

Discussion Paper Series – CRC TR 224

Discussion Paper No. 101
Project B 05

Segmentation versus Agglomeration:
Competition between Platforms with Competitive Sellers

Heiko Karle ¹
Martin Peitz ²
Markus Reisinger ³

June 2019

¹ Department of Economics, Frankfurt School of Finance & Management, Adickesallee 32-34, 60322 Frankfurt am Main, Germany; E-Mail: h.karle@fs.de

² Department of Economics and MaCCI, University of Mannheim, 68131 Mannheim, Germany; EMail: Martin.Peitz@gmail.com; also affiliated with CEPR, CESifo, and ZEW

³ Department of Economics, Frankfurt School of Finance & Management, Adickesallee 32-34, 60322 Frankfurt am Main, Germany. E-Mail: m.reisinger@fs.de; also affiliated with CESifo

Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Segmentation versus Agglomeration: Competition between Platforms with Competitive Sellers*

Heiko Karle[†] Martin Peitz[‡] Markus Reisinger[§]

June 2019

Abstract

For many products, platforms enable sellers to transact with buyers. We show that the competitive conditions among sellers shape the market structure in platform industries. If product market competition is tough, sellers avoid competitors by joining different platforms. This allows platforms to sustain high fees and explains why, for example, in some online markets, several homogeneous platforms segment the market. Instead, if product market competition is soft, agglomeration on a single platform emerges, and platforms fight for the dominant position. These insights give rise to novel predictions. For instance, market concentration and fees are negatively correlated in platform industries, which inverts the standard logic of competition.

Keywords: intermediation, two-sided markets, market structure, price competition, endogenous segmentation

JEL-classification: L13, D43.

*We received helpful comments from Mark Armstrong, Özlem Bedre-Defolie, Paul Belleflamme, Luis Cabral, Emilio Calvano, Jay Pil Choi, Alex de Cornière, Reyer Gerlagh, Michael Grubb, Hanna Halaburda, Ali Hortaçsu, Maarten Janssen, Johannes Johnen, Bruno Jullien, Damian Kozbur, Yassine Lefouili, Marc Möller, Nick Netzer, Georg Nöldeke, Amedeo Piolatto, Sven Rady, Pierre Regibeau, Armin Schmutzler, Tim Simcoe, Timothy Wyndham, Julian Wright, Ali Yurukoglu, and participants at various conferences and seminars. Martin Peitz gratefully acknowledges support by the Deutsche Forschungsgemeinschaft through PE 813/2-2 and CRC TR 224.

[†]Department of Economics, Frankfurt School of Finance & Management, Adickesallee 32-34, 60322 Frankfurt am Main, Germany; E-Mail: h.karle@fs.de.

[‡]Department of Economics and MaCCI, University of Mannheim, 68131 Mannheim, Germany; E-Mail: Martin.Peitz@gmail.com; also affiliated with CEPR, CESifo, and ZEW.

[§]Department of Economics, Frankfurt School of Finance & Management, Adickesallee 32-34, 60322 Frankfurt am Main, Germany. E-Mail: m.reisinger@fs.de; also affiliated with CESifo.

1 Introduction

In many industries, platforms offer intermediation services and play the essential role of enabling transactions between buyers and sellers—more prominently so, with the migration of trade from physical venues to the Internet. The market structure for intermediation services, however, differs considerably across industries.

While, for example, Ebay is by far the most popular online auction portal in the U.S.,¹ other markets are often more segmented, and two (or more) platforms have significant market shares. For example, in the housing market, two major platforms perform the main bulk of matching landlords and tenants in e.g., the UK and Germany.² Other examples for market segmentation include the used car market in which multiple platforms are active (such as Autotrader, Cargurus, and Carsdirect in the U.S., or mobile.de and Autoscout24 in Germany),³ or the market for hospitality services in which Airbnb and Homeaway share the market.

In this paper, we provide a theoretical framework to examine these differences in platform market structure. We find that the market structure is shaped by the competitive conditions in the product market. These conditions are responsible for the fees set by platforms and, thus, for platforms' profits and the number of active platforms.

As is well-known from the theoretical literature, platform markets have the tendency to tip due to positive cross-group external effects between buyers and sellers (i.e., each buyer benefits from more sellers on the same marketplace, and vice versa). This has been shown in the seminal work by Caillaud and Jullien (2001, 2003) and rationalizes the phenomenon of *market agglomeration*, in which all users locate on a single platform.⁴ However, in several industries, two or more platforms have non-negligible market shares, and users join different platforms. The existing literature explains *market segmentation* with platforms offering differentiated matching services (e.g., Rochet and Tirole, 2003; Armstrong, 2006).

In the above examples, and more broadly for many Internet platforms, there is little room for service differentiation—that is, platforms offer services that often appear to be

¹See Bajari and Hortaçsu (2003) or Hasker and Sickles (2010).

²In the UK, these are the portals Rightmove and Zoopla, with together more than 175 million visits per month, which is by far larger than the aggregated number of visits of all other portals (see <https://hoa.org.uk/advice/guides-for-homeowners/i-am-selling/rightmove-zoopla-which-is-best/>, last accessed June 4, 2019). In Germany, the two portals Immobilienscout24 and Immowelt have a joint market share of more than 85% (<http://immobiliencommunity.de/2016/10/27/immobilienscout24-preise/>, last accessed June 4, 2019).

³See, for example, <https://www.digitaltrends.com/cars/best-used-car-websites/> and <https://www.dtgv.de/tests/5875/> for comparisons of the different portals in the two countries (both sites last accessed June 4, 2019).

⁴Using field experiments on Ebay and Yahoo Auctions, Brown and Morgan (2009) find evidence for market tipping on online auction sites.

quite the same. Therefore, it appears a puzzle how competing platforms share the market and earn positive profits.

Our answer to this puzzle is that multiple homogeneous platforms can serve the role of relaxing competition between sellers in the product market. In a nutshell, if sellers decide to be active on different platforms, some buyers will not be informed about all offers, which, in turn, relaxes competition between sellers. Platforms benefit from this provision of endogenous segmentation by charging sellers larger fees. Thus, multiple homogeneous platforms earn positive profits.

We identify the competitive conditions in the product market as the key driver of the arising market structure. If product market competition is soft (e.g., because sellers offer highly differentiated products), agglomeration forces dominate. Then, platforms follow a strategy of “play hard and fight it out” to become the dominant platform, which leads to low fees (at least in the short term, when the number of platforms is exogenous). If, instead, product market competition is tough, multiple platforms segment the market to relax seller competition. Platforms then “play soft” and charge high fees.

Dudey (1990) and Ellison and Fudenberg (2003) demonstrate that under tough product market competition, sellers benefit from allocating at different marketplaces. In those papers, however, marketplaces are not managed by platforms and do not charge fees to sellers. Our analysis advances this literature by confirming that market segmentation can arise even with fee-setting platforms. Yet, we find that fees are strictly positive under market segmentation.

Overall, our paper provides testable predictions of how the competitive environment faced by sellers drives the equilibrium market structure and the platforms’ equilibrium choice of listing fees. Tough competition between sellers implies high platform fees and profits. Therefore, the correlation between competition in the product market and competition in the market for intermediation services is negative. In addition, a low market concentration in platform markets due to multiple active platforms goes together with high listing fees. This implies that the relation between the Hirshman-Herfindahl Index and the markup is reversed in platform markets versus standard oligopoly markets.

In the next section, we present descriptive evidence for market segmentation using data from German online real estate platforms.⁵ In Web Appendix A, we provide further evidence from the market for Spanish holiday homes and the daily deals market in the U.S.. We also point to an example outside e-commerce to which our theory applies (i.e., the market for modem standards in the 1990s).

In our baseline model, multiple platforms compete on listing fees charged to sellers. Buyers prefer platforms with many sellers, and vice versa. Sellers offer a single product

⁵We note that this is purely motivating evidence and not intended as a formal empirical test of our theory and its predictions.

that belongs to one out of many different product categories, and there are multiple sellers within the product category competing with each other. To present the results in the simplest way, we focus on the case with only two platforms and two sellers per category. All of our results extend to a general number of platforms and sellers. After platforms set their fees, sellers and buyers decide simultaneously which platform to join and, thus, play a coordination game. We show that the selection criterion of coalition-proofness, in combination with profit dominance of sellers, gives clear-cut equilibrium predictions in this coordination game. This allows us to establish necessary and sufficient conditions when either agglomeration or segmentation emerges.

A tipping equilibrium prevails if the degree of competition between sellers is low. Buyers are then informed about all offers, implying that sellers are in competition with each other. However, demand is also higher as all buyers are on the same platform. The effect of increased demand dominates increased competition. Platforms compete fiercely to win the market, which leads to a Bertrand-style competition between platforms, and their listing fees are driven down to marginal cost.⁶

By contrast, if competition between sellers in a product category is sufficiently intense, sellers prefer to be active on different platforms. Buyers will split between the two platforms and do not become informed about all offers. Thus, platforms segment the market, and competition between sellers is relaxed. This finding is in line with the example of the German housing market (see Section 2). Segmentation then allows platforms to obtain strictly positive profits. If a platform were to deviate from the associated equilibrium listing fees by charging a slightly lower fee, sellers would not have an incentive to switch to this platform, as this would intensify competition among them.

If the degree of competition between sellers is moderate, the equilibrium in listing fees is in mixed strategies, and platforms segment the market with positive probability. Confirming the result described above, if the degree of competition between sellers gets larger, the probability for segmentation increases, and so does the expected profit of platforms, as they charge higher fees.

While our baseline model features single-homing of users on both sides, we subsequently allow for multi-homing buyers and sellers and show that our solution to the puzzle that multiple platforms share the market carries over. We find that platform profits are non-monotonic in the share of multi-homing buyers, exhibiting an inverted U-shape relationship. Instead, platform profits are unambiguously lower if sellers can multi-home because multi-homing makes agglomeration more likely. Existing literature with differentiated platforms has shown that seller multi-homing allows competing plat-

⁶Agglomeration, therefore, does not imply that a platform acts as a monopolist. Instead, another constraining platform is present, but this platform has a negligible market share. We discuss this in more detail in Section 8.

forms to exert monopoly power over sellers, which possibly increases platform profits. By contrast, we find that seller multi-homing may affect the market structure and has, thereby, a different effect: due to multi-homing, sellers may profitably deviate from segmentation by becoming active on both platforms and, thus, making offers to all buyers. This might render segmentation unstable. Then, agglomeration occurs, and platforms receive lower profits.

We also allow for per-transaction fees and revenue shares as alternative price instruments and show the robustness of our results. Endogenizing the platform fee structure, we find that, in a segmentation equilibrium, per-transaction fees are dominated by a combination of revenue shares and listing fees. This resembles the fee structure commonly used by trading platforms such as Amazon Marketplace.⁷ In addition, we show that revenue shares are a more-important source of platform profits relative to listing fees if the heterogeneity of product categories is larger.

From a welfare perspective, segmentation is inefficient. The reason is that matching quality is lower, as buyers are not informed about all offers, and the deadweight loss is higher than under agglomeration due to higher product market prices. As a consequence for competition policy, restraints such as exclusive dealing contracts, which platforms may impose on sellers, are welfare-reducing, as they prevent seller multi-homing and, thus, are likely to induce segmentation.

In the remainder of this section, we discuss the related literature. In Section 2, we provide descriptive evidence for market segmentation using data from real estate platforms in Germany and point to additional examples. In Section 3, we set out the baseline model and, in Section 4, characterize the equilibrium. In Section 5, we analyze the effects of multi-homing by buyers and sellers. In Section 6, we allow for alternative price instruments and consider an alternative equilibrium selection criterion. In Section 7, we discuss policy implications and empirical predictions. Section 8 concludes. All proofs are relegated to the Appendix and Web Appendix H.

Related Literature. Our paper contributes to the literature on competition in two-sided markets, pioneered by Caillaud and Jullien (2001, 2003), Rochet and Tirole (2003, 2006), and Armstrong (2006). Caillaud and Jullien (2001, 2003) analyze homogeneous platforms and show that the market tips to one platform under relatively general conditions. Rochet and Tirole (2003, 2006) and Armstrong (2006), by contrast, consider sufficiently differentiated platforms such that no tipping occurs. The focus of these papers

⁷For example, the recommended plan for sellers on Amazon Marketplace entails a monthly subscription fee of \$39.99 and a referral fee, which is a percentage charge varying by product category (i.e., a revenue share); for the majority of categories, it ranges between 8% and 15%. However, there is no per-transaction fee (see https://sellercentral.amazon.com/gp/help/external/200336920?language=en_US&ref=efph_200336920_cont_201822160, last accessed June 4, 2019).

is on cross-group externalities between two user groups but not on competition between users within a group. Armstrong (2006) considers seller competition in an extension, and shows that platforms may restrict seller competition to obtain higher profits. In contrast to our paper, in his framework, all platforms are active due to exogenous differentiation.

A few papers in the two-sided markets literature analyze competition between sellers. Nocke, Peitz and Stahl (2007), Galeotti and Moraga-González (2009), and Gomes (2014) analyze platform ownership, search, and optimal auction design, respectively, but consider a monopoly platform, whereas Belleflamme and Toulemonde (2009) study competition between a for-profit and a not-for-profit platform. Dukes and Gal-Or (2003) and Hagiu (2006) consider competition between for-profit platforms and analyze either exclusivity contracts or price commitment by platforms.⁸ None of these papers analyzes how the market structure depends on seller competition.⁹

Ellison, Fudenberg, and Möbius (2004) consider competition between two auction sides. They derive conditions for sellers to be active on different platforms, as this lowers the seller-buyer ratio on each platform and leads to higher prices. Ellison and Fudenberg (2003) provide general conditions such that tipping does not occur in markets with cross-group external effects. The key difference to our paper is that they do not consider fee-setting by platforms (i.e., fees are zero in their setup).¹⁰

The literature on firms' location decisions analyzes the benefits and costs of clustering from a different angle. For example, Dudey (1990) shows that sellers prefer agglomeration in one marketplace over segmentation, as lower product prices are more than offset by increased demand. Stahl (1982) demonstrates that a similar effect arises if buyers are attracted by a greater variety of goods. Church and Gandal (1992) analyze a related model applied to the software market. In contrast to our paper, marketplaces are open platforms in the sense that access is free. Instead, we are interested in markets with fee-setting platforms and the resulting market structure.¹¹

Our paper also contributes to work on price comparison websites. Baye and Morgan (2001) show how homogeneous sellers obtain positive profits, even if a website informs buyers about all prices. Sellers still cater to their local market, in which buyers are

⁸In line with the previous literature, Hagiu (2006) shows that if commitment is not possible and users single-home, an agglomeration equilibrium with zero profits emerges.

⁹An exception is Halaburda, Piskorski and Yilidrim (2018) who consider a matching market with heterogeneous users. In contrast to our paper, they obtain segmentation due to sorting as a result of user heterogeneity.

¹⁰Ellison, Fudenberg, and Möbius (2004), in their Section 7, briefly analyze platform pricing. However, since they do not make assumptions on equilibrium selection in the coordination game between sellers and buyers, they do not provide a unique mapping from fees to market structure.

¹¹An exception is Gehrig (1998), who considers Hotelling competition between marketplaces and competition on the circle (Salop, 1979) between sellers. He shows that agglomeration equilibria may emerge (with positive platform profits), despite platform differentiation.

not informed about all prices. This leads to price dispersion. This result has been tested empirically (e.g., Brown and Goolsbee, 2002; Baye and Morgan, 2004) and the theoretical framework has been extended (e.g., Ronayne, 2019). In contrast to these papers, we focus on competition between websites in addition to competition between sellers.

2 Descriptive Evidence for Segmentation

In this section, we provide descriptive evidence for market segmentation, using data from German online real estate platforms for sales of single-family houses. We make the following observations: (i) more than one platform carries a positive volume of trade; (ii) the matching services that platforms offer appear to be homogenous (i.e., neither horizontally nor vertically differentiated); (iii) a large fraction of sellers single-home and, in particular, each platform hosts some single-homing sellers. In Web Appendix A, we demonstrate very similar findings in the market for *rental* apartments in Germany. Furthermore, we make the same observations in two other markets: the market for renting holiday homes in Spain, and the market for daily deals in the United States.

In many countries, housing offers for sale are predominantly posted on internet portals. This holds for any type of building (new vs. old construction; apartment vs. single-family home). In Germany, the two leading platforms are Immobilienscout24 and Immowelt. Both platforms charge listing fees that depend on the time window the offer will be listed. Immobilienscout24 charges approximately 70 Euro for a basic 2-week offer for sale, while Immowelt charges approximately 45 Euro. A listing for one month is available at approximately 120 Euro at Immobilienscout24, while a 4-week listing is available at approximately 70 Euro at Immowelt.¹²

The two platforms are very similar in appearance and allow for the same type of qualifiers in the search queries.¹³ In addition, platforms do not serve different specific audiences (i.e., they do not cater to separate submarkets): first, in our dataset, the price range of offers appears to be similar on the two platforms, which provides evidence that there is no self-selection of sellers into platforms based on price and associated quality. Thus, there is no indication for vertical differentiation. Second, both platforms host sellers from all product categories, that is, none of the platforms has specialized in a particular subset of categories (i.e., regional or style of buildings). Therefore, in our data set, we do not see evidence for horizontal differentiation between platforms.

We document the prevalence of seller single-homing relative to multi-homing. For

¹²We obtained price information for December 2017 on Immobilienscout24 and Immowelt from <https://www.test-der-immobilienboersen.de/immobilienboersen/>, last accessed June 4, 2019.

¹³See <https://www.immobilienscout24.de/> and <https://www.immowelt.de/>, both last accessed June 4, 2019.

this purpose, we generated a dataset by taking a snapshot of a particular segment of the housing market. Specifically, we carried out a search for selling single-family homes in German cities with more than 100,000 inhabitants. These are 125 cities in total. We use the search criterion “distance to the center less than 3 kilometers”. We treat a property listed on only one platform as a single-homing offer, and a property listed on both platforms as a multi-homing offer.¹⁴ The descriptive statistics are reported in Table 1. The first two lines give the absolute number of offers by sellers on either platform in a city. The last three lines report the shares of single-homing sellers on either platform and the share of multi-homing sellers in a city (“SH” stands for single-homing, “MH” for multi-homing, and we abbreviate Immobilienscout24 by Immoscout).

Table 1: Descriptive Statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
Sellers Immoscout	125	32.92	18.73	4	115
Sellers Immowelt	125	24.57	13.63	5	106
Share of SH Sellers Immoscout	125	0.489	0.153	0.105	0.795
Share of SH Sellers Immowelt	125	0.346	0.155	0.088	0.886
Share of MH Sellers	125	0.165	0.094	0	0.523

As shown in the table, Immobilienscout24 hosts on average more sellers than Immowelt. This turns out to be true for larger and smaller cities.¹⁵ Yet, for all sizes, there are some examples for which the opposite holds—that is, Immowelt hosts more sellers than Immobilienscout24.¹⁶ Both platforms are active in all cities, which shows that tipping does not occur. The table also documents that the share of multi-homing sellers is limited.

Figure 1 reports the share of multi-homers and single-homers on each platform in ascending order of the share of multi-homers in the 125 cities. The reading of the figure is as follows. The bright vertical bars represent the share of single-homing offers on Immobilienscout24 and the dark gray vertical bars the share of single-homing offers on Immowelt. As both bars exist in all cities, there is no city in which one platform hosts all sellers. Depending on the city, between 0 and 52.3% of sellers multi-home. In approximately 75% of cities, less than 20% of sellers multi-home, and, in more than 90%, less than 25% multi-home. This provides evidence of segmentation and that seller

¹⁴Some owners place offers themselves, whereas others use one or, in very few cases, multiple agencies for placing an offer.

¹⁵The correlation coefficient between city size and the share of single-homing sellers on both platforms is not significantly different from zero.

¹⁶For example, for the two largest German cities, Berlin and Hamburg, there are 12, respectively 13, listings on Immobilienscout24 but 13, respectively 23, on Immowelt.

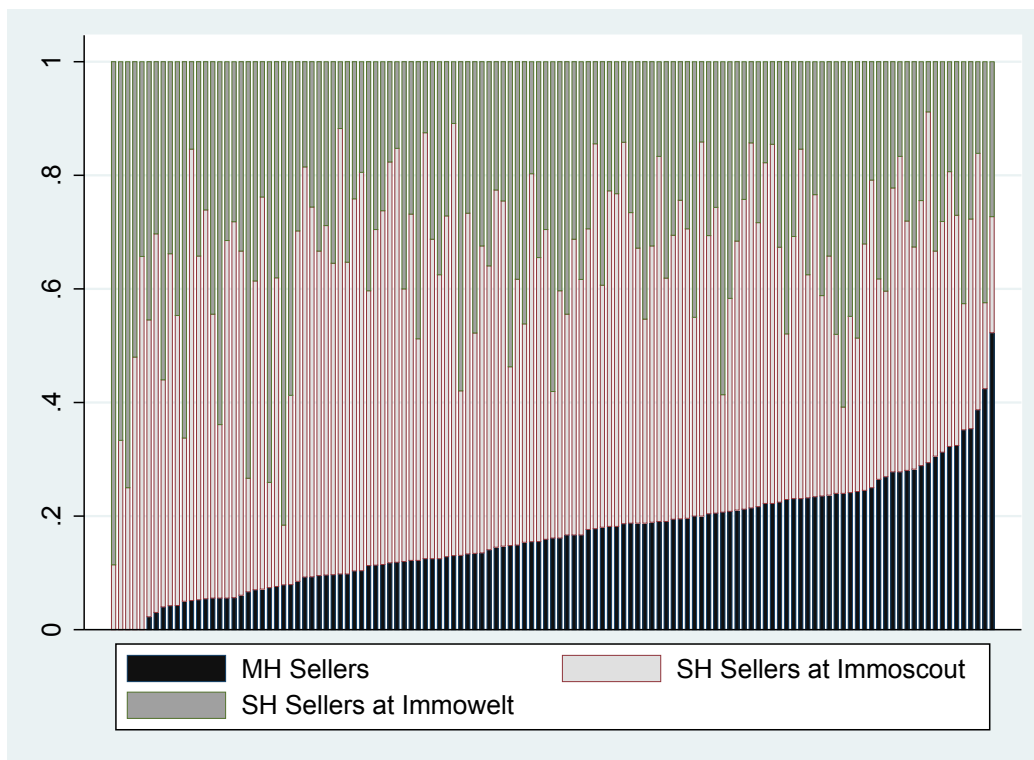


Figure 1: German housing market: Single- and multi-homing sellers.

multi-homing is modest.

Finally, we address two potential concerns: first, it could be that multi-homing of offers is primarily observed in cities with a large number of offers. This would imply that multi-homing is more prominent than what is suggested by the unweighted average of 16.5% over 125 cities (cf. Table 1). To address this concern, we test for the correlation between the multi-homing share and the number of offers per city. We find that there is no correlation—the correlation coefficient is -0.001 —and, hence, the share of multi-homing is not systematically associated with the number of offers in a city. In fact, the average share of multi-homing offers in the total sample is 16.48%.

Second, it could be that the vast majority of buyers are active on both platforms. This would imply that a seller can reach most buyers with a single listing. Unfortunately, no data are available to us that document the overlap on the buyer side. However, the German Federal Cartel Office analyzed the market of online housing platforms (due to a merger) around 3 years ago. In the report, the presence and relevance of single-homing consumers was explicitly mentioned (see Bundeskartellamt, 2016). Furthermore, the fact that some sellers are multi-homing indicates that they expect to reach additional potential buyers when being active on both platforms—this would not be profitable (given the additional fees charged by the second platform) if many buyers multi-homed. These points suggest that multi-homing of buyers is not a first-order issue in this market.

3 The Setup

We consider markets in which buyers and sellers trade via platforms. In what follows, we describe the three types of actors—platforms, sellers, and buyers.

Platforms. Two homogeneous platforms A and B offer listing services to sellers. The platforms enable transactions between sellers of products or services and their prospective buyers. To be listed on platform i , a seller has to pay a listing fee f_i , $i \in \{A, B\}$. Such listing fees are prevalent in markets in which platforms cannot or do not monitor the sale of a product (such as the housing or rental market). Buyers can access platforms for free.¹⁷ For simplicity, we assume that all platform costs are zero.

Sellers. Sellers have to decide which, if any, platform to join. In the baseline model, they cannot be active on both platforms (i.e., sellers single-home)—in Section 5.2, we show that our results carry over to the case with multi-homing sellers.¹⁸ The product of each seller belongs to a product category. There is a mass 1 of such categories, indexed by $k \in [0, 1]$.

For simplicity, we assume that there are two sellers in each product category.¹⁹ Sellers are symmetric and obtain a per-buyer profit π^d in duopoly. If only one seller is listed on the platform, the seller makes a monopoly profit π^m per buyer, with $\pi^m \geq \pi^d$. We denote the symmetric equilibrium duopoly price by p^d and the monopoly price by p^m . According to our formulation, the per-buyer profit in duopoly and monopoly is independent of the number of buyers. At the end of this section, we mention several microfoundations that fulfill this property.²⁰ However, we explain at the end of Section 4 that our qualitative results hold more generally.

In the baseline model, categories are independent.²¹ This captures the fact that, although platforms usually list many items (a continuum in our model), there is competition between only a few of them. For example, a price comparison website often has thousands or even millions of listed products, but only a few items match a buyer's search request and are displayed to the buyer. Similarly, housing platforms are host to many houses and apartments, but a buyer seeking a house of a particular size in her preferred city is not interested in listings in other categories.

¹⁷We discuss the case of two-sided pricing in Web Appendix G.

¹⁸In some industries, single-homing is a natural assumption. For example, in the market for private accommodations, apartment owners have difficulties synchronizing the calendars when they are active on more than one platform. This favors single-homing. Another example is the modem market for Internet access in which each ISP can use only one modem for technological reasons.

¹⁹In Web Appendix F, we show that all our results carry over to the situation with a general number of sellers per category and a general number of platforms.

²⁰Our formulation is also in line with that used in the empirical literature on market entry, starting with the classic work by Bresnahan and Reiss (1990, 1991).

²¹We discuss several possible interactions between categories in Web Appendix G.

Buyers. Each buyer single-homes—that is, she decides to be active on at most one platform. In Section 5.1, we provide an analysis with multi-homing buyers and demonstrate that our main insights remain valid. Each buyer is interested in a single product category and derives a positive gross utility only from products in this category—see, e.g., Burguet, Caminal, and Ellman (2016) for a similar structure. There is mass 1 of buyers per product category. When visiting a platform, a buyer becomes informed about her preferred product category and the price of all products listed on the platform.²² If a platform lists sellers’ products from a fraction $\alpha \in [0, 1]$ of all categories, a buyer expects to find a product from her preferred category with probability α .

A buyer’s (indirect) utility depends on whether one or two sellers are listed in her preferred category. Prior to observing her idiosyncratic taste realization within this category, the buyer obtains an expected utility of V^d if she expects two sellers to be listed in her preferred category. If she expects only one seller to be listed, her expected utility is $V^m < V^d$. The reason for this inequality is twofold: first, if two sellers are listed, they charge the duopoly price p^d , which, in many instances, is less than p^m . Second, if sellers are differentiated, a buyer will find a product closer to her preferences or may enjoy greater variety if two sellers are listed instead of only one.

Timing. The timing is as follows:

1. Platforms A and B set listing fees f_A and f_B , respectively.
2. Sellers and buyers make a discrete choice between platforms A and B , and the outside option (normalized to zero).
3. Sellers in each category set product prices.
4. Buyers observe all offers on the platform they are visiting and make their purchasing decisions.

We make three remarks about our setup. First, according to our timing, sellers decide where to list before setting prices on the product market. This is the relevant timing in most applications because the choice of platform is typically longer-term than the pricing decision. Hence, sellers set prices after learning about the number of competitors in the product market. In addition, listing fees are often paid on a subscription basis, which makes them lumpy, while prices charged by the sellers are flexible.

Second, listing fees do not enter the pricing decisions of sellers in the third stage because they are “fixed” costs for sellers (which are, in addition, sunk when sellers set prices). As we show in Section 6.1, our results still hold with per-transaction fees and revenue shares, which affect sellers’ pricing decisions.

²²The assumption that a buyer learns her preferred category only after deciding which platform to visit is only made to simplify the analysis. All results would also hold if buyers knew their preferred category already at the outset.

Third, we do not impose a particular model of buyer-seller interaction for the subgames starting in stage 3 and, instead, use a reduced-form approach. We provide a microfoundation at the end of this section and two further microfoundations in Web Appendix B.

Payoffs. The profit of platform i is the number of sellers active on platform i multiplied by the listing fee f_i . The profit of a seller who is listed on platform i is $\beta_i\pi - f_i$, where β_i is the fraction of buyers in the seller's category that are active on platform i , and π is either π^m if the rival seller is not listed on platform i or π^d if the rival also lists on platform i . As mentioned above, the utility of a buyer is V^d or V^m and, thus, depends on the number of sellers listed in the buyer's preferred category; the utility is 0 if none of those sellers is listed on the platform where the buyer is active.

Solution Concept. Our solution concept is subgame perfect Nash equilibrium. We assume the following tie-breaking rule. If buyers expect one seller in each category to list on platform A and the other seller on platform B , half of the buyers in each category join platform A and the other half platform B . A natural interpretation is that each buyer mixes with equal probability to be active on either platform A or B . Since there is a continuum of buyers, both platforms will, in fact, be patronized by one half of the buyers.²³ As we point out below, this assumption is not crucial for the results and can be relaxed, allowing for unequal distributions of buyers in the case of indifference.

In the second stage, buyers and sellers face a coordination game on which platform(s) to be active, which may lead to a multiplicity of equilibria. To deal with this well-known issue in two-sided markets, we impose the refinement of coalition-proofness (see e.g., Bernheim, Peleg, and Whinston, 1987a, 1987b). That is, we select only Nash equilibria that are stable against deviations by coalitions of sellers and buyers; and, within the coalition, no subset of sellers and buyers benefits from a further deviation.²⁴ In addition, when coalition-proofness is not sufficient to obtain equilibrium uniqueness, we select equilibria that are profit-dominant for sellers. We will show that the joint application of these refinements leads to a unique equilibrium outcome in stage 1.

A justification of the refinement is that the outcome is equivalent to the outcome of a sequential game in which sellers decide which platform to join before buyers do, as considered, for example, by Hagiu (2006), and sellers play a coalition-proof Nash

²³Another interpretation is that platforms are differentiated by different platform designs but that this differentiation is negligibly small. For example, platforms are differentiated along a Hotelling line, and the transport cost parameter t goes to zero. This means that buyers ex ante have lexicographic preferences, in the sense that they prefer the platform with a larger number of sellers. Buyers decide according to their preference for different platform designs only if they expect this number to be the same across platforms.

²⁴In our game, in stage 2, a coalition-proof Nash equilibrium is equivalent to a Strong Nash equilibrium (Aumann, 1959), which ignores deviations by subcoalitions. This is due to buyers being ex ante identical and sellers benefiting from the presence of more buyers.

equilibrium. In Section 6.2, we analyze the mirror case, in which the payoff-dominant equilibrium for buyers is selected, and demonstrate that the main insights of our analysis will be unchanged. In Web Appendix C, we discuss under which conditions one or the other refinement is more likely, and how our equilibrium selection mechanism might operate in reality.

Summary statistic. As will become clear in the next section, the key summary statistic for our equilibrium characterization is the ratio π^d/π^m , which is an inverse measure of the degree of product market competition and takes values in $[0, 1]$. It is determined by the buyer-seller interaction in stages 3 and 4.

Microfoundation of the Buyer-Seller Interaction. Buyers' choices in stage 4 and sellers' pricing decisions in stage 3 are straightforward: in the fourth stage, depending on the number of listed sellers in her preferred product category, a buyer buys nothing, or a certain amount of one or both products, according to her demand function. In the third stage, sellers set p^d in case they face a competitor in their product category on the platform and p^m in case of monopoly.

In the next paragraph, we provide a simple microfoundation of the buyer-seller interaction based on the Hotelling model and determine per-buyer profits π^d and π^m . In Web Appendix B, we provide two additional examples. First, we analyze a representative consumer model with linear demand (Bowley, 1924, or Singh and Vives, 1984). Other discrete-choice and representative consumer models—for instance, with CES or logit demand—would work as well. Second, we analyze a simple model of thin markets in which there is only a small number of buyers and capacity-constrained sellers (as in the housing market). Another microfoundation that we do not develop here but that also fits our assumptions are models of sequential product search (e.g., Wolinsky, 1986, and Anderson and Renault, 1999).

Consider Hotelling competition in each product category. Each seller is located at one of the extreme points of the unit interval in a particular category—i.e., a seller j is characterized by its category k_j and its location l_j on the unit interval, $(k_j, l_j) \in [0, 1] \times \{0, 1\}$. The buyers' valuation of a product at the ideal location in the preferred category equals v . If a buyer likes category k and is located at x_k (with $(k, x_k) \in [0, 1] \times [0, 1]$), her utility from buying one unit of seller j 's product in this product category is $v - t|x_k - l_j| - p_{lj}$ where $t > 0$ captures the degree of product differentiation. Her utility is zero for products in all categories that are not equal to k . Price competition among Hotelling duopolists leads to equilibrium prices $c+t$ and equilibrium profits $\pi^d = t/2$ per unit mass of buyers.²⁵ A monopoly seller sets price $p^m = (v + c)/2$, and its profit is $\pi^m = (v - c)^2/(4t)$ per unit mass of buyers if the market is not fully covered. This is the case if $t \geq (v - c)/2$. In this

²⁵The upper bound on t is $2(v - c)/3$, as the buyer who is indifferent between both sellers would not obtain a positive utility otherwise.

parameter range, $p^m \leq p^d$. For $t < (v - c)/2$, there is full coverage, and the monopolist sets $p^m = v - t$. Its profit is $\pi^m = v - t - c$.

In the Hotelling model, the ratio π^d/π^m is therefore given by $2t^2/(v - c)^2$ if $(v - c)/2 \leq t \leq 2(v - c)/3$ and by $t/[2(v - t - c)]$ if $t < (v - c)/2$. It follows that $\pi^d/\pi^m \geq 1/2$ for $t \geq (v - c)/2$, and vice versa. That is, if products are sufficiently differentiated, twice the duopoly profit is larger than the monopoly profit.

4 Segmentation versus Agglomeration

In this section, we characterize the equilibrium of the 4-stage game. In particular, we provide conditions for segmentation or agglomeration to be an equilibrium outcome.

In the last section, we analyzed stages 3 and 4. We now turn to the location decisions of buyers and sellers in stage 2. Here, multiple Nash equilibria may exist, given the listing fees set by platforms in the first stage. We first determine the set of Nash equilibria in stage 2. We then explain how our equilibrium selection criteria ensure a unique prediction. A detailed analysis is provided in Web Appendix D.

There exist two types of Nash equilibria. In the first one, all buyers and all active sellers are on one platform and trade takes place only on this platform. In the second one, both platforms are active and each one hosts half of sellers and buyers.

We start by describing the equilibria that can occur in the first type. If a platform charges a fee below π^d , an *agglomeration equilibrium* exists in which both sellers (and all buyers) list on this platform. In addition, if a platform charges a fee between π^d and π^m , an equilibrium exists in which one seller in each category is active on this platform (and all buyers are on this platform). We call an equilibrium of this type *stand-alone equilibrium*.

In the second equilibrium type—i.e., the *segmentation equilibrium*—each platform hosts one seller in all categories.²⁶ The equilibrium exists only if sellers obtain non-negative profits, which implies that both fees cannot be above $\pi^m/2$ and no seller on platform i prefers to be active on platform $-i$. The latter is ensured by the condition $\pi^m/2 - f_i \geq \pi^d/2 - f_{-i}$. This implies that the segmentation equilibrium exists if and only if $f_i \leq \max\{(\pi^m - \pi^d)/2 - f_{-i}, \pi^m/2\}$.

The set of Nash equilibria is visualized in Figure 2—we focus on the relevant range (f_A, f_B) with $f_A \leq \pi^m$ and $f_B \leq \pi^m$ because a fee above π^m leads to zero demand and, in the equilibrium of the full game, no platform will set such a fee. The left panel of the figure displays the case $\pi^d/\pi^m < 1/2$, and the right panel displays the opposite

²⁶We show in Web Appendix D that there does exist a segmentation equilibrium with a different seller composition.

case. In the figure, agglomeration on platform i is denoted by AGG_i (and by AGG_{AB} if agglomeration on each platform is an equilibrium); the stand-alone equilibrium is denoted by STA_i ; and segmentation is denoted by SEG . As can be seen in the left panel, there are regions in which three equilibrium configurations coexist.

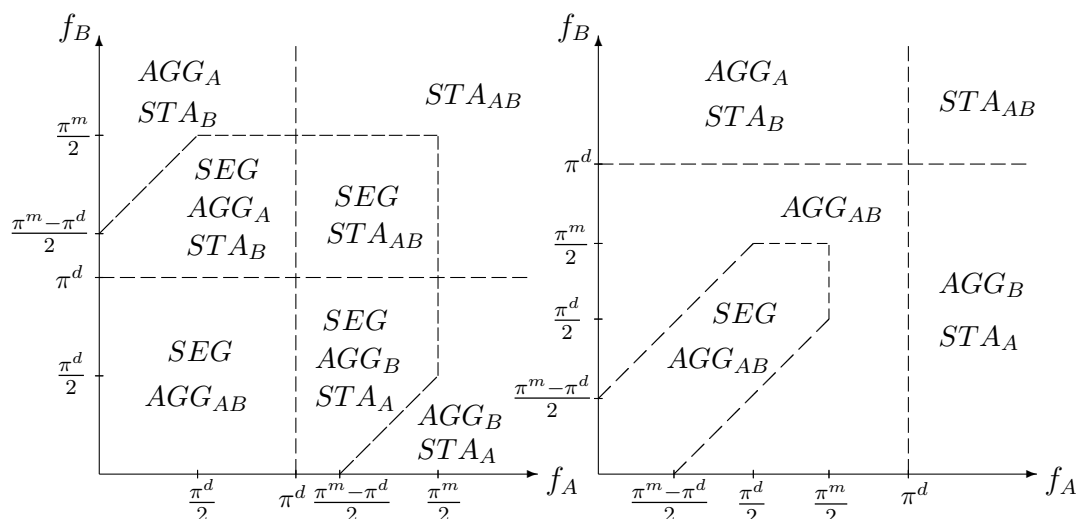


Figure 2: Possible equilibrium configurations in stage 2 : $\pi^d/\pi^m < 1/2$ on the left-hand side and $\pi^d/\pi^m \geq 1/2$ on the right-hand side

We turn to the equilibrium selection accomplished through our refinement. First, consider the situation in which two agglomeration (or two stand-alone) equilibria exist, and, thus, sellers and buyers need to coordinate on which platform to be active. Applying coalition-proofness eliminates the multiplicity of agglomeration equilibria off the diagonal. The reason is that a coalition of sellers and buyers will always choose to be active on the platform with the lower fee. The same reasoning holds if there is a multiplicity of stand-alone equilibria.

Second, consider the situation in which the segmentation equilibrium or the agglomeration co-exist with the stand-alone equilibrium. The stand-alone equilibrium is then never coalition-proof. The reason is that all inactive sellers can then form a coalition with all buyers and trade on the inactive platform. This deviation is neutral for the buyers but strictly profitable for the inactive sellers. Therefore, when co-existing with another equilibrium, the stand-alone equilibrium is always eliminated.

Third, we turn to the region in which the agglomeration and segmentation equilibrium co-exist. The joint use of coalition-proofness and profit dominance of sellers singles out a unique equilibrium in this case.²⁷ First, for $\pi^d/\pi^m \geq 1/2$, the segmentation equilibrium is not stable to the deviation of a coalition of sellers and buyers who are active on a platform with a weakly higher fee. If this coalition switches to the rival platform,

²⁷In Web Appendix D, we show that the two refinements are never in conflict with each other.

buyers are better off because they observe the offers of all sellers, and sellers are weakly better off because they now serve all buyers instead of only half of them. Thus, all segmentation equilibria are eliminated when $\pi^d/\pi^m \geq 1/2$, as can be seen in the right panel of Figure 3. Instead, if $\pi^d/\pi^m < 1/2$, the same argument singles out a unique equilibrium only if $f_i - f_{-i} \geq \pi^m/2 - \pi^d$. Then, the coalition of sellers and buyers active on the platform with the higher fee can profitably deviate to the rival platform. By contrast, if $f_i - f_{-i} < \pi^m/2 - \pi^d$, coalition-proofness has no bite, as the deviation is no longer profitable for sellers. Using seller dominance, however, now singles out a unique equilibrium because segmentation is more profitable than agglomeration for sellers. Hence, as illustrated in the left panel of Figure 3, segmentation is the unique equilibrium if f_i and f_{-i} are sufficiently close to each other (and lower than $\pi^m/2$). The region for segmentation shrinks as π^d gets larger and vanishes if $\pi^d \rightarrow \pi^m/2$.

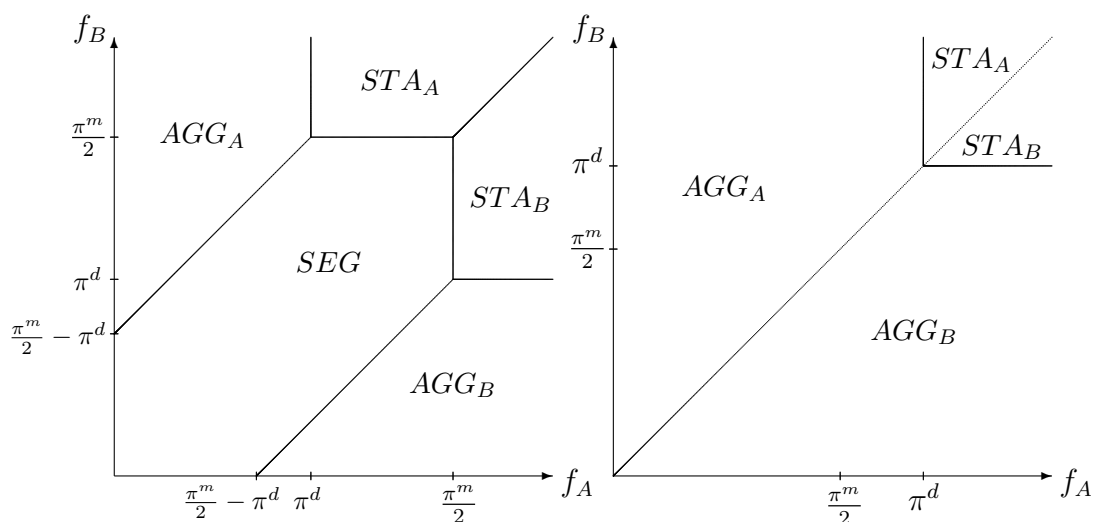


Figure 3: Selected equilibrium configurations in stage 2: $\pi^d/\pi^m < 1/2$ on the left-hand side and $\pi^d/\pi^m \geq 1/2$ on the right-hand side

We now turn to platform pricing in the first stage. Although platforms are homogeneous, the Bertrand logic does not necessarily apply because sellers may benefit from segmentation, which implies that a platform does not necessarily attract all sellers and buyers when undercutting the rival's fee. In four propositions, we characterize the equilibrium listing fees for the different parameter regions and provide precise conditions for platforms to sustain positive fees.

If the ratio of duopoly to monopoly profits is large (i.e., $\pi^d/\pi^m \geq 1/2$), agglomeration occurs. From a seller's point of view, the effect that agglomeration reduces profits due to competition is dominated by the demand expansion effect that all buyers (instead of only half of them) observe the seller's offer. Since each platform attracts the entire demand by setting a fee lower than its rival, platforms "play hard" and fight fiercely to

become dominant. Thus, in this region, the standard Bertrand argument applies, and homogeneous platforms charge fees equal to marginal cost in equilibrium.

Proposition 1. Agglomeration. *If $\pi^d/\pi^m \geq 1/2$, in equilibrium, the listing fees are $f_A^* = f_B^* = 0$, and platforms' profits are $\Pi_A^* = \Pi_B^* = 0$.*

By contrast, if the ratio of duopoly to monopoly profits is small (i.e., $\pi^d/\pi^m \leq 1/4$), segmentation occurs. Sellers avoid competition by listing on different platforms, which, in turn, is exploited by platforms. To see this, suppose that both platforms charge a fee of zero. If π^d is lower than $\pi^m/2$, sellers choose to segment. But then a platform can raise its fee slightly without reducing its demand. Thus, the platform with the higher fee remains active and raises strictly positive profits.

Proposition 2. Segmentation. *If $\pi^d/\pi^m \leq 1/4$, in the unique equilibrium, the listing fees are $f_A^* = f_B^* = \pi^m/2$, and platform profits' are $\Pi_A^* = \Pi_B^* = \pi^m/2$.*

The proposition shows that platforms not only obtain a strictly positive profit, but even extract the *entire* surplus from sellers. The argument is as follows. If a platform deviates from the equilibrium listing fee $f_i^* = \pi^m/2$ to a listing fee slightly below π^d , this induces sellers and buyers to agglomerate on the deviating platform. The deviant platform then obtains a profit of $2\pi^d$. Instead, the equilibrium profit is $\pi^m/2$, which is larger than $2\pi^d$ if $\pi^d/\pi^m \leq 1/4$. Hence, no platform has an incentive to deviate from the listing fee $\pi^m/2$ —platforms “play soft” and do not fight for the dominant position. To sum up, if competition between sellers is sufficiently intense, platforms obtain positive profits by inducing sellers to segment the market. Interestingly, fierce competition among sellers enables platforms to sustain high profits in equilibrium.

In the intermediate range $1/4 < \pi^d/\pi^m < 1/2$, platforms randomize over listing fees. The intuition for the non-existence of a pure-strategy equilibrium in this range is as follows: for any fee set by platform i , platform $-i$'s best response is to either set a fee that is lower by a discrete amount to induce agglomeration or to set a fee that is higher by a discrete amount leading to segmentation. This creates a cycle in best responses. Suppose that platform i sets a relatively high fee. Platform $-i$'s best response is then to set a lower fee, so as to just induce agglomeration. The best response of platform i is to lower its fee slightly and induce segmentation again. This sequence of best responses continues until the fee of platform i reaches such a low level that platform $-i$, instead of setting a lower fee, prefers to set a fee higher than that of platform i , so as to just induce segmentation. In turn, platform i 's best response is to reduce its fee slightly to induce agglomeration, and so on. Therefore, the sequence continues and does not converge.

The logic behind the mixed-strategy equilibrium in the range $1/4 < \pi^d/\pi^m < 1/2$ is reminiscent of, but distinct from, Bertrand-Edgeworth cycles. In the latter, the best-response dynamic involves a marginal undercutting of the rival's fee, as long as fees

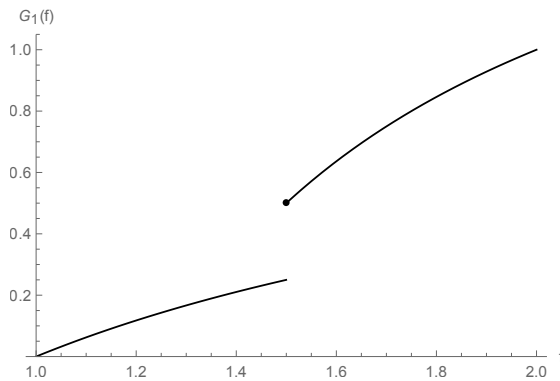


Figure 4: First mixed-strategy equilibrium: Cumulative distribution function with parameters $\pi^m = 5$ and $\pi^d = 2$.

are sufficiently high (see, for example, Edgeworth, 1925; Maskin and Tirole, 1988). By contrast, in our model, for any fee charged by the rival, the best response is to set a fee that is higher or lower by a discrete amount.²⁸ In fact, the range of subscription fees over which platforms mix can be divided into two intervals, a lower and an upper one. In the lower interval, fees are set with the intention to induce agglomeration. In the upper interval, fees are set with the intention to induce segmentation. This leads to mass points in the mixing distribution and potentially disjoint mixing sets.

In the region of $3/8 \leq \pi^d/\pi^m < 1/2$, the upper bound of the lower interval in which a platform aims to induce agglomeration coincides with the lower bound of the upper interval in which a platform aims to induce segmentation. This implies that platforms randomize over a convex set.

Proposition 3. Probabilistic segmentation and agglomeration with listing fees chosen from a convex set. *If $3/8 \leq \pi^d/\pi^m < 1/2$, there is a unique mixed-strategy equilibrium, in which platforms set fees in the domain $f_i \in [\pi^m - 2\pi^d, 2\pi^m - 4\pi^d]$. The mixing probability is characterized by the cumulative distribution function*

$$G_1(f) = \begin{cases} \frac{f - (\pi^m - 2\pi^d)}{f + 1/2(\pi^m - 2\pi^d)}, & \text{if } f \in [\pi^m - 2\pi^d, 3/2\pi^m - 3\pi^d]; \\ \frac{2f - 5/2(\pi^m - 2\pi^d)}{f - 1/2(\pi^m - 2\pi^d)}, & \text{if } f \in [3/2\pi^m - 3\pi^d, 2\pi^m - 4\pi^d], \end{cases}$$

with a mass point at $f = 3/2\pi^m - 3\pi^d$, which is chosen with probability $1/4$. The expected profit is $\Pi_A^* = \Pi_B^* = 3\pi^m/2 - 3\pi^d$.

The cumulative distribution function $G_1(f)$ is illustrated in Figure 4 using values $\pi^m = 5$ and $\pi^d = 2$. The mass point is at the fee that separates the two intervals. Therefore, setting such a fee induces segmentation with probability (almost) 1. Since the

²⁸In this respect, our equilibrium also differs from those found in papers in the search literature, such as Varian (1980) or Janssen and Moraga-González (2004).

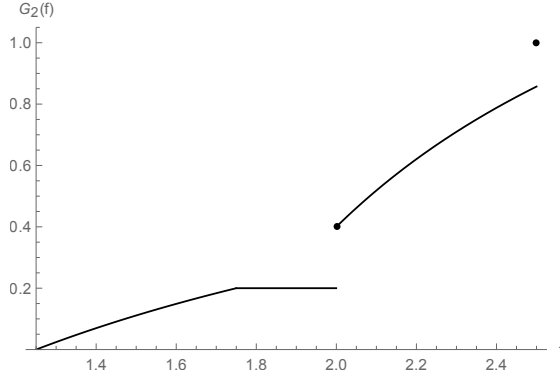


Figure 5: Second mixed-strategy equilibrium: Cumulative distribution function with parameters $\pi^m = 5$ and $\pi^d = 7/4$.

event that both platforms choose this fee occurs with strictly positive probability, the expected equilibrium profit in this regime must equal $3/2\pi^m - 3\pi^d$.

The highest fee that platforms can charge to obtain positive demand is $\pi^m/2$. If π^d/π^m is at the lower bound of the mixing region of Proposition 3 (i.e., $\pi^d/\pi^m = 3/8$), the highest fee in the mixing range, $2\pi^m - 4\pi^d$, reaches this level. It follows that if π^d/π^m is lower, the equilibrium will be different. In particular, as a fee of $\pi^m/2$ must be the upper bound, probability mass will be shifted to this point, and the distribution will entail a mass point at the highest fee. In addition, the best response to this highest fee (i.e., the largest fee in the lower interval) no longer coincides with the fee that induces segmentation with probability (almost) 1. The latter fee is the lowest one in the upper interval, and the support of the mixing region becomes non-convex. This is stated in Proposition 4.

Proposition 4. Probabilistic segmentation and agglomeration with listing fees chosen from a non-convex set. *If $1/4 < \pi^d/\pi^m < 3/8$, there is a unique mixed-strategy equilibrium, in which platforms set fees in the domain $f_i \in [\pi^m/4, \pi^d) \cup [3\pi^m/4 - \pi^d, \pi^m/2]$. The mixing probability is characterized by the cumulative distribution function*

$$G_2(f) = \begin{cases} \frac{f-1/4\pi^m}{f+1/2(\pi^m-2\pi^d)}, & \text{if } f \in [\pi^m/4, \pi^d); \\ \frac{2f-1/4\pi^m-3/2(\pi^m-2\pi^d)}{f-1/2(\pi^m-2\pi^d)}, & \text{if } f \in [3\pi^m/4 - \pi^d, \pi^m/2); \\ 1, & \text{if } f = \pi^m/2; \end{cases}$$

with two mass points, one at the highest fee in the support $f = \pi^m/2$, which is chosen with probability $(3/4\pi^m - 2\pi^d)/\pi^d$, and the other at the lower bound of the upper interval $f = 3\pi^m/4 - \pi^d$, which is chosen with probability $(2\pi^d - 1/2\pi^m)/\pi^m$. The expected profit is $\Pi_A^* = \Pi_B^* = 3\pi^m/4 - \pi^d$.

Figure 5 illustrates $G_2(f)$ in the second mixing regime using values $\pi^m = 5$ and

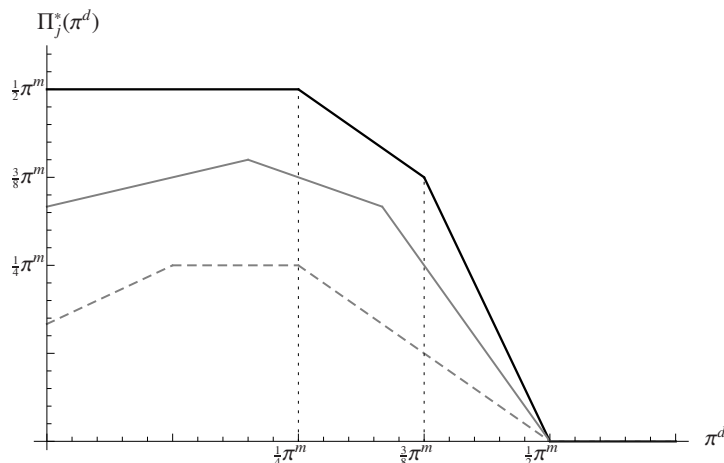


Figure 6: Expected platform profit Π_j^* as a function of π^d . The solid black line is relevant for Section 4 in which consumers are single-homing. The solid gray line and the dashed gray line are relevant for Section 5.1 and correspond to the scenario in which sellers price discriminate and the fraction of multi-homing buyers is $\lambda = 1/3$ (solid gray line) and $\lambda = 2/3$ (dashed gray line), respectively.

$\pi^d = 7/4$. The support of the distribution is then $[5/4, 7/4] \cup [2, 5/2]$. The intuition for the lower mass point (at $f = 3\pi^m/4 - \pi^d$) is the same as that in the first mixed regime. The intuition for the mass point at $f = \pi^m/2$ is, as explained above, that $\pi^m/2$ is an upper bound on profits in any mixing equilibrium.

As follows from Proposition 4, the gap between the two intervals widens as π^d falls. In the limit, as $\pi^d \rightarrow \pi^m/4$, all probability mass is shifted to $\pi^m/2$. As π^d falls, expected fees rise continuously, as do platform profits. The expected equilibrium platform profit is a continuous function but has kinks at the boundary points of the four regions, as displayed in Figure 6. In this figure, only the solid black curve—i.e., the highest one—is relevant for this section; the other two curves correspond to a situation with buyer multi-homing, which we discuss in Section 5.1.

From the analysis, it is easy to see that the assumption of buyers splitting evenly between platforms when being indifferent is not crucial for the results. If this split is more in favor of platform i , the pure-strategy segmentation equilibrium exists for a smaller range: as this equilibrium is less attractive for platform $-i$, this platform has a stronger deviation incentive. However, for any asymmetric split, if π^d is sufficiently small, a deviation to agglomeration is not profitable for platform $-i$. Thus, an asymmetric segmentation equilibrium prevails.

A welfare comparison of the different possible outcomes includes buyers' surplus. There are two reasons why segmentation is worse for buyers than agglomeration. First, because buyers are not informed about all offers, they, on average, buy products with a greater mismatch than when they are informed about all offers. Second, if $p^m > p^d$,

buyers suffer from the higher price under monopoly, implying that the quantities bought by buyers under segmentation are lower than under agglomeration. As a consequence, whereas platforms enjoy profits when they induce segmentation with positive probability, buyers suffer from this market structure, and welfare tends to be lower. In Section 7, we discuss some policy implications that arise from our analysis.

We can express the equilibrium regions in terms of the underlying parameter describing seller competition in the microfoundation laid out in the previous section. In the Hotelling model, a lower degree of product differentiation t decreases the sufficient statistic π^d/π^m . We obtain that the agglomeration region applies if $2(v-c)/3 \geq t \geq (v-c)/2$, the first mixing region if $(v-c)/2 > t \geq 3(v-c)/7$, the second mixing region if $3(v-c)/7 > t > (v-c)/3$, and the segmentation region if $(v-c)/3 \geq t \geq 0$.

Finally, we note that our results do not rely on a constant per-buyer profit. If this profit was not constant, the equilibrium characterization would be more involved, as the boundaries of the regions then depend on the mass of buyers on each platform in addition to the profit per buyer. However, under standard regularity assumptions on demand, the qualitative results are the same as in our analysis.

5 Multi-Homing

In the baseline model, we focus on the case in which both buyers and sellers are single-homing. In this section, we consider multi-homing of buyers and sellers. We will show that in both cases, our qualitative results continue to hold.

5.1 Multi-Homing of Buyers

Suppose that a fraction $\lambda \in [0, 1]$ of buyers joins both platforms. A natural reason is that buyers incur heterogeneous time costs to be active on a second platform. Then, only buyers with sufficiently low time costs are active on both platforms. A higher λ here corresponds to lower time costs in the population.²⁹

Multi-homing of buyers affects seller profits. In fact, a seller will never obtain the monopoly profit when a positive fraction λ of buyers multi-home and is therefore informed about both offers. In a segmentation equilibrium, half of the single-homing buyers are active on platform A and the other half on platform B . Because there is a mass $1 - \lambda$ of single-homing buyers, each platform has a total buyer mass of $(1 + \lambda)/2$, out of which $(1 - \lambda)/2$ are single-homers and λ are multi-homers. Suppose that sellers do not know which buyers single-home and which ones multi-home and, thus, set a single price in the

²⁹For example, if a distribution of time costs among buyers first-order stochastically dominates another one, the latter distribution leads to a larger fraction λ of multi-homing buyers.

product market. The equilibrium price will depend on λ because multi-homers' demand may differ from that of single-homers' demand. Therefore, in a segmentation equilibrium, we can write the *average* seller profit per buyer (assuming constant marginal costs c of sellers) as

$$\pi(\lambda) \equiv (p(\lambda) - c) \left(\frac{1 - \lambda}{1 + \lambda} D^m(p(\lambda)) + \frac{2\lambda}{1 + \lambda} D^d(\mathbf{p}(\lambda)) \right).$$

In this expression, $D^m(\cdot)$ is the seller's monopoly demand per buyer, which depends only on the seller's own price—i.e., $p(\lambda)$ —and $D^d(\cdot)$ is duopoly demand per buyer, which depends on the seller's price and that of its competitor, abbreviated by the vector $\mathbf{p}(\lambda)$. A seller receives a demand of $D^m(\cdot)$ from a single-homing buyer and that of $D^d(\cdot)$ from a multi-homing one. The demand weights are the masses of both groups, adjusted by the total buyer mass per platform $(1 + \lambda)/2$. It is evident that $\pi(0) = \pi^m$ and $\pi(1) = \pi^d$. Naturally, $\pi'(\lambda) \leq 0$, which implies that for all $\lambda \in [0, 1]$, $\pi(\lambda) \in [\pi^d, \pi^m]$.³⁰ Since the share of multi-homing and single-homing buyers affects seller competition, the average per-buyer profit is a function of λ , denoted in reduced form by $\pi(\lambda)$.

The equilibrium with multi-homing buyers is characterized by the following proposition.

Proposition 5. *All results of Propositions 1 through 4 carry over to the case of buyer multi-homing, after replacing $\pi^m/2$ by $(1 + \lambda)\pi(\lambda)/2$.*

The proposition states that the qualitative results of the previous section remain valid if some, but not all buyers multi-home. Even though segmentation does not give monopoly power to sellers, it nevertheless lowers the competitive pressure because some buyers are not informed about all sellers' offer, and platforms exploit this.

Do platforms benefit from buyer multi-homing? If we are in the range of the agglomeration equilibrium, nothing changes compared to buyer single-homing because platforms are engaged in Bertrand competition. However, this is not true for the regions in which the segmentation equilibrium occurs with positive probability. There are two opposing forces. First, sellers benefit from a demand-expansion effect as they have access to multi-homing buyers, regardless of the platform they are active on. In equilibrium, each platform provides access to mass $(1 + \lambda)/2$ of buyers instead of $1/2$ when all buyers single-home. Second, there is a countervailing competition effect, as multi-homing buyers are informed about both offers. The average per-buyer profit is then $\pi(\lambda) < \pi^m$.³¹ The next proposition states that either effect can dominate.

³⁰In an agglomeration equilibrium, a seller's profit is unchanged since all buyers see both offers. This leads to a per-buyer profit of π^d for each seller.

³¹We observe that multi-homing consumers exert a positive externality on single-homing ones. As product prices are lower with more multi-homers, single-homers benefit as well. This implies that devices which foster multi-homing, such as metasearch engines, also benefit consumers who do not use them.

Proposition 6. *Suppose that sellers' products are differentiated and there is an extensive demand margin, that is, the product market is not fully covered in monopoly at price $p^m(\lambda)$. Then, under segmentation,*

- (i) *platform profits strictly increase with λ in the vicinity of $\lambda = 0$.*
- (ii) *platform profits in the vicinity of $\lambda = 1$ are below those at $\lambda = 0$.*

It follows from the proposition that, when segmentation occurs with positive probability, under mild conditions, platform profits are non-monotonic in λ —i.e., they are first increasing and then decreasing. For the microfoundations that satisfy the conditions of this proposition, we obtain that profits as function of λ are inversely U-shaped.³²

The intuition behind the result is as follows: if λ is close to zero, an increase in λ has only a second-order effect on the seller's price. The reason is that $p(0)$ is chosen optimally, given that the seller is a monopolist in its product market, and, by the Envelope Theorem, a small increase in λ affects the optimal price only by a negligible amount. Instead, the demand increase is of first order, which implies that the demand-expansion effect dominates the competition effect if sellers' products are differentiated.³³ Hence, an increase in the number of multi-homing buyers leads to higher platform profits if only few buyers multi-home.

By contrast, for λ close to 1, the opposite result holds. For $\lambda \rightarrow 1$, we have $(1 + \lambda)\pi(\lambda)/2 \rightarrow \pi^d$ as almost all consumers observe both offers. This implies that there is no longer a difference between the segmentation and the agglomeration equilibrium. Platforms can no longer exploit that they reduce seller competition, and Bertrand style competition between platforms occurs. Therefore, platforms obtain zero profits and, thus, are worse off than with single-homing buyers.

The exact shape how platform profits change in λ depends on the concrete demand function. This is not the case if sellers can distinguish between single-homing and multi-homing buyers in the price-setting stage and, thus, price discriminate between the two groups. This applies if sellers track the behavior of buyers on the web and obtain the information whether a specific buyer is a single- or a multi-homer. In this case, sellers set price p^m to a single-homer and price p^d to a multi-homer; hence, the average seller profit per buyer is $(1 - \lambda)/(1 + \lambda)\pi^m + 2\lambda/(1 + \lambda)\pi^d$. Then, a platform's profit is $(1 - \lambda)/2\pi^m + \lambda\pi^d$, which is strictly decreasing in λ , whenever the segmentation equilibrium emerges with

³²In the Hotelling model, the proposition does not apply because the product market is fully covered in monopoly whenever segmentation occurs with positive probability. In this case, profits are globally decreasing in the share of multi-homing consumers.

³³If products are homogeneous, we cannot pursue the analysis in the same way because $D^d(\mathbf{p}(0))$ changes discontinuously in prices. Competition plays out as in a situation in which each seller has some captive buyers (i.e., the single-homing ones) but buyers informed about both prices (i.e., the multi-homing ones) purchase the cheaper product—see, e.g., Varian's (1980) model of sales. In the resulting (mixed-strategy) equilibrium, expected seller profits decrease in the share of multi-homers, regardless of the level of λ .

positive probability. Thus, platforms are unambiguously worse off the larger is λ . The expected platform profit for $\lambda = 1/3$ and $\lambda = 2/3$ is depicted in Figure 6 (see Section 4).

5.2 Multi-Homing of Sellers

In this section, we consider the effects of seller multi-homing. Two pricing scenarios by a multi-homing seller are possible; either a multi-homing seller sets the same price on both platforms (uniform pricing) or it can set different prices on the two platforms (price discrimination). We start with the first scenario. In some markets, sellers cannot price discriminate among platforms. This is the case if platforms impose a most-favored nation clause in their contracts with sellers, which forces multi-homing sellers to set the same price on the platforms. Such contracts have been in place, for example, in the e-book or the online hotel bookings market.³⁴ In other markets, platforms lead the consumer traffic to the website of the seller, and the seller cannot or, for reputation reasons, does not want to condition its price on the consumer's lead-in site. As we show towards the end of the section, the analysis with price discrimination is a special case of that with uniform pricing.

In contrast to buyers, sellers need to pay for being active on a platform. Therefore, even without any exogenous costs of using a second platform, sellers do not necessarily find it profitable to multi-home—introducing such costs for a second listing would not change the main result. We investigate the conditions under which seller multi-homing affects the platform market structure and whether platforms benefit. This focus is different from that in previous literature on two-sided markets (see, e.g., Armstrong, 2006, Hagiu, 2006, or Belleflamme and Peitz, 2019), which investigated the effect of seller multi-homing on the price structure but took the platform market structure as given.

With multi-homing sellers, in the second stage new potential equilibrium configurations may occur. First, both sellers in a category may multi-home. In that case, all buyers are exposed to both offers, implying that each seller receives the duopoly profit π^d per buyer. The profit per buyer is then equivalent to the profit when both sellers agglomerate on one platform. But, in the latter case, sellers have to pay only one listing fee. Therefore, the configuration in which both sellers multi-home is never coalition-proof.

Second, a configuration is possible in which one seller in a category single-homes and the other one multi-homes—a situation we refer to as partial multi-homing. If in one half of the categories, the single-homing seller is on platform A and in the other half on platform B , buyers are indifferent between both platforms and are willing to split evenly between the platforms. In this situation, competition in the product market is

³⁴See, for example, *U.S. v. Apple Inc.*, 12 Civ. 2826 (2015) on the e-book market, and *Skyscanner Ltd. v. Competition and Markets Authority*, 1226/2/12/14 (2014), on the online hotel booking market.

asymmetric. In each category, half of buyers are active on the platform in which only the multi-homing seller is present and observe only the offer of this seller, whereas the other half observes the offers of both sellers. Let us denote the per-buyer profit of the multi-homing seller by π^{MH} and that of the single-homing seller by π^{SH} . As the multi-homing seller can act as a monopolist to one half of the buyers but faces competition for the other half, its uniform price p^{MH} will be between p^m and p^d . The same holds for the price of the single-homing seller p^{SH} , as this seller faces competition from a rival who has some “exclusive” buyers and, therefore, will set a price higher than p^d . For the sellers’ profits, we assume that

$$\pi^d \leq \pi^{SH} \leq \pi^{MH} \leq \pi^m,$$

which follows from the sellers’ pricing decisions. The next proposition characterizes the equilibrium with multi-homing sellers.

Proposition 7. *Suppose that multi-homing sellers cannot price discriminate. Then,*

- *for $\pi^d/\pi^m \geq 1/2$ and $\pi^d/\pi^m \leq 1/4$, the equilibrium is the same as the one characterized in Propositions 1 and 2, respectively.*
- *For $3/8 \leq \pi^d/\pi^m < 1/2$, the equilibrium is the same as the one characterized in Proposition 3 if $\pi^{MH} \leq 3/2\pi^m - 2\pi^d$. Similarly, for $1/4 < \pi^d/\pi^m < 3/8$, the equilibrium is the same as the one characterized in Proposition 4 if $\pi^{MH} \leq 3/4\pi^m$.*
- *Instead, for (i) $3/8 \leq \pi^d/\pi^m < 1/2$ and $\pi^{MH} > 3/2\pi^m - 2\pi^d$ and for (ii) $1/4 < \pi^d/\pi^m < 3/8$ and $\pi^{MH} > 3/4\pi^m$, respectively, in equilibrium, platforms set fees of $f_A^* = f_B^* = 0$, and sellers play an agglomeration equilibrium if $\pi^d > \pi^{SH}/2$ and a partial multi-homing equilibrium if $\pi^d \leq \pi^{SH}/2$.*

The proposition shows that for some parameter constellations, the equilibrium derived in Propositions 1 to 4 remains unchanged. Foremost, if competition between sellers is relatively fierce, the segmentation equilibrium still exists. Although sellers can multi-home, doing so would reduce their profits by too large an amount; hence, they prefer segmentation. Platforms exploit this by extracting the entire seller surplus. Hence, our insight that segmentation leads to high platform profits, even though platforms are homogeneous, is robust to seller multi-homing.

The proposition also shows that the mixed-strategy equilibrium, which involves segmentation with some probability and features positive platform profits, emerges for a smaller parameter range than in the case of single-homing sellers. It is replaced by an equilibrium in which platforms charge zero listing fees. Thus, we obtain the unambiguous result that platforms set (weakly) lower fees to sellers and earn (weakly) lower profits

if the latter can multi-home instead of single-home. This contrasts with the standard intuition in the two-sided markets literature with two-sided pricing, which finds that platforms exert monopoly power on the seller side and, in equilibrium, may set higher fees to sellers and earn higher profits under seller multi-homing.

The intuition behind our result is as follows: if sellers can multi-home, segmentation may break down because sellers have an additional deviation possibility from the segmentation equilibrium. Instead of being active only on the other platform, they can now join both platforms. This deviation is particularly profitable if π^{MH} is large. As a result, platforms can no longer charge high fees. The homogeneity of the platforms then drives fees and profits down to zero.

Interestingly, this also implies that agglomeration is *more* likely if sellers can multi-home. The general notion in the antitrust economics of platform markets is that multi-homing inhibits market tipping because it is more likely that multiple platforms will obtain positive demand (see, e.g., Evans and Schmalensee, 2007). In our model, a different mechanism is at work—that is, the possibility of multi-homing can break the segmentation equilibrium in which multiple platforms are active.

In addition, a partial multi-homing equilibrium occurs under some conditions.³⁵ In particular, if π^{SH} and π^{MH} are relatively large, neither the single-homing nor the multi-homing seller has an incentive to deviate to an agglomeration or a segmentation equilibrium. The partial multi-homing equilibrium is in between pure agglomeration and pure segmentation and has features of both equilibria. While buyers segment, half of them are still informed about both offers due to the multi-homing of one seller in each category. In contrast to the pure segmentation equilibrium, platforms cannot exploit this in equilibrium. The intuition is similar to the one developed for the agglomeration equilibrium: when slightly undercutting the listing fee of the rival, a platform can get the single-homing seller in *each* category (and not only in one half of the categories). This leads to an agglomeration equilibrium on the platform with the lower fee, which gives this platform an upward jump in demand, and is, therefore, always profitable. Hence, the standard Bertrand logic applies, and fees are driven down to zero.³⁶

So far, we assumed that a multi-homing seller sets the same price on each platform. If price discrimination is possible, the seller sets p^d on the platform where the rival is also present and p^m on the platform where the seller is in a monopoly position. Using the notation above, this implies that $\pi^{MH} = (\pi^d + \pi^m)/2$ and $\pi^{SH} = \pi^d$. Hence, the situation with price discrimination is a special case of the analysis above. We then obtain

³⁵Partial multi-homing of sellers can often be observed on price comparison websites. Some sellers list their offers on several platforms at the same time, whereas others use only one.

³⁶In Web Appendix E, we show how the partial multi-homing equilibrium generalizes to the case with more than two sellers.

the following corollary:³⁷

Corollary 1. *Suppose that multi-homing sellers can price discriminate. Then,*

- *for $\pi^d/\pi^m \geq 1/2$ and $\pi^d/\pi^m < 3/8$, the equilibrium is the same as the one characterized in Propositions 1, 2, and 4, respectively;*
- *for $3/8 \leq \pi^d/\pi^m < 1/2$, the equilibrium is the same as the one characterized in Proposition 3 if $3/8 \leq \pi^d/\pi^m \leq 2/5$ but for $2/5 < \pi^d/\pi^m < 1/2$, platforms set fees of $f_A^* = f_B^* = 0$, and an agglomeration equilibrium occurs.*

The corollary shows that also with price discrimination, an agglomeration equilibrium emerges for a larger parameter range. As platforms obtain zero profits in this equilibrium, they are hurt from the possibility of multi-homing also with price discrimination. In contrast to the case with uniform pricing, an equilibrium with partial multi-homing does not occur. The intuition is that single-homing sellers obtain a per-buyer profit of π^d , regardless of whether segmentation or agglomeration occurs. These sellers are therefore better off when tipping occurs (as they then interact with the double amount of buyers), which destroys the partial multi-homing equilibrium.

Finally, we note that although we analyzed multi-homing of buyers and sellers separately, a combination of the two gives similar insights. In particular, fierce competition between sellers (i.e., π^d close to zero) will drive sellers away from agglomeration to segmentation.

6 Generalizations

In this section, we generalize the baseline model by considering alternative pricing instruments on the seller side (Section 6.1) and briefly discuss the alternative selection criterion of buyer-preferred equilibrium (Section 6.2). We show that our results are robust to these extensions. In Web Appendices F and G, we discuss, in addition, the robustness of our results to a general number of platforms and sellers, the case of two-sided pricing, as well as some further extensions.

6.1 Platform Pricing Instruments

6.1.1 Per-Transaction Fees and Revenue Shares

In the baseline model, we consider the case in which platforms charge listing fees to sellers. This pricing instrument is the only feasible one if platforms cannot monitor the

³⁷The corollary can be easily shown by inserting the values $\pi^{MH} = (\pi^d + \pi^m)/2$ and $\pi^{SH} = \pi^d$ in Proposition 7. Therefore, we omit the proof.

transaction between buyers and sellers, as is the case, e.g., in housing markets. However, in other markets, monitoring is possible at relatively low costs. For example, price comparison websites usually charge per-transaction fees (in terms of per-click fees) and many booking services and marketplaces, such as Amazon Marketplace, ask for a percentage of the price charged by sellers—that is, they engage in revenue-sharing.³⁸ In this section, we show that our results are robust to any of those two pricing instruments.

Per-transaction fees. Suppose that the game is the same as the one laid out in Section 3 but that platforms instead of charging listing fees demand a fee per transaction, denoted by ϕ_i , $i \in \{A, B\}$. That is, every time a consumer buys a product from a seller, the seller has to pay ϕ_i to the platform. A listing fee constitutes a fixed cost for the seller and, therefore, does not affect the pricing choice in the product market. By contrast, a per-transaction fee increases the marginal cost of each seller, and will affect the price that the seller charges. We denote the resulting duopoly equilibrium price in the product market by $p^d(\phi_i)$, with $\partial p^d(\phi_i)/\partial \phi_i > 0$, and the associated demand by $D^d(\phi_i)$. The resulting duopoly profit (assuming a constant marginal cost of c) is $\pi^d(\phi_i) = D^d(\phi_i)(p^d(\phi_i) - \phi_i - c)$, with $\partial \pi^d(\phi_i)/\partial \phi_i \leq 0$.³⁹ Similarly, in the monopoly case, the resulting price is $p^m(\phi_i)$, with $\partial p^m(\phi_i)/\partial \phi_i \geq 0$,⁴⁰ the demand is $D^m(\phi_i)$, and the profit is $\pi^m(\phi_i) = D^m(\phi_i)(p^m(\phi_i) - \phi_i - c)$, with $\partial \pi^m(\phi_i)/\partial \phi_i \leq 0$. We maintain the assumption from the main model that $\pi^d(\phi_i)/\pi^m(\phi_i) \leq 1$ for all i .

In addition, we assume that an increase in the per-transaction fee reduces the monopoly profit by more than the duopoly profit, and the same holds true for the monopoly demand compared to duopoly demand; that is,

$$\frac{\partial \pi^m(\phi_i)}{\partial \phi_i} \leq \frac{\partial \pi^d(\phi_i)}{\partial \phi_i} \leq 0 \quad \text{and} \quad \frac{\partial D^m(\phi_i)}{\partial \phi_i} \leq \frac{\partial D^d(\phi_i)}{\partial \phi_i} \leq 0.$$

These properties hold in standard oligopoly models, including those in our examples.

We can then solve the model as in the case with listing fees. The details are provided in the proof of Proposition 8 in Web Appendix H. As we demonstrate there, also with per-transaction fees, our selection criterion singles out a unique type of equilibrium in stage 2.⁴¹ Turning to the full game, with per-transaction fees, platforms cannot extract

³⁸For example, as reported by Hunold et al. (2018), the online travel agents Booking and Expedia usually charge hotels a base commission rate of 10% to 15%. For the fees charged by Amazon Marketplace, see footnote 7.

³⁹The inequality in $\partial \pi^d(\phi_i)/\partial \phi_i$ is only weak because in covered markets (as, for example, in the Hotelling model), an increase in ϕ_i leads to an increase in the product price by the same amount without affecting equilibrium demand, implying that profits are unchanged.

⁴⁰The weak inequality here is due to the fact that in markets with rectangular demand, the monopoly price is independent of cost.

⁴¹We recall that we assumed that buyers decide on which platform to be active according to the expected number of sellers. This assumption is natural in the context of listing fees, as these fees do

the full profit from sellers. Nevertheless, we can formulate the analogue to a listing fee of $\pi^m/2$, which is the highest profit a platform can make in a segmentation equilibrium. With per-transaction fees, we therefore denote by $\phi^m \equiv \arg \max_{\phi_i} \phi_i D^m(\phi_i)/2$. We obtain the following result:

Proposition 8. *If $\pi^d(0)/\pi^m(0) \geq 1/2$, in equilibrium, both platforms set $\phi_A^* = \phi_B^* = 0$, and buyers and sellers agglomerate on either platform A or platform B. If $\pi^d(0)/\pi^m(0) < 1/2$ and either*

$$\frac{\tilde{\phi} D^d(\tilde{\phi})}{\phi^m D^m(\phi^m)} \leq \frac{1}{4},$$

where $\tilde{\phi}$ is defined by $\pi^d(\tilde{\phi}) = \pi^m(\phi^m)/2$ or $\pi^d(\phi) < \pi^m(\phi^m)/2$ for all ϕ , in the unique equilibrium, both platforms set $\phi_A^* = \phi_B^* = \phi^m$, and buyers and sellers segment. If $\pi^d(0)/\pi^m(0) < 1/2$ and

$$\frac{\tilde{\phi} D^d(\tilde{\phi})}{\phi^m D^m(\phi^m)} > \frac{1}{4},$$

there is a unique mixed-strategy equilibrium with similar properties as those in case of listing fees, and agglomeration and segmentation occur with positive probability.

The mixed-strategy equilibrium is fully characterized in the proof of Proposition 8 in Web Appendix H. As is evident from the proposition, the outcome with per-transaction fees resembles that with listing fees. First, if competition between sellers is weak (that is, the ratio of duopoly to monopoly profit is relatively high), a pure-strategy agglomeration equilibrium results with either type of fees, and platforms compete each other down to fees equal to marginal cost. We note that the conditions for the agglomeration equilibrium to occur coincide in Propositions 1 and 8: with listing fees, the condition is $\pi^d/\pi^m \geq 1/2$, which is the same as that with per-transaction fees, $\pi^d(0)/\pi^m(0) \geq 1/2$.

Second, if competition between sellers is fierce, segmentation occurs. Here, platforms set the fee equal to ϕ^m to obtain the largest per-buyer profit—similar to the case of listing fees. The conditions also have a similar interpretation. With listing fees, the condition for the segmentation equilibrium to exist is $\pi^d/\pi^m \leq 1/4$, as a platform should have no incentive to attract both sellers and all buyers in each category instead of only one seller and half of the buyers. With per-transaction fees, the condition is $[\tilde{\phi} D^d(\tilde{\phi})]/[\phi^m D^m(\phi^m)] \leq 1/4$, which rests on the same idea: given the rival's fee ϕ^m , in

not affect product market prices. If buyers were to infer product market prices from platforms' pricing decisions, a platform could attract all buyers by offering a lower per-transaction fee (or revenue share), which would destabilize the segmentation equilibrium. In the real world, however, buyers often do not observe the platforms' pricing decisions and, therefore, cannot make such inferences. As mentioned in Section 3, our analysis and equilibrium refinement then applies to a sequential model in which sellers decide which platform to join before buyers do, and buyers (when deciding which platform to join) only observe the sellers' listing decisions but neither their nor the platforms' prices.

any category, a platform would attract both sellers and all buyers with a fee of $\tilde{\phi}$. The condition precludes that such a deviation is profitable.

Finally, in the remaining region, a unique mixed-strategy equilibrium exists. As we show in the proof of Proposition 8, this mixed-strategy equilibrium has properties similar to those with listing fees, involving mixing either on a convex or on a non-convex set.

We illustrate the result with the Hotelling example. For $t \geq (v - c)/2$, the condition $\pi^d(0)/\pi^m(0) \geq 1/2$ is fulfilled, and a pure-strategy agglomeration equilibrium occurs with fees equal to 0. By contrast, for $t \leq (v - c)/4$, the above segmentation equilibrium is the unique equilibrium. In the intermediate range, the mixed-strategy equilibrium occurs. Therefore, mixing occurs for a larger range of parameters than with listing fees—in the latter case, a mixed-strategy equilibrium emerges only for $(v - c)/3 < t < (v - c)/2$.

Revenue shares. Another pricing instrument that platforms often use is a percentage fee on the revenue made by sellers. For example, application platforms such as the App Store or Google Play usually charge a percentage fee of 30% on the seller's revenue.⁴²

Such revenue sharing can also be incorporated into our model. Suppose, again, that the game proceeds as laid out above, but that each platform $i \in \{A, B\}$ extracts a revenue share $r_i \in [0, 1]$ on each transaction it enables. The seller's profit is then

$$\pi^d(r_i) = [(1 - r_i)p^d(r_i) - c] D^d(r_i)$$

in duopoly, and

$$\pi^m(r_i) = [(1 - r_i)p^m(r_i) - c] D^m(r_i)$$

in monopoly, where $D^m(r_i)$ (respectively, $D^d(r_i)$) is the demand in the seller's monopoly solution (respectively, the sellers' duopoly solution) if platform i demands revenue share r_i . Applying the Implicit Function Theorem, it is easy to show that under standard assumptions on demand, $p^d(r_i)$ and $p^m(r_i)$ are increasing in r_i , as long as costs are strictly positive; if $c = 0$, prices are independent of r_i . In addition, profits and demands are decreasing in r_i .

We impose that

$$\frac{\partial \pi^m(r_i)}{\partial r_i} \leq \frac{\partial \pi^d(r_i)}{\partial r_i} \leq 0 \quad \text{and} \quad \frac{\partial D^m(r_i)}{\partial r_i} \leq \frac{\partial D^d(r_i)}{\partial r_i} \leq 0,$$

which is fulfilled in our examples. The model with revenue sharing can be analyzed in the same way as the one with per-transaction fees. Although the conditions for the boundaries of the equilibrium regions have to be modified, the results closely resemble

⁴²See https://www.theregister.co.uk/2018/08/29/app_store_duopoly_30_per_cent/ for the App Store, and <https://support.google.com/googleplay/android-developer/answer/112622?hl=de> for Google Play, last accessed June 4, 2019.

those with per-transaction fees. Also under revenue sharing, platforms do not extract the entire profits of sellers in the segmentation equilibrium if $c > 0$: platforms set $r = r^m$, with $r^m = \arg \max_{r_i} r_i p^m(r_i) D^m(r_i) / 2$ —that is, they obtain the highest profit compatible with segmentation in stage 2. Thus, our main insight carries over to the setting with revenue sharing between platforms and sellers.

6.1.2 Endogenous Fees

In this section, we endogenize the fee structure in a (pure-strategy) segmentation equilibrium, given that all three types of fees can be used, and demonstrate how the fee structure depends on the product market conditions. At the end of this section, we relate our results to platform revenue models.

We extend our model to allow for heterogeneous categories. Specifically, we assume that per-consumer demand in category k is $v_k D(\cdot)$, where category value v_k is distributed on $[\underline{v}, \bar{v}]$, with a continuously differentiable density function $h(v)$ and an associated cdf $H(v)$ if $\underline{v} < \bar{v}$. Thus, categories are ranked with \bar{v} being the most valuable one and \underline{v} the least valuable one. The multiplicative interaction between v and $D(\cdot)$ provides enough structure to show our results for general demand functions that satisfy the following three assumptions:

- (i) There exists a choke price, denoted by \bar{p} , such that $D(p) = 0$ for all $p \geq \bar{p}$;
- (ii) $D'(p) + D''(p)p < 0$ for all $p < \bar{p}$;
- (iii) $D'''(p)$ is positive or not too negative.

Assumption (i) states that demand becomes zero if the price a seller sets is very high. Assumption (ii) ensures that the seller's maximization problem in the product market has an interior solution. Assumption (iii), together with Assumption (ii), guarantees that the same holds true for a platform's maximization problem.

As above, we denote the listing fee of platform i by $f_i \geq 0$, the per-transaction fee by $\phi_i \geq 0$, and the revenue share by $r_i \in [0, 1]$. In the first stage, the platforms simultaneously choose the levels of the three fees.

As mentioned above, we focus on the pure-strategy segmentation equilibrium—this equilibrium always exists if competition between sellers is fierce (i.e., the duopoly profit is sufficiently low). Regarding the other equilibria, we note the following: first, in the mixed-strategy equilibrium, the fees set by the platforms have a similar structure as in the pure-strategy segmentation equilibrium but are lower in absolute terms—we confirmed this by numerical simulation. Second, analyzing the pure-strategy agglomeration equilibrium with an endogenous fee structure does not yield additional insights because, for the same reason as in the baseline model, all three fees will be zero in equilibrium.

In a segmentation equilibrium (i.e., in each category, there is one seller on platform

$i \in \{A, B\}$), the profit of a seller in category k active on platform i is

$$\beta_i v_k D(p(\phi_i, r_i)) [(1 - r_i)p(\phi_i, r_i) - \phi_i - c] - f_i,$$

where, as above, β_i is the fraction of buyers active on platform i . Note that f_i does not affect the price a seller charges, but ϕ_i and r_i may do so; the revenue share r_i is neutral if $c = 0$. Platform i 's profit is accordingly $\beta_i v_k D(p(\phi_i, r_i)) (r_i p(\phi_i, r_i) + \phi_i) + f_i$. We obtain the result that, in the segmentation equilibrium, platforms will never use all three instruments but only a tariff consisting of listing fees and revenue shares.

Proposition 9. *In the (pure-strategy) segmentation equilibrium, both platforms $i \in \{A, B\}$ optimally set the per-transaction fee ϕ_i equal to zero.*

The intuition behind this result is as follows: a per-transaction fee and a revenue share have a distortionary effect on the product market price. The difference between the two fees is that a per-transaction fee increases a seller's marginal cost by an absolute amount (equal to the level of the fee), whereas, with a revenue share, the payment a seller makes to the platform is conditional on the seller's product price. A revenue share therefore allows the seller to adjust its price to the product market conditions in a less distortionary way. As a consequence, if revenue shares are available, platforms optimally set the per-transaction fee to zero. The listing fee is positive in equilibrium (as long as $\underline{v} > 0$ and $c > 0$) because it does not distort the product price.

We next determine how the tariff is shaped by the product market conditions of sellers. We start with two special cases: in the first case, all categories are identical and, in the second case, sellers' have zero marginal costs.

Proposition 10. *(i) If $\underline{v} = \bar{v}$ and $c > 0$, then $f_i = \underline{v} D(p^m) (p^m - c) / 2$ and $r_i = 0$. (ii) If $c = 0$ and $\bar{v} > \underline{v}$, then $f_i = 0$ and $r_i = 1$.*

In both cases, platforms extract the entire profits from sellers in the pure-strategy segmentation equilibrium. In case (i), in which categories are homogeneous, we are back in our baseline model. Platforms then use only listing fees—that is, revenue-shares are zero in equilibrium. As listing fees do not distort the equilibrium price, only this instrument is used to appropriate sellers' profits. Thus, our baseline model is consistent with endogenous fee setting, that is, even if we allowed for different fees in the baseline model, platforms will endogenously use only listing fees. This result points to the property that a small amount of heterogeneity between categories implies that listing fees are more important relative to revenue shares, as we will show in the next proposition.

Case (ii) of Proposition 10 shows that if sellers' marginal costs are zero but product categories are heterogeneous, platforms extract the sellers' profit by relying on revenue-shares only—that is, listing fees will be zero. The intuition is that if sellers have zero

marginal costs, revenue shares do not distort the seller’s pricing decision—that is, sellers in all categories charge the undistorted monopoly price. By contrast, with heterogeneous categories, fixed fees cannot fully extract each seller’s profit.⁴³

The following comparative-statics results shed light on the question how product market conditions affect the fee structure.

Proposition 11. *(i) A mean-preserving spread of the distribution of v (that changes its support) leads to an increase in r_i and a decrease in f_i . (ii) An increase in c leads to a decrease in r_i and affects f_i non-monotonically with f_i increasing for small c but decreasing for large c .*

Part (i) of the proposition states that, as already alluded to above, listing fees are large and revenue shares small if categories are similar whereas the reverse holds true if categories are different in value.

Part (ii) shows that the larger are marginal costs of sellers, the lower are revenue shares. This result is intuitive because the distortionary effect of revenue shares increases with the level of marginal costs. By contrast, listing fees change non-monotonically with c . There are two opposing effects on listing fees when sellers’ marginal costs rise. First, revenue shares become more distortionary, which implies that platforms do better relying relatively more on listing fees. Second, seller’s profits fall when their costs rise, which implies that platforms have to lower their listing fees. If c increases from a small level, the first effect dominates, whereas if c increases from a high level, the second effect prevails. We can show numerically that the relationship between c and f_i is indeed an inverted U-shaped one.

Platform revenue models. We relate our results of this section to various platform revenue models, which occur in different market environments. As pointed out above, the following two dimensions are important drivers of the fee structure: (i) observability of transactions and (ii) heterogeneity of product categories.

First, in platform markets in which the value of the transaction is observable, our result is consistent with the fact that per-transaction fees are rare. Platforms usually charge a tariff consisting of a fixed fee and a revenue share (such as Amazon Marketplace, as reported in the Introduction). This example is also consistent with our model prediction that if product categories of different value are offered, revenue shares become the most important profit source of platforms. Second, if platforms can observe whether a transaction was initiated but cannot observe the value of the transaction (e.g., because there is uncertainty whether the transaction was consummated), they set a tariff consisting of a per-transaction fee and a fixed fee. The perhaps most prominent example is

⁴³If $\underline{v} = \bar{v}$ and $c = 0$, there is a continuum of combinations of listing fee and revenue share that allow a platform to fully extract the sellers’ profits.

the market of price comparison websites, where tariffs include a per-click fee,⁴⁴ which is an absolute monetary amount and, presuming a constant conversion rate, equivalent to a per-transaction fee. Our model can be used to predict that per-click fees are the more prominent relative to fixed fees the more heterogeneous product categories are.⁴⁵ Third, if no monitoring of transactions is possible, platforms can only set pure listing fees as, for example, in the housing market.

6.2 Buyer-Preferred Equilibrium

In this subsection, we show how our results change if we use the concept of payoff-dominance of buyers (instead of sellers) in the second stage, in addition to coalition-proofness. Because $V^d > V^m$, buyers prefer an agglomeration equilibrium over a segmentation or stand-alone equilibrium. This implies that whenever the refinement of coalition-proofness alone does not suffice to select a unique equilibrium, it is payoff-dominant for buyers to be in an agglomeration equilibrium whenever it exists—as above, the two refinements are never in conflict with each other.

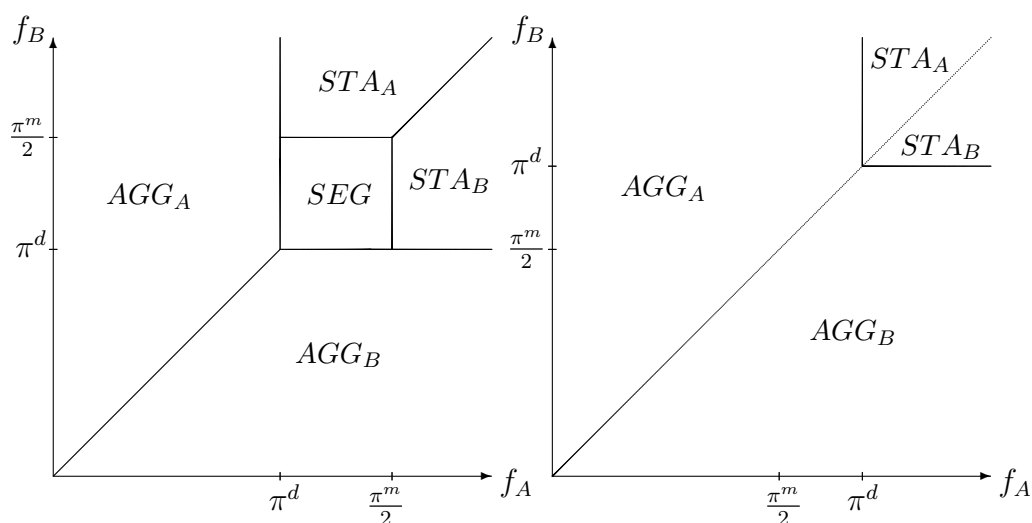


Figure 7: Selected equilibrium configurations with payoff-dominance of buyers

Figure 7 shows the different equilibrium regions with this selection rule. As long as at least one platform sets a fee below π^d , an agglomeration equilibrium exists and will be selected. However, if both fees are larger than π^d , an agglomeration equilibrium does not exist, as sellers would obtain negative profits. The selected equilibrium is then the same

⁴⁴For instance, Amelio et al. (2018, p. 659) report that “retailers typically pay a pay-per-click fee for the comparison shopping services.”

⁴⁵While this section does not contain this result, it follows the same intuition as part (i) of Proposition 11. The analysis is available from the authors upon request.

as in Section 4 because buyers are indifferent between the segmentation and a stand-alone equilibrium, and, if both exist, only the segmentation equilibrium is coalition-proof.

Turning to the first stage, there is now always an equilibrium in which platform fees are equal to 0. Given that platform i sets $f_i = 0$, an agglomeration equilibrium exists and will be selected. Hence, platform $-i$ cannot do better than to also set $f_{-i} = 0$. However, this equilibrium is not the only one if $\pi^d/\pi^m \leq 1/4$. By the same logic as in Section 4, if each platform charges a fee of $\pi^m/2$, no platform has a profitable deviation. Therefore, a segmentation equilibrium in which platforms extract the entire profits from sellers also exists and is profit-dominant for platforms. Invoking profit-dominance in stage 1 (which is implied by coalition-proofness), the same segmentation equilibrium as in Section 4 emerges. Therefore, our result does not hinge on the selection criterion in stage 2.

If a buyer-preferred equilibrium is selected, no mixed-strategy equilibrium exists. The reason is as follows: if $\pi^d/\pi^m > 1/4$, a platform has an incentive to deviate from the equilibrium candidate $f_A = f_B = \pi^m/2$ and to set a fee below π^d to induce agglomeration. The best response of the rival platform is then to undercut this fee slightly, as it cannot induce segmentation with a higher fee (in contrast to the case with a seller-preferred equilibrium in stage 2). Then, the standard Bertrand argument applies, leading to zero fees in equilibrium. Therefore, if the buyer-preferred equilibrium is chosen, we obtain $f_A^* = f_B^* = 0$ if $\pi^d/\pi^m > 1/4$ and $f_A^* = f_B^* = \pi^m/2$ if $\pi^d/\pi^m \leq 1/4$.

7 Policy Implications and Predictions

7.1 Policy Implications

Our paper has implications that can guide policy makers in regulating platform markets. These implications rest on the insight that market segmentation has undesirable welfare properties: segmentation leads to lower total welfare and consumer welfare than agglomeration due to less choice for buyers *and* higher product prices. However, platforms benefit from segmentation, as this allows them to extract rents from sellers. The conflict between what is in the interest of platforms and what is in the interest of society may justify policy intervention.

We discuss two policy instruments—a ban of exclusive contracts and price caps. These instruments are often discussed, not only in the context of platform markets but also generally, as means to spur welfare. Our main question is whether these instruments are effective in avoiding inefficient segmentation.⁴⁶ We find that the first instrument is

⁴⁶While our model focuses on segmentation to reduce seller competition, segmentation can also result from inherent differentiation between platforms. Platforms then cater to different buyer (and possibly seller) tastes, which, all else equal, is welfare-increasing.

effective, but the second is not.

Ban of exclusive contracts. A widely-discussed issue in platform markets is the use of exclusive contracts.⁴⁷ Such contracts restrict sellers to offering their products exclusively on one platform (in exchange for a favorable deal on the fee charged by the platform). Therefore, these exclusive contracts rule out seller multi-homing. As shown in Section 5.2, an agglomeration equilibrium is more likely to arise if sellers can multi-home instead of being forced to single-home. Thus, our paper provides a new rationale for why exclusive contracts are welfare-decreasing: they help sellers commit to a single platform, thereby sustaining market segmentation. As a consequence, a ban of exclusive contracts is an effective tool to reduce the possibility of inefficient segmentation.⁴⁸

Price caps on listing fees. A policy instrument to curb firms' market power is to set price caps. At first glance, this might also look attractive as a way to tame the market power of platforms vis-a-vis sellers because platforms charge strictly positive listing fees only in a segmentation equilibrium. However, the fundamental problem is that even with a low cap, sellers go for segmentation if they obtain higher profits with this configuration. Thus, caps on listing fees do not destabilize segmentation and are merely a rent-shifting device. This shows that, while price caps are intended to increase demand and thereby welfare, they are not effective in inducing listing decisions on platforms that lead to pro-competitive seller behavior.

7.2 Predictions

Our theory also leads to novel predictions that are empirically testable.

Correlation between market concentration and fee levels in platform markets. Our theory predicts a negative correlation between market concentration and the level of listing fees in platform markets.⁴⁹ More precisely, for a given number of available platforms, our theory predicts that the relation between market concentration, in terms of market share, and the level of the (average) listing fee is negative. In markets that feature agglomeration, platform fees are lower than in markets in which platforms have a more equal market share. This reverses the prediction of standard theory. Our prediction is testable, for example, by analyzing cross-industry or cross-country variation in the data.

⁴⁷For example, in the video game industry, console platforms often impose console exclusivity, which prevents game developers from selling a similar game on rival consoles (see Lee, 2013, for an in-depth analysis). As another example, trading platforms sometimes require 'special' offers by sellers to be exclusive on them.

⁴⁸In practice, this positive effect needs to be weighed against potential welfare-enhancing features of exclusive contracts, such as spurring or protecting investment by the parties (Bernheim and Whinston, 1998; Segal and Whinston, 2000; de Meza and Selvaggi, 2007).

⁴⁹The negative correlation between market concentration and platform charges is not only predicted in the baseline model but also in all extensions presented in this paper.

To make a meaningful comparison, one would need to condition on the market characteristics and then consider the correlation between the Hirshman-Herfindahl Index (HHI) of active firms and the level of the listing fee. There would be support for our theory if platform fees were larger in markets with a smaller HHI.

Correlation between market characteristics in the product market and concentration in platform markets. Our theory provides a prediction concerning the relation between the conditions in the intermediated product market and the market outcome in the platform market. Specifically, if competition in the intermediated market is weak, the platform market will be more concentrated (measured by the HHI). Possible sources for different degrees of competitiveness in the product market are manifold. They could be due to differences in the degree of product differentiation or differences of the ratio between sellers and buyers in thin markets. The prediction can be tested, for example, by looking at different regional markets within the same industry and country or by considering different broadly defined product categories. As platforms often operate country-wide and across broad product categories, the same number of platforms may well be present, but concentration may vary across different regions or categories. If a product market has characteristics that are unfavorable to strong competition between sellers—i.e., profits are relatively high even if sellers compete—our theory predicts agglomeration. Thus, reduced competition among sellers tends to lead to market tipping.

As an example, consider price comparison websites. Suppose that buyers consider a purchase within a broad product category and do not yet know which specific product they like. In broad product categories in which there is little room for differentiation between sellers' offers, competition between sellers is intense, and, thus, it is likely that sellers segment. The opposite holds in the broad product categories in which product differentiation between sellers is pronounced.

8 Conclusion

In this paper, we have proposed a theory of competing platforms that enable trade between buyers and sellers. Platforms are homogeneous and charge fees to sellers, and sellers compete in the product market. We have analyzed how the competitive conditions in the seller market affect platform market structure.

Can multiple platforms co-exist and earn positive profits even if there is no differentiation between them? We show that the function of multiple platforms as an endogenous segmentation device for competing sellers can explain such an outcome. Sellers choose to be active on different platforms to avoid fierce competition. Platforms exploit this by setting positive fees, and obtain strictly positive profits. Thus, multiple homogeneous

platforms have a positive market share. Such a segmentation equilibrium exists if competition between sellers is sufficiently strong. If, by contrast, there is little competition between sellers, the standard intuition is confirmed: the equilibrium features agglomeration, and platform fees are low. As a consequence of these results, the relation between market concentration and fees is negative. Platform fees are low in concentrated markets but high if market shares are similar.

Our main insights are robust to several different model formulations—namely, they do not depend on the possibility of buyer and seller multi-homing, the pricing instruments available to platforms, and the number of platforms and sellers. In addition, our framework generates several policy implications and predictions that are empirically testable.

In our analysis, we did not consider entry of platforms. If entry costs are negligible, our analysis applies and agglomeration appears in a favorable light compared to segmentation. In reality, agglomeration then corresponds to a situation in which one platform has high market share (such as Ebay for online auctions), while several competitors (such as Ubid and Catawiki) have small market shares.⁵⁰ Our analysis suggests the following interpretation: a single platform carries most of the trade, but fees are below monopoly levels since small competitors impose a competitive constraint on the large platform. Thus, the presence of small competitors keeps the leading platform at bay.⁵¹ If, instead, platforms incur substantial entry costs, either only one platform will enter (akin to agglomeration) or there will be segmentation such that several platforms will enter and jointly carry all the trade. The welfare comparison between agglomeration versus segmentation would then need to be reconsidered because the single platform in the agglomeration situation operates as a monopolist, charges a high fee, and possibly reduces the number of sellers.

We placed our analysis in a static context and focused on platform pricing and subsequent subscription decisions of buyers and sellers. We leave extensions such as dynamic platform competition and the platforms' innovation incentives for future research.

Appendix

Proof of Proposition 1. First, we consider the case $\pi^d/\pi^m \geq 1/2$. From the discussion towards the beginning of Section 4, we know that an agglomeration equilibrium on platform i exists only if $f_i \leq \pi^d$ and $f_i \leq f_{-i}$. The latter condition occurs because, in the

⁵⁰Ubid is operating as an online auction site since 1997 currently allowing professional sellers to offer consumer merchandise through auctions and at fixed price; see <http://www.ubid.com/>, last accessed June 4, 2019. Catawiki runs online auctions mainly on collectibles since 2011; see <https://www.catawiki.com/>, last accessed June 4, 2019.

⁵¹This is in line with the policy conclusion by Brown and Morgan (2009) that competition authorities should scrutinize the acquisition of small online auction platforms by bigger ones.

second stage, sellers and buyers will coordinate on the equilibrium in which they all join the platform with the lower fee. This implies that, in the first stage, for all $f_{-i} \leq \pi^d$, platform i gains from undercutting the competing platform. This induces agglomeration on platform i . The standard Bertrand logic then applies and platforms set $f_A^* = f_B^* = 0$ in equilibrium.

Second, we consider the case $\pi^d/\pi^m < 1/2$. It is evident from the discussion before the proposition that a segmentation equilibrium is played if the two fees are the same or similar. As a consequence, in a candidate agglomeration equilibrium, the inactive platform must set a sufficiently high fee. But this platform then has the incentive to lower its fee (but still keep it strictly positive) and induce segmentation. By doing so, it obtains a strictly positive profit, as half of the sellers join the platform. This argument holds for any fees (f_A, f_B) that induce agglomeration in the second stage. Hence, no agglomeration equilibrium exists for $\pi^d/\pi^m < 1/2$. \square

Proof of Proposition 2. Consider the case $\pi^d/\pi^m < 1/2$. In a segmentation equilibrium, a seller on platform i obtains a profit of $\pi^m/2 - f_i$. Therefore, the highest possible fee that a platform can charge equals $\pi^m/2$, leaving sellers with zero profits. We first determine the conditions under which an equilibrium with listing fees $\pi^m/2$ exists. If both platforms charge $f_i = \pi^m/2$, segmentation occurs in stage 2. This holds because the seller's gross profit under agglomeration is π^d , which is less than the listing fee.

Suppose that platform i deviates to induce an agglomeration equilibrium in the second stage. To do so, it has to charge $f_i^{dev} = \pi^d - \epsilon$, where $\epsilon > 0$ can be arbitrarily small. Since all buyers will agglomerate on platform i if all sellers do, sellers earn a small positive profit when agglomerating on platform i but zero in the segmentation equilibrium. The deviation profit of platform i is then (letting $\epsilon \rightarrow 0$) $\Pi_i^{dev} = 2\pi^d$ since it obtains π^d from each seller. Therefore, a deviation is not profitable if $\pi^m/2 \geq 2\pi^d$ or $\pi^d/\pi^m \leq 1/4$. It follows that in this region, a segmentation equilibrium with listing fees $(f_A^*, f_B^*) = (\pi^m/2, \pi^m/2)$ is the unique equilibrium. Each platform's equilibrium profit is $\pi^m/2$.

There cannot exist a segmentation equilibrium in which platforms charge fees less than $\pi^m/2$. The reason is that a platform could increase its fee slightly, still induce segmentation, and obtain a higher profit. \square

Proof of Proposition 3. We first show the *non-existence of a pure-strategy equilibrium*. Consider the region $1/4 < \pi^d/\pi^m < 1/2$. From the discussion at the beginning of Section 4, we know that in this region a segmentation equilibrium will be played in the second stage if both platforms charge the same listing fees (conditional on these fees being lower than $\pi^m/2$, which will always be fulfilled in equilibrium). From Proposition 2, it follows that platforms cannot extract the full profit from sellers because this would give each platform an incentive to deviate to a lower fee and induce agglomeration. We proceed by

first determining the highest fee that platforms could charge to make such a downward deviation unprofitable. Suppose that both platforms charge a fee of $\pi^m/2 - x$. Each platform's profit is then $\pi^m/2 - x$, and a seller's profit is x . If platform i deviates in order to attract all sellers and buyers in the second stage, it must set a fee such that $\pi^d - f_i^{dev} > x$. The deviation fee $f_i^{dev} = \pi^d - x - \epsilon$ leads to a deviation profit of $2\pi^d - 2x$ (letting $\epsilon \rightarrow 0$). Such a deviation is unprofitable if $\pi^m/2 - x \geq 2\pi^d - 2x$ or, equivalently, $x \geq 2\pi^d - \pi^m/2$. Hence, with x equal to $2\pi^d - \pi^m/2$, platforms prevent such a deviation. The resulting fee is then $f_i = \pi^m/2 - x = \pi^m - 2\pi^d \equiv \hat{f}_i$, and so is the platform's profit.

To determine if listing fees $\hat{f}_i = \hat{f}_j = \pi^m - 2\pi^d$ can constitute an equilibrium, we need to check if a platform has an incentive to deviate by charging a higher listing fee (upward deviation). Suppose that platform i charges $\hat{f}_i = \pi^m - 2\pi^d$ and platform j charges a deviation fee $f_j^{dev} > \hat{f}_i$ such that segmentation is still the continuation equilibrium. To induce segmentation, we must have $\pi^m/2 - f_j^{dev} > \pi^d - \hat{f}_i = 3\pi^d - \pi^m$ (i.e., a seller's profit with segmentation must be higher than with agglomeration). Thus, the highest possible listing fee is $f_j^{dev} = 3\pi^m/2 - 3\pi^d - \epsilon = 3(\pi^m/2 - \pi^d) - \epsilon$, which yields a larger platform profit than $\hat{f}_i = 2(\pi^m/2 - \pi^d)$. Hence, a profitable upward deviation exists.

It follows that there is no equilibrium in pure strategies in the range of $1/4 < \pi^d/\pi^m < 1/2$. The candidate equilibrium, which prevents downward deviations was $\hat{f}_i = \hat{f}_j = \pi^m - 2\pi^d$, but then an upward deviation is profitable. In turn, for all listing fees above $\pi^m - 2\pi^d$, a downward deviation is profitable.

Randomization domain. From the analysis above, we know that in the range $1/4 < \pi^d/\pi^m < 1/2$ for each f_j , platform i has two best-response candidates: an upper best-response candidate, denoted by f_i^{br+} , which is higher than f_j by a discrete amount and induces segmentation, and a lower best-response candidate, denoted by f_i^{br-} , which is lower than f_j by a discrete amount and induces agglomeration. We will now show that there is a unique f_j so that platform i obtains the same profit with either best-response candidate. In addition, both candidates are increasing in f_j . Due to platform symmetry, this allows us to derive the randomization domain.

Suppose that platform j sets a fee f_j . We now derive the best response of platform $i \neq j$ —for a graphical illustration of the best-response functions, see Web Appendix H. The upper best response f_i^{br+} is the largest fee compatible with segmentation. At this fee, sellers weakly prefer segmentation to agglomeration on j , which implies that the inequality $\pi^m/2 - f_i^{br+} \geq \pi^d - f_j$ is binding.⁵² This leads to a profit of $f_i^{br+} = \pi^m/2 - \pi^d + f_j$. Instead, the optimal lower best response f_i^{br-} is the largest fee compatible with agglomeration on platform i . This can only occur if the seller profit with agglomeration is larger than with segmentation, which implies $\pi^d - f_i^{br-} > \pi^m/2 - f_j$. The lowest upper

⁵²We presume that if sellers are indifferent between segmentation and agglomeration, they choose segmentation. As we will show below, this is consistent with equilibrium behavior.

bound of platform i 's profit is then $2f_i^{br^-} = 2(\pi^d - \pi^m/2 + f_j)$.

The two profits reported above are equal at $f_j = 3/2\pi^m - 3\pi^d$. Thus, if $f_j \leq 3/2\pi^m - 3\pi^d$, platform i prefers the upper to the lower best-response candidate, whereas for $f_j > 3/2\pi^m - 3\pi^d$, the opposite holds true. Platform i 's best response to $f_j \leq 3/2\pi^m - 3\pi^d$ is to induce segmentation by setting its fee equal to $f_i^{br^+} = \pi^m/2 - \pi^d + f_j$, which is increasing in f_j . Vice versa, for $f_j > 3/2\pi^m - 3\pi^d$, platform i 's profit from the lower best-response candidate is larger than that from the upper best-response candidate because $2(\pi^d - \pi^m/2 + f_j) > \pi^m/2 - \pi^d + f_j$ for $\pi^d < \pi^m/2$. This implies that its best response to $f_j > 3/2\pi^m - 3\pi^d$ is $f_i^{br^-} = \pi^d - \pi^m/2 + f_j$, which is also increasing in f_j . Hence, platform i 's highest fee that constitutes a best-response to any f_j is $2\pi^m - 4\pi^d$; it is the best response to $f_j = 3/2\pi^m - 3\pi^d$. By symmetry, this leads to an upper interval of the randomization domain equal to $f_i \in [3/2\pi^m - 3\pi^d, 2\pi^m - 4\pi^d]$. Analogously, the minimum of platform i 's best response to any f_j is given by $\pi^m - 2\pi^d$. This leads to a lower interval of the randomization domain equal to $f_i \in [\pi^m - 2\pi^d, 3/2\pi^m - 3\pi^d]$.

We can write the best-response functions as follows: let $\delta \equiv \pi^m/2 - \pi^d$. Denote $\underline{f} \equiv 2\delta$, $\tilde{f} \equiv 3\delta$, and $\bar{f} \equiv 4\delta$. Thus, the domain over which platforms mix is $[\underline{f}, \bar{f}] = [2\delta, 4\delta]$. For $i, j \in \{A, B\}$ and $i \neq j$, the platforms' best response functions are given by⁵³

$$f_i^{br}(f_j) = \begin{cases} f_j + \delta, & \text{if } f_j \in [\underline{f}, \tilde{f}]; \\ f_j - \delta - \epsilon, & \text{if } f_j \in (\tilde{f}, \bar{f}], \end{cases}$$

where $-\epsilon$ stands for an incremental reduction. Thus, the mixed-strategy equilibrium features $(f_i, f_j) \in [\pi^m - 2\pi^d, 2\pi^m - 4\pi^d]^2$. The expected profit is $3\pi^m/2 - 3\pi^d$ because, when charging a fee equal to this profit, a platform induces segmentation with a probability of (almost) 1. In the mixed-strategy equilibrium, the highest listing fee is $2\pi^m - 4\pi^d$. However, to ensure participation of sellers, the highest fee a platform can charge (in a segmentation equilibrium) is $\pi^m/2$. Therefore, the equilibrium determined above is only valid if $2\pi^m - 4\pi^d \leq \pi^m/2$ or, equivalently, $\pi^d/\pi^m \geq 3/8$.

Mixing probabilities. We derive the mixing probabilities in Web Appendix H—they are characterized by the cumulative distribution function

$$G_1(f) = \begin{cases} \frac{f-2\delta}{f+\delta}, & \text{if } f \in [2\delta, 3\delta]; \\ \frac{2f-5\delta}{f-\delta}, & \text{if } f \in [3\delta, 4\delta]. \end{cases}$$

⁵³As sellers choose segmentation in the second stage when being indifferent between segmentation and agglomeration, the best response to f_j infinitesimally above \tilde{f} is \underline{f} , and the best response to $f_j = \tilde{f}$ is \bar{f} . Hence, the boundaries of the mixing region are well defined. By contrast, if sellers chose agglomeration when indifferent, the upper bound would not be well defined, as \bar{f} is never a best response (only $\bar{f} - \epsilon$ is). Therefore, sellers choosing segmentation as a continuation equilibrium when indifferent is consistent with equilibrium play of the full game.

As the distribution is not absolutely continuous with respect to the Lebesgue measure, it fails to have a density. We define a generalized density, which is a generalized function (which comprises a Dirac delta) such that integration against this function yields the cumulative distribution function from above. The generalized density is given by $g_1(f) = G'_1(f) + \delta^D(f - 3\delta)/4$, where $G'_1(f) = 3\delta/(f + \delta)^2$ if $f \in [2\delta, 3\delta)$ and $3\delta/(f - \delta)^2$ if $f \in [3\delta, 4\delta]$, and $\delta^D(f - f_0)$ denotes the Dirac delta, which is 0 everywhere except for f_0 where it is ∞ . Inserting $\delta = \pi^m/2 - \pi^d$ yields the result stated in the proposition. \square

References

- [1] Amelio, A., T. Buettner, C. Hariton, G. Koltay, P. Papandropoulos, G. Sapi, T. Valletti, and H. Zenger (2018): "Recent Developments at DG Competition: 2017/2018," *Review of Industrial Organization*, 53, 653-679.
- [2] Anderson, S. and R. Renault (1999): "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model," *RAND Journal of Economics*, 30, 719-735.
- [3] Armstrong, M. (2006): "Competition in Two-Sided Markets," *RAND Journal of Economics*, 37, 668-691.
- [4] Aumann, R. (1959): Acceptable Points in General Cooperative N-Person Games, in *Contributions to the Theory of Games IV*, Princeton University Press, Princeton, New Jersey.
- [5] Bajari, P. and A. Hortaçsu (2003): "The Winner's Curse, Reserve Prices, and Endogenous Entry: Empirical Insights from eBay Auctions," *RAND Journal of Economics*, 34, 329-355.
- [6] Baye, M.R. and J. Morgan (2001): "Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets," *American Economic Review*, 91, 454-474.
- [7] Baye, M.R. and J. Morgan (2004): "Price Dispersion in the Lab and on the Internet: Theory and Evidence," *RAND Journal of Economics*, 35, 449-466.
- [8] Belleflamme, P. and M. Peitz (2019): "Platform Competition: Who Benefits from Multihoming?," *International Journal of Industrial Organization*, 64, 1-26.
- [9] Belleflamme, P. and E. Toulemonde (2009): "Negative Intra-Group Externalities in Two-Sided Markets," *International Economic Review*, 50, 245-272.

- [10] Bernheim, D., B. Peleg and M. Whinston (1987a): “Coalition-Proof Nash Equilibria I. Concepts,” *Journal of Economic Theory*, 42, 1-12.
- [11] Bernheim, D., B. Peleg and M. Whinston (1987b): “Coalition-Proof Nash Equilibria II. Applications,” *Journal of Economic Theory*, 42, 13-29.
- [12] Bernheim, D. and M. Whinston (1998): “Exclusive Dealing,” *Journal of Political Economy*, 106, 64-103.
- [13] Bowley, A. L. (1924): *The Mathematical Groundwork of Economics*. Oxford: Oxford University Press.
- [14] Bresnahan, T. F. and P. C. Reiss (1990): “Entry in Monopoly Markets,” *Review of Economic Studies*, 57, 531-553.
- [15] Bresnahan, T. F. and P. C. Reiss (1991): “Entry and Competition in Concentrated Markets,” *Journal of Political Economy*, 99, 977-1009.
- [16] Brown, J.R. and A. Goolsbee (2002): “Does the Internet Make Markets More Competitive? Evidence from the Life Insurance Industry,” *Journal of Political Economy*, 110, 481-507.
- [17] Brown, J. and J. Morgan (2009): “How Much Is a Dollar Worth? Tipping versus Equilibrium Coexistence on Competing Online Auction Sites,” *Journal of Political Economy*, 117, 668-700.
- [18] Bundeskartellamt (2016): “Market Power of Platforms and Networks,” Working Paper B6-113/15.
- [19] Burguet, R., R. Caminal and M. Ellman (2016): “In Google we Trust?,” *International Journal of Industrial Organization*, 39, 44-55.
- [20] Caillaud, B. and B. Jullien (2001): “Competing Cybermediaries,” *European Economic Review*, 45, 797-808.
- [21] Caillaud, B. and B. Jullien (2003): “Chicken and Egg: Competition Among Intermediation Service Providers,” *RAND Journal of Economics*, 34, 309-328.
- [22] Church, J. and N. Gandal (1992): “Network Effects, Software Provision, and Standardization,” *Journal of Industrial Economics*, 40, 85-103.
- [23] de Meza, D. and M. Selvaggi (2007): “Exclusive Contracts Foster Relationship-Specific Investments,” *RAND Journal of Economics*, 38, 85-97.
- [24] Dudey, M. (1990): “Competition by Choice: The Effect of Consumer Search on Firm Location Decisions,” *American Economic Review*, 80, 1092-1104.

- [25] Dukes, A. and E. Gal-Or (2003): “Negotiations and Exclusivity Contracts for Advertising,” *Marketing Science*, 22, 222-245.
- [26] Edgeworth, F. (1925): “The Pure Theory of Monopoly,” in: *Papers Relating to Political Economy*, Vol. 1. London: MacMillan, 111-142.
- [27] Ellison, G. and D. Fudenberg (2003): “Knife-Edge or Plateau: When Do Market Models Tip?,” *Quarterly Journal of Economics*, 118, 1249-1278.
- [28] Ellison, G., D. Fudenberg and M. Möbius (2004): “Competing Auctions,” *Journal of the European Economic Association*, 2, 30-66.
- [29] Evans, D. and R. Schmalensee (2007): “The Industrial Organization of Markets with Two-Sided Platforms,” *Competition Policy International*, 3, 151-179.
- [30] Galeotti, A. and J.L. Moraga-González (2009): “Platform Intermediation in a Market for Differentiated Products,” *European Economic Review*, 53, 417-428.
- [31] Gehrig, T. (1998): “Competing Markets,” *European Economic Review*, 42, 277-310.
- [32] Gomes, R. (2014): “Optimal Auction Design in Two-Sided Markets,” *RAND Journal of Economics*, 45, 248-272.
- [33] Hagiu, A. (2006): “Pricing and Commitment by Two-Sided Platforms,” *RAND Journal of Economics*, 37, 720-737.
- [34] Halaburda, H., M. J. Piskorksi and P. Yilidirim (2018): “Competing by Restricting Choice: The Case of Search Platforms,” *Management Science*, 64, 3574-3594.
- [35] Hasker, K. and R. Sickles (2010): “eBay in the Economic Literature: Analysis of an Auction Marketplace,” *Review of Industrial Organization*, 37, 3-42.
- [36] Hunold, M., R. Kesler, U. Laitenberger and F. Schlütter (2018): “Evaluation of Best Price Clauses in Online Hotel Bookings,” *International Journal of Industrial Organization*, 61, 542-571.
- [37] Janssen, M. and J.-L. Moraga-González (2004): “Strategic Pricing, Consumer Search and the Number of Firms,” *Review of Economic Studies*, 71, 1089-1118.
- [38] Lee, R. (2013): “Vertical Integration and Exclusivity in Platform and Two-Sided Markets,” *American Economic Review*, 103, 2960-3000.
- [39] Maskin, E. and J. Tirole (1988): “A Theory of Dynamic Oligopoly, II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles,” *Econometrica*, 56, 571-599.
- [40] Nocke, V., M. Peitz and K. Stahl (2007): “Platform Ownership,” *Journal of the European Economic Association*, 5, 1130-1160.

- [41] Rochet, J.-C. and J. Tirole (2003): "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, 1, 990-1029.
- [42] Rochet, J.-C. and J. Tirole (2006): "Two-Sided Markets: A Progress Report," *RAND Journal of Economics*, 37, 645-667.
- [43] Ronayne, D. (2019): "Price Comparison Websites," *Warwick Economics Research Paper Series* 1056.
- [44] Salop, S.C. (1979): "Monopolistic Competition with Outside Goods," *Bell Journal of Economics*, 10, 141-156.
- [45] Segal, I. and M. Whinston (2000): "Exclusive Contracts and Protection of Investments," *RAND Journal of Economics*, 31, 603-633.
- [46] Singh, N. and X. Vives (1984): "Price and Quantity Competition in a Differentiated Duopoly," *RAND Journal of Economics*, 15, 546-554.
- [47] Stahl, K. (1982): "Differentiated Products, Consumer Search, and Locational Oligopoly," *Journal of Industrial Economics*, 31, 97-103.
- [48] Varian, H.R. (1980): "A Model of Sales," *American Economic Review*, 70, 651-659.
- [49] Wolinsky, A. (1986): "True Monopolistic Competition as a Result of Imperfect Information," *Quarterly Journal of Economics*, 101, 493-511.

Segmentation versus Agglomeration: Competition between Platforms with Competitive Sellers

Web Appendix

Heiko Karle[†]Martin Peitz[‡]Markus Reisinger[§]

June 2019

This Web Appendix consists of eight sections. In Section A, we provide additional descriptive evidence for segmentation (page 2ff.). In Section B, we provide two further microfoundations for the buyer-seller interaction in stages 3 and 4 of the game (page 8ff.). In Section C, we informally discuss how our equilibrium selection mechanism might operate in reality (page 11ff.). In Section D, we explain in detail the functioning of our equilibrium selection criterion in stage 2 (page 13ff.). In Section E, we analyze equilibria in the model with seller multi-homing and more than two sellers (page 16ff.). In Section F, we show the robustness of our results in the baseline model to a general number of platforms and sellers (page 19ff.). In Section G we consider two-sided pricing and some further extensions (page 22ff.). Finally, in Section H, we provide the proofs that were omitted in the main body of the paper—these are the final part of Proposition 3 and the proofs of Propositions 4 to 11—as well as the proofs of the propositions contained in Sections E and F of this Web Appendix—i.e., the proofs of Propositions 12 and 13 (page 26ff.).

[†]Department of Economics, Frankfurt School of Finance & Management.

[‡]Department of Economics and MaCCI, University of Mannheim.

[§]Department of Economics, Frankfurt School of Finance & Management.

A More Descriptive Evidence for Segmentation

The purpose of this appendix is to present three further empirical examples with the features described in Section 2. In each example, we provide information on platform pricing and report fractions of multi-homing and single-homing sellers on the two leading platforms.

Before doing so, we would like to point out that our framework also applies to industries beyond e-commerce. A case in point are industry standards; for example, the modem standard for end-user Internet access in the 1990s: Augereau, Greenstein, and Rysman (2006) find that two different, but functionally equivalent modem standards were used by Internet Service Providers (ISPs, which would be the sellers in our model) despite positive effects of standardization. This helped ISPs reduce competition (by creating switching costs for consumers). As a result, the two modem standards obtained similar market shares and, thus, segmented the market.

Example 1: Platforms in the German rental market. As a first example, we follow up on the German housing market and consider the same two platforms as in Section 2 (i.e., Immobilienscout24 and Immowelt) but now focus on the rental market. We demonstrate that segmentation is also much more common than agglomeration in the rental market, that is, our finding for the housing market is not an exception but commonly occurs. We keep this example short, as most industry details were already discussed in Section 2.

In the rental market, the two platforms charge listing fees that again depend on the time window of the listed offer. Immobilienscout24 charges approximately 50 Euro for a basic 2-week offer for rent, while Immowelt charges approximately 40 Euro. A listing for one month is available at approximately 80 Euro at Immobilienscout24, while a 4-week listing is available at approximately 60 Euro at Immowelt.

In the rental market, we generated a dataset by carrying out a search for rental apartments in the 125 German cities with more than 100,000 inhabitants. We use the following search criteria: “at least 3 rooms”; “at least 100 m^2 ”; and “distance to the center less than 1 kilometer” for the 10 biggest cities and “distance to the centre less than 3 kilometers” for the remaining cities. The descriptive statistics are reported in Table 2.¹

The table shows that Immobilienscout24 is on average larger than Immowelt also in the rental market.² Yet, for all city sizes, there are some examples for which the opposite

¹We removed two cities as outliers from our sample: the first is Leipzig because it is known to be a city with many empty apartments. In fact, Leipzig had on both platforms a number of offers 10 times larger than cities of comparable size (214 on Immobilienscout24 and 178 on Immowelt). The presumption is that many offers are on the platform for a long time and sellers obtain discounts, which is not the case in other cities. The second city we excluded is Salzgitter, which does not have a single offer on any of the two platforms, given our search criteria.

²This is again true for larger and smaller cities—i.e., the correlation coefficient between city size and

Table 2: Descriptive Statistics: German Rental Market

Variable	Obs	Mean	Std. Dev.	Min	Max
Sellers Immoscout	123	19.98	21.53	0	142
Sellers Immowelt	123	10.63	13.32	0	98
Share of SH Sellers Immoscout	123	0.614	0.240	0	1
Share of SH Sellers Immowelt	123	0.274	0.240	0	1
Share of MH Sellers	123	0.112	0.106	0	0.444

holds and Immowelt hosts more sellers than Immobilienscout24.³ There are a few cities in which one platform is not active conditional on our search criteria.

Figure 8 reports the share of multi-homers and single-homers on each platform in ascending order of the share of multi-homers in the 123 cities. The reading of the figure

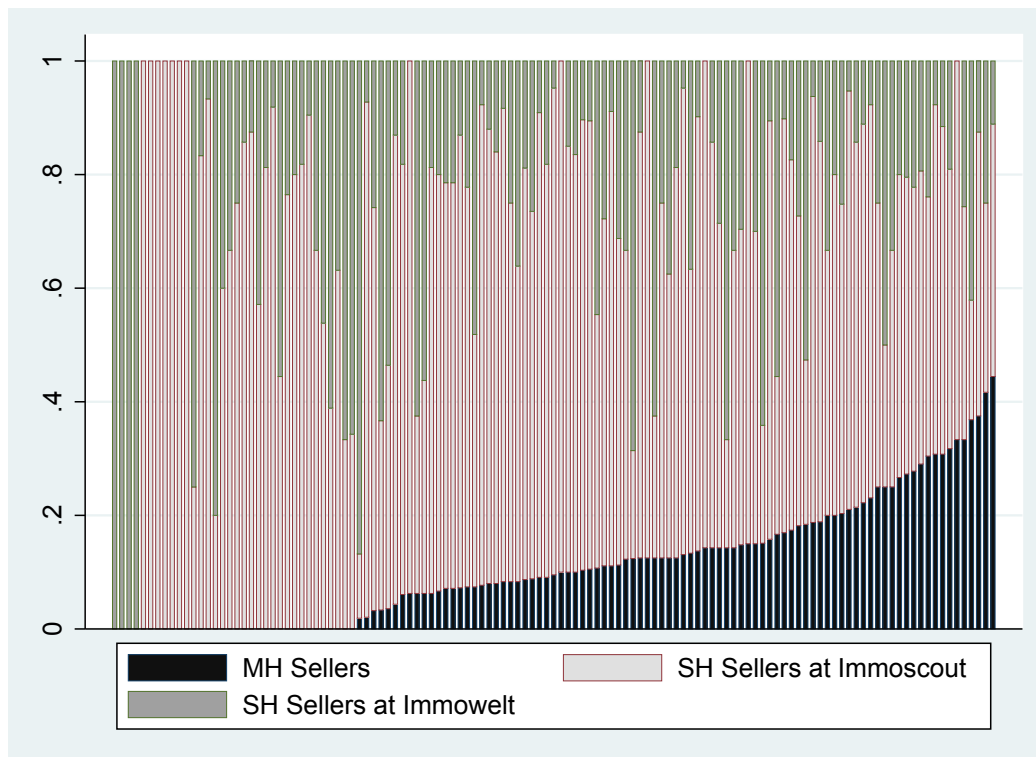


Figure 8: German rental market: Single- and multi-homing sellers.

is the same as for the figure in Section 2. In contrast to the housing market, in the rental market, in some cities all sellers are on one platform. In our sample, this appears in 11 out of 123 cities (7 cities in favor of Immobilienscout24 represented by the bright vertical bars and 4 cities in favor of Immowelt represented by the dark gray vertical bars). Depending on the city, between 0 and 44.4% sellers multi-home. In approximately 80% of the cities,

the share of single-homing sellers is not significantly different from zero.

³For example, for the largest German city Berlin, there are 11 listings on Immobilienscout24 but 22 on Immowelt.

less than 20% of sellers multi-home. This again provides evidence that segmentation occurs more often than agglomeration and that seller multi-homing is modest also in the rental market.

Example 2: Platforms for rural Spanish holiday homes. To rent out holiday homes, the use of online intermediaries appears to be essential for owners, as this is the predominant form by which tenants look for vacation places. A clearly defined submarket are vacations in a rural setting. Spain is a country with a booming rural holiday tourism,⁴ and we use the renting of rural vacation homes in Spain as our second example. The market leaders as intermediaries are Escapadarural and Toprural (the latter is part of Homeaway since 2012). According to its own websites, Escapadarural has more than 16,000 listed properties, while Toprural claims to have “thousands” of listed properties. The market has several similar features as that in Example 1: platforms match offers to consumers, and consumers typically have rather specific needs and there is only a limited number of offers satisfying those needs. Tenants are typically non-locals and are unlikely to have information about how the relative number of offerings on the two platforms differs across destinations. One difference to Example 1 is that sellers are repeatedly active on a platform, as they regularly rent out the same place.

Escapadarural and Toprural charge monthly listing fees as part of a subscription. Their amount depends on the type of seller (in particular, whether he rents out a single or multiple units) and the services provided by the platform (in particular, the visibility given to a particular property). In 2018, listing fees are not made public by the two platforms. According to a blog entry in 2016, Toprural published different monthly fees ranging from 13.75 Euros per month to 333.33 Euros, and Escapadarural charged listing fees that were a bit lower than those of Toprural.⁵ A difference between the two portals is that Escapadarural also offers a free rudimentary listing service. Finally, we note that although both platforms charge monthly fees, sellers need to sign a long-term contract (usually for one year).

On the buyer side, differences between platforms’ matching services appear to be negligible: buyers can use similar search categories, and the presentation of each property appears to be similar. The price range of offers is also very similar on the two platforms in our dataset, and both platforms were active in all Spanish provinces. Thus, there are no direct signs of vertical or horizontal differentiation between platforms.

In terms of absolute number of listings, Escapadarural is much larger. In our sample,

⁴For example, in recent years the newspapers Guardian and Telegraph in the UK featured stories about rural vacation in Spain.

⁵Blog entry by a marketing consultancy specialized in rural tourism, <https://www.demomentsomtres.com/es/turismo-rural-analisis-de-toprural-y-escapada-rural/>, accessed June 4, 2019.

the mean of offers per destination is 21.02 on Escapadrural but only 6.12 on Toprural. Despite the difference in listing numbers, we observe that Toprural is an active competitor in all submarkets, which implies that there is no tipping. The conjecture is that several listings on Escapadarural are rarely rented.⁶ We confirmed this constructing an alternative dataset with searches for apartments and houses in “beautiful villages”, where the market supposedly has a higher turnover—we then do not observe noticeable differences in size between the two platforms.⁷

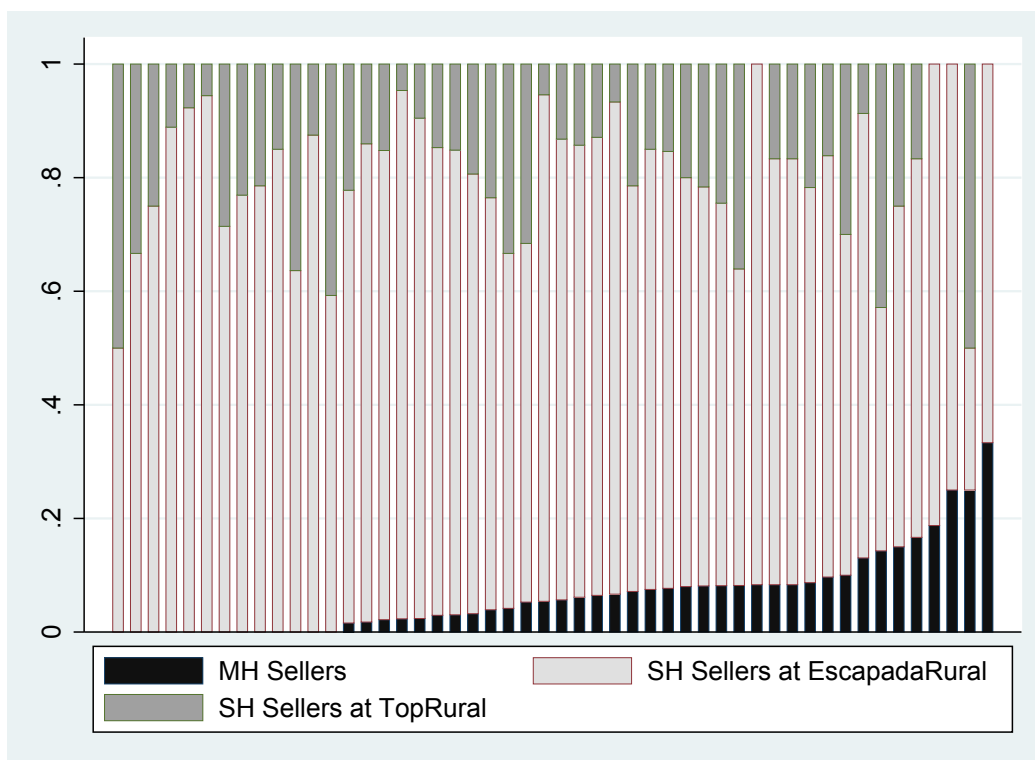


Figure 9: Ratio of multihoming Sellers.

As in our first example, we investigate the prevalence of seller single- and multihoming. For this purpose, we consider the hypothetical demand of a group of four people who want to rent an entire rural house for a week in one of the 50 provinces in Spain. In this and all other searches, we focus on properties with photos as those without any photo are arguably not a plausible alternative.⁸ In popular provinces (with more than 20 listings), we added the qualifier “swimming pool” to reduce the number of listings. We did a search for renting the house in the first week of September with the request being

⁶In addition, as Escapadarural is cheaper than Toprural and sellers usually need to make a commitment for one year, for owners who rent out their apartment or house only for a few months per year, Escapadarural is the more-economic option.

⁷For a more detailed description, see below.

⁸Escapadarural has some listings without photos; however, these listings look inactive and appear at the end of any search query.

made around five weeks before the travel dates.⁹

Findings of our search are reported in Figure 9. The figure reflects the fact that Escapadarural has many more listings; this holds in most provinces. We find that in 46 out of 50 provinces there are single-homing offers on both platforms and the fraction of multi-homing sellers is less than 20% in 47 out of 50 provinces.

One concern could be that the search is defined too broadly, as consumers may search for particular locations rather than whole provinces. The question then is whether seller segmentation can still be observed in very narrowly defined markets. To shed some light on this issue, we constructed another dataset. Here, we searched for any type of property in small villages that are known for their beauty with a flexible date.¹⁰ The number of offers is quite balanced over the two portals. We again find evidence that overall there is segmentation and the data are compatible with the idea that, in many villages, sellers are concerned with seller competition.¹¹

Finally, as in Example 1, we do not have data on consumer behavior, but we see the fact that some sellers multi-home as indication that the fraction of single-homing consumers is not negligible.

Example 3: Daily deals platforms in the United States. We use the daily deals market in the United States as a third example for segmentation. Sellers in the daily deals market offer special deals in a metropolitan area to subscribers. The most prominent categories are restaurants and beauty or fitness services. Sellers listing a deal on a platform specify the product or service offered, the price, duration, and discount rate. Consumers who buy a deal voucher can redeem it later from the seller offering it. In the period between 2010 to 2012, the daily deals market in the U.S. was dominated by two platforms, Groupon and LivingSocial. Data on this period was used in the papers by Li, Shen and Bart (2017) and Li and Zhu (2018), and we use their findings to provide evidence for segmentation.

As their pricing model, both platforms take a revenue share from sellers. This is usually a 50/50 split between the platform and the seller, with few exceptions in which either the platform or the sellers obtain a slightly higher share.

Groupon and LivingSocial can be seen as being close competitors during this period. While Groupon tended to serve consumers who were, on average, younger than those on LivingSocial, there was a large overlap in the age distribution (according to Comscore

⁹This week just falls outside of the standard vacation period in Spain. We also did a search for the last week of August and found similar results. Yet, a larger fraction of properties were already booked. The data are available upon request.

¹⁰We used the list provided by <https://www.lospueblosmasbonitosdeespana.org>, which contained 68 villages, last accessed June 4, 2019.

¹¹In our sample, there are 45 villages with more than three offers. We observe pure seller single-homing on either platform in ten villages and partial seller multi-homing in 22 villages. In the remaining 13 villages, there are no single-homing sellers on one of the two platforms; of these, in only three villages all sellers have agglomerated on a single platform.

Media Metrix, April 2011). This suggests that platforms were not too differentiated and catered to similar buyer profiles.

The importance of seller competition in this market is reflected by the fact that Groupon lists only a fraction of the deals proposed by sellers and, thus, excludes a certain fraction of them.¹² Therefore, the platform actively manages seller participation. As Li, Shen and Bart (2018, p. 1861) observed, “[w]hile more variety may often attract more consumers [...], it can also create competition between deals in the same category, which then may decrease sales of a deal.” Through the lens of our model, when Groupon and Livingsocial actively manage seller participation, each of them should have an incentive to limit the number of listings.

Li and Zhu (2018) have microdata on buyer and seller behavior on Groupon and Livingsocial for the three-year period 2010 to 2012. We present their descriptive evidence on the extent of single- and multihoming in Figure 10.¹³ It is evident that most sellers as well as buyers are single-homers. Li and Zhu (2018) consider a seller to multi-home if he offers a deal on one platform and has previously listed another deal on the competing platform in the same product category. As can be seen in the upper graph of Figure 10, the fraction of sellers on Livingsocial that also list on Groupon (which is the larger platform) is between zero and 12 percent. There is little multi-homing also on the buyer side. Even though the market was quickly expanding in the 2010 to 2012 period, single-homing is prevalent throughout on both sides. In particular, between around 10 and 20 percent of all buyers multi-home. Depending on the month, between around 50 and 70 percent of all buyers single-home on Groupon, whereas between around 15 and 40 percent single-home on Livingsocial. Overall, we view this as strong evidence that a large fraction of buyers and sellers single-home.

Finally, we note that the platforms merged in October 2016. However, both platforms are still active nowadays (managed by the joint owner Groupon). This implies that the owner viewed it as more profitable to keep both platforms alive instead of integrating them in one brand label and enjoying the agglomeration benefits. This is consistent with the idea that segmentation allows for higher rent extraction from sellers due to reduced competition.

¹²According to Bari Weiss, “Groupon’s \$6 billion gambler” Wall Street Journal, December 18, 2010, Groupon accepted only one out of every eight proposed deals.

¹³We are very grateful that Hui Li and Feng Zhu generated the figure for us and allowed us to include it here. The figure is a condensed version of Figures 2 and 5 in Li and Zhu (2018).



Figure 10: Ratio of multi-homing sellers and multi-homing buyers.

B Further Microfoundations of the Buyer-Seller Interaction

In this section, we present two further microfoundations for the buyer-seller interaction in stages 3 and 4: these are price competition with a representative consumer and a thin market with only a small number of buyers and (capacity-constrained) sellers. The latter example does not fulfill all assumptions set out in Section 3 (as there is only a finite number of buyers). However, because only π^d and π^m are relevant for our results, we can restate the model so that it is in line with this example. For both examples, we also express the equilibrium region determined in Section 4 as a function of the underlying

parameters.

Example 1: Price competition with a representative consumer with linear demand and differentiated products.

Suppose that buyers with the same preferred category have an indirect utility function of $q_1 + q_2 - 1/2(q_1^2 + q_2^2) - \gamma q_1 q_2 - p_1 q_1 - p_2 q_2$, with $\gamma \in [0, 1]$ expressing the degree of substitutability between products. This is a representative consumer setting in which each buyer obtains utility from positive quantities of each product in her preferred category. Maximizing this utility function with respect to q_1 and q_2 , we obtain the inverse demand functions $p_i = 1 - q_i - \gamma q_{-i}$, $i \in \{1, 2\}$. Inverting this demand system yields the direct demand functions $q_i = (1 - \gamma - p_i + \gamma p_{-i}) / (1 - \gamma^2)$, for $i \in \{1, 2\}$.

Duopoly equilibrium profit per buyer is $\pi^d = [(1 - \gamma)(1 - c)^2] / [(1 + \gamma)(2 - \gamma)^2]$. For a monopolist, the direct demand is $q_i = 1 - p_i$ and the per-buyer profit is $\pi^m = (1 - c)^2 / 4$. Thus, the ratio π^d / π^m is given by

$$\frac{4(1 - \gamma)}{(1 + \gamma)(2 - \gamma)^2},$$

which is above 1/2 if γ is lower than approximately 0.62.

In this example, the boundaries of the regions are affected by $\gamma \in [0, 1)$, with a higher γ implying less differentiation and fiercer competition. Using the results of Section 4, we obtain, that the agglomeration region applies approximately for $\gamma \leq 0.62$, the first mixing region for $0.62 < \gamma \leq 0.74$, the second mixing region for $0.74 < \gamma < 0.85$, and the segmentation region for $\gamma \geq 0.85$.

Example 2: Thin markets.

In this example, we consider capacity-constrained sellers that each can offer only one unit of a product—the analysis can be extended to allow for sellers with a finite number of products to sell.

Suppose that, in each category, there are two sellers and finitely many buyers $M_B > 1$. The example differs from the baseline model, as there is no continuum of buyers. To keep the exposition simple, suppose that there are two buyer types with valuation $R \in \{\underline{R}, \bar{R}\}$, with $0 \leq \underline{R} < \bar{R}$. The ratio of \bar{R} -types is $\rho \in (0, 1)$. Sellers observe buyers' valuations in the price-setting stage.¹⁴

Consider, first, the case in which both sellers and all buyers are located on platform i . If, for example, there are $M_B = 2$ buyers on platform i , there are four pairs of willingness-to-pay that the sellers can encounter: $(\underline{R}, \underline{R})$, (\underline{R}, \bar{R}) , (\bar{R}, \underline{R}) and (\bar{R}, \bar{R}) . For any pair with fewer \bar{R} -type buyers than sellers, the unique equilibrium is that sellers set $p^* = \underline{R}$ because

¹⁴This simplifying assumption implies that sellers can observe whether or not they are located in a market with sufficiently many \bar{R} -type buyers when they set their prices. Yet, in general, it suffices for our argument that prices are increasing in the number of buyers.

of Bertrand competition. Only if there are at least as many \bar{R} -type buyers as sellers—i.e. (\bar{R}, \bar{R}) realizes—there is the unique equilibrium that sellers set $p^* = \bar{R}$. The probability of this event equals ρ^2 . The expected profit of each seller is then $\pi(2, 2) = \rho^2 \bar{R} + (1 - \rho^2) \underline{R}$. More generally, denoting the probability that the number of high-type buyers is larger than the number of sellers (i.e., $Pr\{\#\}_{\{R_i = \bar{R}\}_{i=1}^{M_B}} \geq 2\}$), by $Q(M_B, 2)$, we obtain

$$Q(M_B, 2) = \sum_{k=2}^{M_B} \binom{M_B}{k} \rho^k (1 - \rho)^{M_B - k}.$$

The expected profit can then be written as $\pi(M_B, 2) = Q(M_B, 2) \bar{R} + (1 - Q(M_B, 2)) \underline{R}$, which corresponds to π^d in the baseline model.

If, instead, one seller per category locates on platform A and the other on platform B , and each buyer joins with probability $1/2$ platform A and with probability $1/2$ platform B (which will happen in equilibrium), the expected profit of a seller on platform i is $\pi(l, 1) = Q(l, 1) \bar{R} + (1 - Q(l, 1)) \underline{R}$ if $l \in \{1, \dots, M_B\}$ buyers join platform i , as $q(l, 1)$ is the probability that there is at least one \bar{R} -type buyer among the l buyers. The probability that $l \in \{1, \dots, M_B\}$ buyers locate on platform i is given by

$$P(l) = \binom{M_B}{l} \left(\frac{1}{2}\right)^l \left(\frac{1}{2}\right)^{M_B - l}.$$

Overall, the expected profit of a single seller located on a platform can then be written as $\sum_{l=1}^{M_B} P(l) \pi(l, 1)$. This expression corresponds to $\pi^m/2$ —that is, the monopoly profit of a seller when reaching each buyer with probability $1/2$. Table 3 illustrates how the ratio π^d/π^m depends on the number of buyers per category. (Note that π^d and π^m depend on the number of buyers M_B .)

M_B	1	2	3	4	5	6	7	8	9	10
π^d/π^m	0	0.1333	0.2367	0.3162	0.3769	0.4228	0.4570	0.4821	0.5002	0.5129

The ratio π^d/π^m as a function of the number of buyers M_B per category and parameter values $\rho = 1/4$, $\underline{R} = 0$ and $\bar{R} = 1$.

Table 3: Thin Markets

In this example, the number of buyers and the probability for a buyer being of high valuation are the drivers for the platform market structure. With two buyers and $\underline{R} = 0$, agglomeration occurs if the probability of high-type buyers is $\rho \geq 4/5$, whereas segmentation occurs if $\rho \leq 4/9$. A larger probability for the high type reduces competition, as it is more likely that both sellers face a high-type buyer, which leads to an increase in π^d . Regarding the number of buyers, in our numerical example with $\rho = 1/4$ and $\underline{R} = 0$,

agglomeration occurs when there are more than eight buyers (cf. Table 3), whereas segmentation occurs for three or less buyers. The first mixing region prevails for five to eight buyers and the second mixing region for four buyers. When the probability of the high type decreases, the boundaries for all regions shift upward, which implies that segmentation becomes more likely.

C Discussion of Equilibrium Refinement

To obtain a unique equilibrium in the second stage of the game in which buyers and sellers choose on which platform to be active, we impose the refinements of coalition-proofness and profit dominance of sellers. The question may arise how this selection mechanism operates in the reality. In this section, we address this question and also discuss under which conditions payoff dominance of buyers might be better suited as a selection mechanism.

First, as mentioned in the main body of the paper, a justification for profit dominance of sellers as a refinement is that sellers decide which platform to join before buyers do.¹⁵ This is a realistic assumption in markets in which platforms enable or facilitate trade between buyers and sellers. Those platforms can profitably operate only when securing deals with sellers listing their products on the platform in the first place. Buyers make their decisions only afterwards. In the real world, this is often a dynamic process in which the number of sellers and buyers change over time.¹⁶ However, sellers are often fewer in numbers and can coordinate their listing decisions more easily than buyers.

By contrast, markets in which it is more likely that buyers decide which platform to join before sellers do are better described by profit dominance of buyers. This is usually the case in industries, in which the main benefit of users is to enjoy content provided by platforms but platforms obtain revenues via advertisements or sales of products. Buyers then visit the platform foremost because of the content available. Examples of such platforms are social networks, such as Facebook, or video sharing platforms, such as YouTube or Vimeo. Users are attracted primarily because they can interact with their friends or watch videos, respectively, and the advertisements are merely a by-product for them (but are the revenue source of the platform). Therefore, it is natural that users move first in these markets, and coordinate on the equilibrium which is best for them. Advertisers or sellers of products then move second.

¹⁵For a general discussion how sequentiality in decisions solves the coordination problem in industries with network effects, see Farrell and Klemperer (2007).

¹⁶In some non-digital markets in which sellers list in directories or magazines, such as yellow pages, their decision must be taken before those of consumers. Similarly, as pointed out by Hagi (2006), in the software or video game industry, many application or game developers join a platform before buyers do due to the long-term development process.

Second, a natural question is how a coalition of agents coordinate their decisions. Coordination within a group can be achieved in a relatively easy way (and only such coordination is required when buyers and sellers move sequentially). For instance, on the seller side, in most markets, some sellers offer their products through an agency (as e.g., in the housing market). As this agency is responsible for the products of many sellers, it acts as a pivotal seller and smaller sellers follow the pivotal ones in their listing decisions. On the buyer side, word-of-mouth communication or rating reports of previous purchasers allow buyers to transmit information between them, and then coordinate their decisions.¹⁷

Coordination between the two groups of users can be achieved through interaction between important players in each group or through mediation of the platforms. We start with the first case. An important agent on the seller side can be an agency responsible for the decision of many sellers or a multi-product firm;¹⁸ on the buyer side, an important agent can be a marquee buyer, who influences other buyers. If the decisions of marquee buyers become public, other users from both sides will follow, which allows to achieve coordination between the two groups.

The second case is that a platform directly manages the coordination between both groups. An important way to do so is through advertising. For instance, large platforms often announce their market shares, thereby inducing users to follow the decisions of previous users.¹⁹ Another possibility is that platforms actively support coordination. For example, platforms may grant discounts to prestigious or well-known sellers to attract buyers. This then also attracts more unknown sellers, thereby facilitating coordination on agglomeration. An example of this pricing practice is demonstrated by Pashigian and Gould (1998) and Gould, Pashigian, and Prendergast (2005) for the case of shopping malls. They show that shopping malls offer rent subsidies to brand-name anchor stores but a rent premium to small tenants, which are less known. Platforms may also assist in fostering segmentation. This can be done by committing to limits in the number of sellers per category, in the extreme, by guaranteeing a seller exclusivity in his category—a practice that can be observed in the video game console industry.

Finally, as explained above, by requiring coalition-proofness, we do not leave room for user miscoordination. In a dynamic context in which users occasionally revise their decisions in a staggered manner, such miscoordination may be rather persistent. This

¹⁷For a recent formal exposition how word-of-mouth affects buyer migration between platforms, see Biglaiser et al. (2018).

¹⁸An example is a large fashion manufacturer listing on an apparel platform, or a hotel chain listing on a hotel booking platform.

¹⁹An example, discussed by Koski and Kretschmer (2002), is the software vendor Oracle, which brings together users of software with certified application developers. Oracle regularly mentioned in its advertising campaign that 98 out of the Fortune 100 companies use Oracle technologies.

opens another avenue to explain segmentation that we do not explore in this paper.²⁰

D Equilibrium Selection in Stage 2

In the first step, we determine all Nash equilibria in stage 2, given fees (f_i, f_{-i}) . In a second step, we show how the refinement of coalition-proofness plus profit dominance of sellers singles out an essentially unique equilibrium. The qualification to the uniqueness is that if $f_i = f_{-i}$, under some conditions two payoff-equivalent equilibria exist. As will become evident below, these equilibria are agglomeration either on platform A or B , or stand-alone either on platform A or B , dependent on the level of f_i . However, as the equilibria are payoff-equivalent, selecting among them is not necessary.

Suppose that the mass of sellers differs across platforms— α_i sellers are on platform i and $\alpha_{-i} < \alpha_i$ sellers are on platform $-i$. Then all buyers will join platform i . This implies that sellers on platform $-i$ have a profitable deviation to either go to platform i or be inactive for any $f_{-i} > 0$. It follows that in equilibrium either one platform has no sellers and no buyers, or $\alpha_i = \alpha_{-i}$, which makes buyers indifferent and induces them to split equally between the two platforms under our tie-breaking rule.

We start with the situation, in which there is trade on only one platform. As sellers are homogeneous across categories, there cannot be an equilibrium in which sellers in different categories follow different strategies. The reason is that if it is profitable for one or both sellers in some categories to list on the platform with a positive volume of trade, this must also be true for sellers in the remaining categories. There can be two equilibrium configurations in which only one platform carries a positive volume of trade.

The first configuration is an agglomeration equilibrium, in which all sellers and all buyers agglomerate on one platform. A seller's profit is then π^d . Hence, an equilibrium with agglomeration on platform i exists if $f_i \leq \pi^d$, independent of f_{-i} . The second equilibrium configuration is a stand-alone equilibrium, in which in each category only one seller is active on platform i and all buyers go to platform i . This configuration occurs if $\pi^m \geq f_i > \pi^d$, independent of f_{-i} . This equilibrium cannot occur with $f_i < \pi^d$, as in this case both sellers in each category prefer to be active on platform i .

We now turn to the equilibrium configuration, in which $\alpha_i = \alpha_{-i}$. The following three types of seller compositions give rise to $\alpha_i = \alpha_{-i}$.

- (i) In each category, one seller lists on platform i and one seller lists on platform $-i$.
- (ii) In 1/2 of the categories, both sellers list on platform i and in the other half both sellers list on platform $-i$.

²⁰On competing one-sided platforms, see Cabral (2011).

- (iii) In $1/2$ of the categories, one seller lists on platform i and in the other half one seller lists on platform $-i$.

In addition, any convex combination of these three seller compositions (i.e., mixing between the three types) leads to $\alpha_i = \alpha_{-i}$. Note that it can never be an equilibrium that in fewer than $1/2$ of the categories both or one seller list on platform i and platform $-i$. The reason is that non-active sellers have a profitable deviation to become active. This is because platform fees must be such that the resulting profits are higher than the listing fees as otherwise there can be no categories in which sellers are willing to list.

We now show that types (ii) and (iii) can never occur in equilibrium. Consider case (ii). Since in $1/2$ of the categories, both sellers are active on platform i , we must have $f_i \leq \pi^d/2$. If a seller active on platform $-i$ then deviates to platform i , its profit changes from $\pi^d/2 - f_{-i}$ to $\pi^m/2 - f_i$. By a similar argument, if a seller deviates from platform i to platform $-i$, its profit changes from $\pi^d/2 - f_i$ to $\pi^m/2 - f_{-i}$. This implies that case (ii) can only be an equilibrium if $\pi^d/2 - f_{-i} \geq \pi^m/2 - f_i$ and $\pi^d/2 - f_i \geq \pi^m/2 - f_{-i}$. Since $\pi^d/\pi^m < 1$, both conditions cannot jointly hold, implying that there must be a profitable deviation. Similarly, in case (iii) platform fees must be smaller than $\pi^m/2$, which implies that non-active sellers have a profitable deviation to list on the platform in which the competitor is not active. Since those two types cannot be an equilibrium configuration, mixing among the three types can be excluded by the same arguments.

As a consequence, the configuration in which both platforms are active can only be such that each platform is host to one seller in each category. This equilibrium can only occur if platform fees are below $\pi^m/2$, and no seller has an incentive to deviate and become active on the other platform. The latter condition implies

$$\frac{\pi^m}{2} - f_i \geq \frac{\pi^d}{2} - f_{-i}$$

Rewriting this condition, we obtain that a segmentation equilibrium exists if and only if

$$f_i \leq \min \left\{ \frac{\pi^m - \pi^d}{2} + f_{-i}, \frac{\pi^m}{2} \right\}. \quad (1)$$

As illustrated in Figure 2, for any combination of listing fees (f_i, f_{-i}) with $f_i \leq \pi^m$ and $f_{-i} \leq \pi^m$, multiple equilibria exist in stage 2.

Finally, we note that for all (f_i, f_{-i}) a no-trade Nash equilibrium exists in which neither buyers nor sellers participate on either platform. However, this no-trade equilibrium is not coalition-proof whenever some other equilibrium exists. We therefore disregard it in the following discussion.

In the second step, we demonstrate how our selection rule singles out a unique equi-

librium (unique subject to the qualification above). We start with the cases in which only a single equilibrium configuration exists (i.e., agglomeration or stand-alone) but multiple equilibria occur because agents can coordinate on either platform. First, consider the case in which there are the two equilibria, where all sellers (and buyers) agglomerate on platform A or platform B . If both platforms charge a fee below π^d and $f_i \neq f_{-i}$, coalition-proofness implies that all sellers and buyers coordinate on the platform with the lower fee, that is, they agglomerate on platform i if $f_i \leq f_{-i}$.²¹ Within this coalition, there is also no subcoalition that can improve by being active on platform $-i$. Similarly, if both platforms charge a fee larger than π^d and the two equilibria in which only half of the sellers are active on platform A or on platform B exist (i.e., the stand-alone equilibria), then sellers choose platform i if $f_i \leq f_{-i}$.

Now we turn to cases in which multiple equilibrium configurations exist. First, consider the case in which agglomeration and stand-alone equilibria exist. From the arguments above, this occurs if one platform, say platform $-i$, charges a fee below π^d whereas the other one charges a fee above π^d . However, the stand-alone equilibrium is then not coalition-proof because a coalition consisting of all buyers and all inactive sellers has a profitable deviation. If all these agents choose to be active on platform $-i$, then buyers are indifferent (as one seller per category is then active on each platform) but the profits of the formerly inactive sellers strictly increase from 0 to $\pi^m - f_i > 0$. In addition, no subcoalition can deviate and be strictly better off. By the same argument, if stand-alone and segmentation equilibria exist, a stand-alone equilibrium is not coalition-proof, whereas a segmentation equilibrium is. To see this, note that for these equilibrium configurations to co-exist, we must have that $\pi^d/\pi^m < 1/2$ and that both fees are between π^d and $\pi^m/2$. Thus, no coalition of sellers has the incentive to deviate from a segmentation equilibrium.²²

Finally, we turn to the region, in which segmentation and agglomeration equilibria exist. The profit of each seller in an agglomeration equilibrium on platform i is $\pi^d - f_i$. By contrast, in a segmentation equilibrium, the profit of a seller is either $\pi^m/2 - f_i$ or $\pi^m/2 - f_{-i}$ dependent on which platform the seller is active. Let us first look at the case $\pi^d/\pi^m \geq 1/2$. It is evident that a coalition of all sellers active on the platform with the higher fee, say platform i (i.e., $f_i \geq f_{-i}$), and all buyers on this platform have a profitable deviation to switch to platform $-i$. After such a deviation, the sellers are (weakly) better off because $\pi^d - f_{-i} \geq \pi^m/2 - f_i$ due to the fact that $\pi^d/\pi^m \geq 1/2$ and $f_{-i} \leq f_i$, and

²¹This is also the profit-dominant equilibrium for sellers.

²²In these regions, profit-dominance of sellers either selects the same equilibrium as coalition-proofness, or profit-dominance has no bite as some sellers prefer stand-alone over agglomeration (or segmentation) whereas others have the opposite preference. Hence, coalition-proofness and profit-dominance are not in conflict with each other.

buyers are better off because they observe the offers of both sellers and not only one. Therefore, for $\pi^d/\pi^m \geq 1/2$, the segmentation equilibrium is eliminated.²³

We now turn to the case $\pi^d/\pi^m < 1/2$. We first show that a similar mechanism as that in the previous paragraph does only partly work then. In particular, sellers on platform i (i.e., the platform with the higher fee) prefer agglomeration on platform $-i$ over segmentation if and only if $\pi^d - f_{-i} \geq \pi^m/2 - f_i$ or

$$f_i \geq \frac{\pi^m}{2} - \pi^d + f_{-i}.$$

If this inequality holds, the segmentation equilibrium is not coalition-proof because sellers and buyers on platform i can profitably deviate and agglomerate on platform $-i$. This shrinks the range for the segmentation equilibrium. In particular, for fees below π^d , the equilibrium exists for $f_i \leq (\pi^m - \pi^d)/2 + f_{-i}$, whereas it survives the refinement only if $f_i < \pi^m/2 - \pi^d + f_{-i}$.²⁴ If $f_i < \pi^m/2 - \pi^d + f_{-i}$, the refinement of coalition-proofness has no bite. However, the refinement of profit-dominance for sellers then selects the segmentation equilibrium as the unique equilibrium. In particular, the inequality ensures that sellers on platform i are better off in the segmentation equilibrium than in the agglomeration equilibrium, and the condition $\pi^d/\pi^m < 1/2$ guarantees that also sellers on platform $-i$ prefer segmentation over agglomeration because $\pi^m/2 - f_{-i} > \pi^d - f_{-i}$.

Therefore, our equilibrium refinement selects the following equilibrium, given any (f_i, f_{-i}) with $f_i \leq \pi^m$ and $f_{-i} \leq \pi^m$:

(i) If $\pi^d/\pi^m \geq 1/2$, then

- for $f_i, f_{-i} \geq \pi^d$, the equilibrium is STA_i if $f_i \leq f_{-i}$;
- for all other values, the equilibrium is AGG_i if $f_i \leq f_{-i}$.

(ii) If $\pi^d/\pi^m < 1/2$, then

- for $f_i, f_{-i} \leq \pi^d$, the equilibrium is SEG if $f_{-i} < \pi^m/2 - \pi^d + f_i$, and AGG_i if $f_{-i} \geq \pi^m/2 - \pi^d + f_i$.
- for $f_i, f_{-i} \in (\pi^d, \pi^m/2]$, the equilibrium is SEG .
- for $f_i > \pi^d$ and $f_{-i} > \pi^m/2$, the equilibrium is STA_i if $f_i \leq f_{-i}$.

²³Again, profit-dominance of sellers also eliminates the segmentation equilibrium.

²⁴If both fees are above π^d there is no restriction because the agglomeration equilibrium does not exist then.

E Seller Multi-Homing with Multiple Sellers

In Section 5.2, we considered the case of seller multi-homing with two sellers per product category and we showed that, under some conditions, a partial multi-homing equilibrium can occur in which one seller in each product category multi-homes and the other seller single-homes. Half of the single-homing sellers are active on platform A and the other half on platform B . In this section, we characterize partial multi-homing equilibria with a general number of sellers $M \geq 2$.²⁵ As in Section 5.2, we consider the case with two platforms. Following the main analysis of the baseline model, we assume that multi-homing sellers set a uniform price across both platforms—that is, they do not price discriminate. The equilibrium refinement in the second stage is the same as in the main model (i.e., we select the seller-preferred, coalition-proof equilibrium).

In addition, we also impose an equilibrium selection criterion in the first stage. With a general number of sellers, the equilibrium in the fee-setting game between platforms may not be unique. Then, as a refinement, we assume that platforms choose the profit-dominant equilibrium.

With $M \geq 2$ and the possibility of seller multi-homing, a seller's profit per buyer depends on the size of three seller groups—the single-homing sellers on either platform and the multi-homing sellers—since they determine how competition between sellers plays out. To state the profits concisely, we write the per-buyer profit of a single-homing seller on platform i as $\pi_i^{SH}(l_i^{SH}, l_{-i}^{SH}, l^{MH})$, where l_i^{SH} is the number of single-homing sellers on platform i , l_{-i}^{SH} is the number of single-homing sellers on platform $-i$, and l^{MH} is the number of multi-homing sellers. Since more intense competition leads to lower profits, we assume that π_i^{SH} is strictly decreasing in the first and third argument, constant in the second argument for $l^{MH} = 0$, and strictly decreasing in the second argument for $l^{MH} > 0$. In particular, an increase in the number of single-homing sellers on the competing platform $-i$ makes competition on that platform more intense. This then spills over into competition on platform i because multi-homing sellers set the same price on both platforms. In addition, we assume that $\pi_i^{SH}(l/2 - \delta, l/2 + \delta, M - l) > \pi_i^{SH}(l/2 + \delta, l/2 - \delta, M - l)$, with, for l even, $2 \leq l \leq M$ and $\delta = 1, 2, \dots, l/2$, and, for l odd, $1 \leq l \leq M$ and $\delta = 1/2, 3/2, \dots, l/2$. That is, given that l sellers single-home, the profit of a single-homing seller is larger when being active on the platform with fewer active sellers. This assumption is natural because the single-homing seller then competes directly only with a smaller number of other sellers (as buyers on that platform only observe the offers of a smaller number of sellers). It is also fulfilled by our

²⁵As explained in Section 5.2, full multi-homing will never occur in equilibrium, as the per-buyer profit is then the same as in an agglomeration equilibrium, but sellers pay two fees in the partial multi-homing equilibrium instead of only a single fee in the agglomeration equilibrium.

microfoundations.²⁶

Similarly, we denote the *average* per-buyer profit of a multi-homing seller by $\pi^{MH}(l_i^{SH}, l_{-i}^{SH}, l^{MH})$, which, for $l^{MH} > 0$, is assumed to be strictly decreasing in all arguments.²⁷

Finally, in case all active buyers and sellers are on the same platform, we denote the per-buyer profit of a seller competing with $m - 1$ others sellers in its product category by $\pi(m)$, with $m \in \{1, \dots, M\}$. In terms of the notation of the baseline model, $\pi(1) = \pi^m$ and $\pi(2) = \pi^d$. We also put some structure on this per-buyer profit by making the following assumption: the difference $\pi(m) - \pi(m + 1) \geq 0$ is decreasing in m —that is, the negative impact on profits due to a larger number of competitors falls with the number of competitors. This is a natural assumption which is fulfilled by many demand functions (including the ones in our examples). We can then show the following result:²⁸

Proposition 12. *A partial multi-homing equilibrium exhibits the following properties:*

- (i) *Platform fees are zero.*
- (ii) *In all product categories, $M - 1$ sellers are multi-homing and one seller is single-homing. In half of the categories, the single-homing seller is active on platform A and, in the other half, on platform B . Buyers split evenly between platforms.*

The proposition shows that, similar to the case with only two sellers, if a partial multi-homing equilibrium occurs, platform fees are zero. The intuition is the same as the one outlined in Section 5.2. If listing fees were positive, one platform can slightly undercut the rival's fee, thereby inducing all single-homing sellers to list on its platform and induce agglomeration. This is always profitable as it increases the number of sellers on the platform. The second statement of the proposition extends the partial multi-homing result of Proposition 7. With a general number of sellers, a partial multi-homing equilibrium features multi-homing of all but one seller. As a special case, in Proposition 7, we obtained the result that with two sellers, a partial multi-homing equilibrium involves one multi-homing and one single-homing seller. The proposition of this section shows now that any additional seller will be multi-homing.

The intuition is as follows: if more than one seller single-homes on the one platform and none single-homes on the other, in half of the categories on platform A and in the other on B , at least one of these sellers has a profitable deviation to switch to the rival platform, as this reduces competition. If, instead, in each category the same number of sellers single-home on both platforms, either one single-homing seller in each category

²⁶Loosely speaking, the assumption implies that 'direct' competition is stronger than 'indirect' competition.

²⁷As explained in the proof of Proposition 7, the per-buyer profit that a multi-homing seller obtains per buyer can differ across platforms if the platforms differ in the number of single-homing sellers they host. Therefore, $\pi^{MH}(\cdot)$ is the *average* per-buyer profit.

²⁸The proof of this proposition can be found in Web Appendix H.

prefers to multi-home to benefit from increased demand or, as we show in the proof of Proposition 12, competition between sellers is sufficiently intense such that platforms will set positive fees in the first stage and induce a segmentation equilibrium (with positive probability). As a consequence, a partial multi-homing equilibrium features multi-homing of all but one sellers and platform fees of zero.

In the proposition, we do not state the conditions for the partial multi-homing equilibrium to exist. Yet, these conditions are similar to those in Proposition 7, adjusted to M sellers. For example, the equilibrium exists only if the single-homing seller in each product category has no incentive to multi-home, which holds if $\pi(M) \leq \pi_i^{SH}(1, 0, M - 1)/2$. This condition is the generalization of condition $\pi^d \leq \pi^{SH}/2$ of Section 5.2, as $\pi(M) = \pi^d$ if $M = 2$.

The condition demonstrates that the extent of multi-homing is governed by the degree of competition between sellers if all but one seller multi-home. Let us explore this in more detail. The intuition why, in the partial multi-homing equilibrium, a single-homing seller has no incentive to multi-home is *not* that it would face a larger number of competitors. As all other sellers multi-home, this number is unchanged. Yet, the multi-homing sellers compete with the single-homer on platform i only for half of the consumers. This makes them less aggressive in their pricing decision than if the single-homer were also active on the other platform. In the latter case, prices of multi-homing sellers would be lower, which may dominate the effect that the seller reaches more buyers. Overall, the incentive to be the only single-homing seller in a partial multi-homing equilibrium tends to be stronger if there are fewer sellers. For example, with $M = 2$, the multi-homing seller is a monopolist on the platform on which the single-homer is not active. Then, in many instances, the multi-homing seller has an incentive to set a high price to extract surplus from buyers on this platform. By contrast, if there is a large number of multi-homing sellers, an additional one will not affect their prices to a large extent, and the condition for the partial multi-homing equilibrium to exist is more difficult to satisfy. As a consequence, the extent of multi-homing is driven by the total number of sellers. If this number is small, multi-homing is more likely to occur in equilibrium than when the number is large.

This result is consistent with, for example, the market for flight search engines. For each route, there is usually only a limited number of flights available, and most airlines multi-home. In this market, several platforms, such as Kayak, Expedia, or Skyscanner, operate with sizable market shares.²⁹

Finally, we note that platform profits in the partial multi-homing are always zero. As we will explain in the last subsection of Web Appendix G, a reason why platforms obtain positive profits in case of multi-homing could be that each platform has some loyal

²⁹See, for example, European Commission (2017).

consumers. There, we relate our discussion to comparison shopping platforms, in which consumer loyalty is prevalent.

F General Number of Sellers and Platforms

In this section, we extend our baseline model to a finite number of platforms and sellers per category. Suppose that there are M sellers (per category) and N platforms, with $M, N > 1$. As in Web Appendix E, we denote the per-buyer profit of a seller competing with $m - 1$ others sellers by $\pi(m)$, with $\pi(m) \geq \pi(m + 1) \geq 0 \forall m \in \{1, \dots, M - 1\}$ —in terms of the notation of the baseline model, $\pi(1) = \pi^m$ and $\pi(2) = \pi^d$. All other assumptions and the equilibrium refinement in the second stage are the same as in main model.

In addition, as in Web Appendix E, we impose an equilibrium selection criterion in the first stage. With a general number of platforms and sellers, the equilibrium in the fee-setting game between platforms may not be unique. Then, as a refinement, we assume that platforms choose the profit-dominant equilibrium.

The main differences from our baseline model are twofold: first, with a general number of sellers and platforms, the number of sellers is no longer necessarily a multiple of the number of platforms. The question is, therefore, how sellers, in order to make buyers indifferent, allocate if multiple platforms carry a positive volume of trade. Second, it may be optimal for platforms in the first stage to exclude sellers via the choice of their listing fees. As we will demonstrate below, this may occur in a segmentation equilibrium.

Following the same structure as with different pricing instruments, we characterize in the next proposition the regions in which the different types of equilibria exist, thereby pointing out the analogy to the simpler baseline model.³⁰ To write the proposition in the most concise form, we define k as the largest integer, such that $M \geq kN$. For example, if $M = 11$ and $N = 4$, then $k = 2$.

Proposition 13. *Consider the case in which $M \geq N$:*

If

$$\frac{\pi(k + 1)}{\pi(k)} \geq \frac{1}{N}, \quad (2)$$

in equilibrium $f_i^ = 0, \forall i \in \{1, \dots, N\}$, platform profits are 0, and there is positive trade only on a subset of platforms.*

If for some $l \in \{1, \dots, k\}$

$$\frac{\hat{m}\pi(\hat{m})}{l\pi(l)} \leq \frac{1}{N}, \quad (3)$$

³⁰The proof of this proposition can be found in Web Appendix H.

with $\hat{m} \in \arg \max_{l < m \leq M} m\pi(m)$, in the unique profit-dominant equilibrium $f_i^* = \pi(l^*)/N$, with $l^* \in \arg \max_l l\pi(l)$ for all $l \in \{1, \dots, k\}$ that satisfy (3), platform profits are $\Pi_i^* = l^*\pi(l^*)/N \forall i \in \{1, \dots, N\}$, and all platforms carry a positive volume of trade.

If neither (2) nor (3) is satisfied, there is a unique profit-dominant mixed-strategy equilibrium, in which platforms make positive profits.

Consider the case in which $M < N$: In equilibrium, $f_i^* = 0 \forall i \in \{1, \dots, N\}$, and platform profits are 0.

If there are at least as many sellers (in each category) as platforms, the proposition demonstrates that the qualitative features of the equilibrium are similar to those in the baseline model. If competition between sellers is relatively moderate, a seller's profit when one additional seller joins the platform falls only by a small amount (i.e., $\pi(k+1)/\pi(k)$ is relatively large), which implies that condition (2) is satisfied. Thus, equilibrium platform fees are zero.

In analogy to the baseline model, this equilibrium prevails if sellers prefer to be active only on a subset of platforms, given that all platforms charge zero fees. To relate this to condition (2), note that k is the largest number of sellers, so that all platforms have positive trade volume, and each one hosts k sellers (so that buyers are willing to split between platforms). Condition (2) states that such a configuration will not emerge in the second stage, as sellers have an incentive to deviate.³¹ With trade occurring only on a subset of platforms, no platform can charge a strictly positive fee as it loses its buyers and sellers to a competitor with zero fee.

In contrast to the baseline model, such an equilibrium does not necessarily lead to *full* agglomeration, as it may be optimal for some sellers to locate on one platform and other sellers on another. Nevertheless, only a subset of platforms carry a positive volume of trade, which implies at least partial agglomeration, and equilibrium fees of zero. This must also be the equilibrium outcome if the number of platforms exceeds the number of sellers in a category, as it implies that at least one platform will not have a positive volume of trade.

By contrast, in a pure-strategy segmentation equilibrium, all platforms carry a positive volume of trade. In analogy to the baseline model, this equilibrium occurs if competition between sellers is intense. From condition (3), the equilibrium exists if, in each category, every platform hosts l sellers, and no platform can obtain a higher profit by attracting a larger number of sellers (where attracting a number \hat{m} is the most profitable one among these deviations). The condition for a segmentation equilibrium to exist in the model with a general number of platforms and sellers resembles that of the baseline model. In the baseline model, we have $l = 1$ and $\hat{m} = 2$, and, thus, condition (3) is

³¹In the baseline model, we have $k = 1$, and, thus, condition (2) is equivalent to $\pi^d/\pi^m \geq 1/2$.

equivalent to $\pi^d/\pi^m \leq 1/4$.

The key difference from the baseline model is that the segmentation equilibrium may lead to the *exclusion* of some sellers—that is, the equilibrium number of sellers on a platform, l^* , may be less than k . If $M > kN$, this must be the case, as a segmentation equilibrium involves at least $M - kN$ inactive sellers. However, even if $M = kN$, it can be optimal for platforms to charge such a high fee that some sellers prefer to stay inactive. The reason is that becoming active increases competition and, therefore, would not allow the seller to recover the fee. In addition, with a general number of sellers, a segmentation equilibrium may involve more than one seller in each category on a platform if this allows platforms to obtain a higher profit. As in the baseline model, platforms extract the entire profit from all active sellers.

Finally, in the region in which neither condition (2) nor condition (3) holds, a mixed-strategy equilibrium occurs. The intuition and the properties are the same as in the baseline model.

In our analysis, we consider the situation with a given number of sellers M with positive profit (gross of the listing fee). Instead, if sellers incurred a fixed entry cost F , $\pi(m) - F$ would become negative for m sufficiently large, as more intense competition drives down margins. Then, even if platforms charge zero fees, no platform would be host to an unlimited number of sellers. Yet, considering a game with free entry yields similar results to those of Proposition 13. The conditions for the equilibrium regions differ, but the qualitative results that buyers and sellers may segment and that platforms obtain positive profits continue to hold. If there is a finite m' as the solution to $\pi(m')/N - F > 0$ and $\pi(m'+1)/N - F < 0$, the region in which the agglomeration equilibrium exists shrinks and eventually vanishes as the number of available sellers M becomes sufficiently large. The reason for this result is that with zero fees, each platform would host m' sellers. A seller's profit is then strictly positive, which gives each platform an incentive to increase its fee. To sum up, with entry, pure agglomeration cannot occur when the number of sellers that may enter is large and, thus, segmentation becomes more likely.

G Two-Sided Pricing and Further Extensions

G.1 Two-Sided Pricing

In the baseline model, we considered the situation in which platforms can set fees only to sellers. This is a common practice among most trading platforms. A main reason is that buyers are often uncertain about whether or not they want to buy a product, and they first inform themselves on the platform about available offers and product characteristics. Thus, charging a subscription fee will deter many buyers. In addition, some buyers often

obtain only a small surplus, and so platforms can charge only a very small fee to keep these buyers on board. With small transaction costs from each payment (e.g., due to fraud), it is more effective to charge sellers who are usually fewer in numbers.

Apart from these justifications for not charging buyers, which are outside the model, we can demonstrate that the segmentation equilibrium derived in the baseline model is robust to two-sided pricing (that is, platforms set a subscription fee to buyers, f_b , on top of the listing fee to sellers, f_s), provided that negative fees are not possible.³²

We focus on the situation $\pi^d/\pi^m \leq 1/4$, in which the pure-strategy segmentation equilibrium exists with one-sided pricing. Platforms then set a seller fee $f_s = \pi^m/2$ and extract the full seller surplus. With two-sided pricing, a fee combination of $(f_s = \pi^m/2, f_b = 0)$ for both platforms is no longer an equilibrium under the refinement of coalition-proofness and seller-dominant equilibrium. To see this, suppose that platform $-i$ sets $(f_s = \pi^m/2, f_b = 0)$. Platform i can then set fees equal to $(f_s = \pi^d - \epsilon, f_b = V^d - V^m)$ and attract all sellers and buyers because sellers obtain a profit of $\epsilon > 0$ on platform i instead of 0 on platform $-i$. Therefore, the coalition of all sellers and buyers on platform $-i$ is better off by moving to platform i , as buyers are indifferent and obtain a payoff of V^m on both platforms.³³ The profit of platform i is then (almost) equal to $2\pi^d + V^d - V^m$. Although we are in the region with $\pi^d/\pi^m \leq 1/4$, the profit from deviating is larger than $\pi^m/2$, as $4\pi^d + 2(V^d - V^m) > \pi^m$ due to the fact $V^d + \pi^d > V^m + \pi^m$.

Determining the equilibrium for the range $\pi^d/\pi^m \leq 1/4$, we obtain (following arguments similar to those in the baseline model) that the unique equilibrium under our refinement is $(f_s^* = 0, f_b^* = V^m)$ and segmentation occurs. With these fees, no platform can attract more sellers since this would lead to competition between them and, therefore, to a reduction in sellers' profits. Instead of extracting the sellers' profits, platforms do not leave surplus to buyers. Importantly, though, despite this difference in fees between one-sided and two-sided pricing, the main intuition for the segmentation equilibrium to occur is the same: sellers avoid competition by being active on both platforms, and platforms exploit this role of segmenting the market by charging strictly positive fees.

If in stage 2 buyers and sellers play the equilibrium that buyers prefer (in addition to coalition-proofness), the equilibrium fees would be the same as in the case of one-sided pricing—that is, $(f_s^* = \pi^m/2, f_b^* = 0)$. Setting a strictly positive fee to buyers can never

³²With negative fees, a divide-and-conquer strategy can destabilize the segmentation equilibrium. Under divide-and-conquer, a deviating platform sets a sufficiently low fee on one side to ensure that this side participates for sure. It can then use the fee on the other side to extract surplus on that side. In particular, a platform deviating from the segmentation equilibrium can attract sellers with negative fees and extract the full surplus generated on the buyer side. However, such negative fees are usually not feasible, as they generate losses for platforms from otherwise uninterested participants who inflate participation levels without generating any transaction opportunities.

³³It can be shown that setting fees equal to $(f_s = \pi^d - \epsilon, f_b = V^d - V^m)$ is, indeed, the most profitable deviation.

be profitable for a platform, as then all buyers prefer the rival platform. Given this, the same arguments as in Section 6.2 apply. Although buyers prefer agglomeration, platforms avoid this in equilibrium by setting listing fees to sellers above π^d .

G.2 Further extensions

In our analysis, we assumed that there is no interdependence between categories and that buyer behavior is not heterogeneous. In this section, we discuss what happens in richer settings that allow for these features.

G.2.1 Interdependence between Categories

Competition across categories

In our analysis, we assumed that each buyer is interested in exactly one product category. Suppose, instead, that a buyer receives a positive gross utility from buying a product in a category other than her preferred one (which, by construction, is less than from products in her preferred category). Then, products in different categories are substitutes. As a consequence, the demand for a product in the preferred category may be lower if prices in different categories are lower. Sellers will take this into account in their pricing decisions, implying that products from different categories may impose competitive constraints. As a consequence, prices will be weakly lower than in our model. This leads to lower values of π^m and, possibly, of π^d . Hence, in the segmentation equilibrium, platforms' fees will be weakly lower, and, depending on the profit ratio π^d/π^m , the regions for the segmentation and agglomeration equilibrium will be affected by competition across platforms. However, the main effects driving the results are still present, and our main insights are robust.

We can restate our main conclusions in a different setting, in which there is only one product (and, thus, one seller) per product category. With this simplification, since sellers do not compete, the market necessarily features agglomeration. Introducing competition between categories, the equilibrium switches to segmentation if segmentation sufficiently reduces seller competition. In such a segmentation equilibrium, half of the categories are listed on platform *A* and the other half on platform *B*. This result is in the same spirit as our findings in the baseline model.

Platforms with diseconomies

Another source of interdependence between categories could be platform diseconomies in the number of categories. Platforms experience such diseconomies if the number of product categories on a platform has a direct and negative effect on buyer utility. Such congestion externalities can be present because, for instance, it becomes more costly for a buyer to find the preferred product category as the number of categories increases.

This implies that a buyer experiences a utility loss if the number of listed categories is large. If platforms can endogenously choose this number, then, to avoid repelling buyers, they may not list some categories in equilibrium. This implies that even in an agglomeration equilibrium, listing fees may be positive, as lowering the fee to zero induces the participation of sellers in all categories, which is not attractive to buyers.³⁴

Alternatively, the number of product categories may affect buyer utility indirectly. This happens if the optimal presentation of products on a platform depends on the product category. Suppose that a platform has to commit to a unique format for presenting products (e.g., to avoid confusing buyers). Then, if very different product categories are listed, the presentation format is not optimal for some products, and, thus, the utility of buyers who prefer these products is reduced. As above, a consequence is that some products will be delisted.³⁵

G.2.2 Heterogeneous Buyer Behavior

In our model, buyers are *ex ante* identical. In particular, they do not prefer one platform over the other. Although this assumption ensures that platforms are fully homogeneous *ex ante* and, therefore, strengthens our theoretical contribution, it may not be in line with the consumer behavior observed in some markets. For example, a fraction of buyers may be loyal to a platform. A market in which consumer loyalty is prevalent is the market for price comparison platforms, such as flight search engines or comparison shopping websites. In a recent study, the UK competition authority (Competition & Market Authority, 2017) found that in this market around 58% of consumers use only one price comparison platform when searching for a particular product or service, mainly due to loyalty reasons. Similarly, some buyers may decide very quickly on which platform to be active and, therefore, assign themselves randomly to a platform. Suppose that each platform has a fraction $\beta/2$ of loyal buyers and that all other buyers are shoppers—i.e., they join the platform that offers the highest expected utility as in our baseline model. To reach loyal buyers, sellers have to list on both platforms. Thus, if sellers can multi-home, in an equilibrium that corresponds to an agglomeration equilibrium, both platforms set a positive listing fee equal to the duopoly profit that a seller earns from loyal buyers so as to induce seller multi-homing. By contrast, in an equilibrium that corresponds to a segmentation equilibrium—i.e., each platform attracts half of the shoppers—sellers

³⁴One could also imagine that product categories differ by the probability of being the preferred category. Then, popular categories will be listed, but less popular categories will be delisted.

³⁵An example in which a platform did not cater well to buyer tastes with its presentation of particular product categories is the market for handmade and vintage items on Ebay. Newer platforms, such as Etsy and Dawanda, offered sellers the opportunity to offer more information, and this led to a quick migration of buyers and sellers to these new platforms. This suggests that Ebay was subject to diseconomies in the number of listed product categories and lost out to newcomers.

single-home along the equilibrium path because the insights from Section 5.2 still hold.

H Relegated Proofs

This section contains the relegated parts of the proof of Proposition 3 and the (full) proofs of Propositions 4 to 13.

Relegated Parts of the Proof of Proposition 3. In the main body of the paper, we derived the best-response function $f_i^{br}(f_j)$, with $i, j \in \{A, B\}$, $i \neq j$. Figure 11 depicts the two best-response functions (using the definition from the main body of the paper that $\delta = \pi^m/2 - \pi^d$). The blue line represents $f_A^{br}(f_B)$ and the red line $f_B^{br}(f_A)$. In the lower part of $f_A^{br}(f_B)$, starting at $f_A^{br}(0) = \delta$ and ending at $f_A^{br}(3\delta) = 4\delta$, platform A's best response induces segmentation. In this case, $f_A^{br}(f_B)$ corresponds to the line separating the segmentation region (*SEG*) from the agglomeration region on platform B (*AGG_B*) in Figure 3, that is, platform A just induces segmentation. Instead, in the upper part of $f_A^{br}(f_B)$ —that is, for all $f_B > 3\delta$ —platform A's best response leads to agglomeration. The function $f_A^{br}(f_B)$ is therefore just above the line separating the segmentation region (*SEG*) from the agglomeration region on platform A (*AGG_A*) in Figure 3. The best response of platform B, $f_B^{br}(f_A)$, can be derived in the same way.³⁶

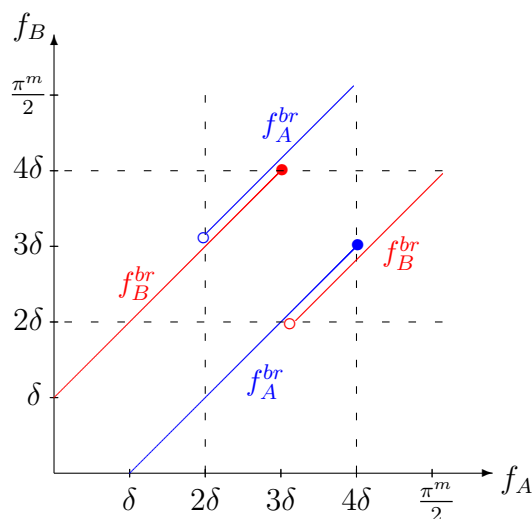


Figure 11: Best-response functions for $3/8 \leq \pi^d/\pi^m < 1/2$

We now derive the mixing probabilities. From the part of the proof in the main body of the paper, we know that in the range $3/8 \leq \pi^d/\pi^m < 1/2$, platforms set fees $f_i, f_j \in [\pi^m - 2\pi^d, 2\pi^m - 4\pi^d]$, and the expected profit is $\Pi_A^* = \Pi_B^* = 3\pi^m/2 - 3\pi^d$.

³⁶In the mixed-strategy equilibrium, both platforms set fees in the range $[2\delta, 4\delta]$, which is indicated by the dashed lines.

All fees in the mixing domain must give an expected profit of 3δ (with $\delta = \pi^m/2 - \pi^d$), as otherwise platforms would not be indifferent between these fees. We need to distinguish between two intervals, a lower and an upper one. The lower interval consists of fees $f_i \in [2\delta, 3\delta)$ and the upper interval consists of fee $f_i \in [3\delta, 4\delta]$. The reason for this distinction is that in the lower interval, sellers may agglomerate on platform i (i.e., this happens if $f_j > f_i + \delta$) but will never agglomerate on platform j . That is, if f_i is in this lower interval, platform i will always obtain a positive profit. By contrast, if f_i is an element of the upper interval, with some probability sellers will choose to agglomerate on platform j —this occurs if platform j charges $f_j < f_i - \delta$ —and platform i obtains no profit. Platform i 's profit can then be written as

$$\Pi_i(f_i, f_j) = \begin{cases} 0, & \text{if } f_i \in (f_j + \delta, 4\delta] \wedge f_j \in [2\delta, 3\delta); \\ f_i, & \text{if } f_i \in [\max\{2\delta, f_j - \delta\}, \min\{f_j + \delta, 4\delta\}] \wedge f_j \in [2\delta, 4\delta]; \\ 2f_i, & \text{if } f_i \in [2\delta, f_j - \delta) \wedge f_j \in (3\delta, 4\delta]. \end{cases}$$

Let us start with the case in which platform i charges a fee in the lower interval—that is, $f_i \in [2\delta, 3\delta)$. Denote the cumulative density function with which platform j mixes by $G_1(f_j)$. Platform i 's profit with a fee in this lower interval is then given by (replacing f_i by f)

$$G_1(f + \delta)f + (1 - G_1(f + \delta))2f.$$

In equilibrium, this expression must be equal to 3δ , yielding

$$G_1(f + \delta)f + (1 - G_1(f + \delta))2f = 3\delta. \quad (4)$$

This equation determines the mixing probabilities of platform j in its upper interval. This is because only if platform j sets a fee above $f + \delta$ (which happens with probability $1 - G_1(f + \delta)$), sellers will agglomerate on platform i . Such a fee must necessarily be in the upper interval.

If platform i charges a fee in the upper interval—that is, $f_i \in [3\delta, 4\delta]$ —its profit is

$$G_1(f - \delta)0 + (1 - G_1(f - \delta))f = 3\delta. \quad (5)$$

This equation determines the mixing probability in the lower interval.

Let us first look at (4). We can substitute $h \equiv f + \delta$ to get

$$G_1(h)(h - \delta) + (1 - G_1(h))2(h - \delta) = 3\delta. \quad (6)$$

Recall that (4) was relevant for f in the lower range and, since $h = f + \delta$, these are

exactly the fees in the upper interval. Solving (6) for $G_1(h)$ gives

$$G_1(h) = \frac{2h - 5\delta}{h - \delta}. \quad (7)$$

It is easy to check that $G_1(4\delta) = 1$.

Now we turn to (5). Here, we can substitute $h \equiv f - \delta$ representing that h is now in the lower interval. We obtain

$$(1 - G_1(h))(h + \delta) = 3\delta. \quad (8)$$

Solving (8) for $G_1(h)$ gives

$$G_1(h) = \frac{h - 2\delta}{h + \delta}. \quad (9)$$

It is easy to check that $G_1(2\delta) = 0$. Using (7) and (9), we obtain $\lim_{G_1(h) \searrow 3\delta} = 1/2$ and $\lim_{G_1(h) \nearrow 3\delta} = 1/4$. This implies the existence of a mass point with mass $1/4$ at a fee equal to 3δ .³⁷ \square

Proof of Proposition 4. As shown in the proof of the previous proposition, a pure-strategy equilibrium in the region $1/4 < \pi^d/\pi^m < 1/2$ does not exist. Furthermore, for $3/8 \leq \pi^d/\pi^m < 1/2$, there exists a mixed-strategy equilibrium which has an upper bound of the randomization domain equal to $2\pi^m - 4\pi^d$. This equilibrium cannot exist in the range $1/4 < \pi^d/\pi^m < 3/8$ because $2\pi^m - 4\pi^d$ would then be larger than $\pi^m/2$. We next derive the randomization domain of the mixed-strategy equilibrium with an upper bound of $\pi^m/2$.

Randomization domain. Suppose that platform j sets $f_j = \pi^m/2$. The best response of platform i is then to set f_i to induce agglomeration in the second stage. To do so, it needs to set $f_i = \pi^d - \epsilon$. The best response of platform j is to marginally reduce its fee to $\pi^m/2 - \epsilon$ and induce a segmentation again, and so on. This goes on until platform i sets the lowest fee in the randomization domain, denoted by \underline{f}^l . This is the fee at which platform i is better off by raising its fee to the highest fee $\pi^m/2$ and induce segmentation instead of marginally reducing it to induce agglomeration. Its segmentation profit is then $\pi^m/2$. Hence, the lowest fee \underline{f}^l is given by $2\underline{f}^l = \pi^m/2$ or, equivalently, $\underline{f}^l = \pi^m/4$. This fee makes sellers exactly indifferent between agglomeration on platform i and segmentation if platform j charges a fee such that $\pi^d - \underline{f}^l = \pi^m/2 - f_j$ or, equivalently, $f_j = 3\pi^m/4 - \pi^d$.

Finally, note that a fee $\pi^d - \epsilon$ (i.e., the fee that induces agglomeration if the rival platform charges the highest fee) is strictly lower than $3\pi^m/4 - \pi^d$ (i.e., the fee at which the rival stops lowering its fee and instead raises the fee to the highest one) since we are

³⁷Intuitively, equation (5) requires a sufficiently high probability of $f - \delta$ being close to 3δ since, as otherwise setting f close to 4δ would lead to zero profit “too often”.

in the range $\pi^d/\pi^m < 3/8$. Therefore, the upper bound of the lower interval is below the lower bound of the upper interval. It follows that there are two disjoint sets of mixing intervals. The upper one $[3\pi^m/4 - \pi^d, \pi^m/2]$, in which each fee is a best response to a fee in the lower interval $[\pi^m/4, \pi^d]$. In turn, each fee in the lower interval is a best response to a fee in the upper interval.

To summarize the above analysis, in the range $1/4 < \pi^d/\pi^m < 3/8$, there is a mixed-strategy equilibrium with fees $f_i \in [\pi^m/4, \pi^d] \cup [3\pi^m/4 - \pi^d, \pi^m/2]$. For any chosen fee, the expected profit in this range must be $3\pi^m/4 - \pi^d$. As above, this is because setting a fee equal to $3\pi^m/4 - \pi^d$ induces segmentation with a probability of (almost) 1.

Mixing Probabilities. Let $\eta \equiv \pi^d - \pi^m/4$, $\delta \equiv \pi^m/2 - \pi^d$ and $\epsilon > 0$ but infinitesimally small. Denote $\underline{f}^l \equiv \pi^m/4$, $\bar{f}^l \equiv \pi^d$, $\underline{f}^u \equiv 3\pi^m/4 - \pi^d$, and $\bar{f}^u \equiv \pi^m/2$ such that the domain of interest can be expressed as $f_i \in [\underline{f}^l, \bar{f}^l] \cup [\underline{f}^u, \bar{f}^u]$. Using η and δ , the mixing domain can be written as $f_i \in [\eta + \delta, 2\eta + \delta] \cup [\eta + 2\delta, 2\eta + 2\delta]$. For $i, j \in \{A, B\}$ and $i \neq j$, the corresponding best response function is given by

$$f_i^{br}(f_j) = \begin{cases} f_j + \delta, & \text{if } f_j \in [\underline{f}^l, \bar{f}^l]; \\ f_j + \eta, & \text{if } f_j = \underline{f}^u; \\ f_j - \delta - \epsilon, & \text{if } f_j \in (\underline{f}^u, \bar{f}^u]. \end{cases}$$

We know that all fees in the mixing domain must give an expected profit of $\underline{f}^u = (3/4)\pi^m - \pi^d = \eta + 2\delta$.

We now proceed analogously to the proof of Proposition 3. If platform i charges a fee in the lower interval—that is, $f_i \in [\underline{f}^l, \bar{f}^l]$ —we obtain an equation analogous to (4), given by

$$G_2(f + \delta)f + (1 - G_2(f + \delta))2f = \eta + 2\delta. \quad (10)$$

This equation determines the mixing probabilities in the upper range.

In case platform i charges a fee in the upper range, that is, $f_i \in [\underline{f}^u, \bar{f}^u]$, the equation is

$$G_2(f - \delta)0 + (1 - G_2(f - \delta))f = \eta + 2\delta. \quad (11)$$

This equation determines the mixing probability in the lower range.

Let us first look at (10). Substituting $h \equiv f + \delta$ and solving for $G_2(h)$ gives

$$G_2(h) = \frac{2h - 4\delta - \eta}{h - \delta}.$$

It is easy to check that $\lim_{f \searrow \underline{f}^u} G_2(f) = \lim_{f \searrow \eta + 2\delta} G_2(f) = \eta/(\eta + \delta)$. Moreover, $\lim_{f \nearrow \bar{f}^u} G_2(f) = \lim_{f \nearrow 2\eta + 2\delta} G_2(f) = 3\eta/(2\eta + \delta) < 1$. The latter implies the existence of a mass point with

mass $1 - 3\eta/(2\eta + \delta) = (\delta - \eta)/(2\eta + \delta)$ at a fee equal to $\bar{f}^u = 2(\eta + \delta)$.³⁸

Consider (11). We substitute $h \equiv f - \delta$. Thus, h is now in the lower range. Solving for $G_2(h)$ gives

$$G_2(h) = \frac{h - \eta - \delta}{h + \delta}.$$

It is easy to check that $G_2(\underline{f}^l) = G_2(\eta + \delta) = 0$, whereas $\lim_{f \nearrow \bar{f}^l} G_2(f) = \lim_{f \nearrow 2\eta + \delta} G_2(f) = \eta/(2(\eta + \delta))$. Note that $\lim_{f \nearrow \bar{f}^l} G_2(f) = \eta/(2(\eta + \delta)) < \eta/(\eta + \delta) = \lim_{f \searrow \underline{f}^u} G_2(f)$, which implies the existence of a second mass point with mass $\eta/(2(\eta + \delta))$ at a fee equal to $\underline{f}^u = \eta + 2\delta$.

The resulting mixing probability is characterized by a cumulative distribution function of

$$G_2(f) = \begin{cases} \frac{f - \eta - \delta}{f + \delta}, & \text{if } f \in [\eta + \delta, 2\eta + \delta); \\ \frac{2f - \eta - 4\delta}{f - \delta}, & \text{if } f \in [\eta + 2\delta, 2(\eta + \delta)); \\ 1, & \text{if } f = 2(\eta + \delta). \end{cases}$$

The corresponding generalized density is given by

$$g_2(f) = G'_2(f) + \frac{\eta}{2(\eta + \delta)} \delta^D(f - (\eta + 2\delta)) + \frac{\delta - \eta}{2\eta + \delta} \delta^D(f - (2(\eta + \delta))),$$

where

$$G'_2(f) = \begin{cases} \frac{\eta + 2\delta}{(f + \delta)^2}, & \text{if } f \in [\eta + \delta, 2\eta + \delta); \\ \frac{\eta + 2\delta}{(f - \delta)^2}, & \text{if } f \in [\eta + 2\delta, 2(\eta + \delta)), \end{cases}$$

and $\delta^D(f - f_0)$ denotes the Dirac delta. Replacing η and δ by their respective definitions yields the result stated in the proposition. \square

Proof of Proposition 5. In stage 4, both multi-homing and single-homing buyers make their optimal buying decisions, given the prices charged by sellers in the third stage. In the third stage, the pricing equilibrium in the product market may be different than with single-homing buyers. If both sellers in a category are on the same platform, they will still charge a price of p^d in equilibrium and obtain a profit of π^d per buyer. Similarly, if only one seller is active, this seller sets its price equal to p^m and earns π^m per buyer. However, if sellers segment and one is active on platform A and the other one on platform B , sellers no longer charge p^m . As explained in the main text, the reason is that a fraction λ of buyers

³⁸Intuitively, in order to satisfy (10), there must be a positive probability of inducing an agglomeration equilibrium and receiving $2f$ in the lower range even for $f = \bar{f}^l$. This is achieved by a mass point at $h = \bar{f}^u$.

(i.e., the multi-homers) are informed about both offers. Therefore, the price charged by a seller depends on how many buyers are informed about both offers. We denote the price charged by a seller in this situation by $p(\lambda)$, with $p(\lambda) \in [\min\{p^d, p^m\}, \max\{p^d, p^m\}]$, and the respective per-buyer profit by $\pi(\lambda)$.

Turning to the second stage, we know that profits in an agglomeration and a stand-alone equilibrium are unchanged. This is not true for the segmentation equilibrium. If sellers segment, the total number of buyers for each seller is $(1 + \lambda)/2$. The profit of a seller active on platform i is then $\pi(\lambda)(1 + \lambda)/2 - f_i$. If the seller deviates and becomes active on platform $-i$, it obtains a profit of $\pi^d(1 + \lambda)/2 - f_{-i}$. It follows that there is no deviation incentive if

$$f_i \leq \min \left\{ (\pi(\lambda) - \pi^d) \frac{(1 + \lambda)}{2} + f_{-i}, \pi(\lambda) \frac{(1 + \lambda)}{2} \right\}.$$

In contrast to the case with single-homing buyers where the relevant condition was given by (1) in Web Appendix D, the buyer mass $1/2$ is now replaced by $(1 + \lambda)/2$ and the monopoly profit π^m is replaced by $\pi(\lambda)$. Proceeding in the same way as in Web Appendix D, we obtain that in the second stage there is unique equilibrium and the conditions for the agglomeration, the segmentation, and stand-alone equilibrium to occur are still the same as given there, with $\pi^m/2$ replaced by $\pi(\lambda)(1 + \lambda)/2$.

We can now move to the first stage. Following the same argument as in the proof of Proposition 1, we obtain that in the range $\pi^d \geq \pi(\lambda)(1 + \lambda)/2$ an agglomeration equilibrium with fees $f_i = f_{-i} = 0$ is the unique equilibrium. Similarly, if both platforms charge a fee of $\pi(\lambda)(1 + \lambda)/2$, the only equilibrium is that sellers segment, and a platform's profit equals $\pi(\lambda)(1 + \lambda)/4$. A platform has no incentive to deviate from this fee combination, if

$$\pi^d \leq \pi(\lambda) \frac{1 + \lambda}{4}.$$

Hence, in this range, the unique equilibrium involves $f_i = f_{-i} = \pi(\lambda)(1 + \lambda)/2$ and a segmentation equilibrium occurs.

It is evident that the regions are the same as in case where $\lambda = 0$ with the difference that $\pi^m/2$ is replaced by $\pi(\lambda)(1 + \lambda)/2$. The same logic applies for the region

$$\pi(\lambda) \frac{1 + \lambda}{2} > \pi^d > \pi(\lambda) \frac{1 + \lambda}{4}.$$

By following the same steps as in the proofs of Propositions 1 through 4, we obtain the same results as in those propositions. \square

Proof of Proposition 6. From Proposition 5, with multi-homing consumers, platform profits strictly increase in the share of multi-homing consumers λ if and only if $\partial (\pi(\lambda)(1 + \lambda)/2)$

$/\partial\lambda > 0$, where $\pi(\lambda)$ is given by

$$(p(\lambda) - c) \left(\frac{1 - \lambda}{1 + \lambda} D^m(p(\lambda)) + \frac{2\lambda}{1 + \lambda} D^d(\mathbf{p}(\lambda)) \right).$$

We start with the case in which λ is close to 0. Taking the derivative of $\pi(\lambda) (1 + \lambda) / 2$ with respect to λ and letting $\lambda \rightarrow 0$, we obtain, using the Envelope Theorem,

$$\frac{p(0) - c}{2} [2D^d(\mathbf{p}(0)) - D^m(p(0))]. \quad (12)$$

As $p(0) > c$, the sign of (12) depends on the sign of $2D^d(\mathbf{p}(0)) - D^m(p(0))$. If products become homogeneous, in duopoly each firm sells half of the monopoly quantity, given that the price is unchanged. In this case, $2D^d(\mathbf{p}(0)) - D^m(p(0)) \rightarrow 0$. In addition, if there is no extensive demand margin, for example, because the market is fully covered in monopoly at price $p(0)$, then $2D^d(\mathbf{p}(0)) - D^m(p(0)) = 0$. However, if products are differentiated and an extensive margin exists, $2D^d(\mathbf{p}(0)) - D^m(p(0)) > 0$, which implies that platform profits increase.

We now turn to the case $\lambda \rightarrow 1$. Then, $\pi(\lambda) ((1 + \lambda)/2) \rightarrow \pi^d$. As the segmentation equilibrium emerges with positive probability only if $\pi(\lambda) ((1 + \lambda)/2) > \pi^d$, it never occurs at $\lambda = 1$. As a consequence, agglomeration occurs, which implies that platforms obtain zero profits. Therefore, their profits are strictly lower than in the situation in which segmentation occurs with positive probability. By continuity of the model, the result also holds true in the vicinity of $\lambda = 1$. \square

Proof of Proposition 7. Stage 4 works in the same way as without multi-homing: given sellers' prices, buyers make their optimal purchasing decisions. In stage 3, due to the possibility of multi-homing, new competition situations can occur. As mentioned in the main text, these are that, in a category, either both sellers multi-home or that only one seller multi-homes whereas the other single-homes on platform i . In the former situation, regardless of the distribution of buyers, all buyers are informed about both sellers' offers. It follows that sellers in the third stage will set a price of p^d , leading to a profit of $\pi^d - f_A - f_B$ for both sellers. In the latter situation, sellers compete in an asymmetric way, as the multi-homing seller reaches all buyers, whereas the single-homing seller reaches only buyers on platform i . If platform i is host to $x \in (0, 1)$ buyers, we denote the prices set by the sellers in the third-stage equilibrium by $p^{SH}(x)$ for the single-homing seller and by $p^{MH}(x)$ for the multi-homing seller. The per-buyer profits are $\pi^{SH}(x)$ and $\pi^{MH}(x)$, respectively, which implies that the profits of the two sellers are $x\pi^{SH}(x) - f_i$ and $\pi^{MH}(x) - f_A - f_B$.³⁹

³⁹The per-buyer profit of the multi-homing seller, $\pi^{MH}(x)$, is a weighted average of the profit obtained

We turn to the second stage. We first determine the conditions under which the different equilibrium configurations determined in the game with single-homing are still Nash equilibria with multi-homing. First, as before, agglomeration on platform i is an equilibrium if $f_i \leq \pi^d$. As there is no buyer on the other platform, the possibility to multi-home does not change the outcome. The same holds true for the stand-alone equilibrium on platform i , which is a Nash equilibrium whenever $\pi^d < f_i \leq \pi^m$. Turning to the segmentation equilibrium, in addition to the deviations considered in Web Appendix D, a seller can now also choose to multi-home. This is not profitable if and only if $\pi^m/2 - f_i \geq \pi^{MH} - f_i - f_{-i}$, where $\pi^{MH} \equiv \pi^{MH}(1/2)$. Therefore, the conditions under which a segmentation equilibrium exists are more demanding than in the case of single-homing; they are given by

$$f_i \leq \min \left\{ \frac{\pi^m - \pi^d}{2} + f_{-i}, \frac{\pi^m}{2} \right\} \quad \text{and} \quad f_{-i} \geq \pi^{MH} - \frac{\pi^m}{2}. \quad (13)$$

In addition to these equilibria which involve single-homing of sellers, there can also be equilibria which involve multi-homing along the equilibrium path. One is a full multi-homing equilibrium in which both sellers multi-home and buyers split evenly on platforms. This is a Nash equilibrium if no seller has an incentive to deviate to single-homing—that is, $\pi^d - f_i - f_{-i} \geq \pi^{SH}/2 - f_i$ and, thus, in equilibrium, $f_{-i} \leq \pi^d - \pi^{SH}/2$.

In addition, there can be a partial multi-homing equilibrium with the following structure: in each category, one seller multi-homes and the other one single-homes. A single-homing seller is active on platform A in half of the categories and on platform B in the other half of the categories. Buyers are indifferent, as each platform has, in expectation, the same number of sellers in the buyers' preferred category and, therefore, buyers will split evenly. A multi-homing seller's profit is $\pi^{MH} - f_A - f_B$ and the profit of a seller single-homing on platform i is $\pi^{SH}/2 - f_i$.

We determine the conditions under which this configuration is a Nash equilibrium. First, any single-homing seller must earn non-negative profits—that is, $\pi^{SH}/2 - f_i \geq 0$. Second, it must be optimal for any such sellers to single-home on platform i instead of single-homing on platform $-i$ —that is, $\pi^{SH}/2 - f_i \geq \pi^{SH}/2 - f_{-i}$. Third, single-homing must be better than multi-homing for this seller—that is, $\pi^{SH}/2 - f_i \geq \pi^{MH} - f_i - f_{-i}$. Moreover, the multi-homing seller must be better off with multi-homing than with single-homing on platform i or $-i$. These conditions are satisfied if $\pi^{MH} - f_i - f_{-i} \geq \pi^d/2 - f_i$ and $\pi^{MH} - f_i - f_{-i} \geq \pi^m/2 - f_{-i}$.

The same conditions must also hold with f_i and f_{-i} interchanged because, in the partial multi-homing equilibrium, one half of the single-homing sellers are on platform A

from buyers on platform i who are informed about both offers and the profit from buyers on platform $-i$ who only observe the multi-homing seller's offer.

and the other half on platform B . Importantly, this implies that $\pi^{SH}/2 - f_{-i} \geq \pi^{SH}/2 - f_i$. Taken together with the condition $\pi^{SH}/2 - f_i \geq \pi^{SH}/2 - f_{-i}$ (which has been derived above), this shows that a partial multi-homing equilibrium can only exist if $f_i = f_{-i}$.⁴⁰ Using $f_i = f_{-i}$ together with all other conditions derived above, we obtain partial multi-homing is a Nash equilibrium if and only if

$$f_i = f_{-i}, \quad f_i \leq \frac{\pi^{SH}}{2}, \quad \text{and} \quad \pi^d - \frac{\pi^{SH}}{2} \leq f_i \leq \pi^{MH} - \frac{\pi^m}{2}. \quad (14)$$

As follows from (13) and (14), the partial multi-homing equilibrium and the segmentation equilibrium co-exist if and only if $f_i = f_{-i} = \pi^{MH} - \pi^m/2$.

Next, we apply our refinement. First, it is easy to see that the full multi-homing equilibrium is never coalition-proof. Take the coalition of all sellers and all buyers on platform i , and consider a deviation in which all buyers go to platform $-i$ and sellers single-home on platform $-i$. Then buyers get the same utility as with full multi-homing but sellers are better off as they receive a profit of $\pi^d - f_{-i} > \pi^d - f_i - f_{-i}$. Therefore, full multi-homing never survives our refinement.

Turning to the partial multi-homing equilibrium, we determine the conditions for coalition-proofness of this equilibrium. First, single-homing sellers on platform i can form a coalition with buyers on platform i and deviate to be active only on platform $-i$. Buyers are then better off, as they are informed about all offers on platform $-i$ and sellers compete in all categories whereas sellers are only better off if $\pi^d - f_{-i} > \pi^{SH}/2 - f_i$. As the partial multi-homing Nash equilibrium only exists for $f_i = f_{-i}$, we obtain that such a deviation is not profitable if $\pi^{SH}/2 \geq \pi^d$. Second, multi-homing sellers can form a coalition with all buyers on the platform where the sellers are monopolists (platform i , say) and single-home on platform $-i$. Buyers are better off, as all sellers compete on platform $-i$, whereas the originally multi-homing sellers are better off if and only if $\pi^d - f_{-i} > \pi^{MH} - f_i - f_{-i}$. Therefore, this deviation is not profitable if $f_i \leq \pi^{MH} - \pi^d$. (The other deviations by sellers do not involve coalitions and, therefore, are already captured by the conditions for the Nash equilibrium to exist.) We now combine these conditions with the ones derived in (14). Since coalition-proofness requires $\pi^{SH}/2 \geq \pi^d$, the lower bound on f_i derived in (14) would be weakly negative and, thus, can be replaced by zero. In addition, from (14), the upper bound on f_i is $\min\{\pi^{SH}/2, \pi^{MH} - \pi^m/2\}$. To sum up, the partial multi-homing equilibrium is coalition-proof only if

$$f_i = f_{-i}, \quad \frac{\pi^{SH}}{2} \geq \pi^d, \quad \text{and} \quad f_i \leq \min\left\{\frac{\pi^{SH}}{2}, \pi^{MH} - \frac{\pi^m}{2}\right\}. \quad (15)$$

⁴⁰Therefore, in the f_A - f_B -diagram presented in Figure 2, the partial multi-homing equilibrium can only exist on the 45-degree line.

Next, we determine whether the partial multi-homing equilibrium is in fact selected in the second stage, given that other equilibrium configurations are coalition-proof as well. From above, we know that it exists together with the segmentation equilibrium if and only if $f_i = f_{-i} = \pi^{MH} - \pi^m/2$. The segmentation equilibrium is then also coalition-proof and preferred by sellers over the agglomeration equilibrium if $\pi^m/2 > \pi^d$ (see Web Appendix D). The profit of a seller in the segmentation equilibrium is $\pi^m/2 - f_i = \pi^m - \pi^{MH}$. Instead, in the partial multi-homing equilibrium, a single-homing seller's profit is $\pi^{SH}/2 - f_i = (\pi^{SH} + \pi^m)/2 - \pi^{MH}$, which is strictly below the one in the segmentation equilibrium. A multi-homing seller's profit is $\pi^{MH} - 2f_i = \pi^m - \pi^{MH}$ and, therefore, the same as in the segmentation equilibrium. It follows that the segmentation equilibrium profit-dominates the partial multi-homing equilibrium. Hence, if $\pi^m/2 > \pi^d$, the partial-multi-homing equilibrium is selected in stage 2 only if (15) holds, with the strengthening of the last condition to $f_i \leq \pi^{SH}/2$ and $f_i < \pi^{MH} - \pi^m/2$.

If instead $\pi^m/2 \leq \pi^d$, the partial-multi-homing equilibrium may co-exist with the agglomeration equilibrium. Because the partial multi-homing equilibrium exists only if $\pi^{SH}/2 \geq \pi^d$ and both fees are the same, the single-homing seller is better off in the partial multi-homing equilibrium. Since $\pi^{MH} \geq \pi^{SH}$, the multi-homing seller is better off as well. Hence, if $\pi^m/2 \leq \pi^d$, the partial multi-homing equilibrium will be chosen in the second stage whenever (15) is fulfilled.

If the partial multi-homing equilibrium does not exist, for $f_i, f_{-i} \geq \pi^{MH} - \pi^m/2$, the same analysis to select an equilibrium as in Web Appendix D applies, as in this case the same equilibria exist as without multi-homing. If instead one or both fees are lower than $\pi^{MH} - \pi^m/2$, it follows from (13) that a segmentation equilibrium does not exist. However, we know from the analysis in Section D (see also Figure 2) that in this region either an agglomeration or a stand-alone equilibrium prevails, depending on parameters. It follows that, off the diagonal, there is a unique equilibrium in the second stage even with seller multi-homing, given our selection criterion.

We turn to the first stage. Let us first consider the case $\pi^d/\pi^m \geq 1/2$. This implies that $\pi^d > \pi^{SH}/2$, as $\pi^m > \pi^{SH}$. Therefore, the partial multi-homing equilibrium does not exist in this case. It follows that the analysis of the proof of Proposition 1 applies, leading to $f_A = f_B = 0$ in equilibrium, and buyers and sellers play an agglomeration equilibrium in the second stage.

Second, consider the case $\pi^d/\pi^m \leq 1/4$. In the pure-strategy segmentation equilibrium of Proposition 2, platforms set $f_A = f_B = \pi^m/2$. As $\pi^m/2 > \pi^{MH} - \pi^m/2$, due to the fact that $\pi^m > \pi^{MH}$, the segmentation equilibrium exists in this case. From the analysis of the second stage, it follows that the partial multi-homing equilibrium does not exist then, and from the proof of Proposition 2 it follows that the pure-strategy segmentation equilibrium is the unique equilibrium in this case. This establishes the first

part of the proposition.

Turning to the range $1/2 > \pi^d/\pi^m > 1/4$, we first consider the situation in which $\pi^d > \pi^{SH}/2$, that is, the partial multi-homing equilibrium does not exist. We know from above that for $\pi^{MH} \leq \pi^m/2$, the segmentation equilibrium exists, which implies that the equilibrium is the same mixed-strategy equilibrium as the one characterized in Propositions 3 and 4. By contrast, for $\pi^{MH} > \pi^m/2$, a segmentation equilibrium does not exist. We will now check under which conditions the possibility to multi-home breaks the mixed-strategy equilibrium of Propositions 3 and 4. This equilibrium exists if the cycle of best responses described in the proofs of these propositions works in the same way if sellers can multi-home. However, this cycle no longer operates if one of the fees in the mixing range is below $\pi^{MH} - \pi^m/2$. The reason is as follows: suppose that platform i sets a fee below $\pi^{MH} - \pi^m/2$. Platform $-i$'s best response in case of single-homing sellers was to set a higher fee to induce segmentation. However, inducing segmentation is no longer possible with multi-homing sellers. As a consequence, the best response of platform $-i$ to a listing fee of f_i below $\pi^{MH} - \pi^m/2$ is to undercut this fee slightly to induce an agglomeration equilibrium on platform $-i$ in the second stage. The lowering of fees then leads to the agglomeration equilibrium with $f_A = f_B = 0$.

It remains to be checked under which conditions the lowest fee in the mixing range is below $\pi^{MH} - \pi^m/2$. Starting with the first mixing region, we obtain that this holds if $\pi^{MH} - \pi^m/2 > \pi^m - 2\pi^d$ or, equivalently,

$$\pi^{MH} > \frac{3\pi^m}{2} - 2\pi^d.$$

If this inequality holds, then π^{MH} is also larger than $\pi^m/2$, implying that any equilibrium features $f_A = f_B = 0$ and agglomeration prevails in the second stage. Instead, if $\pi^{MH} \leq 3\pi^m/2 - 2\pi^d$, the unique equilibrium is the mixed-strategy one, as reported in Proposition 3.

Proceeding in the same way for the second mixing region, we obtain that for

$$\pi^{MH} > \frac{3\pi^m}{4}$$

any equilibrium features $f_A = f_B = 0$ and agglomeration, whereas for $\pi^{MH} \leq 3\pi^m/4$, the unique equilibrium is the mixed-strategy one, as reported in Proposition 4.

Second, we consider the situation $\pi^d \leq \pi^{SH}/2$. The partial multi-homing equilibrium then exists for fees $f_A = f_B$, with $f_i \leq \pi^{SH}/2$ and $f_i < \pi^{MH} - \pi^m/2$, $i \in \{A, B\}$. The profit of each platform is $3/2f_i$. However, each platform then has an incentive to lower its fee slightly. This induces agglomeration (as the segmentation equilibrium does not exist for $f_i < \pi^{MH} - \pi^m/2$). The resulting profit of the platform with the lower fee (platform

i , say) is then $2f_i$. Hence, undercutting is profitable. As a consequence, if a partial multi-homing equilibrium exists in the full game, it can only occur with $f_A = f_B = 0$. But, then, the same mechanism as described for the case $\pi^d > \pi^{SH}/2$ occurs. The mixed-strategy equilibrium is the unique equilibrium in the first stage for the same region as in case $\pi^d > \pi^{SH}/2$. In the other region, in equilibrium, platforms set their fees equal to zero. However, in contrast to the case above, for $\pi^d \leq \pi^{SH}/2$, buyers and sellers in the second stage play the partial multi-homing equilibrium. \square

Proof of Proposition 8. The third and fourth stage play out similarly as in the case with listing fees. In the fourth stage, buyers buy according to their demand functions, that is, a buyer active on platform i either faces a seller price of $p^m(\phi_i)$ or $p^d(\phi_i)$, depending on the number of sellers on platform i , and then buys the respective number of goods $D^m(\phi_i)$ or $D^d(\phi_i)$. In stage 3, a seller on platform i sets $p^m(\phi_i)$ or $p^d(\phi_i)$, depending on the number of rival sellers (either 0 or 1) active on the platform in the seller's product category.

We turn to the second stage. Following the same arguments as in case of listing fees, there potentially exist three types of equilibria with per-transaction fees: agglomeration equilibria, segmentation equilibria, and stand-alone equilibria. In an agglomeration equilibrium on platform i , a seller's profit is $\pi^d(\phi_i)$, whereas in a segmentation equilibrium, the seller's profit is $\pi^m(\phi_i)/2$. In a stand-alone equilibrium on platform i , the profit of an active seller is $\pi^m(\phi_i)$ and the one of an inactive seller is 0. However, since platforms charge per-transaction fees, if ϕ_i is such that $D^m(\phi_i) > 0$ and thereby also $\pi^m(\phi_i) > 0$, also the inactive seller in each category could make a positive profit by becoming active on platform i . The reason is that for $D^m(\phi_i)$ to be positive, ϕ_i must be below the intercept of the demand curve. This implies that also in duopoly sellers will charge prices such that $D^d(\phi_i) > 0$, leading to $\pi^d(\phi_i) > 0$. Since platforms in the first stage will never optimally charge a fee which leads to zero demand for sellers, as this implies zero profits also for platforms, we can restrict attention to those subgames in the second stage in which fees satisfy $D^d(\phi_i) > 0$.⁴¹ For such fees, a stand-alone equilibrium does not exist in the second stage and, thus, will never occur along the equilibrium path of the full game.

Next, we determine the equilibrium that is played in the second stage, given our selection criterion. First, consider the case $\pi^d(0)/\pi^m(0) \geq 1/2$. Due to the assumption $\partial\pi^m(\phi_i)/\partial\phi_i \leq \partial\pi^d(\phi_i)/\partial\phi_i \leq 0$, the condition $\pi^d(0)/\pi^m(0) \geq 1/2$ implies that $\pi^d(\phi_i)/\pi^m(\phi_i) \geq 1/2 \forall \phi_i$. In this case, a segmentation equilibrium cannot exist in the second stage. The reason is that a coalition of all sellers and buyers on the platform with the higher fee have the incentive to deviate to the rival platform. It follows that for

⁴¹If the demand is unbounded (as, for example, with CES demand), implying that there is no demand intercept, this argument holds true independent of the level of the fee.

$\pi^d(0)/\pi^m(0) \geq 1/2$ only an agglomeration equilibrium exists.

Second, suppose that $\pi^d(0)/\pi^m(0) < 1/2$. Then, for $\phi_i, \phi_{-i} > 0$ but small enough, we have $\pi^d(\phi_{-i}) < 1/2\pi^m(\phi_i)$. In this case, the segmentation equilibrium is the unique equilibrium selected by our refinement. To see this, note that in a segmentation equilibrium, sellers on platform i obtain a profit of $1/2\pi^m(\phi_i)$ and those on platform $-i$ a profit of $1/2\pi^m(\phi_{-i})$. A seller active on the platform with the larger fee—for instance, platform i , so that $\phi_i \geq \phi_{-i}$ —has no profitable deviation from this configuration if $1/2\pi^m(\phi_i) \geq 1/2\pi^d(\phi_{-i})$. This implies that for $\pi^m(\phi_i) \geq \pi^d(\phi_{-i})$, segmentation is a Nash equilibrium. In addition, agglomeration is a Nash equilibrium for all fees ϕ_i such that $D^d(\phi_i) > 0$. Therefore, multiple Nash equilibria exist in this range. Applying coalition-proofness, it is evident from the same arguments as in the previous paragraph that the segmentation equilibrium is eliminated if $\pi^d(\phi_{-i}) \geq 1/2\pi^m(\phi_i)$. Thus, if fees are such that $\pi^d(\phi_{-i}) \geq 1/2\pi^m(\phi_i)$, the unique equilibrium selected by our refinement is the agglomeration equilibrium on platform i because the segmentation equilibrium is not coalition-proof.⁴²

Instead, for $\pi^d(\phi_{-i}) < 1/2\pi^m(\phi_i)$, coalition-proofness does not destroy the segmentation equilibrium. Applying, in addition, profit dominance of sellers, selects the segmentation equilibrium as the unique equilibrium. The reason is that a seller's profit in a segmentation equilibrium is at least $1/2\pi^m(\phi_i)$, which is larger than the one in the agglomeration equilibrium, where a seller obtains only $\pi^d(\phi_{-i})$. Since the condition $\pi^d(\phi_{-i}) < 1/2\pi^m(\phi_i)$ is stronger than $\pi^d(\phi_{-i}) \leq \pi^m(\phi_i)$ (i.e., the condition for a segmentation equilibrium to exist), the segmentation equilibrium is selected by our refinement, whenever the condition holds.

Given our refinement, the equilibrium in the second stage is summarized as follows: suppose that $\phi_i \geq \phi_{-i}$. If $\pi^d(0)/\pi^m(0) \geq 1/2$, an agglomeration equilibrium on platform $-i$ occurs. If, by contrast, $\pi^d(0)/\pi^m(0) < 1/2$, the segmentation equilibrium is played for $1/2\pi^m(\phi_i) > \pi^d(\phi_{-i})$ and agglomeration on platform $-i$ occurs for $1/2\pi^m(\phi_i) \leq \pi^d(\phi_{-i})$.

We turn to the first stage. Following the same arguments as in the proof of Proposition 1, it is evident that for $\pi^d(0)/\pi^m(0) \geq 1/2$, the unique equilibrium implies $(\phi_A^*, \phi_B^*) = (0, 0)$, as sellers will coordinate on the platform with the lower per-transaction fee. This establishes the first part of the proposition.

For $\pi^d(0)/\pi^m(0) < 1/2$, we establish next the constraints under which a pure-strategy segmentation equilibrium exists. The highest platform profits that can be obtained in a segmentation equilibrium is reached with fees $\phi_A = \phi_B = \phi^m$. Then, platform i can induce agglomeration only by setting a fee ϕ_i such that $\pi^d(\phi_i) \geq \pi^m(\phi^m)/2$. There-

⁴²Note that because $\partial\pi^m(\phi_i)/\partial\phi_i \leq \partial\pi^d(\phi_i)/\partial\phi_i \leq 0$, we can have $\pi^d(0)/\pi^m(0) < 1/2$ but $\pi^d(\phi_{-i}) \geq 1/2\pi^m(\phi_i)$ if fees are sufficiently high. Then, a segmentation equilibrium is played if fees are close to zero but an agglomeration one for high fees.

fore, if $\pi^d(\phi_i) < \pi^m(\phi^m)/2$ for all ϕ_i , a pure-strategy segmentation equilibrium emerges. In addition, denoting by $\tilde{\phi}$ the largest fee ϕ_i such that $\pi^d(\phi_i) \geq \pi^m(\phi^m)/2$ (as in the proposition), a deviation to $\phi_i = \tilde{\phi}$ so as to induce agglomeration is not profitable if $\phi^m D^m(\phi^m)/2 \geq 2\tilde{\phi}D^d(\tilde{\phi})$, or, equivalently, $1/4 \geq \tilde{\phi}D^d(\tilde{\phi})/(\phi^m D^m(\phi^m))$. This establishes the second part of the proposition, which reports equilibrium transaction fees $(\phi_A^*, \phi_B^*) = (\phi^m, \phi^m)$.

Finally, by the same arguments as in Section 4, there is no pure-strategy equilibrium in the range such that $\pi^d(0)/\pi^m(0) < 1/2$ and $\phi^m D^m(\phi^m) < 4\tilde{\phi}D^d(\tilde{\phi})$. In this case, the mixed-strategy equilibrium can be obtained in a similar way as in the proofs of Propositions 3 and 4. In particular, there will again be two regions, one in which mixing occurs on a convex set and the other in which mixing occurs on a non-convex set. Let us characterize the mixed-strategy equilibrium in each of those two regions.

In the region in which mixing occurs on a convex set, we denote the upper and the lower bound of the range by $\bar{\phi}$ and $\underline{\phi}$, respectively. A platform must be indifferent between setting $\bar{\phi}$ and $\underline{\phi}$, which leads to

$$2D^d(\underline{\phi})\underline{\phi} = \frac{D^m(\bar{\phi})\bar{\phi}}{2}. \quad (16)$$

In addition, following the same steps as in the proof of Proposition 3, there exists a fee, denoted by $\hat{\phi}$, in the interior of the randomization domain, which induces segmentation with probability (almost) 1. At this fee, sellers are indifferent between agglomeration and segmentation if one platform charges $\hat{\phi}$ and the other $\underline{\phi}$, which yields

$$\pi^d(\underline{\phi}) = \frac{\pi^m(\hat{\phi})}{2}. \quad (17)$$

The same holds if one platform charges $\hat{\phi}$ and the other $\bar{\phi}$, which yields

$$\frac{\pi^m(\bar{\phi})}{2} = \pi^d(\hat{\phi}). \quad (18)$$

The three equations (16), (17), and (18) determine the three fees $\hat{\phi}$, $\bar{\phi}$, and $\underline{\phi}$ and, thus, the mixing range. By our assumption that π^m falls to a larger extent than π^d with an increase in the per-transaction fee and that the same relation holds true for D^m and D^d , the three fees $\hat{\phi}$, $\bar{\phi}$, and $\underline{\phi}$ are uniquely determined.

The best-response function $\phi_i(\phi_j)$ is implicitly defined by

$$\frac{\pi^m(\phi_i)}{2} = \pi^d(\phi_j) \text{ for } \phi_j = [\underline{\phi}, \hat{\phi}],$$

and

$$\pi^d(\phi_i) = \frac{\pi^m(\phi_j)}{2} \text{ for } \phi_j = (\hat{\phi}, \bar{\phi}).$$

Using these best responses and determining expected profits, we derive the mixing probabilities. We obtain that, in equilibrium,

$$G(\phi) = \frac{\xi(\phi)D^m(\xi(\phi)) - \hat{\phi}D^m(\hat{\phi})}{\xi(\phi)D^m(\xi(\phi))} \quad \text{if } \phi = [\underline{\phi}, \hat{\phi}], \quad (19)$$

with $\xi(\phi) \equiv (\pi^m)^{-1}(2\pi^d(\phi))$, and

$$G(\phi) = \frac{4\psi(\phi)D^d(\psi(\phi)) - \hat{\phi}D^m(\hat{\phi})}{4\psi(\phi)D^d(\psi(\phi)) - \psi(\phi)D^m(\psi(\phi))} \quad \text{if } \phi = (\hat{\phi}, \bar{\phi}), \quad (20)$$

with $\psi(\phi) \equiv (\pi^d)^{-1}(\pi^m(\phi)/2)$. The mixing probabilities given by (19) and (20), together with the equations determining $\hat{\phi}$, $\bar{\phi}$, and $\underline{\phi}$ characterize the mixed-strategy equilibrium, which exists if $\bar{\phi} \leq \phi^m$.

To see that $G(\underline{\phi}) = 0$, note that, from (17), we can write $\xi(\underline{\phi}) = \hat{\phi}$. Inserting this into (19) yields $G(\underline{\phi}) = 0$. Similarly, from (18), we can deduce that $\psi(\bar{\phi}) = \hat{\phi}$. Inserting this into (20) yields $G(\bar{\phi}) = 1$. To show that there is a mass point at $\phi = \hat{\phi}$, we can use (19) and (20) to get

$$\lim_{\phi \nearrow \hat{\phi}} G(\phi) = \frac{\bar{\phi}D^m(\bar{\phi}) - \hat{\phi}D^m(\hat{\phi})}{\bar{\phi}D^m(\bar{\phi})}$$

and

$$\lim_{\phi \searrow \hat{\phi}} G(\phi) = \frac{4\underline{\phi}D^d(\underline{\phi}) - \hat{\phi}D^m(\hat{\phi})}{4\underline{\phi}D^d(\underline{\phi}) - \underline{\phi}D^m(\underline{\phi})}$$

Using (16), which implies that $\bar{\phi}D^m(\bar{\phi}) = 4\underline{\phi}D^d(\underline{\phi})$, it is evident that the numerator of the right-hand side of the previous two equations is the same. Comparing the denominators, we obtain $\bar{\phi}D^m(\bar{\phi}) = 4\underline{\phi}D^d(\underline{\phi}) > 4\underline{\phi}D^d(\underline{\phi}) - \underline{\phi}D^m(\underline{\phi})$. Therefore, the denominator of $\lim_{\phi \nearrow \hat{\phi}} G(\phi)$ is larger than the one of $\lim_{\phi \searrow \hat{\phi}} G(\phi)$, which yields $\lim_{\phi \nearrow \hat{\phi}} G(\phi) < \lim_{\phi \searrow \hat{\phi}} G(\phi)$. Hence, there is a mass point at $\phi = \hat{\phi}$.

If $\bar{\phi} > \phi^m$, this equilibrium cannot exist, as a platform will never find it optimal to set a higher per-transaction fee than ϕ^m . In this region, we obtain an equilibrium with a non-convex randomization domain. Following the proof of Proposition 4, the lower interval is given by $[\phi', \tilde{\phi}]$, where ϕ' is implicitly defined by $2D^d(\phi')\phi' = D^m(\phi^m)\phi^m/2$, and $\tilde{\phi}$ is defined as in the proposition. The upper interval is $[\phi'', \phi^m]$, where ϕ'' is implicitly defined by $\pi^d(\phi') = \pi^m(\phi'')/2$.⁴³

The existence of mass points at $\phi = \phi''$ and $\phi = \phi^m$ can be shown as above. The

⁴³Due to our assumptions on the shape of the profit and demand functions, all boundaries are unique.

mixing probabilities can be derived in the same way as in the case with a convex set. They are given by

$$\tilde{G}(\phi) = \frac{\xi(\phi)D^m(\xi(\phi)) - \phi''D^m(\phi'')}{\xi(\phi)D^m(\xi(\phi))} \quad \text{if } \phi = [\phi', \tilde{\phi}],$$

$$\tilde{G}(\phi) = \frac{4\psi(\phi)D^d(\psi(\phi)) - \phi''D^m(\phi'')}{4\psi(\phi)D^d(\psi(\phi)) - \psi(\phi)D^m(\psi(\phi))} \quad \text{if } \phi = [\phi'', \phi^m],$$

and

$$\tilde{G}(\phi) = 1 \quad \text{if } \phi = \phi^m.$$

□

Proof of Proposition 9. Consider first the case that $\underline{v} = \bar{v}$. In a (pure-strategy) segmentation equilibrium, sellers in all categories will be active. To extract the maximal revenue from each seller, the platform maximizes its profits by avoiding to charge a fee ϕ_i , as a positive per-transaction fee would reduce the surplus to be shared between seller and platform and the platform can extract the full surplus through the listing fee.

Consider now the case that $\underline{v} < \bar{v}$. We first note that in a (pure-strategy) segmentation equilibrium with positive listing fees, sellers in all categories will be active. Suppose that sellers from a positive mass of categories are inactive. Then, there exists a category \hat{v} in the interior of the distribution, i.e. $\hat{v} \in (\underline{v}, \bar{v})$, such that the sellers in this category are indifferent between participating and not participating, given the fees charged by both platforms. Hence, one seller is active on platform i and the other seller on platform j in all categories $v \in [\hat{v}, \bar{v}]$, but both sellers are inactive in categories $v \in [\underline{v}, \hat{v})$. Then, in the second stage, the coalition of all buyers on platform j and one seller in categories just below \hat{v} could go to platform i . Buyers are then better off as they can interact with sellers from more categories. Also the sellers of the coalition are better off. The reason is that in the segmentation configuration in which they were inactive, their profit from the interaction with the buyers was just not large enough to cover the listing fee of platform i . After the deviation of the coalition, they face a mass 1 of buyers instead of 1/2, which implies that forming the coalition is strictly better for them. Because this argument holds for all cut-off values $\hat{v} > \underline{v}$, in a segmentation equilibrium with positive listing fees there must be full seller participation.

Noting that the price of a seller active on platform i in a segmentation equilibrium is $p^m(\phi_i, r_i)$, platform i 's profit (omitting the argument of p^m) can be written as

$$\int_{\underline{v}}^{\bar{v}} \left\{ \frac{1}{2}vD(p^m)[p^m r_i + \phi_i] + f_i \right\} h(v)dv.$$

By the same arguments as in the baseline model, in the (pure-strategy) segmentation

equilibrium, platforms set fees such that the profit of sellers in \underline{v} are fully extracted. This implies

$$\frac{1}{2}\underline{v}D(p^m) [p^m(1 - r_i) - \phi_i - c] - f_i = 0. \quad (21)$$

For all sellers with $v > \underline{v}$, being active on platform i is beneficial, which implies

$$\frac{1}{2}vD(p^m) [p^m(1 - r_i) - \phi_i - c] - f_i \geq 0 \quad \forall v \geq \underline{v}. \quad (22)$$

From (22), the first-order condition for p^m is given by

$$D(p^m)(1 - r_i) + D'(p^m) [p^m(1 - r_i) - \phi_i - c] = 0. \quad (23)$$

We will now show that for any fee combination (f_i, ϕ_i, r_i) with $f_i > 0$, $\phi_i > 0$, and $r_i > 0$, there exists another combination denoted by (f'_i, ϕ'_i, r'_i) with $f'_i > 0$, $\phi'_i = 0$, and $r'_i > 0$, such that the platform obtains a strictly higher profit with the fee combination (f'_i, ϕ'_i, r'_i) than with (f_i, ϕ_i, r_i) .⁴⁴ To do so, we consider a change in the fees that leaves p^m unchanged and ensures that the seller in the lowest category is still indifferent between being active or not. We can use (23) to show how ϕ_i and r_i must be adjusted to leave p^m unchanged. Differentiating (23) with respect to ϕ_i and r_i and rearranging yields

$$\frac{dr_i}{d\phi_i} = -\frac{D'(p^m)}{D(p^m) + D'(p^m)p^m} < 0. \quad (24)$$

The inequality in (24) results from (23), because (23) implies that the denominator of (24) is strictly negative due to the fact that $(D(p^m) + D'(p^m)p^m)(1 - r_i) = D'(p^m)(\phi_i + c) < 0$.

Second, to keep the seller in category \underline{v} indifferent, from (21), we must have that

$$\frac{1}{2}\underline{v}D(p^m) [-p^m dr_i - d\phi_i] - df_i = 0. \quad (25)$$

Dividing (25) by $d\phi_i$ yields

$$\frac{1}{2}\underline{v}D(p^m) \left[-p^m \frac{dr_i}{d\phi_i} - 1 \right] - \frac{df_i}{d\phi_i} = 0.$$

Using (24) and rearranging yields

$$\frac{df_i}{d\phi_i} = -\frac{1/2\underline{v}[D(p^m)]^2}{D(p^m) + D'(p^m)p^m} > 0. \quad (26)$$

This implies that an increase in ϕ_i , which is compensated by a reduction in r_i so that p^m stays unchanged, implies that f_i can be increased.

⁴⁴This holds regardless of the fee combination of platform $-i$.

We can now determine the effect of the change in the fee structure on platform i 's profit. This is given by

$$\int_{\underline{v}}^{\bar{v}} \left\{ \frac{1}{2} v D(p^m) [dr_i p^m + d\phi_i] + df_i \right\} h(v) dv.$$

Considering a marginal reduction in ϕ_i and dividing by $d\phi_i$ yields

$$\int_{\underline{v}}^{\bar{v}} \left\{ \frac{1}{2} v D(p^m) \left[-\frac{dr_i}{d\phi_i} p^m - 1 \right] - \frac{df_i}{d\phi_i} \right\} h(v) dv.$$

Inserting the values from (24) and (26), and rearranging yields

$$-\frac{1/2 [D(p^m)]^2}{D(p^m) + D'(p^m) p^m} \int_{\underline{v}}^{\bar{v}} \{v - \underline{v}\} h(v) dv > 0,$$

where the inequality comes from the fact that all categories have a value v larger than \underline{v} . Since this holds for any value of ϕ_i , the profit-maximizing ϕ_i equals zero. \square

Proof of Proposition 10. From Proposition 9, we know that in the segmentation equilibrium $\phi_i = 0$ for $i \in \{A, B\}$ and sellers in all categories will participate. As a consequence, platform i 's profit is given by

$$\int_{\underline{v}}^{\bar{v}} \left\{ \frac{1}{2} v D(p^m(r_i)) p^m(r_i) r_i + f_i \right\} h(v) dv, \quad (27)$$

with

$$f_i = \frac{1}{2} \underline{v} D(p^m(r_i)) [p^m(r_i)(1 - r_i) - c]. \quad (28)$$

Inserting (28) in (27) and denoting the expected value of v , which is $\int_{\underline{v}}^{\bar{v}} v h(v) dv$, by $E[v]$, we can write platform i 's profit as

$$\frac{1}{2} \{E[v] D(p^m(r_i)) p^m(r_i) r_i + \underline{v} D(p^m(r_i)) [p^m(r_i)(1 - r_i) - c]\}. \quad (29)$$

The maximization problem of a seller in category v is given by

$$\max_p \frac{1}{2} v D(p) [p(1 - r_i) - c] - f_i.$$

The price that a seller charges, denoted by $p^m(r_i)$ is then determined by the first-order

condition.⁴⁵ Hence, $p^m(r_i)$ is implicitly determined by

$$D'(p^m(r_i)) [p^m(r_i)(1 - r_i) - c] + D(p^m(r_i))(1 - r_i) = 0. \quad (30)$$

Due to the multiplicative interaction between v and the demand, the monopoly price is the same in all categories.

Taking the total derivative of (30) with respect to p^m and r_i to determine how p^m changes in r_i , we obtain

$$\text{sign} \left\{ \frac{dp^m}{dr_i} \right\} = \text{sign} \left\{ - (D'(p^m(r_i))p^m(r_i) + D(p^m(r_i))) \right\}.$$

It is evident that, for any $c > 0$, the first-order condition (30) can only be fulfilled if $D'(p^m(r_i))p^m(r_i) + D(p^m(r_i)) < 0$. This implies that $dp^m/dr_i > 0$, that is, if the platform demands a higher revenue share, sellers will respond with higher product prices.

The profit-maximizing revenue share is determined as follows. Maximizing platform i 's profit by taking the derivative of (29) with respect to r_i and using the Envelope Theorem yields (omitting the argument of p^m)

$$(E[v] - \underline{v}) D(p^m)p^m + E[v]r_i \frac{dp^m}{dr_i} (D'(p^m)p^m + D(p^m)) = 0. \quad (31)$$

The second-order condition is given by

$$(D'(p^m)p^m + D(p^m)) \left((2E[v] - \underline{v}) \frac{dp^m}{dr_i} + E[v]r_i \frac{d^2p^m}{d(r_i)^2} \right) + E[v]r_i \left(\frac{dp^m}{dr_i} \right)^2 (2D'(p^m) + D''(p^m)p^m). \quad (32)$$

It is possible to check that $d^2p^m/d(r_i)^2$ is positive if $D'''(p^m)$ is positive or not highly negative, which is fulfilled because of Assumption (iii). Moreover, from Assumption (ii), $2D'(p^m) + D''(p^m)p^m < 0$ and from (30) we know that $D'(p^m)p^m + D(p^m) < 0$. Taken this together yields that (32) is strictly negative. Therefore, the profit-maximizing value of r_i is implicitly defined by (31).

We are now in a position to determine r_i and f_i in the two special cases $\underline{v} = \bar{v}$ and $c = 0$. We start with the former. If $\underline{v} = \bar{v}$, then $E[v] = \underline{v}$. This implies that the first term of the left-hand side of (31) equals zero. Therefore, as the profit function is concave, at the optimal r_i , the second term of the left-hand side must also be zero. Because $E[v] > 0$, $dp^m/dr_i > 0$, and $D'(p^m)p^m + D(p^m) < 0$, this can only hold if $r_i = 0$. Using this in (28), we obtain that the profit-maximizing listing fee is $f_i = \underline{v}/2D(p^m(0)) [p^m(0) - c]$.

We turn to the case $c = 0$. From (30), the first-order condition of a seller can then be

⁴⁵Due to Assumption (ii), the second-order condition is satisfied.

written as $(1 - r_i)(D'(p^m)p^m + D(p^m)) = 0$. This implies that p^m is implicitly defined by the equation $D'(p^m)p^m + D(p^m) = 0$ and is therefore independent of r_i . Platform i can then ensure that a seller in each category is active and extract the entire seller surplus by setting $r_i = 1$ and $f_i = 0$. As p^m is not distorted by this fee combination, the platform induces sellers to maximize industry profits and appropriates it entirely. Therefore, this fee combination constitutes the equilibrium in a (pure-strategy) segmentation equilibrium.⁴⁶ \square

Proof of Proposition 11. We start with case (i). From the proof of Proposition 10, we know that the optimal r_i is given by

$$(E[v] - \underline{v}) D(p^m)p^m + E[v]r_i \frac{dp^m}{dr_i} (D'(p^m)p^m + D(p^m)) = 0. \quad (33)$$

Considering a mean-preserving spread in which the support of the distribution changes implies that $E[v]$ stays constant but \underline{v} falls and \bar{v} rises. Therefore, (33) is affected by such a mean-preserving spread only via the change in \underline{v} . As the derivative of (33) with respect to r_i is negative due to the second-order condition, we obtain that

$$\text{sign} \left\{ \frac{dr_i}{d\underline{v}} \right\} = \text{sign} \{ -D(p^m)p^m \} < 0.$$

Hence, r_i increases with a mean-preserving spread of v that changes the support of the distribution.

Turning to the change in f_i , we also know from the proof of Proposition 10 that, in equilibrium, a seller in category \underline{v} obtains a profit of zero, which implies that

$$f_i = \frac{1}{2} \underline{v} D(p^m(r_i)) [p^m(r_i)(1 - r_i) - c] \quad (34)$$

and that p^m is implicitly defined by

$$D'(p^m(r_i)) [p^m(r_i)(1 - r_i) - c] + D(p^m(r_i))(1 - r_i) = 0. \quad (35)$$

Taking the total differential of (34) with respect to f_i and \underline{v} and using the fact that, due to (35), we can ignore terms involving dp^m/dr_i , yields

$$\frac{df_i}{d\underline{v}} = \frac{1}{2} D(p^m(r_i)) [p^m(r_i)(1 - r_i) - c] - \frac{\underline{v} D(p^m)p^m}{2} \frac{dr_i}{d\underline{v}} > 0.$$

⁴⁶This can also be seen from (31). Because $dp^m/dr_i = 0$, the second term of the left-hand side equals zero. Instead, the first term is strictly positive, which implies that r_i will be set at the highest possible level, which still ensures that sellers in all categories are active (i.e., $r_i = 1$ and $f_i = 0$).

Hence, f_i falls with a mean preserving spread in the distribution of v that lowers \underline{v} .

We turn to a change in c . Proceeding in the same as above, we can differentiate (33) with respect to c . Again, using the second-order condition for r_i , we obtain (dropping arguments of the demand function)

$$\text{sign} \left\{ \frac{dr_i}{dc} \right\} = \text{sign} \left\{ (E[v] - \underline{v}) (D'p^m + D) \frac{dp^m}{dc} + E[v]r_i \left[(2D' + D''p^m) \frac{dp^m}{dr_i} \frac{dp^m}{dc} + \frac{d^2p^m}{dr_i dc} (D'p^m + D) \right] \right\}.$$

The first term in curly brackets is negative because $E[v] - \underline{v} > 0$, $\text{sign} \{dp^m/dc\} = \text{sign} \{-D'\} > 0$ by (35), and $D'p^m + D < 0$, where the last inequality comes from the first-order condition of a seller given by (30). The first term in square brackets is negative as well because $dp^m/dr_i > 0$ (as shown in the proof of Proposition 10), $dp^m/dc > 0$, and $2D' + D''p^m < 0$ due to Assumption (ii). Finally, one can show that, if D''' is positive or not highly negative, $d^2p^m/(dr_i dc)$ is positive. This implies that also the second term in square brackets is negative as $D'p^m + D < 0$. It follows that $dr_i/dc < 0$.

Finally, we determine how f_i changes with c . From (34), we obtain⁴⁷

$$\frac{df_i}{dc} = -\frac{\underline{v}D(p^m)}{2} \left(1 + p^m \frac{dr_i}{dc} \right). \quad (36)$$

We can determine the value of dr_i/dc from the second-order condition of the platform's maximization problem, given in the proof of Proposition 10 and from the expression in $\text{sign} \{dr_i/dc\}$, provided above in this proposition. This yields

$$\frac{dr_i}{dc} = -\frac{(E[v] - \underline{v}) (D'p^m + D) \frac{dp^m}{dc} + E[v]r_i \left[(2D' + D''p^m) \frac{dp^m}{dr_i} \frac{dp^m}{dc} + \frac{d^2p^m}{dr_i dc} (D'p^m + D) \right]}{(D'p^m + D) \left(2E[v] - \underline{v} + E[v]r_i \frac{d^2p^m}{d(r_i)^2} \right) + E[v]r_i \left(\frac{dp^m}{dr_i} \right)^2 (2D' + D''p^m)}. \quad (37)$$

We start with the case $c \rightarrow 0$. In this case, we know from the proof of Proposition 10 that $D'p^m + D = dp^m/dr_i = 0$. This implies that the numerator and the denominator of the right-hand side of (37) are zero. Applying L'Hospital's rule, we obtain that the derivative of the numerator with respect to c is $(E[v] - \underline{v}) (2D' + D''p^m) (dp^m/dc)^2 < 0$, when letting $c \rightarrow 0$. Doing the same for the denominator yields 0. It follows that $dr_i/dc \rightarrow -\infty$ as $c \rightarrow 0$. Therefore, the right-hand side of (36) is positive, which implies $df_i/dc > 0$.

We now turn to the case in which c gets large. As there is a choke price \bar{p} above which demand equals zero and $dp^m/dr_i > 0$, we know that, when c approaches this choke price,

⁴⁷Because of the Envelope Theorem, we can ignore the terms dp^m/dc and $(dp^m/dr_i)(dr_i/dc)$.

r_i will no longer react to c because otherwise a seller's demand would be equal to zero. This implies that $dr_i/dc \rightarrow 0$ as $c \rightarrow \bar{p}$. As a consequence, the right-hand side of (36) becomes negative, which implies that $df_i/dc < 0$. \square

Proof of Proposition 12. Regarding statement (i) of the proposition, we show that platform fees are zero in any partial multi-homing equilibrium. To do so, we first consider all possible configurations in the second stage that involve partial multi-homing of sellers. We then show that only platform fees of zero give rise to any such partial multi-homing equilibrium. To simplify the exposition of the proof, we focus on configurations in which all M sellers per category are active. As we argue below, the same arguments apply to the case in which only a subset of sellers is active.

The first configuration with partial multi-homing is one in which the same number of sellers single-home across all categories. Specifically, in each category, $M - l$ sellers multi-home (with l being an even number), $l/2$ sellers single-home on platform A , and $l/2$ sellers single-home on platform B . As each platform then hosts the same number of sellers, buyers split evenly on both platforms. It is easy to check that there is no other partial multi-homing configuration that can potentially be sustained as an equilibrium in the second stage, given that the same seller allocation obtains in all categories.

In this configuration, the profit of a multi-homing seller is given by

$$\pi^{MH}(l/2, l/2, M - l) - f_A - f_B. \quad (38)$$

Suppose that multi-homing sellers from a positive mass of product categories form a coalition with all buyers on platform $-i$ and the coalition decides to be active only on platform i . That is, each of these sellers withdraws from platform $-i$ and all buyers move to platform i . The resulting profit of a deviant seller is then

$$\pi(M - l/2) - f_i \quad (39)$$

because all buyers are on platform i and the sellers active on this platform are the $M - l$ multi-homing sellers and the $l/2$ single-homing sellers. Therefore, the deviation is not profitable for the seller if (38) is larger than (39) or, equivalently, $\pi^{MH}(l/2, l/2, M - l) - f_{-i} - \pi(M - l/2) \geq 0$.

We show that the buyers of platform $-i$ are indifferent between forming the coalition with deviating sellers joining platform i and staying with the status quo because they face the same prices and have access to the same number of sellers in both situations. To see this, consider first the outcome after the deviation. As there are no buyers on platform $-i$, all sellers on platform i face the same maximization problem, regardless whether they single-home or multi-home. This problem is given by $\max_p (p - c)D(\mathbf{p}(M - l/2))$, where

$\mathbf{p}(M - l/2)$ abbreviates the price vector set by all $M - l/2$ sellers. Now consider the outcome before the deviation. On both platforms, there are $M - l$ multi-homing sellers and $l/2$ single-homing sellers. The maximization problem of a multi-homing seller can therefore be written as $\max_p (p - c) [1/2D(\mathbf{p}(M - l/2)) + 1/2D(\mathbf{p}(M - l/2))]$ because on each platform, a mass $1/2$ of buyers is active, and the demand of buyers is determined by the price vector of the $M - l/2$ active sellers, respectively. Similarly, the maximization problem of a single-homing seller is $\max_p 1/2(p - c)D(\mathbf{p}(M - l/2))$. It is easy to see that the resulting equilibrium prices are the same, and buyers observe the offers of $M - l/2$ sellers with and without the deviation. Therefore, buyers are indifferent.

From the analysis of the preceding paragraph, it follows that the multi-homing seller faces the same number of competitors per platform with and without the deviation and that the equilibrium prices are the same. Therefore, $\pi^{MH}(l/2, l/2, M - l) = \pi(M - l/2)$, which implies that the deviation is profitable for all $f_{-i} > 0$. As a consequence, the postulated partial multi-homing configuration can only be sustained in the subgame following $f_A = f_B = 0$.

Next, we turn to configurations in which the number of single-homing sellers on a platform varies across product categories. Because a partial multi-homing equilibrium can only emerge if buyers are willing to split evenly, this implies that the single-homing sellers list in such a way that the distribution of sellers across categories is the same on both platforms. Only if this holds, a buyer expects to interact with the same number of sellers in her preferred category when deciding which platform to join. This outcome can be achieved in the following way: a number $M - l$ of sellers multi-home, and l sellers single-home (with l being either even or odd). In half of the categories, the l single-homing sellers list on platform A ; in the other half, they list on platform B . This structure follows that of Proposition 7, in which $M = 2$ and $l = 1$. Applying similar arguments as in the proof of Proposition 7, this configuration can only occur if $f_i = f_{-i}$. If $f_i \neq f_{-i}$, the single-homing sellers on the platform with the higher fee can profitably deviate to the rival platform as they face the same competitive conditions but pay a lower listing fee. Suppose now that $f_i = f_{-i} > 0$. Consider a deviation by platform i to reduce its fee slightly. By the argument from above, it then attracts all buyers and sellers, and obtains a profit of Mf_i .⁴⁸ By contrast, when keeping its fee at the same level as the rival platform, its profit is only $(M - l/2)f_i$. Therefore, the deviation is strictly profitable. As this holds for all strictly positive fees, the partial multi-homing configuration can only be sustained with $f_A = f_B = 0$.

Finally, a partial multi-homing configuration can also emerge via a combination of the

⁴⁸In the same way as in the proof of Proposition 7, we can show that in the range in which the partial multi-homing equilibrium exists, sellers choose agglomeration in case one platform sets a lower fee than the other.

two scenarios above. For example, $M-l$ sellers multi-home and l sellers single-home. Out of the l single-homing sellers, in each product category, l' sellers single-home on platform A and l' sellers single-home on B , with $l' < l/2$. The remaining $l-l'$ single-homing sellers list in such a way that, in half of all categories, they are active on platform A and, in the other half, on platform B . A similar argument as that given in the last paragraph can then be applied. A platform can attract some sellers by lowering its fee slightly in case $f_i = f_{-i} > 0$. This argument holds regardless of the exact combination of the two scenarios above. Hence, also in this case the partial multi-homing configuration can only be sustained with $f_A = f_B = 0$.

We focused on cases in which all sellers are active. However, if only a subset of sellers is active, the configurations which ensure partial multi-homing must be of the same structure as those described when all sellers are active. Otherwise, buyers would not be willing to split evenly on the platforms. Then, the same arguments as those described above apply. As a consequence, in an equilibrium with partial multi-homing, platform fees are zero. This proves statement (i) of the proposition.

Turning to statement (ii), we will show that only one of the different partial multi-homing configurations can occur as an equilibrium in the second stage.

We note that, because platform fees are zero in a partial multi-homing equilibrium, all sellers will be active. We start with the configuration in which in each category $M-l$ sellers multi-home, $l/2$ sellers single-home on platform A , and $l/2$ sellers single-home on B . As shown above, the profit of a single-homing seller can then be written as $\pi(M-l/2)/2$. Consider a deviation of one single-homing seller in a positive mass of product categories who is active on platform i . These sellers form a coalition with all buyers and become single-homers on platform $-i$. Buyers are then better off as they are exposed to $M-l/2+1$ offers of sellers in the respective product categories and face lower prices. Instead, sellers are only better off if $\pi(M-l/2+1) > \pi(M-l/2)/2$. Therefore, the deviation is not profitable if

$$\frac{\pi(M-l/2)}{2} \geq \pi(M-l/2+1). \quad (40)$$

We now show that, given condition (40), the combination $f_A = f_B = 0$ is not an equilibrium in the first stage. Consider $f_A = f_B = 0$ and suppose that platform i increases its fee to ϵ , with $\epsilon > 0$ but small. Then, in the second stage, either an agglomeration or a segmentation configuration will emerge. A configuration with multi-homing cannot emerge because, from the arguments above, we know that partial multi-homing only emerges in case both fees are the same, and full multi-homing is dominated for sellers by a configuration in which all buyers and sellers are active on platform $-i$ (i.e., the platform with the lower fee). If an agglomeration configuration emerges after platform

i has increased its fee, all buyers and sellers will be active on platform $-i$ and a seller's profit is $\pi(M)$.

We next look at the segmentation configuration and start with the case in which M is an even number. Then, a seller's profit when being active on platform i is $\pi(M/2)/2 - \epsilon$. It follows that all sellers prefer segmentation over agglomeration if $\pi(M/2)/2 - \epsilon \geq \pi(M)$. In addition, segmentation emerges only if no seller has a profitable deviation from this configuration. The most profitable deviation for a seller is to form a coalition with all buyers and another single seller in each category and move to platform $-i$. Buyers are then strictly better off as they face an additional seller in each category than without the deviation. The deviating seller's profit is $\pi(M/2 + 1)$. Therefore, the deviation is not profitable if

$$\frac{\pi(M/2)}{2} - \epsilon \geq \pi(M/2 + 1). \quad (41)$$

Comparing (40) and (41), it is easy to see that, for ϵ sufficiently small, the former implies the latter under our assumption that the fall in profit due to a larger number of sellers decreases in the number of sellers: the assumption implies that $\pi(M/2) - \pi(M/2 + 1)$ is larger than $\pi(M - l/2) - \pi(M - l/2 + 1)$ for all $l < M$ (i.e., for all cases in which partial multi-homing occurs). As a consequence, if $\pi(M - l/2)/2 > \pi(M - l/2 + 1)$, then $\pi(M/2)/2 > \pi(M/2 + 1)$ and there exists an ϵ in the vicinity of zero such that $\pi(M/2)/2 - \epsilon > \pi(M/2 + 1)$. Therefore, platform i has a profitable deviation to raise its fee slightly, as the platform continues to carry a positive volume of trade and, thus, obtains a strictly positive profit.

We turn to the case in which M is odd. Then, an equilibrium with zero listing fees in the first stage and partial multi-homing exists. If platform i increases its fee slightly, a segmentation configuration cannot occur. The reason is as follows: the way to support a segmentation configuration is to ensure that buyers expect the same number of sellers in each category. With M being odd, this can only work if sellers in different categories make different listing decisions. For example, $(M - 1)/2$ sellers in all categories list on platform A , $(M - 1)/2$ sellers in all categories list on platform B , and the remaining seller in each category lists on platform A in half of the categories and on B in the other half of categories. However, if platform i charges a higher fee than platform $-i$, the remaining sellers active on platform i have an incentive to move to platform $-i$. They face the same number of competitors (i.e., $(M - 1)/2$) but pay a lower listing fee. Therefore, a platform cannot profitably deviate and an equilibrium in which platforms charge zero listing fees exists.

However, if (41) holds, there exists also an equilibrium with strictly positive listing fees. Suppose each platform is host to $(M - 1)/2$ sellers (i.e., one seller is inactive) and both platforms charge the same strictly positive fees. Then, slightly lowering the fee by ϵ is

not profitable for platform i if sellers and buyers will still play a segmentation equilibrium (instead of an agglomeration equilibrium) in the second stage. Indeed, forming a coalition of one seller in each category and all buyers and moving to platform i is not profitable for the sellers if

$$\frac{\pi((M-1)/2)}{2} - \epsilon \geq \pi((M+1)/2). \quad (42)$$

By the same argument as above, (41) implies (42) given the assumption that $\pi(m) - \pi(m+1)$ is falling in m . As (40) implies (41), we obtain that if (40) holds, this deviation is not profitable. As a consequence, an equilibrium with strictly positive fees exists. In addition, this equilibrium profit-dominates that with zero listing fees. Therefore, the equilibrium with zero fees will not be selected by profit-dominance in the first stage. It follows that employing this equilibrium selection criterion destroys the partial multi-homing configuration.

By the same argument, also partial multi-homing configurations comprised of a combination of the two pure scenarios outlined in the first part of the proof cannot emerge as equilibria of the full game. Under the condition that single-homing sellers have no incentive to multi-home, platforms can, also in this case, charge strictly positive fees and induce a segmentation equilibrium in the second stage with positive profits for the platforms.

We exemplify this with the following analysis. Consider the situation in which $M-l$ sellers multi-home, with $l \geq 3$ being an odd number. In half of the categories, there are $(l+1)/2$ single-homers on platform A and $(l-1)/2$ single-homers on platform B , whereas in the other half, it is the other way round. Then, among the l single-homers $l-1$ split even on both platforms, but the remaining single-homer in a category makes a listing decision that depends on the category he is in. Hence, this scenario involves a combination of the two pure scenarios described above. Following the analysis in which single-homing sellers make the same listing decisions in each category, we can derive the condition such that a single-homing seller in each category has no incentive to deviate from the partial multi-homing equilibrium by forming a coalition with all buyers and be active on the other platform. We obtain a condition similar to the one in (40), which is

$$\frac{\pi^{SH}((l+1)/2, (l-1)/2, M-l)}{2} \geq \pi(M-(l+1)/2). \quad (43)$$

Because the profit of a seller is falling in the number of competitors, $\pi^{SH}((l+1)/2, (l-1)/2, M-l) \leq \pi^{SH}((l-1)/2, (l-1)/2, M-l)$. In addition, a configuration with $(l-1)/2$ single-homers on each platform and $M-l$ multi-homers implies that competition on both platforms works in the same way. As shown above in this proof, this yields $\pi^{SH}((l-1)/2, (l-1)/2, M-l) = \pi(M-l+(l-1)/2) = \pi(M-(l+1)/2)$. Therefore,

(43) implies

$$\frac{\pi(M - (l + 1)/2)}{2} \geq \pi(M - (l + 1)/2). \quad (44)$$

In the same way as above, we can now show that (44) implies that platforms have a profitable deviation from setting zero fees in the first stage. To simplify the exposition, we focus on the case with M even.⁴⁹ Suppose again that platform i increases its fee to ϵ . Then, in the second stage, a segmentation configuration emerges if condition (41) holds, that is,

$$\frac{\pi(M/2)}{2} - \epsilon \geq \pi(M/2 + 1). \quad (45)$$

Comparing (44) and (45), we can apply a similar argument as above under the assumption that the fall in profit due to a larger number of sellers decreases in the number of sellers. In particular, for ϵ sufficiently small, the assumption implies that for all $l < M$ (i.e., for all cases with partial multi-homing), if (44) is fulfilled, then (45) is fulfilled as well. Therefore, platform i has a profitable deviation to raise its fee slightly, which destroys the partial multi-homing equilibrium.

We finally consider configurations in which single-homing sellers in different categories make different listing decisions. Recall from above that these configurations emerge if $M - l$ sellers multi-home, l sellers single-home, and among the single-homing sellers, in half of the categories, they list on platform A and, in the other half, they list on platform B . Suppose that $l \geq 2$. The profit of a single-homing seller active on platform i is $\pi_i^{SH}(l, 0, M - l)/2$ because in the category of the seller all single-homing sellers list on platform i . Consider a deviation of this seller to switch from single-homing on platform i to single-homing on platform $-i$. The resulting profit can then be written as $\pi_{-i}^{SH}(l - 1, 1, M - l)/2$. By symmetry of the platforms, this profit is equal to $\pi_i^{SH}(1, l - 1, M - l)/2$. Hence, the deviation is profitable if

$$\frac{\pi_i^{SH}(1, l - 1, M - l)}{2} > \frac{\pi_i^{SH}(l, 0, M - l)}{2},$$

which holds by our assumption made at the outset. As a consequence, a partial multi-homing configuration with $l \geq 2$ can not constitute an equilibrium of the game.

The same argument does not hold true with $l = 1$, as then the deviation profit is $\pi_{-i}^{SH}(0, 1, M - 1)/2$, which is equal to $\pi_i^{SH}(1, 0, M - 1)/2$. The single-homing sellers in different categories could also deviate by forming a coalition with all buyers and be active only on one platform. As buyers then observe the offers of all M sellers, which implies that a seller's profit is $\pi(M)$, such a deviation is equivalent to one in which all single-homing sellers become multi-homers. This deviation, however, is not profitable if

⁴⁹The case with M odd also works in the same way as above.

$\pi_i^{SH}(1, 0, M - 1)/2 \geq \pi(M)$. This condition can be fulfilled as the competitive pressure in the product market is larger if all M sellers compete for all buyers as compared to the case in which only $M - 1$ sellers compete for all buyers, but one seller interacts with only half of the buyers. As prices are lower in the first case, the single-homing seller may benefit from reducing competition, but selling to only half of the buyers.

We turn to deviations of multi-homing sellers. First, a multi-homing seller can deviate to become a single-homing seller. The most profitable way to do so is to list on the platform on which the other single-homing seller is not present, yielding a profit of $\pi_i^{SH}(1, 1, M - 2)/2$. Therefore, this deviation is not profitable if $\pi^{MH}(1, 0, M - 1) \geq \pi_i^{SH}(1, 1, M - 2)/2$. Second, multi-homing sellers (one per category from a set of product categories of positive mass) can also form a coalition with all buyers and list on the platform on which the single-homing seller is present. (Buyers are not willing to form a coalition with the sellers and be active on the other platform, as in this case, buyers from the platform of the single-homing seller obtain a lower utility because they face only $M - 1$ sellers instead of M .) The seller's profit is then $\pi(M)$. However, $\pi(M)$ is strictly less than $\pi^{MH}(1, 0, M - 1)$, as the competitive pressure is higher, but the number of buyers observing the seller's listing is still the same.

As a consequence, a partial multi-homing equilibrium with the features that sellers in different categories make different listing decisions and that there is only one single-homing seller in each category can be sustained in the second stage if the conditions $\pi_i^{SH}(1, 0, M - 1)/2 \geq \pi(M)$ and $\pi^{MH}(1, 0, M - 1) \geq \pi_i^{SH}(1, 1, M - 2)/2$ are jointly satisfied. This is indeed possible. \square

Proof of Proposition 13. The third and the fourth stage play out in a similar way as in the baseline model. In the fourth stage, buyers make their buying decisions to maximize utility, and in the third stage, sellers set their product prices, conditional on the number of sellers in their product category on the platform.

In the first part of the proof, we determine the conditions under which $f_i = 0, \forall i \in \{1, \dots, N\}$, is an equilibrium of the full game. Note that, given $f_i = 0, \forall i \in \{1, \dots, N\}$, as long as in the equilibrium of the second stage at least one platform does not carry any trade, then no platform can profitably deviate by increasing its fee in the first stage. The reason is that, if a platform carried a positive volume of trade in the equilibrium with zero fees, then, after the deviation, sellers and buyers active on this platform would form a coalition and move to one of the platforms with a fee of zero. If the deviating platform carries no trade, a higher fee cannot make this platform better off since it will not attract any buyers and sellers.

To determine the condition under which a platform could profitably deviate from $f_i = 0$, given that all other platforms charge a fee of zero, we distinguish between the

cases $M = kN$ and $M \neq kN$. Recall that k is the largest integer such that $M \geq kN$.

First, we analyze the case $M = kN$. We know from above that, given zero fees, a platform only has an incentive to deviate to a strictly positive fee if all platforms carry a positive volume of trade. The latter can only occur if each platform hosts $k = M/N$ sellers. This leads to a profit per seller of $\pi(k)/N \geq 0$. The most profitable deviation by a coalition in the second stage is then that one seller moves to another platform together with all buyers (as those benefit from the additional seller). The seller's profit is then $\pi(k+1)$. It follows that for

$$\pi(k+1) < \frac{\pi(k)}{N}, \quad (46)$$

the deviation is not profitable for the seller, and an equilibrium exists in which all N platforms carry a positive volume of trade. To the contrary, if the condition is not fulfilled—i.e., $\pi(k+1) \geq \pi(k)/N$ as in (2)—no equilibrium candidate in stage 2 with N platforms carrying positive volumes of trade exists. Then, for $f_i = 0, \forall i \in \{1, \dots, N\}$, only a subset of platforms will carry a positive volume of trade, which implies that no platform can profitably deviate from these fees. This proves the first part of the proposition for $M = kN$.

Second, consider the case $M \neq kN$. We start by demonstrating that there can never be a coalition-proof equilibrium in which sellers in different categories split differently on the platforms. To see this, consider the case in which all sellers and all platforms are active. Buyers are then only indifferent between platforms if each one is on average host to M/N sellers. To achieve this, we can split the mass of categories in N segments, each with a mass $1/N$. In each segment, a platform has either k or $k+1$ sellers in the respective categories, according to the following two rules. First, in each segment, a number $N(k+1 - M/N)$ of platforms is host to k sellers and a number $N(M/N - k)$ is host to $k+1$ sellers. Then, in each segment of categories, all M sellers are active. Second, we allocate to each platform k sellers in $N(k+1) - M$ segments and $k+1$ sellers in $M - N(k)$ segments. Then, summing up over the categories, the average number of sellers on each platform is M/N .⁵⁰

However, such a distribution is not coalition-proof. Take one segment of categories and consider all sellers who are active on a platform with $k+1$ sellers in their categories. Take as a coalition one seller in each category within the segment together with all buyers on the seller's platform. This coalition has an incentive to go to a platform with only k

⁵⁰For example, if $M = 11$ and $N = 4$, we split the categories in 4 segments, each one with mass $1/4$. In the first one of these segments, platform 1 is host to 2 sellers in all categories in the segment, whereas platforms 2, 3, and 4, are host to 3 sellers. In the second segment, platform 2 is host to 2 sellers and all others platforms are host to 3 sellers, whereas in the third (fourth) segment, platform 3 (4) is host to 2 sellers and the other platforms are host to 3 sellers. Then, each platform has on average $11/4$ and sellers and buyers are indifferent.

sellers. Per category, the deviating seller then obtains a profit (excluding the listing fee) of $2\pi(k+1)/N$, whereas without the deviation the profit is only $\pi(k+1)/N$. In addition, also the buyers benefit as they now expect more sellers on the platform than before (i.e., the expected number of sellers is $k+1$ instead of M/N). It follows that there exists a profitable deviation from such an asymmetric equilibrium.

The same argument holds if only a subset of sellers is active. Therefore, if listing fees are symmetric on all platforms, the equilibrium in the second stage must be symmetric across all categories.

We now determine for the case $M \neq kN$, under which conditions $f_i = 0, \forall i \in \{1, \dots, N\}$, is an equilibrium in the first stage. Suppose first that only a subset of platforms has a positive volume of trade. In this situation, sellers and buyers in the second stage choose either agglomeration (that is, all buyers and all active sellers are on one platform) or another distribution in which all platforms with positive market share host the same number of sellers in all categories. The selected equilibrium depends on the profits that sellers obtain and the numbers M and N .⁵¹ In this situation, following the same arguments as in the case $M = kN$, no platform can profitably increase its fee.

Instead, suppose that all platforms carry a positive volume of trade. Proceeding analogously to the case $M = kN$, we obtain inequality (46) also for the case $M \neq kN$.

We now show that if (46) holds, an equilibrium with zero fees will not be selected by profit-dominance in the first stage. Suppose that (46) holds and all platforms charge strictly positive fees. Then, slightly lowering the fee is not profitable for a platform, as sellers and buyers will still play a segmentation equilibrium in the second stage, in which all platforms have a positive market share and are host to k sellers. As a consequence, an equilibrium with strictly positive fees exists. However, as $M \neq kN$, an equilibrium in which $f_i = 0, \forall i \in \{1, \dots, N\}$ always exists since at least one platform will not carry a positive trade volume. Yet, the latter equilibrium is profit-dominated. Therefore, the equilibrium with strictly positive fees will be selected in the first stage, whenever the two equilibria co-exist. To sum up the analysis so far, in case $M \geq N$, a pure-strategy equilibrium in the first stage with $f_i = 0, \forall i \in \{1, \dots, N\}$ exists and is selected if and only if condition (2) is satisfied.

From the preceding arguments, it also follows that for $M < N$, the unique equilibrium involves $f_i = 0, \forall i \in \{1, \dots, N\}$, as in any equilibrium in the second stage, only a subset of platforms can carry a positive volume of trade.

In the second part of the proof, we turn to the segmentation equilibrium with positive fees. From above, we know that for an equilibrium with positive fees to exist, all platforms

⁵¹Suppose, for example, that there are 10 sellers per category, 3 platforms, and $\pi(10) > 0$. Then, in the first equilibrium type, one platform is host to 10 sellers, whereas in the second equilibrium type, two platforms host 5 sellers each.

must carry a positive volume of trade. This implies that each platform will have a mass of $1/N$ buyers. Suppose that in a segmentation equilibrium candidate, each platform hosts $l \in \{1, \dots, k\}$ sellers. Then, a platform i can charge at most $f_i = \pi(l)/N$, leading to a platform profit of $\Pi_i = l\pi(l)/N$ and zero profits to sellers. Consider a coalition of one seller on a platform $j \neq i$ together with *all* buyers (i.e., not only those on platform j but the whole buyer mass of 1). If this coalition deviates to platform i , all buyers benefit as they now face $l + 1$ sellers instead of only l . The seller instead benefits only if $\pi(l + 1) > \pi(l)/N$. In addition, it is easy to check that this coalition leads to the tightest condition for a segmentation equilibrium to exist in the second stage, as a deviation involving more than one seller (per category) leads to lower seller profits. It follows that segmentation is an equilibrium in the second stage if and only if $\pi(l)/N \geq \pi(l + 1)$.

We turn to the first stage and check if a platform has a profitable deviation from the equilibrium candidate $f_i = \pi(l)/N$. Consider the deviation in which platform i sets a fee slightly below $\pi(l + 1)$. It then attracts $l + 1$ sellers and, thereby, also all buyers. Hence, the deviating platform's profit is $(l + 1)\pi(l + 1)$. Therefore, this deviation is not profitable if $l\pi(l)/N \geq (l + 1)\pi(l + 1)$. This condition is (weakly) stronger than $\pi(l)/N \geq \pi(l + 1)$, which was derived for the candidate segmentation equilibrium to exist in the second stage. Thus, we can focus on the first stage.

By the same logic as in the previous paragraph, a platform can also deviate to any fee slightly below $\pi(m)$, with $l < m \leq M$. The most profitable deviation is therefore to set a fee of $\pi(\hat{m})$, with $\hat{m} \in \arg \max_{l < m \leq M} m\pi(m)$. It follows that if there exists some $l \in \{1, \dots, k\}$ such that (3) holds, a pure-strategy segmentation equilibrium exists in which platforms charge $f_i = \pi(l)/N$. Applying profit-dominance in the first stage, platforms choose $f_i = \pi(l^*)/N$, such that $l^* \in \arg \max_{l \in \{1, \dots, k\}} l\pi(l)$ subject to condition (3).

In the third part of the proof, we show that if (2) is not satisfied, this does not imply that (3) holds (i.e., there can be a region in which neither condition is satisfied). To see this, note that for $\pi(k)/N > \pi(k + 1)$, condition (2) is not satisfied. We now turn to condition (3). Suppose first that $l = k$. Then, (3) holds if $k\pi(k)/N \geq \hat{m}\pi(\hat{m})$ or $\pi(k)/N \geq \hat{m}\pi(\hat{m})/k$. We know that $\hat{m} > k$, which implies that \hat{m} must be at least $k + 1$. Inserting $\hat{m} = k + 1$ into $\pi(k)/N \geq \hat{m}\pi(\hat{m})/k$ yields $\pi(k)/N \geq (k + 1)\pi(k + 1)/k$. It is easy to see that for

$$\pi(k + 1) < \frac{\pi(k)}{N} < \frac{(k + 1)\pi(k + 1)}{k}$$

neither (2) nor (3) is satisfied. Since the right-hand side of (3) is at least as high as $(k + 1)\pi(k + 1)$ (due to the fact that \hat{m} is chosen to maximize $m\pi(m)$ with respect to m), this also holds if $\hat{m} \neq k + 1$.

A similar argument obtains for the case in which $l \neq k$. Rewriting (3), we obtain

$\pi(l)/N \geq \hat{m}\pi(\hat{m})/l$. The left-hand side of the previous inequality is larger than the right-hand side of (2). However, $\pi(k)/N > \pi(k+1)$ does not rule out that there exists an \hat{m} , such that $\pi(l)/N < \hat{m}\pi(\hat{m})/l$ holds. In this case, again neither (2) nor (3) is satisfied.

Showing that there is a unique mixed-strategy equilibrium in this case follows the same arguments as in the proofs of Propositions 3 and 4. Platform profits in the mixed-strategy equilibrium are strictly positive. This follows because, if $\pi(k+1) \geq \pi(k)/N$ is violated, each platform sets a strictly positive fee even if all other platforms charge a fee of zero. Therefore, setting a fee equal to zero is not part of the mixing domain. As a consequence, the mixed-strategy equilibrium profit-dominates the pure-strategy equilibrium with $f_i^* = 0, \forall i \in \{1, \dots, N\}$, which exists for the case $M \neq kN$. Therefore, the mixed-strategy equilibrium is always selected in the first stage. \square

References

- [1] Augereau, A., S. Greenstein and M. Rysman (2006): “Coordination versus Differentiation in a Standards War: 56K Modems,” *RAND Journal of Economics*, 37, 887-909.
- [2] Biglaiser, G., J. Cremér and A. Veiga (2018): “Migration between Platforms,” Working Paper, University of Toulouse.
- [3] Cabral, L. (2011): “Dynamic Price Competition with Network Effects,” *Review of Economic Studies*, 78, 83-111.
- [4] Competition & Markets Authority (2017): “Digital Comparison Tools: Consumer Research,” available at: <https://assets.publishing.service.gov.uk/media/58e224f5e5274a06b3000099/dcts-consumer-research-final-report.pdf>.
- [5] European Commission (2017): “Case M.8416 - The Price-line Group / Momondo Group Holdings,” available at: http://ec.europa.eu/competition/mergers/cases/decisions/m8416_913_3.pdf.
- [6] Farrell, J. and P. Klemperer (2007): “Coordination and Lock-In: Competition with Switching Costs and Network Effects,” in: Armstrong, M. and R. Porter (eds.), *Handbook of Industrial Organization*, III, Elsevier, North-Holland, 1967-2072.
- [7] Gould, E.D., B.P. Pashigian, and C.J. Prendergast (2005): “Contracts, Externalities, and Incentives in Shopping Malls,” *Review of Economics and Statistics*, 87, 411-422.
- [8] Hagiu, A. (2006): “Pricing and Commitment by Two-Sided Platforms,” *RAND Journal of Economics*, 37, 720-737.
- [9] Koski, H. and T. Kretschmer (2002): “Entry, Standards, and Competition: Firm Strategies and the Diffusion of Mobile Telephony,” *ETLA Discussion Paper* 842.
- [10] Li, H., Q. Shen, and Y. Bart (2018): “Local Market Characteristics and Online-to-Offline Commerce: An Empirical Analysis of Groupon,” *Management Science*, 64, 1860-1878.
- [11] Li, H. and F. Zhu (2018): “Multi-Homing and Platform Competition: A Natural Experiment in the Daily Deals Market,” Working Paper, Harvard Business School.
- [12] Pashigian, B.P. and E.D. Gould (1998): “Internalizing Externalities: The Pricing of Space in Shopping Malls,” *Journal of Law and Economics*, 41, 115-142.