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Sequential Trading With Coarse Contingencies

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Abstract

We consider a dynamic pure exchange economy in which agents have a coarse perception of the future and, in particular, may be unaware of some risks. As awareness of these risks emerges, markets have to re-open to allow the agents to re-optimize and purchase insurance. The paper provides an irrelevance theorem, showing that if unforeseen shocks are purely idiosyncratic and agents become aware of them before they occur, then unawareness does not affect equilibrium consumption. Unawareness thus matters only if it concerns aggregate shocks. Building on this insight, we highlight several interesting implications for economies with unforeseen aggregate shocks: the agents' failure to spread the cost of insurance efficiently across time, heterogeneous consumption growth rates, systematically biased price expectations and the possibility of unexpected default.

Keywords: coarse perceptions, unforeseen risks, sequential trading, default.

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1 Introduction

In recent years, the literature on unawareness has made substantive strides, including foundational contributions in decision theory and epistemics, as well as a growing number of applications.[1] In this paper, we study the role of unawareness in a dynamic competitive economy. We believe the setting to be a fertile one as the implications of full rationality, including the ability of agents to anticipate all future contingencies, are nowhere else more striking and well-known. Thus, if markets are complete ex ante, it is known that all trading will take place at a moment in time, with no need for markets to ever reopen. If markets open gradually over time, economic activity will unfold gradually as well. Yet, everything proceeds in line with prior expectations so that, inevitably, the equilibria of the dynamic economy mimic those of the static, once-and-for-all economy.

In our setup, awareness of new risks emerges gradually over time. As the agents "scramble" to re-optimize and purchase insurance, it is immediate that markets have to reopen. What may be more surprising is our central result, which shows that under certain conditions unawareness has no effect on consumption and, in a limited sense, prices. In particular, though the agents rush to buy insurance as their awareness grows, they never deviate from their consumption plans. Moreover, consumption plans coincide with what the agents would have chosen if they anticipated all contingencies ex ante. Unawareness is thus seen to be irrelevant both descriptively and normatively.

Our irrelevance theorem hinges on three main assumptions: (i) awareness of a shock emerges before the shock is actually realized; (ii) once awareness emerges, the shock becomes contractible, that is, the agents can purchase insurance against it; and finally (iii) the unforeseen shocks are purely idiosyncratic: they affect the distribution but not the size of the pie. Assumption (ii) requires little motivation. Assumption (i) is more substantive, but we believe not unreasonable. On a purely descriptive level, it captures naturally a situation in which people are aware of immediate risks but not

[1] On the theory front, see Heifetz, Meier, and Schipper [22, 23], Ghirardato [13], Mukerji [36], Epstein, Marinacci, and Seo [8], Kochov [28], and Karni and Vierø [26], among others. For applications, see Heifetz, Meier, and Schipper [24], Auster [2], or Galanis [12]. A trove of ideas and other references can also be found in the surveys by Dekel, Lipman, and Rustichini [7] and Schipper [39].

distant ones. Additionally, while people sometimes "learn the hard way," they also learn by observing others, by studying history and works of fiction, or simply by taking time to think about the future. Assumption (i) is meant to capture such situations, in which conception occurs before, rather than after, the fact.[2] Taking assumptions (i) and (ii) for granted, assumption (iii) delineates the scope of our irrelevance theorem: *unawareness matters if it concerns aggregate risks, not idiosyncratic ones.*

The theorem rests on two classical results in general equilibrium: the fact that idiosyncratic risks are fully insured when markets are (dynamically) complete and, consequently, that such risks are fairly priced.[3] Given these building blocks, the theorem does not take much technical prowess. Its contribution is conceptual and lies in showing how these well-known results manifest themselves in a model of unawareness, paying specific attention to the modeling choices regarding unawareness that make this possible.[4]

To elaborate, note that under assumptions (i) and (ii), the economies we study are dynamically complete. Additionally, under assumption (i), the agents are always able to purchase insurance in advance of a shock. The main problem therefore is that agents who are initially unaware of a shock may fail to save appropriately and spread the cost of insurance over time. This is where *the fair pricing* of idiosyncratic risks kicks in. Specifically, it implies that a mean-zero idiosyncratic risk (conditional on the aggregate endowment) has no effect on equilibrium wealth and, consequently, on an agent's savings. The problem, of course, is that mean-zero risks would be an odd restriction to impose on the *exogenous environment*. However, we claim that there is a "dual" assumption on *perceptions* that is both interesting and natural. Specifically, we assume that unaware agents act as if they take the first moment of their endowment (conditional on the aggregate one) as the truth and ignore the remaining mean-preserving spread. As explained in Section 2.3, the assumption,

[2] See Becker and Mulligan [4] for a stimulating discussion and other examples of such learning.

[3] That is, the relative price of consumption in any two states in which the aggregate endowment is constant is equal to the relative probability of those states; there is no "risk adjustment."

[4] Our theorem is also intimately linked to a famous result of Cass and Shell [6], which, based on the same building blocks, shows that extrinsic risks do not matter in an economy with complete markets. The connection is discussed in Section 3.1, where we state a lemma that bridges the gap between the two results.

which we call *correct in expectation*,[5] is intimately linked to the question whether agents are *aware of their unawareness*. The latter is a key turning point in much of the unawareness literature and, by what we have just seen, in determining whether awareness of idiosyncratic risks matters in a general equilibrium setting with complete markets.

Taking our irrelevance theorem as a point of departure, our next step is to investigate how *unforeseen aggregate risks* may affect the economy. A preview of the findings follows.

Inefficiencies and heterogeneous growth (rates).

In Section 4.1, we start with a simple example of how an unforeseen aggregate shock may generate an inefficiency relative to the benchmark of full rationality. The logic is immediate given the discussion of our irrelevance theorem. Except in some special circumstances, which we detail at the end of Section 3.4, aggregate shocks, which cannot be fully insured, will always affect the agents' equilibrium wealth. Fully aware agents would save in anticipation of the shock, while the unaware agents of our paper do not. If the shock affects agents heterogeneously, there will be unrealized gains from trade.

Simple as it is, the example contains an important empirical implication on top of its normative one. As we recall in Section 3.1, the full insurance of idiosyncratic risks (under complete markets and full rationality) means that agents' consumption levels move in lock-step with the aggregate endowment and hence with each other. This latter, *cross-sectional* implication, which is necessary but not sufficient for efficiency, is remarkable in that it does not require precise knowledge of the agents' preference parameters. Consequently, any failure of that implication is a robust refutation of efficiency and this is precisely what our example delivers. We also note that an enormous empirical literature has sought to test that implication of efficiency, interpreting any failures thereof as evidence of incomplete markets or *uninsurable idiosyncratic risks*.[6]

[5] Contemporaneously with us, the correct in expectation assumption appeared in the work of Teeple [41]. Jehiel's [25] notion of analogy equilibrium entails a closely related, but ultimately distinct, assumption as well.

[6] The literature is surveyed in Guvenen [21]. We note that in practice it is common to assume homothetic utility and test the more specific finding that agents' consumption levels grow *at the same rate*.

By comparison, our example shows how such failures can arise due to *unforeseen aggregate risks.*

Prices of long-term assets and intertemporal arbitrage.

Another empirical implication concerns prices. As is well-known, *prudent* agents *increase* their savings when they become aware of a new shock to future wealth. In section 4.2, we show that this leads to an *unexpected* increase in the price of long-term assets, which are now seen to provide insurance against risks that were not previously priced. Notably, the observed price patterns cannot arise under any model with fully aware agents as that would present an opportunity for *intertemporal arbitrage*. Unawareness is thus seen to impose "limits on arbitrage," a finding made robust by the fact that assumption (i) seeks to restrict rather than amplify the effects of unawareness.[7]

Unexpected Default.

Finally, we show how an unforeseen aggregate shock may lead to *unexpected* or *involuntary default*. To understand what we mean by this, we note that, outside the parameters of our irrelevance theorem, the economy will typically *not* evolve according to a single "equilibrium of prices, plans, and price expectations" in the sense of Radner [37]. At best, the economy will transition from one such equilibrium to another as awareness of new shocks emerges and the agents rush to re-optimize. It turns out however that even that is not guaranteed - the economy may arrive at a juncture at which there is no market clearing price that allows the agents to honor their obligations. The example, which we develop fully in Section 4.3, is technically simple and can be reduced to a static one in Arrow and Hahn [1, p.119], whereby the endowment lies outside the Edgeworth box and market-clearing prices fail to exist.[8] Our contribution is to show how such a situation can occur within a fully-fledged model of unawareness.

The example complements the well-known work of Modica, Rustichini, and Tallon [35], where, too, default arises as a result of unawareness. In Modica et al. [35],

[7] See Gromb and Vayanos [17] for a survey of the large and active literature on the limits of arbitrage.

[8] See also Green [16] and more recently Ben-Ami and Geanakoplos [5].

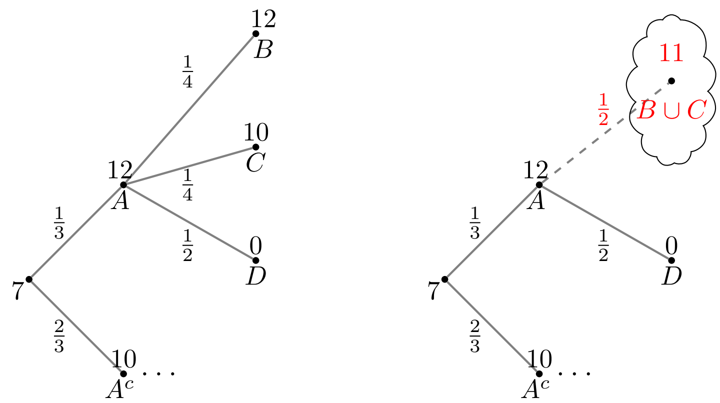

Figure 1: On the left is true or objective environment; on the right, an agent's perception thereof.

however, default arises because agents do not understand the payoff implications of their asset positions. In our model, the agents borrow short-term which, by virtue of assumption (i), means that they fully understand the obligations they undertake. Yet, default arises because the agents fail to predict the prices at which they can roll over their debt.

2 The Model

Throughout the paper, time is discrete and varies over a finite horizon: $t \in \{0, 1, 2, ..., K\}$. There are $N > 1$ agents, indexed by i. We assume that there is a single consumption good, which is infinitely divisible.[9] Next is an example describing how we model unawareness.

2.1 Perceptions at a point in time: Example

The left-hand side of Figure 1 depicts the objective environment and the true endowment of some agent. The former consists of an event tree T, or simply a tree, describing which events occur when, and a probability measure μ describing the probabilities of these events. One reads the figure as follows. Event A occurs with

[9]All our results in Section 3 carry over to the case multiple commodities.

probability $\frac{1}{3}$ in period 1; if so, one of three subevents, B, C, and D, will occur in period 2. In the initial period, the agent's endowment is 7; in period 1, it is 12 if A happens, and 10 if A doesn't happen, etc. The right-hand side of Figure 1 depicts the agent's perception of the environment and of his endowment. We note that the agent is aware of the one-step-ahead contingencies A and A^c. This is assumption (i) mentioned in the introduction. At the same time, the agent, while aware of the event $B \cup C$ in $t = 2$, is unaware of the finer contingencies B and C that comprise it. This is why $B \cup C$ appears as a node in the agent's subjective tree $\hat{T}$. Looking at the figure, we also note that the agent assigns the correct belief to any event he foresees. Finally, being unaware of the finer contingencies B and C, the agent perceives his endowment as being constant conditional on $B \cup C$. In the figure, that constant value is 11 or the conditional expectation of his true endowment given $B \cup C$. When this is true, we say that the agent is correct in expectation. The assumption captures naturally a situation in which the agent is unaware of some mean-preserving shock to his endowment. Further discussion of the assumption is found in Section 2.3.

2.2 Mathematical Formalism

An *event tree*, or simply a *tree*, T is a rooted tree, with the edges directed away from the root and each terminal node having the same number of predecessors. Presently, we focus on the case in which that number is K, the same as the number of periods. "Shorter" trees will only arise when we consider the subtrees of a tree. We write $s \in T$ if s is a node of T and let S_t be the set of nodes $s \in T$ with exactly t predecessors. One can think of a node $s_t \in S_t$ as an event taking place in period t. The root of T is denoted by 0. Any node $s \in T$ can be identified with the terminal nodes that succeed it, that is, with a subset of S_K. Then, $s_t \subset s_{t+k}$ if and only if s_{t+k} succeeds s_t. Since, given $t < K$, every terminal node $s_K \in S_K$ has exactly one predecessor $s_t \in S_t$, we can view S_t as a partition of S_K. In that interpretation, S_0 is the trivial partition of *the set* S_K of terminal nodes and each S_{t+1} is a refinement of S_t. In measure-theoretic terms, we can thus think of a tree T as a state space S_K endowed with a filtration $\{S_t\}_{t=0}^K$. A probability measure μ on a tree T is a probability measure on the measure space $(S^K, 2^{S^K})$.

A tree $\hat{T}$ is a *coarsening* of T if, for each t, $\hat{S}^t$ can be identified with a partition of S^K coarser than S^t. If so, every node $\hat{s} \in \hat{T}$ can be identified with a subset of S_K. Moreover, for each node $s \in T$, there is a unique node $\phi(s) \in \hat{T}$ such that $s \subset \phi(s)$. If s is a strict subset of $\phi(s)$, we say that s is *unforeseen* and that $\phi(s)$ is *coarse*. If, on the other hand, $s = \phi(s)$, we say that s is foreseen and write $s \in T \cap \hat{T}$. Given a coarsening $\hat{T}$ of T, a function $f : T \to \mathbb{R}$ is $\hat{T}$-*measurable* if $f(s) = f(s')$ whenever $\phi(s) = \phi(s')$. We note that any function $\hat{f} : \hat{T} \to \mathbb{R}$ can be viewed as a $\hat{T}$-measurable function on T. Depending on the setup, we may thus write either $\hat{f}(s)$ or $\hat{f}(\hat{s})$. Given functions $f : T \to \mathbb{R}$, $\hat{f} : \hat{T} \to \mathbb{R}$, the equality $f = \hat{f}$ should also be clear. It implies in particular that f is $\hat{T}$-measurable. Finally, a function $f : T \to \mathbb{R}$ may also be referred to as a *process* and be depicted as the sequence $(f_0, f_1, ..., f_K)$ where f_t is the function $s_t \mapsto f(s_t)$.

2.3 Perceptions

Let the objective environment be given by the tuple $(T, \mu, (y^i)_i)$, where T is an event tree, μ is a probability measure on T, and $y^i : T \to \mathbb{R}_+$ is agent i's true endowment process. We model the agents' perceptions of the environment by a tuple $(\hat{T}, \hat{\mu}, (\hat{y}^i)_i)$, where $\hat{T}$ is a coarsening of T, describing the events foreseen by the agents, $\hat{\mu}$ is a belief on $\hat{T}$, and $\hat{y}^i : \hat{T} \to \mathbb{R}_+$ is agent i's perception of his endowment process $y^i : T \to \mathbb{R}_+$. We note the assumed homogeneity of perceptions with regards to foreseen events, given by $\hat{T}$, and their likelihoods, given by $\hat{\mu}$. The assumption means that risk sharing and intertemporal smoothing are the only reasons for trade; speculation is not a factor. We further discipline the model by assuming that agents assign the correct belief to any event they foresee, that is, $\mu(s) = \hat{\mu}(\phi(s))$ for every $s \in T$. More interesting and integral to our analysis are the next three assumptions, which we list explicitly whenever assumed.

Assumption 1 (One-Step-Ahead Awareness (OSAA)) *All period-1 nodes of T are foreseen, that is, $s_1 = \phi(s_1)$ for all $s_1 \in T$.*

OSAA was discussed extensively in the introduction. As noted there, it ensures that agents are able to adapt before a shock hits. This is clearly necessary if one is interested in how far sequential markets can go toward remedying the effects of unawareness. On the other hand, the assumption works against in the second part of the paper where we show how unawareness can make a difference *despite* OSAA.

Assumption 2 (Correct in Expectation (CE)) *For every i, $\hat{y}^i_t = \mathbb{E}_\mu[y^i_t \,|\, S_t]$ for every t or, in more compact notation, $\hat{y}^i = \mathbb{E}_\mu[y^i \,|\, \hat{T}]$.*

CE requires that an agent's perception of his endowment at a node $\hat{s} \in \hat{T}$ be equal to the respective conditional expectation of the true endowment y^i. An immediate implication is that if a node $s \in T$ is foreseen, then the agent knows his endowment at that node. In Figure 1, for instance, the agent knows that his endowment at node A, which he foresees, is 12. The general point is that, under CE, perceptions are tethered to the truth, not arbitrary. Obviously, this will play a role in our irrelevance theorem, though, as we discuss in Section 3.1, the exact role is somewhat subtle. Another important aspect of CE is that it reflects an agents who is *unaware of his unawareness*. Indeed, consider an agent whose perception is given by $\hat{T}$ but who suspects that $\hat{T}$ is not "the end of the story" as there could be additional risks lurking in the background. Even when he knows the conditional expectation of y^i given $\hat{T}$, such an agent may, out of cautiousness, adopt a perception that is purposefully lower. In this respect, a lesson from our analysis is that awareness of one's unawareness may, under certain circumstances, be welfare-decreasing.

The final assumption simply describes the case when unforeseen shocks are purely idiosyncratic, which means that they have no effect on the aggregate endowment.

Assumption 3 (Measurable Aggregate Endowment (MAE)) *The aggregate endowment $Y := \sum_i y^i$ is $\hat{T}$-measurable.*

2.4 Expanding Awareness

Our interest is in situations in which awareness improves over time. To that end, we first explain how we model the fact that perceptions change over time. Consider a node s of the objective tree T and let T^s be the corresponding continuation tree. Likewise, let $\mu^s = \mu(\,\cdot\,|s)$ the Bayes posterior of μ and $y^{i,s} : T^s \to \mathbb{R}_+$ be the restriction of i's endowment process $y^i : T \to \mathbb{R}$ to T^s. When the agents find themselves at the node s, they face a continuation environment (T^s, μ^s) just as at the beginning of time they face the environment (T, μ). This means that we can define perceptions at s in a manner analogous to what we did in the previous section. In particular, we let $\hat{T}^s$ be a coarsening of T^s, describing the events foreseen at node s, and let $\hat{y}^{i,s} : \hat{T}^s \to \mathbb{R}_+$ be i's perception of $y^{i,s}$. As before, beliefs $\hat{\mu}^s$ are simply the restriction of μ^s to $\hat{T}^s$. Finally, expanding awareness is captured by assuming that the trees $\hat{T}^{s_t}$ become progressively finer over time so that if $s_t, s_{t'}, s_{t''}$ are successive nodes of T ($t < t' < t''$ and $s_t \supset s_{t'} \supset s_{t''}$) and $s_{t''} \in \hat{T}^{s_t}$, then $s_{t''} \in \hat{T}^{s_{t'}}$. We do not make this assumption explicit from now on.

2.5 Radner Equilibrium Under Full Awareness

We proceed by defining the dynamic exchange economy under full awareness and the standard notion of Radner equilibrium. This will lay the groundwork for the study of unawareness.

Definition 1 *An economy with an initial distribution of assets, or simply an economy, is a tuple* $\mathcal{E}(\overline{a}) := ((\overline{a}^i)_i, T, \mu, (y^i)_i, (u_i)_i, \beta)$ *where*

1. $\overline{a}^i \in \mathbb{R}$ *are the financial savings (debt, if* $\overline{a}^i < 0$*) which agent* i *brings to the first period of the economy. We restrict ourselves to the case of* $\sum_i \overline{a}^i = 0$.

2. $(T, \mu, (y^i)_i)$ *is the objective event tree, beliefs, and endowment processes for each agent.*

3. *Each agent* i*'s utility of consumption process* $c^i : T \to \mathbb{R}_+$ *is given by*

$$U^i(c_0^i, c_1^i, ...) = \mathbb{E}_\mu \sum\nolimits_t \beta^t [u_i \circ c_t],$$

where $u_i : \mathbb{R}_+ \to \mathbb{R}$ *is a strictly increasing, strictly concave, differentiable utility index and* $\beta > 0$ *is a discount factor.*

If initial savings are zero, $\overline{a}^i = 0$ for all i, we may omit them and write simply $\mathcal{E} = (T, \mu, (y^i)_i, (u_i)_i, \beta)$. We may also write c for $(c^i)_i$, y for $(y^i)_i$, etc.

The financial market is such that at each node a market opens for next-period contingent claims. In a Radner equilibrium of such an economy, the agents execute trades in the available Arrow securities subject to expectations about future prices and their own future behavior. We normalize the price of the consumption good to be 1 at each node. We use $q(s_{t+1})$ to denote the price of the s_{t+1} Arrow security prevailing at the node s_t preceding s_{t+1}. (Since each node has a unique predecessor, we can suppress s_t from the definition of $q(s_{t+1})$.) Likewise, $a^i(s_{t+1})$ is the quantity of s_{t+1} Arrow securities purchased by agent i at the node s_t preceding s_{t+1}. As Definition 2 below is one of a Radner equilibrium from the perspective of the initial node, $q(s_1)$ should be viewed as an actual market clearing price at the initial node, whereas $q(s_{t+1}), t > 0$, is an expected price. Similarly, $a^i(s_1)$ are agent i's actual purchases of the s_1 Arrow security, while $a^i(s_{t+1}), t > 0$, are his expected purchases of the s_{t+1} security. We remark that q and a^i are not defined at the root of T, but for expositional simplicity we depict them as functions on T rather than $T \setminus \{0\}$.

Definition 2 *A **Radner Equilibrium (RE)** (q, a, c) of an economy $\mathcal{E}(\overline{a})$ consists of a consumption and savings plan for each agent, $c^i, a^i : T \to \mathbb{R}$, along with prices $q : T \to \mathbb{R}$ such that*

1. *For each agent i, the plans c^i and a^i maximize i's preferences subject to the budget constraints (one at each node):*

$$c^i(0) + \sum_{s_1} q(s_1)a^i(s_1) \leq y^i(0) + \overline{a}^i$$
$$c^i(s_t) + \sum_{s_{t+1} \supset s_t} q(s_{t+1})a^i(s_{t+1}) \leq y^i(s_t) + a^i(s_t) \quad \text{for all } t > 0 \text{ and } s_t$$

2. *Markets clear: $\sum_i c^i \leq \sum_i y^i$ and $\sum_i a^i = 0$.*[10]

If complete markets for the state-contingent delivery of consumption are open in $t = 0$, the appropriate solution concept is that of an Arrow-Debreu equilibrium.

[10] We remark that in order to simplify the exposition, our definition of an economy does not specify the available trading arrangements. Rather, we make those clear through the appropriate solution concept.

For future reference, we remind the reader of the concept and its relation to RE. Thus, an **Arrow-Debreu equilibrium (ADE)** (p, c) of an economy $\mathcal{E}(\overline{a})$ consists of consumption plans $c^i : T \to \mathbb{R}_+, i \in N$, and prices $p : T \to \mathbb{R}_+$, which we again normalize so that $p_0 = 1$, such that (i) markets clear: $\sum_i c^i_t \leq \sum_i y^i_t$ for all t, and (ii) for every agent i, the consumption plan c^i maximizes i's utility subject to the *single* budget constraint

$$\sum_s p(s)c^i(s) \leq \sum_s p(s)y^i(s) + \overline{a}^i.$$

Given an ADE (p, c), one can construct a RE (q, a, c) with the same consumption allocation by letting

$$q(s_t) = \frac{p(s_t)}{p(s_{t-1})} \qquad a^i(s_1) = \sum_{s_t \subset s_1} \frac{p(s_t)}{p(s_1)}[c^i(s_t) - y^i(s_t)],$$

where s_{t-1} is the predecessor of s_t. It follows that a RE exists whenever an ADE does. In the present context, a sufficient (but not necessary) condition for the latter is that initial savings be zero. Given a RE, it is also straightforward to construct an ADE with the same consumption allocation, from where it follows that RE are efficient.[11]

Finally, we recall that RE are self-fulfilling in that expected prices and the agents' plans remain an equilibrium as time unfolds. To state this formally, let $\overline{a} = (\overline{a}^i)_i$ be the savings with which the agents enter node s_t and define the continuation economy at that node as

$$\mathcal{E}^{s_t}(\overline{a}) := ((\overline{a}^i)_i, T^{s_t}, \mu^{s_t}, (y^{i,s_t})_i, (u_i)_i, \beta).$$

Then, any RE (q, a, c) of $\mathcal{E}(a)$ induces a RE $(q^{s_t}, (a^{i,s_t}, c^{i,s_t})_i)$ of $\mathcal{E}^{s_t}((a^i(s_t))_i)$. As before, we write a^{s_t} for $(a^{i,s_t})_i$, etc.

[11] Details and formal proofs can be found in Mas-Colell, Whinston, and Green [33, Ch.19].

2.6 Radner Equilibrium under Coarse Contingencies

We now develop a natural extension of RE which incorporates unforeseen contingencies. The key observation is that at any point in time, the agents' perceptions constitute a well-defined economy to which we can apply the standard notion of RE. However, since the agents' perceptions can change unexpectedly from one period to the next, the RE at a node need not be a continuation of the original equilibrium but may involve a complete revision of prices and plans. To formalize these ideas, start with a true economy $\mathcal{E}$ (one without initial savings) and let $\hat{\mathcal{E}}^D := \{\hat{T}^s, \hat{y}^s\}_{s\in T}$ describe the evolution of the agents' perceptions. We refer to the pair $(\mathcal{E}, \hat{\mathcal{E}}^D)$ as *a dynamic economy with unawareness* or simply a *dynamic economy*. At time $t = 0$, the agents perceive the economy as $\hat{\mathcal{E}} = (\hat{T}, \hat{\mu}, \hat{y}^i, (u_i)_i, \beta)$. If there are unforeseen contingencies, $\hat{\mathcal{E}}$ is distinct from $\mathcal{E}(a)$, yet it is a well-defined economy with a well-defined RE in the sense of Definition 2. This RE gives us the level of savings $a_1^i(s_1)$ with which agent i enter node s_1. These savings and the perceptions $(\hat{T}^{s_1}, \hat{y}^{s_1})$ at s_1 gives us another perceived but well-defined economy,

$$\hat{\mathcal{E}}^{s_1}(a_1^i(s_1)) = ((a_1^i(s_1)), \hat{T}^{s_1}, \hat{\mu}^{s_1}, \hat{y}^{s_1}, (u_i)_i, \beta),$$

to which we can apply the concept of RE once again. Iterating the argument gives the following definition:

Definition 3 *A* ***Radner Equilibrium in Perceptions (REP)*** *of a dynamic economy $(\mathcal{E}, \hat{\mathcal{E}}^D)$ is a function $s \mapsto (\hat{q}^s, \hat{c}^s, \hat{a}^s)$ such that $(\hat{q}^0, \hat{c}^0, \hat{a}^0)$ is a RE of $\hat{\mathcal{E}}$ and for each $s \in T, s \neq 0$, $(\hat{q}^s, \hat{c}^s, \hat{a}^s)$ is a RE of $\hat{\mathcal{E}}^s(\hat{a}_t^{s'}(s)) = ((\hat{a}^{s'}), \hat{T}^s, \hat{\mu}^s, (\hat{y}^s), (u_i)_i, \beta)$, where s' is the node of T preceding s.*[12]

We observe that, under Definition 3, agents are expected to carry out any obligations they incurred at a prior node even if perceptions and prices change unexpectedly at the time of delivery.

[12]Note that when $s = 0$, we will drop the subscript from $\hat{T}^s$ and $\hat{\mathcal{E}}^s$ and write simply $\hat{T}$ and $\hat{\mathcal{E}}$. On the other hand, we will preserve the subscript and write $(\hat{q}^0, \hat{c}^0, \hat{a}^0)$ when referring to a node the RE at node at $s = 0$. The former facilitates "static" comparisons with the economy under full awareness; the latter is essential in reminding the reader that a REP involves a transition from one equilibrium to another.

3 The Irrelevance Theorem

In a REP, the economy transitions from one equilibrium to another as the agents' awareness improves and prior plans become inadequate. Agent i's actual consumption in a REP is therefore given by the function $c^{i,REP} : T \to \mathbb{R}_+$ such that $c^{i,REP}(s) := \hat{c}^{i,s}(s)$ for all $s \in T$. In other words, from the equilibrium plan $\hat{c}^{i,s} : \hat{T}^s \to \mathbb{R}_+$ at each node s, one retains the contemporaneous consumption level $\hat{c}^{i,s}(s)$, which is actually realized, and discards the plans for the future, which may or may not transpire. We say that a REP is **efficient** if the allocation $c^{REP} = (c^{i,REP})_i$ is efficient in the true economy $\mathcal{E}$. A REP is **consistent** if $c^{i,REP} = \hat{c}^{i,0}$ for every i, that is, if agents never deviate from their original plans. We note that in this case $c^{i,REP} : T \to \mathbb{R}_+$ is $\hat{T}$-measurable.

To state the theorem, we note that a dynamic economy $(\mathcal{E}, \hat{\mathcal{E}}^D)$ will be said to satisfy an assumption, OSAA, CE, or MAE, if the assumption applies to perceptions at each node $s \in T$.

Theorem 1 *Consider a dynamic economy $(\mathcal{E}, \hat{\mathcal{E}}^D)$ satisfying OSAA, CE, and MAE. For every RE of $\mathcal{E}$, there exists a consistent REP of $(\mathcal{E}, \hat{\mathcal{E}}^D)$ that induces the same consumption allocation. Conversely, for every consistent REP of $(\mathcal{E}, \hat{\mathcal{E}}^D)$, there exists a RE of $\mathcal{E}$ that induces the same consumption allocation.*

3.1 Full Insurance and the Pricing of Idiosyncratic Risk

Before proceeding with the formal proof, we decompose the argument in two conceptual steps, relating each one to a well-known result in general equilibrium: the fact that idiosyncratic risks are fully insured when markets are complete and, consequently, that such risks are fairly priced. We start by reminding the reader of the meaning of full insurance.

Lemma 1 (Full Insurance) *If c is an efficient allocation of $\mathcal{E}$, then there are no $s, s' \in T$ and $i, j \in N$ such that $c^i(s) > c^j(s)$ and $c^i(s') < c^j(s')$. In particular, if $Y(s) = Y(s')$, then $c^i(s) = c^i(s')$ for every i.*

This is a textbook result in the study of complete markets. Assuming an interior allocation $c \gg 0$, the proof follows directly from the first-order conditions for Pareto efficiency. An elegant proof of the general case can be found in LeRoy and Werner [30, Thm 15.5.1].[13]

Our next lemma derives an important application of full insurance to economies with unawareness. The proof is simple and mimics closely that of Proposition 3 in Cass and Shell [6], whose work we discuss in further detail at the end of this section. Yet, we found the implications regarding unawareness to be of interest, both in their own right and as a stepping stone to our irrelevance theorem. To state the lemma, observe that under CE and MAE, the true economy $\mathcal{E}$ and the perceived economy $\hat{\mathcal{E}}$ are such that $Y = \hat{Y}$. Since the perceived aggregate endowment $\hat{Y}$ must be $\hat{T}$-measurable, $Y = \hat{Y}$ implies MAE. On the other hand, CE is not implied. Indeed, $Y = \hat{Y}$ is consistent with agents whose perceptions of their individual endowments depart wildly from the truth as long as these departures "cancel out" in the aggregate, in which case:

Lemma 2 *If $\mathcal{E}$ and $\hat{\mathcal{E}}$ are such that $Y = \hat{Y}$, then the two economies have the same set of efficient allocations.*

Proof. By construction, if agent's preferences in the true economy $\mathcal{E}$ are restricted to $\hat{T}$-measurable consumption processes, they coincide with his preference in the perceived economy $\hat{\mathcal{E}}$. When $Y = \hat{Y}$, it is also true that the set of feasible allocations in $\hat{\mathcal{E}}$ is a subset of the set of feasible allocations in $\mathcal{E}$. (The converse is not true since a feasible allocation in $\mathcal{E}$ need not be $\hat{T}$-measurable.) With this in mind, take an allocation c that is efficient in $\mathcal{E}$. By the full insurance result, c is $\hat{T}$-measurable. Since $Y = \hat{Y}$, c is feasible in $\hat{\mathcal{E}}$. If c is not efficient in $\hat{\mathcal{E}}$, then there is a Pareto improvement in $\hat{\mathcal{E}}$. But the latter is also a Pareto improvement in $\mathcal{E}$, a contradiction. Conversely, take an allocation $\hat{c}$ that is efficient in $\hat{\mathcal{E}}$. Suppose $\hat{c}$ is not efficient in

[13] As is well understood, the fact that the agents are risk averse and share a common belief μ are key to the lemma. The common discount factor β plays a role too, though not as essential. Thus, if the discount factors are heterogeneous, one needs to restrict attention to states s and s' that occur within the same period and the conclusions of the lemma will carry through. With respect to our irrelevance theorem, this will complicate the proof of Lemma 4, but we believe both the lemma and the theorem remain true.

$\mathcal{E}$. By the strict convexity of preferences, there exists an allocation c that is efficient in $\mathcal{E}$ and Pareto dominates $\hat{c}$. For example, c could be the outcome of a competitive equilibrium of the true economy with $\hat{c}$ as endowment point. But full insurance and the fact that $Y = \hat{Y}$, c is $\hat{T}$-measurable and hence also a feasible Pareto improvement in $\hat{\mathcal{E}}$. ■

The reader may wonder why Lemma 2 is not the end of the story. The reason is that the lemma concerns behavior only at a point in time. In particular, if CE fails, the agents' mistakes about the first moment of their endowment may be reflected in expected prices and consequently in the particular efficient allocation agreed upon at a moment in time. When at a later stage the agents become aware of those mistakes, they may want to deviate from the original equilibrium and that deviation may imply that the ultimate allocation induced by a REP is inefficient. Appendix A.1 presents an example of this possibility and, hence, of the necessity of CE to attain efficiency.

CE means that agents are correct about the first moment of the distribution of their endowments but may be mistaken about the second. The key question is then whether those mistakes can cause similar inefficiencies. The answer is no, as the omitted risks are not only fully insured, they have no effect on the equilibrium wealth of agents.[14] To see why, consider an economy $\mathcal{E}$ and take two states, s and s', such that $Y(s) = Y(s')$. From full insurance and the first-order conditions for utility maximization, we see immediately that $p(s)/p(s') = \mu(s)/\mu(s')$.[15] It follows that any idiosyncratic risk, holding the aggregate endowment fixed, is fairly priced. In particular, a mean-zero risk will have no effect on the equilibrium wealth of an agent. We summarize this observation in the next lemma.

Lemma 3 *The economies $(T, \mu, (u_i)_i, \beta, (y^i)_i)$ and $(T, \mu, (u_i)_i, \beta, (y^i + z^i)_i)$, where $z^i : T \to \mathbb{R}, i \in N$, are such that $E_\mu[z^i_t \,|\, y^i_t] = 0$ and $\sum_i z^i_t = 0$ for all t, have the same set of Arrow-Debreu equilibria.*

[14] To see why this doesn't follow immediately from Lemma 1, focus on the second statement of the lemma. It says that one's consumption in an efficient allocation is a function of the aggregate endowment. What the statement does not say is that the idiosyncratic risks aren't "baked into" that function.

[15] We are assuming here that at least one agent has an interior optimum, which we establish in Lemma 4 in the next section.

We emphasize that Lemma 3 is a relatively straightforward extension of Cass and Shell's [6, Prop.2] celebrated result that *extrinsic shocks do not matter*.[16] A moment of reflection will also convince the reader that, on a purely mathematical level, our irrelevance theorem follows promptly from Lemma 3 and, hence, Cass and Shell [6]. As we remarked in the introduction, the contribution is therefore largely conceptual and lies in showing how familiar general equilibrium results may manifest themselves in a setting with unawareness. It should further be noted that, while dynamics and the sequential opening of markets are not essential to Cass and Shell [6], or indeed to any setup predicated on full rationality and complete markets, they become so once unawareness is considered. A solid understanding of these aspects will also be essential in the second part of our paper, where we "allow" unawareness to matter by dropping MAE.

3.2 Proof of Theorem 1

Lemma 4 *If c is an efficient allocation of $\mathcal{E}$, there exists an agent i such that $c_i \gg 0$.*

Proof. Let D_i be the set of nodes $s \in T$ such that $c^i(s) = 0$. It follows from Lemma 1 that the sets D_i and D_j are nested for every i and j. If not, there are nodes $s, s' \in T$ and agents i, j such that $0 = c^i(s) < c^j(s)$ and $c^i(s) > c^j(s') = 0$. Since the sets D_i are nested, there is i^* such that $D_{i*} \subset D_j$ for every j. We claim that $D_{i^*} = \emptyset$. If not, there is a node s such that $c^j(s) = 0$ for every j. But this is impossible by the efficiency of c: since $\sum_j y^j(s) > 0$ and utility is strictly increasing in each node, giving an a strictly positive of amount of the good to each agent at node s would be a Pareto improvement. ■

Take a RE (q, a, c) of $\mathcal{E}$ and let (p, c) be the corresponding ADE. By Lemma 1, if $\sum_j y^j(s_t) = \sum_j y^j(s'_t)$, then $c^i(s_t) = c^i(s'_t)$ for all i. By MAE, c is $\hat{T}$-measurable and

[16] Unlike Cass and Shell [6], we consider a dynamic domain and allow the aggregate endowment $\sum_i y^i$ to vary. To eliminate a potential source of confusion, we also note that Cass and Shell [6] define an *extrinsic shock*, also called a *sunspot*, to be one that has no effect on the fundamentals of the economy. By comparison, the z^i shocks in Lemma 3 and the unforeseen shocks in Theorem 1 are not extrinsic in this sense as they affect individual endowments. Yet, Cass and Shell [6, p.209] have noted that their theorem extends to idiosyncratic risks, a conclusion they trace to Malinvaud [32]. See also Balasko [3].

hence feasible in $\hat{\mathcal{E}}$. We are going to find prices $\hat{p}$ such that $(\hat{p}, c)$ is an ADE of $\hat{\mathcal{E}}$. First, we claim that

$$\frac{p(s_t)}{\mu(s_t)} = \frac{p(s'_t)}{\mu(s'_t)} \quad \forall s_t, s'_t \subset \hat{s}_t, \forall \hat{s}_t \in \hat{T}. \tag{3.1}$$

By Lemma 4,$c^{i^*} \gg 0$ for some $i^* \in N$. From the first-order conditions of this agent's optimization problem in the ADE of $\mathcal{E}$,

$$\frac{u'_{i*}(c^{i^*}(s_t))\beta^t \mu(s_t)}{p(s_t)} = \frac{u'_{i*}(c^{i^*}(s'_t))\beta^t \mu(s'_t)}{p(s'_t)} \quad \forall s_t, s'_t.$$

If $s_t, s'_t \subset \hat{s}_t$, then $c^{i^*}(s_t) = c^{i^*}(s'_t)$ and (3.1) follows. Next, take any $\hat{s} \in \hat{T}$ and some $s \subset \hat{s}, s \in T$, and define $\hat{p}(\hat{s}) := \frac{p(s)}{\mu(s \,|\, \hat{s})}$. By (3.1), this definition is independent of the choice of s. We also note that $\hat{p}(\hat{s}) = \sum_{s \subset \hat{s}} p(s)$. To check that $(\hat{p}, c)$ is an ADE of $\hat{\mathcal{E}}$, note that

$$\sum_i c^i \leq \sum_i y^i = \sum_i \hat{y}^i \tag{3.2}$$

The first inequality follows from market clearing in $\mathcal{E}$; the second from MAE. Next, we check that c^i is feasible for agent i given prices $\hat{p}$.

$$\sum_{\hat{s}} \hat{p}(\hat{s})\hat{y}^i(\hat{s}) = \sum_{\hat{s}} \hat{p}(\hat{s}) \sum_{s \subset \hat{s}} \mu(s \,|\, \hat{s}) y^i(s) = \sum_{\hat{s}} \sum_{s \subset \hat{s}} \hat{p}(\hat{s}) \mu(s \,|\, \hat{s}) y^i(s) = \sum_{\hat{s}} \sum_{s \subset \hat{s}} p(s) y^i \tag{3.3}$$

$$= \sum_s p(s) y^i(s) \geq \sum_s p(s) c^i(s) = \sum_{\hat{s}} \hat{p}(\hat{s}) c^i(\hat{s}) \tag{3.4}$$

The first equality follows from CE; the second from a rearrangement of terms; the third from the definition of $\hat{p}$; the inequality from the fact that c^i is feasible agent i in the true economy; and the last equality from $\hat{p}(\hat{s}) = \sum_{s \subset \hat{s}} p(s)$ and the fact that c^i is $\hat{T}$-measurable, which means that c^i can be regarded both as a function on T and

$\hat{T}$ as we have done. To check the optimality of c^i, we first note, based on (3.1), that

$$\frac{p(s_t)}{\mu(s_t)} = \frac{\hat{p}(\hat{s}_t)}{\hat{\mu}(\hat{s}_t)} \quad \forall t, \hat{s}_t \in \hat{T}, s_t \subset \hat{s}_t.$$

Thinking of c^i as a function on both T and $\hat{T}$, we see that

$$\frac{u_i'(c^i(\hat{s}_t))\beta^t \hat{\mu}(\hat{s}_t)}{\hat{p}(\hat{s}_t)} = \frac{u_i'(c^i(s_t))\beta^t \mu(s_t)}{p(s_t)} \quad \forall \hat{s}_t, \forall s_t \subset \hat{s}_t. \tag{3.5}$$

Thus, the first-order conditions for the optimality of c^i in $\hat{E}$ follow from the corresponding first-order conditions in $\hat{\mathcal{E}}$. Since utility is concave, those conditions are also sufficient.

Now, let $(c, \hat{q}, \hat{a})$ be the RE of $\hat{\mathcal{E}}$ corresponding to $(p, \hat{c})$. Logic similar to (3.3) and (3.4) show that $a^i(s_1) = \hat{a}^i(s_1)$ for all i and s_1. Moreover, we know that c is part of a RE of the true continuation economy $\mathcal{E}^{s_1}(a(s_1))$. Analogous arguments then show that c is also part of a RE of $\hat{\mathcal{E}}^{s_1}(a(s_1))$, which moreover has the same asset holdings for period $t = 2$ as that of the true economy. The only difference is that, because some agent may enter s_1 with an obligation, $a^i(s_1) < 0$, their contemporaneous endowment net of that obligation may be negative: $y^i(s_1) + a^i(s_1) < 0$. In general, this raises the question whether a RE of $\mathcal{E}^{s_1}(a(s_1))$ exists. It does because the original economy $\mathcal{E}$ has nonnegative endowments and hence a RE.[17] Iterating all the arguments, we see that $c = c^{REP}$.

To prove the opposite direction, let c be a consistent REP. By definition, c is part of a RE of the economy $\hat{\mathcal{E}}$ and, hence, part of a corresponding ADE $(\hat{p}, c)$. For every c, let $p(s) := \hat{p}(\hat{s})\mu(s \mid \hat{s})$, where $\hat{s}$ is the unique node of $\hat{T}$ such that $s \subset \hat{s}$. Analogous arguments show that (c, p) is an ADE of $\mathcal{E}$ and, hence, c is part of the corresponding RE of $\mathcal{E}$.

3.3 An Irrelevance Result for Prices

As stated, our irrelevance theorem shows the equivalence of equilibrium *allocations* in the true and perceived economies. The proof reveals however that the irrelevance

[17] Recall the remarks following Definition 2 of a RE.

result extends to *prices* as well. To see the full scope of this extension, focus on the implied AD prices, which cover all state-contingent deliveries, and, starting with $\hat{\mathcal{E}}$, consider the price $\hat{p}(\hat{s}_t)$ of a promise to deliver one unit of consumption in $\hat{s}_t$. If $\hat{s}_t$ is coarse, then in reality that promise entails the delivery of one unit of consumption in each objective state $s_t \subset \hat{s}_t$, which means that its price in $\mathcal{E}$ is $\sum_{s_t \subset \hat{s}_t} p(s_t)$. But we have seen that:

$$\hat{p}(\hat{s}_t) = \sum_{s_t \subset \hat{s}_t} p(s_t).$$

More generally, the congruence of consumption plans in the true and perceived economies implies that the stochastic discount factor in the true economy is $\hat{T}$-measurable and coincides with the stochastic discount factor in the perceived economy.

3.4 Consistency vis-à-vis Efficiency

Our irrelevance theorem delivers a REP that is both efficient and consistent. Each property is important in its own right: efficiency is of interest in normative analysis, while consistency is at the heart of many empirical tests as it is intimately linked to the validity of the standard Euler equation.[18] The goal of this section is to examine the interplay between these properties. Our main finding is that an inefficiency will always manifest itself as an inconsistency. As a partial converse, we also show that under suitable conditions, an observed inconsistency is necessarily indicative of an inefficiency. To state the result, recall from Section 3.1 that under CE and MAE the sum of the agents' perceived endowments equals the true aggregate endowment, that is, $Y = \hat{Y}$.

Theorem 2 *If a consistent REP exists, then $Y = \hat{Y}$ and the REP is efficient. Conversely, if $Y = \hat{Y}$ and there is an efficient REP such that $c^{REP} \gg 0$, then the REP is consistent.*[19]

[18]Under consistency, the realized consumption of any agent can be viewed as the outcome of a single utility maximization problem. Hence, a standard Euler equation holds.

[19]We recall that $\hat{Y}$ refers to perceptions at the initial node. Thus, when the equality $Y = \hat{Y}$

Proof. If a REP is consistent, then $\hat{c}^{i,s}(s) = \hat{c}^{i,0}(s)$ for every i and $s \in T$. By market clearing in $\hat{\mathcal{E}}^s$, $\sum_i \hat{c}^{i,s}(s) = Y(s)$ and, by market clearing in $\hat{\mathcal{E}}$, $\sum_i \hat{c}^{i,0}(s) = \hat{Y}(s)$. It follows that $Y = \hat{Y}$. Since $\hat{c}^0$ is efficient in $\hat{\mathcal{E}}$, $\hat{c}^0$ is efficient in $\mathcal{E}$ by Lemma 2. Since the REP is consistent, $c^{REP} = \hat{c}^0$. To prove the second statement, take an efficient REP such that $c^{REP} \gg 0$. Then, as shown in Mas-Colell et al. [33, Ch.16F], there exists $\gamma \in [0,1]^N$ such that $\sum_i \gamma^i = 1$ and

$$\frac{u_i'(\hat{c}^{i,s}(s))}{u_j'(\hat{c}^{j,s}(s))} = \frac{\gamma^i}{\gamma^j} \quad \forall i \in N, s \in T. \tag{3.6}$$

Let v_i be the inverse of u_i' and fix some $j \in N$. From (3.6), we deduce that

$$\hat{c}^{i,s}(s) = v_i(u_j'(\hat{c}^{j,s}(s))\frac{\gamma^j}{\gamma^i}) \quad \forall i \in N. \tag{3.7}$$

Summing over i and using the market clearing condition in $\hat{\mathcal{E}}^s$ gives:[20]

$$\sum_i v_i(u_j'(\hat{c}^{j,s}(s))\frac{\gamma^j}{\gamma^i}) = \sum_i \hat{c}^{i,s}(s) = Y(s). \tag{3.8}$$

Likewise, the efficiency of $\hat{c}^0$ in $\hat{\mathcal{E}}$ implies that there is $\gamma^0 \in [0,1]^N$ such that $\sum_i \gamma^{0,i} = 1$ and an analogue of (3.6) holds. Moreover, since (3.6) holds for the initial node, we get $\gamma = \gamma^0$. Mirroring (3.7) and (3.8), the efficiency of $\hat{c}^0$ in $\hat{\mathcal{E}}$ then implies

$$\sum_i v_i(u_j'(\hat{c}^{j,0}(s))\frac{\gamma^j}{\gamma^i}) = \sum_i \hat{y}^i(s) \quad \forall s \in T.$$

appears in the theorem, it should be understood as applying to the initial node only. A different version of the theorem in which an analogue of $Y = \hat{Y}$ is deduced (assumed) to hold at every node, see Appendix A.2.

[20] To gain some perspective for equation (3.8), recall that, by full insurance, each agent's consumption in an efficient allocation is a function of the aggregate endowment only. The equation, which can be found in Section 8.4 in Ljungqvist and Sargent [31], reconstructs that function explicitly using the "Pareto weights" γ_i.

Since $Y = \hat{Y}$ by assumption,

$$\sum_i v_i(u'_j(\hat{c}^{j,0}(s))\frac{\gamma^j}{\gamma^i}) = \sum_i v_i(u'_j(\hat{c}^{j,s}(s))\frac{\gamma^j}{\gamma^i}).$$

Since v_i and u'_i are strictly decreasing functions, we see that $\hat{c}^{j,s}(s) = \hat{c}^{j,0}(s)$ for all s. Since j was arbitrary, the REP is consistent. ■

The first part of the theorem contains the finding that inefficiencies will always manifest themselves as inconsistencies.[21] This part also shows that with regards to consistency, the sufficient conditions in our irrelevance theorem are tight. In particular, inconsistencies follow whenever perceptions are such that $Y \neq \hat{Y}$. A notable case arises when individual perceptions are such that $\hat{y}^i(\hat{s}) = \min\{y^i(s) : s \subset \hat{s}\}$, which implies that $\hat{y}^i \lneq y^i$ whenever y^i is not $\hat{T}$-measurable.[22] If MAE is maintained, the less extreme assumption that $\hat{y}^i \lneq \mathbb{E}_\mu[y^i \,|\, \hat{T}]$ would likewise imply that $\hat{Y} \lneq Y$. As remarked previously, perceptions such as these arise naturally when agents are aware of their unawareness and, consequently, adopt perceptions that reflect a degree of cautiousness.

The second part of Theorem 2 shows that if $Y = \hat{Y}$ and the REP delivers an interior allocation, then an observed inconsistency is necessarily indicative of an inefficiency. We believe that the technical assumption of an interior allocation can be dispensed with. On the other hand, our next result confirms that the assumption $Y = \hat{Y}$ cannot be dropped.

Theorem 3 *There exist dynamic economies $(\mathcal{E}, \hat{\mathcal{E}}^D)$ such that (i) $Y \neq \hat{Y}$, (ii) there is a unique REP, (iii) the induced allocation is such that $c^{REP} \gg 0$, and (iv) the REP is efficient but not consistent.*

[21] This result relates to one in Kochov [28, Thm 5], showing that if an agent is unaware of some contingency, one can always construct a dynamic decision problem in which the agent will be inconsistent. By our irrelevance theorem, the same is not true in a general equilibrium setting as decision problems cannot be freely chosen but arise endogenously given equilibrium prices. On the other hand, Theorem 2 re-affirms Kochov's conclusion in all cases in which unawareness results in inefficiencies.

[22] Such perceptions have a long pedigree in the literature on unawareness and related phenomena such as ignorance and ambiguity about likelihoods. See Maskin [34], Lehrer [29], Gul and Pesendorfer [19], Kochov [28], and most recently, Guerdjikova and Quiggin [18].

A precise example of an economy satisfying all the conditions in the theorem will be given at the end of Section 4.1. To gain some intuition, recall that our setup is one in which awareness emerges before a shock hits and that, consequently, the agents are able to purchase insurance against it. The main source of inefficiency is therefore the fact that the agents' savings may prove inadequate. In particular, if a newly foreseen aggregate shock changes an agent's wealth (under equilibrium prices), then there would be a precautionary motive not previously accounted for. But if the shock affects all agents symmetrically, there could effectively be no one who can provide the insurance. Then, the only thing the agents can do is "ride out" the shock. Alternatively, if the shock affects the agents' endowments in several periods, its overall effects on their wealth may cancel out. In both cases, the agents' savings decision are the same whether or not they are aware of the shock. Consequently, there is an inconsistency as the agents' consumption levels respond to the shock, but no inefficiency.

4 Implications of Unforeseen Aggregate Shocks

In this section, we maintain OSAA and CE but drop MAE, thus allowing the agents to become gradually aware of aggregate shocks to the economy. Reversing a scenario just discussed in the context of Theorem 3, we start with an example of an *inefficient* REP.

4.1 Savings Mistakes and Heterogeneous Growth (Rates)

There are two agents, both with log utility and discount factor $\beta = 1$. The true and perceived economies are as depicted in Figure 2. We note that the true aggregate endowment Y is not constant in the upper two terminal nodes and that those are nodes are unforeseen by the agents. We also note that since preferences are homothetic, the economy with full awareness has a unique equilibrium in which each agent consumes a constant fraction of the aggregate endowment at every node.[23] In the example, the fractions are $\frac{17}{36}$ for agent 1 and $\frac{19}{36}$ for agent 2. Notably, agent 1, who bears the

[23] See Chapter 8.6.2 in Ljungqvist and Sargent [31].

brunt of the aggregate shock, gives up consumption in each node other than ω^* so as to insure himself against the event in which his endowment is zero. By comparison, when the agents do not foresee the finer contingencies resolving in period 2, they first believe to be fully insured and see no reason to trade in period 0. It is only after moving to node A that agent 1 becomes aware that his future endowment is uncertain. At this point, however, *he can only spread the cost of insurance across the nodes following A, rather than all of them.* In the resulting REP, agent 1 consumes $\frac{15}{36}$ of the aggregate endowment Y at node A and its successors, and $\frac{1}{2}$ of Y at all other nodes.

It is intuitively clear that the REP is inefficient. What might be worth emphasizing is the "robustness" of the inefficiency. In particular, we note that $c^{1,REP}(A) < c^{1,REP}(A^c)$, while $c^{2,REP}(A) > c^{2,REP}(A^c)$. By Lemma 1, this is an inefficiency as long as the agents share a common belief μ and are risk averse; knowledge of the actual μ or the functions $(u_i)_i$ is not required. Alternatively, picking nodes that are not mutually exclusive, we see that $c^{1,REP}(A) < c^{1,REP}(0)$, while $c^{2,REP}(A) > c^{2,REP}(0)$. This is an inefficiency whenever, in addition to the previous conditions, the agents share a common discount factor.[24] Finally, as we remarked in the Introduction, we note that the heterogeneity in growth (growth rates) exhibited by the latter rankings relates to a large empirical literature testing the implications of full insurance, but where the heterogeneity is interpreted as evidence of incomplete markets rather than unawareness.

4.1.1 Proof of Theorem 3

Returning to the proof of Theorem 3, suppose now we change the true economy in Figure 2 so that endowments in the upper two nodes of T are $(15, 15)$ and $(5, 5)$, rather than $(20, 10)$ and $(0, 10)$. This leaves the aggregate endowment unchanged, but distributes the burden of the aggregate shock equally among the agents. As in the discussion of Theorem 3, this is then a case in which no agent can insure the other. In fact, given the complete symmetry of the economy, it is clear that the unique equilibrium of $\mathcal{E}$ is one of no trade. Turning to perceptions, we see that under CE

[24] As explained in ft.13 the need for a common discount factor arises because we are comparing consumption levels at nodes in different time periods.

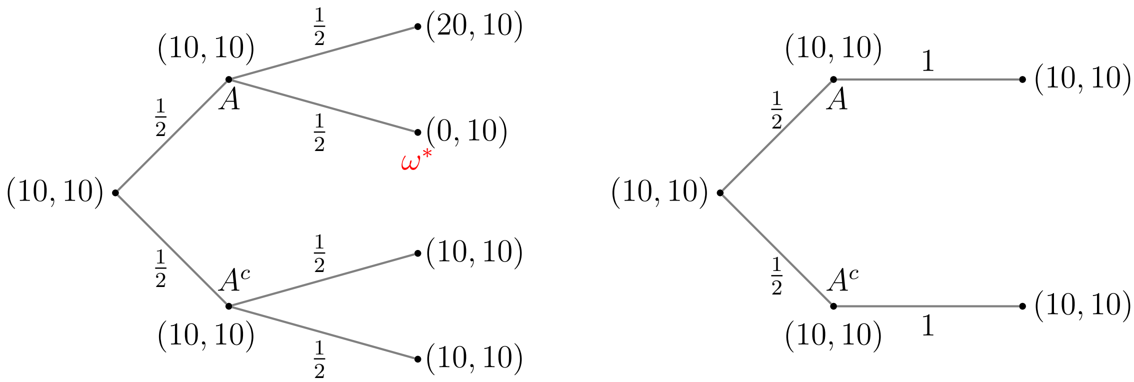

Figure 2: The tree on the left is the economy with perfect awareness, while the tree on the right is the agents' time-0 perception thereof. Endowments, real and perceived, are depicted next to each node.

the perceived economy is once again as depicted in Figure 2. This time however the decision not to trade in period 0 coincides with that under full rationality. Likewise, in period 1, the agents become aware of the aggregate shock but, as the shocks affects them symmetrically, the only thing they can do is to ride it out. Thus, the REP is one of no trade *at any node* and its ultimate allocation coincides with the equilibrium allocation of $\mathcal{E}$. In particular, the REP is efficient. However, since initial plans do not factor in the effects of the aggregate shock, the REP is inconsistent.[25]

4.2 Price Dynamics

As argued in Section 3.3, when shocks to aggregate income are foreseen, not only are consumption plans consistent, but also expectations over future prices are correct. In contrast, if agents are initially unaware of some aggregate shocks, realized prices will typically not coincide with the agents' expectations. Under standard assumptions in the asset pricing literature, this section shows that the departure is systematic and governed by the agents' prudence. Thus, if an agent is prudent, or equivalently, if $u_i''' > 0$, he wants to save more when the future income becomes riskier.[26] This clearly

[25]In contrast to the discussion in Section 4.1, the example suggests that when a REP is inconsistent but efficient, the inconsistency might not be testable as it does not manifest itself in the observed savings decisions in the agents, only in the unrealized plans of the future.

[26]See Kimball [27].

suggests that asset prices for future consumption should increase when new aggregate risks appear. What may complicate matters is the fact that emerging shocks may affect the wealth distribution in the economy, which in turn may affect demand and prices.[27] As in much of the asset pricing literature, we rule out such distributional effects by assuming that agents have identical homothetic utility functions. Together with the fact that concave homothetic functions are automatically prudent, this benchmark case delivers a sharp conclusion: the expected price of consumption at a point in the distant future can only increase as the economy moves closer to that point. To state the result, we let $\phi^{s_t}(s_{t+k})$ be the node in $\hat{T}^{s_t}$ containing $s_{t+k} \in T$. We also note that, as in Section 3.3, we state the result in terms of the implied Arrow-Debreu prices.

Theorem 4 *Consider a dynamic economy $(\mathcal{E}, \hat{\mathcal{E}}^D)$ satisfying OSAA and CE, and in which the agents have identical homothetic utility functions. Take nodes $s_t, s_{t+1}, s_{t+k} \in T$, with $k > 1$, and let Φ be the set of nodes $\hat{s}_{t+k} \in \hat{T}^{s_{t+1}}$ such that $\phi^{s_{t+1}}(s_{t+k}) \subseteq \phi^{s_t}(s_{t+k})$. Then,*

$$\hat{p}^{s_t}(s_{t+k}) \le \hat{p}^{s_t}(s_{t+1}) \sum_{\hat{s}_{t+k} \in \Phi} \hat{p}^{s_{t+1}}(\hat{s}_{t+k}) \ . \tag{4.1}$$

Proof. Agents' utility functions take the form $u(c) = \frac{c^{1-\alpha}-1}{1-\alpha}, \alpha > 0$, with $u(c) = \ln(c)$ if $\alpha = 1$. The s_{t+1} price for one unit of consumption in node s_{t+k} as, respectively, perceived from nodes s_t and s_{t+1} is then given by (Ljungqvist and Sargent [31, Chapter 8.6.2]):

$$\frac{\hat{p}^{s_t}(s_{t+k})}{\hat{p}^{s_t}(s_{t+1})} = \frac{\hat{\mu}^{s_t}(s_{t+k}|s_{t+1}) \left(\sum_i \hat{y}^{i,s_t}(s_{t+k})\right)^{-\alpha} \beta^{k-1}}{\left(\sum_i \hat{y}^{i,s_t}(s_{t+1})\right)^{-\alpha}}$$

and

$$\sum_{\hat{s}_{t+k} \in \Phi} \hat{p}^{s_{t+1}}(\hat{s}_{t+k}) = \frac{\sum_{\hat{s}_{t+k} \in \Phi} \hat{\mu}^{s_{t+1}}(\hat{s}_{t+k}|s_{t+1}) \left(\sum_i \hat{y}^{i,s_{t+1}}(\hat{s}_{t+k})\right)^{-\alpha} \beta^{k-1}}{\left(\sum_i \hat{y}^{i,s_{t+1}}(s_{t+1})\right)^{-\alpha}}.$$

[27] Individual endowments at nodes with a negative aggregate shock become relatively more valuable than those at nodes with a positive shock.

By One-A, we have $\sum_i \hat{y}^{i,s_t}(s_{t+1}) = \sum_i \hat{y}^{i,s_{t+1}}(s_{t+1})$. It thus suffices to compare the numerators of the two expressions. By Jensen's inequality, convexity of $(\cdot)^{-\alpha}$ implies

$$\hat{\mu}^{s_t}(s_{t+k}|s_{t+1}) \left(\sum_i \hat{y}^{i,s_t}(s_{t+k}) \right)^{-\alpha} \leq \sum_{\hat{s}_{t+k} \in \Phi} \hat{\mu}^{s_{t+1}}(\hat{s}_{t+k}|s_{t+1}) \left(\sum_i \hat{y}^{i,s_{t+1}}(\hat{s}_{t+k}) \right)^{-\alpha}.$$

Condition (4.1) then follows. ■

Remark 1 *If we assume that $y^i = y^j$ for all i, j, so that we have a so called representative agent economy, we can drop the assumption of homothetic utility. Then,* (4.1) *holds if and only if $u''' \leq 0$, highlighting the role of prudence.*

To understand equation (4.1), we can think of $\hat{p}^{s_t}(s_{t+k})$ as the implied price, given the equilibrium at s_t, of a long-term asset delivering one unit of consumption in the contingency $\phi^{s_t}(s_{t+k})$. In the standard paradigm of full rationality and self-fulfilling price expectations, the same delivery can be accomplished by purchasing the long-term asset in period $t+1$ and bringing the requisite amount of "money" from period t to $t+1$. Specifically, the cost of that trade (in period t "dollars"), which is depicted on the right hand side of (4.1), should equal $\hat{p}^{s_t}(s_{t+k})$. In our model however, upon arriving in period $t+1$, the agents may become aware of a new risk to the economy and recognize that $\phi^{s_t}(s_{t+k})$ is not a single contingency but one comprised of several subevents. Consequently, both perceptions and the equilibrium in period $t+1$ may change unexpectedly. The gist of Theorem 4 is to show that the price of the long-term asset *increases* relative to what the agents expected it to be, which is $\hat{p}^{s_t}(s_{t+k})/\hat{p}^{s_t}(s_{t+1})$. (Note that the inequality in (4.1) is strict whenever $\phi^{s_t}(s_{t+k}) \neq \phi^{s_{t+1}}(s_{t+k})$.) This is because, under prudence, the newly recognized risks increase demand for savings and because the asset in question is in fact safe from those risks.

The reader may now point out that because our working model is one in which the agents can only trade one-step-ahead securities, Theorem 4 deals with the *implied* prices of long-term assets rather than with *observed* market prices. In other words, how can we guarantee that the implications of Theorem 4 are testable? An obvious answer is to augment the financial structure so that at each point in time the agents can trade any asset whose payoffs are measurable with respect to the agents' perceptions. But this brings up a point that is both subtle and interesting in its own right. By the logic of equation (4.1), agents who purchased the long-term asset in period t would experience a "windfall" relative to the agents who planned to purchase the asset in period $t+1$, or implement the position by some other strategy involving sequential trade. In this way, the availability of long-term assets, which would be redundant in an economy under full rationality and are in fact redundant given the agents' perceptions at a *fixed* point in time, may alter the trajectory of the economy. But if the latter is true, can we still count on the validity of (4.1)? The answer is yes, because, as remarked in the proof of Theorem 4, under homothetic utility prices depend only on the aggregate endowment and the latter is unchanged by the windfall experienced by the holders of long-term assets, which ends up being just a "transfer" among the agents.

We can summarize the discussion as follows. In this section we have shown that otherwise redundant assets might affect the evolution of an economy in which awareness of aggregate shocks emerges over time. Yet, the effects are robust across financial structures in that the prices of long-term assets rise unexpectedly when the relevant perceptions change, which in turn constitutes a windfall for the holders of such assets. Moreover, the observed price pattern cannot arise in any model of full rationality as that would present an opportunity for *intertemporal* arbitrage which fully rational agents can exploit.

4.3 Default

When the irrelevance theorem holds, the economy transitions successfully from one perceived equilibrium to another as the agents' awareness grows. We now show that in the presence of unforeseen shocks to the aggregate endowment this is not guaranteed. The misperceptions of the value of future endowments that led to insufficient saving in Section 4.1 may lead to excessive debt which the agents cannot repay. The economy can then no longer "self-correct" and transition to a new equilibrium. As remarked in the introduction, such possibilities are limited by the assumption of one-step-ahead awareness, but they can nonetheless occur in economies in which agents plan to roll

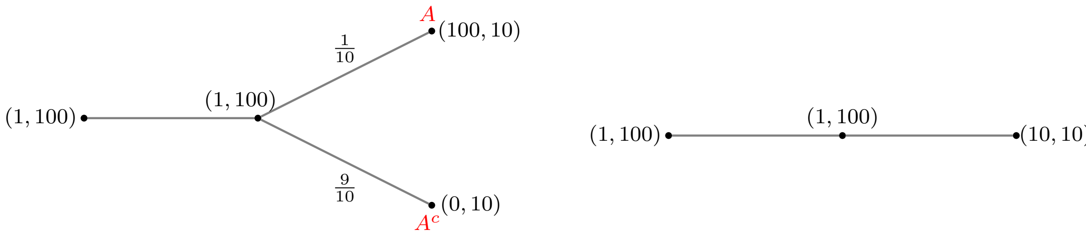

Figure 3: The tree on the left depicts the objective environment, while the tree on the right shows the time $t = 0$ perception.

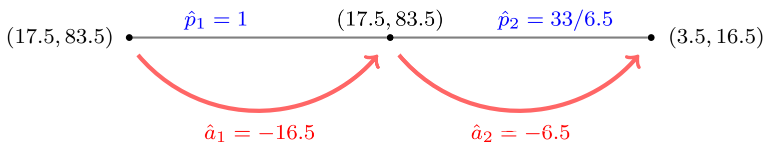

Figure 4: REP in $t = 0$: agent 1 borrows in period 0 and plans to roll over the debt in period 1.

over debt. If emerging risks then decrease the value of future endowments, indebted agents may need to pay off maturing debt out of current endowments, but there may be no market-clearing prices that keep the cost of debt affordable. We illustrate this possibility with an example.

Suppose there are three periods and two agents, both with log utility and a discount factor of 1. The true economy and the period-0 perception are as depicted in Figure 3. In periods 0 and 1, agent 1 is poor, whereas agent 2 is rich. Moreover, according to time-0 perceptions both agents have the same endowment in the last period. Given the perceived endowment structure, the optimal period-0 plan for agent 1 is to shift consumption from the last period to the first two. This plan entails agent 1 taking on debt in period 0 and rolling it over in period 1. Figure 4 depicts the equilibrium. In time 0, agent 1 borrows \$16.5 in order to consume 17.5 units of the consumption good. While this debt exceeds his endowment in period 1, he plans to raise \$33 by selling a claim for 6.5 units of the consumption good in period 2. In this way, the agent can repay his debt and consume 17.5 units of the consumption good in period 1.

Once both agents arrive in period 1, they realize that agent 1's endowment is in fact risky. It is either 100, with probability 1/10, or 0 otherwise. See the left tree in Figure 3. Markets reopen and there are now two relevant Arrow securities, one for each subsequent state. Agent 1 is endowed with 1 unit of the consumption good and owes the other agent 16.5 units. The only way agent 1 can repay this debt is by selling Arrow securities for state A in which agent 1 is rich. Since agent 1's endowment in that state is 100, the price for this asset needs to be weakly greater than $\underline{p} := 0.155$. A calculation shows, however, that at price $\underline{p}$ agent 2's demand for the asset is strictly below 100. Hence, at price $\underline{p}$ agent 1 cannot sell enough units of the asset to repay his debt. Since the same is true for any price above $\underline{p}$, no market clearing price can be found.

The *necessity to default* can be illustrated with an Edgeworth box as in Figure 5. Note first that we can abstract from node A^c, in which agent 1's endowment is 0 and cannot be sold. Second, the fact that agent 1 is in debt in period 1 means that the endowment point lies outside the Edgeworth box. The slope of the budget line passing through the endowment point and the origin describes the lowest relative price of consumption in state A at which agent 1 can repay his debt from period 0. Under homothetic preferences, the contract curve is linear and the marginal rates of substitution are constant at each point on the curve. If an equilibrium without default were to exist, its relative price would be equal to that marginal rate of substitution, which is captured by the slope of the dashed line in Figure 5. This line could be made arbitrarily steep by lowering the probability of state A. If the line is steeper than the line passing through the origin and the endowment point, there will be no spot price that simultaneously clears the market and yields non-negative consumption for both agents.

5 Discussion

In this section, we discuss our framework, its relationship to the literature, and several open problems.

We have already mentioned Modica et al. [35], who were the first to study limited awareness and default in general equilibrium. They, and more recently Teeple [41], develop an equilibrium notion that accounts for the possibility of default by incorpo-

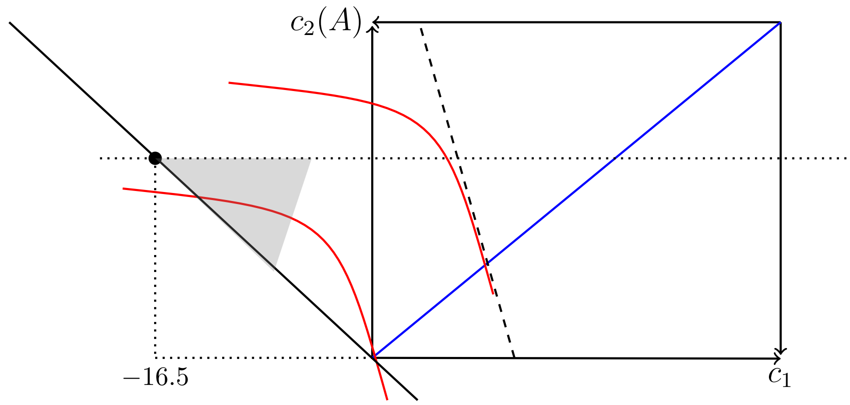

Figure 5: Period-1 Edgeworth box after updating perceptions

rating a settlement rule. We believe that this is equally possible in our framework and that, indeed, it would be interesting to see how the ideas of those papers apply to the present setup.[28] However, our purpose throughout the paper has been to see how far the sequential opening of markets and the standard price mechanism *alone* can go toward remedying the problems caused by unawareness, be it inefficiencies or default. In particular, by showing that these problems persist even in settings in which we have tried to "stack the deck" against unawareness, e.g. by assuming OSAA, we have established the *necessity* of institutional arrangements, such as bankruptcy courts or other procedures for bankruptcy settlement, that go beyond markets. In the same line of thought, our irrelevance theorem identifies conditions under which such institutions may be less essential.

In recent work, Guerdjikova and Quiggin [18] have a proposed a notion of competitive equilibrium in a framework that extends ours along several dimensions. Namely, they allow for (i) asymmetric awareness, (ii) for agents to trade non-measurable assets, and (iii) for agents who are aware of their unawareness and indeed averse to the presence of unforeseen shocks.[29] An interesting lesson from their analysis is that

[28]For other papers that deal with default, though not with unawareness, see Green [15] and Ben-Ami and Geanakoplos [5].

[29]Non-measurable assets are handled in the same way we handle non-measurable endowments – by postulating a measurable perception. Modica et al. [35] is an early example of this approach, while Kochov [28] offers a general treatment. See also Spiegler [40, p.110] and Schipper [38]. With regards to awareness of unawareness, see the discussion in Section 3.4.

the conjunction of (ii) and (iii) may lead endogenously to a setup such as ours in which agents only trade measurable assets. Where our paper is richer is in considering the effects of emerging awareness over time. Developing a tractable setup in which awareness is both asymmetric and changing over time is an interesting and challenging problem. Another question is whether awareness of one's unawareness and the caution that comes with it decrease the likelihood of default. That being said, we have seen that caution entails a departure from CE (Section 2.3 and Section 3.4) and that this may be welfare-decreasing (Section 3.1). Moving away from the competitive price setting, it will be interesting to analyze this tradeoff from a market design perspective.

We must also mention the literature on temporary equilibrium, with which we share the idea that under bounded rationality the economy will transition from one equilibrium to another.[30] While differences abound, both in terms of modeling and conclusions,[31] a key one is that the source of bounded rationality is not explicitly modeled in that literature. Instead, one typically postulates uncertainty about prices which is not pinned down by the physical state of the world. A drawback of this reduced-form approach is that welfare analysis becomes difficult as the concept of welfare itself cannot be dissociated from the market mechanism. By comparison, our framework is explicit about the source of bounded rationality which, as we have illustrated, permits a rich and, in our opinion, interesting array of questions concerning welfare.

Finally, we note a growing recognition that bounded rationality may be essential to understanding various macroeconomic and asset-pricing puzzles. Among others, we refer the reader to the recent work of Farhi and Werning [9], Gabaix [10, 11], and Gul, Pesendorfer, and Strzalecki [20]. Understanding the pros and cons of the various frameworks awaits further development.

[30] See Grandmont [14] and Green's [16] classical existence result.

[31] One curious difference concerns existence. In particular, the main lesson from Green [16] is that a temporary equilibrium fails to exist whenever agents' beliefs about future prices are too disparate. In comparison, we have seen that a REP may fail to exist even when the agents' expectations are in full agreement.

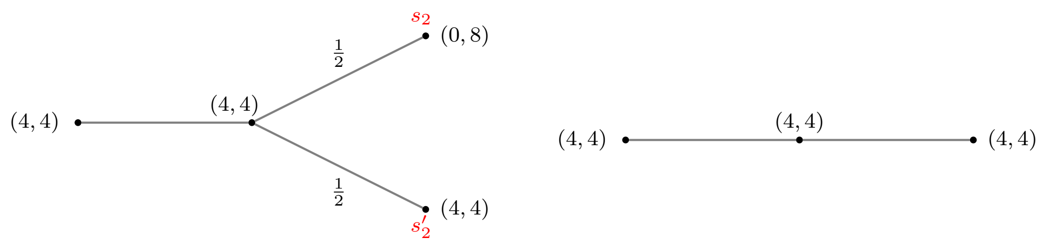

Figure 6: The tree on the left depicts the objective environment, while the tree on the right shows the time $t = 0$ perception.

A Appendix

A.1 Example regarding Lemma 2

Suppose there are three periods and two agents, both with log utility and a discount factor of 1. The true economy and the period-0 perceptions are depicted in Figure 6. Note that $Y = \hat{Y}$, while CE fails for both agents. Given the specified perceptions, it is clear that the equilibrium at time $t = 0$ is one of no trade. Moreover, as stipulated by Lemma 2, the agreed upon allocation is efficient (from the standpoint of the true, as well as perceived, economy). However, in period $t = 1$, perceptions change (so as to coincide with the truth, by OSAA) and so does the equilibrium. The new equilibrium has prices $\hat{q}^{s_1}(s_2) = \hat{q}^{s_1}(s_2') = \frac{1}{2}$, given which agent 1 optimally sells one unit of the s_2' contingent claim and one unit of the consumption good in order to buy three units of the s_2 claim, thereby distributing consumption evenly across the current and future nodes. However, because the agents deviate from the initially agreed upon equilibrium, the resulting consumption allocation, $c^{1,REP} = (4, (3, 3))$ and $c^{2,REP} = (4, (5, 5))$, is inefficient.

A.2 Strong Consistency

Given a node $s \in T$, the consistency of a REP requires that the *contemporaneous consumption level* $\hat{c}^{i,s}(s)$ of any agent i coincide with what in period 0 that agent planned to consume at the node $\hat{s} \in \hat{T}$ that contains s. A stronger notion of consistency is to require that the entire plan formulated at node s be consistent with the agent's plan in period 0. Formally, a REP is *strongly consistent* if $\hat{c}^{i,s}(s') = \hat{c}^{i,0}(s')$ for all i and $s, s' \in T$ such that s' succeeds or is equal to s.[32] It is immediate that under the assumptions of our irrelevance theorem, any REP is strongly consistent, not just consistent. Utilizing strong consistency, one can also establish the following analogue of Theorem 2. (While we leave the relatively straightforward proof to the reader, we note that, unlike consistency, strong consistency is a property of a REP that is necessarily inherited by the implied continuation equilibrium of any of the continuation economies.)

Theorem 5 *If a strongly consistent REP exists, then $Y^s = \hat{Y}^s$ for every node $s \in T$ and the REP is efficient. Conversely, if $Y^s = \hat{Y}^s$ for every $s \in T$ and there is an efficient REP such that $\mathrm{c}^{REP} \gg 0$, then the REP is strongly consistent.*

The first part of Theorem 5 is undoubtedly a more satisfactory converse of our irrelevance theorem in that it establishes the necessity of $Y^s = \hat{Y}^s$ at every node s, not just the initial one. The reason we focused on consistency in the main text is because Theorem 2 delivers the statement that any inefficient REP must fail consistency and because we believe that failures of consistency would be easier to test in practice than failures of strong consistency.

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[32] An equivalent formulation is that $c^{i,REP} = \hat{c}^{i,s}(s')$ for all $s, s' \in T$ such that s' succeeds s.

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