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Pre-Trade Private Investments

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Abstract

This paper investigates the welfare effects of private investments prior to trade. A seller of a durable good can privately invest on changing its quality. After the investment, she receives a take-it-or-leave-it offer from a buyer. Both the seller and the buyer value more goods of higher quality. We obtain that, in equilibrium, the seller mixes the investment choice, adding adverse selection to the exchange. The non-observability of the investment lowers the buyer's payoff without giving the seller additional rents. Notably, adding buyer competition exacerbates the adverse selection and completely eliminates the trade surplus. Partial observability increases the equilibrium investment, makes the seller better off, and lowers the payoff of the buyer.

Keywords: Private Investment, Hold Up Problem, Price Dispersion.

JEL Classifications: D82, D83, D42, L15

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1 Introduction

One of the main determinants of the value of a firm is the set of assets it owns. It is usually assumed that, in the absence of agency problems, owners manage the firm's assets to maximize future profits. Nevertheless, a firm's owner may change her investment choices if she anticipates she may sell it in the near future. Before the sale, she may be less willing to invest in those of the firm's assets that are neither observed by potential buyers nor contractible in the terms of the deal. The anticipation of such behavior may lower the willingness to pay of potential buyers and the resulting trade probability, and hence partially de-incentivize the owner's opportunistic behavior.¹ Similar considerations apply to owners of durable goods, such as cars or houses, who may decide how carefully they maintain them depending on the sale prospects.

This paper studies the welfare implications of private investments by sellers prior to trade. We show that the possibility of private investments destroys some—and very often all—surplus from trade, and tends to make both sellers and buyers worse off. This finding is shown to be robust to many aspects of the problem, such as the social desirability of the investment, or the level of competitiveness of the buyers' market. Our results then identify one channel through which regulations that increase the transparency of a firm's financial status may foster investment and be welfare improving. Both buyers and sellers may benefit from the subsidization of certification agencies or from enlarging the set of financial terms contractible in transaction agreements, as well as from relaxing the legal constraints for information sharing before horizontal acquisitions take place.

The base model consists of a seller, who owns a good, and a buyer. In the first (investment) stage the seller can invest on increasing the good's quality. After the investment, in the second (trade) stage, the buyer makes a take-it-or-leave-it offer, the seller either accepts it or rejects it, and the game ends. Our base model is similar to that in Gul (2001) (see the literature review for a detailed discussion), the main difference being that both the seller and the buyer value the investment (only the seller does in Gul's model.) We focus on the case where the buyer's value for a "lemon" (a good of the lowest quality) is higher than the value that the good has for the seller in the absence of trade.

¹Mathias Kopp, Executive Chairman at Fusion Industries, describes in his blog the limits of due diligence by buyers of small to medium-sized businesses, and states: "at best due diligence allows potential buyers to develop a fragmented and superficial understanding of the acquisition target. Hence, it does not come as a surprise that frequently the acquired business fails to achieve target and deliver expected return." See <https://www.linkedin.com/pulse/limitations-due-diligence-mathias-kopp>.

The base model has a unique equilibrium trade outcome, defined as the equilibrium distribution of investment levels by the seller and the corresponding conditional distribution of accepted offers. As in Gul (2001), it features investment and price dispersion. We find three novel implications derived from the fact that the investment is valuable to both the seller and the buyer. First, the buyer is strictly worse off with respect to the case where the investment is observable, while the seller obtains the same payoff in both cases. This is true even when the equilibrium investment is more efficient when it is unobservable: the negative effect of the endogenous adverse selection dominates. Second, if trade occurs with positive probability, the seller only chooses investment levels where the surplus from trade is strictly positive. This implies that, in some cases, the equilibrium investment is bounded away from zero. Finally, if there is no knowledge of gains from trade—that is, if there is some investment level where the gains from trade are not positive—the equilibrium expected surplus from trade is zero. In this case, the buyer makes offers which are rejected with probability one.

We compare the previous results with the case where the buyer's market is competitive. In this case, the Bertrand-competition between buyers implies that, in any equilibrium, they do not obtain any surplus from trade. This requires the equilibrium adverse selection at the trade stage be more severe than in the monopsonistic case: qualities are more dispersed and the probability of trade is lower. We obtain that the support of the equilibrium price offers by competitive buyers is not connected, featuring an upper interval of high prices, and an isolated low price. Even though buyers are competitive and there is common knowledge of gains from trade, neither the seller nor the buyers obtain any surplus from trading. Competition is then found to be socially harmful, as it lowers surplus from trade. Differently from the monopsonistic case, now the seller would strictly gain if the investment became observable: the holdup problem would disappear, and the seller would obtain all the social value of the investment.

We finally discuss the effect that adding some partial observability of the seller's investment has on the trade outcome. More concretely, we consider the case where the (monopsonistic) buyer is able to observe the mean of the seller's investment policy (interpreted as the amount of assets owned by the firm, or spending in maintenance of the durable good), but not its complete distribution (interpreted as the riskiness of the assets, or what the money was spent on). We show that—with respect to the case where the investment is totally unobservable—the equilibrium investment increases, the seller is strictly better off and the buyer is strictly worse off.

1.1 Related Literature

The literature on the hold-up problem has extensively studied pre-trade investment by a seller when the buyer has all the bargaining power. The focus of this literature is to analyze whether (and when) the inability of a buyer and a seller to write long-term contracts generates (typically observable) underinvestment by the seller (see Grout (1984) and Tirole (1986) for its first formulations). In this literature, the closest paper to ours is Gul (2001). He considers a setting similar to our base model, assuming that the seller's private investment is valuable only for her (and not for the buyer), and analyzes the effect on trade of different bargaining protocols.² When the buyer makes a take-it-or-leave-it offer after the investment stage, he shows that both the seller and the buyer use mixed strategies in equilibrium, and obtain the same payoff as when the investment is observable. Our base model extends the analysis to the case where both the seller and the buyer value the investment. This allows us to determine the adverse selection of private investments, and to show that they are detrimental to the buyer and do not make the seller better off, even when they are more efficient than when are observable. Sometimes, the buyer makes unacceptable offers even though he is certain about trade being beneficial. We further show that a competitive market may not solve the holdup problem and may, in fact, make it more severe, as all trade surplus may vanish even under common knowledge of presence of gains from trade.³

Our model is also related to the literature on private information gathering before trade, as acquiring information is typically costly and may increase the information asymmetry between buyers and sellers (see Section 5.1). The papers in this literature study, among other aspects, the incentives for disclosure, and the effect of information gathering in the contractual terms⁴ between an agent and a principal.⁴ Our analysis of

²In the Gul (2001)'s model the roles of the seller and the buyer are reversed. Its main focus is on analyzing the case where, in the second stage, the seller makes sequential offers to the buyer. He shows that, if the offers are frequent, there are equilibria where the buyer invests efficiently. Similarly, Hermalin and Katz (2009) show that, when the seller and the buyer engage in repeated trade, more information may foster investment.

³Other papers have studied how some observability of the investment choice affects trade. Most saliently, Lau (2008) shows that a positive probability of the buyer's observing the outcome from investment affects the equilibrium investment level non-monotonically. Online Appendix B considers the case where the total size, but not the riskiness, of the asset stock of the firm is perfectly observable of the buyer. We prove that, in this case, the seller is strictly better off, and the buyer strictly worse off.

⁴For example, Shavell (1994) studies information acquisition and voluntary disclosure before transactions,

private investments by a seller differs from these papers in two crucial aspects. First, the seller can choose her valuation for the firm through investment (or de-investment), but her choice is not observed by the buyer.⁵ Second, the choice of the seller has a direct effect on the buyer's valuation. These two features crucially shape the welfare effects of the private investments. Also, instead of focusing on the incentives for disclosure, we analyze how the competitiveness of the buyers' market and the partial observability of the investment shape the trade outcome.

The organization of the paper is as follows. Section 2 presents our base model, and Section 3 the corresponding the equilibrium analysis. In Section 4 we extend our model to analyze the competitive market. Section 5 discusses some extensions of the model and concludes. The Appendix contains the proofs of all lemmas and propositions of the previous sections.

2 Base Model

There are a seller, who owns an indivisible good, and a potential buyer. They play a game divided in two stages. In the first stage, the seller can invest in order to change the quality (or value) of the good. The outcome of the first stage is not observed by the buyer. In the second stage, the buyer makes a take-it-or-leave-it offer to the seller, the seller accepts it or rejects it, and the game ends.

Investment stage: In the first stage of the game, the seller decides the quality of the good $q \in \mathbb{R}_+$, which coincides with her valuation. The cost of choosing a quality q is $K(q)$, where K is an increasing, strictly convex, continuously differentiable function satisfying $K'(0) < 1$ and $\lim_{q \rightarrow \infty} K'(q) > 1$. Throughout the paper $q^* > 0$ denotes the maximizer of

and obtains that voluntary disclosure increases the incentive to acquire information, while compulsory disclosure benefits the sellers in the market. In a similar setting, Dang (2008) shows that the payoffs of the agents are non-monotonic in the information cost, and that trade may not happen in equilibrium even when the agents maintain symmetric equilibrium information. In a different setting, Crémer, Khalil, and Rochet (1998) study how prior information gathering by an agent affects the contract offered by a principal, and find that the agent sometimes randomizes between acquiring full information or not.

⁵Other papers in this literature analyze the case where the principal observes the amount (or precision) of the information gathering, but not its outcome. Examples are Kessler (1998) and Roesler and Szentes (2016). In these models, the agent typically chooses to remain partially uninformed to avoid making the principal more aggressive. Condorelli and Szentes (2016) obtain a similar result in a model where a buyer chooses the distribution of his valuations, the seller being able to see the choice made.

$q - K(q)$ in \mathbb{R}_+ , which is the unique value satisfying $K'(q^*) = 1$ (note that K' is continuous and strictly increasing from $K'(0) < 1$ to $\lim_{q \rightarrow \infty} K'(q) > 1$).

Trade stage: In the second stage, the buyer makes a take-it-or-leave-it offer to the seller, denoted $p \in \mathbb{R}$. The seller decides either to accept it, $a = 1$, or reject it, $a = 0$.

Payoffs: Fix a quality $q \in \mathbb{R}_+$, a price offer $p \in \mathbb{R}$ and an acceptance decision $a \in \{0, 1\}$. The payoff of the seller is

$$\Pi_S(q, p, a) \equiv (1 - a)q + ap - K(q). \quad (2.1)$$

The buyer's value for the good is $U(q)$, where $U : \mathbb{R} \rightarrow \mathbb{R}$ is a continuously-differentiable, strictly increasing function. We assume that $U(q) - K(q)$ is bounded above and, for convenience, we focus on the generic case where $U(0) \neq 0$. The buyer obtains a payoff equal to 0 if he does not purchase it. So, the payoff of the buyer is

$$\Pi_B(q, p, a) \equiv a(U(q) - p). \quad (2.2)$$

The seller and the buyer maximize their expected utility.

Strategies: A strategy of the seller is a random investment level, $F_q \in \Delta(\mathbb{R}_+)$, where F_q is the cumulative distribution function (CDF) of the quality choice, and an acceptance decision α in the trade stage. The acceptance decision α is a function from the set of investment levels in the first stage and the price offer received from the seller to a probability of accepting the offer. A strategy by the buyer is a distribution over price offers $F_p \in \Delta(\mathbb{R})$.

A strategy profile (F_q, α, F_p) is an *equilibrium* if it is a perfect Bayesian equilibrium. (Note that the “perfection” of the equilibrium concept is necessary to guarantee that the seller acceptance decision to an off-path price offer is optimal.) As usual, an *equilibrium outcome* is a joint distribution of investment levels, prices and acceptance decisions generated by some equilibrium. In the model, some multiplicity will arise from the distribution of price offers that are never accepted in equilibrium. It is then convenient to define an (*equilibrium*) *trade outcome* as the joint distribution of investment choices, trade probabilities and distributions of *accepted* price offers for each equilibrium investment level.

3 Equilibrium Analysis

3.1 Preliminary Results

Fix some equilibrium (F_q, α, F_p) for the rest of the section. For any quality choice q chosen in the investment stage, the expected equilibrium continuation payoff of the seller in the second stage (net of investment costs) is equal to

$$W(q) \equiv q + \int_{[q, +\infty)} (p - q) dF_p(p). \quad (3.1)$$

Thus, in the second stage, the seller obtains a rent when the price is above her valuation for the good, and no rent otherwise. The function $W(\cdot)$ is continuous (even if F_p has mass points), so q is in the support of F_q only if it maximizes $W(\tilde{q}) - K(\tilde{q})$ among all $\tilde{q} \in [0, +\infty)$.

We use \mathcal{Q} to denote the support of F_q , and \mathcal{P} to denote the support of F_p . Furthermore, it is useful to use $\hat{\mathcal{P}}$ to denote the intersection of \mathcal{P} and $[\min \mathcal{Q}, \infty)$, which is interpreted as the set of equilibrium prices which the seller is willing to accept for some quality in the support of F_q . The next result provides a characterization of the supports of the equilibrium strategies.

Lemma 3.1. *In any equilibrium, either $\mathcal{Q} = \{q^*\}$, or there exists some $\underline{q} \in [0, q^*)$ such that $\mathcal{Q} = \hat{\mathcal{P}} = [\underline{q}, q^*]$. Consequently, the equilibrium payoff of the seller is $q^* - K(q^*)$.*

Note first that Lemma 3.1 implies that, in equilibrium, there is no over-investment from the seller's perspective: the seller never chooses a quality above her choice in the absence of trade considerations, q^* . This fact is irrespective of the social desirability of the investment or of how valuable the investment is for the buyer. Its logic is similar to the standard hold-up problem. Indeed, notice that $\max \mathcal{P} \leq \max \mathcal{Q}$, since any price above $\max \mathcal{Q}$ is accepted for sure by the seller. Therefore, if the seller chooses $\max \mathcal{Q}$, either she is indifferent on selling the good (if $p = \max \mathcal{Q}$) or strictly willing to keep it. This implies that the equilibrium payoff of the seller is $\max \mathcal{Q}$ is $\max \mathcal{Q} - K(\max \mathcal{Q})$. This payoff is not strictly lower than the minmax payoff of the seller, $q^* - K(q^*)$, only if $\max \mathcal{Q} = q^*$.

The right inclusion in the statement, $\hat{\mathcal{P}} \subset \mathcal{Q}$, is intuitive. Indeed, any price that the buyer is willing to offer should leave the seller with no information rent for some $q \in \mathcal{Q}$. Otherwise, such a price could be slightly decreased ensuring the same acceptance probability, and therefore inducing a profitable deviation. The left inclusion, $\hat{\mathcal{P}} \supset \mathcal{Q}$ is less obvious, and it is related to the absence of gaps in $\hat{\mathcal{P}}$. To see why the result holds assume, for the sake of contradiction, that $(p_1, p_2) \cap \hat{\mathcal{P}} = \emptyset$ for some prices $p_1, p_2 \in \hat{\mathcal{P}} \subset \mathcal{Q}$ with $p_1 < p_2$. Since $p_1, p_2 \in \mathcal{Q}$, the seller is (weakly) willing to choose a quality equal to

p_1 or p_2 over any other quality $q \in (p_1, p_2)$. Additionally, since $(p_1, p_2) \notin \hat{\mathcal{P}}$, we have that $W(q)$ is linear when $q \in [p_1, p_2]$:

$$W(q) = q + \int_q^{q^*} (p - q) dF_p(p) = W(p_1) + \frac{W(p_2) - W(p_1)}{p_2 - p_1} (q - p_1). \quad (3.2)$$

This makes $W(q) - K(q)$ strictly concave for q in $[p_1, p_2]$, but since $W(p_1) - K(p_1) = W(p_2) - K(p_2)$ this implies that $W(q) - K(q) > W(p_1) - K(p_1)$ for all $q \in (p_1, p_2)$. This is a clear contradiction.

Hold-up Problem

Before analyzing of our base model, we first recover the standard hold-up result: if the outcome of the investment is observable by the buyer, the seller undertakes her individually-optimal investment q^* . Given that the seller knows she is going to be “held up” in the second stage, she maximizes her outside option minus the cost of investing. Thus, trade is not beneficial (or detrimental) for the seller.

Proposition 3.1. *If the investment is observable, the seller chooses q^* . If $U(q^*) > q^*$, then the good is transacted at price q^* (for sure), while if $U(q^*) < q^*$ the good is not transacted. The payoff of the buyer is $\max\{U(q^*) - q^*, 0\}$.*

3.2 Trade Outcomes

This section characterizes the unique equilibrium outcome of our model. We focus on the case where $U(q^*) > q^*$, that is, where there are gains from trade when the investment is q^* . It is not difficult to see that when $U(q^*) < q^*$ there is a unique equilibrium trade outcome: as in the case where investment is unobservable, the probability of trade is 0.

Proposition 3.2. *Assume $U(q^*) > q^*$. There is a unique equilibrium outcome. It satisfies $\mathcal{Q} = \hat{\mathcal{P}} = [\underline{q}, q^*]$, where \underline{q} is equal to the lowest quality level such that there are strictly positive gains from trade in $(\underline{q}, q^*]$.⁶ Furthermore,*

1. F_q is continuous in $[\underline{q}, q^*]$ and satisfies $F_q(p) = F'_q(p) (U(p) - p)$.
2. F_p is continuous in $[\underline{q}, q^*]$ and satisfies $F_p(q) = K'(q)$.

The seller's payoff is $q^ - K(q^*)$ and the buyer's payoff is no higher than $\min_{q \in [\underline{q}, q^*]} (U(q) - q)$.*

⁶Formally, \underline{q} is equal to the minimum of $\{q \leq q^* \mid U(q') > q' \text{ for all } q' > q\}$, and equal to 0 if this set is empty. Since, by assumption, $U(0) \neq 0$, either $\underline{q} > 0$ or there is common knowledge of gains from trade, i.e., $U(q) > q$ for all $q \in [0, q^*]$.

Proposition 3.2 establishes the properties of the unique equilibrium outcome. It features randomization of both the seller and the buyer. The seller uses a mixed strategy which guarantees that, with probability one, trade is strictly beneficial in the trade stage. While (by Lemma 3.1) the seller does not gain from trade, the result establishes that the payoff of the buyer is strictly lower than his payoff if the quality was known and belonged to $(\underline{q}, q^*]$.

Equilibrium Pricing. We begin shedding light on part 2 of the proposition by considering the incentives of the seller. Fix some quality level $q \in [\underline{q}, q^*)$. Choosing $q + \varepsilon \leq q^*$ instead of q , for some small $\varepsilon > 0$, increases the payoff of the seller conditional on keeping the good, but lowers her payoff conditional on selling it. On the one hand, the implied increase the investment cost is $K'(q) \varepsilon + O(\varepsilon^2)$. The effect of the additional investment on the second stage is that the seller's payoff increases by ε when she does not sell the good. Thus, the gain from the additional investment is $F_p(q) \varepsilon + O(\varepsilon^2)$. It is then necessarily the case that, in equilibrium, $K'(q) = F_p(q)$.

Note that if $\underline{q} > 0$ then some equilibrium offers are rejected for sure. Indeed, if all equilibrium price offers were in the range $[\underline{q}, q^*]$, the seller would strictly prefer not to invest than choosing quality \underline{q} . This would allow her to obtain a payoff $\mathbb{E}_p[p]$ instead of $\mathbb{E}_p[p] - K(\underline{q})$. In fact, if $\underline{q} > 0$, the probability that the buyer chooses a price which is rejected for sure is equal to $K'(\underline{q}) \in (0, 1)$.

Equilibrium Investment. We now turn our attention to part 1 of Proposition 3.2. The condition ensures that the quality distribution is such that any price offer in $[\underline{q}, q^*]$ gives the buyer the same payoff. To see this, consider an decrease of the price offer from some $p \in (\underline{q}, q^*]$ to $p - \varepsilon \geq \underline{q}$, for some $\varepsilon > 0$ small. As usual, when the quality is lower than $p - \varepsilon$, the buyer still acquires the good, but now at a lower price. The decrease on the total payment of the buyer for these quality levels is $F_q(p - \varepsilon) \varepsilon$. Additionally, since the price is lower, the seller now does not agree to sell the good when $q \in (p - \varepsilon, p]$. The change on the buyer's payoff from the lower trade is $-F'_q(p) (U(p) - p) \varepsilon + O(\varepsilon^2)$. As a result, the buyer is indifferent between offering $p - \varepsilon$ and p , for any small ε , only if $F_q(p) = F'_q(p) (U(p) - p)$.

As we argued before, the equilibrium quality never exceeds q^* , and this is independent of the value of the socially-optimal investment level. Still, if the gains from trade increase (i.e., if $U(q) - q$ weakly increases for each q , while \underline{q} remains the same), the equilibrium investment is higher—in a first-order stochastic sense—and the good is transacted more often.

Payoffs. As argued in Lemma 3.1, the seller’s payoff is independent of whether the investment is observable or not. The buyer’s payoff, instead, is lower when the investment is not observable. Offering q^* is still optimal, but the quality of the good is lower than q^* with probability one. In fact, for any $q \in [0, q^*]$, the standard take-it-or-leave-it logic dictates that the unique equilibrium continuation in the second stage would give him $\max\{U(q) - q, 0\}$. In our equilibrium, a price equal to p is accepted by the seller only if the actual quality choice is weakly lower than p , so the transaction probability is lower and the buyer obtains a good of lower quality than if q was observable. Hence, if the quality was known, the buyer’s payoff would be higher than his payoff in our equilibrium (strictly if the quality was known to be in $(\underline{q}, q^*]$).

To obtain further intuition, consider first the case where there is common knowledge of gains from trade, that is, $U(q) > q$ for all $q \in [0, q^*]$. In this case, the buyer’s payoff is strictly lower than his payoff when the investment is observable, but it is strictly positive. As a result, $\underline{q} = 0$ and all equilibrium offers are accepted with positive probability. Trade is beneficial in the presence of gains from trade, but the adverse selection from the privacy of the investment lowers its efficiency.

Consider now the case where there is no common knowledge of gains from trade. A remarkable consequence of Proposition 3.2 is that, in this case, the buyer does not obtain any surplus from trade. The buyer is willing to make offers arbitrarily close to \underline{q} , which generate an arbitrarily small gain from trade. Trade is, in this case, socially neutral—neither increases nor decreases the ex-ante payoff of the buyer and the seller. The equilibrium effects of the adverse selection are drastic enough that, even though the gains from trade in the trade stage of the game are strictly positive with probability one, the buyer makes offers which are rejected for sure.

Remark 3.1. In equilibrium, the seller investment choice is, with probability one, below the level she would choose in the absence of trade concerns. This does not necessarily imply that the equilibrium investment is less efficient than when investment is observable. Indeed, the socially optimal investment level maximizes $U(q) - K(q)$. Such an investment level is lower than the seller-optimal investment q^* if, for example, the derivative of U is below $K'(0)$. Nonetheless, even if the equilibrium investment is more efficient when it is not observable by the buyer than when it is, the equilibrium surplus from trade is lower. While in the case where the investment is observable the inefficiency arises from the excessive investment by the seller, in the non-observable case such an inefficiency is lower, but there is an additional large inefficiency from the fact that the good is transacted with a lower probability.

Remark 3.2. Proposition 3.2 shares some similarities with Proposition 1 in Gul (2001), who assumes that the buyer does not value the seller’s investment. This corresponds to the case where U is constant (and higher than q^*), so $\underline{q} = 0$. Our exercise identifies the features that can be generalized, and which of them fail when the buyer value for the good depends on its quality. For example, the finding that the equilibrium quality choice of the seller never exceeds q^* is general: it is independent on how valuable the seller’s investment to the buyer. The lowest equilibrium quality may, instead, be bounded away from 0, so not investing may be strictly suboptimal. Even though there is common knowledge of gains from trade in the trade stage, some equilibrium offers are rejected for sure in some circumstances.

Our results indicate that the welfare implications of private investments may be more severe than previously thought. The buyer is strictly worse off (instead equally well off) with respect to the case where the investment is observable. Most saliently, when there is no common knowledge of gains from trade, he does not gain from trading with the seller. That is, even if trade would be socially beneficial for a wide range of investments, and even though the seller only chooses investment levels where the gains from trade are strictly positive, the equilibrium surplus from trade is zero.⁷ Owed to the adverse selection, trade is not socially beneficial.

4 Competitive Offers

All results presented so far rely on the assumption that there is only one buyer in the market. In this section we study the effects of private investments by a seller in a competitive environment. In particular, we consider the opposite case of a monopsonist: now, in the second stage, two buyers (Bertrand-)compete to buy the good.

We analyze the following game, played by a seller and two buyers. As in our base model in Section 2, the seller decides the quality of the good $q \in \mathbb{R}_+$ at cost $K(q)$ in the investment stage (K satisfies the same conditions as in Section 2). Now, in the trade stage, two buyers simultaneously make take-it-or-leave-it offers. The seller decides then to either accept one of them or to keep the good.

⁷In some applications, no investing severely reduces the gains from trade. In this situation, if there are transaction costs (legal fees, taxes,...), the trade surplus from the sale of a good with the lowest quality may be negative.

A strategy of the seller consists on a distribution of quality choices $F_q \in \Delta(\mathbb{R}_+)$ and an acceptance decision $\beta : \mathbb{R}_+ \times \mathbb{R}^2 \rightarrow \Delta(\{0, 1, 2\})$ (where 0 means not selling the good, while $i \in \{1, 2\}$ means selling it to buyer i), which now depends on the quality chosen and the price offered by each of the buyers. A strategy of buyer $i \in \{1, 2\}$ is a distribution of prices $F_p^i \in \Delta(\mathbb{R})$. Given an outcome $(q, p_1, p_2, b) \in \mathbb{R}_+ \times \mathbb{R}^2 \times \{0, 1, 2\}$, the payoff of the seller is

$$\Pi_S(q, p_1, p_2, b) \equiv \mathbb{1}_{b=0} q + \mathbb{1}_{b=1} p_1 + \mathbb{1}_{b=2} p_2 - K(q) ,$$

and the payoff of the i -buyer, for each $i \in \{1, 2\}$, is

$$\Pi_B^i(q, p_1, p_2, b) \equiv \mathbb{1}_{b=i} (U(q) - p_i) ,$$

where U satisfies the same conditions as in Section 2.

A strategy profile $(F_q, \beta, F_p^1, F_p^2)$ is a *competitive equilibrium* if it is a perfect Bayesian equilibrium. For a given equilibrium $(F_q, \beta, F_p^1, F_p^2)$, it will be useful to use $F_p = F_p^1 F_p^2$ and \mathcal{P} as, respectively, the distribution of the maximal offer and its support. Note that F_p is the relevant distribution to determine the seller's incentives.

To avoid trivial competitive equilibria, we will focus on the case where following assumption holds:

Assumption 1. $q^* - K(q^*) > U(0)$, that is, if the seller anticipates a price equal to $U(0)$, the seller prefers investing optimally and keeping the good.

The proof of Proposition 4.1 shows that if Assumption 1 strictly fails (i.e., whenever $q^* - K(q^*) < U(0)$), the unique competitive equilibrium features no investment and a single price offer equal to $U(0)$. Differently from the monopsony case, if $q^* - K(q^*) < U(0)$ then competitive buyers give enough equilibrium rents to the seller to deter her from investing. In this case, trade happens for sure. Note that the quality chosen by the seller may differ from the socially optimal: 0 may not maximize $U(q) - K(q)$ among all $q \geq 0$.⁸

The following result characterizes the set of competitive equilibria of our model. We focus on the case of common knowledge of gains from trade as it is more illustrating of the effects of introducing buyer competition (see Remark 4.1 below):

⁸For example, in the extreme case where the investment of the seller is invaluable for the buyer (so U is equal to some $\bar{u} > q^* - K(q^*)$), buyers offer \bar{u} for sure in the unique equilibrium, and the seller does not invest. This indicates that in settings where the uninformed part does not value the investment, such as Gul (2001), the competitive equilibrium is trivial.

Proposition 4.1. *Assume Assumption 1 holds and $U(q) > q$ for all $q \in [0, q^*]$. Then, there is some $\underline{q}^{\min} \in (U(0), q^*]$ such that for each $\underline{q} \in [\underline{q}^{\min}, q^*]$ there is a competitive equilibrium $(F_q, \beta, F_p^1, F_p^2)$ satisfying*

1. $\mathcal{Q} = \{0\} \cup [\underline{q}, q^*]$ and the payoff of the seller is $q^* - K(q^*)$;
2. $\mathcal{P} = \{U(0)\} \cup [\underline{q}, q^*]$ and the payoff of both buyers is 0; and
3. parts 1 and 2 of Proposition 3.2 apply to F_q and F_p in $[\underline{q}, q^*]$.

Furthermore, all competitive equilibria satisfy the above properties for some $\underline{q} \in [\underline{q}^{\min}, q^*]$.

Proposition 4.1 establishes that the equilibrium distribution of transaction prices is non-degenerated even under perfect competition. Under Assumption 1, there is no equilibrium without investment: the seller's payoff in a putative equilibrium without investment (equal to $U(0)$) would be lower than the payoff from investing optimally and keeping the good. Instead, in any equilibrium, the supports of the value choices and price offers feature two regions: a higher region with properties that resemble the monopsonist outcome in Proposition 3.2, and a low isolated value.

No gains from trade for the buyers. We first shed light on an important property of all equilibria: buyers do not obtain any gain from trade. This result is expected in Bertrand competition settings where the quality is observable: in these settings, all buyers offer their valuation for the good. In our setting, the quality is endogenous and unknown to the buyers, and this generates a distribution of equilibrium price offers.

An intuition for the proof of the result is obtained in two steps. The first illustrates why buyers obtain the same payoff in any equilibrium. To see this, note that a buyer can offer a price slightly higher than the maximum price offer the other buyer is willing to make. Such a price is accepted for sure, and so the buyer can secure a payoff no lower than the other buyer's payoff. The second step consists on assuming, with the aim of reaching a contradiction, that the buyers' payoff is not zero in some equilibrium. In this case, the minimum offer a buyer is willing to make must be accepted with a positive probability, and so the minimal offers made by each buyers in equilibrium must coincide. Also, it must be that when both buyers make such minimal offer, each of them obtains the good with positive probability. It is then clear that each buyer has the incentive to over-price the other buyer: offering a price slightly higher than the minimum offer significantly increases the probability of the offer being accepted, and hence buyers have a profitable deviation.

No gains from trade for the seller. We proceed illustrating why, even though the buyers' market is competitive, the seller's payoff is equal to his reservation value from not trading. We first claim that, in any equilibrium, the maximum offered price cannot be strictly higher than the highest investment. Heuristically, if there was an equilibrium with $\max \mathcal{P} > \max \mathcal{Q}$, both a price $p \in (\max \mathcal{Q}, \max \mathcal{P})$ and a price equal to $\max \mathcal{P}$ would be accepted with positive probability and, conditional on being accepted, the expected buyer's valuation for the good would be the same in both cases (equal to $\mathbb{E}_q[U(q)]$). Hence, offering p would give a buyer a strictly positive payoff, which would contradict our previous result that the payoff of the buyers is 0 in any equilibrium. We then have that $\max \mathcal{P} \leq \max \mathcal{Q}$. Consequently, notice that if the seller chooses quality $\max \mathcal{Q}$, she is weakly unwilling to sell the good, and so she obtains $\max \mathcal{Q} - K(\max \mathcal{Q})$. We conclude that $\max \mathcal{Q} = q^*$ and that the seller's payoff is equal to $q^* - K(q^*)$. Even though the market is competitive and there is common knowledge of gains from trade, the seller does not benefit from trading.

Low, isolated price. There is a salient property of the equilibrium outcome: the existence of a low, isolated price. We now illustrate why this feature is necessary for ensuring that any offer above $\min \mathcal{P}$ (which is accepted with positive probability) gives a buyer a non-positive payoff in equilibrium.

Notice that buyers obtain a payoff equal to 0 only if $\min \mathcal{P} \geq U(\min \mathcal{Q})$: otherwise, offering a price in the set $(\min \mathcal{Q}, U(\min \mathcal{Q})) \cap (\min \mathcal{P}, U(\min \mathcal{Q}))$ would give a buyer a strictly positive payoff. Consider the seller's incentive to invest. It is clear that the choice of a quality $q \in (0, \min \mathcal{P}]$ is strictly dominated, from the seller's perspective, by choosing 0—in both cases the seller's payoff in the second stage of the game is equal to $\mathbb{E}_p[p]$, but the investment cost of choosing q is strictly higher than choosing 0. Hence, it is necessary that $\min \mathcal{Q} = 0$, $\min \mathcal{P} \geq U(0)$ and, defining $\underline{q} \equiv \min(\mathcal{Q} \setminus \{0\})$, that $\underline{q} > \min \mathcal{P}$. Furthermore, since a price in $[\min \mathcal{P}, \underline{q}]$ is only accepted by the seller, in equilibrium, when the quality is 0, we have $\min \mathcal{P} = U(0)$. We then conclude that, since by Assumption 1 we have that F_p cannot be degenerated at $U(0)$, $\mathcal{P} \setminus \{U(0)\}$ is not empty and $\min(\mathcal{P} \setminus \{U(0)\}) \geq \underline{q} > U(0)$.⁹ The “very low, isolated price” $U(0)$ serves as an equilibrium “punishment” for the seller if she chooses a low investment 0. Choosing $q = 0$ saves investment costs, while choosing $q = \underline{q}$ is costly but increases the value of keeping the good if the price offer is low.

⁹The proof of the proposition shows that, in fact, $\min(\mathcal{P} \setminus \{U(0)\}) = \underline{q}$. The argument is similar to the ones given after Lemma 3.1.

Equilibrium multiplicity. Finally, notice that Proposition 4.1 establishes that there is some multiplicity of competitive equilibria. This multiplicity is, nevertheless, limited. The arguments used in Section 3 apply in $[\underline{q}, q^*]$: the seller's payoff is the same for all quality choices in this region, and the buyers' payoffs are also zero for all prices between \underline{q} and q^* . The multiplicity of equilibria arises because there is a non-degenerated range of values of \underline{q} that support an equilibrium, $[\underline{q}^{\min}, q^*]$, which is determined as follows. Recall that both 0 and \underline{q} are optimal choices for the seller in equilibrium. Hence, the payoff function of the seller in the trade stage, $W(\cdot)$ (see equation (3.1)), must satisfy:

$$K(\underline{q}) = W(\underline{q}) - W(0) = F_p(U(0)) (\underline{q} - U(0)). \quad (4.1)$$

Assumption 1 guarantees that there is some value $F_p(U(0)) < 1$ such that the previous equation holds when $\underline{q} = q^*$. Similarly, when \underline{q} approaches $U(0)$, the right hand side of equation (4.1) is lower than its left hand side for all $F_p(U(0)) \leq F_p(\underline{q}) = K'(\underline{q})$, so no equilibrium exists. The value \underline{q}^{\min} is then obtained as the solution of the previous equation when $F_p(U(0))$ is replaced by $K'(\underline{q})$. Thus, a value $F_p(U(0)) \leq K'(\underline{q})$ satisfying the previous equation exists if and only if $\underline{q} \in [\underline{q}^{\min}, q^*]$.

Remark 4.1. Recall that Proposition 3.2 establishes that, in the absence of common knowledge of gains from trade, the payoff of the monopsonist is zero. In this case, it is not difficult to see that the trade outcomes equilibria in the model with a monopsonist buyer are also trade outcomes in the model with buyer competition. In the proof of Proposition 4.1 we argue that this is indeed the case when Assumption 1 holds and there is no common knowledge of gains from trade.

The Effect of Competition

To see the effect of adding competition in the buyers' market, we now compare the equilibrium outcome of the competitive market in Proposition 4.1 with the one for the monopsonistic market in Proposition 3.2 (when Assumption 1 holds and $U(q) > q$ for all $q \in [0, q^*]$).

We begin by noting that the expected "highest" price is the same in a monopsonistic and a competitive market: $\mathbb{E}_p[p] = q^* - K(q^*)$. This is a consequence of the fact that, in both cases, both 0 and q^* are optimal choices for the seller in equilibrium. The distributions of price offers are, however, remarkably different. In a monopsonistic market, the buyer chooses the price offer using a distribution with an interval (connected) support, with prices ranging from the seller's value for the good in the trade stage if she does not invest, 0, to the value when she invests optimally, q^* . Alternatively, if the buyers' market

is competitive the price distribution is disconnected: there is a low, isolated price, and a range of high prices. Hence, price dispersion may not necessarily be an indicator of the lack of competitiveness of the buyers' market. Instead, a unimodal price distribution—in contrast of a bimodal one—may be.

We continue with the observation that, in comparison with the monopsonist case, the equilibrium distribution of quality levels when the buyers' market is competitive is more extreme. To state this more precisely, fix some $\underline{q} \in [q^{\min}, q^*]$, and let $F_q^c(\cdot)$ and $F_q^m(\cdot)$ denote, respectively, the distribution of investment choices in the (unique) equilibrium outcome when there is a monopolist buyer, and the distribution of investment choices in the (unique for the fixed value \underline{q}) equilibrium outcome when the buyer market is competitive. Then, it is easy to see that there exists some $\hat{q} \in (0, \underline{q})$ such that $F_q^c(q) \geq F_q^m(q)$ for all $q \leq \hat{q}$ and $F_q^c(q) \leq F_q^m(q)$ for all $q \geq \hat{q}$. Hence, $F_q^c(\cdot)$ and $F_q^m(\cdot)$ satisfy the so-called *single crossing condition*, a standard measure of dispersion. The additional dispersion of quality values in the competitive market lowers the buyers' payoff from offering intermediate prices, ensuring that no price gives them a positive expected surplus from trade.

The previous observations imply that, independently degree of competitiveness of the market, trade does not make the seller better off. In both the monopsonistic and competitive cases, the highest equilibrium price is q^* and the seller is willing to invest optimally, so her payoff is equal to $q^* - K(q^*)$. This has an important consequence: in the presence of common knowledge of gains from trade, buyers' competition lowers the social surplus with respect to the monopsonist case (where the seller obtains a positive surplus from trade).¹⁰ The additional adverse selection required to lower the buyers' surplus from trade destroys all gains from trade. Compared to the monopsonistic case, competition lowers the rents obtained by the buyers and does not make the seller better off. Furthermore, the seller strictly benefits from making the investment observable (or contractible) in the competitive case: if investment was observable, she would obtain

$$\max_q (U(q) - K(q)) \geq U(q^*) - K(q^*) > q^* - K(q^*).$$

Remarkably, things are quite different when Assumption 1 fails. In this case, when

¹⁰There are other reasons for why more competition may be detrimental for social welfare. For example, Hauswald and Marquez (2006) show that more competition between sellers may lower their incentive to acquire information, while Polevoy, Smorodinsky, and Tennenholtz (2014) show in a different model that more competition between sellers may lower their incentive to provide precise information. We focus on increased competition between buyers and its effect on the endogenous adverse selection generated by the unobservable investment by the seller.

the buyers' market is competitive, the seller uses a pure strategy in equilibrium (she does not invest at all) and there is no price dispersion (of accepted prices). The absence of adverse selection at the trade stage ensures that trade occurs for sure, and that the seller strictly gains from trade. This contrasts with the case where there is a monopsonistic buyer, where there is a significant equilibrium dispersion of qualities and prices. Hence, low quality goods and low prices may indicate that the buyers' market is competitive.

5 Discussion and Conclusions

5.1 Discussion

Partial observation of the investment: In some situations, the amount of physical and financial assets owned by a firm may be verifiable in a due diligence by potential buyers, but the riskiness of their return may be difficult to assess. For example, a national bank acquiring a regional bank may be able to assess the volume its loans, but their riskiness (which depends on many idiosyncratic details that have to be evaluated case-by-case) may be very costly to determine.¹¹ Also, the amount of information that can be shared before a horizontal acquisition is typically severely limited both because of strategic concerns (the buyer can use the information against the seller if he does not purchase) and different legal constraints.¹² Similar considerations apply for the sale of durable goods, such as cars or houses; buyers only partially observe the investment (some spending is verifiable, for example, through receipts), but not its appropriateness.

Appendix B illustrates the effect of introducing partial observability in our model. We extend our model by assuming that buyers can see the value of the firm's investment, but not its riskiness.¹³ To do this, we separate the seller's investment and the product's

¹¹According to Damodaran (2012), "accounting statements do a reasonably good job of categorizing the assets owned by a firm, a partial job of assessing the values of these assets and a poor job of reporting uncertainty about asset values."

¹²Until the an acquisition deal is closed, antitrust laws treat the two firms as independent companies. Therefore the amount of information that can be made available through a due diligence in a horizontal acquisition is severely limited in practice, as it could serve the firms to coordinate pricing or engage in other joint activities.

¹³See Lau (2008) for a model of stochastic observation of the investment. Our setting may correspond, for example, to a horizontal acquisition of firms in the same market. In this case, the acquiring firm may have limited access to some information because of legal constraints (anti-trust laws apply before the deal is closed) or strategic concerns (the information can be used by the buyer to unfairly compete in the

quality. Thus, the seller decides the distribution of investments $\bar{k} \in \mathbb{R}$, and the distribution of returns F_k with $\mathbb{E}[k|F_k] = \bar{k}$, but the buyer observes only \bar{k} . Before the trade stage, a realized investment k is realized (drawn from F_k), observed only by the seller before the trade stage begins. The resulting quality for the good is $K^{-1}(k)$, and the cost of the investment is k .¹⁴

We show that the additional observability of the investment increases the seller's equilibrium investment choice, which may be above her optimal choice in the absence of trade, $K(q^*)$. Also, she adds a considerable level of riskiness to the investment, even though this would be suboptimal in the absence of trade considerations (since K^{-1} is strictly concave). While, in our base model, she sometimes chooses a high investment level to increase her payoff when the offer made by the buyer is low, she now increases the riskiness of the investment to sell the firm when its outcome is low and to keep it when it is high. In equilibrium, and unlike the previous cases, the seller strictly benefits from trade, while the buyer is strictly worse off than he would have been had the investment and its riskiness been observable, or had the investment been completely unobservable.

Learning: Investing can be reinterpreted, in some settings, as learning about the quality of the good, which is also a costly and private process. Adapting our investment model to accommodate learning requires making different assumptions on the investment technology, since the outcome of a learning process is stochastic and constrained by the Bayes' rule. The early version of this paper in Dilmé (2017) analyzes a variation of our base model where, before meeting the buyer, the seller can learn about its valuation for the good through a costly process of information acquisition.¹⁵ We obtain that the equilibrium outcome is similar to the one we obtain in our base model: (1) after the learning stage the support of the seller's valuations is an interval, and (2) the buyer randomizes the price offer in the support of valuations.

This variation of our model allows us to investigate how the trade outcome depends

future if no deal is reached). Instead, the acquiring firm may have access to some aggregate statistics.

¹⁴Producing a good of quality q requires investing k solving $K^{-1}(k) = q$. Hence, as in our model, the cost of producing a good of quality q is $K(q)$.

¹⁵Even though our base model is written in terms of the seller undertaking the private investment and then meeting a buyer, it can easily be rewritten as a model where the buyer first learns about his valuation of the good and then he meets the seller (and the seller makes the offer). In order to preserve the interpretations as close as possible to our base model, we assume that it is the seller who learns about her valuation of the good. An example of seller's information gathering is given by the sale of firms by entrepreneurs who initially patent a business idea, but they are unsure on how profitable it is.

on the cost of information gathering. For example, it has been claimed that recent technological innovations and public policies may have significantly lowered the cost of learning about products. In the model, in the limit where learning becomes costless, the pricing policy of the buyer becomes extreme: either he offers a very high price (to ensure trade) or he offers a very low price (to obtain the good only when its quality is low). Such an extreme pricing policy lowers the equilibrium value of information gathering and, in consequence, the seller chooses to remain partially uninformed. When the gains from trade are large, the buyer offers prices that are close to the seller's highest valuation to increase the probability of transaction. In this instance, the seller obtains all information rents, and trade happens with probability one. When gains from trade are moderate, the buyer becomes less aggressive than he would be if the seller was perfectly informed about her valuation for the good and offers a high price which, in turn, lowers the incentive of the seller to gather information. Now, the seller is better off when the information-gathering costs are low (but positive) than when she is fully informed. Finally, when the gains from trade are low, the buyer offers prices that are close to the seller's lowest valuation. In the limit, the price distribution coincides with that in the case where the seller is fully informed about her valuation for the good, but the probability of trade is noticeably lower. In this case, the seller is found to be better off when the cost of information gathering is moderate as opposed to very low or very high. This reduces the severity of the equilibrium adverse selection. Our analysis highlights that understanding the incentives in markets with endogenous adverse selection is important to determine the effectiveness of some policies regarding the accessibility of consumers to product information or the privacy of their learning process (monitored sometimes by the sellers using, for example, cookies).

5.2 Conclusions

Private investments may have an important impact on investment choices and trade outcomes. For example, when a sale is likely to occur soon, owners of firms or durable goods may tend to invest less in assets or components that are not observable by potential buyers, or to increase their riskiness. The anticipation of the owner's incentives may make buyers more cautious, lowering their willingness to pay and, as a result, the probability of trade. The additional caution of the buyers may lower the owner's incentive to underinvest. As we have seen, both investment and price dispersion are likely to take place, exacerbating the effects of the adverse selection and lowering the resulting probability of trade.

Our results indicate that the possibility of private investments tends to make both sellers and buyers worse off.¹⁶ On the one hand, buyers suffer from the adverse selection arising from private investments, since sellers gain private information about the value of their goods at the moment of trade. On the other hand, sellers suffer from the additional precaution of the buyers: even if the buyers' market is competitive, the anticipation that price offers are only accepted by the seller when the good has a low value severely diminishes expected trade surplus. We obtain that the seller benefits from trade only when the size of the investment, but not its riskiness, is observable. Thus, our model indicates that regulations regarding the transparency or verifiability of a firm's financial status—or the set of contractible terms in a firm's transaction—, or regarding the buyers' accessibility to the previous history of durable goods, may be welfare improving. Making the investment observable will benefit buyers when they have bargaining power, and will benefit sellers when buyers are competitive. Such actions may not only prevent sellers' suboptimal behavior; they may also increase the probability of efficient trade.

¹⁶Our paper shows that this is robust to different specifications. The early version of this paper in Dilmé (2017) obtains results similar to those in Sections 3 and 4 in the case where the seller's investment is stochastic and takes the form of a sequential process. This may apply, for example, to sequential investments in durable goods (such as houses or cars) or to sequential learning.

A Proofs of the Results

Notation for the Proofs and Preliminary Results

Before beginning the proofs, we introduce some notation and two auxiliary results which are going to simplify the posterior arguments.

The following notation is going to be convenient in the proofs to simplify the notation and shorten the expressions. Fix an equilibrium (F_q, α, F_p) . We let Π_S^* and Π_B^* be, respectively, the equilibrium payoffs of the seller and the buyer. Also, we let $\Pi_S(q)$ and $\Pi_B(p)$ denote, respectively, the payoff of the seller if (on or off the path of play) she chooses a valuation q (given α and F_p) and the payoff of the buyer if he offers (on or off the path of play) a price p (given F_q and α). As in the main text, we let \mathcal{Q} and \mathcal{P} be, respectively, the supports of F_q and F_p , and $\hat{\mathcal{P}}$ be the support of F_p intersected with $[\min \mathcal{Q}, \infty)$. We finally use $\hat{\mathcal{P}}'$ to denote the intersection of $\hat{\mathcal{P}}$ with $F_q^{-1}((0, 1])$, which is equal to $\hat{\mathcal{P}}$ if $F_q(\min \mathcal{Q}) > 0$ and $\hat{\mathcal{P}} \setminus \{\min \mathcal{Q}\}$ if $F_q(\min \mathcal{Q}) = 0$.¹⁷

We first establish a useful property of $\hat{\mathcal{P}}'$. If it is not stated explicitly, the auxiliary lemmas in the proofs of the results in Section 3 presume a fixed equilibrium (F_q, α, F_p) and determine some of its properties.

Lemma A.1. *For any $p \in \hat{\mathcal{P}}'$ there exists a sequence $(p_n)_n$ in $\hat{\mathcal{P}}'$ converging to p such that $\Pi_B(p_n) = \Pi_B^*$ for all $n \in \mathbb{N}$.*

Proof. Assume, for the sake of contradiction, that there is some $p \in \hat{\mathcal{P}}'$ such that there is no sequence $(p_n)_n$ in $\hat{\mathcal{P}}'$ converging to p satisfying that $\Pi_B(p_n) = \Pi_B^*$ for all $n \in \mathbb{N}$. This implies that there is some $\varepsilon > 0$ such that $\Pi_B(p') < \Pi_B^*$ for all $p' \in [p - \varepsilon, p + \varepsilon)$. Since $p \in \hat{\mathcal{P}}$, we have $x \equiv F_p((p + \varepsilon)^-) - F_p((p - \varepsilon)^-) > 0$. Then, we have that $\hat{\Pi}_B \equiv \mathbb{E}_{p'}[\Pi_B(p') | p' \in [p - \varepsilon, p + \varepsilon)] < \Pi_B^*$. Therefore,

$$\Pi_B^* = x \hat{\Pi}_B + (1 - x) \mathbb{E}_{p'}[\Pi_B(p') | p' \notin [p - \varepsilon, p + \varepsilon)] .$$

If $x = 1$ we have a clear contradiction. If, instead, $x < 1$ we have that $\mathbb{E}_{p'}[\Pi_B(p') | p' \notin [p - \varepsilon, p + \varepsilon)] > \Pi_B^*$. Define then the distribution

$$\hat{F}_p(p') = \begin{cases} \frac{1}{1-x} F_p(p') & \text{if } p' < p - \varepsilon , \\ 1 - \frac{1}{1-x} (1 - F_p(p + \varepsilon)) & \text{if } p' \in [p - \varepsilon, p + \varepsilon) , \\ 1 - \frac{1}{1-x} (1 - F_p(p')) & \text{if } p' \geq p + \varepsilon , \end{cases}$$

¹⁷Notice that if $F_q(\min \mathcal{Q}) = 0$ then a price offer equal to $\min \mathcal{Q}$ is rejected with probability one in equilibrium. This is the case when there is no common knowledge of gains from trade (see Proposition 3.2).

Notice that, indeed, \hat{F}_p is a distribution, since

$$\hat{F}_p(p - \varepsilon) - \hat{F}_p((p - \varepsilon)^-) = 1 - \frac{1}{1-x} (1 - F_p((p + \varepsilon)^-)) - \frac{1}{1-x} F_p((p - \varepsilon)^-) = 0 ,$$

so it is weakly increasing and belongs to $[0, 1]$. Notice also that $\mathbb{E}_{p'}[p' | \hat{F}_p] = \mathbb{E}_{p'}[\Pi_B(p') | p' \notin [p - \varepsilon, p + \varepsilon]] > \Pi_B^*$. Thus, the seller has a profitable deviation (by playing \hat{F}_p instead of F_p), which is a contradiction. \square

We now show that any pair of the value and price distributions where there is common knowledge of gains from trade in equilibrium (as it is the case in the equilibria described in Proposition 3.2) generates a unique trade outcome:

Lemma A.2. *Let (F_q, α, F_p) be an equilibrium satisfying that $\Pr(U(q) > q | F_q) = 1$. Define*

$$\hat{\alpha}(q, p) = \begin{cases} 0 & \text{if } q < p, \\ 1 & \text{if } q \geq p. \end{cases} \quad (\text{A.1})$$

Then, $(F_q, \hat{\alpha}, F_p)$ is also an equilibrium, and has the same trade outcome as (F_q, α, F_p) .

Proof. An acceptance decision α is sequentially optimal if and only if $\alpha(q, p) = 1$ when $q < p$ and $\alpha(q, p) = 0$ when $q > p$. It is then clear that $(F_q, \hat{\alpha})$ is a best response to F_p .

We now prove that F_p is a best response to $(F_q, \hat{\alpha})$. The payoff the buyer obtains from offering any price p such that $F_q(p) - F_q(p^-) = 0$ (that is, where F_q does not have a mass point) is the same for both (F_q, α) and $(F_q, \hat{\alpha})$. Assume that F_q has a mass point at p (and so $U(p) > p$) and $\alpha(p, p) < \hat{\alpha}(p, p) = 1$. Take a sequence of prices $(p_n)_n$ strictly decreasing towards p . We have

$$\begin{aligned} \lim_{n \rightarrow \infty} \Pi_B(p_n) &= F_q(p) \mathbb{E}[U(q) - q | q \leq p] \\ &> \underbrace{F_q(p^-) \mathbb{E}[U(q) - q | q < p] + (F_q(p) - F_q(p^-)) \alpha(p, p) (U(p) - p)}_{= \Pi_B(p)}, \end{aligned} \quad (\text{A.2})$$

where we used that $U(p) > p$. The previous expression illustrates that the buyer's payoff (in the equilibrium (F_q, α, F_p)) is no lower than $F_q(p) \mathbb{E}[U(q) - q | q \leq p]$, and that by offering a price equal to p he obtains a strictly lower payoff. Hence, the equilibrium probability with which he offers a price p must be 0 (that is, $F_p(p) - F_p(p^-) = 0$). As a result, F_p is a best response by the buyer to both $(F_q, \hat{\alpha})$ and (F_q, α) . Hence, $(F_q, \hat{\alpha}, F_p)$ is an equilibrium.

Finally, note that $(F_q, \hat{\alpha}, F_p)$ generates the same trade outcome as (F_q, α, F_p) . Indeed, it generates the same distribution of investments. Also, for each p , either the probability

that the seller chooses the quality to be p is 0, or the buyer chooses the price to p with probability 0, or $\alpha(p, p) = 1 = \hat{\alpha}(p, p)$. Hence, each price is offered and accepted with the same probability under both (F_q, α, F_p) and $(F_q, \hat{\alpha}, F_p)$. \square

Proof of Lemma 3.1

Proof. Fix an equilibrium (F_q, α, F_p) . We begin establishing the continuity of Π_S and another property of $\hat{\mathcal{P}}'$.

Lemma A.3. Π_S is continuous, so $\Pi_S(q) = \Pi_S^*$ for all $q \in \mathcal{Q}$. Furthermore, for all $p \in \hat{\mathcal{P}}'$ we have that $F_q(p) \mathbb{E}_q[U(q) - p | q \leq p] = \Pi_B^*$ and $U(p) - p \geq 0$.

Proof. **Proof of the continuity of $\Pi_S(\cdot)$.** To prove that $\Pi_S(q)$ is continuous for all $q \in [0, +\infty)$, fix some $q \in [0, +\infty)$, and notice that

$$\begin{aligned} \Pi_S(q) &= F_p(q) q + (1 - F_p(q)) \mathbb{E}_p[p | p > q] - K(q) \\ &= F_p(q^-) q + (1 - F_p(q^-)) \mathbb{E}_p[p | p \geq q] - K(q), \end{aligned} \quad (\text{A.3})$$

where $F_p(q^-) \equiv \lim_{\tilde{q} \nearrow q} F_p(\tilde{q})$. The equality holds because, whenever the price equals q (which happens with probability $F_p(q) - F_p(q^-)$), the seller is indifferent on selling the firm or not. Take a sequence $q_n \rightarrow q$. Assume first that the sequence is strictly increasing. In this case, using the second equality in equation (A.3), we have

$$\begin{aligned} \lim_{n \rightarrow \infty} \Pi_S(q_n) &= \lim_{n \rightarrow \infty} (F_p(q_n^-) q_n + (1 - F_p(q_n^-)) \mathbb{E}_p[p | p \geq q_n] - K(q_n)) \\ &= F_p(q^-) q + (1 - F_p(q^-)) \mathbb{E}_p[p | p \geq q] - K(q) = \Pi_S(q). \end{aligned}$$

Assume now that the sequence is, instead, decreasing. In this case, the result holds using the first equality of equation (A.3). Then, all $q \in \mathcal{Q}$ give the same payoff to the seller.

Proof that $U(p) - p \geq 0$ for all $p \in \hat{\mathcal{P}}'$. To see this, notice that for any $p \in \mathbb{R}$ we have

$$\begin{aligned} \Pi_B(p) &= F_q(p^-) \mathbb{E}_q[U(q) - p | q < p] + (F_q(p) - F_q(p^-)) \alpha(p, p) (U(p) - p) \\ &= F_q(p) \mathbb{E}_q[U(q) - p | q \leq p] - (F_q(p) - F_q(p^-)) (1 - \alpha(p, p)) (U(p) - p), \end{aligned} \quad (\text{A.4})$$

where recall that $\alpha(q, p)$ is the probability with which the seller accepts the offer p when her value for the firm is q (notice that, when $q = p$, she is indifferent). Assume, for the sake of contradiction, that there is some $p \in \hat{\mathcal{P}}'$ such that $U(p) < p$. By Lemma A.1 and the assumption that U is continuous, we have that there exists some p' such that both

$\Pi_B(p') = \Pi_B^*$ and $U(p') < p'$. Let $p'' < p'$ be such that $U(\tilde{p}) < \tilde{p}$ for all $\tilde{p} \in [p'', p']$. Then

$$\begin{aligned} \Pi_B^* = \Pi_B(p') &\leq F_q(p') \mathbb{E}_q[U(q) - p' | q < p'] \\ &< F_q(p'') \mathbb{E}_q[U(q) - p'' | q \leq p''] \leq \Pi_B(p''), \end{aligned} \quad (\text{A.5})$$

The first inequality holds by because the second term on the right hand side of the first equality in equation (A.4) (applied to p' instead of p) is negative. The second inequality holds because $U(\tilde{p}) - \tilde{p} < 0$ for all $\tilde{p} \in [p'', p']$ and $p'' < p'$. The last inequality holds because the second term on the right hand side of equation (A.4) for p'' is negative. This contradicts that $\Pi_B(p') = \Pi_B^*$.

Proof that $F_q(p) \mathbb{E}_q[U(q) - p | q \leq p] = \Pi_B^*$ for all $p \in \hat{\mathcal{P}}'$. Fix some $p \in \hat{\mathcal{P}}'$. Let $p_n \rightarrow p$ be a sequence such that $\Pi_B(p_n) = \Pi_B^*$ for all n (which exists by Lemma A.1). Such a sequence has a subsequence which can be chosen (taking a subsequence if necessary), without loss of generality for our argument, to be either constant, strictly increasing or strictly decreasing. This gives us three cases:

- Assume first that $\Pi_B(p) = \Pi_B^*$ (this is necessarily the case if, for example, p is an isolated point of $\hat{\mathcal{P}}'$). From equation (A.4) we see that if either $\alpha(p, p) = 1$, $F_q(p) - F_q(p^-) = 0$ or $U(p) - p = 0$, then the result clearly holds. Assume otherwise, that is, assume $\alpha(p, p) < 1$, $F_q(p) - F_q(p^-) > 0$ and $U(p) - p > 0$. In this case, offering $p + \varepsilon$, for $\varepsilon > 0$ small enough, dominates offering p , since

$$\Pi_B(p^+) \equiv \lim_{p' \searrow p} \Pi_B(p') = F_q(p) \mathbb{E}_q[U(q) - p | q \leq p], \quad (\text{A.6})$$

which is the same as the right hand side of equation (A.4) when $\alpha(p, p) = 1$. Since $U(p) - p > 0$ we have $\Pi_B(p^+) > \Pi_B(p) = \Pi_B^*$, so $\Pi_B(p_n) > \Pi_B^*$ for some n , which is a contradiction. Therefore, $\Pi_B(p) = \mathbb{E}_q[U(q) - p | q \leq p]$.

- Assume now that the sequence $(p_n)_n$ is strictly decreasing. In this case, by right continuity of F_q , we have that $\Pi_B(p^+) = \Pi_B^*$. Since, by equation (A.6), $\Pi_B(p^+) = F_q(p) \mathbb{E}_q[U(q) - p | q \leq p]$, the result holds.
- Assume finally that the sequence $(p_n)_n$ is strictly increasing, and that $\Pi_B(p) < \Pi_B^*$. In this case we have that

$$\Pi_B^* = \lim_{n \rightarrow \infty} \Pi_B(p_n) \equiv F_q(p^-) \mathbb{E}_q[U(q) - p | q < p]. \quad (\text{A.7})$$

From equation (A.4) we have that if either $\alpha(p, p) = 0$, or $F_q(p) - F_q(p^-) = 0$ or $U(p) - p = 0$, so $\Pi_B(p) = \Pi_B^*$, a contradiction. Assume then that $\alpha(p, p) > 0$, $F_q(p) - F_q(p^-) > 0$ and $U(p) - p > 0$. In this case, $\Pi_B(p) > \Pi_B^*$, which is again a contradiction.

Hence, necessarily, $\Pi_B(p) = F_q(p) \mathbb{E}_q[U(q) - p | q \leq p]$. \square

(Continuation of the proof of Lemma 3.1)

Assume, for the rest of the proof, that $\mathcal{Q} \neq \{q^*\}$. This implies that $\hat{\mathcal{P}}'$ is non-empty: if it was empty, we would have $\Pi_S(q) = q - K(q)$ for all $q \in \mathcal{Q}$, but then choosing q^* would strictly dominate choosing any other value $q \in \mathcal{Q} \setminus \{q^*\}$.

We first show that $\max \mathcal{Q} = \max \hat{\mathcal{P}}' = q^*$. Clearly $\max \hat{\mathcal{P}}' \leq \max \mathcal{Q}$, since offering a price $p > \max \mathcal{Q}$ is dominated by offering $\frac{\max \mathcal{Q} + p}{2}$. Assume then, for the sake of contradiction, that $\max \hat{\mathcal{P}}' < q^*$. Then, if the seller chooses a value $q = \max \hat{\mathcal{P}}'$, she obtains $q - K(q)$ which is strictly smaller than $q^* - K(q^*) \leq \Pi_S^*$. Therefore, it is clear that $\max \hat{\mathcal{P}}' \notin \mathcal{Q}$, so defining $q' \equiv \max\{q \in \mathcal{Q} | q \leq \max \hat{\mathcal{P}}'\}$ we have that $q' < \max \hat{\mathcal{P}}'$. Also, given that $\max \hat{\mathcal{P}}'$ belongs to the support of F_p , there exists (by Lemma A.1) some $p'' \in (\frac{q' + \max \hat{\mathcal{P}}'}{2}, \max \hat{\mathcal{P}}']$ such that $\Pi_B(p'') = \Pi_B^*$. This is a contradiction, since offering $\frac{q' + \max \hat{\mathcal{P}}'}{2} < p''$ dominates offering p'' , since it is a lower price and guarantees the same probability of acceptance. Therefore, $\max \hat{\mathcal{P}}' = q^*$. This implies that $\Pi_S(q) = q - K(q)$ for all $q \geq q^*$, so choosing q^* dominates choosing any $q > q^*$. Hence, $\max \mathcal{Q} = \max \hat{\mathcal{P}}' = q^*$.

We now show that $\hat{\mathcal{P}}' \subset \mathcal{Q}$. To see this assume, for the sake of contradiction, that $p \in \hat{\mathcal{P}}'$ but $p \notin \mathcal{Q}$. This implies, by Lemma A.1 and since \mathcal{Q} is closed, that there is some $p' \notin \mathcal{Q}$ such that $\Pi_B(p') = \Pi_B^*$. Hence, there exists some $\varepsilon > 0$ such that $(p' - \varepsilon, p' + \varepsilon) \cap \mathcal{Q} = \emptyset$. In this case, the buyer gets a strictly higher payoff offering the price $p' - \varepsilon/2$ than offering the price p' , as it induces the same acceptance probability at a lower price; a contradiction.

We continue the proof by showing that $\min \mathcal{Q} = \inf \hat{\mathcal{P}}'$. Notice first that, by the definition of $\hat{\mathcal{P}}'$, we have $\inf \hat{\mathcal{P}}' \geq \min \mathcal{Q}$. Assume then, for the sake of contradiction, that $\inf \hat{\mathcal{P}}' > \min \mathcal{Q}$. In this case, for all $q \in [0, \inf \hat{\mathcal{P}}']$, we have

$$\begin{aligned} \Pi_S(q) &= F_p(q^-) q + (1 - F_p(q^-)) \mathbb{E}_p[p \geq q] - K(q) \\ &= \underbrace{F_p(0^-) 0 + (1 - F_p(0^-)) \mathbb{E}_p[p \geq 0]}_{=\Pi_S(0)} + F_p(0^-) (q - 0) - K(q). \end{aligned}$$

The previous expression shows that $\Pi_S(q)$ is strictly concave for $q \in [0, \inf \hat{\mathcal{P}}']$, and this implies that Π_S has a unique maximizer in $[0, \inf \hat{\mathcal{P}}']$. Let q' denote such a maximizer. Notice that $q' < \inf \hat{\mathcal{P}}'$ (since, by assumption, $\min \mathcal{Q} < \inf \hat{\mathcal{P}}'$) and therefore $\Pi_S(\inf \hat{\mathcal{P}}') < \Pi_S(q')$. This implies that $\min(\mathcal{Q} \setminus \{q'\}) > \inf \hat{\mathcal{P}}'$, but this contradicts that $\hat{\mathcal{P}}' \subset \mathcal{Q}$. Hence $\min \mathcal{Q} = \inf \hat{\mathcal{P}}'$.

The proof is concluded by noticing that the argument in the main text proves that

$\hat{\mathcal{P}}'$ has no gaps. Then, since by the definition of $\hat{\mathcal{P}}'$ we have that either $\hat{\mathcal{P}} = \hat{\mathcal{P}}'$ or $\hat{\mathcal{P}} = \hat{\mathcal{P}}' \cup \{\min \mathcal{Q}\}$, and $\hat{\mathcal{P}}$ is closed, we have $\hat{\mathcal{P}} = [\min \mathcal{Q}, q^*] \subset \mathcal{Q}$, so in fact $\mathcal{Q} = \hat{\mathcal{P}} = [\min \mathcal{Q}, q^*]$. \square

Proof of Proposition 3.1

Proof. Fix an equilibrium. Assume that the seller chooses (on or off the path of play) some value $q \geq 0$ (which is observed by the buyer). If $U(q) \leq q$ then, it is clearly suboptimal for the buyer to make a price offer $p > q$, since it is accepted for sure by the seller and, as a result, the buyer obtains $U(q) - p < 0$ (while offering a price $p < q$ gives him a payoff equal to 0). This implies that the payoff that the seller expects from choosing q is $q - K(q)$. If, instead, $U(q) > q$ then, in the unique continuation play, the buyer offers a price q and transaction happens for sure. To see why, notice that if the buyer offers a price $q + \varepsilon$, for any $\varepsilon > 0$, the seller accepts the offer for sure. Hence, the payoff of the buyer is, at least,

$$\lim_{\varepsilon \searrow 0} U(q) - (q + \varepsilon) = U(q) - q > 0 .$$

This payoff is clearly higher than offering a price strictly below q (which is rejected for sure). Notice that $U(q) - (q + \varepsilon)$ is strictly decreasing in ε . Also, offering q gives the buyer $U(q) - q$ multiplied by the probability of acceptance (note that the seller is indifferent between accepting and rejecting in this case). As a result, the only possible equilibrium play after the seller chooses q is one where the seller offers a price q and transaction happens for sure. Then, the seller obtains again in this case $q - K(q)$. Thus, since the seller obtains $q - K(q)$ for any choice $q \in [0, \infty)$ and q^* is the unique maximizer of $q - K(q)$, the equilibrium probability that the seller chooses a value q such that $q \neq q^*$ is zero, and $\mathcal{Q} = \{q^*\}$.

Assume that $U(q^*) < q^*$. In this case there is no transaction on the path of play, since when the buyer makes an offer which is accepted with positive probability (and therefore weakly higher than q^*), and therefore making such an offer gives him a negative payoff. Conversely assume that $U(q^*) > q^*$. In this case, as we argued before, the buyer offers for sure a price equal to q^* in equilibrium, and the seller accepts the offer also for sure. \square

Proof of Proposition 3.2

Proof. It is convenient to divide the proof into two steps, depending on whether there is common knowledge of gains from trade or not.

Step 1: We first prove the result for the case where there is common knowledge of gains from trade, that is, for the case where $U(q) > q$ for all $q \in [0, q^*]$. As in the statement of Lemma 3.1, we define $\underline{q} \equiv \min \mathcal{Q}$. We divide this step into four sub-steps.

Step 1a. We begin the proof by noticing that if the buyer obtains a positive payoff in equilibrium. To see this, take some price $p \in (\underline{q}, U(\underline{q}))$. Using equation (A.4) we have that, by offering a price slightly below p , the buyer can obtain a payoff arbitrarily close to

$$\underbrace{F_q(p^-)}_{>0} \left(\underbrace{\mathbb{E}_q[U(q)|q < p]}_{\geq U(\underline{q}) > p} - p \right) > 0 .$$

Therefore, the equilibrium probability that the buyer makes an offer that is rejected for sure is zero; that is, $F_p(\underline{q}^-) = 0$. This implies that $\Pi_B^* > 0$ and, by Lemma 3.1, $\mathcal{P} = [\underline{q}, q^*]$. Notice also that $\Pi_S^* = q^* - K(q^*)$.

Step 1b. We now show that $\underline{q} = 0$. Assume, for the sake of contradiction, that $\underline{q} > 0$. In this case, the payoff of the seller from investing \underline{q} is equal to $\mathbb{E}_p[p] - K(\underline{q})$. If the seller instead chooses 0 she obtains a payoff equal to $\mathbb{E}_p[p]$, which is strictly higher than choosing \underline{q} ; a contradiction. So, $\underline{q} = 0$.

Step 1c. We now prove that part 2 of the statement of the proposition holds. We first verify that, indeed, $F_p(q) = K'(q)$ for all $q \in [0, q^*]$ makes the seller indifferent between choosing any $q \in [0, q^*]$. To see this notice that, if $F_p(q) = K'(q)$, the payoff of the seller from choosing $q \in [0, q^*]$ is

$$\begin{aligned} K'(q) q + \int_q^{q^*} p K''(p) dp - K(q) &= K'(q^*) q^* + \int_q^{q^*} K'(p) dp - K(q) \\ &= K'(q^*) q^* - K(q^*) = q^* - K(q^*) , \end{aligned}$$

where we used that $K'(q^*) = 1$. Now, assume that there is some other distribution F_p that also gives the seller a payoff $q^* - K(q^*)$ for all choices of q in $[0, q^*]$. In this case, we have that, for all q ,

$$\Pi_S^* = F_p(q) q + \int_{(q, q^*]} p F_p(dp) - K(q) = q + \int_{(q, q^*]} (p - q) F_p(dp) - K(q) . \quad (\text{A.8})$$

Let G denote the (not necessarily a probability) distribution of a uniform measure with on $[q, q^*]$ such that $G(p) = p - q$ for all $p \in [q, q^*]$. We can then use the Fubini theorem to switch the order of the terms in the integral term of the right hand side of the second equality of equation (A.8). Using the standard notation for measure theory, and defining

the set $A \equiv \{(x, y) \in (q, q^*]^2 \mid x \leq y\}$, we have

$$\underbrace{\int_{(q, q^*]} \int_{(q, q^*]} \mathbf{1}_A \, dF_p \, dG}_{= \int_{(q, q^*]} (F_p(q^*) - F_p(q)) \, G(dp)} = \int \int \mathbf{1}_A \, d(F_p \times G) = \underbrace{\int_{(v, v^*]} \int_{(q, q^*]} \mathbf{1}_A \, dG \, dF_p}_{= \int_{(q, q^*]} (G(q^*) - G(p^-)) \, F_p(dp)} .$$

where, with some abuse of notation, $G \times F_p$ denotes the product measure from the measures that have G and F_p as distribution functions. From the previous expression we have an integration by parts formula for distributions:

$$\underbrace{\int_{(q, q^*]} G(p^-) \, F_p(dp)}_{= \int_{(q, q^*]} (p-q) \, F_p(dp)} = - \underbrace{\int_{(q, q^*]} F_p(p) \, G(dp)}_{= - \int_{(q, q^*]} F_p(p) \, dp} + \underbrace{F_p(q^*) \, G(q^*) - F_p(q) \, G(q)}_{= q^* - q} ,$$

So, we have

$$\Pi_S^* = - \underbrace{\int_{(q, q^*]} F_p(p) \, dp}_{(*)} + q^* - K(q) .$$

It is then clear that the term $(*)$ is differentiable (since the rest of the terms are differentiable) and, by the Fundamental Theorem of Calculus, its derivative is equal to $F_p(q)$, so

$$K'(q) = F_p(q) .$$

Therefore, part 2 of the statement of Proposition 3.2 holds.

Step 1d. We now prove part 1 of the statement of the proposition. By Lemma A.3 we have that $\mathbb{E}_q[U(q) - p \mid q \leq p] = \Pi_B^*$ for all $p \in [0, q^*]$. We can use again integration by parts (using the same arguments as before) to obtain that, for all $p \in [0, q^*]$ it is the case that

$$\begin{aligned} \Pi_B^* &= \int_0^p (U(\tilde{q}) - p) \, F_q(d\tilde{q}) \\ &= (U(p) - p) \, F_q(p) - (U(0) - p) \, F_q(0) - \int_0^p U'(\tilde{q}) \, F_q(\tilde{q}) \, d\tilde{q} . \end{aligned}$$

It is then clear that $F_q(\cdot)$ is continuous in $[0, p]$ and, using the Fundamental Theorem of Calculus, that it is also differentiable in $(0, p)$. Hence, from the previous expression, $F_q(\cdot)$ satisfies the condition in part 1 of the statement. The proof is concluded observing that, by Lemma A.2, the uniqueness of the value and price distributions (showed above) implies the uniqueness of the trade outcome.

Step 2: We now prove the result for the case where there is no common knowledge of gains from trade, that is, for the case where $U(q) \leq q$ for some $q \in [0, q^*]$. We divide this part of the proof in 3 cases:

1. Consider first the case where $U(q^*) < q^*$. Fix some equilibrium. By Lemma 3.1 we have that $\max \mathcal{Q} \leq q^*$. Assume, for the sake of contradiction, that $\underline{q} \equiv \min \mathcal{Q} < q^*$. This implies, by Lemmas 3.1 and A.1, that the set $\{p \in [\underline{q}, q^*] \mid \Pi_B(p) = \Pi_B^*\}$ is dense in $[\underline{q}, q^*]$. Let $p'' \in (\underline{q}, q^*)$ be such that $U(\bar{p}) < \bar{p}$ for all $\bar{p} \in [p'', q^*]$. Let $p' \in (p'', p^*)$ be such that $\Pi_B(p') = \Pi_B^*$. In this case, equation (A.5) holds, that is, $\Pi_B(p'') > \Pi_B(p')$, and so we have a contradiction. So, if $U(q^*) < q^*$, we have $\mathcal{Q} = \{q^*\}$. Since offering a price equal to q^* is, in this case, suboptimal if it is accepted with positive probability, we have that there is no trade in equilibrium.
2. Consider now the case where $U(q^*) > q^*$. Fix an equilibrium. Using the same arguments as in the proof of Proposition 3.2 we have that, in this case, there is no equilibrium where $\mathcal{Q} = \{q^*\}$. Hence, by Lemma 3.1, there is some $\underline{q} < q^*$ such that $\mathcal{Q} = \mathcal{P} = [\underline{q}, q^*]$. As we show in Step 1d, the buyer is indifferent on offering any price $p \in [\underline{q}, q^*]$ only if F_q is continuous on $[\underline{q}, q^*]$ and $F_q(p) = F'_q(p)(U(p) - p)$ for all $p \in (\underline{q}, q^*)$. So, for each $q \in (\underline{q}, q^*)$, we have

$$F_q(q) = \exp\left(-\int_q^{q^*} \frac{1}{U(\tilde{q}) - \tilde{q}} d\tilde{q}\right). \quad (\text{A.9})$$

Let q^\dagger be the lowest quality level such that there are strictly positive gains from trade in (q^\dagger, q^*) (see footnote 6 for the formal definition). We want to show that $\underline{q} = q^\dagger$. Assume first $q^\dagger > 0$. Given that the derivative of U is bounded on $[q^\dagger, q^*]$ (since U is continuously differentiable), we have that the term on the right hand side of the previous equation (A.9) tends to 0 as q tends to q^\dagger from above, that is, $\lim_{q \searrow q^\dagger} F_q(q) = 0$. Therefore, $\underline{q} \geq q^\dagger$. Assume then, for the sake of contradiction, that $\underline{q} > q^\dagger$. In this case, by offering a price in $(\underline{q}, U(\underline{q}))$ the buyer obtains a strictly positive payoff, so $\Pi_B^* > 0$, and therefore $F_p(\underline{q}) = 0$ (since offering a price $p < \underline{q}$ gives the buyer a payoff equal to 0). Nevertheless, this implies that

$$\Pi_S^* = \Pi_S(\underline{q}) = \mathbb{E}_p[p] - K(\underline{q}) < \mathbb{E}_p[p] = \Pi_S(0),$$

and this is a contradiction. Hence, we have that $\underline{q} = q^\dagger$, $F_q(q^\dagger) = 0$, and $\Pi_S^* = 0$. In this case, it is clear that $q^\dagger \geq 0$. Applying the same argument as before, we have again that $\underline{q} = q^\dagger$, $F_q(q^\dagger) = 0$, and $\Pi_S^* = 0$.

Since, in equilibrium, the probability that the seller chooses a value q such that $U(q) - q > 0$ is one, it is easy to show (proceeding analogously as in the proof of Lemma A.2) that, if an equilibrium exists, the trade outcome is independent of the equilibrium. It is then only left to show that an equilibrium exists. We do this explicitly showing that $(F_q^*, \hat{\alpha}, F_p^*)$ is an equilibrium, with F_q^* characterized by equation

(A.9), \hat{a} by equation (A.1), and F_p^* such that

$$F_p^*(q) = \begin{cases} 0 & \text{if } q < 0, \\ K'(\underline{q}) & \text{if } q \in [0, \underline{q}), \\ K'(q) & \text{if } q \geq \underline{q}. \end{cases}$$

It is clear that \hat{a} is optimal for the seller. We showed that the buyer obtains a payoff equal to zero from offering any price $p \in [\underline{q}, q^*]$. It is then clear that he obtains the same payoff if he offers any price below \underline{q} (since $\min \mathcal{Q} = \underline{q}$), and a negative payoff if he offers a price strictly above q^* , so F_p^* is a best response to F_q^* . Also, as we show in the proof of Proposition 3.2, all value choices $q \in [\underline{q}, q^*]$ give the seller the same payoff. Furthermore, the seller prefers choosing q^* than any $q > q^*$ (since choosing $q > q^*$ gives her a payoff $q - K(q) < q^* - K(q^*)$), and also it is the case that $\Pi_S(0) > \Pi_S(q)$ for all $q \in [0, \underline{q})$. Thus, we finally verify that choosing 0 is not a profitable deviation for the seller. Note first that

$$\begin{aligned} \Pi_S(0) &= F_p(q^-) 0 + (1 - F_p(q^-)) \mathbb{E}_p[p | p \geq \underline{q}] \\ &= \Pi_S(\underline{q}) - F_p(q^-) (\underline{q} - 0) + K(\underline{q}). \end{aligned}$$

Further, we have that $F_p(\underline{q}) = K'(\underline{q})$. So, $\Pi_S(0) \leq \Pi_S^*$ if and only if

$$K(\underline{q}) \leq K'(\underline{q}) (\underline{q} - 0).$$

This condition is satisfied by the convexity of K and the fact that $K(0) = 0$. Hence, F_q^* is a best response to F_p^* .

3. Consider finally the case $U(q^*) = q^* > 0$. Assume first, for the sake of contradiction, that $\mathcal{Q} \neq \{q^*\}$, so by Lemma 3.1 we have that $\mathcal{Q} = [\underline{q}, q^*]$ for some $\underline{q} < q^*$. In this case, using the previous arguments, we have that parts 1 and 2 of the statement of Proposition 3.2 hold in $[\underline{q}, q^*]$. This is a contradiction, since by equation (A.9) we have that $F_q(q) = 0$ for all $q < q^*$. Thus, $\mathcal{Q} = \{q^*\}$. Assume now that there is trade with positive probability. Following a similar argument as in the proof of Proposition 3.1, we have that, necessarily, $F_p((q^*)^-) < F_p(q^*) = 1$ (that is, no price strictly higher than q^* is offered with positive probability, and q^* is offered with positive probability). Notice that if the seller chooses an quality equal to $q^* - \varepsilon$, for some small $\varepsilon > 0$, she obtains a payoff equal to

$$\begin{aligned} \Pi_S(q^* - \varepsilon) &= F_p(q^* - \varepsilon) (q^* - \varepsilon) + (1 - F_p(q^* - \varepsilon)) \mathbb{E}_p[p | p \in (q^* - \varepsilon, q^*]] - K(q^* - \varepsilon) \\ &\geq F_p((q^*)^-) (q^* - \varepsilon) + (1 - F_p((q^*)^-)) \mathbb{E}_p[p | p \in (q^* - \varepsilon, q^*]] - K(q^* - \varepsilon), \end{aligned}$$

where the inequality holds because $q^* - \varepsilon < \mathbb{E}_p[p | p \in (q^* - \varepsilon, q^*]]$ and $F_p(q^* - \varepsilon) \leq F_p((q^*)^-)$. Since F_p has a mass point at q^* , we have $\mathbb{E}_p[p | p \in (q^* - \varepsilon, q^*]] = q^* + o(\varepsilon)$ as $\varepsilon \rightarrow 0$. Therefore, we have that

$$\Pi_S(q^* - \varepsilon) \geq q^* - K(q^*) + (K'(q^*) - F_p((q^*)^-)) \varepsilon + o(\varepsilon).$$

as $\varepsilon \rightarrow 0$. Since $K'(q^*) = 1 > F_p((q^*)^-)$, the previous expression implies that $\Pi_S(q^* - \varepsilon) > q^* - K(q^*)$ for $\varepsilon > 0$ small enough. Therefore, the seller has a profitable deviation, a contradiction. \square

Proof of Proposition 4.1

Proof. Before proving Proposition 4.1, we introduce some notation and preliminary results presented in Lemmas A.4-A.6 (note that Lemma A.6 is not necessary for proving Proposition 4.1, but it is helpful for the discussion in the main text).

In the results below, we assume some competitive equilibrium $(F_q, \beta, F_p^1, F_p^2)$ (assuming it exists). We use \mathcal{Q} , \mathcal{P}_1 and \mathcal{P}_2 to denote, respectively, the supports of F_q , F_p^1 and F_p^2 . Also, as in the main text, \mathcal{P} is the support of $F_p = F_p^1 F_p^2$. Similar to our definition in our base model, we define $\hat{\mathcal{P}}^i$ as all $p \in \mathcal{P}$ such that $p > \max\{\min \mathcal{Q}, \min \mathcal{P}\}$. We let Π_B^{1*} and Π_B^{2*} denote the payoff of buyer $i \in \{1, 2\}$. Define $\underline{p}_i \equiv \min \mathcal{P}^i$ and $\bar{p}_i \equiv \max \mathcal{P}^i$, where \mathcal{P}^i is the support of F_p^i , for each $i = 1, 2$.

Lemma A.4. *Assume $U(q) > q$ for all $q \in [0, q^*]$. Then, in any competitive equilibrium both buyers obtain a payoff equal to 0.*

Proof. In this proof we assume, for the sake of contradiction, that there is a competitive equilibrium where at least one of the buyers obtains a strictly positive payoff. Assume, without loss of generality, that $\Pi_B^{1*} > 0$ and $\Pi_B^{1*} \geq \Pi_B^{2*}$.

Before beginning the proof, we introduce some useful notation. It is useful to define, for each p ,

$$\bar{A}(p) \equiv F_q(p) \mathbb{E}_q[U(q) - p | q \leq p]. \quad (\text{A.10})$$

Note that, by offering a price slightly higher than p , buyer 1's payoff can be made arbitrarily close to $F_p^2(p) \bar{A}(p)$. Hence, the equilibrium payoff of buyer 1 is no lower than

$F_p^2(p) \bar{A}(p)$. The payoff of buyer 1 from offering price p is

$$\begin{aligned} \Pi_B^1(F_q, \beta, p, F_p^2) &= F_p^2(p^-) \overbrace{F_q(p) \mathbb{E}_{q, p_2}[\beta(1|q, p, p_2) (U(q) - p) | p_2 < q, q \leq p]}^{\equiv A_1(p)} \\ &\quad + (F_p^2(p) - F_p^2(p^-)) \underbrace{F_q(p) (\mathbb{E}_q[\beta(1|q, p, p) (U(q) - p) | q \leq p])}_{\equiv B_1(p)}. \end{aligned} \quad (\text{A.11})$$

The term $A_1(p)$ in the previous expression can be written as

$$A_1(p) = \bar{A}(p) - \overbrace{(F_q(p) - F_q(p^-)) \mathbb{E}_{p_2}[\beta(0|p, p_2, p) | p_2 < p] (U(p) - p)}^{\leq 0 \text{ (since, by Step 0, } F_q(p) - F_q(p^-) > 0 \text{ only if } p \leq q^*)} \leq \bar{A}(p), \quad (\text{A.12})$$

where we used that, by sequential rationality, $\beta(1|q, p, p_2) = 1$ whenever $p > \max\{q, p_2\}$ and $\beta(0|p, p_2, p) + \beta(1|p, p_2, p) = 1$ whenever $p_2 < p$. Similar expressions can be obtained for buyer 2 (and we can define $A_2(p)$ and $B_2(p)$ analogously). Note also that

$$B_1(p) + B_2(p) = \bar{A}(p) - F_q(p) (\mathbb{E}_q[\beta(0|q, p, p) (U(q) - p) | q \leq p]) \leq \bar{A}(p). \quad (\text{A.13})$$

The rest of the proof is divided into 3 steps:

Step 0. Quality is lower than q^* . Note that $\max \mathcal{Q} \leq q^*$. Indeed, choosing any quality strictly higher than q^* gives a strictly lower payoff to the seller both when she sells the good (since the investment cost is larger) and when she keeps the good (since q^* is the optimal investment level in this case). It is then clear that $\underline{p}_1 < U(q^*)$.

Step 1. Same payoffs. We first want to show that the payoffs of the two buyers are the same. To do this, we focus on the price \bar{p}_1 , that is, the maximum of the support of prices offered by buyer 1. There are two cases. The first is where buyer 1 offers \bar{p}_1 with probability 0. By Lemma A.1, there is a strictly increasing sequence converging to \bar{p}_1 giving buyer 1 his equilibrium payoff. This implies that the equilibrium payoff of buyer 1 is

$$\Pi_B^{1*} = F_p^2(\bar{p}_1^-) F_q(\bar{p}_1^-) \mathbb{E}_q[U(q) - \bar{p}_1 | q < \bar{p}_1].$$

By offering a price slightly lower than \bar{p}_1 , buyer 2 can obtain a payoff arbitrarily close to

$$F_q(\bar{p}_1^-) \mathbb{E}_q[U(q) - \bar{p}_1 | q < \bar{p}_1] \geq \Pi_B^{1*}.$$

Hence, since the left hand side of the previous equation is weakly lower than Π_B^{2*} (otherwise buyer 2 would have a profitable deviation), and since $\Pi_B^{1*} \geq \Pi_B^{2*}$, we have $\Pi_B^{1*} = \Pi_B^{2*}$ in the first case. The second case is where buyer 1 offers \bar{p}_1 with positive probability, that

is, $F_p^1(\bar{p}_1) - F_p^1(\bar{p}_1^-) > 0$. In this case, $\Pi_B^{1*} = \Pi_B^1(F_q, \beta, p, F_p^2)$. There are now two sub-cases. In the first sub-case, buyer 2 offers \bar{p}_1 with probability 0. We then have

$$F_p^2(\bar{p}_1) \bar{A}(\bar{p}_1) \leq \Pi_B^{1*} = \Pi_B^{1*}(F_q, \beta, \bar{p}_1, F_p^2) \leq F_p^2(\bar{p}_1^-) \bar{A}(\bar{p}_1) = F_p^2(\bar{p}_1) \bar{A}(\bar{p}_1),$$

where the first inequality arises from the fact that buyer 1 can offer a price slightly higher than \bar{p}_1 and obtain a payoff arbitrarily close to $F_p^2(\bar{p}_1) \bar{A}(\bar{p}_1)$, and the second inequality comes from equations (A.11) and (A.12). We conclude that $\Pi_B^{1*} = F_p^2(p) \bar{A}(\bar{p}_1)$. Nevertheless, by offering a price slightly above \bar{p}_1 , buyer 2 can obtain a payoff arbitrarily close to $\bar{A}(\bar{p}_1) \geq \Pi_B^{1*}$, so $\Pi_B^{1*} = \Pi_B^{2*}$ in this sub-case. The second sub-case occurs when $F_p^2(p) - F_p^2(p^-) > 0$. In this case, both buyers offer \bar{p}_1 with positive probability. Hence, we can write

$$\begin{aligned} & \overbrace{\frac{\Pi_B^1(F_q, \beta, \bar{p}_1, F_p^2)}{F_p^2(\bar{p}_1) - F_p^2(\bar{p}_1^-)} + \frac{\Pi_B^2(F_q, \beta, F_p^1, \bar{p}_1)}{F_p^1(\bar{p}_1) - F_p^1(\bar{p}_1^-)}}^{(*)} \\ & \leq \left(\frac{F_p^1(\bar{p}_1^-)}{F_p^1(\bar{p}_1) - F_p^1(\bar{p}_1^-)} + \frac{F_p^2(\bar{p}_1^-)}{F_p^2(\bar{p}_1) - F_p^2(\bar{p}_1^-)} \right) \bar{A}(\bar{p}_1) + B_1(\bar{p}_1) + B_2(\bar{p}_1) \\ & \leq \underbrace{\left(1 + \frac{F_p^1(\bar{p}_1^-)}{F_p^1(\bar{p}_1) - F_p^1(\bar{p}_1^-)} + \frac{F_p^2(\bar{p}_1^-)}{F_p^2(\bar{p}_1) - F_p^2(\bar{p}_1^-)} \right)}_{(**)} \bar{A}(\bar{p}_1), \end{aligned} \quad (\text{A.14})$$

where the first and second inequalities follow from equations (A.12) and (A.13), respectively. Since both buyers offer \bar{p}_1 with a positive probability, we have $\Pi_B^{1*} = \Pi_B^1(F_q, \beta, \bar{p}_1, F_p^2)$ and $\Pi_B^{2*} = \Pi_B^2(F_q, \beta, F_p^1, \bar{p}_1)$. Also, as we argued, $\Pi_B^{1*} \geq F_p^2(\bar{p}_1) \bar{A}(\bar{p}_1)$ and $\Pi_B^{2*} \geq F_p^1(\bar{p}_1) \bar{A}(\bar{p}_1)$. Hence, we have that the term (*) in the previous expression is weakly higher than

$$\underbrace{\left(\frac{F_p^2(\bar{p}_1)}{F_p^2(\bar{p}_1) - F_p^2(\bar{p}_1^-)} + \frac{F_p^1(\bar{p}_1)}{F_p^1(\bar{p}_1) - F_p^1(\bar{p}_1^-)} \right)}_{(***)} \bar{A}(\bar{p}_1).$$

Simple algebra shows that $(**) > (***)$, and so in this sub-case (by equation (A.14)) we have $\bar{A}(\bar{p}_1) = 0$ and $\Pi_B^{1*} = \Pi_B^{2*} = 0$. Then, in all cases we obtain that the payoffs of the buyers coincide, and we use $\Pi_B^* \equiv \Pi_B^{1*} = \Pi_B^{2*}$ to denote them.

Step 2. Same minimum price. We now show that $\underline{p}_1 = \underline{p}_2$. To see this note that if a buyer offers a price below the minimum of the other buyer's support of price offers, such a price is rejected for sure in equilibrium. Offering such a price is strictly suboptimal since we assume $\Pi_B^* > 0$. Let then \underline{p} denote the common minimum of the supports of F_p^1 and F_p^2 .

Step 3. Zero payoff. We conclude this part of the proof by reaching a contradiction and therefore concluding that, necessarily, $\Pi_B^{1*} = \Pi_B^{2*} = 0$ in any equilibrium. We do this by noting that either $F_p^2(\underline{p}) = 0$ or not. Assume first that $F_p^2(\underline{p}) = 0$. Since \underline{p} is in the support of F_p^1 , there exists a sequence of prices $(p_n)_n$ weakly decreasing towards \underline{p} such that buyer 1 is willing to offer them (i.e., each of them gives him the equilibrium expected payoff). His payoff from offering p_n is given by equation (A.11) (replacing p by p_n). By right-continuity of F_p^2 , we have $\Pi_B^1(F_q, \beta, p_n, F_p^2)$ tends to 0 as $n \rightarrow \infty$, which is a contradiction. Note that this implies that both $F_p^1(\underline{p}) > 0$ and $F_p^2(\underline{p}) > 0$.

Now assume that $F_p^1(\underline{p}), F_p^2(\underline{p}) > 0$. Buyer 1's payoff from offering \underline{p} is equal to

$$\begin{aligned} \Pi_B^* &= F_p^2(\underline{p}) F_q(\underline{p}^-) \mathbb{E}_q[\beta(1|q, \underline{p}, \underline{p}) (U(q) - \underline{p}) | q < \underline{p}] \\ &\quad + F_p^2(\underline{p}) (F_q(\underline{p}) - F_q(\underline{p}^-)) \beta(1|\underline{p}, \underline{p}, \underline{p}) (U(\underline{p}) - \underline{p}). \end{aligned}$$

A similar expression can be obtained for buyer 2. Using the fact that $\beta(1|q, \underline{p}, \underline{p}) + \beta(2|q, \underline{p}, \underline{p}) = 1$ whenever $q < \underline{p}$ (that is, the seller accepts for sure one of the offers when both are equal to \underline{p} and $q < \underline{p}$), we can use the previous expression to find

$$\begin{aligned} \Pi_B^* &< \frac{\Pi_B^*}{F_p^2(\underline{p})} + \frac{\Pi_B^*}{F_p^1(\underline{p})} \\ &= \underbrace{F_q(\underline{p}^-) \mathbb{E}_q[U(q) - \underline{p} | q < \underline{p}] + (F_q(\underline{p}) - F_q(\underline{p}^-)) (1 - \beta(0|\underline{p}, \underline{p}, \underline{p})) (U(\underline{p}) - \underline{p})}_{\equiv (*)}. \end{aligned}$$

Take a sequence $(p_n)_n$ strictly decreasing toward \underline{p} . Buyer 1's payoff from offering p_n is no lower than

$$F_q(\underline{p}^-) \mathbb{E}_q[U(q) - p_n | q < \underline{p}] + (F_q(\underline{p}) - F_q(\underline{p}^-)) (U(q) - p_n).$$

The limit of the previous expression is no smaller than (*) (equal when $\beta(0|\underline{p}, \underline{p}, \underline{p}) = 0$). Hence, buyer 1 has a profitable deviation, which is again a contradiction. Hence, the payoff of both buyers is equal to 0. \square

Lemma A.5. *Assume $U(q) > q$ for all $q \in [0, q^*]$. Then, $\mathbb{E}_q[U(q) - p | q \leq p] \leq 0$ for all $p > \max\{\min \mathcal{Q}, \min \mathcal{P}\}$ and $\mathbb{E}_q[U(q) - p | q \leq p] = 0$ for all $p \in \hat{\mathcal{P}}'$.*

Proof. Step 1. Proof that $\mathbb{E}_q[U(q) - p] \leq 0$ for all $p > \max\{\min \mathcal{Q}, \min \mathcal{P}\}$. Fix $p > \max\{\min \mathcal{Q}, \min \mathcal{P}\}$. As we argued in the proof of Lemma A.4, the payoff of buyer 1 is no lower than $F_p^2(p) \bar{A}(p)$ (where \bar{A} is defined in equation (A.10)). Since by Lemma A.4 the payoff of buyer 1 is 0, and since $F_q(p), F_p^2(p) > 0$, we have that $\mathbb{E}_q[U(q) - p | q \leq p] \leq 0$.

Step 2. Proof that $\mathbb{E}_q[U(q) - p] = 0$ for all $p \in \hat{\mathcal{P}}'$. Recall the decomposition of buyer 1's payoff from offering a price p and the definitions of $A_1(p)$ and $B_1(p)$ in equation (A.11).

Assume first that $\Pi_B^1(F_q, \beta, p, F_p^2) = 0$, and consider the following two cases. The first case occurs if $F_p^2(p) - F_p^2(p^-) = 0$. Then, $\Pi_B^1(F_q, \beta, p, F_p^2) = F_p^2(p^-) \bar{A}(p)$ and therefore, since $F_p^2(p^-) > 0$ (because $p > \min \mathcal{P}$), we have that $\bar{A}(p) = 0$. Given the definition of $\bar{A}(p)$ and since $F_q(p) > 0$ (because $p > \min \mathcal{Q}$), we have $\mathbb{E}_q[U(q) - p | q \leq p] = 0$. Consider now the second case, where $F_p^2(p) - F_p^2(p^-) > 0$. Equation (A.12) implies that $A_1(p) \leq \bar{A}(p)$, and so Step 1 implies $A_1(p) \leq 0$. Hence, from expression (A.11) we have

$$\Pi_B^1(F_q, \beta, p, F_p^2) \leq B_1(p) = (F_p^2(p) - F_p^2(p^-)) F_q(p) (\mathbb{E}_q[\beta(1|q, p, p) (U(q) - p) | q \leq p]) .$$

Since buyer 2 obtains an equilibrium payoff equal to 0 (by Lemma A.4) and offers p with a positive probability (since $F_p^2(p) - F_p^2(p^-) > 0$), we have $\Pi_B^2(F_q, \beta, F_p^1, p) = 0$. Repeating the previous argument, we have two sub-cases: either $\mathbb{E}_q[U(q) - p | q \leq p] = 0$ or $F_p^1(p) - F_p^1(p^-) > 0$. In the first sub-case, the result holds. In the second sub-case, we can write

$$0 = \frac{\Pi_B^1(F_q, \beta, p, F_p^2)}{F_p^2(p) - F_p^2(p^-)} + \frac{\Pi_B^2(F_q, \beta, F_p^1, p)}{F_p^1(p) - F_p^1(p^-)} \leq \overbrace{F_q(p) (\mathbb{E}_q[(1 - \beta(0|q, p, p)) (U(q) - p) | q \leq p])}^{(***)} .$$

We can express the term (***) in the previous equation using an expression similar to equation (A.12) as

$$F_q(p) \overbrace{\mathbb{E}_q[U(q) - p | q \leq p]}^{\leq 0} - \overbrace{(F_q(p) - F_q(p^-)) \beta(0|p, p, p) (U(p) - p)}^{\leq 0} \leq 0 .$$

Consequently, (***) = 0, which implies again that $\mathbb{E}_q[U(q) - p | q \leq p] = 0$. We have shown that if $p \in \hat{\mathcal{P}}'$ is such that $\Pi_B^1(F_q, \beta, p, F_p^2) = 0$ then $\mathbb{E}_q[U(q) - p | q \leq p] = 0$.

Take now any $p \in \hat{\mathcal{P}}'$ (not necessarily satisfying $\Pi_B^1(F_q, \beta, p, F_p^2) = 0$). We can use Lemma A.1 to show that there exists a sequence $(p_n)_n$ converging to p such that $\Pi_B^1(F_q, \beta, p, F_p^2) = 0$. As a result, there is a subsequence $(p_{j_n})_n$ of $(p_n)_n$ which is either strictly increasing, strictly decreasing, or constant. If $(p_{j_n})_n$ is strictly decreasing, then we have

$$0 = \lim_{n \rightarrow \infty} \Pi_B^1(F_q, \beta, p_{j_n}, F_{p_{j_n}}^2) = F_q(p) F_p^2(p) \mathbb{E}_q[U(q) - p | q \leq p] ,$$

so the result holds. If the sequence is constant, then $\Pi_B^1(F_q, \beta, p, F_p^2) = 0$, and so again $\mathbb{E}_q[U(q) - p | q \leq p] = 0$. Finally, if the sequence is strictly increasing, then

$$0 = \lim_{n \rightarrow \infty} \Pi_B^1(F_q, \beta, p_{j_n}, F_{p_{j_n}}^2) = F_q(p^-) F_p^2(p^-) \mathbb{E}_q[U(q) - p | q < p] ,$$

and so $\mathbb{E}_q[U(q) - p | q < p] = 0$. Finally note that

$$0 \geq \mathbb{E}_q[U(q) - p | q \leq p] = \frac{F_q(p^-)}{F_q(p)} \overbrace{\mathbb{E}_q[U(q) - p | q < p]}^{=0} + \overbrace{\frac{F_q(p) - F_q(p^-)}{F_q(p^-)} (U(p) - p)}^{\geq 0} \geq 0,$$

where the first inequality comes from Step 1, and the second inequality comes from the fact that, by Step 0 of the proof of Lemma A.4, we have that $F_q(p) - F_q(p^-) > 0$ only if $p \leq q^*$. Hence, $\mathbb{E}_q[U(q) - p | q \leq p] = 0$ for all $p \in \hat{\mathcal{P}}'$. \square

Lemma A.6. *Assume $U(0) > q^* - K(q^*)$ and $U(q) > q$ for all $q \in [0, q^*]$. Then, in any competitive equilibrium, $\mathcal{Q} = \{0\}$ and $\mathcal{P} = \{U(0)\}$.*

Proof. First notice that there is a competitive equilibrium where $\mathcal{Q} = \{0\}$ and $\mathcal{P}^1 = \mathcal{P}^2 = \mathcal{P} = \{U(0)\}$, for any sequentially rational β (one can take, for example, β defined in equation (A.17) below). Indeed, if $\mathcal{P} = \{U(0)\}$ it is clearly suboptimal for the seller to choose any quality other than 0. Further, if a buyer offers a price lower than $U(0)$, such a price is accepted with probability zero. Also, if a buyer offers a price higher than $U(0)$, such a price is accepted for sure, but it gives him a negative payoff.

Assume first that there is a competitive equilibrium where $\mathcal{Q} = \{0\}$ but assume, for the sake of contradiction, that $\mathcal{P} \neq \{U(0)\}$. If $\max \mathcal{P} > U(0)$ then there is a positive equilibrium probability that a price in $(U(0), \max \mathcal{P}]$ is offered and accepted, but this gives a negative payoff to the buyer who offers it. Alternatively, if $\min \mathcal{P} < U(0)$ then offering a price in $(\min \mathcal{P}, U(0))$ would give a positive payoff to one of the buyers, contradicting Lemma A.4.¹⁸ Hence, if $\mathcal{Q} = \{0\}$, we have $\mathcal{P} = \{U(0)\}$.

Assume then for the rest of the proof, and with the aim of finding a contradiction, that there is an equilibrium where $\mathcal{Q} \neq \{0\}$. We first argue that $\min \mathcal{P} \geq U(\min \mathcal{Q})$. Indeed, if it was the case that $\min \mathcal{P} < U(\min \mathcal{Q})$ then offering a price in $(\max\{\min \mathcal{Q}, \min \mathcal{P}\}, U(\min \mathcal{Q}))$ would give a positive payoff to one of the buyers. This implies $\min \mathcal{Q} = 0$, since otherwise choosing 0 would be a profitable deviation for the seller, as she would obtain a payoff $\mathbb{E}_p[p] > \mathbb{E}_p[p] - K(\min \mathcal{Q})$.

Note now that $\min \mathcal{P} < \max \mathcal{P}$. Indeed, if $\min \mathcal{P} = \max \mathcal{P}$, the payoff of the seller from choosing quality $q > 0$ is

$$\max\{q - K(q), \min \mathcal{P} - K(q)\} < \max\{q^* - K(q^*), \min \mathcal{P}\} = \min \mathcal{P},$$

¹⁸We keep some of the arguments intuitive, and they can be easily formalized using Lemma A.5. For example, if $\min \mathcal{P} < U(0)$ (and $\mathcal{Q} = \{0\}$) then we have $\mathbb{E}[U(q) - p | q \leq p] = U(0) - p > 0$ for any $p \in (\min \mathcal{P}, U(0))$. This contradicts Lemma A.5, since $p > \max\{\min \mathcal{Q}, \min \mathcal{P}\}$.

which is strictly lower than the payoff from choosing quality 0 (the last equality follows from the fact that $\min \mathcal{P} \geq U(\min \mathcal{Q}) \geq U(0) > q^* - K(q^*)$). Nevertheless, we assumed that $\mathcal{Q} \neq \{0\}$, so it is necessarily the case that $\min \mathcal{P} < \max \mathcal{P}$.

Let $\bar{q} \equiv \max \mathcal{Q} > 0$. If $\max \mathcal{P} \leq \bar{q}$ then we have¹⁹

$$\Pi_S^* = \Pi_S(\bar{q}) = \bar{q} - K(\bar{q}) \leq q^* - K(q^*) < U(0) \leq \Pi_S(0) \leq \Pi_S^* ,$$

which is a contradiction. Assume, instead, that $\max \mathcal{P} > \bar{q}$. Since, by Lemma A.4, the payoff of the buyers is 0, it is the case that $\max \mathcal{P} = \mathbb{E}_q[U(q)]$. Also, it is the case that offering a price $p \equiv \frac{\max \mathcal{P} + \max\{\bar{q}, \min \mathcal{P}\}}{2} < \max \mathcal{P}$ ensures a positive probability of acceptance, and conditionally on being accepted gives a payoff equal to $\mathbb{E}_q[U(q)] - p > 0$. Since this is a contradiction, we have that if $U(0) > q^* - K(q^*)$ then $\mathcal{Q} = \{0\}$ and, since the buyers' payoff is zero, we have $\mathcal{P} = \{U(0)\}$, so the result holds. \square

(Continuation of the proof of Proposition 4.1)

Assume for the rest of the proof that Assumption 1 holds (that is, $U(0) < q^* - K(q^*)$ and $U(q) > q$ for all $q \in [0, q^*]$). We split the arguments into five steps. For steps 1 to 3 we fix some equilibrium $(F_q, \beta, F_p^1, F_p^2)$ (assuming it exists).

Step 1: Proof that $\min \mathcal{Q} = 0$, $\min \mathcal{P} = U(0)$, and $\min(\mathcal{Q} \setminus \{0\}) = \min(\mathcal{P} \setminus \{U(0)\}) > U(0)$. The same argument as in the proof of Lemma A.6 implies that $\min \mathcal{Q} = 0$ and $\min \mathcal{P} = U(0)$. Also, it is clear that Assumption 1 implies that $\mathcal{Q} \neq \{0\}$. Furthermore, note that $\min(\mathcal{Q} \setminus \{0\}) > U(0)$. Indeed, if $\mathcal{Q} \cap [0, U(0)] \neq \{0\}$ then (by continuity of $\Pi_S(\cdot)$) there exists some value $q \in (0, U(0)]$ such that $\Pi_S(q) = \Pi_S(0)$, but

$$\Pi_S(q) = \mathbb{E}_p[p] - K(q) < \mathbb{E}_p[p] = \Pi_S(0) ,$$

so we have a contradiction. Therefore, defining $\underline{q} \equiv \min(\mathcal{Q} \setminus \{0\})$, we have that $\underline{q} > U(0)$.

Note that $\mathcal{P} \cap [0, \underline{q}] = \{U(0)\}$. To see this recall that $\min \mathcal{P} = U(0)$ and $\mathcal{Q} \cap [0, \underline{q}] = \{0\}$. Hence, when a buyer offers a price $p \in (U(0), \underline{q})$, such a price is accepted with positive probability. Furthermore, such a price p is only accepted by the seller, in equilibrium, when the quality is 0, so it gives the buyer a strictly negative payoff. Note also that, $\max \mathcal{P} > U(0)$, since otherwise $\mathcal{P} = \{U(0)\}$, but then $\mathcal{Q} = \{q^*\}$ and a buyer profitably deviates by offering a price in $(q^*, U(q^*))$. Define $\underline{p} \equiv \min(\mathcal{P} \setminus \{U(0)\})$. It then follows that $\underline{p} \geq \underline{q}$, since offering a price in $(U(0), \underline{q})$ gives a buyer a strictly negative payoff. Assume

¹⁹As in the previous proofs, we use Π_S^* and $\Pi_S(q)$ to indicate, respectively, the equilibrium payoff of the seller and her payoff if he chooses (on or off the path of play) quality q .

then, for the sake of contradiction, that $\underline{p} > \underline{q}$. In this case, $\underline{p} \in \mathcal{Q}$, since otherwise for any price $p < \underline{p}$ such that $[p, \underline{p}] \cap \mathcal{Q} = \emptyset$ we would have

$$0 \geq \mathbb{E}_q[U(q) - p | q \leq p] > \mathbb{E}_q[U(q) - \underline{q} | q \leq \underline{q}] = 0,$$

where the first inequality and the equality follow from Lemma A.5. Thus, given that $\underline{q}, \underline{p} \in \mathcal{Q}$ (so $\Pi_S(\underline{q}) = \Pi_S(\underline{p}) = \Pi_S^*$) and that $(\underline{q}, \underline{p}) \cap \mathcal{P} = \emptyset$, equation (3.2) holds for $q \in [\underline{q}, \underline{p}]$ with \underline{q} instead of p_1 and \underline{p} instead of p_2 . Similar to the argument that follows the equation, we have that $\Pi_S(\cdot) = W(\cdot) - K(\cdot)$ strictly concave on $[\underline{q}, \underline{p}]$, but this is a contradiction. Hence, $\underline{p} = \underline{q}$.

Step 2: Proof that $\mathcal{Q} = \{0\} \cup [\underline{q}, q^*]$ and $\mathcal{P} = \{U(0)\} \cup [q, q^*]$. We first prove that $\bar{q} \geq \bar{p}$ (where $\bar{q} = \max \mathcal{Q}$ and $\bar{p} = \max \mathcal{P}$). Assume, for the sake of contradiction, that $\bar{q} < \bar{p}$. Since $\bar{q} > \max\{\min \mathcal{Q}, \min \mathcal{P}\}$, Lemma A.4 implies that $\mathbb{E}_q[U(q)] - \bar{q} \leq 0$. Also, since $\bar{p} \in \hat{\mathcal{P}}'$ and $\bar{p} > \bar{q}$, Lemma A.4 implies that $\mathbb{E}_q[U(q) - \bar{p}] = 0$. Nevertheless, this implies that for all $p \in (\bar{q}, \bar{p})$ we have

$$\mathbb{E}_q[U(q) - p | q \leq p] = \bar{p} - p > 0,$$

which contradicts Lemma A.4 since $p > \max\{\min \mathcal{Q}, \min \mathcal{P}\}$. Hence, we have $\bar{p} \leq \bar{q}$.

Since $\bar{p} \leq \bar{q}$ we have that $\bar{q} = q^*$ (since, otherwise, the seller's payoff from choosing $\bar{q} \geq \bar{p}$ is $\bar{q} - K(\bar{p}) < q^* - K(q^*)$). Furthermore, $\max(\mathcal{Q} \setminus \{\bar{q}\}) < \bar{p}$ (since the payoff from choosing \bar{p} is $\bar{p} - K(\bar{p}) < q^* - K(q^*)$). A similar argument to that we used to prove that $\bar{p} \leq \bar{q}$ can be now applied to show that $\bar{p} \leq \max(\mathcal{Q} \setminus \{\bar{q}\})$, so we have a contradiction. Then, $\bar{q} = \bar{p} = q^*$.

The fact that \mathcal{Q} and \mathcal{P} have no gaps in $[\underline{q}, q^*]$ is obtained using arguments similar to those used in the proof of Lemma 3.1. We first prove that $\hat{\mathcal{P}}' \subset \mathcal{Q}$ by assuming, for the sake of contradiction, that $p \in \hat{\mathcal{P}}'$ exists such that $p \notin \mathcal{Q}$. Then, for any $p' < p$ such that $[p', p] \cap \mathcal{Q} = \emptyset$ and $p' > U(0)$, we have

$$0 = \mathbb{E}_q[U(q) - p | q \leq p] = \mathbb{E}_q[U(q) - p | q \leq p'] < \mathbb{E}_q[U(q) - p' | q \leq p'].$$

This contradicts Lemma 3.1, since $p' > \max\{\min \mathcal{Q}, \min \mathcal{P}\}$, and so we have $\hat{\mathcal{P}}' \subset \mathcal{Q}$. The fact that $\mathcal{Q} \setminus \{0\} \subset \hat{\mathcal{P}}'$ and the absence of gaps of \mathcal{Q} in $[\underline{q}, q^*]$ follows the same argument as the one exposed in the main text after Lemma 3.1, since only the incentives of the seller are used in this argument.

Step 3: Existence of q^{\min} and F_p . The proof continues by showing that there is some $q^{\min} \in (U(0), q^*)$ with the property that for all $q \in [q^{\min}, q^*]$ there exists a distribution F_p

(which depends on \underline{q}) such that the seller has a best response F_q with support $\{0\} \cup [\underline{q}, q^*]$, and there is no such distribution if $\underline{q} \notin [\underline{q}^{\min}, q^*]$. Therefore, it is a necessary condition for equilibrium that $\underline{q} \in [\underline{q}^{\min}, q^*]$.

The previous arguments in this proof show that if $U(0) < q^* - K(q^*)$ then, in each competitive equilibrium, there exists some $\underline{q} \in (U(0), q^*]$ such that $\mathcal{Q} = \{0\} \cup [\underline{q}, q^*]$ and $\mathcal{P} = \{U(0)\} \cup [\underline{q}, q^*]$. The same arguments used in the proof of Proposition 3.2 can be used to show that F_q is continuous on $[\underline{q}, q^*]$ and satisfies $F_q(p) = F'_q(p) (U(p) - p)$ (now the “indifference” condition of the buyer is replaced by the result in Lemma A.4 that $\mathbb{E}_q[U(q) - p | q \leq p] = 0$ for all $p \in \hat{\mathcal{P}}'$) and F_p is continuous in $[\underline{q}, q^*]$ and satisfies $F_p(q) = K'(q)$ (and all prices in $[\underline{q}, q^*]$ give the same payoff to the seller).

We now show that there is some $\underline{q}^{\min} < q^*$ such that, if $\underline{q} \in [\underline{q}^{\min}, q^*]$ there is a distribution F_p with support equal to $\{U(0)\} \cup [\underline{q}, q^*]$ that makes optimal for the seller to choose any value in $\{0\} \cup [\underline{q}, q^*]$, and such a distribution does not exist if $\underline{q} < \underline{q}^{\min}$. Fix some $\underline{q} \leq q^*$, and assume that a distribution F_p that makes it optimal for the seller to choose any quality in $\{0\} \cup [\underline{q}, q^*]$ exists. As we showed before, this implies that $F_p(p) = K'(p)$ for all $p \in [\underline{q}, q^*]$. Notice that, since $\Pi_S(0) = \Pi_S(\underline{q}) = \Pi_S^*$, the following equation holds

$$\mathbb{E}_p[p] = \Pi_S(0) = \Pi_S(\underline{q}) = F_p(U(0)) \underline{q} + (1 - F_p(U(0))) \mathbb{E}_p[p | p \geq \underline{q}] - K(\underline{q}).$$

Additionally, we have that

$$\mathbb{E}_p[p] = F_p(U(0)) U(0) + (1 - F_p(U(0))) \mathbb{E}_p[p | p \geq \underline{q}].$$

Then, the previous expressions can be used to obtain

$$F_p(U(0)) (\underline{q} - U(0)) = K(\underline{q}).$$

Furthermore, since $F_p(\underline{q}) = K'(\underline{q})$, we have

$$K'(\underline{q}) = F_p(\underline{q}) \geq F_p(U(0)) = \frac{K(\underline{q})}{\underline{q} - U(0)}. \quad (\text{A.15})$$

Notice that the previous inequality holds when $\underline{q} = q^*$ (since $K'(q^*) = 1$ and, by assumption, $U(0) < q^* - K(q^*)$), and the reverse inequality holds if \underline{q} is close enough to $U(0)$ (since the term on the right hand side of the last equality becomes arbitrarily large). Rearranging the terms we have that

$$\frac{d}{d\underline{q}} ((\underline{q} - U(0)) K'(\underline{q}) - K(\underline{q})) = (\underline{q} - U(0)) K''(\underline{q}) > 0.$$

Therefore, there exists a unique $\underline{q}^{\min} \in (U(0), q^*)$ such that equation (A.15) holds for all $\underline{q} \in [\underline{q}^{\min}, q^*]$, and does not hold when $\underline{q} \in (U(0), \underline{q}^{\min})$. It is also clear that, in any equilibrium, $\mathcal{Q} = \{0\} \cup [\underline{q}, q^*]$ for some $\underline{q} \in [\underline{q}^{\min}, q^*]$ (that is, $\underline{q} \in [\underline{q}^{\min}, q^*]$ is a necessary condition for the seller to be indifferent on offering any price in $[\underline{q}, q^*]$).

Step 4: Necessity. We now show that if there is an equilibrium $(F_q, \beta, F_p^1, F_p^2)$ where $\mathcal{Q} = \{0\} \cup [\underline{q}, q^*]$ for some $\underline{q} \in [\underline{q}^{\min}, q^*]$ (which we showed is a necessary condition), then such an equilibrium satisfying the properties established in the statement of Proposition 4.1. The previous argument shows that there is a unique distribution of maximum prices F_p that makes the seller indifferent on choosing any quality in \mathcal{Q} and weakly unwilling to choose any quality outside \mathcal{Q} . Such distribution satisfies $F_p(q) = K'(q)$ for all $q \in (\underline{q}, q^*)$, so satisfies the properties established in the statement of the proposition. It is then necessary that F_p^1 and F_p^2 are such that $F_p^1 F_p^2 = F_p$. By Lemma A.4, we have that $\mathbb{E}_q[U(q) - p | q \leq p] = 0$ for all $p \in [\underline{q}, q^*]$. It is also clear that $\mathbb{E}_q[U(q) - U(0) | q \leq U(0)] = 0$. This requires that F_q satisfies part 1 of Proposition 3.2 in $[\underline{q}, q^*]$. Note that by Lemma A.4 we have that

$$0 = \mathbb{E}_q[U(q) - \underline{q} | q \leq \underline{q}] = \frac{F_q(0)}{F_q(\underline{q})} (U(0) - \underline{q}) + \frac{F_q(\underline{q}) - F_q(0)}{F_q(\underline{q})} (U(\underline{q}) - \underline{q}). \quad (\text{A.16})$$

To see that $F_q(0) \in [0, F_q(\underline{q})]$ satisfying the previous equation exists, define the following equation for $\gamma \in [0, 1]$ and $q \in [U(\underline{q}), q^*]$:

$$\gamma U(0) + (1 - \gamma) U(q) = q.$$

Notice that, for each $q \in (U(\underline{q}), q^*]$, there is a unique value $\gamma(q) \in [0, 1)$ such that the previous equation holds. Indeed, if $\gamma(q) = 0$ then left hand side is strictly higher than the right hand side, while if $\gamma(q) = 1$ the opposite is true. Hence, for each $\underline{q} \in [\underline{q}^{\min}, q^*]$ we have $F_q(0) = F_q(\underline{q}) \gamma(\underline{q})$.

Step 5: Sufficiency. We finally show that for all $\underline{q} \in [\underline{q}^{\min}, q^*]$ there is an equilibrium $(F_q, \beta, F_p^1, F_p^2)$ where $\mathcal{Q} = \{0\} \cup [\underline{q}, q^*]$ (and so, by the previous argument, such an equilibrium satisfies the properties in the statement of the proposition). We let F_q be the unique distribution that satisfies the properties of the statement in the proposition and condition (A.16) (which we proved to be necessary). We let β be the unique satisfying

$$\beta(i | q, p_1, p_2) \equiv \begin{cases} 0 & \text{if } p_i < \max\{q, p_{\bar{i}}\}, \\ 1 & \text{if } p_i = q > p_{\bar{i}}, \\ \frac{1}{2} & \text{if } p_i = p_{\bar{i}} \geq q, \\ 1 & \text{if } p_i > \max\{p_{\bar{i}}, q\}. \end{cases} \quad (\text{A.17})$$

Note that such β is sequentially optimal for the seller, is such that the seller sells the good for sure when she is indifferent on doing so, and treats both buyers symmetrically. Finally, we let both F_p^1 and F_p^2 be equal to $(F_p)^{1/2}$ (where F_p is the unique distribution obtained in Step 3), and so we have $F_p = F_p^1 F_p^2$. We have to verify that the buyers do not have an incentive to deviate (notice that we have verified above that the seller best-responds to F_p).

Clearly, offering any price in $[0, U(0)]$ gives a buyer a payoff equal to 0. A price in $(U(0), \underline{q})$ is accepted with positive probability in equilibrium only when the quality is 0, so it gives the buyer a negative payoff. By construction, offering a price in $[\underline{q}, q^*]$ gives a buyer a payoff equal to 0.²⁰ Finally, offering a price p strictly above q^* gives a buyer a negative payoff equal to

$$\mathbb{E}_q[U(q)] - p < \mathbb{E}_q[U(q)] - q^* = 0 .$$

where we used that, by construction, $\mathbb{E}_q[U(q)] = q^*$.

Step 6: No common knowledge of gains from trade. Assume now that there is no common knowledge of gains from trade, that is, there is some $q \in [0, q^*]$ such that $U(q) \leq q$. Proceeding as in Step 5 it is easy to show that for all equilibria when the buyer is a monopsonist, described in Proposition 3.2, there is a competitive equilibrium with the same trade outcome. \square

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²⁰For example, offering \underline{q} gives a buyer a payoff equal to $(F_p(q^-) + \frac{1}{2}(F_p(\underline{q}) - F_p(q^-))) \mathbb{E}_q[U(q) - \underline{q} | q \leq \underline{q}]$, which is equal to 0 by Lemma A.5.

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B Online Appendix: Observable Value of the Assets

In this appendix we analyze how partial limitations on the observability of the characteristics of the assets held by a firm affect the owner’s incentives and the trade outcome, discussed in Section 5.1. More concretely, we investigate the seller’s incentive to increase the riskiness of the capital stock of a firm before trade: if the investment is successful, she can keep the firm, while if it is not she may have the option to sell.

We modify our base model in Section 2 as follows. Now, in the investment stage, the seller first chooses the mean investment (or “total size” firm’s assets), \bar{k} , and then the distribution of investment outcomes (or returns), $F_k(\cdot; \bar{k}) \in \Delta(\mathbb{R}_+)$ satisfying $\mathbb{E}_k[k|F_k(\cdot; \bar{k})] = \bar{k}$.²¹ At the beginning of the trade stage, the buyer observes the mean investment and decides a distribution of price offers $F_p(\cdot; \bar{k})$. Finally the seller, after observing the outcome of the investment stage and the price offer of the buyer, decides to accept it or reject it.

When the investment outcome is k , the seller’s value for the firm is $Q(k)$, where $Q(\cdot) \equiv K^{-1}(\cdot)$ and where K satisfies the same properties as in our base model. Notice that since the seller’s valuation for the outcome of the investment outcome $Q(\cdot)$ is strictly concave, the seller would not add any riskiness to the assets of the firm in the absence of trade motives.

It is straightforward to generalize the strategies of the seller and the buyer. The payoffs for the seller and the buyer if the seller chooses a total size of the firm’s assets \bar{k} , a distribution F_k with mean equal to \bar{k} , the realized investment outcome is k , the buyer offers p and the decision of the seller is a , is given by²²

$$\begin{aligned}\Pi_S(\bar{k}, F_k, k, p, a) &\equiv (1 - a) Q(k) + a p - \bar{k} , \\ \Pi_B(\bar{k}, F_k, k, p, a) &\equiv a (U(Q(k)) - p) .\end{aligned}$$

The following result characterizes the equilibria of the model with observable investment. We focus for simplicity on the case where there is common knowledge of gains from trade.

²¹We implicitly assume that there is a rich set of assets accessible to the seller, all of them giving the same expected return (equal to 0).

²²Note that the distribution F_k only influences the payoff of the seller and the buyer through the investment outcome. It is not difficult to verify that, in fact, if \bar{k} is replaced by k on the right hand side of the expression for the payoff of the seller, the set of equilibria remains unchanged. This implies that, except for the observability of the asset size, the new setting is equivalent to our base model.

Proposition B.1. *Assume that the investment mean is observable and that $U(q) > q$ for all $q \in [0, +\infty)$. Then, in any equilibrium, there exists a unique value $\bar{q} > q^*$ such that $\mathcal{Q} = \mathcal{P} = [0, \bar{q}]$. Furthermore, Proposition 3.2 applies replacing $[q, q^*]$ by $[0, \bar{q}]$, now with $F_p(q) = K'(q)/K'(\bar{q})$.*

Proposition B.1 illustrates how the observability of the total size of the firm's assets (but not their riskiness) influences the trade outcome. We obtain that, even though increasing riskiness of the assets would be suboptimal for the seller in the absence of trade considerations (recall that $Q(\cdot)$ is strictly concave), she exposes herself, in equilibrium, to a remarkably large risk. The riskiness is large enough that there is a positive probability that the firm loses all its value (it is easy to verify that $F_q(0) > 0$), and also there is a positive probability that the firm ends up being more valuable than its value under the seller-optimal investment level, q^* .

The intuition for the result is obtained as follows. As we argued before, we separate the problem of the seller into two parts. The first consists on choosing the total size of the firm's assets, \bar{k} , and the second consists on choosing its riskiness (or distribution conditional on the mean). The proof of Proposition B.1 shows that, in fact, for each choice \bar{k} , there is a unique continuation trade outcome, with distributions denoted using $(F_k(\cdot; \bar{k}), F_p(\cdot; \bar{k}))$ (with $\mathbb{E}_k[k|F_k(\cdot; \bar{k})] = \bar{k}$). Furthermore, the pair $(F_k(\cdot; \bar{k}), F_p(\cdot; \bar{k}))$ is such that Proposition B.1 applies replacing \bar{q} by $Q(\hat{k}(\bar{k}))$, for some continuous and strictly increasing function $\hat{k}(\cdot)$ such that $\hat{k}(\bar{k}) > \bar{k}$ for all $\bar{k} > 0$, and $F_p(q; \bar{k}) = K'(q)/K'(Q(\hat{k}(\bar{k})))$. Using \bar{k}^{no} to denote the unique value satisfying $\hat{k}(\bar{k}^{\text{no}}) = k^* \equiv K(q^*)$, we have that the distributions $F_q(\cdot; \bar{k}^{\text{no}})$ and $F_p(\cdot; \bar{k}^{\text{no}})$ coincide with those of the equilibrium outcome when the total size of the firm's assets is not observable, described in Proposition 3.2.

Fix a choice of the total size of the firm's assets \bar{k} (and given the continuation play of the buyer, $F_p(\cdot; \bar{k})$). It is not difficult to see (see the proof of Proposition B.1) that, since $\bar{k} \in \text{supp}(F_q(\cdot; \bar{k}))$, it is optimal for the seller not to add any riskiness. More formally,

$$\mathbb{E}_p[\max\{Q(\bar{k}), p\}|F_p(\cdot; \bar{k})] = \mathbb{E}_k[\mathbb{E}_p[\max\{Q(k), p\}|F_p(\cdot; \bar{k})]|F_k(\cdot; \bar{k})] . \quad (\text{B.1})$$

The result is intuitive: if the realization \bar{k} would give the seller a payoff lower than her continuation value, the seller could simply replace such a realization by a lottery with mean equal to \bar{k} . Consequently, the first part of the seller's problem consists on maximizing $W(\bar{k}; \bar{k}) - \bar{k}$ over all $\bar{k} \in \mathbb{R}_+$, where $W(k; \bar{k})$ is the payoff of the seller in the second stage of the game if total size of the firm's assets is \bar{k} (so the price distribution is $F_p(\cdot; \bar{k})$) and realized value of the investment is k .²³

²³Notice that the definition of $W(k; \bar{k})$ is analogous to the definition of W in equation (3.1), now defined as

Clearly, $W(k; \bar{k})$ is increasing in both k and \bar{k} : a higher realization of the investment gives a higher value to the seller from rejecting the buyers' offer, while a higher value of \bar{k} increases (in a first-order stochastic sense) the distribution of prices. Hence, we have that for any \bar{k} and \bar{k}' such that $\bar{k} < \bar{k}^{\text{no}} < \bar{k}' \leq k^*$

$$W(\bar{k}; \bar{k}) - \bar{k} < W(\bar{k}; \bar{k}^{\text{no}}) - \bar{k} = W(\bar{k}^{\text{no}}; \bar{k}^{\text{no}}) - \bar{k}^{\text{no}} = W(\bar{k}'; \bar{k}^{\text{no}}) - \bar{k}' < W(\bar{k}'; \bar{k}') - \bar{k}' .$$

Therefore, any optimal choice of the total size of the firm's assets when it is observable, \bar{k}^0 , is strictly higher than \bar{k}^{no} , so it is the case that $\hat{k}(\bar{k}^0) > \hat{k}(\bar{k}^{\text{no}}) = k^*$.

Differently from the case where the total size of the firm's assets is not observable, the seller strictly benefits from trade: if trade was not feasible, she would obtain $Q(k^*) - k^*$, while her equilibrium payoff is no lower than $W(k^*; k^*) - k^* > Q(k^*) - k^*$. The observability of the investment size makes the buyer, instead, strictly worse off. When the total size of the firm's assets is unobservable, he obtains a payoff equal to $F_q^{\text{no}}(0) (U(0) - 0)$, where F_q^{no} is the equilibrium distribution in the case where the total size of the firm's assets is not observable specified in Proposition 3.2. In the case that it is observable, his payoff is $F_q^0(0) (U(0) - 0)$, where now F_q^0 is the distribution characterized in Proposition B.1. Since both F_q^0 and F_q^{no} follow the same equation (with $F_q^{\text{no}}(q^*) = F_q^0(Q(\hat{k}(\bar{k}^0))) = 1$) and since $q^* < Q(\hat{k}(\bar{k}^0))$, we have that $F_q^{\text{no}}(0) > F_q^0(0)$. Hence, the additional investment undertaken by the seller (notice that F_q^0 first-order dominates F_q^{no}) affects negatively the payoff of the buyer, even when investment is desirable from his perspective. As in the previous parts of the paper, this is owed to the increase in the adverse selection.

Remark B.1. The proof of Proposition B.1 shows that $F_q(\cdot; \bar{k}^{\text{no}})$ dominates, in a first-order stochastic sense, $F_q(\cdot; \bar{k}^0)$, that is, the observability of the total size of the investment (but not its riskiness) increases the investment outcome with respect to our base model. This result is reminiscent of the result in Lau (2008) showing that a positive probability that the uninformed party observes the investment affects non-monotonically the equilibrium investment level. In our case, when $q^* > 0$ we cannot rank (in a first-order stochastic sense) $F_q(\cdot; \bar{k}^0)$ and the investment choice of the seller in the fully-observable case (which, by Proposition 3.1, is degenerated at q^*). Nevertheless, it is worth noting that, when $q^* = 0$, the investment is equal to 0 in both the fully-observable and the non-observable cases, while its support is $[0, Q(\hat{k}(\bar{k}^0))]$, with $Q(\hat{k}(\bar{k}^0)) > 0$, when only the total size of the capital stock is observable, so in this case we also obtain a non-monotonicity result.

a function of the investment level instead of the valuation of the seller.

Proof of Proposition B.1

Proof. We begin with a technical but useful result:

Lemma B.1. *Fix some F_k , and let $\mathcal{K} \subset \mathbb{R}_+$ be its support and \bar{k} its mean. Take any closed set $\mathcal{K}^\dagger \subset \mathcal{K}$ such that $\bar{k} \in (\min \mathcal{K}^\dagger, \max \mathcal{K}^\dagger)$ and some $\varepsilon > 0$. Let $\mathcal{K}_\varepsilon^\dagger \equiv \{k \in \mathcal{K} \mid d(k, \mathcal{K}^\dagger) \leq \varepsilon\}$, where $d(k, \mathcal{K}^\dagger) = \min_{k' \in \mathcal{K}^\dagger} |k - k'|$. Then, there exist a distribution $F_k^{\varepsilon^\dagger}$ with support contained in $\mathcal{K}_\varepsilon^\dagger$ and some $\bar{\lambda} > 0$ such that, for all $\lambda \in [-\bar{\lambda}, \bar{\lambda}]$, $\frac{F_k + \lambda F_k^{\varepsilon^\dagger}}{1 + \lambda}$ is a distribution with mean equal to \bar{k} .*

Proof. Let $m \in (0, 1)$ be such that

$$m \mathbb{E}_k[k \mid k < \bar{k}, \mathcal{K}_\varepsilon^\dagger, F_k] + (1-m) \mathbb{E}_k[k \mid k \geq \bar{k}, \mathcal{K}_\varepsilon^\dagger, F_k] = \bar{k} .$$

Note that m exists because, given our assumptions on \mathcal{K}^\dagger , the first expectation on the left hand side of the previous expression is strictly below \bar{k} , and the second strictly above \bar{k} . We now define $F_k^{\varepsilon^\dagger}$ as

$$F_k^{\varepsilon^\dagger}(k) \equiv \begin{cases} \frac{m}{M} \int_{\mathcal{K}_\varepsilon^\dagger \cap [0, k]} F_k(dk) & \text{if } k \in [0, \bar{k}) , \\ \frac{m}{M} \int_{\mathcal{K}_\varepsilon^\dagger \cap [0, \bar{k})} F_k(dk) + \frac{1-m}{M} \int_{\mathcal{K}_\varepsilon^\dagger \cap [\bar{k}, k]} F_k(dk) & \text{if } k \in [\bar{k}, \infty) , \end{cases}$$

where $M \leq 1$ is such that $\lim_{k \rightarrow \infty} F_k^{\varepsilon^\dagger}(k) = 1$. It is clear that $F_k^{\varepsilon^\dagger}$ has support equal to $\mathcal{K}_\varepsilon^\dagger \cap \mathcal{K}$ and is such that $\mathbb{E}_k[k \mid F_k^{\varepsilon^\dagger}] = \mathbb{E}_k[k \mid F_k]$.

Let $\lambda \in [-\frac{M}{m}, \frac{M}{m}]$. Note that $F_k(k) + \lambda F_k^{\varepsilon^\dagger}(k)$ is increasing in k on $[0, \bar{k})$. Indeed, take $k_1, k_2 \in [0, \bar{k})$ such that $k_1 > k_2$. Then

$$F_k(k_1) + \lambda F_k^{\varepsilon^\dagger}(k_1) - (F_k(k_2) + \lambda F_k^{\varepsilon^\dagger}(k_2)) = F_k(k_1) - F_k(k_2) + \lambda \underbrace{(F_k^{\varepsilon^\dagger}(k_1) - F_k^{\varepsilon^\dagger}(k_2))}_{\leq \frac{m}{M} (F_k(k_1) - F_k(k_2))} \geq 0 .$$

A similar argument proves that when $\lambda \in [-\frac{M}{1-m}, \frac{M}{1-m}]$ then $F_k(k) + \lambda F_k^{\varepsilon^\dagger}(k)$ is increasing in k on $[0, \bar{k})$. Hence, defining $\bar{\lambda} \equiv \min\{\frac{M}{m}, \frac{M}{1-m}\}$, we have that $\frac{F_k + \lambda F_k^{\varepsilon^\dagger}}{1 + \lambda}$ is a distribution with the same expectation as F_k for all $\lambda \in [-\bar{\lambda}, \bar{\lambda}]$: it is increasing and belongs to $[0, 1]$. \square

We divide the rest of the proof into 4 steps. In Steps 1 to 3, we exogenously fix the mean investment at some value $\bar{k} \in \mathbb{R}_{++}$ and we characterize the continuation play.²⁴ In

²⁴As it is mentioned in the main text, it is convenient to think the strategy of the seller in the first stage of the game as she choosing first the size and then its riskiness. So, we now “exogenously” fix \bar{k} and then characterize the corresponding game where the seller’s strategy in the first stage consists on choosing $F_k(\cdot; \bar{k})$ with $\mathbb{E}_k[k \mid F_k(\cdot; \bar{k})] = \bar{k}$.

these steps, we will use (F_k, α, F_p) to denote a continuation play such that $\mathbb{E}_k[k|F_k] = \bar{k}$, and \mathcal{K} and \mathcal{P} to denote the supports of F_k and F_p , respectively. We use $F_q(\cdot)$ to denote $F_k(Q^{-1}(\cdot))$ and \mathcal{Q} to denote $Q(\mathcal{K})$. As in the main text, we use $W(k; \bar{k})$ to denote the payoff of the seller in the second stage of the game if mean investment is \bar{k} (so the price distribution is $F_p(\cdot; \bar{k})$) and realized investment outcome is k . Notice that it coincides with $W(Q(k))$ (which is the payoff of the seller in the second stage, and it is defined in equation (3.1)), replacing F_p by $F_p(\cdot; \bar{k})$. Therefore, as we argue after equation (3.1) (and by the continuity of Q), $W(\cdot; \bar{k})$ is continuous. In Step 4 we discuss the properties of the optimal choice of \bar{k} .

Step 1. Characterization of optimality. Fix any closed $\mathcal{K}^\dagger \subset \mathcal{K}$ such that $\bar{k} \in (\min \mathcal{K}^\dagger, \max \mathcal{K}^\dagger)$. Fix also some $\varepsilon > 0$. Then, letting $\mathcal{K}_\varepsilon^\dagger$, $F_k^{\varepsilon^\dagger}$ and $\bar{\lambda} > 0$ denote the same as in Lemma B.1, we have that for all $\lambda \in (-\bar{\lambda}, \bar{\lambda})$ the following holds:

$$\mathbb{E}_k [W(k; \bar{k}) | \frac{1}{1+\lambda} (F_k + \lambda F_k^{\varepsilon^\dagger})] = \frac{1}{1+\lambda} (\mathbb{E}_k [W(k; \bar{k}) | F_k] + \lambda \mathbb{E}_k [W(k; \bar{k}) | F_k^{\varepsilon^\dagger}]).$$

Since choosing F_k is optimal, it is necessarily the case that the derivative of the previous expression with respect to λ at $\lambda = 0$ is equal 0, that is,

$$\begin{aligned} 0 &= \frac{d}{d\lambda} \mathbb{E}_k [W(k; \bar{k}) | \frac{1}{1+\lambda} (F_k + \lambda F_k^{\varepsilon^\dagger})] |_{\lambda=0} = -\mathbb{E}_k [W(k; \bar{k}) | F_k] + \mathbb{E}_k [W(k; \bar{k}) | F_k^\dagger] \\ &= \Pi_S(F_k^{\varepsilon^\dagger}) - \Pi_S^*. \end{aligned}$$

Thus, $\Pi_S(F_k^{\varepsilon^\dagger}) = \Pi_S^*$.

Fix now some k_1, k_2 satisfying $k_1 < \bar{k} < k_2$, and define

$$F_k^\dagger(k; k_1, k_2) \equiv \begin{cases} 0 & \text{if } k < k_1, \\ \frac{k_2 - \bar{k}}{k_2 - k_1} & \text{if } k \in [k_1, k_2), \\ 1 & \text{if } k \geq k_2. \end{cases}$$

Then, it is easy to verify (from the proof of Lemma B.1) that, whenever $k_1, k_2 \in \mathcal{K}$, then using $\mathcal{K}^\dagger \equiv \{k_1, k_2\}$ and letting $F_k^{\varepsilon^\dagger}$ be the corresponding distribution (defined in the proof of Lemma B.1) we have that

$$\lim_{\varepsilon \rightarrow 0} F_k^{\varepsilon^\dagger}(k) = F_k^\dagger(k; k_1, k_2)$$

for all $k \geq 0$. In particular, since $W(\cdot; \bar{k})$ is continuous, this implies that

$$\Pi_S^* = \frac{k_2 - \bar{k}}{k_2 - k_1} W(k_1; \bar{k}) + \frac{\bar{k} - k_1}{k_2 - k_1} W(k_2; \bar{k}). \quad (\text{B.2})$$

Step 2. Proof that $\mathcal{Q} = \mathcal{P} = [0, \bar{q}]$ for some $\bar{q} > Q(\bar{k})$. We first prove that $\mathcal{K} \neq \{\bar{k}\}$. We do this assuming, for the sake of contradiction, that $\mathcal{K} = \{\bar{k}\}$.²⁵ Arguments paralleling those in the proof of Proposition 3.1 show that, in this case, $\mathcal{P} = \{Q(\bar{k})\}$. Nevertheless, the seller can choose a distribution assigning probability $\frac{1}{2}$ to both $\bar{k} - \varepsilon$ and $\bar{k} + \varepsilon$, for some $\varepsilon \in (0, \bar{k})$. Such a distribution gives the seller a total payoff equal to

$$\frac{1}{2} Q(\bar{k}) + \frac{1}{2} Q(\bar{k} + \varepsilon) - \bar{k} > Q(\bar{k}) - \bar{k} ,$$

so the seller has a profitable deviation.

The fact that $\mathcal{P} \subset \mathcal{Q}$ is proved exactly as in the proof of Lemma 3.1. Let \underline{k} and \hat{k} denote, respectively, $\min \mathcal{K}$ and $\max \mathcal{K}$, so we have that $\underline{q} \equiv \min \mathcal{Q} = Q(\underline{k})$ and $\bar{q} \equiv \max \mathcal{Q} = Q(\hat{k})$. We first prove that \mathcal{P} has no gaps to the left of $Q(\bar{k})$, that is, $\mathcal{P} \cap [q, Q(\bar{k}))$ has no gaps. Assume then, for the sake of contradiction, that there are some $k_1, k'_1 < \bar{k}$, that $Q(k_1), Q(k'_1) \in \mathcal{P}$ and that $(Q(k_1), Q(k'_1)) \cap \mathcal{P} = \emptyset$. In this case equation (3.2) can be rewritten, for all $k \in [k_1, k'_1]$, as

$$W(k; \bar{k}) = W(k_1; \bar{k}) + \frac{W(k_2; \bar{k}) - W(k_1; \bar{k})}{Q(k_2) - Q(k_1)} (Q(k) - Q(k_1)) . \quad (\text{B.3})$$

Hence, since Q is a concave function, $W(\cdot; \bar{k})$ is a concave function in $[k_1, k'_1]$. Nevertheless, proceeding similarly to the arguments used to derive equation (B.2), we have that for any $k \in [k_1, k'_1]$ the following holds

$$W(k; \bar{k}) \leq -\frac{\bar{k} - k}{k_2 - \bar{k}} W(k_2; \bar{k}) + \frac{k_2 - k}{k_2 - \bar{k}} \Pi_S^* . \quad (\text{B.4})$$

The right hand side of the previous equation is a convex function of k which is equal to $W(k_1; \bar{k})$ when $k = k_1$ and equal to $W(k'_1; \bar{k})$ when $k = k'_1$, so we have a contradiction with the fact that $W(\cdot; \bar{k})$ is concave in $[k_1, k'_1]$. A similar argument applies to show that $\mathcal{P} \cap (Q(\bar{k}), \bar{q}]$ has no gaps.

We now prove that \mathcal{P} has no gaps. Let $k_1^* \equiv \sup\{k \in Q^{-1}(\mathcal{P}) | k < \bar{k}\}$ and $k_2^* \equiv \inf\{k \in Q^{-1}(\mathcal{P}) | k > \bar{k}\}$. Assume, for the sake of contradiction, that $k_1^* < \bar{k}$. Assume also first that $\bar{k} \notin \mathcal{K}$, so $k_2^* > \bar{k}$. In this case, the previous argument applies: equation (B.3) (with $k_1 = k_1^*$ and $k_2 = k_2^*$) is valid for all $k \in [k_1^*, k_2^*]$ and equation (B.4) applies for all $k \in [k_1^*, \bar{k}]$, which leads again to a contradiction. Assume then that $\bar{k} \in \mathcal{K}$. In this case, as before, equation (B.3) holds with $k_1 = k_1^*$, $k_2 = \bar{k}$ and for all $k \in [k_1^*, \bar{k}]$. As argued before, this implies that $W(\cdot; \bar{k})$ is strictly concave in $[k_1^*, \bar{k}]$. Let $k_2^\dagger \in \mathcal{K}$ be such that $k_2^\dagger > \bar{k}$ (notice that it exists

²⁵Notice that we are focussing on a strictly positive total size of the firm's assets, $\bar{k} > 0$. If $\bar{k} = 0$ then, necessarily, $\mathcal{K} = \{0\}$ and $F_k(\cdot; 0)$ is a distribution degenerated at 0.

because $k_1^* < \bar{k}$, so the variance of F_k is positive, and its mean is equal to \bar{k}). Nevertheless, using equation (B.2) for $k_1 = k_1^*$ and some $k_2 = k_2^\dagger$ and equation (B.4) for $k_2 = k_2^\dagger$ and any $k \in [k_1^*, \bar{k})$, we can write

$$W(k; \bar{k}) \leq \frac{\bar{k} - k}{\bar{k} - k_1^*} W(k_1^*; \bar{k}) + \frac{k - k_1^*}{\bar{k} - k_1^*} \Pi_S^*$$

for all $k \in [k_1^*, \bar{k})$. The right hand side of the previous equation is a convex function of k in $k \in [k_1^*, \bar{k}]$, equal to $W(k_1^*; \bar{k})$ when $k = k_1^*$ and equal to $W(\bar{k})$ for $k = \bar{k}$. This, again, contradicts that $W(\cdot; \bar{k})$ is concave on $[k_1^*, \bar{k}]$. Similar arguments (see also the proof of Lemma 3.1) prove that $\min \mathcal{P} = \underline{q}$ and $\max \mathcal{P} = \bar{q}$. Hence, we have that $\mathcal{Q} = \mathcal{P} = [\underline{q}, \bar{q}]$.

Assume now, for the sake of contradiction, that $\underline{k} > 0$, that is, $\underline{k} > 0$. Notice then that

$$\begin{aligned} \mathbb{E}_k[W(k; \bar{k}) | F_k^\dagger(k; \underline{k}, \hat{k})] &= \frac{\hat{k} - \bar{k}}{\hat{k} - \underline{k}} \mathbb{E}_p[p | F_p] + \frac{\bar{k} - \underline{k}}{\hat{k} - \underline{k}} Q(\hat{k}) \\ &< \frac{\hat{k} - \bar{k}}{\hat{k} - 0} \mathbb{E}_p[p | F_p] + \frac{\bar{k} - 0}{\hat{k} - 0} Q(\hat{k}) \\ &= \mathbb{E}_k[W(k; \bar{k}) | F_k^\dagger(k; 0, \hat{k})]. \end{aligned}$$

This is a contradiction, since choosing $F_k^\dagger(\cdot; \underline{k}, \hat{k})$ is a best response to F_p , but choosing $F_k^\dagger(\cdot; 0, \hat{k})$ gives a higher payoff to the seller. Hence, $\underline{k} = 0$. Since $\underline{k} < \bar{k}$, we have that necessarily $\hat{k} > \bar{k}$.

Step 3. Distributions. The fact that $F_q(\cdot) \equiv F_k(Q^{-1}(\cdot))$ is continuous in $[0, \bar{q}]$ and satisfies $F_q(p) = F'_q(p) (U(p) - p)$ is obtained requiring the buyer to be indifferent on offering any price in $[0, \bar{q}]$ as in the proof of Proposition 3.2. Notice that, using equation (A.9), we have

$$F_k(k) = \exp\left(-\int_{Q(k)}^{\bar{q}} \frac{1}{U(\tilde{q}) - \tilde{q}} d\tilde{q}\right).$$

Since, by assumption, $U(q) - q$ is bounded above, we have that $\mathbb{E}_k[k | F_k] = \hat{k} + o(1)$ as $\hat{k} \rightarrow \infty$. Therefore, for any \bar{k} there exists a unique $\hat{k}(\bar{k})$ such that the corresponding distribution F_k is such that $\mathbb{E}_k[k | F_k] = \bar{k}$.

We now characterize F_p . Notice that $W(k; \bar{k})$ is necessarily linear in k on $[0, \hat{k}]$. Indeed, as we argued before, the seller is indifferent on choosing any bi-valued distribution F_k^\dagger with support contained in $[0, \hat{k}]$ and mean \bar{k} , that is, $\mathbb{E}_k[W(k; \bar{k}) | F_k^\dagger] = W(\bar{k}; \bar{k})$. This implies that

$$W(k; \bar{k}) = W_0 + W_1 k = Q(k) + \int_{(Q(k), \bar{q}]} (p - Q(k)) dF_p(p), \quad (\text{B.5})$$

for $W_0 = \mathbb{E}[p]$ and some $W_1 \in \mathbb{R}$. We can rewrite the previous equation in terms of $q = Q(k)$ (so $k = K(q)$) as

$$W_0 + W_1 K(q) = q + \int_{(q, \bar{q}]} (p - q) F_p(dp).$$

This equation is analogous to equation (A.8). Hence, we can proceed analogously to the proof of Proposition 3.2 so we obtain that

$$W_1 K'(q) = 1 - (1 - F_p(q)) = F_p(q).$$

Since $F_p(\bar{q}) = 1$, we have $W_1 = K'(\bar{q})^{-1}$.

Step 4. Features of an optimal \bar{k} . We now formalize the argument in the main text to show that, when \bar{k} gives the seller her highest payoff, we have $\hat{k}(\bar{k}) > q^*$. Note, using $W(\cdot; \cdot)$ defined in the main text, we have that the seller maximizes $W(\bar{k}; \bar{k}) - \bar{k}$. Note also that $W(\bar{k}; \bar{k}) > Q(\bar{k})$ for all $\bar{k} > 0$, so the equilibrium payoff of the seller is strictly higher than $Q(k^*) - k^*$.

Let $\hat{k}(\bar{k})$ denote the upper bound of the support of $F_q(\cdot; \bar{k})$ for each $\bar{k} \in \mathbb{R}_+$ (which, from the previous analysis, is unique). Notice that $F_k(\cdot; \bar{k})$ can be written using equation (A.9) as

$$F_k(k; \bar{k}) = F_q(Q(k); \bar{k}) = \exp\left(-\int_{Q(k)}^{Q(\hat{k}(\bar{k}))} \frac{1}{U(\tilde{q}) - \tilde{q}} d\tilde{q}\right).$$

So, taking two values $\bar{k}, \bar{k}' \in \mathbb{R}$ we have

$$F_k(k; \bar{k}) - F_k(k; \bar{k}') = \exp\left(-\int_{Q(\hat{k}(\bar{k}))}^{Q(\hat{k}(\bar{k}'))} \frac{1}{U(\tilde{q}) - \tilde{q}} d\tilde{q}\right) \left(1 - \exp\left(-\int_{Q(k)}^{Q(\hat{k}(\bar{k}'))} \frac{1}{U(\tilde{q}) - \tilde{q}} d\tilde{q}\right)\right).$$

Hence, if for example $\bar{k} < \bar{k}'$, it is necessarily the case that $Q(\hat{k}(\bar{k})) < Q(\hat{k}(\bar{k}'))$, because in this case $F_k(k; \bar{k}')$ first-order stochastically dominates $F_k(k; \bar{k})$, and the reverse is true in the opposite case. As a result, since U is continuous and differentiable, $\hat{k}(\cdot)$ is a continuous, differentiable and strictly increasing function.

Since $\hat{k}(0) = 0$ and $\hat{k}(k^*) > k^*$, there exists a unique $\bar{k}^{\text{no}} \in (0, k^*)$ such that $\hat{k}(k^*) = k^*$. For such value, the distributions $F_q(\cdot; \bar{k}^{\text{no}})$ and $F_p(\cdot; \bar{k}^{\text{no}})$ coincide with those obtained when the total size of the capital stock of the firm is not observable (characterized in Proposition 3.2), so the the payoff of the seller in this case is

$$W(\bar{k}^{\text{no}}; \bar{k}^{\text{no}}) - \bar{k}^{\text{no}} = Q(k^*) - k^*.$$

Using our previous result that $W(\cdot; \bar{k})$ is linear with slope equal to $K'(Q(\hat{k}(\bar{k})))^{-1}$ (and therefore the slope is equal to $Q'(\hat{k}(\bar{k}))$), we have that $W(\bar{k}; \bar{k})$ can be expressed as follows:

$$W(\bar{k}; \bar{k}) - \bar{k} = Q(\hat{k}(\bar{k})) - (\hat{k}(\bar{k}) - \bar{k}) Q'(\hat{k}(\bar{k})) - \bar{k} .$$

Differentiating the previous equation we obtain

$$\frac{d}{d\bar{k}}(W(\bar{k}; \bar{k}) - \bar{k}) = Q'(\hat{k}(\bar{k})) - \underbrace{1 - (\hat{k}(\bar{k}) - \bar{k}) Q''(\hat{k}(\bar{k})) \hat{k}'(\bar{k})}_{>0} .$$

Notice that if $\hat{k}(\bar{k}) < k^*$ then the right hand side of the previous expression is positive (since, in this case, $Q'(\hat{k}(\bar{k})) > 1$), and therefore $W(\bar{k}; \bar{k}) - \bar{k}$ does not reach its maximum in $[0, \bar{k}^{\text{no}}]$. Notice finally that, as we showed before, $\bar{k} = \hat{k}(\bar{k}) + o(1)$ when $\bar{k} \rightarrow \infty$, so

$$W(\bar{k}; \bar{k}) - \bar{k} < W(\hat{k}(\bar{k}); \bar{k}) - \bar{k} = Q(\hat{k}(\bar{k})) - \bar{k} = Q(\bar{k}) - \bar{k} + o(1)$$

as $\bar{k} \rightarrow \infty$. So, since $\lim_{\bar{k} \rightarrow \infty} (Q(\bar{k}) - \bar{k}) = -\infty$, we have that a finite maximizer of $W(\bar{k}; \bar{k}) - \bar{k}$ strictly higher than \bar{k}^{no} exists. \square