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Persuading an Informed Committee

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Abstract

A biased sender seeks to persuade a committee to vote for a proposal by providing public information about its quality. Each voter has some private information about the proposal's quality. We characterize the sender-optimal disclosure policy under unanimity rule when the sender can versus cannot ask voters for a report about their private information. The sender can only profit from asking agents about their private signals when the private information is sufficiently accurate. For all smaller accuracy levels, a sender who cannot elicit the private information is equally well off.

Keywords: Voting; Bayesian Persuasion; Strategic Voting; Unanimity

1 Introduction

In voting or collective decision making, the persuasion of decision makers through a biased party plays a crucial role. To which extent a biased party can persuade decision makers might depend on how much decision-relevant knowledge they already possess. Consider for example a CEO who tries to convince a board of directors to vote for a new proposal. While the CEO wishes to always implement the proposal to improve short-term firm performance, directors only want to approve the proposal if it increases long-term performance. If directors already have some private knowledge about the long-term effects of the proposal, what is the most promising way to convince them to vote for the proposal? This is the question of this paper.

In our model an information designer requires a unanimous approval of a group of voters to implement a proposal. Depending on the proposal's unknown binary quality, voters either like or

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dislike the proposal. If the quality was known, all voters would agree on the optimal decision. In contrast, the information designer is biased in that she always wants the proposal to be implemented, irrespective of its quality. Each voter receives a private signal about the proposal having a high or low quality and is, according to his private signal, either optimistic or pessimistic. The information designer chooses a disclosure policy: she sends a public recommendation that is correlated with the quality of the proposal. The term *public* means that the information designer cannot make different recommendations to different voters. After having received the recommendation of the information designer, each voter decides on whether to vote for or against the proposal based on his updated belief. Although voters are aware of the information designer's interest in the proposal, they might nevertheless want to follow her recommendation. This is because the recommendation is based on the true quality of the proposal.

The main contribution of this paper is to unveil the extent to which an information designer can persuade informed voters by choosing the optimal disclosure policy. We characterize when the private information of voters restricts the information designer in her scope for persuasion.

In our benchmark case we consider an *omniscient* information designer who can observe the private signal realizations of all voters. We show that the omniscient information designer recommends to vote for the proposal with probability one when the proposal is of high quality. In the state where the proposal is of low quality, she uses a threshold policy: she recommends the proposal with probability one for any number of optimists above a certain cutoff, and recommends the status quo with certainty below the cutoff. The cutoff is such that a pessimistic voter is indifferent between the proposal and the status quo after the recommendation to vote for the proposal.

Next, we consider an *eliciting* information designer who cannot observe the private information of voters but can ask them for reports about their signal realizations. We show that the eliciting information designer cannot implement the optimal disclosure policy from the omniscient benchmark case and is always worse off compared to the omniscient information designer. This is caused by the optimists having a profitable deviation through misreporting to be pessimists. As a consequence, the eliciting information designer has to give sufficient incentives for truthful reporting by providing voters with more information. This limits the scope of the information designer for persuasion. If the probability of receiving the correct signal is below a lower threshold, the eliciting information designer always recommends to vote for the proposal in the state where voters prefer the proposal. In the state in which voters want to implement the status quo, the probability with which she recommends the proposal is stochastic and decreasing in the accuracy of the private information of voters. This optimal policy of the information designer is equivalent to maximizing the probability of a pessimist to vote for the proposal. In contrast, if the probability of receiving the correct signal is above an upper threshold, the information designer's optimal policy is to maximize the probability of an optimist to vote for the proposal.

Finally, we consider a non-eliciting information designer who can neither observe the signal realizations of voters nor ask voters for reports about their private information. If the probability of receiving the correct signal is below the same lower threshold as in the eliciting case, the optimal disclosure policy of an eliciting and of a non-eliciting information designer are equivalent. Thus, an information designer cannot profit from the ability to ask voters for their private information if the accuracy of voter's private information is not sufficiently high.

We find that voters are better off in the presence of a biased information designer compared to the situation in which they have to decide under unanimity rule only based on their private exogenous information as in Feddersen and Pesendorfer (1998).

2 Related Literature

Our paper belongs to the rapidly growing literature on information design (see Rayo and Segal, 2010; Kamenica and Gentzkow, 2011). While in Kamenica and Gentzkow (2011) there is only one agent that is uninformed, we consider persuasion of a committee of agents that is informed.

Amongst the vast emerging literature on information design, the two strands bearing most resemblance to our paper are first, private information on the receiver's side, and second, persuasion of multiple receivers. Multiple papers extend information design to a setting with *many receivers*. In these papers (Taneva, 2016; Bardhi and Guo, 2018; Alonso and Câmara, 2016; Wang, 2015; Chan et al., 2016; Heese and Lauermann, 2017), agents are aware of the payoff types of each other. There is no uncertainty about the committee constellation, and voters possess no private information about the payoff-relevant state of the world. A crucial difference of these papers to our approach is that in our model, the payoff type of each committee member bears information about the state of the world and is private information. If the committee constellation was known, all voters in our model would agree on the same election outcome.

Taneva (2016) extends the approach of a Bayes correlated equilibrium from Bergemann and Morris (2016a) to a class of Bayesian Persuasion problems with multiple receivers. She fully characterizes a binary-binary[1] model with two receivers and shows that the optimal information structure involves public signals or correlated private signals (not conditionally independent signals). Alonso and Câmara (2016) analyze how a biased sender can influence an uninformed heterogeneous com-

[1]Binary states and binary actions.

mittee of voters with a public signal, as in our model. They elicit the scope for persuasion under different voting rules, and show when agents are worse off. Chan et al. (2016) consider persuading a heterogeneous committee under the restriction to minimal winning coalitions. Wang (2015) compares private persuasion (under the restriction of conditionally independent signals) to public persuasion in collective decision making. She shows, that public persuasion performs weakly better and reveals less information than private persuasion. The closest related to our paper is Bardhi and Guo (2018). They analyze persuasion of a heterogeneous committee, and study a unanimous voting rule. They consider two persuasion regimes: general persuasion (conditional on everybody's payoff type), and individual persuasion (conditional only on own payoff type). Persuasion is private in their model: each agent does not see the messages sent to voters, neither under general nor under individual persuasion. They show that under unanimity, a restriction to a public or private persuasion regime is without loss under some assumptions. Heese and Lauermann (2017) consider persuasion of a heterogeneous committee of voters. They show that the information designer can almost surely guarantee the implementation of her preferred outcome in the limit, as the size of the committee grows sufficiently large.

Amongst the papers considering *private information* on the side of the receiver are Kolotilin et al. (2017), Kolotilin (2018), Bergemann and Morris (2016b) and Bobkova (2017). Kolotilin et al. (2017) study persuasion of one privately informed receiver, who is privately informed about his payoff type. They show that eliciting persuasion is equivalent to non-eliciting persuasion under some conditions[2]. Bergemann et al. (2018) consider a similar environment as Kolotilin et al. (2017) but add monetary transfers, which we do not allow in our framework. Kolotilin (2018) considers an information designer who tries to persuade an informed receiver. Persuasion is non-eliciting: the information designer cannot ask the receiver for his type prior to her information disclosure. Bobkova (2017) considers a stream of short-lived and privately informed buyers, that an information designer (seller) seeks to persuade into buying her product. The seller is restricted in her ability to construct experiments, and has to rely on the private information of previous receivers, that she has to elicit truthfully.

To the best of our knowledge, our framework is the first to introduce private information into persuasion of a group. We analyze the problem of an information designer when she first has to squeeze the private information out of multiple agents before she can condition her disclosure policy on it.

The idea of *omniscient* persuasion and *private* persuasion of one privately informed receiver can be found in Bergemann and Morris (2016b). We extend their discussion by providing a comparison

[2]Kolotilin et al. (2017) refer to eliciting and non-eliciting persuasion as public versus private persuasion. See Bergemann and Morris (2018) for a unified terminology, that we follow in this paper.

of the cases in which the information designer is omniscient and in which the information designer has to first elicit the private signals from multiple agents. A unified perspective of the existing literature on Bayesian Persuasion and information design can be found in Bergemann and Morris (2018).

Finally, our paper relates to the literature on information aggregation in strategic voting, following Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998). Feddersen and Pesendorfer (1998) show that when voters vote strategically, voting truthfully according to one's own private information is not an equilibrium. Voters condition their strategy on pivotality events, and a unanimous voting rule is a 'uniquely bad' voting rule: it implements the inferior inefficient outcome with a higher probability than any other majority rule. In the model of Feddersen and Pesendorfer (1998), voters have to make their decision based on their private exogenous information only. In our model, we allow the designer to provide further correlated information to all agents, conditional on the true state of the world.

The paper is organized as follows. First, we introduce our model in section 2. In Section 3.1, we first analyze the benchmark case of an omniscient information designer and characterize her optimal disclosure policy. In the subsequent section, we analyze information design with elicitation and show that an eliciting information designer cannot implement the optimal disclosure policy from the omniscient benchmark case. We characterize the optimal disclosure of an eliciting information designer and establish two equivalence results. In section 3.3 we deal with the analysis of a non-eliciting information and prove the equivalence of optimal disclosure policies of an eliciting and a non-eliciting information designer if the accuracy of voters' signals is below some threshold. The last section 3.4 deals with restricted information design. Unlike the unrestricted information designer, the restricted eliciting information designer can achieve the same expected payoff as in the omniscient benchmark case.

3 Model

There are three voters which have to decide whether to vote for a proposal or for a status quo. An information designer tries to influence voters to vote for the proposal. In the following we use information designer and sender synonymously. When a voter i votes for the proposal we write $a_i = 1$, and $a_i = 0$ for the status quo. When the outcome of the ballot is the proposal we write $a = 1$, and $a = 0$ when the status quo is chosen. For example, under unanimity rule the outcome is $a = 1$ if $a_i = 1$ for all i.

Whether a voter likes or dislikes the proposal depends on an uncertain state of the world $\theta \in$

$\{B, G\}$, where $\Pr(\theta = G) = \frac{1}{2}$. Voters have the following utility function:

$$u_i(a, \theta) = \begin{cases} 1_{\{\theta = G\}} - \frac{1}{2} & \text{if } a = 1 \\ 0 & \text{if } a = 0 \end{cases}$$

Hence, all voters agree on the optimal decision if the state was known: when $\theta = G$ all voters want to implement the proposal, while when $\theta = B$, all agree on the status quo. In contrast, the sender always prefers the proposal over the status quo independent of the state of the world. The sender's utility function is given by $u_S(a) = a$.

Each voter receives a private signal $z_i \in \{b, g\}$ that is correlated with the true state of the world in the following way: $\Pr[z_i = g|\theta = G] = \Pr[z_i = b|\theta = B] = p \in (\frac{1}{2}, 1)$. A voter i with a signal $z_i = g$ (referred to as a *good signal*) is more optimistic about the state of the world being $\theta = G$ than a voter with signal $z_i = b$ (referred to as a *bad signal*). Likewise, a voter with bad signal $z_i = b$ considers $\theta = B$ more likely.[3]

Denote the set of signal realizations by $Z = \{g, b\}^3$ with a typical element $z = (z_1, z_2, z_3) \in Z$. Let $k(z)$ be the number of g-signals in a typical signal realization z. By $z_{-i} \in Z_{-i}$ we refer to the signals of all voters except voter i, where Z_{-i} is the set of all signal realizations except voter i's signal. In the following we use the shortcut k to refer to $k(z)$ and k_{-i} to refer to $k(z_{-i})$.

4 Omniscient Information Design

We first analyze the benchmark case in which the sender is omniscient, i.e., observing each voter's private signal. The sender's problem is then to choose a disclosure policy $d : \Theta \times Z \to \triangle(R)$ with public recommendations $r \in R$ to maximize the probability of the event that all voters vote for the proposal. We restrict the analysis to anonymous disclosure policies, which take only the number of good and bad signals into account and not which voter has which signal.

Assumption 1. *The sender's disclosure policy is anonymous, i.e., the probability of sending any recommendation is the same for all z, z' with $k(z) = k(z')$.*

This assumption allows us to restrict attention to disclosure policies which only condition on the state of the world and on the number of good signal in the population.

The next assumption specifies how a voter behaves under indifference.[4]

[3]For notational convenience we will sometimes use g_i and b_i respectively as a short cut for voter i having received signal $z_i = g$ and $z_i = b$ respectively.

[4]Note the difference to the sender-preferred equilibrium in Kamenica and Gentzkow (2011): they assumed that if indifferent, their agent votes for the sender-preferred outcome, in our case the proposal. We are interested in partial

Assumption 2. *If a voter is indifferent between his actions, he follows the recommendation of the sender.*

The following proposition says that we can restrict the sender to only two recommendations $r \in \{\hat{0}, \hat{1}\}$ without loss of generality for optimality. These two recommendations are direct voting recommendations, $\hat{1}$ in favor of the proposal, and $\hat{0}$ in favor of the status quo.

Proposition 1. *Under unanimity, it is without loss of generality to restrict the message space of the omniscient sender to* $R = \{\hat{0}, \hat{1}\}$.

All omitted proofs are in the appendix. The disclosure policy of the sender is:

$$d : \Theta \times \{0, 1, 2, 3\} \to \triangle\{\hat{0}, \hat{1}\}. \tag{1}$$

That is, we are looking for a vector with 8 components $\{d[\hat{1}|\theta, k]\}_{k \in \{0,1,2,3\}, \theta \in \{B,G\}} \in [0, 1]^8$. After the sender sends her recommendation, voters have to update their belief about θ and only follow the sender's recommendation if this yields a higher expected utility than disobeying.

Since the decision has to be made under unanimity, after recommendation $\hat{1}$ a voter is pivotal with probability one and after recommendation $\hat{0}$ he is never pivotal. This is because in equilibrium after $r = \hat{1}$ all voters follow the recommendation $\hat{1}$ which is why from the perspective of an individual voter his vote determines the outcome. Similarly, a voter will never be pivotal after $r = \hat{0}$ because all other voters already voted against the proposal which in turn makes one single vote irrelevant under unanimity. As a consequence, a voter will always follow the recommendation $\hat{0}$ and follow the recommendation $\hat{1}$ if his obedience constraint holds:

$$\begin{aligned} & \Pr(\theta = G|\hat{1}, z_i) \geq \frac{1}{2} && (OB^{\hat{1}}_{z_i}) \\ \Leftrightarrow \quad & \Pr(\theta = G|z_i) \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, k_{-i} + k(z_i)] \Pr(k_{-i}|\theta = G) \\ \geq \quad & \Pr(\theta = B|z_i) \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = B, k_{-i} + k(z_i)] \Pr(k_{-i}|\theta = B) \end{aligned}$$

implementation, and such one-sided tie-breaking rule pro proposal would be with loss of generality in our setting. We will see that in the optimum the sender sometimes wants voters to vote for the status quo if indifferent to achieve the highest outcome. This is driven by pivotality considerations and does not arise in Kamenica and Gentzkow (2011).

The omniscient sender's maximization problem is then given by:

$$\max_d \Pr(a=1) = \max_d \Pr(\hat{1}) = \sum_{\theta\in\{B,G\}} \sum_{k=0}^{3} d[\hat{1}|\theta,k] \Pr(k|\theta) \Pr(\theta)$$

$$s.t. \quad 0 \le d[r|\theta,k] \le 1, \quad \forall r \in \{\hat{0},\hat{1}\},\ \theta \in \{B,G\},\ k \in \{0,1,2,3\} \tag{2}$$

$$d[\hat{0}|\theta,k] + d[\hat{1}|\theta,k] = 1, \quad \forall \theta \in \{B,G\},\ k \in \{0,1,2,3\} \tag{3}$$

$$\Pr(\theta = G|\hat{1}, z_i) - \frac{1}{2} \ge 0, \qquad \forall z_i \in \{b,g\} \tag{$OB^{\hat{1}}_{z_i}$}$$

The following lemma states that for the sender it is always optimal to send the recommendation to vote for the proposal when $\theta = G$.

Lemma 1. *In any optimal disclosure policy,* $d[\hat{1}|\theta = G, k] = 1$ *for all* k*.*

To give some intuition for Lemma 1, notice that for $\theta = G$ voters agree on the proposal being the more appropriate choice, independent of their private signal. Hence, the sender does not have to convince voters so that she can simply send $\hat{1}$ for $\theta = G$. Assume that the conjecture is false. Then, the sender could increase the probability of implementing the proposal and at the same time relax the voters' obedience constraints by simply increasing $d[\hat{1}|\theta = G, k]$ for those k for which $d[\hat{1}|\theta = G, k] \neq 1$.

The probability of sending a recommendation $\hat{1}$ for the sender is:

$$\Pr(\hat{1}) = 0.5 \Pr(\hat{1}|\theta = B) + 0.5 \underbrace{\Pr(\hat{1}|\theta = G)}_{=1} \tag{4}$$

$$= 0.5 \sum_{k=0}^{3} d[\hat{1}|k, \theta = B] \Pr(k|\theta = B) + 0.5 \tag{5}$$

Now, consider the obedience constraint for the g-type. It is easy to see that it is always satisfied if Lemma 1 holds.

Lemma 2. *The obedience constraint of the* g*-type is satisfied in any disclosure policy in which* $d[\hat{1}|\theta = G, k] = 1$ *for all* k*.*

Proof. The obedience constraint of the g-type is:

$$0.5p \ge 0.5(1-p) \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = B, k_{-i}+1] \Pr(k_{-i}|\theta = B) \tag{6}$$

Note that due to feasibility of the disclosure policy, $d[\hat{1}|\theta,k] \leq 1$ for all θ and all k. Thus,

$$\sum_{k_{-i}=0}^{2} \underbrace{d[\hat{1}|\theta=B,k_{-i}+1]}_{\leq 1} \Pr(k_{-i}|\theta=B) \tag{7}$$

$$\leq \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|\theta=B) = 1$$

Using this in the obedience constraint, we see that it is always satisfied as $0.5p \geq 0.5(1-p) \geq RHS$ always holds. ■

The next lemma states the disclosure policy if all voters have a g-signal:

Lemma 3. *In any optimal disclosure policy d, it holds that $d[\hat{1}|\theta=B,k=3]=1$.*

Proof. The probability $d[\hat{1}|\theta=B,k=3]$ does not show up in the disclosure policy of the b-type, as it only applies when all voters have a g-signal. Therefore, it has no effect on the obedience of the b-type. For the g-type, by Lemma 2 the obedience constraint of the g-type holds in any disclosure policy that sends recommendation $\hat{1}$ whenever the state is $\theta=G$. Therefore, setting $d[\hat{1}|\theta=B,k=3]$ increases the probability of the proposal being implemented without harming any obedience constraints. ■

Using the above findings, the maximization problem of the omniscient unrestricted designer becomes:

$$\max_{d} \quad \frac{1}{2}\sum_{k=0}^{3} d[\hat{1}|\theta=B,k]\Pr(k|\theta=B) + \frac{1}{2}\Pr(k=3|\theta=B) + \frac{1}{2} \tag{8}$$

$$d[\hat{0}|\theta,k] + d[\hat{1}|\theta,k] = 1, \quad \forall\theta\in\{B,G\},\ k\in\{0,1,2,3\} \tag{9}$$

$$s.t. \quad (1-p) \geq p\sum_{k_{-i}=0}^{2} d[\hat{1}|\theta=B,k_{-i})]\Pr(k_{-i}|\theta=B) \tag{$OB_b^{\hat{1}}$}$$

By using that $p\Pr(k_{-i}|\theta=B) = \Pr(k|\theta=B)\frac{3-k}{3}$ we can rewrite $OB_b^{\hat{1}}$:

$$(1-p) \geq p\sum_{k_{-i}=0}^{2} d[\hat{1}|\theta=B,k_{-i}]\Pr(k_{-i}|\theta=B)\binom{2}{k_{-i}} \tag{10}$$

$$\Leftrightarrow \quad (1-p) \geq \sum_{k=0}^{3} d[\hat{1}|\theta=B,k]\Pr(k|\theta=B)\frac{3-k}{3}. \tag{11}$$

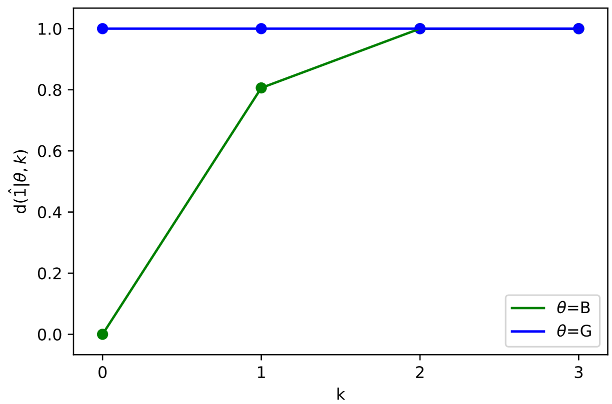

Figure 1: optimal disclosure policy for $p = 0.7$.

Increasing $d[\hat{1}|\theta = B, k]$ for any $k \in \{0, 1, 2\}$ affects differently the obedience constraint and objective function of the designer. While an increase in $d[\hat{1}|\theta = B, k]$ for any particular $k \in \{0, 1, 2\}$ is weighted by the sender with $\Pr(k|\theta = B)$, the b-type weights this increase with $\Pr(k|\theta = B)\frac{3-k}{3}$. In the terminology of the fractional knapsack[5], this means that different k have different value-weight ratios. As a consequence, it will matter for which k the sender will increase the probability of sending recommendation $\hat{1}$ until the constraint binds. The next proposition states that the optimal disclosure policy of the omniscient sender is a monotone threshold policy.

Proposition 2. *The unique optimal disclosure policy of the omniscient sender is a monotone cutoff policy with*

$$d[\hat{1}|\theta = G, k] = 1 \ \forall k, \quad d[\hat{1}|\theta = B, k] \begin{cases} = 1 & \text{if} \quad k > \tilde{k}, \\ \in [0, 1] & \text{if} \quad k = \tilde{k}, \\ = 0 & \text{if} \quad k < \tilde{k}, \end{cases} \tag{12}$$

where $\tilde{k}$ is such that $OB_b^{\hat{1}}$ binds.

Figure 1 shows the optimal policy of the omniscient sender for $p = 0.7$. If $\theta = G$, she sends with the certainty the recommendation $\hat{1}$ irrespective of the number of g-signals. If $\theta = B$, the sender uses a monotone cutoff policy, where she sends $\hat{1}$ with certainty whenever there are at least two voters with a g-signal, mixes whenever there is one voter with a g-signal, and never sends $\hat{1}$ when all have a b-signal.

[5] See appendix for the terminology.

5 Eliciting Information Design

In the previous section the sender could construct any experiment on the true state of the world and was able to see the private signal realizations of each voter. In this section we assume that the sender cannot see the private information of the voters, but can elicit it in an incentive compatible way. When the sender is eliciting, honesty constraints arise, and we have to check for double deviations: if an agent misreports, does he have a profitable deviation? After misreporting, the agent should not be strictly better of from any possible action after the misreport.

Each voter i sends a message to the sender, a report about his private signal realization: $\hat{z}_i \in \{\hat{g}, \hat{b}\}$. The complete profile of reported signals is then given by $\hat{z} \in \hat{Z}$. We employ the same notation as above, with the restriction, that now the sender conditions not on the number of g-signals in the true signal realizations z, but on the number of $\hat{g}$-reports in the reported signal realization $\hat{z}$. Hence, $k(\hat{z})$ denotes now the number of $\hat{g}$-reports in the reported signal realization $\hat{z}$.

As for the omniscient sender, only two signals suffice for the sender to achieve her highest implementable payoff.

Proposition 3. *Under unanimity, it is without loss of generality to restrict the message space of the eliciting sender to* $R = \{\hat{0}, \hat{1}\}$.

The sender commits to a disclosure policy $d : k \in \{0, 1, 2, 3\} \rightarrow \Delta\{\hat{0}, \hat{1}\}$. Let $U(z_i, \hat{z}_i, a_i(\hat{0}, z_i), a_i(\hat{1}, z_i))$ be the expected utility of a voter with signal z_i, who reports being type $\hat{z}_i$, and votes with probability $a_i(\hat{0}, z_i))$ for the proposal after recommendation $\hat{0}$, and with probability $a_i(\hat{1}, z_i))$ for the proposal after recommendation $\hat{1}$.

We refer to a disclosure policy d of an eliciting sender as *implementable* if and only if it satisfies the obedience and the honesty constraints.

The next observation establishes, that the omniscient sender is strictly better off than the eliciting sender.

Observation 1. *The optimal disclosure policy of the omniscient sender is not implementable when the sender is eliciting.*

It is straightforward that with the optimal disclosure policy in Proposition 2 the g-type will have a profitable deviation from misreporting $\hat{b}$ and following the recommendation. Let $U(z_i, \hat{z}'_i, a_i(\hat{0}, z_i), a_i(\hat{1}, z_i))$ be the expected utility of a voter with signal z_i, who reports being type $\hat{z}'_i$, and votes with probability $a_i(\hat{0}, z_i))$ for the proposal after recommendation $\hat{0}$, and with probability $a_i(\hat{1}, z_i))$ for the proposal after recommendation $\hat{1}$. If the g-type is truthful and obedient, his expected utility is

$$\begin{aligned}
U(g_i, \hat{g}_i, 0, 1) = & \Pr(\theta = G|g_i)\frac{1}{2} \\
& - \Pr(\theta = B|g_i)\frac{1}{2}\Pr(k_{-i} = 2|\theta = B) \\
& - \Pr(\theta = B|g_i)\frac{1}{2}\sum_{k_{-i}=0}^{1} d[\hat{1}|\theta = B, k_{-i}+1]\Pr(k_{-i}|\theta = B).
\end{aligned}$$

Consider the following deviation: misreport $\hat{b}$ and follow the recommendation. Then, a g-type prevent the event in which all voters have reported a g-signal, the state is $\theta = B$ and the sender sends $\hat{1}$ with probability 1. In all other states, the misreporting does not matter for the disclosure policy of the sender when $\theta = G$ because when $\theta = G$ the sender sends $\hat{1}$ with probability one for all $k \in \{0, 1, 2, 3\}$. When $\theta = B$, the g-type voter profits from misreporting since, $d[\hat{1}|\theta = B, k]$ is decreasing k. Hence, the misreporting g-type will receive recommendation $\hat{1}$ with a (weakly) smaller probability than when being honest in the unfavorable state $\theta = B$. This follows because the disclosure policy is a cutoff policy: misreporting a g-signal gets a more 'favorable' cutoff than when reporting truthfully. The voter's expected payoff when being dishonest is given by

$$\begin{aligned}
U(g_i, \hat{b}_i, 0, 1) = & \Pr(\theta = G|g)\frac{1}{2} \\
& - \Pr(\theta = B|g_i)\frac{1}{2}\sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = B, k_{-i}]\Pr(k_{-i}|\theta = B).
\end{aligned}$$

Message $\hat{1}$ is sent less frequently if $\theta = B$, hence $U(g_i, \hat{b}_i, 0, 1) > U(g_i, \hat{g}_i, 0, 1)$. The omniscient sender is strictly better of than the eliciting sender under information design. This is in line with the literature. Bergemann and Morris (2016b) show that the implementable set of equilibria is larger for an omniscient than an eliciting sender in a bank run game with one sender and one receiver. Similarly, Bobkova (2017) shows that an omniscient sender has a strictly higher probability of selling a good to a buyer when the sender is omniscient than when she is eliciting.

Since an optimal disclosure policy of the omniscient sender is not implementable for the eliciting sender, we need to solve his maximization problem by taking into account the honesty constraints. The obedience constraint of the g-type after being truthful is

$$U(g_i, \hat{g}_i, 0, 1) = \Pr(\theta = G|g_i) \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, 1 + k_{-i}] \Pr(k_{-i}|\theta = G) \qquad (OB_g^{\hat{1}})$$

$$- \Pr(\theta = B|g_i) \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = B, 1 + k_{-i}] \Pr(k_{-i}|\theta = B) \geq 0 = U(g_i, \hat{g}_i, 0, 0).$$

The obedience constraint of the b-type after being truthful is

$$U(b_i, \hat{b}_i, 0, 1) = \Pr(\theta = G|b_i) \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, k_{-i}] \Pr(k_{-i}|\theta = G) \qquad (OB_b^{\hat{1}})$$

$$- \Pr(\theta = B|b_i) \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = B, k_{-i}] \Pr(k_{-i}|\theta = B) \geq 0 = U(b_i, \hat{b}_i, 0, 0).$$

The honesty constraint of a g-type who is obedient is then given by

$$U(g_i, \hat{g}_i, 0, 1) = \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, 1 + k_{-i}] \Pr(k_{-i}|\theta = G)p \qquad (H_g)$$

$$- d[\hat{1}|\theta = B, 1 + k_{-i}] \Pr(k_{-i}|\theta = B)(1 - p)$$

$$\geq \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, k_{-i}] \Pr(k_{-i}|\theta = G)p$$

$$- d[\hat{1}|\theta = B, k_{-i}] \Pr(k_{-i}|\theta = B)(1 - p) = U(g_i, \hat{b}_i, 0, 1).$$

The honesty constraint of a b-type who is obedient is then given by

$$U(b_i, \hat{b}_i, 0, 1) = \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, k_{-i}] \Pr(k_{-i}|\theta = G)(1 - p) \qquad (H_b)$$

$$- d[\hat{1}|\theta = B, k_{-i}] \Pr(k_{-i}|\theta = B)p$$

$$\geq \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, k_{-i} + 1] \Pr(k_{-i}|\theta = G)(1 - p)$$

$$- d[\hat{1}|\theta = B, k_{-i} + 1] \Pr(k_{-i}|\theta = B)p = U(b_i, \hat{g}_i, 0, 1).$$

Note that after recommendation $r = \hat{0}$, a voter is never pivotal and hence it does not matter whether he follows or disobeys the recommendation after misreporting. That is, $U(z_i, \hat{z}_i, 1, 1) =$

$U(z_i, \hat{z}_i, 0, 1)$. If a voter is not obedient after recommendation $\hat{1}$, i.e., $a_i(\hat{1}, z_i)) = 0$, then his expected utility is simply $U(z_i, \hat{z}_i, 0, 0) = U(z_i, \hat{z}_i, 1, 0) = 0$. This takes care of all double-deviations, since the obedience constraints guarantee a non-negative payoff.

The maximization problem of the unrestricted eliciting sender is

$$\max_{d} \sum_{\theta \in \{B,G\}} \sum_{k=0}^{3} d[\hat{1}|\theta, k] \Pr(k|\theta) \Pr(\theta) \tag{13}$$

$$\text{s.t.} \quad 0 \leq d[r|\theta, k] \leq 1, \quad \forall r \in \{\hat{0}, \hat{1}\}, \theta \in \{B, G\}, k \in \{0, 1, 2, 3\} \tag{14}$$

$$d[\hat{1}|\theta, k] + d[\hat{0}|\theta, k] = 1 \quad \forall \theta \in \{B, G\}, k \in \{0, 1, 2, 3\} \tag{15}$$

$$U(g_i, \hat{g}_i, 0, 1) \geq U(g_i, \hat{g}_i, 0, 0) = 0 \tag{$OB_g^{\hat{1}}$}$$

$$U(b_i, \hat{b}_i, 0, 1) \geq U(b_i, \hat{b}_i, 0, 0) = 0 \tag{$OB_b^{\hat{1}}$}$$

$$U(g_i, \hat{g}_i, 0, 1) \geq U(g_i, \hat{b}_i, 0, 1) \tag{H_g}$$

$$U(b_i, \hat{b}_i, 0, 1) \geq U(b_i, \hat{g}_i, 0, 1). \tag{H_b}$$

Lemma 4. *If $OB_b^{\hat{1}}$ and H_g hold, then $OB_g^{\hat{1}}$ is satisfied.*

Next, we reformulate the Primal of the eliciting sender.

$$\max_{\substack{\{d[\hat{1}|\theta,k] \geq 0\} \\ \theta \in \{B,G\} \\ k \in \{0,1,2,3\}}} \sum_{\theta \in \{B,G\}} \sum_{k=0}^{3} d[\hat{1}|\theta, k] \Pr(k|\theta) \Pr(\theta) \tag{16}$$

$$\text{s.t.} \quad \sum_{k=0}^{2} \frac{3-k}{3} (d[\hat{1}|\theta = B, k] \Pr(k|\theta = B) - d[\hat{1}|\theta = G, k]) \Pr(k|\theta = G) \leq 0, \tag{$OB_b^{\hat{1}}$}$$

$$\sum_{k=1}^{3} \frac{k}{3} ((d[\hat{1}|\theta = G, k-1] - d[\hat{1}|\theta = G, k]) \Pr(k|\theta = G) \tag{H_g}$$
$$- (d[\hat{1}|\theta = B, k-1] - d[\hat{1}|\theta = B, k]) \Pr(k|\theta = B)) \leq 0,$$

$$\sum_{k=0}^{2} \frac{3-k}{3} ((d[\hat{1}|\theta = G, k+1] - d[\hat{1}|\theta = G, k]) \Pr(k|\theta = G) \tag{H_b}$$
$$- (d[\hat{1}|\theta = B, k+1] - d[\hat{1}|\theta = B, k]) \Pr(k|\theta = B)) \leq 0,$$

$$d[\hat{1}|\theta, k] - 1 \leq 0 \quad \forall \theta \in \{B, G\}, k \in \{0, 1, 2, 3\}. \tag{17}$$

Observation 2. *The information designer's maximization problem is equivalent to maximizing*

$\frac{1}{2}\left(\Pr(\hat{1}|b)+\Pr(\hat{1}|g)\right)$.

After rewriting $\frac{1}{2}\left(\Pr(\hat{1}|b)+\Pr(\hat{1}|g)\right)$ into

$$\sum_{k=0}^{3}\left(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B)\frac{1}{2}+d[\hat{1}|\theta=G,k]\Pr(k|\theta=G)\frac{1}{2}\right)\underbrace{\left(\frac{k}{3}+\frac{3-k}{3}\right)}_{=1} \tag{18}$$

one can directly see that maximizing the ex-ante type is equivalent to the objective function of the eliciting designer.

Proposition 4. *The optimal disclosure policy of the eliciting information designer for* $p \leq \underline{p}$ *is* $\forall k$,

$$d[\hat{1}|\theta=G,k]=1,\quad d[\hat{1}|\theta=B,k]=\frac{(1-p)}{p},$$

where $\underline{p}=\frac{1}{\sqrt{2}}$.

The optimal disclosure policy of the eliciting sender for this interval of accuracy levels does not condition on the information reported by voters. The eliciting sender send $\hat{1}$ in each state of the world with a constant probability, i.e., independent of how many g-signals there were reported. Figure 2 shows the optimal disclosure policy of the eliciting sender for $p=0.6$.

Proposition 5. *The optimal disclosure policy of the eliciting information designer for* $\underline{p} \leq p \leq \overline{p}$ *is*

$$d[\hat{1}|\theta=B,k]=\begin{cases}\frac{(1-p)}{p}(2p-1) & \text{if} \quad k=0\\ 2(1-p) & \text{if} \quad k\neq 0\end{cases},\quad d[\hat{1}|\theta=G,k]=\begin{cases}0 & \text{if} \quad k=0\\ 1 & \text{if} \quad k\neq 0,\end{cases} \tag{19}$$

where $\underline{p}=\frac{1}{\sqrt{2}}$ *and* $\overline{p}=\frac{1+\sqrt{13}}{6}$.

In contrast to the previous proposition, for this intermediate interval of accuracy levels, the sender starts to use the information reported by voters. That is, when the private information of voters' is more accurate, the sender conditions her disclosure policy on the number of reported g-signals. Moreover, the eliciting sender's optimal disclosure policy is monotone, i.e., she increases the probability with which she sends the recommendation $\hat{1}$ when the number of reported g-signals increases. Figure 3 shows the disclosure policy for $p=0.75$. If $\theta=G$, with at least one $\hat{g}$-report the sender increases the probability of sending $\hat{1}$ from 0 to 1. Similarly, she sends $\hat{1}$ in $\theta=B$ more often when there is at least one $\hat{g}$-report.

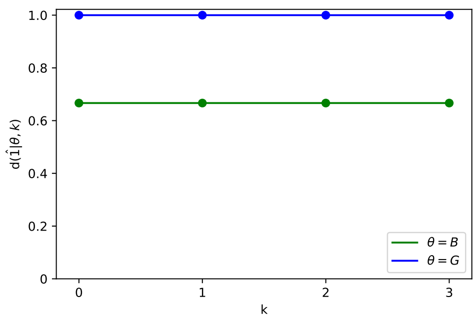

Figure 2: optimal policy for $p = 0.6$.

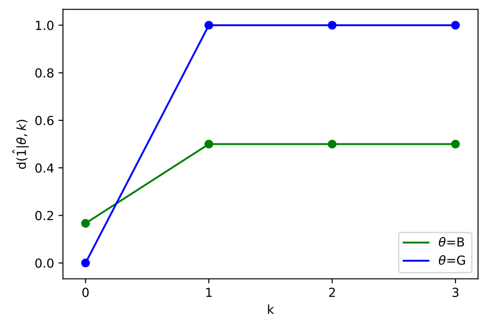

Figure 3: optimal policy for $p = 0.75$.

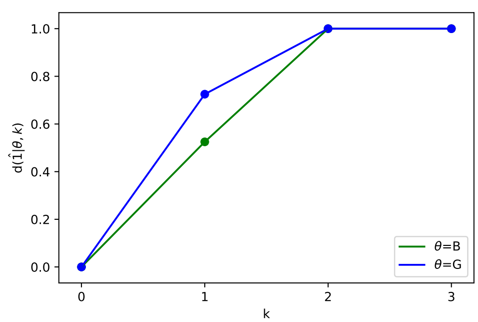

Figure 3.4: optimal policy $p = 0.9$.

Proposition 6. *The optimal disclosure policy of the eliciting information designer for* $\overline{p} \leq p < 1$ *is*

$$d[\hat{1}|\theta = B, k] = \begin{cases} 0 \quad \textit{if} \qquad k = 0, \\ \frac{(p-\frac{1}{2})(3-p)}{2(2p-1)} \quad \textit{if } k = 1, \\ 1 \quad \textit{if} \quad k \in \{2, 3\}, \end{cases} \qquad d[\hat{1}|\theta = G, k] = \begin{cases} 0 \quad \textit{if} \qquad k = 0, \\ \frac{(p-\frac{1}{2})(p+2)}{2(2p-1)} \quad \textit{if } k = 1, \\ 1 \quad \textit{if} \quad k \in \{2, 3\}, \end{cases}$$

for $\overline{p} = \frac{1+\sqrt{13}}{6}$.

As for the previous interval of accuracy levels, the sender makes use of the information reported by voters: she changes the probability with which she sends $\hat{1}$ depending on how many g-signals were reported. Moreover, the sender uses a monotone disclosure policy. Figure 3.4 shows the optimal disclosure policy of the eliciting sender for $p = 0.9$. Note the bang-bang structure of the optimal disclosure policy: In both states of the world, the eliciting sender recommends $\hat{1}$ with certainty whenever there are at least two $\hat{g}$-reports, she mixes between $\hat{0}$ and $\hat{1}$ when there is exactly one $\hat{g}$-report, and never recommends $\hat{1}$ when there is no $\hat{g}$-report.

Proposition 7. *For $p \leq \underline{p}$, the information designer's optimal disclosure policy is equivalent to maximizing* $\Pr(\hat{1}|b)$.

Proposition 8. *For $\overline{p} \leq p < 1$, the information designer's optimal disclosure policy is equivalent to maximizing* $\Pr(\hat{1}|g)$.

While Proposition 7 says that the information designer maximizes the probability of a b-type to vote for the proposal if $p \leq \underline{p}$, Proposition 8 says that her optimal disclosure policy is equivalent to maximizing the probability of a g-type to vote for the proposal if $p \geq \overline{p}$. Note that the expected utility of the sender is strictly decreasing in the accuracy of the voters' private signals, that is, the more convinced the voters, the less scope for persuasion. This is depicted in Figure 5.

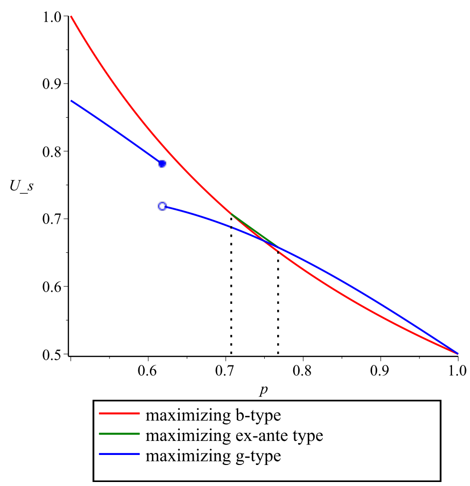

Figure 5: expected payoff of the eliciting sender.

6 Non-Eliciting Information Design

In this section the sender cannot ask voters for their private information. In this case the sender is not able to condition her disclosure policy on the private signals or reports of the voters. The following lemma restricts the sender's message set.

Lemma 5. *Under unanimity, it is without loss of generality for optimality to restrict the message space of the non-eliciting sender to* $R = \{\hat{0}, \widehat{01}, \hat{1}\}$.

As before, obedient voters vote for the proposal after $\hat{1}$ and for the status quo after $\hat{0}$. After $\widehat{01}$, only g-type voters vote for the proposal, and the b-types reject the proposal. To give some intuition for Lemma 3, observe that there exist only only three possible cases that can occur after any recommendation: either both types weakly favor the proposal, only the g-type favors for the proposal, or both types strictly dislike the proposal. A good-type never votes against the proposal while a voter with a bad signal strictly prefers the proposal. This is because a voter with a g-signal is more optimistic about $\theta = G$ than a b-type voter. As a consequence, the above three recommendations are sufficient to capture all the possible cases that can occur.

The disclosure policy of a non-eliciting sender is:

$$d : \Theta \to \Delta\{\hat{0}, \widehat{01}, \hat{1}\}. \tag{20}$$

The obedience constraints for each type of voter after $\hat{1}$ are given by:

$$U_i(g_i, a_i(\hat{1}, g_i) = 1) = \sum_{\theta \in \{B,G\}} d[\hat{1}|\theta] \Pr(\theta|g_i)(1_{\theta=G} - \frac{1}{2}) \geq 0 = U_i(g_i, a_i(\hat{1}, g_i) = 0)$$

$$U_i(b_i, a_i(\hat{1}, b_i) = 1) = \sum_{\theta \in \{B,G\}} d[\hat{1}|\theta] \Pr(\theta|b_i)(1_{\theta=G} - \frac{1}{2}) \geq 0 = U_i(b_i, a_i(\hat{1}, b_i) = 0)$$

The obedience constraints for each type of voter after $\widehat{01}$ are given by:

$$\begin{aligned} U_i(g_i, a_i(\widehat{01}, g_i) = 1) = \sum_{\theta \in \{B,G\}} & d[\widehat{01}|\theta] \Pr(k = 3|\theta) \Pr(\theta)(1_{\theta=G} - \frac{1}{2}) \\ & \geq 0 = U_i(g_i, a_i(\hat{1}, g_i) = 0), \end{aligned}$$

$$\begin{aligned} U_i(b_i, a_i(\widehat{01}, b_i) = 1) = \sum_{\theta \in \{B,G\}} & d[\widehat{01}|\theta] \Pr(k = 2|\theta) \Pr(\theta)(1_{\theta=G} - \frac{1}{2}) \\ & \leq 0 = U_i(b_i, a_i(\hat{1}, b_i) = 0). \end{aligned}$$

The sender's maximization problem becomes

$$\max_{d} \sum_{\theta \in \{B,G\}} (d[\hat{1}|\theta] + d[\widehat{01}|\theta] \Pr(k=3|\theta)) \Pr(\theta) \tag{21}$$

$$\text{s.t.} \quad 0 \le d[r|\theta] \le 1, \quad \forall r \in \{\hat{0}, \widehat{01}, \hat{1}\}, \theta \in \{B,G\} \tag{22}$$

$$d[\hat{1}|\theta] + d[\widehat{01}|\theta] + d[\hat{0}|\theta] = 1 \quad \forall \theta \in \{B,G\} \tag{23}$$

$$U_i(g_i, a_i(\hat{1}, g_i) = 1) \ge U_i(g_i, a_i(\hat{1}, g_i) = 0) = 0 \tag{$OB_g^{\hat{1}}$}$$

$$U_i(b_i, a_i(\hat{1}, b_i) = 1) \ge U_i(b_i, a_i(\hat{1}, b_i) = 0) = 0 \tag{$OB_b^{\hat{1}}$}$$

$$U_i(g_i, a_i(\widehat{01}, g_i) = 1) \ge U_i(g_i, a_i(\widehat{01}, g_i) = 0) \tag{$OB_g^{\widehat{01}}$}$$

$$U_i(b_i, a_i(\widehat{01}, b_i) = 1) \le U_i(b_i, a_i(\widehat{01}, b_i) = 0) \tag{$OB_b^{\widehat{01}}$}$$

The next result shows the solution to the above problem of a non-eliciting information designer.

Proposition 9. *The optimal disclosure policy of the non-eliciting sender for $p \le \tilde{p}$ is*

$$d[\hat{1}|\theta = G] = 1, \qquad d[\hat{1}|\theta = B] = \frac{1-p}{p},$$

$$d[\hat{0}|\theta = G] = 0, \qquad d[\hat{0}|\theta = B] = \frac{2p-1}{p}.$$

The optimal disclosure policy of the non-eliciting sender for $p \ge \tilde{p}$ is

$$d[\hat{1}|\theta = G] = 1 - \frac{(2p-1)(1-p)^3}{(p^4 - (1-p)^4)}, \qquad d[\hat{1}|\theta = B] = 1 - \frac{(2p-1)p^3}{p^4 - (1-p)^4}$$

$$d[\widehat{01}|\theta = G] = \frac{(2p-1)(1-p)^3}{p^4 - (1-p)^4}, \qquad d[\widehat{01}|\theta = B] = \frac{(2p-1)p^3}{p^4 - (1-p)^4},$$

where $\tilde{p} = \sqrt[4]{\frac{1}{2}}$.

Note that for any p, the optimal disclosure policy of the designer never contains more than two messages. When comparing the optimal disclosure policy of an eliciting and non-eliciting information designer, it becomes apparent that they are equivalent for $p \le \underline{p} = \frac{1}{\sqrt{2}}$. The eliciting information designer has no advantage from asking voters about their private information when the accuracy of signals is sufficiently small.

Corollary 6. *Let* $\underline{p} = \frac{1}{\sqrt{2}}$*. For* $p \leq \underline{p}$*, the eliciting sender's optimal disclosure policy is equivalent to the non-eliciting sender's optimal disclosure policy.*

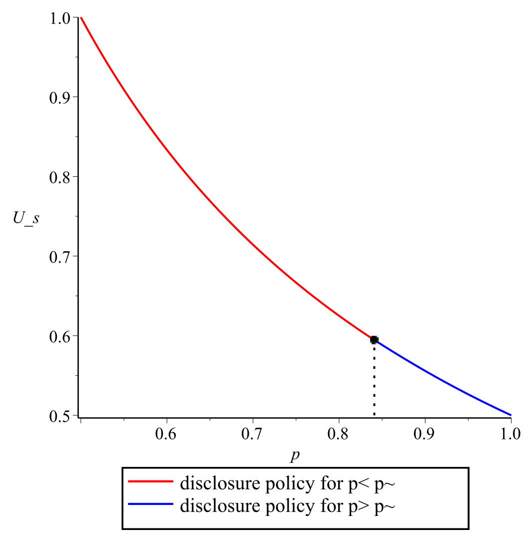

Figure 6: non-eliciting sender's expected payoff.

7 Further Remarks

7.1 Eliciting Sender: Comparison to Feddersen and Pesendorfer (1998)

Next, consider the error probabilities of a wrong decision when $\theta = B$ (i.e., $\Pr(a = 1|\theta = B)$) and implementing the status quo when $\theta = G$ (i.e., $\Pr(a = 0|\theta = B)$) under the optimal disclosure policy d of the sender.

We compare the probabilities of making each type of error in our setting to the corresponding probabilities of making each type of error in Feddersen and Pesendorfer (1998), where voters have to act based on their private signals without any coordination device or sender, and the decision rule is unanimity. Applied to our setting, the following is a Nash equilibrium in their model: a voter with a g-signal always votes in favor of the reform, and a b-signal voter in favor of the reform with probability $\frac{\sqrt{p}\left(p+\sqrt{p(1-p)}-1\right)}{p^{\frac{3}{2}}-(1-p)^{\frac{3}{2}}}$.

First, the probability of choosing the proposal when $\theta = B$ in our setting is bigger for all $p \in (\frac{1}{2}, 1)$ (Figure 7). Second, the probability of choosing the status quo when $\theta = G$ in our setting is smaller for all $p \in (\frac{1}{2}, 1)$ (Figure 8).

Taken together, while in Feddersen and Pesendorfer (1998) the proposal is more often implemented when $\theta = B$, in our setting the proposal is more often implemented when $\theta = G$. A

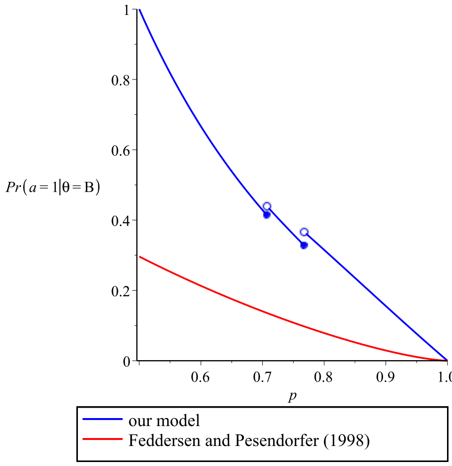

Figure 7: error probabilities if $\theta = B$.

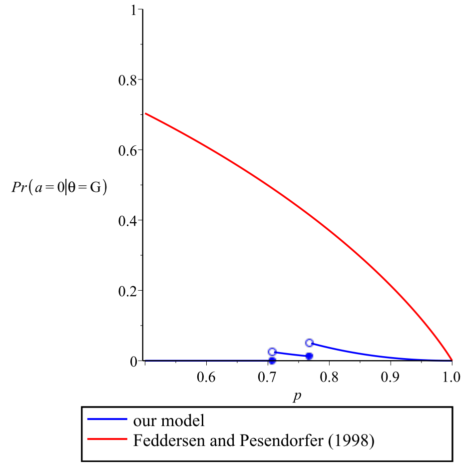

Figure 8: error probabilities if $\theta = G$.

manipulative information designer strictly decreases the probability of rejecting the proposal when the proposal is efficient while increasing the probability of rejecting the status quo when the status quo is efficient.

Overall, the ex-ante type (a voter whose private signal has not yet realized) receives a strictly higher expected payoff in our model than in Feddersen and Pesendorfer (1998).

Proposition 10. *Under unanimity, voters have a strictly higher expected utility with an eliciting information designer than in a symmetric equilibrium as in Feddersen and Pesendorfer (1998).*

Hence, even with a manipulative sender, voters are better off in expectations compared to the situation in which they have to decide on their own under unanimity.

7.2 Omniscient Sender: One Agent with Multiple Signals

In this section we are comparing our previous results from Section 3.1 to the case where the omniscient sender faces only one voter which receives a signal $s \in \{0, 1, 2, 3\}$. One possible interpretation for this setting is the following: Imagine voters could communicate with each other and exchange their private signals before the information designer sends her public recommendation. In this case every voter has exactly the same information, i.e., knows how many g-signals there are. As a consequence, the sender's problem is equivalent to persuading one representative voter that can possibly have four different signals which correspond to the number of g-signals. Formally, the sender's problem becomes

$$\max_d \sum_{\theta \in \{B,G\}} \sum_{k=0}^{3} d[\hat{1}|\theta, k] \Pr(k|\theta) \Pr(\theta) \tag{24}$$

$$\Pr(\theta = G|\hat{1}, s) - \frac{1}{2} \geq 0 \quad \forall s \in \{0, 1, 2, 3\}. \tag{$OB^{\hat{1}}$}$$

The obedience constraint of the representative voter can be rewritten into

$$\Pr(\theta = G|\hat{1}, s) - \frac{1}{2} \geq 0 \tag{25}$$

$$\Leftrightarrow \quad \frac{\Pr(\theta = G, \hat{1}, s)}{\Pr(\hat{1}, s)} \geq \frac{1}{2} \tag{26}$$

$$\Leftrightarrow \quad \frac{Pr(\hat{1}|\theta = G, s) \Pr(\theta = G, s)}{\Pr(\hat{1}, s)} \geq \frac{1}{2} \tag{27}$$

$$\Leftrightarrow \quad \frac{d[\hat{1}|\theta = G, k] \Pr(k|\theta = G) \Pr(\theta = G)}{\Pr(\hat{1}, k)} \geq \frac{1}{2} \tag{28}$$

$$\Leftrightarrow \quad d[\hat{1}|\theta = G, k] \Pr(k|\theta = G) \geq d[\hat{1}|\theta = B, k] \Pr(k|\theta = B) \tag{29}$$

In contrast to before, the omniscient sender who faces only one agent which has one out of four signal realizations, has to take four obedience constraints into account, one for each $s \in \{0, 1, 2, 3\}$. Due to the perfect alignment of interests of the sender and the representative voter in $\theta = G$, the omniscient sender will optimally send $\hat{1}$ in $\theta = G$ with certainty for each $s \in \{0, 1, 2, 3\}$. Simple calculations show that the voter's obedience constraint is slack if the omniscient sender also sends $\hat{1}$ with probability one for $s \in \{2, 3\}$. For $s \in \{0, 1\}$ she will choose $d[\hat{1}|\theta = B, k]$ such that the obedience constraint just binds. That is

$$3p^2(1-p) = d[\hat{1}|\theta = B, k = 1] 3p(1-p)^2 \tag{30}$$

$$\Leftrightarrow \quad d[\hat{1}|\theta = B, k = 1] = \frac{(1-p)}{p} \tag{31}$$

and

$$p^3 = d[\hat{1}|\theta = B, k = 0](1-p)^3 \tag{32}$$

$$\Leftrightarrow \quad d[\hat{1}|\theta = B, k = 0] = \frac{(1-p)^3}{p^3}. \tag{33}$$

Hence, when the omniscient sender faces only one representative voter who knows the number of g-signals, she optimally uses the following disclosure policy

$$d[\hat{1}|\theta = G, k] = 1 \ \forall k, \quad d[\hat{1}|\theta = B, k] = \begin{cases} 1 & \text{if} \quad k \in \{2,3\} \\ \frac{(1-p)}{p} & \text{if} \quad k = 1 \\ \frac{(1-p)^3}{p^3} & \text{if} \quad k = 0. \end{cases} \tag{34}$$

The omniscient sender's expected payoff in this case is given by $\frac{1}{2} + \frac{1}{2}(2-2p)$. This is always lower than the omniscient sender's expected payoff facing three informed voters who cannot communicate. Since the omniscient sender's optimal disclosure policy in $\theta = G$ is exactly the same in both settings and the total probability with which she sends $\hat{1}$ in $\theta = B$ is smaller when voters know the signals of each other, voters are better off if they know each other's signals.

8 Conclusion

We study a biased sender who tries to persuade three voters to vote for a proposal by sending public recommendations. Voters receive some private signals about the quality of the proposal and only want to approve the proposal if it is of high quality. We characterize the optimal disclosure policy under unanimity rule of a 1. omniscient, 2. eliciting, and 3. non-eliciting sender. We find that the eliciting sender can only profit from her ability to ask voters for their private signals when the accuracy of their private information is sufficiently high. Whenever the accuracy level is below some lower threshold the eliciting sender is just equally well of as the non-eliciting sender who cannot ask voters for reports about their private signals.

We show that depending on the accuracy level of the private signals of voters the optimal disclosure policy of the eliciting sender solves two other related maximization problems: For accuracy levels below some lower threshold, the eliciting sender maximizes the probability that a pessimistic voter votes for the proposal. For accuracy levels above some upper threshold, the eliciting sender maximizes the probability of an optimistic voter approving the proposal. Voters are better off in the presence of a biased informed designer than in a setting where they have to vote under unanimity based on their private exogenous information only as in Feddersen and Pesendorfer (1998).

In this work we consider a sender who knows the true quality of the proposal. Extending the analysis to a restricted sender who is uninformed about the true quality and can only send public recommendations on the basis of the reports made by voters is a potential avenue for future research.

A Appendix

A.1 Proof of Proposition 1

Proof. We show that given any outcome of disclosure policy d', the sender can implement an outcome equivalent policy d consisting of only two recommendations $R = \{\hat{0}, \hat{1}\}$. Message $\hat{0}$ is the recommendation to vote with probability 1 for the status quo irrespective of the private signal, and $\hat{1}$ is the recommendation to vote with probability 1 for proposal irrespective of the private signal.

Consider any arbitrary disclosure policy of the sender $d' : \Theta \times \{0, \ldots, 3\} \to \Delta(R')$, with R' being any arbitrary message set. Denote $r' \in R'$ an element of the message space. Let $a(r', z_i)$ be the probability of a voter i with signal z_i voting for the proposal after seeing recommendation r' under disclosure policy d'.[6]

Now consider a filtering d of the original information disclosure policy d' of the following form.

$$d : R' \times K \to \Delta[\hat{0}, \hat{1}].$$

The new disclosure policy takes the realized message r' in the original disclosure policy and the number of g-signals k of the voters, and maps them into a binary voting recommendation. With slight abuse of notation, denote by $d(\hat{1}|r', k)$ the probability of sending recommendation $\hat{1}$ in favor of the proposal.

Consider the following construction for the new disclosure policy d:

$$d(\hat{1}|r', k) = a(r', g)^k a(r', b)^{3-k}. \tag{35}$$

It is immediate that this policy yields the same expected utility to the sender (if implementable), as the probability with which she sends signal $\hat{1}$ corresponds to the probability with which her preferred outcome would have been elected under the original disclosure policy d'.

Furthermore, this disclosure policy is designed to be proportional to the original probability of being pivotal for both voter types. In particular, for $z_i = b$ and $k \in \{0, 1, 2\}$, we have $\Pr(piv|r, k_{-i} = k, b) = a(r', g)^k a(r', b)^{2-k}$ and hence,

$$d(\hat{1}|r', k) = a(r', b) \Pr(piv|r', k_{-i} = k).$$

Similarly, for $z_i = g$ and $k \in \{0, 1, 2\}$, we have $\Pr(piv|r, k_{-i} = k, g) = a(r', g)^{k-1} a(r', b)^{3-k}$ and

[6] We assume that voters with the same signal react symmetrically to the same recommendation, $a_i(r', z_i) = a_j(r', z_j)$ if $z_i = z_j$. Hence, we drop index i.

thus,

$$d(\hat{1}|r', k+1) = a(r', g)\Pr(piv|r', k_{-i} = k).$$

It is left to show that the obedience constraints are also satisfied under the new disclosure policy d'. After public recommendation $\hat{0}$ no voter is ever pivotal. It suffices to show, that both private information types have an incentive to follow the recommendation $\hat{1}$ by voting for the proposal. The obedience constraint of a voter with private signal z_i is:

$$\Pr(\theta = G|\hat{1}, z_i, piv) \geq \frac{1}{2} \quad \forall z_i \in \{g_i, b_i\} \tag{36}$$

This can be rewritten into

$$\begin{aligned} &\Pr(\theta = G|z_i)\Pr(\hat{1}|\theta = G, z_i)\underbrace{\Pr(piv|\hat{1}, \theta = G, z_i)}_{=1} \\ \geq &\Pr(\theta = B|z_i)\Pr(\hat{1}|\theta = B, z_i)\underbrace{\Pr(piv|\hat{1}, \theta = B, z_i)}_{=1}. \end{aligned}$$

Thus, the two obedience constraints can be expressed as

$$p\Pr(\hat{1}|\theta = G, g_i) \geq (1-p)\Pr(\hat{1}|\theta = B, g_i), \tag{$OB_g^{\hat{1}}$}$$

$$(1-p)\Pr(\hat{1}|\theta = G, b_i) \geq p\Pr(\hat{1}|\theta = B, b_i). \tag{$OB_b^{\hat{1}}$}$$

We can invoke the construction of the new disclosure policy d from d' to rewrite:

$$\begin{aligned} \Pr(\hat{1}|\theta, g_i) &= \sum_{r' \in R'} \Pr(\hat{1}, r'|\theta, g_i) & (37) \\ &= \sum_{r' \in R'} \sum_{k_{-i}=0}^{2} \Pr(\hat{1}, r', k_{-i}|\theta, g_i) & (38) \\ &= \sum_{r' \in R'} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|\theta, g_i)\Pr(r'|k_{-i}, \theta, g_i)\Pr(\hat{1}|r', k_{-i}, \theta, g_i) & (39) \\ &= \sum_{r' \in R'} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|\theta, g_i)\Pr(r'|k_{-i}, \theta, g_i) d(\hat{1}|r', k = k_{-i}+1) & (40) \end{aligned}$$

Analogously, for $z_i = b$ we have

$$\Pr(\hat{1}|\theta, b_i) = \sum_{r' \in R'} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|\theta, b_i) \Pr(r'|k_{-i}, \theta, b_i) d(\hat{1}|r', k = k_{-i}) \tag{41}$$

We can use this to rewrite the obedience constraints under the new disclosure policy:

$$\sum_{r' \in R'} \sum_{k_{-i}=0}^{2} p \Pr(k_{-i}|\theta = G, g_i) \Pr(r'|k_{-i}, \theta = G, g_i) d(\hat{1}|r', k = k_{-i} + 1)$$
$$\geq \sum_{r' \in R'} \sum_{k_{-i}=0}^{2} (1-p) \Pr(k_{-i}|\theta = B, g_i) \Pr(r'|k_{-i}, \theta = B, g_i) d(\hat{1}|r', k = k_{-i} + 1)$$

$$\Leftrightarrow \sum_{r' \in R'} \sum_{k_{-i}=0}^{2} d(\hat{1}|r', k = k_{-i} + 1)[p \Pr(k_{-i}|\theta = G, g_i) \Pr(r'|k_{-i}, \theta = G, g_i)$$
$$-(1-p) \Pr(k_{-i}|\theta = B, g_i) \Pr(r'|k_{-i}, \theta = B, g_i)] \geq 0. \tag{$OB_g^{\hat{1}}$}$$

Using the construction of the new disclosure policy, we can rewrite

$$\sum_{r' \in R'} a(r', g_i) \sum_{k_{-i}=0}^{2} \Pr(piv|r, k_{-i}, b_i)[p \Pr(k_{-i}|\theta = G, g_i) \Pr(r'|k_{-i}, \theta = G, g_i)$$
$$-(1-p) \Pr(k_{-i}|\theta = B, g_i) \Pr(r'|k_{-i}, \theta = B, g_i)] \geq 0. \tag{42}$$

Analoguously, for $z_i = b$ we have

$$\sum_{r' \in R'} a(r', b_i) \sum_{k_{-i}=0}^{2} \Pr(piv|r', k_{-i}, b_i)[(1-p) \Pr(k_{-i}|\theta = G, b_i) \Pr(r'|k_{-i}, \theta = G, b_i)$$
$$-p \Pr(k_{-i}|\theta = B, b_i) \Pr(r'|k_{-i}, \theta = B, b_i)] \geq 0. \tag{$OB_b^{\hat{1}}$}$$

Note that the original disclosure policy d' is implementable, i.e. the obedience constraint holds for each type $z_i \in \{g, b\}$ and each message $r' \in R'$, when $a_i(r', z_i) > 0$. That is,

$$\sum_{k_{-i}=0}^{2} \Pr(piv|r', k_{-i}, \theta = G)[p \Pr(k_{-i}|\theta = G, g_i) \Pr(r'|k_{-i}, \theta = G, g_i)$$
$$-(1-p) \Pr(k_{-i}|\theta = B, g_i) \Pr(r'|k_{-i}, \theta = B, g_i)] \geq 0. \qquad (OB_g^{r'})$$

$$\sum_{k_{-i}=0}^{2} \Pr(piv|r', k_{-i}, \theta = G)[(1-p) \Pr(k_{-i}|\theta = G, b_i) \Pr(r'|k_{-i}, \theta = G, b_i)$$
$$-p \Pr(k_{-i}|\theta = B, b_i) \Pr(r'|k_{-i}, \theta = B, b_i)] \geq 0. \qquad (OB_b^{r'})$$

The inner sums in the obedience constraints under the new disclosure policy d, $OB_b^{\hat{1}}$ and $OB_g^{\hat{1}}$, correspond to the original obedience constraints, $OB_g^{r'}$ and $OB_b^{r'}$, under the former disclosure policy d'. This establishes that the filtering d satisfies both obedience constraints and yields the same payoff to the designer.

■

A.2 Proof of Lemma 1

Proof. We prove this by contradiction. Assume that the disclosure policy d is optimal and that there exists k' such that $d[\hat{1}|\theta = G, k'] \neq 1$. Then, construct a new disclosure policy d' that is equal to the old disclosure policy d for all $k \neq k'$ and θ. For $k = k'$ and $\theta = G$, it sends recommendation $\hat{1}$ with probability 1. That is $d'[\hat{1}|\theta = B, k] = d[\hat{1}|\theta = B, k]\ \forall k'$, $d'[\hat{1}|\theta = G, k] = d[\hat{1}|\theta = G, k]\ \forall k' \neq k$ and $d'[\hat{1}|k'] = 1 \neq d[\hat{1}|k']$. Next we check whether the new disclosure policy d' still fulfills the voters' obedience constraints. Note that if the recommendation $\hat{0}$ was sent, no voter is ever pivotal, which is why this doesn't influence the obedience constraints. Sending the recommendation $\hat{1}$ for any k when $\theta = G$ increases the left hand side of the voters' obedience constraints and thus makes them easier to satisfy. The new disclosure policy relaxed the voters' obedience constraints and sends $\hat{1}$ with a strictly higher probability.

■

A.3 Proof of Proposition 2

Proof. The only part of above proposition that we have not proven is $d[\hat{1}|\theta = B, k \neq 3]$. In the following we will use the greedy algorithm (Dantzig, 1957) to solve this problem. We have a fractional knapsack problem of the following form:

Find $0 \le x_k = d[\hat{1}|\theta = B, k] \le 1$ for $k \in \{0,1,2\}$ s.t.

1) $\sum_{k=0}^{2} x_k w_k \le \frac{(1-p)}{p}$ holds and

2) $\sum_{k=0}^{2} x_k v_k$ is maximized,

where $w_k = \frac{3-k}{3}\Pr(k|\theta = B)$ and $v_k = \Pr(k|\theta = B) \cdot 1$. We refer to w_k as the weight and to v_k as the value of k.

Next we calculate the value-per-weight ratio $\rho_k = \frac{v_k}{w_k}$ for $k \in \{0,1,2\}$:

$$\rho_k = \frac{\Pr(k|\theta = B)}{\frac{3-k}{3}\Pr(k|\theta = B)} = \frac{3}{3-k} \tag{43}$$

Variable ρ_k is increasing in k. Hence, if we sort the k's by decreasing ρ_k, we get the following order $2, 1, 0$. How much probability mass we can place on each k until the obedience constraint of the b-signal voter is binding, will depend on the accuracy level of the voters' private signals p.

■

A.4 Proof of Proposition 3

Proof. Consider the same filtering of the original disclosure policy d' into d as in the proof of Proposition 1:

$$d(\hat{1}|r', k) = a(r', g)^k a(r', b)^{3-k}.$$

It remains to be shown that this disclosure policy d satisfies the honesty constraints for each type. The proof of optimality for the designer, and the validity of the obedience constraints were already established in Proposition 1. We prove that the above filtering satisfies the honesty constraints of the g-type. The argument for the b-type is accordingly, and therefore omitted.

Expected utility in equilibrium with d and d'. First, we show that the old disclosure policy d' and new disclosure policy d yields exactly the same expected utility to the g-type in equilibrium, when reporting truthfully. Let R be the message set of the designer with d'.[7]

$$EU(g_i, \hat{g}_i; d') = \sum_{r \in R} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{g}_i, k_{-i}; d') a_i(r, g)^{k_{-i}+1} a_i(r, b)^{2-k_{-i}} q(r, g_i, \hat{g}_i, k_{-i}), \quad (44)$$

where $q(r, g_i, \hat{g}_i, k_{-i}) = E[\theta|r, g_i, \hat{g}_i, k_{-i}] - \frac{1}{2}$ is the expected net utility from implementing the proposal if a g-type voter reported truthfully, k_{-i} others also have a g-signal and the designer sent recommendation r. The factor $a_i(r, g)^{k_{-i}+1} a_i(r, b)^{2-k_{-i}}$ accounts for the probability of being pivotal $(a_i(r, g)^{k_{-i}} a_i(r, b)^{2-k_{-i}})$ times the probability of the g-type voter i voting for the reform $a_i(r, g)$ in equilibrium.

Next, consider the expected utility under the new disclosure policy d. Whenever the designer sends recommendation $\hat{1}$ (which happens with probability $a_i(r, g)^{k_{-i}+1} a_i(r, b)^{2-k_{-i}}$ if recommendation r would have been sent in d') the reform is implemented.

$$\begin{aligned} EU(g_i, \hat{g}_i; d) &= \sum_{r \in R} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{g}_i, k_{-i}; d') d(\hat{1}|r, k = k_{-i} + 1) q(r, g_i, \hat{g}_i, k_{-i}) \\ &= \sum_{r \in R} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{g}_i, k_{-i}; d') a_i(r, g)^{k_{-i}+1} a_i(r, b)^{2-k_{-i}} q(r, g_i, \hat{g}_i, k_{-i}). \quad (45) \end{aligned}$$

This coincides with the utility under the original disclosure policy in Equation 44, $EU(g_i, \hat{g}_i; d) = EU(g_i, \hat{g}_i; d')$.

Expected utility from misreporting in d'. Next, consider the utility of a g-voter who reports $\hat{b}$ in the original disclosure policy d'. To account for double deviations, we denote by $\tilde{a}_i(r, g_i, \hat{b}_i)$ his action after observing r when reporting $\hat{b}$.

$$\begin{aligned} EU(g_i, \hat{b}_i; d') = \sum_{r \in R} &\max_{\tilde{a}_i(r, g_i, \hat{b}_i) \in [0,1]} \tilde{a}_i(r, g_i, \hat{b}_i) \cdot \quad (46) \\ &\sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{b}_i, k_{-i}; d') a_i(r, g)^{k_{-i}} a_i(r, b)^{2-k_{-i}} q(r, g_i, \hat{b}_i, k_{-i}). \end{aligned}$$

For simplicity of notation, denote by $EU(r|g_i, \hat{b}_i)$ the optimal utility after misreporting and observing recommendation r. Note that $EU(r|g_i, \hat{b}_i) \geq 0$ for all r, as a voter can always derive zero

[7] For convenience of notation, we assume that R is finite.

utility by voting against the reform. Hence,

$$EU(g_i, \hat{b}_i; d') = \sum_{r \in R} EU(r|g_i, \hat{b}_i). \tag{47}$$

Expected utility from misreporting in d. Finally, consider the expected utility after misreporting in the new disclosure policy d. The voter optimizes over his action $\tilde{a}(\hat{1}, g_i, \hat{b}_i)$ after recommendation $\hat{1}$ after misreporting. For simplicity, we assume that the voter votes for the reform with probability $\tilde{a}(\hat{1}, g_i, \hat{b}_i)$ after both recommendations $\hat{1}$ and $\hat{0}$, as his utility after recommendation $\hat{0}$ yields utility 0 irrespective of his action. The difference to d' is that he might not know which r lead to the recommendation $\hat{1}$.

$$\begin{aligned} EU(g_i, \hat{b}_i; d) = &\max_{\tilde{a}(\hat{1}, g_i, \hat{b}_i) \in [0,1]} \tilde{a}(\hat{1}, g_i, \hat{b}_i) \cdot \\ &\sum_{r \in R} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{b}_i, k_{-i}; d') \underbrace{a_i(r,g)^{k_{-i}} a_i(r,b)^{3-k_{-i}}}_{=\Pr(\hat{1}|r, k=k_{-i})} q(r, g_i, \hat{b}_i, k_{-i}). \end{aligned} \tag{48}$$

If the voter knew which r of the original disclosure policy d' led to recommendation $\hat{1}$, he would be better off: he could adapt his voting decision $\tilde{a}(\hat{1}, r, g_i, \hat{g}_i)$ to each r (instead of choosing the same $\tilde{a}(\hat{1}, g_i, \hat{g}_i)$ for all r that led to $\hat{1}$). Thus,

$$\begin{aligned} EU(g_i, \hat{b}_i; d) \leq &\sum_{r \in R} \max_{\tilde{a}(\hat{1}, r, g_i, \hat{b}_i) \in [0,1]} \tilde{a}(\hat{1}, r, g_i, \hat{b}_i) \cdot \\ &\sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{b}_i, k_{-i}; d') \underbrace{a_i(r,g)^{k_{-i}} a_i(r,b)^{3-k_{-i}}}_{=d(\hat{1}|r, k=k_{-i})} q(r, g_i, \hat{b}_i, k_{-i}) \end{aligned} \tag{49}$$

$$\begin{aligned} = &\sum_{r \in R} a_i(r,b) \max_{\tilde{a}(\hat{1}, r, g_i, \hat{b}_i) \in [0,1]} \tilde{a}(\hat{1}, r, g_i, \hat{b}_i) \cdot \\ &\sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{b}_i, k_{-i}; d') a_i(r,g)^{k_{-i}} a_i(r,b)^{2-k_{-i}} q(r, g_i, \hat{b}_i, k_{-i}), \end{aligned} \tag{50}$$

where the last inequality follows by putting the non-negative factor $a_i(r, b)$ outside the maximum. But then note that the maximization problem point-wise after each r is exactly the same as in Equation 46 for the original disclosure policy, $EU(r|g_i, \hat{b}_i)$, which is non-negative. Hence, the optimal deviation utility is bounded above by

$$EU(g_i, \hat{b}_i; d) \leq \sum_{r \in R} \underbrace{a_i(r,b)}_{\in[0,1]} \underbrace{EU(r|g_i, \hat{b}_i)}_{\geq 0} \tag{51}$$

$$\leq \sum_{r \in R} EU(r|g_i, \hat{b}_i) \tag{52}$$

$$= EU(g_i, \hat{b}_i; d'). \tag{53}$$

The payoff after a misreport and an optimal best response is weakly lower than in the original disclosure policy. Note that the original disclosure policy d' by assumption satisfied all constraints, including the honesty constraint of the g-type. Hence, we established that the honesty constraint of the g-type holds by proving

$$EU(g_i, \hat{g}_i; d) = EU(g_i, \hat{g}_i; d') \geq EU(g_i, \hat{b}_i; d') \geq EU(g_i, \hat{b}_i; d).$$

■

A.5 Proof of Lemma 4

Proof. First, we rewrite the honesty constraint of g-type to sum over k instead of k_{-i}.

$$\sum_{k=1}^{3} \frac{k}{3}((d[\hat{1}|\theta = G, k-1] - d[\hat{1}|\theta = G, k]) \Pr(k|\theta = G) \\ - (d[\hat{1}|\theta = B, k-1] - d[\hat{1}|\theta = B, k]) \Pr(k|\theta = B)) \leq 0 \tag{H_g}$$

$$\Leftrightarrow \quad \underbrace{\sum_{k=1}^{3} \frac{k}{3}(d[\hat{1}|\theta = B, k] \Pr(k|\theta = B) - d[\hat{1}|\theta = G, k]) \Pr(k|\theta = G)}_{OB_g^{\hat{1}}}$$

$$\leq \quad \underbrace{\sum_{k=1}^{3} \frac{k}{3}(d[\hat{1}|\theta = B, k-1] \Pr(k|\theta = B) - d[\hat{1}|\theta = G, k-1] \Pr(k|\theta = G)}_{*}.$$

Observe that we have rewritten H_g such that the LHS of H_g is just $OB_g^{\hat{1}}$. We rewrite $(*)$, i.e., the RHS of H_g, into

$$\sum_{k=0}^{2} \frac{k+1}{3}(d[\hat{1}|\theta = B, k] \Pr(k+1|\theta = B) - d[\hat{1}|\theta = G, k] \Pr(k+1|\theta = G). \tag{54}$$

Next, we subtract $\sum_{k=0}^{2} \frac{3-k}{3}(d[\hat{1}|\theta = B, k] \Pr(k|\theta = B) - d[\hat{1}|\theta = G, k]) \Pr(k|\theta = G) \leq 0$, which is just $OB(b) \leq 0$ rewritten, and get

$$\begin{aligned}\sum_{k=0}^{2}(d[\hat{1}|\theta = B, k](\frac{3-k}{3} \Pr(k+1|\theta = B) - \frac{k}{3} \Pr(k|\theta = B)) \\ - d[\hat{1}|\theta = G, k](\frac{3-k}{3} \Pr(k+1|\theta = G) - \frac{k}{3} \Pr(k|\theta = G)).\end{aligned} \tag{55}$$

Rewriting yields

$$\sum_{k=0}^{2}(d[\hat{1}|\theta = B, k]p^{2-k}(1-p)^k(1-2p) - d[\hat{1}|\theta = G, k]p^k(1-p)^{2-k}(2p-1) \leq 0.$$

Thus, we have that for all $k \leq 2$ the expression above is negative. Note that $(*) - OB_b^{\hat{1}}$ is the sum of these negative terms and is thus also negative. Moreover, $OB_g^{\hat{1}} \leq 0$ implies that $(*) \leq 0$. Since $OB_g^{\hat{1}} \leq (*)$ by equation (54), we have that $OB_g^{\hat{1}} \leq 0$, which proves the lemma.

■

A.6 Proof of Proposition 4

Proof. First, we take the Dual of the Primal and get

$$\min_{\substack{\lambda_{OB_b^{\hat{1}}} \geq 0,\ \lambda_{H_g} \geq 0,\ \lambda_{H_b} \geq 0 \\ \{\mu_{\theta,\,k} \geq 0\}_{\theta \in \{B,G\},\ k \in \{0,1,2,3\}}}} \quad \sum_{k=0}^{3} \mu_{\theta=B,\,k} + \sum_{k=0}^{3} \mu_{\theta=G,\,k} \tag{56}$$

$$\begin{aligned}\text{s.t.} \quad & p^{3-k}(1-p)^k \frac{1}{2}\left(-\binom{3}{k} + (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b})\binom{2}{k} + \lambda_{H_g}\binom{2}{k-1}\right) \\ & - \lambda_{H_g} p^{2-k}(1-p)^{k+1}\binom{2}{k} - \lambda_{H_b} p^{4-k}(1-p)^{k-1}\binom{2}{k-1} \\ & + \mu_{\theta=B,\,k} \ \geq \ 0 \quad \forall k \in \{0,1,2,3\}\end{aligned} \tag{57}$$

$$p^k(1-p)^{3-k}\frac{1}{2}\left(-\binom{3}{k}-(\lambda_{OB_b^{\hat{1}}}-\lambda_{H_b})\binom{2}{k}-\lambda_{H_g}\binom{2}{k-1}\right)\right) \tag{58}$$
$$+\lambda_{H_g}p^{k+1}(1-p)^{2-k}\binom{2}{k}+\lambda_{H_b}p^{k-1}(1-p)^{4-k}\binom{2}{k-1}$$
$$+\mu_{\theta=G,\,k}\ \geq\ 0 \quad \forall k\in\{0,1,2,3\}$$

Let $\{d[\hat{1}|\theta,k]\geq 0\}_{\theta\in\{B,G\},k\in\{0,1,2,3\}}$ be a feasible disclosure policy for the primal, and $\{\vec{\lambda},\vec{\mu}\}$ feasible vector for the dual. Necessary and sufficient conditions for them to be optimal are

$$\lambda_{OB_b^{\hat{1}}}\cdot\left(\sum_{k=0}^{2}\binom{2}{k}(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B)\frac{1}{2}-d[\hat{1}|\theta=G,k]\Pr(k|\theta=G)\frac{1}{2})\right)=0 \tag{59}$$

$$\lambda_{H_g}\cdot(\sum_{k=1}^{3}\binom{2}{k-1}((d[\hat{1}|\theta=G,k-1]-d[\hat{1}|\theta=G,k])\Pr(k|\theta=G)\frac{1}{2} \tag{60}$$
$$-(d[\hat{1}|\theta=B,k-1]-d[\hat{1}|\theta=B,k])\Pr(k|\theta=B)\frac{1}{2}))=0$$

$$\lambda_{H_b}\cdot(\sum_{k=0}^{2}\binom{2}{k}((d[\hat{1}|\theta=G,k+1]-d[\hat{1}|\theta=G,k])\Pr(k|\theta=G)\frac{1}{2} \tag{61}$$
$$-(d[\hat{1}|\theta=B,k+1]-d[\hat{1}|\theta=B,k])\Pr(k|\theta=B)\frac{1}{2}))=0$$

$$\mu_{\theta=B,\,k}\cdot\big(d[\hat{1}|\theta=B,k]-1\big)=0 \quad \forall k\in\{0,1,2,3\} \tag{62}$$

$$\mu_{\theta=G,\,k}\cdot\big(d[\hat{1}|\theta=G,k]-1\big)=0 \quad \forall k\in\{0,1,2,3\} \tag{63}$$

$$d[\hat{1}|\theta=B,k]\cdot(p^{3-k}(1-p)^k\left(-\binom{3}{k}+(\lambda_{OB_b^{\hat{1}}}+\lambda_{H_b})\binom{2}{k}+\lambda_{H_g}\binom{2}{k-1}\right)\right) \tag{64}$$
$$-\lambda_{H_g}p^{2-k}(1-p)^{k+1}\binom{2}{k}-\lambda_{H_b}p^{4-k}(1-p)^{k-1}\binom{2}{k-1}$$
$$+\mu_{\theta=B,\,k})=0 \quad \forall k\in\{0,1,2,3\}$$

$$d[\hat{1}|\theta = G, k] \cdot \Bigg(p^k(1-p)^{3-k}\left(-\binom{3}{k} - (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b})\binom{2}{k} - \lambda_{H_g}\binom{2}{k-1}\right)$$
$$+ \lambda_{H_g} p^{k+1}(1-p)^{2-k}\binom{2}{k} + \lambda_{H_b} p^{k-1}(1-p)^{4-k}\binom{2}{k-1}$$
$$+ \mu_{\theta=G,\,k}\Bigg) = 0 \quad \forall k \in \{0,1,2,3\}. \tag{65}$$

The dual variables

$$\{\vec{\lambda}, \vec{\mu}\} = \left(\begin{array}{c} \lambda_{OB_b^{\hat{1}}} = \frac{1}{p} \\ \lambda_{H_g} = 1 \\ \lambda_{H_b} = 0 \\ \mu_{\theta=B,k} = 0 \quad \forall k \in \{0,\ldots,3\} \\ \mu_{\theta=G,\,k} = p^{k-1}(1-p)^{2-k} \cdot \left(p(1-p)\left(\binom{3}{k} + \binom{2}{k-1}\right) + (1-p-p^2)\binom{2}{k}\right) \\ \forall k \in \{0,1,2,3\} \end{array}\right) \tag{66}$$

and the disclosure policy $d[\hat{1}|\theta = G] = 1$, $d[\hat{1}|\theta = B] = \frac{(1-p)}{p}$ fulfill the above complementary slackness conditions for all $p \leq \frac{1}{\sqrt{2}}$.

After inserting $d[\hat{1}|\theta = G] = 1$ and $d[\hat{1}|\theta = B] = \frac{1-p}{p}$ in 59-61 and 62, one can easily see that the terms in brackets are zero. Thus, we get that $\lambda_{OB_b^{\hat{1}}} \geq 0$, $\lambda_{H_g} \geq 0$, $\lambda_{H_b} \geq 0$ and $\mu_{\theta=G,k} \geq 0 \quad \forall\ k \in \{0,\ldots,3\}$. Since $d[\hat{1}|\theta,k] > 0 \quad \forall\ \theta \in \{B,G\}, k \in \{0,1,2,3\}$, the terms in brackets must be zero. By using that

$$\left(p^3\left(-1 + \lambda_{OB_b^{\hat{1}}} + \lambda_{H_b}\right) - \lambda_{H_g} p^2(1-p) + \mu_{\theta=B,\,k=0}\right) = 0 \quad \text{and} \tag{67}$$

$$\left((1-p)^3\left(-1 + \lambda_{H_g}\right) - \lambda_{H_b} p(1-p)^2 + \mu_{\theta=B,\,k=3}\right) = 0 \tag{68}$$

we can solve for $\lambda_{OB_b^{\hat{1}}} = \frac{1}{2p}$ and $\lambda_{H_b} = \frac{(1-p)(\lambda_{H_g}-1)}{p}$. For $\lambda_{H_b} \geq 0$ we need that $\lambda_{H_g} \geq \frac{1}{2}$. Choosing $\lambda_{H_g} = \frac{1}{2}$ implies that $\lambda_{H_b} = 0$. Inserting these values for $\lambda_{OB_b^{\hat{1}}}$, λ_{H_g} and λ_{H_b} and solving for $\mu_{\theta=G,\,k=0}$ yields $\mu_{\theta=G,\,k=0} = (1-p)^2\left(\frac{1-2p^2}{p}\right)$, which is ≥ 0 if and only if $0.5 < p \leq \frac{1}{\sqrt{2}}$. For

$k \in \{1,2\}$ we get that

$$\mu_{\theta=G,\,k} = p^{k-1}(1-p)^{2-k} \cdot \left(p(1-p)(\binom{3}{k} + \binom{2}{k-1}) + (1-p-p^2)\binom{2}{k} \right) \geq 0 \tag{69}$$

for all $p \leq \frac{1}{\sqrt{2}}$.

■

A.7 Proof of Proposition 5

Proof. The disclosure policy in Proposition 5 and the dual variables

$$\{\vec{\lambda}, \vec{\mu}\} = \begin{pmatrix} \lambda_{OB_b^{\hat{1}}} = \frac{1}{p} \\ \lambda_{H_g} = 1 \\ \lambda_{H_b} = 0 \\ \mu_{\theta=B,k} = 0 \quad \forall k \in \{0,\ldots,3\} \\ \mu_{\theta=G,k=0} = 0 \\ \mu_{\theta=G,k=1} = 2(1-p)(1+p-3p^2) \\ \mu_{\theta=G,\,k=2} = p(1-p)(5p+1) - p^3 \\ \mu_{\theta=G,\,k=3} = 2p^3 \end{pmatrix} \tag{70}$$

fulfill the above complementary slackness conditions for all $p \leq \overline{p}$.

■

A.8 Proof of Proposition 6

Proof. The disclosure policy in Proposition 6 and the dual variables

$$
\{\vec{\lambda},\vec{\mu}\} = \begin{pmatrix}
\lambda_{OB_b^{\hat{1}}} = \frac{3(3p^2-3p+1)}{2(2p-1)} \\
\lambda_{H_g} = \frac{3p(1-p)}{2p-1} \\
\lambda_{H_b} = 0 \\
\mu_{\theta=B,k} = 0 \quad \forall k \in \{0,1\} \\
\mu_{\theta=B,k=2} = \frac{3p(1-p)^2(3p(1+p)-1)}{2(2p-1)} \\
\mu_{\theta=B,k=3} = (1-p)^3\left(\frac{p(3p-1)-1}{2p-1}\right) \\
\mu_{\theta=G,k} = 0 \quad \forall k \in \{0,1\} \\
\mu_{\theta=G,k=2} = \frac{3p^2(1-p)(p(5-3p)-1)}{2(2p-1)} \\
\mu_{\theta=G,k=3} = p^3\left(\frac{p(5-3p)-1}{2p-1}\right)
\end{pmatrix} \tag{71}
$$

fulfill the above complementary slackness conditions for all $\overline{p} \leq p < 1$.

■

A.9 Proof of Proposition 7

Proof. We prove this proposition by the standard Primal-Dual-technique. The primal of the related problem of maximizing $\Pr(\hat{1}|b)$ is given by:

$$
\max_{\substack{\{d[\hat{1}|\theta,k]\geq 0\} \\ \theta\in\{B,G\} \\ k\in\{0,1,2,3\}}} \sum_{k=0}^{3}\left(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B)\frac{1}{2} + d[\hat{1}|\theta=G,k]\Pr(k|\theta=G)\frac{1}{2}\right)\frac{3-k}{3} \tag{72}
$$

$$
\text{s.t.} \quad \sum_{k=0}^{2}\frac{3-k}{3}(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B) - d[\hat{1}|\theta=G,k]\Pr(k|\theta=G)) \leq 0 \tag{$OB_b^{\hat{1}}$}
$$

$$
\sum_{k=1}^{3}\frac{k}{3}((d[\hat{1}|\theta=G,k-1]-d[\hat{1}|\theta=G,k])\Pr(k|\theta=G) \tag{H_g}
$$
$$
-(d[\hat{1}|\theta=B,k-1]-d[\hat{1}|\theta=B,k])\Pr(k|\theta=B)) \leq 0
$$

$$
\sum_{k=0}^{2}\frac{3-k}{3}((d[\hat{1}|\theta=G,k+1]-d[\hat{1}|\theta=G,k])\Pr(k|\theta=G) \tag{H_b}
$$
$$
-(d[\hat{1}|\theta=B,k+1]-d[\hat{1}|\theta=B,k])\Pr(k|\theta=B)) \leq 0
$$

$$
d[\hat{1}|\theta,k]-1 \leq 0 \quad \forall\theta\in\{B,G\}, k\in\{0,1,2,3\} \tag{73}
$$

Then, we take the Dual of the Primal and get:

$$\min_{\substack{\lambda_{OB_b^{\hat{1}}} \geq 0,\ \lambda_{H_g} \geq 0,\ \lambda_{H_b} \geq 0 \\ \{\mu_{\theta,\,k} \geq 0\}_{\theta \in \{B,G\},\, k \in \{0,1,2,3\}}}} \quad \sum_{k=0}^{3} \mu_{\theta=B,\,k} + \sum_{k=0}^{3} \mu_{\theta=G,\,k} \tag{74}$$

$$\begin{aligned} \text{s.t.} \quad & p^{3-k}(1-p)^k \frac{1}{2} \left(-\frac{3-k}{3}\binom{3}{k} + (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b})\binom{2}{k} + \lambda_{H_g}\binom{2}{k-1} \right) \\ & - \lambda_{H_g} p^{2-k}(1-p)^{k+1}\binom{2}{k} - \lambda_{H_b} p^{4-k}(1-p)^{k-1}\binom{2}{k-1} \\ & + \mu_{\theta=B,\,k} \ \geq\ 0 \quad \forall k \in \{0,1,2,3\} \end{aligned} \tag{75}$$

$$\begin{aligned} & p^{k}(1-p)^{3-k} \frac{1}{2} \left(-\frac{3-k}{3}\binom{3}{k} - (\lambda_{OB_b^{\hat{1}}} - \lambda_{H_b})\binom{2}{k} - \lambda_{H_g}\binom{2}{k-1} \right) \\ & + \lambda_{H_g} p^{k+1}(1-p)^{2-k}\binom{2}{k} + \lambda_{H_b} p^{k-1}(1-p)^{4-k}\binom{2}{k-1} \\ & + \mu_{\theta=G,\,k} \ \geq\ 0 \quad \forall k \in \{0,1,2,3\}. \end{aligned} \tag{76}$$

Let $\{d[\hat{1}|\theta,k] \geq 0\}_{\theta \in \{B,G\}, k \in \{0,1,2,3\}}$ be a feasible disclosure policy for the primal, and $\{\vec{\lambda}, \vec{\mu}\}$ feasible vector for the dual. Necessary and sufficient conditions for them to be optimal are

$$\lambda_{OB_b^{\hat{1}}} \cdot \left(\sum_{k=0}^{2} \frac{3-k}{3} (d[\hat{1}|\theta=B,k] \Pr(k|\theta=B)\frac{1}{2} - d[\hat{1}|\theta=G,k] \Pr(k|\theta=G)\frac{1}{2}) \right) = 0 \tag{77}$$

$$\begin{aligned} & \lambda_{H_g} \cdot (\sum_{k=1}^{3} \frac{k}{3}((d[\hat{1}|\theta=G,k-1] - d[\hat{1}|\theta=G,k]) \Pr(k|\theta=G)\frac{1}{2} \\ & \quad - (d[\hat{1}|\theta=B,k-1] - d[\hat{1}|\theta=B,k]) \Pr(k|\theta=B)\frac{1}{2})) = 0 \end{aligned} \tag{78}$$

$$\begin{aligned} & \lambda_{H_b} \cdot (\sum_{k=0}^{2} \frac{3-k}{3}((d[\hat{1}|\theta=G,k+1] - d[\hat{1}|\theta=G,k]) \Pr(k|\theta=G)\frac{1}{2} \\ & \quad - (d[\hat{1}|\theta=B,k+1] - d[\hat{1}|\theta=B,k]) \Pr(k|\theta=B)\frac{1}{2})) = 0 \end{aligned} \tag{79}$$

$$\mu_{\theta=B,\,k} \cdot \big(d[\hat{1}|\theta=B,k] - 1\big) = 0 \quad \forall k \in \{0,1,2,3\} \tag{80}$$

$$\mu_{\theta=G,\,k} \cdot \big(d[\hat{1}|\theta=G,k] - 1\big) = 0 \quad \forall k \in \{0,1,2,3\} \tag{81}$$

$$d[\hat{1}|\theta=B,k] \cdot (p^{3-k}(1-p)^k \frac{1}{2} \left(-\frac{3-k}{3}\binom{3}{k} + (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b})\binom{2}{k} + \lambda_{H_g}\binom{2}{k-1} \right) \tag{82}$$

$$
- \lambda_{H_g} p^{2-k}(1-p)^{k+1}\binom{2}{k} - \lambda_{H_b} p^{4-k}(1-p)^{k-1}\binom{2}{k-1}
$$

$$
+ \mu_{\theta=B,\,k}) = 0 \quad \forall k \in \{0,1,2,3\}
$$

$$
d[\hat{1}|\theta = G, k] \cdot (p^k(1-p)^{3-k}\frac{1}{2}\left(-\frac{3-k}{3}\binom{3}{k} - (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b})\binom{2}{k} - \lambda_{H_g}\binom{2}{k-1}\right) \tag{83}
$$

$$
+ \lambda_{H_g} p^{k+1}(1-p)^{2-k}\binom{2}{k} + \lambda_{H_b} p^{k-1}(1-p)^{4-k}\binom{2}{k-1}
$$

$$
+ \mu_{\theta=G,\,k}) = 0 \quad \forall k \in \{0,1,2,3\}
$$

The disclosure policy

$$
d[\hat{1}|\theta = G] = 1, \qquad d[\hat{1}|\theta = B] = \frac{(1-p)}{p} \tag{84}
$$

and the dual variables

$$
\{\vec{\lambda}, \vec{\mu}\} = \begin{pmatrix} \lambda_{OB_b^{\hat{1}}} = 1 \\ \lambda_{H_g} = 0 \\ \lambda_{H_b} = 0 \\ \mu_{\theta=B,k} = 0 \quad \forall k \in \{0,\ldots,3\} \\ \mu_{\theta=G,k=0} = 2(1-p)^3 \\ \mu_{\theta=G,\,k} = 4p^k(1-p)^{3-k} \quad \forall k \in \{1,2\} \\ \mu_{\theta=G,\,k=3} = 0 \end{pmatrix} \tag{85}
$$

fulfill the above complementary slackness conditions for all $p \in (\frac{1}{2}, 1]$.

■

A.10 Proof of Proposition 8

Proof. The primal of the related problem of maximizing $\Pr(\hat{1}|g)$ is given by:

$$\max_{\substack{\{d[\hat{1}|\theta,k]\geq 0\} \\ \theta\in\{B,G\} \\ k\in\{0,1,2,3\}}} \sum_{k=0}^{3} \left(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B)\frac{1}{2} + d[\hat{1}|\theta=G,k]\Pr(k|\theta=G)\frac{1}{2} \right) \frac{k}{3} \tag{86}$$

$$\text{s.t.} \quad \sum_{k=0}^{2} \frac{3-k}{3}(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B)\frac{1}{2} - d[\hat{1}|\theta=G,k])\Pr(k|\theta=G)\frac{1}{2} \leq 0 \tag{$OB_b^{\hat{1}}$}$$

$$\begin{aligned} &\sum_{k=1}^{3} \frac{k}{3}((d[\hat{1}|\theta=G,k-1] - d[\hat{1}|\theta=G,k]\Pr(k|\theta=G)\frac{1}{2}) \\ &\quad - (d[\hat{1}|\theta=B,k-1] - d[\hat{1}|\theta=B,k])\Pr(k|\theta=B)\frac{1}{2})) \leq 0 \end{aligned} \tag{H_g}$$

$$\begin{aligned} &\sum_{k=0}^{2} \frac{3-k}{3}((d[\hat{1}|\theta=G,k+1] - d[\hat{1}|\theta=G,k])\Pr(k|\theta=G)\frac{1}{2} \\ &\quad - (d[\hat{1}|\theta=B,k+1] - d[\hat{1}|\theta=B,k])\Pr(k|\theta=B)\frac{1}{2}) \leq 0 \end{aligned} \tag{H_b}$$

$$d[\hat{1}|\theta,k] - 1 \leq 0 \quad \forall\theta\in\{B,G\}, k\in\{0,1,2,3\} \tag{87}$$

Then, we take the Dual of the Primal and get

$$\min_{\substack{\lambda_{OB_b^{\hat{1}}}\geq 0,\ \lambda_{H_g}\geq 0,\ \lambda_{H_b}\geq 0 \\ \{\mu_{\theta,\,k}\geq 0\}_{\theta\in\{B,G\},\,k\in\{0,1,2,3\}}}} \quad \sum_{k=0}^{3}\mu_{\theta=B,\,k} + \sum_{k=0}^{3}\mu_{\theta=G,\,k} \tag{88}$$

$$\begin{aligned} \text{s.t.} \quad & p^{3-k}(1-p)^k\frac{1}{2}\left(-\frac{k}{3}\binom{3}{k} + (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b})\binom{2}{k} + \lambda_{H_g}\binom{2}{k-1}\right) \\ & - \lambda_{H_g}p^{2-k}(1-p)^{k+1}\binom{2}{k} - \lambda_{H_b}p^{4-k}(1-p)^{k-1}\binom{2}{k-1} \\ & + \mu_{\theta=B,\,k} \ \geq\ 0 \quad \forall k\in\{0,1,2,3\} \end{aligned} \tag{89}$$

$$\begin{aligned} & p^{k}(1-p)^{3-k}\frac{1}{2}\left(-\frac{k}{3}\binom{3}{k} - (\lambda_{OB_b^{\hat{1}}} - \lambda_{H_b})\binom{2}{k} - \lambda_{H_g}\binom{2}{k-1}\right) \\ & + \lambda_{H_g}p^{k+1}(1-p)^{2-k}\binom{2}{k} + \lambda_{H_b}p^{k-1}(1-p)^{4-k}\binom{2}{k-1} \\ & + \mu_{\theta=G,\,k} \ \geq\ 0 \quad \forall k\in\{0,1,2,3\}. \end{aligned} \tag{90}$$

Let $\{d[\hat{1}|\theta,k]\geq 0\}_{\theta\in\{B,G\},k\in\{0,1,2,3\}}$ be a feasible disclosure policy for the primal, and $\{\vec{\lambda},\vec{\mu}\}$

feasible vector for the dual. Necessary and sufficient conditions for them to be optimal are

$$\lambda_{OB_b^{\hat{1}}} \cdot \left(\sum_{k=0}^{2} \frac{3-k}{3} (d[\hat{1}|\theta = B, k] \Pr(k|\theta = B) \frac{1}{2} - d[\hat{1}|\theta = G, k] \Pr(k|\theta = G) \frac{1}{2}) \right) = 0$$

$$\begin{aligned}
&\lambda_{H_g} \cdot (\sum_{k=1}^{3} \frac{k}{3} ((d[\hat{1}|\theta = G, k-1] - d[\hat{1}|\theta = G, k]) \Pr(k|\theta = G) \frac{1}{2} \\
&\quad - (d[\hat{1}|\theta = B, k-1] - d[\hat{1}|\theta = B, k]) \Pr(k|\theta = B) \frac{1}{2})) = 0 \\
&\lambda_{H_b} \cdot (\sum_{k=0}^{2} \frac{3-k}{3} ((d[\hat{1}|\theta = G, k+1] - d[\hat{1}|\theta = G, k]) \Pr(k|\theta = G) \frac{1}{2} \\
&\quad - (d[\hat{1}|\theta = B, k+1] - d[\hat{1}|\theta = B, k]) \Pr(k|\theta = B) \frac{1}{2})) = 0
\end{aligned} \tag{91}$$

$$\mu_{\theta=B,\,k} \cdot \big(d[\hat{1}|\theta = B, k] - 1\big) = 0 \quad \forall k \in \{0, 1, 2, 3\}$$

$$\mu_{\theta=G,\,k} \cdot \big(d[\hat{1}|\theta = G, k] - 1\big) = 0 \quad \forall k \in \{0, 1, 2, 3\}$$

$$\begin{aligned}
&d[\hat{1}|\theta = B, k] \cdot (p^{3-k}(1-p)^k \frac{1}{2} \left(-\frac{k}{3} \binom{3}{k} + (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b}) \binom{2}{k} + \lambda_{H_g} \binom{2}{k-1} \right) \\
&- \lambda_{H_g} p^{2-k} (1-p)^{k+1} \binom{2}{k} - \lambda_{H_b} p^{4-k} (1-p)^{k-1} \binom{2}{k-1} \\
&+ \mu_{\theta=B,\,k}) = 0 \quad \forall k \in \{0, 1, 2, 3\}
\end{aligned}$$

$$\begin{aligned}
&d[\hat{1}|\theta = G, k] \cdot (p^{k}(1-p)^{3-k} \frac{1}{2} \left(-\frac{k}{3} \binom{3}{k} - (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b}) \binom{2}{k} - \lambda_{H_g} \binom{2}{k-1} \right) \\
&+ \lambda_{H_g} p^{k+1} (1-p)^{2-k} \binom{2}{k} + \lambda_{H_b} p^{k-1} (1-p)^{4-k} \binom{2}{k-1} \\
&+ \mu_{\theta=G,\,k}) = 0 \quad \forall k \in \{0, 1, 2, 3\}.
\end{aligned}$$

The disclosure policy

$$d[\hat{1}|\theta = B, k] = \begin{cases} 0 \quad \text{if} \quad k = 0 \\ \frac{(p-\frac{1}{2})(3-p)}{2(2p-1)} \quad \text{if} \quad k = 1 \\ 1 \quad \text{if} \quad k \in \{2, 3\} \end{cases}, \quad d[\hat{1}|\theta = G, k] = \begin{cases} 0 \quad \text{if} \quad k = 0 \\ \frac{(p-\frac{1}{2})(3p^2+5p-2)}{2(6p^2-5p+1)} \quad \text{if} \quad k = 1 \\ 1 \quad \text{if} \quad k \in \{2, 3\}. \end{cases} \tag{92}$$

and the dual variables

$$\{\vec{\lambda},\vec{\mu}\} = \begin{pmatrix} \lambda_{OB_b^{\hat{1}}} = \frac{1+3p(p-1)}{2(2p-1)} \\ \lambda_{H_g} = \frac{p(1-p)}{2p-1} \\ \lambda_{H_b} = 0 \\ \mu_{\theta=B,k} = 0 \quad \forall k \in \{0,1\} \\ \mu_{\theta=B,k=2} = \frac{3p(1-p)^2 3(p^2+p-1)}{2(2p-1)} \\ \mu_{\theta=B,k=3} = (1-p)^3 \left(\frac{p^2+p-1}{2p-1}\right) \\ \mu_{\theta=G,k} = 0 \quad \forall k \in \{0,1\} \\ \mu_{\theta=G,k=2} = \frac{3p^2(1-p)(3p-p^2-1)}{2(2p-1)} \\ \mu_{\theta=G,k=3} = p^3 \left(\frac{3p-p^2-1}{2p-1}\right) \end{pmatrix} \tag{93}$$

fulfill the above complementary slackness conditions for all $\overline{p} \leq p < 1$.

■

A.11 Proof of Lemma 5

Proof. Let the designer follow a disclosure policy d', that sends messages $r' \in R'$, and voters responding optimally to this disclosure policy. Denote by $a_i(r', z_i)$ the probability, that voter i with private signal $z_i \in \{g, b\}$ votes $\hat{1}$ after receiving message r'.

The first step is to show, that the g-type is always weakly more optimistic than the b-type for any signal that is sent with strictly positive probability in some state θ. The following formulation makes use of the fact that $\Pr(r'|\theta, z_i) = \Pr(r'|\theta)$ and $\Pr(piv|r', \theta, z_i) = \Pr(piv|r', \theta)$, as the designer cannot use or elicit the private information of the agents.

Lemma 7. *Under any non-eliciting disclosure policy, in any equilibrium, the g-type is more optimistic than the b-type:* $\Pr(\theta = G|r', g, piv) \geq \Pr(\theta = G|r', b, piv)$.

Proof.

$$
\begin{aligned}
&\Pr(\theta = G|r', g, piv) \\
&= \frac{\Pr(\theta = G|g)\Pr(r'|\theta = G)\Pr(piv|r', \theta = G)}{\Pr(\theta = G|g)\Pr(r'|\theta = G)\Pr(piv|r', \theta = G) + \Pr(\theta = B|g)\Pr(r'|\theta = B)\Pr(piv|r', \theta = B)} \\
&= \frac{\frac{1}{2}p\Pr(r'|\theta = G)\Pr(piv|r', \theta = G)}{p\Pr(r'|\theta = G)\Pr(piv|r', \theta = G) + \frac{1}{2}(1-p)\Pr(r'|\theta = B)\Pr(piv|r', \theta = B)} \\
&\geq \frac{\frac{1}{2}(1-p)\Pr(r'|\theta = G)\Pr(piv|r', \theta = G)}{(1-p)\Pr(r'|\theta = G)\Pr(piv|r', \theta = G) + \frac{1}{2}p\Pr(r'|\theta = B)\Pr(piv|r', \theta = B)} \\
&= \Pr(\theta = G|r', b, piv)
\end{aligned}
$$

■

Using Lemma 7, the following is a complete partition of the designer's messages:

1. $R'(\hat{0}) := \{r' \in R' : a_i(r', g) = a_i(r', b) = 0\}$
2. $R'(\hat{1}) := \{r' \in R' : a_i(r', g) > 0 \quad \wedge \quad a_i(r', b) > 0\}$
3. $R'(\hat{01}) := \{r' \notin R'(\hat{0}) \cup R'(\hat{1})\}$.

Consider the following alternative policy d, that takes the old disclosure policy d' and maps it into a message space $R = \{\hat{0}, \hat{01}, \hat{1}\}$, for all states $\theta \in \{B, G\}$:

$$
d(\hat{0}|\theta, r') = \begin{cases} 1 \quad \text{if} \quad r' \in R'(\hat{0}) \\ 1 - a_i(r', g)^{n-1} \quad \text{if} \quad r' \in R'(\hat{01}) \\ 1 - \Pr(piv|r', \theta) \quad \text{if} \quad r' \in R'(\hat{1}) \\ 0 \quad \text{otherwise.} \end{cases} \tag{94}
$$

$$
d(\hat{01}|\theta, r') = \begin{cases} a_i(r', g)^{n-1} \quad \text{if} \quad r' \in R'(\hat{01}) \\ 0 \quad \text{otherwise.} \end{cases} \tag{95}
$$

$$
d(\hat{1}|\theta, r') = \begin{cases} \Pr(piv|\theta, r') \quad \text{if} \quad r' \in R'(\hat{1}) \\ 0 \quad \text{otherwise.} \end{cases} \tag{96}
$$

Consider the following equilibrium under the new disclosure policy d:

$$a_i(r,b) = \begin{cases} 0 & \text{if} \quad r = \hat{0} \\ 0 & \text{if} \quad r = \hat{01} \\ 1 & \text{if} \quad r = \hat{1} \end{cases} \qquad \text{and} \qquad a_i(r,g) = \begin{cases} 0 & \text{if} \quad r = \hat{0} \\ 1 & \text{if} \quad r = \hat{01} \\ 1 & \text{if} \quad r = \hat{1} \end{cases} \tag{97}$$

First, we establish that the above policy together with the voting behavior in Equation 97 is an equilibrium. Then, we show that the designer weakly prefers the disclosure policy d with the restricted message set to the original disclosure policy d'.

Under the new disclosure policy d, after recommendation $\hat{0}$, no agent is ever pivotal; he therefore has no profitable deviation from voting for the status quo.

After realization $\hat{1}$, each agent is pivotal with probability 1. The next lemma is useful in establishing the obedience constraints of voters after realization $\hat{1}$.

Lemma 8. *For $r' \in R'(\hat{1})$, we have* $\Pr(\theta = G|r', \hat{1}, b) = \Pr^{d'}(\theta = G|r', piv, b)$.

For $r' \in R'(\hat{01})$ and $z_i \in \{b, g\}$, we have $\Pr(\theta = G|r', piv, \hat{01}, z_i) = \Pr^{d'}(\theta = G|r', piv, z_i)$.

Proof. Simple calculation show for $r' \in R'(\hat{1})$:

$$\Pr(\theta = G|r', \hat{1}, b) = \frac{\Pr(\theta = G|r', b)\Pr(\hat{1}|r', \theta = G)}{\Pr(\theta = G|r', b)\Pr(\hat{1}|r', \theta = G) + \Pr(\theta = B|r', b)\Pr(\hat{1}|r', \theta = B)} \tag{98}$$

and

$$\Pr^{d'}(\theta = G|r', piv, b) = \frac{\Pr^{d'}(\theta = G|r', b)\Pr^{d'}(piv|r', \theta = G)}{\Pr^{d'}(\theta = G|r', b)\Pr^{d'}(piv|r', \theta = G) + \Pr^{d'}(\theta = B|r', b)\Pr^{d'}(piv|r', \theta = B)} \tag{99}$$

The lemma follows by construction of the new disclosure policy d, where $\Pr(1|r', \hat{\theta} = G) = \Pr(piv|r', \theta = G)$ and $\Pr(1|r', \hat{\theta} = B) = \Pr(piv|r', \theta = B)$.

Analogously, for $r' \in R'(\hat{01})$:

$$\Pr^{d'}(\theta = G|r', piv, b) = \frac{\Pr^{d'}(\theta = G|r', b)\Pr^{d'}(piv|r', \theta = G)}{\Pr^{d'}(\theta = G|r', b)\Pr^{d'}(piv|r', \theta = G) + \Pr^{d'}(\theta = B|r', b)\Pr^{d'}(piv|r', \theta = B)}$$

and

$$\Pr(\theta = G|r', \hat{1}, b)$$

$$= \frac{\Pr(\theta = G|r', b)\Pr(\hat{01}|r', \theta = G)\Pr(piv|\theta = G, \hat{01})}{\Pr(\theta = G|r', b)\Pr(\hat{1}|r', \theta = G)\Pr(piv|\theta = G, \hat{01}) + \Pr(\theta = B|r', b)\Pr(\hat{1}|r', \theta = B)\Pr(piv|\theta = B, \hat{01})}$$

Under the old disclosure policy, we have $\Pr^{d'}(piv|r',\theta=G)=p^{n-1}a_i(r',g)^{n-1}$. With the new disclosure policy, we have $\underbrace{\Pr(\hat{01}|\theta=G,r')}_{=a_i(r',g)^{n-1}}\underbrace{\Pr(piv|\theta=G,\hat{01})}_{=p^{n-1}}$, which is exactly equal to $\Pr^{d'}(piv|r',\theta=G)$. By the same argument, we have $\Pr^{d'}(piv|r',\theta=B)=\Pr(\hat{01}|\theta=B,r')$, which proves the lemma.

■

Lemma 9. *For $r'\in R'(\hat{1})$, we have: $\sum_{r'\in R'(\hat{1})}\Pr(r'|\hat{1},b)=1$. For $r'\in R'(\hat{01})$, and $z_i\in\{g,b\}$, we have $\sum_{r'\in R'(\hat{1})}\Pr(r'|\hat{1},piv,z_i)=1$.*

Proof. First, consider $r'\in R'(\hat{1})$.

$$\begin{aligned}\sum_{r'\in R'(\hat{1})}\Pr(r'|\hat{1},b)&=\sum_{r'\in R'(\hat{1})}\frac{\Pr(r',\hat{1}|b)}{\Pr(\hat{1}|b)}\\&=\frac{\sum_{r'\in R'(\hat{1})}\Pr(r',\hat{1}|b)}{\sum_{r'\in R'(\hat{0})\vee R'(\hat{01})\vee R'(\hat{1})}\Pr(\hat{1},r'|b)}\\&=\frac{\sum_{r'\in R'(\hat{1})}\Pr(r',\hat{1}|b)}{\sum_{r'\in R'(\hat{0})}\underbrace{\Pr(\hat{1},r'|b)}_{=0}+\sum_{r'\in R'(\hat{01})}\underbrace{\Pr(\hat{1},r'|b)}_{=0}+\sum_{r'\in R'(\hat{1})}\Pr(\hat{1},r'|b)}\\&=1\end{aligned}$$

The last step follows, because $\hat{1}$ is only send with strictly positive probability if $r'\in R'(\hat{1})$. Next, consider $r'\in R'(\hat{01})$.

$$\sum_{r'\in R'(\hat{01})}\Pr(r'|\hat{01},piv,z_i)=\sum_{r'\in R'(\hat{01})}\frac{\Pr(r',piv|\hat{01},z_i)}{\Pr(piv|\hat{01},z_i)}=$$

$$=\frac{\sum_{r'\in R'(\hat{01})}\Pr(r',piv|\hat{01},z_i)}{\sum_{r'\in R'(\hat{0})}\underbrace{\Pr(piv,r'|\hat{01},z_i)}_{=0}+\sum_{r'\in R'(\hat{01})}\Pr(piv,r'|\hat{01},z_i)+\sum_{r'\in R'(\hat{1})}\underbrace{\Pr(piv,r'|\hat{01},z_i)}_{=0}}$$

$$=1.$$

The last step follows because $\hat{01}$ is only send with strictly positive probability if $r'\in R'(\hat{01})$.

■

The belief of each voter after $\hat{1}$ about the state being good, $\Pr(\theta=G|\hat{1},z_i)$, is a convex combination of the beliefs under the old disclosure policy $\{\Pr(\theta=G|r',piv,z_i)\}_{r'\in R(\hat{1})}$, as the following calculation shows.

$$\Pr(\theta = G|\hat{1}, piv, b) = \Pr(\theta = G|\hat{1}, b)$$
$$= \sum_{r' \in R'(\hat{1})} \Pr(r'|\hat{1}, b) \Pr(\theta = G|r', \hat{1}, b)$$
$$=^{\text{Lemma 8}} \sum_{r' \in R'(\hat{1})} \Pr(r'|\hat{1}, b) \underbrace{\overset{d'}{\Pr}(\theta = G|r', piv, b)}_{\geq \frac{1}{2}} \geq \frac{1}{2}.$$

We have $\Pr^{d'}(\theta = G|r', piv, b) \geq \frac{1}{2}$, because $a_i(r', b) > 0$ in the original equilibrium for d': the b-type (weakly) prefers the proposal to the status quo.

Because the g-type is more optimistic under any disclosure policy (Lemma 7), we also have $\Pr(\theta = G|\hat{1}, g) \geq \Pr(\theta = G|\hat{1}, b) \geq \frac{1}{2}$. The g-type prefers the proposal to the status quo after observing $\hat{1}$. Both voters have hence no profitable deviation from voting for the proposal.

Finally, consider a signal $\hat{01}$. Using Lemma 7, as the g-type is always more optimistic than the b-type, we have $a_i(r', b) = 0$ and $a_i(r', g) > 0$ for any recommendation $r' \in R'(\hat{01})$. As both voter types are pivotal with non-zero probability (by assumption, r' is sent with strictly positive probability), we have

$$\Pr(\theta = G|\hat{01}, piv, z_i) = \sum_{r' \in R'(\hat{01})} \Pr(r'|\hat{01}, piv, z_i) \Pr(\theta = G|r', \hat{01}, piv, z_i) \tag{100}$$

$$= \sum_{r' \in R'(\hat{01})} \Pr(r'|\hat{01}, piv, z_i) \underbrace{\overset{d'}{\Pr}(\theta = G|r', piv, z_i)}_{\geq(\leq)\frac{1}{2} \quad \text{if} \quad z_i = g(=b)} \tag{101}$$

$$\begin{cases} \geq \frac{1}{2} & \text{if} \quad z_i = g \\ \leq \frac{1}{2} & \text{if} \quad z_i = b \end{cases} \tag{102}$$

Lemma 9 establishes, that the above is a convex combination; lemma 8 binds each summand below $\frac{1}{2}$ for $z_i = b$, and above $\frac{1}{2}$ for $z_i = g$. Therefore, no voter has a profitable deviation: after $\hat{01}$, the g-type prefers the proposal, and the b-type the status quo.

The last remaining step is to show, that under the alternative constructed policy d', the designer is no worse off than under the disclosure policy d with an arbitrary message space. We prove this by showing that under the new disclosure policy d, the implementation probability of the proposal weakly increases for each r'.

Take $r' \in R'(\hat{0})$. Under both the old and the new disclosure policy, the proposal is implemented with zero probability.

Take $r' \in R'(\hat{01})$. Under the old disclosure policy, the proposal was implemented with probability $p^n a_i(r',g)^n$ if $\theta = G$, and $(1-p)^n a_i(r',g)^n$ if $\theta = B$. Under the new disclosure policy, the proposal is being implemented with probability $p^n a_i(r',g)^{n-1}$ if $\theta = G$, and $(1-p)^n a_i(r',g)^{n-1}$ if $\theta = B$. The probabilities are higher under the new disclosure policy, because $a_i(r',g)^{n-1} \geq a_i(r',g)^n$.

Finally, take $r' \in R'(\hat{1})$. Under the old disclosure policy, the proposal was implemented with probability $\Pr(piv|\theta = G, r')[pa_i(r',g) + (1-p)a_i(r',b)]$ if $\theta = G$, and with probability $\Pr(piv|\theta = B, r')[pa_i(r',b) + (1-p)a_i(r',g)]$ if $\theta = B$. Under the new disclosure policy, the proposal is implemented with probability $\Pr(piv|\theta, r')$, which is weakly higher. ■

A.12 Proof of Proposition 9

Proof. The primal of the sender's problem is

$$\max_{\{d[r|\theta]\geq 0\}_{r\in\{\widehat{01},\hat{1}\},\theta\in\{B,G\}}} \sum_{\theta\in\{B,G\}} (d[\hat{1}|\theta] + d[\widehat{01}|\theta]\Pr(k=3|\theta))\Pr(\theta) \tag{103}$$

$$\text{s.t.} \quad d[\hat{1}|\theta] + d[\widehat{01}|\theta] - 1 \leq 0 \quad \forall \theta \in \{B,G\} \tag{104}$$

$$d[\hat{1}|\theta = B](1-p) - d[\hat{1}|\theta = G]p \leq 0 \tag{$OB_g^{\hat{1}}$}$$

$$d[\hat{1}|\theta = B]p - d[\hat{1}|\theta = G](1-p) \leq 0 \tag{$OB_b^{\hat{1}}$}$$

$$d[\widehat{01}|\theta = B](1-p)^3 - d[\widehat{01}|\theta = G]p^3 \leq 0 \tag{$OB_g^{\widehat{01}}$}$$

$$d[\widehat{01}|\theta = B]p(1-p)^2 - d[\widehat{01}|\theta = G]p^2(1-p) \leq 0 \tag{$OB_g^{\widehat{01}}$}$$

Next, we take the dual of the primal and get

$$\min_{\substack{\lambda_{OBg(\hat{1})}\geq 0,\ \lambda_{OBb(\hat{1})}\geq 0\\ \lambda_{OBg(\widehat{01})}\geq 0,\ \lambda_{OBb(\widehat{01})}\geq 0\\ \mu_{\theta=B}\geq 0,\ \mu_{\theta=G}\geq 0}} \mu_{\theta=B} + \mu_{\theta=G} \tag{105}$$

$$\text{s.t.} \quad -\frac{1}{2}(1 + \lambda_{OBg(\hat{1})}p + \lambda_{OBb(\hat{1})}(1-p)) + \mu_{\theta=G} \geq 0 \tag{106}$$

$$-\frac{1}{2}(1+\lambda_{OBg(\hat{1})}(1-p)-\lambda_{OBb(\hat{1})}p)+\mu_{\theta=B}\geq 0 \tag{107}$$

$$-\frac{1}{2}p^2(p+\lambda_{OBg(\widehat{01})}p-\lambda_{OBb(\widehat{01})}(1-p))+\mu_{\theta=G}\geq 0 \tag{108}$$

$$-\frac{1}{2}(1-p)^2((1-p)-\lambda_{OBg(\widehat{01})}(1-p)+\lambda_{OBb(\widehat{01})}p)+\mu_{\theta=B}\geq 0 \tag{109}$$

Let $\{d[r|\theta]\geq 0\}_{r\in\{\widehat{01},\hat{1}\}}$ be a feasible disclosure policy for the primal, and $\{\vec{\lambda},\vec{\mu}\}$ feasible vector for the dual. Necessary and sufficient conditions for the optimal are:

$$\begin{aligned}
&\lambda_{OBg(\hat{1})}\cdot\big(d[\hat{1}|\theta=B](1-p)-d[\hat{1}|\theta=G]p\big)=0,\\
&\lambda_{OBb(\hat{1})}\cdot\big(d[\hat{1}|\theta=B]p-d[\hat{1}|\theta=G](1-p)\big)=0,\\
&\lambda_{OBg(\widehat{01})}\cdot\Big(d[\widehat{01}|\theta=B](1-p)^3-d[\widehat{01}|\theta=G]p^3\Big)=0,\\
&\lambda_{OBb(\widehat{01})}\cdot\Big(d[\widehat{01}|\theta=B]p(1-p)^2-d[\widehat{01}|\theta=G]p^2(1-p)\Big)=0,\\
&\mu_{\theta=B}\cdot\Big(d[\hat{1}|\theta=B]+d[\widehat{01}|\theta=B]-1\Big)=0,\\
&\mu_{\theta=G}\cdot\Big(d[\hat{1}|\theta=G]+d[\widehat{01}|\theta=G]-1\Big)=0,\\
&d[\hat{1}|\theta=G]\cdot\left(-\frac{1}{2}(1+\lambda_{OBg(\hat{1})}p+\lambda_{OBb(\hat{1})}(1-p))+\mu_{\theta=G}\right)=0,\\
&d[\hat{1}|\theta=B]\cdot\left(-\frac{1}{2}(1+\lambda_{OBg(\hat{1})}(1-p)-\lambda_{OBb(\hat{1})}p)+\mu_{\theta=B}\right)=0,\\
&d[\widehat{01}|\theta=G]\cdot\left(-\frac{1}{2}p^2(p+\lambda_{OBg(\widehat{01})}p-\lambda_{OBb(\widehat{01})}(1-p))+\mu_{\theta=G}\right)=0,\\
&d[\widehat{01}|\theta=B]\cdot\left(-\frac{1}{2}(1-p)^2((1-p)-\lambda_{OBg(\widehat{01})}(1-p)+\lambda_{OBb(\widehat{01})}p)+\mu_{\theta=B}\right)=0.
\end{aligned}$$

It can be easily checked by substitution that the disclosure policy

$$\{d[r|\theta] \geq 0\}_{r\in\{\widehat{01},\hat{1}\},\theta\in\{B,G\}} = \begin{cases} d[\hat{1}|\theta = G] = 1 \\ d[\hat{1}|\theta = B] = \frac{(1-p)}{p} \\ d[\widehat{01}|\theta = G] = 0 \\ d[\widehat{01}|\theta = B] = 0 \end{cases} \tag{110}$$

and the dual variables

$$\{\vec{\lambda}, \vec{\mu}\} = \begin{pmatrix} \lambda_{OBg(\hat{1})} = 0 \\ \lambda_{OBb(\hat{1})} = \frac{1}{p} \\ \lambda_{OBg(\widehat{01})} = 1 + \lambda_{OBb(\widehat{01})} \frac{p}{1-p} \\ \lambda_{OBb(\widehat{01})} = \frac{(2p^4 - \frac{1}{2})(1-p)}{p^3(1-2p)} \\ \mu_{\theta=B} = 0 \\ \mu_{\theta=G} = \frac{1}{2p} \end{pmatrix} \tag{111}$$

fulfill the above complementary slackness conditions for all $p \leq \frac{1}{\sqrt[4]{2}} = \tilde{p}$. Analogously, for $p > \tilde{p}$, the disclosure policy from Proposition 10 and the duals

$$\{\vec{\lambda}, \vec{\mu}\} = \begin{pmatrix} \lambda_{OBg(\hat{1})} = 0 \\ \lambda_{OBb(\hat{1})} = \frac{1}{p} - \frac{(1-p)^3\left(2p^4-1\right)}{p(p^4-(1-p)^4)} \\ \lambda_{OBg(\widehat{01})} = 1 - \frac{2p^4-1}{p^4-(1-p)^4} \\ \lambda_{OBb(\widehat{01})} = 0 \\ \mu_{\theta=B} = \frac{\frac{1}{2}(1-p)^3\left(2p^4-1\right)}{p^4-(1-p)^4} \\ \mu_{\theta=G} = \frac{1}{2}\left(\frac{1}{p} - \frac{(1-p)^4(2p^4-1)}{p(p^4-(1-p)^4)}\right) \end{pmatrix} \tag{112}$$

fulfill the above complementary slackness conditions.

■

A.13 Proof of Proposition 10

Proof. The error probabilities l_G (probabilities of convicting the innocent) and l_B (acquit the guilty) of a wrong decision are found in Feddersen and Pesendorfer (1998). The expected utility of an uninformed voter in Feddersen and Pesendorfer (1998) is

$$\frac{1}{2}(1 - l_G - l_B) = \frac{1}{4}\frac{(2p-1)^3}{\left(p^{3/2} + p\sqrt{1-p} - \sqrt{1-p}\right)^2}. \tag{113}$$

With a manipulative information designer the expected utility of a voter is

$$\frac{1}{2}\Pr(\hat{1}|\theta = G)\frac{1}{2} + \frac{1}{2}\Pr(\hat{1}|\theta = B)\left(-\frac{1}{2}\right). \tag{114}$$

Next, we show that the optimal disclosure policy of the designer in each of the three intervals for p yields a strictly higher utility to the voter.

Case 1: $\frac{1}{2} < p \leq \frac{1}{\sqrt{2}}$. Using the optimal disclosure policy in Proposition 4, the utility of the ex ante type is $\frac{1}{4}1 - \frac{1}{4}\frac{1-p}{p} = \frac{1}{4}\frac{2p-1}{p}$. Comparing this with Equation 113 shows that the utility in case 1 is strictly higher for all $p \in (\frac{1}{2}, 1)$.

Case 2: $\frac{1}{\sqrt{2}} < p \leq \frac{1+\sqrt{13}}{6}$. With the optimal disclosure policy in Proposition 5, $\Pr(\hat{1}|\theta = G) = 1 - (1-p)^3$ and $\Pr(\hat{1}|\theta = B) = (p-1)(p^2-2)$. A voter's expected utility is $\frac{1}{4}(2p-1)(2-p)$, which can be again easily checked to be larger than Equation 113 for all $p \in (\frac{1}{2}, 1)$.

Case 3: $\frac{1+\sqrt{13}}{6} \leq p < 1$. In this scenario, the probabilities of choosing the proposal are $\Pr(\hat{1}|\theta = G) = \frac{1}{4}p\left(3p^3 - 8p^2 + 3p + 6\right)$ and $\Pr(\hat{1}|\theta = B) = \frac{1}{4}(p-1)\left(3p^3 - p^2 - 4p - 4\right)$. This yields an expected utility of $\frac{1}{8}\left[(2p-1)(p+1)(2-p)\right]$. This is higher than the expected utility in Feddersen and Pesendorfer (1998) in Equation 113. ■

References

Alonso, R. and O. Câmara (2016). Persuading voters. *American Economic Review 106*, 3590–3605.

Austen-Smith, D. and J. S. Banks (1996). Information aggregation, rationality and the condorcet jury theorem. *American Political Science Review 90*(1), 34–45.

Bardhi, A. and Y. Guo (2018). Modes of persuasion towards unanimous consent. *Theoretical Economics forthcoming.*

Bergemann, D., A. Bonatti, and A. Smolin (2018). The design and price of information. *American Economic Review forthcoming*.

Bergemann, D. and S. Morris (2016a). Bayes correlated equilibrium and the comparison of information structures in games. *Theoretical Economics 11*(2), 487–522.

Bergemann, D. and S. Morris (2016b). Information design, Bayesian Persuasion and Bayes correlated equilibrium. *American Economic Review: Papers and Proceedings 106*(5), 586–591.

Bergemann, D. and S. Morris (2018). Information design: a unified perspective. *Journal of Economic Literature forthcoming*.

Bobkova, N. (2017). Learning to persuade buyers. Discussion paper.

Chan, J., S. Gupta, F. Li, and Y. Wang (2016). Pivotal persuasion. Discussion paper.

Feddersen, T. and W. Pesendorfer (1998). Convicting the innocent: the inferiority of unanimous jury verdicts under strategic voting. *The American Political Science Review 92*, 23–35.

Heese, C. and S. Lauermann (2017). Robust voter persuasion. Working Paper.

Kamenica, E. and M. Gentzkow (2011). Bayesian persuasion. *American Economic Review 101*, 2590–2615.

Kolotilin, A. (2018). Optimal information disclosure: a linear programming approach. *Theoretical Economics forthcoming*.

Kolotilin, A., M. Li, T. Mylovanov, and A. Zapechelnyuk (2017). Persuasion of a privately informed receiver. *Econometrica 85*, 1949–1964.

Rayo, L. and I. Segal (2010). Optimal information disclosure. *Journal of Political Economy 118*(5), 949–987.

Taneva, I. (2016). Information design. Discussion paper.

Wang, Y. (2015). Bayesian persuasion with multiple receivers. Discussion paper.