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Information Design and Career Concerns

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Abstract

This paper studies the interplay between information and incentives in principal-agent relationships with career concerns. I derive conditions for when more precise information about performance or more uncertainty about the agent's ability lead to stronger incentives due to career concerns, absent any ad-hoc restrictions on the production technology or set of information structures. A key condition for deriving these comparative statics is how effort changes the informativeness of performance signals regarding ability. However, more sophisticated information revelation technologies that are implicitly ruled out in the literature overturn commonly held assertions regarding information design and career concerns.

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1 Introduction

The focus of this paper is a principal-agent relationship where there is uncertainty about the agent's ability, and the agent has a preference for being perceived as of high ability. This motive is often called career concerns, and provides incentives beyond those derived from any existing wage contract, as by exerting effort the agent can manipulate the belief about his ability. The strength of this incentive is affected by two types of information, namely observable performance measures, and prior uncertainty about the agent's ability. The overall aim of this paper is to provide a better understanding of the interplay between information and incentives in such an environment.

The main contribution of this paper is a derivation of substantive conditions under which more or less information about performance or ability leads to stronger incentives related to career concerns. When are career-concerns incentives increased by evaluating an agent based on a more informative signal of performance? When are incentives depressed when more information about the agent's ability becomes available? I derive interpretable assumptions which make each of these comparative statics true. The fundamental novelty is that I do not impose a particular production technology or parametric restrictions on the information that can be revealed; both are maintained in most of the literature, yet the comparative statics with respect to information are sensitive to either type of restriction.

The considerations at the core of this paper are relevant to the problem of a principal who exploits career concerns as part of her incentive scheme, and is able to design information revelation. Consider the following examples. (*i*) A manager evaluates her subordinate through a performance review which will be revealed to an (internal) labor market. What aspects of the worker's performance should she emphasize? Should he be evaluated based on the performance of a colleague? If so, should the benchmark be someone with a similar background (reducing uncertainty about ability), or conducting a similar task (reducing uncertainty about performance)? (*ii*) A promotion or tenure decision is based on the perception of the agent's ability (thereby creating career concerns). Should the decision maker commit ex-ante to use or disregard certain demographic information that is correlated with ability? My findings can be considered as a foundation for transparency in organizations with respect to measures of performance, and opaqueness regarding certain information about the agent's ability, such as demographics. That said, the goal of this paper is not to provide the details of an optimal information disclosure policy for these examples, but address more generally what type of information is conducive to career-concerns incentives.

To think of career concerns, consider a situation where an agent exerts effort once, after which a performance measure such as output is realized. Output is informative about the agent's ability, and observed by a player called the market. The market pays the agent a wage equal to the perceived ability, thereby generating career concerns. Effort is unobserved, so the market belief is based on a conjecture about the agent's behavior. The agent then can try to manipulate the market belief, which he does by trading off the expected change in the perception of his ability against the effort cost. In equilibrium, the agent finds it not worthwhile to influence the market belief any further, and typically a positive effort is then sustained.

To introduce information design, suppose there is a principal who to some extent controls the market's information. First, consider the informativeness of performance measures. Output is the most informative signal of performance, but the principal can commit to garble it in an unrestricted way. The market no longer observes output, but whatever signal is chosen by the principal. Second, to study the effect of uncertainty regarding the agent's ability, suppose that the principal cannot garble output. What she can do is to disclose or withhold a signal of the agent's ability, potentially observable by the market together with output. Given the question of what type of information maximizes career-concerns incentives, the principal is endowed with the objective of maximizing effort.

It seems intuitively reasonable that more information about performance and uncertainty about ability both increase career-concerns incentives. After all, if the market does not observe any signal of performance or if it knows the agent's ability, it is impossible for the agent to manipulate its belief and career-concerns incentives disappear. Beyond these extreme cases, the effect of information is less obvious when no particular restrictions on signals are imposed. One might argue that garbling information about performance leads to less variability in terms of the wage paid, and therefore weakens incentives to exert more effort. My analysis shows that this is true, but only in a relatively weak sense.

As far as information about performance is concerned, suppose that effort shifts the distribution of output upwards (in some appropriate sense), and that output is a favorable signal of ability. It turns out that these natural ordering conditions are not sufficient for effort to be maximized by full revelation of output. Under the additional assumption that higher effort increases the informativeness of output about ability, such a result can be derived. In fact, if one can find two effort levels such that for the higher effort output is less informative, then there exists a cost of effort function such that career-concerns incentives are maximized by a noisy signal of performance.

As for the uncertainty about ability, I derive conditions under which career-concerns incentives are reduced when the market observes a signal of ability (in addition to output). It is still required that effort increases the informativeness of output, and the signal of ability needs to satisfy an ordering condition which formalizes the idea that it is a favorable signal of ability, for any effort conjecture and realized output. I then use the result to look at the effect of relative performance evaluation.

In the main exposition, incentives are derived only from career concerns. The comparative statics are shown to remain true if in addition the agent is motivated by an explicit wage contract. On the other hand, once the agent randomizes over effort there exist more sophisticated and somewhat non-obvious information revelation technologies that condition the signal on the agent's effort. If beneficial, such mechanisms garble information about output and therefore overturn the result that full revelation of performance measures maximizes career-concerns incentives.

There is a sizable literature addressing questions similar to mine in various contexts. The main novelty is that I do not make parametric restrictions on the statistical environment, in particular on the production technology or how information can be revealed to the market. This allows me to identify substantive conditions that determine how information affects career-concerns incentives. For example, most of the literature considers a parametric setting without any complementarity between effort and ability in the production technology, and restricts the way information can be released. Yet the comparative statics with respect to information can be overturned once one allows for effort to interact with ability in the production technology, even when maintaining the restrictions on how information can be revealed.

The main obstacle to my analysis is the richness of all possible information structures. This, together with the strategic interaction between agent and market, makes a direct analysis in terms of signals intractable. To that end, an alternative representation of the payoff effect due to career concerns is developed. It provides an indirect means of representing information structures that is more conducive to performing comparative statics with respect to the amount of information available.

The model is introduced in Section 2, and Section 3 develops the main results on how information affects career-concerns incentives in the class of pure strategy equilibria (in terms of the agent's effort). Extensions allowing for wage contracts, mixed strategy equilibria, and more sophisticated information revelation technologies are considered in Section 4. Section 5 provides a discussion of the results and how they relate to the literature.

1.1 Literature

The directly related literature is discussed here, and an overview of applied theory work featuring information design questions in the presence of career concerns can be found at the end of Section 5.

Career concerns. The idea of career concerns was introduced formally by Holmström (1999). In a dynamic situation where a principal is not able to offer long term contracts and output contingent wages, incentives can still be generated endogenously. Due to competition for the agent, future wages depend on the belief about the agent's ability, providing incentives today. Holmström (1999) identifies a tractable way of modeling career concerns through the normal-linear model, on which most applications of career concerns are based. While the focus is on the dynamic effort profile for a given information structure, one can perform comparative statics with respect to the noisiness of the performance signal and the uncertainty about the agent's ability and show that the former is detrimental, and the latter beneficial for incentives. Here I explore how general these results are, and show what substantive assumptions are implicit in the normal-linear model that generate them.

Information design. Through examples, Dewatripont, Jewitt and Tirole (1999*a*) show that a more accurate performance measure might not raise career-concerns incentives (which does not occur in Holmström's setting). They then show that such a comparative static is indeed true when the baseline performance measure is a favorable signal of ability, and its distribution is shifted by effort in a certain way. I explain in Section 5 why their results do not imply that the most informative performance measure maximizes career-concerns incentives. Based on a similar (quasi-) static model, I additionally look at the effect of uncertainty regarding the agent's ability and apply the results to look at the effect of relative performance evaluation, which was investigated by Meyer and Vickers (1997) within the normal-linear setting.

2 Model

There are three players: the principal (she), the agent (he), and a market (it).

Actions. The agent exerts effort e that affects the joint distribution of his ability θ, a signal of ability η called "ability estimate", and output q. This joint distribution is denoted by $\hat{F}(\theta, \eta, q \mid e)$. Let $\Theta \subset \mathbb{R}$ be the set of abilities, $\Xi \subset \mathbb{R}$ the set of possible realizations of the ability estimate, and $Q = \{q_1, ..., q_M\} \subset \mathbb{R}$ be a finite set of outputs with $q_1 < ... < q_M$. The set of effort levels E is compact. The marginal distribution over output given effort, denoted $F(q \mid e)$, is continuous in e and has full support. The market pays a wage $w \in \mathbb{R}$ to the agent.

The principal decides what information the market gets to observe in form of a signal s, through an information structure H.

- In Section 3.2, she garbles output. Formally, the principal selects an information structure H that specifies for each q a distribution over the signal $s \in \mathbb{R}$, so $H = \{H(s \mid q)\}_{q \in Q}$.[1] The ability estimate is irrelevant, in that no player observes it.

- In Section 3.3, she cannot garble output so the market observes q. What she can decide is whether the market observes only output, or output and the ability estimate. Formally, H specifies whether $s = q$ or $s = (q, \eta)$.

Information and timing. The timing is summarized as follows.

1. The principal publicly commits to an information structure H.

2. The agent exerts effort e, unobserved by the principal and market.

3. Ability θ, ability estimate η, output q and signal s are realized, and signal s is disclosed to the market.

4. The market pays wage w to the agent.

That the agent has no private information about his ability when taking effort is a standard assumption in the career-concerns literature, where ability is usually independent of effort. The above formulation allows for depreciation or learning-by-doing effects.

Payoffs. The principal wants to maximize (the expectation of some increasing function of) effort, but any stated result will be true if the objective is to maximize expected output. The agent maximizes the expectation of $u_A(w, e) = w - c(e)$, where $c(e)$ is nondecreasing and continuous. The market

[1] The signal space is taken to be $\mathbb{R}$ for convenience, and in fact without loss of generality as any equilibrium can be implemented through a direct recommendation mechanism where the principal recommends a wage to the market, and the recommendation is incentive compatible. At any rate, the results are unaffected by the choice of the signal space as long as it is rich enough to fully reveal output.

pays the agent his expected ability, perhaps due to Bertrand competition, so formally it maximizes the expectation of $-(w-\theta)^2$ conditional on its information which comprises the signal and a conjecture about the agent's behavior.

Equilibrium. The principal's strategy specifies an information structure, which is mapped by the agent's strategy into a randomization over effort. The market's strategy specifies for each information structure and signal realization a wage payment. An equilibrium is a Perfect Bayesian Equilibrium (PBE) that in addition induces a PBE in each subgame following a choice of H. In line with the principal-agent literature, attention will be restricted to the principal's preferred equilibrium.

In a mixed strategy equilibrium, the agent draws effort from $\sigma \in \Delta(E)$ on-path. A pure strategy equilibrium is a mixed strategy equilibrium where σ assigns probability one to some e on-path. Except in Sections 4.2 and 4.3, attention is restricted to pure strategy equilibria. In that case, the problem of the principal can be written as

$$\max_{H,e} e \text{ s.t. } e \in \arg\max_{\tilde{e}} \mathbb{E}_s[w(s \mid H, e) \mid H, \tilde{e}] - c(\tilde{e}),\ w(s \mid H, e) = \mathbb{E}(\theta \mid H, s, e).$$

The constraints reflect the agent's and market's optimality conditions, respectively ($\mathbb{E}_s[\cdot \mid H, \tilde{e}]$ is the expectation with respect to s under $(H, \tilde{e})$).

In most of the analysis I consider the subgame following an arbitrary H, and loosely refer to the PBE induced by the continuation strategies as "equilibrium".

Interpretation of the model. Taken literally, the proposed model represents a two period interaction where in each period, a principal offers a short term contract specifying an upfront (and hence output independent) wage. Ability θ represents the surplus from hiring the agent in the second period, and said surplus is divided through a linear sharing rule between the agent and second period principal (with Bertrand competition as a special case). Since the first period principal cannot offer an output contingent contract, she wants to maximize effort (as it maximizes expected output under an assumption made later). She does so through a disclosure policy about information regarding the agent.

Comment on the production technology. The production side of the economy is specified by $\hat{F}(\theta, \eta, q \mid e)$. A more concrete primitive would be a stochastic production function, $\Gamma(q \mid \theta, \eta, e)$, and the distribution of ability and ability estimate, $\Phi(\theta, \eta \mid e)$. One can then derive $\hat{F}(\theta, \eta, q \mid e)$ from $\Gamma(q \mid \theta, \eta, e)$ and $\Phi(\theta, s\eta \mid e)$.

3 Analysis

Section 3.1 provides a short discussion of the normal-linear specification of career concerns, deriving commonly believed results about how information affects incentives. Sections 3.2 and 3.3 present the main results regarding the informativeness of performance measures and prior uncertainty about ability, respectively.

3.1 The normal-linear example

In this example only, it is assumed that $Q = \mathbb{R}$. Output is additive in ability and effort, so

$$q = \theta + e.$$

When garbling output, the principal can only add normally distributed noise ϵ to the realized output and select its variance. Simple information structures such as partitions of q are not allowed. The market observes the signal

$$s = q + \epsilon,$$

where $\epsilon \sim N(0, \sigma_\epsilon^2)$ and σ_ϵ^2 is controlled by the principal. Ability is normally distributed, $\theta \sim N(\mu_\theta, \sigma_\theta^2)$, and independent of ϵ. Straightforward calculations show that if the market expects effort e^*, the wage satisfies (the information structure is written as $H(\sigma_\epsilon^2)$)

$$w(s \mid H(\sigma_\epsilon^2), e^*) = \alpha(\sigma_\theta^2, \sigma_\epsilon^2, e^*) + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\epsilon^2} \cdot s,$$

where $\alpha(\sigma_\theta^2, \sigma_\epsilon^2, e^*)$ is independent of the actual effort exerted. Given that $\mathbb{E}[s \,|\, e] = \mu_\theta + e$, when exerting e the agent receives expected payoff

$$\mathbb{E}[u_A \,|\, e] = \alpha(\sigma_\theta^2, \sigma_\epsilon^2, e^*) + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\epsilon^2} \cdot (\mu_\theta + e) - c(e).$$

Note that marginal incentives to exert effort, $\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\epsilon^2}$, do not depend on the market's conjecture e^*. With a differentiable strictly convex cost function, an interior equilibrium effort is characterized by the first order condition

$$c'(e^*) = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\epsilon^2}.$$

It is immediate that less noise about output (lower σ_ϵ^2) leads to a higher equilibrium effort. As for the uncertainty about ability, suppose that ability θ and ability estimate η are jointly normal. Disclosing η to the market then amounts to reducing σ_θ^2, which is detrimental.

Beyond the normal-linear example, it is not clear under what conditions career-concerns incentives are raised by fully revealing performance measures or increasing uncertainty about the agent's ability. A general analysis is not straightforward for two reasons:

1. The principal optimizes over the set of all information structures.

2. For a given garbling of output H, it is not transparent how $w(s \,|\, H, \hat{e})$ depends on the market conjecture $\hat{e}$.

In the normal-linear example, one restricts attention to a parametric class of information structures so that one can derive a closed form solution for the wage schedule, which eliminates either problem.

3.2 Information about performance

In the following example, output is a favorable signal of ability and effort increases output, yet a noisy disclosure of output maximizes equilibrium effort within the class of pure strategy equilibria.

e	$\delta(e)$	$f(q_H \mid e)$	$c(e)$
e_L	1	1/3	0
e_M	2	1/2	$1/6 + 2\epsilon$
e_H	1	2/3	$1/2 + \epsilon$

Table 1: Details for Example 1

Example 1 *Let $E = \{e_L, e_M, e_H\}$ and $Q = \{q_L, q_H\}$. Since output is binary, the expected wage under effort e, information structure H, and market conjecture $\hat{e}$ can be written as*

$$\mathbb{E}_s[w(s \mid H, \hat{e}) \mid q_L] + f(q_H \mid e)\{\mathbb{E}_s[w(s \mid H, \hat{e}) \mid q_H] - \mathbb{E}_s[w(s \mid H, \hat{e}) \mid q_L]\}.$$

The information structure affects incentives only through the term in curly brackets. When output is not garbled, this term equals $\delta(\hat{e}) := \mathbb{E}(\theta \mid q_H, \hat{e}) - \mathbb{E}(\theta \mid q_L, \hat{e})$. The primitives are specified in Table 1, where ϵ is a positive number sufficiently close to zero. Under full revelation, the unique equilibrium is e_L. If the market conjecture is e_M, the agent wants to deviate to e_H, and if e_H is conjectured the agent wants to deviate to e_L.

Yet e_M can be sustained by a noisy information structure H that maps q_L into signal s_L, and q_H into a uniform randomization over s_L and s_H. Given that $\delta(e_M) = 2$, one can verify that $\mathbb{E}_s[w(s \mid H, e_M) \mid q_H] - \mathbb{E}_s[w(s \mid H, e_M) \mid q_L] = 3/2$. This sufficiently decreases incentives to deviate to e_H, yet e_M remains preferable to e_L thus implementing e_M.

In Example 1, better information about output leads to stronger incentives *given the market's conjecture*. So effort is maximized by full revelation for a fixed conjecture, although I show later that even this result is particular to the binary output case. The problem is that once the conjecture is adjusted to the higher effort level, from e_M to e_H, incentives under full revelation are not strong enough to sustain e_H because the wage is less sensitive to the output realization. Given that e_H is not sustainable in equilibrium, a garbling is beneficial because it eliminates the incentive to deviate upwards starting from e_M. If $\delta(e)$ was nondecreasing, dampening incentives would not be necessary to maximize effort. Intuitively, a decreasing $\delta(e)$ reflects a reduced informativeness of output about ability.

In Proposition 1 below, I provide conditions under which full revelation maximizes effort *in equilibrium*, not just for a given conjecture. As suggested by Example 1, some condition on how effort affects the informativeness of output is needed.

Definition 1 *For $e_1, e_2 \in E$, say that output is more informative about ability under e_2 than under e_1 if $\forall \hat{q} \in Q$,*

$$\mathbb{E}_q \left[\mathbb{E}(\theta \,|\, q, e_2) - \mathbb{E}(\theta \,|\, q, e_1) \,|\, e_2 \right] \geq \mathbb{E}_q \left[\mathbb{E}(\theta \,|\, q, e_2) - \mathbb{E}(\theta \,|\, q, e_1) \,|\, q \leq \hat{q}, e_2 \right]. \quad (1)$$

Say that ***effort increases the informativeness of output about ability*** *if (1) is satisfied $\forall e_1, e_2 \in E$ with $e_2 > e_1$.*

Why condition (1) is related to the informativeness of output will be explained later (see Lemma 2 and the discussion following it). A simple sufficient condition for effort to increase the informativeness of output about ability is that $\mathbb{E}(\theta \,|\, q, e)$ is supermodular.

Say that $F(q \,|\, e)$ is ordered according to the monotone likelihood ratio property (MLRP) if for any e_1, e_2 with $e_2 > e_1$, $f(q \,|\, e_2)/f(q \,|\, e_1)$ is nondecreasing in q. The main result regarding the informativeness of performance measures follows.

Proposition 1 *Assume that*

- *$F(q \,|\, e)$ is ordered according to the MLRP,*
- *$\mathbb{E}(\theta \,|\, q, e)$ is nondecreasing in q, and*
- *effort increases the informativeness of output about ability.*

A best pure strategy equilibrium exists, and is achieved by fully revealing output.

The first two assumptions are natural ordering conditions. The relevant concept of effort increasing (the distribution of) output is the MLRP, and output is a favorable signal of ability as $\mathbb{E}(\theta \mid q, e)$ is nondecreasing in q. As for the informativeness condition, one can verify that in Example 1, output is most informative about ability under the intermediate effort e_M.

Whether effort makes output a more informative signal about ability will depend on the application, as the following example illustrates. Suppose effort and output are binary, so the agent can either work or shirk, and succeed or fail. First consider a task that is difficult in that the agent always fails if he shirks, but succeeds with a positive probability that is increasing in ability when working.[2] Under shirking the outcome is completely uninformative about ability, but not so when the agent works. The opposite is true for a mundane task where the agent always succeeds when working, yet there is some chance of failure when shirking that is decreasing in the agent's ability. For such a task, informativeness of the outcome is decreasing in effort. Indeed, the "intuitive" result that full revelation of output maximizes effort then fails in that a noisy signal of output maximizes effort for some cost of effort function, as discussed in the next paragraph.

Necessity of the conditions. Given the abstract environment, it is natural to ask to what extent the conditions in Proposition 1 are necessary. Attention is restricted to primitives $\hat{F}(\theta, \eta, q \mid e)$ that satisfy the first two assumptions, and these are called *ordered statistical environments*. So the question is, to what extent is it necessary that effort increases the informativeness of output. Fixing the statistical environment $\hat{F}(\theta, \eta, q \mid e)$, the only remaining primitive is the cost function $c(e)$.

Proposition 2 *Take any ordered statistical environment. Suppose that there exist effort levels e_1, e_2 with $e_2 > e_1$ such that output is more informative about ability under e_1 than under e_2, but not more informative under e_2 than under e_1. If $f(q \mid e_2)/f(q \mid e_1)$ is increasing in q, then there exists a cost function such that the best pure strategy equilibrium is achieved through an information structure that is not fully revealing.*

[2] That this example violates the full support assumption is immaterial. One can assume that with small probability, the observation is drawn from a noise distribution that is independent of the actual output.

Propositions 1 and 2 do not provide a tight characterization in that the informativeness of output about ability might not be comparable for a given pair of effort levels. This gap is related to the incompleteness of the Blackwell ordering of informativeness.

The following discussion develops the techniques used to show Proposition 1, and provides a sketch of the proof. A fully revealing information structure is denoted by H_{FR}.

A representation of information structures. The wage paid by the market depends on the realized signal s, the information structure H, and market conjecture $\hat{e}$. Let $t(q \mid H, \hat{e})$ denote the expected wage the agent receives given $(H, \hat{e})$ if output q is realized,

$$t(q \mid H, \hat{e}) = \mathbb{E}_s[w(s \mid H, \hat{e}) \mid q].$$

$t(\cdot \mid H, \hat{e})$ will be referred to as the *transfer schedule* induced by $(H, \hat{e})$. After each output realization the agent faces a lottery over wages, which can be identified with its mean under risk neutrality.

If output is garbled, one would expect that roughly speaking the agent is made better off (relative to full revelation) when realized output indicates low ability, and the opposite when it indicates high ability. E.g. if output is binary, any garbling will sometimes lead to a signal that leaves the market to some extent uncertain about whether output is high or low. Conditional on the low output being realized, the garbling is preferable to full revelation for the agent.

This suggests that $t(q \mid H, \hat{e})$ lies above (below) $t(q \mid H_{\text{FR}}, \hat{e})$ for low (high) values of q, as illustrated Figure 1. In what follows, I define the sense in which $t(\cdot \mid \cdot, \hat{e})$ becomes "flatter" as one garbles information. The significance of this is that flatter transfer schedules depress incentives, albeit in a relatively weak sense.

Implementability: a necessary condition. For the fully revealing information structure, the transfer schedule satisfies $t(q \mid H_{\text{FR}}, \hat{e}) = \mathbb{E}(\theta \mid q, \hat{e})$,

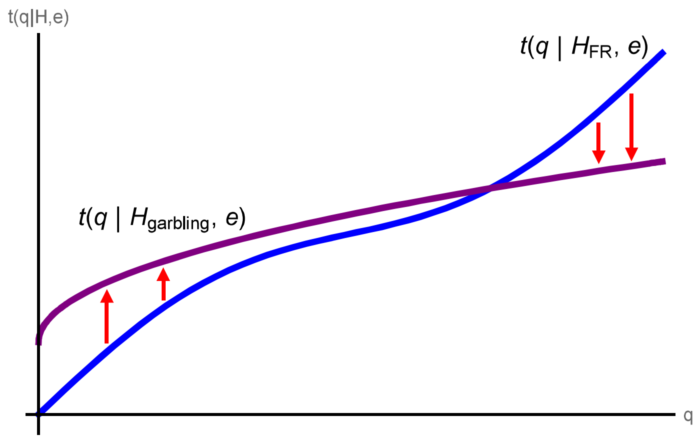


Figure 1: Effect of a garbling

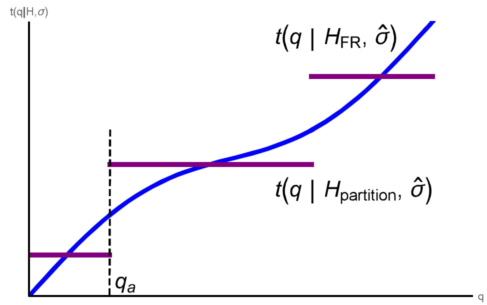
$\forall q$, given conjecture $\hat{e}$. For any other information structure H, one can say that output q gets "subsidized" if $t(q\,|\,H_{\text{FR}},\hat{e}) > \mathbb{E}(\theta\,|\,q,\hat{e})$, and "taxed" if the inequality is reversed. As discussed, it should be the case that lower outputs tend to be subsidized, and the opposite for higher outputs. This is formalized as follows.

Lemma 1 *Fix any information structure H and conjecture $\hat{e}$. If $\mathbb{E}(\theta\,|\,q,\hat{e})$ is nondecreasing in q, then*

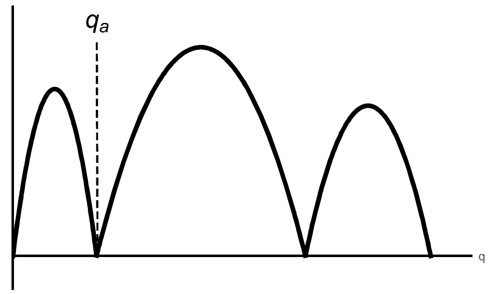
$$\sum_{i=1}^{j} t(q_i\,|\,H,\hat{e})f(q_i\,|\,\hat{e}) \geq \sum_{i=1}^{j} t(q_i\,|\,H_{FR},\hat{e})f(q_i\,|\,\hat{e}) \qquad (2)$$
$$\forall j = 1,...,M, \text{ with equality for } j = M.$$

The result says that for any garbling, the "cumulative subsidy up to j", $\sum_{i=1}^{j}[t(q_i\,|\,H,\hat{e}) - t(q_i\,|\,H_{\text{FR}},\hat{e})]f(q_i\,|\,\hat{e})$, has to be non-negative, and vanish for the highest possible output. The "with equality for $j = M$" condition has to be satisfied since the expected wage has to coincide with the expected ability. From now on, Equation (2) will be called the "cumulative subsidy condition".

Figure 2 illustrates the result of Lemma 1. Information structure $H_{\text{partition}}$ corresponds to an "interval" partition, where each element of the partition is an adjacent set of outputs. In this special case, the cumulative subsidy is zero at each of the break points such as q_a. The market never confounds outputs



(a) $t(q \mid H_{\text{partition}}, \hat{\sigma})$, $t(q \mid H_{\text{FR}}, \hat{\sigma})$



(b) Cumulative subsidy

Figure 2: Illustration of the cumulative subsidy condition

weakly below q_a with those above it, so it pays a "fair" wage conditional on q being weakly below q_a.

Moving beyond partitions, consider first the case of binary output. After any signal realization, the market is to some extent uncertain as to which output generated it, and pays an intermediate wage. This subsidizes (taxes) the low (high) output, and when weighted by the relative frequency of the two outputs, the subsidy and tax are of the same magnitude. The same reasoning applies if output is not binary, yet every signal is generated by at most two outputs. For a more general information structure where every signal can be generated by an arbitrary subset of outputs, consider any signal s'. It turns out that one can replace signal s' with a finite number of new signals, where each new signal is generated by two outputs only, and leads to exactly the same wage as signal s'.[3] The bottom line is that after the translation to such an information structure that has the same joint distribution over wages and outputs, each signal generates a subsidy and tax in a way that satisfies the cumulative subsidy condition.

Implementability: a sufficient condition. The next step is to ask which transfer schedules can be implemented by some information structure.

[3] Given the posterior distribution over output conditional on s', start out by pooling the highest and lowest output into signal s^1 in such a way that s^1 leads to $w(s')$. This can be done until the mass of one of the two outputs is exhausted, say the highest output. Now start pooling the second highest output with the lowest one into signal s^2, again generating $w(s')$. This procedure can be continued until one exhausts the probability mass of all outputs, indeed it terminates after at most M steps.

Satisfying the cumulative subsidy condition of Lemma 1 turns out not to be sufficient.

Lemma 2 *Take an arbitrary function* $g : Q \to \mathbb{R}$*. Under conjecture* $\hat{e}$*, there exists an information structure* H *with* $t(\cdot \mid H, \hat{e}) = g(\cdot)$ *if (i)* $g(q)$ *and* $\mathbb{E}(\theta \mid q, \hat{e})$ *are nondecreasing in* q*, and (ii)* $\forall j = 1, ..., M$*,*

$$\sum_{i=1}^{j} g(q_i) f(q_i \mid \hat{e}) \geq \sum_{i=1}^{j} t(q_i \mid H_{FR}, \hat{e}) f(q_i \mid \hat{e}), \text{ with equality for } j = M.$$

The added condition in Lemma 2 is that the transfer schedule is nondecreasing, under which one can construct an information structure that implements it. It is fairly immediate that a strictly decreasing transfer schedule is not implementable, since given that output is a favorable signal of ability, it would imply that the market is systematically fooled in that it pays a lower wage for outputs which indicate high ability. While there exist information structures that lead to a locally decreasing transfer schedule, intuitively one can ignore them as they discourage effort by (sometimes) penalizing high output.[4]

Using Lemmas 1 and 2, I now explain why condition (1) is related to the informativeness of output about ability. For each effort level, output corresponds to some experiment about ability. Under condition (1), Lemma 2 implies that some garbling of output under effort e_2 leads to the same transfer schedule as fully revealing output under effort e_1. Conversely, if condition (1) is violated, then Lemma 1 implies that no garbling of output under effort e_2 leads to the same transfer schedule as fully revealing output under effort e_1.

Information and incentives: partial equilibrium. The cumulative subsidy condition of Lemma 1 defines a sense in which garbling output makes a transfer schedule "flatter". The effect on incentives is the content of the following lemma, where $U_A(e \mid H, \hat{e}) := \mathbb{E}_q[u_A(t(q \mid H, \hat{e}), e) \mid e]$ is the agent's

[4]The following example shows that some locally decreasing transfer schedules are implementable. Take $Q = \{q_1, q_2, q_3\}$, and an information structure $\tilde{H}$ that reveals q_2 and pools $\{q_1, q_3\}$ into the same signal. For a given conjecture $\hat{e}$, one typically has $\mathbb{E}(\theta \mid q_2, \hat{e}) \neq \mathbb{E}(\theta \mid q \in \{q_1, q_3\}, \hat{e})$, which leads to a nonmonotone transfer schedule as either $t(q_2 \mid \tilde{H}, \hat{e}) > t(q_1 \mid \tilde{H}, \hat{e}) = t(q_3 \mid \tilde{H}, \hat{e})$ or $t(q_2 \mid \tilde{H}, \hat{e}) < t(q_1 \mid \tilde{H}, \hat{e}) = t(q_3 \mid \tilde{H}, \hat{e})$.

expected payoff when exerting effort e under information structure H and market conjecture $\hat{e}$.

Lemma 3 *Assume that* $\mathbb{E}(\theta \,|\, q, e)$ *is nondecreasing in* q *and* $F(q \,|\, e)$ *is ordered according to the MLRP. Fix* $\hat{e}$ *and* H. *Then* $\forall e \leq \hat{e}$,

$$U_A(\hat{e} \,|\, H_{FR}, \hat{e}) - U_A(e \,|\, H_{FR}, \hat{e}) \geq U_A(\hat{e} \,|\, H, \hat{e}) - U_A(e \,|\, H, \hat{e}).$$

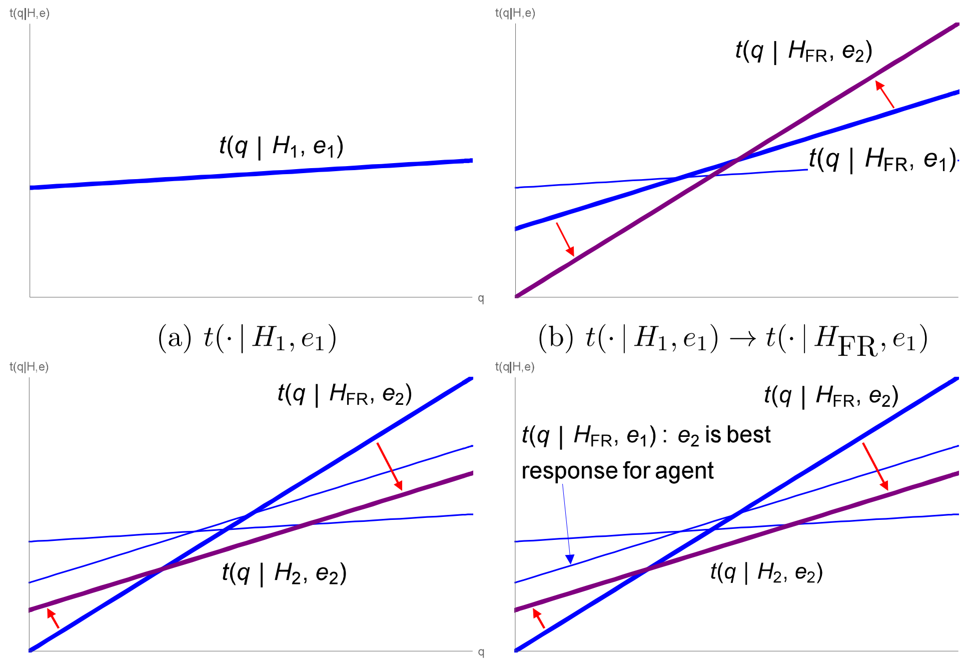
Lemma 3 states that for a given market conjecture $\hat{e}$, moving from an arbitrary garbling to full revelation makes $\hat{e}$ more attractive compared to any lower effort. Better information about output and effort are complementary, albeit in a weak sense. It is for example *not* true that for two arbitrary effort levels $e_H > e_L$, the higher effort e_H becomes more attractive when moving to full revelation, unless $e_H = \hat{e}$.

Nevertheless, Lemma 3 can be seen as a partial equilibrium version of the result that under appropriate ordering conditions, noisy disclosure of output reduces effort due to career-concerns incentives. For if some e^* is an equilibrium effort under a garbling H, then Lemma 3 implies that the agent wants to deviate (weakly) upwards if output gets fully revealed, yet the market's conjecture remains at e^*. Below, it will be shown how this can be developed into an equilibrium result when effort makes output more informative about ability.

That $F(q \,|\, e)$ has to satisfy the MLRP in Lemma 3 is due to the absence of restrictions on how output can be garbled. If one were to restrict attention to information structures H such that the transfer schedule corresponding to full revelation is everywhere steeper, then the concept of first order stochastic dominance (FOSD) would be sufficient to deliver the same conclusion.[5] Yet this would rule out even simple information structures like partitions.

Proof of Proposition 1: a sketch. The idea behind Proposition 1 will

[5] $t(\cdot \,|\, H_{\text{FR}}, \hat{e})$ is everywhere steeper than $t(\cdot \,|\, H, \hat{e})$ if $\forall q_2 > q_1$, $t(q_2 \,|\, H_{\text{FR}}, \hat{e}) - t(q_1 \,|\, H_{\text{FR}}, \hat{e}) \geq t(q_2 \,|\, H, \hat{e}) - t(q_1 \,|\, H, \hat{e})$. In the normal-linear example, all transfer schedules are linear and hence can be ordered by steepness, but in any case the normal distribution satisfies the MLRP. Another example is when output is binary (and the set of information structures is unrestricted), yet in this case FOSD implies the MLRP.



(a) $t(\cdot \mid H_1, e_1)$

(b) $t(\cdot \mid H_1, e_1) \to t(\cdot \mid H_{\text{FR}}, e_1)$

(c) $t(\cdot \mid H_{\text{FR}}, e_1) \to t(\cdot \mid H_{\text{FR}}, e_2)$

(d) $t(\cdot \mid H_{\text{FR}}, e_2) \to t(\cdot \mid H_2, e_2)$

Figure 3: Illustration of the proof of Proposition 1

be explained in a series of steps, which are illustrated in Figure 3. The result is shown by contradiction, starting with an arbitrary information structure H_1 and equilibrium effort e_1. It will be shown that either e_1 can be also implemented by full revelation, or there exists an alternative (possibly noisy) information structure that implements an effort $e_2 > e_1$. Therefore, the best effort across all pure strategy equilibria (which can be shown to exist) must be achieved by full revelation. The sketch is as follows:

(a) Start with the equilibrium that involves a garbled information structure H_1 and effort level e_1. This leads to transfer schedule $t(\cdot \mid H_1, e_1)$.

(b) Suppose that the principal moves to full revelation of output, but the market still conjectures that the agent exerts e_1, so that the transfer schedule changes to $t(\cdot \mid H_{\text{FR}}, e_1)$. By Lemma 3, the agent wants to deviate weakly upwards to some effort level $e_2 \geq e_1$. If $e_2 = e_1$, this implies that H_{FR} also implements e_1. Going forward, the case $e_2 > e_1$ is considered.

(c) The goal is to show that e_2 can be implemented by some information structure. Observe that H_{FR} provides incentives that are too strong, since under the informativeness condition $t(q \,|\, H_{\text{FR}}, e_2) = \mathbb{E}(\theta \,|\, q, e_2)$ might induce even more effort (but not less) than $t(\cdot \,|\, H_{\text{FR}}, e_1) = \mathbb{E}(\theta \,|\, q, e_1)$.

(d) By Lemma 2 there exists some information structure H_2 that induces a transfer schedule that coincides with $t(\cdot \,|\, H_{\text{FR}}, e_1)$ up to a constant. Since the constant does not affect incentives, e_2 is a best response to $t(\cdot \,|\, H_2, e_2)$.

That effort makes output a more informative signal about ability is used in steps (c) and (d). In conjunction with Lemma 2, the assumption implies that when the market conjectures e_2, then $t(\cdot \,|\, H_{FR}, e_1)$ can be induced by some information structure H_2 up to a constant.

On the cardinality of Q. Proposition 1 holds for any measurable $Q \subset \mathbb{R}$, as shown in Proposition 9 in Appendix A.1 under a mild regularity condition.[6] In particular, one can verify that the normal-linear example satisfies the assumptions of Proposition 1. The MLRP holds since $F(q \,|\, e) = N(\mu_\theta + e, \sigma_\theta^2)$, and $\mathbb{E}(\theta \,|\, q, e) = q - e$ which is increasing in q and (trivially) supermodular.

Corollary 1 *In the normal-linear example, full revelation leads to the highest effort among all pure strategy equilibria.*

3.3 Information about ability

To study the effect of uncertainty about ability on incentives, it is assumed that output is not garbled but the principal can decide whether to disclose *ability estimate* η to the market. Intuitively one would expect that there is less of an incentive to try to fool the market, as disclosure of η reduces the uncertainty about ability, and therefore it becomes harder to manipulate the market belief. The following example shows that this is not true in general.

Example 2 *Let $\Theta = \{0, 1\}$, $E = \{e_L, e_H\}$, and $Q = \{q_L, q_H\}$. The task is one of two "types", η_0 and η_1, and the agent does not know the task type.*

[6] Attention is restricted to information structures that induce transfer schedules with at most countably many discontinuities.

The probability of high ability $\theta = 1$ is 2/3, as is the probability of facing task η_1. Ability and task type are independently distributed, and unaffected by effort. Effort e_L leads to q_L, for any θ and η.[7] *When e_H is exerted, then q_H is generated if $(\theta, \eta) \in \{(0, \eta_0), (1, \eta_1)\}$, that is when ability and task "match", while the remaining combinations of (θ, η) lead to q_L.*

Suppose that the market conjectures $\hat{e} = e_H$. When exerting e_H, the agent receives an expected wage of $Pr(\theta = 1) = 2/3$. If he deviates to e_L, output q_L is realized and he receives

$$Pr(\theta = 1 \,|\, q = q_L, \hat{e} = e_H) = \frac{Pr(\theta = 1) \cdot Pr(\eta = \eta_0)}{Pr(\theta = 1) \cdot Pr(\eta = \eta_0) + Pr(\theta = 0) \cdot Pr(\eta = \eta_1)} = \frac{1}{2}.$$

If $c(e_H) - c(e_L) > \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$, then e_H is not an equilibrium.

Now suppose that the actual task gets revealed to the market through signal $\eta \in \{\eta_0, \eta_1\}$, together with output (the agent still does not know which type of task he is facing). Given conjecture $\hat{e} = e_H$, the agent still receives an expected wage of $Pr(\theta = 1) = 2/3$ when exerting e_H. If the market observes q_L and $\eta = \eta_0$ ($\eta = \eta_1$), given its conjecture of $\hat{e} = e_H$ it believes the agent is of ability $\theta = 1$ ($\theta = 0$). So the agent receives an expected wage of $Pr(\eta = \eta_0) = \frac{1}{3}$ when deviating to e_L. If $c(e_H) - c(e_L) \leq \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$, effort e_H is indeed an equilibrium.

In summary, if $\frac{1}{6} < c(e_H) - c(e_L) \leq \frac{1}{3}$, strictly higher effort is generated if the market observes the task.

Let $G(\eta \,|\, q, e)$ be the distribution of η conditional on (q, e). Say that the ability estimate η is a *favorable signal of ability* if (i) $\mathbb{E}(\theta \,|\, q, \eta, e)$ is nondecreasing in η, and (ii) for $\forall q \in Q$ and $e_2 > e_1$, $G(\eta \,|\, q, e_2)$ is dominated by $G(\eta \,|\, q, e_1)$ according first order stochastic dominance. So η is good news about ability, and higher effort makes lower ability estimates more likely conditional on any output realization.

In Example 2, η is not a favorable signal of ability. In particular, $\mathbb{E}(\theta \,|\, q, \eta, e)$ is not nondecreasing in η. Given a market conjecture of e_H, $\eta = \eta_1$ is good

[7] The violation of the full support assumption on $F(q \,|\, e)$ is not critical for the conclusion.

news about ability if q_H is realized, but bad news conditional on q_L. In the example, ability is to some extent "horizontal" since the lower ability type $\theta = 0$ does better at task η_0. Still, ability is "vertical" in that the market is willing to pay a higher wage to the high ability type $\theta = 1$. Disclosing information about the horizontal aspect reduces effort.

Proposition 3 *Assume that*

- $F(q \mid e)$ *is ordered according to the MLRP,*
- *effort increases the informativeness of output about ability, and*
- η *is a favorable signal of ability.*

Equilibrium effort is higher if the ability estimate η *is not disclosed to the market.*

The proposition makes a comparison between disclosing q and (q, η). It is easy to verify that when η is a favorable signal of ability, then this also applies to $\gamma(\eta)$ for any nondecreasing function γ. So disclosing η through a monotone partition is not beneficial either.

Relative performance evaluation. In reality, a principal can use relative performance evaluation (RPE) when two (or more) agents are present. This means that agent A^i is evaluated not only based on his own performance, q^i, but also on the performance of agent A^j, q^j. Intuitively, q^j contains information both about agent i's performance and ability.

For example, let agents i and j share a similar educational background which correlates their abilities. They are assigned to different divisions, and their performance is influenced by shocks idiosyncratic to their division. In this setting, the following two properties are plausibly satisfied. (a) Better performance by j reflects favorably on i. (b) A given performance by i is less impressive the more effort i exerted, and leads to a negative inference about j's ability (which makes it more likely that q_j takes on a low value). Then, $\eta = q^j$ is a favorable signal of i's ability and disclosure of η reduces effort under the conditions of Proposition 3.

Another case is where both agents work in the same division and are exposed to common productivity shocks. In such a case, one can expect the following two conditions to be satisfied. (a) Better performance by j reflects unfavorably on i. (b) For a given performance by i, higher effort by i indicates that the division faced a difficult environment, which makes it more likely that q_j takes on a low value. To formalize conditions (a) and (b), say that η *reduces uncertainty about performance* if (i) $\mathbb{E}(\theta \,|\, q, \eta, e)$ is nonincreasing in η, and (ii) for $\forall q \in Q$ and $e_2 > e_1$, $G(\eta \,|\, q, e_2)$ is dominated by $G(\eta \,|\, q, e_1)$ according first order stochastic dominance. The following result provides conditions under which disclosure of $\eta = q_j$ leads to more effort.

Proposition 4 *Assume that*

- $F(q \,|\, \eta, e)$ *is ordered according to the MLRP in* e*,*
- *effort increases the informativeness of output about ability for each* η*,*
- η *reduces uncertainty about performance, and*
- *the marginal of* η *does not depend on* e*.*

Equilibrium effort is higher if (q, η) *is disclosed to the market.*

That effort increases the informativeness of output about ability for each η means that condition (1) in Definition 1 needs to be satisfied conditional on each realization of η, and the same is true for the MLRP.

Since (q^i) is a garbling of (q^i, q^j), one might wonder why Proposition 4 is not a corollary of Proposition 1. However, the multidimensional baseline signal (q^i, q^j) will in general not satisfy the assumptions of Proposition 1.[8]

4 Extensions

Several extensions are considered. In Section 4.1, the principal can also design a wage contract. Section 4.2 studies mixed strategy equilibria, while Section

[8]The set of possible output realizations is $Q^i \times Q^j$. One would need to find a linear order on $Q^i \times Q^j$ such that all assumptions of Proposition 1 are satisfied with respect to that order.

4.3 allows for a more general information revelation technology that becomes available when the agent randomizes.

4.1 Wage contracts

So far, incentives were only derived from career concerns. Suppose now that there is also an explicit wage contract $\hat{w}$, offered by the principal. The agent maximizes the expectation of $\hat{u}_A(\hat{w}, w, e) = \hat{w} + w - c(e)$, where w is the market wage. Notice that the objectives of effort and profit maximization do not necessarily coincide anymore.

Garbling output raises a conceptual issue about the wage contract. I assume that the principal can achieve a separation between payments and information revelation. This requires that the market does not observe the wage payment $\hat{w}$, and the principal can base $\hat{w}$ on the actual output realization even when q is garbled. An alternative would be that information about the actual output realization gets lost and the wage payment has to be based on the signal realization (in which case it is immaterial whether the market observes $\hat{w}$ or not). Notice that in the latter scenario the principal is more constrained in terms of the wage contracts she can offer when garbling output. So if fully revealing output is optimal under my scenario, this remains true when the wage contract has to be based on the signal.

For now, take a wage contract $\hat{w} : Q \to \mathbb{R}$ as given. The next result shows that effort is still maximized by fully revealing output and hiding the ability estimate. Basically, the two types of incentives enter the agent's payoff in an additive way, and the disclosure policy has no effect on the incentives from the wage contract.

Proposition 5 *For any wage contract $\hat{w}$, effort is maximized by fully revealing output under the assumptions of Proposition 1 and not disclosing the ability estimate under the assumptions of Proposition 3.*

Now, instead of taking the wage contract as given, let the principal design $\hat{w}$ and H to maximize profits subject to a limited liability constraint $\hat{w} \geq 0$.

Proposition 6 gives conditions when full revelation will be part of the principal's solution, under the following assumption. Say there are *decreasing returns to signal jamming* if the set of effort levels is an interval $E = [\underline{e}, \overline{e}]$, and the payoff under full revelation given any market conjecture $\hat{e}$, $\mathbb{E}_q[\mathbb{E}(\theta \mid q, \hat{e}) \mid e] - c(e)$, is concave in e.

Proposition 6 *Assume that there are decreasing returns to signal jamming and $f(q_M|e)$ is concave.*

- *Take an arbitrary garbling of output and wage contract that lead to some effort $\tilde{e}$. Under the assumptions of Proposition 1, there exists an alternative wage contract $\hat{w}^*$ that together with full revelation implements $e^* \geq \tilde{e}$ and leads to a lower expected wage payment, that is $\mathbb{E}[\hat{w}^* \mid e^*] \leq \mathbb{E}[\hat{w} \mid \tilde{e}]$.*
- *Take an arbitrary wage contract that leads to some effort $\tilde{e}$ when (η, q) is disclosed. Under the assumptions of Proposition 3, there exists an alternative wage contract $\hat{w}^*$ that together with just disclosing q implements $e^* \geq \tilde{e}$ and leads to a lower expected wage payment, that is $\mathbb{E}[\hat{w}^* \mid e^*] \leq \mathbb{E}[\hat{w} \mid \tilde{e}]$.*

Career concerns deliver "free incentives" for the principal, and providing incentives through a wage contract is costly at the margin. Combining a disclosure policy that maximizes career concerns with a wage contract that only rewards the highest output (which is optimal due to the MLRP) generates more effort (and hence output) at a lower cost.

4.2 Mixed strategy equilibrium

Notice that for a given information structure, a pure strategy equilibrium might not exist. Such garblings were implicitly ruled out above. To extend the analysis to mixed strategy equilibria, a modified notion of effort increasing the informativeness of output about ability is introduced.

Definition 2 *For any mixed strategy* $\sigma \in \Delta(E)$*, let* $\bar{e}(\sigma) = sup(support(\sigma))$*. Say that* ***effort increases the informativeness of output about ability allowing for mixed strategies*** *if* $\forall \sigma, e \geq \bar{e}(\sigma)$*,*

$$\mathbb{E}_q \left[\mathbb{E}(\theta \mid q, e) - \mathbb{E}(\theta \mid q, \sigma) \mid e \right] \geq \mathbb{E}_q \left[\mathbb{E}(\theta \mid q, e) - \mathbb{E}(\theta \mid q, \sigma) \mid q \leq q_j, e \right] \ \ \forall q_j \in Q. \tag{3}$$

Clearly, if effort increases the informativeness of output about ability allowing for mixed strategies, then effort increases the informativeness of output about ability.

Proposition 7 *Assume that*

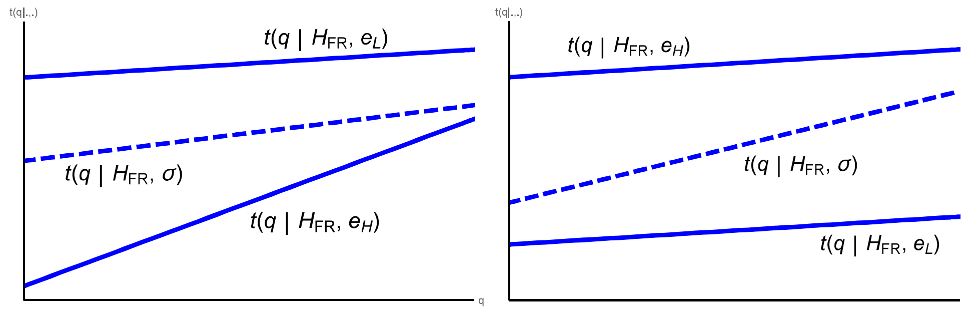
- $F(q \mid e)$ *is ordered according to the MLRP,*
- $\mathbb{E}(\theta \mid q, e)$ *is nondecreasing in* q*, and*
- *effort increases the informativeness of output about ability allowing for mixed strategies.*

A best mixed strategy equilibrium exists, is achieved by fully revealing output, and is pure. More precisely, for any information structure $\hat{H}$ *with equilibrium* $\hat{\sigma}$*, there exists an equilibrium under full revelation with effort* $e^* \geq \bar{e}(\hat{\sigma})$*.*

The main complication is that on the set of mixed strategies, the MLRP will not be satisfied in general.[9] Therefore, one cannot conclude from Lemma 3 that starting from any mixed strategy equilibrium, the agent wants to deviate upwards when moving to full revelation when the market conjecture is fixed.

Another issue is that the notion of how effort increases the informativeness of output needs to be strengthened. Remember that when restricting attention to pure strategies, supermodularity of $E(\theta \mid q, e)$ was a simple sufficient condition. This does not guarantee that effort increases the informativeness when allowing for mixed strategies.

[9] More precisely, if $F(q \mid e)$ satisfies the MLRP given the natural order on E, then unless E or Q are binary there is in general no linear order on $\Delta(E)$ such that the MLRP is satisfied with respect to that order.



(a) Good news effect of high output (b) Bad news effect of high output

Figure 4: Illustration of the effect of mixing on the transfer schedule: $t(q \,|\, H_{FR}, \sigma)$ lies in between $t(q \,|\, H_{FR}, e_L)$ and $t(q \,|\, H_{FR}, e_H)$, and under MLRP a higher q is more indicative of high effort. In panel 4a, this leads to a steeper transfer schedule under σ, and incentives are the strongest when the market entertains conjecture σ. In panel 4b, this leads to a flatter transfer schedule, and incentives are the strongest when the market entertains conjecture e_H.

To see this, consider a non-degenerate mixed strategy equilibrium under full revelation. Given the MLRP of $F(q \,|\, e)$, high output is indicative of high effort, and whether this effect leads to a favorable inference about ability or not depends on how $\mathbb{E}(\theta \,|\, q, e)$ varies with e. This is illustrated in Figure 4, where σ is a randomization over (e_L, e_H). In panel 4a, $\mathbb{E}(\theta \,|\, q, e)$ is increasing in e so that there is a "good news" effect of high output about ability, in that for higher output it is more likely that the agent exerted e_H which leads to an favorable inference about θ. This makes the transfer schedule steeper under σ than under e_L or e_H, and therefore output more informative. Conversely, $\mathbb{E}(\theta \,|\, q, e)$ is decreasing in e in panel 4b, and the "bad news" effect of output flattens the transfer schedule under σ, making output most informative under e_H.

The preceding discussion suggests the following easily verifiable sufficient condition for effort to increase the informativeness of output about ability when allowing for mixed strategies.

Lemma 4 *If $\mathbb{E}(\theta \,|\, q, e)$ is supermodular in (q, e) and nonincreasing in e, and $F(q \,|\, e)$ is ordered according to the MLRP, then effort increases the informativeness of output about ability allowing for mixed strategies.*

Apart from providing easily verifiable conditions for effort to increase the informativeness of output about ability when allowing for mixed strategies, Lemma 4 shows that mixed strategy equilibria are not relevant in the normal-linear model.

Corollary 2 *In the normal-linear example, effort is maximized by full revelation within the class mixed strategy strategy equilibria.*

The informativeness condition can be dispensed with when output is binary. All that matters for incentives is by how much the expected wage differs across output realizations, and garbling decreases the wage difference.

Proposition 8 *Assume that*

- $F(q \mid e)$ *is ordered according to the MLRP,*
- $\mathbb{E}(\theta \mid q, e)$ *is nondecreasing in* q*, and*
- *output is binary:* $M = 2$*.*

Fully revealing output is optimal. More precisely, for any information structure $\hat{H}$ *with equilibrium* $\hat{\sigma}$*, there exists an equilibrium under full revelation with mixed strategy* σ^**, and* σ^* *dominates* $\hat{\sigma}$ *according to the MLRP.*

To my knowledge, only Dewatripont, Jewitt and Tirole (1999*b*) considered mixed strategies in the presence of career concerns, but the context is different. Effort is multidimensional, and the agent randomizes over the task he exerts effort on. The overall amount of effort is constant, and the randomization across tasks makes output a less accurate signal of ability. Here, the agent randomizes over the overall amount of effort, and how this affects the informativeness of output is a priori not obvious as Figure 4 shows.

4.3 Mediated information structures

This section allows for a generalized communication protocol between the principal and agent. Instead of just garbling output, the principal also commits to make a secret effort recommendation to the agent which is unobserved by the market.

Formally, an information structure consists of $(\mathcal{M}, \alpha_{\mathcal{M}}, H)$, where $\mathcal{M}$ is a message space, $\alpha_{\mathcal{M}} \in \Delta(\mathcal{M})$ is a distribution over the message space and H specifies for each combination of $q \in Q$ and $m \in \mathcal{M}$ a signal distribution $H(\cdot \mid m, q) \in \Delta(\mathbb{R})$. The timing is now as follows.

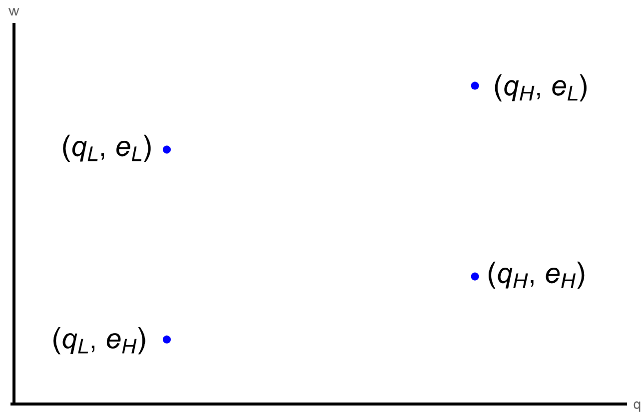
1. The principal publicly commits to an information structure $(\mathcal{M}, \alpha_{\mathcal{M}}, H)$.
2. The principal draws a realization from $\alpha_{\mathcal{M}}$ and privately communicates it to the agent.
3. The agent exerts effort e, unobserved by the principal and market.
4. Ability θ, output q and signal s are realized, and signal s is disclosed to the market.
5. The market pays wage w to the agent.

Any information structure in this communication protocol will be called a *mediated*, while the standard mechanisms up to now are called *conventional*. Since the principal can commit to communicate the realization of $\alpha_{\mathcal{M}}$ truthfully, it is without loss of generality to look at equilibria where the agent is obedient. That is, $\mathcal{M} = E$ and $\alpha_{\mathcal{M}} = \sigma$, where σ is the equilibrium distribution over effort.

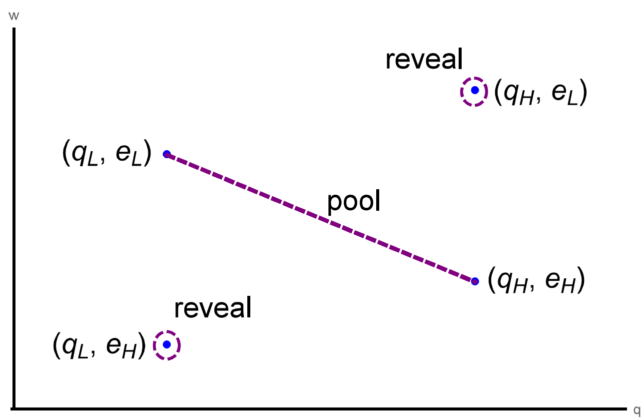
Clearly, a mediated mechanism can improve upon a conventional one only when the agent plays a randomized strategy. Notice that compared to a mixed strategy equilibrium induced by a conventional mechanism, the agent need not be indifferent between all effort levels in the support of his strategy; it is possible that he strictly prefers the recommended effort.

A mediated information structure allows the principal to correlate the effort and signal, *conditional on* q. The following example illustrates how the principal can benefit from this.

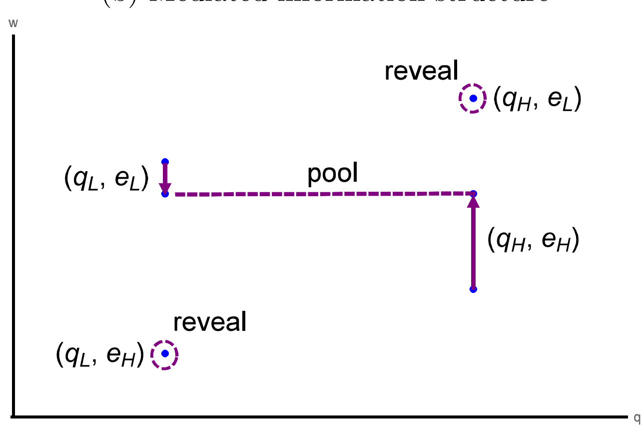
Example 3 *Let* $E = \{e_L, e_H\}$ *and* $Q = \{q_L, q_H\}$*, the primitives are specified in Table 2. Incentives are driven by the difference in expected wage across*



(a) Wages under full revelation



(b) Mediated information structure



(c) Wages under mediated information structure

Figure 5: Illustration of a mediated information structure

e	$\mathbb{E}(\theta \mid q_L, e)$	$\mathbb{E}(\theta \mid q_H, e)$	$f(q_H \mid e)$	$c(e)$
e_L	4	5	1/3	0
e_H	0	2	2/3	1

Table 2: Details for Example 3

signal s	outcomes inducing s	wage $w(s)$
s_1	(q_L, e_H)	0
s_2	$\{(q_L, e_L), (q_H, e_H)\}$	3
s_3	(q_H, e_L)	5

Table 3: Mediated equilibrium in Example 3: $\sigma(e_L) = \sigma(e_H) = 1/2$

outcomes q_L and q_H. In order to sustain e_H, this difference needs to be at least 3. Under full revelation, e_L is the unique equilibrium as conditional on e_H, the wage difference is $2 - 0 = 2 < 3$. Since the conditions of Proposition 7 apply, this is the best outcome that can be achieved through conventional mechanisms even when considering mixed equilibria. Table 3 summarizes a mediated equilibrium where the agent randomizes with equal probability between e_L and e_H. Using (q, e) to denote a combination of an output realization and an effort recommendation, then (q_L, e_H) and (q_H, e_L) get revealed (leading to wages of 0 and 5, respectively), while $\{(q_L, e_L), (q_H, e_H)\}$ get pooled into the same signal (leading to a wage of 3). The agent's difference in expected wages across q_L and q_H equals $5 - 3 = 2$ after recommendation e_L, and $3 - 0 = 3$ after recommendation e_H, making the respective recommendations incentive compatible.

Figure 5 illustrates the mechanics of the example. In equilibrium, the agent randomizes between e_L and e_H, and panel (a) plots the wages the agent would get under full revelation and either effort conjecture. Panel (b) describes how the mediated information structure reveals different output realizations and effort recommendations. Since $\mathbb{E}(\theta \mid q_L, e_L) > \mathbb{E}(\theta \mid q_H, e_H)$, pooling $\{(q_L, e_L), (q_H, e_H)\}$ rewards q_H after recommendation e_H, and punishes q_L after recommendation e_L, compared to full revelation. By increasing the difference in the wage across the two outputs, incentives are raised after *either* recommendation as illustrated in panel (c). The example specifies a cost function that makes the effort recommendations incentive compatible, yet can only sustain e_L with conventional mechanisms.

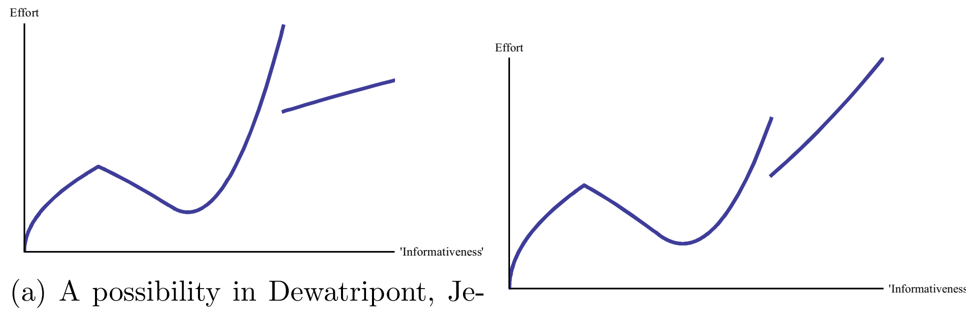
When a mediated mechanism improves upon the best conventional one, it will do so by not fully revealing output. Mediated mechanisms allow the principal to correlate the signal s with effort, *conditional on* q. Therefore, q is not anymore sufficient for (q, s) when estimating θ, as is the case in a conventional mechanism. Section 5 has some further discussion on this. Ruling out the optimality of mediated mechanisms appears elusive. However, one can show that if $\mathbb{E}(\theta \mid q, e)$ is independent of e (say because there is no residual uncertainty about θ given q), then there is no benefit from mediated mechanisms. Some characterization of which transfer schedules can be induced by mediated mechanisms would be helpful to make further progress, but this is beyond the scope of this paper.

5 Discussion

This section discusses several assumptions and attempts to put the results of this paper into perspective.

Full equilibrium analysis vs local comparative statics. Dewatripont, Jewitt and Tirole (1999*a*) show in their Proposition 5.2 that for a given market conjecture, adding more noise to the signal locally decreases incentives (see also Ogaku (2017) for a similar result).[10] Since their result holds for any market conjecture, they interpret it as a local comparative static around equilibrium. At first sight, this seems to show that effort is increasing in the informativeness of the signal, and maximized by fully revealing output (when the first-order approach is valid). The catch is that the local comparative static result relies on several assumptions about the baseline signal to which noise is being added. Even if output satisfies those assumptions, it will never be true that *every*

[10]Lemma 3 can be seen as a non-local analogue in that *any* effort below $\hat{e}$ is shown to become less attractive. Besides the MLRP, Dewatripont, Jewitt and Tirole (1999*a*) assume that θ and q are affiliated conditional on e, which is stronger than $\mathbb{E}(\theta \mid q, e)$ being nondecreasing in q.



(a) A possibility in Dewatripont, Jewitt and Tirole (1999a)

(b) Result of Proposition 1

Figure 6: Comparison with Dewatripont, Jewitt and Tirole (1999a)

garbling of output (when treated as the baseline signal) satisfies them, unless output is binary. Therefore, the approach in Dewatripont, Jewitt and Tirole (1999a) shows that starting from a certain set of information structures (to which full revelation belongs), adding noise decreases effort. Since not all garblings belong to this set, a situation like in Figure 6a can arise. Adding noise decreases effort when starting from full revelation, yet a noisy signal is optimal. Example 4 in Appendix A.1 shows that this can arise within the normal-linear example once there are complementarities between effort and ability in the production function, even if the principal is restricted to information structures that add normally distributed noise to output.

Full revelation is a global optimum under the conditions of Proposition 1. This does not rule out that making the signal Blackwell-less informative can raise effort, as illustrated schematically in Figure 6b. In fact, unless output is binary this arises in any statistical environment. Since when there are at least three possible output realizations, some garblings lead to transfer schedules that are locally decreasing. Adding even more noise can eliminate the nonmonotonicity in the transfer schedule and raise incentives. Finally, Figure 6 highlights that the maximal effort can be discontinuous in the information structure; the reason is that the set of equilibria is not lower hemicontinuous.

On the full support assumption. The market's expectation about the agent's ability is pinned down by Bayes' rule when every output realization is on-path, which obviates the need to specify off-path beliefs. One might think that the full revelation result of Proposition 1 remains true even when the full support assumption is violated as long as one can find nearby primitives with full support (and where the assumptions of Proposition 1 are satisfied). Since full revelation does weakly better than any other information structure along a sequence of primitives, this should be also true in the limit. The problem is that the equilibrium correspondence is not upper hemicontinuous in general once one imposes a refinement. A previous version of this paper defined a refinement in the spirit of sequential equilibrium and had an example where such a reversal arises in the limit.[11]

General risk attitudes. The agent's risk neutrality with respect to the market belief is critical. To illustrate why e.g. Proposition 1 fails, suppose the agent maximizes the expectation of $u_A(w, e) = \mathbb{1}\{w \geq w^t\} - c(e)$. The agent is only concerned whether the belief is above a threshold w^t, say because the reward is a promotion and hence indivisible. Once Q has three or more elements, full revelation fails to maximize effort in general. Suppose there is an output $\tilde{q}$ such that effort increases (decreases) the probability of outputs $q > \tilde{q}$ ($q < \tilde{q}$). Under full revelation of q, there will be a threshold level q^* such that the agent gets promoted only if output is above q^*. If say q^* is above $\tilde{q}$, the principal can typically induce more effort by recommending a promotion not just for output levels above q^*, but also for some adjacent output levels below q^*. This is incentive compatible as the market is unaware of the output realization that induced the recommendation.

More generally, garbling output creates a mean-preserving contraction of the marginal distribution of wages. While the marginal distribution over wages does not pin down the transfer schedule and hence incentives, one can show that under risk neutrality even the "best" transfer schedule that can be derived from the new marginal distribution over wages depresses incentives, in the sense of Lemma 3. The idea is that when wages are less dispersed, there is less leeway to reward good over bad performance.

[11] The refinement coincides with sequential equilibrium when E is finite and one restricts attention to information structures with finitely many signals. In the example, even information structures that only garble off-path outputs are inferior.

With general risk attitudes, transfer schedules should specify how output realizations are related to the agent's expected *utility*. But a garbling of output does not cause a mean-preserving contraction of the distribution of utilities. For example, if the agent is risk averse then pooling information insures the agent. By selectively pooling high output realizations, the principal might raise the reward associated with high output through the insurance effect, thereby generating stronger incentives.[12]

Relationship to Bayesian Persuasion. Consider a joint distribution of three real valued random variables (θ, q, s), where θ and s are independent conditional on q. It is well known that the marginal distribution of $\mathbb{E}(\theta \mid s)$ is a mean-preserving contraction of the marginal distribution of $\mathbb{E}(\theta \mid q)$ (see Blackwell (1953)). Gentzkow and Kamenica (2016) apply this result to a typical Bayesian Persuasion problem.

In my setting, the result fully characterizes the set of marginal wage distributions that can be induced by some information structure for a given market conjecture $\hat{e}$. However, the link from information structure to market behavior cannot be treated in isolation. Two information structures can induce the same marginal distribution of wages, yet different transfer schedules, and therefore different incentives for the agent. In the language of Bayesian Persuasion, the sender (the principal) designs disclosure about a state (output) in order to influence the action (wage) of the receiver (market). The variation here is that the joint distribution of state and receiver action influences the preferences of a third player (agent), whose action is taken secretly before the state is realized. The third player's action affects both the distribution of the state and the receiver's optimal action.

Mediated equilibria. In a conventional mechanism with associated mixed strategy equilibrium σ, q is a sufficient statistic for (q, s) when estimating θ,

[12] My results are not "discontinuous" when the agent is "almost risk neutral" in the sense that on a certain bounded interval of wages the risk premium of every lottery that is less risky than the one induced by full revelation is arbitrarily close to zero: If effort is finite, then the full revelation result remains generically true. If effort is continuous and a certain concavity assumption on the payoffs under full revelation is satisfied, then full revelation is almost optimal.

that is $\mathbb{E}(\theta \mid q, s, \sigma) = \mathbb{E}(\theta \mid q, \sigma)$. In a mediated mechanism this is not true because conditional on q, s provides information about the realization from σ. Unless $\mathbb{E}(\theta \mid q, e)$ is independent of e, q is not a sufficient statistic for (q, s) when estimating θ. Indeed, one can show that if $\mathbb{E}(\theta \mid q, e)$ is constant in e, there is no benefit from mediated mechanisms.

An alternative perspective is that the principal chooses a direct recommendation policy that maps output into a randomized wage. Let T be the set of recommendation policies. A conventional mechanism that induces a mixed strategy equilibrium can be seen as an element from $\Delta(E) \times T$ which satisfies the agent's and market's obedience constraints. A mediated mechanism is an element from $\Delta(E \times T)$, and the correlation between effort and recommendation policy allows to relax the obedience constraints.[13]

Literature. A brief summary of the applied theory literature that features both career-concerns incentives and information design questions follows.

Ratchet effect. In a dynamic principal-agent relationship with uncertainty about the agent's ability, career concerns arise when the principal cannot commit to long-term contracts. Future contracts are influenced by current performance, and a forward-looking agent takes this into account which is described as the ratchet effect. A tractable formalization was introduced by Gibbons and Murphy (1992), where the relevant uncertainty is normally distributed, the agent has CARA preferences, and the principal offers linear contracts. This has been used to study the effects of relative performance comparisons across agents (Meyer and Vickers (1997)), optimal job design (Meyer (1995) and Meyer, Olsen and Torsvik (1996)), and optimal duration of employment (Auriol, Friebel and Pechlivanos (2002)). Each of the preceding examples can be posed as a question of optimal information disclosure about performance

[13] Similar ideas appear in Rahman and Obara (2010), Rahman (2012), and Krähmer (2017). In Rahman and Obara (2010), random effort recommendations make the monitoring technology richer in a multi-agent moral hazard problem. Conditioning the contract on the effort recommendation and keeping it secret can increase the set of implementable allocations substantially. In Rahman (2012) and Krähmer (2017), the principal elicits an agent's privately observed signal whose distribution she can affect through some variable k. Secretly randomizing over k and correlating it with the contract makes it easier to elicit the signal from the agent.

and ability in the presence of career concerns.

Fully dynamic models. Most career-concerns applications are static in that effort is exerted once. Moav and Neeman (2010) point out that more informative performance measures, while initially conducive to inducing output, remove uncertainty about ability in the future. This is detrimental for incentives when ability is persistent. Hörner and Lambert (2018) consider information design within a dynamic version of the normal-linear model. They determine the optimal way to disclose past and present performance, and compare the amount of information revealed when the market has access to previously disclosed information and when it does not.

Other work. In Bar-Isaac (2007), an agent can acquire an asset with unknown productivity. This provides him with an incentive device to exert effort if the market is not able to ascertain whether output was produced because of effort or because of the high productivity of the asset. To maintain these incentives, the agent does not wish to reveal information about the source of high output. In Jeon (1996), a principal has to assign workers to teams. Uncertainty about ability differs across agents, and team composition affects the informativeness of output regarding the ability of the team members. Dewatripont, Jewitt and Tirole (1999*b*) study multi-tasking with career concerns, in particular the effect of transparency about the agent's focus on incentives. Bar-Isaac and Ganuza (2008) consider how different recruitment and training policies affect the career concerns of agents. Recruitment policies affect the type uncertainty, and training policies affect the informativeness of output, hence these are questions of information design. The effect of information on career-concerns incentives has also been studied in relational contract settings (Mukherjee, 2008*b*, 2010) and in matching markets (Mukherjee (2008*a*)). The setting in Wolitzky (2012) is similar to that in Section 4.1 where I consider wage contracts, in particular Proposition 6, except that the principal's wage contract and disclosure policy are unobserved by the market. In both Boleslavsky and Kim (2017) and Rodina and Farragut (2018), an agent wishes to create the impression that he has produced high output, which can be seen as a special case of the current model by defining ability to equate to output.

The transparency result of Proposition 1 does not apply in the settings they consider, and some qualitative features of the optimal garbling are derived.

6 Conclusion

This paper derives substantive conditions under which full revelation of performance measures and more uncertainty about the agent's ability lead to stronger career-concerns incentives. A key assumption for deriving these comparative statics is that effort increases the informativeness of output regarding ability. The results provide a foundation for two principles regarding the evaluation of agents within organizations: transparency with respect to measures of performance, and opaqueness regarding certain information about the agent's ability. The following three issues have not been addressed, however.

First, when the conditions for full revelation of performance measures are violated, it is natural to ask what features the optimal garbling has. The same can be said for mediated mechanisms, because they are not fully revealing when improving upon conventional disclosure policies.

Second, comparative statics with respect to information about performance or ability are considered separately. Disclosing certain types of information might affect both types of information simultaneously. Plausibly, a tradeoff arises between the forces identified here. Conceptually more interesting is the case where the principal can control information about performance and ability *simultaneously*, that is, she can garble (q, η).

Third, it would be desirable to understand the effect of information beyond the (quasi-) static model considered here.[14] In fact, one might wonder how my results should be interpreted given that career concerns arise in dynamic situations. The main change is that information released at some point is also available in the future, so that e.g. increasing the informativeness of performance measures today has (beyond the direct effect) an indirect effect

[14]One complication in a (fully) dynamic model is the presence of asymmetric information between the principal and the agent, at least off the equilibrium path (which one has to solve in order to determine on-path behavior). Some progress on this has been made by Cisternas (2017).

of reducing the uncertainty about ability tomorrow. One possible answer is that the static approach separates these two effects and allows to sign them in a setting without particular assumptions on the statistical environment. A natural conjecture is that with dynamics it remains optimal to withhold information about ability when the signal structure in the stage game satisfies appropriate conditions as derived here, yet full revelation of performance measures requires stronger assumptions. A second interpretation is that the static setting approximates situations where a worker frequently changes employers: knowing that the agent will move on, the principal designs disclosure myopically in order to extract as much effort as possible today.[15]

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A Appendix

A.1 Proofs for Section 3

Proof of Proposition 1 The proof is split in two parts: first it is shown that effort is maximized by full revelation, and then that a maximal effort exists.

Part 1: effort is maximized by full revelation. Define $\mathrm{BR}(t(\cdot))$ as the set of utility maximizing effort levels for the agent when he faces transfer schedule $t(\cdot)$. Take an arbitrary information structure $\hat{H}$ that induces effort $\hat{e}$, so that $\hat{e} \in \mathrm{BR}(t(\cdot \mid \hat{H}, \hat{e}))$. By Lemma 3, there exists an effort level $e_1 \geq \hat{e}$ that is a best response to transfer schedule $t(\cdot \mid H_{\mathrm{FR}}, \hat{e})$.

Starting with e_1, define a sequence of effort levels through $e_{n+1} = \xi(e_n)$, where $\xi(e) := \max\left(\mathrm{BR}(t(\cdot \mid H_{\mathrm{FR}}, e))\right)$. The maximum of $\mathrm{BR}(\cdot)$ exists by the Maximum Theorem. That sequence is nondecreasing is due to the assumption that effort makes output a more informative signal about ability. An equivalent statement of the assumption is as follows:

Assumption 1 *For any $e_L, e_H \in E$ with $e_H > e_L$, $\forall j = 1, ..., M$*

$$\sum_{i=1}^{j} \left\{E(\theta \mid q_i, e_L) - E(\theta \mid q_i, e_H) + \kappa\right\} f(q_i \mid e_H) \geq 0,$$

where $\kappa = E(\theta \mid e_2) - E_q[E(\theta \mid q, e_1) \mid e_2]$.

An immediate implication of Assumption 1 is that $\forall j = 1, ..., M$

$$\sum_{i=1}^{j} [t(q_i \mid H_{\mathrm{FR}}, e_L) + \kappa] f(q_i \mid e_H) \geq \sum_{i=1}^{j} t(q_i \mid H_{\mathrm{FR}}, e_H) f(q_i \mid e_H).$$

Since $e_n \in \mathrm{BR}(t(\cdot \mid H_{\mathrm{FR}}, e_{n-1})) = \mathrm{BR}(t(\cdot \mid H_{\mathrm{FR}}, e_{n-1}) + \kappa)$, it follows from Lemma 3 that some $e_{n+1} \geq e_n$ is a best response to transfer schedule $t(\cdot \mid H_{\mathrm{FR}}, e_n)$.

It was already shown that $e_1 \geq \hat{e}$, and given the sequence $\{e_n\}$ is non-decreasing, its limit e^* (which exists by compactness of E) satisfies $e^* \geq \hat{e}$. Effort e^* is an equilibrium because $e_{n+1} \in \mathrm{BR}(t(\cdot \mid H_{\mathrm{FR}}, e_n))$ implies $e^* \in \mathrm{BR}(t(\cdot \mid H_{\mathrm{FR}}, e^*))$ by the Maximum Theorem. This shows that the maximal effort across all information structures (if it exists) has to be achieved by full revelation.

Part 2: a maximal effort exists. The set of all possible pure strategy effort levels, E^*, is non-empty since for the information structure that reveals no information has an equilibrium with $e = \min E$. Let $\bar{e} = \sup E^*$, which is finite by compactness of E. There exists a sequence of information structures $\{H_n\}$ and associated equilibrium effort levels $\{e_n\}$ with $e_n \to \bar{e}$. By the first part of the proof, one can find another sequence $\{\tilde{e}_n\}$ with $\tilde{e}_n \geq e_n$, and each $\tilde{e}_n$ is induced by H_{FR}. Again, by the Maximum theorem it is true that $\bar{e}$ is an equilibrium under full revelation. ∎

Proof of Proposition 2 Since output is more informative under e_1 than under e_2, but not vice versa,

$$\sum_{i=1}^{j} \{\mathbb{E}(\theta \,|\, q_i, e_2) - \mathbb{E}(\theta \,|\, q_i, e_1) + \kappa\} \, f(q_i \,|\, e_1) \geq 0, \, \forall j, \tag{4}$$

with equality at $j = M$, where $\kappa = \mathbb{E}(\theta \,|\, e_1) - \mathbb{E}_q[\mathbb{E}(\theta \,|\, q, e_2) \,|\, e_1]$. Moreover, for some j the inequality is strict. Since the strict MLRP is satisfied, $f(q \,|\, e_2)/f(q \,|\, e_1)$ is increasing.[16] This implies

$$0 < \mathbb{E}_q[\mathbb{E}(\theta \,|\, q, e_2) \,|\, e_2] - \mathbb{E}_q[\mathbb{E}(\theta \,|\, q, e_2) \,|\, e_1] < \mathbb{E}_q[\mathbb{E}(\theta \,|\, q, e_1) \,|\, e_2] - \mathbb{E}_q[\mathbb{E}(\theta \,|\, q, e_1) \,|\, e_1]. \tag{5}$$

The first inequality is true because $\mathbb{E}(\theta \,|\, q, e_2)$ is nondecreasing in q and since $f(q \,|\, e_2)$ dominates $f(q \,|\, e_1)$ according to the MLRP. The second inequality follows from Lemma 5.

Consider the following cost function

$$c(e) = \begin{cases} 0 & \text{if } e \leq e_1, \\ \xi & \text{if } e_1 < e \leq e_2, \\ \infty & \text{if } e > e_2, \end{cases}$$

[16] One can get the same conclusion without the strict MLRP if Equation (4) is a strict inequality $\forall j < M$.

where $\xi > 0$ satisfies

$$\xi \in (\mathbb{E}_q[\mathbb{E}(\theta \,|\, q, e_2) \,|\, e_2] - \mathbb{E}_q[\mathbb{E}(\theta \,|\, q, e_2) \,|\, e_1], \mathbb{E}_q[\mathbb{E}(\theta \,|\, q, e_1) \,|\, e_2] - \mathbb{E}_q[\mathbb{E}(\theta \,|\, q, e_1) \,|\, e_1]).$$

Given this cost function, under full revelation a pure equilibrium can only involve e_1 or e_2. However, given that the market conjectures e_1, the agent wants to deviate to e_2, and given a conjecture of e_2, the agent wants to deviate to e_1. On the other hand, if the market conjectures e_1, then by Equation (4) and Lemma 2 one can implement a transfer schedule that coincides with $\mathbb{E}(\theta \,|\, q, e_2)$ up to a constant, and therefore makes e_1 a best response.[17] ∎

Proof of Lemma 1 Fix a possibly mixed conjecture $\hat{\sigma}$. In inequality j in Equation (2),

$$F(q_j \,|\, \hat{\sigma})\mathbb{E}_q[\mathbb{E}_s[\mathbb{E}_\theta[\theta \,|\, s, \hat{\sigma}] \,|\, q] \,|\, q \leq q_j, \hat{\sigma}]$$

is compared with

$$F(q_j \,|\, \hat{\sigma})\mathbb{E}_q[\mathbb{E}_\theta[\theta \,|\, q, \hat{\sigma}] \,|\, q \leq q_j, \hat{\sigma}].$$

Notice that $\mathbb{E}_\theta[\theta \,|\, s, \hat{\sigma}] = \mathbb{E}_q[\mathbb{E}_\theta[\theta \,|\, q, s, \hat{\sigma}] \,|\, s, \hat{\sigma}] = \mathbb{E}_q[\mathbb{E}_\theta[\theta \,|\, q, \hat{\sigma}] \,|\, s, \hat{\sigma}]$, where the first equality follows from the law of iterated expectations, and the second from s being a garbling of q. So one can write

$$\begin{aligned} &\mathbb{E}_q[\mathbb{E}_s[\mathbb{E}_\theta[\theta \,|\, s, \hat{\sigma}] \,|\, q] \,|\, q \leq q_j, \hat{\sigma}] \\ &= \mathbb{E}_q[\mathbb{E}_s[\mathbb{E}_q[\mathbb{E}_\theta[\theta \,|\, q, \hat{\sigma}] \,|\, s, \hat{\sigma}] \,|\, q] \,|\, q \leq q_j, \hat{\sigma}] \\ &= \mathbb{E}_s[\mathbb{E}_q[\mathbb{E}_\theta[\theta \,|\, q, \hat{\sigma}] \,|\, s, \hat{\sigma}] \,|\, q \leq q_j, \hat{\sigma}]. \end{aligned}$$

On the other hand,

$$\begin{aligned} &\mathbb{E}_q[\mathbb{E}_\theta[\theta \,|\, q, \hat{\sigma}] \,|\, q \leq q_j, \hat{\sigma}] \\ &= \mathbb{E}_s[\mathbb{E}_q[\mathbb{E}_\theta[\theta \,|\, q, \hat{\sigma}] \,|\, s, q \leq q_j, \hat{\sigma}] \,|\, q \leq q_j, \hat{\sigma}]. \end{aligned}$$

[17] For the specified cost function, no pure equilibrium exists under full revelation. Suppose that in addition there exists a third effort level e_0 with $e_0 < e_1$ such that output is more informative under both e_1 and e_2 than under e_0. Then $c(e)$ can be modified at $e = \min E$ such that e_0 is an equilibrium under H_{FR}.

Since $\mathbb{E}_\theta[\theta \mid q, \hat{\sigma}]$ is nondecreasing in q,

$$\mathbb{E}_q[\mathbb{E}_\theta[\theta \mid q, \hat{\sigma}] \mid s, \hat{\sigma}] \geq \mathbb{E}_q[\mathbb{E}_\theta[\theta \mid q, \hat{\sigma}] \mid s, q \leq q_j, \hat{\sigma}],$$

with equality for $j = M$. $\blacksquare$

Proof of Lemma 2 Fix a possibly mixed conjecture $\hat{\sigma}$. Since $\hat{\sigma}$ will be kept fixed throughout, the following shorthands are introduced: $f_i = f(q_i \mid \hat{\sigma}), t_i = t(q_i \mid H_{FR}, \hat{\sigma})$, and $g_i = g(q_i)$. Define $\Delta_i = g_i - t_i$, and let $J = \{j : \Delta_j > 0\}$ be the set of all output realizations after which the expected wage under the transfer schedule $g(\cdot)$ is higher than under the fully informative signal. If J is empty, the supposition of the lemma implies that $\forall i$, $g(q_i) = t(q_i \mid H_{FR}, \hat{\sigma})$ so that the fully revealing information structure implements $g(\cdot)$.

Step 1: A building block in the construction of the information structure that implements $g(\cdot)$ will be a collection of vectors satisfying certain properties summarized as follows. There exists a collection of vectors $\{\alpha^j\}_{j \in J}$, where each α^j is an $M \times 1$ vector, and satisfies the following properties:

- α^j is nonnegative,
- $\alpha^j_i = 0$ for $i < j$ and $\alpha^j_j = 1$,
- if $i > j$ and $\alpha^j_i > 0$, then $\Delta_i \leq 0$,
- $\sum_{i=1}^M \alpha^j_i f_i \Delta_i = 0$,
- the collection of α^j's satisfies $\sum_{j \in J} \alpha^j = e_M$.[18]

Since each α^j is nonnegative, the last property implies that $\forall i, j$, $\alpha^j_i \in [0, 1]$. Indeed, the elements of α^j will correspond to certain probabilities in the information structure.

The following algorithm is used to show that a collection of vectors $\{\alpha^j\}_{j \in J}$ with the desired properties exists. The algorithm consists of $|J|$ steps and its starting point is an $M \times 1$ vector of "budgets" $b^1 = e_M$ that at each step k

[18] e_M is an $M \times 1$ vector of 1's.

gets updated from b^k to b^{k+1}. By construction, b^k will be nonnegative at each step k. The algorithm moves from the highest to the lowest element in J, so let $j(k)$ be the k-th highest element in J, that is $j(1) > ... > j(|J|)$. Let $\xi(b^k, j(k))$ be the largest element such that

$$\sum_{i=j(k)}^{\gamma} b_i^k f_i \Delta_i \geq 0, \forall \gamma = j(k), ..., \xi(b^k, j(k)),$$

which exists since $\Delta_{j(k)} > 0$. So either $\xi(b^k, j(k)) = M$, or $\sum_{i=j(k)}^{\xi(b^k,j(k))+1} \Delta_i f_i b_i^k < 0$.

Step k in the algorithm starts with the nonnegative vector of budgets b^k, and uses it to define the vector $\alpha^{j(k)}$ as

$$\alpha_i^{j(k)} = \begin{cases} 0 & \text{for } i < j(k), \\ b_i^k & \text{for } j(k) \leq i \leq \xi(b^k, j(k)), \\ \frac{\sum_{h=j(k)}^{\xi(b^k,j(k))} b_h^k f_h \Delta_h}{-f_{\xi(b^k,j(k))+1} \Delta_{\xi(b^k,j(k))+1}} & \text{for } i = \xi(b^k, j(k)) + 1, \\ 0 & \text{for } i > \xi(b^k, j(k)) + 1. \end{cases}$$

If $\xi(b^k, j(k)) = M$, only the first two cases apply. At the end of step k, b^{k+1} is defined as $b^k - \alpha^{j(k)}$. For b^{k+1} to be nonnegative, it needs to be shown that $\alpha^{j(k)} \leq b^k$. This is obviously true for all elements except $i = \xi(b^k, j(k)) + 1$, which is relevant when $\xi(b^k, j(k)) < M$. Then by the definition of $\xi(b^k, j(k))$

$$\sum_{i=j(k)}^{\xi(b^k,j(k))+1} b_i^k f_i \Delta_i < 0,$$

and $f_{\xi(b^k,j(k))+1} \Delta_{\xi(b^k,j(k))+1} b^k_{\xi(b^k,j(k))+1} < 0$. This implies that $\alpha^{j(k)}_{\xi(b^k,j(k))+1} \leq b^k_{\xi(b^k,j(k))+1}$.

Now all the desired properties of the collection $\{\alpha^j\}_{j \in J}$ will be verified. At step k, $\alpha^{j(k)}$ gets defined and is obviously nonnegative for $i \neq \xi(b^k, j(k)) + 1$. For $i = \xi(b^k, j(k)) + 1$ (which is relevant when $\xi(b^k, j(k)) < M$), this follows from $\sum_{h=j(k)}^{\xi(b^k,j(k))} b_h^k f_h \Delta_h \geq 0$ and $f_{\xi(b^k,j(k))+1} \Delta_{\xi(b^k,j(k))+1} < 0$. By construction

$\alpha_i^j = 0$ for $i < j$, and $\alpha_j^j = 1$ since the algorithm moves from the highest to the lowest element in J so that at each step k, $b_{j(k)}^k = 1 = \alpha_{j(k)}^{j(k)}$. This implies that at step k_1 which defines $\alpha^{j(k_1)}$, $b_{j_2}^{k_1} = 0$ for $j_2 > j(k_1)$. Therefore if $i > j$ and $\alpha_i^j > 0$, then $\Delta_i \leq 0$. The next property is that $\sum_{i=1}^{M} \alpha_i^j f_i \Delta_i = 0$, which for any step k with $\xi(b^k, j(k)) < M$ is satisfied as can be seen from the construction of $\alpha_{\xi(b^k,j(k))+1}^{j(k)}$. When at some step k it is that $\xi(b^k, j(k)) = M$, a violation of $\sum_{i=1}^{M} \alpha_i^j f_i \Delta_i = 0$ would imply that $\sum_{i=j(k)}^{M} b_i^k f_i \Delta_i > 0$. But this leads to a contradiction as for the first $\tilde{k}$ that such a violation can be found it is that

$$\sum_{k=1}^{\tilde{k}} \sum_{i=j(k)}^{M} b_i^k f_i \Delta_i = \sum_{i=j(\tilde{k})}^{M} f_i \Delta_i > 0.$$

This would contradict the supposition of the lemma that

$$\sum_{i=1}^{j} g_i f_i \geq \sum_{i=1}^{j} t_i f_i \qquad \forall j = 1, ..., M, \text{ with equality for } j = M.$$

The final property is that $\sum_{j \in J} \alpha^j = e_M$. Given that $b^{|J|+1} = e_M - \sum_{j \in J} \alpha^j$, this is equivalent to showing that at the the end of the final step $k = |J|$, $b^{|J|+1} = 0$. A violation of this would imply that at the final step $k = |J|$, $\xi(b^k, j(k)) < M$ and $b_i^{|J|+1} > 0$ for (possibly several) i with $\Delta_i < 0$. On the one hand, it was previously shown that $\forall j$, $\sum_{i=1}^{M} \alpha_i^j f_i \Delta_i = 0$, so

$$\sum_{k=1}^{|J|} \left(\sum_{i=1}^{M} \alpha_i^{j(k)} f_i \Delta_i \right) = 0.$$

But this generates a contradiction, since using that $b^{|J|+1} = e_M - \sum_{j \in J} \alpha^j$,

$$\sum_{k=1}^{|J|} \sum_{i=1}^{M} \alpha_i^{j(k)} f_i \Delta_i = \sum_{i=1}^{M} \left(\sum_{k=1}^{|J|} \alpha_i^{j(k)} \right) f_i \Delta_i = \sum_{i=1}^{M} \left(1 - b_i^{|J|+1} \right) f_i \Delta_i = - \sum_{i=1}^{M} b_i^{|J|+1} f_i \Delta_i > 0.$$

Step 2: The information structure will be supported on $|J| \cdot M$ signals, so define the set of signals $\{s_i^j\}_{\{j \in J, i=1,\ldots,M\}}$ consisting of arbitrary numbers that are all distinct.

For $j \in J$ and $i \notin J$, set $\Pr(s_i^j \,|\, q_i) = \alpha_i^j$, and $\Pr(s_i^j \,|\, q_j) = \beta_i^j$, where $\beta_i^j = 0$ if $\alpha_i^j = 0$, and if $\alpha_i^j > 0$ then β_i^j is given by

$$\beta_i^j = \frac{\alpha_i^j f_i(t_i - g_i)}{f_j(g_i - t_j)}. \tag{6}$$

$\beta_i^j \geq 0$ since $\alpha_i^j > 0$ implies both that $\Delta_i = g_i - t_i \leq 0$ and $i > j$, so that $g_i \geq g_j > t_j$.

For $j \in J$, set $\Pr(s_j^j \,|\, q_j) = 1 - \sum_{i \neq j} \beta_i^j$. This is nonnegative since for $i > j$, β_i^j satisfies

$$\beta_i^j f_j(g_i - t_j) + \alpha_i^j f_i \Delta_i = 0.$$

Summing over $i > j$ gives

$$\sum_{i>j} \beta_i^j f_j(g_i - t_j) + \sum_{i>j} \alpha_i^j f_i \Delta_i = 0.$$

By the construction from step 1, $\sum_{i>j} \alpha_i^j f_i \Delta_i = -f_j \Delta_j$. So

$$f_j \Delta_j = \sum_{i>j} \beta_i^j f_j(g_i - t_j) \geq \sum_{i>j} \beta_i^j f_j \Delta_j,$$

where the inequality follows from g being nondecreasing. Since $f_j \Delta_j > 0$, $\sum_{i>j} \beta_i^j = \sum_{i \neq j} \beta_i^j \leq 1$.

All probabilities are specified, except for two cases. The first is $\Pr(s_i^{j_1} \,|\, q_{j_2})$ where $j_1, j_2 \in J$, $j_1 \neq j_2$, and the second is $\Pr(s_{i_1}^j \,|\, q_{i_2})$ where $i_1, i_2 \notin J$, $i_1 \neq i_2$, and $j \in J$. All of these are set to zero. For this to constitute a valid information structure, it remains to show that every output is mapped with probability 1 into some set of signals. For $j \in J$, it was just shown that

$$\sum_{s \in S} \Pr(s \,|\, q_j) = \sum_{i=1}^{M} \Pr(s_i^j \,|\, q_j) = \Pr(s_j^j \,|\, q_j) + \sum_{i \neq j} \Pr(s_i^j \,|\, q_j) = 1 - \sum_{i \neq j} \beta_i^j + \sum_{i \neq j} \beta_i^j = 1.$$

For $i \notin J$,

$$\sum_{s \in S} \Pr(s \mid q_i) = \sum_{j \in J} \Pr(s_i^j \mid q_i) = \sum_{j \in J} \alpha_i^j = 1.$$

This completes the specification of H.

Step 3: It now remains to show that H implements the transfer schedule $g(\cdot)$. There are two types of signals $s \in S$. Either $s = s_i^j$ for $i \notin J$ and $j \in J$, or $s = s_j^j$ for $j \in J$. Some signal $s' \notin S$ might get never realized, more formally $\Pr(s' \mid q) = 0$, $\forall q \in Q$. Any such signal plays no role under the assumption that the marginal distribution over q has full support for all effort levels, as no deviation by the agent can induce such a signal realization.

Any signal of the form s_i^j for $i \notin J$ and $j \in J$ gets only induced by outputs q_i and q_j. The wage associated with such a signal is (suppressing notation for the information structure and effort conjecture $\hat{\sigma}$

$$w(s_i^j) = \frac{f_i \alpha_i^j t_i + f_j \beta_i^j t_j}{f_i \alpha_i^j + f_j \beta_i^j} = g_i,$$

as can be seen from the definition of β_i^j in Equation (6). Any remaining signal has to be of the form s_j^j for $j \in J$. Such a signal realization can only be induced by q_j, so output gets revealed and $w(s_j^j) = t_j$.

To show that transfer schedule $g(\cdot)$ is implemented, first outputs of the form $q = q_i$ with $i \notin J$ are considered. For any such output

$$\mathbb{E}[w(s) \mid q_i] = \sum_{j \in J} \alpha_i^j w(s_i^j) = \sum_{j \in J} \alpha_i^j g_i = g_i.$$

For any output of the form $q = q_j$ with $j \in J$,

$$\begin{aligned}
\mathbb{E}[w(s) \mid q_j] = &\ \textstyle\sum_{i \notin J} \beta_i^j w(s_i^j) + \left(1 - \sum_{i \notin J} \beta_i^j\right) w(s_j^j) \\
= &\ \textstyle\sum_{i \notin J} \beta_i^j g_i + \left(1 - \sum_{i \notin J} \beta_i^j\right) t_j \\
= &\ \textstyle t_j + \sum_{i \notin J} \beta_i^j \left(g_i - t_j\right) \\
= &\ \textstyle t_j + \sum_{i \notin J} \alpha_i^j \frac{f_i}{f_j} \left(t_i - g_i\right),
\end{aligned}$$

where the final equality follows from the definition of β_i^j in Equation (6). After subtracting g_j from both sides,

$$(\mathbb{E}[w(s)\,|\,q_j] - g_j)f_j = -\sum_{i \notin J} \alpha_i^j f_i \Delta_i - f_j \Delta_j,$$

and since the right hand side is zero by construction of α^j (remember that $\alpha_j^j = 1$) it follows that $\mathbb{E}[w(s)\,|\,q_j] = g_j$. ∎

Let $u_A(w, \sigma) := \mathbb{E}_\sigma[u_A(w, e)]$, define the notation $U(t, \sigma) := \mathbb{E}_q[u_A(t(q), \sigma)\,|\,\sigma]$ as the expected payoff the agent gets when being rewarded according to transfer schedule t and playing the mixed strategy σ. Also, $c(\sigma) = \mathbb{E}_\sigma[c(e)]$.

Lemma 5 *Fix a mixed strategy σ, and suppose that two functions $t_1(q), t_2(q)$ (not necessarily transfer schedules induced by some information structure) satisfy*

$$\sum_{i=1}^{j} t_2(q_i) f(q_i\,|\,\sigma) \geq \sum_{i=1}^{j} t_1(q_i) f(q_i\,|\,\sigma) \qquad (7)$$
$$\forall j = 1, ..., M, \text{ with equality for } j = M.$$

For any $\tilde{\sigma}$ that is dominated by σ according to the MLRP,

$$U(t_1, \sigma) - U(t_1, \tilde{\sigma}) \geq U(t_2, \sigma) - U(t_2, \tilde{\sigma}),$$

and

$$\mathbb{E}[t_2(q)\,|\,\tilde{\sigma}] \geq \mathbb{E}[t_1(q)\,|\,\tilde{\sigma}].$$

For any $\tilde{\sigma}$ that dominates σ according to the MLRP,

$$U(t_1, \sigma) - U(t_1, \tilde{\sigma}) \leq U(t_2, \sigma) - U(t_2, \tilde{\sigma}),$$

and

$$\mathbb{E}[t_2(q)\,|\,\tilde{\sigma}] \leq \mathbb{E}[t_1(q)\,|\,\tilde{\sigma}].$$

Proof of Lemma 5 First consider the case where $\tilde{\sigma}$ is dominated by σ according to the MLRP.

$$U(t_2, \sigma) - U(t_2, \tilde{\sigma}) = \sum_{i=1}^{M} t_2(q_i)[f(q_i \,|\, \sigma) - f(q_i \,|\, \tilde{\sigma})] - [c(\sigma) - c(\tilde{\sigma})].$$

Defining $L_i = f(q_i \,|\, \tilde{\sigma})/f(q_i \,|\, \sigma)$, one can write $f(q_i \,|\, \sigma) - f(q_i \,|\, \tilde{\sigma}) = f(q_i \,|\, \sigma)(1 - L_i)$. By the MLRP supposition, L_i is weakly decreasing.

$$\begin{aligned}
&\sum_{i=1}^{M} t_2(q_i)[f(q_i \,|\, \sigma) - f(q_i \,|\, \tilde{e})] - [c(\sigma) - c(\tilde{\sigma})] \\
&= \sum_{i=1}^{M} t_2(q_i) f(q_i \,|\, \sigma)(1 - L_i) - [c(\sigma) - c(\tilde{\sigma})] \\
&= \sum_{i=1}^{M} t_2(q_i) f(q_i \,|\, \sigma)(1 - L_M + \sum_{j=i+1}^{M} L_j - L_{j-1}) - [c(\sigma) - c(\tilde{\sigma})] \\
&= \sum_{i=1}^{M} t_2(q_i) f(q_i \,|\, \sigma)(1 - L_M) + \sum_{i=1}^{M} \sum_{j=i+1}^{M} t_2(q_i) f(q_i \,|\, \sigma)(L_j - L_{j-1}) - [c(\sigma) - c(\tilde{\sigma})].
\end{aligned}$$

But the first term is $\mathbb{E}[t_2(q) \,|\, \sigma](1 - L_M) = \mathbb{E}[t_1(q) \,|\, \sigma](1 - L_M)$ by Equation (7). The only term where the transfer schedule appears is

$$\begin{aligned}
&\sum_{i=1}^{M} \sum_{j=i+1}^{M} t_2(q_i) f(q_i \,|\, \sigma)(L_j - L_{j-1}) \\
&= \sum_{j=2}^{M} (L_j - L_{j-1}) \sum_{i=1}^{j-1} t_2(q_i) f(q_i \,|\, \sigma).
\end{aligned}$$

Since $L_j - L_{j-1} \leq 0$, it follows from Equation (7) that

$$U(t_1, \sigma) - U(t_1, \tilde{\sigma}) \geq U(t_2, \sigma) - U(t_2, \tilde{\sigma}).$$

Canceling out cost terms and using that $\mathbb{E}[t_1(q) \,|\, \sigma] = \mathbb{E}[t_2(q) \,|\, \sigma]$ gives

$$\mathbb{E}[t_2(q) \,|\, \tilde{\sigma}] \geq \mathbb{E}[t_1(q) \,|\, \tilde{\sigma}].$$

In the other case where $\tilde{\sigma}$ MLRP dominates σ, the proof can be taken verbatim except that L_i is weakly increasing, and this is why the inequality is reversed. $\blacksquare$

Proof of Lemma 3 This follows from Lemma 5. Fix an arbitrary e^* and information structure H. Let $\sigma = e^*$ and $\tilde{\sigma} = e$. Since $e \leq e^*$ the MLRP condition is satisfied. Define $t_2(\cdot)$ as $t(\cdot \,|\, H, e^*)$, and $t_1(\cdot)$ as $t(\cdot \,|\, H_{\text{FR}}, e^*)$. By Lemma 1, the supposition in Equation (7) is satisfied. $\blacksquare$

Proposition 9 *Suppose all assumptions in Proposition 1 are satisfied, $Q \subset \mathbb{R}$ is measurable, and $F(q \,|\, e)$ has continuous density. Restrict attention to information structures that induce a transfer schedule with a countable number of discontinuities in equilibrium. Then full revelation maximizes effort among all pure strategy equilibria.*

Proof of Proposition 9 Start with information structure H and equilibrium $\hat{e}$. Lemma 1 implies that $\forall \hat{q}$, $\int_{-\infty}^{\hat{q}} t(q \,|\, H, \hat{e}) \, \mathrm{d}F(q \,|\, \hat{e}) \geq \int_{-\infty}^{\hat{q}} t(q \,|\, H_{\text{FR}}, \hat{e}) \, \mathrm{d}F(q \,|\, \hat{e})$, with equality for $\hat{q} = +\infty$. As argued momentarily, one can show that Lemma 3 applies so that $e_1 := \max \text{BR}(t(\cdot \,|\, H_{\text{FR}}, \hat{e})) \geq \hat{e}$. As in the proof of Proposition 1, one can construct a sequence $\{e_n\}$ with limit $e^* \geq \hat{e}$, and e^* is an equilibrium under full revelation.

It remains to show that Lemma 3 applies. Suppose that for some $\hat{e}$ and two transfer schedules with countably many discontinuities, t_1 and t_2, satisfy $\forall \hat{q}$, $\int_{-\infty}^{\hat{q}} t_2(q) \, \mathrm{d}F(q \,|\, \hat{e}) \geq \int_{-\infty}^{\hat{q}} t_1(q) \, \mathrm{d}F(q \,|\, \hat{e})$, with equality for $\hat{q} = +\infty$. It needs to be shown that for any $e \leq \hat{e}$,

$$\delta := [U(t_1, \hat{e}) - U(t_1, e)] - [U(t_2, \hat{e}) - U(t_2, e)] \geq 0.$$

Let $L(q) = f(q \,|\, e)/f(q \,|\, \hat{e})$, which is nonincreasing, and define $\Delta(q) := t_2(q) - t_1(q)$. Since $U(t_1, \hat{e}) = U(t_2, \hat{e})$, δ can be written as

$$\delta = \int_{-\infty}^{+\infty} \Delta(q) L(q) \, \mathrm{d}F(q \,|\, \hat{e}) = \sum_{i=1}^{+\infty} \int_{I_i} \Delta(q) L(q) \, \mathrm{d}F(q \,|\, \hat{e}),$$

where I_i is an interval on which $\Delta(q)$ is continuous, and I_i lies below I_{i+1}. To show that $\sum_{i=1}^{+\infty} \int_{I_i} \Delta(q)L(q)\ \mathrm{d}F(q \,|\, \hat{e})$ is nonnegative, it is claimed that $\forall j$,

$$\sum_{i=1}^{j} \int_{I_i} \Delta(q)L(q)\ \mathrm{d}F(q \,|\, \hat{e}) \geq 0. \tag{8}$$

Equation (8) is equivalent to

$$\sum_{i=1}^{j} \int_{I_i} \Delta(q)\frac{L(q)}{L(q_j)}\ \mathrm{d}F(q \,|\, \hat{e}) \geq 0.$$

Since $\Delta(q)$ is continuous within each interval,

$$\sum_{i=1}^{j} \int_{I_i} \Delta(q)\frac{L(q)}{L(q_j)}\ \mathrm{d}F(q \,|\, \hat{e}) = \sum_{i=1}^{j} \frac{L(q_i)}{L(q_j)} \int_{I_i} \Delta(q)\ \mathrm{d}F(q \,|\, \hat{e}),$$

where $q_i \in I_i$. It is now claimed that

$$\sum_{i=1}^{j} \frac{L(q_i)}{L(q_j)} \int_{I_i} \Delta(q)\ \mathrm{d}F(q \,|\, \hat{e}) \geq \sum_{i=1}^{j} \int_{I_i} \Delta(q)\ \mathrm{d}F(q \,|\, \hat{e}) \geq 0,$$

where the second inequality follows from the supposition that $\forall \hat{q}$, $\int_{-\infty}^{\hat{q}} t_2(q)\ \mathrm{d}F(q \,|\, \hat{e}) \geq \int_{-\infty}^{\hat{q}} t_1(q)\ \mathrm{d}F(q \,|\, \hat{e})$. For $j = 1$, the first inequality is obviously satisfied. If it is satisfied up to k, then for $j = k + 1$,

$$\begin{aligned}\sum_{i=1}^{k+1} \frac{L(q_i)}{L(q_{k+1})} \int_{I_i} \Delta(q)\ \mathrm{d}F(q \,|\, \hat{e}) = &\ \textstyle\sum_{i=1}^{k} \frac{L(q_i)}{L(q_{k+1})} \int_{I_i} \Delta(q)\ \mathrm{d}F(q \,|\, \hat{e}) + \int_{I_{k+1}} \Delta(q)\ \mathrm{d}F(q \,|\, \hat{e}) \\ \geq &\ \textstyle\sum_{i=1}^{k} \int_{I_i} \Delta(q)L(q)\ \mathrm{d}F(q \,|\, \hat{e}) + \int_{I_{k+1}} \Delta(q)\ \mathrm{d}F(q \,|\, \hat{e}).\end{aligned}$$

The inequality follows from $L(q_{k+1}) \leq L(q_k)$. ∎

Proof of Proposition 3 First consider the case where the market observes (q, η), with some equilibrium effort $\hat{e}$. The wage after each realization of (q, η)

satisfies $w(q, \eta, \hat{e}) = \mathbb{E}(\theta \,|\, q, \eta, \hat{e})$. The payoff from a deviation is

$$\begin{aligned} & \mathbb{E}_{(q,\eta)}[\mathbb{E}(\theta \,|\, q, \eta, \hat{e}) \,|\, e] - c(e) \\ = \;& \mathbb{E}_q[\mathbb{E}_\eta[\mathbb{E}(\theta \,|\, q, \eta, \hat{e}) \,|\, q, e] \,|\, e] - c(e). \end{aligned}$$

Now compare this to a situation where only q gets disclosed to the market, and its conjecture is $\hat{e}$. Since the wage satisfies $w(q, \hat{e}) = \mathbb{E}(\theta \,|\, q, \hat{e})$, the agent maximizes

$$\begin{aligned} & \mathbb{E}_q[\mathbb{E}(\theta \,|\, q, \hat{e}) \,|\, e] - c(e) \\ = \;& \mathbb{E}_q[\mathbb{E}_\eta[\mathbb{E}(\theta \,|\, q, \eta, \hat{e}) \,|\, q, \hat{e}] \,|\, e] - c(e). \end{aligned}$$

If η is a favorable signal of ability, then for $e < \hat{e}$,

$$\mathbb{E}_\eta[\mathbb{E}(\theta \,|\, q, \eta, \hat{e}) \,|\, q, \hat{e}] \leq \mathbb{E}_\eta[\mathbb{E}(\theta \,|\, q, \eta, \hat{e}) \,|\, q, e] \tag{9}$$

since $\mathbb{E}(\theta \,|\, q, \eta, \hat{e})$ is nondecreasing in η and $G(\eta \,|\, q, e)$ dominates $G(\eta \,|\, q, \hat{e})$ according to FOSD. Since $\hat{e}$ is an equilibrium when (q, η) is disclosed, this implies that there exists a best response $e_1 \geq \hat{e}$ when only q is disclosed.

Since $F(q \,|\, e)$ is ordered according to the MLRP and effort increases the informativeness of output about ability, there exists a nondecreasing sequence $\{e_n\}$ whose limit $e^* \geq \hat{e}$ is an equilibrium under full revelation when only q is disclosed (see part 1 of the proof of Proposition 1). ∎

Proof of Proposition 4 When η reduces uncertainty about performance, the inequality in (9) is reversed. Retracing the steps of the proof of Proposition 3, this implies that starting from an equilibrium where only q is disclosed with effort $\hat{e}$, the agent wants to deviate to $e_1 \geq \hat{e}$ when (q, η) is disclosed.

Under disclosure of (q, η) and market conjecture $\hat{e}$, the agent's payoff can be written as

$$\mathbb{E}_\eta[\mathbb{E}_q[\mathbb{E}(\theta \,|\, q, \eta, \hat{e}) \,|\, \eta, e]] - c(e),$$

using that the marginal of η does not depend on e.

Since for each η, effort increases the informativeness of output and the

MLRP is satisfied. Therefore, for any $e_H > e_L$,

$$\begin{aligned}\mathbb{E}_q[\mathbb{E}(\theta \,|\, q, \eta, e_H) \,|\, \eta, e_H] - \mathbb{E}_q[\mathbb{E}(\theta \,|\, q, \eta, e_H) \,|\, \eta, e_L] &\geq \\ \mathbb{E}_q[\mathbb{E}(\theta \,|\, q, \eta, e_L) \,|\, \eta, e_H] - \mathbb{E}_q[\mathbb{E}(\theta \,|\, q, \eta, e_L) \,|\, \eta, e_L],\end{aligned}$$

which follows from Lemma 5.

By the same argument as in Proposition 1, one can construct a nondecreasing sequence $\{e_n\}$ whose limit $e^* \geq \hat{e}$ is an equilibrium under full revelation when (q, η) are disclosed. ∎

Example 4 *In the following example, all assumptions of Proposition 5.2 Dewatripont, Jewitt and Tirole (1999*a*) are satisfied, yet full revelation does not maximize effort.*

Let θ and η be two random variables that are independently distributed, and have a standard normal distribution, that is $\theta \sim N(0,1)$ and $\eta \sim N(0,1)$. The production function satisfies

$$q = e + \theta + \alpha(e)e\theta + \beta(e)\eta,$$

where $\alpha(e)$ and $\beta(e)$ are functions that will be specified later, and satisfy

$$(1 + \alpha(e)e)^2 + \beta(e)^2 = k,$$

for some constant k. As long as $\forall e$, $1 + \alpha(e)e > 0$, one has that θ and q are affiliated conditional on e. Also, the marginal over output is $q \,|\, e \sim N(e, k)$, so the MLRP is satisfied. Consider the simple class of signals where $s = q + \epsilon$, with $\epsilon \sim N(0, \sigma_\epsilon^2)$ that is independent of (θ, η). The information structure will be denoted as $H_{\sigma_\epsilon^2}$.

Up to a term that is independent of s and therefore irrelevant,

$$\mathbb{E}(\theta | H_{\sigma_\epsilon^2}, s, e) = s \cdot \frac{1 + \alpha(e)e}{k + \sigma_\epsilon^2}.$$

Given a market conjecture of $\hat{e}$*, the agent maximizes*

$$e \cdot \frac{1+\alpha(\hat{e})\hat{e}}{k+\sigma_\epsilon^2} - c(e).$$

Now consider the following parametrization. $E = \mathbb{R}_+$, $c(e) = e^2/2, k = 4$. *Also,* $\alpha(e) = 0$ *for* $e \le 0.25 + \delta$ *or* $e \ge 0.25 + 3\delta$*, while* $\alpha(e) = 1$ *for* $e \in (0.25+\delta, 0.25+3\delta)$*, for some* $\delta > 0$ *sufficiently small. Under full revelation of* q*, that is when* $\sigma_\epsilon^2 = 0$*, there is a unique pure strategy equilibrium with* $e_{FR} = 0.25$*. On the other hand, if* $\sigma_\epsilon^2 = \frac{0.25-6\delta}{0.25+2\delta}$*, there exists an equilibrium with* $e = 0.25 + 2\delta$*.*

A.2 Proofs for Section 4

Proof of Proposition 5 For a given information structure and market conjecture, after output realization q the agent can expect a wage of

$$\hat{t}(q \,|\, H, e, \hat{w}) = \hat{w}(q) + t(q \,|\, H, e),$$

which consists of wage contract $\hat{w}(q)$ and the career-concerns term $t(q \,|\, H, e)$. The proof of Proposition 1 still applies if one can show the following two properties.

1. $\forall H$, $\sum_{i=1}^{j} \hat{t}(q_i \,|\, H, e, \hat{w}) f(q_i \,|\, e) \ge \sum_{i=1}^{j} \hat{t}(q_i \,|\, H_{\mathrm{FR}}, e, \hat{w}) f(q_i \,|\, e)$, $\forall j$, with $=$ at $j = M$.

2. $\forall e_2 > e_1$, $\sum_{i=1}^{j} [\hat{t}(q_i \,|\, H_{\mathrm{FR}}, e_2, \hat{w}) + \kappa] f(q_i \,|\, e_2) \ge \sum_{i=1}^{j} \hat{t}(q_i \,|\, H_{\mathrm{FR}}, e_1, \hat{w}) f(q_i \,|\, e_2)$, $\forall j$, with $=$ at $j = M$.

Both properties are satisfied iff they are satisfied when $\hat{t}(q \,|\, H, e, \hat{w})$ is replaced by $t(q \,|\, H, e)$, since any term involving $\hat{w}$ cancels out. Lemma 1 implies the first property, and the assumption that effort increases the informativeness of output implies the second property.

Similarly, in the proof of Proposition 3, replace $\mathbb{E}(\theta \,|\, q, \eta, \hat{e})$ with $\mathbb{E}(\theta \,|\, q, \eta, \hat{e}) + \hat{w}(q)$, and the rest of the proof is unchanged. ∎

Proof of Proposition 6 Take any $(H, \tilde{w})$ that induces effort $\tilde{e}$, and let $w^e = \mathbb{E}[\tilde{w} \mid \tilde{e}] \geq 0$ be the expected wage paid to the agent. Consider contracts of the form that pay a wage of zero for all outputs, except a wage r for the highest output q_M. Denote any such contract $w_M(\cdot \mid r) : Q \to \mathbb{R}$.

Let $\text{BR}(H, w, e)$ be the agent's best response under information structure H, wage contract w, and market conjecture e. By construction of $w_M(q \mid w^e/f(q_M \mid \tilde{e}))$ and the limited liability constraint $\tilde{w}(q) \geq 0$, $\forall j = 1, ..., M$,

$$\sum_{i=1}^{j} \tilde{w}(q_i) f(q_i \mid \tilde{e}) \geq \sum_{i=1}^{j} w_M(q_i \mid w^e/f(q_M \mid \tilde{e})) f(q_i \mid \tilde{e}),$$

with equality for $j = M$.

To unify the proof across the two cases, let H_{FR} be the information structure that fully reveals q and does not disclose η.

- When H is a garbling of q, Lemma 5 implies that $\text{BR}(H_{\text{FR}}, w_M(w^e/f(q_M \mid \tilde{e})), \tilde{e}) \geq \tilde{e}$. This follows from setting $\sigma = \tilde{e}$ and

$$t_1(q) = w_M(q \mid w^e/f(q_M \mid \tilde{e})) + t(q \mid H_{\text{FR}}, \tilde{e}), \quad t_2(q) = \tilde{w}(q) + t(q \mid H, \tilde{e}).$$

- When H reveals (q, η), then Equation (9) and Lemma 5 imply the same conclusion.

If $\text{BR}(H_{\text{FR}}, w_M(0), \tilde{e}) \geq \tilde{e}$, then either the proof of Proposition 1 or Proposition 3 implies that full revelation absent any wage payment induces some effort $e^* \geq \tilde{e}$.

If $\text{BR}(H_{\text{FR}}, w_M(0), \tilde{e}) < \tilde{e}$, it follows from the concavity of the agent's objective that any

$$e \in [\text{BR}(H_{\text{FR}}, w_M(0), \tilde{e}), \text{BR}(H_{\text{FR}}, w_M(w^e/f(q_M \mid \tilde{e})), \tilde{e})]$$

can be made optimal by some contract $w_M(t)$ with $t \in [0, w^e/f(q_M \mid \tilde{e})]$. So there exists contract $w_M(\tilde{t})$ that implements $\tilde{e}$ at cost $f(q_M \mid \tilde{e}) \cdot \tilde{t} \leq w^e$. That the agent's objective is concave follows from the concavity of $f(q_M \mid e)$ and the decreasing returns to signal jamming assumption. ∎

Proof of Proposition 7 Take any information structure H that induces a mixed strategy σ. To shorten notation, let $\bar{e} = \bar{e}(\sigma) = \sup(\text{support}(\sigma))$. It will be shown below that when effort increases the informativeness of output about ability when allowing for mixed strategies, then for some constant k, we have that $\forall j = 1, ..., M$ that

$$\sum_{i=1}^{j} [t(q_i \mid H, \sigma) + k] f(q_i \mid \bar{e}) \geq \sum_{i=1}^{j} t(q_i \mid H_{\text{FR}}, \bar{e}) f(q_i \mid \bar{e}), \tag{10}$$

with equality for $j = M$. By continuity, $\bar{e}$ maximizes the agent's effort under transfer schedule $t(q \mid H, \sigma) + k$. Given this, Lemma 5 implies that when the agent faces transfer schedule $t(q \mid H_{\text{FR}}, \bar{e})$, there exists a best response e_1 with $e_1 \geq \bar{e}$.

As in the proof of Proposition 1, construct a sequence $e_{n+1} = \xi(e_n)$, where $\xi(e) := \max(\text{BR}(t(\cdot \mid H_{\text{FR}}, e)))$. By the same argument as in the proof of Proposition 1, this sequence is nondecreasing and the limit $e^* \geq \bar{e}$ is an equilibrium under full revelation.

It remains to show that Equation (10) is satisfied, which is shown using several steps. To save on notation, define $\Delta t(q) = t(q \mid H, \sigma) - t(q \mid H_{\text{FR}}, \sigma)$.

Step 1: Q can be partitioned into P segments, where segment $p \in \{1, ..., P\}$ takes the form

$$Q_p = \{q_i \in Q : m(p-1) + 1 \leq i \leq m(p)\},$$

where $m(0) = 0 < m(1) < ... < m(P) = M$. In words, each segment Q_p consists of adjacent outputs, and any output belongs to some segment. The segments can be picked in a way so that $\forall p = 1, ..., P$,

(a) $\sum_{q \in Q_p} \Delta t(q) f(q \mid \bar{e}) \leq 0$,

(b) $\forall \tilde{q} \in Q_p$, $\sum_{q \in Q_p \wedge q \leq \tilde{q}} \Delta t(q) f(q \mid \bar{e}) \geq \sum_{q \in Q_p \wedge q \leq q_{m(p)}} \Delta t(q) f(q \mid \bar{e})$.

In words, property (a) says that within each segment, the mean of $\Delta t(q)$ under distribution $f(q \mid \bar{e})$ is weakly negative. Property (b) says that the sum of $\Delta t(q)$ from the lowest output within the segment up to an arbitrary output within the segment, and weighted by $f(q \mid \bar{e})$, is minimized by the endpoint of the segment.

To satisfy properties (a) and (b), the segments are defined through the following algorithm. The starting point is $m(0) = 0$. Segment p is defined by by first finding the smallest $\tilde{q} > q_{m(p-1)}$ with

$$\sum_{q_{m(p-1)} < q \leq \tilde{q}} \Delta t(q) f(q \mid \bar{e}) \leq 0.$$

Next, let $q_{m(p)} = \max q \geq \tilde{q}$ such that $\forall q$ with $\tilde{q} \leq q \leq q_{m(p)}$, we have that $\Delta t(q) \leq 0$. By construction, all segments except for possibly the uppermost one, Q_P, satisfy properties (a) and (b).

To show that property (a) holds for Q_P, first it is argued that for any $i_1, i_2 \in \{1, ..., M\}$ and any function $\gamma(q)$, if

$$\sum_{i=i_1}^{j} \gamma(q_i) f(q_i \mid \sigma) \geq 0 \;\; \forall j = i_1, ..., i_2, \text{ with } = \text{ for } j = i_2,$$

then we have

$$\sum_{i=i_1}^{i_2} \gamma(q_i) f(q_i \mid \bar{e}) \leq 0.$$

This follows from Lemma 5 since $f(q \mid \bar{e})$ dominates $f(q \mid \sigma)$ according to the MLRP.

If $P = 1$, so that $Q_P = Q$, then property (a) follows by setting $i_1 = 1$, $i_2 = M$, and $\gamma(q) = \Delta t(q)$. When $P > 1$, Lemma 1 implies that $\sum_{q \in Q_P} \Delta t(q) f(q \mid \sigma) \leq 0$. At the same time, from the construction of the segments it follows that for the smallest element in Q_P, $q_{m(P-1)+1}$, that $\Delta t(q_{m(P-1)+1}) > 0$. Therefore, for some $\tilde{q} \in Q_P$, $\sum_{q \in Q_P, q < \tilde{q}} \Delta t(q) f(q \mid \sigma) > 0$ and $\sum_{q \in Q_P, q \leq \tilde{q}} \Delta t(q) f(q \mid \sigma) \leq 0$.

Define $\alpha \in [0,1]$ such that

$$\sum_{q \in Q_P, q < \tilde{q}} \Delta t(q) f(q \,|\, \sigma) + \alpha \Delta t(\tilde{q}) f(\tilde{q} \,|\, \sigma) = 0.$$

Since $\Delta t(\tilde{q}) < 0$, this implies

$$\sum_{q \in Q_P \wedge q \leq \tilde{q}} \Delta t(q) f(q \,|\, \bar{e}) \leq 0,$$

For P to be the highest segment, it must be that $\tilde{q} = q_M$ and therefore property (a) is satisfied.

To show that property (b) holds for Q_P, take any $\tilde{q} \in Q_P$. If $\sum_{q \in Q_P \wedge q \leq \tilde{q}} \Delta t(q) f(q \,|\, \bar{e}) > 0$, then property (b) is implied by property (a). If $\tilde{q}$ is such that $\sum_{q \in Q_P \wedge q \leq \tilde{q}} \Delta t(q) f(q \,|\, \bar{e}) \leq 0$, then by construction of the segments $\forall q > \tilde{q}$, $\Delta t(q) \leq 0$, and therefore property (b) is satisfied.

Step 2: I claim that for some constant κ,

$$\sum_{i=1}^{j} [t(q_i \,|\, H, \sigma) + \kappa] f(q_i \,|\, \bar{e}) \geq \sum_{i=1}^{j} t(q_i \,|\, H_{\text{FR}}, \sigma) f(q_i \,|\, \bar{e}), \tag{11}$$

with equality for $j = M$.

Given the segments from step 1, by property (a) it is true that $\sum_{q \in Q_p} [t(q \,|\, H, \sigma) + \kappa_p - t(q \,|\, H_{\text{FR}}, \sigma)] f(q \,|\, \bar{e}) = 0$ for some $\kappa_p \geq 0$. Therefore, $\kappa = \sum_{p=1}^{P} \kappa_p$ in order to satisfy Equation (11) with equality for $j = M$. Suppose that Equation (11) is violated for some $\hat{j}$. Let $\hat{p}$ be the segment that $q_{\hat{j}}$ belongs to. By property (b) of the segments, it is true that

$$\sum_{i=1}^{m(\hat{p})} [t(q_i \,|\, H, \sigma) + \kappa] f(q_i \,|\, \bar{e}) < \sum_{i=1}^{j} t(q_i \,|\, H_{\text{FR}}, \sigma) f(q_i \,|\, \bar{e}).$$

At the same time,

$$\sum_{i=1}^{m(\hat{p})} [t(q_i \,|\, H, \sigma) + \sum_{p=1}^{\hat{p}} \kappa_p] f(q_i \,|\, \bar{e}) = \sum_{i=1}^{j} t(q_i \,|\, H_{\text{FR}}, \sigma) f(q_i \,|\, \bar{e}).$$

Since $\kappa \geq \sum_{p=1}^{\hat{p}} \kappa_p$, this leads to a contradiction.

Step 3: Since effort increases the informativeness of output about ability when allowing for mixed strategies, for some $\bar{k}$,

$$\sum_{i=1}^{j}[t(q_i \,|\, H_{\text{FR}}, \sigma) + \bar{k}]f(q_i \,|\, \bar{e}) \geq \sum_{i=1}^{j} t(q_i \,|\, H_{\text{FR}}, \sigma)f(q_i \,|\, \bar{e}),$$

with equality for $j = M$. Using the result of step 2 and setting $k = \kappa + \bar{k}$, we get

$$\sum_{i=1}^{j}[t(q_i \,|\, H, \sigma) + k]f(q_i \,|\, \bar{e}) \geq \sum_{i=1}^{j} t(q_i \,|\, H_{\text{FR}}, \bar{e})f(q_i \,|\, \bar{e}),$$

with equality for $j = M$. ∎

Proof of Lemma 4 Fix an arbitrary mixed strategy σ, and to simplify notation let $\bar{e} = \bar{e}(\sigma) = \sup(\text{support}(\sigma))$. Fix an arbitrary $q_2 > q_1$, and define the likelihood ratio $L_1^2(e) := \frac{f(q_2 \,|\, e)}{f(q_1 \,|\, e)}$, which is nondecreasing in e by the MLRP property. Then

$$\begin{aligned}
\mathbb{E}(\theta \,|\, q_2, \sigma) - \mathbb{E}(\theta \,|\, q_1, \sigma) &= \frac{\mathbb{E}_\sigma[f(q_2 \,|\, e)\mathbb{E}(\theta \,|\, q_2, e)]}{\mathbb{E}_\sigma[f(q_2 \,|\, e)]} - \frac{\mathbb{E}_\sigma[f(q_1 \,|\, e)\mathbb{E}(\theta \,|\, q_1, e)]}{\mathbb{E}_\sigma[f(q_1 \,|\, e)]} \\
&= \frac{\mathbb{E}_\sigma[f(q_1 \,|\, e)L_1^2(e)\mathbb{E}(\theta \,|\, q_2, e)]}{\mathbb{E}_\sigma[f(q_1 \,|\, e)L_1^2(e)]} - \frac{\mathbb{E}_\sigma[f(q_1 \,|\, e)\mathbb{E}(\theta \,|\, q_1, e)]}{\mathbb{E}_\sigma[f(q_1 \,|\, e)]} \\
&= \frac{\mathbb{E}_\sigma[f(q_1 \,|\, e)L_1^2(e)\{\mathbb{E}(\theta \,|\, q_2, e) - \mathbb{E}(\theta \,|\, q_1, e)\}]}{\mathbb{E}_\sigma[f(q_1 \,|\, e)L_1^2(e)]} \\
&+ \frac{\mathbb{E}_\sigma[f(q_1 \,|\, e)L_1^2(e)\mathbb{E}(\theta \,|\, q_1, e)]}{\mathbb{E}_\sigma[f(q_1 \,|\, e)LR(e)]} - \frac{\mathbb{E}_\sigma[f(q_1 \,|\, e)\mathbb{E}(\theta \,|\, q_1, e)]}{\mathbb{E}_\sigma[f(q_1 \,|\, e)]}.
\end{aligned}$$

By supermodularity,

$$\mathbb{E}(\theta \,|\, q_2, \bar{e}) - \mathbb{E}(\theta \,|\, q_1, \bar{e}) \geq \frac{\mathbb{E}_\sigma[f(q_1 \,|\, e)L_1^2(e)\{\mathbb{E}(\theta \,|\, q_2, e) - \mathbb{E}(\theta \,|\, q_1, e)\}]}{\mathbb{E}_\sigma[f(q_1 \,|\, e)L_1^2(e)]}.$$

Since $L_1^2(e)$ is nondecreasing and $\mathbb{E}(\theta \,|\, q_1, e)$ is nonincreasing in e, we have

$$0 \geq \frac{\mathbb{E}_\sigma[f(q_1 \,|\, e)L_1^2(e)\mathbb{E}(\theta \,|\, q_1, e)]}{\mathbb{E}_\sigma[f(q_1 \,|\, e)L_1^2(e)]} - \frac{\mathbb{E}_\sigma[f(q_1 \,|\, e)\mathbb{E}(\theta \,|\, q_1, e)]}{\mathbb{E}_\sigma[f(q_1 \,|\, e)]}.$$

The bottom line is that $\mathbb{E}(\theta \mid q_2, \bar{e}) - \mathbb{E}(\theta \mid q_1, \bar{e}) \geq \mathbb{E}(\theta \mid q_2, \sigma) - \mathbb{E}(\theta \mid q_1, \sigma)$. Therefore,

$$\mathbb{E}_q \left[\mathbb{E}(\theta \mid q, \bar{e}) - \mathbb{E}(\theta \mid q, \sigma) \mid \bar{e} \right] \geq \mathbb{E}_q \left[\mathbb{E}(\theta \mid q, \bar{e}) - \mathbb{E}(\theta \mid q, \sigma) \mid q \leq q_j, \bar{e} \right].$$

∎

Proof of Proposition 8 With $Q = \{q_1, q_2\}$, define $\delta(H, \sigma) = t(q_2 \mid H, \sigma) - t(q_1 \mid H, \sigma)$ for arbitrary H and σ. By Lemma 1, $\delta(H_{\text{FR}}, \sigma) \geq \delta(H, \sigma)$, with strict inequality if H is not fully revealing. Also, for any $\delta \in [0, \delta(H_{\text{FR}}, \sigma)]$, $\exists \hat{H}$ such that $\delta(\hat{H}, \sigma) = \delta$. The agent maximizes $\delta f(q_2 \mid e) - c(e)$ up to a constant, which is supermodular in (e, δ).

Take information structure H_0 that is not fully revealing and induces mixed strategy σ_0. If $\delta(H_{\text{FR}}, \bar{e}(\sigma_0)) \leq \delta(H_0, \sigma_0)$, then for some $\alpha > 0$, the strategy that plays $\bar{e}(\sigma_0)$ with probability α and σ_0 with probability $1 - \alpha$ is an equilibrium under full revelation, and this strategy dominates σ_0 according to the MLRP. If $\delta(H_{\text{FR}}, \bar{e}(\sigma_0)) > \delta(H_0, \sigma_0)$, then $\bar{e}(\sigma_0)$ is induced by some $\bar{H}$.

Let $e_1 \geq \bar{e}(\sigma_0)$ be the largest pure strategy that can be induced by some information structure, which is denoted H_1. Let δ_1 be the largest δ such that e_1 is a best response. If e_1 cannot by induced by H_{FR}, then $\delta_1 < \delta(H_{\text{FR}}, e_1)$. By supermodularity, some effort $e_2 > e_1$ is also a best response under δ_1. Since e_2 cannot be induced by any information structure, it must be that

$$\delta(H_{\text{FR}}, e_1) > \delta_1 > \delta(H_{\text{FR}}, e_2).$$

For some $\beta \in (0, 1)$, the mixed strategy $\sigma(\beta)$ that plays e_1 and e_2 with respective probabilities probabilities β and $1 - \beta$ satisfies $\delta(H_{\text{FR}}, \sigma(\beta)) = \delta_1$, and therefore induces $\sigma(\beta)$ which dominates σ_0 according to the MLRP. ∎