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Strategic Respondents and Turnout Implications

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Polls and Elections: Strategic Respondents and Turnout Implications*

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Abstract

This paper studies the effect of pre-election polls on the participation decision of citizens in a large, two-candidate election, and the resulting incentives for the poll participants. Citizens have private values and voting is costly and instrumental. The environment is ex ante symmetric and features aggregate uncertainty about the distribution of preferences. Citizens base their participation decision on their own preferences and on the information provided in the poll. If all participants answer the poll truthfully, the underdog effect implies that the supporters of the trailing candidate turn out at higher rates than the supporters of the leader of the poll. This effect yields incentives for the poll participants to misrepresent their preferences to encourage the voters who have the same preferences to turn out. If poll participants are strategic, however, there does not exist an equilibrium in which the poll conveys any information. Thus, in the limit, the majority candidate wins the election almost surely, regardless of voters' posterior beliefs.

JEL Classification: D72, D83

Keywords: Costly voting; Polls; Aggregate Uncertainty; Underdog Effect

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1 Introduction

Polls matter. They receive widespread attention in the media, and they are perceived to have an influence on voting behavior and turnout. As an example, on 23 June 2016, the United Kingdom decided to leave the European Union with a 52% majority. By contrast, opinion polls released on the eve of the referendum predicted that 51% of voters would support remaining (cf. Wells (2016)). Later, several sources, such as Low (2016), voiced concern that many citizens might not have voted because they had believed that "Brexit" would be defeated. Government regulations restricting the timing of the publication of polls in the run-up to elections reflect the perceived influence of polls. Several countries prohibit the publication of opinion polls in specific circumstances, usually quite close to Election Day itself. In a study of policies that address the publication of voter polls in 133 countries, Frankovic et al. (2018) find that 60% of these countries ban the publication of polls before elections for a certain period of time, called the blackout period. In France, for example, the blackout period is currently set at two days.

Generally, turnout decisions are strategic and depend on the citizens' beliefs about the preferences and turnout decisions of other voters.[1] Polls inform citizens about these preferences and allow rational voters to update their beliefs. Therefore, polls can influence elections by updating beliefs, which, in turn, affect the incentives of poll participants. The aim of this paper is to analyze the effect of the information provided through polls on voter turnout, the incentives of the poll participants, and the implications for regulations. To this end, I study and build upon the canonical model of costly voting as introduced by Palfrey and Rosenthal (1983). I add the feature that there is aggregate uncertainty about the distribution of preferences. This captures the observation that voters rarely know with certainty whether they are part of the majority. Finally, I introduce polls that inform citizens about the preferences of others.

In more detail, I consider a large election taking place between candidates A and B. Citizens have private values, and there is aggregate uncertainty about the preference distribution, which is governed by the state of the world. In state α, the probability that a randomly drawn voter supports candidate A is $q > \frac{1}{2}$, and in state β, the probability that a randomly drawn voter supports candidate A is $1 - q$, where each state is equally likely. Thus, ex ante, it is equally likely that a random voter prefers candidate A or B. Each voter knows his or her own preferences, but does not know whether he or she is part of the majority. An opinion poll conducted prior to the election publishes its results of preferences for each candidate as expressed by those participating in the poll. Voters in the electorate use the information from the poll as well as their own preference type to update their beliefs about the state of the world. It is assumed that a known and finite number of citizens, randomly drawn from the population, participate in the opinion poll and indicate their preferred candidate. So, the sample of citizens surveyed in the pre-election poll is

[1] For empirical evidence, see Agranov et al. (2018), Klor and Winter (2018), Morton et al. (2015), Blais et al. (2006), and Cantoni et al. (2019).

representative.[2]

Voting is costly and voluntary, and costs are drawn from a smooth distribution that has bounded support $[0, \bar{c}]$ and a strictly positive density everywhere on the support.

The election is decided by majority voting. The candidate with the most votes wins the election, and ties are broken randomly.

I solve the game backwards, starting with the voting equilibrium. For any given polling outcome, and for any strategy the poll participants pursue, voters hold some posterior belief about the state of the world. I show that there always exists a voting equilibrium with strictly positive participation rates by both groups. Focusing on large elections, i.e., taking the limit as the size of the population goes to infinity, I show that there exists a unique limit of the ratio of participation rates. This limit ratio reflects the *underdog effect*: the expected minority candidate participates at higher rates.[3] Mentioned informally decades ago in Palfrey and Rosenthal (1983), the underdog effect has been observed repeatedly in the experimental literature, e.g. Levine and Palfrey (2007). As formalized in Ledyard (1984), a vote for the expected minority candidate is pivotal with higher probability because it pushes the election closer to a tie; by contrast, a vote for the majority candidate pushes the likely outcome further away from a tie. Consequently, because a voter's perceived benefit of voting increases with his or her belief in the likelihood that his or her vote will be pivotal, those supporting the perceived underdog in the race have a higher incentive to vote, implying higher participation rates.[4] Importantly, I show that this underdog effect is monotonic in the sense that the limit ratio of participation rates is monotonic in the posterior beliefs. However, the underdog effect is only *partial*: the limit ratio of participation rates is closer to one than the ratio of the respective population shares of A and B supporters. Consequently, in the limit, the majority candidate almost surely wins the election. Notably, this holds for any posterior belief voters might hold, including the case in which any aggregate uncertainty about the distribution of preferences is resolved and, thus, the state of the world is known.

The solution of the voting stage has interesting implications if polls are answered truthfully. Due to the partial underdog effect, the margin of victory decreases, but the majority candidate still wins the election in the limit almost surely. Yet, prior research has found that poll participants do not necessarily answer truthfully[5]. So, I next consider the strategic behavior of participants in the polling stage of the election by initially assuming that all participants behave in strategic ways. By contrast, suppose, that all poll participants answer truthfully and that the electorate believe this to be the case. In such a situation,

[2]To isolate effects, the citizens participating in the poll are excluded from voting in the election, but it is assumed that they derive the same utility from the election outcome as eligible voters who share their preferences.

[3]Note that if there are no polls, supporters of both candidate A and B have the same probability of turning out.

[4]Note that the underdog effect is related to the free-riding problem in public goods games because voting is comparable to contributing to the public good.

[5]See, for example, Clarke and Whiteley (2016) who are concerned with false answers regarding voting intention, or Keeter and Samaranayake (2007) and Hopkins (2009) who consider the Bradley effect.

posterior beliefs are monotonic in the poll's margin, and, thus, so is the underdog effect. However, given this monotonicity, there cannot exist an equilibrium with any information transmission. In equilibrium, the poll is uninformative. Intuitively, misrepresenting preferences is profitable because it simultaneously stimulates the participation of voters who have the same preferences and discourages the participation of opponent voters. In Appendix C, I consider an extension in which a fixed share of poll participants is prescribed to answer the poll truthfully and the other poll participants are strategic. Then, it holds again that the majority candidate almost surely wins the election in the limit. The direction of the underdog effect will depend on the share of exogenously truthful poll participants. If this share is larger than one half, the poll is informative. Else, the poll is not informative. In any case, the strategic poll participants have an incentive to misrepresent their preferences.

In conclusion, in the limit, polls do not prevent the almost sure election of the majority candidate. If the poll result is informative, it stimulates participation of voters who support the minority candidate. However, the higher participation rate is not sufficient to overturn the election outcome. In contrast, if poll participants are strategic, the poll is uninformative and voters behave as if there were no poll in the first place.

Goeree and Großer (2007) study the effect of exogenously truthful information on the distribution of preferences. They find that this information (for example, as provided by truthful polls) is detrimental for welfare because, as a result, both candidates are equally likely to win. In their paper, all voters face homogenous voting costs, which are set such that turnout is incomplete and positive. For voters to be willing to employ mixed strategies and, thus, to be indifferent between abstaining and voting, given the homogenous cost, the expected benefit of voting needs to coincide for all voter types, implying equal pivot probabilities for votes for either candidate. Since a vote for the trailing candidate has a higher probability of being pivotal, pivot probabilities (and, thus, expected voting benefits) can only be equal if the expected vote shares coincide. As a result, models with homogeneous voting costs (such as the model of Goeree and Großer (2007)) observe a *full underdog effect* because the minority's heightened participation has to completely offset the majority's advantage in equilibrium. By contrast, if one assumes, as I do, that the distribution of costs is smooth, the underdog effect must be partial. Intuitively, if the underdog effect were to fully compensate for the majority's advantage such that expected vote shares would be equal, the pivot probabilities would be equal for the two groups. However, in my model, if the probability of being pivotal would indeed be the same for both groups, the participation rates would be equal, and, thus, vote shares would necessarily be strictly different. This contrasts with Goeree and Großer (2007), where the vote shares can be the same if the pivot probabilities are the same. Overall, while they show how drastic the effects of polls can be, I demonstrate that the negative welfare effects do not carry over when considering a slightly different model framework. Coming back to the "Brexit" example, my results can be interpreted to demonstrate that in equilibrium, and in a large election, polls should not have been of concern for the election outcome.

However, polls generally do matter in the sense that they reduce the margin of victory in elections (or referenda) compared to the actual advantage of the majority candidate (or alternative) through the partial underdog effect. This effect has consequences if the margin of victory or vote shares themselves matter.

The remainder of this paper is organized as follows: Section 2 gives an overview of the related literature. Section 3 introduces the model, and section 4 analyzes the voting equilibrium. Section 5 analyzes the polling equilibrium, and section 6 concludes. All omitted proofs appear in Appendix A. Appendix B contains the properties of posterior beliefs. Appendix C considers a poll with a share of exogenously truthful participants.

2 Related Literature

The theory of costly participation in a two-candidate election was introduced by Palfrey and Rosenthal (1983), Ledyard (1984), and Palfrey and Rosenthal (1985), who explore the paradox of not voting and give conditions for equilibria with positive turnout for given candidate platforms. Palfrey and Rosenthal (1983) characterize multiple equilibria in a setting in which voting costs are identical for all voters. Nöldeke and Peña (2016) provide missing proofs. Ledyard (1984) considers spatial preferences of voters and a smooth cost distribution. He characterizes the voting equilibrium, showing that if candidates can freely set their platforms, the welfare maximizing platform is chosen by all candidates, and there is no turnout in equilibrium. Palfrey and Rosenthal (1985) also consider fixed platforms and allow for different distributions of costs across groups. They show that in large elections, only voters with negative or zero costs of voting turn out.

The partial underdog effect has been identified in the literature studying idiosyncratic uncertainty about voters' preferences, assuming, as I do, a smooth distribution of costs. Herrera et al. (2014) contrast the voting systems of majority and proportional representation in a setting with population uncertainty; they characterize the differences in turnout. Krishna and Morgan (2015) give conditions under which simple majority rule selects the utilitarian candidate.[6] In a model of ethical voting, Evren (2012) assumes that a fraction of agents is altruistic, and that there is aggregate uncertainty about the expected share of altruists for supporters of either candidate. While selfish agents abstain from voting, altruistic agents turn out if their private voting cost is outweighed by their vote's contribution to the welfare of society.

Myatt (2017) studies protest voting in a setting in which voting is not costly, but the possibility to protest bears opportunity costs, and voting for the opponent potentially influences policy. He observes an "offset" effect which is directly related to the underdog effect. The anticipation of a larger protest reduces the motivation of like-minded agents to join the protest—thus reducing the size of the protest, but not fully compensating for the increase in enthusiasm. I show that the partial underdog effect exists for any posterior

[6] Grüner and Tröger (2019) study utilitarian-optimal voting rules if voting is costly.

belief that can be induced by a pre-election poll if there is aggregate uncertainty about the distribution of preferences and voters only expect to be part of the majority or the minority, but do not know this with certainty.

Aggregate uncertainty about the distribution of preferences in costly voting models has been studied by Goeree and Großer (2007), Taylor and Yildirim (2010a), and Myatt (2015). In similar papers, Goeree and Großer (2007) and Taylor and Yildirim (2010a) both contrast the effects that occur in two different environments. The first is an environment in which voters are informed about the distribution of preferences; the second is an environment in which there is aggregate uncertainty about this distribution and the prior over the state distribution is symmetric. Voting costs are homogeneous for all voters. Their papers differ in that Goeree and Großer (2007) consider two states of the world and focus on small elections, whereas Taylor and Yildirim (2010a) allow for finitely many states of the world and consider small and large elections. Under their common assumptions on costs, the underdog effect implies that expected vote shares are equal. In the informed case, this results in a toss-up election in expectation. In the case with aggregate uncertainty about the preference distribution, the symmetric prior yields identical participation rates for both types of voters, such that the majority candidate is more likely to win the election. Goeree and Großer (2007) and Taylor and Yildirim (2010a) thus conclude that information provision that resolves the aggregate uncertainty is unambiguously detrimental to voters' welfare since it decreases the probability of the majority candidate winning the election. My modeling of aggregate uncertainty about the preference distribution follows Goeree and Großer (2007). I show that their conclusion about the welfare implications of information is sensitive to assumptions on the distribution of costs. My work is distinct in two other aspects: I consider the incentives of poll participants, and I do not require the poll to perfectly reveal the state of the world.

In Myatt (2015), the probability that candidate A is preferred by a randomly drawn voter is given by p, which is itself a random variable with mean $\bar{p}$ and density $f(p)$. He studies the response of turnout and the election outcome to, amongst others, varying assumptions on costs, the importance of the election, the preference intensities, or the perceived popularity of candidates. He also finds that there exists an underdog effect, which is complete if costs are the same for all voters, and partial if the cost distribution is smooth. Our two models are not nested. First, Myatt (2015) assumes full support on $[0, 1]$ for the density f. Further, in Myatt (2015), reducing the aggregate uncertainty about the distribution of preferences corresponds to decreasing the variance; if the uncertainty is resolved, voters' beliefs coincide with the mean $\bar{p}$. By contrast, in my model, if the state of the world is known, the probability of preferring A is either q or $1 - q$, but never their mean.[7] Further, Myatt (2015) does not consider the incentives faced by poll participants but only studies the impact of different beliefs about preferences on voting.[8]

[7] A similar argument is made in Agranov et al. (2018).

[8] Cvijanović et al. (2020) study corporate voting, building on Myatt (2015).

This paper is also related to the literature on information provision in elections and on signaling in elections.

Burke and Taylor (2008) study polls with signaling incentives, assuming that the same voters participate in the poll and in the election. There is only idiosyncratic uncertainty about voters' preferences, and voting costs are the same for all voters. They find that truthful reporting is an equilibrium in the pre-election poll for a three-person electorate and low voting costs. This holds because in the case of a two-citizen majority, if preferences are known, there is no underdog effect for sufficiently low voting costs, and therefore, the majority is more likely to win. The incentive to truthfully reveal preferences in the case of a two-citizen majority dominates the incentive to misrepresent in all other cases. For general n, Burke and Taylor (2008) derive sufficient conditions for the non-existence of a truthful reporting equilibrium. Finally, they show that if a truthful equilibrium exists, it is welfare enhancing because the minority is discouraged from participating in the election. My model marries the incentive considerations in a poll that is intended to inform the electorate about the prevailing preferences with the assumption that the distribution of these preferences is initially unknown. Then, truthtelling cannot be an equilibrium of the polling stage.

Another subject of study of the roles played by polls concerns their ability to serve as signaling and coordination devices, or a means to inform politicians about the desired policy.

Hummel (2011) proposes a model of polling in sequential elections, in which the winner of the first election faces a third candidate, and finds incentives to misrepresent preferences to increase the winning probability of one's favorite candidate in the second election. Piketty (2000) analyzes a similar sequential election, in which there are no polls, but voters use their votes in the first election round to communicate their preferences—thereby trading off sincere with strategic voting. Hummel (2014) considers a three-candidate election, in which the third candidate is supported by a minor party; he explains why third party candidates achieve better results in pre-election polls than in elections.

Meirowitz (2005) and Morgan and Stocken (2008) analyze the incentives of poll participants if candidates use the information revealed in the poll to select policy platforms. To be more precise, Morgan and Stocken analyze a setting in which a policy maker polls the constituents, who differ in terms of information they have and ideology they hold, about their preferred policy, and provide conditions for full information aggregation. Relatedly, Battaglini (2017) and Ekmekci and Lauermann (2019) study information aggregation through informal elections, such as public protests.

Communication in committees prior to a binding vote has also been modeled through straw votes (a full poll). Coughlan (2000) and Austen-Smith and Feddersen (2006) give conditions for full information revelation in a non-binding straw vote for a Condorcet jury setting. Gerardi and Yariv (2007) allow for general communication protocols; they show that the set of equilibrium outcomes is invariant to the voting institution, as long as it is non-unanimous.

The incentives of poll participants and the effects of polls, or exogenous information release, on voters' beliefs have been studied in the experimental literature as well. Agranov et al. (2018) study the effect of the release of exogenous information by testing the model proposed by Goeree and Großer (2007). They do not observe an underdog effect and find that information about the distribution of preferences does not reduce welfare. Agranov et al. (2018) argue that the data can be explained by assuming that voters have preferences to vote for the winner.[9] Klor and Winter (2018) also consider exogenous information; they find that close polls stimulate turnout, and that the effect is greater for majority voters because of false beliefs about the probability of casting a pivotal vote. Morton et al. (2015) employ a natural experiment featuring exit polls in France; their findings show that the publication of exit polls while the election was ongoing led to a decrease in turnout by 11%, and an increase in bandwagon voting, i.e., voting for the expected winner of the election. Großer and Schram (2010) find that polls stimulate turnout, and that this is driven by undecided voters. Blais et al. (2006) examine the impact of polls in the 1988 Canadian election. They find that the polls affected the beliefs about the outcome of the election and voting itself by discouraging turnout of supporters of a party that was not considered likely to win. They do not observe a bandwagon effect. Because I abstract from voter preferences that prescribe that participants want to vote for the winner, I avoid the effect described by Blais et al. (2006) that would counteract some of my results. Cantoni et al. (2019) conduct a field experiment to elicit the beliefs of individuals about others' planned participation in a public protest and the effects on turnout. The authors find that there is strategic substitutability related to the underdog effect in the sense that turnout is stimulated if and only if others are believed not to participate.

Methodically, this paper is related to the seminal work of Myerson (1998a), Myerson (1998b), and Myerson (2000) on population uncertainty. It is also related to Krishna and Morgan (2012), who study welfare properties of majority voting in a two-candidate election with common values and population uncertainty, in which the state of the world indicates which candidate is more competent.

3 The Model

Two candidates, A and B, vie for election. Citizens have independent private values. An A supporter receives a utility of $v > 0$ if and only if A is elected, and zero otherwise, and a B supporter receives a utility of $v > 0$ if and only if B is elected, and zero otherwise. There is aggregate uncertainty about the distribution of preferences that is governed by two states, $\omega \in \Omega = \{\alpha, \beta\}$. In state α, the probability that a randomly drawn citizen prefers A is $\Pr(A|\alpha) = q > \frac{1}{2}$, while, in state β, the probability that a randomly drawn

[9] Callander (2007) shows in a theoretical model that preferences to vote for the winner can result in the so-called *bandwagon effect*.

citizen prefers A is $\Pr(A|\beta) = 1 - q$. The number of eligible voters is finite but uncertain, and it is Poisson distributed with mean n. Hence, the probability that the electorate consists of k citizens is $e^{-n}\frac{n^k}{k!}$. This induces an extended Poisson game as introduced by Myerson (1998a).

Voting is costly and voluntary. Each citizen can decide between the actions "vote for A", "vote for B", and "abstain". If a citizen chooses to vote for one of the candidates, he or she incurs a voting cost c. The voting cost is distributed according to the cumulative distribution function F with density f that is strictly positive on its support $[0, \bar{c}]$, with $\bar{c} \geq v$.[10] Further, F is assumed to be differentiable. Costs are drawn independently for each individual citizen and, thus, do not depend on preferences. The candidate who obtains the majority of votes wins, and ties are broken by the toss of a fair coin. Prior to the election, but after the state and preferences have been realized, an opinion poll is conducted. To this end, m independently drawn citizens are asked which candidate they prefer. Then, the poll result is published in the form of the pair $\tau = (\tau_A, \tau_B)$, where τ_i denotes the number of poll participants who indicated a preference for candidate i, for $i \in \{A, B\}$. For tractability, the m participants of the poll are assumed not to take part in the main election. That is, they will not belong to the electorate. They will, however, have the same preferences over the election outcome as the members of the electorate, and so, have the same stakes in the election, absent the cost of voting.

Thus, the overall timing is as follows: Nature draws the number of voters and the state of the world, preferences are determined by independent draws from the state-dependent Bernoulli distribution, the pre-election poll is conducted and published, and, finally, the election is held.

I will consider symmetric Perfect Bayesian equilibria, in which all supporters of the same candidate employ the same strategy.

4 Voting Equilibrium

This section addresses the equilibrium of the election stage and collects its properties. I take as given the citizens' posterior beliefs about the state of the world. First, I derive the existence of voting equilibria and show that for all n, participation rates are equal if and only if the posterior beliefs coincide with the prior beliefs. Then, I turn to the analysis of large elections. I show that the limit ratio of participation rates is unique and that it reflects the underdog effect, which is monotonic in the posterior beliefs. Finally, I show that the majority candidate almost surely wins the election in the limit, independently of posterior beliefs.

[10] This condition is sufficient, but not necessary to guarantee positive and incomplete turnout.

4.1 Equilibrium Existence

Observe first that given the assumption that voting is costly, for every supporter of candidate i, voting for candidate j is strictly dominated by abstention. Thus, if a citizen chooses to vote, he or she will vote for his or her preferred candidate. Voting is always sincere.

So, a citizen trades off voting for his or her favorite candidate against abstaining. To that end, a citizen will contrast the expected benefit of his or her vote with the associated costs. His or her vote will directly benefit him or her only if the vote changes the outcome of the election, i.e., only if his or her vote is *pivotal*. A vote for candidate A is pivotal in two cases: 1) if both candidates are tied, that is, if there are $2k$ other voters, where k are voting for A and k are voting for B, and 2), if candidate A is exactly one vote behind, that is, if there are $2k+1$ other voters, where k are voting for A and $k+1$ are voting for B. Piv_A denotes the event that a vote for candidate A is pivotal, analogously, Piv_B denotes the event that a vote for B is pivotal.

Upon observing their own preference type, citizens do not hold uniform priors. That is, a citizen of type i, for $i \in \{A, B\}$, holds the prior $\Pr(\omega|i)$ for $\omega \in \{\alpha, \beta\}$. After observing a poll result τ, the posterior belief of a citizen of type i that the state is ω is denoted by $\Pr(\omega|i, \tau)$. The properties of these beliefs are derived in Appendix B.
The expected benefit of voting for an i supporter is thus given by

$$\Pr(\alpha|i, \tau) \cdot \Pr(Piv_i|\alpha) \cdot v + \Pr(\beta|i, \tau) \cdot \Pr(Piv_i|\beta) \cdot v.$$

A citizen will vote for his or her preferred candidate if and only if the expected benefit of voting is weakly larger than his or her voting cost c. Since the expected benefit of voting is independent of c, there exist cost cutoffs c_A^*, c_B^*,[11] satisfying

$$\Pr(\alpha|A, \tau) \cdot \Pr(Piv_A|\alpha) \cdot v + \Pr(\beta|A, \tau) \cdot \Pr(Piv_A|\beta) \cdot v = c_A^* \quad (1)$$

$$\Pr(\alpha|B, \tau) \cdot \Pr(Piv_B|\alpha) \cdot v + \Pr(\beta|B, \tau) \cdot \Pr(Piv_B|\beta) \cdot v = c_B^*.\text{[12]} \quad (2)$$

The cost cutoffs determine the voters' participation decision. Thus, the equilibrium strategy for a voter of type i, for $i \in \{A, B\}$, will now be identified by the cutoff cost c_i^*. A voting equilibrium is a pair of cutoff costs (c_A^*, c_B^*) such that it is optimal for a citizen of type i with cost $c \leq c_i^*$ to turn out and vote for candidate i if all other citizens in the electorate pursue this strategy.
Let p_A denote the probability that an A supporter chooses to vote for A, and analogously, let p_B denote the probability that a B supporter chooses to vote for B. Then, the probability that an i supporter abstains is $1 - p_i$. I will call p_A and p_B the participation rates

[11] The cost cutoffs may depend on τ. The dependence is omitted in the notation for the sake of readability.

[12] The probability that a vote is pivotal is larger if the election is expected to be close. The above equations imply that close elections induce higher turnout because higher pivot probabilities increase the cost cutoffs.

of A and B supporters, respectively.
Given the pair of cost cutoffs, these participation rates are

$$p_A := F(c_A^*),\ p_B := F(c_B^*).$$

Since the size of the electorate follows a Poisson distribution with mean n, the number of votes for candidate i conditional on the state of the world ω is distributed according to a Poisson distribution with mean denoted by $\lambda(i|\omega), i \in \{A, B\}, \omega \in \{\alpha, \beta\}$. Note that the number of votes for candidate i is independent of the number of votes for candidate j conditional on the state, cf. Myerson (2000).
These means, which I also call expected conditional votes, are given by

$$\begin{aligned}
\lambda(A|\alpha) &:= n \cdot q \cdot p_A, \\
\lambda(A|\beta) &:= n \cdot (1-q) \cdot p_A, \\
\lambda(B|\alpha) &:= n \cdot (1-q) \cdot p_B, \\
\lambda(B|\beta) &:= n \cdot q \cdot p_B.
\end{aligned}$$

Since in an extended Poisson game, the pivot probabilities depend only on the expected conditional votes for either candidate, I can now calculate these probabilities. As mentioned above, a vote is pivotal if it either creates a tie or breaks a tie. Let T be the event of a tie, let T_{-1}^A be the event that candidate A is one vote behind, and T_{-1}^B be the event that candidate B is one vote behind. The probabilities of these events are given by

$$\begin{aligned}
\Pr(T|\omega) &= e^{-\lambda(A|\omega)-\lambda(B|\omega)} \cdot \sum_{k=0}^{\infty} \frac{\lambda(A|\omega)^k}{k!} \cdot \frac{\lambda(B|\omega)^k}{k!}, \\
\Pr(T_{-1}^A|\omega) &= e^{-\lambda(A|\omega)-\lambda(B|\omega)} \cdot \sum_{k=1}^{\infty} \frac{\lambda(A|\omega)^{k-1}}{(k-1)!} \cdot \frac{\lambda(B|\omega)^k}{k!}, \\
\Pr(T_{-1}^B|\omega) &= e^{-\lambda(A|\omega)-\lambda(B|\omega)} \cdot \sum_{k=1}^{\infty} \frac{\lambda(A|\omega)^k}{k!} \cdot \frac{\lambda(B|\omega)^{k-1}}{(k-1)!}.
\end{aligned}$$

Therefore, the probability that an A supporter's vote for A is pivotal in state ω is

$$\Pr(Piv_A|\omega) = \frac{1}{2}\Pr(T|\omega) + \frac{1}{2}\Pr(T_{-1}^A|\omega),$$

and the probability that a B supporter's vote for B is pivotal for B in state ω is

$$\Pr(Piv_B|\omega) = \frac{1}{2}\Pr(T|\omega) + \frac{1}{2}\Pr(T_{-1}^B|\omega).$$

The existence of a voting equilibrium is now established in the following proposition.

Proposition 1 (Existence)**.** *There exists an equilibrium of the election stage; that is, there exist equilibrium cutoff costs (c_A^*, c_B^*) for every pair $\tau = (\tau_A, \tau_B)$. All voting equilibria* *involve interior participation rates, i.e., $p_A, p_B \in (0, 1)$.*

Proof. Recall that $p_A := F(c_A^*)$ and $p_B := F(c_B^*)$. Further, $\lambda(i|\omega)$, for $i \in \{A, B\}$ and $\omega \in \{\alpha, \beta\}$, are functions of p_A, p_B, and the pivot probabilities are functions of $\lambda(i|\omega)$. This defines the pivot probabilities as functions of the cost cutoffs. Note that all above functions are continuous, and so are the cost cutoffs as functions of the pivot probabilities.

Define $h, g : [0, \bar{c}] \times [0, \bar{c}] \to [0, \bar{c}] \times [0, \bar{c}]$, with $c_A^* =: h(c_A, c_B)$, $c_B^* =: g(c_A, c_B)$. Since $[0, \bar{c}] \times [0, \bar{c}]$ is a compact convex subset of $\mathbb{R}^2$, Brouwer's fixed-point theorem guarantees the existence of cutoff costs c_A^*, c_B^* that simultaneously satisfy $c_A^* = h(c_A^*, c_B^*)$ and $c_B^* = g(c_A^*, c_B^*)$. This establishes the existence of the voting equilibrium.

To see that the equilibrium cutoffs must be interior, i.e., $c_A^*, c_B^* \in (0, \bar{c})$, assume first, by contradiction and without loss of generality, that $c_A^* = 0$. Then, $p_A = 0$. In this case, an A supporter is pivotal if either no B supporter or if exactly one B supporter shows up at the ballot. But then, the probability of a vote being pivotal for candidate A is strictly positive in both states, implying a positive cost cutoff c_A^*, which is the desired contradiction. Secondly, by equation (1), and since $\bar{c} \geq v$, $c_A^* < \bar{c}$, so
$p_A = F(c_A^*) = 1$ is impossible. □

For the equilibrium analysis, it is crucial to understand the influence of the voters' beliefs about the state of the world on their participation rates. Consider a sequence of equilibria with the corresponding sequence of equilibrium participation rates $(p_A(n), p_B(n))_n$. Lemma 1 establishes that along all equilibrium sequences, participation rates coincide if and only if the voters hold their prior beliefs. Intuitively, since the prior beliefs are symmetric, both A and B supporters have the same incentives to participate in the election.[13] Therefore, the participation rates must coincide.

Lemma 1. *Along all equilibrium sequences, the participation rates of A and B supporters coincide for all n if and only if voters hold their prior beliefs. That is, for all n, $p_A(n) = p_B(n)$ if and only if $\Pr(\alpha|A, \tau) = \Pr(\alpha|A) = q$ and $\Pr(\beta|B, \tau) = \Pr(\beta|B) = q$.*

4.2 Pivot Probabilities

Having established the common properties of all voting equilibria, let me now turn to the limiting case of a large election. That is, for the remainder of the paper, I assume that n goes to infinity. I start with rather technical results which will allow the application of an approximation of the pivot probabilities, making the model more tractable. All subsequent results will rely on this approximation.

Lemma 2 establishes that in the limit, as n goes to infinity, the participation rates must go to zero. To see why this is the case, suppose that, to the contrary, participation rates

[13] Note that upon learning his or her own preferences, any voter believes to be in the majority. For this, see also the derivations in Appendix B.

are strictly positive in the limit. Then, the probability of being pivotal goes to zero in the limit; with this, the gross benefit of voting also disappears. But this means that any randomly drawn voter would be better off by abstaining than by voting, contradicting positive participation rates.

Lemma 2. *As $n \to \infty$, the participation rates p_A, p_B go to zero along every sequence of equilibria, that is,* $\limsup_{n\to\infty} p_A(n) = \limsup_{n\to\infty} p_B(n) = 0$.

Lemma 3 reveals that the participation rates converge slowly enough to zero such that expected conditional votes nevertheless go to infinity as n grows large. For this result, it is essential that zero is in the support of the cost distribution. To see this, suppose total turnout were finite in the limit. Then, the pivot probabilities would have strictly positive limits, yielding a strictly positive gross benefit of voting. If costs can arbitrarily become close to zero, there will be voters who are better off voting than abstaining—implying strictly positive participation rates and contradicting finite turnout as n goes to infinity. However, if costs are bounded away from zero, even with a strictly positive gross benefit of voting, abstaining might be more profitable than voting, yielding finite turnout.

Lemma 3. *As $n \to \infty$, the expected conditional votes $\lambda(A|\alpha)$, $\lambda(B|\alpha)$, $\lambda(A|\beta)$, $\lambda(B|\beta)$ go to infinity. That is,* $\liminf_{n\to\infty} \lambda(i|\omega) = \infty$ *for $i \in \{A, B\}$ and $\omega \in \{\alpha, \beta\}$. Further, the participation rates are of the same order of magnitude, that is,* $\liminf_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 0$ *and* $\limsup_{n\to\infty} \frac{p_A(n)}{p_B(n)} < \infty$.

Given these properties in large elections, the pivot probabilities can be approximated by employing modified Bessel functions (cf. Abramowitz and Stegun (1965)), as suggested by Myerson (2000).

Lemma 4. *As $n \to \infty$,*

$$\Pr(Piv_A|\omega) \approx \frac{1}{2}\frac{e^{-\left(\sqrt{\lambda(A|\omega)}-\sqrt{\lambda(B|\omega)}\right)^2}}{\sqrt{4\pi\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}\left(1+\sqrt{\frac{\lambda(B|\omega)}{\lambda(A|\omega)}}\right), \tag{3}$$

$$\Pr(Piv_B|\omega) \approx \frac{1}{2}\frac{e^{-\left(\sqrt{\lambda(A|\omega)}-\sqrt{\lambda(B|\omega)}\right)^2}}{\sqrt{4\pi\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}\left(1+\sqrt{\frac{\lambda(A|\omega)}{\lambda(B|\omega)}}\right). \tag{4}$$

4.3 Underdog Effect

Based on these approximations, I can prove three important results: (i) the limit ratio of participation rates reflects the underdog effect; (ii) this limit ratio is unique in large elections; and (iii) the underdog effect is monotonic in beliefs, meaning that the limit ratio of participation rates is monotonic in the posteriors.

The underdog effect captures that the supporters of the expected underdog (i.e., the voters who are expected to be in the minority) participate with higher probability than the supporters of the expected leader. This is already a well-known result given the assumption that there is only idiosyncratic uncertainty about voters' preferences. Evren (2012), Myatt (2015), and Myatt (2017) also observe this result.

Consider the posterior probabilities $\Pr(\alpha|\tau), \Pr(\beta|\tau)$. Supporters of candidate A are the expected minority if and only if $\Pr(\alpha|\tau) < \Pr(\beta|\tau)$.[14]

Proposition 2 (Underdog effect)**.** *Fix some posterior probabilities* $\Pr(\alpha|\tau)$*,* $\Pr(\beta|\tau)$*. Along all equilibrium sequences,*

1. *if* $\Pr(\alpha|\tau) > \Pr(\beta|\tau)$*,* $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$*, and*
2. *if* $\Pr(\alpha|\tau) < \Pr(\beta|\tau)$*,* $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$*.*

For an intuition, consider the effect of a vote for the expected underdog compared to the expected leader. A vote for the leader increases the expected margin between the candidates and pushes the election further away from a tie. In contrast, a vote for the expected underdog decreases the margin and increases the probability of an election toss-up. Thus, a vote for the underdog is pivotal with higher probability, yielding a higher expected benefit of voting for the supporters of the underdog. Consequently, the supporters of the expected minority candidate turn out at higher rates. Further note that the turnout decision is related to a public goods problem, since (costly) voting is comparable to contributing to the public good. Therefore, just as in the public goods problem, there is an incentive to free ride on the participation of like-minded voters. The underdog effect can be interpreted as a situation in which the free-riding problem is less pronounced among the minority.

Proposition 3 establishes the uniqueness of the limit ratio of participation rates.

Proposition 3 (Uniqueness)**.** *The limit of the ratio of participation rates,* $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$*, is unique.*

I can now prove that in a large election, the underdog effect that is reflected by the unique limit ratio of participation rates is actually monotonic in the posterior beliefs.

Proposition 4 (Monotonicity)**.** *In a large election, the limit of the ratio of participation rates of the* A *supporters relative to* B *supporters,* $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$*, is strictly decreasing in* $\Pr(\alpha|\tau)$*.*

Consider two poll results τ and τ', and suppose that the state is more likely to be α upon observing τ, than when observing τ'. Then, the limit of the ratio of participation rates of A supporters over those of B supporters is lower under posterior beliefs induced by τ than under posterior beliefs induced by τ'. Intuitively, as posterior beliefs shift toward state α, supporters of candidate A become increasingly optimistic of their victory, leading them to adopt relatively ever lower cost thresholds compared to supporters of candidate B. This monotonicity will be the main driver of poll participants' incentives.

[14]If $\Pr(\alpha|\tau) = \Pr(\beta|\tau)$, then Lemma 1 has bite.

4.4 Election Outcome

Because the underdog effect favors the expected minority—attenuating the expected majority candidate's advantage through relatively lower turnout probabilities of the expected majority—one might worry about the implications for election outcomes. That is, one might now worry that the underdog will be more likely to win the election because of this effect. Indeed, in Goeree and Großer (2007), the underdog effect yields a toss-up election, in which both candidates are equally likely to win.

By contrast, in my model, the *partial underdog compensation* result by Herrera et al. (2014) carries over to the present setting with aggregate uncertainty about the distribution of preferences. The advantage of the majority candidate is only partially attenuated by the increased turnout of the minority. Therefore, in both states of the world, the majority candidate wins the election almost surely. The result holds for any fixed posterior belief induced by any poll. In particular, the result is independent of the poll size, the polling outcome, and the poll participants' strategies.

Proposition 5. *As $n \to \infty$, in each state of the world, the majority candidate will win the election almost surely, regardless of the poll result. That is, the probability that, in the limit, A will win in state α, and candidate B will win in state β, is 1.*

For an intuition, assume that the state of the world is α, and hence, candidate A is the majority candidate. If posterior beliefs (mistakenly) indicate that the state of the world is more likely to be β, i.e., $\Pr(\beta|\tau) > \frac{1}{2}$, it holds that $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$. So, candidate A will win the election with probability one because A is preferred by the majority and, at the same time, A supporters turn out at higher rates.

The more intricate case is one in which beliefs accurately reflect that the state of the world is more likely to be α, i.e., $\Pr(\alpha|\tau) > \frac{1}{2}$. Here, in the limit, the underdog effect leads B supporters to turn out at higher rates. Proposition 4 implies that the limit of the ratio of participation rates is monotonic in the beliefs. Therefore, p_A is lowest relative to p_B if the beliefs are such that citizens are convinced that the state of the world is α—that is, if the aggregate uncertainty about the state of the world is completely resolved. Yet, this is equivalent to a model setup in which the state of the world is known from the outset. For this setup it has already been shown that the underdog effect is only partial (e.g., Herrera et al. (2014)). Intuitively, if the underdog effect were fully compensating such that expected vote shares would be equal, both groups of voters would have the same cost cutoffs because of equal pivot probabilities. Given that beliefs favor one state of the world, this then contradicts equal expected vote shares. Finally, if the majority candidate almost surely wins at the relatively lowest participation rates of the majority, he or she will win for all intermediate cases as well.

5 Polling Equilibrium

Having analyzed the equilibrium of the voting subgame for any induced posterior belief, let me now focus on the polling stage. To isolate effects, I first assume that all citizens participating in the opinion poll answer truthfully such that τ represents the participants' true underlying preferences. Understanding the way voters react to exogenously truthful information about the state of the world is necessary to understand poll participants' incentives.

5.1 Truthful Reporting

Assuming truthful reporting in the poll, how do different poll results translate into posterior beliefs and, eventually, into participation rates? Consider first the posterior beliefs that are induced if a poll of fixed size is assumed to be answered truthfully, its result being given by (τ_A, τ_B). The derivations in Appendix B reveal that a lead for candidate A in the poll induces posteriors according to which state α is more likely than state β, and that these beliefs are monotonic in the poll's margin. More formally, $\Pr(\alpha|\tau) > \Pr(\beta|\tau)$ if and only if $\tau_A > \tau_B$; and $\Pr(\alpha|\tau)$ is increasing in the poll's margin $\tau_A - \tau_B$ (Claim 3). Lastly, if the poll is balanced such that $\tau_A = \tau_B$, posterior beliefs are the same as if no poll had been released. Applying now Lemma 1 and Proposition 2 yields that if $\tau_A = \tau_B$, A and B supporters will turn out at the same rates, and if the poll favors candidate A, A supporters will turn out at lower rates than B supporters, and vice versa. This is summarized in Corollary 1.

Corollary 1. *Fix a poll of size m. Assume that polled agents state their preferences truthfully and that the poll result is given by $\tau = (\tau_A, \tau_B)$. Then,*

1. *If no poll is conducted ($m = 0$), or if $\tau_A = \tau_B$, then $p_A(n) = p_B(n)$ for all n.*
2. *If $\tau_A > \tau_B$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$.*
3. *If $\tau_A < \tau_B$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$.*

To summarize, the supporters of the trailing candidate turn out at higher rates. Finally, applying Proposition 4 yields that in a large election, this relation is monotonic in the poll result: As the perceived support of candidate A in comparison to candidate B increases—which is measured by an increasing margin $\tau_A - \tau_B$—the participation rate of A supporters relative to the participation rate of B supporters decreases. The result is summarized in Corollary 2.

Corollary 2. *Fix the poll size $m = \tau_A + \tau_B$, and assume that poll participants state their preferences truthfully. Then, in a large election, the limit of the ratio of participation rates of the A supporters relative to B supporters is strictly increasing in the margin of the poll $\tau_B - \tau_A$, i.e., $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$ is a strictly increasing function of $\tau_B - \tau_A$.*

Importantly, these results imply that each vote in the poll affects the participation rates unambiguously. A vote for candidate A in the poll decreases the participation rate of A supporters relative to the participation rate of B supporters in the limit.

5.2 Incentives

In Section 5.1, the analysis was based on the premise that citizens participating in the pre-election poll state their preferences truthfully. Suppose now that all poll participants are strategic. Consider the incentives of an A supporter who is questioned by a pollster. Even though poll participants are excluded from the main election, they still have the same stakes concerning the election winner. Therefore, the A supporter seeks to maximize the probability that A wins, which is increasing in $\frac{p_A}{p_B}$. If the A supporter assumes that all other poll participants answer truthfully, Corollary 1 and Corollary 2 together imply that an additional vote for A in the poll decreases the limit of $\frac{p_A}{p_B}$ in the election. This reveals an incentive to misrepresent the preferences in the poll and yields that truthtelling cannot be an equilibrium. Intuitively, if an A supporter claims to prefer candidate B in the poll, he or she increases free-riding among the B supporters, and simultaneously decreases free-riding among the A supporters, shifting $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$ in a favorable direction.

Likewise, there cannot exist an equilibrium in which everybody misrepresents their preferences because this would be understood and the true preferences could be worked out by the electorate, giving an incentive to deviate to revealing preferences truthfully.

Say that there is information transmission if the voters update their beliefs about the state of the world after observing the poll's publication such that $\Pr(\alpha|\tau) \neq \Pr(\beta|\tau)$, and call the corresponding equilibrium of the polling stage informative. Proposition 6 states that there does not exist an informative equilibrium of the polling stage. Consequently, there can only exist the babbling equilibrium and the poll is discarded by the electorate.

Proposition 6. *The babbling equilibrium is the unique equilibrium of the polling stage.*

The implications of Proposition 6 are as follows: If it is reasonable to assume that all citizens participating in a poll are strategic, the poll result does not convey any information. Thus, eligible voters can ignore it. As a result, the beliefs of the electorate coincide with their prior beliefs. Lemma 1 implies that in this case, for all n and along all equilibrium sequences, the probability of turning out to vote is the same for both A and B supporters. Therefore, by the law of large numbers, in each state, the probability that the majority candidate wins goes to one as n grows large. With this in mind, if the poll participants behave strategically—behavior which undermines the purpose of the poll—the results do not reduce the probability that the majority candidate will be elected.

Appendix C extends this result by assuming that a fixed share of poll participants is exogenously truthful. Again, strategic poll participants never have an incentive to truthfully reveal their preferences. Yet, the poll is informative if and only if the share of truthful agents is strictly larger than one-half. In any case, Proposition 5 applies, and, in the limit, the majority candidate almost surely wins the election.

6 Conclusion

This paper analyzes the effect of pre-election polls on election outcomes. I analyze how information revealed through polls affects the participation decision of citizens in large elections, and the incentives this yields for poll participants. My analysis relies on a framework with aggregate uncertainty about the distribution of preferences, in which the poll is conducted to resolve the aggregate uncertainty; voting is voluntary and costly, and the cost is drawn from a smooth cost distribution.

My main findings are that for any posterior belief about the state of the world induced by the poll (i) there exists a unique limit of the ratio of participation rates. This limit ratio (ii) reflects the underdog effect, and (iii) it is monotonic in the posterior belief. However, the limit ratio of participation rates is (iv) closer to one than the ratio of the respective population shares of supporters of candidates A and B, such that, in the limit, the majority candidate almost surely wins for any given belief. Given the underdog effect and its monotonicity in beliefs, (v) citizens participating in the poll always have an incentive to avoid truthfully reporting their preferences. There does not exist an equilibrium in which the poll provides any information transmission.

My findings contrast with those of Goeree and Großer (2007), who study the effect of exogenously truthful information about the state of the world in a framework in which voting costs are homogenous. This assumption on the cost of voting implies that the authors obtain a *full underdog effect*, where the increased turnout by the minority completely offsets the majority's initial advantage. Because the cost of voting is set such that turnout is incomplete and positive, voters are employing mixed strategies in equilibrium. To be willing to mix, the expected benefit of voting needs to equal the voting cost. Therefore, given homogeneous costs for all voters, expected benefits of voting coincide for all voters. These can only be the same if the expected vote shares coincide, yielding the full underdog effect.[15]

However, if the cost distribution is smooth, and its support is bounded below by zero, cost cutoffs are interior. If expected vote shares would coincide, the participation rates would also be the same, contradicting equal vote shares, because there is a strict majority. The underdog effect can therefore only be *partial*. This has already been observed for idiosyncratic preference uncertainty, e.g. Herrera et al. (2014), and under different forms of aggregate uncertainty, e.g. Evren (2012) or Myatt (2015). Note that these assumptions on the voting costs allow the inclusion of voters who vote because of a sense of duty or ethical reasons. Thus, the assumptions capture the potential for voting costs to differ

[15]The same result is obtained if costs are continuously distributed but bounded away from zero. Then, only those voters with a cost realization at the lower bound turn out to vote, and the argument boils down to the one with fixed costs. This is the case in Taylor and Yildirim (2010b).

across voters.

The full underdog effect implies that both candidates are equally likely to win the election, supporting the conclusion that polls are detrimental to welfare. Goeree and Großer (2007) conclude that their results may explain why several countries impose a black-out period prior to elections. By contrast, I show that the partial underdog effect does not result in such a toss-up election. Rather, for any posterior belief induced by the poll, the majority candidate almost surely wins in the limit. This includes the extreme cases where the poll is either uninformative—and the game is as if no poll were conducted in the first place—or perfectly reveals the state of the world. Consequently, my work demonstrates that the conclusions of Goeree and Großer (2007) on the possibly drastic effect of polls do not hold if the model framework is slightly altered.

While, in my model, polls do not overturn election outcomes, polls still matter because the partial underdog effect has real implications. Affecting the turnout margin and vote shares, the partial underdog effect implies that referenda or elections will be closer than they truly are. If vote shares themselves have policy implications, this effect of polls might be concerning. This is an interesting topic which will be left for future research.

Appendix A Proofs

A.1 Preliminaries

For the subsequent analysis, it is useful to express the pivot probabilities in terms of modified Bessel functions (cf. Abramowitz and Stegun (1965) and Krishna and Morgan (2012)), which are defined as

$$I_0(z) = \sum_{k=0}^{\infty} \frac{(\frac{z}{2})^k (\frac{z}{2})^k}{k! \cdot k!}, \; I_1(z) = \sum_{k=1}^{\infty} \frac{(\frac{z}{2})^{k-1} (\frac{z}{2})^k}{(k-1)! \cdot k!}.$$

Reformulating the pivot probabilities yields for all $\omega \in \{\alpha, \beta\}$:

$$\begin{aligned}
\Pr(T|\omega) &= e^{-\lambda(A|\omega)-\lambda(B|\omega)} \cdot I_0\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right), \\
\Pr(T^A_{-1}|\omega) &= e^{-\lambda(A|\omega)-\lambda(B|\omega)} \cdot \sqrt{\frac{\lambda(B|\omega)}{\lambda(A|\omega)}} \cdot I_1\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right), \\
\Pr(T^B_{-1}|\omega) &= e^{-\lambda(A|\omega)-\lambda(B|\omega)} \cdot \sqrt{\frac{\lambda(A|\omega)}{\lambda(B|\omega)}} \cdot I_1\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right).
\end{aligned}$$

So,

$$\Pr(Piv_A|\omega) = \frac{1}{2}e^{-\lambda(A|\omega)-\lambda(B|\omega)}\left[I_0\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right) + \sqrt{\frac{\lambda(B|\omega)}{\lambda(A|\omega)}} \cdot I_1\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right)\right],$$
$$\Pr(Piv_B|\omega) = \frac{1}{2}e^{-\lambda(A|\omega)-\lambda(B|\omega)}\left[I_0\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right) + \sqrt{\frac{\lambda(A|\omega)}{\lambda(B|\omega)}} \cdot I_1\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right)\right].$$

For $z \to \infty$, Abramowitz and Stegun (1965) show that $I_0(z) \approx \frac{e^z}{\sqrt{2\pi z}} \approx I_1(z)$.[16]

A.2 Proof for Section 4.1

Proof of Lemma 1.
"If"
Recall that citizens update their beliefs about the state of the world upon observing their own type. It is derived in Appendix B that $\Pr(\alpha|A)$ $=\Pr(\beta|B) = q$.
Assume that the voters' posterior beliefs coincide with these prior beliefs, that is, $\Pr(\alpha|A,\tau) = q = \Pr(\beta|B,\tau)$.
Recall that the cost cutoffs are defined as follows

$$\Pr(\alpha|A,\tau) \cdot \Pr(Piv_A|\alpha) \cdot v + \Pr(\beta|A,\tau) \cdot \Pr(Piv_A|\beta) \cdot v = c_A^*(n),$$
$$\Pr(\alpha|B,\tau) \cdot \Pr(Piv_B|\alpha) \cdot v + \Pr(\beta|B,\tau) \cdot \Pr(Piv_B|\beta) \cdot v = c_B^*(n).$$

Suppose without loss of generality that along some subsequence, there exists n s.t. $p_A(n) > p_B(n) \Leftrightarrow c_A^*(n) > c_B^*(n)$. Suppressing the dependence on n,

$$\begin{aligned}
c_A^* - c_B^* &= q \cdot v \cdot [\Pr(Piv_A|\alpha) - \Pr(Piv_B|\beta)] + (1-q) \cdot v \cdot [\Pr(Piv_A|\beta) - \Pr(Piv_B|\alpha)] \\
&= \frac{1}{2} \cdot q \cdot v \cdot \Big\{ I_0\left(2\sqrt{n^2 q(1-q)p_A p_B}\right) \cdot \left(e^{-n[qp_A+(1-q)p_B]} - e^{-n[(1-q)p_A+qp_B]}\right) \\
&+ I_1\left(2\sqrt{n^2 q(1-q)p_A p_B}\right) \cdot \sqrt{\frac{1-q}{q}} \cdot \left(\sqrt{\frac{p_B}{p_A}}e^{-n[qp_A+(1-q)p_B]} - \sqrt{\frac{p_A}{p_B}}e^{-n[(1-q)p_A+qp_B]}\right)\Big\} \\
&+ \frac{1}{2}(1-q) \cdot v \cdot \Big\{ I_0\left(2\sqrt{n^2 q(1-q)p_A p_B}\right) \cdot \left(e^{-n[(1-q)p_A+qp_B]} - e^{-n[pqp_A+(1-q)p_B]}\right) \\
&+ I_1\left(2\sqrt{n^2 q(1-q)p_A p_B}\right) \cdot \sqrt{\frac{q}{1-q}} \cdot \left(\sqrt{\frac{p_B}{p_A}}e^{-n[(1-q)p_A+qp_B]} - \sqrt{\frac{p_A}{p_B}}e^{-n[qp_A+(1-q)p_B]}\right)\Big\}.
\end{aligned}$$

Rearranging yields

$$\begin{aligned}
&c_A^* - c_B^* \\
=&\frac{1}{2} \cdot v \Big\{ (2q-1) \cdot I_0\left(2\sqrt{n^2 q(1-q)p_A p_B}\right) \cdot \left(e^{-n[qp_A+(1-q)p_B]} - e^{-n[(1-q)p_A+qp_B]}\right) \\
&+ I_1\left(2\sqrt{n^2 q(1-q)p_A p_B}\right) \cdot \sqrt{q(1-q)} \cdot \left[\left(\sqrt{\frac{p_B}{p_A}} - \sqrt{\frac{p_A}{p_B}}\right) \cdot \left(e^{-n[qp_A+(1-q)p_B]} + e^{-n[(1-q)p_A+qp_B]}\right)\right]\Big\} \\
<&0,
\end{aligned}$$

[16]Suppose x_n and y_n are functions of n. Then, $x_n \approx y_n$ indicates that $\lim_{n\to\infty} \frac{x_n}{y_n} = 1$

contradicting the assumption that $c_A^*(n) > c_B^*(n)$.
The inequality holds, since $e^{-n[qp_A+(1-q)p_B]} < e^{-n[(1-q)p_A+qp_B]}$, $q > \frac{1}{2}$, and $\sqrt{\frac{p_B}{p_A}} < \sqrt{\frac{p_A}{p_B}}$ because of $p_A > p_B$.
$c_B^*(n) > c_A^*(n)$ is analogous. Thus, for all equilibrium sequences and for all n, $p_A(n) = p_B(n)$ if $\Pr(\alpha|A,\tau) = \Pr(\alpha|A) = \Pr(\beta|B) = \Pr(\beta|B,\tau)$.

"Only if"
Assume now that for all n, $p_A(n) = p_B(n) =: p$.

$$\begin{aligned}
0 &= c_A^*(n) - c_B^*(n) \\
&= \Pr(\alpha|A,\tau) \cdot \frac{1}{2} e^{-np} \cdot \left[I_0\left(2np\sqrt{q(1-q)}\right) + \sqrt{\frac{1-q}{q}} \cdot I_1\left(2np\sqrt{q(1-q)}\right) \right] \\
&\quad + \Pr(\beta|A,\tau) \cdot \frac{1}{2} e^{-np} \cdot \left[I_0\left(2np\sqrt{q(1-q)}\right) + \sqrt{\frac{q}{1-q}} \cdot I_1\left(2np\sqrt{q(1-q)}\right) \right] \\
&\quad - \Pr(\alpha|B,\tau) \cdot \frac{1}{2} e^{-np} \cdot \left[I_0\left(2np\sqrt{q(1-q)}\right) + \sqrt{\frac{q}{1-q}} \cdot I_1\left(2np\sqrt{q(1-q)}\right) \right] \\
&\quad - \Pr(\beta|B,\tau) \cdot \frac{1}{2} e^{-np} \cdot \left[I_0\left(2np\sqrt{q(1-q)}\right) + \sqrt{\frac{1-q}{q}} \cdot I_1\left(2np\sqrt{q(1-q)}\right) \right] \\
&= \frac{1}{2} e^{-np} \cdot \Big[I_0\left(2np\sqrt{q(1-q)}\right) \cdot \left(\Pr(\alpha|A,\tau) + \Pr(\beta|A,\tau) - \Pr(\alpha|B,\tau) - \Pr(\beta|B,\tau)\right) \\
&\quad + I_1\left(2np\sqrt{q(1-q)}\right) \cdot \left(\sqrt{\frac{1-q}{q}} \Pr(\alpha|A,\tau) + \sqrt{\frac{q}{1-q}} \Pr(\beta|A,\tau) - \sqrt{\frac{q}{1-q}} \Pr(\alpha|B,\tau) - \sqrt{\frac{1-q}{q}} \Pr(\beta|B,\tau) \right) \Big] \\
&= \frac{1}{2} e^{-np} \cdot I_1\left(2np\sqrt{q(1-q)}\right) \\
&\quad \cdot \left(\sqrt{\frac{1-q}{q}} \Pr(\alpha|A,\tau) + \sqrt{\frac{q}{1-q}} \Pr(\beta|A,\tau) - \sqrt{\frac{q}{1-q}} \Pr(\alpha|B,\tau) - \sqrt{\frac{1-q}{q}} \Pr(\beta|B,\tau) \right),
\end{aligned}$$

where the last step follows from

$$\Pr(\alpha|A,\tau) + \Pr(\beta|A,\tau) - \Pr(\alpha|B,\tau) - \Pr(\beta|B,\tau) = 0.$$

Now,

$$\begin{aligned}
&\sqrt{\frac{1-q}{q}} \Pr(\alpha|A,\tau) + \sqrt{\frac{q}{1-q}} \Pr(\beta|A,\tau) - \sqrt{\frac{q}{1-q}} \Pr(\alpha|B,\tau) - \sqrt{\frac{1-q}{q}} \Pr(\beta|B,\tau) \\
=&\sqrt{\frac{1-q}{q}} \cdot [\Pr(\alpha|A,\tau) - \Pr(\beta|B,\tau)] + \sqrt{\frac{q}{1-q}} \cdot [\Pr(\beta|A,\tau) - \Pr(\alpha|B,\tau)] \\
=&\sqrt{\frac{1-q}{q}} \cdot [\Pr(\alpha|A,\tau) - \Pr(\beta|B,\tau)] + \sqrt{\frac{q}{1-q}} \cdot [(1 - \Pr(\alpha|A,\tau)) - (1 - \Pr(\beta|B,\tau))] \\
=&\sqrt{\frac{1-q}{q}} \cdot [\Pr(\alpha|A,\tau) - \Pr(\beta|B,\tau)] + \sqrt{\frac{q}{1-q}} \cdot [\Pr(\beta|B,\tau) - \Pr(\alpha|A,\tau)] \\
=&[\Pr(\alpha|A,\tau) - \Pr(\beta|B,\tau)] \cdot \left(\sqrt{\frac{1-q}{q}} + \sqrt{\frac{q}{1-q}} \right).
\end{aligned}$$

This term is zero if and only if $\Pr(\alpha|A,\tau) = \Pr(\beta|B,\tau)$. Since $I_1(\cdot) > 0$, it follows that $p_A(n) = p_B(n) = 0$ holds if and only if $\Pr(\alpha|A,\tau) = \Pr(\beta|B,\tau)$. In Appendix B, I show that this is equivalent to $\Pr(\tau|\alpha) = \Pr(\tau|\beta)$. Thus, the participation rates coincide only if the poll is uninformative and the posterior beliefs are equal to the prior beliefs about the state of the world. $\square$

A.3 Proofs for Section 4.2

Proof of Lemma 2.
Assume, by contradiction, that, along some subsequence, $\lim_{n\to\infty} c_A^*(n) > 0$, implying $\lim_{n\to\infty} p_A(n) > 0$. Then,

$$\lim_{n\to\infty} \Pr(\alpha|A,\tau)\cdot\Pr(Piv_A|\alpha)\cdot v + \Pr(\beta|A,\tau)\cdot\Pr(Piv_A|\beta)\cdot v > 0,$$

suppressing the dependence on n for notational simplicity.

Since $\lim_{n\to\infty} p_A(n) > 0$, $\lambda(A|\alpha), \lambda(A|\beta) \to \infty$.

If $\lim_{n\to\infty} \sqrt{\lambda(A|\omega)\cdot\lambda(B|\omega)} < \infty$, $\Pr(Piv_A|\omega) \to 0$, since $e^{-\lambda(A|\omega)-\lambda(B|\omega)} \to 0$, $I_0\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right)$ is finite and $e^{-\lambda(A|\omega)-\lambda(B|\omega)}\sqrt{\frac{\lambda(B|\omega)}{\lambda(A|\omega)}} \to 0$.

If $\lim_{n\to\infty} \sqrt{\lambda(A|\omega)\cdot\lambda(B|\omega)} = \infty$, the modified Bessel functions can be approximated by

$$I_0\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right) \approx \frac{e^{2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}{\sqrt{2\pi\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right)}} \approx I_1\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right),$$

yielding

$$\Pr(Piv_A|\omega) \to \frac{1}{2}\frac{e^{-\left(\sqrt{\lambda(A|\omega)}-\sqrt{\lambda(B|\omega)}\right)^2}}{\sqrt{4\pi\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}\left(1+\sqrt{\frac{\lambda(B|\omega)}{\lambda(A|\omega)}}\right).$$

This probability converges to 0 in both states of the world, since the denominator is unbounded, whereas the numerator is bounded.
But then, in any case,

$$\lim_{n\to\infty} \Pr(\alpha|A,\tau)\cdot\Pr(Piv_A|\alpha)\cdot v + \Pr(\beta|A,\tau)\cdot\Pr(Piv_A|\beta)\cdot v = 0,$$

contradicting $\lim_{n\to\infty} p_A(n) > 0$. □

Proof of Lemma 3.
Recall that

$$\begin{aligned}
\lambda(A|\alpha) &= n\cdot q\cdot p_A,\\
\lambda(A|\beta) &= n\cdot(1-q)\cdot p_A,\\
\lambda(B|\alpha) &= n\cdot(1-q)\cdot p_B,\\
\lambda(B|\beta) &= n\cdot q\cdot p_B.
\end{aligned}$$

Assume, by contradiction, that it is not true that $\forall i \in \{A,B\}\ \forall\omega\in\{\alpha,\beta\}$,

$\liminf_{n\to\infty} \lambda(i|\omega) = \infty$.
Suppose first that along some subsequence, $\lambda(A|\alpha), \lambda(A|\beta), \lambda(B|\alpha), \lambda(B|\beta) < \infty$ as $n \to \infty$, i.e., the expected number of votes for each candidate is finite in each state.
Then, along this subsequence, the pivot probabilities are strictly positive in every state: $\Pr(Piv_A|\omega) > 0, \Pr(Piv_B|\omega) > 0\ \forall\omega$.

This implies that $\lim_{n\to\infty} c_i^*(n) > 0$ for $i \in \{A, B\}$, and, given the assumptions on the cdf F and the corresponding density f, the participation rates p_A, p_B must remain strictly positive in the limit as $n \to \infty$: $\lim_{n\to\infty} p_i(n) > 0$ for $i \in \{A, B\}$. But then, expected turnout must go to infinity for every candidate in every state as $n \to \infty$—a contradiction.

Suppose now that along some subsequence, $\lambda(A|\alpha) < \infty$ and $\lambda(B|\alpha) \to \infty$ as $n \to \infty$.
Given the definitions of $\lambda(\cdot|\omega)$, this implies that $\lambda(A|\beta) < \infty$ and $\lambda(B|\beta) \to \infty$ as $n \to \infty$.
Consider $\lim_{n\to\infty} \frac{\lambda(B|\beta)}{\lambda(A|\alpha)}$:

$$\lim_{n\to\infty} \frac{\lambda(B|\beta)}{\lambda(A|\alpha)} = \lim_{n\to\infty} \frac{n \cdot q \cdot p_B(n)}{n \cdot q \cdot p_A(n)} = \lim_{n\to\infty} \frac{p_B(n)}{p_A(n)}.$$

Since $\lim_{n\to\infty} p_i(n) = 0$, $i \in \{A, B\}$, a Taylor expansion of F around zero yields

$$\begin{aligned}
\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} &= \lim_{n\to\infty} \frac{F(c_A^*(n))}{F(c_B^*(n))} \\
&\approx \lim_{n\to\infty} \frac{F(0) + f(0) \cdot (c_A^*(n) - 0) + \frac{1}{2} f'(0)(c_A^*(n) - 0)^2 + ...}{F(0) + f(0) \cdot (c_B^*(n) - 0) + \frac{1}{2} f'(0)(c_B^*(n) - 0)^2 + ...} \\
&\approx \lim_{n\to\infty} \frac{f(0) \cdot c_A^*(n)}{f(0) \cdot c_B^*(n)} \\
&= \lim_{n\to\infty} \frac{c_A^*(n)}{c_B^*(n)}.
\end{aligned}$$

By the assumption above, $\lim_{n\to\infty} \frac{\lambda(B|\beta)}{\lambda(A|\alpha)} = \infty$ and this implies

$$\lim_{n\to\infty} \frac{c_B^*(n)}{c_A^*(n)} = \infty.$$

For ease of exposition, define for the following step $z := \lambda(A|\alpha)$ and $y := \lambda(B|\beta)$ and observe that $\lambda(A|\beta) = \frac{1-q}{q} z$ and $\lambda(B|\beta) = \frac{q}{1-q} y$. Since $\lambda(B|\beta)/\lambda(A|\alpha) \to \infty$, $y/z \to \infty$.
Working towards a contradiction, derive an expression for $\lim_{n\to\infty} \frac{c_B^*(n)}{c_A^*(n)}$.

$$
\begin{aligned}
&\lim_{n\to\infty}\frac{c^*_B(n)}{c^*_A(n)}\\
=&\lim_{n\to\infty}\frac{\Pr(\alpha|B,\tau)\cdot\Pr(Piv_B|\alpha)+\Pr(\beta|B,\tau)\cdot\Pr(Piv_B|\beta)}{\Pr(\alpha|A,\tau)\cdot\Pr(Piv_A|\alpha)+\Pr(\beta|A,\tau)\cdot\Pr(Piv_A|\beta)}\\
=&\lim_{\frac{y}{z}\to\infty}\frac{\Pr(\alpha|B,\tau)\frac{1}{2}\frac{e^{-(\sqrt{z}-\sqrt{y})^2}}{\sqrt{4\pi\sqrt{zy}}}\left(1+\sqrt{\frac{z}{y}}\right)+\Pr(\beta|B,\tau)\frac{1}{2}\frac{e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2}}{\sqrt{4\pi\sqrt{zy}}}\left(1+\frac{1-q}{q}\sqrt{\frac{z}{y}}\right)}{\Pr(\alpha|A,\tau)\frac{1}{2}\frac{e^{-(\sqrt{z}-\sqrt{y})^2}}{\sqrt{4\pi\sqrt{zy}}}\left(1+\sqrt{\frac{y}{z}}\right)+\Pr(\beta|A,\tau)\frac{1}{2}\frac{e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2}}{\sqrt{4\pi\sqrt{zy}}}\left(1+\frac{q}{1-q}\sqrt{\frac{y}{z}}\right)}\\
=&\lim_{\frac{y}{z}\to\infty}\frac{\Pr(\alpha|B,\tau)e^{-(\sqrt{z}-\sqrt{y})^2}\left(1+\sqrt{\frac{z}{y}}\right)+\Pr(\beta|B,\tau)e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2}\left(1+\frac{1-q}{q}\sqrt{\frac{z}{y}}\right)}{\Pr(\alpha|A,\tau)e^{-(\sqrt{z}-\sqrt{y})^2}\left(1+\sqrt{\frac{y}{z}}\right)+\Pr(\beta|A,\tau)e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2}\left(1+\frac{q}{1-q}\sqrt{\frac{y}{z}}\right)}\\
=&\lim_{\frac{y}{z}\to\infty}\frac{\Pr(\alpha|B,\tau)\left(1+\sqrt{\frac{z}{y}}\right)+\Pr(\beta|B,\tau)e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2+(\sqrt{z}-\sqrt{y})^2}\left(1+\frac{1-q}{q}\sqrt{\frac{z}{y}}\right)}{\Pr(\alpha|A,\tau)\left(1+\sqrt{\frac{y}{z}}\right)+\Pr(\beta|A,\tau)e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2+(\sqrt{z}-\sqrt{y})^2}\left(1+\frac{q}{1-q}\sqrt{\frac{y}{z}}\right)}\\
\leq&1.
\end{aligned}
$$

The last step follows from $\lim_{\frac{y}{z}\to\infty} e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2+(\sqrt{z}-\sqrt{y})^2}=0$ and $Pr(\alpha|A,\tau)>Pr(\alpha|B,\tau)$. The second step follows from $y\to\infty$, allowing to apply the approximation of the modified Bessel functions.
Overall, $\lim_{n\to\infty}\frac{c^*_B(n)}{c^*_A(n)}\leq 1$ contradicts $\lim_{n\to\infty}\frac{c^*_B(n)}{c^*_A(n)}=\infty$.
The case in which $\lambda(B|\omega)<\infty$ and $\lambda(A|\omega)\to\infty$ is analogous.
Therefore, it must be the case that expected turnout goes to infinity for each candidate and in each state when n goes to infinity.
From the last part of the proof it follows immediately that it is not possible that along some subsequence either $\liminf_{n\to\infty}\frac{p_A(n)}{p_B(n)}=0$ or that $\limsup_{n\to\infty}\frac{p_A(n)}{p_B(n)}=\infty$, proving the lemma. $\square$

Proof of Lemma 4.
Since Lemma 3 implies that $2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\to\infty$ as $n\to\infty$, by Abramowitz and Stegun (1965), it holds that

$$
I_0\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right)\approx\frac{e^{2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}{\sqrt{2\pi\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right)}}\approx I_1\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right).
$$

Therefore, the pivot probabilities can indeed be approximated by

$$\Pr(Piv_A|\omega) \approx \frac{1}{2}\frac{e^{-\left(\sqrt{\lambda(A|\omega)}-\sqrt{\lambda(B|\omega)}\right)^2}}{\sqrt{4\pi\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}\left(1+\sqrt{\frac{\lambda(B|\omega)}{\lambda(A|\omega)}}\right),$$
$$\Pr(Piv_B|\omega) \approx \frac{1}{2}\frac{e^{-\left(\sqrt{\lambda(A|\omega)}-\sqrt{\lambda(B|\omega)}\right)^2}}{\sqrt{4\pi\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}\left(1+\sqrt{\frac{\lambda(A|\omega)}{\lambda(B|\omega)}}\right).$$

□

A.4 Proofs for Section 4.3

Proof of Proposition 2.
Before commencing the proof, let me state some preliminary claims.
Preliminaries:
By the Taylor expansion from the proof of Lemma 3,

$$\lim_{n\to\infty}\frac{p_A(n)}{p_B(n)} = \lim_{n\to\infty}\frac{c_A^*(n)}{c_B^*(n)}.$$

Thus, suppressing the dependence of p_i on n, for $n \to \infty$,

$$\frac{p_A}{p_B} \approx \frac{Pr(\alpha|A,\tau)e^{-(\sqrt{nqp_A}-\sqrt{n(1-q)p_B})^2}\left(1+\sqrt{\frac{1-q}{q}}\sqrt{\frac{p_B}{p_A}}\right)+Pr(\beta|A,\tau)e^{-(\sqrt{n(1-q)p_A}-\sqrt{nqp_B})^2}\left(1+\sqrt{\frac{q}{1-q}}\sqrt{\frac{p_B}{p_A}}\right)}{Pr(\alpha|B,\tau)e^{-(\sqrt{nqp_A}-\sqrt{n(1-q)p_B})^2}\left(1+\sqrt{\frac{q}{1-q}}\sqrt{\frac{p_A}{p_B}}\right)+Pr(\beta|B,\tau)e^{-(\sqrt{n(1-q)p_A}-\sqrt{nqp_B})^2}\left(1+\sqrt{\frac{1-q}{q}}\sqrt{\frac{p_A}{p_B}}\right)}.$$

If $qp_A \neq (1-q)p_B$, $\left(\sqrt{nqp_A}-\sqrt{n(1-q)p_B}\right)^2$ diverges for $n\to\infty$, and
if $(1-q)p_B \neq qp_B$, $\left(\sqrt{n(1-q)p_A}-\sqrt{nqp_B}\right)^2$ diverges for $n\to\infty$, given that $q\neq\frac{1}{2}$.
This implies that for $n\to\infty$

$$p_A > p_B \Rightarrow e^{-\left(\sqrt{nqp_A}-\sqrt{n(1-q)p_B}\right)^2+\left(\sqrt{n(1-q)p_A}-\sqrt{nqp_B}\right)^2} \to 0, \tag{5}$$
$$p_A < p_B \Rightarrow e^{-\left(\sqrt{n(1-q)p_A}-\sqrt{nqp_B}\right)^2+\left(\sqrt{nqp_A}-\sqrt{n(1-q)p_B}\right)^2} \to 0. \tag{6}$$

I will now prove the proposition.
Assume without loss that candidate A is the underdog, that is, $\Pr(\alpha|\tau) < \Pr(\beta|\tau)$ and suppose, by contradiction, that $\lim_{n\to\infty}\frac{p_A(n)}{p_B(n)} \leq 1$.

The assumption that $Pr(\alpha|\tau) < Pr(\beta|\tau)$ together with Lemma 1 imply that for all n, $p_A(n) \neq p_B(n)$. Therefore, it must hold that for n sufficiently large, $p_A(n) < p_B(n)$.

By (6), as $n \to \infty$,

$$\frac{p_A(n)}{p_B(n)} \to \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \frac{\frac{\sqrt{qp_A(n)}+\sqrt{(1-q)p_B(n)}}{\sqrt{qp_A(n)}}}{\frac{\sqrt{qp_A(n)}+\sqrt{(1-q)p_B(n)}}{\sqrt{(1-q)p_B(n)}}}$$
$$= \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \sqrt{\frac{1-q}{q}} \cdot \sqrt{\frac{p_B(n)}{p_A(n)}}.$$

However,

$$\frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} = \frac{Pr(A|\alpha)}{Pr(B|\alpha)} \cdot \frac{Pr(B|\alpha)Pr(\tau|\alpha) + Pr(B|\beta)Pr(\tau|\beta)}{Pr(A|\alpha)Pr(\tau|\alpha) + Pr(A|\beta)Pr(\tau|\beta)}$$
$$= \frac{q}{1-q} \cdot \frac{(1-q)Pr(\tau|\alpha) + q \cdot Pr(\tau|\beta)}{q \cdot Pr(\tau|\alpha) + (1-q)Pr(\tau|\beta)}$$
$$> \frac{q}{1-q},$$

where the last inequality holds, since given Claim 1,

$$Pr(\alpha|\tau) < Pr(\beta|\tau) \;\Leftrightarrow\; Pr(\tau|\alpha) < Pr(\tau|\beta)$$
$$\Rightarrow \quad \frac{(1-q)Pr(\tau|\alpha) + q \cdot Pr(\tau|\beta)}{q \cdot Pr(\tau|\alpha) + (1-q)Pr(\tau|\beta)} > 1.$$

Therefore,

$$\frac{p_A(n)}{p_B(n)} \to \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \sqrt{\frac{1-q}{q}} \cdot \sqrt{\frac{p_B(n)}{p_A(n)}}$$
$$> \sqrt{\frac{q}{1-q}} \cdot \sqrt{\frac{p_B(n)}{p_A(n)}}$$
$$> 1,$$

a contradiction to $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} \leq 1$!
The proof for the case in which B is the underdog is analogous and therefore omitted. □

Proof of Proposition 3.
From Proposition 2 and Lemma 1, along all equilibrium sequences, either $p_A(n) = p_B(n)$ for all n, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$ or $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$.

If $p_A(n) = p_B(n)$, the claim obviously holds.

Next, assume that in equilibrium, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$.

Then, for n sufficiently large, $p_A(n) < p_B(n)$ and by (6), as $n \to \infty$,

$$\begin{aligned}
\frac{p_A(n)}{p_B(n)} &\approx \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \frac{\frac{\sqrt{qp_A(n)}+\sqrt{(1-q)p_B(n)}}{\sqrt{qp_A(n)}}}{\frac{\sqrt{(1-q)p_B(n)}+\sqrt{qp_A(n)}}{\sqrt{(1-q)p_B(n)}}} \\
&= \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \sqrt{\frac{1-q}{q}}\sqrt{\frac{p_B(n)}{p_A(n)}} \\
\Rightarrow \lim_{n\to\infty}\left(\frac{p_A(n)}{p_B(n)}\right)^{\frac{3}{2}} &= \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \sqrt{\frac{1-q}{q}}.
\end{aligned}$$

Since the left-hand side is strictly increasing in $\frac{p_A(n)}{p_B(n)}$ and the right-hand side is independent of $\frac{p_A(n)}{p_B(n)}$, the limit of the ratio of participation rates is unique for any equilibrium sequence that satisfies $\lim_{n\to\infty}\frac{p_A(n)}{p_B(n)} < 1$.

Analogously, the limit of the ratio of participation rates is unique for any equilibrium sequence that satisfies $\lim_{n\to\infty}\frac{p_A(n)}{p_B(n)} > 1$. $\square$

Proof of Proposition 4.
Consider two poll results τ, τ' such that $\Pr(\alpha|\tau') < \Pr(\alpha|\tau)$. Abusing notation, denote by p'_A, p'_B the equilibrium participation rates if the poll result is τ' and by p_A, p_B the participation rates if the poll result is τ. Recall that given some beliefs, the limit of the ratio of participation rates is unique. Thus, the limit of the ratio of participation rates is monotonic in the beliefs if and only if $\Pr(\alpha|\tau') < \Pr(\alpha|\tau)$ implies $\lim_{n\to\infty}\frac{p_A(n)}{p_B(n)} < \lim_{n\to\infty}\frac{p'_A(n)}{p'_B(n)}$.

Assume, by contradiction, that there exist poll results τ, τ' with $\Pr(\alpha|\tau') < \Pr(\alpha|\tau)$ such that $\lim_{n\to\infty}\frac{p_A(n)}{p_B(n)} \geq \lim_{n\to\infty}\frac{p'_A(n)}{p'_B(n)}$.

<u>Case 1</u>: $\Pr(\alpha|\tau') < \Pr(\alpha|\tau) < \frac{1}{2}$.
Then, by Proposition 2, $\lim_{n\to\infty}\frac{p'_A(n)}{p'_B(n)} > 1$. Thus, by (5) and by the Taylor approximation around zero,

$$\begin{aligned}
&\lim_{n\to\infty}\frac{p'_A(n)}{p'_B(n)} \leq \lim_{n\to\infty}\frac{p_A(n)}{p_B(n)} \\
\Leftrightarrow \frac{\Pr(\beta|A,\tau')}{\Pr(\beta|B,\tau')} \cdot \sqrt{\lim_{n\to\infty}\frac{p'_B(n)}{p'_A(n)}} &\leq \frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)} \cdot \sqrt{\lim_{n\to\infty}\frac{p_B(n)}{p_A(n)}} \\
\Leftrightarrow \frac{\frac{\Pr(\beta|A,\tau')}{\Pr(\beta|B,\tau')}}{\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)}} &\leq \sqrt{\frac{\lim_{n\to\infty}\frac{p_B(n)}{p_A(n)}(x)}{\lim_{n\to\infty}\frac{p'_B(n)}{p'_A(n)}}} \leq 1.
\end{aligned}$$

By Claim 2, $\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)}$ is strictly decreasing in $\Pr(\alpha|\tau)$, meaning that the left hand side of the above inequality is strictly larger than 1—a contradiction!

Case 2: $\Pr(\alpha|\tau') < \frac{1}{2} \leq \Pr(\alpha|\tau)$
Then, by Proposition 2, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} \leq 1$ for all n and $\lim_{n\to\infty} \frac{p'_A(n)}{p'_B(n)} > 1$, a contradiction.

Case 3: $\Pr(\alpha|\tau') \leq \frac{1}{2} < \Pr(\alpha|\tau)$
Then, $\lim_{n\to\infty} \frac{p'_A(n)}{p'_B(n)} \geq 1$ for all n and $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$, a contradiction.

Case 4: $\frac{1}{2} < \Pr(\alpha|\tau') < \Pr(\alpha|\tau)$
By Proposition 2 and by (6),

$$\lim_{n\to\infty} \frac{p'_A(n)}{p'_B(n)} \leq \lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$$

$$\Leftrightarrow \frac{\Pr(\alpha|A,\tau')}{\Pr(\alpha|B,\tau')} \cdot \sqrt{\lim_{n\to\infty} \frac{p'_B(n)}{p'_A(n)}} \leq \frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)} \cdot \sqrt{\lim_{n\to\infty} \frac{p_B(n)}{p_A(n)}}$$

$$\Leftrightarrow \frac{\frac{\Pr(\alpha|A,\tau')}{\Pr(\alpha|B,\tau')}}{\frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)}} \leq \sqrt{\frac{\lim_{n\to\infty} \frac{p_B(n)}{p_A(n)}(x)}{\lim_{n\to\infty} \frac{p'_B(n)}{p'_A(n)}}} \leq 1.$$

However, by Claim 2, $\frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)}$ is strictly decreasing in $\Pr(\alpha|\tau)$, implying that $\frac{\frac{\Pr(\alpha|A,\tau')}{\Pr(\alpha|B,\tau')}}{\frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)}} > 1$—a contradiction!
Thus, the limit of the ratio of participation rates is monotonic in the beliefs. □

A.5 Proof for Section 4.4

Proof of Proposition 5.
Candidate A is the majority candidate in state α and candidate B is the majority candidate in state β. As $n \to \infty$, by the law of large numbers, the majority candidate wins the election in each state if and only if

$$\frac{1-q}{q} < \lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < \frac{q}{1-q}.$$

If $\tau_A = \tau_B$ or if no poll is considered, for all n, $p_A(n) = p_B(n) =: \hat{p}(n)$. Then, for all n, $\frac{p_A(n)}{p_B(n)} = 1$ and the result holds.

If $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$, by (6), as $n \to \infty$,

$$\begin{aligned}
\frac{q}{1-q} \cdot \frac{p_A(n)}{p_B(n)} &\approx \frac{q}{1-q} \cdot \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \sqrt{\frac{1-q}{q}}\sqrt{\frac{p_B(n)}{p_A(n)}} \\
&> \frac{q}{1-q} \cdot \sqrt{\frac{1-q}{q}}\sqrt{\frac{p_B(n)}{p_A(n)}} \\
&= \sqrt{\frac{q}{1-q}}\sqrt{\frac{p_B(n)}{p_A(n)}} \\
&> 1,
\end{aligned}$$

where the third to last step follows from $\frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} > 1$, which is derived in Appendix B, and the last step follows because $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$ and $q > \frac{1}{2}$ by assumption.
Finally, if $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$, by (5), as $n \to \infty$,

$$\begin{aligned}
\frac{q}{1-q} \cdot \frac{p_B(n)}{p_A(n)} &\to \frac{q}{1-q} \cdot \frac{Pr(\beta|B,\tau)}{Pr(\beta|A,\tau)} \cdot \sqrt{\frac{1-q}{q}}\sqrt{\frac{p_A(n)}{p_B(n)}} \\
&> \frac{q}{1-q} \cdot \sqrt{\frac{1-q}{q}}\sqrt{\frac{p_A(n)}{p_B(n)}} \\
&= \sqrt{\frac{q}{1-q}}\sqrt{\frac{p_A(n)}{p_B(n)}} \\
&> 1,
\end{aligned}$$

since $\frac{Pr(\beta|B,\tau)}{Pr(\beta|A,\tau)} > 1$.
Therefore, as $n \to \infty$, candidate A wins in state α and candidate B wins in state β with probability 1, so the majority candidate is elected almost surely. □

A.6 Proof for Section 5.2

Proof of Proposition 6.
Following the derivations in Appendix C, in particular Proposition 7, and setting $\gamma = 0$ immediately yields that the unique equilibrium strategy prescribes poll participants to reveal their preferences truthfully with probability $\frac{1}{2}$ ($\mu = \frac{1}{2}$). Consequently, $\Pr(\alpha|\tau) = \Pr(\beta|\tau)$ and babbling is the unique equilibrium of the polling stage.

□

Appendix B Posterior Beliefs

This section is concerned with the derivation of posterior beliefs about the state of the world, firstly after observing one's own preference type, and secondly after additionally observing the result of the pre-election poll. Further, useful properties of related conditional probabilities will be derived.

Priors upon observing the preference type

Recall that it is assumed that $\Pr(\alpha) = \Pr(\beta) = \frac{1}{2}$ and that $\Pr(A|\alpha) = q = \Pr(B|\beta)$, where the $\Pr(A|\alpha)$ indicates the probability that a randomly drawn citizen prefers candidate A over B given that the state is α. Learning about his or her own preferences, a citizen updates his or her beliefs about the state of the world as follows:

$$
\begin{aligned}
\Pr(\omega = \alpha|A) = \frac{\Pr(\omega = \alpha, A)}{\Pr(A)} &= \frac{q}{q \cdot \frac{1}{2} + (1-q)\frac{1}{2}} = q, \\
\Pr(\beta|A) &= 1 - q, \\
\Pr(\alpha|B) &= 1 - q, \\
\Pr(\beta|B) &= q.
\end{aligned}
$$

Posteriors after observing the poll

Additionally observing the pre-election poll result $\tau = (\tau_A, \tau_B)$ yields the posterior beliefs

$$
\begin{aligned}
\Pr(\alpha|A,\tau) &= \frac{\Pr(\alpha,\tau,A)}{\Pr(\tau,A)} \\
&= \frac{\Pr(A,\tau|\alpha)\cdot\Pr(\alpha)}{\Pr(A,\tau|\alpha)\cdot\Pr(\alpha) + \Pr(A,\tau|\beta)\cdot\Pr(\beta)} \\
&= \frac{\Pr(\tau|\alpha)\cdot\Pr(A|\alpha)\cdot\Pr(\alpha)}{\Pr(\tau|\alpha)\cdot\Pr(A|\alpha)\cdot\Pr(\alpha) + \Pr(\tau|\beta)\cdot\Pr(A|\beta)\cdot\Pr(\beta)},
\end{aligned}
$$

where $\Pr(\tau|\omega)$ denotes the posterior probability that the state is ω if the poll result is τ. For $q > \frac{1}{2}$,

$$\Pr(\alpha|A,\tau) > \Pr(\alpha|B,\tau), \ \ \Pr(\beta|B,\tau) > \Pr(\beta|A,\tau),$$

and

$$\Pr(\alpha|A,\tau) - \Pr(\alpha|B,\tau) = \Pr(\beta|B,\tau) - \Pr(\beta|A,\tau).$$

The following relation will be prove useful:

Claim 1.

$$\Pr(\alpha|A,\tau) < \Pr(\beta|B,\tau) \ \Leftrightarrow \ \Pr(\tau|\alpha) < \Pr(\tau|\beta) \ \Leftrightarrow \ \Pr(\alpha|\tau) < \Pr(\beta|\tau)$$

Proof.

$$\frac{\Pr(\alpha|A,\tau)}{\Pr(\beta|B,\tau)} = \frac{\Pr(\tau|\alpha)}{\Pr(\tau|\beta)} \cdot \frac{(1-q)\cdot\Pr(\tau|\alpha) + q\cdot\Pr(\tau|\beta)}{q\cdot\Pr(\tau|\alpha) + (1-q)\cdot\Pr(\tau|\beta)}$$

$$\Rightarrow \frac{\Pr(\alpha|A,\tau)}{\Pr(\beta|B,\tau)} \begin{cases} = 1 \;\Leftrightarrow\; \Pr(\tau|\alpha) = \Pr(\tau|\beta) \\ > 1 \;\Leftrightarrow\; \Pr(\tau|\alpha) > \Pr(\tau|\beta) \\ < 1 \;\Leftrightarrow\; \Pr(\tau|\alpha) < \Pr(\tau|\beta), \end{cases}$$

and further,

$$\Pr(\alpha|\tau) = \frac{\Pr(\tau|\alpha)\cdot\Pr(\alpha)}{\Pr(\tau)} = \frac{\Pr(\tau|\alpha)}{2\cdot\Pr(\tau)},$$

$$\Rightarrow \Pr(\alpha|\tau) < \Pr(\beta|\tau) \Leftrightarrow \Pr(\tau|\alpha) < \Pr(\tau|\beta).$$

□

The following claim is used in the proof of Proposition 4.

Claim 2. $\frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)}$ *and* $\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)}$ *are strictly decreasing in* $\Pr(\alpha|\tau)$.

Proof. By the derivations above and since $\Pr(\alpha) = \Pr(\beta)$,

$$\frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)} = \frac{q}{1-q}\frac{\Pr(\tau|\alpha)(1-q) + \Pr(\tau|\beta)q}{\Pr(\tau|\alpha)q + \Pr(\tau|\beta)(1-q)}$$
$$\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)} = \frac{1-q}{q}\frac{\Pr(\tau|\alpha)(1-q) + \Pr(\tau|\beta)q}{\Pr(\tau|\alpha)q + \Pr(\tau|\beta)(1-q)}.$$

Since $\Pr(\tau|\omega) = 2\Pr(\omega|\tau)\Pr(\tau)$ for $\omega \in \{\alpha,\beta\}$, and $\Pr(\beta|\tau) = 1 - \Pr(\alpha|\tau)$,

$$\frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)} = \frac{q}{1-q}\frac{\Pr(\alpha|\tau)(1-2q) + q}{\Pr(\alpha|\tau)(2q-1) + (1-q)}$$
$$\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)} = \frac{1-q}{q}\frac{\Pr(\alpha|\tau)(1-2q) + q}{\Pr(\alpha|\tau)(2q-1) + (1-q)}.$$

Since

$$\frac{d}{d\Pr(\alpha|\tau)}\frac{\Pr(\alpha|\tau)(1-2q) + q}{\Pr(\alpha|\tau)(2q-1) + (1-q)} = \frac{1-2q}{(\Pr(\alpha|\tau)(2q-1) + 1 - q)^2} < 0,$$

the claim follows, since $q > \frac{1}{2}$. □

Truthfully answered polls

Assuming now that the poll was answered truthfully, the posterior beliefs become

$$\begin{aligned}
\Pr(\alpha|\tau) &= \frac{\binom{\tau_A+\tau_B}{\tau_A}\cdot q^{\tau_A}\cdot(1-q)^{\tau_B}}{\binom{\tau_A+\tau_B}{\tau_A}\cdot\left(q^{\tau_A}\cdot(1-q)^{\tau_B}+(1-q)^{\tau_A}\cdot q^{\tau_B}\right)},\\
&= \frac{1}{1+(\frac{q}{1-q})^{\tau_B-\tau_A}},\\
\Pr(\beta|\tau) &= \frac{1}{1+(\frac{q}{1-q})^{\tau_A-\tau_B}}.
\end{aligned}$$

Thus, $\tau_A > \tau_B \;\Leftrightarrow\; \Pr(\alpha|\tau) > \Pr(\beta|\tau)$ and $\Pr(\alpha|\tau) = \Pr(\beta|\tau)$ if and only if $\tau_A = \tau_B$.

Claim 3. *The posterior probability* $\Pr(\alpha|\tau)$ *is increasing in* $\tau_A - \tau_B$.

Proof.

$$\frac{d}{d(\tau_A-\tau_B)}\Pr(\alpha|\tau) = \frac{(\frac{q}{1-q})^{(\tau_B-\tau_A)}\log(\frac{q}{1-q})}{\left((\frac{q}{1-q})^{(\tau_B-\tau_A)}+1\right)^2} > 0.$$

□

$$\begin{aligned}
\Pr(\alpha|A,\tau) &= \frac{\binom{\tau_A+\tau_B}{\tau_A}\cdot q^{\tau_A}\cdot(1-q)^{\tau_B}\cdot q\cdot\frac{1}{2}}{\binom{\tau_A+\tau_B}{\tau_A}\cdot\frac{1}{2}\cdot\left(q^{\tau_A}\cdot(1-q)^{\tau_B}\cdot q+(1-q)^{\tau_A}\cdot q^{\tau_B}\cdot(1-q)\right)},\\
&= \frac{1}{1+(\frac{q}{1-q})^{-\tau_A+\tau_B-1}},\\
\Pr(\beta|A,\tau) &= \frac{1}{1+(\frac{q}{1-q})^{\tau_A-\tau_B+1}},\\
\Pr(\alpha|B,\tau) &= \frac{1}{1+(\frac{q}{1-q})^{-\tau_A+\tau_B+1}},\\
\Pr(\beta|B,\tau) &= \frac{1}{1+(\frac{q}{1-q})^{\tau_A-\tau_B-1}}.
\end{aligned}$$

Note that $\Pr(\omega|A) = \Pr(\omega|A,\tau)$ if and only if either $\tau_A = \tau_B$ or $q = \frac{1}{2}$. This implies that a balanced poll where $\tau_A = \tau_B$ has the same effect as if no poll was published at all.

Appendix C Polls with Exogenously Truthful Participants

In this extension, I prescribe a share $\gamma > 0$ of poll participants to always state their preferences truthfully.[17] The share $1-\gamma$ is strategic, and states preferences truthfully with probability $\mu \in [0,1]$. I show that the majority candidate is elected with probability one in the limit. Further, I demonstrate how the underdog effect extends, and how this affects the incentives of the strategic share of poll participants.

Fix the strategy $\mu \in [0,1]$, and denote the probability that a poll participant states a preference for candidate i in state ω by $\Pr(\text{“}i\text{”}|\omega)$. Define $\kappa := \Pr(\text{“}A''|\alpha)$. Then,

$$\begin{aligned}
\Pr(\text{“}A\text{”}|\alpha) &= q \cdot (\gamma \cdot 1 + (1-\gamma)\cdot\mu) + (1-q)\cdot(1-\gamma)\cdot(1-\mu), \\
&=: \kappa, \\
&= \Pr(\text{“}B\text{”}|\beta), \\
\Pr(\text{“}B\text{”}|\alpha) &= q \cdot (1-\gamma)\cdot(1-\mu) + (1-q)\cdot(\gamma\cdot 1 + (1-\gamma)\cdot\mu), \\
&= 1-\kappa, \\
&= \Pr(\text{“}A\text{”}|\beta).
\end{aligned}$$

Note that $\kappa = q$ if and only if $\gamma = 1$ or $\mu = 1$, and $\kappa < q$ else.
The posterior beliefs after observing the poll become

$$\begin{aligned}
\Pr(\alpha|A,\tau) &= \frac{\binom{\tau_A+\tau_B}{\tau_A}\cdot\kappa^{\tau_A}\cdot(1-\kappa)^{\tau_B}\cdot q\cdot\frac{1}{2}}{\binom{\tau_A+\tau_B}{\tau_A}\cdot\frac{1}{2}\cdot\left[\kappa^{\tau_A}\cdot(1-\kappa)^{\tau_B}\cdot q + (1-\kappa)^{\tau_A}\cdot\kappa^{\tau_B}\cdot(1-q)\right]}, \\
&= \frac{1}{1+(\frac{\kappa}{1-\kappa})^{\tau_B-\tau_A}\cdot\frac{1-q}{q}}, \\
\Pr(\beta|A,\tau) &= \frac{1}{1+(\frac{\kappa}{1-\kappa})^{\tau_A-\tau_B}\cdot\frac{q}{1-q}}, \\
\Pr(\alpha|B,\tau) &= \frac{1}{1+(\frac{\kappa}{1-\kappa})^{\tau_B-\tau_A}\cdot\frac{q}{1-q}}, \\
\Pr(\beta|B,\tau) &= \frac{1}{1+(\frac{\kappa}{1-\kappa})^{\tau_A-\tau_B}\cdot\frac{1-q}{q}}.
\end{aligned}$$

Lemma 5 reveals that the underdog effect depends on κ. For any fixed μ, if $\kappa > \frac{1}{2}$, as before, supporters of the candidate obtaining the higher vote count in the poll turn out at lower rates. Intuitively, as $\kappa > \frac{1}{2}$, a vote for A is more likely to occur if the state is α, and, thus, leads voters to update their beliefs toward α. However, if $\kappa < \frac{1}{2}$, the effect is reversed. Supporters of the candidate with the higher vote count in the poll turn out at higher rates because voters understand that $\tau_A > \tau_B$ actually implies that the state is more likely to be β. If $\kappa = \frac{1}{2}$ or $\tau_A = \tau_B$, the poll is not informative, and both groups turn out at equal rates.

[17] If $\gamma = 0$, all poll participants are strategic. Plugging in $\gamma = 0$ in the analysis below yields the proof for Proposition 6.

Lemma 5.

1. *Let $\kappa > \frac{1}{2}$. Then,*
   (a) *if $\tau_A > \tau_B$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$,*
   (b) *if $\tau_A = \tau_B$, $p_A(n) = p_B(n)\ \forall n$,*
   (c) *and if $\tau_A < \tau_B$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$.*
2. *Let $\kappa = \frac{1}{2}$. Then, $p_A(n) = p_B(n)\ \forall n$.*
3. *Let $\kappa < \frac{1}{2}$. Then,*
   (a) *if $\tau_A > \tau_B$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$,*
   (b) *if $\tau_A = \tau_B$, $p_A(n) = p_B(n)\ \forall n$,*
   (c) *and if $\tau_A < \tau_B$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$.*

Proof. Suppose $\kappa > \frac{1}{2}$ and $\tau_A > \tau_B$.
Assume, by contradiction, that as $n \to \infty$, $p_A(n) > p_B(n)$.[18] By (5), as $n \to \infty$,

$$\frac{p_A(n)}{p_B(n)} \to \frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)} \cdot \sqrt{\frac{q}{1-q}} \cdot \sqrt{\frac{p_B(n)}{p_A(n)}}.$$

Claim 4. $\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)} \cdot \sqrt{\frac{q}{1-q}} < 1$.

Proof.

$$\begin{aligned} &\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)} \cdot \sqrt{\frac{q}{1-q}} < 1 \\ \Leftrightarrow\ &\frac{1 + (\frac{\kappa}{1-\kappa})^{\tau_A-\tau_B} \cdot \frac{1-q}{q}}{1 + (\frac{\kappa}{1-\kappa})^{\tau_A-\tau_B} \cdot \frac{q}{1-q}} < 1 \\ &\Leftrightarrow \sqrt{\frac{q}{1-q}} - 1 < \left(\frac{\kappa}{1-\kappa}\right)^{\tau_A-\tau_B} \cdot \left(\frac{q}{1-q} - \sqrt{\frac{1-q}{q}}\right). \end{aligned}$$

The last statement is true because $\left(\frac{\kappa}{1-\kappa}\right)^{\tau_A-\tau_B} > 1$ given that $\kappa > \frac{1}{2}$ and $\tau_A > \tau_B$ by assumption, and because $\frac{q}{1-q} - \sqrt{\frac{1-q}{q}} > \sqrt{\frac{q}{1-q}} - 1$.

□

This yields a contradiction because $\frac{p_A(n)}{p_B(n)} > 1$ by assumption, but
$\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)} \cdot \sqrt{\frac{q}{1-q}} \cdot \sqrt{\frac{p_B(n)}{p_A(n)}} < 1$.
The proofs of parts 1 c), 3 a) and 3 c) are analogous.
For the proofs of parts 1 b), 2 and 3 b), observe that if either $\tau_A = \tau_B$ or $\kappa = \frac{1}{2}$, $\Pr(\omega|i,\tau) = \Pr(\omega|i)$. Hence, by Lemma 1, the result obtains. □

[18] By Lemma 1, $p_A(n) = p_B(n)$ can be excluded.

Similarly, the direction of the monotonicity of the underdog effect in the poll margin depends on κ.

Corollary 3.

1. *If $\kappa > \frac{1}{2}$, in a large election, the limit of the ratio of participation rates of the A supporters relative to B supporters is strictly increasing in the margin of the poll $\tau_B - \tau_A$, i.e., $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$ is a strictly increasing function of $\tau_B - \tau_A$.*

2. *If $\kappa < \frac{1}{2}$, in a large election, the limit of the ratio of participation rates of the A supporters relative to B supporters is strictly decreasing in the margin of the poll $\tau_B - \tau_A$, i.e., $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$ is a strictly decreasing function of $\tau_B - \tau_A$.*

The result immediately follows by observing that given $\tau_A > \tau_B$, $\Pr(\alpha|\tau) > \frac{1}{2}$ if and only if $\kappa > \frac{1}{2}$, and given $\tau_A < \tau_B$, $\Pr(\alpha|\tau) < \frac{1}{2}$ if and only if $\kappa > \frac{1}{2}$.

How does this affect the incentives of the share $1-\gamma$ of poll participants who are strategic? As it turns out, the optimal strategy μ^* depends on the share of exogenously truthful poll participants, γ.

Proposition 7. *Fix the share of exogenously truthful poll participants, γ.*

1. *If $\gamma > \frac{1}{2}$, $\mu^* = 0$. That is, all strategic poll participants misrepresent their preferences to be exactly the opposite of their true preferences. Since $\kappa > \frac{1}{2}$, the poll is informative.*

2. *If $\gamma = \frac{1}{2}$, $\mu^* = 0$. Since $\kappa = \frac{1}{2}$, the poll is not informative.*

3. *If $\gamma < \frac{1}{2}$, $\mu^* = \frac{1-2\gamma}{2(1-\gamma)}$. Since $\kappa = \frac{1}{2}$, the poll is not informative.*

Proof. Note first that $\frac{d\kappa}{d\mu} > 0$, $\frac{d\kappa}{d\gamma} > 0$ and that $\kappa = \frac{1}{2}$ if $\gamma = \frac{1}{2}$ and $\mu = 0$.

Case 1: $\gamma > \frac{1}{2}$.
Then, $\kappa > \frac{1}{2}$ for all $\mu \in [0, 1]$. By Lemma 5 and Corollary 3, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$ is a strictly increasing function of $\tau_B - \tau_A$. Therefore, it is optimal for all strategic poll participants to play $\mu^* = 0$ and claim to have the exact opposed preferences. Since $\kappa > \frac{1}{2}$, the poll is informative.

Case 2: $\gamma = \frac{1}{2}$.
Then, $\kappa \geq \frac{1}{2}$ and the inequality is strict if $\mu = 0$. Suppose, by contradiction, that all other poll participants play according to $\mu > 0$. Since $\kappa > \frac{1}{2}$, by Lemma 5 and Corollary 3, it is optimal for an individual poll participant to deviate to $\mu = 0$, thereby increasing the relative participation rate of like-minded voters. So, $\mu > 0$ cannot be part of an equilibrium.

In contrast, if all other poll participants play according to $\mu = 0$, $\kappa = \frac{1}{2}$. Then, the poll is not informative and in particular, voters do not take the poll into account. Thus, $\mu^* = 0$ is the equilibrium best response.

Case 3: $\gamma < \frac{1}{2}$.
Then, there exists a unique μ^* such that $\kappa = \frac{1}{2}$ if and only if $\mu = \mu^* = \frac{1-2\gamma}{2(1-\gamma)}$. Suppose that $\mu < \mu^*$. Then, $\kappa < \frac{1}{2}$ and by Lemma 5 and Corollary 3, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$ is a strictly decreasing function of $\tau_B - \tau_A$. Thus, $\mu = 1$ is an optimal deviation. If $\mu > \mu^*$, then, $\kappa > \frac{1}{2}$ and $\mu = 0$ is an optimal deviation. Finally, if $\mu = \mu^*$, the poll is uninformative and does not affect the voters' beliefs. Then, $\mu = \mu^*$ is the equilibrium best response. □

Proposition 7 reveals that the poll is informative if and only if the share of exogenously truthful poll participants is strictly larger than one-half. While the strategic poll participants will again misrepresent their preferences, the truthful response of the majority of poll participants allows the electorate to derive some information from the poll.

Finally, Corollary 4 states that utilitarian efficiency also holds in a large election if poll participants play mixed behavioral strategies.

Corollary 4. *In the limit, the majority candidate wins the election with probability 1.*

Proof. The result follows by repeating the same steps as in the proof of Proposition 5, and observing that

$$\frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)} = \frac{1 + (\frac{\kappa}{1-\kappa})^{\tau_B - \tau_A} \cdot \frac{q}{1-q}}{1 + (\frac{\kappa}{1-\kappa})^{\tau_B - \tau_A} \cdot \frac{1-q}{q}} > 1.$$

□

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