Discussion Paper Series – CRC TR 224

Discussion Paper No. 235
Project C 02

Falling Natural Rates, Rising Housing Volatility and the Optimal Inflation Target

Klaus Adam [1]
Oliver Pfäuti [2]
Timo Reinelt [3]

November 2020

[1] University of Mannheim, Germany; E-mail: adam@uni-mannheim.de
[1] University of Mannheim, Germany; E-mail: oliver.pfaeuti@gess.unimannheim.de
[1] University of Mannheim, Germany; E-mail: timo.reinelt@gess.uni-mannheim.de

Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Falling Natural Rates, Rising Housing Volatility and the Optimal Inflation Target

Klaus Adam, Oliver Pfäuti, Timo Reinelt*

November 4, 2020

Abstract

The decline in natural interest rates in advanced economies over the past decades has been accompanied by a significant increase in the volatility of housing prices. We show that the monetary policy implications of these macroeconomic trends depend—in the presence of a lower-bound constraint on nominal rates—on the source of increased housing price volatility. If housing price expectations are rational, increased housing price volatility reflects more volatile housing demand shocks. Under optimal monetary policy, average inflation then increases only minimally, as average natural rates fall and housing shocks become more volatile. Instead, if housing price volatility is partly due to speculative housing price beliefs, as suggested by survey data, then lower natural rates endogenously trigger larger fluctuations in subjective housing price beliefs and housing prices. A belief-driven increase in housing price volatility causes also the natural rate of interest to become more volatile. This exacerbates the lower-bound problem, especially when average natural rates are low. Under optimal monetary policy, average inflation then rises much more strongly following a fall in natural rates, rationalizing larger increases in the inflation target.

1 Introduction

The persistent fall in natural interest rates and long-term growth rates represent some of the most troubling macroeconomic trends in advanced economies over the past decades (Holston et al. (2017), Del Negro et al. (2017) and Fujiwara et al. (2016)). These trends have received considerable attention in policy circles and in the academic literature and even led to a revival of the secular stagnation hypothesis, 75 years after Hansen (1939) has coined the term to describe periods with low interest rates, slow economic growth and dampened aggregate demand (Summers (2014)).

*Department of Economics, University of Mannheim, adam@uni-mannheim.de, oliver.pfaeuti@gess.uni-mannheim.de, timo.reinelt@gess.uni-mannheim.de. The authors gratefully acknowledge financial support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 (Project C02).

This paper documents that the fall in long-term growth rates and natural interest rates has been associated with additional macroeconomic trends that are particularly troubling from a monetary policy perspective: the U.S. and other advanced economies experienced also a considerable increase in the volatility of housing prices. In addition, there are indications that the *volatility* of natural rates and the volatility of housing investment rates have also increased over time.

The goal of this paper is to assess how these macroeconomic trends affect the conduct of optimal monetary policy and the optimal inflation target that monetary policy should pursue. We do so by considering a sticky price model featuring a lower-bound constraint for nominal interest rates and a housing sector. The model is consistent with lower growth rates triggering lower natural rates, which in turn increase the volatility in housing markets. Yet, the monetary policy implications of increased housing market volatility depends crucially on the economic drivers responsible for the increase in housing price volatility.

If housing prices are driven purely by structural shocks, which may have increased in size over time, then average inflation rises only mildly under optimal monetary policy when average natural rates fall. Instead, if housing price fluctuations are partly driven by speculative housing price beliefs, as suggested by survey data on consumers' capital gain expectations in housing markets, then average inflation must rise considerably more as natural rates fall. This is so because belief-driven increases in the volatility of housing prices also increase the volatility of the natural rate. This exacerbates the lower-bound problem for monetary policy, requiring it to promise higher inflation when being at the lower bound constraint, even though higher inflation has adverse welfare consequences. As we show, it also makes it optimal for monetary policy to lean against housing demand shocks, unlike in a setting with rational housing expectations.

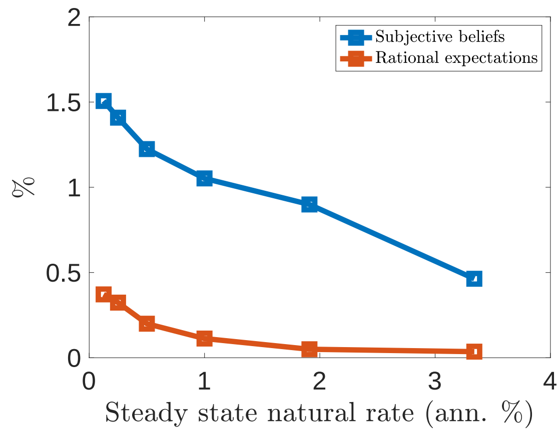
To analyze monetary policy in the presence of speculative private sector expectations, we consider a setting in which households and firms are *internally rational*, i.e., maximize utility (or profits) conditional on their subjective beliefs about variables outside of their control (Adam and Marcet (2011)). Internal rationality encompasses fully rational expectations as a special case, when subjective beliefs coincide with the objective outcomes generated by the model.

Our analysis considers both a setting with rational housing price expectations and a setting in which subjective beliefs give rise to extrapolation of past housing price movements into the future, following Adam et al. (2016), and in line with survey evidence on consumers' capital gain expectations in housing markets. Extrapolation generates housing prices that are characterized by momentum as well as mean reversion, featuring occasional and long-lived boom-bust patterns, consistent with the empirically observed behavior of housing prices (Glaeser and Nathanson (2017)).[1] To make the deviations from rational expectations as parsimonious as possible, we assume throughout the paper that expectations about variables other than housing prices (and associated rental rates) are rational at all times.

Internally rational agents condition their optimal consumption and investment plan on

[1]Experimental evidence on housing price expectations (Armona et al. (2019)) and survey evidence on stock price expectations (Greenwood and Shleifer (2014), Adam et al. (2017)) provide additional support for the extrapolative nature of housing price expectations implied by our subjective belief setup. Soo (2018) constructs a housing sentiment index based on media coverage of housing news, documenting the extrapolative nature of house price expectations.

Figure 1: Average Inflation under Optimal Monetary Policy



their subjective beliefs about future housing price developments. Therefore, the real interest rate that is consistent with the objectively efficient consumption allocation – the natural interest rate – will respond to changes in subjective housing price beliefs. Since the subjective housing price beliefs become endogenously more volatile as the average natural rate falls, as we explain further in the main part of the paper, a fall in natural rates triggers larger volatility of the natural rate. This exacerbates the lower bound problem for monetary policy in a setting where natural rates are low and induce higher average inflation rates under Ramsey optimal monetary policy.

We calibrate the model such that it matches the observed standard deviation of the price-to-rent ratio and the natural interest rate in the United States before 1990. When moving from a relatively high average value for the natural rate, as observed before the 1990s, to the lower average level observed thereafter, the rational expectations model generates an increase in the volatility of the price-to-rent ratio that is much smaller than in the data and no additional volatility for the natural rate.[2] As a result, even when the average natural rates falls to a level of 0.25% per year, the average inflation rate under optimal monetary policy increases only to 0.32% in our calibrated model. Under rational expectations and optimal monetary policy, lower natural rates thus do not justify significant increases in the inflation target, see figure 1.

In contrast, the model with subjective housing price beliefs matches the bulk of the observed increase in housing volatility following the decline in natural rates after 1990, without having to resort to increased volatility of housing demand shocks. This is the case because lower natural rates amplify belief-driven fluctuations in housing prices and thereby magnify momentum effects in prices. The more volatile housing beliefs contribute to a considerable increase in the volatility of the natural rate, close to the one estimated for the U.S. after 1990. In response, average inflation increases substantially more under Ramsey optimal monetary policy compared to a setting with rational expectations. For an average

[2]Under rational expectations, housing-price-to-rent ratios are solely driven by housing preference shocks, which themselves do not affect the natural rate.

natural rate of 0.25% per year, optimal monetary policy gives rise to an average inflation rate of 1.41%, i.e., to a much higher optimal inflation target than under rational expectations, see figure 1.

Eggertsson et al. (2019) and Andrade et al. (2018) also study the implications of lower natural rates for monetary policy. Consistent with our findings, they show that a substantial increase in the inflation target is a promising monetary instrument to deal with the zero lower bound (ZLB) problem. We add to their work by studying Ramsey optimal policy, by considering a model featuring a housing sector, and by allowing for the presence of subjective beliefs.

Earlier work discussed Ramsey optimal monetary policy in the presence of a ZLB constraint, but abstracted from housing markets and the presence of subjective beliefs. A seminal early contribution is due to Eggertsson and Woodford (2003), which shows how commitment to future inflation can help ameliorating the effects of not being able to lower nominal rates further at the ZLB. The one-time nature of the shock in their analysis precluded, however, a discussion of the effects of the ZLB on average inflation. Adam and Billi (2006) and Coibion et al. (2012) discuss Ramsey optimal monetary policy under rational expectations in a setting where repeated shocks cause the ZLB to be occasionally binding. They find that ZLB episodes tend to be short and infrequent under optimal policy, so that average inflation increases by a few basis points only, compared to a setting that ignores the existence of the ZLB constraint. The present paper shows that this conclusion is altered in a model with housing, when the average natural interest rate falls and the resulting increase in the volatility of housing prices causes also increases in the volatility of the natural rate.

As we show, the implications of asset price booms and busts for monetary policy generally depend on the source of these fluctuations. Iacoviello (2005) introduces housing markets into a monetary business cycle model and shows that output and inflation volatility are almost independent of whether or not the monetary authority responds to house prices; a finding documented for asset prices more generally in Bernanke and Gertler (2001) and Gilchrist and Leahy (2002). In Iacoviello (2005), house price fluctuations are driven by structural shocks and are thus efficient. In contrast, Adam and Woodford (2020) analyze optimal monetary policy in a setting that allows for deviations from efficient house price fluctuations. As beliefs differ from rational expectations, a monetary tightening in response to an increase in house prices becomes optimal in the presence of positive housing subsidies. The same is true for the present setting. In the presence of subjective beliefs, optimal monetary policy leans against housing demand shocks, while this fails to be the case in a setting with rational housing prices. Caines and Winkler (2020) also find that leaning against housing price increases can be optimal when beliefs are distorted. We add to this literature by taking into account the ZLB constraint on nominal interest rates and by considering a more parsimonious deviation from rational expectations that allows only for deviations of housing price beliefs.

More generally, Kaplan et al. (2020) argue that subjective beliefs about future house prices were the most important driver of housing prices in the U.S. economy during the boom and bust phase around the Great Recession. Adam et al. (2012) find that subjective housing price dynamics can explain the dynamics of housing booms and current accounts in the G7 economies up until the Great Recession. Adelino et al. (2017) document that rising home prices and increased housing price expectations were the main contributor to the observed increase in U.S. mortgage debt before the Great Recession.[3] Section 5.3 in the present paper shows that the housing price expectations of U.S. housholds are inconsistent with the RE hypothesis. Taken together these contributions render credibility to the notion that subjective housing price beliefs play an important role for housing price dynamics, making it important to consider the monetary policy implications of such fluctuations in the presence of a ZLB constraint.

The present paper abstracts from a number of economic features that can generate additional effects on the optimal inflation target, e.g., cash-distortions as analyzed in Khan et al. (2003), nominal wage rigidity considered in Benigno and Ricci (2011), or the relative price trends recently studied in Adam and Weber (2019, 2020).

The rest of the paper is structured as follows. Section 2 presents the empirical facts about natural rates, housing prices and housing investment. Section 3 introduces the economic model, which allows for subjective beliefs, and section 4 derives the nonlinear optimal monetary policy problem. The two alternative drivers of house price fluctuations —housing preference shocks and subjective housing price beliefs—are introduced in Section 5. Section 6 presents a quadratic approximation to the monetary policy problem, which allows to gain analytic insights. We calibrate the model in section 7 and present our main results in section 8. Section 9 concludes.

2 Natural Rates and Housing Prices in Advanced Economies

2.1 Natural Rates: Declining Levels and Rising Volatility

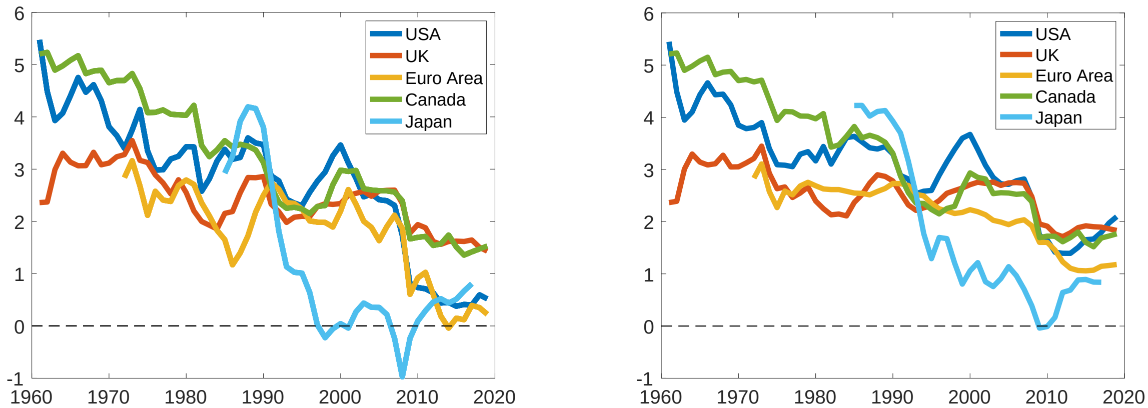
Natural real interest rates and long-term growth rates have displayed a steady downward trend in the G7 economies over the past decades. Figure 2 illustrates these trends using the estimated natural rates as well as long-term growth rates (both annualized) of Holston et al. (2017) and Fujiwara et al. (2016).

While these facts have received considerable attention in the literature, the evolution of the volatility of the natural rate over time has received virtually no attention. Figure 3 depicts the estimated standard deviation of the natural rate pre and post 1990 after taking out a linear time trend. The figure reports the point estimates (the colored bars), the 90% confidence bands (black lines), and the p-values for the null hypothesis that the volatility has not changed from pre to post 1990. The point estimates have increased over time in all currency areas, except for the U.K. Yet, since estimation uncertainty is relatively large, the increase in the point estimates are not statistically significant, except for the United States and Canada. We take this as tentative evidence that the documented fall in the level of the natural rate is associated with an increase in its volatility in the G7 currency areas.

[3]Further evidence on the important role of expectations as drivers of house prices is documented, e.g., in Case and Shiller (1988), Shiller (2007), Piazzesi and Schneider (2009), Case et al. (2012), and Ben-David et al. (2019).

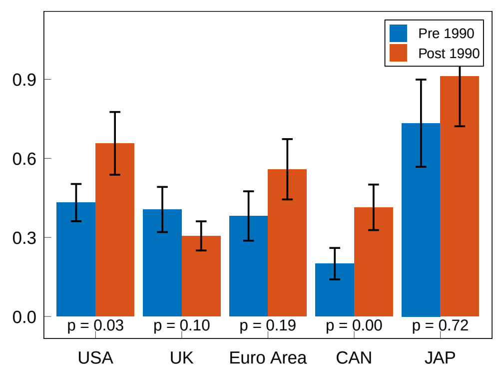
Figure 2: Natural Rates and Long-Term Growth Rates

(a) Natural Rates of Interest (b) Long-Term Growth Rates



Source: Holston et al. (2017) and Fujiwara et al. (2016)

Figure 3: Volatility of Natural Rates



Source: Holston et al. (2017) and Fujiwara et al. (2016) (natural rate estimates). The black lines denote the 90%-confidence bands. The reported p-values are for the null hypothesis that volatility has not changed from pre to post 1990.

2.2 The Rising Volatility of Housing Prices

We now show that the volatility of housing prices has increased markedly in the G7 economies over the period 1970-2019. To the best of our knowledge, we are the first to document this development.

It is generally difficult to estimate the volatility of housing prices in a precise manner because housing prices tend to be rather volatile and also tend to display a high degree of persistence over time. Table 1 considers the behavior of the annual price-to-rent (PR) ratios in the G7 economies and reports the autoregressive coefficient of an estimated AR(1) process in the PR ratios. The table reports estimates for the whole sample, as well as for the first and last 30 years of the sample period.[4] It shows that the price-to-rent ratios are overall

[4]The numbers below the estimated coefficients are the 95% confidence intervals.

highly persistent and for some countries the point estimates are very close to 1, especially after 1990.

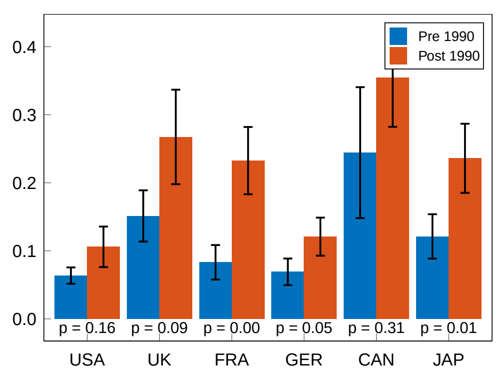
Table 1: Annual Persistence of Price-to-Rent Ratios

Sample	USA	Japan	Germany	France	UK	Canada
All Periods	0.90	0.95	0.97	0.98	0.97	0.99
	[0.79, 1.01]	[0.85, 1.05]	[0.92, 1.03]	[0.92, 1.05]	[0.88, 1.07]	[0.95, 1.04]
Pre 1990	0.81	0.89	0.88	0.89	0.84	0.97
	[0.52, 1.11]	[0.59, 1.19]	[0.63, 1.13]	[0.68, 1.10]	[0.47, 1.21]	[0.80, 1.15]
Post 1990	0.91	0.99	0.98	0.97	0.96	0.99
	[0.75, 1.06]	[0.93, 1.06]	[0.87, 1.08]	[0.86, 1.07]	[0.86, 1.07]	[0.90, 1.08]

Source: OECD database, own calculations. The 95%-confidence intervals are reported in brackets.

To deal with these features of housing price behavior, we compare the volatility of the price-to-rent ratio over 30-year long subsamples, i.e., for the years 1970-1989 and 1990-2019. Figure 4 reports the estimated standard deviation of the G7 price-to-rent ratios for the first and second subsample, together with 90%-confidence bands (black lines), and the p-value of the null-hypothesis that the standard deviations have not changed across the two subsamples.[5] The point estimate of the standard deviation of the PR ratio has increased in all G7 economies. Appendix B.1 shows that these results are robust to using different sample split points. This said, due to the large estimation uncertainty, not all increases in the point estimates are statistically significant at conventional significance levels: at the 10% confidence level, 4 of the 6 observations are statistically significant.

Figure 4: Standard Deviation of the Price-to-Rent Ratio, 1970-1989 versus 1990-2019



Source: OECD database. The black lines denote the 90%-confidence bands. The p-values are for the null hypothesis the standard deviation has not changed pre to post 1990.

Panel (a) in Figure 5 reports further evidence on the standard deviation of the residential investment to output ratio (RIY-ratio) in the first and second part of the sample. It shows

[5] The reported standard deviations are expressed in terms of % deviation from the sample mean of the considered period, so as to be consistent with the theoretical model developed later on. Considering instead the absolute standard deviation across the two subperiods leads to identical conclusions, see Appendix B.1.

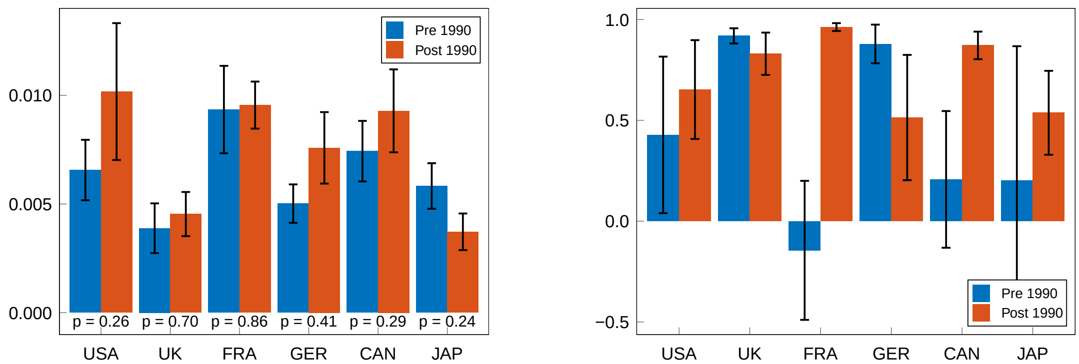
that all countries, except for Japan, saw the point estimate for the standard deviation of the RIY-ratio increase, which suggests that the increase in the volatility of the PR ratio documented above was associated with an increase in the volatility of the RIY ratio, even if the increase in investment tends not to be statistically significant. Appendix B.1 shows that these results are again quite robust to using different sample split points.

Panel (b) in Figure 5 depicts the time-series correlation between the PR-ratio and the RIY-ratio in the two sample periods. While the correlation in the first sample period is statistically insignificant for three of the considered countries, the correlation is very positive and statistically significant in the latter sample period in all countries. This is suggestive of the fact that more volatile PR-ratios in the second half of the sample have been associated with stronger association between housing investment and housng prices.

Figure 5: Housing Volatility Increased Over the Last Decades.

(a) Standard Deviation of Residential Investment Pre and Post 1990.

(b) Correlation of Price-to-Rent Ratios and Residential Investment Pre and Post 1990.



Source: Holston et al. (2017) and Fujiwara et al. (2016) (natural rate estimates) and OECD database (PR and RIY ratios), own calculations. The black lines denote the 90%-confidence bands. The p-values correspond to the test whether or not the values changed from pre to post 1990.

3 A Sticky Price Model with Housing

We build in our analysis on the sticky price model with housing and subjective beliefs developed in Adam and Woodford (2020). We augment it by considering a rental market for housing and by explicitly incorporating a zero lower bound constraint for nominal interest rates.[6] We also allow for more general forms of belief distortions in the private sector, including subjective beliefs that imply that private decisionmakers may not know the equilibrium

[6]The existence of an occasionally binding ZLB constraint critically complicates the nature of the optimal policy problem because the consumption Euler equation is then an implementability constraint that needs to be taken into account when designing optimal monetary policy. When the ZLB is assumed to be never binding, one can ignore the consumption Euler equation when determining Ramsey optimal monetary policy, because interest rates can be determined ex-post to make the Euler equation hold.

mapping from fundamentals to market outcomes.[7] Admitting these more general forms of belief distortions allows the model to generate realistic amounts of housing price volatility, without having to resort to the presence of large housing preference shocks, and it allows replicating important features of the structure of housing forecast errors, as observed in survey data.

Following Adam and Marcet (2011), we consider an economy populated by internally rational decisionmakers. Households (firms) maximize utility (profits), but entertain a potentially subjective probability measure $\mathcal{P}$, which assigns probabilities to all external variables, i.e., to all variables that agents take as given in their decision problem. These variables include fundamental shocks, as well as competitive market prices (wages, goods prices, housing prices and rents). The setup delivers rational expectations in the special case when $\mathcal{P}$ is the rational probability measure.

The economy is made up of identical infinitely-lived households, each of which maximizes the following objective function[8]

$$U \equiv E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \left[\tilde{u}(C_t; \xi_t) - \int_0^1 \tilde{v}(H_t(j); \xi_t) dj + \tilde{\omega}(D_t + D_t^R; \xi_t) \right], \tag{1}$$

subject to a sequence of flow budget constraints

$$\begin{aligned} & C_t + B_t + (D_t - (1-\delta) D_{t-1}) \frac{q_t^u}{\tilde{u}_C(C_t; \xi_t)} + k_t + R_t D_t^R = \\ & \tilde{d}(k_t; \xi_t) \frac{q_t^u}{\tilde{u}_C(C_t; \xi_t)} + \int_0^1 w_t(j) H_t(j) dj + \frac{B_{t-1}}{\Pi_t}(1 + i_{t-1}) + \frac{\Sigma_t}{P_t} + \frac{T_t}{P_t}, \end{aligned} \tag{2}$$

where C_t is an aggregate consumption good, $H_t(j)$ is the quantity supplied of labor of type j and $w_t(j)$ the associated real wage, D_t the stock of owned houses, D_t^R the units of rented houses, $\delta \in [0, 1]$ the housing depreciation rate, q_t^u the real price of houses in marginal utility units, defined as

$$q_t^u \equiv q_t \tilde{u}_C(C; \xi_t),$$

where q_t is the real house price in units of consumption. The variable q_t^u provides a measure of whether housing is currently expensive or inexpensive, in units that are particularly relevant for determining housing demand. The variable k_t denotes investment in new houses and $\tilde{d}(k_t; \xi_t)$ the resulting production of new houses.[9] The variable $B_t \equiv \widetilde{B}_t / P_t$ denotes the real value of nominal government bond holdings $\widetilde{B}_t$, P_t the nominal price of consumption, $\Pi_t = P_t / P_{t-1}$ the inflation rate, i_t the nominal interest rate, R_t the real rental rate for housing units, and ξ_t is a vector of exogenous disturbances, which may induce random shifts in the functions $\tilde{u}$, $\tilde{v}$, $\tilde{\omega}$ and $\tilde{d}$. The variable T_t denotes nominal lump sum transfers (taxes

[7] The setup in Adam and Woodford (2020) assumed absolute continuity between subjective and objective beliefs over arbitrary finite horizons, which implies that agents know the equilibrium mapping from fundamentals to market outcomes. The present setting nest this setup as a special case.

[8] It cannot be common knowledge to households that they are representative whenever $\mathcal{P}$ deviates from the rational measure.

[9] We consolidate housing production into the household budget constraint. It would be equivalent to have instead a separate housing production section that is owned by households.

if negative) from the government and Σ_t nominal profits accruing to households from the ownership of firms.

Households discount future payoffs at the rate $\beta \in (0,1)$. Since our model is formulated in terms of growth-detrended variables, the discount rate β jointly captures the time preference rate $\widetilde{\beta} \in (0,1)$ and the steady-state growth rate of marginal utility. Letting $g_c \geq 0$ denote the steady-state growth rate of consumption in non-detrended terms, we have

$$\beta \equiv \widetilde{\beta} \frac{\tilde{u}_C(C(1+g_c);\underline{\xi})}{\tilde{u}_C(C;\underline{\xi})}, \tag{3}$$

where $\underline{\xi}$ denotes the steady state value of the disturbance ξ_t. When the growth rate g_c of the economy falls, then the discount rate β increases because marginal utility falls less fast. We can thus capture the fall in economic growth rates via a simple increase in the time discount rate β. In line with the empirical evidence, the increase in the discount rate β will cause safe interest rates to fall.

The aggregate consumption good is a Dixit-Stiglitz aggregate of each of a continuum of differentiated goods,

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \tag{4}$$

with an elasticity of substitution $\eta > 1$. We further assume isoelastic functional forms

$$\begin{aligned}
\tilde{u}(C_t;\xi_t) &\equiv \frac{C_t^{1-\tilde{\sigma}^{-1}} \bar{C}_t^{\tilde{\sigma}^{-1}}}{1-\tilde{\sigma}^{-1}}, \\
\tilde{v}(H_t(j);\xi_t) &\equiv \frac{\lambda}{1+\nu} \left(H_t(j)\right)^{1+\nu} \bar{H}_t^{-\nu}, \\
\tilde{\omega}(D_t + D_t^R;\xi_t) &\equiv \xi_t^d \left(D_t + D_t^R\right), \\
\tilde{d}(k_t;\xi_t) &\equiv \frac{A_t^d}{\tilde{\alpha}} k_t^{\tilde{\alpha}},
\end{aligned} \tag{5}$$

where $\tilde{\sigma}, \nu > 0$, $\tilde{\alpha} \in (0,1)$ and $\{\bar{C}_t, \bar{H}_t, \xi_t^d, A_t^d\}$ are bounded exogenous and positive disturbance processes which are among the exogenous disturbances included in the vector ξ_t.

Our specification includes two housing related disturbances, namely ξ_t^d which captures shocks to housing preferences and A_t^d shocks to the productivity in the construction of new houses. We impose linearity in the utility function (5) as this greatly facilitates the characterization of optimal policy, with rented and owned housing units being perfect substitutes. We could introduce a weight on rental units relative to housing units that would allow us to perfectly match the average price-to-rent ratio we observe in the data. But since this does not change any other results, we abstract from such a scaling parameter and assign equal weight to housing and renting in the utility.

The housing demand shock ξ_t^d evolves according to

$$\xi_t^d / \underline{\xi}^d = \left(\xi_{t-1}^d / \underline{\xi}^d\right)^{\rho_\xi} e^{\varepsilon_t^d}, \tag{6}$$

where $\underline{\xi}^d$ is the steady state value of the housing demand disturbance, ε_t^d an i.i.d. innovation satisfying $E[e^{\varepsilon_t^d}] = 1$, and $|\rho_\xi| < 1$ captures the persistence of housing demand disturbances.

Each differentiated good is supplied by a single monopolistically competitive producer; there is a common technology for the production of all goods, in which (industry-specific) labor is the only variable input,

$$y_t(i) = A_t f(h_t(i)) = A_t h_t(i)^{1/\phi}, \tag{7}$$

where A_t is an exogenously varying technology factor, and $\phi > 1$. The Dixit-Stiglitz preferences (4) imply that the quantity demanded of each individual good i will equal[10]

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\eta}, \tag{8}$$

where Y_t is the total demand for the composite good defined in (4), $p_t(i)$ is the (money) price of the individual good, and P_t is the price index,

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}}, \tag{9}$$

corresponding to the minimum cost for which a unit of the composite good can be purchased in period t. Total demand is given by

$$Y_t = C_t + k_t + g_t Y_t, \tag{10}$$

where g_t is the share of the total amount of composite goods purchased by the government, treated here as an exogenous disturbance process.

3.1 Household Optimality Conditions

Internally rational households choose state-contingent sequences for the choice variables $\{C_t, H_t(j), D_t, D_t^R, k_t, B_t\}$ so as to maximize (1), subject to the budget constraints (2), taking as given their beliefs about the processes $\{P_t, w_t(j), q_t^u, R_t, i_t, \Sigma_t/P_t, T_t/P_t\}$, as determined by the (subjective) measure $\mathcal{P}$.

We shall be particularly interested in the policy implications generated by subjective housing price beliefs. To insure that an optimum exists in the presence of potentially distorted beliefs about the housing price q_t^u, we require housing choices to lie in some compact choice set $D_t \in [0, D^{\max}]$, where $D^{\max} < \infty$ is an arbitrarily large but finite upper bound on the quantity of housing the household can purchase. We choose $D^{\max}$ large enough, such that it will never bind in equilibrium.

The first order conditions give rise to an optimal labor supply relation

$$w_t(j) = \frac{\tilde{v}_H(H_t(j); \xi_t)}{\tilde{u}_C(C_t; \xi_t)}, \tag{11}$$

[10]In addition to assuming that household utility depends only on the quantity obtained of C_t, we assume that the government also cares only about the quantity obtained of the composite good defined by (4), and that it seeks to obtain this good through a minimum-cost combination of purchases of individual goods.

a consumption Euler equation

$$\tilde{u}_C(C_t;\xi_t) = \beta E_t^{\mathcal{P}}\left[\tilde{u}_C(C_{t+1};\xi_{t+1})\frac{1+i_t}{P_{t+1}/P_t}\right], \tag{12}$$

an equation characterizing optimal investment in new houses

$$k_t = \left(A_t^d q_t^u \frac{C_t^{\tilde{\sigma}^{-1}}}{\bar{C}_t^{\tilde{\sigma}^{-1}}}\right)^{\frac{1}{1-\tilde{\alpha}}}, \tag{13}$$

an optimality condition for rental units

$$\xi_t^d = R_t \tilde{u}_C(C_t,\xi_t), \tag{14}$$

and a set of conditions determining the optimal housing demand D_t :

$$\begin{array}{ll} q_t^u < \xi_t^d + \beta(1-\delta)E_t^{\mathcal{P}} q_{t+1}^u & \text{if } D_t = D^{\max} \\ q_t^u = \xi_t^d + \beta(1-\delta)E_t^{\mathcal{P}} q_{t+1}^u & \text{if } D_t \in [0, D^{\max}] \\ q_t^u > \xi_t^d + \beta(1-\delta)E_t^{\mathcal{P}} q_{t+1}^u & \text{if } D_t = 0. \end{array} \tag{15}$$

With rational expectations, the upper and lower holding bounds never bind.[11] We are, however, interested in how the presence of belief distortions about future housing values affect equilibrium outcomes. With subjective housing price expectations, the holding bounds in equation (15) can potentially bind under the *subjectively* optimal plans. This explains why an internally rational household can hold subjective housing price expectations, even if it holds rational expectations about the preference shocks ξ_T^d in equation (15).[12]

Forward-iterating on equation (12), which holds with equality under all belief-specifications, delivers a present-value formulation of the consumption Euler equation

$$\tilde{u}_C(C_t;\xi_t) = \lim_{T\to\infty} E_t^{\mathcal{P}}\left[\tilde{u}_C(C_T;\xi_T)\beta^T \prod_{k=0}^{T-t}\frac{1+i_{t+k}}{P_{t+k+1}/P_{t+k}}\right], \tag{16}$$

which will be convenient to work with, especially under subjective belief specifications. Household choices must also satisfy the transversality constraint

$$\lim_{T\to\infty}\beta^T E_t^{\mathcal{P}}\left(\tilde{u}_C(C_T;\xi_T)B_T + D_T q_T^u\right) = 0. \tag{17}$$

Optimal household behavior under potentially distorted beliefs is jointly characterized by equations (11) and (13)-(17).

[11] The upper bound $D^{\max}$ has been chosen sufficiently large for this to be true. The lower bound is never reached because the housing production function satisfies Inada conditions.

[12] See Adam and Marcet (2011) for a detailed discussion of this point.

3.2 Optimal Price Setting by Firms

The producers in each industry fix the prices of their goods in monetary units for a random interval of time, as in the model of staggered pricing introduced by Calvo (1983) and Yun (1996). Producers use the representative households' subjectively optimal consumption plans to discount profits and are assumed to know the product demand function (8). They need to formulate beliefs about the future price levels P_T, industry wages $w_T(j)$, aggregate demand Y_T, and productivity A_T.

Let $0 \leq \alpha < 1$ be the fraction of prices that remain unchanged in any period. A supplier i in industry j that changes its price in period t chooses its new price $p_t(i)$ to maximize

$$E_t^{\mathcal{P}} \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi\left(p_t(i), P_T, w_T(j), Y_T, A_T\right), \tag{18}$$

where $E_t^{\mathcal{P}}$ denotes the expectations of price setters conditional on time t information, which are identical to the expectations held by consumers. Firms discount random nominal income in period T using households' subjective stochastic discount factor $Q_{t,T}$, which is given by

$$Q_{t,T} = \beta^{T-t} \frac{\tilde{u}_C\left(C_T, \xi_T\right)}{\tilde{u}_C\left(C_t, \xi_t\right)} \frac{P_t}{P_T}.$$

The term α^{T-t} in equation (18) captures the probability that a price chosen in period t will not have been revised by period T, and the function $\Pi\left(p_t(i), ...\right)$ indicates the nominal profits of the firm in period t, as discussed next.

Profits are equal to after-tax sales revenues net of the wage bill. Sales revenues are determined by the demand function (8), so that (nominal) after-tax revenue equals

$$\left(1 - \tau_t\right) p_t(i) Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\eta}.$$

Here τ_t is a proportional tax on sales revenues in period t, $\{\tau_t\}$ is treated as an exogenous disturbance process, taken as given by the monetary policymaker. We assume that τ_t fluctuates over a small interval around a non-zero steady state level $\underline{\tau}$. We allow for exogenous variations in the tax rate in order to include the possibility of "pure cost-push shocks" that affect the equilibrium pricing behavior while implying no change in the efficient allocation of resources.

The labor demand of firm i at a given industry-specific wage $w_t(j)$ can be written as

$$h_t(i) = \left(\frac{Y_t}{A_t}\right)^{\phi} p_t(i)^{-\eta\phi} P_t^{\eta\phi}, \tag{19}$$

which follows from (7) and (8). Using this, the nominal wage bill is given by

$$P_t w_t(j) h_t(i) = P_t w_t(j) \left(\frac{Y_t}{A_t}\right)^{\phi} p_t(i)^{-\eta\phi} P_t^{\eta\phi}.$$

Subtracting the nominal wage bill from the above expression for nominal after tax revenue, we obtain the function $\Pi\left(p_t(i), P_T, w_T(j), Y_T, A_T\right)$ used in (18).

Each of the suppliers that revise their prices in period t chooses the same new price p_t^*, that maximizes (18). The first-order condition with respect to $p_t(i)$ is given by[13]

$$E_t^{\mathcal{P}} \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi_1 \left(p_t(i), P_T, w_T(j), Y_T, A_T\right) = 0.$$

The equilibrium choice p_t^*, which is the same for each firm i in industry j, is the solution to this equation. Letting p_t^j denote the price charged by firms in industry j at time t, we have $p_t^j = p_t^*$ in periods in which industry j resets its prices and $p_t^j = p_{t-1}^j$ otherwise.

Under the assumed isoelastic functional forms, the optimal choice has a closed-form solution

$$\left(\frac{p_t^*}{P_t}\right)^{1+\eta(\phi-1)} = \frac{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \frac{\eta}{\eta-1} \phi w_T(j) \left(\frac{Y_T}{A_T}\right)^{\phi} \left(\frac{P_T}{P_t}\right)^{\eta\phi+1}}{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} (1-\tau_T) Y_T \left(\frac{P_T}{P_t}\right)^{\eta}}. \tag{20}$$

The price index evolves according to a law of motion

$$P_t = \left[(1-\alpha)\, p_t^{*1-\eta} + \alpha P_{t-1}^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{21}$$

as a consequence of (9). The equilibrium inflation in any period is characterized by

$$\left(\frac{P_t}{P_{t-1}}\right)^{\eta-1} = \frac{1-(1-\alpha)\left(\frac{p_t^*}{P_t}\right)^{1-\eta}}{\alpha}. \tag{22}$$

The welfare loss from price adjustment frictions can be captured by price dispersion, which is defined as

$$\Delta_t \equiv \int_0^1 \left(\frac{p_t^j}{P_t}\right)^{-\eta(1+\omega)} dj \geq 1, \tag{23}$$

where

$$\omega \equiv \phi(1+\nu) - 1 > 0$$

is the elasticity of real marginal cost in an industry with respect to industry output.

Using equation (21) together with the fact that the relative prices of the industries that do not change their prices in period t remain the same, one can derive a law of motion for the price dispersion term Δ_t of the form

$$\Delta_t = h(\Delta_{t-1}, P_t/P_{t-1}), \tag{24}$$

with

$$h(\Delta_t, P_t/P_{t-1}) \;\equiv\; \alpha \Delta_t \left(\frac{P_t}{P_{t-1}}\right)^{\eta(1+\omega)} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{P_t}{P_{t-1}}\right)^{\eta-1}}{1-\alpha}\right)^{\frac{\eta(1+\omega)}{\eta-1}}.$$

[13]Note that supplier i's profits in (18) are a concave function of the quantity sold $y_t(i)$, since revenues are proportional to $y_t(i)^{\frac{\eta-1}{\eta}}$ and hence concave in $y_t(i)$, while costs are convex in $y_t(i)$. Moreover, since $y_t(i)$ is proportional to $p_t(i)^{-\eta}$, the profit function is also concave in $p_t(i)^{-\eta}$. The first-order condition for the optimal choice of the price $p_t(i)$ is the same as the one with respect to $p_t(i)^{-\eta}$; hence the first-order condition with respect to $p_t(i)$ is both necessary and sufficient for an optimum.

As is commonly done, we assume that the initial degree of price dispersion is small ($\Delta_{-1} \sim O(2)$).

Equations (20), (22), and (24) jointly define a short-run aggregate supply relation between inflation, output and house prices (via the aggregate demand equation (10) and (13)), given the current disturbances ξ_t, and expectations regarding future wages, prices, output, consumption and disturbances. Equation (24) describes the evolution of the costs of price dispersion over time.

For future reference, we remark that all firms together make total profits equal to

$$\frac{\Sigma_t}{P_t} = (1 - \tau_t) Y_t - w_t H_t, \tag{25}$$

where $w_t H_t = \int_0^1 w_t(j) H_t(j) dj$.

3.3 Government

The government consumes goods $g_t Y_t$, imposes a sales tax τ_t, issues nominal bonds $\widetilde{B}_t \equiv P_t B_t$, and pays for lump sum transfers T_t to households. The government budget constraint is given by

$$B_t = B_{t-1} \frac{1 + i_{t-1}}{P_t / P_{t-1}} + \frac{T_t}{P_t} + (g_t - \tau_t) Y_t.$$

For simplicity, we assume that lump sum transfers (taxes if negative) are set such that they keep real government debt constant at some initial level B_{-1}. This implies that government transfers are given by

$$\frac{T_t}{P_t} = -(g_t - \tau_t) Y_t + B_{t-1} \left(1 - \frac{1 + i_{t-1}}{P_t / P_{t-1}} \right). \tag{26}$$

3.4 Market Clearing Conditions

Using (10) and (13), one can express the market clearing condition for the consumption/investment good as

$$Y_t = \frac{C_t + \Omega_t C_t^{\frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}}}}{1 - g_t}, \tag{27}$$

where

$$\Omega_t \equiv \left(A_t^d \bar{C}_t^{-\tilde{\sigma}^{-1}} q_t^u \right)^{\frac{1}{1-\tilde{\alpha}}} > 0 \tag{28}$$

is a term that depends on exogenous shocks and belief distortions in the housing market only, see equation (15). The previous two equations implicity define a function

$$C_t = C(Y_t, q_t^u, \xi_t), \tag{29}$$

which delivers the market clearing consumption level, for a given output level Y_t, given housing prices q_t^u and given exogenous disturbances ξ_t.

The market clearing condition for housing is

$$D_t = (1 - \delta) D_{t-1} + \tilde{d}(k_t; \xi_t), \tag{30}$$

and rental market clearing requires

$$D_t^R = 0. \tag{31}$$

Labor market clearing requires that the supply of labor of type j in (11) is equal to labor demand of industry j, which is given by (19), as all firms in the industry charge the same price. This delivers

$$w_t(j) = \frac{\tilde{v}_H(H_t(j);\xi_t)}{\tilde{u}_C(C_t;\xi_t)} = \frac{\lambda\,(H_t(j))^\nu\,\bar{H}_t^{-\nu}}{C_t^{-\tilde{\sigma}^{-1}}\bar{C}_t^{\tilde{\sigma}^{-1}}} = \lambda\frac{\bar{H}_t^{-\nu}}{\bar{C}_t^{\tilde{\sigma}^{-1}}}\left(\frac{Y_t}{A_t}\right)^{\nu\phi} C_t^{\tilde{\sigma}^{-1}}\left(\frac{p_t^j}{P_t}\right)^{-\nu\eta\phi}, \tag{32}$$

where $p_t^j = p_t^*$ in periods where industry j can adjust prices and $p_t^j = p_{t-1}^j$ otherwise.

3.5 Internally Rational Expectations Equilibrium

We are now able to define an *internally rational expectations equilibrium (IREE).*

Definition 1 *An internally rational expectations equilibrium is a bounded stochastic process for* $\{Y_t, C_t, k_t, D_t, \{w_t(j)\}, p_t^*, P_t, \Delta_t, q_t^u, i_t\}_{t=0}^\infty$ *satisfying the aggregate supply equations (20), and (22), the law of motion for the evolution of price distortions (24), the household optimality conditions (13), (15), (16), and the market clearing conditions (27), (30) and (32), where the latter has to hold for all industries* j*.*

The equilibrium features ten variables (counting the continuum of wages as a single variable) that must satisfy nine conditions, leaving one degree of freedom to be determined by monetary policy.[14] In the special case with rational expectations, $E_t^{\mathcal{P}}[\cdot] = E_t[\cdot]$, the IREE is a rational expectations equilbrium (REE).

Given the equilibrium outcomes, the remaining model variables can be determined as follows. Equilibrium profits are given by equation (25) and equilibrium taxes by equation (26). Equilibrium labor supply $H_t(j)$ follows from equation (11) for each type of labor j. Equilibrium bond holdings satisfy $B_t = B_{-1}$ and equilibrium inflation is

$$\Pi_t \equiv P_t/P_{t-1}.$$

Equilibrium rental units follow from equation (31) and equilibrium rental prices from equation (14).

4 The Nonlinear Optimal Policy Problem

We shall consider Ramsey optimal policies in which the policymaker chooses the sequence of policy rates, prices and allocations to maximize household utility under rational expectations, subject to the constraint that prices and allocations constitute an Internally Rational Expectations Equilibrium. The policymaker thus maximizes utility under a probability measure that is different from the one entertained by households, whenever the latter hold distorted beliefs. Benigno and Paciello (2014) refer to such a policymaker as being 'paternalistic'.

[14] The transversality condition (17) must also be satisfied in equilibrium, but is not imposed as an equilibrium condition, as it will hold for all belief specifications considered below.

The objective of the policymaker is to maximize household utility. Using equation (8) to express the relative quantities demanded of the differentiated goods each period as a function of their relative prices and the linear dependence of utility on the stock of assets, we can write the utility flow to the representative household in the form

$$u(Y_t, q_t^u; \xi_t) - v(Y_t; \xi_t)\Delta_t + \bar{\xi}_t^d \frac{A_t^d}{\tilde{\alpha}} k_t^{\tilde{\alpha}},$$

with

$$\begin{aligned} u(Y_t, q_t^u; \xi_t) &\equiv \tilde{u}(C(Y_t, q_t^u, \xi_t); \xi_t) \\ v(y_t^j; \xi_t) &\equiv \tilde{v}(f^{-1}(y_t^j/A_t); \xi_t), \end{aligned}$$

where Δ_t, defined in equation (23), captures the misallocations from price dispersion. The term

$$\bar{\xi}_t^d \equiv \sum_{T=t}^{\infty} E_t[(1-\delta)^{T-t} \beta^{T-t} \xi_T^d]$$

captures the present value contribution from new housing investment. We can use (13), and (29) to express k_t in terms of Y_t, q_t^u and exogenous shocks. Hence, we can express the policy maker's objective of maximizing (1) under rational expectations, as maximizing

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t, q_t^u; \xi_t),$$

where the flow utility is given by

$$\begin{aligned} U(Y_t, \Delta_t, q_t^u; \xi_t) \equiv & \frac{\bar{C}_t^{\tilde{\sigma}^{-1}} C(Y_t, q_t^u, \xi_t)^{1-\tilde{\sigma}^{-1}}}{1-\tilde{\sigma}^{-1}} \\ & - \frac{\lambda}{1+\nu} \bar{H}_t^{-\nu} \left(\frac{Y_t}{A_t}\right)^{1+\omega} \Delta_t \\ & + \frac{A_t^d \bar{\xi}_t^d}{\tilde{\alpha}} \Omega(q_t^u, \xi_t)^{\tilde{\alpha}} \; C(Y_t, q_t^u, \xi_t)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}} \tilde{\sigma}^{-1}}, \end{aligned} \tag{33}$$

which is a monotonically decreasing function of Δ given Y, q^u and ξ and where $\Omega(q^u, \xi)$ is the function defined in (28). The only endogenous variables that are thus relevant for evaluating the policymaker's objective function are Y_t, Δ_t and q_t^u. Note that the policymaker holds rational expectations about these variables and maximizes expected utility under the rational probability measure, while agents might hold subjective beliefs.

The non-linear optimal monetary policy problem is then given by

$$\max_{\{Y_t, q_t^u, p_t^*, w_t(j), P_t, \Delta_t, i_t \geq 0\}} \quad E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t, q_t^u; \xi_t) \tag{34}$$

subject to

$$
\begin{aligned}
\left(\frac{p_t^*}{P_t}\right)^{1+\eta(\phi-1)} &= \frac{E_t^{\mathcal{P}}\sum_{T=t}^{\infty}(\alpha)^{T-t}Q_{t,T}\frac{\eta}{\eta-1}\phi w_T(j)\left(\frac{Y_T}{A_T}\right)^{\phi}\left(\frac{P_T}{P_t}\right)^{\eta\phi+1}}{E_t^{\mathcal{P}}\sum_{T=t}^{\infty}(\alpha)^{T-t}Q_{t,T}(1-\tau_T)Y_T\left(\frac{P_T}{P_t}\right)^{\eta}} && (35)\\
w_t(j) &= \lambda\frac{\bar{H}_t^{-\nu}}{\bar{C}_t^{\tilde{\sigma}^{-1}}}\left(\frac{Y_t}{A_t}\right)^{\phi\nu}C\left(Y_t,q_t^u,\xi_t\right)^{\tilde{\sigma}^{-1}}\left(\frac{p_t^*}{P_t}\right)^{-\eta\phi\nu} && (36)\\
(P_t/P_{t-1})^{\eta-1} &= \frac{1-(1-\alpha)\left(\frac{p_t^*}{P_t}\right)^{1-\eta}}{\alpha} && (37)\\
\Delta_t &= h(\Delta_{t-1},P_t/P_{t-1}) && (38)\\
\tilde{u}_C(C(Y_t,q_t^u,\xi_t);\xi_t) &= \lim_{T\to\infty}E_t^{\mathcal{P}}\left[\tilde{u}_C(C_T;\xi_T)\beta^T\prod_{k=0}^{T-t}\frac{1+i_{t+k}}{P_{t+k+1}/P_{t+k}}\right] && (39)\\
q_t^u &= \xi_t^d+\beta(1-\delta)E_t^{\mathcal{P}}q_{t+1}^u && (40)
\end{aligned}
$$

where the initial price level P_{-1} and the initial price dispersion Δ_{-1} are given. Equation (36) insures that wages clear current labor markets. Similarly, by setting $C_t = C(Y_t, q_t^u, \xi_t)$ on the left-hand side of the consumption Euler equation (39), we impose market clearing for output goods in period t. Similarly, setting q_t^u equal to the value defined in (40) insures market clearing in the housing market.[15] Firms' subjective expectations about future wages and households' subjectively optimal consumption plans for the future, however, will generally not be consistent with labor market or goods market clearing in the future, whenever beliefs deviate from rational ones.

To be able to analyze the policy problem (34) further, it will be necessary to be more specific about the beliefs $\mathcal{P}$ entertained by households and firms about external variables entering their decision problem.

5 Housing Prices Dynamics: Alternative Views

We shall now consider two alternative belief settings that give rise to different drivers for housing price dynamics.[16]

The first setting we consider is entirely standard and assumes that agents hold rational expectations (RE) about all variables. Housing prices will then exclusively be driven by housing demand shocks. While this is a useful benchmark to consider, the assumption of rational housing price expectations is not supported by empricial evidence available from survey expectations about future housing prices, as we show in section 5.3 below. We therefore consider a second setting that allows for subjective housing price and rental rate

[15] This holds as long as $D^{\max}$ is chosen sufficiently large, such that it never binds along the equilibrium path.

[16] Recall from our earlier discussion that firms must hold beliefs about future values of P_t, $w_t(j)$, Y_t and that households must hold beliefs about future values of $(P_t, w_t(j), q_t^u, R_t, i_t, \Sigma_t/P_t, T_t/P_t)$. Both actors must additionally hold beliefs about the fundamental shocks entering their decision problem. We always assume that these beliefs are rational.

beliefs. We maintain the assumption of rational expectations for all other variables to make a minimal deviation from the standard setting. The setting with subjective housing beliefs will align well with the properties of survey expectations, as we document in section 5.3. Housing prices will then depend on two economic forces: housing demand shocks and subjective optimism/pessimism about future housing prices.

5.1 Housing Prices under Rational Expectations

The following lemma summarizes the housing price dynamics under rational expectations (RE):[17]

Lemma 1 *Under RE, the equilibrium housing price is*

$$q_t^{u,RE} = \bar{\xi}_t^d, \tag{41}$$

where

$$\bar{\xi}_t^d \equiv \sum_{T=t}^{\infty} E_t[(1-\delta)^{T-t}\, \beta^{T-t} \xi_T^d] \tag{42}$$

captures the present value contribution to household utility from new housing investment. The price-to-rent (PR) ratio is

$$PR_t^{RE} = \frac{q_t^{u,RE}}{\xi_t^d}. \tag{43}$$

To a first-order approximation, we have

$$\widehat{q}_t^{u,RE} = \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\rho_\xi} \widehat{\xi}_t^d, \tag{44}$$

$$\widehat{PR}_t^{RE} = \left(\frac{\beta(1-\delta)\left(\rho_{\xi^d}-1\right)}{1-\beta(1-\delta)\rho_{\xi^d}} \right) \widehat{\xi}_t^d \tag{45}$$

where hatted variables denote percent deviations from steady state.

Proof. See Appendix C. ■

Equations (44) and (45) show that housing prices display large and persistent variations over time, whenever housing demand shocks $\widehat{\xi}_t^d$ display large and persistent variations. In fact, housing demand shocks are a commonly used modelling approach for generating large and persistent housing price fluctuations in the RE literature (Iacoviello (2005)). The approach pre-supposes that the observed housing prices fluctuations are in fact efficient, as they reflect preference fluctuations.[18]

[17] Under the RE hypothesis, households hold rational expectations about the prices $\{P_t, w_t(j), q_t^u, R_t, i_t\}$, firms hold rational expectations about $\{P_t, w_t(j), Y_t\}$, and all actors hold rational expectations about the exogenous fundamentals. Household expectations about $\{\Sigma_t/P_t, T_t/P_t\}$ are given by equations (25) and (26), evaluated with rational output expectations and the optimal choices for $\{H_t, k_t, B_t\}$.

[18] Under RE, this is also the case in the present setup: actual housing prices fluctuate as much as efficient housing prices.

Equations (44) and (45) show that lower natural rates, i.e., a higher value for β, increase the volatility of house prices, in line with the empirical evidence. Yet, the effect of lower natural interest rates on housing price volaility is bounded, because the coefficients premultiplying ξ_t^d in equations (44) and (45) is never larger than one, when $\rho_\xi \in (0,1)$. As we shall see, this will cause the predicted increase in housing price volatility, when considering a drop in the natural rate from pre-1990 levels to post-1990 levels, to fall short of the empirically observed increase. To match the increase in housing price volatility under RE one thus has to assume that the volatility of housing demand shocks has increased over the period in over which natural rates decreased.

5.2 Housing Prices with Subjective Housing Price Beliefs

Our second belief setting considers a single deviation from the rational expectations assumption, which allows for the presence of subjective capital gain expectations in housing markets and associated subjective beliefs for rental prices for an arbitrarily long but finite amount of time $t \leq \bar{T} < \infty$.[19] We will discuss the empirical plausibility of the subjective belief specification in the next subsection.

To isolate the effects of speculative housing and rent price expectations, we continue to assume that households hold rational expectations about all other prices at all times, i.e., about $\{P_t, w_t(j), i_t\}$. Likewise, firms hold rational expectations about $\{P_t, w_t(j), Y_t\}$ all times and all actors continue to hold rational expectations about the exogenous fundamentals. Beliefs about profits and lump sum taxes, $\{\Sigma_t/P_t, T_t/P_t\}$ continue to be determined by equations (25) and (26), evaluated with rational output expectations and the subjectively optimal choices for $\{H_t, k_t, B_t\}$.

The introduction of subjective housing price expectations is motivated by the observation that reconciling survey expectations about future return expectations with the actual behavior of future returns requires introducing some departure from the rational expectations hypothesis (Adam et al. (2017)). These departures could take the form of extrapolative expectations (Barberis et al. (2015), Adam and Merkel (2019)), learning about underlying trends in price growth (Adam et al. (2016)), or learning from life-time experience (Collin-Dufresne et al. (2016), Nagel and Xu (2019), Malmendier and Nagel (2011, 2016)).[20] We show in Section 5.3 that our subjective belief model matches observed patterns in survey expectations about future housing prices.

We employ the subjective belief setup of Adam et al. (2016) and consider households

[19] The fact that agents will eventually hold rational housing and rental price expectations can be interpreted as agents learning to make rational predictions in the long-run. On a technical level, this will insure that the transversality condition holds for agents' subjectively optimal plans.

[20] Adam et al. (2020) show that this discrepancy between survey return expectations and actual return behavior cannot be explained away by assuming that survey repondents confound beliefs and preferences when answering the survey questions and report, for instance, risk-neutral or pessimistically tilted expectations. Armona et al. (2019) provide experimental evidence for extrapolative expectation formation about house prices. Soo (2018) documents the extrapolative nature of housing expectations based on a constructed housing sentiment index.

who perceive (for all periods $t \leq \bar{T}$) risk-adjusted price growth to follow

$$\frac{q_t^u}{q_{t-1}^u} = b_t + \varepsilon_t, \tag{46}$$

where b_t is an unobserved persistent growth component given by

$$b_t = b_{t-1} + \nu_t$$

and $\varepsilon_t \sim iiN(0, \sigma_\varepsilon^2)$ and $\nu_t \sim iiN(0, \sigma_\nu^2)$ are independent transitory shocks. Adam et al. (2016) show that Bayesian belief updating implies that subjective conditional expectations are given by:[21]

$$E_t^{\mathcal{P}} \left(\frac{q_{t+1}^u}{q_t^u} \right) = \beta_t, \tag{47}$$

where β_t evolves according to

$$\beta_t = \min \left\{ \beta_{t-1} + \frac{1}{\alpha} \left(\frac{q_{t-1}^u}{q_{t-2}^u} - \beta_{t-1} \right), \beta^U \right\}, \tag{48}$$

with $1/\alpha$ denoting the Kalman gain, which depends on the subjectively perceived values for $(\sigma_\varepsilon^2, \sigma_\nu^2)$, and where the upper bound $\beta^U < (\beta(1-\delta))^{-1}$ insures that optimism is bounded from above, so as to keep subjectively expected utility finite.[22]

Equation (48) implies that subjective optimism rises (falls), when agents observe risk-adjusted capital gains that exceed (fall short of) their prior expectations. Since realized housing prices q_t^u will depend positively on expected future housing price growth β_t, this setup can generate persistent housing booms (busts) from endogenous belief dynamics: as housing prices increase (decrease), they trigger rising (falling) optimism and therefore future housing price increases (decreases), consistent with the empirical observation that capital gains in housing markets display considerable auto-correlation over time.

Associated with the subjective housing price expectations are subjectively optimal consumption plans, as determined by the Euler equation (16). These subjective consumption plans satisfy the transversality condition (17), as shown in Appendix C.1, when agents' housing price expectations are rational in the arbitrarily distant future. We insure this by assuming that from some (arbitarily distant) period $\bar{T} < \infty$ onwards, housing price beliefs are given by

$$q_t^u = \bar{\xi}_t^d \quad \text{for all } t \geq \bar{T}, \mathcal{P} \text{ almost surely,}$$

instead of by equation (46), where $\bar{\xi}_t^d$ is the housing price under rational expectations.

The subjectively optimal consumption plans determine, according to equation (14), the subjective rental price expectations that households must entertain together with their subjective housing price expectations. These subjective rental price expectations are required to

[21] This assumes that agents entertain normally distributed conjugate prior belief about the unobserved persistent component b_t.

[22] The bounding function in (48) is a special case of the bounding function used in equation (27) in Adam et al. (2016), which is obtained by setting $\beta^L = \beta^U$.

make the household's subjective beliefs consistent with internal rationality.[23] The long-run beliefs about housing price and rental rate beyond period $\bar{T}$ are rational, with housing price expectations being given by equation (41).

The following lemma derives the equilibrium PR-ratio and housing price under the considered subjective belief specification:

Lemma 2 *With subjective housing price expectations, the equilibrium housing price for periods $t < \bar{T}$ is given by*

$$q_t^{u,\mathcal{P}} = \frac{1}{1 - \beta(1-\delta)\beta_t} \xi_t^d, \tag{49}$$

where β_t evolves according to (48). The percent deviation of housing prices from steady state is

$$\widehat{q}_t^{u,\mathcal{P}} = \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\widehat{\xi}_t^d + \frac{\beta(1-\delta)(\beta_t - 1)}{1-\beta(1-\delta)\beta_t} \tag{50}$$

$$= \widehat{q}_t^{u,RE} + \frac{\beta(1-\delta)(\beta_t-1)}{1-\beta(1-\delta)\beta_t} + \frac{(1-\beta(1-\delta))\,\beta(1-\delta)\,(\beta_t - \rho_\xi)}{(1-\beta(1-\delta)\beta_t)\,(1-\beta(1-\delta)\rho_\xi)}\widehat{\xi}_t^d, \tag{51}$$

where hatted variables denote percent deviations from steady-state values.
Therefore, we can express the expected housing price under subjective beliefs as follows

$$E_t^{\mathcal{P}}\left[\widehat{q}_{t+1}^{u,\mathcal{P}}\right] = E_t\left[\widehat{q}_{t+1}^{u,RE}\right] + (\beta_t - 1)\left[1 + \frac{\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\right] \tag{52}$$

$$+ \left(1-\beta(1-\delta)\rho_\xi - (1-\beta(1-\delta)\beta_t)\,\rho_\xi\right)\frac{(1-\beta(1-\delta))}{(1-\beta(1-\delta)\beta_t)\,(1-\beta(1-\delta)\rho_\xi)}\widehat{\xi}_t^d, \tag{53}$$

which becomes

$$E_t^{\mathcal{P}}\left[\widehat{q}_{t+1}^{u,\mathcal{P}}\right] = E_t\left[\widehat{q}_{t+1}^{u,RE}\right] + (\beta_t - 1)\left[1 + \frac{\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\left(1 + \widehat{\xi}_t^d\right)\right] \tag{54}$$

as $\rho_\xi \to 1$.

Proof. See Appendix C. ■

The lemma shows that housing prices under subjective beliefs fluctuate because of fluctuations in the fundamental shocks $\widehat{\xi}_t^d$ and because of fluctuations in the subjective beliefs β_t.

In the special case without subjective belief fluctuations ($\beta_t = 1$ for all $t < \bar{T}$) and with a unit root in the fundamental shocks ($\rho_\xi \to 1$), the housing price under subjective beliefs is equal to the one under rational expectations at all times, in particular $\widehat{q}_t^{u,\mathcal{P}} = \widehat{q}_t^{u,RE}$. Since

[23]Households' subjective beliefs are otherwise inconsistent with the household first-order conditions. This shows how internal rationality imposes restrictions on what rational agents can plausibly believe about external variables.

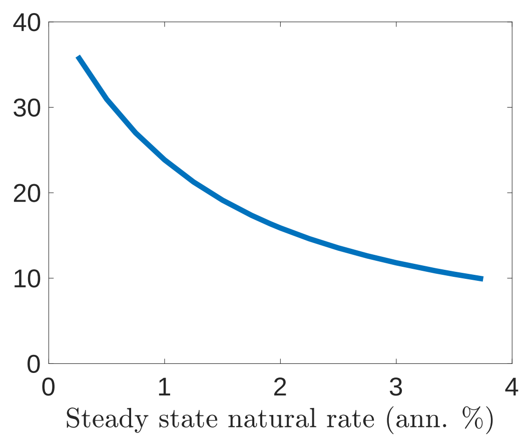
ρ_ξ will be close to one in the calibration used later on, the subjective belief setup would generate close-to-rational expectations absent belief-fluctuations.[24]

In general, fluctuations in housing price expectations generate additional housing price fluctuations, which then feed back into housing price expectations. Adam et al. (2016) show that these fluctiations take the form of persistent boom-bust cycles, in line with the observed dyanmics of housing prices in the data.

As the steady-state natural interest rate falls, i.e., as the discount factor β moves closer to one, the subjective belief dynamics will display more instability and thereby generate more variable housing prices. This is illustrated in Figure 6, which depicts the standard deviation of the housing price q_t^u as a function of the steady-state natural interest rate, using the calibrated subjective belief model presented later on. The reason for the downward sloping pattern of housing price volatility is that housing prices become more sensitive to belief revisions as the term $\beta(1-\delta)$ in the denominator of equation (49) moves closer to one: the denominator $1 - \beta(1-\delta)\beta_t$ then fluctuates around numbers that are closer to zero, so that any given revision of beliefs has larger pricing implications. This again feeds back into larger belief revisions.

In contrast to the RE model, the subjective belief model will thus be able to generate a sufficiently strong increase in house price volatility, following a drop in the average natural rate, without having to resort to changes in the volatility of housing demand shocks ξ_t^d.[25]

Figure 6: Unconditional standard deviation of housing prices q_t^u



[24]A setting with constant growth expectations $\beta_t = 1$ is in fact a special setting of the subjective belief setup. It requires that initial beliefs (1) assign probability one to $b_0 = 1$ and (2) probability one to $\sigma_\nu^2 = 0$. Bayesian updating then implies $1/\alpha = 0$ in equation (48). More generally, however, if prior beliefs are such that $\sigma_\nu^2 > 0$, then $\alpha < \infty$ and realized growth rates of housing prices will influence subjective expectations about future housing price growth.

[25]Section 7 shows that the subjective belief model is indeed capable of generating the observed volatility in the data.

5.3 Empirical Evidence on Housing Price Expectations

In this section, we show that survey expectations about housing prices are inconsistent with full-information rational expectations. Our model under subjective beliefs, on the other hand, matches the empirical findings. Notably, the model's quantitative predictions, given the calibration outlined in Section 7, align well with their empirical counterparts, even though none of these moments was targeted in our calibration strategy.

First of all, we find that expected capital gains are positively correlated with current price-to-rent ratios. For actual capital gains, on the other hand, we find a negative relationship. This difference in the cyclicality of actual versus expected capital gains is at odds with the rational expectations hypothesis. Second, we show that ex post forecast errors about house prices are positively correlated with ex ante revisions in expectations. This finding points towards a sluggish belief adjustment, consistent with our model under subjective beliefs. Again, these findings are inconsistent with full-information rational expectations as in that case forecast errors are unpredictable.

5.3.1 Cyclicality of Actuals versus Expected Capital Gains

We obtain data on housing price expectations from the Survey of Consumers from the University of Michigan. Survey respondents report how much they expect housing prices to grow over the next four quarters.[26] Data on nominal housing prices is taken from the Case-Shiller House Price Index.

Table 2: Expected vs. actual capital gains

	$\hat{c} \cdot 10^3$	$\hat{\mathbf{c}} \cdot 10^3$	bias $\cdot 10^3$ $-E(\hat{\mathbf{c}} - \hat{c})$	p-value $H_0 : c = \mathbf{c}$
Nominal Housing Prices				
Mean	0.607	-0.462	0.023	0.000
Median	0.187	-0.462	0.106	0.0571
Real Housing Prices				
Mean	0.607	-0.532	0.022	0.000
Median	0.187	-0.532	0.105	0.0351

We follow Adam et al. (2017) and regress expected and realized capital gains on the level of the PR ratio and test whether the two coefficients are equal, which would be true under rational expectations. Furthermore, we correct for small sample bias. Thus, we run the following two regressions

$$E_t^{\mathcal{P}} \left[CG_{t+4} \right] = a + c \cdot PR_t + u_t \tag{55}$$

$$CG_{t+4} = \mathbf{a} + \mathbf{c} \cdot PR_t + \mathbf{u}_t \tag{56}$$

[26] The exact variable we use for the expected gross growth rate is the answer to the following question: *By about what percent do you expect prices of homes like yours in your community to go (up/down), on average, over the next 12 months?*

and test whether c and $\mathbf{c}$ are equal. Here, $E_t^{\mathcal{P}}[CG_{t+4}] \equiv \frac{HP_{t+4}}{HP_t}$ and $CG_{t+4} \equiv \frac{HP_{t+4}}{HP_t}$ denote the expected and actual capital gains for the next four quarters. We do this for nominal housing prices as well as for real housing prices for which we divide nominal housing prices by the GDP deflator.

Table 2 shows the results. While a high PR ratio is usually associated with low future capital gains, the expected capital gains are higher when PR ratios are high. From the last column we observe that this difference in the cyclicality of actual versus expected capital gains is statistically significant. We therefore reject the rational expectations hypothesis.

To compare these results with the model predictions, we re-run regressions (55) and (56) on model-simulated data. For the expected capital gain expectation four periods ahead, we use the fact that households in our model perceive housing-price growth to follow a random walk. Therefore, $E_t^{\mathcal{P}}[CG_{t+4}] = \beta_t^4$. Actual capital gains are given by $CG_{t+4} = \frac{q_{t+4}^u}{q_t^u}$, and the PR ratio is given by $PR_t = \frac{q_t^u}{\xi_t^d}$ as shown in Appendix C.

Table 3 shows the results for different average natural rates. We observe the same pattern in the model as we observed in the data. Actual capital gains are countercyclical with respect to price-to-rent ratios, while expected capital gains are procyclical. The point estimates lie in the ballpark of the empirical estimates, especially for relatively low natural rates.

Table 3: Expected vs. actual capital gains

	Model				Michigan Survey	
Specification	$\underline{r}^n = 3.34\%$	$\underline{r}^n = 1.91\%$	$\underline{r}^n = 1\%$	$\underline{r}^n = 0.25\%$	Mean	Median
$\widehat{c} \cdot 10^3$	1.000	0.601	0.395	0.220	0.607	0.187
$\widehat{\mathbf{c}} \cdot 10^3$	-1.815	-1.161	-0.756	-0.527	-0.462	-0.462

5.3.2 Sluggish Adjustment of Housing Price Expectations

Following Coibion and Gorodnichenko (2015), we regress forecast errors about the *level* of housing prices on the revisions in these forecasts. We therefore compute the expected level of housing prices by multiplying the expected housing-price growth rates, $E_t^{\mathcal{P}}\left[g_{t+4}^h\right]$ with the current level

$$E_t^{\mathcal{P}}[HP_{t+4}] = \left(1 + E_t^{\mathcal{P}}\left[g_{t+4}^h\right]\right) \cdot HP_t, \tag{57}$$

where HP denotes real house prices. We consider average and median responses as our measure of expected house price growth. In the model, we focus on house prices in terms of marginal utility of consumption, denoted by q_t^u. If we assume that marginal consumption—which is indeed relatively stable over time in the data—is constant, the model and the data coincide up to a constant scaling factor.

Forecast errors in house price expectations are given by $Error_t \equiv HP_{t+4} - E_t^{\mathcal{P}}[HP_{t+4}]$, and revisions are computed as $Revisions_t \equiv E_t^{\mathcal{P}}[HP_{t+4}] - E_{t-1}^{\mathcal{P}}[HP_{t+3}]$.

We run the following regression

$$Error_t = \alpha + \beta_{CG} \cdot Revisions_t + \epsilon_t, \tag{58}$$

where the coefficient of interest, β_{CG}, carries the subscript CG as a shorthand notation for Coibion and Gorodnichenko (2015).

Table 4 shows the results. In the first two columns, we see the empirical regression coefficients for two different sample periods. The last four columns show the model-implied coefficients for different average natural rates. Both empirical estimates are positive and highly statistically significant, even though the sample size is quite small. We control for serial correlation and heteroskedasticity in the error term by using the Newey-West estimator including four lags.[27]

We observe that the estimates are quite close to the model-implied ones, especially for relatively low natural rates. These results are robust to using median survey expectations instead of averages as well as other specifications, such as using an instrumental-variable regression in which we instrument forecast revisions with monetary policy shocks obtained via high-frequency identification (see Appendix D.1 for the robustness of our results). Altogether, the data—consistently with our model under subjective beliefs—indicates that if consumers revise their house price expectations, they do so insufficiently.

The positive sign of β_{CG} is consistent with previous findings on survey expectations, concerned with other macroeconomic variables such as output, inflation or unemployment (see, e.g., Coibion and Gorodnichenko (2015), or Angeletos et al. (2020)). Bordalo et al. (2018) provide similar evidence for a number of variables, including residential investment and new housing starts. Note, $\beta_{CG} \neq 0$ is inconsistent with full-information rational expectations as forecast errors should not be predictable by forecast revisions. The regression coefficient β_{CG} coming from the model under rational expectations is therefore equal to zero.

Table 4: Sluggish Adjustment of Housing Price Expectations

	2007 - 2019	2010 - 2019	$\underline{r}^n = 3.34\%$	$\underline{r}^n = 1.91\%$	$\underline{r}^n = 1\%$	$\underline{r}^n = 0.25\%$
β_{CG}	2.14^{***}	1.54^{***}	0.73	1.15	1.71	2.96
	(0.540)	(0.509)				

Note: The first two columns show the empirical estimates for different samples. The last four columns show the model-implied regression coefficients for different levels of the average natural rates.
Significance levels: $^{***}: p < 0.01$, $^{**}: p < 0.05$, $^{*}: p < 0.1$

In Appendix D, we show that these results are robust to using median instead of mean expectations, to using monthly data instead of quarterly data, and to an IV approach in which we instrument the forecast revisions. Additionally, we divide the survey respondents into three groups based on their income and show that we observe very similar patterns of forecast errors in all three groups.

6 Quadratic Approximation of the Policy Problem

One can obtain analytic insights into the nature of the nonlinear optimal monetary policy problem by considering a quadratic approximation to the objective (34) and a linear approx-

[27] The results are robust to using different lag lengths.

imation to the constraints (35)-(40), keeping the lower-bound constraint on nominal interest rates in its nonlinear form.[28]

Appendix A shows that under the considered belief setting, the Lagrangian of the optimal policy problem can be approximated as follows:

$$\max_{\{\pi_t, y_t^{gap}, \widehat{q}_t^u, i_t \geq \underline{i}\}} \min_{\{\varphi_t, \lambda_t\}} \tag{59}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ -\frac{1}{2}\left(\Lambda_\pi \pi_t^2 + \Lambda_y \left(y_t^{gap}\right)^2\right)$$

$$+ \varphi_t \left[\pi_t - \kappa_y y_t^{gap} - \kappa_q \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) - u_t - \beta E_t \pi_{t+1}\right] \tag{60}$$

$$+ \lambda_t \left[y_t^{gap} - \lim_T E_t y_T^{gap} + \varphi E_t \sum_{k=0}^{\infty} \left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n,RE} \right) + \frac{C_q}{C_Y} \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \right] \tag{61}$$

$$- \varphi_{-1} \pi_0 - \lambda_{-1} \left(\varphi \pi_0 - y_0^{gap} - \frac{C_q}{C_Y} \left(\widehat{q}_0^u - \widehat{q}_0^{u*}\right) \right) \Bigg\}.$$

The variable π_t denotes inflation, y_t^{gap} the output gap, i_t nominal interest rates, and $\widehat{q}_t^u - \widehat{q}_t^{u*}$ the housing price gap. The housing price gap is the difference between the housing price $\widehat{q}_t^u$ and its efficient welfare-maximizing level $\widehat{q}_t^{u*}$, which (in nonlinear terms) is given by[29]

$$q_t^{u*} = \overline{\xi}_t^d. \tag{62}$$

Nominal interest choices are subject to an effective lower bound $i_t \geq \underline{i}$, where the lower bound $\underline{i} < 0$ is expressed in terms of deviation from the interest rate in a zero-inflation steady state. For the case with a zero lower bound on nominal rates, we have $\underline{i} = -(1-\beta)/\beta$.

Constraint (60), which features the Lagrange multiplier φ_t, is the New Keynesian Phillips Curve for our model with housing and depends on the housing price gap. The coefficients $\kappa_q < 0$ and $\kappa_y > 0$ are defined in appendix A.3 and imply that a positive housing price gap has a negative cost-push effects. This is so because high housing prices increase housing investment. For a given output gap, higher housing investment raises the marginal utility of non-housing consumption, thereby depressing wages and marginal production costs.

Constraint (61), which features the Lagrange multiplier λ_t, is the linearized (and forward-iterated) IS equation. The coefficients $C_q < 0$ and $C_Y > 0$ are the derivatives of the function $C(\cdot)$ defined in (29) with respect to q^u and Y, respectively, evaluated at the efficient steady state. The long-run output gap expectations $\lim_T E_t y_T^{gap}$ in the IS curve are the ones

[28]This will deliver a valid second-order approximation to the problem for small shock disturbances, whenever (i) the steady-state Lagrange multipliers associated with the constraints are of order $O(1)$, which is the case when the steady state output distortion $\Theta \equiv \log\left(\frac{\eta}{\eta-1}\frac{1-g}{1-\underline{\tau}}\right)$ is of order $O(1)$, and (ii) the gap between the steady-state interest rate and the lower bound, i.e., $\frac{1}{\beta} - 1$, is also of $O(1)$. Eggertsson and Singh (2019) compare the exact solution of the New Keynesian model with lower bound to the solution of the linear-quadratic approximation with lower bound and show that the quantitative deviations are modest, even for extreme shocks of the size capturing the 2008 recession in the U.S..

[29]See the derivation in appendix C.2.

associated with a setting in which agents hold rational housing expectations.[30] The initial Lagrange multipliers $(\varphi_{-1}, \lambda_{-1})$ capture initial pre-commitments.

Interestingly, the expectations showing up in the monetary policy problem (59) are all rational. Therefore, subjective housing price expectations affect the monetary policy problem solely through their effects on housing price gaps. These gaps are determined differently under RE and under subjective beliefs, see section 5.

An interesting novel feature of our setup is that the housing price gap also enters the IS equation (61), which is key for understanding the quantitative results presented later on. The RE natural interest rate $r_t^{n,RE}$ entering the IS equation is thereby defined in the usual way: it is the real interest rate consistent with the optimal consumption level in a setting with flexible prices and rational expectations.[31] Importantly, the restrictions that the IS constraint (61) imposes on monetary policy choices depend on the belief setting, as we discuss next.

Rational Housing Price Expectations. With fully rational expectations, it follows from lemma 1 and equation (62) that

$$\widehat{q}_t^u = \widehat{q}_t^{u*}. \tag{63}$$

The housing price gap is thus always zero. In particular, the housing price gap is independent of monetary policy and economic shocks. While policy can affect the level of raw housing prices $(\widehat{q}_t)$, it cannot affect the housing price in marginal utility units $(\widehat{q}_t^u)$. This will also be true for the setting with subjective housing beliefs and explains why one can ignore utility contributions from the housing price gap in the objective function of problem (59), despite the fact that the nonlinear utility function (34) depends on q_t^u.

Equation (63) implies that under RE the IS equation simplifies to

$$y_t^{gap} = \lim_T E_t y_T^{gap} - E_t \left(\sum_{k=0}^{\infty} \varphi \left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n,RE} \right) \right),$$

so that setting real interest rates equal to natural rates each period, i.e., choosing

$$i_t - E_t \pi_{t+1} = r_t^{n,RE} \quad \text{for all } t, \tag{64}$$

causes the IS equation to be consistent with a constant output gap $y_t^{gap} = \lim_T E_t y_T^{gap}$ for all t, as in the simple New Keynesian model without housing. In the presence of a zero lower bound constraint on nominal rates, however, it will generally not be optimal (or even impossible) to implement (64) at all times.

The RE housing investment gap $\widehat{k}_t - \widehat{k}_t^*$ is given by[32]

$$\widehat{k}_t - \widehat{k}_t^* = \frac{\tilde{\sigma}^{-1} C_Y}{1 - \tilde{\alpha}} y_t^{gap}, \tag{65}$$

[30] Recall that we assume housing expectations to be rational in the long-run for all our belief settings. Our numerical solution approach solves for the long-run expectations together with the state-contingent optimal policy functions.

[31] See appendix A.4 for the precise definition.

[32] Equation (65) follows from linearizing equation (13) and using the linearized version of equation (29) to substitute consumption.

which shows that housing investment under RE is purely driven by movements in the output gap. This is so because the house price gap is 0, so that additional output will be allocated with constant shares to housing and non-housing consumption. For the calibration used later on, we have $\frac{\tilde{\sigma}^{-1}C_Y}{1-\tilde{\alpha}} > 0$, so that the output gap comoves positively with the housing investment gap.

Subjective housing price expectations. With subjective housing price beliefs, the efficient level of housing prices continues to be given by equation (62), but housing prices are now jointly determined by equations (48) and (50).[33] As a result, the housing price gap $\widehat{q}_t^u - \widehat{q}_t^{u*}$ will generally differ from zero, with the housing price gap exceeding (falling short of) zero, whenever agents' subjective capital gain expectations β_t are larger than one (smaller than ρ_ξ).[34]

With subjective housing price beliefs, a policy that sets real interest rates equal to the RE natural real rate $r_t^{n,RE}$ ceases to deliver a constant output gap. In particuarl, the IS equation then implies

$$y_t^{gap} = \lim_T E_t y_T^{gap} - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right),$$

which shows that high house price gap ($\widehat{q}_t^u - \widehat{q}_t^{u*} > 0$) will then be associated with a more positive output gap (recall that $C_q/C_Y < 0$): high housing prices stimulate housing investment and thus output.

The following lemma derives the natural rate $r_t^{n,\mathcal{P}}$ that - in a setting with subjective beliefs - causes the IS equation to be consistent with a constant output level. As in the case with RE, it will generally not be optimal (or not even feasible) to set interest rates equal to this natural rate level at all times in the presence of a lower bound constraint on nominal rates:

Lemma 3 *Define the natural rate under subjective beliefs as*

$$r_t^{n,\mathcal{P}} \equiv r_t^{n,RE} - \frac{1}{\varphi}\frac{C_q}{C_Y}\left(\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) - E_t\left(\widehat{q}_{t+1}^u - \widehat{q}_{t+1}^{u*}\right)\right) \qquad \text{for all } t, \tag{66}$$

where $E_t[\cdot]$ denotes the rational expectations operator. When real interest rates are equal to $r_t^{n,\mathcal{P}}$ for all t, the IS equation is consistent with

$$y_t^{gap} = \lim_T E_t y_T^{gap} \qquad \text{for all } t.$$

Proof. See Appendix C. ■

Equation (66) generalizes the natural interest rate definition under RE to a setting with potentially subjective beliefs. In the special case with a constant housing price gap, we have

[33] Since the equations do not depend on policy, one can again treat the housing price gap as an exogenous process, as in the case with RE.

[34] In the intermediate range $\beta_t \in (\rho_\xi, 1)$, the ranking depends also on other paramaters. ρ_ξ will be close to one in our calibration.

$r_t^{n,\mathcal{P}} = r_t^{n,RE}$. More generally, predictable fluctuations in the housing price gap will contribute to fluctuations in the natural rate of interest. To the extent that housing prices and thus the housing price gap becomes more volatile as the average natural rate falls, natural rate volatility will go up in line with housing price volatility, as is the case in the data.

Consider, for instance, a setting where the housing price gap is high but expected to go down over time. We then have $E_t\left(\widehat{q}^u_{t+1} - \widehat{q}^{u*}_{t+1}\right) < \left(\widehat{q}^u_t - \widehat{q}^{u*}_t\right)$ and the natural rate under subjective beliefs will exceed its RE level. Conversely, if the housing price gap is low or even negative, but expected to rebound over time, then natural rates will be lower than under RE.

With subjective beliefs, the housing investment gap is given by

$$\widehat{k}_t - \widehat{k}^*_t = \frac{\tilde{\sigma}^{-1}C_Y}{1-\tilde{\alpha}} y^{gap}_t + \frac{1+\tilde{\sigma}^{-1}C_q}{1-\tilde{\alpha}}\left(\widehat{q}^u_t - \widehat{q}^{u*}_t\right). \tag{67}$$

Unlike in the case with RE, the housing investment gap is also driven by housing prices. Given the calibration considered later on, which implies $\frac{1+\tilde{\sigma}^{-1}C_q}{1-\tilde{\alpha}} > 0$, a housing price boom will go hand in hand with a housing investment boom. Therefore, larger housing price volatility translates into larger housing investment volatility, as tends to be the case in the data.

In order to solve the problem in (59), we recursify the problem as proposed in Marcet and Marimon (2019) and solve for the associated value functions and optimal policies. Details of the recursive formulation can be found in Appendix C.4.

7 Calibration

We calibrate the model to the pre-1990 period in the U.S., matching salient features of the behavior of natural interest rates and housing prices. We then test the model by considering its predictions for lower natural rate levels, as observed post 1990.

Table 5 summarizes the model parameterization. The quarterly discount factor β is chosen such that the steady-state natural rate equals the pre-1990 average of the U.S. natural rate of 3.34%, as estimated by Holston et al. (2017). The interest rate elasticity of output φ, the slope of the Phillips curve κ_y, and the welfare weight $\frac{\Lambda_y}{\Lambda_\pi}$ are taken from table 2 in Adam and Billi (2006). The Phillips Curve coefficient κ_q and the ratio C_q/C_y are set as in Adam and Woodford (2020).[35] Details of the calibration are spelled out in Appendix C.6.

We now dicuss parameterization of the exogenous shock processes. We assume that the RE natural rate follows an AR(1) process with

$$r_t^{n,RE} = \rho_r r_{t-1}^{n,RE} + \epsilon^n_t, \tag{68}$$

where $\epsilon^n_t \sim iiN(0, \sigma^2_{r^n})$. We set $\rho_{r^{n,RE}} = 0.8$ following Adam and Billi (2006) and the persistence of housing demand shocks $\rho_{\xi^d} = 0.99$, following Adam and Woodford (2020). The mark-up shock is set to a constant value, so as to economize on the number of state

[35] The calibration target for the ratio C_q/C_y is the ratio of residential fixed investment over the sum of nonresidential fixed investment and personal consumption expenditure, which is on average approximately equal to 6.3% in the US. This and the remaining parameters then imply $\kappa_q = -0.0023$.

Table 5: Model Parameterization

Parameter	Value	Source/Target
		Preferences and technology
β	0.9917	Average U.S. natural rate pre 1990
φ	1	Adam and Billi (2006)
κ_y	0.057	Adam and Billi (2006)
$\frac{\Lambda_y}{\Lambda_\pi}$	0.007	Adam and Billi (2006)
κ_q	-0.0023	Adam and Woodford (2020)
$\frac{C_q}{C_Y}$	-0.29633	Adam and Woodford (2020)
δ	0.03/4	Adam and Woodford (2020)
		Exogenous shock processes
ρ_{r^n}	0.8	Adam and Billi (2006)
σ_{r^n}	0.2940% (RE)	Adam and Billi (2006)
	0.1394% (subj beliefs)	
ρ_{ξ^d}	0.99	Adam and Woodford (2020)
σ_{ξ^d}	0.0233 (RE)	Std. dev. of price-to-rent ratio pre 1990
	0.0165 (subj. beliefs)	
		Subjective belief parameters
α	1/0.007	Adam et al. (2016)
β^U	1.0031	Max percent deviation of PR-ratio from mean

variables in the model.[36] This is justified by the fact that - given their size and persistence - mark-up shocks tend to be quantitatively irrelevant for driving the economy towards the lower bound constraint (Adam and Billi (2006)). Specifically, setting $u_t = 0$ implies that the model collapses under RE to the setup in Adam and Billi (2006) for the case where one abstracts from mark-up shock disturbances.

The standard deviations of the innovations to the housing preference and natural rate shocks are set differently under RE and subjective beliefs, because housing volatility and natural rate volatility differ across the two specifications.

Under RE, we set σ_{ξ^d} so that the model matches the unconditional standard devation of the percent deviation of the price-to-rent ratio of 6% observed over the period 1970-1990 in the U.S..[37] The standard deviation of the innovation to the natural rate, $\sigma_{r^n,RE}$, is set equal to the value in Adam and Billi (2006). The implied annual unconditional standard deviation of the natural rate is then 1.96%.[38]

For the subjective belief model, we choose again ε_t^d to match the unconditional standard deviation of the price-to-rent ratio. This is achieved by simulating equations (48) and (49) and using $PR_t^{\mathcal{P}} = q_t^{u,\mathcal{P}}/\xi_t^d$ to compute the price-to-rent ratio. Doing so requires specifying

[36]Solving for optimal policy still requires solving an optimization problem featuring six state variables.

[37]This calibration is based on equation (45). The reported standard deviation refers to ratio of the housing price over quarterly rent.

[38]We prefer this calibration approach to matching the standard deviation estimated in Holston et al. (2017), because Adam and Billi (2006) identify the standard deviation of the natural rate in a way that is consistent with our structural model, while Holston et al. (2017) use an empirically motivated model.

the subjective belief parameters α and β^U, which enter equation (48). We set $\alpha = 1/0.007$ following Adam et al. (2016) and determine σ_{ξ^d} and β^U jointly such that (1) the volatility of the price-to-rent ratio is 6% and (2) the simulated data matches the maximum deviation of the price-to-rent ratio from its sample mean, which is a statistic that identifies β^u. This yields $\beta^U = 1.0031$ and $\sigma_{\xi^d} = 0.165$. Note that the innovations to the housing demand disturbance are less volatile than under RE because part of the fluctuations in housing prices are now generated by fluctuations in subjective beliefs.

It only remains to determine $\sigma_{r^{n,RE}}$ for the subjective belief model. We choose its value, such that the generalized natural rate for the subjective belief model, as defined in equation (66), has the same volatility as the natural rate in the RE model. This yields $\sigma_{r^{n,RE}} = 0.1393\%$, which is lower than under RE, because belief fluctuations also contribute to fluctuations in the natural rate.

Figure 7 compares the predictions of the RE and subjective belief model for various steady-state levels of the natural rate of interest. Since the steady-state level of the natural rate is purely a function of the time discount factor factor, we vary the discount factor accordingly. As discussed before, variations in the discount factor may well be driven by variations in the long-term growth rate of the economy.

Panel (a) in figure figure 7 depicts the standard deviation of the price-to-rent ratio and panel (b) the standard deviation of the natural rate.[39] The dots in the figures report the data values for the pre- and post 1990 U.S. sample, in which the average natural rate was equal to 3.34% and 1.91%, respectively.[40] Since the model has been calibrated to the pre-1990 period, the RE and subjective belief model both match the pre-1990 data point.

The subjective belief model also performs quite well in matching the post-1990 outcome, despite the fact that it has not been targeted in the calibration. In particular, the standard deviation of the price-to-rent ratio and the standard deviation of the natural rate endgoenously increase as the natural rate falls, with the magnitudes roughly matching the increase observed in the data. In contrast, the RE model produces no increase in the volatility of the natural rate and an insufficiently strong increase in the volatility of the price-to-rent ratio. Matching the increase in housing price volatility under RE would require increasing the volatility of housing demand shocks. However, under RE these shocks are irrelevant for monetary policy and inflation outcomes, allowing us to ignore volatility increases in housing preference shocks. Matching the increase in the natural rate volatility would require increasing the σ_{r^n}.

8 Quantitative Findings

This section discusses the quantitative impliations of falling natural rates for optimal monetary policy. We start with a discussion of the inflation target implications, then discuss the effects of shocks for optimal stabilization policy under subjective beliefs and RE.

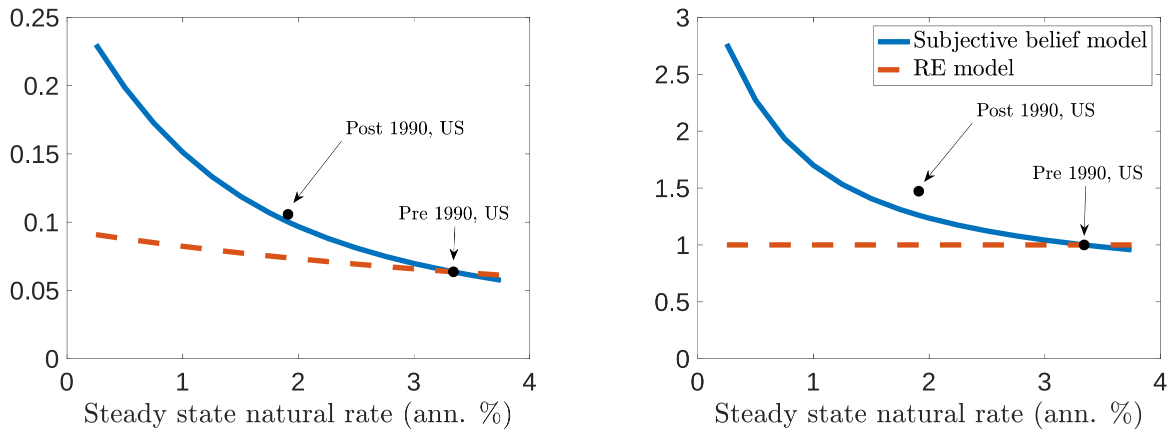
[39]These predictions follow directly from the calibration and are independent of policy.

[40]The reported increase in the standard deviation of the natural rate is based on the estimates in Holston et al. (2017).

Figure 7: Standard deviation of Price-to-Rent Ratio and Natural Rate

(a) Standard deviation of price-to-rent ratio (relative to corresponding mean)

(b) Standard deviation of the natural rate relative to case with $\underline{r}^{n,RE} = 3.34\%$



8.1 Optimal average inflation

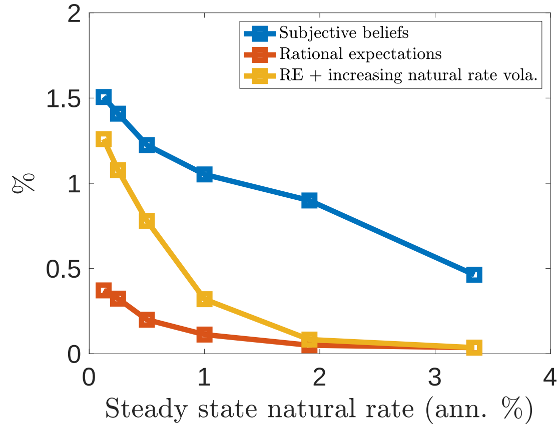
Figure 8 depists the optimal inflation target for different levels of the natural rate of interest, where changes in the natural rate are brought about by varying the discount factor β.[41] The figure shows that a fall in the natural rate leads only to a small increase in the optimal inflation under rational expectations, as indicated by the red line. Even though lower natural rates trigger larger housing price fluctuations, these increased fluctuations are efficient, thus do not require a monetary policy response. In addition, the natural rate of interest is - under RE - independent of housing fluctuations. Therefore, even at very low levels of the natural rate, the annualized optimal inflation rate is barely above zero. This shows that the findings in Adam and Billi (2006) are surprisingly robust towards assuming a lower average natural rate of interest.

Under subjective beliefs, the optimal inflation target is over all higher and also rises more strongly as the natural rate falls. In fact, a fall in the steady-state natural rate to 0.25% causes the optimal inflation target to increase to 1.41%. Already at the pre-1990 average of the natural rate (3.34%), the inflation target with subjective beliefs is larger than under RE for any considered level of the natural rate. This is case even though the volatility of the natural rate is calibrated at this point to be equal across the RE and subjective belief models. This shows that fluctuations in the natural rate that are induced by belief fluctuations affect the inflation target more than fluctuations in the natural rate induced by other disturbances.

To illustrate this point, the yellow line in figure 8 shows the optimal inflation rate under rational expectations, when we set the volatility of the (exogenous) natural rate in the RE model such that it matches the volatility of implied by the subjective belief model at each considered natural rate level. While the optimal inflation rate increases relative to the

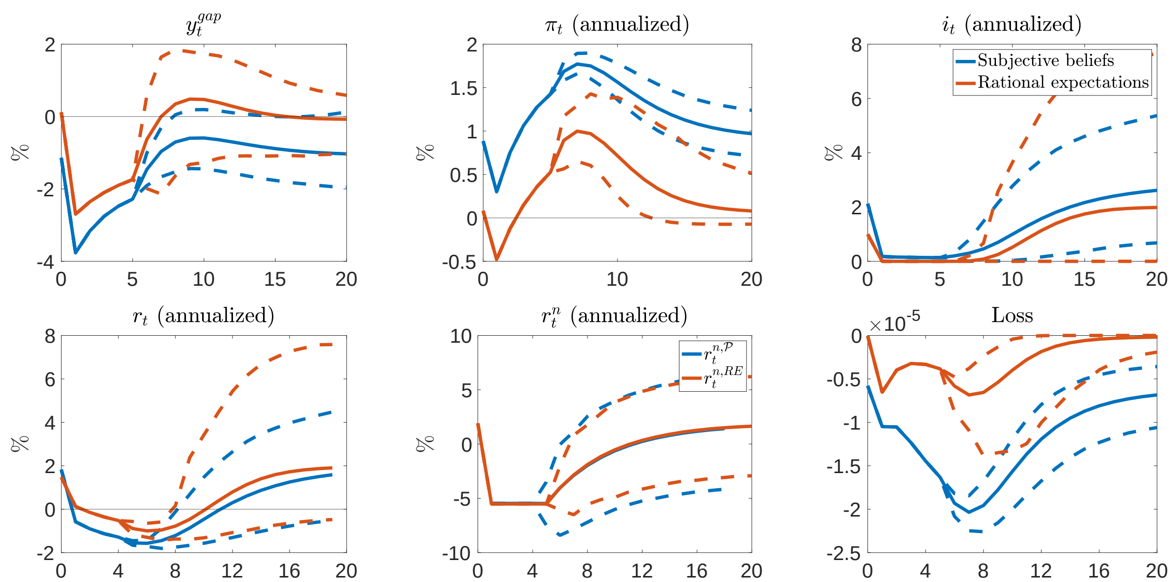
[41]Recall that a falling steady-state growth rates is associated with a rise in the discount factor and thus a decrease in the steady-state real interest rate.

Figure 8: Average Inflation under Optimal Monetary Policy



benchmark RE setting, the level of the optimal inflation target still falls short of the one implied by subjective beliefs.

Figure 9: Zero lower bound episode



8.2 Optimal Policy Responses at the Lower Bound

To understand why the optimal inflation target is higher under subjective beliefs than under rational expectations, we consider a specific shock episode that pushes the economy towards its zero-lower-bound constraint. We do so by considering a steady-state natural rate of 1.91%, which is the post-1990 mean for the United States.

We initialize the state variables of the economies with RE and subjective beliefs at their respective ergodic means. We then cause an initial fall in the natural rate and keep the natural rate at this level for 6 quarters, with no other shocks occuring during this period, see the panel in the center in the lower row of figure 9. After quarter 6, all shocks operate again as usual. We then report the mean response of the economy (solid lines in figure 9), as well as the 1st and 99th percentiles of the response distribution (dashed lines in figure 9).

The fall in the natural rate depresses the output gap and inflation. The monetary policymaker lowers the nominal rate to counteract this and indeed, the ZLB starts binding in both economies. The loss, measured by $-(\Lambda_\pi \pi_t^2 + \Lambda_y (y_t^{gap})^2)$, increases in both cases. After that, however, the two economies evolve quite differently. First of all, the higher inflation rate under subjective beliefs induces a lower real rate. This allows the economy to escape the lower bound faster. Especially in a "worst-case scenario", indicated by the lower dashed lines, the optimal policy response in the economy under subjective beliefs is to raise interest rates shortly after the natural rate starts recovering. The economy under rational expectations, on the other hand, is kept at the lower bound for a longer time. This is true even though the potential worst-case scenario is worse under subjective beliefs as housing demand shocks may exacerbate the situation, as indicated by the evolution of r_t^n.

While the economy escapes the lower bound faster under subjective beliefs, the hike in nominal rates is more pronounced under rational expectations if the recovery is supported by favorable shocks, indicated by the upper dashed lines. This is mirrored in the faster recovery of the real rate under rational expectations, which is especially driven by the large increase if future shocks are positive (move the economy away from the bound).

The slower recovery of the real rate under subjective beliefs is nevertheless accompanied by a greater loss than in a RE setting. This illustrates that the lower bound poses a more severe problem under subjective beliefs and that it is optimal to allow for a substantially higher inflation rate.

8.3 Leaning Against Housing Demand Shocks

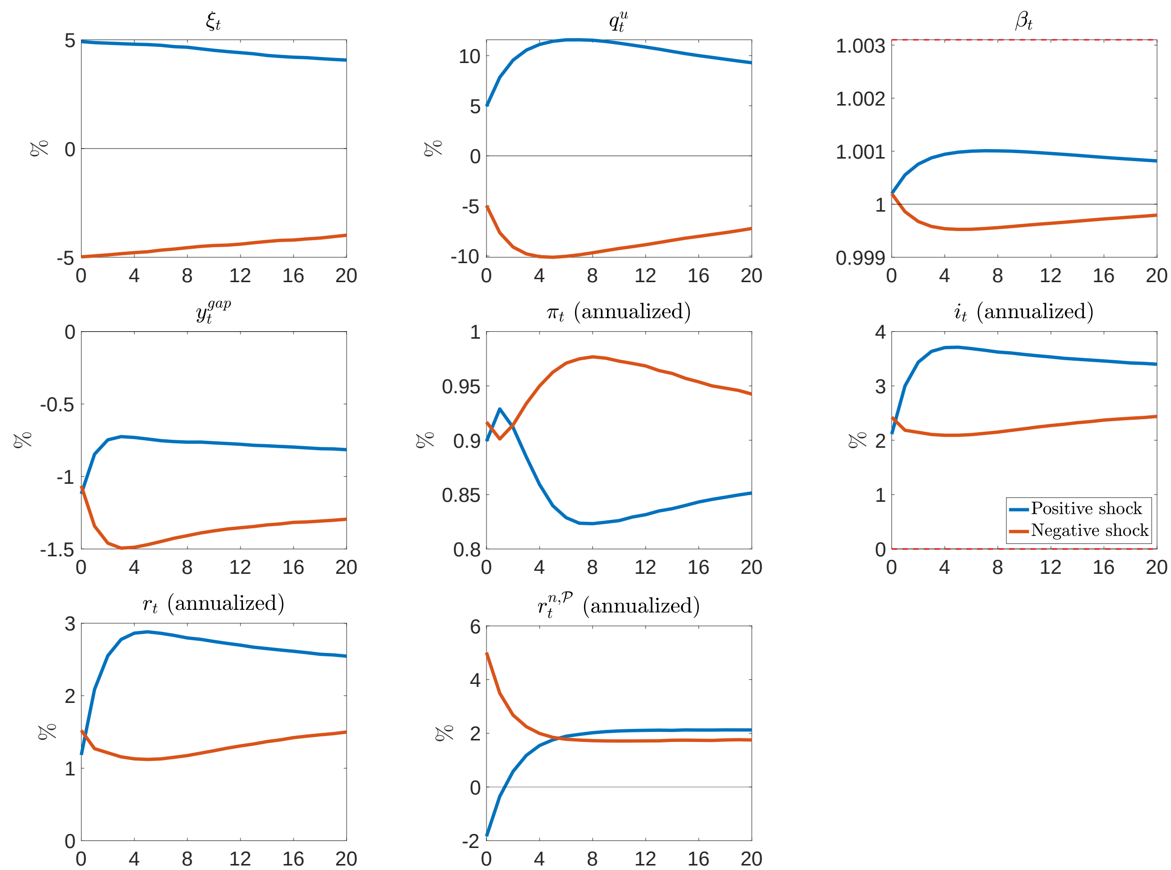
We now examine the optimal monetary policy response to housing demand shocks.[42]

Under RE, the housing demand shocks only affect the housing price, but leave nominal rate, the output gap and inflation unaffected. In contrast, under subjetive beliefs, it becomes optimal to lean against housing demand shocks, with the optimal response being asymmetric when faced with positive and negative shocks.

The top row in figure 10 shows the response of housing-related variables. A positive housing demand shock increases house prices and housing price expectations. As anticipated, our subjective belief formulation generates momentum in housing prices. In this specific scenario, the initial shock pushes housing prices up by about 5%, with belief momentum generating another 5% approximately. As actual house price increases start to fall short of the expected house price increases, the housing boom slows down and house prices eventually reverts direction.

[42] As before, we initialize the economy at its ergodic mean. We then hit the economy with one-time shocks and average the subsequent response over the possible future shock realizations. As before, the impulse responses assume a state natural rate of 1.91%, which is the post-1990 mean for the United States.

Figure 10: Impulse responses to a housing preference shock



Higher housing prices push up housing investment, which increases the output gap. As discuss before the increase in housing prices and investment increases marginal utility of consumption, hence, dampens wages and marginal costs. The result of a positive housing demand shock is thus a disinflationary housing boom episode.

Optimal monetary policy leans strongly against the housing price increase. In particular, the policy response is much stronger than when policy faced with a negative housing demand shock.

9 Conclusion

This paper documents new facts about the changing volatility patterns in housing markets and the natural rate. In advanced economies, the standard deviation of the price-to-rent ratio and of the natural rate increased have both increase as the average levels of the natural rate fell.

We examine the implications of these macroeconomic trends for monetary policy in a New Keynesian model featuring a housing market and a zero lower bound on nominal interest rates. Lower natural rates trigger larger volatility in house prices and investment. The policy implications of these developments depend on the source of the increased housing price volatility. If agents hold rational house price expectations, house price fluctuations are driven by efficient housing demand shocks and optimal policy implies that average inflation should rise only by very little, following a fall in natural rates. Instead, if housing price volatility is driven by speculative beliefs about future housing prices, then falling natural rates require a much stronger increase in average inflation under optimal policy. Larger housing price fluctuations endogenously trigger more volatility in natural interest rates, which exacerbates the lower bound problem. Ameliorating the adverse effects of the lower bound then requires implementing on average higher inflation rates.

A Quadratic Approximation of the Policy Problem

This appendix derives the linear-quadratic approximation to the nonlinear policy problem in section 4.

A.1 Optimal Dynamics and the Housing Price Gap

It will be convenient to determine the welfare-maximizing level of output and the welfare-maximizing housing price under flexible prices, so as to express output and housing prices in terms of gaps relative to these maximizing values. We thus define (Y_t^*, q_t^{u*}) as the values (Y_t, q_t^u) that maximize $U(Y_t, 1, q_t^u; \xi_t)$, which are implicitly defined by[43]

$$U_Y(Y_t^*, 1, q_t^{u*}; \xi_t) = U_{q^u}(Y_t^*, 1, q_t^{u*}; \xi_t) = 0.$$

In particular, we have

$$q_t^{u*} = \overline{\xi}_t^d, \tag{69}$$

as shown in Appendix C.2. We have

$$\widehat{q}_t^{u,RE} = \widehat{q}_t^{u*}, \tag{70}$$

which shows that housing price fluctuations are indeed efficient under RE.

Under subjective beliefs, it follows from equations (44) and (50) that

$$\widehat{q}_t^{u,\mathcal{P}} - \widehat{q}_t^{u*} = \left(\frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\beta_t} - \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\rho_\xi} \right) \widehat{\xi}_t^d + \frac{\beta(1-\delta)(\beta_t - 1)}{1-\beta(1-\delta)\beta_t}. \tag{71}$$

Again, for the case where $\beta_t = 1$ and with persistent housing demand shocks ($\rho_\xi \to 1$), the housing price gap under subjective beliefs is equal to the housing price gap under RE. Belief fluctuations, however, now contribute to fluctuations in the housing price gap.

For the real house price gap, $\widehat{q}_t - \widehat{q}_t^*$, this implies

$$\widehat{q}_t - \widehat{q}_t^* = \left(1 + \widetilde{\sigma}^{-1} C_q\right) \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) + \widetilde{\sigma}^{-1} C_Y y_t^{gap}. \tag{72}$$

A.2 Quadratically Approximated Welfare Objective

A second-order approximation to the utility function delivers[44]

$$\frac{1}{2} U_{\widehat{Y}\widehat{Y}} \left(\widehat{y}_t - \widehat{y}_t^*\right)^2 + \frac{1}{2} U_{\widehat{q}^u \widehat{q}^u} \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)^2 + \frac{1}{2} \underline{\gamma}^* h_{22} \pi_t^2 + t.i.p.,$$

where $t.i.p.$ denotes terms independent of policy and $\underline{\gamma}^*$ is the Lagrange multiplier associated with equation (38) at the optimal steady state.The dependence of the objective function on inflation follows from a second-order approximation of the constraint (38), which allows

[43]The optimal path for $\{Y_t^*, q_t^{u*}\}$ can then be used to determine optimal dynamics for the remaining variables. In particular, equation (29) determines C_t^*, equation (13) determines k_t^* and thus D_t^*, and equation (7) determines H_t^*.

[44]See Appendix C.3 for a detailed derivation.

expressing the second-order utility losses associated with price distortions Δ_t as a function of squared inflation terms.

Since the fluctuations in the housing price gap, $\widehat{q}_t^u - \widehat{q}_t^{u*}$, are either constant (with RE) or determined independently of policy (under subjective beliefs, see (71)), the endogenous part of the loss function can be written as

$$\sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left(\Lambda_\pi \pi_t^2 + \Lambda_y \left(y_t^{gap} \right)^2 \right),$$

where $y_t^{gap} \equiv \widehat{y}_t - \widehat{y}_t^*$ denotes the output gap; the log-difference of output from its dynamically optimal value.

A.3 New Keynesian Phillips Curve

We now linearize equations (35)-(37) to derive the linearized Phillips curve. The condition for the equilibrium wage (36) in period T in industry j in which firms last updated their prices in period t is given by

$$w_T(j) = \tilde{w}_T(j) \left(\frac{p_t^j}{P_t} \right)^{-\eta\phi\nu} \left(\frac{P_T}{P_t} \right)^{\eta\phi\nu},$$

where

$$\tilde{w}_T(j) \equiv \lambda \frac{\bar{H}_T^{-\nu}}{\bar{C}_T^{\tilde{\sigma}^{-1}}} \left(\frac{Y_T}{A_T} \right)^{\phi\nu} C\left(Y_T, q_T^u, \xi_T \right)^{\tilde{\sigma}^{-1}}.$$

Since the firms' expectations about $w_T(j)$ and P_T are rational, their expectations about $\tilde{w}_T(j)$ are rational as well.

Using the expression for $w_T(j)$, noting that $p_t(i) = p_t^j = p_t^*$, and writing out $Q_{t,T}$, it follows that

$$\left(\frac{p_t^*}{P_t} \right) = \left(\frac{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{\eta}{\eta-1} \phi \bar{C}_T^{\tilde{\sigma}^{-1}} C_T^{-\tilde{\sigma}^{-1}} \tilde{w}_T(j) \left(\frac{Y_T}{A_T} \right)^{\phi} \left(\frac{P_T}{P_t} \right)^{\eta(1+\omega)}}{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \bar{C}_T^{\tilde{\sigma}^{-1}} C_T^{-\tilde{\sigma}^{-1}} (1-\tau_T) Y_T \left(\frac{P_T}{P_t} \right)^{\eta-1}} \right)^{\frac{1}{1+\omega\eta}}. \tag{73}$$

Log-linearizing equation (73) delivers[45]

$$\widehat{p}_t^* - \widehat{P}_t = \frac{1-\alpha\beta}{1+\omega\eta} \left\{ \widehat{\tilde{w}}_t(j) + \phi \left(\widehat{y}_t - \widehat{A}_t \right) - \widehat{\tau}_t - \widehat{y}_t + \alpha\beta E_t^{\mathcal{P}} \left[\frac{1+\omega\eta}{1-\alpha\beta} \left(\widehat{p}_{t+1}^* - \widehat{P}_{t+1} + \pi_{t+1} \right) \right] \right\}. \tag{74}$$

[45] This follows from the the fact that in steady state, we have $p^* = P$, so that

$$\frac{\eta}{\eta-1} \phi \bar{C}^{\tilde{\sigma}^{-1}} C^{-\tilde{\sigma}^{-1}} \tilde{w}(j) \left(\frac{Y}{A} \right)^{\phi} = \bar{C}^{\tilde{\sigma}^{-1}} C^{-\tilde{\sigma}^{-1}} (1-\tau) Y.$$

The steady state value of the numerator in (73) is thus given by $\frac{1}{1-\alpha\beta} \frac{\eta}{\eta-1} \phi \bar{C}^{\tilde{\sigma}^{-1}} C^{-\tilde{\sigma}^{-1}} \tilde{w}(j) \left(\frac{Y}{A} \right)^{\phi}$ and the steady state value of the denominator by $\frac{1}{1-\alpha\beta} \bar{C}^{\tilde{\sigma}^{-1}} C^{-\tilde{\sigma}^{-1}} (1-\tau) Y$.

As the expectation in (74) is only about variables about which the private agents hold rational expectations, we can replace $E_t^{\mathcal{P}}[\cdot]$ with $E_t[\cdot]$.[46] Therefore, (37) can be used in period t and $t+1$, which in its linearized form is given by

$$\widehat{p}_t^* - \widehat{P}_t = \frac{\alpha}{1-\alpha}\pi_t.$$

Substituting $\widehat{\widetilde{w}}_t(j)$ with the linearized version of the equilibrium condition (36) delivers the linearized New Keynesian Phillips Curve:

$$\pi_t = \kappa_y y_t^{gap} + \kappa_q \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) + \beta E_t \pi_{t+1} + u_t, \tag{75}$$

where the coefficients κ are given by

$$\kappa_y = \frac{1-\alpha}{\alpha}\frac{1-\alpha\beta}{1+\omega\eta}\left(k_y - f_y\right) > 0$$
$$\kappa_q = -\frac{1-\alpha}{\alpha}\frac{1-\alpha\beta}{1+\omega\eta} f_q < 0,$$

with $k_y = \partial \log k / \partial \log y,\ f_y = \partial \log f / \partial \log y,\ f_q = \partial \log f / \partial \log q^u$, such that

$$k_y - f_y = \omega + \tilde{\sigma}^{-1}\frac{\left(1-\underline{g}\right)\underline{Y}}{\underline{C} + \frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}}\underline{k}} = \omega + \tilde{\sigma}^{-1} C_Y > 0$$
$$f_q = \tilde{\sigma}^{-1}\frac{\frac{\underline{k}}{1-\tilde{\alpha}}}{\underline{C} + \frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}}\underline{k}} = -\tilde{\sigma}^{-1} C_q > 0,$$

where $C_q \equiv \frac{q^u}{C}\frac{\partial C}{\partial q^u}$ and $C_Y \equiv \frac{Y}{C}\frac{\partial C}{\partial Y}$, and where the functions $f\left(Y, q^u; \xi\right) \equiv \left(1-\tau\right)\bar{C}^{\tilde{\sigma}^{-1}} Y C\left(Y, q^u; \xi\right)^{-\tilde{\sigma}^{-1}}$ and $k\left(y; \xi\right) \equiv \frac{\eta}{\eta-1}\lambda\phi\frac{\bar{H}^{-\nu}}{A^{1+\omega}} Y^{1+\omega}$ are the same as in Adam and Woodford (2020), for the current period in which markets clear and the internally rational agents observe this.

The cost-push shock u_t is given by

$$u_t = \frac{\left(1-\alpha\right)\left(1-\alpha\beta\right)}{\alpha\left(1+\omega\eta\right)}\left(\Theta + \widehat{\tau}_t - \widehat{g}_t\right),$$

where

$$\widehat{\tau}_t = -\log\left(\frac{1-\tau_t}{1-\bar{\tau}_t}\right)$$
$$\widehat{g}_t = -\log\left(\frac{1-g_t}{1-\bar{g}_t}\right)$$

define deviations of τ_t and g_t from their second-best steady state values.

As in the standard New Keynesian model, a linearization of (38) implies that the state variable Δ_t is zero to first order under the maintained assumption that initial price dispersion satisfies $\Delta_{-1} \sim O(2)$. This constraint, together with the assumption that the Lagrange multipliers are of order $O(1)$, thus drops out of the quadratic formulation of the optimal policy problem. The second-order approximation of (38) is, however, important to express the quadratic approximation of utility in terms of inflation.

[46] The subjective consumption plans showing up in the stochastic discount factor drop out at this order of approximation.

A.4 Linearized IS Equation with Potentially Non-Rational Housing Price Beliefs

We now linearize constraint (39). One difficulty with this constraint is that it features the limiting expectations of the subjectively optimal consumption plan on the right hand side. Generally, this would require solving for the subjectively optimal consumption paths, which is generally difficult.

Under our beliefs specifications, housing prices beliefs are rational in the limit. This insures that we do not have to solve for the subjectively optimal consumption plan, instead can derive the IS equation directly in terms of the output gap.

We can now define the natural rate of interest:

Definition 2 *The natural rate $r_t^{n,RE}$ is the one implied by by the consumption Euler equation (12) or (39), rational expectations, and the welfare-maximizing consumption levels under flexible prices $\{C_t^*\}$. It satisfies*

$$\tilde{u}_C(C_t^*;\xi_t) = \beta E_t \left[u_C(C_{t+1}^*;\xi_t)(1 + r_{t+k}^{n,RE}) \right]. \tag{76}$$

Using the previous definition, we obtain the linearized Euler equation under potentially subjective housing prices beliefs:

Lemma 4 *For the considered belief specifications, the log-linearized household optimality condition (39) implies for all t*

$$y_t^{gap} = \lim_T E_t y_T^{gap} - E_t \left(\sum_{k=0}^{\infty} \varphi \left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n,RE} \right) \right) - \frac{C_q}{C_Y} \left(\widehat{q}_t^u - \widehat{q}_t^{u*} \right), \tag{77}$$

where $\lim_T E_t y_T^{gap}$ is the (rational) long-run expectation of the output gap, and $\varphi \equiv -\frac{\tilde{u}_c}{\tilde{u}_{cc} C} \frac{1}{C_Y} > 0$. The coefficients $C_q < 0$ and $C_Y > 0$ are the ones defined in the derivation of the linearized Phillips Curve.

Proof. See Appendix C. ■

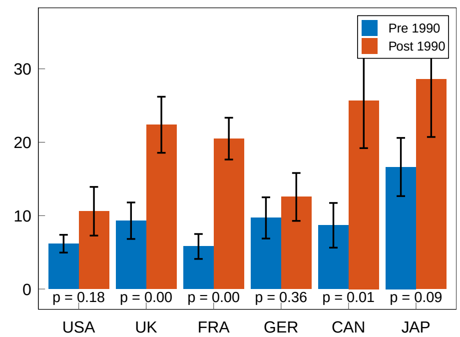
B Online Appendix - Not for publication

B.1 Robustness of Empirical Results

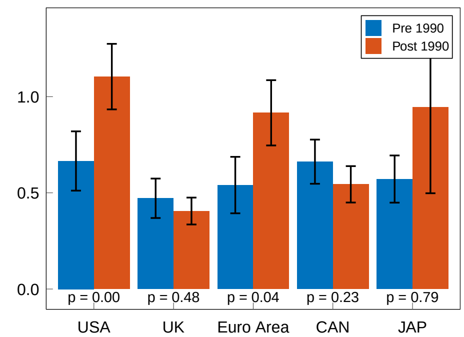
Panel (a) in Figure 11 confirms that not only the standardized volatility of the price-to-rent ratio increased post 1990 compared to the period before 1990, but also the absolute standard deviations of the price-to-rent ratio increased. Panel (b) shows that the volatility of the (non-detrended) natural rate increased in all currency areas, except for the UK.

Figure 11: Absolute Standard Deviation of the Price-to-Rent Ratio Pre and Post 1990.

(a) Absolute Standard Deviation of the Price-to-Rent Ratio Pre and Post 1990.



(b) Standard Deviation of Natural Rate Pre and Post 1990.

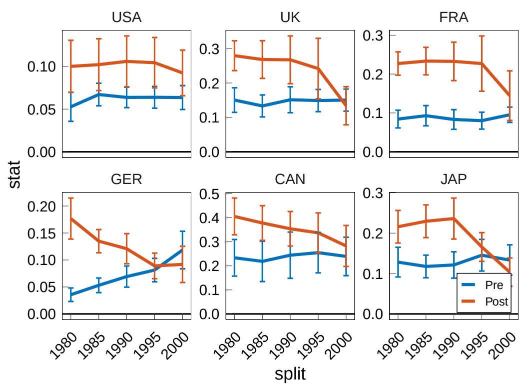


Source: Holston et al. (2017) and Fujiwara et al. (2016) (natural rate estimates), OECD database, own calculations. The black lines denote the 90%-confidence bands. The p-value corresponds to the test whether or not the values changed from pre to post 1990.

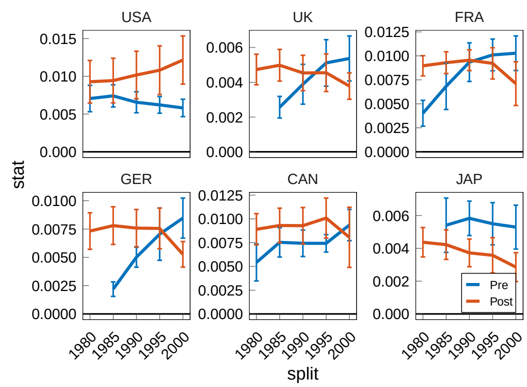
Figure 12 shows that the documented increase in housing volatility is not specific to the chosen periods, but also holds when choosing another cutoff year. This is especially visible for the US where independent of the cutoff, price-to-rent ratios and residential investment always exhibit larger volatility in more recent years compared to previous periods.

Figure 12: Housing Volatility Increased Over the Last Decades.

(a) Standard Deviation of the Price-to-Rent Ratios for Different Sample Splits.



(b) Standard Deviation of Residential Investment for Different Sample Splits.



Source: OECD database, own calculations. The black lines denote the 90%-confidence bands.

C Proofs

Proof of Lemma 1

Proof. Result (41) follows from iterating forward on (15). Log linearizing (41), we have

$$\widehat{q}_t^u = \widehat{\overline{\xi}}_t^d,$$

and log-linearizing (6) delivers

$$\widehat{\xi}_t^d = \rho_\xi \widehat{\xi}_{t-1}^d + \varepsilon_t^d.$$

Since the steady-state value of $\underline{\overline{\xi}}^d$ is

$$\underline{\overline{\xi}}^d = \frac{\underline{\xi}^d}{1-\beta(1-\delta)},$$

the log-linearization of (42) delivers

$$\begin{aligned}
\widehat{\overline{\xi}}_t^d &= (1-\beta(1-\delta))\left[\widehat{\xi}_t^d + \beta(1-\delta)E_t\widehat{\xi}_{t+1}^d + \ldots\right] \\
&= (1-\beta(1-\delta))\left[\widehat{\xi}_t^d + \beta(1-\delta)\rho_\xi\widehat{\xi}_t^d + \ldots\right] \\
&= (1-\beta(1-\delta))\sum_{T=t}^{\infty}\left(\beta(1-\delta)\rho_\xi\right)^{T-t}\widehat{\xi}_t^d \\
&= \widehat{\xi}_t^d \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\rho_\xi}.
\end{aligned}$$

The results for the price-to rent ration follow by noticing that equation (14) implies

$$PR_t \equiv \frac{q_t}{R_t} = \frac{q_t^u}{\xi_t^d}. \tag{78}$$

■

Proof of Lemma 2

Proof. From equation (15), which has to hold with equality in equilibrium, and equation (47) we get

$$q_t^{u,\mathcal{P}} = \frac{1}{1-\beta(1-\delta)\beta_t}\xi_t^d$$

The percent deviation of housing prices from the steady state, in which $\beta_t = 1$ and $\xi_t^d = \underline{\xi}^d$, is then given by

$$
\begin{aligned}
\widehat{q}_t^{u,\mathcal{P}} &= \frac{\frac{1}{1-\beta(1-\delta)\beta_t}\xi_t^d - \frac{1}{1-\beta(1-\delta)}\underline{\xi}^d}{\frac{1}{1-\beta(1-\delta)}\underline{\xi}^d} \\
&= \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\frac{\xi_t^d}{\underline{\xi}^d} - 1 \\
&= \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\left(1+\widehat{\xi}_t^d\right) - 1 \\
&= \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\widehat{\xi}_t^d + \frac{\beta(1-\delta)(\beta_t-1)}{1-\beta(1-\delta)\beta_t} \qquad (79)
\end{aligned}
$$

Note, that we can decompose the housing price under subjective beliefs into the housing price under RE and terms that are driven by beliefs:

$$
\widehat{q}_t^{u,\mathcal{P}} = \widehat{q}_t^{u,RE} + \frac{\beta(1-\delta)\left(\beta_t-1\right)}{1-\beta(1-\delta)\beta_t} + \frac{\left(1-\beta(1-\delta)\right)\left(\beta(1-\delta)\left(\beta_t-\rho_\xi\right)\right)}{\left(1-\beta(1-\delta)\beta_t\right)\left(1-\beta(1-\delta)\rho_\xi\right)}\widehat{\xi}_t^d. \qquad (80)
$$

Note, that

$$
E_t^{\mathcal{P}}\left[q_{t+1}^{u,\mathcal{P}}\right] = \beta_t q_t^{u,\mathcal{P}}.
$$

Therefore, a log-linear approximation around the optimal steady state, in which $\underline{\beta} = 1$, yields

$$
E_t^{\mathcal{P}}\left[\widehat{q}_{t+1}^{u,\mathcal{P}}\right] = \widehat{q}_t^{u,\mathcal{P}} + \left(\beta_t - 1\right).
$$

From this, we can add and subtract on the right-hand side

$$
E_t\left[\widehat{q}_{t+1}^{u,RE}\right] = \rho_\xi\widehat{\xi}_t^d\frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\rho_\xi},
$$

which, after plugging in the expression from (79), delivers

$$
\begin{aligned}
E_t^{\mathcal{P}}\left[\widehat{q}_{t+1}^{u,\mathcal{P}}\right] = {}& E_t\left[\widehat{q}_{t+1}^{u,RE}\right] + \left(\beta_t-1\right)\left[1+\frac{\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\right] \\
&+ \left(1-\beta(1-\delta)\rho_\xi - \left(1-\beta(1-\delta)\beta_t\right)\rho_\xi\right)\frac{\left(1-\beta(1-\delta)\right)}{\left(1-\beta(1-\delta)\beta_t\right)\left(1-\beta(1-\delta)\rho_\xi\right)}\widehat{\xi}_t^d.
\end{aligned}
$$

In the limit $\rho_\xi \to 1$, this boils down to

$$
E_t^{\mathcal{P}}\left[\widehat{q}_{t+1}^{u,\mathcal{P}}\right] = E_t\left[\widehat{q}_{t+1}^{u,RE}\right] + \left(\beta_t-1\right)\left[1+\frac{\beta(1-\delta)}{1-\beta(1-\delta)\beta_t}\left(1+\widehat{\xi}_t^d\right)\right]
$$

This proves the Lemma. ∎

Proof of Lemma 4

Proof. Log-linearizing equation (39) around the optimal steady state delivers

$$\tilde{u}_{CC}C\widehat{c}_t + \tilde{u}_{C\xi}\underline{\xi}\widehat{\xi}_t = E_t^{\mathcal{P}}\sum_{k=0}^{\infty}\tilde{u}_C\left(i_{t+k} - \pi_{t+1+k}\right) + \lim_{T\to\infty}E_t^{\mathcal{P}}\left(\tilde{u}_{CC}C\widehat{c}_T + \tilde{u}_{C\xi}\underline{\xi}\widehat{\xi}_T\right),$$

and log-linearizing (76) gives

$$\tilde{u}_{CC}C\widehat{c}_t^* + \tilde{u}_{C\xi}\underline{\xi}\widehat{\xi}_t = E_t\sum_{k=0}^{\infty}\tilde{u}_C r_{t+k}^{n,RE} + \lim_{T\to\infty}E_t\left(\tilde{u}_{CC}C\widehat{c}_T^* + \tilde{u}_{C\xi}\underline{\xi}\widehat{\xi}_T\right).$$

Subtracting the previous equation from (81) delivers

$$\widehat{c}_t - \widehat{c}_t^* = E_t^{\mathcal{P}}\sum_{k=0}^{\infty}\frac{\tilde{u}_C}{\tilde{u}_{CC}C}\left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n,RE}\right) + \lim_{T\to\infty}E_t^{\mathcal{P}}\left(\widehat{c}_{T+1} - \widehat{c}_{T+1}^*\right), \tag{81}$$

where we used $E_t^{\mathcal{P}}\xi_T = E_t\xi_T$ and $E_t^{\mathcal{P}}\widehat{c}_{T+1}^* = E_t\widehat{c}_{T+1}^*$, which hold because agents hold rational expectations about fundamentals.

In all periods in which the subjectively optimal plan is consistent with market clearing in the goods sector, the plan satisfies equation (29). Log-linearizing equation (29) delivers

$$\widehat{c}_t = C_Y\widehat{y}_t + C_q\widehat{q}_t^u + C_\xi\widehat{\xi}_t, \tag{82}$$

where $\widehat{\xi}_t$ is a vector of exogenous disturbances (involving $A_t^d, \bar{C}_t, g_t$). Evaluating this equation at the optimal dynamics defines the optimal consumption gap $\widehat{c}_t^*$:

$$\widehat{c}_t^* \equiv C_Y\widehat{y}_t^* + C_q\widehat{q}_t^{u*} + C_\xi\widehat{\xi}_t.$$

Subtracting the previous equation from (82) delivers

$$\begin{aligned}\widehat{c}_t - \widehat{c}_t^* &= C_Y\left(\widehat{y}_t - \widehat{y}_t^*\right) + C_q\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)\\ &= C_Y y_t^{gap} + C_q\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)\end{aligned} \tag{83}$$

Since the current consumption market in period t clears, equation (83) holds in period t and can be used to substitute the consumption gap on the l.h.s. of equation (81). Similarly, since housing price expectations are rational in the limit, the consumption market also clears in the limit under the subjectively optimal plans, i.e., equation (29) holds for $t \geq T'$. We can thus use equation (83) also to substitute the consumption gap on the r.h.s. of equation (81). Using the fact that housing price expectations are rational in the limit ($\lim_T E_t^P\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) = 0$), we obtain

$$y_t^{gap} = \lim_T E_t^{\mathcal{P}}y_T^{gap} - E_t\left(\sum_{k=0}^{\infty}\varphi\left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n,RE}\right)\right) - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right).$$

Since we assumed that agents' beliefs about profits and taxes are given by equations (25) and (26), respectively, evaluated using rational income expectations, the household holds rational expectations about total income. This can be seen by substituting (25) and (26) into the budget constraint (2). We thus have $\lim_T E_t^{\mathcal{P}} y_T^{gap} = \lim_T E_t y_T^{gap}$ in the previous equation, which delivers (77). ■

Proof of Lemma 3

Proof. Under the proposed policy that sets $i_t - E_t\pi_{t+1}$ equal to the natural rate defined in equation (66), we have

$$
\begin{aligned}
y_t^{gap} &= \lim_T E_t y_T^{gap} - E_t\left(\sum_{k=0}^{\infty}\varphi\left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n,RE}\right)\right) - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \\
&= \lim_T E_t y_T^{gap} - E_t\left(\sum_{k=0}^{\infty}\varphi\left(r_{t+k}^{n,RE} - \frac{1}{\varphi}\frac{C_q}{C_Y}\left(\left(\widehat{q}_{t+k}^u - \widehat{q}_{t+k}^{u*}\right) - E_{t+k}\left(\widehat{q}_{t+k+1}^u - \widehat{q}_{t+k+1}^{u*}\right)\right) - r_{t+k}^{n,RE}\right)\right) \\
&\quad - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \\
&= \lim_T E_t y_T^{gap} + E_t\left(\sum_{k=0}^{\infty}\left(\frac{C_q}{C_Y}\left(\left(\widehat{q}_{t+k}^u - \widehat{q}_{t+k}^{u*}\right) - \left(\widehat{q}_{t+k+1}^u - \widehat{q}_{t+k+1}^{u*}\right)\right)\right)\right) - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \\
&= \lim_T E_t y_T^{gap} + E_t\left(\frac{C_q}{C_Y}\left(\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) - \lim_k E_t\left(\widehat{q}_{t+k+1}^u - \widehat{q}_{t+k+1}^{u*}\right)\right)\right) - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \\
&= \lim_T E_t y_T^{gap} + \left(\frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)\right) - \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) \\
&= \lim_T E_t y_T^{gap},
\end{aligned}
$$

which proves that with this policy, the output gap is indeed constant, and $r^{n,\mathcal{P}}$ is the real rate that implies a constant output gap. ■

C.1 Transversality Condition Satisfied with Subjective Housing Price Beliefs

This appendix shows that under the considered subjective belief specifications, the optimal plans satisfy the transversality constraint (17). Since $D_t \in [0, D^{\max}]$ and $E_t^{\mathcal{P}} q_T^u = E_t\bar{\xi}_T^d$ for $T \geq T'$, we have $\lim_{T\to\infty}\beta^T E_t^{\mathcal{P}}\left(D_T q_T^u\right) = 0$. We thus only need to show that $\lim_{T\to\infty}\beta^T E_t^{\mathcal{P}}\frac{\bar{C}_T^{\tilde{\sigma}^{-1}}}{C_T^{\tilde{\sigma}^{-1}}}B_T = 0$. Combining the budget constraint (2) with (25) and (26) we obtain

$$C_t + B_t + \left(D_t - (1-\delta)D_{t-1} - \tilde{d}(k_t;\xi_t)\right) q_t^u \frac{C_t^{\tilde{\sigma}^{-1}}}{\bar{C}_t^{\tilde{\sigma}^{-1}}} + k_t = (1-g_t)\,Y_t + B_{t-1}.$$

For $t \geq T'$ the subjectively optimal plans satisfy market clearing in the housing market, i.e.,

$$D_t - (1-\delta)D_{t-1} - \tilde{d}(k_t;\xi_t) = 0$$

so that the budget constraint implies

$$C_t + B_t + k_t = (1 - g_t)\, Y_t + B_{t-1}. \tag{84}$$

Furthermore, for $t \geq T'$ subjectively optimal plans also satisfy market clearing for consumption goods, i.e.,

$$C_t + k_t = (1 - g_t)\, Y_t.$$

It thus follows that the subjectively optimal debt level B_t in the budget constraint (84) is constant under the subjectively optimal plan, after period $t \geq T'$. Furthermore, the expectations about Y_t in the budget constraint (84) is rational under the assumed lump sum transfer expectations, so that the household's subjective consumption expectations are the same as in a rational expectations equilibrium. (The subjectively optimal investment decisions k_t are driven by rational housing price expectations). Since the limit expectations $\bar{C}_T^{\tilde{\sigma}^{-1}} / C_T^{\tilde{\sigma}^{-1}}$ are bounded in the rational expectations equilibrium, it follows that $\lim_{T \to \infty} \beta^T E_t^{\mathcal{P}} \frac{\bar{C}_T^{\tilde{\sigma}^{-1}}}{C_T^{\tilde{\sigma}^{-1}}} B_T = 0$.

C.2 Optimal House Price Absent Price Rigidities

The following derivation closely follows Adam and Woodford (2020). We obtain $U_{q^u}\left(Y_t, \Delta_t, q_t^u, \xi_t\right)$ from differentiating equation (33) with respect to q_t^u and set it equal to 0:

$$\begin{aligned}
U_{q^u}\left(Y_t, \Delta_t, q_t^u, \xi_t\right) &= \bar{C}_t^{\tilde{\sigma}^{-1}} C_{q^u}\left(Y_t, q_t^u, \xi_t\right) C\left(Y_t, q_t^u, \xi_t\right)^{-\tilde{\sigma}^{-1}} \\
&\quad + A_t^d \overline{\xi}_t^d \frac{\partial \Omega\left(q_t^u, \xi_t\right)}{\partial q_t^u} \Omega\left(q_t^u, \xi_t\right)^{\tilde{\alpha}-1} C\left(Y_t, q_t^u, \xi_t\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\tilde{\sigma}^{-1}} \\
&\quad + \frac{\tilde{\sigma}}{1-\tilde{\alpha}} A_t^d \overline{\xi}_t^d \Omega\left(q_t^u, \xi_t\right)^{\tilde{\alpha}} C\left(Y_t, q_t^u, \xi_t\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\tilde{\sigma}^{-1}-1} C_{q^u}\left(Y_t, q_t^u, \xi_t\right) = 0,
\end{aligned}$$

where

$$\frac{\partial \Omega\left(q_t^u, \xi_t\right)}{\partial q_t^u} = \frac{1}{q_t^u} \frac{1}{1-\tilde{\alpha}} \Omega\left(q_t^u, \xi_t\right),$$

and when defining $\chi \equiv \frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}} - 1$, we get

$$C_{q^u}(Y_t, q_t^u; \xi_t) \equiv \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial q^u} = \frac{-\frac{1}{q_t^u} \frac{1}{1-\tilde{\alpha}} \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{1 + (1+\chi)\, \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi}}.$$

Taking everything together, we get

$$U_{q^u}\left(Y_t, \Delta_t, q_t^u, \xi_t\right) = \frac{\frac{1}{q_t^u} \frac{1}{1-\tilde{\alpha}} \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{1 + (1+\chi)\, \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi}} \bar{C}_t^{\tilde{\sigma}-1} \left(\frac{\overline{\xi}_t^d}{q_t^u} - 1 \right).$$

In order for U_{q^u} to be zero, we need to have that

$$q_t^{u*} = \overline{\xi}_t^d,$$

as stated in equation (69).

C.3 Quadratically Approximated Welfare Objective

This derivation follows Adam and Woodford (2020). In the optimal steady state, we have $U_Y = U_{q^u} = U_{Yq^u} = 0$, as well as $U_\Delta + \underline{\gamma}\,(\beta h_1 - 1) = 0$. Given the assumption $\Delta_{-1} \sim O(2)$, it follows $\Delta_t \sim O(2)$ for all $t \geq 0$. Additionally, we have $h_2 \equiv \frac{\partial h(\Delta,\Pi)}{\partial \Pi} = 0$ at the optimal steady state. Therefore, a second-order approximation of the contribution of the variables $(Y_t, \Delta_t, q_t^u, \Pi_t, \xi_t)$ to the utility of the household yields

$$\frac{1}{2}U_{\widehat{Y}\widehat{Y}}\left(\widehat{y}_t - \widehat{y}_t^*\right) + \frac{1}{2}U_{\widehat{q}^u\widehat{q}^u}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) + \frac{1}{2}\underline{\gamma}^* h_{22}\pi_t^2 + t.i.p.,$$

where $t.i.p.$ contains all terms independent of policy. Under rational expectations, we have that $(\widehat{q}_t^u - \widehat{q}_t^{u*}) = 0$ and is thus constant and independent of (monetary) policy. Under subjective beliefs, $(\widehat{q}_t^u - \widehat{q}_t^{u*})$ is purely driven by beliefs β_t and housing demand shocks ξ_t^d, see equation (71), both independent of policy. Therefore, we include $\frac{1}{2}U_{\widehat{q}^u\widehat{q}^u}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)$ in $t.i.p.$.

The term $U_{\widehat{Y}\widehat{Y}}$ is given by $U_{\widehat{Y}\widehat{Y}} \equiv Y\frac{\partial}{\partial Y}\left(U_{\widehat{Y}}\right) \equiv Y\frac{\partial}{\partial Y}\left(YU_Y\right) = \underline{Y}^* U_Y + \left(\underline{Y}^*\right)^2 U_{YY}$. At the optimal steady state, we have

$$\begin{aligned}
\Lambda_\pi &= -\frac{1}{2}\underline{\gamma}^* h_{22} > 0 \\
\Lambda_y &= -\frac{1}{2}\left(\underline{Y}^*\right)^2 U_{YY} > 0,
\end{aligned}$$

where

$$\begin{aligned}
U_{YY} &= -\tilde{\sigma}^{-1}\left(1-\underline{g}\right)\underline{\bar{C}}^{\tilde{\sigma}^{-1}} C\left(\underline{Y}, \underline{q}^u, \underline{\xi}\right)^{-\tilde{\sigma}^{-1}-1} C_Y \frac{\underline{Y}^*}{C\left(\underline{Y}, \underline{q}^u, \underline{\xi}\right)} \\
&\quad - \frac{\lambda}{1+\nu}\left(1+\omega\right)\omega\frac{\underline{\bar{H}}^{-\nu}}{\underline{A}^{1+\omega}}\underline{Y}^{\omega-1} < 0 \\
h_{22} &= \frac{\alpha\eta\left(1+\omega\right)\left(1+\omega\eta\right)}{1-\alpha} > 0 \\
\underline{\gamma}^* &= \frac{U_\Delta}{1-\alpha\beta} < 0,
\end{aligned}$$

with

$$U_\Delta = -\frac{\underline{Y}^*\left(1-\underline{g}\right)}{1+\omega}\left(\frac{\bar{C}^{\tilde{\sigma}^{-1}}}{C\left(\underline{Y}^*, \underline{q}^{u*}, \underline{\xi}\right)}\right)^{\tilde{\sigma}^{-1}} < 0.$$

C.4 Recursifying the Optimal Policy Problem with Lower Bound

We apply the techniques of Marcet and Marimon (2019) to recursify the optimal policy problem with forward-looking constraints (59). We thereby assume that the Lagrangian defined by problem (59) satisfies the usual duality properties that allow interchanging the order of maximization and minimization, which we then show numerically to hold. The value function for $t = T'$ is given by the RE value function $W^{RE}(\cdot)$. For $t \leq T'$ we have a value function $W_t(\cdot)$ which satisfies the following recursion:

$$
\begin{aligned}
& W_t(\varphi_{t-1}, \mu_{t-1}, u_t, r_t^{n,RE}, \beta_t, \xi_t^d, q_{t-1}^u) \\
= \; & \max_{\left(\pi_t, y_t^{gap}, i_t \geq \underline{i}\right)} \min_{(\varphi_t, \lambda_t)} -\frac{1}{2}\left(\Lambda_\pi \pi_t^2 + \Lambda_y \left(y_t^{gap}\right)^2\right) \\
& + \left(\varphi_t - \varphi_{t-1}\right)\pi_t - \varphi_t\left(\kappa_y y_t^{gap} + \kappa_q\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) + u_t\right) \\
& + \lambda_t\left[y_t^{gap} - \lim_T E_t y_T^{gap} + \varphi\left(i_t - E_t \sum_{k=0}^{\infty} r_{t+k}^{n,RE}\right) + \frac{C_q}{C_Y}\left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right)\right] \\
& + \mu_{t-1}\varphi\left(i_t - \pi_t\right) + \gamma_t\left(i_t - \underline{i}\right) \\
& + \beta E_t\left[W_{t+1}(\varphi_t, \underbrace{\beta^{-1}\left(\lambda_t + \mu_{t-1}\right)}_{=\mu_t}, u_{t+1}, r_{t+1}^{n,RE}, \beta_{t+1}, \xi_{t+1}^d, q_t^u\right]
\end{aligned} \tag{85}
$$

where the next period state variables (β_{t+1},q_t^u) are determined by equations (48) and (49) and $(\widehat{q}_t^u - \widehat{q}_t^{u*})$ is determined by equation (71). Here we assume that $r_t^{n,RE}$ follows a Markov process, such that the term $E_t \sum_{k=0}^{\infty} r_{t+k}^{n,RE}$ showing up in the current-period return can be expressed as a function of the current state $r_t^{n,RE}$. The future state variables $(\varphi_t, \mu_t, \beta_{t+1}, q_t^u)$ are predetermined in period t. The expectation about the continuation value is thus only over the exogenous states $(u_{t+1}, r_{t+1}^{n,RE}, \xi_{t+1}^d)$. The endogenous state variable φ_{t-1} is simply the lagged Lagrange multiplier on the NK Phillips curve with housing. The endogenous state variable μ_{t-1} is given for all $t \geq 0$ by

$$\mu_t = \beta^{-(t+1)}\left(\lambda_0 + \mu_{-1}\right) + \beta^{-t}\lambda_1 + ... + \beta^{-1}\lambda_t.$$

The initial values (φ_{-1}, μ_{-1}) are given at time zero and equal to zero in the case of time-zero-optimal monetary policy.

For periods $t < T'$, where T' is the period from which housing price expectations are rational and the lower bound constraint ceases to bind, the value functions depend on time, thereafter they are time-invariant. Likewise for sufficiently large T', the value functions $W_t(\cdot)$ and $W_{t+1}(\cdot)$ will become very similar.

We can numerically solve for the value function $W_t(\cdot)$ by value function iteration, starting with $W_{T'}$ which is the value function associated with the LQ problem with RE and without lower bound.

C.5 Optimal Targeting Rule

Differentiating (85) with respect to $\{\pi_t, y_t^{gap}, i_t\}$ yields:

$$
\begin{aligned}
\frac{\partial W_t}{\partial \pi_t} &= -\Lambda_\pi \pi_t + \left(\varphi_t - \varphi_{t-1}\right) - \mu_{t-1}\varphi = 0 \\
\frac{\partial W_t}{\partial y_t^{gap}} &= -\Lambda_y y_t^{gap} - \varphi_t \kappa_y + \lambda_t = 0 \\
\frac{\partial W_t}{\partial i_t} &= \gamma_t + \lambda_t \varphi + \mu_{t-1}\varphi = 0 \text{ and } \gamma_t\left(i_t - \underline{i}\right) = 0.
\end{aligned}
$$

Combining these first-order conditions, we can derive the following targeting rule which characterizes optimal monetary policy

$$\Lambda_\pi \pi_t + \frac{\Lambda_y}{\kappa_y}\left(y_t^{gap} - y_{t-1}^{gap}\right) + \frac{\lambda_{t-1}}{\kappa_y} + \mu_{t-1}\left(\varphi + \frac{1}{\kappa_y}\right) + \frac{\gamma_t}{\varphi\kappa_y} = 0,$$

where γ_t is the Lagrange multiplier associated with the lower bound on interest rates. If the lower bound on the nominal interest rate does not bind in the current period, we have $\gamma_t = 0$. Furthermore, if the lower bound has not been binding up to period t, the IS equation has not posed a constraint for the monetary policymaker. Thus, $\lambda_{t-1} = \lambda_{t-k} = 0$ for all $k = 0, 1, ..., t$. For an initial value of $\mu_{-1} = 0$, it follows that $\mu_{t-1} = 0$. The targeting rule then collapses to

$$\Lambda_\pi \pi_t + \frac{\Lambda_y}{\kappa_y}\left(y_t^{gap} - y_{t-1}^{gap}\right) = 0,$$

which is the same as in Clarida et al. (1999).

The Lagrange multiplier $\gamma_t \leq 0$ captures the cost of a currently binding lower bound. If $\gamma_t < 0$, the optimal policy requires a compensation in the form of a positive output gap or inflation. The multipliers λ_{t-1} and μ_{t-1} capture promises from past commitments when the lower bound was binding.

Another way to express equation (86) is to write it as

$$\Lambda_\pi \pi_t + \frac{\Lambda_y}{\kappa_y}\left(y_t^{gap} - y_{t-1}^{gap}\right) + \frac{1}{\varphi\kappa_y}\left[\gamma_t - \frac{1+\beta+\varphi\kappa_y}{\beta}\gamma_{t-1} + \frac{\gamma_{t-2}}{\beta}\right] = 0. \tag{86}$$

House prices do not enter the optimal target criterion directly but larger fluctuations in house prices make the lower bound bind more often and for a longer period of time. The optimal policy, thus, requires larger compensations in terms of positive output gaps and inflation. To implement this, the nominal interest rate needs to be kept longer at the lower bound.

C.6 Calibration of C_q/C_Y

To calibrate C_q/C_Y, the ratio of the consumption elasticities to housing prices and income, respectively, note that from appendix "Second-Order Conditions for Optimal Allocation" in Adam and Woodford (2020), we have

$$C_{q^u}(Y_t, q_t^u; \xi_t) \equiv \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial q^u} = \frac{-\frac{1}{q_t^u}\frac{1}{1-\tilde{\alpha}}\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}{1 + (1+\chi)\,\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi}}$$

where $\chi \equiv \frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}} - 1$. In our formulation, we have defined

$$\begin{aligned} C_q &\equiv \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial \ln q_t^u} \\ &= \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial q_t^u}\frac{\partial q_t^u}{\partial \ln q_t^u} \\ &= C_{q^u}(Y_t, q_t^u; \xi_t)\frac{q_t^u}{C_t} \end{aligned}$$

so that we have

$$C_q = -\frac{\frac{1}{1-\widetilde{\alpha}}\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}{C(Y_t, q_t^u, \xi_t) + (1+\chi)\,\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}.$$

From the appendix in Adam and Woodford (2020) we also have

$$C_Y(Y_t, q_t^u, \xi_t) \equiv \frac{\partial C_Y(Y_t, q_t^u, \xi_t)}{\partial Y_t} = \frac{1-g_t}{1+\Omega(q_t^u, \xi_t)\,(1+\chi)\,C(Y_t, q_t^u, \xi_t)^{\chi}}$$

so that in our notation

$$C_Y \equiv \frac{\partial C_Y(Y_t, q_t^u, \xi_t)}{\partial \ln Y_t} = \frac{(1-g_t)\,Y_t}{C(Y_t, q_t^u, \xi_t) + \Omega(q_t^u, \xi_t)\,(1+\chi)\,C(Y_t, q_t^u, \xi_t)^{\chi+1}}.$$

We then have

$$\begin{aligned}
\frac{C_q}{C_Y} &= \frac{\frac{-\frac{1}{1-\widetilde{\alpha}}\Omega(q_t^u,\xi_t)C(Y_t,q_t^u,\xi_t)^{\chi+1}}{C(Y_t,q_t^u,\xi_t)+(1+\chi)\Omega(q_t^u,\xi_t)C(Y_t,q_t^u,\xi_t)^{\chi+1}}}{\frac{(1-g_t)Y_t}{C(Y_t,q_t^u,\xi_t)+\Omega(q_t^u,\xi_t)(1+\chi)C(Y_t,q_t^u,\xi_t)^{\chi+1}}} \\
&= -\frac{1}{1-\widetilde{\alpha}}\frac{\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}{(1-g_t)\,Y_t}.
\end{aligned}$$

In the steady state, we have $\overline{Y}(1-\overline{g}) = \overline{C} + \overline{\Omega C}^{\chi+1}$, which says that privately consumed output $\overline{Y}(1-\overline{g})$ is divided up into consumption $\overline{C}$ and resources invested in the housing sector, $\overline{\Omega C}^{1+\chi}$. We thus have that

$$\begin{aligned}
\frac{\overline{\Omega C}^{\chi+1}}{\overline{Y}(1-\overline{g})} &= 1 - \frac{\overline{C}}{\overline{Y}(1-\overline{g})} \\
&= 1 - \frac{\overline{C}}{\overline{C} + \overline{\Omega C}^{\chi+1}} \\
&= 1 - \frac{1}{1+\overline{\Omega C}^{\chi}}.
\end{aligned}$$

Following Adam and Woodford (2020), we set this to the share of housing investment to total consumption, $\overline{\Omega C}^{\chi}$, equal to 6.3%, so that in steady state we have

$$\frac{C_q}{C_Y} = -\frac{1}{1-\widetilde{\alpha}}\left(1 - \frac{1}{1.063}.\right)$$

Finally, following Adam and Woodford (2020), we set the long-run elasticity of housing supply equal to five, which implies $\widetilde{\alpha} = 0.8$, so that

$$\frac{C_q}{C_Y} = -5\left(1 - \frac{1}{1.063}\right) \approx -0.29633.$$

From this, it follows that

$$
\begin{aligned}
C_Y &= \frac{(1-g)Y}{C+(1+\chi)\,\Omega C^{\chi+1}} \\
&= \frac{C+k}{C+\frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}}k} \\
&= \frac{1+\frac{k}{C}}{1+\frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}}\frac{k}{C}} \\
&= \frac{1+0.063}{1+5\cdot 0.063} \\
&= 0.80836
\end{aligned}
$$

and hence,

$$C_q = -0.29633 \cdot 0.80836 = -0.23954.$$

D Survey Expectations and Model Expectations

To see how we construct $Revisions_t$ from the model, given our belief-formation, note that we have $E_t^{\mathcal{P}}\left[q_{t+1}^u\right] = \beta_t q_t^u$ and

$$E_{t-1}^{\mathcal{P}}\left[q_{t+1}^u\right] = E_{t-1}^{\mathcal{P}}\left[E_t^{\mathcal{P}}\left[q_{t+1}^u\right]\right]$$

by the law of iterated expectations. It then follows that

$$
\begin{aligned}
E_{t-1}^{\mathcal{P}}\left[q_{t+1}^u\right] &= E_{t-1}^{\mathcal{P}}\left[\beta_t q_t^u\right] \\
&= \beta_{t-1} E_{t-1}^{\mathcal{P}}\left[q_t^u\right] \\
&= \beta_{t-1}^2 q_{t-1}^u,
\end{aligned}
$$

where the second line follows from β_t being a random walk. Similarly, for four and five periods ahead expectations, we have

$$
\begin{aligned}
E_t^{\mathcal{P}}\left[q_{t+4}^u\right] &= \beta_t^4 q_t^u \\
E_{t-1}^{\mathcal{P}}\left[q_{t+4}^u\right] &= \beta_{t-1}^5 q_{t-1}^u.
\end{aligned}
$$

Revisions in expectation in the model are thus given by

$$Revisions_t \equiv E_t^{\mathcal{P}}\left[q_{t+4}^u\right] - E_{t-1}^{\mathcal{P}}\left[q_{t+4}^u\right] = \beta_t^4 q_t^u - \beta_{t-1}^5 q_{t-1}^u. \tag{87}$$

D.1 Robustness of Survey Expectations and Model-Implied Expectations

Forecast Errors and Housing Prices An equivalent version of the test from Adam et al. (2017) that we presented in Section 5.3 is proposed by Kohlhas and Walther (2018). In this case, we regress forecast errors on the level of the current housing price. We therefore estimate

$$Error_t = \alpha + \beta_{AMB} \cdot HP_t + \epsilon_t. \tag{88}$$

Table 6 shows the results. In the data, we find a negative, statistically significant coefficient when considering the whole period from 2007 to 2019. If we restrict the sample to start in 2010, we still find a negative, but not significant, estimate. In general, we can state that consumers tend to become too optimistic when they observe high housing prices, inconsistent with rational expectations (as in Adam et al. (2017), Kohlhas and Walther (2018) and Angeletos et al. (2020)). The last four columns show the model-implied coefficients. We see that the model predicts, consistent with the data, a negative sign.

Table 6: Forecast Errors and Housing Prices

	2007 - 2019	2010 - 2019	$\underline{r}^n = 3.34\%$	$\underline{r}^n = 1.91\%$	$\underline{r}^n = 1\%$	$\underline{r}^n = 0.25\%$
β_{AMB}	-0.36**	-0.08	-0.07	-0.07	-0.08	-0.11
	(0.147)	(0.109)				

Note: The first two columns show the empirical estimates for different samples. The last four columns show the model-implied regression coefficients for different levels of the average natural rates.
Significance levels: $^{***}: p < 0.01$, $^{**}: p < 0.05$, $^{*}: p < 0.1$

Median Expectations. Table 7 shows our results when using median expectations instead of averages. Overall, the results are robust to this change.

Table 7: Survey Estimates using Median Expectations (One-Year Ahead Expectations)

Data:	RHP 2007-19	RHP 2010-19
β_{CG}	2.72***	2.04***
	(0.519)	(0.602)
β_{AMB}	-0.31^{*}	0.00
	(0.162)	(0.119)

Significance levels: $^{***}: p < 0.01$, $^{**}: p < 0.05$, $^{*}: p < 0.1$

Instrumental Variable Regression To make sure that the forecast revisions are not correlated with the error term in regression (58), we follow Coibion and Gorodnichenko (2015) and use an IV approach to estimate it, using monetary policy shocks as an instrument.

Table 8 shows the first-stage F-statistic is about 20. The results show that our results are robust to using this IV approach. The estimated coefficients are positive and significantly different from 0.

Table 8: Instrumental Variable Regression

	Survey Average		Survey Median	
	b	p-value	b	p-value
Michigan, 1yr	2.851	0.0230	3.834	0.010
1^{st} stage F-stat.	21.88		17.78	

Significance levels: $^{***}: p < 0.01$, $^{**}: p < 0.05$, $^{*}: p < 0.1$

Monthly Data In the following, we show that our results are robust if we move from quarterly frequency to monthly frequency. To compute real house prices, we again use the Case-Shiller Home Price Index but deflate it with the CPI instead of the GDP deflator. The revisions will, consistent with our main exercise, be the change in the expectations from one quarter to the next. Table 9 shows the results.

Table 9: Survey Estimates using Monthly Data

Data:	RHP 2007-19	RHP 2010-19
Mean Expectations		
β_{CG}	2.13^{***}	1.29^{***}
	(0.323)	(0.391)
β_{AMB}	-0.329^{***}	-0.079
	(0.098)	(0.067)
Median Expectations		
β_{CG}	2.60^{***}	1.74^{***}
	(0.337)	(0.435)
β_{AMB}	-0.278^{***}	-0.007
	(0.105)	(0.072)

Significance levels: $^{***}: p < 0.01$, $^{**}: p < 0.05$, $^{*}: p < 0.1$

Nominal Forecast Errors We now show that our results are robust to using nominal forecast errors and revisions in the expectations about nominal housing prices instead of real forecast errors and revisions in expectations about real housing prices. Given the trend in the level of the nominal house prices, however, we still regress the forecast error on the real house price to estimate β_{AMB}. Table 10 shows the results at the quarterly and monthly frequency.

Table 10: Survey Estimates using Nominal Housing Price Data

Data:	NHP 2007-19 (quarterly)	NHP 2010-19 (quarterly)	NHP 2007-19 (monthly)	NHP 2010-19 (monthly)
Mean Expectations				
β_{CG}	2.17***	1.49***	2.02***	1.24***
	(0.503)	(0.508)	(0.418)	(0.387)
β_{AMB}	-0.34**	-0.05	-0.288***	-0.06
	(0.156)	(0.121)	(0.090)	(0.068)
Median Expectations				
β_{CG}	2.77***	2.02***	2.63***	1.78***
	(0.498)	(0.606)	(0.423)	(0.452)
β_{AMB}	-0.29	0.04	-0.23**	0.03
	(0.175)	(0.131)	(0.100)	(0.073)

Significance levels: *** : $p < 0.01$, ** : $p < 0.05$, * : $p < 0.1$

Expectations across the income distribution. We now divide the survey respondents into three groups, according to their reported income. Table 11 shows the results. The first column gives the estimated coefficients from a panel regression, using income-tertile-specific fixed effects. The next three column looks at the three groups separately, in increasing order. We see that our results are robust, and additionally that the results do not systematically differ along the income distribution. In fact, we cannot reject in any case the nullhypothesis that the regression coefficients differ across income groups.

Table 11: Survey Estimates for Different Income Groups

	Panel (monthly)	Bottom 33% (monthly)	Middle 33% (monthly)	Top 33% (monthly)
Mean Expectations				
β_{CG}	1.98***	1.98***	2.04***	1.91***
	(0.085)	(0.345)	(0.341)	(0.320)
β_{AMB}	-0.31***	-0.32***	-0.33***	-0.29***
	(0.024)	(0.110)	(0.107)	(0.102)
Median Expectations				
β_{CG}	2.49***	2.71***	2.50***	2.28***
	(0.090)	(0.327)	(0.339)	(0.329)
β_{AMB}	-0.26***	-0.25**	-0.27**	-0.27**
	(0.026)	(0.117)	(0.111)	(0.107)

Significance levels: *** : $p < 0.01$, ** : $p < 0.05$, * : $p < 0.1$

Optimists vs. pessimists In the following, we test whether our results are driven by either optimists or pessimists. To do so, we re-run the CG and the AMB regressions as before, but instead of considering mean or median expectations, we look at the 25^{th} and the 75^{th} percentile, respectively. For this, we use monthly data and consider real and nominal house prices. Table 12 shows the results. We observe the expected difference in the magnitude of the coefficients (higher β_{CG} and less negative β_{AMB} for the pessimists). Nevertheless, the signs are the same in all cases, and we reject the rational expectations hypothesis.

Table 12: Survey Estimates: Pessimists (25^{th} percentile) vs. Optimists (75^{th} percentile)

	25^{th} pct, Real HP (monthly)	25^{th} pct, Nominal HP (monthly)	75^{th} pct, Real HP (monthly)	75^{th} pct, Nominal HP (monthly)
β_{CG}	2.45^{***}	2.49^{***}	1.86^{***}	1.80^{***}
	(0.357)	(0.453)	(0.320)	(0.341)
β_{AMB}	-0.25^{**}	-0.20^{*}	-0.39^{***}	-0.35^{***}
	(0.109)	(0.105)	(0.095)	(0.083)

Significance levels: $^{***}: p < 0.01$, $^{**}: p < 0.05$, $^{*}: p < 0.1$

Information set concerns A potential concern with the stated result that house price growth expectations are higher in times of higher current house prices might be that households are not aware of the level of current house prices. In order to account for this, we re-run the regression. But instead of using the current house price as the independent variable, we use the household's perceived change of the value of its own home over the last year, taken from the Survey of Consumers. More specifically, we include the share of households that state that their own home increased in value as well as the share of households that say their house value decreased over the last 12 months as independent variables. The first column of Table 13 shows the results. The two coefficients $\widehat{\beta}_{Up}$ and $\widehat{\beta}_{Down}$ correspond to the share of households that state that their house value went up or down, respectively. We see that house price expectations are higher after periods in which many people think their own house value increased, and vice-versa. In the second column, we report the results if we additionally control for lagged perceived house price growth in the previous period.

To further strengthen this result, we use monthly data from the Survey of Consumers by region. The four regions are *North Central, North East, South, and West*. This leaves us with much more observations. Furthermore, we can control for time- and region-specific fixed effects. The last three columns of Table 13 show that the sign of the coefficients are unaltered. Only the exact values change across specification.

Table 13: Expected and Perceived House Prices

Dependent Variable: $E_t^{\mathcal{P}}\left[\Delta HP_{t,t+4}\right]$		
Data	Panel	Panel
$\widehat{\beta}_{Up}$	0.026^{***}	0.044^{***}
	(0.0038)	(0.0055)
$\widehat{\beta}_{Down}$	-0.041^{***}	-0.037^{***}
	(0.0042)	(0.0062)
N	636	636
Time FE	×	✓
Region FE	✓	✓

Significance levels: $^{***}: p < 0.01$, $^{**}: p < 0.05$, $^{*}: p < 0.1$

References

Adam, K. and R. M. Billi (2006): "Optimal monetary policy under commitment with a zero bound on nominal interest rates," *Journal of Money, Credit and Banking*, 1877–1905.

Adam, K. and A. Marcet (2011): "Internal rationality, imperfect market knowledge and asset prices," *Journal of Economic Theory*, 146, 1224–1252.

Adam, K., A. Marcet, and J. Beutel (2017): "Stock price booms and expected capital gains," *American Economic Review*, 107, 2352–2408.

Adam, K., A. Marcet, and P. Kuang (2012): "House Price Booms and the Current Account," in *NBER Macroeconomics Annual 2011*, ed. by D. Acemoglu and M. Woodford, 77 – 122.

Adam, K., A. Marcet, and J. P. Nicolini (2016): "Stock market volatility and learning," *The Journal of Finance*, 71, 33–82.

Adam, K., D. Matveev, and S. Nagel (2020): "Do Survey Expectations of Stock Returns Reflect Risk Adjustments?" *Journal of Monetary Economics*.

Adam, K. and S. Merkel (2019): "Stock price cycles and business cycles," *CEPR Discussion Paper No. 13866*.

Adam, K. and H. Weber (2019): "Optimal Trend Inflation," *American Economic Review*, 109(2), 702–737.

——— (2020): "Estimating the Optimal Inflation Target from Trends in Relative Prices," ECB Working Paper No. 2370.

Adam, K. and M. Woodford (2020): "Robustly Optimal Monetary Policy in a New Keynesian Model with Housing," Tech. rep., National Bureau of Economic Research.

Adelino, M., A. Schoar, and F. Severino (2017): "Dynamics of housing debt in the recent boom and bust," *NBER Macroeconomics Annual*, 32, 261–311.

Andrade, P., J. Galí, H. L. Bihan, and J. Matheron (2018): "The optimal inflation target and the natural rate of interest," Tech. rep., National Bureau of Economic Research.

Angeletos, G.-M., Z. Huo, and K. Sastry (2020): "Imperfect expectations: Theory and evidence," in *NBER Macroeconomics Annual 2020, volume 35*, University of Chicago Press.

Armona, L., A. Fuster, and B. Zafar (2019): "Home price expectations and behaviour: Evidence from a randomized information experiment," *The Review of Economic Studies*, 86, 1371–1410.

Barberis, N., R. Greenwood, L. Jin, and A. Shleifer (2015): "X-CAPM: An extrapolative capital asset pricing model," *Journal of financial economics*, 115, 1–24.

Ben-David, I., P. Towbin, and S. Weber (2019): "Expectations During the US Housing Boom: Inferring Beliefs from Actions," Tech. rep., National Bureau of Economic Research.

Benigno, P. and L. Paciello (2014): "Monetary Policy, Doubts and Asset Prices," *Journal of Monetary Economics*, 64, 85–98.

Benigno, P. and L. A. Ricci (2011): "The Inflation-Output Trade-Off with Downward Wage Rigidities," *American Economic Review*, 101, 1436–1466.

Bernanke, B. S. and M. Gertler (2001): "Should central banks respond to movements in asset prices?" *american economic review*, 91, 253–257.

Bordalo, P., N. Gennaioli, Y. Ma, and A. Shleifer (2018): "Over-reaction in macroeconomic expectations," Tech. rep., National Bureau of Economic Research.

Caines, C. and F. Winkler (2020): "Asset Price Learning and Optimal Monetary Policy," *Working Paper*.

Calvo, G. A. (1983): "Staggered prices in a utility-maximizing framework," *Journal of monetary Economics*, 12, 383–398.

Case, K. E. and R. J. Shiller (1988): "The behavior of home buyers in boom and post-boom markets," Tech. rep., National Bureau of Economic Research.

Case, K. E., R. J. Shiller, and A. Thompson (2012): "What have they been thinking? Home buyer behavior in hot and cold markets," Tech. rep., National Bureau of Economic Research.

Clarida, R., J. Gali, and M. Gertler (1999): "The science of monetary policy: a new Keynesian perspective," *Journal of economic literature*, 37, 1661–1707.

Coibion, O. and Y. Gorodnichenko (2015): "Information rigidity and the expectations formation process: A simple framework and new facts," *American Economic Review*, 105, 2644–78.

Coibion, O., Y. Gorodnichenko, and J. Wieland (2012): "The Optimal Inflation Rate in New Keynesian Models: Should Central Banks Raise Their Inflation Targets in Light of the Zero Lower Bound?" *Review of Economic Studies*, 79, 1371–1406.

Collin-Dufresne, P., M. Johannes, and L. A. Lochstoer (2016): "Parameter learning in general equilibrium: The asset pricing implications," *American Economic Review*, 106, 664–98.

Del Negro, M., D. Giannone, M. P. Giannoni, and A. Tambalotti (2017): "Safety, liquidity, and the natural rate of interest," *Brookings Papers on Economic Activity*, 2017, 235–316.

Eggertsson, G. B., N. R. Mehrotra, and J. A. Robbins (2019): "A model of secular stagnation: Theory and quantitative evaluation," *American Economic Journal: Macroeconomics*, 11, 1–48.

Eggertsson, G. B. and S. R. Singh (2019): "Log-linear Approximation versus an Exact Solution at the ZLB in the New Keynesian Model," *Journal of Economic Dynamics and Control*, 105, 21–43.

Eggertsson, G. B. and M. Woodford (2003): "Zero bound on interest rates and optimal monetary policy," *Brookings papers on economic activity*, 2003, 139–233.

Fujiwara, S., Y. Iwasaki, I. Muto, K. Nishizaki, and N. Sudo (2016): "Developments in the natural rate of interest in apan," *Bank of Japan Review*, October 2016-E-12.

Gilchrist, S. and J. V. Leahy (2002): "Monetary policy and asset prices," *Journal of monetary Economics*, 49, 75–97.

Glaeser, E. L. and C. G. Nathanson (2017): "An extrapolative model of house price dynamics," *Journal of Financial Economics*, 126, 147–170.

Greenwood, R. and A. Shleifer (2014): "Expectations of returns and expected returns," *The Review of Financial Studies*, 27, 714–746.

Hansen, A. H. (1939): "Economic progress and declining population growth," *The American economic review*, 29, 1–15.

Holston, K., T. Laubach, and J. C. Williams (2017): "Measuring the natural rate of interest: International trends and determinants," *Journal of International Economics*, 108, S59–S75.

Iacoviello, M. (2005): "House prices, borrowing constraints, and monetary policy in the business cycle," *American economic review*, 95, 739–764.

Kaplan, G., K. Mitman, and G. L. Violante (2020): "The Housing Boom and Bust: Model Meets Evidence," *Journal of Political Economy*, 128, 3285–3345.

Khan, A., R. G. King, and A. L. Wolman (2003): "Optimal Monetary Policy," *Review of Economic Studies*, 70, 825–860.

Kohlhas, A. and A. Walther (2018): "Asymmetric Attention," *Available at SSRN 3161628*.

Malmendier, U. and S. Nagel (2011): "Depression babies: do macroeconomic experiences affect risk taking?" *The Quarterly Journal of Economics*, 126, 373–416.

——— (2016): "Learning from inflation experiences," *The Quarterly Journal of Economics*, 131, 53–87.

Marcet, A. and R. Marimon (2019): "Recursive contracts," *Econometrica*, 87, 1589–1631.

Nagel, S. and Z. Xu (2019): "Asset pricing with fading memory," Tech. rep., National Bureau of Economic Research.

Piazzesi, M. and M. Schneider (2009): "Momentum traders in the housing market: Survey evidence and a search model," *American Economic Review*, 99, 406–11.

Shiller, R. J. (2007): "Understanding recent trends in house prices and home ownership," Tech. rep., National Bureau of Economic Research.

Soo, C. K. (2018): "Quantifying Sentiment with News Media across Local Housing Markets," *The Review of Financial Studies*, 31, 3689–3719.

Summers, L. H. (2014): "US economic prospects: Secular stagnation, hysteresis, and the zero lower bound," *Business economics*, 49, 65–73.

Yun, T. (1996): "Nominal price rigidity, money supply endogeneity, and business cycles," *Journal of monetary Economics*, 37, 345–370.