A Structural Investigation of Quantitative Easing

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Abstract

Did the Federal Reserves' Quantitative Easing (QE) in the aftermath of the financial crisis have macroeconomic effects? To answer this question, we estimate a large-scale DSGE model over the sample from 1998 to 2020, including data of the Fed's balance sheet. We allow for QE to affect the economy via multiple channels that arise from several financial frictions. Our nonlinear Bayesian likelihood approach fully accounts for the zero lower bound on nominal interest rates. We find that QE increased output by about 1.2 percent, reflecting a net increase in investment of nearly 9 percent accompanied by a 0.7 percent drop in aggregate consumption. Both, government bond and capital asset purchases effectively improved financing conditions. Especially capital asset purchases significantly facilitated new investment and increased the production capacity. Against the backdrop of a fall in consumption, supply side effects dominated, leading to a disinflationary effect of about 0.25 percent annually.

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1 Introduction

The recent outbreak of the COVID-19 virus in Europe and United States has caused severe turmoil on financial markets, last seen during the Global Financial Crisis (GFC) in 2007/08. The Federal Reserve (henceforth Fed) responded – then and today – by quickly lowering its policy rate to levels near zero, providing liquidity to financial markets and purchasing assets in large scale. The latter, so the commonly held view, can be seen as a substitute for conventional monetary policy, if the short-term interest rate is constrained by the zero lower bound (ZLB)[1]. Such conclusions are reassuring as developed economies increasingly find themselves in a "new normal" that is characterized by a low natural rate of interest and low inflation, both of which increase the likelihood of future ZLB episodes (Williams, 2016; Kiley, 2018). And indeed, over the past decade, several studies showed that large-scale asset purchases (LSAPs) – the primary policy instrument at the ZLB – were effective in easing financing conditions through compressing term, credit and liquidity premia.[2] Against this backdrop, unconventional monetary policy measures in general, and LSAPs in particular, have become a permanent part of central banks' toolkits.

Despite their prominent role, however, little is known about the macroeconomic impact of LSAPs, in particular, their effect on output, inflation and aggregate investment. The empirical evidence that aims at answering this question remains, by and large, limited to evidence from Vector Autoregression models (including, e.g. Kapetanios et al., 2012; Baumeister and Benati, 2013; Gambacorta et al., 2014; Weale and Wieladek, 2016; Boeckx et al., 2017). While some studies, such as Andres et al. (2004); Gertler and Karadi (2011, 2013); Chen et al. (2012) and Carlstrom et al. (2017), developed structural models to study the effects of QE, none of the studies is estimated over the relevant sample period due to the challenges posed by the ZLB and the resulting nonlinear estimation. As such, a structural investigation of quantitative easing is yet absent.

In this paper, we close this gap by estimating a large-scale DSGE model over the sample from 1998 to 2020 including data of the Fed's balance sheet. Our model incorporates different channels of the aforementioned literature thereby allowing QE to affect the economy via multiple channels. First, three financial frictions ensure limits to arbitrage between short and long-term assets. On the household side, we assume portfolio adjustment costs for patient households and further assume that impatient households are segmented from the market for short-term assets by being restricted to borrow in long-term private loans as in Chen et al. (2012). On the banks' side, an agency problem creates an endogenous constraint to the bank's net worth by limiting their ability to obtain funds from households. As a result, the balance sheet of the banking sector becomes a critical determinant of the cost of credit. These assumptions give rise to an extranormal term premium on government bonds, and similarly, extranormal credit risk premia on private loans and capital claims. Central bank asset purchases compress these risk spreads, thereby easing financing conditions for firms and households in our

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[1] To name a few, see e.g. Hamilton and Wu (2012); Gertler and Karadi (2013); Kiley (2018); Doniger et al. (2019); Bernanke (2020); Sims and Wu (2020)

[2] See e.g. Gagnon et al. (2011); Krishnamurthy and Vissing-Jorgensen (2011); d'Amico et al. (2012); Bauer and Neely (2014); Swanson (2017) for the US, or e.g. Altavilla and Giannone (2017); Altavilla et al. (2019); Eser et al. (2019) for the euro area

model. Importantly, through the portfolio rebalancing channel, these risk spreads are compressed even if the specific asset under consideration is itself not purchased (d'Amico et al., 2012). Specifically, purchases of private capital assets directly affect the real economy by increasing investment.[3] Moreover, through the portfolio rebalancing, also risk premia on private loans fall, thereby stimulating consumption of impatient households. Purchases of government bonds similarly reduce the credit risk premia on capital and private loans, albeit by to a lesser extent. Ultimately, by purchasing government bonds, the central bank frees up balance sheet capacity of the banks, which can then increase their supply of credit. Finally, central bank liquidity provisions directly affect a bank's supply of credit by easing it's incentive compatibility constraint.

Our nonlinear Bayesian likelihood approach fully accounts for the ZLB on nominal interest rates. In the context of large-scale DSGE models, the solution, filtering and estimation of models with occasionally binding constraints poses a host of computational challenges. In order to overcome these challenges, we use the solution method developed in Boehl (2020b) together with the proposed nonlinear Bayesian filter and smoother. The linearized model is solved with the ZLB as an endogenous occasionally binding constraint. To allow sampling from possibly multi-modal, disjoined and high-dimensional posterior distribution, we apply a tempered version of the differential evolution Monte Carlo Markov Chain method which uses a large number of chains (ter Braak, 2006; ter Braak and Vrugt, 2008). Different to e.g. Chen et al. (2012) or Carlstrom et al. (2017), we do not have to cut the sample before the GFC when the short-term rate reached the ZLB. Instead, our nonlinear Bayesian likelihood approach allows us to structurally assess the effects of QE through the lens of a large-scale DSGE model during a period where the QE measures were taken, but the ZLB was binding. This allows us to take a far more in-debt account of the effects of these programs.

We find that between 2009 to 2015, QE increased output by about 1.2. According to our results, this reflects a net increase in investment of nearly 9 percent, that was accompanied by a 0.7 percent drop in aggregate consumption. While emergency liquidity provision measures sharply lowered the credit spread at the onset of the crisis by around 100 basis points, their macroeoconomic effects were negligible due to their short-lived nature. Purchases of private capital securities, on the other hand, were the most expansionary by significantly facilitating new investment. Both, government bond and private security purchases were effective in improving borrowing conditions for firms, but also for households, thereby increasing investment and the productive capacity. This led to a mild disinflationary effect of about 0.25 percent annually.

Expansionary financial shocks can be disinflationary if supply effects dominate demand effects. Demand effects dominate, for instance, in models of Curdia and Woodford (2010) and Gertler and Karadi (2011). Contrary, in models with a cost channel, expansionary financial shocks can be deflationary as lower financing costs are passed on to prices. This is the case in models where firms borrow in advance of production to pay wages, as in Christiano et al. (2005) and Ravenna and Walsh (2006), or capital as in Fiore and Tristani (2013). Quantitative easing, in turn, can be interpreted as such an expan-

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[3]Following Gertler and Karadi (2013), we model the Fed's purchases of mortgage backed securities as the central bank buying claims on the productive capital stock. Despite this discrepancy, our model captures the link between the financial and real economy as balance sheet policies of the central bank affect extranormal credit risk and term premia and thereby lending rates.

sionary financial shock that substantially lowers long-term interest rates. The resulting surge in investment activates what we call the *cost channel of QE* in our model even in the absence of a loan-in-advance constraint for firms as in Carlstrom et al. (2017). In the presence of costs associated with changes in the degree of capital utilization, the higher capital stock resulting from an increase in investment induces a lower capital utilization and thereby pushes down the associated marginal costs. This mechanism is similar to Acharya et al. (2020), who find that cheap credit to impaired firms has a disinflationary effect by creating excess production capacity. Consistent with prevailing supply effects, Barth III and Ramey (2001); Chowdhury et al. (2006) and Abbate et al. (2016) find evidence in favor of a cost channel. Abbate et al. (2016), for instance, show that – using a vector autoregression model with sign restrictions – financial shocks that lower firms' funding costs and increase credit growth and stock prices indeed reduce inflation in the short run. Similarly, using Italian firm-level data on output prices and interest rates paid on debt, Gaiotti and Secchi (2006) find the cost channel to be proportional to the working capital. A different yet related channel is proposed by Gilchrist et al. (2017). Using granular micro-data, the authors show that firms' with binding liquidity constraints increased prices during the GFC, while unconstrained firms lowered them. To rationalize this empirical finding, Gilchrist et al. (2017) build a theoretical model where firms price goods above marginal costs in order to hedge against the risk of relying on costly external finance. Against this backdrop, our results suggest that aggregate supply channels dominated in determining the response of inflation to LSAPs.

Our results are robust to the specification of the household side the related direct channel of QE to consumption. We test this by estimating the representative agent version of the model. In this model vintage, investment increases more in response to a QE shock than in our benchmark model. However, the resulting increase in the production capacity drives utilization further down, which actually induces an even larger fall in inflation and aggregate consumption. In a second vintage, we replace the impatient households by hand-to-mouth consumers inspired by Kaplan et al. (2018). Different to our impatient households, hand-to-mouth consumers do not optimize but simply consume their period labor income, which more closely ties consumption to investment. In this version, QE does not affect consumption directly, but only indirectly via the labor income. For this model vintage, aggregate consumption again falls stronger than in our benchmark model, because falling wages dominate any labor income gains from increased labor supply. The channel described above that exerts downward pressure on inflation is unaltered leaving the inflation response consistent with our benchmark model.

We further test whether our results are also robust to the model choice by estimating the Carlstrom et al. (2017) model the more recent data sample and the same methodology as for our benchmark mode. We confirm the fall in consumption to be a robust consequence of QE. In contrast to our benchmark model, however, in the estimated CFP model demand effects of LSAPs appear to outweigh their supply effects. We take this result with a considerable degree of caution due to several caveats related to the suitability of the CFP model in its original form for estimation on a post-GFC sample. For instance, we find that this model lacks any demand side shock that allows a strict co-movement of consumption and investment.

The rest of the paper is organized as follows. Section 2 presents the core of our large-scale dynamic general equilibrium model. The nonlinear Bayesian estimation methodology is explained in Section 3. Our posterior estimates are discussed in Section 4, together

with an empirical analysis of the GFC and its aftermath. Section 5 then presents our main results in the form of counterfactual analysis of the Fed's unconventional monetary policy measures. Subsequently, in Section 6, we show that our results are robust with respect to several model features. Finally, Section 7 concludes.

2 Model

To study the macroeconomic effects of quantitative easing, we build a large-scale New Keynesian model featuring patient and impatient households, banks, firms as well as a fiscal and a monetary authority. Patient households consume, supply labor and save. Their savings can take the form of short-term bank deposits, private securities backed by firm's capital and government bonds, where the latter two are long-term assets and subject to convex portfolio adjustment costs. Instead, impatient households borrow long-term private loans from banks besides consuming and supplying labor. Similarly to Chen et al. (2012), households are thus split into "savers" and "borrowers". Banks are modeled as in Gertler and Karadi (2011). They collect deposits from patient households, which they lend on to impatient households, intermediate good producers and the government in the form of long-term bonds and loans. A moral hazard problem constrains their leverage, which creates limits to arbitrage needed for QE to have an effect on the banks balance sheet.

The production sector consists of three types of firms for reasons of tractability. Intermediate good producers employ labor and capital to produce their goods. Each period, after producing their output, they sell their used capital stock to the capital goods producers. The latter repair it and invest in new capital. At the end of the period, capital is re-sold to the intermediate good producers which use it for production in the next period. Intermediate goods are purchased by retailers which repackage them and sell them with a markup as final goods. Similarly, labor is differentiated by a union with monopoly power that faces nominal rigidities.

The government consumes final goods, collects taxes, and issues long-term government bonds. Monetary policy sets the short-term interest rate according to a Taylor-type rule which is constrained by the ZLB. In line with the literature on estimated DSGE models, our model also includes standard features such as habit formation in consumption, investment adjustment costs, variable capital utilization, nominal rigidities as in Calvo (1983) in both, price and wage setting, as well as price and wage indexation.

We model large-scale asset purchases of treasury bonds and private capital assets to follow exogenous AR(2) processes. This way, we allow for anticipation and stock effects without necessarily having to specify a policy rule for unconventional monetary policy. Arguably, the measures of QE came as much as a surprise to the US economy as the crisis did. At the same time, once in action the future path of these measures was public information. Our outlined setup approximates this structure. Liquidity injections by the central bank to financial intermediaries are also exogenous.

Finally, time is discrete and one period in the model represent one quarter. Below, we will describe the financial sector and the household structure in more detail, while we refer the reader to Appendix B in the Online Appendix for a full description of the model.

2.1 The household structure

The model is populated by two types of households. We assume a continuum of impatient households with mass χ and a continuum of patient households of mass $1-\chi$. In the spirit of Gertler and Karadi (2013), we impose that a constant fraction f of the patient household works as banker, whereas the remaining fraction $1-f$ consists of workers who – like impatient households – supply labor to the intermediate good producers. While workers receive their wage income every period, bankers reinvest their gains in asset holdings of the bank over several periods. Only when a banker (exogenously) exits the banking sector, she contributes to the patient households' income by bringing home the accumulated profits. Perfect consumption insurance within patient households ensures that workers and bankers face the same consumption stream. The expected lifetime utility of any household i is given by

$$U_t = E_0 \sum_{t=0}^{\infty} \beta_i^t \left(\frac{(C_{i,t} - hC_{i,t-1})^{1-\sigma_c} - 1}{1 - \sigma_c} \right) exp\left(\frac{\sigma_c - 1}{1 + \sigma_l} L_{i,t}^{1+\sigma_l} \right) \tag{1}$$

where parameters β_i, h, σ_c, and σ_l are, respectively, the discount factor, the degree of external habit formation in consumption, the coefficient of relative risk aversion, and a weight on the disutility of labor. The discount factor has a household-specific subscript i, because we assume the discount factor of impatient households β_m to be smaller than the discount factor of patient households, i.e. $\beta_m < \beta_p$. Finally. $C_{i,t}$ and $L_{i,t}$ denote consumption and hours worked of household $i \in \{m,p\}$, respectively.[4]

2.2 Patient households

The patient household earns the real wage, W_t, for her supplied labor, $L_{p,t}$. She can save in one-period bank deposits, D_t, that pay an interest rate, R_t^d, in government bonds, $B_{h,t}$, that yield an interest rate, R_t^b, and in capital assets $K_{h,t}$ with an associated interest rate, R_t^k. These interest rates are already in real terms, i.e. accounting for inflation. Capital claims, just like government bonds, are modeled as long-term assets. As for all stock variables, we use the end-of-period notation, so that D_t denotes the household's deposits at the end of period t. The return on deposits, $R_t^d = v_{u,t}R_t$, includes a disturbance term, $v_{u,t}$ which drives a wedge between the risk-free real rate and the return on deposits. We assume $v_{u,t}$ to follow an AR(1) process in logs. Smets and Wouters (2007) label this shock a risk-premium shock which they interpret as variations in the confidence in the banking system. Patient households spend their funds on consumption $C_{p,t}$, and save in new deposits, bonds and capital. Savings in government bonds and capital are, as in e.g. Chen et al. (2012), subject to portfolio adjustment costs with adjustment parameters κ_b and κ_k. The budget constraint of patient households, in real terms, reads

$$C_{p,t} + \frac{D_t}{R_t^d} + Q_t[K_{h,t} + \frac{1}{2}\kappa_k(K_{h,t} - K_h)^2] + Q_t^b[B_{h,t} + \frac{1}{2}\kappa_b(B_{h,t} - B_h)^2]$$
$$= D_{t-1} + W_t L_{p,t} + R_t^k Q_{t-1} K_{h,t-1} + R_t^b Q_{t-1}^b B_{h,t-1} - T_t + \mathfrak{P}_t. \tag{2}$$

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[4]For the ease of notation, we only use subscripts $i \in \{m,p\}$ indicating which type of household is meant if necessarily needed.

T_t denotes lump sum taxes raised by the government to finance government spending, and $\mathfrak{P}_t$ are profits of monopolistic firms and banks that accrue to the patient households. Maximizing (1) subject to the patient household's budget constraint (2) and rearranging the first order conditions yields the well-known Euler equation, a condition for the optimal supply of labor and two no-arbitrage conditions:

$$1 = \beta_p E_t \left[\exp \left(\frac{\sigma_c - 1}{1 + \sigma_l} (L_{p,t+1}^{1+\sigma_l} - L_{p,t}^{1+\sigma_l}) \right) \left(\frac{C_{p,t+1} - hC_{p,t}}{C_{p,t} - hC_{p,t-1}} \right)^{-\sigma_c} \right] R_t^d, \quad (3)$$

$$W_t^h = (C_{p,t} - hC_{p,t-1}) L_{p,t}^{\sigma_l}, \quad (4)$$

$$E_t R_{t+1}^k = R_t^d [1 + \kappa_k (K_{h,t} - K_h)], \quad (5)$$

$$E_t R_{t+1}^b = R_t^d [1 + \kappa_b (B_{h,t} - B_h)]. \quad (6)$$

Ultimately, the no-arbitrage conditions (5) and (6) specify the patient household's optimal holdings of both capital claims and government bonds.

2.3 Impatient households

There is a fraction of χ impatient households which consume, supply labor and borrow long-term private loans from the banks which gives rise to the following budget constraint in real terms

$$C_{m,t} + R_t^p Q_{t-1}^p B_{m,t-1}^p = W_t L_{m,t} + Q_t^p B_{m,t}^p, \quad (7)$$

where R_t^P and Q_t^P denote, respectively, the interest rate and price of private loans $B_{m,t}^p$. Price and yield of private loans are related through

$$R_t^p = \frac{\xi + \kappa_p Q_t^p}{Q_{t-1}^p},$$

where ξ is the coupon (or redemption) and κ_p denotes the decay factor (Woodford, 1998, 2001). Maximizing (1) subject to the impatient households budget constraint (7) yields, after some rearranging, an Euler equation for the optimal borrowing and a condition for the labor supply of impatient households:

$$1 = \beta_m E_t \left[\exp \left(\frac{\sigma_c - 1}{1 + \sigma_l} (L_{m,t+1}^{1+\sigma_l} - L_{m,t}^{1+\sigma_l}) \right) \left(\frac{C_{m,t+1} - hC_{m,t}}{C_{m,t} - hC_{m,t-1}} \right)^{-\sigma_c} \right] R_t^p, \quad (8)$$

$$W_t^h = (C_{m,t} - hC_{m,t-1}) L_{m,t}^{\sigma_l}. \quad (9)$$

2.4 Banks

The banking sector draws on Gertler and Karadi (2013) with the extensions that we outline below. Banks collect deposits D_t from patient households and – together with their own net worth N_t – use these funds to extend loans $B_{b,t}^p$ to impatient households, purchase capital securities from intermediate good producers, $K_{b,t}$, and purchase government bonds $B_{b,t}$. Given these financial operations, the balance sheet of a representative bank then follows as

$$Q_t K_{b,t} + Q_t^b B_{b,t} + Q_t^p B_{b,t}^p = N_t + D_t + L_t^q, \quad (10)$$

where L_t^q denotes exogenous emergency liquidity injections by the Federal Reserve. While it can be argued that this might be an ad hoc way of modeling such injections, the provision of central bank liquidity was a very important funding source when interbank market dried up during the height of financial crisis. As such, these operations were essential in preserving market functioning and preventing cascading fire sales which, ultimately, might have led to a credit crunch (Bernanke, 2008; Fleming, 2012). In our model, these liquidity injections directly support bank lending by increasing banks' net worth and easing their financial constraints as shown below. Despite their relatively short duration, these liquidity injections have been sizable, and their effects have not yet been assessed empirically in a structural context. For simplicity, we assume that central bank liquidity is lent at a zero nominal interest rate (i.e. their real rate equals $R_t^L = 1/\Pi_{t+1}$, where Π_t denotes gross inflation). Banks retain their earnings and add it to their current net worth. This gives rise to the following law of motion for the bank's net worth

$$
N_t = R_t^k Q_{t-1} K_{b,t-1} + R_t^b Q_{t-1}^b B_{b,t-1} + R_t^p Q_{t-1}^p B_{b,t-1}^p
$$
$$
- R_{t-1}^d D_{t-1} - R_{t-1}^L L_{t-1}^q. \tag{11}
$$

Note that while the interest rate on deposits raised in period $t-1$ is determined in the same period, the return of assets is risky and only determined after the realization of shocks at the beginning of period t.

Bankers continue accumulating their individual net worth until they (involuntarily) exit the business, which occurs randomly with exogenous probability, $1 - \theta$. Conversely, bankers continue their operations with probability θ. Draws from this lottery are i.i.d. and do not depend on the banker's history. When a banker leaves the sector, she adds her terminal wealth, V_t, to the wealth of the patient household she is member of. Therefore, bankers seek to maximize the expected discounted terminal value of their wealth

$$
\begin{aligned}
V_t &= \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta_p^{i+1} \frac{\Lambda_{p,t+1+i}}{\Lambda_{p,t+i}} N_{t+1+i}, \\
&= \max E_t \left[\beta_p \frac{\Lambda_{p,t+1}}{\Lambda_{p,t}} (1-\theta) N_{t+1} + \theta V_{t+1} \right], \tag{12}
\end{aligned}
$$

where $\Lambda_{p,t}$ denotes the patient household's Lagrangian multiplier with respect to the budget constraint.

Banks operate under perfect competition. If financial intermediation was frictionless, the risk adjusted return on the bank's asset should equal the return on deposits. As in Gertler and Karadi (2013), however, bankers can divert a fraction of their assets and transfer it to their respective households. If they do so, their depositors will withdraw their remaining funds and force the bank into bankruptcy. This moral hazard/costly enforcement problem creates an endogenous limit to the amount of deposits that households are willing to supply. While the latter ensures that bankers earn a strictly positive excess return, it also creates limits to arbitrage, as bankers can not scale-up their balance sheet to arbitrage away any price differences. In order to prevent a banker from diverting a fraction of assets, households keep their deposits at a bank only as long as the bank's continuation value is higher or equal to the amount that the bank can divert. Formally, the latter condition is given by the following incentive compatibility constraint of the

bank

$$V_t \geq \lambda_{k,t}Q_tK_{b,t} + \lambda_b Q_t^b B_{b,t} + \lambda_p Q_t^p B_{b,t}^p - \lambda_L L_t^q, \tag{13}$$

where λ_j for $j \in \{k, p, b\}$ denotes the respective fraction of capital claims, government bonds or private loans that the bank can diverted. Following Dedola et al. (2013) and Gelain and Ilbas (2017), we allow $\lambda_{k,t}$ to be time-varying, formally following an AR(1) process in logs with mean λ_k. Ultimately, this shock triggers variations in the divertiblity of capital assets and can be interpreted as variations in the trust depositors have in the quality of banks' capital assets.

In steady state, the λ_j's for $j \in \{k, p, b\}$ determine – together with other estimated parameters such as the discount factor of patient households, β_p, and the trend growth rate, γ – the returns on capital claims and government bonds, R^k and R^b, as well as the private loan rate R^p. We set the prior mean of the relevant parameters such that the respective excess returns over the deposit rate are based on data for the US treasury rate, Gilchrist and Zakrajšek (2012) corporate spread and mortgages rates for private households. This results, a priori, in $\lambda_b < \lambda_p < \lambda_k$, which intuitively can be motivated by the fact that, in general, the collateral value of government bonds is higher than that of mortgage loans and capital claims.[5] The reason is that treasury bonds enjoy higher credit ratings and are subject to less liquidity risks than mortgage loans or capital claims. Finally, the last term in the incentive constraint is due to the assumption that liquidity injections serve to relax the incentive constraint of banks.

To solve the bank problem, let an initial guess of the value function be of the form

$$V_t = \nu_{k,t}Q_tK_{b,t} + \nu_{b,t}Q_t^b B_{b,t} + \nu_{p,t}Q_t^p B_{b,t}^p + \nu_{n,t}N_t + \nu_{L,t}L_t^q, \tag{14}$$

where $\nu_{k,t}, \nu_{b,t}, \nu_{p,t}, \nu_{d,t}$, and $\nu_{L,t}$ are time-varying coefficients. Maximizing (14) with respect to $K_{b,t}, B_{b,t}$ and $B_{b,t}^p$ subject to (13) yields the following first order conditions for capital claims, governments bonds, private loans, and μ_t, the Lagrangian multiplier on the incentive compatibility constraint

$$\nu_{k,t} = \lambda_{k,t}\frac{\mu_t}{1+\mu_t}, \tag{15}$$

$$\nu_{b,t} = \lambda_b\frac{\mu_t}{1+\mu_t}, \tag{16}$$

$$\nu_{p,t} = \lambda_p\frac{\mu_t}{1+\mu_t}, \tag{17}$$

$$\nu_{n,t}N_t + \nu_{L,t}L_t^q = (\lambda_{k,t} - \nu_{k,t})Q_tK_{b,t} + (\lambda_b - \nu_{b,t})Q_t^b B_{b,t} + (\lambda_p - \nu_{p,t})Q_t^p B_{b,t}^p. \tag{18}$$

Given that the incentive compatibility constraint binds[6], we can rewrite this last equation as

$$Q_tK_{b,t} = \frac{\nu_{b,t} - \lambda_b}{\lambda_{k,t} - \nu_{k,t}}Q_t^b B_{b,t} + \frac{\nu_{p,t} - \lambda_p}{\lambda_{k,t} - \nu_{k,t}}Q_t^p B_{b,t}^p + \frac{\nu_{L,t} + \lambda_L}{\lambda_{k,t} - \nu_{kt}}L_t^q + \frac{\nu_{n,t}}{\lambda_{k,t} - \nu_{k,t}}N_t. \tag{19}$$

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[5]In a similar vein, Meeks et al. (2017) use the same approach to distinguish between the collateral values of loans and asset-backed securities.

[6]The constraint binds in the neighborhood of the steady state. For convenience, we make the assumption that it is binding throughout all experiments.

Intuitively, (19) states that banks' demand for capital claims decreases in λ_j for $j \in \{k, p, b\}$, which regulate the tightness of the incentive constraint with respect to capital claims, mortgage loans over government bonds. Central bank liquidity injections L_t^q, on other hand, support the demand for capital claims.

Substituting the demand for capital claims into (14), and combining the result with (15) one can write the terminal value of the banker as a function of its net worth

$$V_t = (1 + \mu_t)\nu_{nt}N_t + [(1 + \mu_t)\nu_{L,t} + \mu_t\lambda_L]L_t^q \tag{20}$$

A higher continuation value, V_t, is associated with a higher shadow value of holding an additional marginal unit of assets, or put differently, with a higher shadow value of marginally relaxing the incentive compatibility constraint. Then, define the bank's stochastic discount factor as

$$\Omega_t \equiv \frac{\Lambda_{p,t}}{\Lambda_{p,t-1}}\left[(1 - \theta) + \theta(1 + \mu_t)\nu_{n,t}\right], \tag{21}$$

and substitute (20) into the Bellman equation (12). Using the law of motion for net worth (11), one can then write the value function as

$$\begin{aligned}
V_t =& \beta_p E_t\Bigg[\Omega_{t+1}((R_{t+1}^k - R_t^d)Q_tK_{b,t} + (R_{t+1}^b - R_t^d)Q_t^bB_{b,t} \\
& + (R_{t+1}^p - R_t^d)Q_t^pB_{b,t}^p + (R_t^d - R_{L,t})L_t^q + R_t^dN_t)\Bigg] \\
& + \beta_p E_t\left[\frac{\Lambda_{p,t+1}}{\Lambda_{p,t}}\theta[(1 + \mu_{t+1})\nu_{L,t+1} + \mu_{t+1}\lambda_L]L_{t+1}^q\right]
\end{aligned}$$

Finally, verifying the initial guess for the value function yields

$$\nu_{k,t} = \beta_p E_t\Omega_{t+1}(R_{t+1}^k - R_t^d), \tag{22}$$

$$\nu_{b,t} = \beta_p E_t\Omega_{t+1}(R_{t+1}^b - R_t^d), \tag{23}$$

$$\nu_{p,t} = \beta_p E_t\Omega_{t+1}(R_{t+1}^p - R_t^d), \tag{24}$$

$$\nu_{n,t} = \beta_p E_t\Omega_{t+1}R_t^d \tag{25}$$

$$\nu_{L,t} = \beta_p E_t\left[\Omega_{t+1}(R_t^d - R_t^L) + \theta\rho_{cbl}\frac{\Lambda_{p,t+1}}{\Lambda_{p,t}}[(1 + \mu_{t+1})\nu_{L,t+1} + \mu_{t+1}\lambda_L]\right], \tag{26}$$

where the last equality follows from the fact that L_t^q follows an AR(1) in logs with persistence parameter ρ_{cbl}.

2.5 Monetary policies and the ZLB

In response to the Great Recession, the Federal Reserve cut its policy rate to essentially zero. We model conventional monetary policy as a standard reaction function with the central bank responding to deviations of inflation from it's target, the output gap and its growth rate

$$\frac{R_t^s}{R^n} = \left(\frac{R_{t-1}^s}{R^n}\right)^\rho\left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_t^*}\right)^{\phi_y}\left(\Delta\left(\frac{Y_t}{Y_t^*}\right)\right)^{\phi_{dy}}\right]^{1-\rho}v_{r,t}, \tag{27}$$

10

with the ZLB constraint

$$R_t^n = \max\left\{\bar{R}, R_t^s\right\},\tag{28}$$

where we refer to the unconstrained nominal rate R_t^s as the notional (or shadow) rate. Y_t^* denotes the potential output and $\Delta\left(\frac{Y_t}{Y_t^*}\right)$ denotes the growth in the output gap. The parameter ρ_R expresses an interest rate smoothing motive by the central bank over the notional rate and ϕ_π, ϕ_y and ϕ_{dy} are feedback coefficients. The max-operator in (28) reflects the zero lower bound (ZLB) on the nominal interest rate R_t^n, which we take into account in our estimation procedure. For this purpose, $\bar{R}$ denotes the exact level at which the ZLB binds.[7] When the economy is away from the ZLB, the stochastic process $v_{r,t}$ – which follows an AR(1) in logs – represents a regular interest rate shock. However, when the nominal interest rate is zero, $v_{r,t}$ may not directly affect the level of the nominal interest rate. Instead, $v_{r,t}$ affects the expected path of the notional rate, first through it's own persistence and, second, through the persistence in the notional rate, and therefore alter the expected duration of the lower bound spell. At the ZLB, it can hence be viewed as a forward guidance shock.

At the onset of the Financial crisis, the Federal Reserve injected large amounts of liquidity into the financial sector which is known as credit easing. We capture these emergency liquidity injections with as an exogenous variable that eases banks' incentive compatibility constraint (13) and thereby stimulates lending. In our estimation, we feed the time series data on these liquidity injections into the model and assume that the associated process follow AR(1) process. That is, formally

$$\tilde{L}_t = \rho_{cbl}\tilde{L}_{t-1} + \epsilon_{CBL,t},\tag{29}$$

where $\tilde{L} \equiv \frac{L_t^q}{P_t Y_t}$ denotes central bank liquidity as percentage of GDP.

When the policy rate hit the ZLB in December 2008, the Federal Reserve further started its large scale asset purchase program, under which it purchased different debt instruments in order to suppress credit and term premia. In our analysis, we divide these purchases into private (capital) security purchases and government bond purchases, both which we assume to follow an AR(2) process in logs.

$$\tilde{K}_{cb,t} = \rho_{k,1}\tilde{K}_{cb,t-1} + \rho_{k,2}\tilde{K}_{cb,t-2} + \epsilon_{QEK,t},\tag{30}$$

$$\tilde{B}_{cb,t} = \rho_{b,1}\tilde{B}_{cb,t-1} + \rho_{b,2}\tilde{B}_{cb,t-2} + \epsilon_{QEB,t}.\tag{31}$$

Similar to the liquidity injections, $\tilde{K}$ and $\tilde{B}$ denote, respectively, the central banks capital claim and government bond purchases as a fraction of GDP. The advantage of an AR(2) process is that it can capture the hump-shaped response of the asset purchases, thereby also ensuring anticipation or stock effects at the moment the announcement was made.

3   Estimation and Methodology

The fact that our sample includes a long episode where the ZLB binds poses a host of technical challenges. These are related to the solution, filtering and estimation of the

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[7]Given that, empirically, the Federal Funds rate remained strictly above zero, we choose $\bar{R}$ to be slightly above one in our estimation. Moreover, due to the fact that the Fed never implemented negative rates, we use the term "zero lower bound" and "effective lower bound" interchangeably.

model in the presence of an occasionally binding constraint (OBC). While solution methods for models with OBCs exists – as do nonlinear filters – the satisfactory combination of both in the context of a large-scale DSGE model is computationally very expensive and was so far deemed impossible. In this section, we first briefly sketch the set of novel methods proposed by Boehl (2020b) that allow us to estimate such high dimensional models in the presence of a binding ZLB. Subsequently, this section describes our choices with regard to the data.

3.1 Solution method

Throughout this paper we apply the solution method for OBCs presented in Boehl (2020b). We refer to the original paper for details. The model is linearized around its steady state balanced growth path and thereby implicitly detrended. Respecting the ZLB, the original model with the variable vector $\mathbf{y}_t$ can be represented as a piecewise one-sided first-order auxiliary model with

$$\mathbf{N} \begin{vmatrix} \mathbf{v}_t \\ \mathbf{w}_{t-1} \end{vmatrix} + \mathbf{h} \max \left\{ \mathbf{p} \begin{vmatrix} E_t \mathbf{v}_{t+1} \\ \mathbf{w}_t \end{vmatrix} + \mathbf{m} \begin{vmatrix} \mathbf{v}_t \\ \mathbf{w}_{t-1} \end{vmatrix}, \bar{r} \right\} = E_t \begin{vmatrix} \mathbf{v}_{t+1} \\ \mathbf{w}_t \end{vmatrix}, \tag{32}$$

where $\begin{vmatrix} \mathbf{v}_t \\ \mathbf{w}_{t-1} \end{vmatrix} = S \begin{vmatrix} \mathbf{y}_t \\ \boldsymbol{\varepsilon}_t \end{vmatrix}$ is an auxiliary representation. $\mathbf{w}_{t-1}$ contains all the (latent) state variables augmented by the current shocks, and $\mathbf{v}_t$ contains all forward looking variables. $\mathbf{N}$ is the system matrix and $\bar{r}$ is the minimum value of the constrained variable r_t (here, the nominal interest rate). The constraint is included with $r_t = \max \left\{ \mathbf{p} \begin{vmatrix} E_t \mathbf{v}_{t+1} \\ \mathbf{w}_t \end{vmatrix} + \mathbf{m} \begin{vmatrix} \mathbf{v}_t \\ \mathbf{w}_{t-1} \end{vmatrix}, \bar{r} \right\}$. The vector $\mathbf{h}$ contains the effects of r_t onto all other variables. Further, denote by the two integer values k and l respectively the expected duration of the ZLB spell and the expected number of periods before the ZLB binds.

It can be shown that the rational expectations solution to (32) for the state s periods ahead, $(\mathbf{v}_{t+s}, \mathbf{w}_{t+s-1})$, can be expressed in terms of $\mathbf{w}_{t-1}$ and the expectations on k and l as

$$L_s(l, k, \mathbf{w}_{t-1}) = \mathbf{N}^{\max\{s-l,0\}} \hat{\mathbf{N}}^{\min\{l,s\}} S(l, k, \mathbf{w}_{t-1}) \tag{33}$$

$$+ (\mathbf{I} - \mathbf{N})^{-1} (\mathbf{I} - \mathbf{N}^{\max\{s-l,0\}}) \mathbf{h} \bar{r} \tag{34}$$

$$= \begin{vmatrix} \mathbf{v}_{t+s} \\ \mathbf{w}_{t+s-1} \end{vmatrix}, \tag{35}$$

where $\hat{\mathbf{N}} = (\mathbf{I} - \mathbf{h} \otimes \mathbf{p})^{-1} (\mathbf{N} + \mathbf{h} \otimes \mathbf{m})$ and

$$S(l, k, \mathbf{w}_{t-1}) = \left\{ \mathbf{v}_t : \mathbf{Q} \mathbf{N}^k \hat{\mathbf{N}} \begin{vmatrix} \mathbf{v}_t \\ \mathbf{w}_{t-1} \end{vmatrix} = -\mathbf{Q}(\mathbf{I} - \mathbf{N})^{-1} (\mathbf{I} - \mathbf{N}^k) \mathbf{h} \bar{r} \right\}. \tag{36}$$

Here, $\mathbf{Q} = \begin{vmatrix} \mathbf{I} & -\boldsymbol{\Omega} \end{vmatrix}$ for $\mathbf{v}_t = \boldsymbol{\Omega} \mathbf{w}_{t-1}$ represents the linear rational expectations solution of the unconstrained system as e.g. given by Blanchard and Kahn (1980) or Klein (2000).

Finding the equilibrium values of (l, k) must be done numerically. One advantage of the above representation is that the simulation of anticipated equilibrium paths can be avoided when iterating over (l, k). The resulting transition function is a nonlinear

state-space representation.[8]

3.2 Filtering and Estimation Method

Likelihood inference requires a nonlinear Bayesian filter (An and Schorfheide, 2007). Given the high dimensionality of our model, the particle filter is not feasible.[9] To fill this gap, Boehl (2020b) introduces the transposed-ensemble Kalman filter (TEnKF) which is a hybrid of the particle filter and Kalman filter. For the transition $t - 1 \rightarrow t$ an *ensemble* of particles is sampled from the state distribution at $t - 1$. Instead of re-sampling (particle filter), the TEnKF applies statistical linearization to update the state estimate represented by the ensemble to match each new observation vector. This allows to efficiently approximate the distribution of states for large-scale nonlinear systems with only a few hundred particles instead of several million or billion, as for the particle filter, which is computationally advantageous.[10] Boehl (2020b) also proposes a nonlinear path-adjustment smoother (NPAS) for high-dimensional nonlinear models, which we use obtain the smoothed/historic shock innovations.

We sample from the posterior distribution using a tempered version of the differential evolution Monte Carlo Markov chain method (ter Braak, 2006; ter Braak and Vrugt, 2008, DE-MCMC). The DE-MCMC sampler is a subclass of ensemble MCMC methods. Instead of using a single Markov chain (as e.g. the Metropolis algorithm), such ensemble samplers use a large number of chains (as well called *ensemble*). Proposals for each iteration are generated based on the state of the previous ensemble instead of an explicit proposal distribution.. The DE-MCMC sampler is hence self-tuning and the ensemble structure make massive parallelization straightforward. This is in particular important as both the simulation and the filtering step are computationally expensive. The combination of DE-MCMC with tempering (similar to Herbst and Schorfheide, 2014) has the advantage that it is very robust to local maxima and odd-shaped or bimodal distributions.[11]

3.3 Data and calibration

In order to quantify the effects of the large scale asset purchases and liquidity injections taken in the aftermath of the Financial Crisis of 2008-09, we estimate our model on data from 1998:I to 2019:IV. Importantly, and different to earlier papers that also attempt to assess the effects of large-scale asset purchases, such as Chen et al. (2012)

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[8]We use the implementations of Boehl (2020a,c), which for the model presented here will solve for the nonlinear state-space representation of about 80.000 particles draws per processor and second.

[9]The *inversion filter* used in Guerrieri and Iacoviello (2017) and discussed in Cuba-Borda et al. (2019) is also not an option as it is not a Bayesian filter and ignores uncertainty on the initial states and the observations. This may not be crucial for small-scale models without endogenous state variables as in Atkinson et al. (2019), but is important given the high dimensionality of our model and our relatively short data sample.

[10]For all estimations and for the numerical analysis we use an ensemble of 350 particles. For our model, the evaluation of the likelihood for one parameter vector would then take 1-2 seconds on a single CPU.

[11]We initialize the posterior ensemble with 200 draws sampled from the prior distribution. We then use 8 temperature steps with each 200 iterations. Finally, we let the sampler 2500 run iterations, of which we keep the last 500 ensembles. The posterior parameter distribution is hence represented by $500 \times 200 = 10000$ parameter sets.

and Carlstrom et al. (2017), we also include the ZLB period and data on the Fed's balance sheet in our nonlinear estimation procedure. We find that this is crucial to properly assess the effects of QE.[12] In a longer sample, which includes the Great Moderation, a downward trend in the nominal interest rate severely distorts the analysis.[13] Stretching the sample further back in time as in Carlstrom et al. (2017), on the other hand, bears the risk of misspecification given the large amount of parameters we estimate and, concomitantly, a risk of biasing the effects of QE. For this, we chose the sample to adequately capture the current low interest rate environment and potential structural changes in the economy, like a flatter Phillips curve.[14]

We use a total of eleven observables in the estimation. GDP, consumption, investment and wages are all in real terms and calculated as per capita growth rates. Likewise, we use average weekly hours worked multiplied by the employment level and divided by civilian noninstitutional population to measure the per capita labor supply. Due to artificial dynamics in the civilian noninstitutional population series that arise from irregular updating (Edge et al., 2013), we use a 4-quarter trailing moving average instead. Inflation is measured as the log differences of the GDP deflator. We include the Federal Funds Rate (FFR) and the Gilchrist and Zakrajšek (2012, henceforth GZ) spread as quarterly rates. The latter is an average credit spread on corporate bonds, very similar to Moody's BAA spread, yet considers the entire spectrum of credit ratings from "single D" to "triple A". While the dynamics of both spreads is very similar over our sample (the correlation of quarterly data reaches 0.9), the GZ spread peaks higher during the Financial crisis but remains less elevated thereafter than the BAA spread, both which is preferred by the model.

For the unconventional monetary policies, we use three observables that we feed into the model as exogenous policy shocks. First, we take the total face value of U.S. Treasury securities held by the Federal Reserve divided by nominal GDP as measure for the Fed's government bond purchases. Second, we add the current face value of mortgage-backed obligations held by Federal Reserve to the net portfolio holdings of the Commercial Paper Funding Facility, both as a fraction of nominal GDP, and use it as a measure for purchases of private capital securities. Third, to measure the Fed's emergency liquidity injections in 2008/09, we add – in line with Fleming (2012) – the central bank liquidity swaps, the current face value of federal agency obligations held by Federal Reserve, the term auction credit held by the Federal Reserve and other loans held by the Federal Reserve, all as a fraction of nominal GDP. The latter mainly includes the Asset-backed Commercial Paper Money Market Mutual Fund Liquidity Facility and the Primary Dealer Credit Facility. Figure 1 shows those three time series.

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[12]In Appendix I, we show that using the pre-crisis sample from 1983:I until 2008:III may severely bias the effects of these measures.

[13]Boehl and Strobel (2020) show that in the estimation of the Smets and Wouters (2007) model on data from 1983-2019, the downwards trend in the nominal rate is reflected by a artificially elevated consumption series that even in the GFC does not drop below steady-state.

[14]There is an extensive literature that documents a fall in the natural rate of interest and trend inflation since the 1980s (see e.g. Laubach and Williams, 2003; Brand et al., 2018, 2020). Similarly, the semi-structural estimates of Laubach and Williams (2003) and Brand et al. (2020) indicate weak relationships between inflation and economic slack, in line with structural estimates of Del Negro et al. (2015) and Kulish et al. (2017). Finally, a similar flattening has been documented for the wage Phillips curve by e.g. Daly and Hobijn (2014).
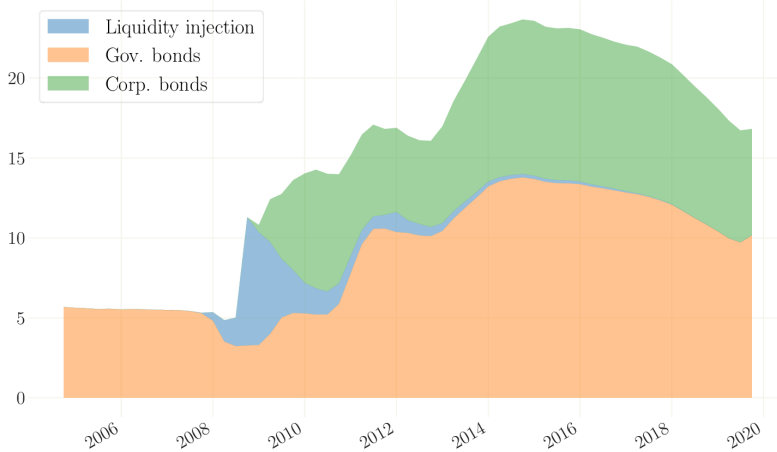
Figure 1: Unconventional monetary policy measures: The Fed's balance sheet expansion.
Note: All variables are in % of GDP. See Appendix A for more details on their construction.

To facilitate the nonlinear filtering, we assume small measurement errors for all variables with a variance that is 0.01 times the variance of the respective series. Since the Federal Funds rate is perfectly observable (though on higher frequency) we divide the measurement error variance here again by 100. Except for labor supply, the data is not demeaned as we assume the non-stationary model follows a balanced growth path that we estimate in line with Smets and Wouters (2007). The measurement equations and a detailed description of the data, as well as its treatment and sources is delegated to Appendix A.

Finally, we fix several parameters prior to estimating the others. In line with Smets and Wouters (2007), we set the depreciation rate to $\delta = 0.025$, the steady state government share in GDP to $G/Y = 0.18$, and the curvature parameters of the Kimball aggregators for prices and wages to $\epsilon_p = \epsilon_w = 10$. The steady state markup in the labor market is set to $\lambda_w = 1.1$. We set the decay factor for both government bonds and private loans to 0.975, which implies an average maturity of 40 quarters. The quarterly coupon/repayment is set to 0.04. Also, we calibrate the empirical lower bound of the nominal interest rate for the U.S. to 0.05% quarterly. Setting it exactly to zero would imply that the ZLB never binds in our estimations, as the observed FFR remained strictly above zero. Our choice therefor maintains that the ZLB is considered binding throughout the period from 2009:Q1 to 2015:Q4. More precisely, it holds that $\bar{r} = -100(\frac{\bar{\pi}}{(\beta\gamma)^{-\sigma_c}} - 1) + 0.05$. Lastly, the Fed's treasury holdings as percentage of GDP have neither been zero nor constant in the years preceding the Financial crisis (see Figure 1). In order to not account these holdings as QE measures, we fix the mean of the central bank's treasury holdings in the measurement equation to $\frac{\overline{B}_{cb}}{\overline{Y}} = 5.5\%$, and assume this was also the case prior to 2003:I (the first data point observed).

15

	Prior			Posterior				
	dist.	mean	std	mean	std	mode	5%	95%
σ_c	normal	1.500	0.375	0.908	0.033	0.876	0.854	0.959
σ_l	normal	2.000	0.750	1.160	0.351	1.046	0.581	1.721
β_{tpr}	gamma	0.250	0.100	0.201	0.058	0.186	0.105	0.292
h	beta	0.700	0.100	0.799	0.033	0.804	0.748	0.853
S''	normal	4.000	1.500	5.119	0.754	4.181	3.949	6.370
ι_p	beta	0.500	0.150	0.232	0.067	0.155	0.128	0.343
ι_w	beta	0.500	0.150	0.426	0.118	0.648	0.222	0.617
α	normal	0.300	0.050	0.210	0.012	0.200	0.189	0.229
ζ_p	beta	0.500	0.100	0.870	0.025	0.861	0.828	0.911
ζ_w	beta	0.500	0.100	0.747	0.046	0.728	0.668	0.817
Φ_p	normal	1.250	0.125	1.331	0.071	1.364	1.218	1.445
ψ	beta	0.500	0.150	0.810	0.067	0.841	0.696	0.919
ϕ_π	normal	1.500	0.250	1.323	0.207	1.542	0.962	1.610
ϕ_y	normal	0.125	0.050	0.171	0.022	0.168	0.136	0.207
ϕ_{dy}	normal	0.125	0.050	0.181	0.041	0.239	0.111	0.250
ρ	beta	0.750	0.100	0.829	0.031	0.850	0.777	0.879
$\overline{\gamma}$	normal	0.440	0.050	0.399	0.029	0.414	0.346	0.441
$\overline{\pi}$	gamma	0.625	0.100	0.624	0.058	0.688	0.534	0.718
$\bar{l}$	normal	0.000	2.000	1.246	0.488	0.701	0.416	2.013
κ_τ	gamma	0.300	0.100	0.287	0.081	0.188	0.166	0.425
PAC	gamma	2.000	4.000	0.495	1.189	0.132	0.107	0.450
LEV	normal	3.000	1.000	4.312	0.405	4.433	3.635	4.986
θ	beta	0.950	0.050	0.815	0.034	0.747	0.761	0.870
λ_{cbl}	gamma	3.000	3.000	0.229	0.207	0.086	0.001	0.519
χ	beta	0.300	0.100	0.190	0.059	0.133	0.099	0.284
$\overline{termspread}$	gamma	0.500	0.100	0.594	0.106	0.427	0.418	0.773
$\overline{ppremium}$	gamma	0.100	0.030	0.083	0.020	0.110	0.049	0.114
$\overline{spread}$	normal	0.500	0.100	0.428	0.051	0.563	0.340	0.513

Table 1: Estimation results for the baseline model: model parameters

4   The Macroeconometric Context of Quantitative Easing

In this section we present our estimation results and use the estimated benchmark model to give a general account of the US dynamics from 1998 to 2019. The in-depth analysis of the quantitative easing measures is deferred to Section 5.

4.1   Priors and parameter estimates

Our priors and posterior estimates are shown in Tables 1 and 2. All prior distributions are characterized by their mean and standard deviation. The priors for the parameters that pertain to the real economy are chosen in line with Smets and Wouters (2007, henceforth SW).[15] The upper part of Table 1 – the parameters from σ_c to $\bar{l}$ – displays

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[15]In contrast to Gust et al. (2017), the computational efficiency of our approach allows us to use the same priors as in SW instead of using tighter prior standard deviations.

		Prior			Posterior			
	dist.	mean	std	mean	std	mode	5%	95%
ρ_r	beta	0.500	0.200	0.566	0.086	0.525	0.421	0.700
ρ_g	beta	0.500	0.200	0.887	0.081	0.944	0.738	0.965
ρ_i	beta	0.500	0.200	0.734	0.048	0.651	0.656	0.810
ρ_z	beta	0.500	0.200	0.951	0.025	0.954	0.915	0.993
ρ_p	beta	0.500	0.200	0.617	0.098	0.624	0.462	0.777
ρ_w	beta	0.500	0.200	0.735	0.070	0.707	0.625	0.852
ρ_u	beta	0.500	0.200	0.896	0.013	0.906	0.873	0.917
ρ_{lk}	beta	0.500	0.200	0.934	0.027	0.936	0.893	0.977
ρ_{cbl}	beta	0.500	0.200	0.762	0.036	0.762	0.704	0.826
$\mathrm{root}_{b,1}$	beta	0.500	0.200	0.903	0.049	0.786	0.835	0.982
$\mathrm{root}_{b,2}$	beta	0.500	0.200	0.880	0.054	0.973	0.794	0.966
$\mathrm{root}_{k,1}$	beta	0.500	0.200	0.901	0.041	0.917	0.833	0.967
$\mathrm{root}_{k,2}$	beta	0.500	0.200	0.926	0.037	0.902	0.870	0.982
μ_p	beta	0.500	0.200	0.450	0.139	0.293	0.195	0.649
μ_w	beta	0.500	0.200	0.517	0.102	0.485	0.345	0.679
ρ_{gz}	normal	0.500	0.250	0.570	0.188	0.463	0.325	0.943
σ_g	inv. gamma	0.100	0.250	0.260	0.029	0.235	0.214	0.307
σ_z	inv. gamma	0.100	0.250	0.345	0.035	0.313	0.282	0.395
σ_r	inv. gamma	0.100	0.250	0.147	0.031	0.164	0.096	0.194
σ_i	inv. gamma	0.100	0.250	0.610	0.086	0.766	0.485	0.760
σ_p	inv. gamma	0.100	0.250	0.235	0.070	0.190	0.125	0.341
σ_w	inv. gamma	0.100	0.250	0.691	0.074	0.687	0.570	0.807
σ_u	inv. gamma	0.100	0.250	0.482	0.086	0.454	0.339	0.617
σ_{lk}	inv. gamma	0.100	0.250	0.272	0.040	0.244	0.208	0.339
σ_{cbl}	inv. gamma	0.100	0.250	0.956	0.084	0.966	0.816	1.090
σ_{qeb}	inv. gamma	0.100	0.250	0.197	0.014	0.186	0.175	0.220
σ_{qek}	inv. gamma	0.100	0.250	0.176	0.015	0.183	0.152	0.198

Table 2: Estimation results for the baseline model: shock processes

the estimates of the parameters inherited from the SW backbone of our model. We compare the posterior estimates for these parameters, among others, to the estimates of Boehl and Strobel (2020, henceforth BS), who apply the same methodology on the same sample in a pure SW-type framework (and extended versions). This comparison seems to be more obvious than the comparison with the results of Kulish et al. (2017), who use a different methodology on a longer data sample. The bottom part of Table 1 contains the new parameters for the financial sector, which are central for the transmission of the large-scale asset purchases. We discuss these parameters mainly in Section 6.1.

Our estimates of the parameters inherited from SW are much aligned with those from BS. The finding that intertemporal elasticity of substitution, σ_c, is close to unity (and well below the prior mean) is widely shared in the literature (see e.g. Smets and Wouters, 2007; Gelain and Ilbas, 2017; Kulish et al., 2017; Boehl and Strobel, 2020). The posterior mean of β_{tpr}, the time preference rate, is close to the estimates by SW, BS, and others. Similarly, BS also find a very high degree of habit formation in their re-estimation of the SW model on the crisis sample (0.833 in BS vs. 0.799 here) as well as substantial investment adjustment costs ($S'' = 5.287$ in BS vs 5.119 here). Both the price and wage

Phillips Curves are estimated to be quite flat, with an estimated price Calvo parameter of $\zeta_p = 0.87$ in line with BS and Kulish et al. (2017). Similarly, values for wage and price indexation ι_p and ι_w, as well as the the fixed cost parameter, Φ_p are in a standard range, albeit notably higher than e.g. SW due to the more recent sample. The estimated feedback coefficients of the policy rule as well as the interest rate smoothing parameter, ρ, match the estimates in BS.

We follow Kulish et al. (2017) and BS in the choice of our prior for the common trend $\bar{\gamma}$ and opt for a tighter prior of this parameter than in SW. Arguably, the economy deviated strongly and persistently from its steady state during the Great Recession. In order to dampen the data's pull of the parameter down to the sample mean, we therefore prefer the tight prior as well. This circumvents unrealistically low estimates of the trend growth rate which would imply implausibly high levels of consumption and output after 2008. Our estimates of $\bar{l}$ and mean inflation, $\bar{\pi}$ are in line with BS.[16] The effects of the priors on the macroeconomic dynamics are illustrated in the impulse response functions in Figures 6 and C.15.

For the non-SW part of our model, we estimate the steady state leverage of financial intermediaries, LEV, their survival rate θ, the sensitivity of the incentive constraint to liquidity injections λ_{CBL}, and the feedback coefficient for government debt in the tax rule, κ_τ. Additionally, we estimate the AR(2) shock processes for the QE measures, which are identified independently of the model choice. For the prior of LEV we choose a normal distribution centered around 3, and with a standard deviation of 1. Intuitively, a high leverage LEV implies a high initial vulnerability of the financial system to shocks that affect its asset prices or funding costs. Conversely, it also implies that the small steady state net worth that is associated with a high leverage will be replenished (reduced) faster by increased (compressed) excess returns on banks' assets such that the financial system reverses more quickly to its original state. Depending on the nature of the shock this can come either with an amplification or an attenuation of the effects on the real economy. As we are not aware of prior estimates of this parameter in the context of a structural model, we opt for choosing a rather wide prior.[17] For θ, we choose a beta distribution for the prior with a mean of 0.95, corresponding to an expected time horizon of the bankers of 5 years. This value is only slightly below the value of 0.975 in Gertler and Karadi (2011), but the standard deviation of 0.05 allows for significant departures from the prior mean. Generally, a higher θ is associated with a higher stochastic discount factor of financial intermediaries and therefore shapes the persistence of the effects of QE shocks. Our estimations however suggest that the data favors large values for LEV and a relatively small θ. As we will discuss below, both values point towards rather smaller effects of the QE measures. For λ_{CBL} we set a wide prior around a mean of 3. The shape and width of the prior reflect our agnostic approach to the effect of central bank liquidities injections. For $\lambda_{CBL} = 0$, liquidity injections have no effect at all. In fact, the estimate for λ_{CBL} of 0.229 is quite low, allowing only for limited effects of the Fed's

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[16]$\bar{\pi}$ not only is the constant in the measurement equation for inflation, but also the Fed's inflation target and the long-run trend in inflation. We, however, refrain from setting the prior value to 2% p.a. because we assume that the original SW prior better reflects the long-run trend while still being flexible enough to allow for lower estimates if necessary. Moreover, note that the Fed targets headline inflation while, in the estimation, we use GDP deflator.

[17]Villa (2016) estimates a version of the model by Gertler and Karadi (2011) on US data, but calibrates LEV and θ to fixed values.
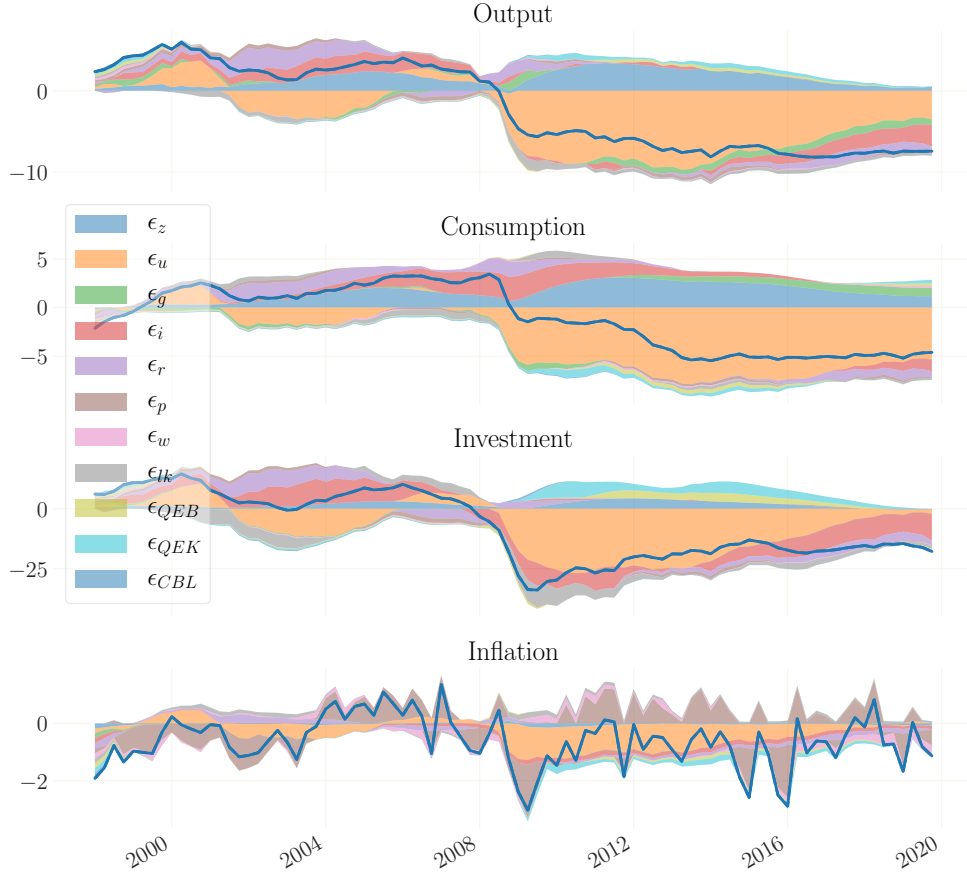
Figure 2: Estimated baseline model. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 1000 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.

liquidity provision program.

4.2 A decomposition of the dynamics

Figures 2 and 3 show the historical decompositions of a selection of smoothed states into the contributions of the different shocks. The effect of each shock is normalized as suggested by Boehl and Strobel (2020). This means that each shock reflects the exact contribution, independently of any ordering effects that might occur in nonlinear models. The overall picture of the historical decomposition points towards variations in the risk premium shock, ϵ_t^u, as the most important driver of the observables. This financial shock affects banks' funding conditions and has a demand shock type effect on the real economy in our model. Besides its role in the GFC, it was also identified as a main driver in the pre-crisis period by Smets and Wouters (2007). As such, the high risk premium can
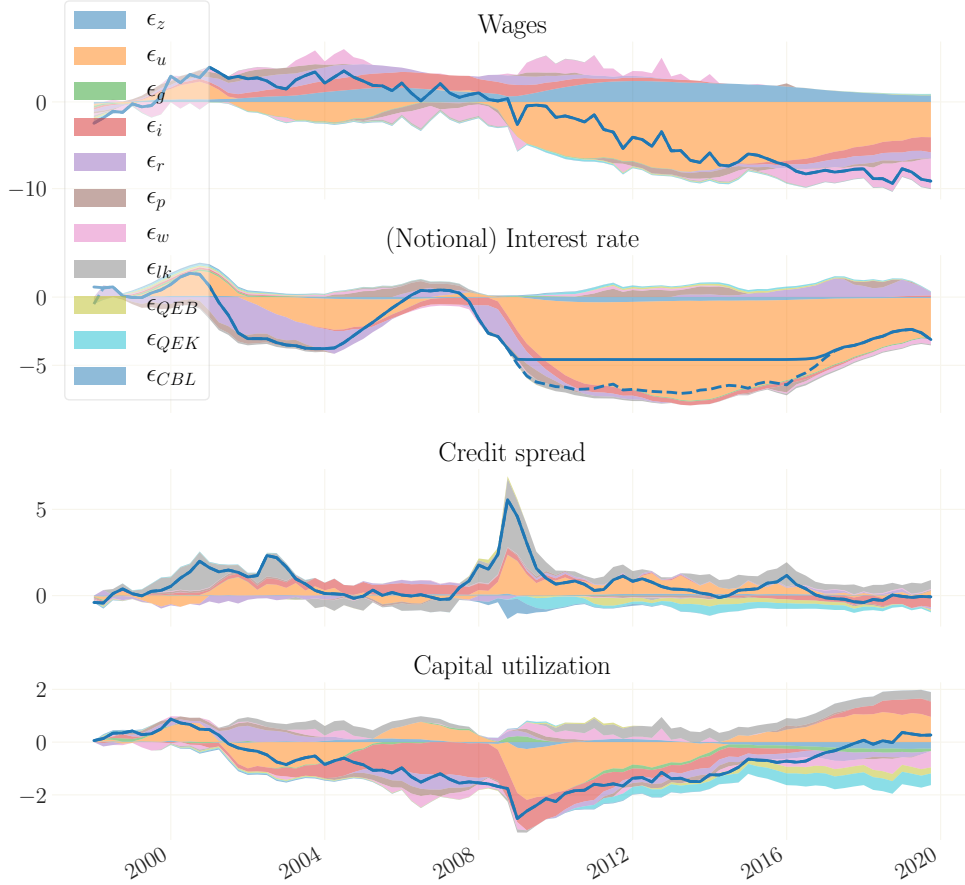
Figure 3: Estimated baseline model. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 1000 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.

not only explain the 2008-2009 drop in consumption, but can also account for a large share of the fall in investment. Ultimately, the lasting effect of an elevated risk premium resonates in a high estimate of ρ_u of 0.896. The plunge in investment is further supported by a fall in the efficiency of investment, ϵ_t^i. To our surprise, the other financial shock ϵ_t^{lk} to the divertibility of capital assets, $\lambda_{k,t}$, that we adopted from Dedola et al. (2013), substantially raises the credit spread during the financial crisis but only plays a minor role in driving macroeconomic dynamics.[18]

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[18]We also experimented with various other financial shocks, such as the capital quality shock in Gertler and Karadi (2011).We find that the risk-premium shock together with the investment-specific technology shock, the latter which has also been interpreted as a financial shock (see, e.g. Justiniano et al., 2011; Gust et al., 2017; Kulish et al., 2017), to deliver the most robust results and are preferred in terms of the marginal data density (see Appendix N).

The finding of a flat Phillips Curve is mirrored in the dynamics of inflation. While the risk premium shock persistently depresses price level dynamics, the crumb of inflation in 2009 is explained by exogenous movements in the firms' price markup. In fact, this process governs almost all high-frequency movements in inflation. Consequently, exogenous impulses to the price markup process are rather short-lived with an estimate of $\rho_p = 0.617$. As expected, the technology level z_t is a driver of the low-frequency dynamics, which is reflected in a very high estimate of ρ_z. A similar result holds for the process of government spending, which only plays a secondary role.[19]

The parameters for the exogenous processes of the QE measures are identified independently of the model parameters and are entirely driven by the time series of the central bank's balance sheet that are fed into the estimation. In the next section, we provide an in-depth account of the effects of QE and discuss their impact key macroeconomic variables in more detail.

Finally, note that our estimate of the notional rate differs conceptually from those inferred from affine term structure models, such as Krippner (2013) or Wu and Xia (2016). Our estimate reflects the implications of a Taylor-type policy rule and is not based on information of the cross-section of yields. The estimate thus provides a counterfactual, indicating by how much lower the short-term rate would have been given the levels of inflation and the output gap.[20] As we assume that at the ZLB, the Fed considers the shadow rate for interest rate smoothing rather than the nominal rate, the shadow rate directly enters the model. It thus is an important measure as it drives expectation dynamics. At this point, we also want to highlight that our model matches expected durations at the ZLB from survey data surprisingly well.[21] Importantly – and different from Kulish et al. (2017) – we do neither target these expectation nor feed them in to our estimation procedure. They are instead determined endogenously by our solution method.

5   The Quantitative Effects of QE

In response to the GFC, the Federal Reserve undertook several measures in unprecedented scale to address disruptions on financial markets and later to further ease the monetary stance once the ZLB was reached. Among the first such programs are the Term Auction Facility Program (TAF), the Term Asset-Backed Securities Loan Facility (TALF), as well as the primary dealer and other broker-dealer credit programs, which were aimed at addressing elevated pressures in short-term funding markets at the height of the financial crisis in 2007/08. We summarize these measures under the umbrella of "emergency liquidity injections".

In order to provide further monetary accommodation, the Federal Reserve turned to programs of large-scale asset purchases. The first of these programs, later dubbed "QE1",

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[19]In principle, our model additionally allows for forward guidance shocks at the ZLB. However, we find that, in the absence of additional data input such as, e.g., term premia, nonlinear filters do not perform reliably well in identify forward guidance shocks at the ZLB. For a related discussion, see Boehl and Strobel (2020).

[20]Note that this ignores the general equilibrium effect of such lower rates: if rates would have been lower, inflation and output would have been higher, and, in turn, the Federal Funds Rate would have been higher than our shadow rate.

[21]Figure D.17 in the Appendix illustrates the course of the number of expected periods at the ZLB.

lasted from December 2008 to March 2010 and included net purchases of $1.25 trillion in mortgage-backed securities (MBS), $175 billion in agency securities and roughly $300 billion in treasury. After a brief pause, a second round of purchases (called "QE2") in November 2010 involved net-purchases of $600 billion plus reinvestment of the proceeds from the earlier MBS purchases in longer term government Treasury securities. After yet another brief pause, the Fed started what is now known as Operation Twist, a portfolio shift of the size of $600 billion from short and medium-term Treasuries with a maturity up to three years, to long-term Treasuries with maturities of six years and above. Our analysis does not explicitly incorporate the effects of Operation Twist, as our data series of government bonds pools Treasuries of all maturities and hence does not account for effectively sterilized actions. Lastly, QE3 started in September 2012 and lasted until December 2014. It further increased the Fed's balance sheet through net-purchases of $95 billion each months – $45 billion in long-term treasury bonds and $40 billion in MBS.

In this section, we investigate the macroeconomic effects of these liquidity provisions and large-scale asset purchases through the lens of our estimated model. As touched upon in Section 2.4, we assume that liquidity provisions directly affect a bank's supply of credit by easing it's incentive compatibility constraint. MBS and commercial paper purchases under QE1 and government bond purchases under QE2, on the other hand, can be interpreted as central bank intermediation (Gertler and Karadi, 2013). The financial frictions in our model give rise to an extranormal term premium on government bonds, and similarly, extranormal credit risk premia on private loans and capital claims. Central bank asset purchases compress these risk spreads, thereby easing financing conditions for firms and households. Importantly, through the portfolio rebalancing channel, these risk spreads are compressed even if the specific asset under consideration is itself not purchased.

We illustrate by the means of counterfactual experiments how output, inflation and the other key macroeconomic variables would have evolved in the absence of the liquidity injections and LSAP programs. While all these measures have prevented a further contraction in investment and hence output, we find LSAPs to have suppressed aggregate consumption by around 0.7 percentage point. Moreover, by increasing the production capacity, LSAPs created additional downward pressure on prices. Although the latter result may appear counterintuitive at first, we show that (positive) financial shocks can have disinflationary effects if aggregate supply effects dominate the aggregate demand.

5.1 The Effects of the QE Measures

What would have happened to the economy if the Fed did not expand its balance sheet in this unprecedented way? We answer this question by looking at counterfactual experiments in which we isolate the effects of each of the balance sheet policies taken. These counterfactuals are summarized in Figures 4 and 5. The red-dashed line on the left indicates the mean of the respective smoothed observable or state. The right side of each Figure then shows the net contribution of the policy measures by adding one measure at the time.

Starting with the liquidity provisions in dashed-blue, denoted CBL in Figures 4 and 5, we observe a sharp fall in the credit spread of around 100 basis points in the beginning of 2009. Despite this large effect on financial conditions, output only increased marginally driven by an increase in consumption of borrowing (i.e. impatient) households and an increase in investment. In the model, liquidity injections trigger a relaxation of the bank's
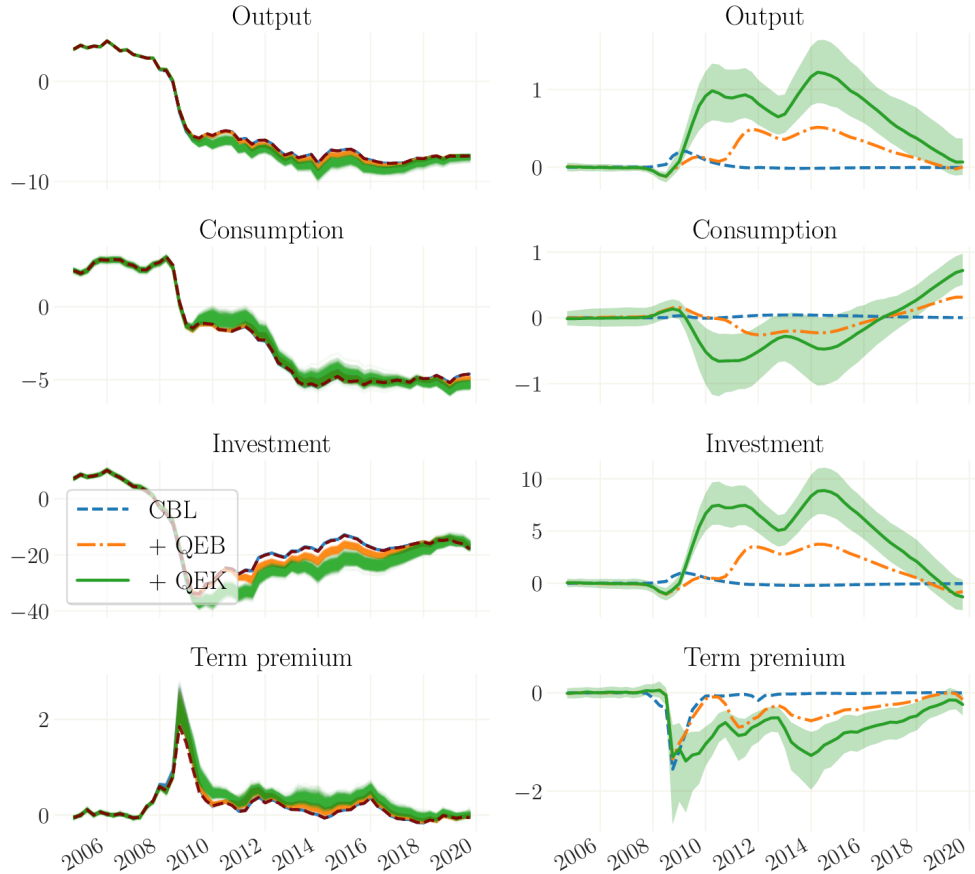
Figure 4: Left: counterfactual simulations without the QE measures. Right: net contribution of each QE measure. Effects in both graphs are cumulative.
Note: Means over 1000 simulations drawn from the posterior. Annualized measures where applicable.

financial constraint which is associated with a broad increase in banks' supply of credit. Their overall macroeconomic effect, however, remained limited: the peak output increase was 0.20%. With the injections reaching around 8% of GDP, their peak output multiplier is only roughly 0.025. Our findings contrast those of Del Negro et al. (2017), who report large effects of liquidity injections on the real economy. One potential reason for this discrepancy is the high persistence of the liquidity shock in their model. Yet Figure 1 suggests and our estimation confirms that the liquidity injections were relatively short-lived, with our posterior mean estimate for persistence parameter $\rho_{cbl} = 0.762$ being substantially smaller than the calibrated value of 0.953 in Del Negro et al. (2017).[22]

We argue, therefore, that the effects of liquidity injections have to be assessed against

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[22]To be precise, Del Negro et al. (2017) model the liquidity shock to be very persistent and with the central bank liquidity policy to respond to the deviations of this persistent shock from its steady state.
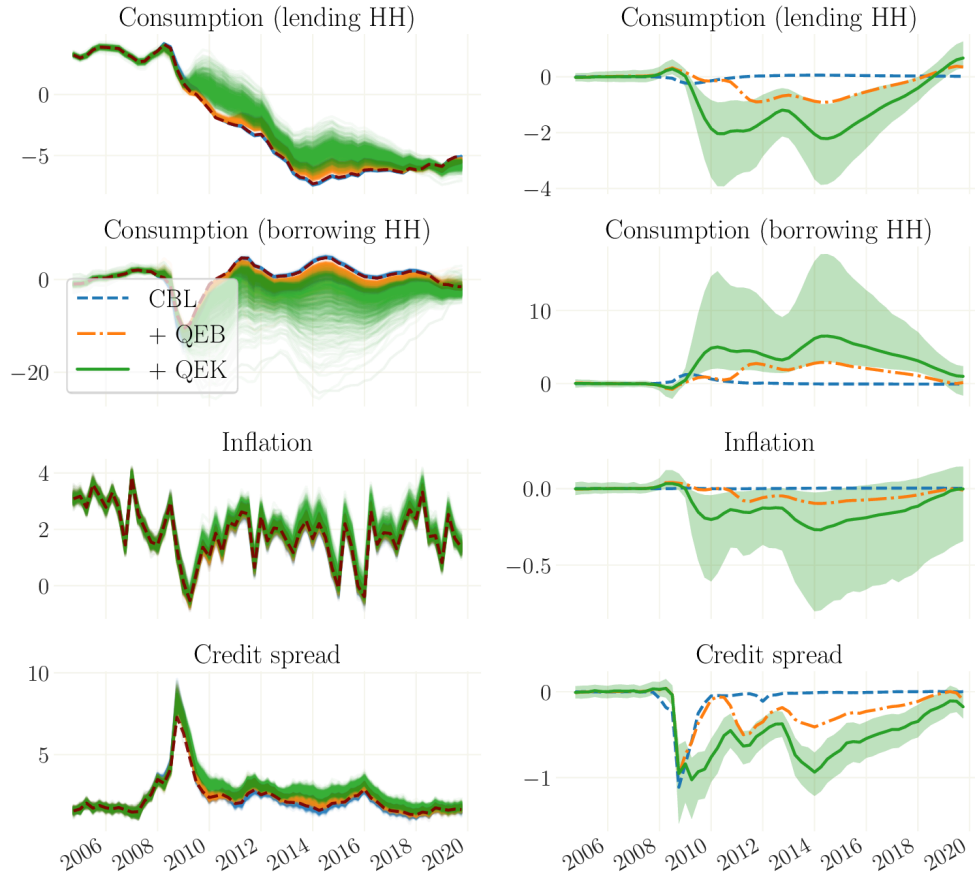
Figure 5: Left: counterfactual simulations without the QE measures. Right: net contribution of each QE measure. Effects in both graphs are cumulative. Measures of spreads and inflation are annualized. *Note:* Means over 1000 simulations drawn from the posterior. Annualized measures where applicable.

the backdrop of a smaller magnitude and duration relative to the LSAP programs (as can be seen in Figure 1). In line with the intuition from Equation (19), liquidity injections support bank lending to firms which, in turn, translates into higher investment.[23] The simultaneous fall in the credit spread spills over to the interest rate on private mortgages, which leads to an increase in lending to impatient households, thereby boosting their consumption. However, due to an initial mild fall in inflation the real rate increases, causing a fall in consumption of patient households, which pushes down aggregate consumption in the short run. Total output increases nonetheless, boosting labor demand and wages. The latter, in turn supports a pick-up of inflation in the longer term.

In the last quarter of 2008, the Fed started purchasing private securities in order to

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[23]Figure C.14 in the Appendix provides further details on how central bank liquidity injections transmit by showing the posterior impulse response functions.

further ease financing conditions. These holdings reached 9,7% of GDP in 2015 before the Fed slowly started to reduce its position. The securities purchases consisted largely of mortgage-backed securities (MBS), but also included the smaller and more shortly-lived commercial paper funding facility (CPFF) program. Other than the liquidity provisions, LSAPs were very persistent with both roots of each AR(2) process around 0.9. The solid-green line in Figures 4 and 5 indicates that the effects the private capital security purchases are substantial: investment and output were 7% and 0.8% higher, respectively, and the effect persisted over roughly six years before gradually dissipating. In our model, capital securities purchasing basically reflect central bank intermediation (see Gertler and Karadi, 2013), which compressed corporate credit spreads very persistently. The resulting broad appreciation of long-term debt prices via the no-arbitrage conditions bolsters banks' balance sheets and increases their net worth; an effect Brunnermeier and Sannikov (2016) called a "stealth recapitalization" of banks. The increase in banks' net worth, in turn, stimulates lending to impatient households and the government, i.e. the portfolio rebalancing channel (see e.g. Gagnon et al., 2011; d'Amico et al., 2012).

Notwithstanding the positive effects of LSAPs on economic activity, we observe an inconvenient effect of private security purchases on inflation. Our counterfactual suggests that inflation would have been 0.2% higher in the absence of private security purchases, with the 95% credible set even includes a fall of inflation by 0.7%. Although such disinflationary effects may seem counterintuitive at first, the following section will explain the channels of why this result is, in fact, not that surprising, and discusses its robustness together with other empirical literature that support this finding.

At the end of 2010, the Federal Reserve embarked in large-scale purchases of treasury bonds in its second round of QE, which – after a short pause – got extended in September 2012 by the QE3 program. Overall the government bond holdings of the Fed reached close to 14% of GDP. In line with earlier studies, we find purchases of assets that entail some credit risk to be more effective in compressing the excess return on capital assets than purchases of government bonds.[24] The latter translates into a stronger response of investment, consumption of borrowing households and ultimately output. Our counterfactual analysis suggests these treasury purchases to have lifted GDP by 0.5% in 2012 (orange dash-dotted line in Figures 4 and 5). Again, this effect was driven by an increase in investment of more than 4% and consumption of borrowing households of around 3%. The effect on aggregate consumption is, however, more benign, as consumption of savers (i.e. patient households) slightly falls. The reason for the latter is again a mild disinflationary effect of the government bond purchases, which leads to an increase in the real deposit rate. Additionally, the rise in the return on capital increases impatient households' marginal benefits of investing over and above their marginal costs of foregone consumption. Contrary, borrowing households see their risk spread decline (via the portfolio rebalancing channel) by more than the fall in inflation, which stimulates their consumption.

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[24]Krishnamurthy and Vissing-Jorgensen (2011), for instance, find that default risk/default risk premium for corporate bonds fell in response to QE1, thus lowering corporate bond yields. Relatedly, Caballero and Farhi (2018) show that central bank purchases of risky assets (in contrast to save government bonds) in a swap for safe assets (central bank reserves) can boost the economy and lower risk premia.
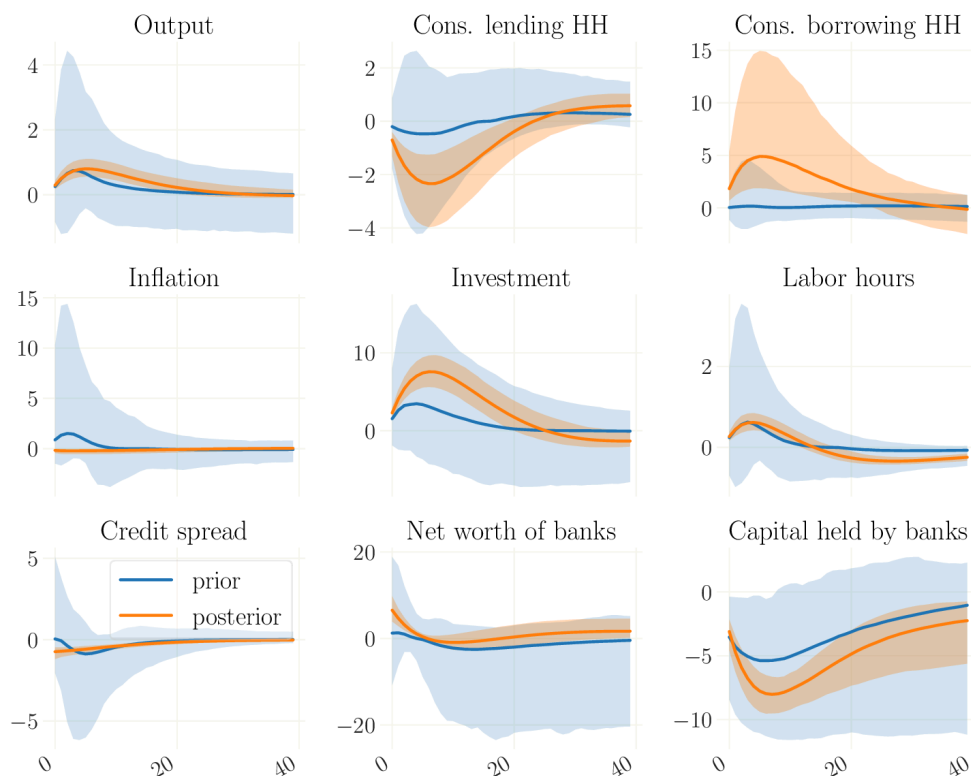
Figure 6: Prior and posterior impulse response functions of a shock to private security purchases. .
Note: Posterior IRFs in orange, with 95% credible set. Posterior sample obtained from 1000 draws from the posterior distribution. Prior IRFs in blue. Solid lines represent the mean. For each draw, the strength of the shock is chosen such that the peak of the corresponding stochastic process matches the peak of the empirical time series. See Appendix F for details. The prior sample is obtained from 2000 draws to account for the strong heterogeneity of effects. Prior draws adjusted for high autoregressive coefficients with mean/std of 0.9 and 0.05 because AR-coefficients are identified independently from the model. Annualized measures where applicable.

5.2 The disinflationary effect of Quantitative Easing

As displayed in Figure 5, we find that the LSAPs had an unpleasant effect on inflation and consumption. This implies that, at a time when inflation was still depressed by the persistent effect of the GFC, the QE measures exerted additional downward pressure on the price level. The disinflationary effects are particularly true for the purchases under QE1, which – over the course of the program – induced inflation to drop an additional 0.25% with the maximum effect in the 95% credible set even exceeding 0.7% p.a. While the main goal of QE in the US was to create favorable financing conditions and to stimulate lending[25], our finding of a disinflationary effect of QE is highly policy relevant. Specifically, in the US, LSAPs create a trade-off for policymakers between the stabilizing

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[25]See the Fed's statement from November 25th, 2008.

prices and employment. But the results may be even more important for the Euro Area, where the asset purchase programme (APP) of the ECB was explicitly undertaken to stabilize inflation and inflation expectations at a time when fears of deflation surged.[26]

It is important to note that this result stands in stark contrast to the effects that occur when sampling from the prior distribution. To illustrate this, Figure 6 shows the response functions to an impulse in private security purchases at both our prior and posterior mean. The former suggests a peak effect on inflation of 2%, with the 95% credible set even allowing to peak at 14%. Against this backdrop, we see our posterior result as a strong case for the data to prefer a disinflationary effect of the Fed's Quantitative Easing measures.

To understand this effect, let us recognize that in our model, LSAPs can be interpreted as expansionary financial shock that loosens liquidity and balance sheet constraints. The response of inflation depends, inter alia, on whether aggregate demand or supply effects dominate. For instance, inflation may rise if demand effects dominate, as is the case in Curdia and Woodford (2010) and Gertler and Karadi (2011). Contrary, if supply side effects dominate, expansionary financial shocks may in fact be disinflationary as lower financing costs may be passed on to prices.

According to our prior predictive analysis, the priors we choose, give an edge to demand side effects on unconventional monetary policy. An easing of borrowing conditions for households and firms stimulates aggregate spending. Higher aggregate demand, in turn, increases factor demand, thereby raising factor prices. Elevated marginal costs are, over time, passed on to consumers in form of higher prices. Higher inflation, in turn, further lowers real interest rates which amplifies the initial increase in aggregate demand. In fact and to the best of our knowledge, such effects also prevail in all common models of QE, including the original Gertler and Karadi (2013) framework which lends as a blueprint for our benchmark model and its derivatives.[27]

Notwithstanding, our estimation favors the dominance of the supply side channels of QE. To formally understand the mechanism in our model, recall that firms use a standard Cobb-Douglas production function of the form $Y_t = F(Z_t, L_t, \bar{K}_t)$, where $\bar{K}_t = U_t K_{t-1}$ is the effective capital in period t. That is, in each period, firms must take their capital stock as given but can decide over the utilization rate U_t. As in Smets and Wouters (2007), the costs for changes in the utilization rate are given by a function $\Psi(U_t)$ that increases proportionally when U_t deviates from its steady-state value. Thus, the marginal product of effective capital, $\widehat{mpk}_t$ only depends on the capital utilization rate and can, up to a first-order approximation, be expressed as

$$\widehat{mpk}_t = \frac{\psi}{1-\psi} \hat{u}_t, \tag{37}$$

where $\hat{x}_t$ represents a variable X_t in percentage deviations from its steady state. The parameter ψ depends positively on the elasticity of the capital utilization adjustment cost function and is defined on $(0,1)$. Moreover, since marginal costs $\widehat{mc}_t$ is linked to the

marginal factor product, we have

$$\widehat{mc}_t = (1 - \alpha)\hat{w}_t - \hat{z}_t + \alpha \frac{\psi}{1 - \psi}\hat{u}_t, \tag{38}$$

i.e. marginal costs increase in real wages $\hat{w}_t$ and the rate of capital utilization $\hat{u}_t$ and decrease in total factor productivity, $\hat{z}_t$.

For simplicity, let us picture the purchases of capital assets by the central banks as an exogenous shock on the capital stock (via increased investment). For a given aggregate demand, an increase in the capital stock will mechanically decrease the utilization rate and, correspondingly, the marginal product of effective capital. Holding wages constant, this directly translates into lower marginal costs and, in turn, prices and inflation via the New Keynesian Phillips curve. Additionally, the decrease in consumption by patient households supports a slight decline in wages via the wealth effect on the labor supply. This adds to the downward pressure on inflation.

Importantly, note that this mechanism does not hinge on the presence of time-varying capital utilization. In fact, the rise in the capital stock would even in the absence of varying capital utilization lead to a fall in marginal cost.[28]

The notion that a loosening of financial constraints is associated with disinflationary tendencies is quite common in the literature. For instance, Boehl and Lieberknecht (2020) show that at the ZLB, the costs of external financing (i.e. credit spread) can dominate firms' price setting which results in a flatter Philips curve. Similarly, in models with a cost channel where firms borrow in advance of production to pay wages, as in Van Wijnbergen (1983); Christiano et al. (2005) and Ravenna and Walsh (2006), or capital as in Fiore and Tristani (2013), lower credit spreads (and hence financing costs) are associated with a decrease in firms' marginal costs which is passed on to consumers in the form of lower prices. Consistent with prevailing supply effects, Barth III and Ramey (2001); Chowdhury et al. (2006) and Abbate et al. (2016) find evidence in favor of a cost channel. Abbate et al. (2016), for instance, show that – using a vector autoregression model with sign restrictions – financial shocks that lower firms' funding costs and increase credit growth and stock prices actually reduce inflation in the short run. Similarly, using Italian firm-level data on output prices and interest rates paid on debt, Gaiotti and Secchi (2006) find the cost channel to be proportional to the working capital. A different yet related channel is proposed by Gilchrist et al. (2017). Using granular micro-data, the authors show that firms' with binding liquidity constraints increased prices during the GFC, while unconstrained firms did lowered them. To rationalize this empirical finding, Gilchrist et al. (2017) build a theoretical model where firms price goods above marginal costs in order to hedge against the risk of relying on costly external finance.

As discussed above, in our model, we identify a *cost channel of QE* that materializes via the capital utilization cost, which decrease following a QE-induced rise in investment. This mechanism is similar to Acharya et al. (2020), who find that cheap credit to impaired firms has a disinflationary effect by creating excess production capacity. Against this backdrop, our results suggest that aggregate supply channels dominated in determining the response of inflation to LSAPs.

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[28]See Appendix L for the historical decompostions of the model without capital utilization.
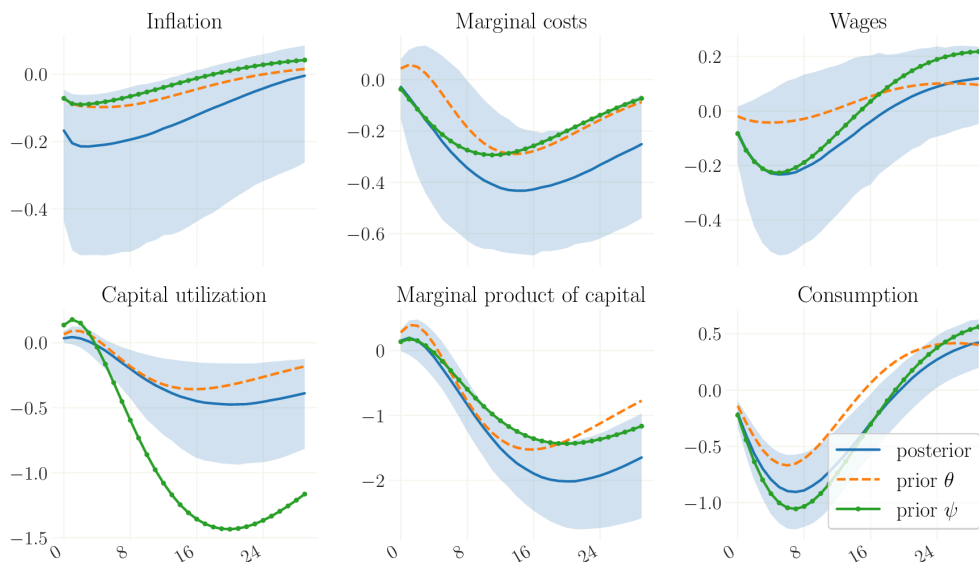
Figure 7: Impulse response functions with respect to a shock of capital securities purchases in blue, sampled from the posterior distribution with 95% credible set. In orange impulse responses at the posterior mean with $\theta = 0.95$. In green impulse responses at the posterior mean with $\psi = 0.5$.

Note: Posterior sample obtained from 1000 draws from the posterior distribution, solid line represent the mean. For each draw, the strength of the shock is chosen such that the peak of the corresponding stochastic process matches the peak of the empirical time series. See Appendix F for details. Annualized measures where applicable.

6 Robustness

In this section, we first analyze which parameters are the root cause for the disinflationary effects and the fall in consumption resulting from LSAPs in section 6.1. We then show that our main results are also robust to specific model assumption by estimating different model vintages in section 6.2. Lastly, we present the effects of LSAPs through the lens of an estimated version of the model by Carlstrom et al. (2017) and discuss the differences to our benchmark results.

6.1 Model parameters - Inspecting the mechanism

As shown in the previous section, according to (38), the effect of an increase in the utilization rate on marginal costs depends, ceteris paribus, positively on ψ and α. The estimate of α additionally governs the relative weight of the marginal product of capital and wages in the marginal costs. The blue line and shades in Figure 7 show the posterior IRFs to capital asset purchases together with their 95% credible sets. To highlight the effect of the parameter estimates on inflation dynamics, the green lines show the IRFs for the case in which ψ is set to their prior mean.

The increased capital stock induced by higher investments leads to a fall in capital utilization, $\hat{u}_t$, and, correspondingly, the marginal product of capital. In addition with the decline in wages this implies a proportional decline in marginal costs and, ultimately,
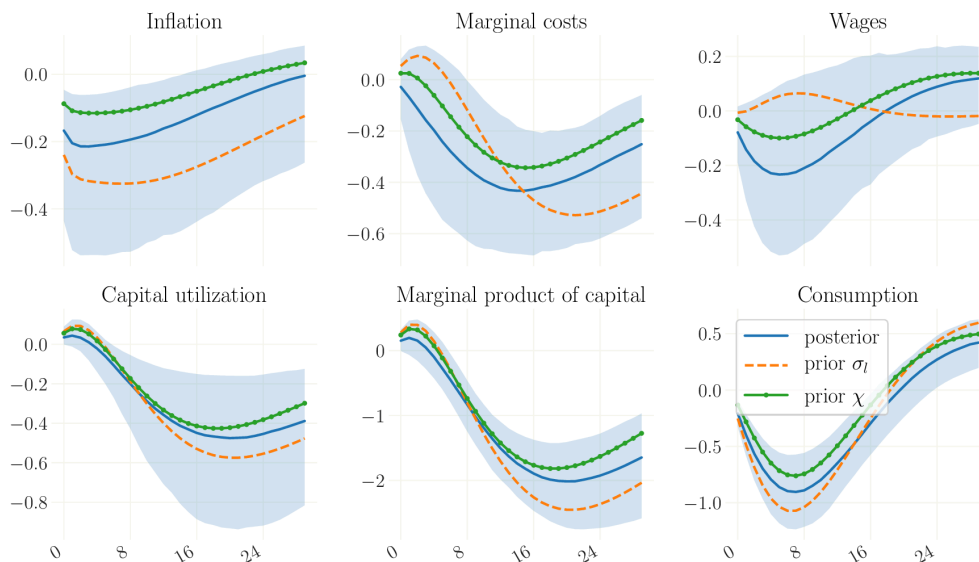
Figure 8: Impulse response functions with respect to a shock of capital securities purchases in blue, sampled from the posterior distribution with 95% credible set. In orange impulse responses at the posterior mean with $\sigma_l = 2$. In green impulse responses at the posterior mean with $\chi = 0.3$.

Note: Posterior sample obtained from 1000 draws from the posterior distribution, solid line represent the mean. For each draw, the strength of the shock is chosen such that the peak of the corresponding stochastic process matches the peak of the empirical time series. See Appendix F for details. Annualized measures where applicable.

inflation. As can be seen in Figure 7, our estimate of ψ supports a disinflationary effect of the asset purchasing program in our benchmark model. The posterior mean value of ψ is 0.810 with a standard deviation of 0.067 (see Table 1) lies between the estimate of a plain Smets and Wouters (2007) model on the same sample using the same methodology (see Table E.3 in Appendix E) and the one reported in Kulish et al. (2017). Contrary, for the capital share, our posterior mean estimate of $\alpha = 0.210$ dampens the disinflationary effect compared to a simulation in which α is replaced by its prior mean of 0.3. However, this effect is rather mild. The posterior mean value is notably higher than those from either a plain SW model, those reported by Kulish et al. (2017) and those reported for a selection of models in Boehl and Strobel (2020), which all range between 0.16-0.174.

For the financial sector, we estimate the survival probability of banks, θ, and the steady state leverage ratio, LEV. Figure 7 illustrates the effects of θ on the transmission of central bank capital purchases. The posterior mean estimate of $\theta = 0.815$ lies well below the parameter's prior mean of 0.95. The lower survival probability is associated with a smaller share of incumbent bankers in this sectors, whose excess returns are affected by current and past shocks. Therefore, in the model a lower θ somewhat insulates the sector as a whole from the effects of reduced profitability of bank assets. Overall, the impulse to investment is strengthened by a lower survival probability of bankers, and therefore aggregate output as well. The stronger expansion of the capital stock again lowers marginal costs inflation. Hence, the low estimate of θ supports the disinflationary

effects of unconventional monetary policy. The estimate of *LEV* above its prior mean is associated with a relatively attenuated expansion of investment and output in response to LSAPs. Inflation is, however, barely affected. The attenuation of these shocks associated with a high steady state leverage ratio can be explained by the fact that the compressed excess returns on bank assets quickly undo the gains in net worth triggered by the central banks asset purchases (see Equation (11)). This reduction in the profitability on assets drives this process faster, when the asset position is relatively large compared to the net worth of banks. As the gains in the financial sector are more short-lived, the investment response is attenuated.[29]

On the household side, the preference parameters σ_c and σ_l affect the transmission of unconventional monetary policy shocks. Figure 8 compares the IRFs sampled from the posterior with IRFs in which the inverse of the Frisch elasticity, σ_l, is set to its prior mean. Our estimate of σ_l is below its prior mean. The low slope of the labor supply schedule comes with a stronger but more short-lived fall of wages in response to central bank capital asset purchases. This translates to a qualitatively similar pattern in marginal costs and, since inflation is strongly forward looking, is associated with a attenuated decrease in inflation compared to the case, in which σ_l is replaced by its prior mean. As discussed in section 3, a posterior mean estimate of $\sigma_c = 0.908$ is in close proximity to values commonly found in the literature. A high coefficient of relative risk aversion at its prior mean implies a low sensitivity to changes in the real rate and establishes a positive co-movement of household spending and labor hours via the non-separabilities in preferences (see Equations (3) and (8)). In contrast, the posterior value slightly below unity reduces the latter channel and even slightly reverses its effect, such that, in sum, aggregate spending declines. In turn, the fall in consumption causes an expansion of the labor supply schedule and reduces real wages. Consequently, the low estimate of σ_c substantially weakens the demand effect of quantitative easing and supports its disinflationary implications. Importantly, this parameter is well identified, which adds to our argument that the data favors disinflationary effects of LSAPs. Also on the household side of the model, the fraction of impatient households, χ affects the response of consumption, real wages and inflation to unconventional monetary policy measures. We show the counterfactual simulation with χ at its prior mean as well in Figure 8. The households profit from lower funding costs and increase their spending in response to quantitative easing. Our estimate of this parameter is at 0.190 and therefore considerably below its prior mean. For a higher value of χ, the reduction in consumption and inflation is dampened. Notably, given the other parameter estimates, it would require a fraction of borrowing households as high as 0.5 to generate roughly neutral effects of LSAPs on inflation and consumption – a value which is far away from what the data favors.

6.2 *Model features*

In Gertler and Karadi (2013), the representative household is a "saver", with those savings in form of bank deposits and investments into both, corporate and government

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[29]Note, that a higher *LEV* is also associated with an amplification of risk premium shocks, which affect financial intermediaries primarily via their funding costs. This is important, as risk premium shocks drive the bulk of the business cycle. Higher leverage ratios can therefore be associated with a deeper recession following the GFC.

bonds. In response to a QE shock (private security or government bond purchases), household consumption only increases to the extent that the real deposit rates falls. The model, however, does not allow QE to directly affect consumption. For this reason, our baseline model features a second type of households, namely the impatient, that can borrow using long-term private mortgages to finance his consumption. This channel is comparable to the one presented in Chen et al. (2012) and allows for a direct impact of loser financing conditions on private consumption. In particular, after a QE shock, impatient households will see their borrowing costs falling, which stimulates their consumption and thereby, possibly, aggregate consumption. As discussed in the previous section, we estimate the fraction of impatient households to equal $\chi = 0.195$.

In order to show that our results do not hinge on this additional channel of QE, we propose two alternative specifications of our benchmark in this section. First, we estimate the representative agent version of our model, which we call the RANK version, by setting the share of impatient households $\chi = 0$. This model vintage does not allow for indebted households and, as a result, QE does not affect household consumption directly, but only to the extent that it affects the real deposit rate. Second, we estimate a model in which we replace the fraction of impatient households with "hand-to-mouth" (H2M) consumers inspired by Kaplan et al. (2018) in the spirit of Campbell and Mankiw (1989, 1991). Neither in this model vintage, denoted H2M, does QE affect consumption directly. Instead, any effects on consumption have to come from the response of labor income (i.e. real wages and/or labor supply) and again the real deposit rate.

Figure 9 repeats the counterfactual analysis of section 5.1 for the RANK model. The overall effects of the UMP measures seem to be consistent even in the absence of the above described channel. Despite a similar increase in investment increases of around 10%, the overall effect on output is more muted, peaking at around 0.5%. The reason for this substantially smaller output effect lies in the twice as large decline in the price level, now reaching on average -0.5%, and the accompanied stronger fall in consumption that weighs on aggregate demand. While most parameter estimates remain, by and large, unchanged, a handful of parameter estimates reflects these dynamics.[30] Specifically, in σ_l falls from 1.231 in the benchmark model to 0.654 in order to rationalize the stronger fall in consumption with an simultaneous increase in labor supply. The reduced overall effectiveness of QE in stimulating the economy is also mirrored in (i) higher investment adjustment costs which mutes the investment response, (ii) a lower value for the portfolio adjustment costs which implies less limits to arbitrage and (iii) a higher leverage which facilitates a speedy normalization of the financial sector and therefore implies a weaker expansionary impulse for the real economy.

Similarly, Figures 10 and 11 illustrate the counterfactual of how key macroeconomic series would have evolved in the absence of the UMPs through the lens of the H2M vintage of our model. As is the case in our benchmark model, all three UMPs stimulated investment, with the largest effect – an increase of about 7% – coming from the MBS purchases. However, despite this larger response of investment, the overall response of output remains muted relative to our benchmark model. The reason again is a larger fall in aggregate consumption because both, Ricardian and H2M households reduce their consumption. In line with the patient households in our benchmark model, Ricardian

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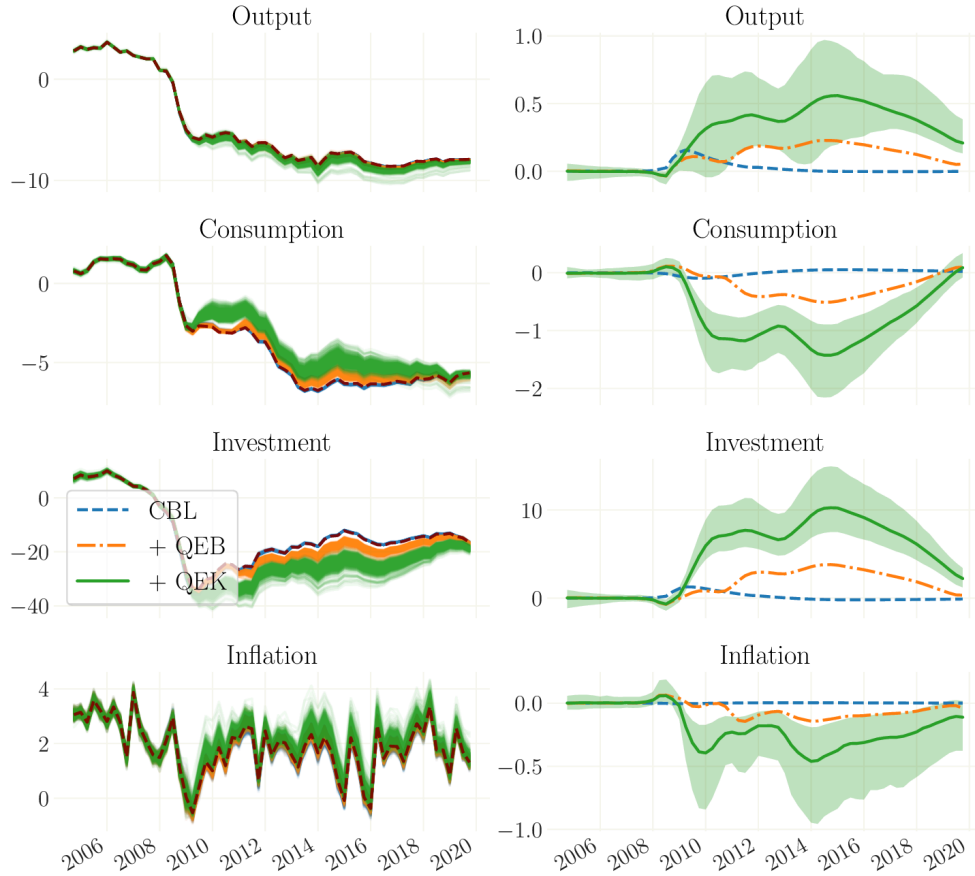[30]See Table H.4 in Appendix H for more details.

Figure 9: RANK specification of benchmark model. Left: counterfactual simulations without the QE measures. Right: net contribution of each QE measure. Effects in both graphs are cumulative.
Note: Means over 1000 simulations drawn from the posterior. Annual measures where applicable.

households see an increase in the real deposit rate. The latter is even amplified by the larger fall in inflation by -0.5% on average, with the 95% credible set including a fall in the price level of up to 1%. On the other hand, consumption of H2M households actually falls because falling wages reduce their labor income.

Again, these dynamics are reflected in a small set of parameter estimates that differ notably from the benchmark estimation. In order to square the rising labor input induced by QE with an even stronger fall in consumption than in the RANK model, the estimation prefers yet a lower value of $\sigma_l = 0.332$ (instead of 1.231 in the benchmark model and 0.654 in the RANK vintage). Also – as is the case in the RANK model – higher investment adjustment costs, S'' and lower portfolio adjustment costs PAC reduce the effectiveness of QE. Finally, we estimate a substantial share of H2M households $\chi = 0.348$, which aligns
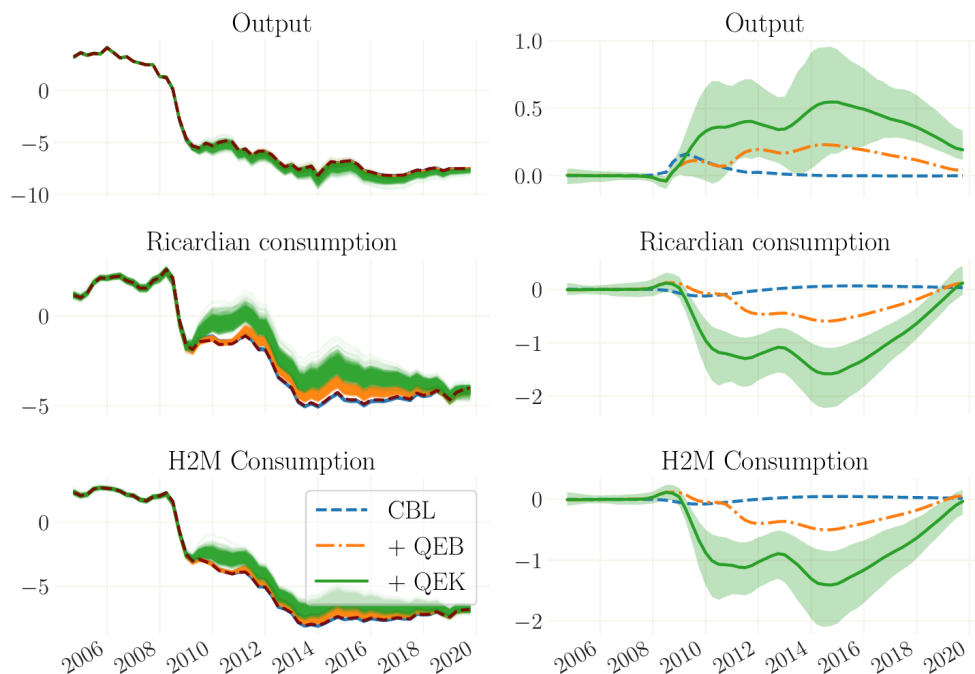
Figure 10: H2M specification of benchmark model. Left: counterfactual simulations without the QE measures. Right: net contribution of each QE measure. Effects in both graphs are cumulative. *Note:* Means over 1000 simulations drawn from the posterior. Annual measures where applicable.

previous estimates of the literature.[31] Yet note that despite this estimate being larger than our estimated share of impatient households in the benchmark model, both types of households are fundamentally different which makes a direct comparison impossible.

To conclude, both model vintages above show that our results do not hinge on the inclusion of impatient households in the model. In fact, if anything, we find the inclusion of market segmentation on the household side imply larger effects on economic activity and *less* deflationary effects, together with a *smaller* decline in aggregated consumption. We therefore regard the disinflationary effect presented in the benchmark model as a conservative estimate, which indicates a lower bound.

6.3 Model choice

Besides the framework proposed by Gertler and Karadi (2013) – which serves as a backbone of our financial sector – two other widely used models to study the effects of quantitative easing include Chen et al. (2012) and Carlstrom et al. (2017). Chen et al. (2012) basically add market segmentation and transaction cost to an otherwise standard Smets and Wouters (2007)-type model in line with Andres et al. (2004), such that QE directly affects consumption of restricted households. Our model captures this channel

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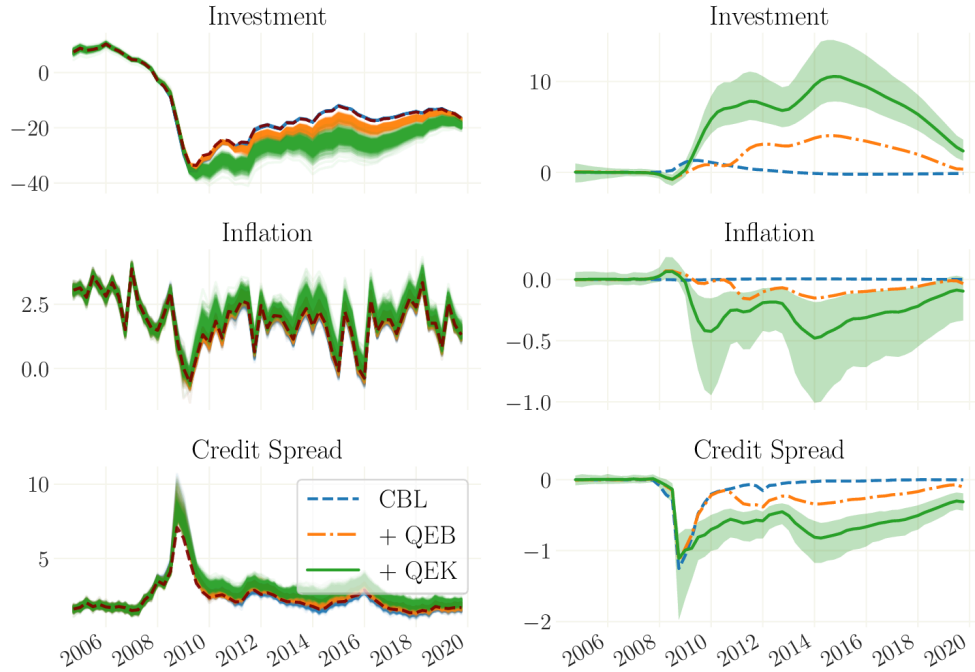
[31]See, e.g., Kaplan et al. (2014).

Figure 11: H2M specification of benchmark model. Left: counterfactual simulations without the QE measures. Right: net contribution of each QE measure. Effects in both graphs are cumulative. Measures of spreads and inflation are annualized.
Note: Means over 1000 simulations drawn from the posterior. Annual measures where applicable.

as impatient households have to borrow using long-term private loans, but are effectively restricted from short-term assets. As discussed above, this feature allows QE to affect consumption directly rather than via its effect on the real deposit rate. Carlstrom et al. (2017, henceforth CFP), on the other hand, assume that households are segmented from long-term debt markets and can only save in bank deposits. A hold-up problem – similar in spirit to our bank's moral hazard problem – together with portfolio adjustment costs further limits the financial intermediary from arbitraging yield gaps between short and long-term debt. Lastly, as firms' have to issue long-term debt in oder to finance their capital demand, central bank asset purchases have real effects.

While this model shares several features with ours, there are two important differences. First, the price (and hence yield) of corporate and government bonds is assumed to be same, which restricts us from separating the effects of MBS and government bond purchases. Second, the model does not include a shock that moves consumption and investments into the same direction, which renders the estimation of the GFC difficult, as we will discuss below.[32]

In order to gauge whether our results are also robust to the model choice, we replicate

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[32]In addition, it is not straightforward to add liquidity injections to the model in a comparable fashion due to the reduced form moral hazard problem.
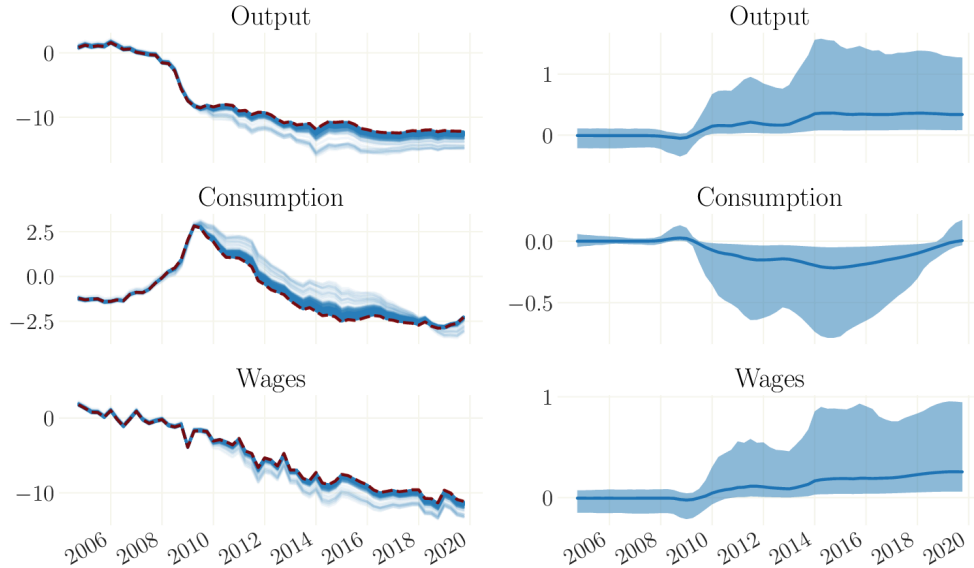
Figure 12: CFP model. Left: counterfactual simulations without the QE measures. Right: net contribution of each QE measure. Effects in both graphs are cumulative.
Note: Means over 1000 simulations drawn from the posterior. Annual measures where applicable.

Carlstrom et al. (2017) by estimating their model using their priors, but our nonlinear Bayesian likelihood approach and the same sample period including the ZLB.[33]

We find the sample choice to affect the posterior parameter estimates in a non-negligible way, confirming our intuition of the importance of structural changes in the economy. For instance, we find a substantially steeper wage Phillips curve, despite a notable flatter price Phillips curve. But more broadly, various of our parameter estimates are even outside the 95% credible set reported in CFP. Besides this, the most important difference lies in the estimate of ψ_n, the adjustment costs parameter on net worth, which is the most important driver of QE. We find $\psi_n = 0.14$ instead of 0.7850 reported in CFP, which substantially lowers the effectiveness of QE. The reason is that lower net worth adjustment cost imply less limits to arbitrage.

Let us study the counterfactuals of how the key macroeconomic variables would have evolved in the absence of unconventional monetary policy measures in the CFP model. As evident from Figures 12 and 13, MBS and government purchases jointly compress the term premium, albeit only a little, with the peak effect of around -10 basis points. The reason for the latter is our low estimate for the net worth adjustment cost $\psi_n = 0.14$ which govern the limits to arbitrage. The small reduction in the term premium, in turn, translates into similarly muted responses of investment and output. Consumption, on the other hand, falls as it does in our model. Even though the real deposit rate falls, the increase in the return on capital raises the marginal benefits to invest over and above

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[33]See Appendix M for more details on the data and the parameter estimates.
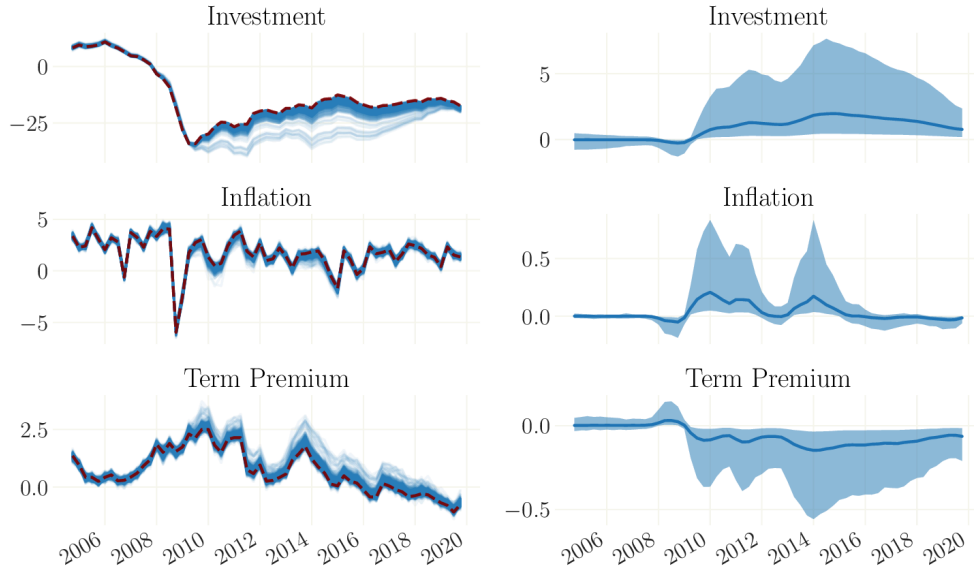
Figure 13: CFP model. Left: counterfactual simulations without the QE measures. Right: net contribution of each QE measure. Effects in both graphs are cumulative.
Note: Means over 1000 simulations drawn from the posterior. Annual measures where applicable.

the marginal benefit to consume. So far, the dynamics are consistent with our reported results, although the magnitude is significantly reduced. Contrary to our analysis above, however, inflation seems to fall only initially, but turns positive later on. The latter can be explained by increasing marginal costs. Specifically, without capital utilization, higher investment increases the marginal product of capital. In order to keep the ratio of marginal products constant, also labor demand has to increase mechanically, leading to an increase in wages. Both rising wages and the return on capital which – at first order – equals the marginal product of capital in this model, push up marginal cost and ultimately inflation.

Thus, in contrast to our benchmark model, the demand effects of LSAPs outweigh their supply effects in the estimated CFP model. However, we take this result with a considerable degree of caution. In particular, we argue that the CFP model in its original form is not well suited to study the GFC because it lacks a shock that can explain the joint decline of consumption and investment in the Great Recession. In fact, the historical decomposition suggests that through the lens of the CFP model, the GFC was driven by a negative monetary policy shock. At the same time, CFP do not include consumption as an observable. Following their approach, we see that the smoothed state of consumption suggests that household spending actually peaked in the direct aftermath of the GFC.[34] Both observations let us conclude that the CFP model would require some substantial

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[34]See Figures M.35 and M.36 in Appendix M. Adding consumption as an observable together with an additional demand shock such as a government spending shock does not resolve this problem since then, consumption is solely explained by this additional shock.

adjustments to facilitate a meaningful empirical analysis of macroeconomic dynamics in the US on a sample that includes the GFC and its aftermath.

7   Conclusion

After several years of recovery from the Global Financial Crisis, the Federal Reserve embarked again in large-scale asset purchases once the policy rate reached the ZLB in spring 2020. Although the commonly held view points towards expansionary effects of Quantitative Easing, its macroeconomic impact remains unclear.

Using a nonlinear Bayesian likelihood approach that fully accounts for the ZLB on nominal interest rates, we are the first to provide a structural investigation of the macroeconomic effects of QE. Our large-scale New Keynesian model includes several channels for QE to have macroeconomic effects and is estimated over the sample from 1998 until 2019 including data of the Federal Reserves balance sheet.

According to our results, the QE programs in the aftermath of GFC had sizable expansionary effects raising output by around 1.2%. This effect was by and large driven by a large increase investment of around 9 percent. Aggregate consumption, on the other hand, contracted by 0.7%. This finding is robust across models and different vintages with a representative agent or heterogeneous agents.

The surge in investment raises the production capacity which led to a mild disinflationary effect of 0.25 percent. Different to conventional monetary policy, supply side effects of LSAPs seem to dominate the aggregate demand effects, consistent with a cost channel of monetary policy. Our finding of a disinflationary effect of QE is highly policy relevant. In the US, LSAPs create a trade-off for policymakers between the stabilizing prices and employment. For the Euro Area, the result may be even more pressing as its asset purchase programme (APP) was explicitly undertaken with the goal to stabilize inflation and inflation expectations at a time when fears of deflation surged.

In times when central banks in developed countries review the robustness of their monetary policy strategies against the backdrop of a higher likelihood of future ZLB episodes, our novel results provide a useful guidance of the macroeconomic effects of large-scale asset purchases. What role wealth or income inequality plays in this context is an important question we leave for future research.

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Appendix A   Data

Our eleven observables are mapped into the model equations using the following measurement equations:

$$\text{Real GDP growth} = \overline{\gamma} + (y_t - y_{t-1}),$$
$$\text{Real consumption growth} = \overline{\gamma} + (c_t - c_{t-1}),$$
$$\text{Real investment growth} = \overline{\gamma} + (i_t - i_{t-1}),$$
$$\text{Real wage growth} = \overline{\gamma} + (w_t - w_{t-1}),$$
$$\text{Labor hours} = \bar{l} + l_t,$$
$$\text{Inflation} = \overline{\pi} + \pi_t,$$
$$\text{Federal funds rate} = (\frac{\overline{\pi}}{\beta\gamma^{-\sigma_c}} - 1) * 100 + r_t,$$
$$\text{GZ-spread} = \overline{spread} + E_t[\tilde{r}^k_{t+1} - r_t],$$
$$\text{Gov. bonds purchases} = \frac{\overline{B}_{cb}}{\overline{Y}} + b_{cb,t},$$
$$\text{Corp. bonds purchases} = k_{cb,t},$$
$$\text{Liquidity injections} = l^q_t.$$

These observables are constructed as follows:

- GDP: ln(GDP/GDPDEF/CNP16OV_ma)*100

- CONS: ln((PCEC-PCEDG) / GDPDEF / CNP16OV_ma)*100

- INV: ln((GPDI+PCEDG) / GDPDEF / CNP16OV_ma)*100

- LAB: ln(13*AWHNONAG * CE16OV / CNP16OV_ma)*100

- INFL: ln(GDPDEF)*100

- WAGE: ln(COMPNFB / GDPDEF)*100

- FFR: FEDFUNDS/4

- GZ Spread: GZSpread/4

- CB GBonds: TREAST/GDP

- CB CBonds: (WSHOMCB+WCPFF)/GDP

- CB Liquidity: (WACBS + FEDDT + TERAUCT + OTHLT)/GDP

We take log changes for GDP, CONS, INV and WAGE in our measurement equations. Except the GZ spread, which can be downloaded from the Federal Reserves homepage[35], all data is downloaded from the FRED database of the St. Louis Fed, with the above mnemonics representing:

- GDP: GDP - Gross Domestic Product, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED

- GDPDEF: Gross Domestic Product: Implicit Price Deflator , Index 2012=100, Quarterly, Seasonally Adjusted , FRED

- CNP16OV: Civilian noninstitutional population, Thousands of Persons, Quarterly, Seasonally Adjusted, FRED

- CNP16OV_ma: a four-quarter trailing average of CNP16OV

- PCEC: Personal Consumption Expenditures, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED

- PCEDG: Personal Consumption Expenditures: Durable Goods, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED

- GPDI: Gross Private Domestic Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED

- AWHNONAG: Average Weekly Hours of Production and Nonsupervisory Employees: Total private, Hours, Quarterly, Seasonally Adjusted, FRED

- CE16OV: Employment Level, Thousands of Persons, Quarterly, Seasonally Adjusted, FRED

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[35]https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/files/ebp_csv.csv

- COMPNFB, Nonfarm Business Sector: Compensation Per Hour, Index 2012=100, Quarterly, Seasonally Adjusted, FRED

- FEDFUNDS: Effective Federal Funds Rate, Percent, FRED

- GZ Spread: A corporate bond credit spread with a high information content for economic activity constructed by Gilchrist and Zakrajšek (2012) using secondary market prices of senior unsecured bonds issued by a large representative sample of US non-financial firms. Quarterly, Percent, Federal Reserve Board.

- TREAST: U.S. Treasury securities held by the Federal Reserve: All Maturities, Millions of Dollars, Quarterly, Not Seasonally Adjusted, FRED

- WSHOMCB: Current face value of mortgage-backed obligations held by Federal Reserve Banks: All Maturities, Millions of Dollars, Quarterly, Not Seasonally Adjusted, FRED

- WCPFF: Net Portfolio Holdings of Commercial Paper Funding Facility LLC, Billions of Dollars, Quarterly, Not Seasonally Adjusted, FRED

- WACBS: Central Bank Liquidity Swaps, Millions of Dollars, Quarterly, Not Seasonally Adjusted, FRED

- FEDDT: Federal agency debt securities held by the Federal Reserve: All Maturities, Millions of Dollars, Quarterly, Not Seasonally Adjusted, FRED

- TERAUCT: Term auction credit held by the Federal Reserve: All Maturities, Millions of Dollars, Quarterly, Not Seasonally Adjusted, FRED

- OTHLT: Other loans held by the Federal Reserve: All Maturities, Millions of Dollars, Quarterly, Not Seasonally Adjusted, FRED

Appendix B   Full model

For our analysis, we extend a standard medium scale model as in Smets and Wouters (2007) and financial intermediates as in Gertler and Karadi (2013), patient and impatient households as well as unconventional monetary policy tools. Time is discrete, and one period in the model represents one quarter. The model features households, banks, intermediate good producers, capital good producers, retailers, labor unions, a fiscal and a monetary authority. The model includes habit formation in consumption, investment adjustment costs, variable utilization of productive capital and nominal rigidities that give rise to a price and wage Phillips curves as in Smets and Wouters (2007)to enhance the empirical plausibility of the model dynamics.

Appendix B.1   The household structure

The model is populated by two types of households. There is a continuum of impatient households with mass χ and a continuum of patient households of mass $1 - \chi$. In the spirit of Gertler and Karadi (2013), we assume that a constant fraction f of each patient household's members works as banker, whereas the remaining fraction $(1 - f)$

consists of workers who – like impatient households – supply labor to the intermediate good producers. While workers receive their wage income every period, bankers reinvest their gains in asset holdings of the bank over several periods. Only when a banker (exogenously) exits the banking sector, she contributes to the patient households' income by bringing home the accumulated profits. Perfect consumption insurance within patient households ensures that workers and bankers face the same consumption stream. The expected lifetime utility of any household i is given by

$$U_t = E_0 \sum_{t=0}^{\infty} \beta_i^t \left(\frac{(C_{i,t} - hC_{i,t-1})^{1-\sigma_c} - 1}{1 - \sigma_c} \right) exp \left(\frac{\sigma_c - 1}{1 + \sigma_l} L_{i,t}^{1+\sigma_l} \right) \tag{B.1}$$

where parameters β_i, h, σ_c, and σ_l are, respectively, the discount factor, the degree of external habit formation in consumption, the coefficient of relative risk aversion, and a weight on the disutility of labor. The discount factor has a household-specific subscript i, because we assume the discount factor of impatient households β_m to be smaller than the discount factor of patient households, i.e. $\beta_m < \beta_p$. Finally. $C_{i,t}$ and $L_{i,t}$ denote consumption and hours worked of household $i \in \{m, p\}$, respectively.[36]

Appendix B.2   Patient households

The patient household earns the real wage, W_t, for her supplied labor, $L_{p,t}$. She can save in one-period bank deposits, D_t, that pay an interest rate, R_t^d, in government bonds, $B_{h,t}$, that yield an interest rate, R_t^b, and in capital assets $K_{h,t}$ with an associated interest rate, R_t^k. These interest rates are already in real terms. Capital claims, just like government bonds, are modeled as long-term assets. As for all stock variables, we use the end-of-period notation, so that D_t denotes the household's deposits at the end of period t. The return on deposits, $R_t^d = v_{u,t}R_t$, includes a disturbance term, $v_{u,t}$ which drives a wedge between the risk-free real rate and the return on deposits. We assume $v_{u,t}$ to follow an AR(1) process in logs. Patient households spend their funds on consumption $C_{p,t}$, and save in new deposits, bonds and capital. Savings in government bonds and capital are subject to portfolio adjustment costs with adjustment parameters κ_b and κ_k. The budget constraint of patient households, in real terms, reads

$$C_{p,t} + \frac{D_t}{R_t^d} + Q_t[K_{h,t} + \frac{1}{2}\kappa_k(K_{h,t} - K_h)^2] + Q_t^b[B_{h,t} + \frac{1}{2}\kappa_b(B_{h,t} - B_h)^2]$$
$$= D_{t-1} + W_t L_{p,t} + R_t^k Q_{t-1}K_{h,t-1} + R_t^b Q_{t-1}^b B_{h,t-1} - T_t + \mathfrak{P}_t. \tag{B.2}$$

T_t denotes lump sum taxes raised by the government to finance government spending, and $\mathfrak{P}_t$ are profits of monopolistic firms and banks that accrue to the patient households. Maximizing (B.1) subject to the patient household's budget constraint (B.2) and rearranging the first order conditions yields the Euler equation, a condition for the optimal

---

[36]For the ease of notation, we only use subscripts $i \in \{m, p\}$ indicating which type of household is meant if necessarily needed.

supply of labor and two no-arbitrage conditions:

$$1 = \beta_p E_t \left[\exp\left(\frac{\sigma_c - 1}{1 + \sigma_l}(L_{p,t+1}^{1+\sigma_l} - L_{p,t}^{1+\sigma_l}) \right) \left(\frac{C_{p,t+1} - hC_{p,t}}{C_{p,t} - hC_{p,t-1}} \right)^{-\sigma_c} \right] R_t^d, \qquad \text{(B.3)}$$

$$W_t^h = (C_{p,t} - hC_{p,t-1})L_{p,t}^{\sigma_l}, \qquad \text{(B.4)}$$

$$E_t R_{t+1}^k = R_t^d[1 + \kappa_k(K_{h,t} - K_h)], \qquad \text{(B.5)}$$

$$E_t R_{t+1}^b = R_t^d[1 + \kappa_b(B_{h,t} - B_h)]. \qquad \text{(B.6)}$$

Ultimately, the no-arbitrage conditions (B.5) and (B.6) specify the patient household's optimal holdings of both capital claims and government bonds.

$$K_{h,t} = K_h + E_t \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \left(\frac{R_{t+1}^k - R_t^d}{\kappa_k} \right), \qquad \text{(B.7)}$$

$$B_{h,t} = B_h + E_t \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \left(\frac{R_{t+1}^b - R_t^d}{\kappa_b} \right). \qquad \text{(B.8)}$$

Appendix B.3 Impatient households

There is a fraction of χ impatient households which consume, supply labor and borrow long-term private loans from the banks which gives rise to the following budget constraint in real terms

$$C_{m,t} + R_t^p Q_{t-1}^p B_{m,t-1}^p = W_t L_{m,t} + Q_t^p B_{m,t}^p, \qquad \text{(B.9)}$$

where R_t^P and Q_t^P denote, respectively, the interest rate and price of private loans $B_{m,t}^p$. Price and yield of private loans are related through

$$R_t^p = \frac{\xi + \kappa_p Q_t^p}{Q_{t-1}^p},$$

where ξ is the coupon (or redemption) and κ_p denotes the decay factor (Woodford, 1998, 2001). Maximizing (B.1) subject to the impatient households budget constraint (B.9) yields, after some rearranging, an Euler equation for the optimal borrowing and a condition for the labor supply of impatient households:

$$1 = \beta_m E_t \left[\exp\left(\frac{\sigma_c - 1}{1 + \sigma_l}(L_{m,t+1}^{1+\sigma_l} - L_{m,t}^{1+\sigma_l}) \right) \left(\frac{C_{m,t+1} - hC_{m,t}}{C_{m,t} - hC_{m,t-1}} \right)^{-\sigma_c} \right] R_t^p, \qquad \text{(B.10)}$$

$$W_t^h = (C_{m,t} - hC_{m,t-1})L_{m,t}^{\sigma_l}. \qquad \text{(B.11)}$$

Appendix B.4 Firm sectors

The model contains three types of firms. Intermediate goods are produced by perfectly competitive firms, which use capital and labor as inputs for production. Monopolistically competitive retailers buy a continuum of intermediate goods, and assemble them into a final good. Nominal frictions make the retailers optimization problem dynamic. Additionally, a capital producing sector buys up capital from the intermediate good producer, repairs it, and builds new capital, which it sells to the intermediate good sector again. Investment in new capital is subject to investment adjustment costs.

Intermediate good producers

Intermediate good producers are in monopolistic competition, employ labor and capital services from households, and set their prices as markups over the marginal cost. Firm i produces according to the Cobb-Douglas Function

$$Y_{m,t}(i) = e^{z_t} \overline{K}_t(i)^\alpha \left(\gamma^t L_t(i) \right)^{1-\alpha} - \gamma^t \overline{\Phi}. \tag{B.12}$$

Here, $Y_{m,t}(i)$ are intermediate goods, z_t is a TFP-shock, $\overline{K}_t(i)$ is the effective capital used in production defined as $\overline{K}_t(i) = U_t(i)K_{t-1}(i)$, and parameter α is the output elasticity with respect to effective capital. γ^t represents the labor-augmenting growth rate in the economy, and $\overline{\Phi}$ is the fixed cost of production.

At the end of each period the intermediate good producer sells the capital stock that it used for production to the capital producer which repairs the capital, and purchases the capital stock that it is going to use in the next period from the capital producer. To finance the purchase of the new capital at the price Q_t per unit, it issues a claim for each unit of capital it acquires to banks, which trade at the same price. The interest rate the firm has to pay on the loans is R_t^k. Under the assumption that the competitive firms make zero profits, the interest rate on their debt will just equal the realized ex-post return on capital. Furthermore, we assume that the firm incurs costs of capital utilization that are proportional to the amount of capital used, $\Psi(U_t)P_{m,t}K_{t-1}$. This assumptions for the utilization costs are set to match the setting in Smets and Wouters (2007). In steady state it holds that $a(\cdot) = 0$. The operating profit of firm i is therefore

$$P_{m,t}(i)Y_t(i) - W_t L_t(i) - a(U_t(i))P_{m,t}K_{t-1}(i).$$

Note that $P_{m,t}$, the price of the intermediate good, represents the marginal cost from the viewpoint of final good producers, which purchase intermediate goods. Each period the firm determines its optimal capital purchase by maximizing

$$E_t[\beta \Lambda_{t,t+1}(-R_{t+1}^k Q_t K_t(i) + P_{m,t+1}(i)[Y_{m,t+1}(i) - \Psi(U_t(i))K_t(i)] - W_{t+1}L_{t+1}(i) + (1-\delta)Q_{t+1}K_t(i))]$$

with respect to $K_t(i)$. As all firms make the same decisions, we can drop the index i. In optimum the ex-post return then is as follows

$$R_{t+1}^k = \frac{P_{m,t+1}[\alpha \frac{Y_{m,t+1}}{K_t} - \Psi(U_{t+1})] + (1-\delta)Q_{t+1}}{Q_t}. \tag{B.13}$$

Additionally, the choices for optimal labor input and optimal capital utilization yield the first order conditions

$$W_t = P_{m,t}(1-\alpha)\frac{Y_{m,t}}{L_t}. \tag{B.14}$$

$$\Psi'(U_t)K_{t-1} = \alpha\frac{Y_{m,t}}{U_t} \Leftrightarrow \Psi'(U_t) = \alpha\frac{Y_{m,t}}{\overline{K}_t} \tag{B.15}$$

Capital good producers

Capital evolves according to the following law of motion

$$K_t = (1 - \delta)K_{t-1} + v_{i,t}\left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right)I_t, \tag{B.16}$$

where δ is the depreciation rate and the function $S(\cdot)$, indicates a cost of adjusting the level of investment. In steady state it holds that $S = 0, S' = 0$, and $S'' > 0$. and $v_{i,t}$ follows an AR(1) process in logs. The capital good producer's role in the model is to isolate the investment decision that becomes dynamic through the introduction of convex investment adjustment costs, which is a necessary feature to generate variation in the price of capital. Capital good producers buy the used capital, restore it and produce new capital goods. Since capital producers buy and sell at the same price, the profit they make is determined by the difference between the quantities sold and bought, i.e. investment. Capital producers bear the resource costs associated with changes in investment. They choose the optimal amount of investment to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_{0,t}\left\{Q_t\left(1 - S\left(\frac{I_t(k)}{I_{t-1}(k)}\right)\right)v_{i,t} - 1\right\}I_t.$$

The first order condition of the capital producer reads

$$
\begin{aligned}
1 &= Q_t v_{i,t}\left(1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) \\
&\quad + E_t\left\{\Lambda_{t,t+1}Q_{t+1}v_{i,t+1}S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\}
\end{aligned}
$$

Final good producers

Final good producers buy the goods produced by the intermediate good producers and sell them to final good producers. They act under monopolisitic competition. Each period, retailers firms face a constant probability of being able to optimally adjust their prices, ζ_p. Those firms, which cannot optimally adjust their prices in a given period, index their prices to a weighted average of last periods inflation and steady state inflation. These assumptions give rise to the following maximization problem for firm i

$$\max_{P_t(i)} E_t \sum_{s=0}^{\infty} (\beta\zeta_p)^s \frac{\Lambda_{t,t+s}}{\Pi_{t,t+s}}\left[P_t(i)\Pi_{l=1}^s(\Pi_{t+l-1}^{\imath_p}\Pi^{1-\imath_p}) - MC_{t+s}\right]Y_{t+s}(i) \tag{B.17}$$

$$s.t. \quad \frac{Y_{t+s}(i)}{Y_{t+s}} = G'^{-1}\left(\frac{P_t(i)\Pi_{l=1}^s(\Pi_{t+l-1}^{\imath_p}\Pi^{1-\imath_p})}{P_{t+s}}\tau_{t+s}\right). \tag{B.18}$$

$P_t(i)$ is the price set by firm i, $\Pi_{t,t+s}$ is the accumulated change in the aggregate price level between periods t and $t+s$, $\Lambda_{t,t+s}$ is the stochastic discount factor of the firm, Y_{t+s} is the demand by final good firms for intermediate goods. Parameter $\imath_p$ is the degree of price indexation. Function G governs how the relative price of firm i affects the amounts

of goods it can sell. We make the same assumptions on G as Smets and Wouters (2007).[37] Furthermore, $\tau_{t+s} \equiv \int_0^1 G'\left(\frac{Y_{t+s}(i)}{Y_{t+s}}\right) \frac{Y_{t+s}(i)}{Y_{t+s}} di$. The aggregate price index is in this case given by

$$
P_t = [(1-\zeta_p)(P_t^*)G'^{-1}\left[\frac{P_t^* \tau_t}{P_t}\right] + \zeta_p \Pi_{t-1}^{\iota_p} \Pi^{(1-\iota_p)} P_{t-1} G'^{-1}\left[\frac{\Pi_{t-1}^{\iota_p} \Pi^{(1-\iota_p)}) P_{t-1} \tau_t}{P_t}\right] \quad (B.19)
$$

where P_t^* is the optimal price in period t. The price markup set by final goods producers, $v_{p,t}$, is time-varying and subject to markup shocks, $v_{p,t}$, which follow an AR(1)-process in logs.

Retailers

Retailers act under perfect competition. They buy the goods from final good producers, bundle them in final goods. and sell them to the public. Their maximization problem reads

$$
\max_{Y_t, Y_t(i)} P_t Y_t - P_t(i) Y_t(i) \quad (B.20)
$$

$$
s.t. \left[\int_0^1 G\left(\frac{Y_t(i)}{Y_t}, \lambda_{p,t}\right) di\right] = 1. \quad (B.21)
$$

Unions and Labor Packers

The supply of labor to intermediate good firms is organized by unions and labor packers. Households of both types supply labor to a labor union, which differentiates the labor services and sets wages. Unions act in monopolistic competition with each other and set their wages, $W_t(i)$, as a markup over the average marginal rate of substitution between consumption and leisure of households. We assume that the wage setting process, in the same way as price setting, is subject to a Calvo type friction, and that unions, which cannot adjust their wages in a given period, index their last wage to a weighted average of last periods inflation and steady state inflation. Labor packers buy the labor services from unions, bundle them, and provide them to intermediate good firms at the wage W_t. Thus, the maximization problem of labor packers is

$$
\max_{L_t, L_t(i)} W_t L_t - W_t(i) L_t(i) \quad (B.22)
$$

$$
s.t. \left[\int_0^1 G_w\left(\frac{L_t(i)}{L_t}, \lambda_{w,t}\right) di\right] = 1, \quad (B.23)
$$

where L_t is labor provided by labor packers to intermediate good firms, $L_t(i)$ denotes the labor services sold by unions to labor packers, and G_w is the labor aggregator on which we make the same assumptions as on G. The wage markup, $\lambda_{w,t}$, is time-varying and subject to wage markup shocks, $v_{w,t}$, which follows an AR(1)-process in logs.

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[37] That is that $G' > 0$, $G'' < 0$, and $G(1) = 1$. As shown by Kimball (1995), the assumptions on G imply that the demand for a good is decreasing in its relative price, and that the elasticity of demand for a good increases with its relative price, which in turn implies a higher persistence of aggregate inflation dynamics.

Labor unions observe the average marginal rate of substitution between consumption and leisure of all households and charge a markup on top of it. Their maximization problem is

$$\max_{W_t(i)} E_t \sum_{s=0}^{\infty} (\beta\zeta_w)^s \frac{\Lambda_{t,t+s}}{\Pi_{t,t+s}} \left[W_t(i)\Pi_{l=1}^s (\Pi_{t+l-1}^{1_w}\Pi^{1-1_w}) - W_{t+s}^h \right] L_{t+s}(i) \qquad (B.24)$$

$$s.t. \frac{L_{t+s}(i)}{L_{t+s}} = G_w'^{-1} \left(\frac{W_t(i)\Pi_{l=1}^s(\Pi_{t+l-1}^{1_w}\Pi^{1-1_w})}{W_{t+s}} \tau_{t+s}^w \right). \qquad (B.25)$$

$W_t(i)$ is the wage set by union, i. W_t^h is the households' marginal rate of substitution between consumption and leisure (i.e. the wage that would prevail in the absence of wage setting power). Parameter $\imath_w$ is the degree of wage indexation, and $\tau_{t+s}^w \equiv \int_0^1 G_w' \left(\frac{L_{t+s}(i)}{L_{t+s}} \right) \frac{L_{t+s}(i)}{L_{t+s}} di$. The aggregate wage index is in this case given by

$$W_t = [(1-\zeta_w)(W_t^*)G_w'^{-1} \left[\frac{W_t^*\tau_t^w}{W_t} \right] + \zeta_w\Pi_{t-1}^{1_w}\Pi^{(1-1_w)})W_{t-1}G_w'^{-1} \left[\frac{\Pi_{t-1}^{1_w}\Pi^{(1-1_w)})W_{t-1}\tau_t^w}{W_t} \right], \qquad (B.26)$$

where W_t^* is the optimal wage set by labor unions in period t.

Appendix B.5   Banks

The banking sector builds on Gertler and Karadi (2013) with some extensions that we outline below. Banks collect deposits D_t from patient households and – together with their own net worth N_t – use these funds to extend loans $B_{b,t}^p$ to impatient households, purchase capital securities from intermediate good producers, $K_{b,t}$, and purchase government bonds $B_{b,t}$. Given these financial operations, the balance sheet of a representative bank then follows as

$$Q_t K_{b,t} + Q_t^b B_{b,t} + Q_t^p B_{b,t}^p = N_t + D_t + L_t^q, \qquad (B.27)$$

where L_t^q denotes exogenous emergency liquidity injections by the Federal Reserve. For simplicity, we assume that central bank liquidity is lent at a zero nominal interest rate (i.e. their real rate equals $R_t^L = 1/\Pi_{t+1}$, where Π_t denotes gross inflation). Banks retain their earnings and add it to their current net worth. This gives rise to the following law of motion for the bank's net worth

$$N_t = R_t^k Q_{t-1} K_{b,t-1} + R_t^b Q_{t-1}^b B_{b,t-1} + R_t^p Q_{t-1}^p B_{b,t-1}^p$$
$$-R_{t-1}^d D_{t-1} - R_{t-1}^L L_{t-1}^q. \qquad (B.28)$$

Note that while the interest rate on deposits raised in period $t-1$ is determined in the same period, the return of assets is risky and only determined after the realization of shocks at the beginning of period t.

Bankers continue accumulating their net worth until they (involuntarily) exit the business, which occurs randomly with exogenous probability, $1 - \theta$. Conversely, bankers continue their operations with probability θ. Draws from this lottery are i.i.d. and do not depend on the banker's history. When a banker leaves the sector, she adds her terminal

wealth, V_t, to the wealth of the patient household she is member of. Therefore, bankers seek to maximize the expected discounted terminal value of their wealth

$$
\begin{aligned}
V_t &= \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta_p^{i+1} \frac{\Lambda_{p,t+1+i}}{\Lambda_{p,t+i}} N_{t+1+i}, \\
&= \max E_t \left[\beta_p \frac{\Lambda_{p,t+1}}{\Lambda_{p,t}} (1-\theta) N_{t+1} + \theta V_{t+1} \right], \quad\quad (B.29)
\end{aligned}
$$

where $\Lambda_{p,t}$ denotes the patient household's Lagrangian multiplier with respect to the budget constraint.

Banks operate under perfect competition. If financial intermediation was frictionless, the risk adjusted return on the bank's asset should equal the return on deposits. As in Gertler and Karadi (2013), however, bankers can divert a fraction of their assets and transfer it to their respective households. If they do so, their depositors will withdraw their remaining funds and force the bank into bankruptcy. This moral hazard/costly enforcement problem creates an endogenous limit to the amount of deposits that households are willing to supply. While the latter ensures that bankers earn a strictly positive excess return, it also creates limits to arbitrage, as bankers can not scale-up their balance sheet to arbitrage away any price differences. In order to prevent a banker from diverting a fraction of assets, households keep their deposits at a bank only as long as the bank's continuation value is higher or equal to the amount that the bank can divert. Formally, the latter condition is given by the following incentive compatibility constraint of the bank

$$
V_t \geq \lambda_{k,t} Q_t K_{b,t} + \lambda_b Q_t^b B_{b,t} + \lambda_p Q_t^p B_{b,t}^p - \lambda_{cbl} L_t^q, \quad\quad (B.30)
$$

where λ_j for $j \in \{k, p, b\}$ denotes the respective fraction of capital claims, government bonds or private loans that the bank can diverted. Following Dedola et al. (2013) and Gelain and Ilbas (2017), we allow $\lambda_{k,t}$ to be time-varying, formally following an AR(1) process in logs with mean λ_k. Ultimately, this shock triggers variations in the divertiblity of capital assets and can be interpreted as variations in the trust depositors have in the quality of banks' capital assets.

In steady state, the λ_j's for $j \in \{k, p, b\}$ determine – together with other estimated parameters such as the discount factor of patient households, β_p, and the trend growth rate, γ – the returns on capital claims and government bonds, R^k and R^b, as well as the private loan rate R^p. The respective excess returns over the deposit rate imply that $\lambda_b < \lambda_p < \lambda_k$, which intuitively can be motivated by the fact that, in general, the collateral value of government bonds is higher than that of mortgage loans and capital claims.[38] Finally, the last term in the incentive constraint is due to the assumption that liquidity injections serve to relax the incentive constraint of banks.

To solve the bank problem, let an initial guess of the value function be of the form

$$
V_t = \nu_{k,t} Q_t K_{b,t} + \nu_{b,t} Q_t^b B_{b,t} + \nu_{p,t} Q_t^p B_{b,t}^p + \nu_{n,t} N_t + \nu_{L,t} L_t^q, \quad\quad (B.31)
$$

where $\nu_{k,t}, \nu_{b,t}, \nu_{p,t}, \nu_{d,t}$, and $\nu_{L,t}$ are time-varying coefficients. Maximizing (B.31) with

---

[38] In a similar vein, Meeks et al. (2017) use the same approach to distinguish between the collateral values of loans and asset-backed securities.

respect to $K_{b,t}$, $B_{b,t}$ and $B_{b,t}^p$ subject to (B.30) yields the following first order conditions for capital claims, governments bonds, private loans, and μ_t, the Lagrangian multiplier on the incentive compatibility constraint

$$\nu_{k,t} = \lambda_{k,t} \frac{\mu_t}{1 + \mu_t}, \tag{B.32}$$

$$\nu_{b,t} = \lambda_b \frac{\mu_t}{1 + \mu_t}, \tag{B.33}$$

$$\nu_{p,t} = \lambda_p \frac{\mu_t}{1 + \mu_t}, \tag{B.34}$$

$$\nu_{n,t} N_t + (\lambda_{cbl} + \nu_{L,t}) L_t^q = (\lambda_{k,t} - \nu_{k,t}) Q_t K_{b,t} + (\lambda_b - \nu_{b,t}) Q_t^b B_{b,t} + (\lambda_p - \nu_{p,t}) Q_t^p B_{b,t}^p. \tag{B.35}$$

Given that the incentive compatibility constraint binds[39], we can rewrite the last equation above as

$$Q_t K_{b,t} = \frac{\nu_{b,t} - \lambda_b}{\lambda_{k,t} - \nu_{k,t}} Q_t^b B_{b,t} + \frac{\nu_{p,t} - \lambda_p}{\lambda_{k,t} - \nu_{k,t}} Q_t^p B_{b,t}^p + \frac{\nu_{L,t} + \lambda_L}{\lambda_{k,t} - \nu_{kt}} L_t^q + \frac{\nu_{n,t}}{\lambda_{k,t} - \nu_{k,t}} N_t. \tag{B.36}$$

Intuitively, (B.36) states that banks' demand for capital claims decreases in λ_j for $j \in \{k, p, b\}$, which regulate the tightness of the incentive constraint with respect to capital claims, mortgage loans over government bonds. Central bank liquidity injections L_t^q, on other hand, support the demand for capital claims.

Substituting the demand for capital claims into (B.31), and combining the result with (15) one can write the terminal value of the banker as a function of its net worth

$$V_t = (1 + \mu_t) \nu_{nt} N_t + [(1 + \mu_t) \nu_{L,t} + \mu_t \lambda_L] L_t^q \tag{B.37}$$

A higher continuation value, V_t, is associated with a higher shadow value of holding an additional marginal unit of assets, or put differently, with a higher shadow value of marginally relaxing the incentive compatibility constraint. Then, define the bank's stochastic discount factor as

$$\Omega_t \equiv \frac{\Lambda_{p,t}}{\Lambda_{p,t-1}} \big[(1 - \theta) + \theta(1 + \mu_t) \nu_{n,t} \big], \tag{B.38}$$

and substitute (B.37) into the Bellman equation (B.29). Using the law of motion for net worth (B.28), one can then write the value function as

$$\begin{aligned} V_t = & \beta_p E_t \Big[\Omega_{t+1} ((R_{t+1}^k - R_t^d) Q_t K_{b,t} + (R_{t+1}^b - R_t^d) Q_t^b B_{b,t} \\ & + (R_{t+1}^p - R_t^d) Q_t^p B_{b,t}^p + (R_t^d - R_{L,t}) L_t^q + R_t^d N_t) \Big] \\ & + \beta_p E_t \Big[\frac{\Lambda_{p,t+1}}{\Lambda_{p,t}} \theta [(1 + \mu_{t+1}) \nu_{L,t+1} + \mu_{t+1} \lambda_L] L_{t+1}^q \Big] \end{aligned}$$

---

[39]The constraint binds in the neighborhood of the steady state. For convenience, we make the assumption that it is binding throughout all experiments.

Finally, verifying the initial guess for the value function yields

$$\nu_{k,t} = \beta_p E_t \Omega_{t+1}(R_{t+1}^k - R_t^d), \tag{B.39}$$

$$\nu_{b,t} = \beta_p E_t \Omega_{t+1}(R_{t+1}^b - R_t^d), \tag{B.40}$$

$$\nu_{p,t} = \beta_p E_t \Omega_{t+1}(R_{t+1}^p - R_t^d), \tag{B.41}$$

$$\nu_{n,t} = \beta_p E_t \Omega_{t+1} R_t^d \tag{B.42}$$

$$\nu_{L,t} = \beta_p E_t \left[\Omega_{t+1}(R_t^d - R_t^L) + \theta \rho_{cbl} \frac{\Lambda_{p,t+1}}{\Lambda_{p,t}} [(1 + \mu_{t+1})\nu_{L,t+1} + \mu_{t+1}\lambda_L] \right], \tag{B.43}$$

where the last equality follows from the fact that L_t^q follows an AR(1) in logs with persistence parameter ρ_{cbl}.

Appendix B.6 Policy and market clearing

The policy makers in our model are the central bank as conductor of interest rate policy and QE, and the government sector.

Monetary Policy

We model conventional monetary policy as a standard reaction function with the central bank responding to deviations of inflation from it's target, the output gap and its growth rate

$$\frac{R_t^s}{R^n} = \left(\frac{R_{t-1}^s}{R^n}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_y} \left(\Delta\left(\frac{Y_t}{Y_t^*}\right)\right)^{\phi_{dy}} \right]^{1-\rho_R} v_{r,t}, \tag{B.44}$$

with the ZLB constraint

$$R_t^n = \max\left\{\bar{R}, R_t^s\right\}, \tag{B.45}$$

where we refer to the unconstrained nominal rate R_t^s as the notional (or shadow) rate. Y_t^* denotes the potential output and $\Delta\left(\frac{Y_t}{Y_t^*}\right)$ denotes the growth in the output gap. The parameter ρ_R expresses an interest rate smoothing motive by the central bank over the notional rate and ϕ_π, ϕ_y and ϕ_{dy} are feedback coefficients. The max-operator in (B.45) reflects the zero lower bound (ZLB) on the nominal interest rate R_t^n, which we take into account in our estimation procedure. For this purpose, $\bar{R}$ denotes the exact level at which the ZLB binds.[40] When the economy is away from the ZLB, the stochastic process $v_{r,t}$ – which follows an AR(1) in logs – represents a regular interest rate shock. However, when the nominal interest rate is zero, $v_{r,t}$ may not directly affect the level of the nominal interest rate. Instead, $v_{r,t}$ affects the expected path of the notional rate, first through it's own persistence and second through the persistence in the notional rate, and therefore alter the expected duration of the lower bound spell. At the ZLB, it can hence be viewed as a forward guidance shock.

We capture emergency liquidity injections with as an exogenous variable that eases banks' incentive compatibility constraint (B.30) and thereby stimulates lending. In our

---

[40]Given that, empirically, the Federal Funds rate remained strictly above zero, we choose $\bar{R}$ to be slightly above one in our estimation. Moreover, due to the fact that the Fed never implemented negative rates, we use the term "zero lower bound" and "effective lower bound" interchangeably.

estimation, we feed these liquidity injections into the model and assume they follow AR(1) process in logs. That is, formally

$$\tilde{L}_t = \rho_{cbl}\tilde{L}_{t-1} + \epsilon_{CBL,t}, \tag{B.46}$$

where $\tilde{L} \equiv \frac{L_t^q}{P_t Y_t}$ denotes central bank liquidity as percentage of GDP.

In our analysis, we divide large scale asset purchases into private (capital) security purchases and government bond purchases, both which we assume to follow an AR(2) process in logs.

$$\tilde{K}_{cb,t} = \rho_{k,1}\tilde{K}_{cb,t-1} + \rho_{k,2}\tilde{K}_{cb,t-2} + \epsilon_{QEK,t}, \tag{B.47}$$

$$\tilde{B}_{cb,t} = \rho_{b,1}\tilde{B}_{cb,t-1} + \rho_{b,2}\tilde{B}_{cb,t-2} + \epsilon_{QEB,t}. \tag{B.48}$$

Similar to the liquidity injections, $\tilde{K}$ and $\tilde{B}$ denote, respectively, the central banks capital claim and government bond purchases as a fraction of GDP.

Fiscal policy and market clearing

Government spending, G_t, is exogenous and follows an AR(1) process

$$G_t \quad = Ge^{g_t}, \tag{B.49}$$

$$\text{and} \quad g_t \quad = \rho_g g_{t-1} + \epsilon_t^g, \tag{B.50}$$

where G is the steady state government consumption, ρ_g is the autocorrelation of government consumption, and ϵ_t^g is a shock to government spending. The government finances its expenditures, by issuing government bonds, which are bought by banks and the central bank, as well as by raising lump sum taxes, T_t. Taxes follow a simple feedback rule, such that they are sensitive to the level of public debt,

$$T_t = T + \kappa_b(B_{t-1} - B), \tag{B.51}$$

where T and B are the steady state levels of tax revenue and government debt, respectively. κ_b is set to ensure that the real value of debt grows a rate smaller than the gross real rate on government debt. As shown by Bohn (1998), this rule is a sufficient condition to guarantee the solvency of the government. Bonds are modeled analogously to private securities by impatient households.

$$R_t^b = \frac{\xi + \kappa_b Q_t^b}{Q_{t-1}^b}. \tag{B.52}$$

The flow budget constraint of the government reads

$$G_t + R_t^b Q_{t-1}^b B_{t-1} \quad = Q_t^b B_t + T_t \tag{B.53}$$

Finally, the aggregate resource constraint in real terms reads

$$Y_t = C_t + I_t + G_t + a(U_t)K_{t-1}, \tag{B.54}$$

where the last term on the right hand side of the equation marks the resource costs of

adjusting the utilization of installed capital.

Appendix B.7 Linearized Equilibrium conditions

This subsection briefly presents the linearized equilibrium conditions. Small letters denote the log-deviation of the corresponding variable from its steady state value.

Non-financial part of the economy

With the exception of equations (B.59), (B.60), the equation for the return on capital and the considerations of household heterogeneity, the equations in this subsections are identical to those of the model by Smets and Wouters (2007).

Equation (B.55 and B.56) are the Euler equation for consumption for patient and impatient households. The presence of habit formation justifies the presence of lagged consumption in the equation.

$$
c_t^p = \frac{h/\gamma}{(1+h/\gamma)}c_{t-1}^p + \frac{1}{1+h/\gamma}E_t[c_{t+1}^p] + \frac{(\sigma_c-1)(W^hL/C)}{\sigma_c(1+h/\gamma)}(l_t^p - E_t[l_{t+1}^p])...
$$
$$
-\frac{(1-h/\gamma)}{(1+h/\gamma)\sigma_c}(r_t^n - E_t[\pi_{t+1}] + v_{u,t}), \tag{B.55}
$$

$$
c_t^m = \frac{h/\gamma}{(1+h/\gamma)}c_{t-1}^m + \frac{1}{1+h/\gamma}E_t[c_{t+1}^m] + \frac{(\sigma_c-1)(W^hL/C)}{\sigma_c(1+h/\gamma)}(l_t^m - E_t[l_{t+1}^m])...
$$
$$
-\frac{(1-h/\gamma)}{(1+h/\gamma)\sigma_c}(r_{t+1}^p). \tag{B.56}
$$

The deposit rate is given by

$$
r_t^d = rr_t + v_{u,t}, \tag{B.57}
$$

and the relationship between the real and the nominal interest rate is given by the Fisher equation

$$
rr_t = r_t^n - E_t[\pi_{t+1}]. \tag{B.58}
$$

The linearized conditions that relate to the patient households' choices of capital assets and government bonds are

$$
\kappa_K K_h k_{h,t} = (R^k - R)(-r_t^d) + R^k r_{t+1}^k - R^d r_t^d, \tag{B.59}
$$
$$
\kappa_B B_h b_{h,t} = (R^b - R)(-r_t^d) + R^b r_{t+1}^b - R^d r_t^d. \tag{B.60}
$$

The linearized link between price and return on private bonds reads

$$
-R^p(r_t^p + q_{t-1}^p) = \kappa_p q_t^p. \tag{B.61}
$$

The efficient wage equals the marginal rate of substitution between consumption and leisure for both household types reads

$$
w_t^h = \frac{1}{(1-h\gamma)}(c_t^m - h\gamma c_{t-1}^m) + \sigma_l l_t^m, \tag{B.62}
$$

$$w_t^h = \frac{1}{(1 - h\gamma)}(c_t^p - h\gamma c_{t-1}^p) + \sigma_l l_t^p. \tag{B.63}$$

Economy-wide consumption and labor are the aggregates of the corresponding household specific variables

$$c_t = \chi c_t^m + (1 - \chi)c_t^p \tag{B.64}$$

$$l_t = \chi l_t^m + (1 - \chi)l_t^p \tag{B.65}$$

Equation (B.66) is the linearized first order condition for investment

$$i_t = \frac{1}{1 + \overline{\beta}}[i_{t-1}] + \frac{\overline{\beta}}{1 + \overline{\beta}}E_t[i_{t+1}] + \frac{1}{(1 + \overline{\beta})\gamma^2 S''}q_t^k \tag{B.66}$$

where $\overline{\beta} = \beta\gamma^{(1-\sigma_c)}$. The dynamics of investment are governed by Tobins q. S'' is the steady state value of the second derivative of the investment adjustment cost function. The accumulation equation of physical capital reads

$$k_t = (1 - \delta)/\gamma k_{t-1} + (1 - (1 - \delta)/\gamma)\hat{i}_t + (1 - (1 - \delta)/\gamma)(1 + \overline{\beta})\gamma^2 S''v_{i,t}. \tag{B.67}$$

The marginal cost of the firms and the marginal product of capital are given by (B.68) and (B.69), while (B.70) is the equation for the return on capital.

$$mc_t = w_t - z_t + \alpha(l_t - \overline{k}_t) \tag{B.68}$$

$$mpk_t = w_t - \overline{k}_t + l_t \tag{B.69}$$

$$R^k r_t^k = MC * MPK(mc_t + y_t - \overline{k}_t) + (1 - \delta)q_t - R^k q_{t-1}. \tag{B.70}$$

The relation between physical capital and effective capital is given by (B.71). Here, parameter ψ is the elasticity of the capital utilization adjustment cost function and normalized to be between zero and one.

$$\overline{k}_t = u_t + k_{t-1}. \tag{B.71}$$

$$u_t = \frac{1 - \psi}{\psi}mpk_t. \tag{B.72}$$

(B.73) is the aggregate production function, and (B.74) is the aggregate resource constraint.

$$y_t = \Phi(\alpha\overline{k}_t + (1 - \alpha)l_t + z_t) \tag{B.73}$$

$$y_t = \frac{G}{Y}g_t + \frac{C}{Y}c_t + \frac{I}{Y}i_t + \frac{R^k K}{Y}u_t \tag{B.74}$$

$$\pi_t = \frac{\overline{\beta}}{1+\textbf{1}_p\overline{\beta}}E_t\pi_{t+1} + \frac{\textbf{1}_p}{1+\textbf{1}_p\overline{\beta}}\pi_{t-1} + \frac{(1-\zeta_p\overline{\beta})(1-\zeta_p)}{(1+\overline{\beta}\textbf{1}_p)\zeta_p((\Phi-1)\epsilon_p+1)}(w_t - z_t + \alpha l_t - \alpha k_t)$$
$$\text{(B.75)}$$

Equation (B.75) is the New Keynesian Phillips curve. The last term in parenthesis corresponds to the marginal cost of production. As we employ the Kimball aggregator, the sensitivity of inflation to fluctuations in marginal cost is affected by the market power of firms, represented by the steady state price markup, $(\Phi - 1)$.[41] Furthermore, the curvature of the Kimball aggregator, ϵ_p, affects the adjustment of prices to marginal cost, since a higher ϵ_p implies a higher degree of strategic complementarity in price setting and dampens the price adjustment to shocks.

$$w_t = \frac{1}{1+\overline{\beta}\gamma}(w_{t-1} + \textbf{1}_w\pi_{t-1}) + \frac{\overline{\beta}\gamma}{1+\overline{\beta}\gamma}E_t[w_{t+1} + \pi_{t+1}] - \frac{1+\textbf{1}_w\overline{\beta}\gamma}{1+\overline{\beta}\gamma}\pi_t$$
$$+ \frac{(1-\zeta_w\overline{\beta}\gamma)(1-\zeta_w)}{(1+\overline{\beta}\gamma)\zeta_w((\lambda_w-1)\epsilon_w+1)}(w_t^h - w_t)$$
$$\text{(B.76)}$$

Equation (B.76) is the Wage Phillips curve. w_t^h is the wage that would prevail in the absence of market power by unions. Therefore, $(w_t - w_t^h)$ is the wage markup. Analogous to equation (B.75), the terms λ_w and ϵ_w represent the steady state wage markup and the curvature of the Kimball aggregator for labor services.

The financial sector

The variable μ_t is the Lagrangian multiplier on the incentive constraint of financial intermediaries.

Linearizing the banks' first order conditions for capital assets, bonds and private loans yields

$$\frac{\lambda_k}{\lambda_k - \nu_k}(\hat{\nu}_{k,t} - \hat{\lambda}_{k,t}) = \hat{\mu}_t, \tag{B.77}$$

$$\hat{\nu}_{b,t} = \hat{\nu}_{k,t} - \hat{\lambda}_{k,t}, \tag{B.78}$$

$$\hat{\nu}_{b,t} = \hat{\nu}_{p,t}. \tag{B.79}$$

The first order condition for the Lagrangian multiplier of the incentive constraint can be linearized as

$$(\hat{\lambda}_{k,t} - \hat{\nu}_{kt})K_b + (\lambda_k - \nu_k)K_b(q_t + k_{b,t})$$
$$= Q^b B_b(\nu_b\hat{\nu}_{b,t} + (\nu_b - \lambda_b)(q_t^b + b_{b,t})) + Q^p B_b^p(\nu_p\hat{\nu}_{p,t} + (\nu_p - \lambda_p)(q_t^p + b_{b,t}^p)) \tag{B.80}$$
$$+ (\nu_L + \lambda_L)L_t^q + \nu_n N(\hat{\nu}_{n,t} + n_t)$$

Note: as the steady state of central bank liquidity injections is zero, L_t^q denotes absolute deviations in the calculations above instead of log-deviations.

---

[41] Note, that in equilibrium, the fixed cost parameter is related to the steady state price markup by a zero profit condition.

The shadow value of capital assets, government bonds and private loans and net worth for financial intermediaries are

$$\nu_k \hat{\nu}_{k,t} = \beta \Omega E_t(\hat{\Omega}_{t+1}(R^k - R^d) + R^k r^k_{t+1} - R^d r^d_t) \tag{B.81}$$

$$\nu_b \hat{\nu}_{b,t} = \beta \Omega E_t(\hat{\Omega}_{t+1}(R^b - R^d) + R^b r^b_{t+1} - R^d r^d_t) \tag{B.82}$$

$$\nu_p \hat{\nu}_{p,t} = \beta \Omega E_t(\hat{\Omega}_{t+1}(R^p - R^d) + R^p r^p_{t+1} - R^d r^d_t) \tag{B.83}$$

$$\hat{\nu}_{n,t} = E_t \hat{\Omega}_{t+1} + r^d_t \tag{B.84}$$

Note, that we can omit the shadow of central bank liquidity injections for financial intermediaries as it does not affect the rest of the equilibrium dynamics.

Acknowledging that $\frac{\Lambda_{p,t}}{\Lambda_{p,t-1}} R^d_t = 1$, we can linearize the stochastic discount factor of financila intermediaries as

$$\Omega \hat{\Omega}_t \;\; = \Omega r^d_{t-1} + \theta((1 + \mu)\nu_n \hat{\nu}_{n,t} + \nu_n \mu \hat{\mu}_t) \tag{B.85}$$

The remaining equations are the linearized law of motion of bankers, and the linearized equation, defining the starting capital that new bankers have, when they enter the sector

$$Nn_t \;\; = \frac{\theta}{\gamma}[(R^k - R)K_b(q_{t-1} + k_{b,t-1}) + R^k K_b r^k_t + (R^b - R)Q^b B_b(q^b_{t-1} + b_{b,t-1}) + Q^b B_b R^b r^b_t$$

$$+ B^p_b(q^p_{t-1} + b^p_{b,t-1}) + Q^p B^p_b R^p r^p_t + (R - R^L)L^q_{t-1} - R^d D r^d_{t-1} + R^d Nn_{t-1}] + Nn_t \tag{B.86}$$

and

$$N_n n_{n,t} = \frac{\omega}{\gamma}(K_b(q_{t-1} + k_{b,t-1}) + Q^b B_b(q^b_{t-1} + b_{b,t-1}) + Q^p B^p_b(q^p_{t-1} + b^p_{b,t-1})) \tag{B.87}$$

Policy and exogenous processes

The fiscal sector can be summarized by the linearized budget constraint, which already entails the tax rule (B.88), and the linearized return on long-term bonds, (B.89).

$$\frac{R^b Q^b B}{\gamma}(r^b_t + q^b_{t-1} + b_{t-1}) * Gg_t - \kappa_b b_{t-1}\frac{B}{\gamma} = Q^b B(q^b_t + b_t), \tag{B.88}$$

$$-R^b(r^b_t + q^b_{t-1}) = \kappa_b q^b_t \tag{B.89}$$

Equation (B.90) is the linearized Taylor rule in terms of the shadow interest rate.

$$r^n_t = max\{0, \rho r^s_{t-1} + (1 - \rho)(\phi_\pi \pi_t + \phi_y y_t + \phi_{dy}(y_t - y_{t-1})) + v_{r_t}\}, \tag{B.90}$$

$$\tilde{L}_t = \rho_{cbl}\tilde{L}_{t-1} + \epsilon_{CBL,t}, \tag{B.91}$$

where $\tilde{L} \equiv \frac{L^q_t}{P_t Y_t}$ denotes central bank liquidity as percentage of GDP.

In terms of the overall capital (bond) stock, central bank asset purchases can be written as

$$x_{k,t} = k_t - \frac{K_b}{K}k_{b,t} - \frac{K_h}{K}k_{h,t}, \tag{B.92}$$

$$x_{b,t} = b_t - \frac{B_b}{B} b_{b,t} - \frac{B_h}{B} b_{h,t}. \tag{B.93}$$

In order to match them with the observables, which are defined relative to GDP, they can be rewritten as

$$\tilde{K}_{cb,t} = x_{k,t} K/(4Y), \tag{B.94}$$

$$\tilde{B}_{cb,t} = x_{b,t} B/(4Y). \tag{B.95}$$

Finally, the stochastic drivers of our model are the following eleven processes

$$v_{u,t} = \rho_u v_{u,t-1} + \epsilon_t^u, \tag{B.96}$$
$$z_t = \rho_z z_{t-1} + \epsilon_t^z, \tag{B.97}$$
$$g_t = \rho_g g_{t-1} + \epsilon_t^g + \rho_{gz} z_t, \tag{B.98}$$
$$v_{r,t} = \rho_r v_{r,t-1} + \epsilon_t^r, \tag{B.99}$$
$$v_{i,t} = \rho_i v_{i,t-1} + \epsilon_t^i, \tag{B.100}$$
$$v_{p,t} = \rho_p v_{p,t-1} + \epsilon_t^p - \mu_p \epsilon_{t-1}^p, \tag{B.101}$$
$$v_{w,t} = \rho_w v_{w,t-1} + \epsilon_t^w - \mu_w \epsilon_{t-1}^w, \tag{B.102}$$
$$v_{lk,t} = \rho_{lk} v_{lk,t-1} + \epsilon_t^{lk}, \tag{B.103}$$
$$\tilde{K}_{cb,t} = \rho_{k,1} \tilde{K}_{cb,t-1} + \rho_{k,2} \tilde{K}_{cb,t-2} + \epsilon_{QEK,t}, \tag{B.104}$$
$$\tilde{B}_{cb,t} = \rho_{b,1} \tilde{B}_{cb,t-1} + \rho_{b,2} \tilde{B}_{cb,t-2} + \epsilon_{QEB,t}, \tag{B.105}$$
$$\tilde{L}_t = \rho_{cbl} \tilde{L}_{t-1} + \epsilon_{CBL,t}. \tag{B.106}$$

where the last three processes are the unconventional policy tools employed by the central bank. It holds that $\epsilon_t^k \stackrel{iid}{\sim} N(0, \sigma_k^2)$ for all $k = \{u, z, g, r, i, p, w, lk, QEK, QEB, CBL\}$.

Additional equations

Additional to the equations above, the model features equations for a flex-price-flex-wage equilibrium, which is used to define output under these conditions and consequently the output gap. Also, the codes contain equations to track various excess premia, the leverage ratio, the sum of all unconventional monetary policy measures (e.g. the central banks balance sheet), the notional nominal interest rate, and the equations for the observables.

Appendix C Additional impulse response functions of the benchmark model

Figures C.14 and C.15 illustrate the impulse response functions with respect to a shock of central bank liquidity provisions and a shock of government bond purchases, respectively. The impulse responses of a conventional monetary policy shock a shown in Figure C.16.

Figure C.14: Impulse response functions with respect to a shock of central bank liquidity provision (in orange). Sampled from the posterior distribution with 95% confidence intervals. The prior is not shown because it is far broader than the posterior. Displaying it would render the figure less informative.
Note: Posterior sample obtained from 1000 draws from the posterior distribution. Solid lines represent the mean. For each draw, the strength of the shock matches the peak of the empirical time series. Annualized measures where applicable.
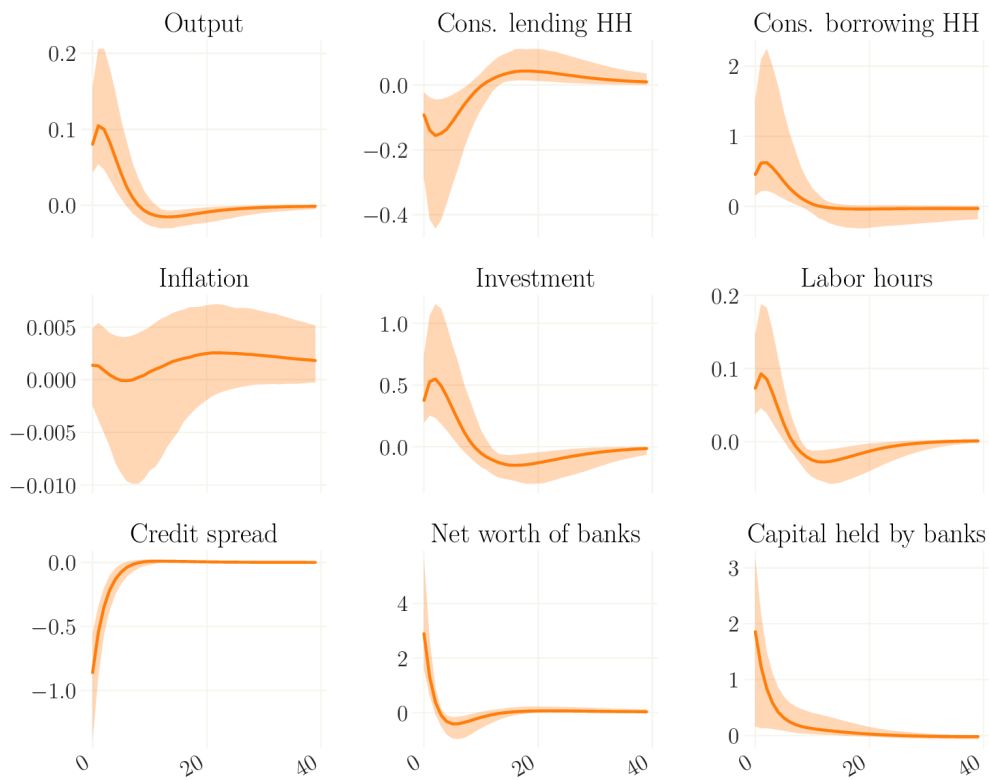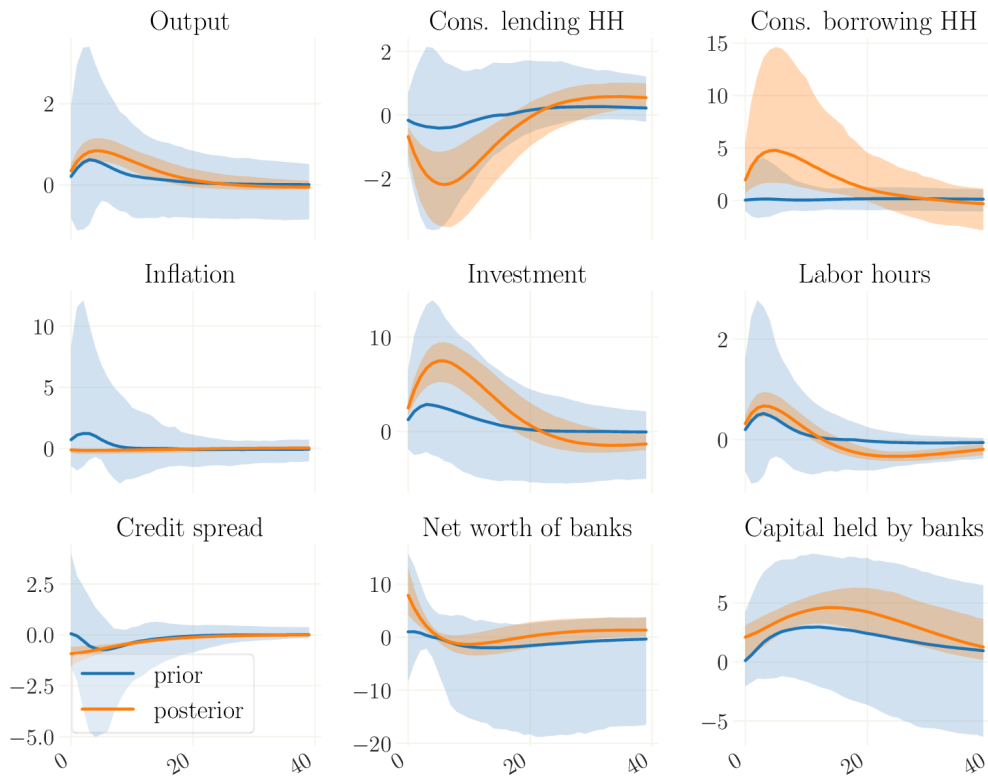
Figure C.15: Impulse response functions with respect to a shock of government bond purchases (in orange). Sampled from the posterior distribution with 95% confidence intervals. In blue: impulse responses sampled from the prior.

Note: Posterior sample obtained from 1000 draws from the posterior distribution. Solid lines represent the mean. For each draw, the strength of the shock is chosen such that the peak of the corresponding stochastic process matches the peak of the empirical time series. See Appendix F for details. The prior sample is obtained from 2000 draws, account for strong heterogeneity of effects. Prior draws adjusted for high autoregressive coefficients with mean/std of 0.9 and 0.05 because AR-coefficients are identified independently from the model.
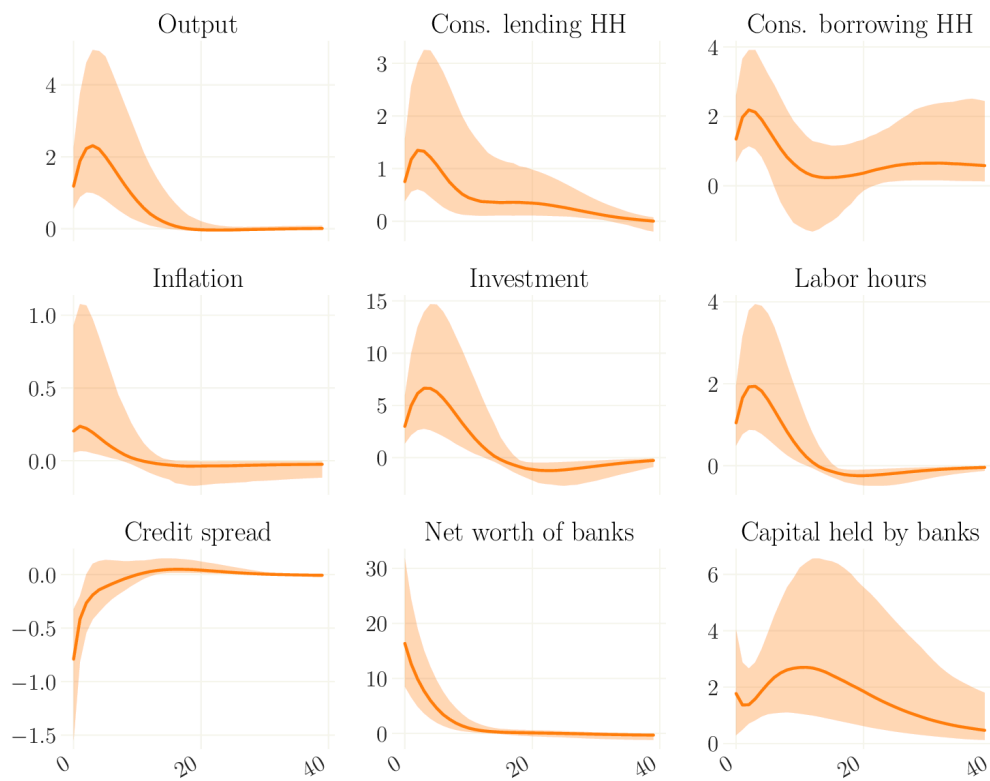
Figure C.16: Impulse response functions to an accommodative 100 basis point monetary policy shock. Sampled from the posterior distribution with 95% confidence intervals.
Note: Posterior sample obtained from 1000 draws from the posterior distribution. Solid lines represent the mean.

Appendix D Model-implied expected ZLB durations

The long duration of the ZLB is largely interpreted by our estimation as an endogenous response of the central bank to the deterioration of fundamentals via the Taylor rule, rather than to an active lower-for-longer policy.

Figure D.17 shows the dynamics and the distribution of the expected duration of the ZLB spell over the sample. The mean expected durations vary between seven and eleven quarters throughout the ZLB years. Although we do not target, nor use any prior information on the actual expectations of market participants on the duration of the ZLB, for the most our results are in close range to expected durations in survey data. The lower panels of Figure D.17 show the distributions of expected ZLB durations at different points in time. In 2009:Q1, most of the probability mass lies on durations of, or higher than, 10 quarters. The same holds for the first quarters of 2012 and 2013, for which survey data shows high expected durations as well. In contrast, for 2011:Q1 when our mean expected duration of seven quarters slightly exceeds the mean implied by the Primary Dealer Survey, the distribution shows that considerable probability mass is allocated to lower expected durations and the survey mean is within the confidence interval of the RANK estimation.

Our results concerning the expected durations of the ZLB are quite similar to those in ?[BS]boehl2020, confirming that the inclusion of the financial sectorá la Gertler and Karadi (2013) does not fundamentally alter the interpretation of business cycle dynamics. BS also provides a comparison to the expected durations obtained by Gust et al. (2017) who obtain an average ZLB spell of merely 3.5 quarters, and those obtained by Kulish et al. (2017), who use survey data to construct priors on expected durations, which they estimate directly. As in BS, but in contrast to the other aforementioned papers, our sample also covers the takeoff from the ZLB. The mean of the smoothed nominal interest rate series leaves the ZLB a year after the actual ZLB period ended. The model therefore interprets the still very low federal fund rate in 2016 to have the same effects on equilibrium dynamics as a binding ZLB. This might capture uncertainty effects that could not explicitly included in our modeling approach.
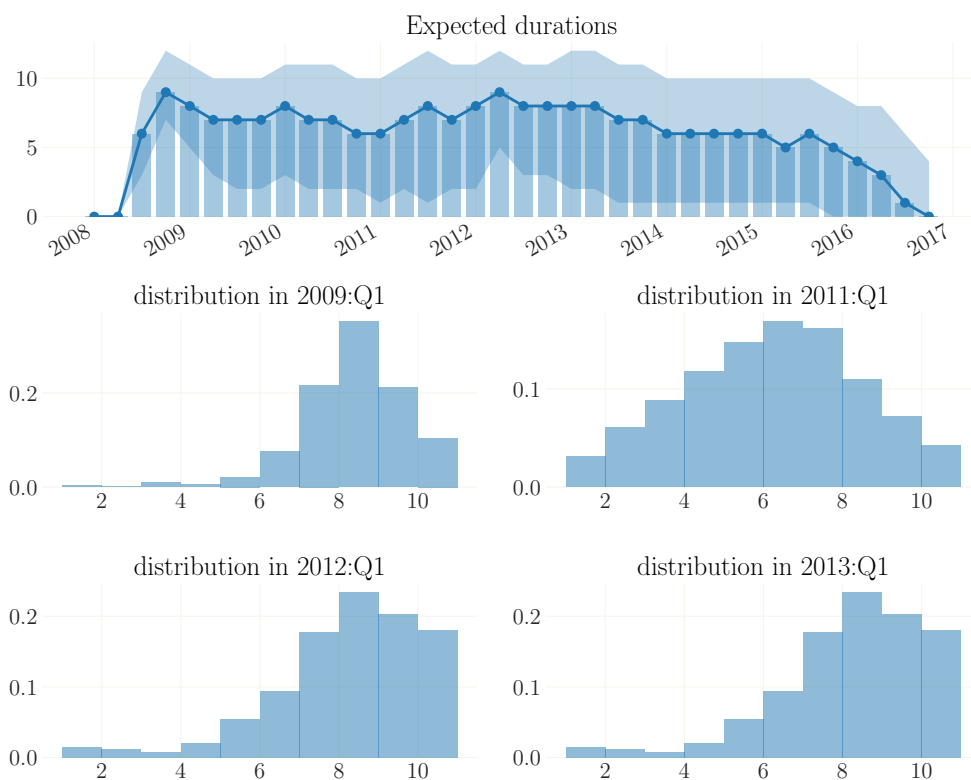
Figure D.17: ZLB durations
Note: Means over 1000 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020).
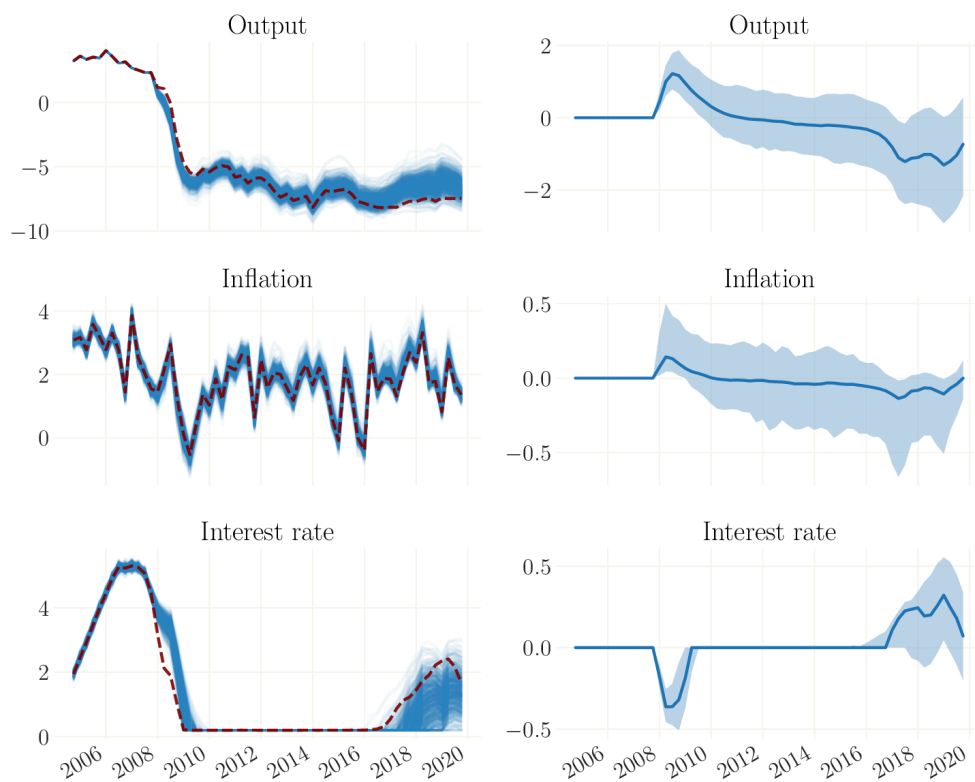
Figure D.18: Decomposition of the net-effect of forward guidance and interest rate measures since 2009. *Note:* Means over 1000 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annualized measures where applicable.

Appendix E   Estimation of plain vanilla Smets and Wouters (2007) model

Table E.3 reproduces the estimates of the Smets and Wouters (2007) model taken from Boehl and Strobel (2020) who use the same sample as for our benchmark model. This is solely done for robustness considerations: we use these estimates as reference points to determine which parameters of our model are identified independently of the financial frictions building block of our model, and which are specific to it.

		Prior			Posterior			
	dist.	mean	std	mean	std	mode	5%	95%
σ_c	normal	1.500	0.375	0.930	0.081	0.882	0.831	1.048
σ_l	normal	2.000	0.750	1.753	0.459	1.315	1.021	2.505
β_{tpr}	gamma	0.250	0.100	0.158	0.055	0.141	0.068	0.244
h	beta	0.700	0.100	0.833	0.027	0.839	0.793	0.878
S''	normal	4.000	1.500	5.287	0.914	4.926	3.731	6.662
ι_p	beta	0.500	0.150	0.192	0.066	0.166	0.086	0.302
ι_w	beta	0.500	0.150	0.371	0.112	0.426	0.186	0.547
α	normal	0.300	0.050	0.168	0.013	0.175	0.146	0.188
ζ_p	beta	0.500	0.100	0.852	0.033	0.840	0.801	0.906
ζ_w	beta	0.500	0.100	0.710	0.044	0.678	0.642	0.786
Φ_p	normal	1.250	0.125	1.254	0.076	1.249	1.128	1.382
ψ	beta	0.500	0.150	0.757	0.080	0.802	0.633	0.893
ϕ_π	normal	1.500	0.250	1.353	0.218	1.512	0.995	1.685
ϕ_y	normal	0.125	0.050	0.207	0.029	0.190	0.157	0.251
ϕ_{dy}	normal	0.125	0.050	0.170	0.040	0.165	0.104	0.235
ρ	beta	0.750	0.100	0.816	0.042	0.833	0.751	0.886
ρ_r	beta	0.500	0.200	0.754	0.088	0.710	0.622	0.909
ρ_g	beta	0.500	0.200	0.918	0.019	0.915	0.886	0.948
ρ_z	beta	0.500	0.200	0.979	0.013	0.982	0.962	0.999
ρ_u	beta	0.500	0.200	0.866	0.022	0.871	0.833	0.902
ρ_p	beta	0.500	0.200	0.639	0.090	0.679	0.496	0.776
ρ_w	beta	0.500	0.200	0.455	0.097	0.369	0.303	0.607
ρ_i	beta	0.500	0.200	0.602	0.127	0.528	0.436	0.854
μ_p	beta	0.500	0.200	0.315	0.121	0.300	0.109	0.502
μ_w	beta	0.500	0.200	0.255	0.090	0.166	0.125	0.391
ρ_{gz}	normal	0.500	0.250	0.607	0.085	0.646	0.463	0.745
σ_g	IG	0.100	2.000	0.222	0.025	0.208	0.180	0.261
σ_u	IG	0.100	2.000	0.681	0.148	0.626	0.456	0.915
σ_z	IG	0.100	2.000	0.399	0.043	0.412	0.329	0.467
σ_r	IG	0.100	2.000	0.106	0.017	0.122	0.080	0.133
σ_p	IG	0.100	2.000	0.184	0.058	0.139	0.101	0.253
σ_w	IG	0.100	2.000	1.272	0.294	1.487	0.818	1.761
σ_i	IG	0.100	2.000	0.881	0.276	1.041	0.365	1.254
$\overline{\gamma}$	normal	0.440	0.050	0.382	0.036	0.386	0.322	0.439
$\overline{l}$	normal	0.000	2.000	0.997	0.634	1.099	0.038	2.103
$\overline{\pi}$	gamma	0.625	0.100	0.632	0.059	0.659	0.539	0.733

Table E.3: Estimation results for the model of Smets and Wouters (2007) estimated from 1998:I – 2019:IV.

Appendix F   The AR(2) processes for exogenous QE

A autoregressive process of second order AR(2) is given by

$$x_t = \rho_1 x_{t-1} + \rho_2 x_{t-2} + \epsilon_t. \tag{F.1}$$

The two autoregressive parameters ρ_1 and ρ_2 can be expressed in terms of the roots of the process

$$\rho_1 = \lambda_1 + \lambda_2, \tag{F.2}$$

$$\rho_2 = -\lambda_1 \lambda_2. \tag{F.3}$$

The implicit two-dimensional system is given by

$$\begin{vmatrix} x_t \\ y_t \end{vmatrix} = \mathbf{A} \begin{vmatrix} x_{t-1} \\ y_{t-1} \end{vmatrix} + \mathbf{b}\epsilon_t, \tag{F.4}$$

where $y_t = x_{t-1}$ is an auxiliary variable and

$$\mathbf{A} = \begin{vmatrix} \lambda_1 + \lambda_2 & -\lambda_1 \lambda_2 \\ 1 & 0 \end{vmatrix}, \tag{F.5}$$

$$\mathbf{b} = \begin{vmatrix} 1 \\ 0 \end{vmatrix}. \tag{F.6}$$

We want to find the impulse response

$$x_{t+s} = \Omega(s)\epsilon_t. \tag{F.7}$$

Assuming $\epsilon_{\bar{t}} = 0$ for all $\bar{t} \neq t$, we can iterate (F.4) forward to express

$$\begin{vmatrix} x_{t+s} \\ y_{t+s} \end{vmatrix} = \mathbf{A}^s \mathbf{b}\epsilon_t. \tag{F.8}$$

Denote the spectral decomposition of $\mathbf{A}$ as $\mathbf{A} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^{-1}$ with $\mathbf{\Lambda} = \text{diag}(\lambda_1, \lambda_2)$ ($\mathbf{A}$ is clearly non-singular for $\lambda_1, \lambda_2 \neq 0$) and acknowledge that

$$\mathbf{A}^k = \mathbf{Q}\mathbf{\Lambda}^k \mathbf{Q}^{-1}. \tag{F.9}$$

A pair of eigenvectors $\{q_1, q_2\}$ of $\mathbf{A}$ is given by

$$q_1 = (\lambda_1, 1), \tag{F.10}$$

$$q_2 = (\lambda_2, 1). \tag{F.11}$$

Let $\mathbf{Q} = \begin{vmatrix} q_1 & q_2 \end{vmatrix}$. Since $\mathbf{\Lambda}$ is diagonal it holds that $\mathbf{\Lambda}^k = \text{diag}(\lambda_1^k, \lambda_2^k)$ and we can rewrite (F.8) as

$$\begin{vmatrix} x_{t+s} \\ y_{t+s} \end{vmatrix} = \mathbf{Q} \begin{vmatrix} \lambda_1^s & 0 \\ 0 & \lambda_2^s \end{vmatrix} \mathbf{Q}^{-1} \mathbf{b}\epsilon_t. \tag{F.12}$$

Inserting **Q** and reduction finally yields for the impulse response in (F.7) that

$$\Omega(s) = \frac{\lambda_1^{1+s} - \lambda_2^{1+s}}{\lambda_1 - \lambda_2}. \tag{F.13}$$

We are looking for the peak response of (F.1). This can be found by setting the first derivative of $\Omega(s)$ to zero. Hence

$$\frac{\partial\Omega(s)}{\partial s} = \frac{\lambda_1^{1+s}\ln\lambda_1 - \lambda_2^{1+s}\ln\lambda_2}{\lambda_1 - \lambda_2} \doteq 0. \tag{F.14}$$

The solution is

$$1 + s = \frac{\ln\left\{\frac{\ln\lambda_1}{\ln\lambda_2}\right\}}{\ln\lambda_1 - \ln\lambda_2}, \tag{F.15}$$

which can directly be inserted into (F.13) to give the peak effect of the AR(2) process. Assume that $\lambda_1, \lambda_2 \in [0, 1]$ to ensure stationarity and to exclude oscillating dynamics. Then both $\ln\lambda_1$ and $\ln\lambda_2$ are defined and smaller than zero. It follows that $\frac{\ln\lambda_1}{\ln\lambda_2} > 0$, ensuring that $\ln\left\{\frac{\ln\lambda_1}{\ln\lambda_2}\right\}$ is defined as well.

Appendix G   Comparing the effects of QE and interest rate policy

There has been an active debate on whether unconventional monetary policy measures can act as a substitute for conventional interest rate policy once the nominal interest rate is constrained by the ZLB.[42] In this section, we briefly compare the effectiveness of the LSAPs with a standard accommodative monetary policy shock using our estimated model.

An accommodative monetary policy shock, which lowers the federal funds rate by 25 basis points (annualized), creates a peak output response of around 0.19%. We ask the question of how large, as a percentage of GDP, asset purchases have to be, in order to achieve a similar effect on output. Figures G.19 and G.20 summarize the result for capital asset and government bond purchases. For each draw from the posterior, the size of the QE shock is chosen such that its peak response on output equals the peak response of a quarterly 25 basis points monetary policy shock.

Before turning to the results, let us stress one important caveat of this comparison. We do not restrict the short term rate to remain unchanged, which – in our model – implies that the central bank raises rates to counter the expansionary effects of QE. The reason for doing so lies in the different model-implied expected durations of the ZLB for each posterior draw which render a comparison difficult.

With this caveat in mind, we find – in line with our previous results – that an expansion of the Fed's balance sheet of 0.63% of GDP through capital asset purchases is needed in order to stimulate output by the same amount as a 25 basis points monetary policy shock. In the case of treasury purchases, this number increases to 1.8% of GDP. To put this numbers into context, in 2019 this translated into roughly 135 and 385.7 Billion Dollar, respectively.

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[42]To name a few, see e.g. Eberly et al. (2019); Doniger et al. (2019); Sims and Wu (2020).

The results for government bond purchases is comparable in size to Doniger et al. (2019), who find that treasuries worth 2% GDP have to be purchased in order to achieve a similar effect on output as a 25bps rate cut using the FRB/US model. Nonetheless, there is a word of caution and the above numbers are to be interpreted carefully. First, across the posterior, there is a large heterogeneity of how large the volume of the purchases must be to get comparable output effects. The corresponding posterior standard deviations of the above numbers are 0.2674% and 0.8284% of GDP for capital asset and treasury purchases, respectively. Second, there are crucial differences in the dynamics of several variables when comparing a standard monetary policy shock with a LSAPs. Specifically, as outlined above, we find LSAPs to exercise downward pressure on prices, consistent with a dominant cost channel, and to induce a non-negligible decline in consumption. Finally, as the standard deviation of the peak-output response of the QE shocks is rather small compared to the standard deviations of the output response to a monetary policy shock (compare Figs. 6, C.15 and C.16), we suspect that this heterogeneity is due to the large uncertainty surrounding the effects of a conventional monetary policy shock in our model. This uncertainty is likely due to the fact that monetary policy shocks do not play a vivid role in our sample, and the characteristics of the respective AR(1) process for monetary policy shocks are hence probably poorly identified.
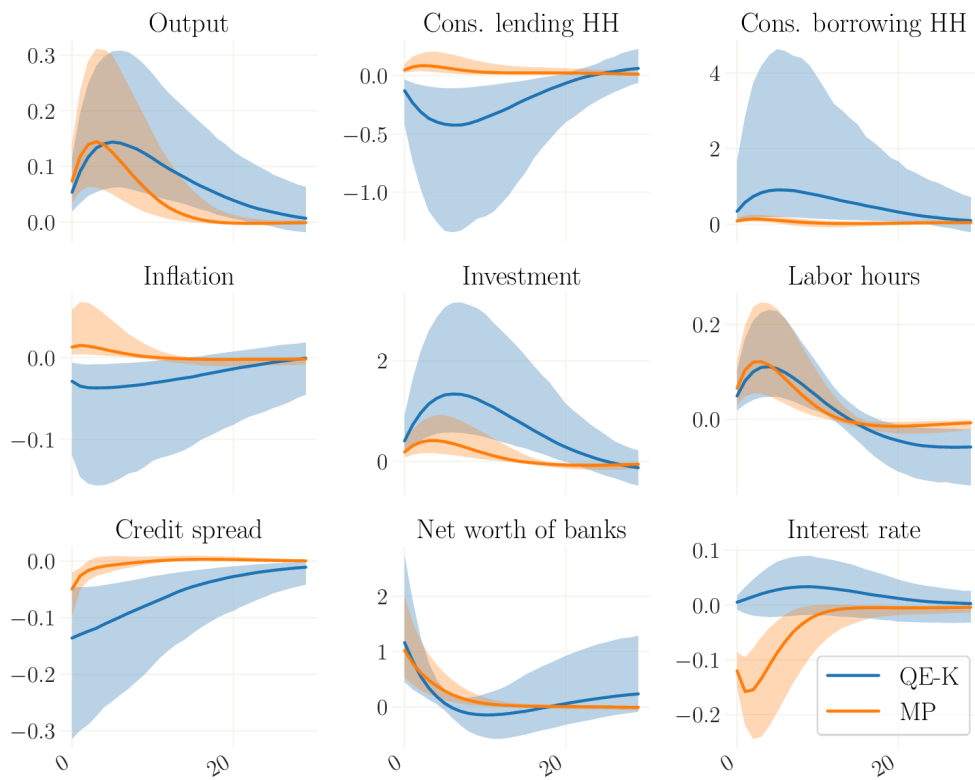
Figure G.19: Impulse response functions to an accommodative annualized 25 basis points monetary policy shock, compared to capital purchases shocks where for each draw from the posterior, the size of each shock is chosen such that the peak effect of QE on output equals the peak effect to the monetary policy shock. Sampled from the posterior distribution with 95% confidence intervals.
Note: Posterior sample obtained from 1000 draws from the posterior distribution. Solid lines represent the mean.

70

Figure G.20: Impulse response functions to an accommodative annualized 25 basis points monetary policy shock, compared to bond purchases shocks where for each draw from the posterior, the size of each shock is chosen such that the peak effect of QE on output equals the peak effect to the monetary policy shock. Sampled from the posterior distribution with 95% confidence intervals.
Note: Posterior sample obtained from 1000 draws from the posterior distribution. Solid lines represent the mean.

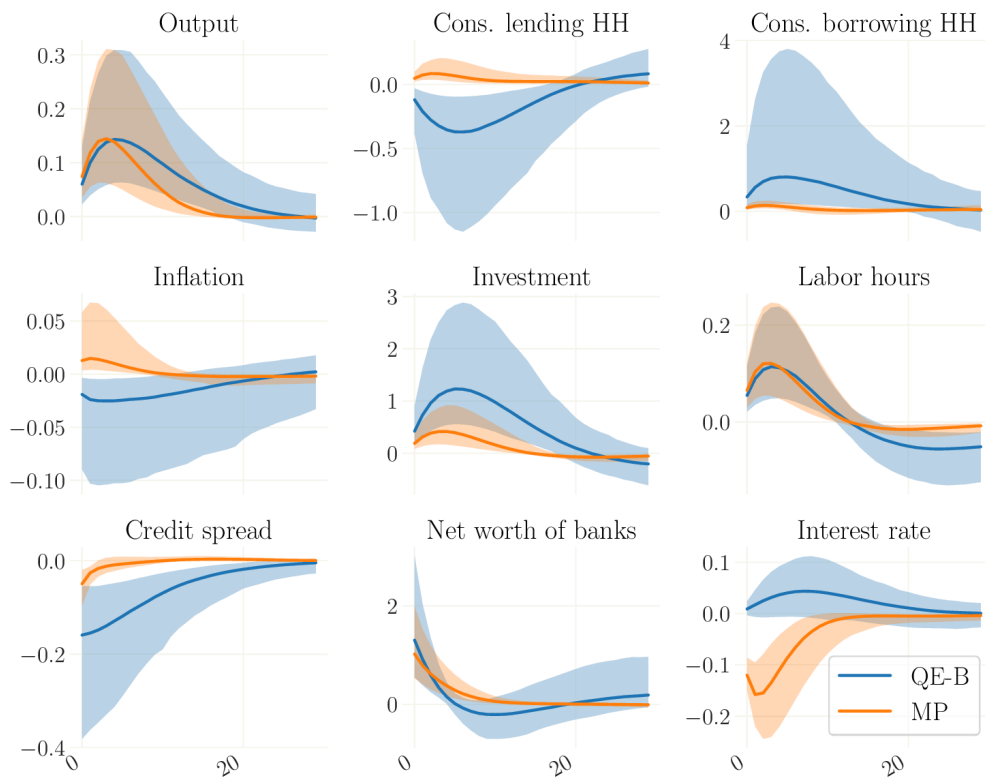## Appendix H Comparing posterior estimates of the RANK and H2M vintages

Tables H.4 and H.5 below compare the parameter estimates of our benchmark model with impatient households with alternative model specifications. The RANK version basically remains closer to Gertler and Karadi (2013) and restricts the household sector to a representative household – the "saver" or patient household in our benchmark model – by setting the fraction of impatient households to be zero, i.e. $\chi = 0$. Instead, the "hand-to-mouth" (H2M) version keeps the heterogeneous agent structure in the household sector, yet restricts the second type to be a "hand-to-mouth" household following, among others, Kaplan et al. (2018). A H2M household consumes all his labor income immediately and does not save.

| | Prior | | | Posterior | | | | | | | | |
| | | | | Benchmark | | | H2M | | | RANK | | |
	dist.	mean	std	mean	std	mode	mean	std	mode	mean	std	mode
σ_c	normal	1.500	0.375	0.908	0.033	0.876	0.861	0.025	0.865	0.848	0.036	0.848
σ_l	normal	2.000	0.750	1.160	0.351	1.046	0.461	0.166	0.396	0.485	0.222	0.590
β_{tpr}	gamma	0.250	0.100	0.201	0.058	0.186	0.155	0.046	0.223	0.158	0.054	0.159
h	beta	0.700	0.100	0.799	0.033	0.804	0.854	0.018	0.866	0.848	0.019	0.876
S''	normal	4.000	1.500	5.119	0.754	4.181	7.524	0.897	7.427	7.055	0.798	8.058
ι_p	beta	0.500	0.150	0.232	0.067	0.155	0.205	0.067	0.180	0.225	0.077	0.227
ι_w	beta	0.500	0.150	0.426	0.118	0.648	0.435	0.120	0.456	0.406	0.117	0.305
α	normal	0.300	0.050	0.210	0.012	0.200	0.185	0.011	0.195	0.183	0.012	0.173
ζ_p	beta	0.500	0.100	0.870	0.025	0.861	0.892	0.020	0.890	0.894	0.020	0.908
ζ_w	beta	0.500	0.100	0.747	0.046	0.728	0.696	0.048	0.671	0.708	0.043	0.740
Φ_p	normal	1.250	0.125	1.331	0.071	1.364	1.305	0.069	1.293	1.305	0.060	1.259
ψ	beta	0.500	0.150	0.810	0.067	0.841	0.773	0.075	0.850	0.789	0.071	0.783
ϕ_π	normal	1.500	0.250	1.323	0.207	1.542	1.158	0.142	0.987	1.160	0.152	1.017
ϕ_y	normal	0.125	0.050	0.171	0.022	0.168	0.202	0.025	0.219	0.193	0.025	0.188
ϕ_{dy}	normal	0.125	0.050	0.181	0.041	0.239	0.166	0.039	0.162	0.178	0.039	0.199
ρ	beta	0.750	0.100	0.829	0.031	0.850	0.782	0.034	0.757	0.787	0.032	0.794
κ_τ	gamma	0.300	0.100	0.287	0.081	0.188	0.291	0.079	0.242	0.291	0.079	0.331
PAC	gamma	2.000	4.000	0.495	1.189	0.132	0.169	0.493	0.062	0.139	0.453	0.059
LEV	normal	3.000	1.000	4.312	0.405	4.433	3.177	0.469	3.487	3.336	0.468	3.433
θ	beta	0.950	0.050	0.815	0.034	0.747	0.820	0.038	0.795	0.806	0.044	0.772
λ_{cbl}	gamma	3.000	3.000	0.229	0.207	0.086	0.113	0.107	0.029	0.100	0.088	0.057
χ	beta	0.300	0.100	0.190	0.059	0.133	0.316	0.085	0.317			
$\overline{termspread}$	gamma	0.500	0.100	0.594	0.106	0.427	0.527	0.099	0.466	0.496	0.092	0.493
$\overline{ppremium}$	gamma	0.100	0.030	0.083	0.020	0.110						
$\overline{spread}$	normal	0.500	0.100	0.428	0.051	0.563	0.482	0.064	0.489	0.502	0.061	0.509
$\overline{\gamma}$	normal	0.440	0.050	0.399	0.029	0.414	0.394	0.022	0.406	0.392	0.019	0.382
$\overline{\pi}$	gamma	0.625	0.100	0.624	0.058	0.688	0.624	0.052	0.636	0.618	0.052	0.564
$\bar{l}$	normal	0.000	2.000	1.246	0.488	0.701	0.826	0.523	0.948	0.710	0.554	0.596
MDD				-580.170 / -578.068			-571.082 / -560.355			-562.600 / -560.337		

Table H.4: Comparison of estimation results for different model vintages. Marginal data density (MDD) given as Modified Harmonic Mean and Laplace Approximations.

| | | Prior | | | Posterior | | | | | | | |
| | | | | Benchmark | | | H2M | | | RANK | | |
	dist.	mean	std	mean	std	mode	mean	std	mode	mean	std	mode
ρ_r	beta	0.500	0.200	0.566	0.086	0.525	0.556	0.071	0.587	0.543	0.073	0.576
ρ_g	beta	0.500	0.200	0.887	0.081	0.944	0.935	0.029	0.924	0.941	0.021	0.955
ρ_i	beta	0.500	0.200	0.734	0.048	0.651	0.551	0.087	0.524	0.601	0.075	0.603
ρ_z	beta	0.500	0.200	0.951	0.025	0.954	0.955	0.024	0.968	0.942	0.024	0.964
ρ_p	beta	0.500	0.200	0.617	0.098	0.624	0.598	0.070	0.537	0.584	0.086	0.587
ρ_w	beta	0.500	0.200	0.735	0.070	0.707	0.578	0.108	0.552	0.682	0.097	0.700
ρ_u	beta	0.500	0.200	0.896	0.013	0.906	0.895	0.015	0.885	0.894	0.015	0.883
ρ_{lk}	beta	0.500	0.200	0.934	0.027	0.936	0.926	0.018	0.944	0.916	0.023	0.923
ρ_{cbl}	beta	0.500	0.200	0.762	0.036	0.762	0.742	0.038	0.735	0.745	0.039	0.756
$\text{root}_{b,1}$	beta	0.500	0.200	0.903	0.049	0.786	0.899	0.044	0.832	0.900	0.049	0.921
$\text{root}_{b,2}$	beta	0.500	0.200	0.880	0.054	0.973	0.890	0.048	0.946	0.887	0.050	0.863
$\text{root}_{k,1}$	beta	0.500	0.200	0.901	0.041	0.917	0.914	0.035	0.884	0.913	0.030	0.923
$\text{root}_{k,2}$	beta	0.500	0.200	0.926	0.037	0.902	0.913	0.033	0.933	0.919	0.028	0.930
μ_p	beta	0.500	0.200	0.450	0.139	0.293	0.239	0.102	0.128	0.258	0.134	0.285
μ_w	beta	0.500	0.200	0.517	0.102	0.485	0.354	0.113	0.314	0.460	0.124	0.502
ρ_{gz}	normal	0.500	0.250	0.570	0.188	0.463	0.492	0.117	0.546	0.486	0.110	0.483
σ_g	inv. gamma	0.100	0.250	0.260	0.029	0.235	0.263	0.024	0.251	0.261	0.024	0.239
σ_z	inv. gamma	0.100	0.250	0.345	0.035	0.313	0.349	0.037	0.300	0.353	0.038	0.317
σ_r	inv. gamma	0.100	0.250	0.147	0.031	0.164	0.146	0.025	0.137	0.151	0.027	0.132
σ_i	inv. gamma	0.100	0.250	0.610	0.086	0.766	0.910	0.215	0.879	0.795	0.165	0.714
σ_p	inv. gamma	0.100	0.250	0.235	0.070	0.190	0.179	0.034	0.183	0.200	0.046	0.197
σ_w	inv. gamma	0.100	0.250	0.691	0.074	0.687	1.056	0.218	1.196	0.845	0.141	0.789
σ_u	inv. gamma	0.100	0.250	0.482	0.086	0.454	0.592	0.097	0.694	0.571	0.093	0.714
σ_{lk}	inv. gamma	0.100	0.250	0.272	0.040	0.244	0.496	0.088	0.487	0.543	0.104	0.542
σ_{cbl}	inv. gamma	0.100	0.250	0.956	0.084	0.966	0.965	0.089	0.972	0.966	0.090	0.962
σ_{qeb}	inv. gamma	0.100	0.250	0.197	0.014	0.186	0.192	0.014	0.193	0.193	0.014	0.189
σ_{qek}	inv. gamma	0.100	0.250	0.176	0.015	0.183	0.174	0.013	0.175	0.173	0.013	0.163

Table H.5: Comparison of estimation results for different model vintages: exogenous processes.

Appendix I Posterior estimates based on the sample until 2008

This section presents posterior estimates for the sample from 1983:I to 2008:III. The starting date is chosen as in Kulish et al. (2017), while the end date reflects last quarter before the ZLB was binding. For this exercise, we remove the three shock processes of the Fed's balance sheet and their respective observables from the estimation. We also exclude all parameters from the estimation that cannot be identified without this data. These are σ_{cbl}, σ_{qeb}, σ_{qek}, ρ_{cbl}, $\text{root}_{b,1}$, $\text{root}_{b,2}$, $\text{root}_{k,1}$, $\text{root}_{k,2}$ and λ_{cbl}. Ignoring λ_{cbl} during the estimation implies that the effects of the central bank liquidity purchases must remain unidentified in this experiment. The resulting posterior estimates are given in Table I.6. While we do not want to discuss the differences in detail, we note that several parameters changed notably, which biases the counterfactual analysis in a non-trivial way.

In order to illustrate this, we use these estimates to conduct a historic decomposition of the data from 1998 to 2020 (i.e. our baseline sample from the main body). For this purpose we set the above parameters, that were excluded in the estimation, to the mean values of the posterior distribution for the benchmark estimation. We presume that this is safe for the parameters related to the exogenous processes for QE, but setting λ_{cbl} to the posterior mean value has implication on the effects of the measures of liquidity provision. The decompositions can be found in Figures I.21 and I.24.

Similarly, we use these estimates to perform counterfactual simulations of the effects of QE. In these simulations, we omit the measures of liquidity because these depend heavily on the parameter λ_{cbl}, which is ineffective in the pre-crisis sample. Regarding QE, however, we find that using the shorter sample and omitting the QE data in the estimation has very strong effects on the counterfactual dynamics. Specifically, some of the remaining smoothened shocks lead to unstable dynamics or non-convergence of our solution method. For our counterfactual analysis, we rejected all simulated trajectories that contain numerical errors, which are 16.4% of the simulated series. The remaining simulations can be found in Figures I.23 and I.24.
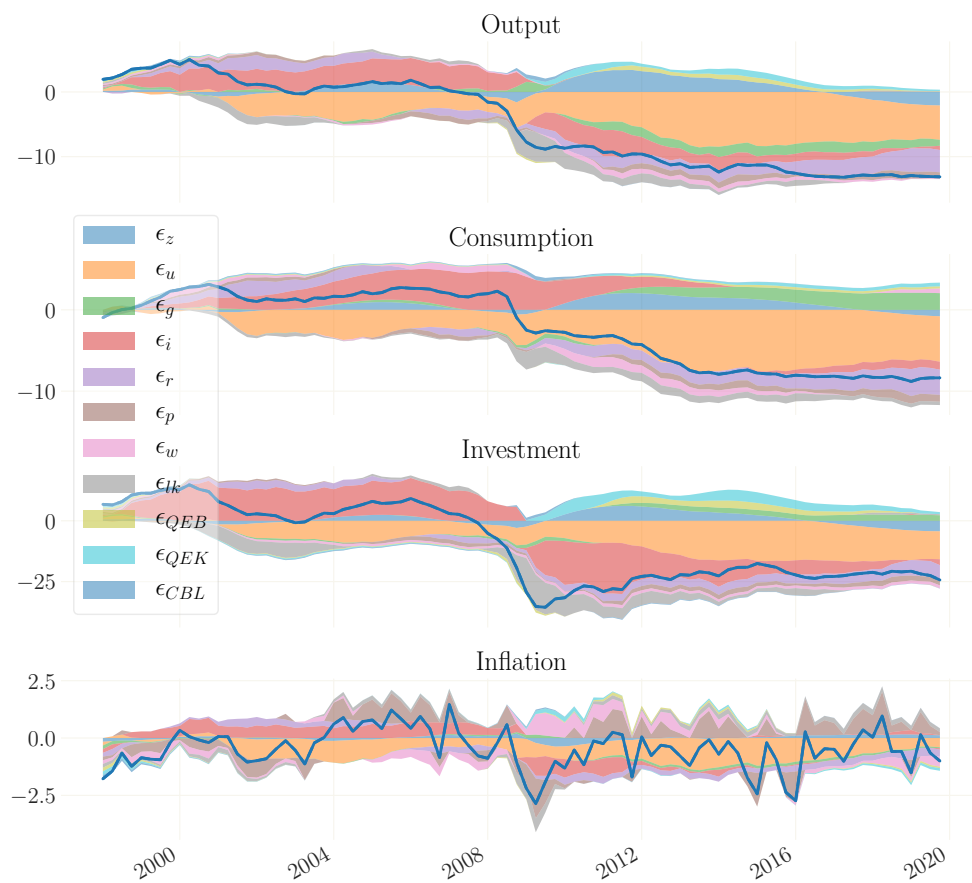
Figure I.21: Decomposition based on the pre-crisis parameter estimates of the baseline model. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 250 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.

	distribution	pst_mean	sd/df	mean	sd	mode	hpd_5	hpd_95	mc_error
σ_c	normal	1.500	0.375	1.334	0.094	1.449	1.185	1.495	0.004
σ_l	normal	2.000	0.750	2.554	0.534	2.713	1.707	3.478	0.020
β_{tpr}	gamma	0.250	0.100	0.243	0.072	0.155	0.122	0.360	0.003
h	beta	0.700	0.100	0.572	0.045	0.542	0.498	0.645	0.002
S''	normal	4.000	1.500	5.446	0.936	5.048	3.938	6.898	0.036
ι_p	beta	0.500	0.150	0.370	0.144	0.343	0.127	0.602	0.006
ι_w	beta	0.500	0.150	0.465	0.135	0.622	0.217	0.664	0.005
α	normal	0.300	0.050	0.222	0.010	0.215	0.205	0.237	0.000
ζ_p	beta	0.500	0.100	0.831	0.072	0.849	0.679	0.911	0.003
ζ_w	beta	0.500	0.100	0.755	0.103	0.768	0.568	0.898	0.004
Φ_p	normal	1.250	0.125	1.485	0.067	1.483	1.377	1.594	0.003
ψ	beta	0.500	0.150	0.468	0.099	0.467	0.326	0.654	0.004
ϕ_π	normal	1.500	0.250	1.695	0.373	1.715	1.190	2.408	0.015
ϕ_y	normal	0.125	0.050	0.194	0.046	0.236	0.115	0.269	0.002
ϕ_{dy}	normal	0.125	0.050	0.212	0.039	0.224	0.150	0.277	0.001
ρ	beta	0.750	0.100	0.744	0.034	0.751	0.690	0.798	0.001
ρ_r	beta	0.500	0.200	0.621	0.103	0.716	0.440	0.778	0.004
ρ_g	beta	0.500	0.200	0.949	0.025	0.958	0.913	0.988	0.001
ρ_i	beta	0.500	0.200	0.885	0.042	0.893	0.825	0.963	0.002
ρ_z	beta	0.500	0.200	0.937	0.019	0.950	0.906	0.970	0.001
ρ_p	beta	0.500	0.200	0.667	0.123	0.697	0.455	0.866	0.005
ρ_w	beta	0.500	0.200	0.831	0.091	0.842	0.696	0.969	0.004
ρ_u	beta	0.500	0.200	0.852	0.044	0.839	0.786	0.930	0.002
ρ_{lk}	beta	0.500	0.200	0.896	0.036	0.880	0.832	0.949	0.001
μ_p	beta	0.500	0.200	0.457	0.153	0.374	0.192	0.685	0.006
μ_w	beta	0.500	0.200	0.537	0.146	0.474	0.306	0.779	0.006
ρ_{gz}	normal	0.500	0.250	0.417	0.074	0.439	0.298	0.542	0.003
σ_g	inv. gamma	0.100	0.250	0.233	0.017	0.252	0.209	0.263	0.001
σ_z	inv. gamma	0.100	0.250	0.339	0.032	0.327	0.284	0.389	0.001
σ_r	inv. gamma	0.100	0.250	0.174	0.041	0.134	0.114	0.239	0.002
σ_i	inv. gamma	0.100	0.250	0.555	0.061	0.552	0.457	0.655	0.002
σ_p	inv. gamma	0.100	0.250	0.157	0.056	0.111	0.077	0.239	0.002
σ_w	inv. gamma	0.100	0.250	0.385	0.080	0.315	0.271	0.514	0.003
σ_u	inv. gamma	0.100	0.250	0.322	0.075	0.313	0.204	0.432	0.003
σ_{lk}	inv. gamma	0.100	0.250	0.298	0.057	0.348	0.190	0.388	0.002
κ_τ	gamma	0.300	0.100	0.319	0.087	0.283	0.172	0.451	0.003
PAC	gamma	2.000	4.000	0.757	2.081	0.112	0.079	0.628	0.092
LEV	normal	3.000	1.000	3.572	0.417	3.846	2.948	4.293	0.016
θ	beta	0.950	0.050	0.917	0.029	0.951	0.876	0.964	0.001
χ	beta	0.300	0.100	0.347	0.090	0.367	0.200	0.492	0.004
$\overline{termspread}$	gamma	0.500	0.100	0.649	0.104	0.568	0.473	0.807	0.004
$\overline{ppremium}$	gamma	0.100	0.030	0.116	0.028	0.109	0.074	0.163	0.001
$\overline{spread}$	normal	0.500	0.100	0.273	0.037	0.320	0.216	0.334	0.001
$\overline{\gamma}$	normal	0.440	0.050	0.460	0.030	0.502	0.414	0.511	0.001
$\overline{\pi}$	gamma	0.625	0.100	0.594	0.067	0.572	0.479	0.699	0.002
$\bar{l}$	normal	0.000	2.000	2.179	0.579	2.383	1.278	3.061	0.022

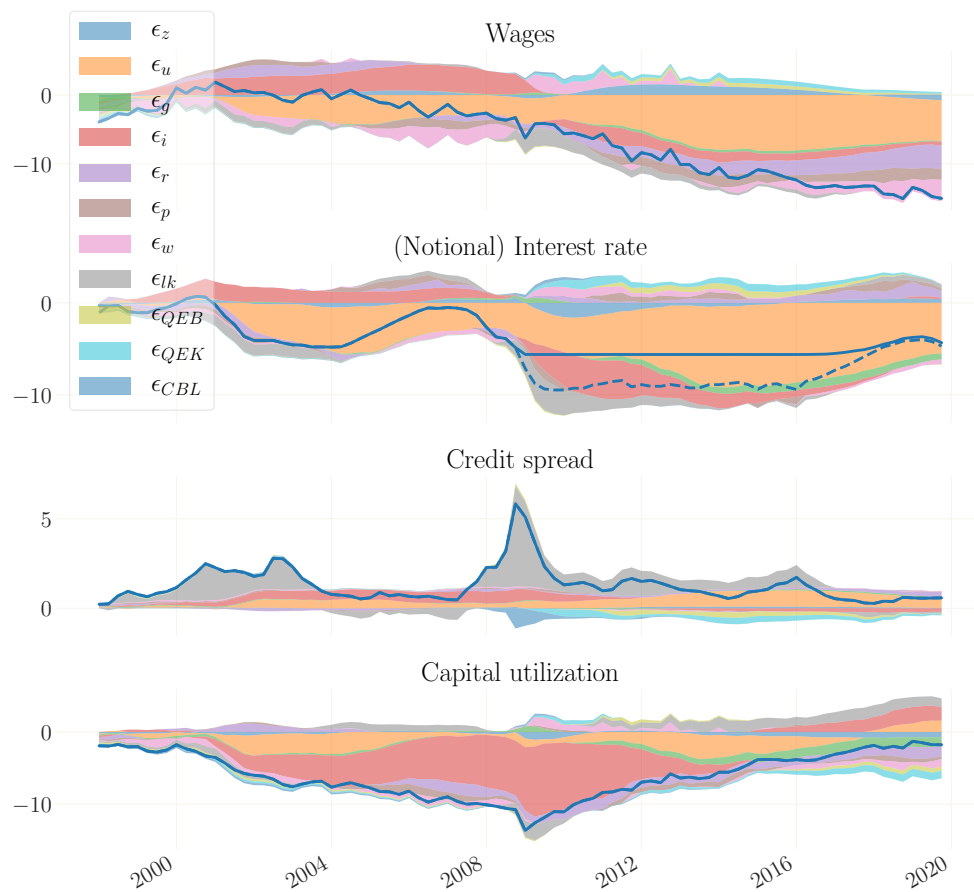Table I.6: Parameter estimates for the sample from 1983Q1 to 2008Q2.

Figure I.22: Decomposition based on the pre-crisis parameter estimates of the baseline model. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 250 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.
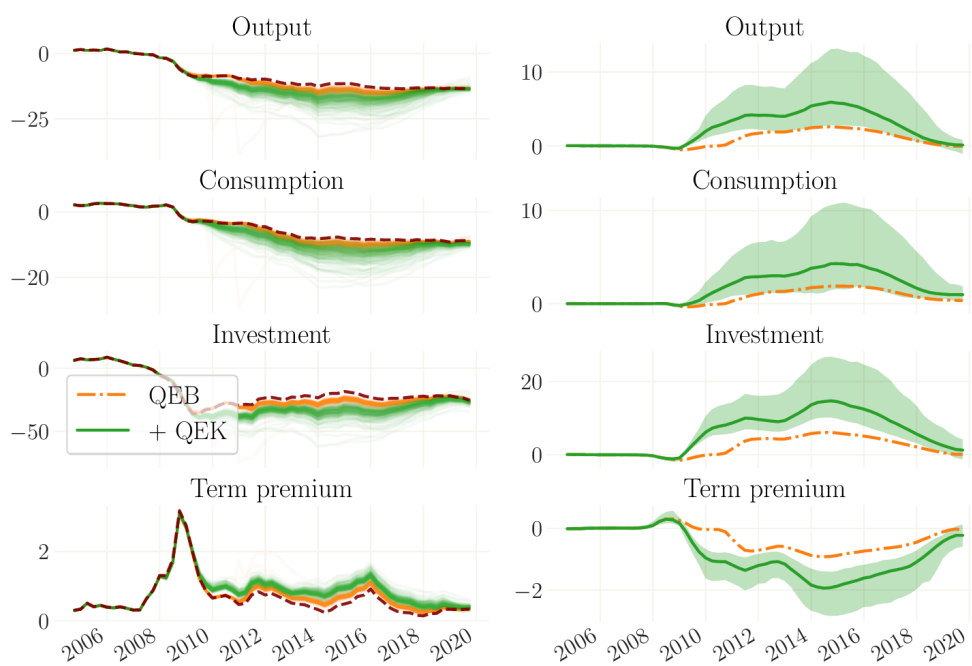
Figure I.23: Counterfactuals using parameter estimates based on the pre-2008 sample. Left: counterfactual simulations without the QE measures. Right: net contribution of each QE measure. Effects in both graphs are cumulative.
Note: Means over 209 out of 250 simulations drawn from the posterior. Annualized measures where applicable.
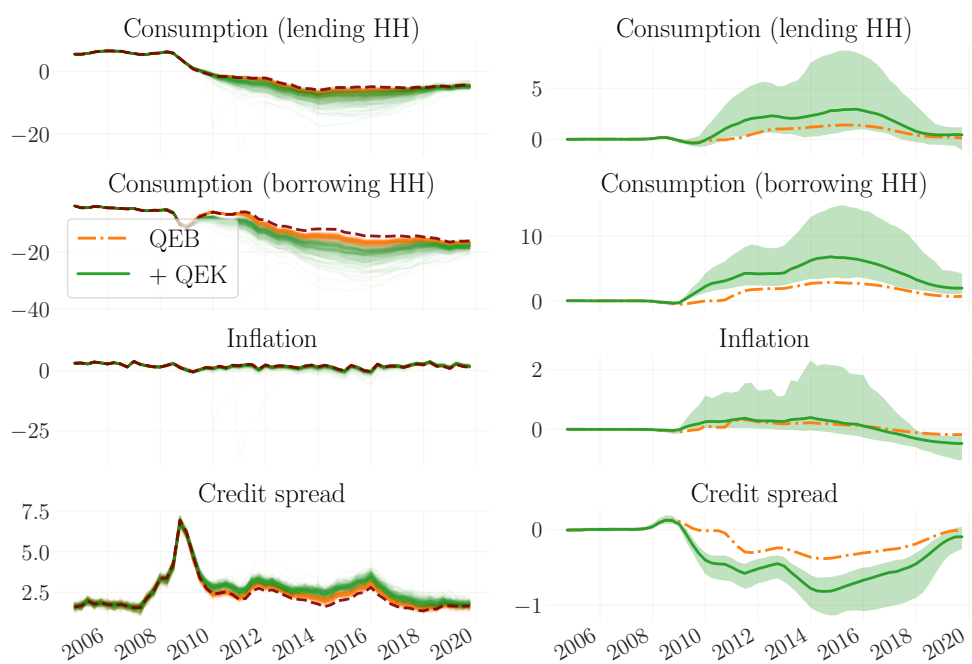
Figure I.24: Counterfactuals using parameter estimates based on the pre-2008 sample. Left: counterfactual simulations without the QE measures. Right: net contribution of each QE measure. Effects in both graphs are cumulative.
*Note:* Means over 209 out of 250 simulations drawn from the posterior. Annualized measures where applicable.
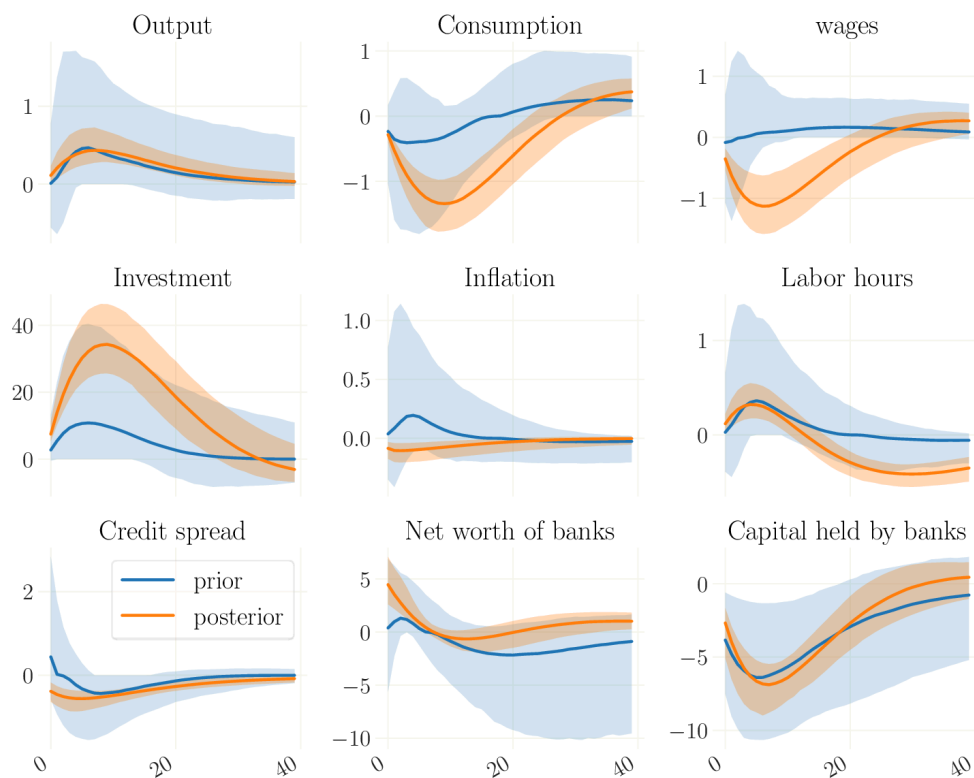
Figure J.25: H2M specification of the benchmark model. Impulse response functions with respect to a shock of MBS purchases (orange). Sampled from the posterior distribution with 95% confidence intervals. In blue: impulse responses sampled from the prior.

Note: Posterior sample obtained from 1000 draws from the posterior distribution. Solid lines represent the mean. For each draw, the strength of the shock is chosen such that the peak of the corresponding stochastic process matches the peak of the empirical time series. See Appendix F for details. The prior sample is obtained from 2000 draws, account for strong heterogeneity of effects. Prior draws adjusted for high autoregressive coefficients with mean/std of 0.9 and 0.05 because AR-coefficients are identified independently from the model. Annual measures where applicable.

Appendix J   Prior and posterior IRFs for RANK and H2M

As with our benchmark model, both the pronounced fall in consumption as well as the disinflationary effect of QE stand in stark contrast to the results implied a priori. To illustrate this, Figures J.26 and J.25 compares the a priori impulse response functions to a capital asset purchase shock with those a posteriori for the RANK and H2M model vintages, respectively.
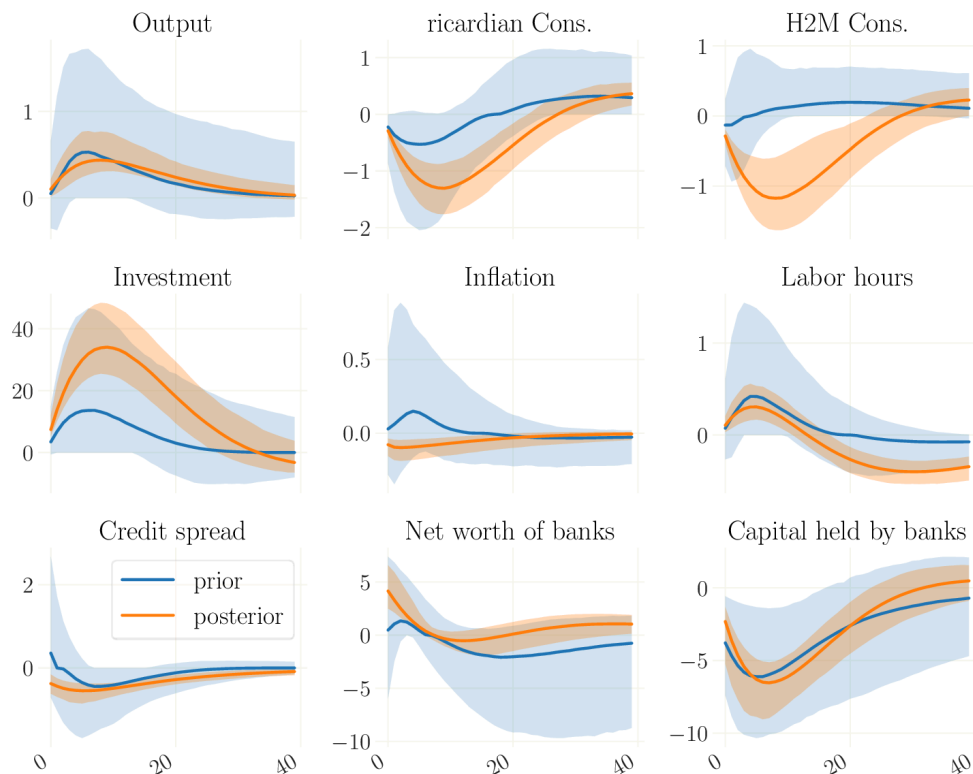
Figure J.26: RANK specification of the benchmark model. Impulse response functions with respect to a shock of MBS purchases (orange). Sampled from the posterior distribution with 95% confidence intervals. In blue: impulse responses sampled from the prior.

Note: Posterior sample obtained from 1000 draws from the posterior distribution. Solid lines represent the mean. For each draw, the strength of the shock is chosen such that the peak of the corresponding stochastic process matches the peak of the empirical time series. See Appendix F for details. The prior sample is obtained from 2000 draws, account for strong heterogeneity of effects. Prior draws adjusted for high autoregressive coefficients with mean/std of 0.9 and 0.05 because AR-coefficients are identified independently from the model. Annual measures where applicable.

Appendix K QE counterfactuals of the model vintage without capital utilization

As mentioned in section 5, the effects of QE on both, inflation and consumption, do not hinge on the presence of capital utilization cost. To see this, note that in the absence of time-variation in the utilization rate, we have $U_t(i) = 1, \forall t$. As a result, the log-linearized marginal costs equation B.68 simply becomes

$$mc_t = w_t - z_t + \alpha(l_t - k_{t-1}).$$

Then, it is easy to see that the larger capital stock induced by QE nonetheless exerts downward pressure on firms' marginal costs. In fact, in the absence of varying utilization,
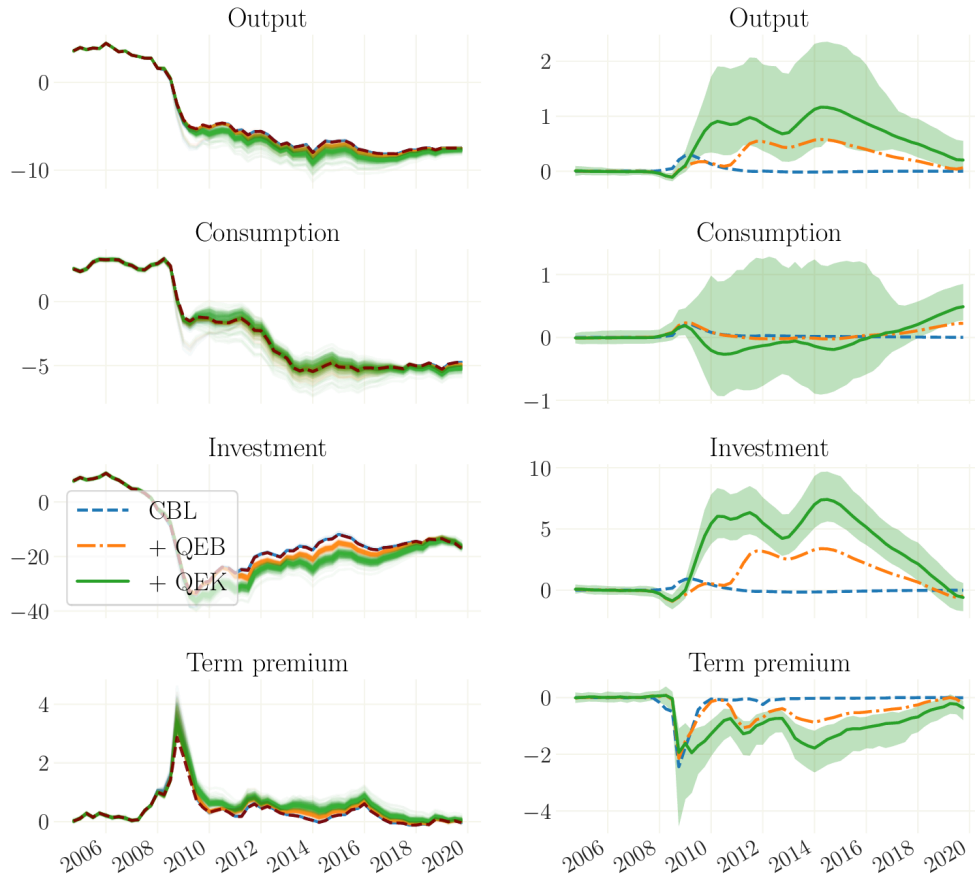
Figure K.27: Benchmark model without capital utilization. Left: counterfactual simulations without the QE measures. Right: net contribution of each QE measure. Effects in both graphs are cumulative. *Note:* Means over 1000 simulations drawn from the posterior. Annual measures where applicable.

firms have no choice to deploy the full capital stock which seems to increase the disinflationary effects of QE even more. Figures K.27 and K.28 show this in the counterfactual simulations.
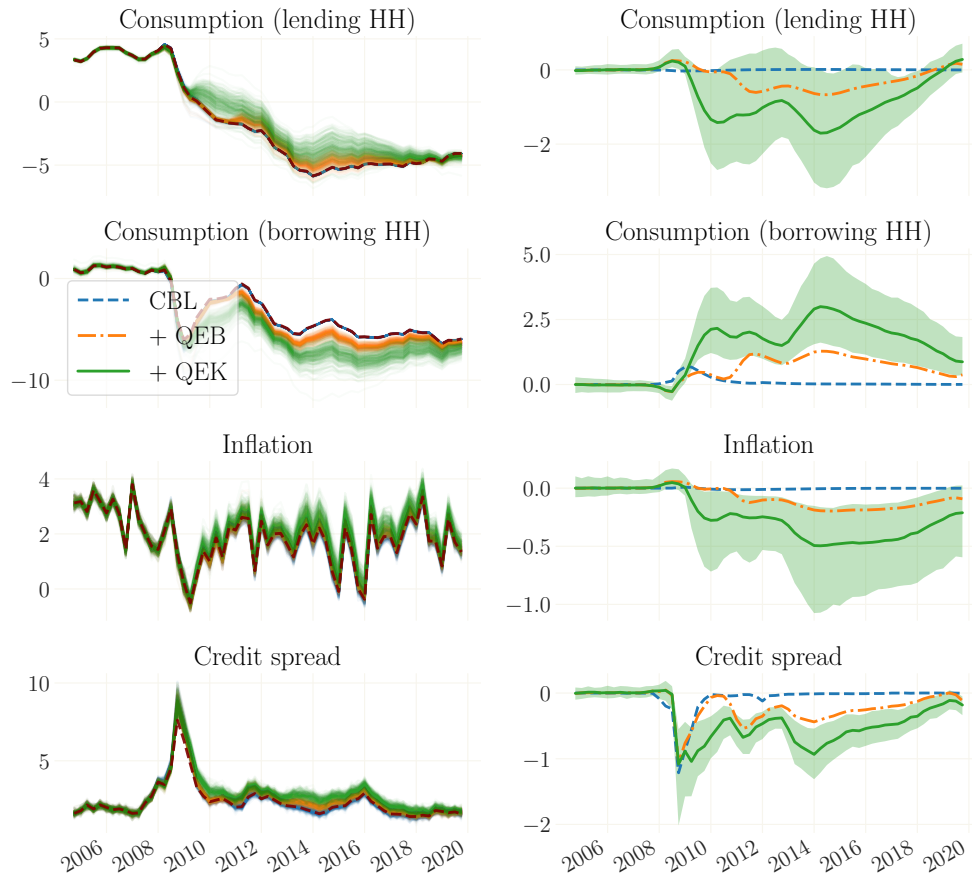
Figure K.28: Benchmark model without capital utilization. Left: counterfactual simulations without the QE measures. Right: net contribution of each QE measure. Effects in both graphs are cumulative. Measures of spreads and inflation are annualized.
Note: Means over 1000 simulations drawn from the posterior. Annual measures where applicable.

Appendix L   Historic shock decompositions for alternative specifications

Finally, there is large degree of consistency in how the different model vintages rationalize the historic dynamics in our sample. For completeness, Figures L.29 until L.32 show the historical decompositions of selected variables for the RANK and H2M model vintages. Moreover, Figures L.33 and L.34 show the historical decompositions for the model without time-varying capital utilization.
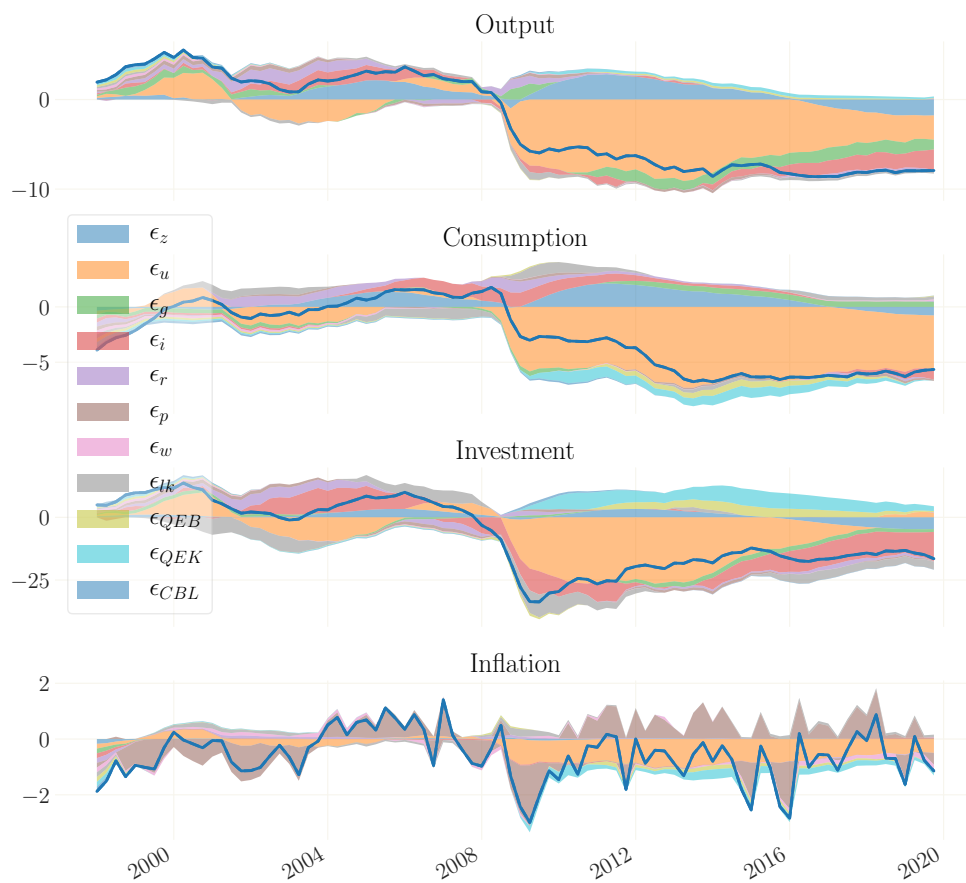
Figure L.29: Estimated RANK version of model. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 1000 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.
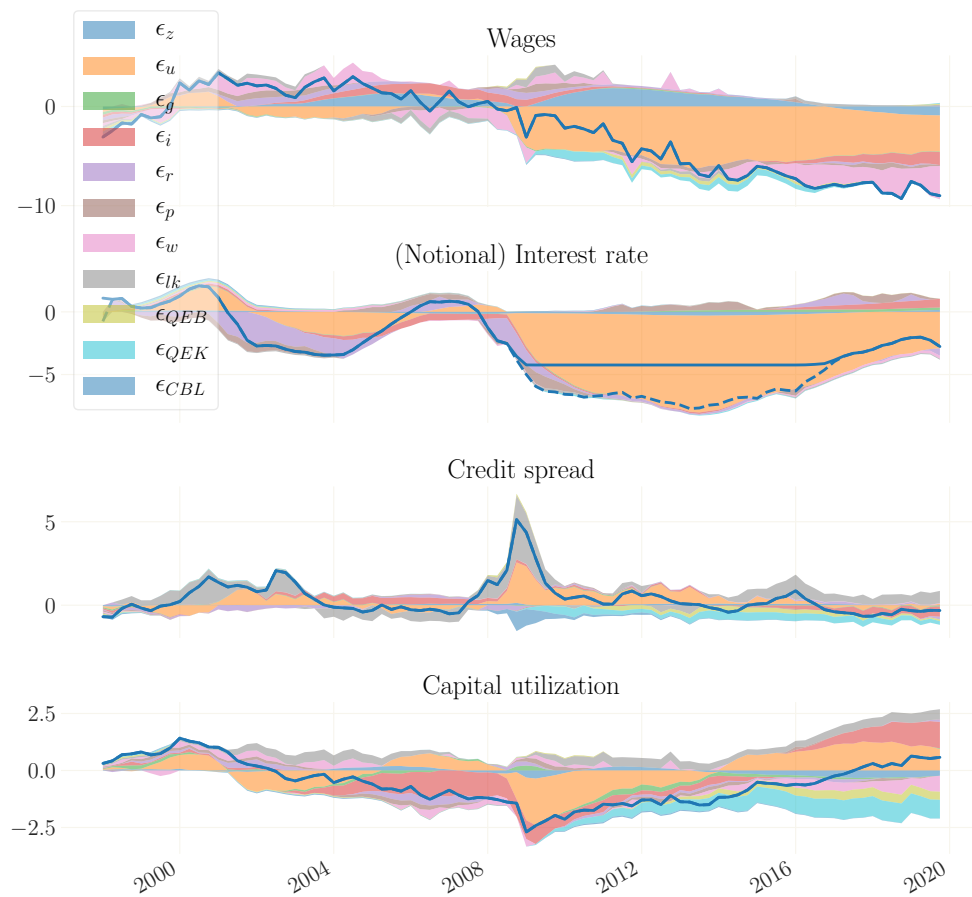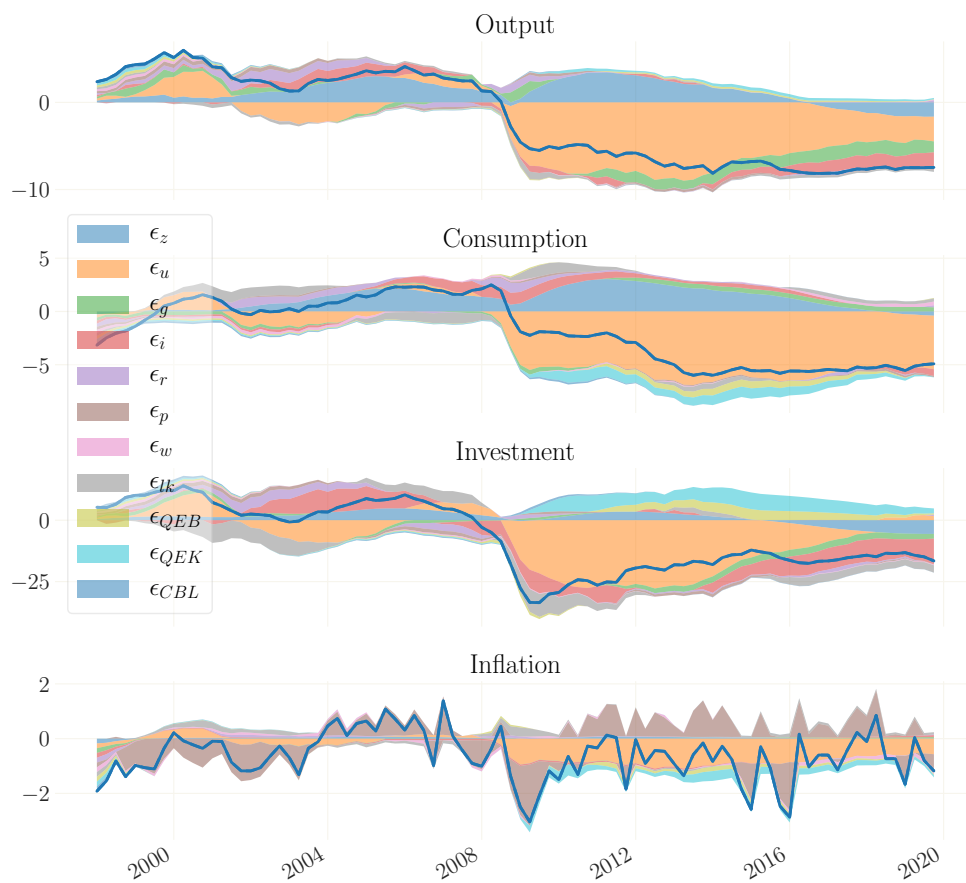
Figure L.30: Estimated RANK version of model. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 1000 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.

Figure L.31: Estimated model with H2M. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 1000 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.
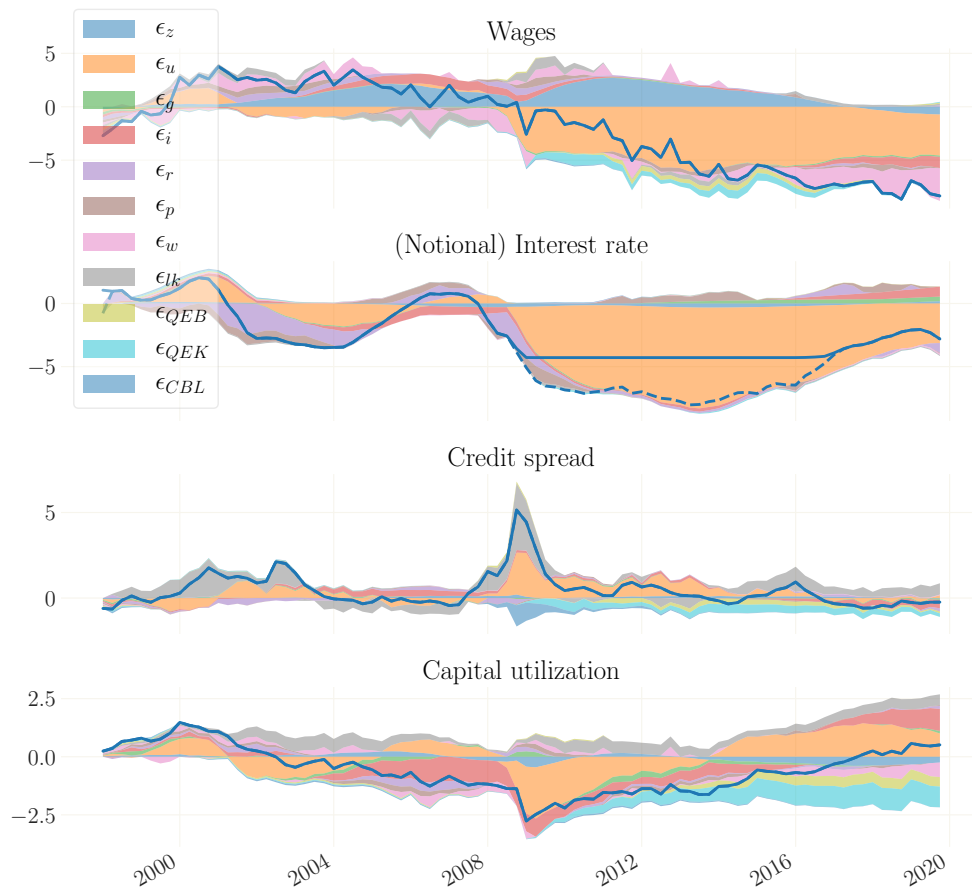
Figure L.32: Estimated model with H2M. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 1000 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.
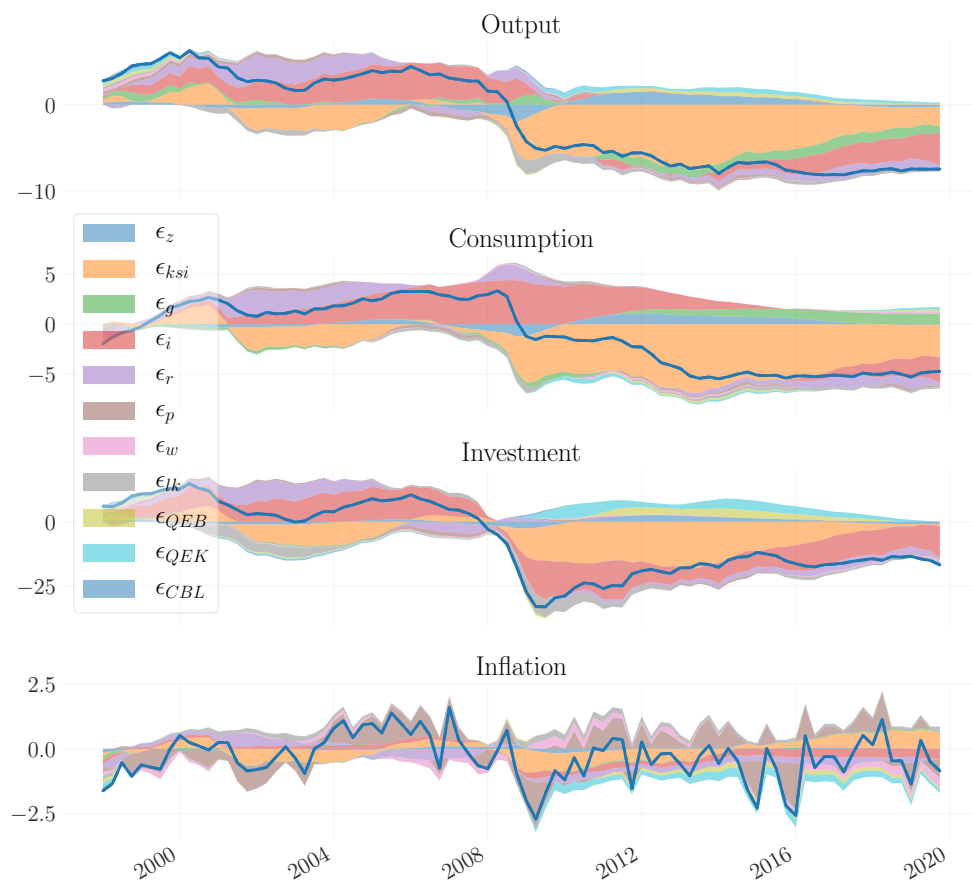
Figure L.33: Estimated benchmark model without capital utilization. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 250 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.
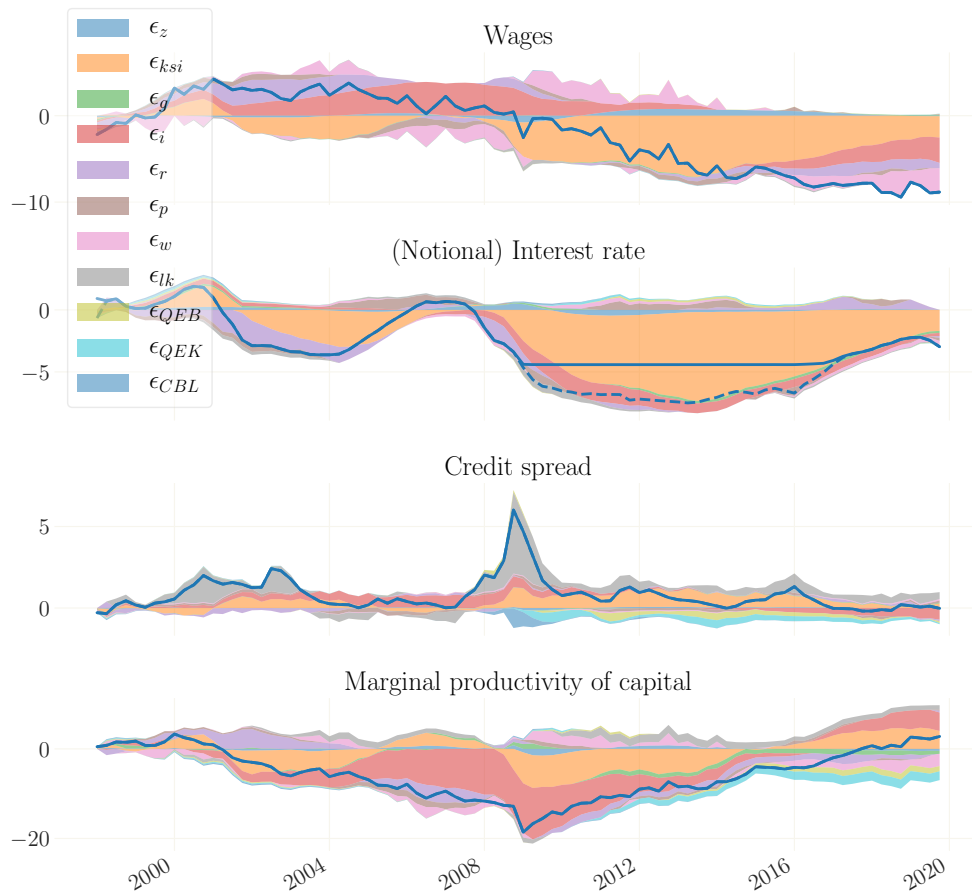
Figure L.34: Estimated benchmark model without capital utilization. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 250 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.

Figure M.35: Estimated CFP model. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 1000 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.
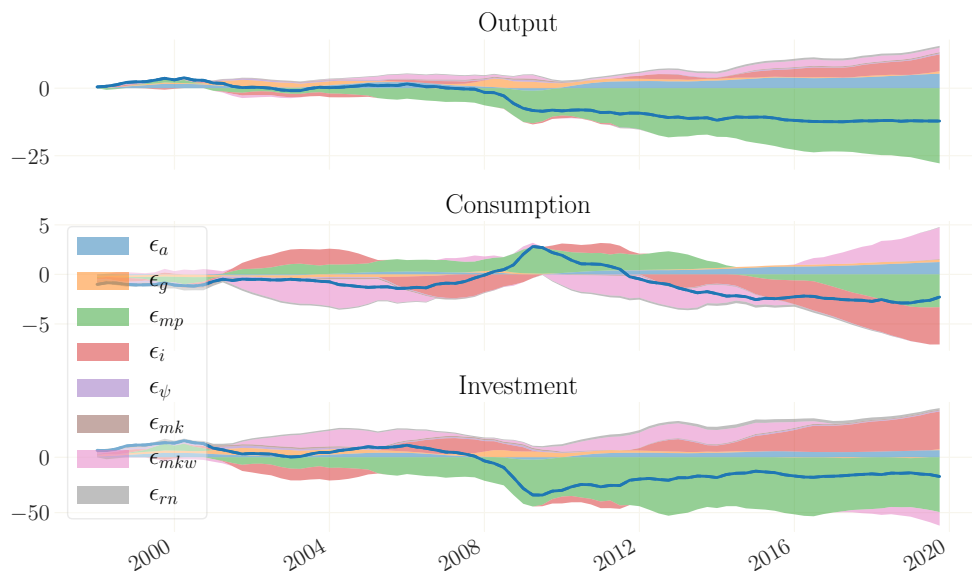
Appendix M   Replicating Carlstrom et al. (2017)

In order to gauge whether our results are robust to the model choice, we also replicate Carlstrom et al. (2017) by estimating their model using our nonlinear Bayesian likelihood approach and the same sample period including the ZLB. Although we use the same sample period as for the estimation or our benchmark model, some observables differ between both models. Specifically, the model includes quarterly changes in PCE inflation, real GDP growth and investment, hours worked and real wages as explained in section Appendix A. Moreover, we lumped all asset purchases of the Fed under the variable "QE tot" by simply adding "CB GBonds" and "CB CBonds". Although we agree that this is a very stylized approach, we do so because the price (and hence the yield) of long-term corporate and government bonds is the same in CFP. Finally, we include the quarterly Federal Funds rate and 10-year treasury term premium from Adrian et al. (2013), the latter which is downloaded from the New York Fed's homepage.[43]

Table M.7 presents our parameter estimates of the CFP model using our sample from 1998:I until 2019:IV and compares these those published by Carlstrom et al. (2017).

For a selection of variables, Figures M.35 and M.36 show the historical decompositions. Finally, Figure M.37 shows the impulse responses to a QE shock.

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[43]https://www.newyorkfed.org/research/data_indicators/term_premia.html

	Prior				Posterior					Carlstrom et al. (2017)		
	dist.	mean	std/df	mean	std	mode	5%	95%	mean	5%	95%
η	gamma	2.0	0.75	2.608	0.376	2.509	2.009	3.214	2.0259	1.2673	2.7526
h	beta	0.6	0.10	0.888	0.019	0.884	0.864	0.915	0.6225	0.5760	0.6687
ψ_n	uniform	5.0	2.89	0.140	0.584	0.045	0.005	0.113	0.7850	0.3389	1.2394
ϕ	gamma	3.0	1.00	5.433	0.995	5.752	3.794	6.917	3.2821	2.1857	4.3639
τ_π	normal	1.5	0.10	1.575	0.086	1.602	1.441	1.718	1.4202	1.2828	1.5493
τ_y	normal	0.5	0.10	0.666	0.070	0.726	0.545	0.779	0.4906	0.3566	0.6292
ρ_i	beta	0.8	0.10	0.895	0.011	0.898	0.875	0.913	0.7712	0.7309	0.8109
ι_p	beta	0.6	0.10	0.311	0.060	0.359	0.212	0.409	0.4175	0.2752	0.5610
ι_w	beta	0.6	0.10	0.611	0.089	0.627	0.470	0.760	0.5110	0.4085	0.6205
κ_{pc}	beta	0.2	0.10	0.045	0.011	0.043	0.026	0.063	0.0860	0.0104	0.1544
κ_w	beta	0.2	0.10	0.060	0.022	0.062	0.026	0.096	0.0002	0.0001	0.0004
ρ_a	beta	0.6	0.20	0.947	0.031	0.963	0.905	0.997	0.9921	0.9841	0.9997
ρ_{mu}	beta	0.6	0.20	0.938	0.014	0.936	0.918	0.961	0.8695	0.8281	0.9122
ρ_ϕ	beta	0.6	0.20	0.985	0.013	0.984	0.967	1.000	0.9821	0.9682	0.9963
ρ_{mk}	beta	0.6	0.20	0.235	0.071	0.225	0.125	0.349	0.6650	0.4945	0.8405
ρ_{mkw}	beta	0.6	0.20	0.198	0.059	0.165	0.098	0.289	0.2059	0.1036	0.3027
ρ_m	beta	0.6	0.20	0.690	0.053	0.644	0.618	0.776	0.1564	0.0646	0.2515
ρ_{rn}	beta	0.6	0.20	0.852	0.036	0.856	0.795	0.914	0.9483	0.9212	0.9751
$root_1$	beta	0.5	0.20	0.916	0.047	0.891	0.845	0.984			
$root_2$	beta	0.5	0.20	0.922	0.046	0.962	0.853	0.991			
σ_a	inv. gamma (df)	0.5	1.00	0.433	0.048	0.457	0.360	0.515	0.6481	0.5936	0.7030
σ_i	inv. gamma (df)	0.5	1.00	5.441	0.693	5.884	4.384	6.574	7.3454	5.5735	9.2124
σ_{mp}	inv. gamma (df)	0.1	1.00	0.131	0.017	0.141	0.104	0.155	0.2151	0.1935	0.2368
σ_{mk}	inv. gamma (df)	0.1	1.00	1.307	0.510	1.228	0.567	2.029	0.2442	0.1830	0.3049
σ_{mkw}	inv. gamma (df)	0.1	1.00	5.894	2.039	6.487	2.713	8.753	0.4840	0.4103	0.5569
σ_{rn}	inv. gamma (df)	0.1	1.00	0.495	0.144	0.433	0.258	0.717	0.1588	0.1179	0.2000
σ_{psi}	inv. gamma (df)	0.5	1.00	11.060	4.174	11.733	4.695	17.854	2.7196	1.9449	3.4826
σ_{qe}	inv. gamma (df)	0.1	2.00	0.277	0.026	0.268	0.234	0.319			

Table M.7: Parameter estimates for Carlstrom et al. (2017) model. For the inverse gamma distribution the same specification (via mean and degrees of freedem) as in dynare is chosen to ensure comparability of results.
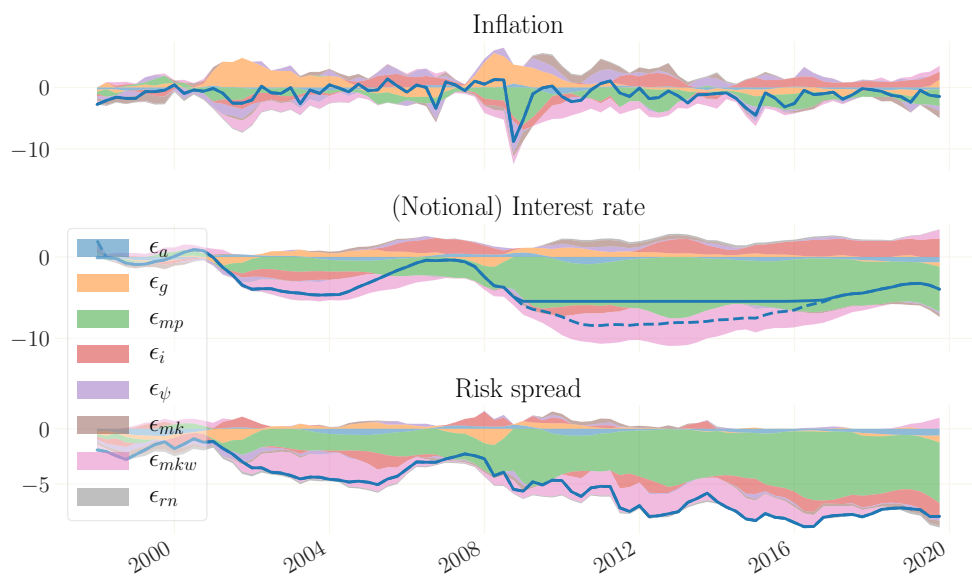
Figure M.36: Estimated CFP model. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 1000 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.

Figure M.37: CFP model. Impulse response functions with respect to a generic QE-shock. Sampled from the posterior distribution with 95% credible set.
Note: Posterior sample obtained from 1000 draws from the posterior distribution. Solid lines represent the mean. For each draw, the strength of the shock is chosen such that the peak of the corresponding stochastic process matches the peak of the empirical time series. See Appendix F for details. Annual measures where applicable.

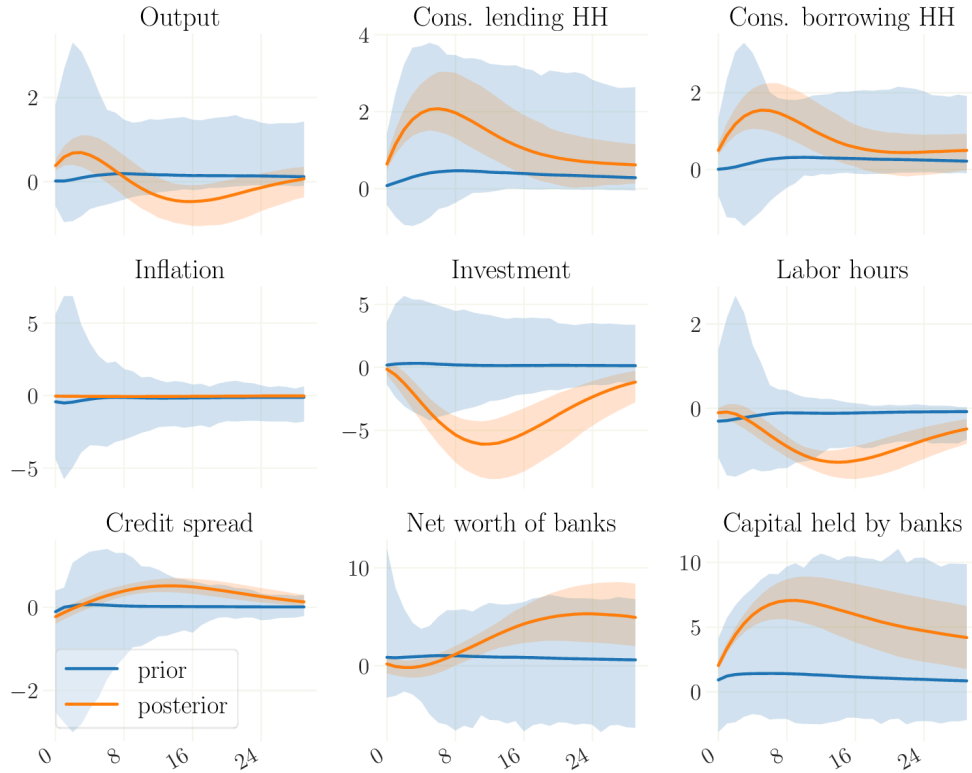## Appendix N Model vintage with capital quality shocks

Figure N.38: Estimated benchmark model with the risk premium shock replaced by the capital quality shock. Prior and posterior impulse response functions with respect to a negative capital quality shock with 95% credible set.
Note: Means over 2000 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.

As an additional exercise, we estimate a version of our benchmark model in which we replace the risk premium shock by the capital quality shock proposed in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). The latter shock reduces the physical capital stock, thereby affecting both the productivity and value of capital. In Gertler and Karadi (2011), capital quality shocks triggers fire sales in the banking sector that raises the credit spread and, ultimately, leads to a simultaneous decline in aggregate consumption and investment. In sum, this shock was calibrated to explain the key events of the GFC. Note, however, that the co-movement in investment and aggregate consumption that is needed to explain the GFC hinges critically on the calibration of θ. Specifically, if one changes θ from 0.972 in Gertler and Karadi (2013) to any value smaller or equal to 0.95, this co-movement disappears.

Through the lens of our estimated model, the this co-movement disappears for the capital quality shock. Yet, as the prior impulse responses in Figure N.38 illustrate, this
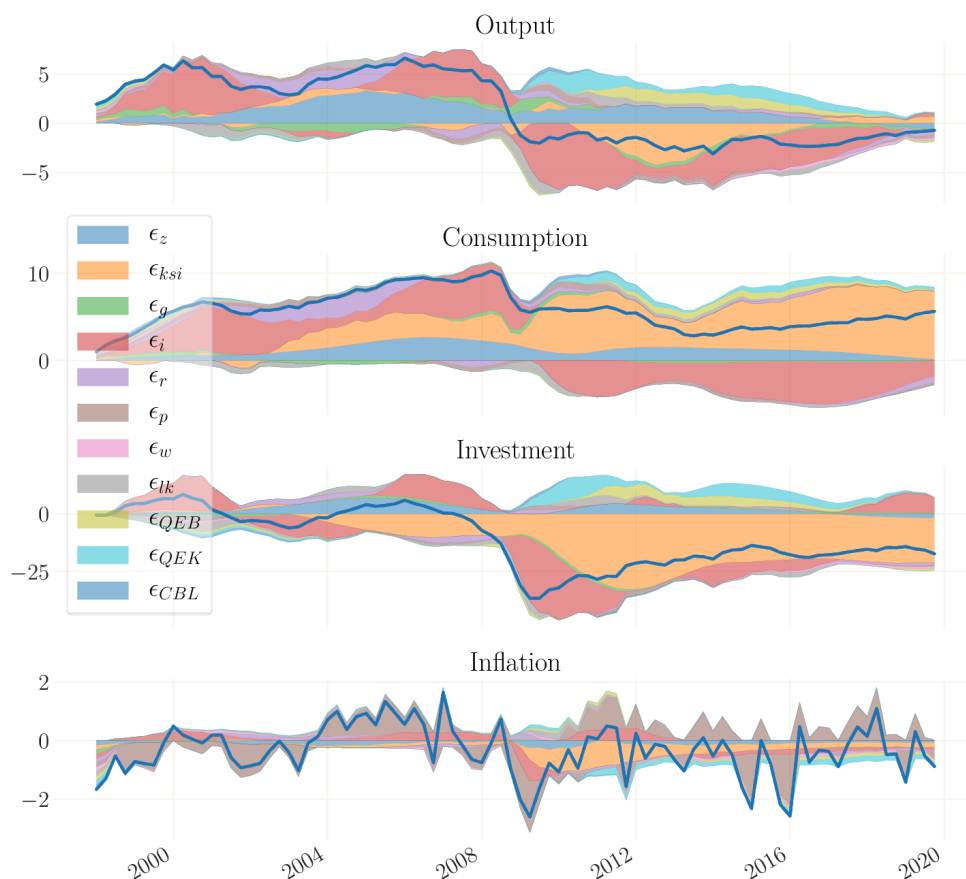
Figure N.39: Estimated benchmark model with the risk premium shock replaced by the capital quality shock. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 2000 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.

result is not hardwired a priori. Figures N.39 and N.39 show that while capital quality shock reduces investment, it actually stimulates aggregate consumption and plays only a limited role for the spike of the credit spread observed in the GFC. Overall, in the context of our benchmark model estimated on crisis data, the capital quality shock has far less explanatory power for the events triggered by the GFC, than the risk premium shock it replaces. As the risk premium shock, which plays a dominant role for macroeconomic dynamics in the benchmark model, is replaced in this variant, other shocks, such as the investment-specific shock, inevitably gain a more prominent role in the decomposition of macroeconomic dynamics. The historical shock decomposition shows that at the mean the effect of the Fed's asset purchases on investment and consumption are positive during the Great Recession. After 2011, the effect of capital purchases on price dynamics turns and becomes deflationary.
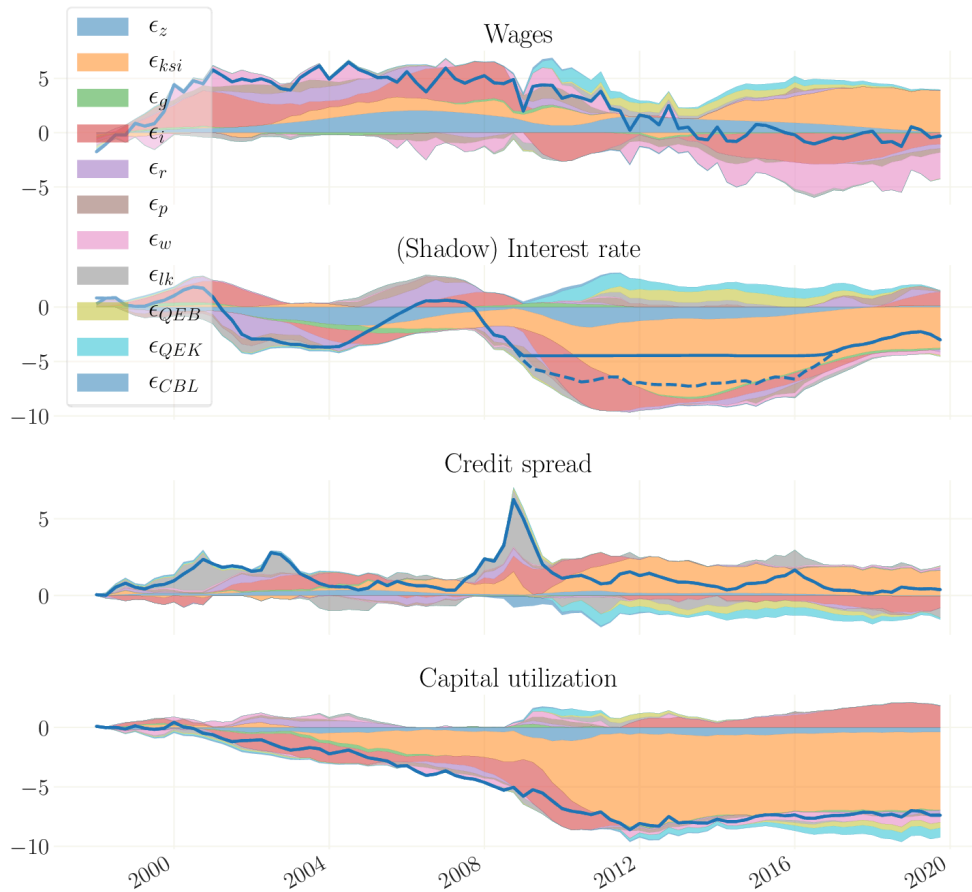
Figure N.40: Estimated benchmark model with the risk premium shock replaced by the capital quality shock. Decomposition of the smoothed states into the contribution of the different shocks.
Note: Means over 2000 simulations drawn from the posterior. The contribution of each shock is normalized and calculated as in Boehl and Strobel (2020). Annual measures where applicable.

These findings should be taken with caution. As it is hard to identify a main common driver for the economic dynamics of the model, the respective smoothed series of economic shocks are less probable compared to our baseline estimation. For this reason, the smoothing procedure via NPAS becomes unstable for some posterior draws. While this could be fixed by increasing the shock search-space of the smoother, it reflects the poor performance of the augmented model to explain the post-2007 dynamics. We hence find that this variant is not well suited for a structural investigation of quantitative easing.