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Optimal Auctions with Signaling Bidders

Olivier Bos *
Martin Pollrich **

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* Panthéon-Assas University (LEMMA), Institut Universitaire de France (IUF) and ZEW-Leibniz Centre for European Economic Research. E-mail: olivier.bos@u-paris2.fr.
** University of Bonn. E-mail: martin.pollrich@uni-bonn.de.

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Olivier Bos† and Martin Pollrich‡

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Abstract

We study optimal auctions in a symmetric private values setting, where bidders' care about winning the object and a receiver's inference about their type. We reestablish revenue equivalence when bidders' signaling concerns are linear, and the auction makes participation observable via an entry fee. With convex signaling concerns, optimal auctions are fully transparent: every standard auction, which reveals all bids yields maximal revenue. With concave signaling concerns there is no general revenue ranking. We highlight a trade-off between maximizing revenue derived from signaling, and extracting information from bidders. Our methodology combines tools from mechanism design with tools from Bayesian persuasion.

Keywords: optimal auctions, revenue equivalence, Bayesian persuasion, information design

JEL classification: D44; D82

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†Panthéon-Assas University (LEMMA), Institut Universitaire de France (IUF) and ZEW-Leibniz Centre for European Economic Research. E-mail: olivier.bos@u-paris2.fr.

‡University of Bonn. E-mail: martin.pollrich@uni-bonn.de.

1 Introduction

Since 1945, the *Hospices de Beaune*[1], in Burgundy (France), organizes an annual wine auction to raise money for local retirement houses and hospitals. In a special segment—the "pièce des Présidents"—some wine is auctioned to raise money for special charity purposes. This segment attracts special attention not the least due to the involvement of celebrities. In the 2017 "pièce des Présidents" auction two barrels of *Corton Clos du Roi Grand Cru* were sold at a total price of €410,000. During the regular auction, the same wine realized prices ranging from €30,000 to €40,000 per barrel. Roughly speaking, public attention increased the price per barrel by 500%.[2]

This is but one example of an auction where bidders have signaling concerns: bidders do not only care about the object at sale but also about how they are perceived. Other examples include art auctions, where bidders signal proficiency and taste to their acquaintances, takeover bidding, where bidders also signal strength to rival companies, and procurement auctions, where suppliers also care for sending strong signals to ensure participation in future tenders.

In this manuscript we study auction design when bidders care about the information conveyed through their own performance. How others perceive the bidders' performance crucially depends on the auction design. For example, in a first-price auction, outsiders observe the winner's bid (via the price) but no other bids. This allows precise inference on the winner's type, but only noisy inference on losers' types. In a second-price auction outsiders observe the highest losing bid, hence inference on *all* bidders remains noisy. With this in mind, the design of the auction affects the possibilities for outside inference, which in turn affects bidding behavior and auction revenue. Our analysis is then looking for the revenue-maximizing auction design when bidders have signaling concerns.

In their seminal contributions Myerson (1981) and Riley and Samuelson (1981) show that—absent signaling concerns—every standard auction yields the same revenue. This needs no longer to be the case when bidders care for signaling.[3] We study an auction environment with independent private values. In addition, bidders preferences depend on the mean of posterior beliefs about their own type. These posteriors are formed conditional on the outcome of the auction: the winner's identity and each bidder's payment. Our main result establishes that whether revenue equivalence obtains depends on the

[1] https://www.beaune-tourism.com/discover/hospices-de-beaune-wine-auction

[2] Similar patterns arose in the previous years. Data for 2016 and 2017 are available at http://hospices-de-beaune.com/index.php?/hospicesdebeaune/content/download/3869/14085/version/1/file/catalogue_resultats_2016.pdf and http://hospices-de-beaune.com/index.php?/hospicesdebeaune/content/download/4248/15476/version/1/file/Vente+des+vins+-+Catalogue+des+r%C3%A9sultats+2017.pdf

[3] Giovannoni and Makris (2014), Bos and Truyts (2019) study environments where different auction formats (and bid disclosure policies) yield different auction revenue. Goeree (2003), Molnar and Virag (2008), Katzman and Rhodes-Kropf (2008) find revenue equivalence in their respective settings.

curvature of the signaling motive in a bidder's utility, and the use of an additional instrument: entry fees. When signaling concerns are linear (i.e. mean posteriors enter linearly in bidders' utility) revenue equivalence obtains between standard auctions that use entry fees. Payments extract (signaling) value from the bidders. The auctioneer uses entry fees to extract *all* signaling value from bidders, independent of whether they end up winning the object. Therefore, as the signaling concerns enter linearly, the distribution of signaling value across auction outcomes does not affect revenue.

Matters are different when the signaling concerns is not linear. When bidders' preferences are convex, revenue increases in the amount of information the auction reveals. An all-pay auction yields maximal revenue, because in the fully separating equilibrium a bidder's payment perfectly reveals her type (maximizing signaling value) and all bidders pay their bid (extracting signaling value). Moreover, every standard auction that additionally reveals *all* bids yields the same revenue, because revealing bids automatically reveals whether a bidder participated.

With concave signaling concerns a general revenue ranking cannot be established. The information revealed during the auction affects revenue in two ways. First, revealing less information about bidders' types increases signaling value, which in turn increases revenue via more aggressive bidding. Second, to extract all signaling value the auctioneer should again charge an entry fee, which, however, reveals additional information about bidders and thereby reduces signaling value. Therefore, if participation is already fully observable it becomes optimal to reveal only the winner's identity. Moreover, if participation in the auction is high enough, charging an entry fee is optimal, because the reduction in signaling value is small.

Ex-post payments play a crucial role in our analysis, because beliefs are formed based on the realized auction outcome. This prevents us from directly applying standard tools from auction theory (e.g., Myerson, 1981), that use interim payments. We adapt methods from Bayesian persuasion to work with distributions over posterior beliefs. This intermediate step allows us to move the entire analysis to the interim stage.

Auctions with signaling concerns have been recently investigated by Giovannoni and Makris (2014) and Bos and Truyts (2019).[4] The former consider auctions that reveal the winner's identity together with four disclosure policies: no information, the highest bid, the second highest bid, or all bids (each together with the respective bidder's identity). In particular, only a (strict) subset of bidders' payments is observable. Bos and Truyts compare second-price and English auctions, that reveal the winner's identity and her payment. Both studies compare specific auction formats and disclosure policies, and establish a failure of revenue equivalence. Our analysis, which covers all standard auctions,

[4]There are also contributions about information transmission comparing specific auction formats followed by oligopoly competition. See, e.g., Goeree (2003), Das Varma (2003), Katzman and Rhodes-Kropf (2008) and von Scarpatetti and Wasser (2010).

provides conditions under which revenue equivalence is restored via the use of an entry fee and optimal bid disclosure.

Our paper is also related to the literature on mechanism design with aftermarkets. Calzolari and Pavan (2006*a*,*b*) study contracting environments where the agent participates in an aftermarket. They find conditions under which no information release to the aftermarket is optimal. Dworczak (2020) analyzes an auction environment with a very general aftermarket. He restricts the analysis to cut-off mechanisms in which the information revealed about the winner only depends on the losers' bids. These mechanisms rule out disclosure of information contained only in the winner's bid, such as the price in a first-price or all-pay auction, which we show is optimal in some cases. Molnar and Virag (2008) also investigate revenue maximizing auctions with upstream competition. Both in Dworczak (2020) and Molnar and Virag (2008) only the winner participates in the aftermarket, and there is no aftermarket if there is no winner. Contrary to our analysis, this eliminates the benefit of releasing information about participation decisions, and the necessity to design information about losing bidders as well.

Information disclosure in auctions has first been analyzed in the setting of affiliated values by Milgrom and Weber (1982). Mechanism design problems with allocative and informational externalities have also been studied by Jehiel and Moldovanu (2000, 2001). The underlying assumption in this strand of literature is that an agent's valuation depends also on other agents' private information (and allocation). In our setting a bidder's utility is affected by the aftermarket's belief about her own valuation, while such beliefs have no impact in the literature on mechanism design with interdependent valuations.

The paper is organized as follows. Section 2 introduces the formal setting. Section 3 studies the case of linear signaling concerns. In Section 4 we derive optimal auctions when the signaling concerns are convex, and in Section 5 analyzes the concave case. We conclude and discuss our results in Section 6.

2 Formal Setting

We consider n bidders, who bid for a single object in an auction, and also care about the inference of an outside observer about their type.

Bidder i's valuation for the object (her 'type'), is denoted V_i, and is assumed i.i.d. and drawn according to a distribution function F with support on $[\underline{v}, \overline{v}] \subset \mathbb{R}_+$. Let $f \equiv F'$ denote the density function, $G \equiv F^{n-1}$ the distribution function of the highest order statistic among $n-1$ remaining valuations and $g \equiv G'$ the corresponding density function. Bidder i's realization of V_i, denoted v_i, is her private information, but the number of bidders and the distribution F are common knowledge.

We consider *standard auctions* in which each bidder submits a (non-negative) bid b_i, the highest bidder wins (ties broken at random), and bidder i's payment p_i depends on the entire vector of bids, i.e., $p_i(b_1, \dots, b_n)$. In addition, the auctioneer controls participation, e.g., via charging an entry fee φ or setting a reserve price r. Entry fees play a prominent role in our analysis. Under a non-zero entry fee a bidder who wishes to submit a bid first has to pay the entry fee. With slight abuse of notation we denote $p_i(b_1, \dots, b_n)$ the final payment that bidder i makes, potentially including the entry fee. In particular, we may have $p_i > 0$ even though bidder i did not win the object.

Each bidder cares about winning the object, and about the inference of an outside observer, the 'receiver', about her type. This receiver can represent, e.g., the general public or press, business contacts or acquaintances of the bidder, or experts related to the object at sale. The receiver observes the outcome of the auction $\mathcal{O} = (i^\star, p_1, \dots, p_n)$, where $i^\star$ is the winner's identity and p_i the payment made by bidder i, and forms a posterior belief about each bidder's type, denoted as $\mu_i(\mathcal{O})$. We assume that a bidder's utility depends on the receiver's belief only through the posterior mean, i.e., the expected value given the posterior distribution.[5,6] Formally, there is an increasing measurable function $\Phi : [\underline{v}, \overline{v}] \to \mathbb{R}_+$, such that the bidder's utility is given by

$$u_i(v_i, \mathcal{O}) = \begin{cases} v_i - p_i + \Phi\left(\mathbb{E}\left(V_i|\mathcal{O}\right)\right), & \text{if } i = i^\star, \\ -p_i + \Phi\left(\mathbb{E}\left(V_i|\mathcal{O}\right)\right), & \text{if } i \neq i^\star. \end{cases}$$

The function Φ represents a reduced form of a (continuation) game in which the receiver chooses an action that directly affects the bidder's payoff.[7] Note that a bidder's utility is not affected by the receiver's belief about other bidders' types. For instance, from an individual bidder's perspective it is equivalent to have either a different or the same receiver for each bidder.

Any standard auction defines a signaling game among bidders and the receiver. We consider symmetric perfect Bayesian equilibrium, consisting of the bidders' bidding strategies $\beta : [\underline{v}, \overline{v}] \to \mathbb{R}_+$ and the receiver's belief $(\mu_1, \dots, \mu_n)$.[8] Each bidder's bidding strategy is optimal, given the other bidders' bidding and the receiver's beliefs. Also, the receiver's

[5] Similar assumptions are made in Giovannoni and Makris (2014), Bos and Truyts (2019) and Molnar and Virag (2008) in the context of auctions with signaling. Dworczak and Martini (2019) use this assumption in the context of Bayesian Persuasion.

[6] Note that we do not assume that a bidder's type v_i *directly* affects the receiver's payoff. The receiver cares about some other characteristic of the bidder, which is correlated with the bidder's type. See also the example at the end of this section.

[7] For example, Giovannoni and Makris (2014) analyze takeover auctions with signaling to an after-market. A bidder's private valuation is interpreted by a post-auction job-market for managers as a signal of its manager's ability to extract revenue from an acquisition. The action taken by the receiver corresponds to the wage offered to the manager, resulting from a competitive wage offer.

[8] To save on notation we do not formalize the bidders' entry decision. Under an effective entry fee each bidder first decides whether to enter the auction. The receiver observes whether a bidder paid φ, and updates her belief accordingly.

beliefs are Bayesian consistent with the bidding strategy. Following the usual approach in mechanism design, we focus on separating equilibria, i.e., where bidders use strictly increasing bidding strategies, and do not emphasize multiplicity of equilibria, selection and refinements.

We conclude this section with an illustrative example for the bidders' utility functions. Suppose the bidder's valuation v is uniformly distributed on $[0,1]$. The receiver cares about the bidder's characteristic θ, given by $\theta = \alpha v + (1-\alpha)s$, where s is uniformly distributed on $[0,1]$, independent of v, and $\alpha \in (0,1)$. The receiver takes an action a to maximize her utility $U_R(a,\theta) = -(a-\theta)^2$. Given the auction outcome $\mathcal{O}$, the receiver chooses $a = \mathbb{E}(\theta|\mathcal{O})$. We have that

$$a = \mathbb{E}(\theta|\mathcal{O}) = \mathbb{E}(\alpha v + (1-\alpha)s|\mathcal{O}) = \alpha\mathbb{E}(v|\mathcal{O}) + \frac{1-\alpha}{2}$$

Defining $\Phi(x) = \alpha x + (1-\alpha)/2$ yields an example of the inference function, as defined above. In the context of our introductory example of the *Hospices de Beaune*, θ may represent a bidder's altruism. Our model assumes that a bidder's altruism is correlated with her valuation for the wine auctioned, where α measures the degree of correlation.

3 Linear Signaling Concerns

In this section, we consider a linear inference $\Phi(v) = \lambda v$, with $\lambda > 0$ the strength of a bidder's signaling concerns. Previous results indicate that even with linear inference revenue equivalence may fail. We show that revenue equivalence holds when the auctioneer uses an entry fee to induce the desired level of participation.

Recall, that we focus on symmetric equilibria in which bidders follow a strictly increasing bidding strategy. This implies that there exists a cut-off type, denoted $\tau \in [\underline{v}, \overline{v}]$, such that a bidder with valuation v participates in the auction if and only if $v \geq \tau$. Our first proposition shows that the revenue of a standard auction can be decomposed into a static component—the revenue of the respective auction without signaling concerns—and a signaling component.

Proposition 1. *Consider a standard auction, in which every bidder follows a strictly increasing bidding strategy, and participates whenever his type is above τ. The revenue in this auction is given by*

$$Rev^{\mathcal{M}}(\tau) + n\left(\lambda\mathbb{E}(V) - \mathcal{W}_{0,\tau}\right) \tag{1}$$

with $Rev^{\mathcal{M}}(\tau)$ the ex ante expected revenue of the respective auction without signaling[9]

[9]See Myerson (1981) and Riley and Samuelson (1981).

and $\mathcal{W}_{0,\tau}$ the expected inference of a bidder who does not receive the object and makes a zero payment.

From the revenue equivalence in the standard setting (e.g., Myerson, 1981) we know that the first term, $Rev^{\mathcal{M}}$, does not depend on the specific auction format. The key insight of Proposition 1 is that the revenue depends on the details of the auction *only* through the inference about bidders who do not win and do not pay. Note that $\mathcal{W}_{0,\tau}$ already differs between first and second-price auctions. In a first-price auction we infer that all losers have a type below the winner's, which is perfectly revealed from his payment. In a second-price auction the winner's payment reveals the second-highest type, and a loser has either exactly this type or a lower type.

To prove Proposition 1 we use standard arguments.[10] We denote $m(v)$ the expected payment and $\mathcal{W}(v)$ the expected value from signaling of a bidder with valuation v. A bidder's expected payoff from mimicking type $\widetilde{v}$ is

$$\Pi(v,\widetilde{v}) = G(\tilde{v})v - m(\tilde{v}) + \mathcal{W}(\tilde{v}). \tag{2}$$

At equilibrium a bidder's payoff is $\Pi(v,v)$ and the following first-order condition holds

$$\frac{\partial}{\partial \widetilde{v}}\Pi(v,\widetilde{v}) = g(\widetilde{v})v - m'(\widetilde{v}) + \mathcal{W}'(\widetilde{v}) = 0 \;\text{ at }\; \widetilde{v} = v, \tag{3}$$

for all participating types, i.e., for all $v \geq \tau$. At equilibrium, a bidder of type τ is indifferent between participating and not, hence

$$\mathcal{W}_{0,\tau} = G(\tau)\tau + \mathcal{W}(\tau) - m(\tau). \tag{4}$$

Using (3) and (4) we express the *interim* expected payment of a bidder as follows

$$m(v) = G(\tau)\tau + \int_{\tau}^{v} g(x)x \,\mathrm{d}x + \mathcal{W}(v) - \mathcal{W}_{0,\tau}. \tag{5}$$

Note that this expected payment depends on the auction format via the signaling component $\mathcal{W}(v) - \mathcal{W}_{0,\tau}$. Taking the expectation yields the expected revenue

$$\begin{aligned} n\int_{\underline{v}}^{\overline{v}} m(v)\,\mathrm{d}F(v) &= n\int_{\tau}^{\overline{v}} m(v)\,\mathrm{d}F(v) \\ &= \underbrace{n\left((1-F(\tau))G(\tau)\tau + \int_{\tau}^{\overline{v}}\int_{\tau}^{v} g(x)x\,\mathrm{d}x\mathrm{d}F(v)\right)}_{=Rev^{\mathcal{M}}(\tau)} \\ &\quad + n\left(\int_{\tau}^{\overline{v}} \mathcal{W}(v)\,\mathrm{d}F(v) - (1-F(\tau))\mathcal{W}_{0,\tau}\right) \end{aligned}$$

[10] For example see Riley and Samuelson (1981) and Krishna (2009).

Finally, note that by law of iterated expectations $F(\tau)\mathcal{W}_{0,\tau} + \int_{\tau}^{\overline{v}} \mathcal{W}(v)\, \mathrm{d}F(v) = \lambda\mathbb{E}(V)$. Therefore,

$$n\int_{\underline{v}}^{\overline{v}} m(v)\, \mathrm{d}F(v) = Rev^{\mathcal{M}}(\tau) + n\big(\lambda\mathbb{E}(V) - \mathcal{W}_{0,\tau}\big).$$

Following Proposition 1 any revenue ranking of auctions is based entirely on $\mathcal{W}_{0,\tau}$, the expected inference of non-participating bidders. In many auction formats it is impossible to tell apart a losing bidder who does not make any payment from a non-participating bidder. This lack of precise inference inflates $\mathcal{W}_{0,\tau}$, benefiting low-type bidders but hurting the auctioneer.

Proposition 2 (Revenue Equivalence)**.** *Consider a standard auction, in which every bidder follows a strictly increasing bidding strategy. When all types above τ participate and participation is observable (e.g. via an entry fee) the revenue equals*

$$Rev^{\mathcal{M}}(\tau) + n\lambda\big(\mathbb{E}(V) - \mathbb{E}(V|V < \tau)\big). \tag{6}$$

Moreover, no other auction yields higher revenue.

To prove Proposition 2 note that $\mathcal{W}_{0,\tau} \geq \lambda\mathbb{E}(V|V < \tau)$, because by assumption all types below τ do not participate in the auction. Furthermore, if participation is observable we get equality in the latter formula. Plugging this value into (1) yields (6).

A commonly applied way for making participation observable is to charge an *entry fee*. This way, an outsider who observes only all payments made during the auction can tell apart losing bidders from non-participating bidders. Note that setting a *reserve price* does not yield maximal revenue, as the outsider cannot distinguish between participating losers and non-participating bidders.

Next we want to compare optimal levels of participation among situations with and without signaling concerns. We focus on auctions with entry fees, as these attain the maximal revenue determined in Proposition 2, with $\tau^\star(\lambda)$ the optimal level of participation under signaling strength λ.[11] Note that $\tau^\star(0) = \tau^{\mathcal{M}}$, where $\tau^{\mathcal{M}}$ denotes the optimal participation cut-off in an auction without signaling concerns.

Corollary 1 (Optimal Participation)**.** *Assume virtual valuations are increasing. We have that*

(i) $\tau^\star(\lambda) \leq \tau^\star(\lambda') < \tau^{\mathcal{M}}$ *for all* $\lambda > \lambda' > 0$.

(ii) There exists $\overline{\lambda}$ *such that* $\tau^\star(\lambda) = \underline{v}$, *for all* $\lambda > \overline{\lambda}$.

[11] In general there may not be a unique optimal level of participation. Our assumption of increasing virtual valuations in Corollary 1 guarantees both existence and uniqueness of $\tau^\star(\lambda)$.

Proof. $Rev^{\mathcal{M}}(\tau)$ is a strictly concave in τ and $\frac{\partial Rev^{\mathcal{M}}}{\partial \tau}(\tau^{\mathcal{M}}) = 0$. Furthermore, the expected inference for a zero-bid at the equilibrium, $\lambda\mathbb{E}(V|V < \tau)$, strictly increases in τ. Hence, (i) follows. When

$$\lambda \frac{\partial}{\partial \tau}\mathbb{E}(V|V < \tau)|_{\tau=\underline{v}} \geq \left(Rev^{\mathcal{M}}\right)'(\underline{v})$$

full participation maximizes revenue in the auction with signaling. The terms $\frac{\partial}{\partial \tau}\mathbb{E}(V|V < \tau)|_{\tau=\underline{v}}$ and $\left(Rev^{\mathcal{M}}\right)'(\underline{v})$ are both strictly positive and finite, hence the inequality holds for λ sufficiently large. □

We conclude this setting with an illustration of our results for the first- and the second-price auction.

Example 1 (Equilibrium of the first-price auction)**.** Consider a first-price auction with entry fee φ. The critical type τ pays the entry fee φ and places a zero bid. Equations (4) and (5) together with $\beta(\tau) = 0$ yields

$$\beta(v) = \int_{\tau}^{v} \frac{xg(x)}{G(v)}dx + \frac{\mathcal{W}(v) - \mathcal{W}(\tau)}{G(v)}, \qquad \forall v > \tau.$$

The signaling value $\mathcal{W}(v)$ can be expressed as follows

$$\begin{aligned}\mathcal{W}(v) &= G(v)v + (1 - G(v))\frac{1}{1 - G(v)}\int_{v}^{\overline{v}} \mathbb{E}(V|\tau < V < x)dG(x) \\ &= G(v)v + \int_{v}^{\overline{v}}\int_{\tau}^{x} \frac{\lambda y}{F(x) - F(\tau)}dF(y)dG(x).\end{aligned}$$

Hence,

$$\beta(v) = \int_{\tau}^{v} \frac{xg(x)}{G(v)}dx + \frac{G(v)v - G(\tau)\tau}{G(v)} + \int_{\tau}^{v}\int_{\tau}^{x} \frac{\lambda y}{F(x) - F(\tau)}dF(y)dG(x).$$

Bos and Truyts (2018, Proposition 1) show that these bidding strategies indeed constitute an equilibrium of the first-price auction.

Example 2 (Non-existence of monotone equilibria in the second-price auction)**.** Consider a second-price auction with two bidders and valuations uniformly distributed on $[0, 1]$. Using equations (4) and (5) we get that

$$\int_{\tau}^{v} \beta(x)dx = \int_{\tau}^{v} xdx + \mathcal{W}(v) - \mathcal{W}(\tau),$$

which implies $\beta(v) = v + \mathcal{W}'(v)$. In the second-price auction we have for $v \geq \tau$

$$\mathcal{W}(v) = \tau\lambda\mathbb{E}(V|V \geq \tau) + \int_{\tau}^{v} \lambda\mathbb{E}(V|V \geq x)dx + (1 - v)\lambda v.$$

Hence,

$$\beta(v) = v + \mathcal{W}'(v) = v + \lambda \frac{3}{2}(1-v)$$

For $\lambda > 2/3$ the bidding strategy is *decreasing* in v, hence an *increasing* equilibrium does not exist. Observe, that the non-existence result does not depend on $\mathcal{W}_{0,\tau}$.

4 Convex Signaling Concerns

In this section, we consider convex signaling concerns, i.e., the case where Φ is strictly increasing and convex. As for the linear case, we can decompose the revenue into the static component $Rev^{\mathcal{M}}(\tau)$ and the signaling component. With convex Φ it becomes impossible to average the signaling value at the *interim* level. Yet, any auction induces a distribution over posterior means that averages to the same mean, hence is a mean-preserving spread. With a convex Φ it is then optimal to 'disclose' as much information as possible, because this increases the signaling value. The maximal amount of information that an auction can disclose corresponds to fully disclosing the types of participating bidders, as no information from non-participating bidders is obtained.

Proposition 3 (Optimal auction under convexity)**.** *Consider a standard auction, in which every bidder follows a strictly increasing bidding strategy, and participates whenever his type is above τ. The revenue in this auction is at most*

$$Rev^{\mathcal{M}}(\tau) + n\left(\int_{\tau}^{\overline{v}} \Phi(v)dF(v) - (1-F(\tau))\Phi(\mathbb{E}[V|V \leq \tau])\right). \tag{7}$$

The all-pay auction exhibits the described equilibrium and attains the revenue bound.

Proof. Following the same steps as for the linear case, the revenue is given by

$$n\int_{\underline{v}}^{\overline{v}} m(v)\ \mathrm{d}F(v) = Rev^{\mathcal{M}}(\tau) + n\left(F(\tau)\mathcal{W}_0 + \int_{\tau}^{\overline{v}} \mathcal{W}(v)\ \mathrm{d}F(v) - \mathcal{W}_{0,\tau}\right). \tag{8}$$

Denote H_τ the distribution over posterior beliefs induced by the selected equilibrium (if types above τ participate). Following the law of iterated expectation, we have

$$F(\tau)\mathcal{W}_{0,\tau} + \int_{\tau}^{\overline{v}} \mathcal{W}(v)\ \mathrm{d}F(v) = \int_{\underline{v}}^{\overline{v}} \Phi(v)dH_\tau(v). \tag{9}$$

Define $H_\tau^{\max}$ the distribution over posterior means as follows

$$H_\tau^{\max}(v) = \begin{cases} 0, & \text{if } \underline{v} \leq v < \mathbb{E}[V|V \leq \tau], \\ F(\tau), & \text{if } \mathbb{E}[V|V \leq \tau] \leq v \leq \tau, \\ F(v), & \text{if } \tau < v \leq \overline{v}. \end{cases} \tag{10}$$

The distribution $H_\tau^{\max}$ has a mass point at $\mathbb{E}[V|V \leq \tau]$ and otherwise corresponds to the prior F for values above τ. $H_\tau^{\max}$ is the distribution over posterior beliefs induced by a rule that fully reveals the type of a participating bidder, and otherwise discloses no further information. Because the observer can never distinguish the types of nonparticipating bidders, we have that $H_\tau^{\max}$ is a mean-preserving spread of H_τ. Convexity of Φ thus implies

$$\int_{\underline{v}}^{\overline{v}} \Phi(v)dH_\tau(v) \leq \int_{\underline{v}}^{\overline{v}} \Phi(v)dH_\tau^{\max}(v) = F(\tau)\Phi(\mathbb{E}[V|V \leq \tau]) + \int_\tau^{\overline{v}} \Phi(v)dF(v). \quad (11)$$

Together with (8) and (9) and our previous observation that $\mathcal{W}_{0,\tau} \geq \Phi(\mathbb{E}[V|V \leq \tau])$ the revenue bound (7) follows.

To show the revenue bound (7) can be attained, consider an all-pay auction with an entry fee $\varphi := G(\tau)\tau + \Phi(\tau) - \Phi(\mathbb{E}[V|V \leq \tau])$. We establish the following equilibrium. A bidder enters whenever $v \geq \tau$ and bids

$$\beta(v) = \int_\tau^v x dG(x) + \Phi(v) - \Phi(\tau).$$

Denote $\Pi(v, \widetilde{v})$ the expected utility of a type v when mimicking the strategy of type $\widetilde{v}$. For $\widetilde{v} \geq \tau$ we have that

$$\begin{aligned}\Pi(v, \widetilde{v}) &= G(\widetilde{v})v + \Phi(\widetilde{v}) - \beta(\widetilde{v}) - \varphi \\ &= G(\widetilde{v})v - G(\tau)\tau - \int_\tau^v x dG(x) + \Phi(\mathbb{E}[V|V \leq \tau])\end{aligned}$$

Hence, for all $v, \widetilde{v} \geq \tau$ it follows that

$$\Pi(v, v) - \Pi(v, \widetilde{v}) = \int_{\widetilde{v}}^v (v - x)dG(x) \geq 0.$$

In addition, for all $v < \tau \leq \widetilde{v}$ we have that $\Pi(v, v) = \Phi(\mathbb{E}[V|V \leq \tau]) > \Pi(v, \widetilde{v})$ and, similarly, $\Pi(\widetilde{v}, \widetilde{v}) > \Phi(\mathbb{E}[V|V \leq \tau])$. The all-pay auction induces the distribution over posterior means $H_\tau^{\max}$. Using the first-part of the proof shows that the revenue coincides with (7). □

Part of our analysis of finding the optimal auction amounts to determining the optimal information structure. We maximize (9) over the set of distributions which are a mean-preserving spread of $H_\tau^{\max}$, where $H_\tau^{\max}$ is the distribution over posterior means of a disclosure policy that reveals the true type v if $v \geq \tau$ and lumps all other types on one signal. Put differently, $H_\tau^{\max}$ arises from disclosing the (maximal) information gathered by running the auction. Dworczak and Martini (2019) provide a solution to this problem, which in our case of a convex signaling function Φ yields full disclosure. But notice an important difference to their analysis: the signaling value of non-participating bidders $\mathcal{W}_{0,\tau}$ negatively enters the auctioneer's objective. In the convex case maximizing (9) coincidentally minimizes $\mathcal{W}_{0,\tau}$ and hence the solution obtains.

Proposition 3 shows that the all-pay auction yields maximal revenue when the bidders' signaling concerns are convex. This is not a property of the payment rule in an all-pay auction, but a consequence of the induced revelation of bidders' types. In general, full disclosure leads to a separation of bidding and signaling incentives, and any auction achieves the upper bound on revenue when the entire vector of bids gets disclosed (and the auction retains its separating equilibrium).

Corollary 2 (Revenue Equivalence with Full Bid Disclosure)**.** *Consider a standard auction, in which every bidder follows a strictly increasing bidding strategy, and participates whenever his type is above τ. If the auction discloses all bids, revenue coincides with* (7)*.*

Any standard auction with full bid disclosure and strictly increasing bid functions of participating bidders leads to the same inference as in the all-pay auction. Hence, a bidder's strategy is pinned down by payoff equivalence, as in the case without signaling concerns. The only difference is that the signaling value is added to the equilibrium bid β, i.e. $\beta(v) = \beta^{\mathcal{M}}(v) + \Phi(v) - \Phi(\mathbb{E}[V|V \leq \tau])$, where $\beta^{\mathcal{M}}$ denotes the bidding strategy for the respective auction without signaling concerns.

Example 3 (First-price auction with full bid disclosure)**.** Reconsider the first-price auction as in Example 1, but now with a convex signaling value $\Phi(\cdot)$. To induce the participation cut-off τ the auctioneer sets the reserve bid to $r(\tau) = \tau + \Phi(\tau)/G(\tau) - \Phi(\mathbb{E}[V|V \leq \tau])/G(\tau)$. The auction gives rise to an equilibrium where bidders use the following increasing bidding strategy

$$\beta(v) = \begin{cases} 0, & \underline{v} \leq v < \tau, \\ \frac{G(v)v - G(\tau)\tau}{G(v)} + \int_\tau^v x d\frac{G(x)}{G(v)} + \frac{\Phi(v)}{G(v)} - \frac{\Phi(\mathbb{E}[V|V \leq \tau])}{G(v)}, & \tau \leq v \leq \overline{v}. \end{cases}$$

As in the case of linear signaling concerns, we can look for the optimal participation threshold $\tau^\star$ in the auction. The bidders' signaling concerns induce the auctioneer to reduce the threshold for participation below the optimal level without signaling.

Corollary 3. *Assume virtual valuations are increasing. In the revenue maximizing auction more bidders participate than if bidders had no signaling concern, i.e. $\tau^\star < \tau^{\mathcal{M}}$.*

Proof. It follows similar steps as the proof of Corollary 1. □

5 Concave Signaling Concerns

In this section, we consider concave signaling concerns. Following our earlier analysis, the revenue of a standard auction at the equilibrium with strictly increasing strategies is

$$Rev^{\mathcal{M}}(\tau) + n \cdot \left(\int_{\underline{v}}^{\overline{v}} \Phi(v) \mathrm{d}H_\tau(v) - \mathcal{W}_{0,\tau} \right).$$

Information disclosure directly affects the second and third components. To extract more signaling value from participating bidders, the auction should reveal less information. At the same time, revealing less information *increases* the outside option, i.e., the signaling value from non-participation $\mathcal{W}_{0,\tau}$.

More precisely, consider first the signaling value, given by the term $\int \Phi(v)\mathrm{d}H_\tau(v)$. Information disclosure amounts to choosing a distribution over posterior means H_τ, where H_τ is a mean-preserving spread of the *minimal information* distribution $H_\tau^{\min}$. The latter distribution is given by the policy that discloses only the winner's identity, if the auction has a winner. Because the inference function Φ is concave, the signaling value is maximal for $H_\tau = H_\tau^{\min}$. In words, the signaling value is maximal when the auction reveals no additional information beyond the winner's identity (which has to be revealed by assumption). Next consider the term $\mathcal{W}_{0,\tau}$, the expected inference of a bidder who does not participate. As before, $\mathcal{W}_{0,\tau}$ is minimal if the auction reveals whether a bidder participated, for instance via charging an entry fee. In contrast to the convex case, maximizing the signaling value and minimizing $\mathcal{W}_{0,\tau}$ conflict with each other. In particular, revealing whether a bidder participated in the auction reveals *more* information than only revealing the winner's identity. In general this yields a non-trivial trade-off without a straightforward solution.

Define H_τ^P the distribution over posterior means that arises from a disclosure policy which reveals the winner's identity and whether a bidder participated in an auction with participation threshold τ. Note that $H_{\underline{v}}^P = H_{\underline{v}}^{\min}$. Moreover, define

$$Rev^P(\tau) = Rev^{\mathcal{M}}(\tau) + n\Big(\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H_\tau^P(v) - \mathbb{E}[V|V < \tau]\Big). \tag{12}$$

Proposition 4 (Optimal auction under concavity)**.** *Consider a standard auction, in which every bidder follows a strictly increasing bidding strategy, and participates whenever his type is above τ.*

(i) If participation is fully observable we have that $Rev(\tau) \leq Rev^P(\tau)$.

(ii) There exists $\tau' > \underline{v}$ such that $Rev(\tau) \leq Rev^P(\tau)$ whenever $\tau < \tau'$.

Proof. Following our previous argument, the revenue in an auction where all bidders with valuation above τ participate is

$$Rev(\tau) = Rev^{\mathcal{M}}(\tau) + n\left(\int_{\underline{v}}^{\overline{v}} \Phi(v)\ \mathrm{d}H_\tau(v) - \mathcal{W}_{0,\tau}\right).$$

With observable participation we have $\mathcal{W}_{0,\tau} = \mathbb{E}[V|V < \tau]$, independent of the specific auction format. Furthermore, because Φ is concave the signaling value $\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H_\tau(v)$ is maximal when only the winner's identity is disclosed in addition, i.e., when $H_\tau = H_\tau^P$. This proves (i).

To prove (ii), note that for every auction in which participation is not fully observable we have $\mathcal{W}_{0,\tau} > \mathbb{E}[V|V < \tau]$. Moreover, for $\tau = \underline{v}$ we have $\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H_{\underline{v}}(v) \leq \int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^{\min}(v)$ by concavity of Φ. Hence, $Rev(\underline{v}) < Rev^P(\underline{v})$. Both $\mathcal{W}_{0,\tau}$ and $\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H_\tau(v)$ are continuous in the participation threshold τ, hence (ii) follows by continuity from the previous assertion.

□

Proposition 4 derives an upper bound for the revenue if either participation is fully observable, or many bidder types participate. This bound stems from a (hypothetical) auction that discloses only the winner's identity, and a list of participating bidders. However to guarantee existence of a separating equilibrium the auction necessarily uses a discriminatory payment rule, which ultimately reveals information beyond the winner's identity. To see this, note that if we had only payments for winner and loser's, there would not exist an equilibrium with strictly increasing bidding strategies. This suggests the revenue bound is too large. We show in the next proposition that the revenue bound is indeed tight.

Proposition 5. *For every $\varepsilon > 0$ there is an auction that exhibits an equilibrium with strictly increasing bidding strategies, for which $Rev(\tau) > Rev^P(\tau) - \varepsilon$.*

Proof. Consider the following variant of a first-price auction: Bidders submit non-negative bids, the bidder submitting the highest bid wins and with exogenous probability $1-\varepsilon$ makes no payment, but pays his own bid with probability ε. Every bidder has to pay the entry fee φ before submitting a bid. The expected profit of a bidder of type v upon entering the auction and bidding as if he was type v' is

$$\Pi(v|v') = G(v')(v - \varepsilon\beta(v')) + \mathcal{W}(v') - \varphi.$$

From the first-order condition we get

$$\beta^\star(v) = \frac{1}{\varepsilon}\beta^{\mathcal{M}}(v) + \frac{\mathcal{W}(v) - \mathcal{W}(\tau)}{\varepsilon G(v)},$$

where $\beta^{\mathcal{M}}$ is the bidding strategy in a first-price auction without signaling and entry fee that induces only types above τ to participate. Note that we have used the fact that $\beta(\tau) = 0$, which is true because in equilibrium type τ only wins the auction when no other bidder enters and is thus not willing to bid a strictly positive amount. Furthermore, to induce participation for all types above τ the fee has to satisfy

$$\mathcal{W}_{0,\tau} = G(\tau)\tau + \mathcal{W}(\tau) - \varphi \quad \Leftrightarrow \quad \varphi = G(\tau)\tau + \mathcal{W}(\tau) - \mathcal{W}_{0,\tau}.$$

The revenue is thus given by

$$\begin{aligned} Rev^{\varepsilon}(\tau) &= n(1-F(\tau))\varphi + (1-F^n(\tau))\mathbb{E}\big[\varepsilon\beta^{\star}(V_1)|V_1 \geq \tau\big] \\ &= n(1-F(\tau))\big(G(\tau)\tau + \mathcal{W}(\tau) - \mathcal{W}_{0,\tau}\big) + \int_{\tau}^{\overline{v}} \beta^{\mathcal{M}}(s) + \frac{\mathcal{W}(s)-\mathcal{W}(\tau)}{G(s)} \mathrm{d}F^n(s) \\ &= Rev^{\mathcal{M}}(\tau) + n(1-F(\tau))(\mathcal{W}(\tau) - \mathcal{W}_{0,\tau}) + n\int_{\tau}^{\overline{v}} \big(\mathcal{W}(s) - \mathcal{W}(\tau)\big) f(s)\mathrm{d}s \\ &= Rev^{\mathcal{M}}(\tau) + n\left[F(\tau)\mathcal{W}_{0,\tau} + \int_{\tau}^{\overline{v}} \mathcal{W}(s) f(s)\mathrm{d}s - \mathcal{W}_{0,\tau}\right]. \qquad (13) \end{aligned}$$

Note that

$$\begin{aligned} \mathcal{W}(s) &= \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{F_k(s|\tau)\cdot\Big(\varepsilon\Phi(s) + (1-\varepsilon)\Phi\big(v_{W,k}\big)\Big) \\ &\qquad + (1-F_k(s|\tau))\cdot\Big(\varepsilon\int_s^{\overline{v}} \Phi\big(v_{L,k}(x)\big)\mathrm{d}F_k(x|\tau) + (1-\varepsilon)\Phi\big(v_{L,k}\big)\Big)\Big\} \\ &= \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{F_k(s|\tau)\Phi\big(v_{W,k}\big) + (1-F_k(s|\tau))\,\Phi\big(v_{L,k}\big)\Big\} \\ &\quad -\varepsilon\sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{F_k(s|\tau)\Big(\Phi(s) - \Phi\big(v_{W,k}\big)\Big) \\ &\qquad +(1-F_k(s|\tau))\Big(\int_s^{\overline{v}} \Phi\big(v_{L,k}(x)\big)\mathrm{d}F_k(x|\tau) - \Phi\big(v_{L,k}\big)\Big)\Big\}, \end{aligned}$$

where $\mathcal{B}_{n-1,F(\tau)}(k-1) \coloneqq \binom{n-1}{k-1}F(\tau)^{n-k}(1-F(\tau))^{k-1}$, $v_{W,k} \coloneqq \mathbb{E}[V_1|V_k \geq \tau > V_{k+1}]$, $v_{L,k} \coloneqq \mathbb{E}[V|V_1 > V \geq V_k \geq \tau > V_{k+1}]$, $v_{L,k}(s) \coloneqq \mathbb{E}[V|V_1 = s, s > V \geq V_k \geq \tau > V_{k+1}]$ and $F_k(v|\tau) \coloneqq \big(\frac{F(v)-F(\tau)}{1-F(\tau)}\big)^{k-1}$ for all $k = 1, \ldots, n$ denotes the conditional probability of the maximum of the $k-1$ other bids if all of these exceed τ. Hence,

$$\begin{aligned}\int_{\tau}^{\overline{v}} \mathcal{W}(s)f(s)\mathrm{d}s &= \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{\tfrac{1}{k}\Phi\big(v_{W,k}\big) + \tfrac{k-1}{k}\Phi\big(v_{L,k}\big)\Big\} \\ &\quad -\varepsilon \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{ \int_{\tau}^{\overline{v}} F_k(s|\tau)\Big(\Phi(s) - \Phi\big(v_{W,k}\big)\Big) f(s)\mathrm{d}s \\ &\quad + \int_{\tau}^{\overline{v}} (1-F_k(s|\tau))\Big(\int_{s}^{\overline{v}} \Phi\big(v_{L,k}(x)\big)\mathrm{d}F_k(x|\tau) - \Phi\big(v_{L,k}\big)\Big) f(s)\mathrm{d}s\Big\} \\ &= \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{\tfrac{1}{k}\Phi\big(v_{W,k}\big) + \tfrac{k-1}{k}\Phi\big(v_{L,k}\big)\Big\} - \varepsilon C,\end{aligned}$$

where by concavity of Φ and compactness of the support of the bidders' valuations we have $C > 0$ and finite. Plugging the above expression back into (13) and noting that $\mathcal{W}_{0,\tau} = \mathbb{E}[V|V < \tau]$ (because participation is observable) yields $Rev^{\varepsilon}(\tau) = Rev^{P}(\tau) - \varepsilon C \to Rev^{P}(\tau)$ as $\varepsilon \to 0$. □

Propositions 4 and 5 reveal a fundamental difference between pure information design and mechanism design with information disclosure. In information design the sender has costless access to all information, while in mechanism design the information is privately held by the agents. In our setting the auctioneer benefits from revealing additional information, namely whether a bidder participated. Such disclosure reduces the value of the bidders' outside option, i.e., the expected signaling value from non-participation. The reduced outside option allows the auctioneer to extract more revenue from bidders. The benefit from revealing the bidders' participation is larger, the lower the participation threshold τ. For $\tau \approx \underline{v}$ it becomes optimal to disclose only the winner's identity and all participation decisions.

6 Conclusion

In this paper, we analyze optimal auctions in an independent private values environment with signaling, i.e. where bidders' care about the perception of a third party. To keep the analysis concise and tractable we focused on linear, convex and concave signaling concerns. The results of Dworczak and Martini (2019) indicate that the disclosure policy maximizing the signaling value for general preferences is a combination of intervals where the type is fully disclosed and intervals on which types are fully pooled. However, it is not straightforward to translate such a disclosure rule into a payment rule for a standard auction. Understanding the polar cases of convexity and concavity allows us to address a preference for the aftermarket that has been studied in the literature on information design, namely where Φ is a distribution function.[12] Under regularity conditions, there is a unique value $\hat{v}$ such that Φ is convex on $[\underline{v}, \hat{v}]$ and concave on $[\hat{v}, \overline{v}]$. Hence, if the participation threshold is sufficiently high we are back in the concave case. Otherwise, maximizing revenue calls for revealing low bids while at the same time pooling higher bids.

A natural follow-up question concerns the extent to which our results can be generalized to a richer class of mechanisms. Beyond mechanism design, that could provide new and exciting perspectives in applied fields such as advertising, marketing science and industrial organization. For instance, the literature on conspicuous consumption (e.g., Bagwell and Bernheim (1996) and Corneo and Jeanne (1997)) studies product markets where the consumption value depends on the belief of a social contact. A profit-maximizing auctioneer will try to exploit this by tailoring its product line and prices to the information revealed by the consumer's choice. That will lead to new insights about consumer behavior and firm strategies that exploit signaling concerns.[13]

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[12] Rayo and Segal (2010) study a sender–receiver game. The receiver chooses a binary action $a \in \{0, 1\}$. Choosing $a = 0$ yields a fixed utility r which is distributed according to some distribution function G. Choosing $a = 1$ yields utility θ, where θ is the sender's private information. The sender wants to maximize the probability of choosing $a = 1$. Hence, the sender's reduced form utility is $G(\mathbb{E}(\theta))$.

[13] See Rayo and Segal (2010) and Friedrichsen (2018) for analyses into that direction.

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