Discussion Paper Series – CRC TR 224

Discussion Paper No. 190
Project B 03

Concentration Screens for Horizontal Mergers

Volker Nocke [1]
Michael D. Whinston [2]

July 2020

[1] Department of Economics, University of Mannheim, and School of Economics, University of Surrey; email: volker.nocke@gmail.com
[2] Department of Economics and Sloan School of Management, MIT; email: whinston@mit.edu

Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Concentration Screens for Horizontal Mergers*

Volker Nocke† Michael D. Whinston‡

July 1, 2020

First version: September 2019

Abstract

Concentration-based screens for horizontal mergers, such as those employed in the US DOJ and FTC *Horizontal Merger Guidelines*, play a central role in merger analysis. However, the basis for these screens, in both form and level, remains unclear. We show that there is both a theoretical and an empirical basis for focusing solely on the change in the Herfindahl index, and ignoring its level, in screening mergers for whether their unilateral effects will harm consumers. We also argue, again both theoretically and empirically, that the levels at which the presumptions currently are set may be too lax, especially with regards to safe harbors.

1 Introduction

Concentration measures play a central role in merger analysis. The current Department of Justice and Federal Trade Commission *Horizontal Merger Guidelines* state various presumptions – both safe harbor presumptions and presumptions of anticompetitive effects – based on the level of the post-merger Herfindahl index and the change that the merger induces in that index (both naively computed, by adding the merging firms' pre-merger shares together). While many other factors come into play in the agencies' analyses, these concentration-based presumptions have a significant impact on agency decisions, especially in screening mergers for further review.[1] Surprisingly, perhaps, the basis for these presumptions in both form and level remains unclear.[2]

In this paper, we examine these presumptions, focusing on a merger's likely unilateral effects.[3] We make two points: First, we show that there is both a theoretical and an empirical basis for focusing solely on the change in the Herfindahl index, and ignoring its level, in screening mergers for whether their unilateral effects will harm consumers. This point has been recognized by others before us (e.g., Shapiro, 2010; Froeb and Werden, 1998) and, indeed, is made in the 2010 *Guidelines* for the case of mergers in differentiated product industries, but is still not yet widely appreciated.[4] Here we go further in demonstrating why this is so theoretically and in providing empirical evidence in support of this proposition. Second, we argue, again

*We thank Nate Miller and Matt Weinberg for generously providing the data we use in Section 4, and for responding so helpfully to our many questions and requests. We also thank Carl Shapiro for sharing his knowledge of *Merger Guidelines* history, Joe Farrell, Luke Froeb, Bernhard Ganglmair, Louis Kaplow, John Kwoka, Craig Peters, Patrick Rey, Nancy Rose, Steve Salop, Mark Satterthwaite, Nicolas Schutz, Carl Shapiro, Paolo Somaini, John Vickers, Ali Yurukoglu and seminar audiences at Berkeley, Mannheim, Stanford, the 2019 Northwestern University antitrust conference, and the New Zealand Competition Commission for comments, and Federico Innocenti and Zi Yang Kang for their excellent RA work. Nocke gratefully acknowledges financial support from the German Research Foundation (DFG) through CRC TR224 (Project B03).

†Department of Economics, University of Mannheim, and School of Economics, University of Surrey; email: volker.nocke@gmail.com

‡Department of Economics and Sloan School of Management, MIT; email: whinston@mit.edu

[1] Moreover, when the agencies challenge mergers in court, these concentration measures are frequently emphasized by the agencies, and often factor significantly in courts' decisions even if other, more dispositive information may be available at that point.

[2] See Schmalensee (1987, pp. 47-50) for one previous discussion of the *Guidelines* concentration thresholds.

[3] Miller and Weinberg (2017) provide evidence that horizontal mergers may also lead to coordinated effects, as noted in the agencies' *Guidelines*.

[4] Shapiro (2010, p. 63, fn 53) notes that "There is no good link between the level of the HHI and unilateral price effects with differentiated products." See also his discussion on pp. 68-9. The 2010 *Guidelines* (p. 21) note that "The Agencies rely

both theoretically and empirically, that the levels at which the presumptions currently are set may be too lax, especially in creating a safe harbor based on the post-merger Herfindahl index, at least unless one is crediting large synergies or a significant presumption that entry, repositioning, or other factors would prevent any anti-competitive effects of the typical merger.[5]

Throughout, we focus on the use of concentration measures at the initial screening stage, when information on margins, diversion ratios and cost synergies is unlikely to be known. We therefore adopt the perspective that there is some standard presumption regarding cost synergies and elasticities that the agency uses at the screening stage.[6] As we discuss later, the standard presumptions regarding elasticities may depend on how narrowly markets are being defined.

The paper is organized as follows. In Section 2, we review the history of concentration screens in the various versions of the *Horizontal Merger Guidelines*.[7]

In Section 3, we examine three canonical models of competition in which one might hope that there would be a clear relationship between equilibrium concentration measures and the effect of a merger on consumer surplus: the Cournot model of output/capacity competition in homogeneous good industries, and the multinomial logit and constant elasticity of substitution models of differentiated product price competition. As in Werden (1996), Froeb and Werden (1998), and Farrell and Shapiro (2010), our focus in this analysis is on the level of marginal cost reduction (the "synergy" or "efficiency gain") required to prevent a merger from harming consumers.[8] We show that this critical level of efficiencies depends in these models on the merging firms' shares, but not on the shares of non-merging firms. In fact, for mergers between symmetric firms in the Cournot model, given the market demand elasticity, the required synergy depends solely on the (naively-computed) change in the Herfindahl index, and not at all on its post-merger level. We also examine how the levels of the required synergies depend on the merging firms' shares. In the Cournot model, with synergies of 3% and common levels of market demand elasticity, consumer harm occurs when the merging firms' shares are much like those in the 1968 *Guidelines*' thresholds. In contrast, the threshold levels of merger-induced change in the Herfindahl index are more lenient, but still restrictive, in the multinomial logit and constant elasticity of substitution models of price competition.

The theoretical models of Section 3 are certainly special. In Section 4, we provide an empirical investigation of how mergers' effects on consumers are related to concentration measures in one industry. We focus on possible mergers in brewing. Using the estimated demand system in Miller and Weinberg (2017), a random-coefficient nested logit demand system that is not covered by our theoretical analysis, and treating each local market separately, we compute for various hypothetical (local) mergers the efficiency improvement that would be required to prevent consumer harm. The results show that, as in the models of Section 3, the required efficiency gain is strongly related to the (naively computed) change in the Herfindahl index and not very related to the level of the post-merger Herfindahl (once one conditions on the change in the Herfindahl). The levels of the merger-induced change in the Herfindahl necessary to prevent consumer harm in these markets generally fall in the range of those we derive in the theoretical models of Section 3. The levels required indicate that if the typical merger in these markets would result in a 3% efficiency gain then many of these hypothetical mergers falling into the current safe harbor, and in particular those with post-merger Herfindahl levels below 1500, would be likely to harm consumers. As well, for a 3% percent efficiency gain, the results indicate that there should perhaps also be a stronger concern expressed in the *Guidelines* about mergers whose post-merger Herfindahl levels are between 1500-2500 and that change the Herfindahl by more than 200.

In Section 5 we provide a discussion of our results and consider whether there are other ways to justify the current *Guidelines'* focus on the level of the Herfindahl index.

much more on the value of diverted sales than on the level of the HHI for diagnosing unilateral price effects in markets with differentiated products." The 2006 *Commentary* on the 1992 *Guidelines* makes a similar point on p.16, noting that for unilateral effects "[t]he concentration of the remainder of the market often has little impact on the answer...."

[5]Kwoka (2017) reaches a similar conclusion about the safe harbor in a study examining outcomes identified in retrospective studies of 9 mergers.

[6]In some cases, information on margins, diversion ratios and cost synergies may still not be known after further investigation.

[7]Throughout, we focus on the U.S. agencies' screening criteria, but similar points apply to many other jurisdictions. For example, the European Commission also has horizontal merger guidelines that adopt thresholds based on the Herfindahl level and its change.

[8]Our analysis is thus complementary to that in Nocke and Schutz (2019) who show that, absent efficiencies, the merger-induced loss in consumer surplus is approximately proportional to the naively-computed change in the Herfindahl index, where the approximations are taken around small market shares and around monopolistic competition conduct.

Anticompetitive presumptions in markets with CR4 >75%:

Acquiring Firm	Acquired Firm
4%	4% or more
10%	2% or more
15% or more	1% or more

Anticompetitive presumptions in markets with CR4 < 75%:

Acquiring Firm	Acquired Firm
5%	5% or more
10%	4% or more
15%	3% or more
20%	2% or more
25% or more	1% or more

Figure 1: Anticompetitive presumptions in the 1968 *Merger Guidelines*

We conclude in Section 6 and note a number of areas of research that would help further strengthen the basis for concentration-based horizontal merger screening thresholds. Overall, given our results, for evaluating unilateral effects of horizontal mergers we see it as likely that the form of current concentration screens should be modified to emphasize more the change in the Herfindahl index and less its post-merger level, as probable that current safe harbors are allowing mergers to proceed that lead to consumer harm, and as possible that current concentration-based presumptions should also be strengthened for certain other categories of mergers.

2 History of the Merger Guideline Concentration Screens

The first version of the *Merger Guidelines* – issued solely by the Department of Justice – appeared in 1968, shortly after the 1963 Philadelphia National Bank decision and roughly contemporaneous with the *Neal Report* on antitrust policy. As described by Shapiro (2010), the 1968 *Guidelines* approach toward horizontal mergers was focused entirely on preventing increases in concentration and it proposed concentration thresholds that were markedly more stringent than those today. Those presumptions, summarized in Figure 1, were largely dependent on the shares of the two merging firms. For mergers in markets in which the four-firm concentration ratio was above 75%, a merger would be blocked if a firm with a 4% share wanted to acquire another firm with a 4% share, and a firm with a 15% share could not acquire a firm with a 1% share.[9] For markets with a four-firm concentration ratio below 75%, the thresholds were not much more lenient: a merger between two 5% firms would be blocked.

The DOJ's 1982 *Guidelines* represented a marked change, with the Herfindahl index (HHI) replacing the four-firm concentration ratio, but more importantly with the level of market concentration having much more importance, and with much more lenient standards.[10] Figure 2 depicts the 1982 screening thresholds, which depend on the naively-computed post-merger level of the HHI (measured out of 10,000) and the naively-computed merger-induced change in HHI (labelled "ΔHHI" in the figure). For example, a merger between two 5% share firms, which would lead to a 50 point increase in the HHI, rather than being challenged became presumptively legal. More specifically, mergers in "unconcentrated" markets with a post-merger HHI below 1000 fell into the (moderately shaded) green zone of the figure, representing mergers that were unlikely to be challenged. In "moderately concentrated" markets, with post-merger Herfindahl indices between 1000 and 1800, a merger was "more likely than not" to be challenged if it fell into the (lightly shaded) yellow zone because △HHI was above 100, while mergers for which △HHI was below 100 fell into the green zone. In "highly concentrated markets" with a post-merger HHI above 1800, mergers whose △HHI were below 50 fell into the green zone, those with △HHI between 50 and 100 fell into the yellow zone, while those with △HHI above 100 fell into the (darkly shaded) red zone that meant that the DOJ was "likely to challenge." The 1992 *Horizontal Merger Guidelines*, issued for the first time jointly by the DOJ and FTC, maintained these presumptions.[11]

[9] Somewhat curiously, the 1968 screens depended on which merger partner was the acquirer.

[10] Shapiro (2010) describes well the other significant innovations in the 1982 *Guidelines*, and the continuing increase over time in consideration of other market factors in analyzing prospective mergers. One factor that may have ameliorated to some extent the more lenient standards was the introduction in the 1982 *Guidelines* of the Horizontal Monopolist Test for market definition, which may have led to narrower market definitions.

[11] The 1992 *Guidelines* did change "more likely to be challenged than not" for the yellow zone to "potentially raise significant

Figure 2: Screening thresholds in the 1982 *Horizontal Merger Guidelines*

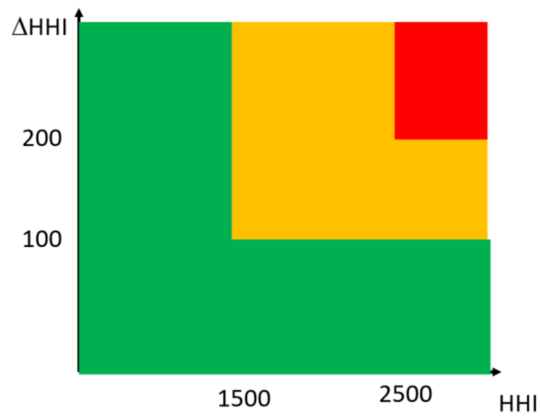

Figure 3: Screening thresholds in the 2010 *Horizontal Merger Guidelines*

Most recently, the 2010 revision of the *Horizontal Merger Guidelines* relaxed these standards. As depicted in Figure 3, it raised the safe harbor level of the HHI from 1000 to 1500, the threshold for considering a market highly concentrated from 1800 to 2500, and the critical levels of ΔHHI in highly concentrated markets from 50 to 100 for the safe harbor, and from 100 to 200 for the presumption of harm (thresholds in moderately concentrated markets were not changed).[12]

Notably, while the theoretical and empirical basis for neither the 1968 *Guidelines* nor the 1982 changes were ever clearly laid out by the agencies, the reason for the change in 2010 was made explicit: the aim was to enhance transparency by making the thresholds conform more closely with actual agency practice (see Shapiro, 2010). So, once again, no explicit economic rationale was offered.

Figure 4 depicts actual FTC enforcement results for those horizontal mergers that received second requests from 1996-2011.[13] A merger received an "enforcement action" if the FTC sought to block or modify it. Evident in the figure is both the strong effect of the level of concentration on the likelihood of enforcement

competitive concerns." The 1982 *Guidelines* also had a presumption of anticompetitive harm, eliminated in the 1992 *Guidelines*, if the acquirer was the leading firm in the industry, had a share of at least 35%, was more than twice as large as the second largest firm, and was acquiring a firm with at least a 1% share.

[12] At the same time, the 2010 revision continued the move of the *Guidelines* away from rigid structural presumptions and toward reliance on a range of evidence of potential anticompetitive effects in making final determinations about whether to initiate an enforcement action.

[13] See "Horizontal Merger Investigation Data: Fiscal Years 1996-2011," Federal Trade Commission, January 2013; available at https://www.ftc.gov/os/2013/01/130104horizontalmergerreport.pdf.

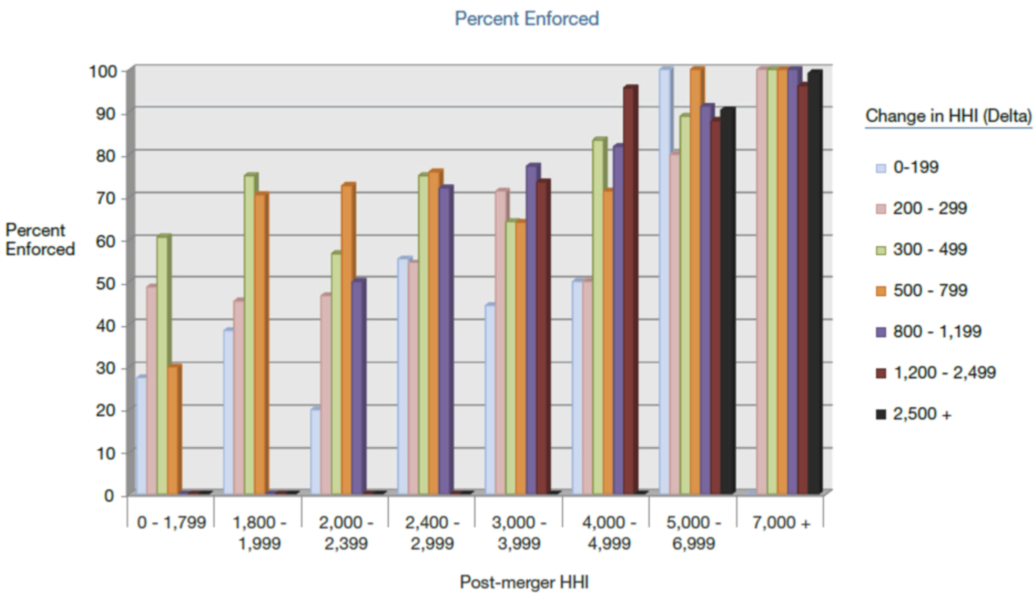

Figure 4: FTC horizontal merger enforcement frequencies for mergers receiving a second request, as a function of the post-merger level of the Herfindahl index and the merger-induced change in the Herfindahl index, 1996-2011. [Source: Federal Trade Commission (2013)]

and the fact that many mergers that fell into the "red zone" anticompetitive presumption nonetheless were approved in the end (without conditions). Because the figure does not break out the change in the Herfindahl index for levels below 200, nor the level of the Herfindahl below 1800, it does not provide evidence on how the FTC treated mergers in the green zone safe harbor that were issued a second request. However, the same FTC report indicates (see Federal Trade Commission, 2013, Table 3.1), that of the 1359 second requests considered in Figure 4, only 29 involved mergers with ΔHHI less than 100 and 11 of those where in markets with a post-merger HHI above 1800 and so may have been in the yellow zone. While the report leaves unclear the number of second requests for mergers falling into the part of the green zone representing unconcentrated markets in which $\triangle$HHI is above 100, this low number likely reflects an important asymmetry in the treatment given to the red zone anticompetitive presumption versus the green zone safe harbor: mergers that fall into the safe harbor, perhaps because of a low post-merger Herfindahl index, are typically simply allowed without further scrutiny, while those that fall into the anticompetitive presumption category are scrutinized further and may be allowed based on other factors.

3 Theoretical Analysis

Analysis of horizontal mergers focuses on weighing the risk of anticompetitive reductions in competition against the prospect for merger-related efficiencies. Concentration screens for mergers must therefore aim to capture, based on firms' market shares, the likely balance of these two effects for the "typical" merger. Since absent any efficiency gains a horizontal merger will generally (weakly) increase prices, any merger screen aimed at preventing consumer harm that would allow some mergers and block others must implicitly be relying on some presumption of the efficiency gain that, on average, should be credited to a typical merger. As such, we focus throughout the paper on how the required efficiency gain is related to measures of concentration.

In general, models of oligopolistic competition need not produce a clean relationship between the effect of a merger and market shares, let alone concentration measures such as the Herfindahl index. In this section, however, we focus theoretically on three models that do, the Cournot model of output/capacity competition in a homogeneous good industry and the multinomial logit and constant elasticity of substitution models of price competition.

3.1 Mergers in the Homogeneous-Goods Cournot Model

Consider an industry with a set $\mathcal{F}$ of firms producing a homogeneous good with constant returns to scale and competing in a Cournot fashion. Let c_f denote the (constant) marginal cost of firm $f \in \mathcal{F}$, and $P(Q)$ inverse demand, where Q is aggregate output. We impose standard assumptions ensuring that there exists a unique Nash equilibrium in quantities: that for any Q such that $P(Q) > 0$, we have $P'(Q) < 0$ and $P'(Q) + QP''(Q) < 0$; moreover, $\lim_{Q\to\infty} P(Q) = 0$.

Let Q^* denote the pre-merger aggregate equilibrium output. For simplicity, we assume that all firms in $\mathcal{F}$ are active before the merger in that $\max_{f\in\mathcal{F}} c_f < P(Q^*)$. The pre-merger market share of firm f, s_f, satisfies

$$s_f = -\frac{P(Q^*) - c_f}{Q^* P'(Q^*)},$$

and the pre-merger Herfindahl index is given by $H = \sum_{f\in\mathcal{F}} s_f^2$.

Consider a merger $M = \{m, n\}$ between firms m and n. Given their pre-merger market shares s_m and s_n, their combined pre-merger market share is $s_M \equiv s_m + s_n$. The naively-computed post-merger Herfindahl index is given by

$$\overline{H} = s_M^2 + \sum_{f\in\mathcal{F}\setminus M} s_f^2,$$

and the naively-computed merger-induced change in the index by $\Delta H \equiv \overline{H} - H = 2s_m s_n$. For reasons that will become clear later, let $H_M \equiv (s_m^2 + s_n^2)/s_M^2$ denote the *within-merger* Herfindahl index, whose value lies between 1/2 and 1, and let

$$c_M = \frac{s_m c_m + s_n c_n}{s_M}.$$

denote the output-weighted average marginal costs of the merger partners prior to the merger. We denote the merged firm's post-merger marginal cost by $\overline{c}_M$.

We seek to relate the merger-induced efficiency gains necessary to make the merger have no effect on consumer surplus — that is, to be "CS-neutral" — to the pre-merger market structure.[14] Recall from Farrell and Shapiro (1990) (see also Nocke and Whinston, 2010) that merger M is CS-neutral if and only if

$$P(Q^*) - \overline{c}_M = [P(Q^*) - c_m] + [P(Q^*) - c_n]. \tag{1}$$

It is instructive to begin with the simple case in which the two merger partners are symmetric: $c_m = c_n \equiv c_M$ and thus $s_m = s_n \equiv s_M/2$. Using equation (1), the merger is CS-neutral if the fractional change in the merger partners' marginal cost satisfies

$$\frac{c_M - \overline{c}_M}{c_M} = \frac{P(Q^*) - c_M}{c_M}. \tag{2}$$

From the merger partners' pre-merger first-order conditions, we have

$$c_M = P(Q^*)\left[1 - \frac{s_M}{2\epsilon}\right],$$

where $\epsilon \equiv -P(Q^*)/[Q^* P'(Q^*)]$ is the pre-merger price elasticity of demand. Substituting for c_M on the right-hand side of equation (2), we obtain

$$\frac{c_M - \overline{c}_M}{c_M} = \frac{\frac{s_M}{2\epsilon}}{1 - \frac{s_M}{2\epsilon}} = \frac{\sqrt{\frac{\Delta H}{2}}}{\epsilon - \sqrt{\frac{\Delta H}{2}}}. \tag{3}$$

[14] Under the regularity conditions we assume, a reduction in a firm's marginal cost expands output and lowers price. Thus, any larger synergy than that required for CS-neutrality will result in the merger benefiting consumers, while any lower synergy will result in the merger harming consumers.

That is, for a given demand elasticity, the required efficiencies are perfectly related to and increasing in the naively-computed *change* in the Herfindahl index, and *completely independent of the level* of the Herfindahl index. Any relationship between consumer harm and the level of the Herfindahl index would therefore need to come through a relationship between the Herfindahl and the elasticity of demand, but in general there is no clear theoretical relationship between the two.[15]

The change in the Herfindahl required to prevent harm to consumers at various levels of the market demand elasticity and efficiency gain are also striking if one views the merging firms achieving a 5% synergy as fairly optimistic for the typical horizontal merger. Table 1 shows these levels, as well as the corresponding market share levels for the merging firms. For example, in a market with a demand elasticity of 1.5, a merger of symmetric firms that results in a 5% synergy would lower consumer surplus if the (naively-computed) change in the Herfindahl exceeds 102, which corresponds to the merging firms having roughly a 7% share. Were the industry symmetric, that would be a market with 13 firms. With a 3% synergy the change in the Herfindahl would need to be below 38 to prevent consumer harm, regardless of the level of the post-merger Herfindahl. This is a level similar to that in the 1968 *Guidelines*. Still, in markets in which the elasticity of demand reaches 2.5, with a 5% synergy some mergers that fall into the anticompetitive presumption category of the 2010 *Guidelines* because they have a post-merger Herfindahl above 2500 and a change in the Herfindahl above 200 would actually be beneficial for consumers (if $\Delta H < 283$).

Table 1: Maximal Level of Individual Shares and ΔH (*10,000) To Prevent Consumer Harm for Various Levels of Cost Synergy in the Cournot Model

		Cost Synergy:						
Demand Elasticity		1%	2%	3%	4%	5%	7.5%	10%
1	Individual shares	1	2	3	4	5	7	9
	ΔH	2	7	17	30	45	97	165
1.5	Individual shares	1	3	4	6	7	10	14
	ΔH	4	17	38	67	102	219	372
2	Individual shares	2	4	6	8	10	14	18
	ΔH	8	30	68	118	181	389	661
2.5	Individual shares	2	5	7	10	12	17	23
	ΔH	12	48	106	184	283	608	1033

Importantly, Table 1 also shows that if elasticity information is available, it should be used to adjust the screening thresholds for the required change in the Herfindahl.[16] This relation between screening thresholds and the demand elasticity also indicates how these screening thresholds should interact with market definition, since the narrower the "market," the higher the market demand elasticity is likely to be.

Proposition 1 shows how condition (3) generalizes to the case of mergers between asymmetric firms:[17,18]

[15] Under the standard regularity conditions we assume, an increase in output lowers the elasticity of demand. However, as is well known, there is no clear relationship between the level of the Herfindahl index and the level of output in a market. For example, an increase in the number of firms will raise output and lower the Herfindahl index, but a reduction in cost for the most efficient firm in the market will raise output but increase the Herfindahl index. As well, the elasticity of demand may affect the number of active firms and thus the Herfindahl index.

[16] Note that there is a relation between the market demand elasticity ϵ and the pre-merger HHI *given* margins: $(p-c)/p = H/\epsilon$, where c is the industry-level average marginal cost. However, recall that we take the perspective that margins are not available at the screening stage, so other *a priori* information on likely elasticities would need to be used here.

[17] Froeb and Werden (1998) derive an equivalent expression. Ordover, Sykes, and Willig (1982) derive an expression for the required cost reduction as a percentage of the pre-merger price.

[18] Note that with asymmetric merger partners this notion of merger-induced efficiencies differs from the notion of "synergies" in Farrell and Shapiro (2010), which only counts reductions in marginal cost below the pre-merger marginal cost of the more efficient merger partner as a merger synergy, and not any benefits from reshuffling production across the merger partners. We believe that the measure we employ here is a more natural benchmark when thinking about the efficiency gains to be credited to a "typical" merger. As Farrell and Shapiro show, however, for a merger to not harm consumers in a Cournot market, it must involve synergies in their sense.

Proposition 1. *For merger M to be CS-neutral, the merger-induced efficiencies have to satisfy:*

$$\frac{c_M - \overline{c}_M}{c_M} = \frac{\left(\sqrt{\frac{\Delta H}{2}}\right)\left(\sqrt{2(1-H_M)}\right)}{\epsilon - \left(\sqrt{\frac{\Delta H}{2}}\right)\left(\frac{H_M\sqrt{2}}{\sqrt{1-H_M}}\right)}. \tag{4}$$

Proof. We have

$$\begin{aligned}
\frac{c_M - \overline{c}_M}{c_M} &= \frac{s_m c_m + s_n c_n - s_M \overline{c}_M}{s_m c_m + s_n c_n} \\
&= \frac{s_m c_m + s_n c_n - s_M[c_m + c_n - P(Q^*)]}{s_m c_m + s_n c_n} \\
&= \frac{s_n[P(Q^*) - c_m] + s_m[P(Q^*) - c_n]}{s_m c_m + s_n c_n} \\
&= \frac{s_n P(Q^*)\frac{s_m}{\epsilon} + s_m P(Q^*)\frac{s_n}{\epsilon}}{s_m P(Q^*)[1 - \frac{s_m}{\epsilon}] + s_n P(Q^*)[1 - \frac{s_n}{\epsilon}]} \\
&= \frac{\frac{2 s_m s_n}{\epsilon}}{s_M\left[1 - \frac{s_m^2 + s_n^2}{s_M \epsilon}\right]} \\
&= \frac{\frac{\Delta H}{s_M}}{[\epsilon - s_M H_M]} \qquad (5) \\
&= \frac{\left(\sqrt{\frac{\Delta H}{2}}\right)\left(\sqrt{2(1-H_M)}\right)}{\epsilon - \left(\sqrt{\frac{\Delta H}{2}}\right)\left(\frac{H_M\sqrt{2}}{\sqrt{1-H_M}}\right)},
\end{aligned}$$

where the first equality follows from the definition of c_M, the second from equation (1), the fourth from the pre-merger first-order conditions, and the last from substituting for s_M using the fact that since

$$\Delta H = (s_M)^2(1 - H_M) \tag{6}$$

we have

$$s_M = \sqrt{\frac{\Delta H}{(1 - H_M)}}.$$

□

Intuitively, one would expect that, holding the change in the Herfindahl index fixed, the required efficiency shrinks as the merging firms become more asymmetric. (When one of the merging firms has zero share, there is no anticompetitive effect of the merger even absent synergies.) The following corollary confirms this.

Corollary 1. *In the Cournot model, the marginal cost reduction required to prevent a reduction in consumer surplus falls with a sum-preserving spread of the merging firms' shares.*

Proof. Substituting for H_M in expression (5), using the fact that (6) implies that

$$H_M = 1 - \frac{\Delta H}{(s_M)^2},$$

which yields

$$\frac{c_M - \overline{c}_M}{c_M} = \frac{\Delta H}{s_M(\epsilon - s_M) + \Delta H}.$$

Holding s_M fixed, the right-hand side is increasing in ΔH, which reaches its maximum when the merging firms are symmetric and is monotonically decreasing as they become more asymmetric. □

3.2 Mergers in Differentiated Goods Industries with Price-setting Competition

We now consider mergers between multiproduct firms offering differentiated goods and competing in prices. There is a set $\mathcal{N}$ of horizontally differentiated products offered by firms in set $\mathcal{F}$. Each product $k \in \mathcal{N}$ is offered by only one firm but each firm $f \in \mathcal{F}$ may offer multiple products, $f \subset \mathcal{N}$. As in the Cournot model analyzed above, we assume that firms have constant returns to scale, with c^k denoting the marginal cost of product k.

We focus on two demand systems: constant elasticity of substitution (CES) and multinomial logit (MNL). Multiproduct-firm price competition with such demands shares a useful feature with the homogeneous-goods Cournot model: the game is *aggregative* in that each firm's profit depends on the strategic choices of its rivals only through a one-dimensional aggregator, and consumer surplus depends only on the value of that aggregator. The difference between the two demand systems is that, under CES demand, total expenditure (including the outside good) is fixed, whereas under MNL demand, total consumption (including the outside good) is fixed.

CES demand. We begin with the case of CES demand. The demand for product $k \in \mathcal{N}$ is given by

$$D^k(p^k; A) = \frac{b^k (p^k)^{-\sigma}}{A},$$

where

$$A \equiv \sum_{j \in \mathcal{N}} b^j (p^j)^{1-\sigma} + A^0,$$

is the value of the aggregator, b^j and p^j are the quality and price of product j, respectively, $\sigma > 1$ denotes the elasticity of substitution, and $A^0 \geq 0$ represents the outside good.[19] Consumer surplus is equal to $\mathrm{CS}(A) = \log A$.

The profit of firm f equals

$$\Pi_f((p^k)_{k \in f}) = \sum_{k \in f} (p^k - c^k) D^k(p^k; A),$$

and therefore depends on the price of any rival's product $j \notin f$ only through the value of the aggregator A. From the first-order conditions of profit maximization, it can be shown that firm f sets the same percentage markup $\mu_f > 0$ on each of its products,[20]

$$\frac{p^j - c^j}{p^j} = \mu_f \quad \forall j \in f, \tag{7}$$

and that firm f's markup μ_f satisfies

$$\sigma \mu_f \left(1 - \frac{\sigma - 1}{\sigma} \frac{T_f}{A}(1 - \mu_f)\right) = 1, \tag{8}$$

where

$$T_f \equiv \sum_{k \in f} b^k \left(c^k\right)^{1-\sigma}$$

is firm f's 'type' (which is equal to the firm's contribution to the aggregator – and thus to consumer surplus – if it were to price all of its products at marginal cost). Equation (8) has a unique solution in μ_f, denoted $m(T_f/A)$. The function $m(\cdot)$ is called the *markup fitting-in function*. It is strictly increasing, $m'(\cdot) > 0$: Firms with higher types (larger T) or facing less competition (lower A) charge higher markups.

As total expenditure on all products, including the outside good, is fixed and equal to one (when normalizing the price of the outside good to one), the market share in revenue of product k equals $s^k \equiv p^k D^k(p^k; A)$.

[19] The parameter σ equals the product-level own-price elasticity (of demand D^j) for a firm that takes the level of the aggregator A as fixed. More generally, the own-price elasticity of a product j considering also the effect on A is $\epsilon^j = \sigma - (\sigma - 1)s^j$, while the aggregate elasticity for the inside goods is $\epsilon = \sigma - (\sigma - 1)(1 - s^0)$, where s^0 is the market share of the outside good.

[20] The equilibrium analysis here follows Nocke and Schutz (2018).

The market share of firm f, s_f, can be shown to satisfy

$$s_f = \sum_{k \in f} s^k = \frac{T_f}{A^*} (1 - \mu_f)^{\sigma - 1} \equiv S\left(\frac{T_f}{A}\right). \tag{9}$$

$S(\cdot)$ is called the *market share fitting-in function*; it is strictly increasing: $S'(\cdot) > 0$. Combining equations (8) and (9), we obtain a monotonic relationship between firm f's markup μ_f and its market share s_f:

$$\sigma \mu_f = \frac{1}{1 - \left(\frac{\sigma - 1}{\sigma}\right) s_f}. \tag{10}$$

The equilibrium aggregator level A^* is the unique solution in A to the market shares (including that of the outside good) adding up to unity:

$$\sum_{f \in \mathcal{F}} S\left(\frac{T_f}{A}\right) + \frac{A^0}{A} = 1.$$

Consider now merger M between firms m and n. The post-merger equilibrium value of the aggregator, $\overline{A}^*$, then satisfies

$$S\left(\frac{\overline{T}_M}{\overline{A}^*}\right) + \sum_{f \notin M} S\left(\frac{T_f}{\overline{A}^*}\right) + \frac{A^0}{\overline{A}^*} = 1,$$

where $\overline{T}_M$ is the merged firm's type. (If the merged firm were to produce exactly the same product lines as the merger partners did jointly before the merger, at the same vector of marginal costs, then we would have $\overline{T}_M = T_m + T_n$.) Hence, the merger is CS-neutral with $\overline{A}^* = A^*$, if $\overline{T}_M$ is such that

$$S\left(\frac{\overline{T}_M}{A^*}\right) = s_m + s_n. \tag{11}$$

As shown in Nocke and Schutz (2019), for merger M to be CS-neutral, it must involve synergies in that $\overline{T}_M > T_m + T_n$.[21]

The following proposition indicates how large the type synergies have to be for the merger not to hurt consumers:

Proposition 2. *With CES demand, for merger M to be CS-neutral, the merger-induced type synergy has to satisfy*

$$\frac{\overline{T}_M}{T_m + T_n} = \frac{s_M \left(\sigma + \frac{s_M}{1 - s_M}\right)^{\sigma - 1}}{s_m \left(\sigma + \frac{s_m}{1 - s_m}\right)^{\sigma - 1} + s_n \left(\sigma + \frac{s_n}{1 - s_n}\right)^{\sigma - 1}}. \tag{12}$$

Proof. From equations (9) and (10), we obtain

$$\frac{T_f}{A^*} = s_f (\sigma - 1)^{1 - \sigma} \left(\sigma + \frac{s_f}{1 - s_f}\right)^{\sigma - 1}. \tag{13}$$

Hence, for merger M to be CS-neutral, the post-merger type $\overline{T}_M$ has to satisfy

$$\frac{\overline{T}_M}{A^*} = s_M (\sigma - 1)^{1 - \sigma} \left(\sigma + \frac{s_M}{1 - s_M}\right)^{\sigma - 1}. \tag{14}$$

Combining (13) and (14), yields equation (12). $\square$

Proposition 2 shows that, similar to the Cournot model, the magnitude of the required type synergies depends only on the pre-merger market shares of the merger partners and not on the concentration in the rest of the industry. As noted by Nocke and Schutz (2019), the proposition implies that a larger merger (i.e., an increase in s_m or s_n and thus in $s_M \equiv s_m + s_n$) requires larger synergies.

While the required type synergy depends on both merger partners' shares, rather than simply on ΔH, a sum-preserving spread of their market shares—which decreases ΔH—does reduce the required synergy:

[21] As $S(0) = 0$ and $S''(\cdot) < 0$, the market share fitting-in function is sub-additive. The result then follows from equation (11).

Corollary 2. *With CES demand, a sum-preserving spread of the merger partners' pre-merger market shares (which decreases the change in the Herfindahl index) reduces the type efficiencies required to prevent consumer harm.*

Proof. This follows from the convexity of $s(\sigma + s/(1-s))^{\sigma-1}$ in s, implying that a sum-preserving spread of s_m and s_n increases the denominator on the r.h.s. of equation (12). $\square$

As the notion of type synergies may be unfamiliar, the following corollary relates the size of the required marginal cost synergies (measured as a percentage change in marginal cost, $\phi^j \equiv (\overline{c}^j - c^j)/c^j$ for $j \in (m \cup n)$) to pre-merger market shares, assuming the merger does not affect the number and qualities of the merger partners' products. The corollary does so for two specific vectors of marginal cost changes:[22] The first is such that all prices remain unchanged, whereas the second is such that all percentage changes in marginal cost are the same.[23]

Corollary 3. *Suppose that the set of products offered—and the associated qualities—are not affected by merger M.*

(i) *With CES demand, for all prices to be unaffected by the merger, the percentage change in the marginal cost of product $j \in m$, ϕ^j, has to satisfy*

$$\phi^j = -\frac{s_M - s_m}{(1-s_m)[\sigma(1-s_M)+s_M]}.$$

(ii) *With CES demand, if the marginal cost of each product $j \in (m \cup n)$ changes by the same fraction ϕ, then for consumer surplus to remain unchanged ϕ has to satisfy*

$$\phi = 1 - \left(\frac{s_M \left(\sigma + \frac{s_M}{1-s_M}\right)^{\sigma-1}}{s_m \left(\sigma + \frac{s_m}{1-s_m}\right)^{\sigma-1} + s_n \left(\sigma + \frac{s_n}{1-s_n}\right)^{\sigma-1}} \right)^{1/(1-\sigma)}.$$

Proof. See Appendix A. $\square$

According to part (i) of the corollary, for all prices to remain unchanged with CES demand, every product of each merger partner must have the same percentage reduction in marginal cost, with the required cost synergy being larger for the smaller merger partner. Part (ii) of the corollary gives the required synergies (i.e., percentage marginal cost changes) when those are the same for all products.

To get an idea of the magnitudes involved, consider a merger among symmetric firms (i.e., $s_m = s_n$). Table 2 depicts the maximal shares and change in the Herfindahl index for a symmetric merger not to harm consumers for various synergy levels, assuming that there is no outside good (i.e., $A^0 = 0$). The upper part of the table measures synergies in terms of percentage change in the merging firms' type (i.e., as $[\overline{T}_M - (T_m + T_n)]/(T_m + T_n)$, with $T_m = T_n$ for the case of a symmetric merger), whereas the lower part measures synergies in terms of percentage change in marginal cost (i.e., as $\phi \times 100$).

Table 2 shows that, compared to the Cournot case, at common levels of the substitution parameter σ, the maximal shares and change in the Herfindahl index are larger with price competition and CES demand for any given cost synergy. Nevertheless, even a merger among relatively small firms would need to entail substantial synergies to prevent consumer harm. For example, with $\sigma = 5$, a symmetric merger between two firms with a market share of 11% each (thus raising the Herfindahl index by 242) requires a 3% reduction in marginal cost (or more than a 10% increase in type) so as not to hurt consumers.

Note, however, that our definition of market share coincides with that of the *Guidelines* only if there is no outside good. If there is an outside good, with share s^0, the critical share levels recorded in Table 2 would need to be adjusted by the factor $1/(1-s^0)$. To get a sense for how this would change the critical shares,

[22] There exists a continuum of vectors of marginal cost changes that leave consumer surplus unchanged.

[23] Compared to the analyses for differentiated product price competition of Werden (1996) and Farrell and Shapiro (2010), our result expresses the required synergies in terms of market shares only, in contrast to their characterizations in terms of margins and diversion ratios and (in Werden, 1996) prices. Those papers' results also focus only on deriving product-specific synergies that keep all prices unchanged, as in part (i) of Corollary 3.

Table 2: Maximal Level of Individual Shares and ΔH (*10,000) To Prevent Consumer Harm for Various Levels of Type Synergy (Upper Panel) and Cost Synergy (Lower Panel) with CES Demand

		Type Synergy:						
σ		1%	2%	3%	4%	5%	7.5%	10%
4	Individual shares	1.3	2.5	3.6	4.6	5.6	7.8	9.7
	ΔH	3.3	12.2	25.6	42.6	62.3	120.6	186.7
5	Individual shares	1.2	2.3	3.4	4.3	5.3	7.3	9.1
	ΔH	2.9	10.8	22.7	37.7	55.3	107.4	166.9
6	Individual shares	1.2	2.2	3.2	4.2	5.1	7.1	8.8
	ΔH	2.7	10.0	21.0	34.9	51.3	99.8	155.3

		Cost Synergy:						
σ		1%	2%	3%	4%	5%	7.5%	10%
4	Individual shares	3.6	6.7	9.4	11.7	13.7	18.0	21.4
	ΔH	26.6	90.1	174.9	272.0	375.9	646.1	913.4
5	Individual shares	4.4	8.0	11.0	13.5	15.8	20.2	23.7
	ΔH	39.4	128.6	242.0	366.9	496.3	818.6	1122.6
6	Individual shares	5.2	9.2	12.5	15.2	17.5	22.1	25.6
	ΔH	54.0	170.0	311.4	461.9	613.8	978.9	1310.4

consider the case of an industry composed of symmetric firms with symmetric products. In this case, the aggregate elasticity formula in footnote 19 implies that $s^0 = (\epsilon - 1)/(\sigma - 1)$, where ϵ is the aggregate price elasticity of the inside goods. For example, if $\epsilon = 1.5$ and $\sigma = 5$, the critical shares in Table 2 would increase by a factor of 1.14. As in the Cournot case, this dependence of screening thresholds on the level of the market demand elasticity shows how screening thresholds should interact with market definition.

MNL demand. In the MNL case, the demand for product k can be written as

$$D^k(p^k; A) = \frac{\exp\left(\frac{b^k - p^k}{\lambda}\right)}{A},$$

where the aggregator A now takes the form

$$A \equiv \sum_{j \in \mathcal{N}} \exp\left(\frac{b^j - p^j}{\lambda}\right) + A^0$$

and $\lambda > 0$ is a price sensitivity parameter.[24] As in the CES case, consumer surplus is equal to $\log A$.

From the first-order conditions of profit maximization, firm f sets the same absolute markup $\mu_f > 0$ on each of its products,[25]

$$p^j - c^j = \mu_f \quad \forall j \in f. \tag{15}$$

As total consumption (including the outside good) is equal to one, firm f's market share is naturally measured in volume (rather than value), and given by

$$s_f \equiv \sum_{j \in f} D^j(p^j; A).$$

[24]The own-price elasticity of a product j is $\epsilon^j = (1 - s^j)p^j/\lambda$, while the aggregate elasticity for the inside goods is $\epsilon = s_0\overline{p}/\lambda$, where s^0 is the market share of the outside good and $\overline{p}$ is the quantity-weighted average price of the inside goods.

[25]The equilibrium analysis here follows again Nocke and Schutz (2018).

The markup and market share fitting-in functions are the unique solutions in μ_f and s_f to the following system of equations:

$$\mu_f = \frac{\lambda}{1 - \frac{T_f}{A}\exp\left(-\frac{\mu_f}{\lambda}\right)}, \tag{16}$$

$$s_f = \frac{T_f}{A}\exp\left(-\frac{\mu_f}{\lambda}\right), \tag{17}$$

where

$$T_f \equiv \sum_{k\in f}\exp\left(\frac{b^k - c^k}{\lambda}\right)$$

is firm f's type.

We are interested in the synergies required for merger M between firms m and n not to harm consumers. First, we state the MNL-analog of Proposition 2:

Proposition 3. *With MNL demand, for merger M to be CS-neutral, the merger-induced type synergy has to satisfy*

$$\frac{\overline{T}_M}{T_m + T_n} = \frac{s_M \exp\left(\frac{1}{1-s_M}\right)}{s_m\exp\left(\frac{1}{1-s_m}\right) + s_n\exp\left(\frac{1}{1-s_n}\right)}, \tag{18}$$

where $s_M \equiv s_m + s_n$ is the naively-computed market share of the merged firm.

Proof. From equations (16) and (17), we obtain

$$\frac{T_f}{A^*} = s_f\exp\left(\frac{1}{1-s_f}\right). \tag{19}$$

Hence, for merger M to be CS-neutral, the post-merger type $\overline{T}_M$ has to satisfy

$$\frac{\overline{T}_M}{A^*} = s_M\exp\left(\frac{1}{1-s_M}\right). \tag{20}$$

Combining (19) and (20), yields equation (18). □

Proposition 3 shows that with MNL demand, the required synergies again do not depend on the level of concentration among outsiders and are larger for larger mergers. In addition, holding s_M fixed, a more asymmetric merger again requires fewer type synergies:

Corollary 4. *With MNL demand, a sum-preserving spread of the merger partners' pre-merger market shares (which decreases the change in the Herfindahl index) reduces the type synergies required to prevent consumer harm.*

Proof. This follows from the convexity of $s\exp(1/(1-s))$ in s, implying that a sum-preserving spread of s_m and s_n increases the denominator on the r.h.s. of equation (18). □

Assuming that the merger affects only marginal costs, the following corollary relates the size of the required (absolute) marginal cost changes to pre-merger market shares:

Corollary 5. *Suppose that the set of products offered—and the associated qualities—are not affected by merger M.*

(i) *With MNL demand, for all prices to be unaffected by the merger, the change in marginal cost of product $j \in m$, Δc^j, has to satisfy*

$$\Delta c^j = -\frac{\lambda(s_M - s_m)}{(1-s_M)(1-s_m)}.$$

(ii) *With MNL demand, if the marginal cost of each product $j \in (m \cup n)$ changes by the same amount Δc, then for consumer surplus to remain unchanged Δc has to satisfy*

$$\Delta c = -\lambda \log \left(\frac{s_M \exp\left(\frac{1}{1-s_M}\right)}{s_m \exp\left(\frac{1}{1-s_m}\right) + s_n \exp\left(\frac{1}{1-s_n}\right)} \right).$$

Proof. See Appendix B. □

Assuming no outside good ($A^0 = 0$), Table 3 depicts the maximal individual shares and change in the Herfindahl index for a symmetric-firm merger at various synergy levels. The upper part of the table measures synergies in terms of percentage change in type (as did the upper part of Table 2 for CES demand).

The lower part of the table measures synergies in terms of percentage change in marginal cost (as did the lower part of Table 2 for CES demand). However, recall from Corollary 5 that – under MNL demand – what is pinned down, for a given price sensitivity parameter λ, are the required absolute rather than relative cost changes. However, in the special case in which all firms and products are symmetric, we can derive a condition for the required *percentage* cost synergy ϕ for a given price elasticity of firm-level demand:

$$\phi = \frac{-\Delta c}{c} = \frac{\lambda}{c} \frac{s}{(1-2s)(1-s)} = \left(\frac{1}{\epsilon_f - 1}\right)\left(\frac{s}{1-2s}\right), \tag{21}$$

where c is the common pre-merger marginal cost and s is the common firm-level market share. The second equality follows from Corollary 5, and the third from equations (15)-(17), yielding $c = p - \lambda/(1-s)$, and the fact that $\epsilon_f = (1-s)p/\lambda$. The individual shares reported in the lower part of the table are the solutions in s (times 100) of equation (21).

Table 3: Maximal Level of Individual Shares and ΔH (*10,000) To Prevent Consumer Harm for Various Levels of Type Synergy (Upper Panel) and Cost Synergy (Lower Panel) with MNL Demand

	Type Synergy:						
	1%	2%	3%	4%	5%	7.5%	10%
Individual shares	1.0	1.9	2.7	3.5	4.3	6.0	7.5
ΔH	1.9	7.0	14.8	24.8	36.5	71.6	112.4

		Cost Synergy:						
ϵ_f		1%	2%	3%	4%	5%	7.5%	10%
4	Individual shares	2.8	5.4	7.6	9.7	11.5	15.5	18.8
	ΔH	16.0	57.4	116.3	187.3	266.3	481.6	703.1
5	Individual shares	3.7	6.9	9.7	12.1	14.3	18.8	22.2
	ΔH	27.4	95.1	187.3	293.8	408.2	703.1	987.7
6	Individual shares	4.5	8.3	11.5	14.3	16.7	21.4	25.0
	ΔH	41.3	138.9	266.3	408.1	555.6	918.4	1250.0

Like for the CES model, Table 3 shows that, at common firm-level elasticities, lower cost synergies are required in the MNL model than in the Cournot model. Nonetheless, as in the CES model, even mergers among small firms would require substantial synergies for the merger not to harm consumers. For example, a merger between two firms with a 10% pre-merger market share each (raising the Herfindahl index by 200) would require type synergies exceeding 10%, and cost synergies exceeding 3% when the firm-level own-price elasticity is 5.

As in the CES case, if there is an outside good, with share s^0, the critical share levels recorded in Table 3 would need to be adjusted by the factor $1/(1-s^0)$. For the MNL case, the elasticity formulas in footnote 24 imply that $s^0 = (1-s^j)\epsilon/\epsilon^j$, where s^j is the market share of each product, ϵ^j is the product-level own-price elasticity, and ϵ is the aggregate price elasticity of the inside goods. Thus, $s^0 \leq \epsilon/\epsilon^j$. For example, if $\epsilon = 1.5$ and $\epsilon^j = 5$, the critical shares in Table 3 would increase by at most a factor of 1.43.

4 Empirical Analysis of Mergers in Brewing

The theoretical results above suggest that the presence of consumer harm from a horizontal merger may be more strongly related to the change in the Herfindahl than to its post-merger level. However, these models are very special, and the results of Section 3 also leave some possibility for the level of the Herfindahl to be related to the presence of consumer harm through its relation to aggregate conditions such as the market elasticity of demand in the Cournot model or the outside good share in the MNL and CES models.

In this section, we take a different approach, by looking empirically at how the synergy required to prevent consumer harm is related to the level and merger-induced change in the Herfindahl index (both naively computed) for various hypothetical mergers in the U.S. brewing industry.

We focus on the brewing industry because markets for beer are local, giving us many hypothetical mergers with varying market shares and market conditions, and because prior work by Miller and Weinberg (2017) has estimated a demand system and marginal costs for the major beer brands. We consider the estimates from Miller and Weinberg's RCNL-1 and RCNL-3 monthly models, random-coefficient nested logit models that are not covered by our analysis in Section 3.[26] We use these demand estimates, Miller and Weinberg's derived region/brand-specific marginal costs, and the values of the exogenous determinants of demand in each region in January 2005 (the first month of the Miller and Weinberg estimation sample) to simulate each possible hypothetical merger among the producers in each of Miller and Weinberg's 39 local markets. Given the five firms in their estimation model, this gives 10 possible mergers in each local market, for a total of 390 hypothetical mergers.

For each possible merger and a given specified synergy for the merging firms (which reduces the pre-merger marginal costs of each of the merging firms' products by the same percentage), we compute the pricing equilibrium and resulting consumer welfare.[27] We do this for various possible synergy levels, and identify the synergy level at which the merger is CS-neutral. As well, we calculate the naively-computed post-merger Herfindahl index and the change in the Herfindahl for that merger, with the shares for this computation including all firms in the market, not just the five firms in the Miller and Weinberg estimation model. We report results based on volume shares in the main text, and provide results based on revenue shares in the Appendix.[28] (Overall, the results are very similar.) We then examine how these two characteristics of mergers are related to the required synergy across our hypothetical mergers.

Figures 5 and 6 plot the results for the RCNL-1 and RCNL-3 models, respectively. Each small symbol represents a merger and its location shows that merger's naively-computed post-merger Herfindahl index and the naively-computed merger-induced change in the Herfindahl. Green crosses indicate mergers whose required efficiency gain is 0-5%; brown squares indicate those with a required gain between 5% and 10%, blue circles between 10% and 15%, and red diamonds above 15%. The visually striking aspect of the figure is that whether a merger would require less than a 5% efficiency gain to avoid harming consumers is highly related to the change in the Herfindahl, and seems nearly unrelated to the level of the post-merger Herfindahl (and, if anything, holding fixed the change, increases in the level of the Herfindahl require *lower* efficiency gains to prevent consumer harm).

Columns (1) and (4) of Table 4 confirm this impression, reporting on the results of a simple linear regression of the synergy required to make a merger CS-neutral on a merger's post-merger naively-computed Herfindahl index (referred to as "hhi" in the table), the change in the naively-computed Herfindahl caused by the merger (referred to as "delta" in the table), and a constant. For both RCNL models, the change in the Herfindahl is strongly significant while the level of the post-merger Hefindahl is insignificant and small

[26] The difference between these two models is in the product attributes that are given random coefficients. In RCNL-1, price, calories, and a constant receive random coefficients that depend on a consumer's income. In RCNL-3, import status and package size (the two key determinants of price) receive random coefficients instead of price. In general, for a given package size, import status is the key product characteristic leading the demand estimates to diverge from the identical cross elasticity across inside goods that characterizes a simple nested logit model (see Miller and Weinberg's Table V for the RCNL-1 model, and Table I.1 in http://www.nathanhmiller.org/Miller%20Weinberg%20(Supplement).pdf).

[27] Miller and Weinberg include only the flagship brands of the five firms in their demand model. With the other brands of these firms implicitly included in the outside good, the price elevation arising in our merger simulations is likely less than would be the price elevation were all of these firms' products included as inside goods.

[28] It is not clear from the *Horizontal Merger Guidelines* which share measures the agencies would be likely to use in a beer merger. The *Guidelines* comment that "Revenues in the relevant market tend to be the best measure of attractiveness to consumers...," but also note that "where one unit of a low-priced product can substitute for one unit of a higher-priced product, unit sales may measure competitive significance better than revenues."

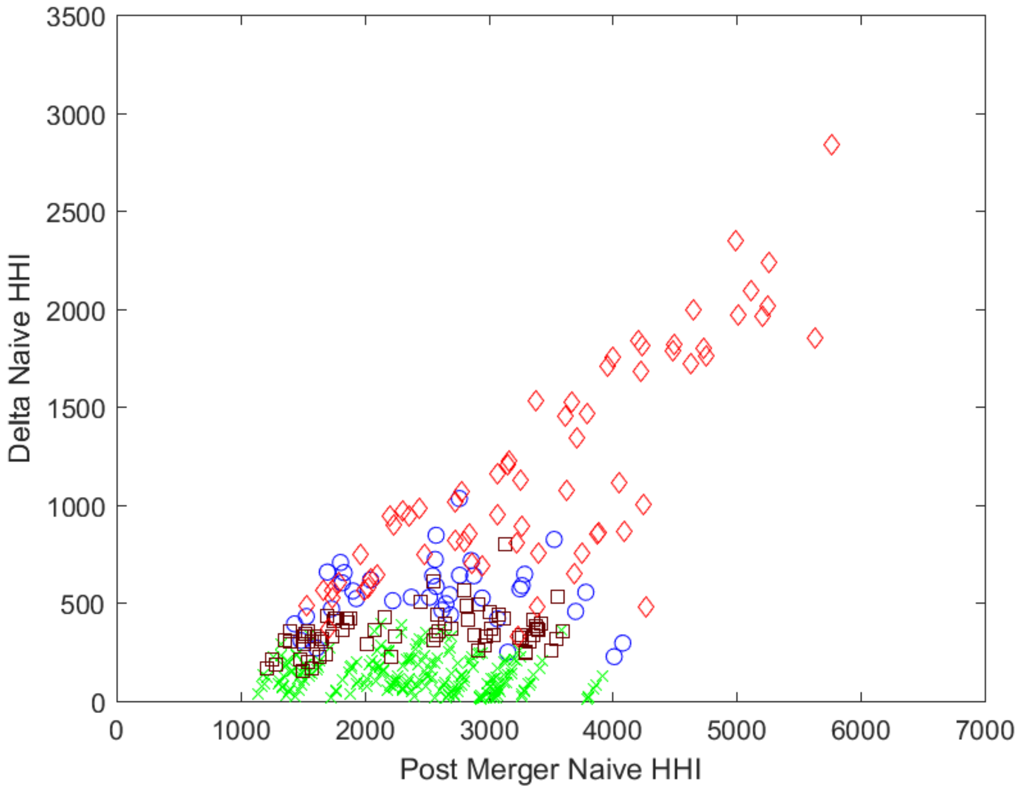

Figure 5: Relationship between the synergy required for a merger to be CS-neutral and the post-merger naively-computed Herfindahl index and its naively computed change, based on the RCNL-1 model and volume shares [green crosses $< 5\%$; brown squares 5-10%, blue circles 10-15%; red diamonds $> 15\%$]

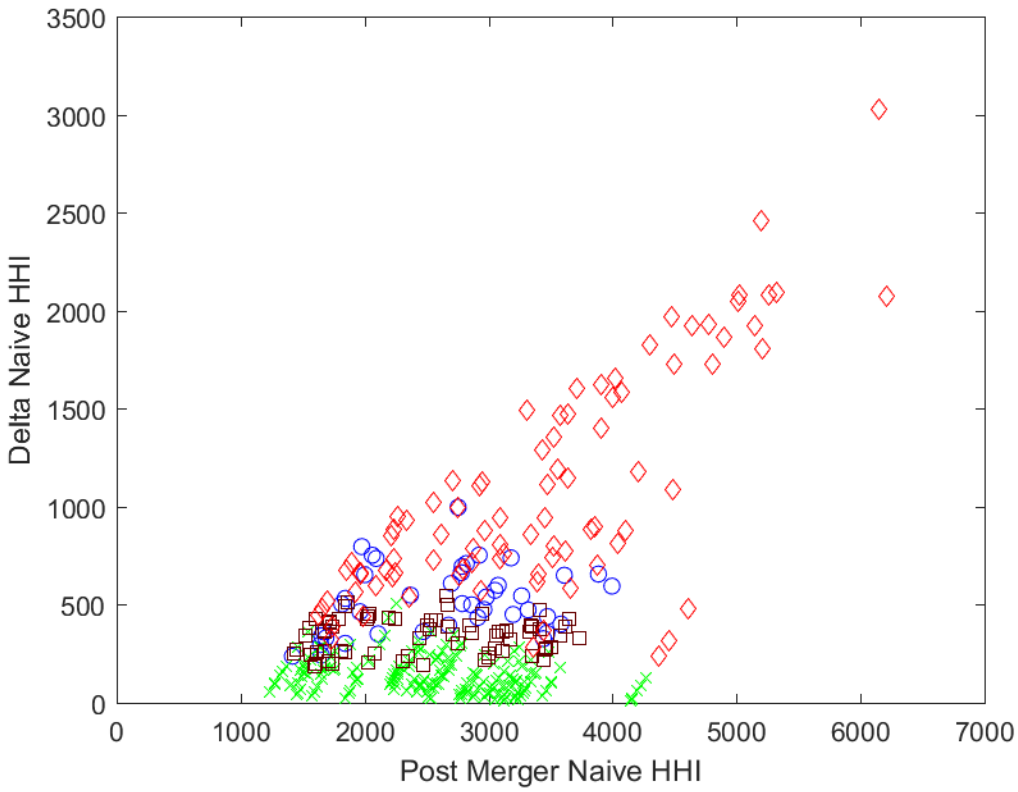

Figure 6: Relationship between the synergy required for a merger to be CS-neutral and the post-merger naively-computed Herfindahl index and its naively computed change, based on the RCNL-3 model and volume shares [green crosses $< 5\%$; brown squares 5-10%, blue circles 10-15%; red diamonds $> 15\%$]

in magnitude. For example, the RCNL-3 estimated coefficient on the post-merger Herfindahl implies that a 1000 point increase in the post-merger Herfindahl causes only a 0.38% increase in the synergy required for consumers to not be harmed. In contrast, the estimated coefficient on the change in the Herfindahl implies that an extra 100 points for the change leads to a 3.1% increase in the required synergy. Note also that the R^2 of both of these regressions is remarkably high, equalling 0.85 in column (1) and 0.83 in column (4).

Columns (2) and (5) of Table 4 explore this relationship further by expanding the specification to include second-order terms in hhi and delta. Columns (3) and (6) then restrict the sample to the 352 mergers for which the post-merger Herfindahl is less than 4000 and the change in the Herfindahl is less than 1000, which is both where most of the data lies and the region where screening and presumption thresholds are likely most relevant. F-tests for all of these estimations strongly reject both the simple linear model and a model in which all terms involving the post-merger Herfindahl index are dropped.

Table 4: Regression of the Required Synergy on Functions of the Herfindahl and the Change in the Herfindahl (Volume-based)

Dependent Variable: Synergy Required to Prevent Consumer Harm

	RCNL-1			RCNL-3		
	(1)	(2)	(3)	(4)	(5)	(6)
hhi	−.013	−.795	−.352	.038	−1.13	−.457
	(.032)	(.202)	(.141)	(.047)	(.292)	(.218)
	[−.41]	[−3.93]	[−2.50]	[.81]	[−3.85]	[−2.09]
delta	2.39	3.21	2.68	3.14	4.18	3.12
	(.062)	(.302)	(.310)	(.089)	(.044)	(.480)
	[38.89]	[10.62]	[8.65]	[35.36]	[9.58]	[6.52]
hhi × delta		−4.44	−4.17		−4.44	−3.83
		(1.38)	(1.01)		(2.00)	(1.55)
		[−3.21]	[−4.15]		[−2.22]	[−2.46]
hhi^2		1.79	.81		2.56	1.06
		(.45)	(.30)		(.65)	(.46)
		[4.00]	[2.73]		[3.96]	[2.30]
delta^2		3.77	9.98		1.71	13.36
		(1.66)	(2.33)		(2.41)	(3.61)
		[2.27]	[4.28]		[.71]	[3.70]
constant	−.002	.077	.036	−.016	.102	.045
	(.008)	(.022)	(.017)	(.011)	(.032)	(.026)
	[−.26]	[3.47]	[2.17]	[−1.47]	[3.20]	[1.75]
Sample	Full	Full	Restricted	Full	Full	Restricted
# Observations	390	390	352	390	390	352
R^2	.85	.86	.82	.83	.84	.77

Notes: Dependent variable measured as 0.01 for 1% synergy, hhi is the naively-computed volume-based post-merger Herfindahl index scaled between 0 and 1, and delta is the naively-computed merger-induced change in the volume-based Herfindahl index scaled between 0 and 1. Standard errors are in parentheses; t-statistics are in square brackets.

In all four regressions, a greater increase in the Herfindahl increases the synergy required for consumers not to be harmed. In contrast, while the post-merger Herfindahl does matter in these second-order specifications, its effect is not monotonic and its magnitude is often small. To see this point, Figures 7 and 8 plot contour lines for the estimates in the restricted samples of columns (3) and (6). We plot contour lines for synergies of 1%, 3%, and 5%. Thus, for example, if a merger has a 3% synergy, those mergers lying above the 3% line

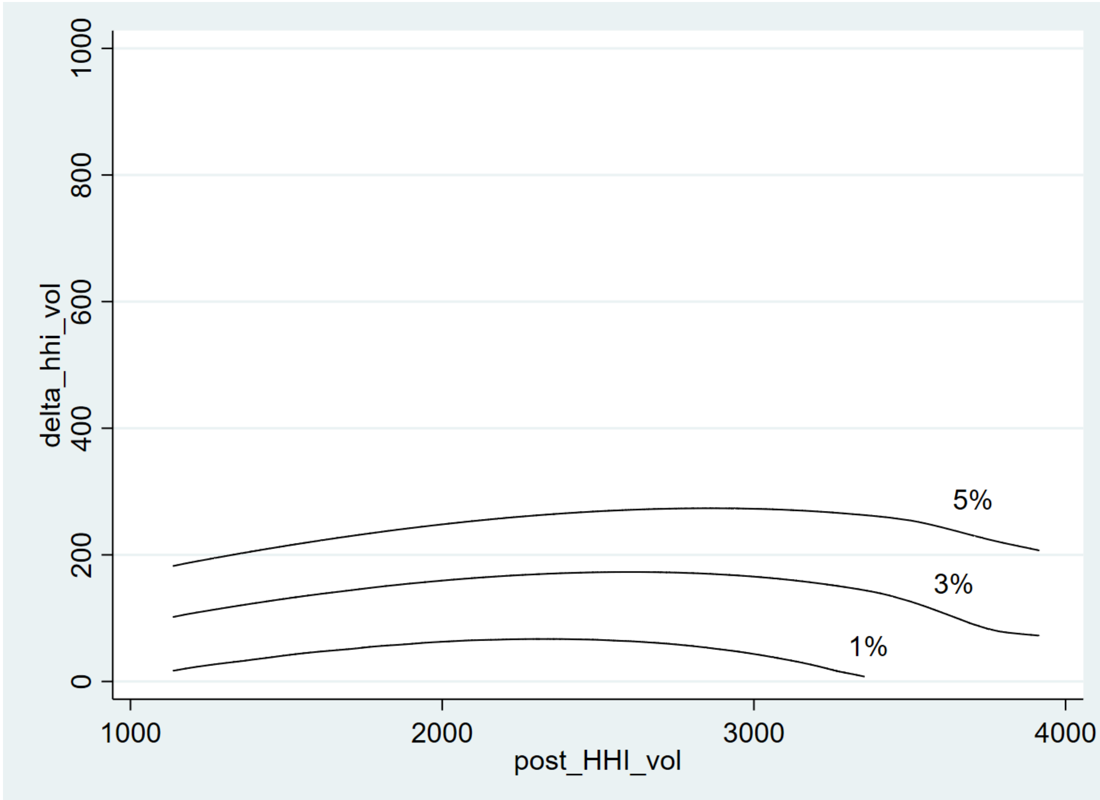

Figure 7: Contour plot showing the combinations of the naively-computed post-merger Herfindahl (labelled here as "post_hhi_vol") and the naively-computed merger-induced change in the Herfindahl ("delta_hhi_vol") that have no effect on consumer surplus if there is a 1%, 3%, and 5% synergy due to the merger. Points above (respectively, below) a contour line correspond to mergers that are expected to harm (respectively, benefit) consumers. Based on estimates in Table 4, column (3).

are CS-decreasing, and those lying below it are CS-increasing.

As can be seen in the two figures, the effect of the post-merger Herfindahl is quite small for Herfindahl levels between 1500 and 2500, where a merger is expected to leave consumers unharmed if the change in the Herfindahl is somewhere in the 150-180 range. For levels of the post-merger Herfindahl both below and above this range, lower changes in the Herfindahl are required for consumers to be unharmed; only at very high levels of concentration do increases in the Herfindahl make a merger much more likely to lead to consumer harm (for a given size of the merger-induced change). Notice, as well, that if a 3% efficiency gain is presumed, any merger that induces an increase of more than 200 in the Herfindahl index is expected to harm consumers, regardless of the level of the post-merger Herfindahl index. The results here suggest a screening standard somewhere in the the middle of those suggested by the theoretical models of Section 3 (compare Tables 1-4).

A different way to evaluate what study of these mergers says about the *Merger Guidelines'* screens is to see what the effect on consumers is of our hypothetical brewing mergers that fall into each of the *Guidelines'* green, yellow, and red zones (recall Figure 3). Table 5 presents these statistics for the case in which mergers result in a 3% synergy.[29] Several clear points come out. First, a very high share of the mergers in which the post-merger Herfindahl is below 1500, which fall in the *Guidelines*' safe harbor, lead to consumer harm: 68% for the RCNL-1 model and 76% in the RCNL-3 model. That part of the safe harbor seems to be too lenient. On the other hand, the safe harbor at higher levels of the Herfindahl does not seem problematic since it applies only to mergers that induce a small change in the Herfindahl index. Likewise, the anticompetitive

[29] One should be cautious in interpreting Table 5, since the characteristics of the hypothetical mergers in our sample may not correspond to the distribution of mergers that would actually be proposed to the agencies. Indeed, even when mergers are profitable (as all are here), which mergers get proposed is the result of both negotiations/bidding among firms in an industry, and the treatment firms expect from the agencies (see, for example, Nocke and Whinston, 2010 and 2013).

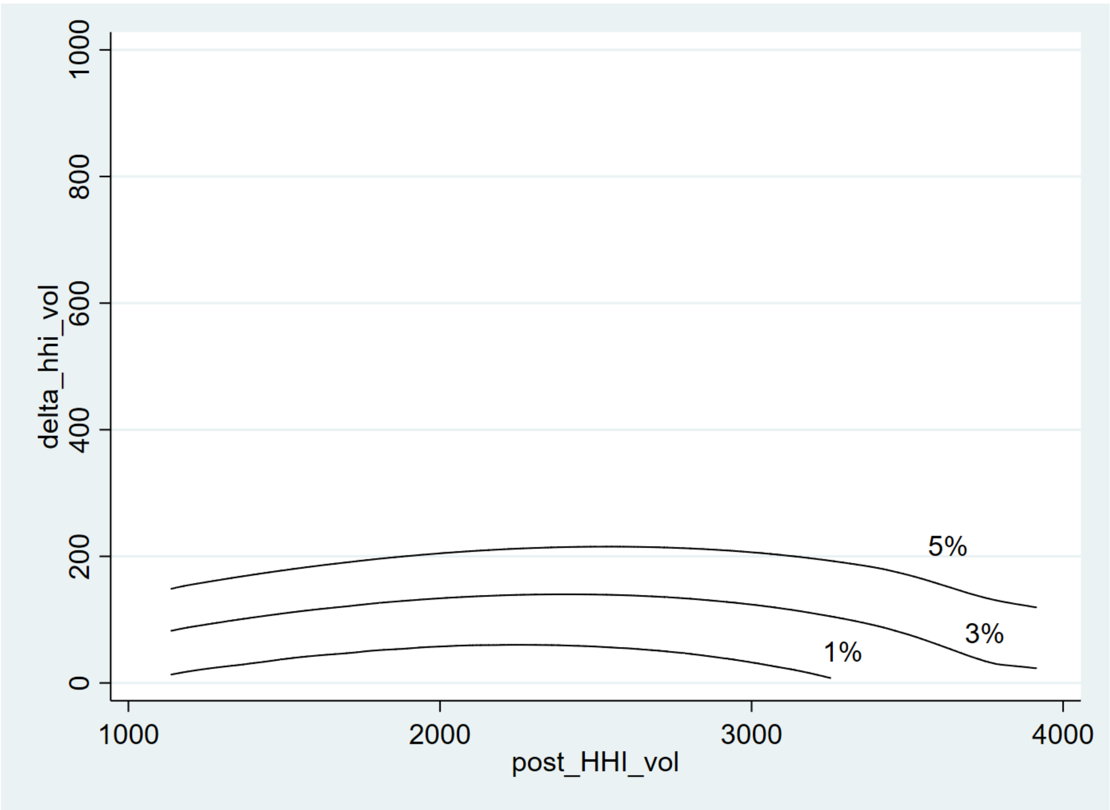

Figure 8: Contour plot showing the combinations of the naively-computed post-merger Herfindahl (labelled here as "post_hhi_vol") and the naively-computed merger-induced change in the Herfindahl ("delta_hhi_vol") that have no effect on consumer surplus if there is a 1%, 3%, and 5% synergy due to the merger. Points above (respectively, below) a contour line correspond to mergers that are expected to harm (respectively, benefit) consumers. Based on estimates in Table 4, column (6).

presumption in the red zone seems justified, as nearly all mergers in this region harm consumers. Finally, the treatment of mergers in the yellow zone – which the *Guidelines*' consider potentially problematic – seems fine for mergers with post-merger Herfindahls above 2500 (since for these, the change in the Herfindahl must be below 200), but may be too lenient for mergers that lead to a post-merger Herfindahl between 1500 and 2500. For these, an increased concern about anticompetitive harm may be appropriate when the change in the Herfindahl is above 200.

Finally, in Table 6 we report the same information under the presumption that mergers lead to a 5% synergy. The table shows that with this larger presumed synergy the *Guidelines'* thresholds would be much more successful at sorting good and bad mergers among this set of brewing mergers, although mergers in which the post-merger Herfindahl is below 1500 still are harming consumers 24-29% of the time.

Table 5: Share of Hypothetical Brewing Mergers with 3% Efficiency Gain That Harm Consumers Under 2010 *Guidelines'* Screening Thresholds (Volume-based)

Merger Guidelines' Screening Zone		**RCNL-1**	**RCNL-3**
Green Zone (Safe Harbor)		*0.22*	*0.34*
	HHI < 1500	0.68	0.76
	HHI $\in (1500, 2500)$ and $\Delta H < 100$	0.00	0.12
	HHI > 2500 and $\Delta H < 100$	0.00	0.15
Yellow Zone		*0.74*	*0.94*
	HHI $\in (1500, 2500)$ and $\Delta H > 100$	0.82	0.94
	HHI > 2500 and $\Delta H \in (100, 200)$	0.36	0.92
Red Zone (Anticompetitive Presumption)		*0.99*	*1.00*

Table 6: Share of Hypothetical Brewing Mergers with 5% Efficiency Gain That Harm Consumers Under 2010 *Guidelines'* Screening Thresholds (Volume-based)

Merger Guidelines' Screening Zone		**RCNL-1**	**RCNL-3**
Green Zone (Safe Harbor)		*0.08*	*0.09*
	HHI < 1500	0.24	0.29
	HHI $\in (1500, 2500)$ and $\Delta H < 100$	0.00	0.00
	HHI > 2500 and $\Delta H < 100$	0.00	0.00
Yellow Zone		*0.40*	*0.47*
	HHI $\in (1500, 2500)$ and $\Delta H > 100$	0.49	0.56
	HHI > 2500 and $\Delta H \in (100, 200)$	0.00	0.04
Red Zone (Anticompetitive Presumption)		*0.85*	*0.94*

5 Discussion

The theoretical and empirical results above indicate that for screening mergers for whether their unilateral effects will harm consumers, the merger-induced change in the (naively-computed) Herfindahl index should play a much more prominent role in screening than the level of the Herfindahl. How might one then understand or justify current practice?

One possibility, of course, is that the prominent role of the level of concentration reflects concerns not over unilateral effects, but rather over coordinated effects, the likelihood of entry and/or repositioning, and other factors. While this is certainly a possibility, the literature awaits a well-articulated analysis that establishes proper screening thresholds for such effects.

Focusing on unilateral effects, another possibility is that current horizontal merger screens reflect not so much an aim to prevent consumer harm, but rather to prevent *significant* consumer harm. In the Cournot model, for example, reducing the number of firms by one has increasingly large price elevation effects the fewer firms are in the market. Formally, the magnitude of the resulting shortfall in consumer surplus depends on the characteristics of the non-merging outsiders, as can be seen by taking the derivative of consumer surplus with respect to the merged firm's post-merger marginal cost, evaluated at the level at which the merger would just be CS-neutral,

$$\frac{dCS(Q^*)}{dQ}\frac{dQ}{d\overline{c}_M} = -\frac{Q^*}{|\mathcal{F}| - \sigma(Q^*)},$$

where $\sigma(Q) \equiv -QP''(Q)/P'(Q) < 1$ is the curvature of inverse demand and $|\mathcal{F}|$ is the pre-merger number of active firms. Hence, at a given pre-merger equilibrium output level Q^*, the shortfall in consumer surplus is smaller the larger is the number of firms. This fact also implies that if the antitrust agencies' goal is to ensure that the post-merger CS-level is at least a fraction x of the pre-merger level, with x strictly less than (but close to) one, then the required merger-induced efficiencies are decreasing in the number of firms. The key force driving this effect is that with fewer rival firms, non-merging firms replace less of any reduction in the merging firms' supply.

Similarly, under price competition with MNL/CES demands, the concentration among outsiders' market shares—akin to the Herfindahl index—comes into play, as the following proposition shows:

Proposition 4. *Assume that the market share of each non-merging firm does not exceed 0.65. Then, with MNL or CES demand, a sum-preserving spread of the market shares of the non-merging firms makes consumer surplus more responsive to a shortfall in the merger-induced efficiencies.*

Proof. See Appendix C. □

We explored this possibility in our brewing merger data by performing a similar analysis to that in Section 4, but instead focusing on the level of synergy required to prevent a merger from causing more than a 5% reduction in consumer surplus. We found evidence of a positive effect of the level of the post-merger Herfindahl when using the RCNL-1 estimates, but not when using the RCNL-3 estimates.

A related possibility is that current practice reflects the need to protect consumers given a limited enforcement budget. In that situation, the agencies would want to focus on the worst mergers for consumers. To explore this avenue, we looked at the relationship between the absolute size of a merger's effect on consumer surplus and the levels of the Herfindahl index, its merger-induced change, and market size for a 3% presumed marginal cost synergy. We found that both the change in the Herfindahl and market size strongly predicted the absolute level of consumer harm from a merger, but there was no significant effect of the level of the post-merger Herfindahl once these other variables were controlled for.[30]

6 Conclusion

In this paper we have explored the use of concentration measures to screen horizontal mergers for unilateral effects. Looking both theoretically and empirically, our results suggest that such screens should most likely focus much more on the merger-induced (naively-computed) *change* in the Herfindahl index than on its post-merger level. As such, they suggest screens closer in form to the 1968 *Guidelines* than to the current ones.

In terms of stringency, our results indicate that if a 3% efficiency gain is a reasonable presumption, then the current safe harbor for mergers in markets with post-merger Herfindahl levels below 1500 is likely allowing many mergers that cause consumer harm. As well, mergers resulting in post-merger Herfindahl levels between 1500 and 2500 and that induce a Herfindahl increase over 200 should perhaps be accorded a greater presumption of harm than is currently reflected in the agencies' *Guidelines*. The current *Guidelines* may be adequate, however, if most mergers result in at least a 5% efficiency gain.

We see several useful directions for further work to refine concentration screens for horizontal mergers. First, further empirical analysis along the lines of that in Section 4 in other markets with different estimated

[30]Another possibility is that a focus on the Herfindahl is appropriate if the authority is concerned about aggregate, rather than consumer, surplus (despite the law's focus on consumer harm). Farrell and Shapiro (1990) and Nocke and Schutz (2019) discuss some aspects of the relationship between the level of the Herfindahl and the aggregate surplus effect of a merger, but we are unaware of any results about the overall relationship.

demand and costs would be very useful. Second, more evidence on the synergies arising in horizontal mergers, especially conditional on market structure, would be extremely valuable. Third, work identifying thresholds for screening mergers for possible consumer harm due to coordinated effects would complement our analysis. Finally, continuing work on merger retrospectives is important, especially aimed at learning both the extent to which entry, repositioning, or other factors on average ameliorate unilateral anti-competitive effects, and the extent to which coordinated effects arise that exacerbate them.

At the same time, of course, concentration screens are just one piece of the merger evaluation puzzle, and are only useful when combined with effective in-depth analysis of mergers deemed to raise possible competitive concerns.

References

[1] Farrell, J. and C. Shapiro (1990), "Horizontal Mergers: An Equilibrium Analysis," *American Economic Review*, 100: 107-126.

[2] Farrell, J. and C. Shapiro (2010), "Antitrust Evaluation of Horizontal Mergers: An Economic Alternative to Market Definition," *B.E. Journal of Theoretical Economics* 10: Article 9.

[3] Froeb, L. M., and G.J. Werden (1998), "A Robust Test for Consumer Welfare Enhancing Mergers Among Sellers of a Homogeneous Product," *Economics Letters* 58: 367-9.

[4] Kwoka, J.E. (2013), "Does Merger Control Work? A Retrospective on U.S. Enforcement Actions and Merger Outcomes," *Antitrust Law Journal* 78: 619-650.

[5] Kwoka, J.E. (2017), "The Structural Presumption and Safe Harbor in Merger Review: False Positives or Unwarranted Concerns?," *Antitrust Law Journal* 81: 837-72.

[6] Miller, N.H. and M.C. Weinberg (2017), "Understanding the Price Effects of the MillerCoors Joint Venture," *Econometrica* 85: 1763-91.

[7] Nocke, V. and N. Schutz (2018), "Multiproduct-Firm Oligopoly: An Aggregative Games Approach," *Econometrica* 86: 523-57.

[8] Nocke, V. and N. Schutz (2019), "An Aggregative Games Approach to Merger Analysis in Multiproduct-Firm Oligopoly," *CEPR Discussion Paper* 12905, Centre for Economic Policy Research.

[9] Nocke, V. and M.D. Whinston (2010), "Dynamic Merger Review," *Journal of Political Economy* 118: 1200-51.

[10] Nocke, V. and M.D. Whinston (2013), "Merger Policy with Merger Choice," *American Economic Review* 103: 1006-33.

[11] Ordover, J.A., A.O. Sykes, and R.D. Willig (1982), "Herfindahl Concentration, Rivalry, and Mergers," *Harvard Law Review* 95: 1857-74.

[12] Report of the White House Task Force on Antitrust Policy (aka *The Neal Report*, May 27, 1969), originally published at 115 Cong. Rec. 11, 13890.

[13] Schmalensee, R. (1987), "Horizontal Merger Policy: Problems and Changes," *Journal of Economic Perspectives* 1: 41-54.

[14] Shapiro, C. (2010), "The 2010 Horizontal Merger Guidelines: From Hedgehog to Fox in Forty Years," *Antitrust Law Journal* 77: 49-108.

[15] U.S. Department of Justice and Federal Trade Commission (2006), *Commentary on the Horizontal Merger Guidelines*, available at https://www.justice.gov/atr/f215200/215247.

[16] U.S. Department of Justice and Federal Trade Commission (2010), *Horizontal Merger Guidelines*, available at https://www.justice.gov/atr/horizontal-merger-guidelines-08192010.

[17] U.S. Federal Trade Commision (2013), "Horizontal Merger Investigation Data: Fiscal Years 1996-2011," available at https://www.ftc.gov/os/2013/01/130104horizontalmergerreport.pdf.

[18] Werden, G.J. (1996), "A Robust Test for Consumer Enhancing Mergers Among Sellers of Differentiated Products," *Journal of Industrial Economics* 44: 409-13.

Appendix

A Proof of Corollary 3

To see part (i), note that the pre-merger marginal cost of product $k \in m$ can be written as

$$c^k = p^k \left[1 - \frac{1}{\sigma(1-s_m) + s_m}\right],$$

where we have used equations (7) and (10). For the merged firm to charge the same prices for all of its products (implying that its post-merger market share is $s_M = s_m + s_n$), the post-merger marginal cost of product k has to satisfy

$$\overline{c}^k = p^k \left[1 - \frac{1}{\sigma(1-s_M) + s_M}\right].$$

Combining, we obtain:

$$\phi^k = \frac{\overline{c}^k - c^k}{c^k} = -\frac{s_M - s_m}{(1-s_m)[\sigma(1-s_M) + s_M]}.$$

To see part (ii), note that:

$$\begin{aligned}
\frac{\overline{T}_M}{T_m + T_n} &= \frac{\sum_{k \in M} b^k \left((1-\phi)c^k\right)^{1-\sigma}}{\sum_{k \in m} b^k \left(c^k\right)^{1-\sigma} + \sum_{k \in n} b^k \left(c^k\right)^{1-\sigma}} \\
&= \frac{(1-\phi)^{1-\sigma} \sum_{k \in M} b^k \left(c^k\right)^{1-\sigma}}{\sum_{k \in m} b^k \left(c^k\right)^{1-\sigma} + \sum_{k \in n} b^k \left(c^k\right)^{1-\sigma}} \\
&= (1-\phi)^{1-\sigma}.
\end{aligned}$$

The assertion then follows from applying Proposition 2.

B Proof of Corollary 5

To see part (i), note that the pre-merger marginal cost of product $k \in m$ can be written as

$$c^k = p^k - \frac{\lambda}{1 - s_m},$$

where we have used equations (15) and (16) For the merger to leave all prices unchanged (implying that the post-merger market share is $s_M = s_m + s_n$), the post-merger marginal cost of product k has to satisfy

$$\overline{c}^k = p^k - \frac{\lambda}{1 - s_M}.$$

Combining, we obtain:

$$\Delta c^k = \overline{c}^k - c^k = \frac{\lambda}{1 - s_m} - \frac{\lambda}{1 - s_M} = \frac{\lambda(s_M - s_m)}{(1 - s_M)(1 - s_m)}.$$

To see part (ii), note that:

$$\begin{aligned}
\frac{\overline{T}_M}{T_m + T_n} &= \frac{\sum_{k \in M} \exp\left(\frac{b^k - c^k - \Delta c}{\lambda}\right)}{\sum_{k \in m} \exp\left(\frac{b^k - c^k}{\lambda}\right) + \sum_{k \in n} \exp\left(\frac{b^k - c^k}{\lambda}\right)} \\
&= \frac{\exp\left(\frac{-\Delta c}{\lambda}\right) \sum_{k \in M} \exp\left(\frac{b^k - c^k}{\lambda}\right)}{\sum_{k \in m} \exp\left(\frac{b^k - c^k}{\lambda}\right) + \sum_{k \in n} \exp\left(\frac{b^k - c^k}{\lambda}\right)} \\
&= \exp\left(\frac{-\Delta c}{\lambda}\right).
\end{aligned}$$

The assertion then follows from applying Proposition 3.

C Proof of Proposition 4

If the post-merger type $\overline{T}_M$ falls short by a small fraction of the level that would restore consumer surplus after the merger, the shortfall in consumer surplus is given by

$$-\frac{dCS(A^*)}{dA}\frac{dA}{d\overline{T}_M}\overline{T}_M = -\frac{\frac{\overline{T}_M}{A^*}S'\left(\frac{\overline{T}_M}{A^*}\right)}{\frac{\overline{T}_M}{A^*}S'\left(\frac{\overline{T}_M}{A^*}\right)+\sum_{f\notin M}\frac{T_f}{A^*}S'\left(\frac{T_f}{A^*}\right)}$$
$$= -\frac{S^{-1}(s_M)S'\left(S^{-1}(s_M)\right)}{S^{-1}(s_M)S'\left(S^{-1}(s_M)\right)+\sum_{f\notin M}S^{-1}(s_f)S'\left(S^{-1}(s_f)\right)} \tag{22}$$

where the first equality follows from applying the implicit function theorem to the adding-up condition

$$S\left(\frac{\overline{T}_M}{A^*}\right)+\sum_{f\notin M}S\left(\frac{\overline{T}_f}{A^*}\right)+\frac{A^0}{A^*}=1.$$

As the number of outsiders is finite, it is straightforward to see that a sum-preserving spread of the outsiders' market shares can be decomposed into a finite number of steps where at each step there is a sum-preserving spread of market shares involving only two outsiders. We now prove that at any such step the denominator on the r.h.s. of equation (22) decreases, from which the result follows.

Let $t_f \equiv T_f/A^*$ and suppose that $t_f > t_g$. We need to show that an increase in t_f and a decrease in t_g such that $S(t_f)+S(t_g)$ remains unchanged induces a reduction in $t_fS'(t_f)+t_gS'(t_g)$. We have:

$$\left.\frac{d\left[t_fS'(t_f)+t_gS'(t_g)\right]}{dt_f}\right|_{S(t_f)+S(t_g)=\text{const.}} = S'(t_f)\left[\frac{t_fS''(t_f)}{S'(t_f)}-\frac{t_gS''(t_g)}{S'(t_g)}\right].$$

As $S'(\cdot)>0$, we thus only need to show that the elasticity of S' is decreasing, i.e.,

$$\frac{d}{dt}\frac{tS''(t)}{S'(t)}<0.$$

From the proof of Proposition 9 in Nocke and Schutz (2019), we have:

$$S'(t) = \frac{1}{t}\frac{S(t)(1-S(t))(1-\alpha S(t))}{1-S(t)+\alpha S(t)^2},$$
$$S''(t) = -\frac{\alpha}{t^2}\frac{(2-S(t))S(t)^2(1-S(t))(1-\alpha S(t))}{[1-S(t)+\alpha S(t)^2]^3},$$

where $\alpha=1$ if demand is of the MNL form and $\alpha=(\sigma-1/\sigma)<1$ if it is of the CES form. It follows that

$$\frac{tS''(t)}{S'(t)} = -\frac{\alpha(2-S(t))S(t)}{[1-S(t)+\alpha S(t)^2]^2}.$$

We thus have

$$\frac{d}{dt}\frac{tS''(t)}{S'(t)}<0$$

if and only if

$$[(2-S(t))S'(t)-S(t)S'(t)][1-S(t)+\alpha S(t)^2] > 2(2-S(t))S(t)[-S'(t)+2\alpha S(t)S'(t)],$$

i.e.,

$$1+\alpha S(t)^3 > 3\alpha S(t)^2.$$

It can easily be verified that this inequality holds, for any $\alpha\in(0,1]$ if $S(t)\le 0.65$.

Table 7: Regression of the Required Synergy on Functions of the Herfindahl and the Change in the Herfindahl (Revenue-based)

Dependent Variable: Synergy Required to Prevent Consumer Harm

	RCNL-1			RCNL-3		
	(1)	(2)	(3)	(4)	(5)	(6)
hhi	.020	−.813	−.635	.082	−1.150	−.834
	(.032)	(.182)	(.170)	(.045)	(.265)	(.265)
	[.62]	[−4.46]	[−3.73]	[1.78]	[−4.34]	[−3.15]
delta	2.30	2.72	2.41	3.01	3.49	2.85
	(.060)	(.270)	(.325)	(.087)	(.392)	(.505)
	[38.53]	[10.09]	[7.43]	[34.81]	[8.90]	[5.65]
hhi × delta		−2.86	−3.58		−2.34	−3.23
		(1.17)	(1.08)		(1.71)	(1.69)
		[−2.44]	[−3.30]		[−1.37]	[−1.92]
hhi^2		1.67	1.37		2.38	1.80
		(.37)	(.36)		(.54)	(.56)
		[4.46]	[3.79]		[4.37]	[3.21]
delta^2		2.76	7.74		.64	9.45
		(1.47)	(2.31)		(2.13)	(3.59)
		[1.88]	[3.35]		[.30]	[2.63]
constant	−.015	.086	.069	−.034	.117	.089
	(.008)	(.022)	(.020)	(.011)	(.032)	(.031)
	[−1.88]	[3.96]	[3.44]	[−2.96]	[3.70]	[2.85]
Sample	Full	Full	Restricted	Full	Full	Restricted
# Observations	390	390	343	390	390	343
R^2	.85	.86	.79	.83	.84	.74

Notes: Dependent variable measured as 0.01 for 1% synergy, hhi is the naively-computed revenue-based post-merger Herfindahl index scaled between 0 and 1, and delta is the naively-computed merger-induced change in the revenue-based Herfindahl index scaled between 0 and 1. Standard errors are in parentheses; t-statistics are in square brackets.

D Empirical Results for Brewing Mergers using Revenue Shares

Here we present the tables and figures for the empirical analysis of Section 4 when markets shares and the Herfindahl index are revenue-based rather than volume-based.

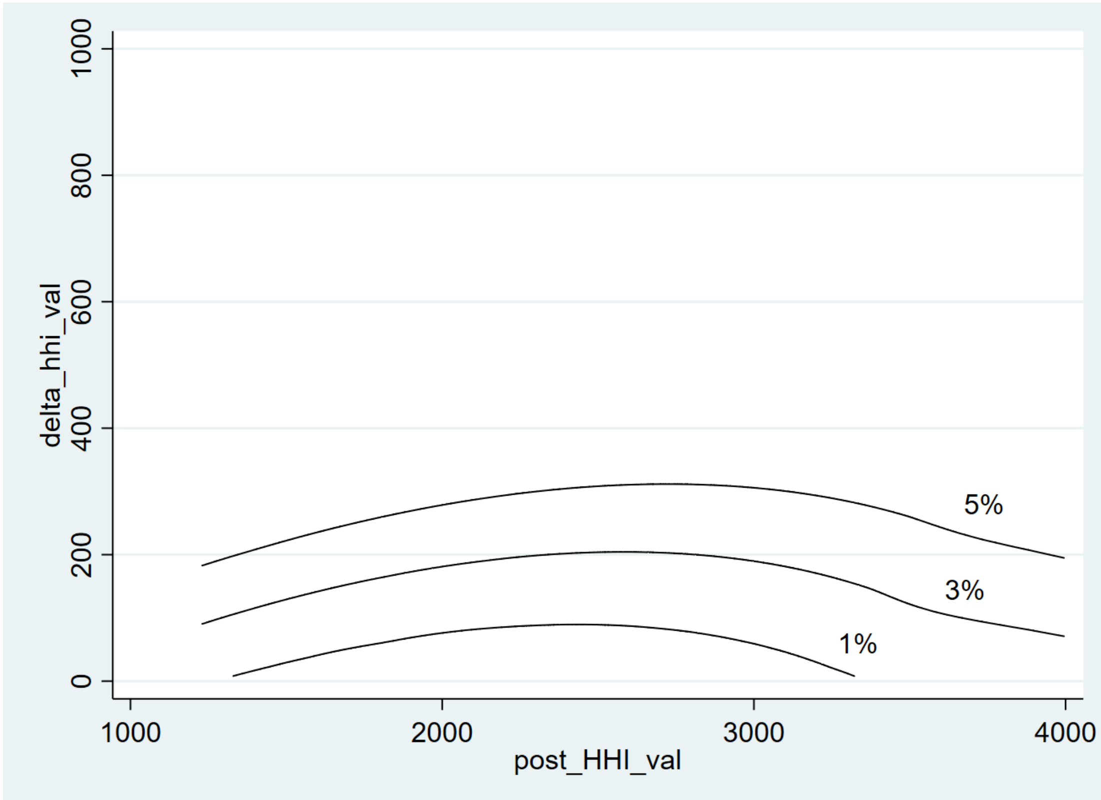

Figure 9: Contour plot showing the combinations of the naively-computed post-merger Herfindahl (labelled here as "post_hhi_val") and the naively-computed merger-induced change in the Herfindahl ("delta_hhi_val") that have no effect on consumer surplus if there is a 1%, 3%, and 5% synergy due to the merger. Points above (respectively, below) a contour line correspond to mergers that are expected to harm (respectively, benefit) consumers. Based on estimates in Table 7, column (5).

Table 8: Share of Mergers with 3% Efficiency Gain That Harm Consumers Under 2010 *Guidelines'* Screening Thresholds (Revenue-based)

Merger Guidelines' Screening Zone		**RCNL-1**	**RCNL-3**
Green Zone (Safe Harbor)		*0.13*	*0.20*
	HHI < 1500	0.61	0.72
	HHI $\in (1500, 2500)$ and $\Delta H < 100$	0.00	0.06
	HHI > 2500 and $\Delta H < 100$	0.00	0.06
Yellow Zone		*0.68*	*0.87*
	HHI $\in (1500, 2500)$ and $\Delta H > 100$	0.80	0.90
	HHI > 2500 and $\Delta H \in (100, 200)$	0.21	0.75
Red Zone (Anticompetitive Presumption)		*0.96*	*1.00*

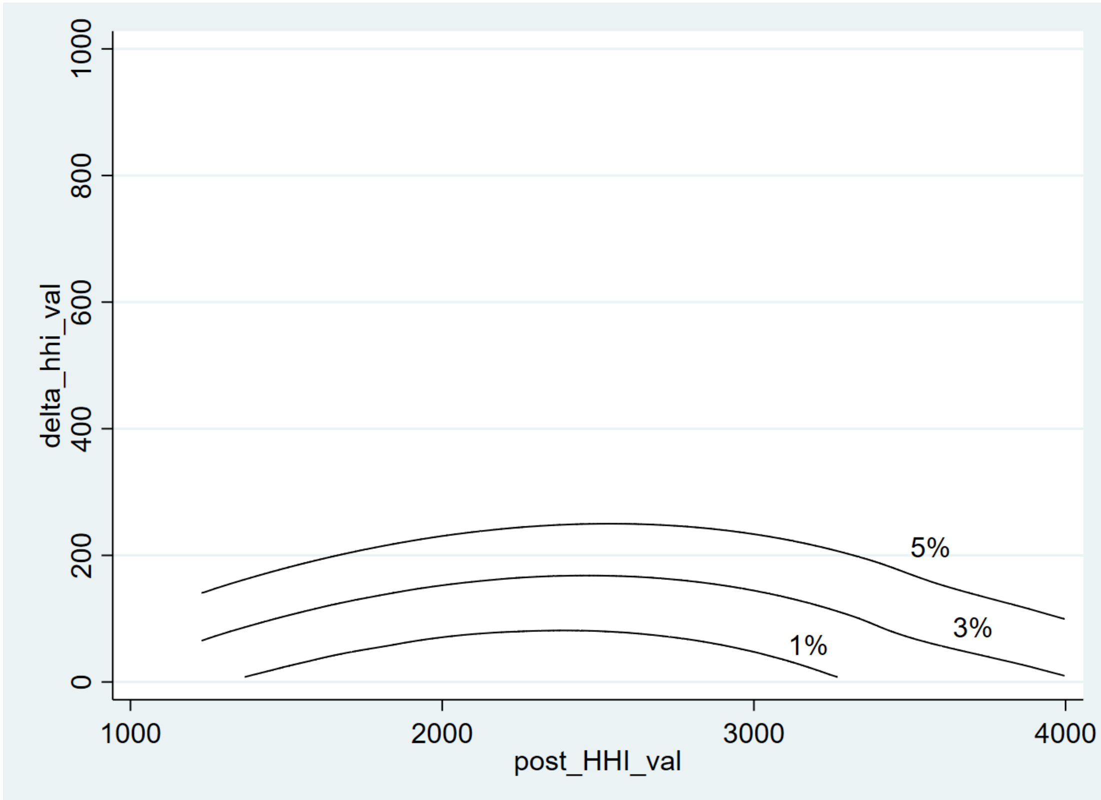

Figure 10: Contour plot showing the combinations of the naively-computed post-merger Herfindahl (labelled here as "post_hhi_val") and the naively-computed merger-induced change in the Herfindahl ("delta_hhi_val") that have no effect on consumer surplus if there is a 1%, 3%, and 5% synergy due to the merger. Points above (respectively, below) a contour line correspond to mergers that are expected to harm (respectively, benefit) consumers. Based on estimates in Table 7, column (6).

Table 9: Share of Mergers with 3% Efficiency Gain That Harm Consumers Under 2010 *Guidelines'* Screening Thresholds (Revenue-based)

Merger Guidelines' Screening Zone		**RCNL-1**	**RCNL-3**
Green Zone (Safe Harbor)		*0.04*	*0.04*
	HHI < 1500	0.17	0.17
	HHI $\in (1500, 2500)$ and $\Delta H < 100$	0.00	0.00
	HHI > 2500 and $\Delta H < 100$	0.00	0.00
Yellow Zone		*0.38*	*0.43*
	HHI $\in (1500, 2500)$ and $\Delta H > 100$	0.48	0.55
	HHI > 2500 and $\Delta H \in (100, 200)$	0.00	0.00
Red Zone (Anticompetitive Presumption)		*0.77*	*0.87*