $\hat{s} \lesseqgtr s_\omega$; thus, what matters for the organizer's incentives to recruit voters is the relation of the cutoff to the median signal in state ω. So, if $s_\alpha < \hat{s} < s_\beta$, then organizer recruits voters only in state α but not in β; if $s_\beta < \hat{s}$, then the organizer recruits voters in both states and if $\hat{s} < s_\alpha$, the organizer recruits voters in neither state.

Remark 1. *The number of potential voters, N, appears in the recruitment effect in Equation* (4) *and as a constraint in the organizer's best reply in Equation* (8). *However, the term N cancels out in the recruitment effect. Moreover, by Assumption (3), the number N is sufficiently large that it is never a binding constraint in the organizer's best reply in Equation* (8). *Therefore, N plays no further role in the analysis.*

4.2 Manipulated Electorates

We study election outcomes when c is small. When c is small, the organizer may recruit many voters, and thus we can compare our result to those for exogenously large elections. To this end, we fix the common prior ρ and some information structure F that satisfies Assumptions (1) and (2). Let $\{G(c)\}_{c>0}$ be a collection of voting games in which, for each game $G(c)$, the prior belief is ρ, the information structure is F, the recruitment cost to the organizer is c, and the number of potential voters, $N(c)$, is some integer that satisfies Assumption (3).

Theorem 2. *Let $\{c_k\}_{k=1,2,\dots}$ be a sequence of positive numbers that converge to* 0*. Then, there is a sequence of symmetric Nash equilibria of $G(c_k)$ such that in both states:*

1. *The probability that policy* a *is implemented converges to* 1*.*
2. *The number of recruited voters increases without bound.*
3. *The organizer's payoff converges to* 1*.*

Theorem 2 states that, as the recruitment cost vanishes and the number of potential voters becomes large, there are equilibria in which policy a is elected with a probability that is arbitrarily close to 1 in *both* states. Moreover, in both states the number of recruited voters becomes large, and the organizer's expected payoff becomes 1. Thus, an endogenously large electorate may lead to the failure of information aggregation, and in the limit, the organizer incurs no cost from the recruitment efforts although he recruits an unbounded number of voters.

As we will see, in all such manipulated equilibria a randomly selected voter supports policy a with a probability strictly larger than 1/2 in both states of the world, and the organizer recruits more voters in state β than in state α.[22] The following two observations provide intuition. First, if the organizer is expected to recruit more voters in state β than in state α, then it is a voting equilibrium that voters support policy a with a probability strictly larger than 1/2 in both states (as in our previous result, Theorem 1). Second, if this is the voters' behavior, then it is optimal for the organizer to recruit more voters in state β than in state α.

Observation 1. If the organizer recruits $\{n_\alpha^k, n_\beta^k\}_{k=1}^{\infty}$ voters in the two states, with $n_\alpha^k \to \infty$, $n_\beta^k \to \infty$, and $0 < \lim_{k\to\infty} n_\alpha^k/n_\beta^k < 1$, then there exists a voting

[22] So, the ratio $\frac{n_\alpha}{n_\beta} < 1$. Still, as we discuss at the end of this section, the ratio is strictly positive in the limit. It may even approach 1.

equilibrium for every (n_α^k, n_β^k) in which the voters use a cutoff strategy $\hat{s}^k$ for which $1 > \lim_{k\to\infty} q_\alpha(\hat{s}^k) > \lim_{k\to\infty} q_\beta(\hat{s}^k) > 1/2$.[23]

Thus, by creating an expectation of an imbalance in the number of voters across states—no matter how small—the organizer can manipulate the election in his favor and induce an equilibrium in which his favorite outcome wins with a probability approaching 1.

Observation 2. If the voters use cutoff strategies $\{\hat{s}^k\}_{k=1}^\infty$ that imply that $1 > \lim_{k\to\infty} q_\alpha(\hat{s}^k) > \lim_{k\to\infty} q_\beta(\hat{s}^k) > 1/2$, and (n_α^k, n_β^k) is an optimal recruitment strategy given $\hat{s}^k$ and c_k for all k as $c_k \to 0$, then it must be that $n_\alpha^k \to \infty$, $n_\beta^k \to \infty$, and $\lim_{k\to\infty} n_\alpha^k / n_\beta^k < 1$.[24]

To see why it is optimal to recruit more voters in state β under the hypothesis that voters support policy a with a probability larger than 1/2 in both states, consider Figure 1. The figure depicts the probability that the majority selects policy a as a function of n for an example with $q_\alpha = 0.7$ and $q_\beta = 0.6$. When n is large, then the curve given q_β is steeper than the curve given q_α; that is, for any given n that is sufficiently large, the marginal benefit of an additional voter is larger in state β. This property holds true for all $1 > q_\alpha > q_\beta > 1/2$ and is a simple consequence of the fact that both functions must approach 1 eventually, with the function for q_β starting from a lower point.

Taken together, the two previous observations imply the following: If the organizer is expected to create an imbalance by recruiting more voters in state β and the voters behave optimally given this expectation, then it is, in fact, optimal for the organizer to recruit more actively in state β. Our proof of the theorem uses a fixed-point argument to show that this loop of best responses can be closed and establishes the existence of a manipulated equilibrium.

Optimal Recruitment Bounds the Critical Likelihood Ratio

As we discuss now, the organizer's recruitment decision is linked to the expected vote shares in such a way that the implied participation rates necessarily keep the inference from being pivotal moderate, for *any* fixed voting strategy in which a has a larger vote share.

In particular, the organizer's optimal recruitment strategy has the following implications for the pivot probabilities in different states: Take any arbitrary cutoff $\hat{s} \in (s_\beta, 1)$—not necessarily an equilibrium—for which the vote share is $1 > q_\omega(\hat{s}) >$

[23] Observation 1 is a special case of our characterization of all voting equilibria in large elections in Theorem 1, as remarked at the end of its proof.

[24] Observation 2 is a special case of our characterization in Lemma 3 in the Appendix.

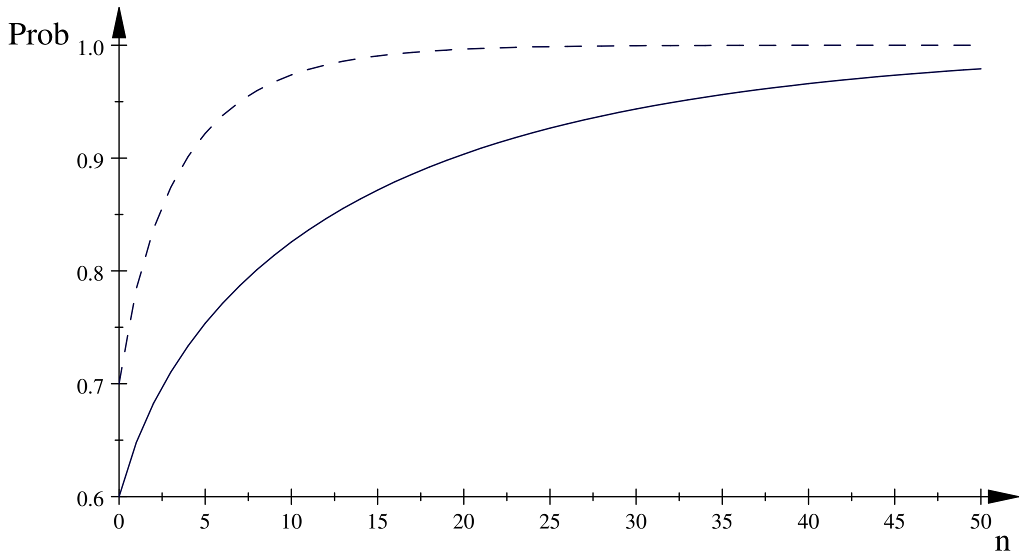

Figure 1: The probability that policy a receives the majority of votes given the number of recruited voter pairs n for $q_\beta = 0.6$ (straight) and for $q_\alpha = 0.7$ (dashed). The curve is steeper for $q_\beta = 0.6$ when n is large, implying a larger marginal benefit.

1/2. Then, the organizer chooses the number of recruited voters, $2n+1$, such that[25] $\Delta(n-1,\omega,\hat{s}) \geq c \geq \Delta(n,\omega,\hat{s})$, meaning (dropping the argument $\hat{s}$),

$$\binom{2n}{n} q_\omega^n (1-q_\omega)^n (2q_\omega - 1) \geq 2c \geq \binom{2n+2}{n+1} q_\omega^{n+1} (1-q_\omega)^{n+1} (2q_\omega - 1).$$

These bounds relate the pivot probability to c in both states, and so they can be used to bound the ratio of the pivot probabilities. Rewriting the bounds and taking their ratio, Lemma 2 in the appendix shows that whenever the voters' cutoff $\hat{s}$ satisfies $1 > q_\omega(\hat{s}) > 1/2$, the ratio of the pivot probabilities is bounded as follows:

$$3q_\beta(1-q_\beta)\frac{2q_\beta - 1}{2q_\alpha - 1} \leq \frac{\binom{2n_\alpha}{n_\alpha}(q_\alpha)^{n_\alpha}(1-q_\alpha)^{n_\alpha}}{\binom{2n_\beta}{n_\beta}(q_\beta)^{n_\beta}(1-q_\beta)^{n_\beta}} \leq \frac{1}{3q_\alpha(1-q_\alpha)}\frac{2q_\beta - 1}{2q_\alpha - 1}. \tag{10}$$

Evaluating both sides shows that the ratio of the pivot probabilities stays bounded away from 0 and ∞ whenever $1 > q_\omega(\hat{s}) > 1/2$.

Importantly, the bounds are independent of c (and n_ω). This is because the organizer's choice of the size of the electorate keeps the pivot probabilities in each state relatively at the same order, and when c vanishes, the relative pivot probabilities stay bounded away from 0 and infinity.

[25] Recall the derivation of the marginal benefit of an additional voter from (9).

Nevertheless, the bounds on the ratio are wide, approaching 0 and ∞ for $q_\beta \to 1$ and $q_\alpha \to 1$, respectively. Thus, while optimality gives us some constraints on n_α and n_β, the integer constraint in the maximization problem still allows for a wide range of pivotality ratios since the ratio can be quite different for n_ω versus $n_\omega + 1$. This range is important in our proof.

We use a version of (10) to derive a bound on the equilibrium ratio n_α/n_β, where Stirling's approximation simplifies the left-hand side. In particular, we show in Lemma 7, located in the appendix, and in the subsequent remark, that the ratio of the number of recruited voters in states α and β stays bounded away from 0 and infinity in the sequence of the manipulated equilibria of Theorem 2. With n_α^k and n_β^k denoting the expected number of recruited pairs of voters in those equilibria given c_k, if n_α^k/n_β^k converges (along some subsequence), then

$$0 < \lim_{k\to\infty} \frac{n_\alpha^k}{n_\beta^k} \leq 1.$$

Preview of Discussion and Extensions. In Section 6, we provide additional results and extensions.

All Equilibria. We complete our analysis of the equilibria with endogenous participation and show that for small c, *every* nontrivial equilibrium is either of the manipulated form discussed here or of one specific form. In particular, there are no equilibria in which information is aggregated. This is immediate: For information to aggregate, voters must support b with probability larger than 1/2 in state β. Hence, the organizer would not recruit additional voters in that state. Our analysis identifies the exact properties of the equilibria that are not fully manipulated.

Election Design. What voting rule is robust to this type of manipulation via asymmetric participation patterns? We introduce and discuss a particular variation of unanimity as a potential safeguard.

Competing Organizers. When two organizers compete, with one wishing to implement a and the other wishing to implement b, then there are two types of equilibria. We show that there are still fully manipulated equilibria but the presence of competition also opens up the possibility for information aggregation.

5 Discussions and Extensions: Exogenous Participation

This and the next sections contain some extensions of the model that (i) complete the analysis, (ii) highlight the robustness of our model to variations of the assumptions, and (iii) add some additional observations.

We split the discussion and extensions into two parts following the previous structure of the two main results. The first part, presented now, considers the case with *exogenous* participation. The second part, presented later, considers the case with *endogenous* participation.

5.1 Robustness: (Aggregate) Population Uncertainty

In the previous analysis, conditional on the state, there is no uncertainty in the number of voters. That the number of voters is exactly known conditional on the state may be too demanding. This may be a concern, especially in the interpretation as the outcome of an organizer who can affect the election size. It may be more realistic to think that the number of voters is uncertain in both states, but, on average, there are more voters in state β than in state α. The first discussion below captures this using a Poisson model of population uncertainty. The second discussion introduces further uncertainty on the aggregate level with the important consequence of increasing the pivotality probability by an order of magnitude.

All Voting Equilibria in a Poisson Model. Myerson (1998a) studied a common value environment analogous to ours in which the number of voters is Poisson distributed. The mean of the Poisson distribution depends on the state and is k and θk, respectively.[26] Myerson (1998a) shows that for all $\theta > 0$, there exists a sequence of equilibria that aggregates information as k becomes large.

In a companion paper, Ekmekci and Lauermann (2018), we study the equilibria of Poisson Elections, which includes the setting by Myerson (1998a). We derive a result that is analogous to Theorem 1: for a large election with a Poisson distributed number of voters, if $\theta \neq 1$, then there are additional, nontrivial interior equilibria that do not aggregate information. In such equilibria, the policy that is best for the voters in the state with the smaller participation rate wins with probability close to 1 independent of the state, in a large election. Thus, information aggregation is guaranteed only when $\theta = 1$ (i.e., when the expected number of voters is the same across states).[27]

We also consider a scenario with voluntary voting so that voters can *abstain*. For that case, we also show the existence of an equilibrium that fails to aggregate information in large elections whenever participation rates are asymmetric.

[26] In our setting, the described Poisson distribution arises as a special case for a mixed recruitment strategy $\tilde{\mathbf{n}}$ for which $\tilde{n}_\alpha(n) = \frac{k^n e^{-k}}{n!}$ and $\tilde{n}_\beta(n) = \frac{(\theta k)^n e^{-\theta k}}{n!}$, provided that $N = \infty$.

[27] Myerson (1998a) notes the existence of non-interior equilibria in which voters support a particular policy independently of their signal, for some parameter constellations. To the best of our knowledge, the existence of interior ("responsive") equilibria that fail to aggregate information in the model by Myerson (1998a) has not been noted.

Aggregate Uncertainty. Several authors have argued that elections may be subject to aggregate uncertainty, even with a large population.[28] In addition to the added realism, aggregate uncertainty is shown to have desirable implications. In particular, with aggregate uncertainty, the probability of a tied election is significantly larger: instead of being exponentially small, the probability is decreasing linearly.[29] Thus, with aggregate uncertainty, the predicted turnout can be significant even if there are voting costs, addressing a common criticism of pivotal voting models.[30]

Since we stressed the differential speeds of convergence of the "recruitment effect" (linear) and the "pivotality effect" (exponential), it is natural to wonder whether our results are robust to aggregate uncertainty. We study this question in our companion paper, Ekmekci and Lauermann (2018). There, we show that our results still hold in the presence of aggregate uncertainty if it is sufficiently small.

Specifically, aggregate uncertainty is modeled by assuming the presence of "noise voters" in addition to the standard voters. In the companion paper, we assume that the difference of the noise votes for a and b is distributed according to a normal distribution with mean 0. The variance is on the order of the expected number of standard voters, that is, there is aggregate uncertainty even in the limit. When the standard deviation of the noise is small, the outcomes of the model with aggregate uncertainty continuously approach the outcomes of a model with no noise.

Unboundedly Informative Signals. Our analysis here has considered boundedly informative signals. In Ekmekci and Lauermann (2018), we also discuss the case with an unboundedly informative signals, that is, $\lim_{s\to 0} \frac{f(s|\alpha)}{f(s|\beta)} = 0$ and $\lim_{s\to 1} \frac{f(s|\alpha)}{f(s|\beta)} = \infty$. We show that the basic insights continue to hold. In particular, information aggregates in all equilibria if $\theta = 1$ and, if $\theta \neq 1$, there exist additional interior equilibria in which information fails to aggregate.[31]

[28] Aggregate uncertainty refers to uncertainty that remains large relative to the total number of voters, even in the limit. Population uncertainty of the Poisson variety does not have this feature. The standard deviation of a Poisson distribution of mean n is $\sqrt{n}$ and hence negligible relative to the total.

[29] The probability of being pivotal is of order n^{-1} rather than e^{-n} for $n \to \infty$; see Evren (2012), Myatt (2017), Edlin, Gelman, and Kaplan (2007), and early contributions by Good and Mayer (1975) and Chamberlain and Rothschild (1981). We thank Christian Hellwig for bringing this literature to our attention.

[30] In addition, a number of authors note that voters will often be socially motivated and do not just consider their own material benefit from the election outcome but also the benefits to others, increasing the benefits of participation even further.

[31] In fact, with an unboundedly informative signal and a Poisson distributed number all equilibria are necessarily interior because there is a strictly positive chance that the population size is 1.

5.2 Election Design: Unanimity with Asymmetric Participation

It is well known that unanimity is a uniquely bad rule for information aggregation, in the sense that the modern Condorcet jury theorem holds for all (super-)majority rules but unanimity; see Duggan and Martinelli (2001) and Feddersen and Pesendorfer (1998). Here, we will see that unanimity is a good rule in the context of asymmetric participation rates. In particular, unanimity can act as a potential safeguard against manipulation by a biased organizer who can choose (commit to) the participation rates ex ante.

Recall our initial example with $n_\alpha(0) = 1$ and $n_\beta(0) = \lambda$, $n_\beta(1) = 1 - \lambda$, meaning there is 1 voter in state α and in state β, there is 1 voter with probability λ and 3 voters with probability $1 - \lambda$. We observed that for λ small enough, there is an equilibrium where all voters vote for a. The critical observation is that when all votes are for a, then the probability of being pivotal is 1 in state α and λ in state β. Therefore, for λ small enough, being pivotal implies that the probability of α is arbitrarily large.

Now, consider voting given the unanimity rule, with any voter being able to veto outcome a as follows: the outcome is b if and only if there is at least one vote for it; if there is no vote for b, the outcome is a. It is easy to see why unanimity upsets the reasoning behind the manipulated equilibrium. If all voters support a with probability 1, then the outcome is a in both states. Hence, in *both* states, each voter is pivotal with probability 1 independent of λ: if a voter chooses to support a as well, the outcome is a, otherwise, if she supports b ("vetoes a"), the outcome is b.

The same argument also applies to any strategy profile for which the outcome is a with probability close to 1 (but not necessarily equal to it). Again, in this case, the probability of being pivotal would be close to 1 in both states, and hence being pivotal would contain no further information.

As a consequence, the fully manipulated outcome will typically not be an equilibrium with asymmetric participation rates.[32] So, given the unanimity requirement, a biased organizer who can choose (commit to) the number of voters in each state would not be able to fully manipulate the outcome. In this sense, unanimity may be a potential "safeguard."

[32] In particular, it will not be an equilibrium if signals are such that a voter believes β is more likely than α conditional on being recruited and having the highest signal, $s = 1$.

6 Discussions and Extensions: Endogenous Participation

We now continue the discussion of the case where participation is *endogenous* and chosen by a biased organizer, subject to sequential optimality. First, we sharpen the result by showing that information aggregation fails in all equilibria. Then, we discuss extensions.

6.1 All Limit Equilibria

This section completes the equilibrium analysis. Theorem 3 characterizes the set of all limiting equilibrium outcomes that are generated by some sequence of equilibria as the organizer's recruitment cost vanishes. The first part of the theorem shows that there are only two types of limit cutoffs: the limit cutoff is either strictly larger than s_β or equal to s_α. (Recall that s_ω is the median signal in state ω, that is, $F(s_\omega|\omega) = 1/2$.) The equilibria with limit cutoff $s^* > s_\beta$ are analogous to the equilibrium outcomes of the equilibria presented in Theorem 2 in which the majority selects policy a with probability 1.

The second part of the theorem characterizes equilibria with $s^* = s_\alpha$. If the following inequality holds,

$$\frac{\rho}{1-\rho}\frac{f(s_\alpha|\alpha)}{f(s_\alpha|\beta)} > 1, \tag{11}$$

then for these equilibria, policy a wins with a probability converging to 1 in state α and b wins with a probability converging to $(1 - F(s_\alpha|\beta)) \in (0, 1)$ in state β.

If Inequality (11) holds, then there are no trivial equilibria in which the organizer recruits no additional voter in either state, for a sufficiently small c.[33] If the inequality fails, there is always a trivial equilibrium in which each voter supports policy a with a probability below 1/2 in both states of the world, which justifies the organizer's strategy to recruit no additional voters.[34]

Theorem 3. *Let $\{c_k\}_{k=1,2,\ldots}$ be a sequence of positive numbers converging to* 0 *and $\{G(c_k)\}_{k=1,2,\ldots}$ be the induced voting games.*

1. *For every limit cutoff s^* of nontrivial equilibria, either $s^* = s_\alpha$ or $s^* > s_\beta$. Therefore, information is never aggregated in the limit.*

[33] This is because, if $n_\alpha = n_\beta = 0$ (there is only one voter in each state), then voters with signals above but close to s_α support policy a, implying a vote share for a larger than 1/2 in state α. Thus, for c small enough, the organizer would choose to recruit additional voters.

[34] Equivalently, information aggregates with *sincere* voting in large electorates if and only if Inequality (11) holds. (Sincere voting means voting for the alternative which is more likely to be correct based on one's individual signal.)

2. *There exists a sequence of nontrivial equilibria with limit cutoff $s^* > s_\beta$ and another sequence with limit cutoff $s^* = s_\alpha$.*

3. *If inequality (11) holds, then along all nontrivial equilibrium sequences with limit cutoff $s^* = s_\alpha$*

   - *The number of recruited voters increases without bounds in state α and is 0 in state β.*
   - *Policy a is implemented in state α with probability converging to 1 and with probability converging to $F(s_\alpha|\beta)$ in state β.*

The proof of the theorem is in the appendix. The characterization of the set of all equilibrium outcomes may be the technically most demanding part of the analysis in the paper.

To give a rough idea of the characterization of equilibria with limit cutoff s_α, note that, for a sequence of cutoffs with limit $s^* \in (s_\alpha, s_\beta)$, the pivotality probability would vanish at an exponential rate in state α but stay positive and equal to 1 in state β. This is because the number of recruited voters increases without bounds in state α and is 0 in state β. Therefore, the critical likelihood ratio explodes and such cutoffs are not equilibria. However, when the cutoff $\hat{s}_k$ converges to s_α at exactly the right speed, then the probability of being pivotal declines linearly in state α, precisely balancing the linearly increasing probability of being recruited in that state, allowing the critical likelihood ratio to remain bounded.

In the appendix, we also discuss the case in which inequality (11) fails. In that case, there is an additional nontrivial equilibrium sequence in which the number of recruited voters in state α remains bounded.

6.2 Robust Election Design

We explore whether election design can be a remedy for an organizer's ability to manipulate election outcomes by inducing asymmetric participation rates. To this end, we discuss a design based on the unanimity rule that provides protection against such manipulation.

Unanimity Rule. We already observed that the unanimity rule can be beneficial with an exogenous asymmetry in participation. With endogenous participation, there is yet another potential benefit. As discussed before, suppose the outcome is a unless one or more voters support b (that is, unless there is a "veto" against it). It is immediate that, under this rule, the organizer recruits no additional voters. This is because an additional voter only changes the outcome if she submits a veto against the organizer's preferred outcome. As a consequence, with this rule, the organizer has no incentives to induce an asymmetric participation pattern. Also, even though just one voter chooses for all, her choice is still preferred by the voters to the fully manipulated equilibrium outcome from Theorem 2 in which the outcome is almost surely a.

Near Unanimity with a Quorum. The unanimity rule has two drawbacks. First, only the information from one voter is included because the organizer will cease recruitment. To remedy this, one may require the organizer to recruit a certain minimal number of voters; that is, to impose a quorum: The organizer has to recruit at least m additional pairs of voters, otherwise the outcome is b.

Second, as noted in the literature, unanimity is generally not a good rule for information aggregation. Intuitively, since one veto already decides the outcome, the outcome cannot reflect the information from much more than one signal.[35] To remedy this second problem, one may reduce the veto power and stipulate that some number $k(m)$ of vetoes are needed for b to win. So, "near unanimity with a quorum" stipulates that the outcome is b if the quorum m fails or more than $k(m)$ vetoes are submitted; otherwise, the outcome is a.[36] Informally, the organizer is required to ask the opinion of at least m agents, and of those, not more than $k(m)$ must be against his preferred alternative. A result by Chakraborty and Ghosh (2003) implies that, with this rule, information is fully aggregated if and only if $m - k(m)$ and m both grow to infinity.[37]

In practice, many decisions by committees require a quorum in order to ensure that a certain minimal number of members is heard. It seems likely that this rule is useful to limit the ability of an organizer to implement policies with the help of a small minority.[38]

[35] See Feddersen and Pesendorfer (1998), Duggan and Martinelli (2001), and Chakraborty and Ghosh (2003). Chakraborty and Ghosh (2003) shows that with unanimity there is a cutoff $\bar{n}$ such that increasing the number of voters beyond $\bar{n}$ does not increase the voters' payoffs. In fact, when symmetric equilibria are considered, increasing the number of voters will generally strictly lower their payoffs.

[36] We thank a referee for suggesting a voting rule along these lines and prodding us to think about it.

[37] This result is reminiscent of the double-largeness requirement for information aggregation in auctions by Pesendorfer and Swinkels (1997a).

[38] Note that, while a quorum is frequently required, it is rare that a voting rule requires an exact number of voters to participate. While such a rule would theoretically also curb the possibility of asymmetric participation, it would seem impractical in most circumstances.

6.3 Multiple Organizers

In this paper, a single organizer makes all the recruitment choices. Suppose that there is a second organizer, whom we refer to as O_1, who prefers that policy b be implemented regardless of the state. O_1 incurs the same marginal recruitment cost as the organizer, whom we refer to as O_0, who prefers that policy a be implemented regardless of the state.

In this scenario, there is always a sequence of manipulated equilibria in which policy a is implemented with a probability that converges to 1 in both states and in which only O_0 recruits voters while O_1 is passive. There is another sequence of manipulated equilibria in which policy b is implemented with a probability that converges to 1 in both states and in which only O_1 is active while O_0 is passive. There is, however, one more sequence of equilibria in which O_0 chooses to recruit many voters in state α, O_1 chooses to recruit many voters in state β, and information is aggregated; that is, the correct policy is implemented with a probability that converges to 1. Therefore, competition among organizers opens up the possibility of information aggregation.

6.4 Costly Voting and Subsidies

Suppose that, in contrast to our model, all citizens can vote but voting is costly. Here, recruitment may correspond to a subsidy by the organizer. Concretely, suppose that there are N citizens and each citizen can vote at a cost r. This cost may correspond to the cost of walking to the voting booth. The organizer can reduce the cost of voting to 0 by paying c; for example, by bussing voters to the voting booth. If the voting costs r are not too small, only the citizens who receive a subsidy actually vote.[39]

Further analysis of costly voting with subsidies may be an interesting extension of the current model, and such analysis may yield a better understanding of exactly what such scenarios may be and when to expect voter subsidies to have substantial effects on voting behavior.

6.5 Information About Voter Turnout

Thus far, we have assumed that voters do not directly observe the realized turnout. Note, however, that being recruited already contains information about overall turnout.

[39] Note that, in fact, r may be quite low since voters will compare r to the probability of being pivotal.

Now, suppose voters observe a public but noisy signal about the realized number of actual voters (say, by observing the outcome of a likely voter survey or seeing the queues on TV) and consider the voting game in which the number of voters n_α and n_β is exogenously fixed. Then, as long as the signal stays boundedly informative as the number of voters grows large, the conclusion of Theorem 1 for large elections continues to hold; that is, whenever $\lim n_\alpha/n_\beta < 1$, there are equilibria of the voting game in which information fails to aggregate. This is because a public signal moves the common prior, but we already know that the failure of information aggregation holds independent of the prior. In particular, it follows that if we take as given the organizer's original recruitment strategies from Theorem 2, the original voter behavior remains close to a best response sufficiently deep into the sequence even if there are public noisy signals.[40] (For this, note that in the manipulated equilibrium of Theorem 2, the ratio of the number of recruited voters is bounded and bounded away from 0.[41])

However, if voters receive noisy signals about the recruitment activity and if we now consider the organizer's optimal recruitment strategy given the noisy signal, then the organizer recruits differently in order to *signal* the state. We do not know how this signaling incentive affects the equilibrium outcome. Signaling may be considered an additional and somewhat different mechanism to affect voting from the one we consider here.

Thus, we believe our results are robust to adding noisy information about voter turnout when we take the original recruitment strategies as given. However, adding such information implies signaling incentives for the organizer that likely lead to a different behavior. We leave this analysis for future research since this additional signaling mechanism is likely to function differently from the mechanism that we focus on here.

6.6 Other Extensions

In the working paper version, we include a few other extensions. In particular, we consider (i) heterogeneous voter preferences, (ii) many states (continuum), (iii) the possibility for abstention, (iv) a simple quorum as a safeguard, and (v) the role of the organizer's private information and recruitment costs.

[40] We do not know how equilibrium looks like with private signals on turnout. In this case, types are two-dimensional (the original signal s and the additional signal about n). This complicates the analysis significantly because voting strategies are no longer characterized by a one-dimensional cutoff.

[41] This is discussed in the text after Theorem 2 and proven in Lemma 7 in the Appendix.

7 Literature Review

Information aggregation in elections with strategic voters has been studied by Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1996, 1997, 1998, 1999a,b), McLennan (1998), Myerson (1998a,b), and Duggan and Martinelli (2001), among others.[42] These papers study equilibrium outcomes with an exogenously large number of voters.

In particular, Feddersen and Pesendorfer (1997) show that in a model with multiple states—and private and common values—under all supermajority rules except the unanimity rule, large electorates aggregate information. Similar to this paper, they provide a complete characterization of all equilibria. The main difference between their model and ours is that here the number of participating voters is selected by a conflicted organizer, so the number of voters participating in the election is endogenously state dependent.

Myerson (1998a) introduces a Poisson model with population uncertainty in which the expected number of voters may be state dependent. He shows that large electorates aggregate information along some sequence of equilibria. In his model, the ratio of the expected number of voters across states is fixed along the sequence as the expected number of voters grows. In our model, similar to Myerson's, the number of voters participating is state dependent. However, the ratio of the number of voters is endogenously determined via the choice of an organizer who incurs a cost for increasing the number of participating voters. A second difference is that we characterize the limiting outcomes of all symmetric equilibria. We show that there is no equilibrium in which information fully aggregates when the number of voters is endogenous and there also exist equilibria in which the organizer's favorite outcome is implemented regardless of the state. We study Poisson models with exogenous population uncertainty in a companion paper, Ekmekci and Lauermann (2018).

Information aggregation fails in our setting because whenever the number of voters depends nontrivially on the state, equilibria exist in which the same policy becomes certain to win in both states. This is driven by the effect of the number of voters on the inference voters make about the state from being pivotal. To the best of our knowledge, this has not been observed before. The literature has identified other circumstances in which information may fail to aggregate. Feddersen and Pesendorfer (1997) show such a failure in an extension (Section 6) when the aggregate distribution of preferences remains uncertain conditional on the realized state. Mandler (2012) demonstrates a similar failure if the aggregate distribution of

[42] For example, Bouton and Castanheira (2012) consider information aggregation with more than two candidates.

signals remains uncertain. In these settings, the effective state is multi-dimensional. Intuitively, this implies an invertibility problem from the relevant order statistic of the vote shares to payoff-relevant states. A recent generalization was made by Barelli, Bhattacharya, and Siga (2017). Bhattacharya (2013) observes the necessity of preference monotonicity for information aggregation.[43] Gul and Pesendorfer (2009) show that information aggregation fails when there is policy uncertainty. Razin (2003) shows that information aggregation fails when voters have a signaling motive to affect the policy choice of the winning candidate. In our setting, conditional on the state, there is no residual aggregate uncertainty about the distribution of signals or preferences, the preferences over policies are monotone in the state, and there is no policy uncertainty.

Methodologically, information aggregation in elections is related to work on large auctions, which were studied, for example, by Wilson (1977), Milgrom (1979), Pesendorfer and Swinkels (1997b, 2000), and Atakan and Ekmekci (2014). Some recent papers consider related questions with state-dependent bidder participation: e.g., Atakan and Ekmekci (2016), Murto and Välimäki (2015), and, in particular, Lauermann and Wolinsky (2017).

The latter paper shows the following: if the number of bidders is exogenous, sufficiently large, and asymmetric across value-states, the auction fails to be competitive; instead, the bidders inevitably pool on a common bid below the expected value. As a consequence, the auction fails to aggregate any information. When bidder participation is endogenized via costly recruitment by an auctioneer, such an asymmetric participation pattern is shown to arise in an equilibrium.

The literatures streams on auctions and voting share the criticality of updating conditional on a certain event. However, the strategic nature of the environments is different, requiring a different analysis, reflecting different economic mechanisms, and implying different results. In particular, voters are in a common interest game, while bidders play a competitive (Bertrand) game. One consequence is that, in an election, information aggregation is the preferred outcome for the voters whereas, in an auction, information aggregation is the worst outcome for the bidders (because the price equals the value). Another difference regards the nature of the effect of asymmetric participation on the critical posterior. In our setting, the smaller participation in one state shifts the critical posterior toward that state. By contrast, in an auction, what matters is the expected value conditional on winning. However, the expected posterior across winning bids has to equal the prior by Bayesian consistency; thus, the critical posterior cannot be moved systematically in the direction

[43] See also Acharya (2016), Bhattacharya (2018), and Ali, Mihm, and Siga (2017).

of a particular state at all bids.[44]

Related studies of voter (non-)participation in elections include especially Feddersen and Pesendorfer (1996), who identify the swing voters' curse when voters can abstain, and the vast literature on costly voting, especially Ledyard (1984), Palfrey and Rosenthal (1985), and Krishna and Morgan (2011, 2012). In these models, the number of votes cast depends on the private signals of the voters. In Feddersen and Pesendorfer (1996), abstention facilitates information aggregation, whereas in Krishna and Morgan (2011), the cost of voting helps to increase (utilitarian) welfare by screening according to preference intensities in a model with common and private values. In Krishna and Morgan (2012), voluntary voting also results in signal-dependent participation, which leads to information aggregation across all equilibria. These models emphasize choice on the voters' side, showing how this can improve election outcomes, whereas our model emphasizes the organizer's ability to affect turnout and how it decreases efficiency. Critically, in these models, the underlying population of eligible voters is assumed to be independent of the state. Our companion paper Ekmekci and Lauermann (2018) allows for abstention in such models where the expected number of voters is state dependent.

A related paper that endogenizes the issues that are voted on by a strategic proposer is Bond and Eraslan (2010). Similar to us, they show that the unanimity rule may be superior to other supermajority voting rules. In their model, voting behavior under different rules has different implications for the proposals put on the table by a strategic proposer. In particular, the unanimity rule disciplines the proposer to make offers preferred by the voters. In contrast, here the alternatives are fixed but the turnout is endogenously determined by a strategic organizer. Moreover, the unanimity rule restricts the organizer's ability to utilize the asymmetry of voter turnout across the states.

Finally, a large body of literature analyzes a conflicted agent's ability to manipulate one or more decision makers to act in favor of the agent's interests, either through using informational tools or by taking actions that directly affect the decision makers' incentives. This includes models of cheap-talk, emanating from Crawford and Sobel (1982), which analyzes a biased sender's ability to transmit information and induce behavior that is beneficial to the sender. Our model shares with these models the feature that the organizer has superior information, no commitment power, and biased preferences. Our model differs from the cheap talk literature

[44] Asymmetric participation affects the expected value conditional on winning differentially across bids at different positions. In Lauermann and Wolinsky (2017), the main finding is driven by the fact that larger participation in the low-value state increases the expected value conditional on winning at low bids but decreases the expected value conditional on winning at high bids.

in that information transmission is not through cheap talk messages. The recent literature on Bayesian persuasion, initiated by Kamenica and Gentzkow (2011) and applied to a voting context by Wang (2013), Alonso and Câmara (2016), and Bardhi and Guo (2018) (among others), assumes that a sender can commit to an information disclosure rule that generates public or private signals. Similar to that literature, we are interested in an agent's ability to induce others to undertake his preferred action. However, the agent's tools are different, and the agent cannot commit.

8 Conclusion

Understanding the performance of voting mechanisms to pick the best alternatives for society has always received attention, dating all the way back to the Athenian leader Cleisthenes and, later, to Condorcet. In this paper, we have studied the ability of voting mechanisms to aggregate dispersed information among voters when the election takes place in the presence of an organizer who has the tools to change the turnout and whose interests are not aligned with those of the voters. Our main result is that such an organizer can influence the election outcomes in his favor and thus prevent information aggregation. This result indicates that although voting mechanisms may be very effective in aggregating information, they may be quite susceptible to manipulation activities by outsiders, and thus may not be robust.

An interesting feature of our model is that small electorates in which the organizer is not allowed to intervene may perform better than large electorates with an organizer (in fact, a single voter would choose better than the electorate). More generally, we discuss how a combination of a participation requirement ("quorum") and a certain generalization of a unanimity requirement can function as a safeguard against manipulation (election design).

The organizer's ability to achieve his desired outcome relies on his ability to recruit many voters. It does not rely on cherry-picking voters who have information supporting his favorite policy or voters who are a priori more inclined to vote for his favorite policy. In practice, however, many of the manipulation schemes involve the use of additional tools, such as the timing of elections or subsidies that target particular voters. Because, in our model, the organizer can affect only the overall turnout and cannot distinguish between voters with different characteristics, our results suggest that the manipulation of elections might be even easier if we afforded the organizer some of the additional targeting possibilities that can be found in practice.

A Appendix

In this appendix, we use Stirling's approximation,

$$\lim_{n\to\infty} \frac{n!}{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n} = 1. \tag{12}$$

Using Stirling's approximation for the pivot probability yields:

$$\begin{aligned}
&\binom{2n}{n}(q)^n(1-q)^n = \frac{(2n)!}{(n!)^2}(q)^n(1-q)^n \\
&\approx \frac{\sqrt{2\pi 2n}\left(\frac{2n}{e}\right)^{2n}}{\left(\sqrt{2\pi n}\left(\frac{n}{e}\right)^n\right)^2}(q)^n(1-q)^n = \frac{(4q(1-q))^n}{\sqrt{\pi}\sqrt{n}}.
\end{aligned} \tag{13}$$

Thus, given a sequence $\left(n_\alpha^k, n_\beta^k\right) \to (\infty,\infty)$ and $\left(\hat{s}^k\right)$ with $\hat{s}^k \in (0,1)$ for all k, the limit of the likelihood ratio of being pivotal is:

$$\lim_{k\to\infty} \frac{\binom{2n_\alpha^k}{n_\alpha^k}\left(q_\alpha\left(\hat{s}^k\right)\right)^{n_\alpha^k}\left(1-q_\alpha\left(\hat{s}^k\right)\right)^{n_\alpha^k}}{\binom{2n_\beta^k}{n_\beta^k}\left(q_\beta\left(\hat{s}^k\right)\right)^{n_\beta^k}\left(1-q_\beta\left(\hat{s}^k\right)\right)^{n_\beta^k}} = \lim_{k\to\infty}\sqrt{\frac{n_\beta^k}{n_\alpha^k}}\left(\frac{\left(4q_\alpha\left(\hat{s}^k\right)\left(1-q_\alpha\left(\hat{s}^k\right)\right)\right)}{\left(4q_\beta\left(\hat{s}^k\right)\left(1-q_\beta\left(\hat{s}^k\right)\right)\right)^{\frac{n_\beta^k}{n_\alpha^k}}}\right)^{n_\alpha^k}. \tag{14}$$

A.1 Proof of Theorem 1 (Large Voting Equilibria)

Proof. By hypothesis, $\frac{n_\alpha^k}{n_\beta^k} = \theta$ for some $\theta > 0$ and all k. Let s^* be a limit point of some sequence of cutoffs $\hat{s}^k \in (0,1)$. Rewriting Stirling's approximation from (14), the critical likelihood is

$$\lim_{k\to\infty} \Phi(\hat{s}^k, \text{piv}, \text{rec}; \mathbf{n}^k, \hat{s}^k) = \frac{\rho}{1-\rho}\frac{f(s^*|\alpha)}{f(s^*|\beta)}\theta \lim_{k\to\infty}\frac{1}{\sqrt{\theta}}\left(\frac{(4q_\alpha(1-q_\alpha))^\theta}{4q_\beta(1-q_\beta)}\right)^{n_\beta^k}, \tag{15}$$

where $\hat{s}^k$ is dropped as an argument from q_ω. Recall that $\hat{s}^k$ is an interior equilibrium if and only if the cutoff satisfies $\Phi(\hat{s}^k,\text{piv,rec}\;; \mathbf{n}^k, \hat{s}^k) = 1$.

Auxiliary Claims. Let s^* be a limit point of some sequence of cutoffs $\hat{s}^k \in (0,1)$.

1. If $s^* = 0$, then $\lim_{k\to\infty}\Phi(\hat{s}^k,\text{piv,rec}\;;\mathbf{n}^k,\hat{s}^k) = 0$ if $\theta > 1$ and $= \infty$ if $\theta \leq 1$.

2. If $s^* = 1$, then $\lim_{k\to\infty}\Phi(\hat{s}^k,\text{piv,rec}\;;\mathbf{n}^k,\hat{s}^k) = 0$ if $\theta \geq 1$ and $= \infty$ if $\theta < 1$.

3. If $s^* = s_\beta$, then $\lim_{k\to\infty}\Phi(\hat{s}^k,\text{piv,rec};\mathbf{n}^k,\hat{s}^k) = 0$.

4. If $s^* = s_\alpha$, then $\lim_{k\to\infty}\Phi(s^k,\text{piv,rec};\mathbf{n}^k,s^k) = \infty$.

Proof of the Claim.

Claim 1, for $s^* = 0$. Abbreviate $F_\omega := F(\cdot|\omega)$ and $f_\omega := f(\cdot|\omega)$. Then, from $q_\omega\left(\hat{s}^k\right) = F_\omega\left(\hat{s}^k\right)$, using the continuity of F_ω and $f_\omega(0) > 0$, the limit on the right-hand side of (15) is

$$\lim_{k\to\infty} \frac{1}{\sqrt{\theta}} \frac{\left(4F_\alpha\left(\hat{s}^k\right)\left(1 - F_\alpha\left(\hat{s}^k\right)\right)\right)^{\theta n_\beta^k}}{\left(4F_\beta\left(\hat{s}^k\right)\left(1 - F_\beta\left(\hat{s}^k\right)\right)\right)^{n_\beta^k}} = \lim_{k\to\infty} \frac{1}{\sqrt{\theta}} \left(\frac{4^\theta\left(\hat{s}^k\right)^\theta\left(f_\alpha(0)\right)^\theta}{4\hat{s}^k f_\beta(0)}\right)^{n_\beta^k}.$$

The limit is $= 0$ if $\theta > 1$ and it is $= \infty$ if $\theta < 1$ because the fraction in the brackets is dominated by the behavior of $\frac{\left(\hat{s}^k\right)^\theta}{\hat{s}^k}$ in these cases and $n_\beta^k \to \infty$. If $\theta = 1$, then the limit is $= \infty$ because then $\frac{4^\theta\left(\hat{s}^k\right)^\theta\left(f_\alpha(0)\right)^\theta}{4\hat{s}^k f_\beta(0)} = \frac{f_\alpha(0)}{f_\beta(0)} > 1$. Now, the claim 1 follows from (15) because $\frac{\rho}{1-\rho}\frac{f(0|\alpha)}{f(0|\beta)}\theta > 0$.

Claim 2, for $s^* = 1$. The argument for $s^* = 1$ is analogous to the one for $s^* = 0$.

Claim 3, for $s^* = s_\beta$. If $s^* = s_\beta$, then $\Phi(\hat{s}^k,\text{piv},\text{rec}\ ;\mathbf{n}^k,\hat{s}^k) \to 0$. This follows from (15): Note that $q_\beta(s_\beta) = \frac{1}{2}$ (from definition of s_β) implies that $4q_\beta(1-q_\beta) = 1$, $q_\alpha(s_\beta) \neq \frac{1}{2}$ implies that $\left(4q_\alpha(1-q_\alpha)\right)^\theta < 1$, and $n_\beta^k \to \infty$.

Claim 4, for $s^* = s_\alpha$. If $s^* = s_\alpha$, then $\Phi(\hat{s}^k,\text{piv},\text{rec};\mathbf{n}^k,\hat{s}^k) \to \infty$. This follows since then $\left(4q_\alpha(1-q_\alpha)\right)^\theta = 1$, $4q_\beta(1-q_\beta) < 1$, and $n_\beta^k \to \infty$.

An implication of the claims is that for every limit point of a sequence of voting equilibria,

$$s^* \notin \{0, s_\alpha, s_\beta, 1\}. \tag{16}$$

Step 1: We show that there always exists a sequence of equilibria along which information is aggregated, for every $\theta > 0$. The continuity of $\Phi(s,\text{piv},\text{rec};\mathbf{n}^k,s)$ in s for every k, the intermediate value theorem, and the claims 3 and 4 together imply that, for all k large enough, there exists some $\hat{s}^k \in (s_\alpha, s_\beta)$ such that $\Phi(\hat{s}^k,\text{piv},\text{rec};\mathbf{n}^k,\hat{s}^k) = 1$. By (16), $\hat{s}^k \to s^* \in (s_\alpha, s_\beta)$ for every convergent subsequence of such cutoffs. Thus, by the weak law of large numbers, along all convergent subsequences, information is aggregated. It follows that information is aggregated for the original sequence as well.

Step 2: We show that if $\theta = 1$, then there is no other equilibrium. Suppose $\hat{s}^k$ is an interior voting equilibrium with limit s^*. By (15), this requires that

$$\lim_{k\to\infty} \frac{4q_\alpha\left(\hat{s}^k\right)\left(1 - q_\alpha\left(\hat{s}^k\right)\right)}{4q_\beta\left(\hat{s}^k\right)\left(1 - q_\beta\left(\hat{s}^k\right)\right)} = 1.$$

From claims 1 and 2, $s^* \notin \{0,1\}$. Therefore, $\hat{s}^k \to s^* \in (0,1)$, which requires $4q_\alpha(s^*)(1-q_\alpha(s^*)) = 4q_\beta(s^*)(1-q_\beta(s^*))$. Now, since $q(1-q)$ is symmetric around its peak at $q=\frac{1}{2}$, and $q_\alpha(s^*) > q_\beta(s^*)$, this requires $q_\alpha(s^*) - 1/2 = 1/2 - q_\beta(s^*)$, which implies that $s^* \in (s_\alpha, s_\beta)$. This and Step 1 prove part 1 of the theorem.

Step 3: Suppose that $\theta < 1$. Given claims 2 and 3 and the continuity of $\Phi(s,\text{piv},\text{rec};\mathbf{n}^k,s)$ in s, the intermediate value theorem implies that, for all k large enough, there exists some $\hat{s}^k \in (s_\beta, 1)$ such that $\Phi(s^k,\text{piv},\text{rec};\mathbf{n}^k,s^k) = 1$. By (16), $s^k \to s^* \in (s_\beta, 1)$ for every convergent subsequence. Thus, by the weak law of large numbers, policy b is implemented in both states for every convergent subsequence of such cutoffs and, hence, for the original sequence.

Step 4: We now argue that there can be no equilibria for which s^* is not in $(s_\beta, 1)$ or (s_α, s_β). Given the observation (16), it only remains to rule out $s^* \in (0, s_\alpha)$. But this cannot hold since, for $0 < s^* < s_\alpha$, we have $\frac{(q_\alpha(s^*)(1-q_\alpha(s^*))^\theta}{q_\beta(s^*)(1-q_\beta(s^*))} > 1$ (by the behavior of $q(1-q)$ on $(0,1)$ together with $\frac{1}{2} > q_\alpha(s^*) > q_\beta(s^*) > 0$ and $\theta < 1$), and, therefore, $\Phi \to \infty$. This and Step 3 prove part 2 of the theorem.

Step 5: The case $\theta > 1$ is symmetric to $\theta < 1$. This proves part 3 of the theorem and finishes the proof.

$\square$

Remark 2. *An alternative argument to prove the existence of an equilibrium sequence with information aggregation utilizes the common interest structure of the voting game, as observed by McLennan (1998). This is discussed in the main text.*

Remark 3. *The proof implies Observation 1 on Page 17 in the main text because Stirling's approximation also works if* $0 < \lim_{k\to\infty} n_\alpha^k/n_\beta^k = \theta < 1$ *but* $n_\alpha^k \neq \theta n_\beta^k$ *along the sequence.*

In addition, a different method of proof can be used to consider sequences for which $\lim_{k\to\infty} n_\alpha^k/n_\beta^k = 1$ *but* $n_\alpha^k < n_\beta^k$ *and to show that information aggregation can fail in such cases as well, depending on the speed at which* $n_\alpha^k/n_\beta^k \to 1$.

A.2 Auxiliary Results: Full Equilibrium

In this part, we explore several properties of the organizer's best-reply correspondence and the critical likelihood ratio that are used in proving the theorems.

Organizer's Best Reply:

The set of mixed strategies for the organizer in the voting game with recruitment cost c is denoted by $\tilde{N}(c)$. Given a generic mixed strategy $\tilde{\mathbf{n}} = (\tilde{n}_\alpha, \tilde{n}_\beta)$, the term $\tilde{n}_\omega(i)$ denotes the probability that the strategy $\tilde{\mathbf{n}}$ assigns to integer i in state ω. The organizer's best-reply correspondence to the voter cutoff s when the recruitment cost is c is denoted by $\eta(s,c) := (\eta_\alpha(s,c), \eta_\beta(s,c)) \subset \tilde{N}(c)$. Thus, $\tilde{\mathbf{n}} \in \eta(s,c)$ if and only if each positive integer that is in the support of $\tilde{n}_\omega$ solves

$$\max_{n \in \left\{0,1,\ldots,\frac{1}{2}(N-1)\right\}} \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} \left(q_\omega\left(s\right)\right)^i \left(1 - q_\omega\left(s\right)\right)^{2n+1-i} - nc.$$

We abuse notation and write $\mathbf{n} \in \eta(s,c)$ if the pure strategy $\mathbf{n}$ is optimal.

We will frequently need to evaluate binomial coefficients. For this, the following observations are useful:

$$\begin{aligned}
\binom{2n+1}{n} &= \frac{(2n+1)\,2n\,(2n-1)\cdots(n+2)\,(n+1)\,n\,(n-1)\cdots 1}{n\,(n-1)\cdots 1\,(n+1)\,n\,(n-1)\cdots 1} \\
&= \frac{(2n+1)\,2n\,(2n-1)\cdots(n+2)\,(n+1)}{(n+1)\,n\,(n-1)\cdots 1}\frac{n\,(n-1)\cdots 1}{n\,(n-1)\cdots 1} \\
&= \frac{2n+1}{n+1}\binom{2n}{n},
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{2}\binom{2n}{n} &= \frac{1}{2}\frac{2n\,(2n-1)\cdots(n+2)\,(n+1)\,n\,(n-1)\cdots .1}{n\,(n-1)\cdots 1 n\,(n-1)\cdots 1} \\
&= \frac{(2n-1)\cdots(n+2)\,(n+1)\,n\,(n-1)\cdots .1}{(n-1)\cdots 1 n\,(n-1)\cdots 1} \\
&= \binom{2n-1}{n-1},
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{2}\binom{2n+2}{n+1} &= \frac{1}{2}\frac{(2n+2)}{(n+1)}\frac{(2n+1)\,2n\,(2n-1)\cdots(n+2)\,(n+1)\,n\,(n-1)\cdots .1}{n\,(n-1)\cdots 1\,(n+1)\,n\,(n-1)\cdots 1} \\
&= \frac{1}{2}\,(2)\,\frac{(2n+1)\,2n\,(2n-1)\cdots(n+2)\,(n+1)\,n\,(n-1)\cdots .1}{n\,(n-1)\cdots 1\,(n+1)\,n\,(n-1)\cdots 1} \\
&= \binom{2n+1}{n}.
\end{aligned}$$

In particular,

$$\binom{2n+2}{n+1} = 2\binom{2n+1}{n} = 2\frac{2n+1}{n+1}\binom{2n}{n}. \tag{17}$$

Properties of $\eta(s,c)$:

Recall the marginal benefit of an additional voter pair,

$$\Delta\left(n-1,\omega,\hat{s}\right)=\frac{1}{2}\binom{2n}{n}\left(q_{\omega}\left(\hat{s}\right)\right)^{n}\left(1-q_{\omega}\left(\hat{s}\right)\right)^{n}\left(2q_{\omega}\left(\hat{s}\right)-1\right).$$

We will therefore often consider the function

$$g\left(n,q\right):=\binom{2n}{n}\left(q\right)^{n}\left(1-q\right)^{n}\left(2q-1\right).$$

Note that, using (17),[45]

$$\frac{g\left(m,x\right)}{g\left(m+1,x\right)}=\frac{1}{4x(1-x)}+\frac{\frac{1}{2}}{2\left(2m+1\right)x(1-x)}>1. \tag{18}$$

This follows from $x(1-x)\leq\frac{1}{4}$. So, $g\left(m,x\right)$ is decreasing in m. Also, $\lim_{m\to\infty}g\left(m,x\right)=0$, as one would expect.

If $\hat{s}$ is such that $q_{\omega}(\hat{s})\leq 1/2$, then $\Delta\left(n-1,\omega,\hat{s}\right)\leq 0$ for every n. Thus, $\eta_{\omega}(\hat{s},c)$ is single-valued with $\tilde{n}_{\omega}(0)=1$.

If $q_{\omega}(\hat{s})>1/2$, then $\Delta\left(n-1,\omega,\hat{s}\right)>0$ and $\Delta\left(n-1,\omega,\hat{s}\right)>\Delta\left(n,\omega,\hat{s}\right)$, and $\lim_{n\to\infty}\Delta(n-1,\omega,\hat{s})=0$, by the corresponding behavior of g. So, for any c small enough, there is a unique n such that $\Delta\left(n-1,\omega,\hat{s}\right)\geq c>\Delta\left(n,\omega,\hat{s}\right)$. Hence, the support of any $\tilde{n}_{\omega}\in\eta_{\omega}$ contains at most two integers, and if it includes two integers, they have to be adjacent.

We prove the following implication of the optimality condition $\Delta(n_{\omega}-1,\omega,x)\geq c\geq\Delta(n_{\omega},\omega,x)$. Here, $n_{\omega}(x,c)$ is some pure best reply of the organizer if the voters use the cutoff strategy x. We drop the arguments occasionally to save notation. Recall that s_{ω} is the median signal with $F(s_{\omega}|\omega)=1/2$.

Lemma 2. *Given any $x\in(s_{\omega},1)$, $c>0$ and $n_{\omega}\in\eta_{\omega}(x,c)$: If $n_{\omega}\geq 1$ then*

$$\frac{2c}{2q_{\omega}-1}\leq\binom{2n_{\omega}}{n_{\omega}}\left(q_{\omega}\right)^{n_{\omega}}\left(1-q_{\omega}\right)^{n_{\omega}}\leq\frac{c}{q_{\omega}\left(1-q_{\omega}\right)\left(2q_{\omega}-1\right)}\frac{\left(n_{\omega}+1\right)}{\left(2n_{\omega}+1\right)}.$$

[45] Rewriting $\frac{\binom{2m}{m}x^{m}(1-x)^{m}}{\binom{2m+2}{m+1}x^{m+1}(1-x)^{m+1}}=\frac{\binom{2m}{m}}{2\frac{2m+1}{m+1}\binom{2m}{m}x(1-x)}=\frac{m+1}{2(2m+1)x(1-x)}=\frac{m+\frac{1}{2}}{4\left(m+\frac{1}{2}\right)x(1-x)}+\frac{\frac{1}{2}}{2(2m+1)x(1-x)}$.

Proof. Rewriting the hypothesis, using (17)

$$\Delta(n_\omega,\omega,x)\leq c\Rightarrow$$
$$\frac{2n_\omega+1}{n_\omega+1}\binom{2n_\omega}{n_\omega}(q_\omega)^{n_\omega+1}(1-q_\omega)^{n_\omega+1}(2q_\omega-1)\leq c\Rightarrow$$
$$(2n_\omega+1)\binom{2n_\omega}{n_\omega}(q_\omega)^{n_\omega}(1-q_\omega)^{n_\omega}\leq\frac{c(n_\omega+1)}{q_\omega(1-q_\omega)(2q_\omega-1)},$$

and

$$\frac{1}{2}\binom{2n_\omega}{n_\omega}(q_\omega)^{n_\omega}(1-q_\omega)^{n_\omega}(2q_\omega-1)\geq c\Rightarrow$$
$$\binom{2n_\omega}{n_\omega}(q_\omega)^{n_\omega}(1-q_\omega)^{n_\omega}\geq\frac{2c}{2q_\omega-1}.$$

Taken together, the claim follows.

□

Using the lemma to bound the numerator and the denominator of the ratio of the pivotality probabilities yields for $n_\omega\in\eta_\omega(s,c)$ that

$$3q_\beta(1-q_\beta)\frac{2q_\beta-1}{2q_\alpha-1}\leq\frac{\binom{2n_\alpha}{n_\alpha}(q_\alpha)^{n_\alpha}(1-q_\alpha)^{n_\alpha}}{\binom{2n_\beta}{n_\beta}(q_\beta)^{n_\beta}(1-q_\beta)^{n_\beta}}\leq\frac{1}{3q_\alpha(1-q_\alpha)}\frac{2q_\beta-1}{2q_\alpha-1},\tag{19}$$

using that $3\leq\frac{2(2n_\beta+1)}{(n_\beta+1)}$ and $\frac{(n_\alpha+1)}{2(2n_\alpha+1)}\leq\frac{1}{3}$.

Lemma 3. *For $x\in(s_\beta,1)$ and for any selection of pure strategy best replies by the organizer to it, $\{n_\alpha(x,c),n_\beta(x,c)\}_{c>0}$, for $c\to0$,*

$$\lim_{c\to0}\frac{n_\beta(x,c)}{n_\alpha(x,c)}=\frac{\ln(4q_\alpha(1-q_\alpha))}{\ln(4q_\beta(1-q_\beta))}>1.$$

Proof. From $x\in(s_\beta,1)$ and $s_\beta>s_\alpha$, it follows that $q_\omega(x)\in(0.5,1)$ for $\omega\in\{\alpha,\beta\}$. So, both sides of (19) are bounded and bounded away from 0. Hence, from (14),

$$\lim_{c\to0}\sqrt{\frac{n_\beta(x,c)}{n_\alpha(x,c)}}\left(\frac{(4q_\alpha(1-q_\alpha))}{(4q_\beta(1-q_\beta))^{\frac{n_\beta(x,c)}{n_\alpha(x,c)}}}\right)^{n_\beta(x,c)}$$

must be bounded and bounded away from 0 as well. This requires that

$$0<\liminf_{c\to0}\frac{n_\beta(x,c)}{n_\alpha(x,c)}\leq\limsup_{c\to0}\frac{n_\beta(x,c)}{n_\alpha(x,c)}\neq\infty,$$

since otherwise, if $\lim_{c\to 0} \frac{n_\beta}{n_\alpha} = 0$, then the ratio (14) vanishes, and if $\lim_{c\to 0} \frac{n_\beta}{n_\alpha} = \infty$, then the ratio explodes. That the ratio (14) is bounded requires therefore that

$$\frac{(4q_\alpha(1-q_\alpha))}{(4q_\beta(1-q_\beta))^{\lim_{c\to 0} \frac{n_\beta(x,c)}{n_\alpha(x,c)}}} = 1.$$

Solving this equation for $\lim_{c\to 0} \frac{n_\beta(x,c)}{n_\alpha(x,c)}$ proves the lemma. □

Critical likelihood ratio when the organizer uses a mixed strategy:

We extend Φ from pure to mixed recruitment strategies (with a slight abuse of notation) and define

$$\Phi(s, \text{piv}, \text{rec}; \tilde{\mathbf{n}}, \hat{s}) := \frac{\rho}{1-\rho} \frac{f(s|\alpha)}{f(s|\beta)} \frac{\sum_{i\geq 0} \tilde{n}_\alpha(i)(\frac{2i+1}{N})\binom{2i}{i} q_\alpha(\hat{s})^i (1-q_\alpha(\hat{s}))^i}{\sum_{i\geq 0} \tilde{n}_\beta(i)(\frac{2i+1}{N})\binom{2i}{i} q_\beta(\hat{s})^i (1-q_\beta(\hat{s}))^i}. \tag{20}$$

Lemma 4. *Fix $\hat{s} \in (0,1)$. For every $s \in [0,1]$,*

$$\max_{\tilde{\mathbf{n}} \in \eta(\hat{s},c)} \Phi(s, piv, rec; \tilde{\mathbf{n}}, \hat{s})$$

exists, and is attained by some pure strategy $\mathbf{n} \in \eta(\hat{s}, c)$. The set of maximizers is independent of s. Similarly,

$$\min_{\tilde{\mathbf{n}} \in \eta(s,c)} \Phi(s, piv, rec; \tilde{\mathbf{n}}, \hat{s})$$

exists, and is attained by some pure strategy $\mathbf{n} \in \eta(\hat{s}, c)$. The set of minimizers is independent of s.

Proof. The function Φ is continuous in $\tilde{\mathbf{n}}$, and the maximum of a continuous function over a compact domain exists.

Independence of the maximizers from s is seen by inspection of the function Φ. The extreme values of Φ are attained by a pure strategy $\mathbf{n}$ because the numerator and denominator of the Equation (20) are linear functions of the weights on two adjacent integers, due to the property of the organizer's best reply correspondence η.

□

Operator $\tilde{\Phi}$:

Definition 1. *Let*

$$\tilde{\Phi} : [0,1] \times \mathbb{R}_+ \rightrightarrows \mathbb{R}_+,$$

with $x \in \tilde{\Phi}(\hat{s}, c)$ *if and only if* $\Phi(\hat{s}, piv, rec; \tilde{\mathbf{n}}, \hat{s}) = x$ *for some* $\tilde{\mathbf{n}} \in \eta(\hat{s}, c)$ *.*

The mapping $\tilde{\Phi}$ takes a cutoff strategy $\hat{s}$ of the voters, calculates the best-reply correspondence of the organizer to $\hat{s}$, and then returns every number that is equal to the critical likelihood ratio of type $\hat{s}$ when all other voters follow the cutoff strategy $\hat{s}$ and the organizer is following a strategy that belongs to the set of best replies to $\hat{s}$. Note that $\tilde{\Phi}$ is well-defined at the boundaries $\hat{s} \in \{0, 1\}$ since then $\eta(\hat{s}, c) = \{0\}$.

Lemma 5. *The correspondence* $\tilde{\Phi}(\hat{s}, c)$ *is convex valued and upper hemicontinuous in its first argument* $\hat{s}$ *for* $\hat{s} \in (0, 1)$.

Proof. The best-reply correspondence, $\eta(\hat{s}, c)$ is upper hemicontinuous in $\hat{s}$—which follows from Berge's maximum theorem—and convex valued. The function $\Phi(\hat{s}, \text{piv}, \text{rec}; \tilde{\mathbf{n}}, \hat{s})$ is continuous in $\tilde{\mathbf{n}}$. Moreover, because the densities $f(\cdot|\omega)$ are continuous for each $\omega \in \{\alpha, \beta\}$, the upper hemicontinuity of the organizer's best-reply correspondence implies that $\tilde{\Phi}$ is upper hemicontinuous. Convex-valuedness of $\tilde{\Phi}$ follows from the convex-valuedness of $\eta(\hat{s}, c)$, continuity of Φ in $\tilde{n}$, and the fact that Φ is single-dimensional.

□

The next lemma is immediate and we skip its proof.

Lemma 6. *An interior signal* $s \in (0, 1)$ *is an equilibrium cutoff signal of* $G(c)$ *if and only if* $1 \in \tilde{\Phi}(s, c)$.

A.3 Proof of Theorem 2

Recall that s_ω satisfies $q_\omega(s_\omega) = F(s_\omega|\omega) = 1/2$.

Our proof strategy is to first show that, for all $\epsilon > 0$ and c small enough, there is some $s(c) > s_\beta + \epsilon$ such that $1 \in \tilde{\Phi}(s(c), c)$. This means there are equilibria in which the voters support policy a with probability more than 1/2 in both states. The second part of the proof shows that, in such equilibria, as c vanishes, a gets selected with probability approaching 1, that the number of voters grows without bound, and that the organizer's payoff approaches 1.

We start by showing the existence of equilibria with a large cutoff, utilizing two claims. We denote by $\max \tilde{\Phi}(s, c)$ the highest element of the image of the correspondence $\tilde{\Phi}$ at (s, c),

$$\max \tilde{\Phi}(s, c) = \max \left\{ x \geq 0 | x \in \tilde{\Phi}(s, c) \right\}.$$

The maximum exists by Lemma 5.

Claim 1:

$$\exists\epsilon > 0 \text{ such that } \lim_{c\to 0} \max \tilde{\Phi}(s_\beta + \epsilon, c) < 1.$$

Claim 2:

$$\exists\epsilon_c > 0, \text{ with } \lim_{c\to 0} \epsilon_c \to 0, \text{ such that } \lim_{c\to 0} \max \tilde{\Phi}(1 - \epsilon_c, c) = \infty.$$

These two findings together with the upper-hemicontinuity and convex-valuedness of $\tilde{\Phi}$ (Lemma 5) imply, via a version of the intermediate value theorem for correspondences,[46] that for all c smaller than a cutoff $\bar{c} > 0$, there is an $s(c) \in (s_\beta + \epsilon, 1 - \epsilon_c)$ such that $1 \in \tilde{\Phi}(s(c), c)$, which delivers the desired result.

Proof of Claim 1:

From the bounds in Lemma 2, it follows that

$$\begin{aligned}
&\max \tilde{\Phi}(s, c) \\
&= \max_{(n_\alpha, n_\beta)\in\eta(s,c)} \frac{\rho f(s|\alpha)}{(1-\rho)f(s|\beta)} \frac{(2n_\alpha + 1)\binom{2n_\alpha}{n_\alpha}(F(s|\alpha)(1 - F(s|\alpha)))^{n_\alpha}}{(2n_\beta + 1)\binom{2n_\beta}{n_\beta}(F(s|\beta)(1 - F(s|\beta)))^{n_\beta}} \\
&\leq \max_{(n_\alpha, n_\beta)\in\eta(s,c)} \frac{\rho f(s|\alpha)}{(1-\rho)f(s|\beta)} \frac{(2n_\alpha + 1)\frac{c}{q_\alpha(1-q_\alpha)(2q_\alpha-1)}\frac{(n_\alpha+1)}{(2n_\alpha+1)}}{(2n_\beta + 1)\frac{2c}{2q_\beta-1}} \\
&= \max_{(n_\alpha, n_\beta)\in\eta(s,c)} \frac{\rho f(s|\alpha)}{(1-\rho)f(s|\beta)} \frac{2n_\alpha + 1}{2n_\beta + 1} \frac{2q_\beta - 1}{2q_\alpha (1 - q_\alpha)(2q_\alpha - 1)} \frac{n_\alpha + 1}{2n_\alpha + 1}.
\end{aligned}$$

The term (n_α, n_β) denotes a pure strategy that puts probability 1 to integers n_α and n_β in states α and β respectively. Applying Lemma 3, we obtain that, for any fixed s such that $q_\beta(s) \in (0.5, 1)$,

$$\lim_{c\to 0} \max \tilde{\Phi}(s, c) \leq \frac{\rho f(s|\alpha)}{(1-\rho)f(s|\beta)} \frac{\ln(4(q_\beta)(1 - q_\beta))}{\ln(4(q_\alpha)(1 - q_\alpha))} \frac{2q_\beta - 1}{4q_\alpha (1 - q_\alpha)(2q_\alpha - 1)}.$$

Note that the right side vanishes for $s \to s_\beta$ from above because $q_\beta(s) \to 1/2$ while $q_\alpha(s)$ stays strictly larger than $1/2$. So, there exists some ε and c^* such that for all $c \leq c^*$,

$$\max \tilde{\Phi}(s_\beta + \varepsilon, c) < 1.$$

This proves Claim 1.

[46] Claim 1 in the appendix of Shimer and Smith (2000) states an appropriate extension of the standard intermediate value theorem to convex valued, upper-hemicontinuous correspondences.

Note also that the number 1 on the right-hand side of the inequality is arbitrary, and the same proof works to show that one can choose ε so that this inequality holds for any positive number.

Proof of Claim 2:

Let $\gamma(q) := 2q(1-q)(2q-1)$, and let $\epsilon_c > 0$ be a number that is close to 0 that satisfies $\gamma(q_\alpha(1-\epsilon_c)) = 2c$. Existence of such an $\epsilon_c > 0$ is guaranteed when c is small, and $\lim_{c\to 0} \epsilon_c = 0$. We show that $\lim_{c\to 0} \max \tilde{\Phi}(1-\epsilon_c, c) = \infty$.

Since $\gamma(q_\omega(s)) = 2\Delta(n-1,\omega,s)$, the definition of $1-\epsilon_c$ is such that, in state α, the organizer with a marginal cost c is indifferent between recruiting no additional voters and recruiting one pair of voters. Note that,

$$\gamma'(q) = 12q(1-q) - 2.$$

and $\gamma'(q) < 0$ for q sufficiently close to 1. Hence, for x close to 1, $q_\beta(x) < q_\alpha(x)$ implies then if $\gamma(q_\alpha(x)) = 2c$, then $\gamma(q_\beta(x)) > 2c$. Hence, the organizer's best reply to the cutoff $1-\epsilon_c$ is that in state β he recruits at least 1 pair and, actually as $1-\epsilon_c \to 1$, exactly one pair. This is because we have $2\Delta(1,\beta,x) = \binom{4}{2} q_\beta(x)^2 (1-q_\beta(x))^2 (2q_\beta(x)-1) < \gamma(q_\alpha(x)) = 2c$ for all x sufficiently close to 1, which follows from $\frac{(1-q_\beta(x))^2}{(1-q_\alpha(x))} \to_{x\to 1} 0$.

Writing down the pivot probability in state β, we get $2q_\beta(x)(1-q_\beta(x))$, and if in state α the organizer recruits no one, then the pivot probability in state α is 1. Because $\lim_{x\to 1} \frac{q_\beta(x)}{q_\alpha(x)} = 1$, and $\lim_{x\to 1} \frac{1-q_\beta(x)}{1-q_\alpha(x)} = \frac{f(1|\beta)}{f(1|\alpha)}$, and that $\frac{f(1|\alpha)}{f(1|\beta)} > 0$, we have that

$$\begin{aligned}\lim_{c\to 0} \max \tilde{\Phi}(1-\epsilon_c, c) &= \lim_{c\to 0} \frac{\rho f(1-\epsilon_c|\alpha)}{(1-\rho) f(1-\epsilon_c|\beta)} \frac{1}{3} \frac{1}{2q_\beta(1-\epsilon_c)(1-q_\beta(1-\epsilon_c))} \\ &= \infty.\end{aligned}$$

Combining Claims 1 and 2 and Lemma 5:

Because $\tilde{\Phi}(s,c)$ is upper-hemicontinuous and convex valued (Lemma 5), and combining this with Claims 1 and 2, it follows via the intermediate value theorem for correspondences (Footnote 46) that there is a $\bar{c} > 0$ and $\epsilon > 0$ such that for every $c < \bar{c}$, there is an $s(c) > s_\beta + \epsilon$ such that $1 \in \tilde{\Phi}(s(c), c)$.

Hence, there is an equilibrium in which the voters support policy a with a probability more than 1/2. However, the theorem makes the stronger claim that the number of recruited voter pairs grows to infinity. We now show this:

Modifying the Proof of Claim 2 to ensure large turnout:

In this part, we modify the second part of the above proof (that is, the proof of Claim 2) to show that ϵ_c can be chosen in such a way that the organizer, when faced with voters using a cutoff $1-\epsilon_c$, is indifferent between $m(c)$ and $m(c)-1$ pairs of voters in state α and recruits $m(c)$ pairs of voters in state β, and that $\lim_{c\to 0} m(c) = \infty$.

The alternative mapping that we consider is $x_m(c)$, defined analogously as the solution to

$$\binom{2m}{m}\left(q_\alpha(x)\right)^m\left(1-q_\alpha(x)\right)^m(2q_\alpha(x)-1) = 2c.$$

As before, for any given c that is sufficiently small, for $x = x_m(c)$

$$\begin{aligned}&\binom{2m}{m}\left(q_\beta(x)\right)^m\left(1-q_\beta(x)\right)^m(2q_\beta(x)-1)\\ &> 2c >\\ &\binom{2m+2}{m+1}\left(q_\beta(x)\right)^{m+1}\left(1-q_\beta(x)\right)^{m+1}(2q_\beta(x)-1).\end{aligned}$$

Thus, we can pick some $\hat{x}_m(c)$ just above $x_m(c)$ such that in state α, the organizer recruits $m-1$ pairs of voters and in state β recruits m pairs of voters. As $c\to 0$, it must be that $\hat{x}_m(c)\to 1$ and similar to before,

$$\lim_{c\to 0}\max\tilde{\Phi}(\hat{x}_m(c),c) = \infty.$$

Now consider a sequence of equilibria (whose existence has been shown) with cutoffs larger than s_β and bounded away from it; i.e., $\lim_{c\to 0} s(c) = s^* > s_\beta$.

If $s_\beta < s^* < 1$ then $\lim_{c\to 0} n_\omega(s(c),c)\to\infty$. If $s^*=1$ then note the following: Recall the function $g(m,x) = \binom{2m}{m}x^m(1-x)^m(2x-1)$ where m is a positive integer and $x\in[0,1]$. By inspection of (18), $g(m,x)$ is decreasing in m and there is some $\varepsilon>0$ such that $g(m,x)$ is decreasing in x in the region where $x>1-\varepsilon$.[47] This property of the function g together with the property for the equilibrium cutoff $s(c)$ that $s(c)\le x_m(c)$, and the hypothesis that $\lim_{c\to 0} s(c) = 1$ together imply that $n_\omega(s(c),c)\ge m(c)$. Since this exercise can be repeated for any arbitrary m, we can pick the sequence $m(c)$ in such a way that it grows unboundedly. Therefore, the resulting equilibrium turnout grows without bound.

Showing that policy a gets selected:

Let $s(c)$ denote the equilibrium cutoff sequence from the previous parts of this

[47] To see that $g(m,x)$ is decreasing in x, note that $\frac{d}{dx}\left(x^m(1-x)^m(2x-1)\right)$ is equal to $x^{m-1}(1-x)^{m-1}\left(m(1-x)(2x-1) - mx(2x-1) + 2x(1-x)\right)$ and the final term is approximately $-m$ for x close to one.

proof. By construction, $1 > s(c) > s_\beta + \varepsilon$ for all c smaller than some $\bar{c} > 0$. We now show that the probability of the majority voting for policy a approaches 1 as $c \to 0$. Without loss of generality, suppose $s(c)$ converges. The claim is obvious if

$$\lim_{c\to 0} s(c) = 1.$$

If not, then

$$1 > \lim_{c\to 0} s(c) \geq s_\beta + \varepsilon$$

implies

$$\lim_{c\to 0} q_\alpha(s(c)) > \lim_{c\to 0} q_\beta(s(c)) > 0.5,$$

and $\lim_{c\to 0} n_\omega(c) \to \infty$. This implies the claim for the second case because the weak law of large numbers applies. Thus, policy a gets implemented as c vanishes for this sequence of equilibria.

Showing that organizer's payoff is 1 in both states:

Let $U_O^c(s, \tilde{\mathbf{n}})$ denote the organizer's payoff in the election in which the marginal recruitment cost is c and the strategy profile is $s, \tilde{\mathbf{n}}$. Consider the equilibrium from the previous part of the proof, $s(c), \tilde{\mathbf{n}}(c)$, and consider following alternative strategy $\bar{n}(c) := (\lfloor \frac{1}{\sqrt{c}} \rfloor, \lfloor \frac{1}{\sqrt{c}} \rfloor)$; i.e., $\bar{n}(c)$ is the strategy in which the organizer invites $\lfloor \frac{1}{\sqrt{c}} \rfloor$ pairs of voters in both states. As $c \to 0$ the recruitment cost incurred by the organizer given strategy $\bar{n}$ vanishes. Moreover, because $c \to 0$, the number of recruited voters goes to ∞, and because $s(c) \to s^* > s_\beta + \epsilon$, by the weak law of large numbers the probability that the majority votes for policy a approaches 1 when the organizer employs strategy $\bar{n}(c)$. Hence, $\lim_{c\to 0} U_O^c(s(c), \bar{n}(c)) \to 1$. Because $\tilde{\mathbf{n}}(c)$ is a best reply to voter cutoff strategy $s(c)$, it has to be that $U_O^c(s(c), \tilde{\mathbf{n}}(c)) \geq U_O^c(s(c), \bar{n}(c))$, for every $c > 0$. Therefore, $\lim_{c\to 0} U_O^c(s(c), \tilde{\mathbf{n}}(c)) = 1$, as well. Since in each state the organizer's payoff is bounded above by 1, and since each state occurs with positive probability, the organizer's payoff conditional on each state converges to 1, as well. This finishes the proof of Theorem 2. $\square$

Showing that the ratio $\frac{n_\alpha(x(c),c)}{n_\beta(x(c),c)}$ is bounded:

Lemma 7. *Suppose* $(x(c), \tilde{\mathbf{n}}(c))$ *is a collection of interior equilibria given* c *and suppose* $n_\alpha(x(c), c)$ *and* $n_\beta(x(c), c)$ *are in the support of* $\tilde{\mathbf{n}}(c)$ *with* $\lim_{c\to 0} x(c) > s_\beta$ *and* $\lim_{c\to 0} n_\alpha(x(c), c) = \lim_{c\to 0} n_\beta(x(c), c) = \infty$*, then*

$$0 < \liminf_{c\to 0} \frac{n_\alpha(x(c), c)}{n_\beta(x(c), c)} \leq \limsup_{c\to 0} \frac{n_\alpha(x(c), c)}{n_\beta(x(c), c)} \leq 1.$$

Proof. If $\lim_{c\to 0} x(c) = x$ such that $q_\omega(x) \in (0.5, 1)$ for $\omega \in \{\alpha, \beta\}$, then the ratio of the number of voters in the two states stays bounded as c goes to 0 by a straightforward adjustment of the proof of Lemma 3 using only the organizer's optimality condition.

By hypothesis, this leaves $\lim_{c\to 0} x(c) = 1$. We show that $\frac{n_\alpha}{n_\beta} \to 1$ in this case. Using approximation from (14),

$$\lim_{c\to 0} \frac{2n_\alpha + 1}{2n_\beta + 1} \frac{\binom{2n_\alpha}{n_\alpha} (q_\alpha)^{n_\alpha} (1 - q_\alpha)^{n_\alpha}}{\binom{2n_\beta}{n_\beta} (q_\beta)^{n_\beta} (1 - q_\beta)^{n_\beta}} = \lim_{c\to 0} \sqrt{\frac{n_\alpha}{n_\beta}} \frac{(4q_\alpha(1 - q_\alpha))^{n_\alpha}}{(4q_\beta(1 - q_\beta))^{n_\beta}}. \tag{21}$$

This approximation stays correct for $\lim_{c\to 0} q_\omega(x(c)) = 1$, $\omega \in \{\alpha, \beta\}$, since we approximate only the binomial terms. Abbreviate $f_\omega := f(\cdot|\omega)$. With $\Delta(c) := 1 - x(c)$, we have $1 - q_\omega(x(c)) \cong \Delta f_\omega(1)$ by the continuity and boundedness of f_ω. Taking logs of the expression on the right-hand side of (21), with $f_\omega = f_\omega(1)$,

$$\begin{aligned}
&\ln \sqrt{\frac{n_\alpha}{n_\beta}} \left(\frac{(4\Delta f_\alpha)^{n_\alpha}}{(4\Delta f_\beta)^{n_\beta}} \right) \\
&= \frac{1}{2} \ln n_\alpha - \frac{1}{2} \ln n_\beta + n_\alpha \ln 4f_\alpha + n_\alpha \ln \Delta - n_\beta \ln 4f_\beta - n_\beta \ln \Delta \\
&= n_\alpha \ln \Delta \left(\frac{\frac{1}{2} \ln n_\alpha}{n_\alpha \ln \Delta} + \frac{\ln 4f_\alpha}{\ln \Delta} + 1 \right) - n_\beta \ln \Delta \left(\frac{\frac{1}{2} \ln n_\beta}{n_\beta \ln \Delta} + \frac{\ln 4f_\beta}{\ln \Delta} + 1 \right).
\end{aligned}$$

From $n_\omega \to \infty$ for $\omega \in \{\alpha, \beta\}$ and $\ln \Delta \to (-\infty)$, the first two terms in each of the brackets vanish, and we have

$$\lim_{c\to 0} \left(\ln \sqrt{\frac{n_\alpha}{n_\beta}} \left(\frac{(4\Delta f_\alpha)^{n_\alpha}}{(4\Delta f_\beta)^{n_\beta}} \right) \right) = \lim_{c\to 0} (n_\alpha - n_\beta) \ln \Delta.$$

<u>Case 1.</u> Suppose $\lim \frac{n_\alpha}{n_\beta} > 1$. Then $\ln \Delta \to (-\infty)$ and $\lim (n_\alpha - n_\beta) = (+\infty)$ imply that

$$\lim_{c\to 0} \left(\ln \sqrt{\frac{n_\alpha}{n_\beta}} \left(\frac{(4\Delta f_\alpha)^{n_\alpha}}{(4\Delta f_\beta)^{n_\beta}} \right) \right) = -\infty.$$

Hence, since the natural logarithm of it diverges to $-\infty$, the limit of the right-hand side of (21) is 0. Note that (21) is proportional to $\tilde{\Phi}$. Therefore, $\lim \tilde{\Phi}(x(c), c) = 0$, in contradiction to $x(c)$ being an interior equilibrium, requiring $1 \in \tilde{\Phi}(x(c), c)$.

<u>Case 2.</u> Suppose $\lim_{c\to 0} \frac{n_\alpha}{n_\beta} < 1$. Then, $\ln \Delta \to (-\infty)$ and $\lim_{c\to 0} (n_\alpha - n_\beta) = (-\infty)$ imply that

$$\lim_{c\to 0} \left(\ln \sqrt{\frac{n_\alpha}{n_\beta}} \left(\frac{(4\Delta f_\alpha)^{n_\alpha}}{(4\Delta f_\beta)^{n_\beta}} \right) \right) = +\infty.$$

Hence, the limit of (21) is $+\infty$. Therefore, $\lim_{c\to 0}\tilde{\Phi}(x(c),c)=\infty$. This is again in contradiction to $x(c)$ being an interior equilibrium.

Thus, if $\lim_{c\to 0} x(c)=1$ then $\lim_{c\to 0}\frac{n_\alpha}{n_\beta}=1$, as claimed. $\square$

A.4 Proof of Theorem 3

This proof characterizes all limit points of nontrivial equilibrium cutoffs. The proof of the theorems proceeds through a sequence of steps that are combined at the end of the section.

To simplify some of the expressions, we sometimes omit the argument c. Moreover, we sometimes omit $s(c)$ as well in expressions like $q_\omega = q_\omega(s(c))$.

We also use the following lemma.

Lemma 8. *Let $\{s(c), n_\alpha(c), n_\beta(c)\}_{c>0}$ be a selection of cutoffs $s(c)$ for the voters, and a pair of integers $(n_\alpha(c), n_\beta(c))$ that are in the support of the organizer's best reply to voter strategy $s(c)$ with recruitment cost c. If $1 > \lim_{c\to 0} s(c) > s_\omega$ for $\omega \in \{\alpha,\beta\}$, then (dropping the dependence of n_ω on c)*

$$\lim_{c\to 0}(2n_\omega+1)\binom{2n_\omega}{n_\omega}q_\omega(s(c))^{n_\omega}(1-q_\omega(s(c)))^{n_\omega}=0 \qquad \text{for } \omega\in\{\alpha,\beta\}.$$

Proof. First, if $1>\lim s(c) > s_\omega$, then $1>\lim q_\omega(s(c)) = q^* > 1/2$ implies $\lim n_\omega(c)=\infty$. By Stirling's approximation (see (13)),

$$\lim_{c\to 0}\frac{\binom{2n_\omega}{n_\omega}}{4^{n_\omega}} = \lim_{c\to 0}\frac{\frac{4^{n_\omega}}{\sqrt{\pi}\sqrt{n_\omega}}}{4^{n_\omega}} = 0. \tag{22}$$

Because $1>q^*>1/2$, we have $4q^*(1-q^*)<1$. This and $n_\omega(c)\to\infty$ imply[48]

$$\begin{aligned}&\lim_{c\to 0}(2n_\omega+1)\binom{2n_\omega}{n_\omega}q_\omega(s(c))^{n_\omega}(1-q_\omega(s(c)))^{n_\omega}\\ &\le \lim_{c\to 0}(2n_\omega+1)(4q_\omega(s(c))^{n_\omega}(1-q_\omega(s(c)))^{n_\omega}=0.\end{aligned}$$

Combining this with (22) delivers the result.

$\square$

<u>Step 1</u>: To show s_α is the only possible limit point of nontrivial equilibria that is not above s_β.

[48] Recall $nx(n)^n \to 0$ for any sequence $x(n)$ with $x(n)\to x^* \in (0,1)$ and $n\to\infty$.

There are 3 cases to consider and rule out: $s^* < s_\alpha$, $s^* \in (s_\alpha, s_\beta)$, and $s^* = s_\beta$. The first two cases are easier to rule out while the last case is more subtle. We deal with the first two cases first.

Case 1. Suppose $s^* < s_\alpha$. If this is true, then the probability that a randomly selected voter supports policy a is strictly less than 1/2 in both states, and the organizer recruits no one. However, this is a trivial equilibrium.

Case 2. Suppose $s_\alpha < s^* < s_\beta$. Then, the organizer recruits no one in state β and many voters in state α as $k \to \infty$. In fact, because $s^* > s_\alpha$, $q_\alpha(s(c)) \to q_\alpha^* > 1/2$, and therefore, in any sequence of equilibria, for any selection of integers $n_\alpha(c)$ that are in the support of the equilibrium recruitment strategy of the organizer, $n_\alpha(c) \to \infty$. Therefore, by Lemma 8 above,

$$(2n_\alpha(c)+1)\binom{2n_\alpha(c)}{n_\alpha(c)} q_\alpha(s(c))^{n_\alpha(c)}(1-q_\alpha(s(c)))^{n_\alpha(c)} \to 0.$$

Therefore, $\max \tilde{\Phi}(s(c), c) \to 0$, which is a contradiction to $s(c)$ being an equilibrium cutoff.

Case 3. Suppose $s^* = s_\beta$. We argue that this cannot be the case either, by showing that

$$\lim_{c \to 0} \left(\max \tilde{\Phi}(s(c), c) \right) = 0.$$

First, note that

$$\lim_{c \to 0} q_\alpha(s(c)) = q_\alpha(s_\beta) > 1/2,$$

and this implies

$$\lim_{c \to 0} (2n_\alpha + 1)\binom{2n_\alpha}{n_\alpha} q_\alpha^{n_\alpha}(1-q_\alpha)^{n_\alpha} = 0.$$

for every sequence of integers $n_\alpha(c)$ in the support of the organizer's best reply, via Lemma 8. Thus, there cannot be any subsequence of cutoffs in which $q_\beta(s(c)) \leq 1/2$. This is because, otherwise, along such a subsequence, $n_\beta(c) = 0$, and hence $\lim \max \tilde{\Phi}(s(c), c) = 0$.

Therefore, consider a subsequence along which $q_\beta > 1/2$ (so, $q_\beta \to 1/2$ from above). Recall that $\max \tilde{\Phi}(s(c), c)$ is attained by some pure strategy that is in $\eta(s(c), c)$. Denote such a pure strategy with a pair of integers (n_α, n_β) that correspond to the integers in the support of the strategy in states α and β, respectively. These integers depend on c, but for the ease of reading we drop the dependence of these integers on c.

We now bound $\lim_{c \to 0} \max \tilde{\Phi}(s(c), c)$ from above, by either putting a lower bound on the multiplication of two terms on the denominator, which is

$$(2n_\beta+1)\binom{2n_\beta}{n_\beta}q_\beta^{n}(1-q_\beta)^{n_\beta},$$

or by directly arguing that

$$\frac{(2n_\alpha+1)\binom{2n_\alpha}{n_\alpha}q_\alpha^{n_\alpha}(1-q_\alpha)^{n_\alpha}}{(2n_\beta+1)\binom{2n_\beta}{n_\beta}q_\beta^{n_\beta}(1-q_\beta)^{n_\beta}}\to 0.$$

For any given $q>1/2$, the function $\gamma(q,n):=(2n+1)\binom{2n}{n}q^n(1-q)^n$ can have at most one peak, when viewed as a function of n. This is because, using (17),

$$\frac{\gamma(q,n+1)}{\gamma(q,n)}=\frac{2n+3}{2n+1}\frac{(2n+2)(2n+1)}{(n+1)^2}q(1-q)=\frac{4n+6}{n+1}q(1-q).$$

A simple calculation shows that the expression for $\frac{\gamma(q,n+1)}{\gamma(q,n)}$ is a strictly decreasing function of n. When q is sufficiently close to $1/2$, $\gamma(q,n)$ is strictly increasing in n at $n=0$, since $\gamma\left(q,0\right)=1$ and $\gamma\left(q,1\right)=6q\left(1-q\right)$. Therefore, for every nonnegative integer N^*, the minimum of $\gamma(q,n)$ in the domain $n\in\{0,1,\cdots,N^*\}$ is attained at one of the extreme points, i.e., either at $n=0$ or $n=N^*$.

We consider two subsequences. First, consider a subsequence for which $\gamma\left(q_\beta,n\right)$ attains its minimum at $n=0$. Then, the claim in Case 3 follows because

$$(2n_\beta+1)\binom{2n_\beta}{n_\beta}q_\beta^{n_\beta}(1-q_\beta)^{n_\beta}\geq\min_{n\in\{0,1,\cdots,N(c)\}}\gamma(q_\beta,n)=\gamma(q_\beta,0)=1.$$

This together with

$$\lim_{c\to 0}(2n_\alpha+1)\binom{2n_\alpha}{n_\alpha}q_\alpha^{n_\alpha}(1-q_\alpha)^{n_\alpha}=0,$$

delivers that $\lim_{c\to 0}\max\tilde{\Phi}(s(c),c)=0$ along such a sequence.

Now, consider an infinite subsequence for which the minimum is attained at $N\left(c\right)$. Also, suppose that, for all c, $n_\alpha(c)\geq n_\beta(c)$. When c is small,

$$\gamma(q_\beta(s(c)),n_\beta(c))\geq\min\{\gamma(q_\beta(s(c)),n_\alpha(c));\gamma(q_\beta(s(c)),0)\}.$$

To see why, suppose $\gamma(q_\beta(s(c)),n_\beta(c))<\gamma(q_\beta(s(c)),0)$. So, $n_\beta(c)$ is larger than the n' at which $\gamma(q_\beta(s(c)),n)$ attains it peak. Thus, γ is decreasing in n at $n_\beta(c)$ and so $n_\alpha(c)\geq n_\beta(c)$ implies that $\gamma(q_\beta(s(c)),n_\beta(c))\geq\gamma(q_\beta(s(c)),n_\alpha(c))$.

So, in this case,

$$\frac{(2n_\alpha+1)\binom{2n_\alpha}{n_\alpha}q_\alpha^{n_\alpha}(1-q_\alpha)^{n_\alpha}}{(2n_\beta+1)\binom{2n_\beta}{n_\beta}q_\beta^{n_\beta}(1-q_\beta)^{n_\beta}} \leq \frac{(2n_\alpha+1)\binom{2n_\alpha}{n_\alpha}q_\alpha^{n_\alpha}(1-q_\alpha)^{n_\alpha}}{(2n_\alpha+1)\binom{2n_\alpha}{n_\alpha}q_\beta^{n_\alpha}(1-q_\beta)^{n_\alpha}}$$
$$= \frac{q_\alpha^{n_\alpha}(1-q_\alpha)^{n_\alpha}}{q_\beta^{n_\alpha}(1-q_\beta)^{n_\alpha}} \to 0;$$

the last line follows from the facts that $q_\alpha(1-q_\alpha) < q_\beta(1-q_\beta)$ and $n_\alpha \to \infty$. Here, $q_\alpha(1-q_\alpha) < q_\beta(1-q_\beta)$ because $q_\alpha > q_\beta \geq 1/2$.

Now the only remaining subsequence is the one along which $n_\alpha(c) < n_\beta(c)$. For such a subsequence, notice that the optimality of the organizer's best reply delivers (see Lemma 2),

$$\binom{2n_\beta}{n_\beta}q_\beta^{n_\beta}(1-q_\beta)^{n_\beta} \geq \frac{2c}{2q_\beta-1},$$

and

$$\binom{2n_\alpha}{n_\alpha}q_\alpha^{n_\alpha}(1-q_\alpha)^{n_\alpha} \leq \frac{c}{2q_\alpha(1-q_\alpha)(2q_\alpha-1)}\frac{n_\alpha+1}{2n_\alpha+1} \leq \frac{c}{2q_\alpha(1-q_\alpha)(2q_\alpha-1)}.$$

Notice that, $2q_\beta - 1 \to 0$, and combining this with $n_\alpha(s(c)) < n_\beta(s(c))$ delivers

$$\lim_{c\to 0}\frac{(2n_\alpha+1)\binom{2n_\alpha}{n_\alpha}q_\alpha^{n_\alpha}(1-q_\alpha)^{n_\alpha}}{(2n_\beta+1)\binom{2n_\beta}{n_\beta}q_\beta^{n_\beta}(1-q_\beta)^{n_\beta}} = 0,$$

which then implies that $\lim_{c\to 0}\max\tilde{\Phi}(s(c),c) = 0$ along such a sequence as well.

Step 2: To show s_α is an attainable limit point.

The proof strategy here is similar to the proof for the existence of manipulated equilibria in Theorem 2.

Using Lemma 8 as before, it is straightforward to show that there exists some small $\bar{\varepsilon} > 0$ such that for every $0 < \varepsilon < \bar{\varepsilon}$, $\lim_{c\to 0}\max\tilde{\Phi}(s_\alpha+\varepsilon, c) = 0$.

We show that there is an $\varepsilon(c) > 0$ with $\lim_{c\to 0}\varepsilon(c) = 0$, such that $\lim_{c\to 0}\max\tilde{\Phi}(s_\alpha+\varepsilon(c), c) = \infty$. Then, the intermediate value theorem for correspondences (Footnote 46) implies that for small c an equilibrium exists that has a cutoff $s(c) \in (s_\alpha+\varepsilon(c), s_\alpha+\varepsilon)$. By the previous step, the limit point of $s(c)$ has to be s_α.

Note that in state β, the organizer recruits no one, for $\varepsilon(c)$ sufficiently small. So our task is to show that $(2n_\alpha+1)\binom{2n_\alpha}{n_\alpha}q_\alpha^{n_\alpha}(1-q_\alpha)^{n_\alpha}$ can be made arbitrarily large, for small c, with cutoff $s_\alpha+\varepsilon(c)$.

Given any integer a and sufficiently small c, let $s(a,c) > s_\alpha$ be such that the organizer is indifferent between recruiting a and $a+1$ pairs of voters when their cutoff strategy is $s(a,c)$. In particular, $g\,(a,x) = 2c$ at $x = q_\alpha(s(a,c))$, with g defined as in the proof of Theorem 2. Since $g\,(a,1/2) = 0$, $g\,(a,q) > 0$ for $q > 1/2$, and g is continuous in q, we can select $s(a,c)$ such that $\lim_{c\to 0} s(a,c) \to s_\alpha$ for every fixed integer a.

Let $s(a) > s_\alpha$ be equal to $\min\{\tilde{s}(a), s_\alpha + \bar{\varepsilon}\}$, where $\tilde{s}(a)$ is the largest signal that has the property that, for every $q \in [1/2, q_\alpha(\tilde{s}(a))]$

$$2 - \frac{a(2q-1)^2}{q(1-q)} \geq 0.$$

For every a, such a $\tilde{s}(a) > s_\alpha$ exists, by inspection of the inequality. Moreover, for sufficiently large a, $s(a) = \tilde{s}(a) < s_\alpha + \bar{\varepsilon}$.

Note that $\lim_{c\to 0} \max \tilde{\Phi}(s(a), c) = 0$ from $s\,(a) > s_\alpha$. Moreover,

$$\lim_{c\to 0} \max \tilde{\Phi}(s(a,c), c) = \lim_{c\to 0} a \binom{2a}{a} q_\alpha(s(a,c))^a (1 - q_\alpha(s(a,c)))^a = a \binom{2a}{a} \left(\frac{1}{2}\right)^a \left(\frac{1}{2}\right)^a,$$

so that from Stirling's approximation,

$$\lim_{a\to\infty} \lim_{c\to 0} \max \tilde{\Phi}(s(a,c), c) = \lim_{a\to\infty} a \frac{4^a}{\sqrt{\pi}\sqrt{a}} \left(\frac{1}{2}\right)^a \left(\frac{1}{2}\right)^a = \infty.$$

Therefore, by the intermediate value theorem for correspondences, for each sufficiently large a, there is a $\bar{c}$ such that for all $c < \bar{c}$, there is $s^*(a,c) \in (s(a,c), s(a))$ such that $1 \in \tilde{\Phi}(s^*(a,c), c)$.

Step 3: To show that an equilibrium sequence exists whose limit point is s_α and for which in state α, the majority selects policy a with probability 1 in the limit.

Note that if $s^* = s_\alpha$, then in state β no one is recruited, and hence there is only one voter for every c. Therefore, as $c \to 0$, the term $(2n_\alpha + 1)\binom{2n_\alpha}{n_\alpha} q_\alpha^{n_\alpha} (1 - q_\alpha)^{n_\alpha}$ converges to a number $k \in (0, \infty)$. Now consider the cutoff $s^*(a,c)$ defined in Step 2, above. Note that the organizer's best reply to this cutoff in state α is to recruit at least a voters, for c small enough. This is because, $s^*(a,c) < \tilde{s}(a)$, where for every $q \in [1/2, q_\alpha(\tilde{s}(a))]$,

$$2 - \frac{a(2q-1)^2}{q(1-q)} \geq 0.$$

The marginal benefit of the organizer at $n = a$ is increasing in q,

$$\frac{d}{dq}\left(\frac{1}{2}\binom{2a}{a}(q(1-q))^a(2q-1)\right) = \frac{1}{2}\binom{2a}{a}(q(1-q))^a[2 - \frac{a(2q-1)^2}{q(1-q)}] > 0, \quad (23)$$

for every $q \in [1/2, q_\alpha(\tilde{s}(a))]$. Hence, the support of the organizer's best reply at state α is bounded from below by a pairs of voters, whenever the voters are using a cutoff between $s(a,c)$ and $\tilde{s}(a)$. Because the equilibrium cutoff $s^*(a,c)$ that we identified is in that interval, the organizer indeed recruits at least a pairs of voters in state α.

Since a is arbitrary, we can construct a sequence of equilibria along which $s(c) \rightarrow s_\alpha$, and the number of voters recruited in state α grows without bound.

Now we show that if $s(c) \rightarrow s_\alpha$ and if the number of voters in state α grows without bound, then the majority selects policy a with a probability that converges to 1. As we stated in the previous paragraph, $n_\beta = 0$ requires that

$$\lim_{c\rightarrow 0}(2n_\alpha + 1)\binom{2n_\alpha}{n_\alpha} q_\alpha^{n_\alpha}(1-q_\alpha)^{n_\alpha} = k \in (0,\infty),$$

where n_α and q_α depend on $s(c)$ and c but we dropped the dependence. Because $n_\alpha \rightarrow \infty$, $s(c) > s_\alpha$. The probability that the majority selects policy a in state α is:

$$\sum_{i=n_\alpha+1}^{2n_\alpha+1}\binom{2n_\alpha+1}{i} q_\alpha^{i}(1-q_\alpha)^{2n_\alpha+1-i}.$$

To show that this probability converges to 1, we use the following lemma.

Lemma 9. *Let* $\{q(c)\}_{c>0}$ *be a selection of probabilities with* $\lim_{c\rightarrow 0} q(c) \rightarrow \frac{1}{2}$, *and* $\{n(c)\}_{c>0}$ *be a selection of integers such that* $\lim_{c\rightarrow 0} n(c) \rightarrow \infty$. *If*

$$\lim_{c\rightarrow 0}(2n(c)+1)\binom{2n(c)}{n(c)} q(c)^{n(c)}(1-q(c))^{n(c)} = k \in (0,\infty),$$

then

$$\lim_{c\rightarrow 0}\sum_{i=0}^{n(c)}\binom{2n(c)+1}{i} q(c)^{i}(1-q(c))^{2n(c)+1-i} = 0.$$

Proof. Pick any pair q, n. Let

$$t(i,n) := \frac{\binom{2n+1}{n+1}q^{n+1}(1-q)^n}{\binom{2n+1}{i}q^i(1-q)^{2n+1-i}} = (\frac{q}{1-q})^{n+1-i}\frac{(2n+1-i)(2n-i)\cdots(n+2)}{n(n-1)\cdots(i+1)}.$$

Note that $t(i,n) > 1$ for $i \leq n$ because $q > 1/2$. Moreover, $t(i,n)$ is decreasing in i.

Pick an arbitrary $\epsilon > 0$. Let $1+\kappa(\epsilon)$ be a lower bound strictly larger than 1 for the term

$$\frac{2n+1-(n(1-\epsilon))}{n(1-\epsilon)+1}.$$

For $i \leq (1-2\epsilon)n$, we have that $t(i,n) \geq (1+\kappa(\epsilon))^{\epsilon n}$. Therefore,

$$\sum_{i=0}^{n} \binom{2n+1}{i} q(n)^i (1-q(n))^{2n+1-i} \leq ((n(1-2\epsilon))(1+\kappa(\epsilon))^{-\epsilon n} + 2\epsilon n) \binom{2n+1}{n} q^{n+1}(1-q)^n.$$

Taking $n \to \infty$, and then using the fact that ϵ was arbitrary, and the fact that

$$(2n(c)+1)\binom{2n(c)}{n(c)} q(c)^{n(c)}(1-q(c))^{n(c)} \to k \in (0,\infty)$$

delivers the result.

$\square$

<u>Step 4:</u> To prove that if inequality (11) holds, then in all sequences of equilibria with limit cutoff s_α the number of voters diverges and information is aggregated.

On the way to a contradiction, suppose that there is an equilibrium sequence with limit cutoff s_α, which has a bounded number of voters in state α, say less than k. Notice that,

$$\liminf_{c \to 0} \sum_{i \geq 0} \tilde{n}_\alpha(c)(i) \times (2i+1)\binom{2i}{i}(q_\alpha(s(c)))(1-q_\alpha(s(c)))^i \geq 1,$$

where $\tilde{n}_\alpha(c)$ is the equilibrium strategy of the organizer. This is because, first, $q_\alpha(s(c)) \to 1/2$, second, $(2i+1)\binom{2i}{i}(1/4)^i$ is strictly increasing in i, and third, $\tilde{n}(c)(i) = 0$ for every $i > k$. Moreover, $\frac{f(s|\alpha)}{f(s|\beta)}$ is continuous in s, and hence, for every $s > s_\alpha$, if (11) holds, then for all s close to s_a,

$$\frac{\rho}{1-\rho}\frac{f(s|\alpha)}{f(s|\beta)} > 1.$$

However, this contradicts the equilibrium requirement that $1 = \Phi(s(c),\text{piv},\text{rec}\,;\tilde{\mathbf{n}}(c), s(c))$. Finally, by Lemma 9, information is aggregated.

<u>Step 5:</u> To prove that if Inequality (11) fails, then there is an equilibrium with limit cutoff s_α and with a bounded number of voters.

Note that $\tilde{\Phi}(s_\alpha, c)$ is single valued for every $c > 0$, and that value is equal to $\frac{\rho}{1-\rho}\frac{f(s_\alpha|\alpha)}{f(s_\alpha|\beta)}$. This is because $\eta(s_\alpha, c)$ has a single element for every $c > 0$, and this single element is a pure strategy that recruits no one in both states.

Hence, if inequality (11) fails, then $\max \tilde{\Phi}(s_\alpha, c) \leq 1$, for every $c > 0$. By the argument in Step 2, there are some a and $\bar{c} > 0$ such that for every $c < \bar{c}$, $\max \tilde{\Phi}(s(a,c), c) > 1$. Therefore, by the intermediate value theorem for correspondences, there is some $s(c) \in [s_\alpha, s(a,c)]$ such that $1 \in \tilde{\Phi}(s(c), c)$. Because

$s(c) < s(a,c)$, and because for all sufficiently small c, $s(a,c) < \tilde{s}(a)$, and because $s(a,c)$ is the cutoff signal to which the organizer's best reply is to recruit at most $a+1$ pairs of voters in state α, the organizer recruits not more than $a+1$ pairs of voters in state α when the voters use the cutoff $s(c)$ (recall that the marginal benefit is increasing in q for $q > \frac{1}{2}$ but close to it, see (23)). Because this is true for every $c < \bar{c}$, and because $\lim_{c\to 0} s(a,c) = s_\alpha$, we can construct a sequence of equilibrium cutoffs that converge to s_α, and along such equilibria, the organizer recruits a bounded number of voters in state α (and no one in state β).

Combining the steps to prove Theorem 3 and the following Remark:

Theorem 3.1 is implied by Step 1. Theorem 3.2 is implied by Theorem 1 (equilibria exist with limit $s^* > s_\beta$) and by Step 2 (equilibria exist with limit $s^* = s_\alpha$). Theorem 3.3 is implied by Step 3 and 4. The remark in the text when (11) fails is implied by Step 5.

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