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Monetary Policy and Speculative Asset Markets

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Abstract

I study monetary policy in an estimated financial New-Keynesian model extended by behavioral expectation formation in the asset market. Credit frictions create a feedback between asset markets and the macroeconomy, and behaviorally motivated speculation can amplify fundamental swings in asset prices, potentially causing endogenous, nonfundamental bubbles. These features greatly improve the power of the model to replicate empirical-key moments. I find that monetary policy can indeed dampen financial cycles by carefully leaning against asset prices, but at the cost of amplifying their transmission to the macroeconomy, and of causing undesirable responses to movements in fundamentals.

Keywords: Monetary policy, nonlinear dynamics, heterogeneous expectations, credit constraints, bifurcation analysis

JEL: E44, E52, E03, C63

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1 Introduction

Since at least the 1990s, a recurring question is whether central banks should "lean against asset prices" to dampen financial cycles, and to mitigate potential spillovers to the economy.[1] This debate has recently gained new momentum given that, despite a long period of historically low interest rates, inflation and real rates in the US and the Euro Area are still persistently low, while at the same time asset prices have increased substantially (Borio et al., 2018; Rungcharoenkitkul et al., 2019). Can expansionary monetary policy fuel financial markets but leave inflation and output unaffected? If so, this could lead to overheated financial markets which in turn can build up risks that again would call for monetary responses.

The debate so far has neglected two potentially important points. First, most of the analysis is based on models with fully rational agents, either with sophisticated equilibrium selection strategies (Galí, 2014; Miao and Wang, 2018), or assuming that non-fundamental volatility in asset prices is stochastic (Bernanke and Gertler, 2000). These approaches can yield misleading policy implications because purely stochastic fluctuations in the asset market do not take into account potential feedback effects from the economy to asset prices, whereas "rational bubbles" entail some counterintuitive implications on the interaction of interest rates and asset prices. The second, potentially neglected, point is that, given these limitations, most policy responses are either tailored towards to control the source of non-fundamental financial cycles, or consider the role of monetary policy in the transmission of financial cycles to the macroeconomy, but not on both.

This paper addresses these points in a microfounded financial New-Keynesian model in which asset prices can have macroeconomic effects through credit frictions. Much in line with the bulk of the literature on financial frictions (Christiano et al., 2010, 2014; Davis and Taylor, 2019), a boom in asset prices improves the financial conditions of firms and acts similar to a supply shock. If firms lever their profits by borrowing from the financial intermediaries, and pledge their equity as collateral,

[1] E.g. Poole (1970); Cecchetti (2000); Borio and Lowe (2002).

borrowing conditions and finance costs depend on the value of collateral offered, which is reflected by asset prices. A non-fundamental increase in asset prices hence reduces the costs of finance and is preceded by a drop in inflation.[2]

I extend this model by a behavioral framework of asset trading. Recent studies on survey evidence of market expectations stressed the relevance of non-fundamental expectations for asset pricing. Greenwood and Shleifer (2014); Adam et al. (2017, 2018) highlight the strong empirical correlation between current and expected returns. This observation is at odds with rational expectations, which would suggest an obverse relationship. Similarly, evidence from the behavioral laboratory (Hommes et al., 2005; Hommes, 2011; Assenza et al., 2013) documents that trading patterns in experimental asset markets can be summarized by simple, heterogeneous forecasting rules instead of complex cognitive mechanisms. In line with these findings, Adam et al. (2016); Winkler (2019) show that introducing bounded rationality into otherwise standard models greatly improves the empirical performance of these models. This result is backed by empirical evidence e.g. from Abbate et al. (2016). The outcomes from this literature challenge the conventional view on asset price targeting: if non-fundamental deflections in asset markets are driven by behavioral motives, they are no longer exogenous, but may be a response to flows in macroeconomic aggregates. Then, episodes of loose monetary policy can indeed stimulate optimistic and extrapolative behavior that then evolves independently, and by itself could become a driver of economic dynamics.

My model of the financial market draws from the literature of behavioral finance (Brock and Hommes, 1998; LeBaron, 2006; Hommes, 2006). This type of model is motivated by first principles, and supported by evidence from the laboratory as well as the empirical facts on asset market expectations. Crucially, this extension to my model allows to capture extreme cases such as herding behavior and features

[2]Another potential channel is a wealth effect that works through aggregate demand (Bernanke and Gertler, 2000): increasing stock prices raise the nominal value of assets held by households, which amplifies consumer demand. Such effect is ruled out in a representative agent framework where seller and buyer are identical and changes in asset prices level out to zero in aggregate.

expectation dynamics can be self-reinforcing and lead to large, persistent swings in asset prices. Dynamics are endogenous in the sense that the steady state might not be stationary but may fluctuate, independently of any structural shocks to the model.

I argue that such endogenous or *chaotic* dynamics are a necessary feature if one wants to study the potential of monetary policy to prevent future financial crisis even before they occur. In contrast, a model in which all fluctuations are exogenous can not assign a credible role in preventing future financial tumult. The model presented here is able to reproduce recurring endogenous financial crisis that originate in dislocations in asset markets, whereas the expectation system of the financial market is closely tied to the outcome of macro-markets. As myopic traders extrapolate trends coming from the latter, and monetary policy accommodatingly responds to falling prices a feedback loop can arise. Since asset prices tend to rise when interest rates fall, an accommodative monetary policy can further boost asset prices. To limit the degrees of freedom that arise from introducing boundedly rational behavior, the expectation of markets other than financial markets are assumed to be rational, conditionally on the outcome of asset markets.

Similar to Winkler (2019), I find that incorporating a behavioral model of speculation in a model with credit friction allows to capture a row of key-moments, not only on the real and monetary side of the economy, but also regarding the dynamics of asset prices and asset price expectations. I estimate the model and my estimation results strongly suggest that asset prices play a non-negligible role for the firms price-setting decision.

I find that a central bank that cares to reduce volatility in inflation tends to amplify fluctuations in asset prices, as these tend to be deflationary. If the degree of endogenous amplification in financial markets is limited – which, as my estimation results suggest, seems to be the regular case – a monetary policy that targets asset prices can dampen excess volatility of financial markets and as such can mitigate the *source* of spillovers. This however amplifies the degree to which this volatility affects the real sector. This amplified transition channel casts serious doubts on the effectiveness of such policy. Instead, policy makers can decide not to fight volatility in asset prices, but to limit the spillovers. The scope for such a policy however is limited as well: recucing the response of inflation to non-fundamental fluctuations in asset prices will amplify the spillover to output, and vice versa. Importantly however, my simulation results suggest indeed that a monetary policy that temporally reacts overly expansionary to deflationary pressures can boost asset prices, thereby amplifying financial cycles. Potentially, this not only causes a deflationary spiral as a second-round effect, but comprises the risk of future financial crisis.

This result stands in contrast to findings from Adam and Woodford (2018), who find a positive role for monetary policy to target housing prices if expectations are not fully rational. These differences could be due to the fact that housing prices more strongly affect the demand side of the economy, and hence rather are inflationary. My analysis goes beyond the work of Winkler (2019) in so far, as that the interaction of boundedly rational markets and monetary policy in my model allows to replicate fully endogenous patterns of financial crisis, that slowly build up, but then burst suddenly, thereby spreading hazard to the economy. I explicitly study the role of monetary policy to prevent such endogenous crisis, or at least its role to dampen their harm. While in such cases it is indeed favorable to dampen the feedback from monetary policy to asset markets, the scope of such policy is again limited by the amplification of the transmission channel of fluctuations from asset markets to the macroeconomy.

This work adds to a larger literature that researches the macroeconomics of asset prices. The bulk of this literature however mainly focusses on rational expectations.[3] Miao et al. (2012) and Miao et al. (2016), building on a Bayesian model with rational stock price bubbles, find that the feedback between asset prices and asset price expectations plays a key role on the formation of a stock price bubble. They report that about 20% of the variance of GDP can be explained through fluctuations in stock prices. Galí (2014) prominently represents a series of studies

[3]For a more complete survey see Allen et al. (2011).

on rational bubbles and monetary policy. A key assumption is, that the non-fundamental part of a bubble grows *proportionally to* the policy rate, which does not align with the common intuition that bubbles can emerge after an episode of too-loose monetary policy. Related to this debate, Assenmacher and Gerlach (2008) find that asset prices react almost instantaneously to monetary surprises, and provide evidence in support of the conventional view that bubbles build up when monetary policy is rather loose. The authors furthermore find that shocks on asset prices can have impact on both, GDP and credit volume and that fluctuations in stock prices can explain about 10% to 15% of the variance of GDP.

The next Section 2 introduces the macroeconomic model. I discuss theoretical insights in Section 3 and present simulation results in Section 4. In-debt policy analysis is provided in 5 whereas Section 6 concludes.

2 Model

I propose a small-scale New Keynesian model with credit frictions as in Bernanke et al. (1999, henceforth BGG) with rational agents, and an asset market with boundedly rational traders. In my model, credit frictions affect firms through the cost channel, giving rise to financial accelerator effects. Additionally, the presence of boundedly rational traders in the asset market can lead to speculative dynamics and amplify the effects on macroeconomic variables in response to fluctuations of asset prices.[4] This section describes the model environment and the equations that characterize the general equilibrium. The next section presents the expectation formation process of boundedly rational (or behavioral) agents in the asset market and discusses the aggregate solution of the model in the presence of both rational and behavioral agents.

The model economy is populated by ex-ante identical households, retail firms and wholesale firms, with the latter being operated by risk-neutral entrepreneurs.

[4]One important advantage of using a parsimonious model over a larger model as for example in Christiano et al. (2005) and Smets and Wouters (2007) is that it allows to isolate and analyze the underlying key economic mechanisms as e.g. in Sims and Wu (2019).

Following Ravenna and Walsh (2006), wages have to be paid before production takes place, implying that wholesale firms are subject to external financing. These firms borrow through financial intermediaries who face a costly state verification (CSV) problem and therefore require a risk premium as a means to insure against firms' default risk.

2.1 Households

Households maximize the expected present value of future utility derived from consumption and leisure. C_t denotes the composite consumption good and H_t are labour hours supplied on a competitive labor market at real wage rate W_t. Furthermore, households can deposit monetary savings D_t at the financial intermediary for which they will receive the gross nominal interest rate R_{t+1} in the next period. The maximization problem for household i is:

$$\max E_t \sum_{s=t}^{\infty} \beta^{s-t} \zeta_{i,t} \left(\ln C_{i,s} - \frac{H_{i,s}^{1+\eta}}{1+\eta} \right) \tag{1}$$

s.t. the budget constraint

$$C_{i,t} + D_{i,t} \leq W_t H_{i,t} + R_t \frac{P_{t-1}}{P_t} D_{i,t-1} + \mu \int_0^{\bar{\omega}_t} \omega H_t / X_t dF(\omega) \ \forall t = 1, 2, ... \tag{2}$$

where η is the relative weight on the disutility of labor. The term $\mu \int_0^{\bar{\omega}_t} \omega H_t / X_t dF(\omega)$ represents "audition costs" that are incurred in case of a firm's default. These costs are assumed to be equally distributed among households. Further details are provided in the next subsection or in Appendix A.

Each household i is subject to an idiosyncratic preference shock $\zeta_{i,t}$. Define $\tilde{d}_{i,t} = \log\left(\frac{\zeta_{i,t}}{\zeta_{i,t+1}}\right)$ and assume that $\tilde{d}_{i,t} = \rho_d d_{t-1} + \epsilon_t^d + \tilde{\epsilon}_{i,t}$. Let ϵ_t^d and all $\tilde{\epsilon}_{i,t}$ be normally distributed with zero mean and standard deviation σ_d. Denote by

$$d_t = \int \tilde{d}_{i,t} di \approx \rho_d d_{t-1} + \epsilon_t^d \tag{3}$$

the aggregate over all preference shocks, which follows an AR(1) structure.

Assumption 1. *Aggregate shock processes are unobservable to agents. Each agent i can only observe his individual realization $\tilde{d}_{i,t}$ but neither the aggregate d_t nor the stochastic innovations $\tilde{\epsilon}_{i,t}$ and ϵ_t^d.*

This information structure is crucial for the expectation formation process of rational agents. In fact, the assumptions on the observability of structural shocks imply that rational agents will not be able to identify the source of economic fluctuations, as either originating from the structural shock or from endogenous movements of model variables.

The maximization problem of the household yields the Euler equation and the labor supply equation:

$$\exp(d_t)C_t^{-1} = E_t\left\{\beta R_{t+1}\frac{P_t}{P_{t+1}}C_{t+1}^{-1}\right\}, \tag{4}$$

$$H_t^{\eta} = \frac{W_t}{C_t}, \tag{5}$$

Finally, the transversality condition is satisfied and the household's budget constraint holds with equality.

2.2 Wholesale and retail firms

The wholesale sector consists of a continuum of perfectly competitive firms indexed by j. Each firm is operated by a risk-neutral entrepreneur. Assume that each firm produces a single good $Y_{j,t}$ using a CRS technology with labor as the only production factor:[5]

$$Y_{j,t} = \omega_{j,t}A_tH_{j,t}, \tag{6}$$

[5] Assume that in the short-run, capital is fixed and labor is the only production factor that can be adjusted.

where $\omega_{j,t}$ is a firm-specific idiosyncratic productivity shock and A_t is an aggregate technology shock where $\ln A_t$ follows an AR(1) process:

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_t^a. \tag{7}$$

The timing of events is as follows. At the beginning of each period, aggregate shocks realize. Then, firms choose their labor input and agree on a loan contract with the financial intermediaries. I assume that workers are paid before production takes places, similar to e.g. Ravenna and Walsh (2006). After workers are paid, firm specific shocks $\omega_{j,t}$ realize. Finally, firms decide on their dividend payments $\Theta_{j,t+1}$ which are distributed at the beginning of the next period.[6]

Denote by X_t the gross markup that retailers charge over the price of wholesale goods. Hence, X_t^{-1} can be interpreted as the relative price, i.e. price of a good in wholesale relative to its price in retail, that a wholesale firm receives for one unit of his goods. Dividends in each period can be written as

$$\Theta_{j,t} = \omega_{j,t-1} A_{t-1} H_{j,t-1} / X_{t-1} - N_{j,t}, \tag{8}$$

The aggregate return on working capital (i.e. the return on investing one monetary unit into labor) is $A_t(X_t W_t)^{-1}$.

Shares of the firms' equity are sold to the financial intermediaries. Given no arbitrage and – for now – assuming rational expectations, the real price $S_{j,t}$ of one share of firm j is

$$S_{j,t} = E_t \left\{ \frac{\Theta_{j,t+1} + S_{j,t+1}}{R_{t+1} \frac{P_t}{P_{t+1}}} \right\}. \tag{9}$$

[6] I assume that dividend payments can be also negative. In this case, a firm obtains funding resources from their shareholders who are willing to increase the firm's equity if the net present value of future dividends given the additional investment is positive. Using this assumption ensures a closed form solution of the model. See e.g. Martin and Ventura (2010) for a similar approach.

Simple example without external financing. In this subsection, I derive the relationship between real aggregates and asset prices. To this end, I abstract from external financing for now to intuitively illustrate the link between asset prices and the aggregate return per unit of labor, $A_t(X_tW_t)^{-1}$.

Recall that each wholesale firm is owned by a risk-neutral entrepreneur who has opportunity cost $R_{t+1}\frac{P_t}{P_{t+1}}$ in period t. Hence, dropping the j-subscript, the wholesale firm's Lagrangian reads

$$\max E_t \sum_{s=t}^{\infty} \left(\prod_{k=t}^{s} \frac{1}{R_k} \frac{P_k}{P_{k-1}} \right) \left[\omega_{s-1} A_{s-1} H_{s-1}/X_{s-1} - N_s \right] - \lambda_s \left(W_t H_s - N_s \right), \tag{10}$$

where $\omega_t A_t H_t / X_t$ is the total return on labor. The first-order condition is

$$H_t/X_t = N_t R_{t+1} \frac{P_t}{P_{t+1}}, \tag{11}$$

Combine Equations 11 and 8 to obtain an expression for expected dividends, $E_t\Theta_{s+1} = R_{t+1}N_t - E_tN_{t+1}$. Using this expression in Equation (9) shows that asset prices reflect the value of equity perfectly

$$S_t = \frac{R_{t+1}\frac{P_t}{P_{t+1}} N_t - E_t N_{t+1} + \frac{E_t\left\{R_{t+2}\frac{P_{t+1}}{P_{t+2}}N_{t+1}\right\}+\ldots}{R_{t+2}\frac{P_{t+1}}{P_{t+2}}}}{R_{t+1}\frac{P_t}{P_{t+1}}} = N_t. \tag{12}$$

Combine Equations 11 and 12 to obtain the optimal labor demand equation, $H_t = R_{t+1}\frac{P_t}{P_{t+1}} S_t X_t$. This result shows that labor demand does not only depend on the wholesale price and the interest rate but also on asset prices.

Full model with external financing. I now consider the wholesale firm's problem with external financing. The external funds that firm j requires is the difference between its working capital $W_tH_{j,t}$ and equity:

$$B_{j,t} = W_t H_{j,t} - N_{j,t}. \tag{13}$$

As in BGG, I assume that external financing is subject to a costly state verification (CSV) problem, whereby banks cannot observe the state of the entrepreneur unless they pay a monitoring cost. Appendix A provides mathematical details. To minimize agency costs, the bank will only pay the fee when the entrepreneur defaults in order to seize his collateral. However, to compensate for possible monitoring cost, the optimal financial contract includes a risk premium over the risk-free nominal interest rate. Define the risk premium as a function $z(\cdot)$ of the individual firm's leverage, $\frac{N_{j,t}}{W_t H_{j,t}}$. Then, the loan rate for external funds, $R^B_{j,t+1}$, is

$$R^B_{j,t+1} = z\left(\frac{N_{j,t}}{W_t H_{j,t}}\right) R_{t+1}, \tag{14}$$

with $z(\cdot) > 1$ and $\frac{\partial z(\cdot)}{\partial N_t} < 0$. The risk premium increases with the leverage ratio because default becomes more likely and, hence, loans are more risky. As shown in Appendix A, optimality requires that the premium on assets is equal to the rate paid on external funds, $R_{t+1}\frac{S_{j,t}}{N_{j,t}} = E_t R^B_{t+1}$. Intuitively, this must hold because otherwise wholesalers would have an incentive to adjust the borrowing volume.

It follows that

$$\frac{S_{j,t}}{N_{j,t}} = z\left(\frac{N_{j,t}}{W_t H_{j,t}}\right). \tag{15}$$

No arbitrage also requires that the rate of return on working capital equals the (real) rate on external funding:

$$\frac{A_t}{X_t W_t} = z\left(\frac{N_{j,t}}{W_t H_{j,t}}\right) R_{t+1}\frac{P_t}{P_{t+1}}. \tag{16}$$

Denote the derivative of $z(\cdot)$ in the steady state as $z'(\cdot) = -\bar{\nu}$. Then, after combining Equations (15) and (16) and log-linearizing, the markup over wholesale prices in log-deviation from its steady state, x_t, can be represented as:

$$x_t = \nu s_t - \psi y_t - (r_{t+1} - \pi_{t+1}) + \frac{1+\eta}{1+\bar{\nu}} a_t \tag{17}$$

where $\nu = \frac{\bar{\nu}}{1-\bar{\nu}}$ is the elasticity of the markup with respect to asset prices, and

$\psi = \frac{1+\eta+\tilde{\nu}}{1-\tilde{\nu}}$ is the elasticity of the markup with respect to output.

The perfectly competitive wholesale firms set identical prices and, hence, offer the same expected return on equity. As a consequence, the stock market evaluation of firms' shares are identical. Given the demand for shares by traders, this determines the level of equity and ultimately also the level of dividend payments. Higher asset prices lower the cost of borrowing which in turn reduces marginal cost and, finally, the price for the wholesale good.

Assume there exists a continuum of retailers that buy the wholesale output from entrepreneurs, taken as given the wholesale price, and slightly differentiate the output at no cost. Differentiation allows each retailer to have some degree of market power. Finally, households purchase and consume CES aggregates of these retail goods.

To motivate price inertia, assume that retail firms are subject to nominal rigidities as in Calvo (1983). The solution of the price setting problem of the retailers is the New Keynesian Phillips curve, expressed in terms of markup X_t.[7]

2.3 Financial intermediation

Assume a continuum of financial intermediaries indexed by k. Households provide deposits to financial intermediaries at deposit rate R^D_{t+1}. Each intermediary receives his share of deposits $D_{k,t}$ and lends a fraction of the deposits to wholesale firms. The rest is invested in the asset market traders act on behalf of the financial intermediary at no cost. Intermediaries further have access to central bank liquidity, which is provided at the nominal deposit rate R_{t+1}. Next period's aggregated real dividends net of seized collateral are expected to be $\hat{E}_{k,t}\Theta_{t+1}$ without a j-subscript, where $\hat{E}_{k,t}$ denotes trader-k's expectations, which do not necessarily have to be the rational. Let $J_{k,t}$ be the fraction of firms whose shares are owned by intermediary k and $B_{k,t}$ the amount of loans to wholesale firms of intermediary k.

Denote by S_t real asset prices aggregated over the firms. In equilibrium, it holds

[7]See Bernanke et al. (1999) for details on this particular solution.

$$R^D_{t+1}\frac{P_t}{P_{t+1}}D_{k,t} = \hat{E}_{k,t}[\Theta_{t+1} + S_{t+1}]J_{k,t} + z^{-1}R^B_{t+1}\frac{P_t}{P_{t+1}}B_{k,t} \tag{18}$$

subject to the constraint $D_{k,t} \geq S_t J_{k,t} + B_{k,t}$ and given z as in Equation (14). Given the central bank interest rate, optimality requires that $R^D_{t+1} = z^{-1}R^B_{t+1} = \frac{\hat{E}_{k,t}\{\Theta_{t+1}+S_{t+1}\}}{S_t}\frac{P_{t+1}}{P_t} = R_{t+1}$.

The different expectations $\hat{E}_{k,t}S_{t+1}$ can be aggregated over the traders, and up to a first order approximation yield

$$S_t = \hat{E}_t\left\{\frac{\Theta_{t+1} + S_{t+1}}{R_{t+1}\frac{P_t}{P_{t+1}}}\right\}. \tag{19}$$

2.4 The central bank

The central bank follows a simple monetary policy rule, according to which it adjusts the current nominal interest rate in response to inflation and possibly also to asset prices. Denote the case in which monetary policy reacts to both inflation and asset prices as asset price targeting (APT) or "leaning against asset prices".[8]

The linearized monetary policy rule is

$$r_{t+1} = \phi_\pi \pi_t + \phi_s s_t. \tag{20}$$

I consider a monetary policy rule in terms of the *level* of asset prices, instead of changes in asset prices. This is important as the level of asset prices directly influences macroeconomic aggregates, and it is also the level that depends on the interest rate and traders' behavioral sentiments.

Since the central bank cannot distinguish fluctuations of asset prices from speculation or changes in fundamentals, it will *always* react to asset prices in the same manner independent of the source of their fluctuations.

[8]For the sake of simplicity, I abstain from a more complex monetary policy rule that includes output, the output gap, growth, or interest rate smoothing.

2.5 General equilibrium

In general equilibrium, the linearized economy is characterised by the following set of equations:[9]

$$
\begin{aligned}
\pi_t &= \beta E_t \pi_{t+1} - \kappa x_t + a_t, && (21)\\
y_t &= E_t y_{t+1} - (r_{t+1} - E_t \pi_{t+1}) + d_t, && (22)\\
x_t &= \nu s_t - \psi y_t - (r_{t+1} - \pi_{t+1}) + \frac{1+\eta}{1+\bar{\nu}} a_t, && (23)\\
r_{t+1} &= \phi_\pi \pi_t + \phi_s s_t, && (24)\\
s_t &= \beta \hat{E}_t s_{t+1} - r_{t+1} + \pi_{t+1}. && (25)
\end{aligned}
$$

Equation (21) is the New-Keynesian Phillips Curve which is derived from staggered price setting. Inflation is dynamically linked to the markup (the inverse marginal costs), with κ being the slope of the Phillips Curve. Equation (22) is the dynamic IS-curve. The connection between the textbook model and the financial sector is represented by Equation (23), which includes an explicit role for asset prices s_t. Equation (24) is the linearized monetary policy rule. (25) is a linearized aggregate no-arbitrage-equation for asset prices arising from Equation (19), with $\hat{E}_t$ the aggregate and not necessarily rational expectations operator left to be specified. The law of motion of the two shock processes is given by the Equations (3) and (7).

3 Expectation formation and theoretical insights

I now relax the assumption of full rationality and establish a law-of-motion for a model with rational and boundedly rational agents. The presence of bounded rationality is a necessary condition for speculative dynamics: define speculation as trading activity where traders seek profits from short-term fluctuations in asset

[9] For simplicity, I abstract from government's economic activity.

prices. In a rational world with full and symmetric information, it would be impossible to reap these profits as all traders will perfectly anticipate any price changes. For this reason I use the terms *boundedly rational* and *speculative* interchangeably throughout this paper.

3.1 Coexistence of rational and boundedly rational agents

Define rational expectations as the benchmark case, given by $\hat{E}_t s_{t+1} = E_t s_{t+1}$. In this case Equations (21) – (25) can be represented as a 3-dimensional system

$$\mathbf{M}\mathbf{x}_t = \mathbf{P}E_t\mathbf{x}_{t+1} + \mathbf{v}_t, \tag{26}$$

with $\mathbf{x}_t = \{\pi_t, y_t, s_t\}$ and $\mathbf{v}_t = \{a_t, d_t\}$. The components of $\mathbf{M}$ and $\mathbf{P}$ are described in Appendix B.

One possible strategy to introduce boundedly rational expectations is to deviate from complete rationality in all markets equally, as e.g. in Evans and Honkapohja (2003) or De Grauwe (2011). However, fully abstracting from rational expectations would make it impossible to identify and analyze the role of speculative asset markets for the model dynamics.

The work of Greenwood and Shleifer (2014); Adam et al. (2017, 2018) on expectation formation on asset markets lends a strong motivation of boundedly rational asset markets. At the same time, the survey data used in these studies hardly justify speculative behavior in aggregate good markets. A further, non-trivial problem with models of boundedly rational expectations is that they come with additional degrees of freedom that typically cannot be disciplined easily. This is closely tied to the critique of the *wilderness of bounded rationality*. Based on these considerations, I introduce boundedly rational expectations in the financial market only.

The interaction of boundedly and fully rational agents can be highly nontrivial. Using a reduced form asset pricing model, Boehl and Hommes (2020) show that speculative dynamics can be amplified in the presence of rational agents who can perfectly predict the behavior of sentiment traders. To obtain a solution of the rational expectations equilibrium with quasi-periodic and potentially chaotic dynamics requires the authors to rely on advanced iterative methods. For the sake of analytical tractability, I instead assume in this paper that rational agents form *conditional model consistent rational* expectations such that they form expectations that correspond to the true expectations, taken fluctuations in asset prices as exogenous. A similar approach is chosen in Adam and Woodford (2018) who consider non-rational beliefs in the housing market in an otherwise rational expectations model conditional on the outcome of the housing market.

Assumptions 1 and 2 together imply that rational agents are unable to distinguish between exogenous shocks and speculation dynamics that cause fluctuations in asset prices:

Assumption 2. *The distribution of agent types is unobservable to any of the agents.*

Let $\mathbf{\tilde{v}_t} = (\tilde{a}_t, \tilde{d}_t)'$ be the exogenous processes that are *perceived* by rational agents. Denote by $E_t \left[\mathbf{x_{t+1}} | \mathbf{\tilde{v}_t}\right]$ the model and observation consistent rational expectations solution of (26) in terms of these perceived shocks. Then the actual law of motion (ALM) of the system is given by

$$\mathbf{M}\mathbf{x}_t = \mathbf{P} \begin{vmatrix} E_t \left[\pi_{t+1} | \mathbf{\tilde{v}_t}\right] \\ E_t \left[y_{t+1} | \mathbf{\tilde{v}_t}\right] \\ \hat{E}_t s_{t+1} \end{vmatrix} + \mathbf{v_t}. \tag{27}$$

The perceived law of motion (PLM) for rational agents is

$$\begin{vmatrix} \mathbf{M} & \mathbf{0_v} \\ \mathbf{0_x} & \boldsymbol{\rho} \end{vmatrix} \begin{vmatrix} \mathbf{x_t} \\ \mathbf{\tilde{v}}_t \end{vmatrix} = \begin{vmatrix} \mathbf{P} & \mathbf{0_v} \\ \mathbf{0_x} & \mathbf{I} \end{vmatrix} E_t \begin{vmatrix} \mathbf{x_{t+1}} \\ \mathbf{\tilde{v}}_{t+1} \end{vmatrix}, \tag{28}$$

with $\boldsymbol{\rho}$ being the diagonal matrix containing the autocorrelation parameters. This simply expresses System (26) in terms of perceived exogenous shocks instead of the real exogenous shocks. Denote $\mathbf{\Omega}$ the (linear) rational expectations solution of

(28).[10] For the belief system of model/observation consistent agents it must hold by definition that

$$\begin{vmatrix} \pi_t \\ y_t \end{vmatrix} = \mathbf{\Omega} \begin{vmatrix} \tilde{a}_t \\ \tilde{d}_t \end{vmatrix} \qquad \text{and} \qquad \begin{vmatrix} E_t[\pi_{t+1}|\tilde{\mathbf{v}}_\mathbf{t}] \\ E_t[y_{t+1}|\tilde{\mathbf{v}}_\mathbf{t}] \end{vmatrix} = \mathbf{\Omega}\boldsymbol{\rho} \begin{vmatrix} \tilde{a}_t \\ \tilde{d}_t \end{vmatrix}. \tag{29}$$

Even without explicitly solving for the perceived shocks $\tilde{\mathbf{v}}_\mathbf{t}$, the relationships in (29) can be used to express the conditional expectations on inflation and output in the next period in terms of inflation and output in the current period:

$$\begin{vmatrix} E_t[\pi_{t+1}|\tilde{\mathbf{v}}_\mathbf{t}] \\ E_t[y_{t+1}|\tilde{\mathbf{v}}_\mathbf{t}] \end{vmatrix} \quad = \quad \mathbf{\Omega}\boldsymbol{\rho}\tilde{\mathbf{v}}_\mathbf{t} \quad = \quad \mathbf{\Omega}\boldsymbol{\rho}\mathbf{\Omega}^{-\mathbf{1}} \begin{vmatrix} \pi_t \\ y_t \end{vmatrix}. \tag{30}$$

Using this result in (27) yields an ALM in terms of the exogenous states, $\mathbf{v}_\mathbf{t}$, and traders' expectations on asset prices, $\hat{E}_t s_{t+1}$.[11]

The ALM can be expressed as a mapping $\mathbf{\Psi} : (\mathbf{\Phi}, \boldsymbol{\phi}) \to \mathbb{R}_{3\times 3}$, where $\mathbf{\Phi}$ is the set of model parameters $(\beta, \nu, \psi, \kappa, \rho_a, \rho_d)$ and $\boldsymbol{\phi}$ the monetary policy parameters (ϕ_ϕ, ϕ_s):

$$\begin{vmatrix} \pi_t \\ y_t \\ s_t \end{vmatrix} = \mathbf{\Psi} \begin{vmatrix} d_t \\ a_t \\ \hat{E}_t s_{t+1} \end{vmatrix}, \tag{31}$$

The system in 31 represents the solution for inflation, output and asset prices as a function of the actual exogenous shocks and boundedly rational beliefs $\hat{E}_t s_{t+1}$.

3.2 Theoretical results

In the absence of any exogenous noise, that is if $\mathbf{v}_t = \mathbf{0}$, the law-of-motion in (29) reduces to

$$\mathbf{x}_t = \mathbf{\Psi}_{\cdot,\mathbf{3}} \hat{E}_t s_{t+1} \qquad \text{and} \qquad s_t = \mathbf{\Psi}_{\mathbf{3},\mathbf{3}} \hat{E}_t s_{t+1}, \tag{32}$$

[10] The derivation of $\mathbf{\Omega}$ is provided in Appendix B

[11] This requires $\mathbf{\Omega}$ to be non-singular. Singularity of $\mathbf{\Omega}$ would mean that either π or y_t need to be independent of $\mathbf{v}_t$, or either a_t or d_t to have no effect on $\mathbf{x}_t$.

where $\mathbf{\Psi}_{3,3}$ can be interpreted as the root of the process that determines s_t.

I calibrate some parameters to values that are commonly found in the literature. Accordingly, the quarterly household discount rate β is set to 0.99, and the autocorrelation parameters of the structural shocks are $\rho_a = 0.9$ and $\rho_d = 0.7$, respectively. Consistent with Woodford (2003), I set the inverse Frisch labor supply elasticity η to 0.3 and firms' probability ω to change prices in a given period to 0.66. Hence, the slope of the Phillips curve is $\kappa = (1-\omega)(1-\beta\omega)/\omega \approx 0.179$. Furthermore, for the baseline scenario, I choose the monetary policy response coefficients to be $\phi_\pi = 1.5$ and $\phi_s = 0$, implying that the central bank does not target asset prices. In Section 4, I provide estimates of key parameters, including an estimate of the derivative of the external financing premium in steady state $z'(\cdot) = -\tilde{\nu}$. I find that $\tilde{\nu} = 0.09$ such that $\psi = \frac{1+\eta+\tilde{\nu}}{1-\tilde{\nu}} \approx 1.52$ and $\nu = \frac{\tilde{\nu}}{1-\tilde{\nu}} \approx 0.099$.

Using the calibration above, I find that $\mathbf{\Psi}_{3,3}$ is positive and smaller but close to a unit-root. This means that the expectations of different agents are strategic complementarities, and implies a non-trivial role of monetary policy for asset prices: assume that boundedly rational traders expect asset prices to increase in the next period. As a consequence of the direct feedback, current asset prices will increase, as captured in Equation . Higher asset prices reduce firms' borrowing cost, which in turn leads firms to lower their prices in aggregate. Finally, in response to the drop in inflation, the central bank cuts the nominal interest rate, thereby further fueling the increase in asset prices.

There exists a large literature that stresses the role of strategic complementaries for asset pricing (c.f. Bulow et al., 1985; Cooper and John, 1988). Previous work has shown that systems in which strategic complementarities can lead to large feedback and amplification effects are prone to dynamic instability.[12] In fact, these systems can exhibit large swings and bubbles. The probability to observe a bubble increases as the characteristic root of the system is closer to unity. Based on this argumentation, one can also interpret $\mathbf{\Psi_{3,3}}$ as a measure of the probability of bubbly

[12] See Hommes (2011) for a review.

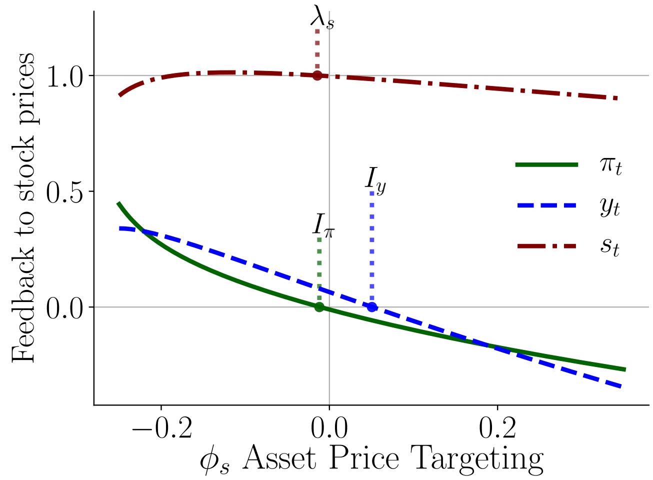


Figure 1: Direct ceteris paribus effect of a 1% change in asset price expectations on output, inflation and asset prices. The effect is shown as a function of the central bank policy coefficient to asset prices. Responses in deviations from steady state.

episodes. Hence, a policy that aims at preventing financial bubbles would reduce the reduces the values of $\mathbf{\Psi_{3,3}}$. If such a policy is either unavailable or undesirable, the second best solution would to minimize $\mathbf{\Psi_{1,3}}$ and $\mathbf{\Psi_{2,3}}$, thereby reducing the spillover effects of asset prices on real activity.

Figure 1 shows the relationship between $\mathbf{\Psi_{:,3}}$ and the monetary policy response coefficient to asset prices ϕ_s. Normalizing the change in asset price expectations to 1 percent, $\mathbf{\Psi_{1,3}}$, $\mathbf{\Psi_{2,3}}$ and $\mathbf{\Psi_{3,3}}$ can be interpreted as the general equilibrium response of π_t, y_t and s_t, respectively. Negative values for ϕ_s imply that the central banks reduces nominal interest rates in response to higher asset prices. Even for the moderate values of ϕ_s that are shown in Figure 1, the effects on π_t, y_t and s_t vary non-negligibly. Denote by λ_s the value of ϕ_s such that $\mathbf{\Psi_{3,3}} = 1$. The increase in current asset prices can be even larger than 1 percent, i.e. $\mathbf{\Psi_{3,3}} > 1$, for ϕ_s ranging between -0.18 and $\lambda_s \approx -0.01$ but is monotonically decreasing for positive values of ϕ_s. This suggests that a moderate degree of asset price targeting can indeed reduce the amplification and feedback effects from higher expected asset prices to macroeconomic variables.

Note that setting $\phi_s = I_y \approx 0.05$ will entirely offset the impact of an increase in expected asset prices on output (that is $\mathbf{\Psi_{2,3}} = 0$). A monetary policy rule with $\phi_s > I_y$ will in fact reduce output in response to a 1% increase in asset price expectations. Furthermore, notice that the equilibrium effect on inflation is zero when $\phi_s = I_\pi \approx -.02$. In this case, the central bank will reduce the nominal interest rate in responds to higher asset prices. Lower interest rates reduce firms' borrowing cost and increase consumer demand, with the latter resulting in higher firm marginal cost. The two effects balance out when $\phi_s = I_\pi \approx -.02$.

When non-fundamental fluctuations in asset prices cause real spillovers, the central bank faces a trade-off. By carefully targeting asset prices, policy makers can reduce the impact of asset price dynamics on either inflation or output, but must accept potentially strong dynamic feedbacks captured by a high $\mathbf{\Psi_{3,3}}$. Alternatively, the central bank can choose to decrease the dynamic feedback, but such policy bears the risk to increase the impact of asset price fluctuations on real variables. The following section quantitatively asses this trade-off in a realistic framework of expectation setting.

4 Quantitative results

This section first presents a row of stylized facts and the data used to fit the model. Secondly, the model is completed by introducing a realistic, behaviorally grounded process for the formation of asset price expectations.. This model is then estimated to match the presented data.

4.1 Data and stylized facts

Episodes with booms and busts in asset markets are a recurrent phenomena. In their database on financial crisis, Laeven and Valencia (2013) find 124 Systemic banking crisis between 1970 to 2007. They report that such crisis are often preceded by credit booms, with an pre-crisis annual credit growth of about 8.3%. The average loss in GDP for these crisis is roughly 20 percent. Similar findings are reported by Miao and Wang (2018); Davis and Taylor (2019). In an analysis of the housing market and equity prices in industrialized economies during the postwar period, the

IMF found that booms in both markets arise frequently (on average every 13–20 years) with entailed drops in prices averaging around 30% and 45% respectively. These busts are associated with losses in output that reflect declines in consumption and investment.

Table 1 summarizes the first and second moments of inflation, output and asset prices in Core-Europe. The data is obtained from the OECD, asset prices are represented by the MSCI-Europe index.[13] The data is quarterly and ranges from 1976Q1 to 2015Q1, hence a total of 158 observations is being used. The time series are deflated by the consumer price index (prices given in 2005 EUR. The HP-Filter is applied to the log of each series with $\lambda = 1600$). I opt for data from the Euro Area instead of US data, because in the US, the effective lower-bound (often called the *zero lower-bound*, ZLB) on interest rates was binding following 2008Q4. It is well understood that the ZLB can heavily affect the economic dynamics (see e.g. **?**). Although it would be technically straightforward to consider the ZLB as an additional source of nonlinearities, it is not clear how this would improve the analysis that is in focus of this paper.

As my empirical analysis is based on the first two moments of the data, the choice of the data source is very robust to a broad class of variations. Other sources will result in comparable stylized facts.[14] Note that the data sample does not include a severe financial crisis. The sample will therefore be used to calibrate the model to normal times, but not to crisis times. Let me summarize the following stylized facts from the data:

i) The standard deviation of asset prices is roughly one order of magnitude higher than the standard deviations of inflation and output.
ii) Inflation is (weakly) countercyclical.
iii) Asset prices and output are positively correlated.
iv) Asset prices and inflation are negatively correlated.

[13] At the time of writing the series on inflation and output is available at `https://data.oecd.org/`. Asset prices are downloaded from `https://www.msci.com/indexes`.

[14] Compare e.g. with Winkler (2019) or Adam et al. (2017).

v) The negative correlation between asset prices and inflation is stronger than the correlation between output and inflation.[15]

Note also that both, the US and the Euro Area have recently seen high asset prices together with low interest rates and low inflation.

	π	y	s
SD	0.0092	0.0104	0.1407
π	1	-0.1734	-0.3867
y	–	1	0.6025
s	–	–	1

Table 1: Standard deviations and cross correlations of inflation, output and real asset prices. Quarterly data for Core-Europe from 1976 to 2014.

The next task is to specify an explicit mechanism of expectation formation for asset markets and to fit the then completed model to this data.

4.2 Expectation formation process

I assume that financial traders follow the *Heterogeneous Agent Switching Model* (Brock and Hommes, 1998, henceforth the BH-model). In this behavioral model, agents switch endogenously between simple forecasting heuristics, depending on the performance of each heuristics. As trading strategies are complementaries (see Bulow et al., 1985), agents have incentive to mimic successful strategies. If the share of traders that use a specific heuristic accumulates to a critical mass, they may be able to outperform rational traders due to the direct positive feedback of expectations on prices.

The choice of the BH-model is driven by a set of empirical properties. Other than in models with rational expectations, the heterogeneous agent switching model can replicate a positive correlation between returns and expected returns, and fat tails of the distribution of asset prices (see e.g. Greenwood and Shleifer, 2014; Adam et al., 2017, 2018). The intuitive mechanism of expectations formation fits well to

[15]Note that this also suggests that the link between asset prices and macroeconomic activity should rather be motivated through the supply side than through the demand side.

the description of *animal spirits* in Keynes (1937). The model is also substantiated by a vast experimental literature (see cf. Hommes, 2006). Further, the BH-model allows to explicit model *endogenous* financial cycles and the endogenous nonlinear propagation of real shocks to the asset market. This propagation goes beyond simple spillover effects: while a decrease in the interest rate will boost asset prices, this can trigger complicated expectations dynamics and extrapolative behavior that result in speculative asset price booms. As Boehl and Hommes (2020) show, these financial cycles can even turn into rare-disaster type financial crisis. The endogenous nature of these dynamics is a central motivation here: it is arguably hard to enrich our understanding of financial cycles and crisis if these events are purely driven by exogenous shocks, as it is often assumed (Christiano et al., 2015; Del Negro et al., 2015; **?**; **?**).

Assume that asset traders are heterogeneous in their forecasting rules. Let there be $H > 1$ simple predictors of future prices and let each predictor $h = 1, 2, \dots, H$ be of the form $\hat{E}_{t,h}s_{t+1} = g_h s_{t-1} + b_h$. Aggregating over the individual optimality conditions (19) yields, up to a first order approximation, the economy wide price for shares S_t. Let $n_{t,h}$ denote the fraction of traders using predictor h at time t, then

$$R_{t+1}S_t = E_t\Theta_{t+1} + \sum_h n_{t,h}\hat{E}_{t,h}S_{t+1}. \tag{33}$$

Assume further that traders take the interest rate R_{t+1} as given.[16] Log-linearization yields an analogue representation to Equation (25), i.e.

$$s_t = \beta\hat{E}_t s_{t+1} - (r_{t+1} - \pi_{t+1}) \qquad \text{with} \qquad \hat{E}_t s_{t+1} = \sum_h n_h \hat{E}_{h,t} s_{t+1}. \tag{34}$$

The fractions $n_{h,t}$ of each predictor are updated according to predictor h's *measure of performance $U_{h,t}$*. In line with Brock and Hommes (1998) I utilize *realized*

[16]Alternatively, the $R_t\frac{P_t}{P_{t+1}}$ could be included explicitly in the performance measure. This does not fundamentally change the dynamics, but leads to a slight asymmetry of the resulting bifurcations.

past profits as the performance measure because it is a good proxy for evolutionary fitness. This incorporates the idea that strategies, that were more successful in the past are more likely to be chosen. Hence,

$$U_{h,t} = (\beta s_t - s_{t-1})(\beta \hat{E}_{t-1,h} s_t - s_{t-1}). \tag{35}$$

The choice of the performance measure is a major determinant of the nonlinear properties of the system. An additional feature captured by realized past profits is the notion that asset markets, due to the positive feedback, reward the prediction of the right sign of the price change to the next period, instead of rewarding an accurate estimate of the price.

The probability that an agent choses predictor h is given by the *multinomial discrete choice model*

$$n_{h,t} = \frac{\exp\{\gamma_U U_{h,t-1}\}}{Z_{t-1}} \qquad \text{and} \qquad Z_{t-1} = \sum_{h=1}^{H} \exp\{\gamma_U U_{h,t-1}\}, \tag{36}$$

where γ_U is called the *intensity of choice*.

Consider a simple 3-type model where agents are either fundamentalists – traders that believe next-period's price will be the fundamental price – or trend extrapolators with a positive and a negative bias, respectively. Assume that the latter two share a common trend-following parameter γ_s and that the positively/negative bias is symmetric by γ_b. The three types are then given by

$$\begin{aligned} \hat{E}_{t,1} s_{t+1} &= 0, \\ \hat{E}_{t,2} s_{t+1} &= \gamma_s s_{t-1} + \gamma_b, \\ \hat{E}_{t,3} s_{t+1} &= \gamma_s s_{t-1} - \gamma_b. \end{aligned} \tag{37}$$

This specification of the mechanism for expectations on asset prices closes the model. It consists of a linear part associated with the macroeconomy and the formation of rational expectations, which is represented by Equation (29). Additionally, it includes a nonlinear part which contains the boundedly rational expectation formation which is given by the performance measure $U_{h,t}$ (Equations 34 and 35), the fractions $n_{h,t}$ and the normalization factor (Equation 36), and the predictors (Equation 37). The behavioral parameters γ_U, γ_s and γ_b remain to be discussed for the next subsection.

4.3 Fitting the model to the data

The remaining free parameters are estimated using the method of simulated moments (MSM, McFadden, 1989). The underlying intuition behind MSM is to minimize some norm of the simulated and empirical moments. To prevent overfitting, I estimate five parameters to fit six moments. As with the generalized method of moments (Hansen, 1982), a weighting matrix is used to corrects for the quality of the moment estimates. This weighting matrix is estimated using by the 2-step procedure. The simulated moments are retrieved from a batch of 100 simulated time series, each of the length of the original data. MSM has the advantage that only the specified moments are targeted and not – as with likelihood-based methods – the complete time series. It is clear that a small-scale model as the one presented in Section 2 can not yield a satisfactory fit to economic times series, as it would be required in the context of likelihood based methods (see e.g. An and Schorfheide, 2007; Del Negro et al., 2007; Smets and Wouters, 2007). For this reason it is useful only focus on the first moments of the data to discipline the model.

As an additional reference point – in particular to provide a robustness check for the estimate of ν – I also consider a rational expectations (RE) version of the model, where asset markets are homogeneous and fully rational but subject to an additional add-hoc exogenous AR(1) process on asset price expectations $E_t s_{t+1}$. This allows for exogenous fluctuations in asset prices in the RE-model and provides an equal number of (and comparable) degrees of freedom as for the alternative model. Thus, for the rational model variant I add the exogenous state v_t^s with

$$v_t^s = \rho_s v_{t-1}^s + \varepsilon_t^s, \qquad \varepsilon_t^s \sim N(0, \sigma_s) \tag{38}$$

to Equation (23).

For both models the elasticity of marginal costs to asset prices, ν, and the standard deviations of the real shocks, σ_d and σ_a are estimated. Additionally, for the RE-model the autocorrelation coefficient and the standard deviation for the expectations shock, ρ_s and σ_s, need to be fitted. For the nonlinear model BH-model the parameters γ_s and γ_b will be estimated and γ_U set to unity. The latter is without lack of generality as the two former parameters already have sufficient degrees of freedom to allow for a rich portfolio of nonlinear dynamics (see Brock and Hommes, 1998).

	γ_s	γ_b	ρ_s	σ_s	ν	σ_a	σ_d
BH-model	1.006	1.229	–	–	0.090	0.001	0.004
RE-model	–	–	0.790	0.018	0.083	0.001	0.004

Table 2: MSM parameter estimates. Simulated moments obtained from 100 batches of simulated time series. For the weighting matrix the 2-step procedure is used.

The parameter values obtained from MSM are displayed in Table 2. Independently of the asset market specification the procedure identifies a value of ν below but close to 0.1. Given that the standard deviation of asset prices is roughly ten times the standard deviation of inflation, the estimate of ν implies a rather large impact of asset price fluctuations on the economy. This also yields a potential explanation for the large share of the variance of GDP explained by fluctuations in asset prices in Assenmacher and Gerlach (2008) and Miao et al. (2012). The standard deviations σ_a and σ_d imply strong fluctuations in aggregate demand, a finding which is in line with recent estimates e.g. from **?**. All in all, the estimate of ν provides a strong motivation to study the role of policy intervention.

The estimate of γ_s is almost unity, a finding which is well in line with experimental results (Hommes, 2013) that report a high degree of trend extrapolation. However, as the parameter is only marginally larger than one, extrapolation by itself is not a source of explosive dynamics.[17] The estimate of the behavioral bias of $\gamma_b = 1.229$ can be seen as rather moderate estimate, relative to the high empirical

[17]Setting γ_s to a value in between .97 and 1 does not notably reduce the goodness of fit.

volatility of asset prices.

	BH-Model		
	π	y	s
SD	0.006 (.001)	0.013 (.002)	0.128 (.039)
π	1	-0.125 (.170)	-0.345 (.090)
y	–	1	0.635 (.140)
s	–	–	1
	RE-Model		
	π	y	s
SD	0.007 (.001)	0.012 (.002)	0.140 (.016)
π	1	-0.130 (.161)	-0.443 (.102)
y	–	1	0.613 (.118)
s	–	–	1

Table 3: Standard deviations and cross correlations for the estimated models. Moments obtained from 100 batches of simulated time series.

Table 3 shows the simulated moments for both, the RE and BH estimation. For both models the moment estimates are close to the original moments of the data. The endogenous amplification of asset prices through the BH model reduces the correlation between asset prices and inflation. Likewise, endogenous amplification explains the high standard deviation of asset prices because it does not one-to-one feed back on inflation and output: an increase in asset prices dampens inflation trough the marginal cost channel and the central bank lowers the interest rate which in turn stimulates demand. This ensures that the correlation between output and asset prices is relatively strong even in the absence of a procyclical dividend component.

To summarize the findings from this section, non-fundamental fluctuations in asset prices, in combination with the link between asset prices and real activity allows to replicate key-moments of the data that are otherwise hard to reconcile. The next Section will discuss potential implications for monetary policy.

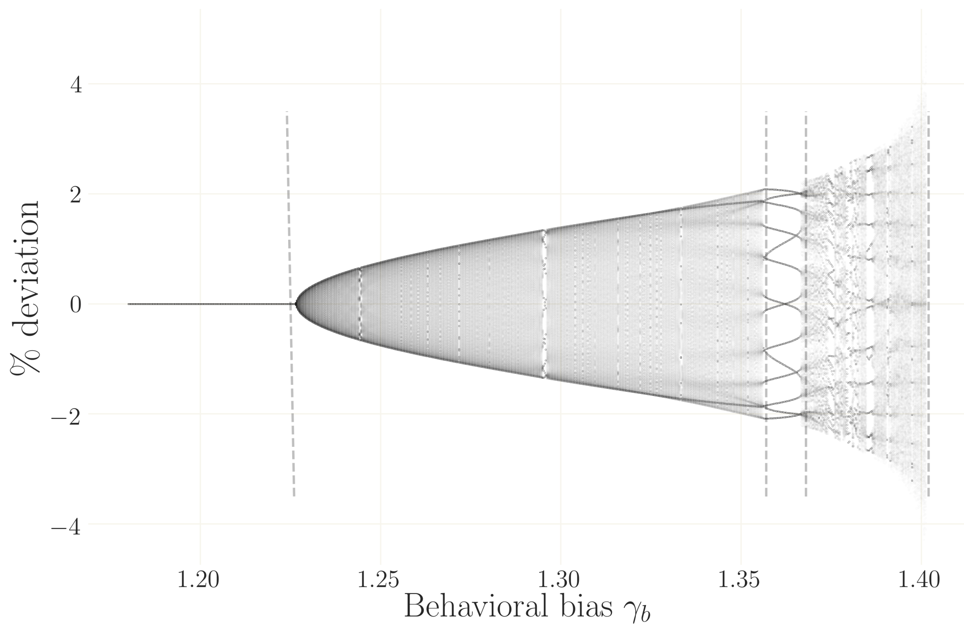


Figure 2: Bifurcation diagram for asset prices output w.r.t. bias parameter γ_b. A primary Hopf bifurcation at $\gamma_b \approx 1.22$ leads to periodic and quasi-periodic dynamics. All other parameters as in Table 2.

5 Asset price targeting

This section turns towards the quantitative policy implications of speculative asset markets. I first explore the dynamic properties of the model, in particular regarding non-trivial, complicated or *chaotic* dynamics that originate in the interplay of goods market and speculative financial markets. I then study potential implications for the role of monetary policy during normal times. Thereafter I take the model to the limit case of endogenous, financial crisis-type dynamics, and discuss the potential role of asset price targeting in mitigating such crisis.

5.1 Dynamic properties

The model presented in the foregoing sections allows for a wide range of dynamics. In the simplest case it nests the standard rational expectations New-Keynesian model in the spirit of Woodford (2003) and Galí (2008), where all dynamics are a linear propagation of exogenous shocks. In the absence of these shocks, such linear model is stationary. While linear dynamics entail some convenient attributes – e.g. closed-form representations of dynamic attributes and a straightforward application of econometric estimation methods – it is not suitable to explore the potentially complex interaction of highly dynamic and potentially overheated financial markets with the goods market. My detailed account of the feedback mechanism between inflation, monetary policy, and speculative financial markets can give rise to complicated, endogenous dynamics. For this reason it is useful to first study the dynamics of this feedback mechanism in the absence of exogenous shocks.

Such complicated dynamics can give rise to endogenous fluctuations, or even chaotic dynamics: prices and aggregates can evolve without the need for exogenous disturbances and potentially do not revert to a stationary steady state. The behavior of such systems is studied in the field of bifurcation theory (see e.g. Arnold et al., 2013). Figure 2 presents the bifurcation diagram of asset prices as a function of the parameter γ_b. For each parameter value on the x-axis, it displays all the points visited in the long-run and in the absence of any additional stochastic noise.[18]

The vertical grey lines separate four different regions of the parameter space. Each region differs in the *type* of dynamics. For values of γ_b below 1.22 the fundamental steady state is stable and unique. In this region, the degree of the bias of sentiment traders is not large enough to impact on asset prices without exogenous shocks, and asset markets are dominated by fundamental beliefs. While exogenous impulses could lead to a temporal increase in the fraction of belief-biased agents – which would protract the response of the impulse – this effect is not strong enough to prevent prices from returning to the fundamental steady state.

A bias larger than 1.22 leads to limit cycles. As beliefs are to a large extent self-fulfilling, in an upswing the fraction of belief-biased agents will grow in time through positive self-enforcement. The natural limit of this feedback process –

[18] For each value at the bifurcation parameter 11.000 iterations are run. A transition phase of 10.000 periods is omitted in the analysis.

the amplitude – is reached when the price equals the bias. Biased agents will not be able to extract any additional profits, and their fraction will thus start to fall again, which reflects in falling asset prices. When the price approaches the value predicted by positively/negatively biased traders, they reduce their long/short position. By that, their profit shrinks and alternative strategies become relatively more attractive.[19] As these alternative beliefs become more widespread, the fraction of positively (negatively) biased traders will again decrease and prices begin to drop again.

For values larger than $\gamma_b \approx 1.37$ cycles become unstable and more erratic. The simulations suggest that the system is close to a *homoclinic orbit*[20]: the zero steady state is globally stable but locally unstable (Ott, 2002; Hommes, 2013). Long periods of stability can then be followed by asset price booms that are hard to predict, and which, through the credit-collateral channel, can be followed by abrupt, severe recessions. For even higher values of γ_b dynamics become explosive because the share of fundamentalists is insufficiently large to stabilize the system.

The bifurcation diagrams for γ_s and γ_U can be found in Appendix C. While amplitudes and periodicity of periodic and quasi-periodic dynamics can differ, the *types* of possible dynamics are the same as for γ_b.

5.2 Asset price targeting in normal times

After this short primer on the type of complicated dynamics that can emerge within the model, let me focus on the role of interest rate policy for the interplay between financial markets and the macroeconomy. Figure 3 shows the bifurcation diagram for the policy parameter ϕ_s. For the estimated values under the assumption of no asset price targeting, ($\phi_s = 0$, gray dashed line), the figure suggests the existence of periodic movements in asset prices with a very small amplitude. This

[19]Figure C.10 illustrates graphically the different forces at work behind the financial limit cycles.

[20]A homoclinic orbit is a trajectory of a flow of a dynamical system which joins a saddle equilibrium point to itself. In the terminology of dynamic system theory, a homoclinic orbit is said to lie in the intersection of the stable manifold and the unstable manifold of an equilibrium, see c.f. Ott (2002).

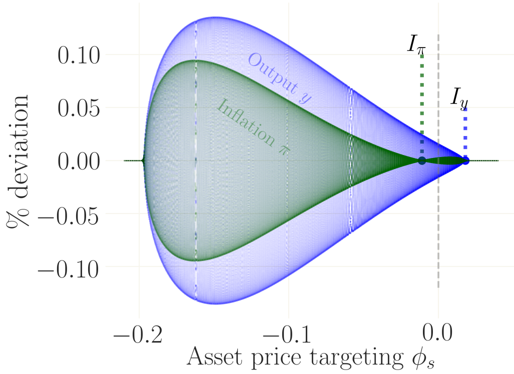


Figure 3: Bifurcation diagram for output (blue/light) and inflation (green/dark) with respect to the policy parameterϕ_s. Output and inflation inherit the financial cycles originating from the asset market. A primary Hopf bifurcation leads to periodic periodic dynamics. The system reverts to stationarity after a second Hopf bifurcation. All other parameters as in Table 2.

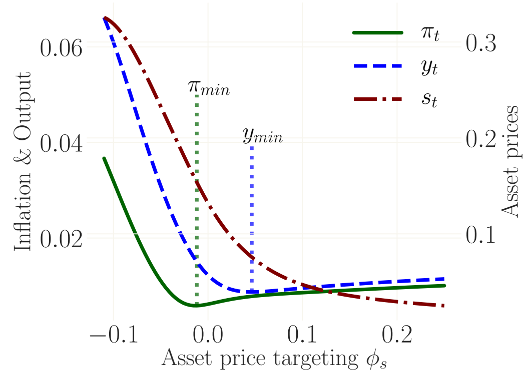


Figure 4: Standard deviations of stochastic simulations as a function of the asset price targeting policy parameter ϕ_s. The economy is driven by exogenous shocks, but endogenously amplified by the nonlinear behavioral process in the asset market.

translates to similar cycles in inflation and output. Comparing these fluctuations to the standard deviations of the data suggests that the data is not driven by endogenous dynamics, but rather by the interplay of exogenous macroeconomic shocks and endogenous amplification. This still implies that speculative agents react endogenously to fluctuations in asset prices, which has the potential to create large non-fundamental swings in asset prices when combined with exogenous shocks.

In Section 3 I show that an increase in ϕ_s can mitigate the direct impact of asset prices on output and reduce the positive feedback of asset price expectations. Figure 3 confirms that, in the absence of stochastic shocks, the central bank can mitigate endogenous speculative dynamics by carefully targeting asset prices. An increase in the interest rate in response to asset prices counteracts the decrease of the rate induced by inflation targeting. Relatively higher rates act as a natural dampener to the expectation feedback. For this reason the amplitude decreases with ϕ_s. It can be shown that at the point $B(\lambda_s)$ of Figure 1, one of the eigenvalues of the law-of-motion crosses the unit circle and the periodic solution turns into a stable fixed point. In the field of bifurcation such point is known as a (inverse) *supercritical Hopf-Bifurcation* (see e.g. Kuznetsov, 2013).

The previous exercise was helpful to understand the pure trading dynamics. Let us turn now to the general case of stochastic simulations. Figure 4 shows the standard deviations of simulations as a function of the policy parameter ϕ_s. Here, the exogenous processes for a_t and u_t are unmute. The dynamics emerge as a combination of the i.i.d. noise and endogenous responses of the financial market to these shocks. The Figure indicates that

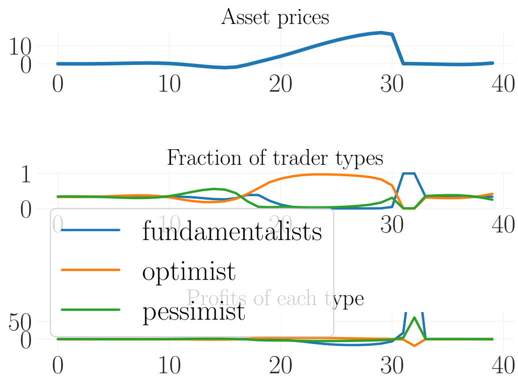
a) an increase of ϕ_s reduces endogenous asset price volatility,

b) an increase of ϕ_s up to $y_{min} \approx 0.018 \approx I_y$ shuts-off the transition of asset price volatility to output, but increases volatility of inflation,

c) an increase of ϕ_s to a value higher than y_{min} leads to additional fluctuations in both, inflation and output.

The "collateral damage" effect in (b) and (c) – i.e. the unwanted increase in the volatility of output and inflation – stems from the fact that a leaning-against-the-wind policy necessarily also reacts to fluctuations in asset prices that are (efficient) general equilibrium responses to exogenous shocks. Explained given the logic of a negative productivity shock, the increase in inflation triggers monetary policy to raise the interest rate. This leads to a deflation of asset prices. An additional response to asset prices will induce economic costs in terms of an incremental drop in output and inflation. The logic for a demand shock works similarly. Hence, the standard result holds that in an economy in which asset prices are not a source of fluctuations, asset price targeting tends to increase volatility in real aggregates. This effect runs in the opposite direction of the stabilizing effect. The results from this section suggest that the optimal sensitivity of monetary policy to asset prices should be bounded by $\pi_{min} \approx -0.01 = I_\pi$ and I_y.

The analysis in Galí (2014) is based on a framework that includes *rational bubbles*. Such rational bubbles grow proportionally to the interest rate. The author hence suggests to actually lower the policy rate when facing asset price bubbles.



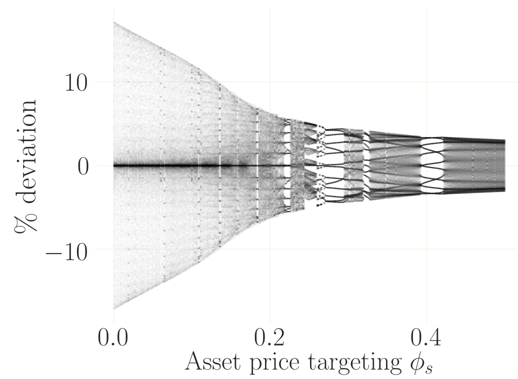


Figure 5: *Top panel:* time series of asset prices during the build-up and bust of a financial bubble. The dynamics are absent any additional shocks. Parameters: $\gamma_b = 2.5$, $\gamma_s = 0.93$ and $\gamma_U = 0.4$. $\nu = 0.09$ as in 2. *Center panel:* during the build-up, a large fraction of traders is optimistic. Optimism vanishes as the bubble peaks. During the crash fundamental beliefs dominate. *Bottom:* profits of each trader type. The fundamental predictor is unfavorable because it is costly.

Figure 6: Bifurcation diagram for asset prices with respect to the policy parameter ϕ_s. $\gamma_b = 2.5$, $\gamma_s = 0.93$ and $\gamma_U = 0.4$. $\nu = 0.09$ as in 2. For low values of ϕ_s the trajectories are close to a homoclinic orbit. For values of ϕ_s close to 0.5 the time series displays periodic and quasi-periodic behavior.

My model suggests that such policy would lead to a considerable increase in output and asset price volatility and, even for small values of the policy's sensitivity, an amplification of the fluctuations in asset prices.

5.3 Asset price targeting and endogenous financial crises

The results from the previous subsection assign no positive role for asset price targeting. The central bank faces a trade-off between fragility of asset prices – the stabilization component of asset price targeting – and the additional volatility caused by such policy. Note that the parameter estimates from Table 2 reflect a sample, in which financial markets remain rather calm for the largest part. For this reason it is an intrusive result that the costs of additional volatility weight higher than the benefits from a stabilization of asset prices. An important question however is, if asset price targeting can help in preventing financial crisis and bubbles in times where financial markets are overheated.

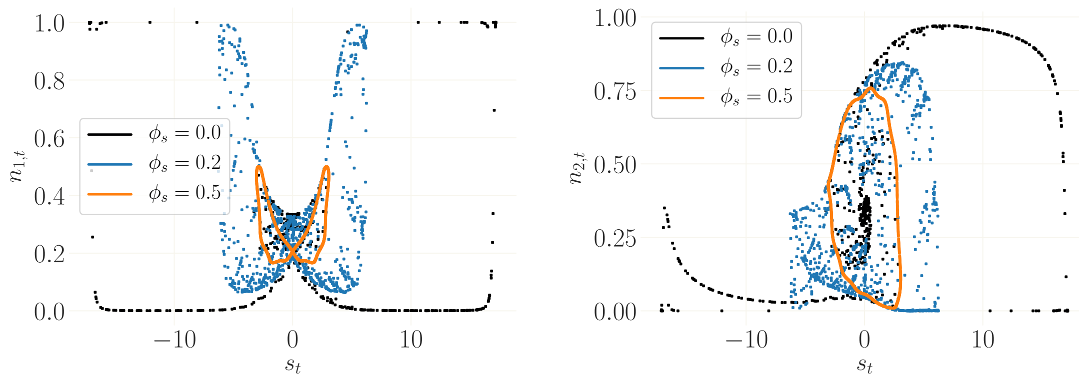


Figure 7: Phase diagrams of the endogenous dynamics. Parameters as in Figure 6. *Left:* asset prices and share of fundamentalists. *Right:* asset prices and share of optimistic traders.

In my model it is straightforward to create an experimental world with endogenous financial crises. While it seems natural to assume that the deep economic parameters – parameters such as η, ν or β – remain time-invariant, the behavioral parameters are likely to change. Assume a regime where the bias of traders γ_b is increased to 2.5. Set $\gamma_s = 0.93$ and γ_U to 0.4. The choice of the latter is more moderate than in the benchmark, but necessary to prevent the system from explosive dynamics given the relatively high bias.[21] The resulting time series – again in the absence of any real shocks – are displayed in the top panel of Figure 9. After long episodes of stability, asset bubbles slowly build up through expectations dynamics (period 17 ff.): more and more traders switch to the positive-biased heuristic. At the same time both sentiment traders extrapolate the positive trend. This process is self enforcing until the effects of bias and extrapolation level out (period 30), and profits of the sentiment traders decrease. At this point a large share of traders switch to the fundamental trading strategy and prices collapse immediately. During the asset bubble, refinancing conditions for firms improve which is reflected in a relative decrease in prices. The central bank responds with lowering interest rates, which stimulates consumption and, in general equilibrium, further fuels financial

[21] The bifurcation diagram for the intensity of choice can be found in Figure C.12 in Appendix Appendix C.

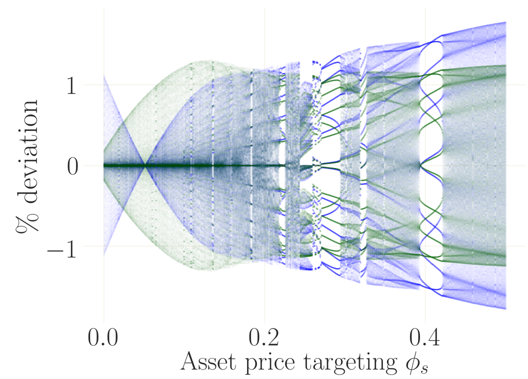


Figure 8: Bifurcation diagram for output (blue/light) and inflation (green/dark) with respect to the policy parameter ϕ_s. Parameters as in Figure 6. Dynamics of inflation and output and output do not inherit the dampening effect of asset price targeting on asset prices. Instead, the amplitude of the dynamics increases with the aggressiveness of the policy.

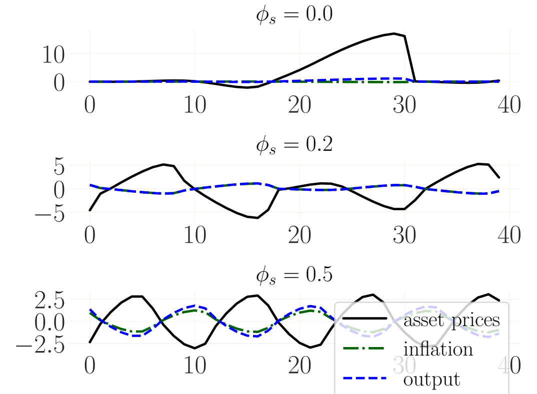


Figure 9: Time series of asset prices, inflation and output for different values of ϕ_s. Endogenous dynamics in the absence of additional stochastic noise. $\gamma_b = 2.5$, $\gamma_s = 0.93$ and $\gamma_U = 0.4$. $\nu = 0.09$ as in 2. *Top panel:* the trajectory is close to an homoclinic orbit. After stable episodes, bubbles slowly emerge en then burst. *Center panel:* for $\phi_s = 0.2$ the time series exibits quasi-periodic dynamics. *Bottom panel:* limit cycles for high values of ϕ_s. Inflation and output are both countercyclical.

markets. The boom in asset prices is accompanied by an episode of growth and low inflation.

Can the central bank prevent such boom-bust cycle in asset prices? Figure 6 shows the bifurcation diagram of asset prices with respect to the policy parameter, taking the crisis setup developed in the previous paragraph as the starting point. An increase of interest rates in response to a surge in asset prices can indeed mitigate the cyclic movement in asset prices: when inflation decreases, the central bank wants to lower rates. If at the same time asset prices are high, the central bank will lower rates by less, thereby partly cutting-off the feedback to financial markets. For the example chosen here, a value of ϕ_s of roughly 0.25 is sufficient to prevent trajectories close to an homoclinic orbit. However, cyclical movements in asset prices remain, although with moderate amplitude. For values of $\phi_s > 0.25$ a higher feedback coefficient to asset prices does not have a notably strong effect on the amplitude of asset prices.

How does this dampening effect translate to the real economy? Figure 8 shows the bifurcation diagram of output and inflation to the policy parameter. The Figure shows clearly that the reduction of endogenous fluctuations does not pass on to the real economy. The increase in the interest rate in response to a boom in asset prices reduces the output response, which in turn further decreases inflation through its effect on wages. Hence, low values of ϕ_s decrease the volatility of output in response to financial cycles, but increase inflation volatility. This effect prevails until ϕ_s reaches the point I_y from Figure 1. Here is the blind spot of the output response on asset price fluctuations: the interest rate response on inflation and asset prices chancel out exactly. An additional increase in interest rates again translates into more volatility in output, which – through the marginal cost channel – further raises inflation volatility.

Figure 7 shows the phase diagrams for different values of ϕ_s. The left panel shows how a reduction of the dynamic feedback translates to the dynamics of the share of fundamentalists. For the case without any asset price targeting, the share of fundamentalists is zero for most times. As prices reach the peak-amplitude, all agents become fundamentalists and prices collapse. The right panel shows how the fraction of optimists evolves. The higher prices are, the more traders are optimistic, which reflects in higher expectations on asset prices, which again drives up their level. At the peak, the fraction tumbles when prices collapse. With moderate/strong leaning against the wind, the reduction in expectations feedback reflects in more moderate cycles. At all times, a positive share of fundamentalists stabilize the market, while the periodic dynamics are mirrored in the periodic fluctuations in the share of optimistic agents.

The two bottom panels of Figure 9 picture the cases of $\phi_s = 0.2$ and $\phi_s = 0.5$. While in the middle panel, financial cycles are dampened, the output response to asset prices is already negative through the real-rate effect. For $\phi_s = 0.5$ the amplitude of the financial cycles is much reduced and close to 2%, but the spillover to output and inflation is rather extreme. This confirms the findings from the previous sections: while asset price targeting can indeed mitigate financial cycles, the respective spillover effects of such policy to inflation and output are highly destructive.

6 Concluding remarks

This paper studies the role of a monetary policy that leans against asset prices in a model in which asset markets are governed by behavioral speculation. Credit frictions create a channel for spillover effects from asset prices to the macroeconomy. I confirm previous findings that emphasize that a causal feedback between asset prices and real activity in combination with speculation in the asset market can help to replicate key-moments of the data on inflation, output and asset prices.

My model does not provide a motive for the inclusion of asset prices themselves into welfare considerations, relevant for the welfare of households is only the impact of asset prices on output and inflation. On this backdrop, I find that the role of a policy that targets asset prices is limited. Targeting asset prices can indeed mitigate the extent to which asset markets are driven by non-fundamental beliefs, but at the same time, such policy amplifies their transmission to the macroeconomy. This argument favors a policy that does not target asset prices, and suggests that any general policy that decouples the real economy from asset markets is worth a detailed study.

I further study the role of such policy in the context of severe endogenous financial crisis that arise from overheated financial markets. Here, the effect of dampening the feedback from monetary policy to asset markets is stronger, which suggests that carefully leaning against the wind may be advantageous if financial markets are severely overheated. However, the scope of such policy is again bounded narrowly by its potential to amplify the transmission of financial crisis to the real economy. Additionally, it remains unclear how a central bank can safely identify whether asset markets are overheated or not.

Lastly, I find that endogenous financial cycles and crisis can be triggered by macroeconomic events. While it may not be favorable to dampen the effects of a series of positive shocks on asset prices, it is important to note that expansionary monetary surprises can also trigger asset price bubbles and can hence comprise hazard to the economy.

Independently of the quantitative results provided in this paper, my analysis suggests that policy institutions may be well-advised to handle tools like asset price targeting with care since such instruments might add a structural link between asset prices and macroeconomic aggregates. An additional link implies the risk of other unforeseeable complications, independently of how closely asset prices and real activity are connected. This is particularly true because asset prices impact solely through signaling effects. This however motivates other macroprudential policies that potentially restricts the degree of speculation or reduces speculative profits in financial markets (i.e. policies such as short-selling constraints or leverage requirements), and indicates that such policy could contribute to overall economic stability. More research in this field is needed. My findings also suggest that neither asset prices nor indices on asset prices are good economic indicators (e.g. credibility, evaluation of competitors). Practitioners should be aware that for asset prices and real activity, causality might run in both directions.

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Appendix A Entrepreneurs' optimization problem

This section follows Bernanke et al. (1999) closely, but instead of assuming idiosyncratic stochastic productivity of capital, I assume idiosyncratic risk in labor productivity. I further assume the contract to be defined in real terms with $Q_{t+1} = R_{t+1}\frac{P_t}{P_{t+1}}$. Firm j's ex post gross return on one unit of labor, ω_j, is i.i.d. across time with a continuous and at least once-differentiable CDF $F(\omega)$ over a non-negative support and with an expected value of 1. I assume that the hazard rate $h(\omega) = \frac{dF(\omega)}{1-F(\omega)}$ is restricted to $h(\omega) = \frac{\partial(\omega h(\omega))}{\partial\omega} > 0$. The optimal loan contract between firms and financial intermediaries is then defined by a gross non-default loan rate, $Z_{j,t+1}$, and a threshold value $\bar{\omega}_{j,t}$ on the idiosyncratic shock $\omega_{j,t}$. For values of the idiosyncratic shock greater or equal than $\omega_{j,t}$, the entrepreneur will be able to repay the loan, otherwise he will default. $\bar{\omega}_{j,t}$ is then defined by

$$\bar{\omega}_{j,t} Q^H_{t+1} H_{j,t} = Z_{j,t+1} B_{j,t}.$$

Dropping firms' subscripts, as in Bernanke et al. (1999) the optimal contract loan contract must then satisfy

$$\left\{ [1 - F(\bar{\omega}_t)]\,\bar{\omega}_t + (1-\mu)\int_0^{\bar{\omega}_t} \omega dF(\omega) \right\} H_t/X_t = Q_{t+1}(W_t H_t - N_t),$$

and the expected return to the wholesaler is (dropping time-subscript of ω_t for better readability)

$$E\left\{ \int_{\bar{\omega}}^{\infty} \omega dF(\omega) - (1 - F(\bar{\omega}))\bar{\omega} \right\} H_t/X_t.$$

Given constant returns to scale, the cutoff $\bar{\omega}$ determines the division of expected gross profits H_t/X_t between borrower and lender. Let me define

$$\mathfrak{F}(\bar{\omega}) = \int_0^{\bar{\omega}} \omega f(\omega) d\omega - \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega$$

to be the expected gross share of profits going to the lender with $\mathfrak{F}'(\bar{\omega}) = 1 - F(\bar{\omega})$ and $\mathfrak{F}''(\bar{\omega}) = -f(\bar{\omega})$. This implies strict concavity in the cutoff value. I define similarly the expected monitoring costs as

$$\mu \mathfrak{G}(\bar{\omega}) = \mu \int_0^{\bar{\omega}} f(\omega) d\omega,$$

with $\mu \mathfrak{G}'(\bar{\omega}) = \mu \omega f(\omega)$. See BGG for the proof that the following result is a non-rationing outcome. The resellers problem of choosing the optimal equity can be solved by maximizing the discounted sum of profits over equity, or by maximizing return on investment and including investment as part of the optimization problem.

Thus,

$$\begin{aligned}\max_{\{H_t\},\{\bar{\omega}_t\},\{N_t\},\{\lambda_t\}} E_t \sum_{s=t}^{\infty} N_t^{-1} \prod_{l=t}^{s} Q_{l+1}^{-1} \left[(1-\mathfrak{F}(\bar{\omega}_s))\, H_s/X_s - N_{s+1} \right] \\ -\lambda_s \left(\left[\mathfrak{F}(\bar{\omega}_t) - \mu\mathfrak{G}(\bar{\omega})\right] H_t/X_t - Q_{t+1}(W_t H_t - N_t) \right).\end{aligned} \tag{A.1}$$

The first-order conditions for this problem can be written as

$$\begin{aligned}
H &: (1-\mathfrak{F}(\bar{\omega}_t))\,(X_t N_t Q_{t+1})^{-1} - \lambda_t \left(\left[\mathfrak{F}(\bar{\omega}_t) - \mu\mathfrak{G}(\bar{\omega})\right]/X_t - Q_{t+1}W_t \right) &&= 0 \\
\bar{\omega} &: \mathfrak{F}'(\bar{\omega}_s)(N_t Q_{t+1})^{-1} - \lambda_t \left[\mathfrak{F}'(\bar{\omega}_t) - \mu\mathfrak{G}'(\bar{\omega})\right] &&= 0 \\
N &: -\frac{S_t}{Q_{t+1}N_t^2} - Q_{t+1}\lambda_t &&= 0 \\
\lambda &: \left[\mathfrak{F}(\bar{\omega}_t) - \mu\mathfrak{G}(\bar{\omega})\right] H_t/X_t - Q_{t+1}(W_t H_t - N_t) &&= 0
\end{aligned}$$

Combining the first three conditions implies a connection between the optimal choice of labor, prices and asset prices. Using the optimality condition for the cutoff value $\bar{\omega}_t$ and rearranging yields

$$\frac{\mathfrak{F}'(\bar{\omega}_t)}{\mathfrak{F}'(\bar{\omega}_t) - \mu\mathfrak{G}'(\bar{\omega}_t)} = \frac{S_t}{Q_{t+1}N_t}$$

where the LHS can be written as a function $\rho(\bar{\omega})$. BGG show that under reasonable assumptions $\rho(\bar{\omega})$ is a mapping from $\bar{\omega}$ to $\mathbb{R}^+$. The inverse of $\rho(\cdot)$ can be used to establish that the premium payed on external funds depends on the return payed on internal funds. As noted in the main body, this is intuitive since the marginal costs of external and internal finance need to be equal. Likewise the risk premium on external funds can be defined to be a function of the leverage ratio (if $N_t = W_t H_t$, the premium is obviously one), which establishes the relationship in the main body.

Appendix B Solving for the rational expectations equilibrium

The System in 26 reads as

$$\underbrace{\begin{bmatrix} 1-\kappa\phi_\pi & -\kappa\psi & \kappa(\nu-\phi_s) \\ \phi_\pi & 1 & \phi_s \\ \phi_\pi & 0 & 1+\phi_s \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} \pi_t \\ y_t \\ s_t \end{bmatrix}}_{\mathbf{x}_t} = \underbrace{\begin{bmatrix} \beta-\kappa & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1-\beta & \beta \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} E_t\pi_{t+1} \\ E_t y_{t+1} \\ E_t s_{t+1} \end{bmatrix}}_{E_t\mathbf{x}_{t+1}} + \mathbf{Q} \underbrace{\begin{bmatrix} v_t^\pi \\ v_t^y \\ 0 \end{bmatrix}}_{\boldsymbol{v}_t}. \tag{B.1}$$

For a sensible range of parameter values the Blanchard-Kahn-Conditions are satisfied. The closest bound for which eigenvalues cross the unit circle is if $\phi_s < -0.305$. Exchanging shocks $\mathbf{v}_t$ by perceived shocks $\tilde{\mathbf{v}}_t$, in expectations it has to hold that

$$\tilde{a}_{t+1} = \rho_a \tilde{a}_t^\pi \tag{B.2}$$

$$\tilde{d}_{t+1} = \rho_d \tilde{d}_t^y. \tag{B.3}$$

Using this form, the PLM can be written by using the system of equations (21) – (25) and by bringing all expectations to the LHS:

$$\beta E_t\pi_{t+1} = \pi_t + \kappa x_t - \tilde{a}_t \tag{B.4}$$

$$E_t\pi_{t+1} + E_t y_{t+1} = r_{t+1} + y_t - \tilde{d}_t \tag{B.5}$$

$$E_t\pi_{t+1} = x_t + \psi y_t + r_{t+1} - \nu s_t - \frac{1+\eta}{1+\tilde{nu}} a_t \tag{B.6}$$

$$0 = -r_{t+1} + \phi_\pi \pi_t + \phi_s s_t \tag{B.7}$$

$$E_t\pi_{t+1} + \beta E_t s_{t+1} = s_t + r_{t+1} \tag{B.8}$$

$$\tilde{a}_{t+1} = \rho_a \tilde{a}_t \tag{B.9}$$

$$\tilde{d}_{t+1} = \rho_d \tilde{d}_t \tag{B.10}$$

Using (B.6) and (B.7) to substitute for r_{t+1} and x_t and rewriting as a matrix yields the System (28). Express this system as

$$\tilde{\mathbf{P}} E_t \tilde{\mathbf{x}}_{\mathbf{t+1}} = \mathbf{M} \tilde{\mathbf{x}}_{\mathbf{t}}.$$

$\tilde{\mathbf{N}} = \tilde{\mathbf{P}}^{-\mathbf{1}} \tilde{\mathbf{M}}$ is the 5×5 matrix which summarizes the dynamics of the perceived law of motion of rational agents. I use eigenvector/eigenvalue decomposition to obtain $\mathbf{\Gamma \Lambda \Gamma^{-1}} = \tilde{\mathbf{N}}$, where $\mathbf{\Lambda}$ is the diagonal matrix $\text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_5)$ of the eigenvalues of $\tilde{\mathbf{N}}$ ordered by size (smallest in modulus first) and $\mathbf{\Gamma}$ the associated eigenvectors, with columns ordered in the same fashion. The expectation system can then be rewritten as

$$\mathbf{\Gamma}^{-\mathbf{1}} E_t \mathbf{x}_{\mathbf{t+1}} = \mathbf{\Lambda \Gamma^{-1}} \mathbf{x}_{\mathbf{t}}.$$

Denote the sub-matrix of $\mathbf{\Lambda}$ that only contains unstable eigenvalues as $\mathbf{\Lambda_u}$, and the associated eigenvectors as $\mathbf{\Gamma_u}^{-\mathbf{1}}$. In order to be consistent with the transversallity condition it must hold that $\mathbf{\Gamma_u^{-1}} E_t \mathbf{x}_{\mathbf{t+1}} = \mathbf{0}$. Using this fact, solve for $E_t \mathbf{x}_{\mathbf{t+1}}$ by

$$E_t \mathbf{x}_{\mathbf{t+1}} = \mathbf{\Gamma}_{\mathbf{u},\mathbf{1:3}}^{-\mathbf{1}} \mathbf{\Gamma}_{\mathbf{u},\mathbf{4:5}} E_t \tilde{\mathbf{v}}_{\mathbf{t+1}} = \mathbf{\Gamma}_{\mathbf{u},\mathbf{1:3}}^{-\mathbf{1}} \mathbf{\Gamma}_{\mathbf{u},\mathbf{4:5}} \boldsymbol{\rho} \tilde{\mathbf{v}}_{\mathbf{t}}.$$

Note that the requirement that $\mathbf{\Gamma}_{\mathbf{u},\mathbf{1:3}}$ is invertible implies the Kuhn-Tucker condition, imposing that $\mathbf{\Gamma}_{\mathbf{u},\mathbf{1:3}}$ is a square matrix with full rank. This means that the number of forward looking variables has to equal the number n_u of unstable eigenvalues $\lambda_i > 1$ of $\tilde{\mathbf{N}}^{-\mathbf{1}}$. Let me define $\bar{\mathbf{\Omega}} = \mathbf{\Gamma}_{\mathbf{u},\mathbf{1:3}}^{-\mathbf{1}} \mathbf{\Gamma}_{\mathbf{u},\mathbf{4:5}}$. The solution from the main body is then $\bar{\mathbf{\Omega}}_{\mathbf{1:2},\mathbf{1:2}}$.[22]

[22]This implies that rational agents do not take asset prices into account when forming expectations. However, a more general approach including the adjustment for measurement errors of projecting three endogenous variables on two shock terms (stochastic indeterminacy) approximately lead to the same Ω. Assuming that agents use OLS to regress $\mathbf{x_t}$ on $\tilde{\mathbf{v}}_{\mathbf{t}}$, $\tilde{\mathbf{\Omega}} = \left(\bar{\mathbf{\Omega}}^{\mathbf{T}} \bar{\mathbf{\Omega}}\right) \bar{\mathbf{\Omega}}^{\mathbf{T}} \in \mathbb{R}^{3\times 2}$ and $\tilde{\mathbf{\Omega}}_{\mathbf{1:2},\mathbf{1:2}} \approx \mathbf{\Omega}$.

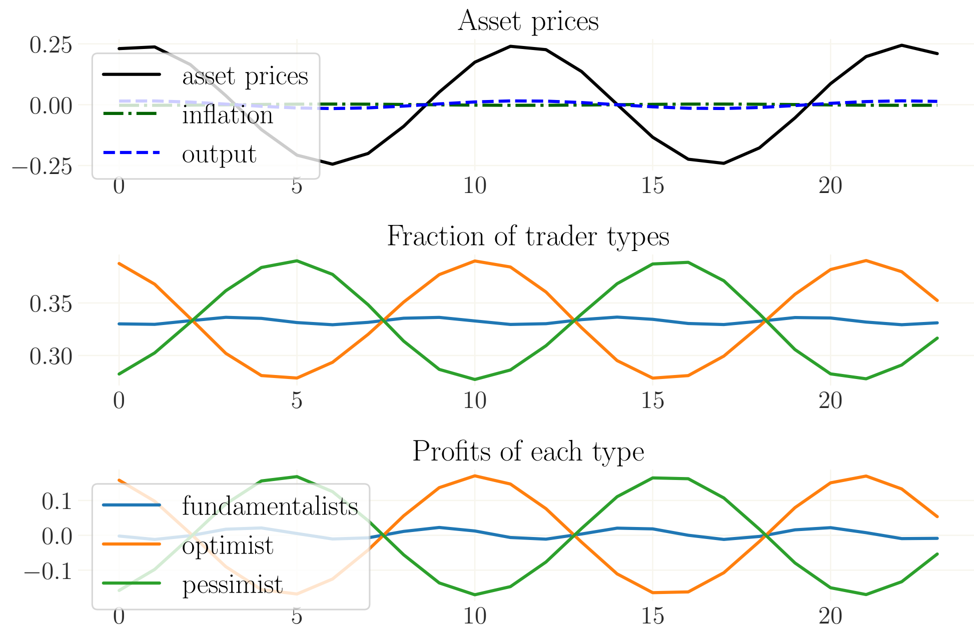

Figure C.10: *Top panel:* time series of asset prices, inflation and output at the estimated parameters (s. Table 2) in the absence of additional shocks. *Middle panel:* switching dynamics of the different trader types. *Bottom panel:* the cyclical component is reflected in changes in profitability of each trader type.

Appendix C Additional figures and bifurcation diagrams

Figures C.11 and C.12 show the bifurcation diagram for γ_s. An increase in trend extrapolation has a similar effect as an increase in bias. The amplitude of cycles increases with γ_s, for parameter values larger than 1.2, the trajectory again approaches a homoclinic orbit. For higher values the system exhibits explosive dynamics. An increase in the behavioral parameters γ_b and γ_s hence implies two effects. The *quantitative* aspect is, that the amplitude increases with increases in the parameters. The *qualitative* aspect is that the type of dynamics can also change. Figure C.10 shows the time series of trader types and profits for each of the times for the limit cycle/financial cycle.

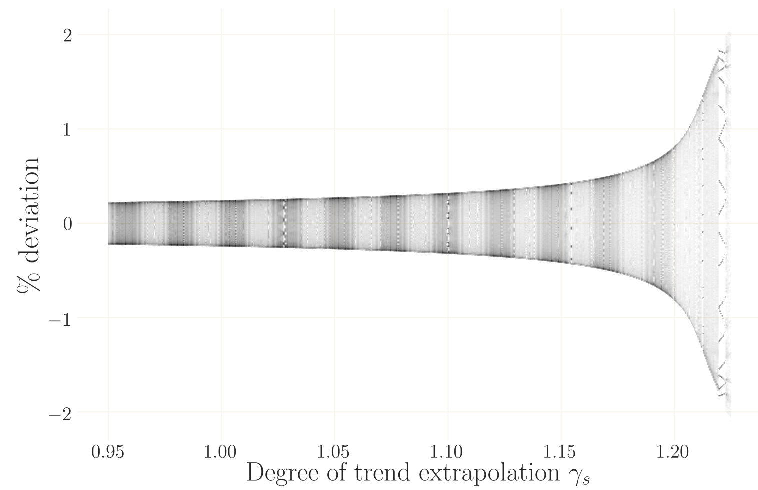

Figure C.11: Bifurcation diagram of inflation and output w.r.t. γ_s. All other parameters as in Table 2.

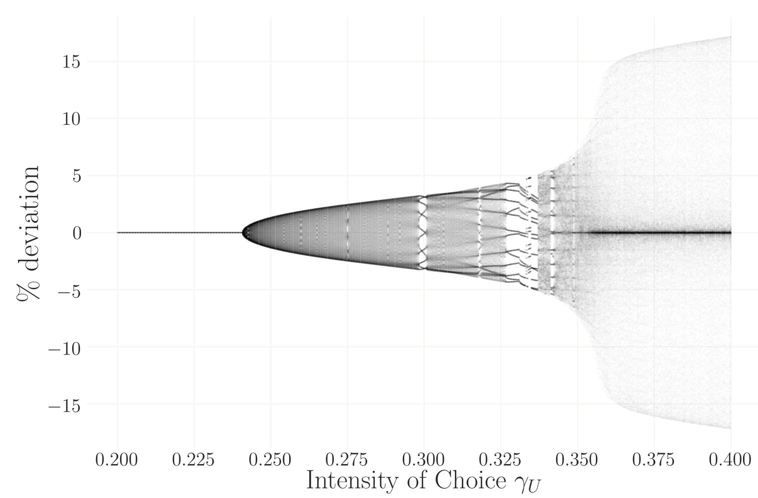

Figure C.12: Bifurcation diagram of inflation and output w.r.t. γ_U. All other parameters as in Table 2.