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Competitive Price Discrimination, Imperfect Information, and Consumer Search

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Competitive price discrimination, imperfect information, and consumer search *

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Abstract

Price discrimination in real-world settings is likely based on *imperfect* information. I analyse a homogenous goods framework where firms receive binary and noisy signals about consumer valuations and consumers engage in sequential search. Firms have no information about consumers' search histories. In this framework, the existence of on-path search can be understood as an imperfect screening device that firms employ to the detriment of consumers. Firm profits and equilibrium prices are highest in the unique symmetric pure-strategy equilibrium with search on the equilibrium path, as compared to any other symmetric pure-strategy equilibrium. The equilibrium with on-path search can only be sustained when search costs are at an intermediate level. At low search costs, an equilibrium is played in which there is no on-path search, but consumers use the threat of searching to ensure low prices. High levels of signal precision are detrimental to consumers by facilitating existence of the equilibrium with on-path search.

Keywords: search, competitive price discrimination, imperfect customer recognition

JEL Classification: D43, D83, L13, L15

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1 Introduction

The issue of price discrimination gains relevance with every year that passes in the digital age. Both the amount of data that consumers generate online and the ability of firms to use this information profitably is rising over time. These ideas, together with the increasing amount of empirical evidence for price discrimination on online markets [1], generate urgency for economists to fully understand how to protect consumers in such settings. A key issue in this endaveour is the notion that the information which firms use to price discriminate is likely *imperfect*. Consumer preferences are both erratic and latent - because of this, the prediction of preferences is noisy by construction. The issue of price discrimination based on imperfect information in *competitive* settings is the focus of this paper.

I study the above issues in a theoretical framework where consumers have to acquire consumption opportunities via sequential search and the size of consumer choice sets is thus endogenous. The analysis of imperfect price discrimination has close ties to the modelling of optimal consumer search decisions, both from an applied and a theoretical standpoint. Firstly, consumer search choices play an important role in real-world settings such as online markets where it is likely that price discrimination is particularly relevant. To understand outcomes on such markets and guide policy, it is thus instructive to consider endogenous search choices. Secondly, price discrimination based on noisy information presents a theoretical explanation of equilibrium search in settings where consumers are fully aware of their preferences before searching and all consumers have strictly positive search costs. This is particularly relevant when considering markets where consumers repeatedly buy the same or similar goods, such as the market for commercial flight tickets.

I formalize the above ideas in the following framework: There is a unit mass of consumers who each want to buy at most one unit of an indivisible and homogenous good. This product is produced by two firms that each have zero marginal costs of producing the good. Consumers have heterogenous valuations for the good. These valuations are private information to each consumer and are drawn independently from the unit interval according to a distribution that is common knowledge. While the first search is costless, searching an additional firm after that incurs strictly positive search costs. Consumers pick the first firm they visit randomly - this is without loss of generality when considering a homogenous goods framework and restricting attention to symmetric equilibria.

[1]Contributions such as Mikians et al. (2012) and Escobari et al. (2019) provide rigorous evidence for this.

When a firm is visited by a consumer, this firm receives an informative binary signal about the true valuation of this consumer. The probability distribution of the signal depends only on the consumer's valuation and nothing else. I define the signal realization which becomes more likely to occur when a consumer's valuation rises as the *high signal* and the other signal as the *low signal*. A firm knows nothing about any consumer's search history. Consumers have perfect recall and firms commit to the prices they have offered to a given consumer. A firm's pure strategy is thus a price tuple (p^L, p^H) that consists of a low signal price p^L that is offered to all consumers who generate the low signal and a high signal price p^H that is offered to the rest. When studying symmetric pure-strategy equilibria, it is without loss of generality to restrict attention to equilibria where $p^L < p^H$.

Initially, I consider a simplified version of this framework with a simple step-function distribution of the signal. I assume that the probability to generate the high signal is $1 - \sigma$ for any agent with a valuation below 0.5 and σ for any agent with a valuation above 0.5. In the following, I refer to consumers with valuation above and below 0.5 as *high-valuation* and *low-valuation consumers*, respectively. The parameter σ, which I call information precision, is the probability that the binary signal correctly identifies whether a consumer is in the upper or the lower half of the valuation spectrum. In this game, there are three potential symmetric pure-strategy equilibrium candidates: Two of these potential equilibrium candidates do not feature search on the equilibrium path, namely the *monopoly equilibrium* and what I refer to as the *search deterrence equilibrium*. In the monopoly equilibrium, firms choose the prices they would offer to consumers when search is prohibitively costly. In the search deterrence equilibrium, the high signal price is set in such a way that the consumers with the highest incentives to search are just indifferent between searching and not searching.

I refer to the unique equilibrium with on-path search as the *search equilibrium*. The search equilibrium is characterized by the following key results: Firstly, no consumer with a valuation above the high signal price p^H can search on the equilibrium path. If this condition were to be violated, there would exist a strictly positive measure of consumers with a valuation above p^H who are offered the high signal price at multiple firms. This group of consumers would create an undercutting motive that eliminates any such pure-strategy equilibrium. Secondly, there exists a non-degenerate interval of prices above the low signal price p^L for which no consumer would move on to search. Moreover, any consumer that arrives after searching would buy the good when offered a price in this interval. These two results shape the structure of the search equilibrium: When setting the high signal price, the sale is not made to any consumer that arrives after searching. By contrast, the sale is made to all consumers who arrive after searching when offering the low signal price.

The monopoly equilibrium exists if and only if low valuation consumers, who have the greatest incentives to search in this equilibrium, do not search on path. This implies that the monopoly equilibrium exists when search costs are comparatively high and when signal precision is relatively low. Perhaps surprisingly, the search equilibrium only exists for intermediate search costs. Existence of this equilibrium requires that low-valuation consumers search on the equilibrium path, while high-valuation consumers do not. This scenario is possible because high valuation consumers have lower incentives to search at equilibrium prices, given that the probability of generating the low signal is smaller for them. If search costs become too small, incentives to search for high valuation consumers become too large to sustain this equilibrium. The search deterrence equilibrium, in which there is no on-path search, exists when search costs are low. Existence of this equilibrium is based on the feature that high-valuation consumers can credibly promise to search at the monopoly high signal price and at prices slightly below this, which requires low search costs. It turns out that only one of the three aforementioned equilibria exists for almost all of the parameter combinations.

I show that the search equilibrium features the highest prices and the highest firm profits of all equilibria. In other words, the presence of equilibrium search coincides with *higher* prices and firm profits, as compared to the equilibria without search. This has two reasons: Firstly, equilibrium search requires that low-valuation and high-valuation consumers systematically differ in their equilibrium search behaviour. Equilibrium search is thus beneficial to firms because it offers them an imperfect screening device. Secondly, equilibrium search essentially offers firms in the industry a "second chance" to make the sale to consumers that could not purchase at an initial price offer. This "second chance" allows firms to generate additional profits from these consumers who would not have purchased the good in an equivalent equilibrium without on-path search.

The effects of an increase in search costs are non-monotonic and depend fundamentally on the equilibrium that is being played and whether the parameter change induces a shift of the equilibrium that is being played. Moreover, the relationship of search costs and the volume of equilibrium search is non-monotonic. Starting off at low levels of search costs for which the search deterrence equilibrium is played, an increase of search costs increases the high signal price while leaving the low signal price unaffected. The high signal price increases because the ability of consumers to restrict firm pricing by threatening to search is reduced. There exists a threshold level of search costs at which an increase of search costs eliminates the search deterrence equilibrium and the search equilibrium will be played. When this happens, both equilibrium prices will jump up discontinuously. In the search equilibrium, a further increase of search costs has no effects on the high signal price but benefits consumers by reducing the equilibrium low signal price. This effect is driven by the fact that any consumer who arrives at a firm after searching generates locally price inelastic demand for this firm at the equilibrium low signal price.

The effects of increases in the signal precision parameter σ on prices are similarly non-monotonic. Higher information precision reduces the low signal price in any equilibrium that is being played. This is because greater signal precision implies that less consumers search on path and that those consumers who generate the low signal have a lower average valuation. In addition, increases of signal precision also reduce the high signal price in the search deterrence equilbrium. This is because a rise in information precision increases the incentives to search for low-valuation consumers. However, high information precision may also be detrimental to consumers. This is because the interval of search costs for which the search equilibrium can be supported as a perfect Bayesian equilibrium expands as information precision increases.

This baseline analysis shows that the effects of search costs and information precision are highly non-monotonic in the markets I describe. Moreover, observing that consumers acquire a large choice set via search is not necessarily an indicator for thriving competition in a market nor a reflection of low search costs. Instead, such an equilibrium outcome may simply represent an imperfect screening device for firms in an industry. To further add credibility to these ideas, I generalize my framework after completing the aforementioned analysis by studying general signal distributions. I show that two conditions are sufficient to reproduce the above results in general settings. The first condition is that the signal distribution is continuously differentiable. The second condition ensures that the set of valuations that would search on the equilibrium path is always convex.

The rest of the paper proceeds as follows. I lay out the related literature in section 2. In section 3, I set up the model and provide initial results. In section 4, I solve the baseline model described above. Section 5 is devoted to the analysis of generalized versions of this model. Section 6 concludes.

2 Related literature

My work connects to the extensive literature that exists on price discrimination, such as Villas-Boas (1999), Fudenberg & Tirole (2000), and Acquisti & Varian (2005). More specifically, my work has ties to the theoretical contributions which study price discrimination based on imperfect information in a monopoly setting, such as Aron et al. (2006), Belleflamme & Vergote (2016), de Cornière & Montes (2017), Koh et al. (2017), and Valletti & Wu (2020). In contrast to all these papers, I study a setting with competition.

In addition, my work is not just a model of price discrimination but also a model of endogenous search. Thus, it is related to the developing strand of theoretical research which connects price discrimination to endogenous consumer search choices. A keystone of this literature is the work of Fabra & Reguant (2020), who study a simultaneous search setting where consumers are heterogenous in their search costs and the quantity of the good they desire. Importantly, all consumers have the same valuation for the good. Firms observe the consumer's desired quantity and update beliefs about the consumer's search behaviour. Price discrimination then occurs based on these notions.

Following the ideas of Fabra & Reguant (2020), other papers have started to examine the role of price discrimination based on ex-ante consumer heterogeneity in search models. Crucially, all these papers study a setting where firms receive information about the search costs of consumers or the size of the choice sets consumers are endowed with. By contrast, I study a setting where firms receive noisy signals about consumer valuations and all consumers are endowed with the same search technology. Mauring (2021) and Atabak (2021) study a setting with shoppers and non-shoppers as defined in Burdett & Judd (1983) and Stahl (1989). Mauring (2021) and Atabak (2021) assume that firms receive imperfect information about the affiliation of a particular consumer to the groups of shoppers and non-shoppers. Armstrong & Vickers (2019) study a setting where firms face exogenously captive and non-captive consumers and receive information about this status. Bergemann et al. (2020) study a homogenous goods setting where competing firms receive imperfect information about a consumer's search technology and the number of price offers a consumer obtains or has previously obtained.

My work is also related to the following papers which study settings where competing firms receive noisy or partial information about consumer valuations. My work fundamentally differs from all these papers in the sense that I study a model with endogenous search, while the following papers do not. Esteves (2014) studies a Hotelling-style framework where firms receive noisy information about the continuous horizontal preference parameters of consumers. Peiseler et al. (2018) consider an infinite repetition of a stage game that is very similar to the one analysed in Esteves (2014). Within this framework, the authors analyse under what conditions collusion can be sustained over time. Clavorà Braulin (2021) studies a horizontal differentiation setting where consumer preferences vary in two dimensions. In Clavorà Braulin (2021), firms have perfect information about the realizations of one dimension of consumer preferences, but not both. Importantly, all these papers assume that the market is fully covered, i.e. that all consumers will purchase the good in equilibrium.

Finally, Belleflamme et al. (2020) study a setting with homogenous goods where firms have access to an imperfect profiling technology. The technology they consider differs fundamentally from the one I consider. In Belleflamme et al. (2020), a firm either knows a consumer's valuation perfectly or knows nothing at all about a consumer. In my setup, a firm knows something about all consumers that visit the firm, but this information is always noisy.

3 Framework

I study a model that incorporates price discrimination based on noisy information into a sequential search setup. There is a unit mass of consumers indexed i, who each want to buy at most one unit of an indivisible good. There are 2 firms indexed $j \in \{1, 2\}$ active in the market who each have a marginal cost equal to 0. A firm is free to offer a different price to any consumer that visits the firm. Goods are homogenous and a given consumer i has a valuation of $\nu_i \in [0, 1]$ for the good of any firm. These valuations are private information to each consumer and are known by the consumer at the beginning of the game. The distribution of these valuations, which is identical for all consumers, is common knowledge. Consumers maximize expected utility. When a consumer i buys the good at price p, the utility of the consumer is:

$$u(\nu_i, p) = \nu_i - p \tag{1}$$

Consumers visit firms by sequential search. After realizing their valuations, consumers randomly visit some firm first. This incurs no costs to the consumers. The firm that is visited first offers a price to the consumer. After having received this price, a consumer decides whether or not she wants to visit an additional firm, i.e. to search. Searching the second firm implies that the consumer incurs a fixed cost equal to $s > 0$. The firm that is visited second then also offers the consumer a price. After this, the consumer decides from which firm to buy the product, or not to buy the good from either firm. The assumption that a consumer randomly chooses which firm to visit first is without loss of generality when considering symmetric equilibria in a setting with homogenous goods.

When a firm j is visited by a consumer i, this firm receives a binary signal $\tilde{\nu}_{i,j} \in \{\tilde{\nu}^L, \tilde{\nu}^H\}$ about the true valuation of this consumer. The signal probability distribution is expressed by $Pr(\tilde{\nu}^H|\nu_i)$. Note that the signal probability distribution only depends on ν_i and nothing else. In particular, the signals $\tilde{\nu}_{i,j}$ and $\tilde{\nu}_{i,-j}$ which the two firms j and $-j$ receive for a given consumer i are independent, conditional on ν_i. I assume that $Pr(\tilde{\nu}^H|\nu_i)$ is a weakly increasing function. I label the signal $\tilde{\nu}^H$, which becomes more likely as a consumer's valuation increases, as the *high signal* and the other signal $\tilde{\nu}^L$ as the *low signal*.

A firm knows nothing about any consumer's search history. In particular, a firm j does not know its' position in the search queue of any consumer. This assumption allows me to put the spotlight on the role of imperfect information about preferences. Consumers are fully aware of all the prices they have received from any firm they have visited. Moreover, when a consumer visits a firm, decides to search, and returns back to the firm, the consumer can purchase the good at the price that was initially offered to her without further cost.

I focus on deriving symmetric pure-strategy perfect Bayesian equilibria. A consumer's pure strategy consists of a search strategy and a purchasing strategy. Sequential rationality implies that the consumer's search strategy must also be optimal off-path, i.e. for prices p_j that the consumer would not be offered if firms play their equilibrium strategies. A firm's pure strategy is a mapping $p : \{\tilde{\nu}^L, \tilde{\nu}^H\} \to \mathbb{R}$. There are no relevant off-path decisions for firms.

Any symmetric pure-strategy equilibrium in the above game is characterized by the following result:

Proposition 1 *In a symmetric pure strategy equilibrium (p^L, p^H) with $p^H > p^L$, the set of consumers that search on the equilibrium path and have a valuation $\nu_i \geq p^H$ must have measure 0.*

Suppose, for a contradiction, that there is a strictly positive measure of consumers that search on the equilibrium path and have $\nu_i \geq p^H$. Because goods are homogenous, such a consumer can only search on the equilibrium path after receiving p^H. No consumer would have incentives to search after receiving the low signal price. Thus, the consumers that search on-path must have received p^H at the first firm they visited - this implies that $Pr(\tilde{\nu}^H|\nu_i) > 0$ must hold true for them. Thus, they would also be offered the price p^H with positive probability at the firm they visit second. This means that there exists a set of consumers with positive measure who receive the high price at both firms and who would buy the good at this price. This creates an undercutting motive which eliminates the equilibrium. Note that there exists no equivalent undercutting motives for the low price p^L because consumers never search when being offered this price.

4 Baseline model:

4.1 Setup and monopoly solution

Consider the following special case of the model outlined above: The valuations of the consumers are uniformly distributed and the signal follows a step-function distribution with precision parameter σ, where the following holds:

$$Pr(\tilde{\nu}^H|\nu_i) = \begin{cases} \sigma & \nu_i \geq 0.5 \\ 1-\sigma & \nu_i < 0.5 \end{cases} \tag{2}$$

I assume that $\sigma > 0.5$, which indicates that the signal is informative. In the following, I refer to the setting I have just described as the "baseline setting".

Pinning down the profits and optimal prices when firms have monopoly power is instructive to understand the competitive equilibrium. Note that monopoly power could be reached, for instance, when $s \to \infty$. I denote $\Pi^M(p_j;\tilde{\nu})$ as the expected profits a firm with monopoly power obtains from a consumer that generates the signal $\tilde{\nu} \in \left\{\tilde{\nu}^L, \tilde{\nu}^H\right\}$ at their firm when offering them a price p_j. In this setting, these functions are strictly concave almost everywhere and optimal prices in such a setting are pinned down by the following lemma:

Lemma 1 *Consider the baseline setting and suppose that $s \to \infty$. The optimal low and high signal prices are given by:*

$$p^{L,M} = arg\max_{p_j} \Pi^M(p_j;\tilde{\nu}^L) = 1/4\sigma \tag{3}$$

$$p^{H,M} = arg\max_{p_j} \Pi^M(p_j;\tilde{\nu}^H) = 0.5 \tag{4}$$

The monopoly low signal price $p^{L,M}$ is falling in σ. A more precise signal, which is reflected by an increase of σ, implies that the average valuation of consumers that generate the low signal decreases. This means that the optimal price that would be offered to these consumers falls. The fact that the high price $p^{H,M}$ is independent of the information precision parameter σ follows from the distributional assumptions on $Pr(\tilde{\nu}^H|\nu_i)$. Notice also that $p^{L,M} < p^{H,M}$ holds true because the signal is informative.

4.2 Search

Having established this, I move on to characterize the competitive equilibrium. The starting point for doing this is pinning down the sequentially rational search behaviour. In what follows, I will drop the index i which refers to a given consumer. Consider a symmetric equilibrium price tuple where the price p^L (the *low signal price*) is set when observing the low signal and the price p^H (the *high signal price*) is set when observing the high signal. Assume, for now, that $p^L < p^H$.

A consumer that has initially received a price $p_j \leq p^L$ will never search, since the gains of search are necessarily 0. Thus, consider the decision situation of a consumer who has initially received a price $p_j \in [p^L, p^H]$. This consumer will find it strictly optimal to search if and only if:

$$Pr(\tilde{\nu}^L|\nu)\max\{\nu - p^L, 0\} + Pr(\tilde{\nu}^H|\nu)\max\{\nu - p_j, 0\} - s > \max\{\nu - p_j, 0\} \tag{5}$$

I assume that a consumer will only search if it is strictly optimal to do so. Any consumer with $\nu \leq p^L$ would never search. Moreover, this equation can be rewritten as:

$$Pr(\tilde{\nu}^L|\nu)\max\{\nu - p^L, 0\} - s > Pr(\tilde{\nu}^L|\nu)\max\{\nu - p_j, 0\} \tag{6}$$

Note that the RHS of this equation is always weakly positive. A consumer with current price offer $p_j \in (p^L, p^H]$ can thus only search if the following necessary condition is satisfied:

$$Pr(\tilde{\nu}^L|\nu)\max\{\nu - p^L, 0\} - s > 0 \iff \nu > p^L + \frac{s}{Pr(\tilde{\nu}^L|\nu)} \tag{7}$$

Notice that this condition is independent of the best price the consumer currently has in hand. A consumer can only search on-path if her valuation satisfies condition (7). Given that the probability to generate the low signal varies discontinuously around $\nu = 0.5$, the structure of this necessary condition varies around this threshold.

Now consider prices $p_j \in (p^H, 1)$, i.e. off-equilibrium prices. A consumer will find it strictly optimal to search if and only if:

$$Pr(\tilde{\nu}^L|\nu)\max\{\nu - p^L, 0\} + Pr(\tilde{\nu}^H|\nu)\max\{\nu - p^H, 0\} - s > \max\{\nu - p_j, 0\} \tag{8}$$

For a consumer with $\nu \leq p^H$, the necessary condition for search to occur at prices $p_j \in (p^H, 1]$ is the same as the necessary condition for on-path search laid out in condition (7). For consumers with $\nu > p^H$, the necessary condition for search at prices $p_j \in (p^H, 1]$ is potentially less strong and given by:

$$\nu > Pr(\tilde{\nu}^L|\nu)p^L + Pr(\tilde{\nu}^H|\nu)p^H + s \tag{9}$$

Having established this, we are ready to pin down sufficient conditions for search. I define a cutoff price function $\hat{p}(\nu)$ such that a consumer with valuation ν will find it strictly optimal to search if and only if the current price p_j satisfies $p_j > \hat{p}(\nu)$. Consider first consumers with a valuation ν that satifies neither necessary condition outlined above. For these consumers, $\hat{p}(\nu) = \infty$ holds, which means that they will never search. The cutoff prices of consumers that fulfil one of the necessary conditions outlined above depend on the equilibrium under consideration. For example, when $p^H \geq s/(1-\sigma) + p^L$, the cutoff price of consumers with $\nu > 0.5$ is given by:

$$\hat{p}(\nu) = p^L + \frac{s}{1-\sigma} \tag{10}$$

When $p^H < s/(1-\sigma) + p^L$, the cutoff price of these consumers is given by:

$$\hat{p}(\nu) = (1-\sigma)p^L + \sigma p^H + s \tag{11}$$

4.3 Equilibria

This section is devoted to the characterization of the symmetric pure-strategy perfect Bayesian equilibria that exist in the baseline setting. I first characterize the types of pure-strategy equilibria that can exist in the following proposition:

Proposition 2 *Consider the baseline setting. A symmetric pure-strategy equilibrium with search on the equilibrium path must satisfy:*

$$p^L < p^H = p^{H,M} \tag{12}$$

A symmetric pure-strategy equilibrium without search on the equilibrium path must satisfy *one of the following expressions:*

$$(p^L, p^H) = (p^{L,M}, p^{H,M}) \tag{13}$$

$$(p^L, p^H) = (p^{L,M}, s/\sigma + p^{L,M}) \tag{14}$$

The above result shows that there are three types of pure-strategy equilibria that are possible in the model that I have outlined. Firstly, there potentially exists an equilibrium with search on the equilibrium path. Such an equilibrium must satisfy condition (12). I show later that there exists a unique candidate for this equilibrium, which I then refer to as the *search equilibrium*. In addition, there are exactly two candidates for a pure-strategy equilibrium without on-path search. I label these equilibria as follows: The price vector $(p^{L,M}, p^{H,M})$ is labelled the *monopoly equilibrium*, and the price vector $(p^{L,D}, p^{H,D}) = (p^{L,M}, s/\sigma + p^{L,M})$ is labelled the *search deterrence equilibrium*. Note also that the low signal price must always be strictly below the high signal price in any symmetric pure-strategy equilibrium.

In the search deterrence equilibrium, the high signal price is set in such a way that consumers with valuations $\nu \in (s/\sigma + p^L, 0.5)$ are exactly indifferent beween searching and not searching.[2] These consumers generate the low signal and are offered the low price with probability $\sigma > 0.5$. This means that they have greater incentives to search at equilibrium prices than consumers with $\nu > 0.5$, which implies that there will be no on-path search in the search deterrence equilibrium. Note further that consumers with $\nu \in (s/\sigma + p^L, 0.5)$ would search and never return if offered a price $p_j > s/\sigma + p^L$.

Having established this, I move on to characterize when each of these equilibria exist. I start with the equilibria that do not feature search on the equilibrium path. To do so, define $\Pi^C(p_j; \tilde{\nu})$ as the expected profits a firm in a competitive setting obtains from a consumer that generates the signal $\tilde{\nu} \in \left\{\tilde{\nu}^L, \tilde{\nu}^H\right\}$ when offering the consumer a price p_j.

Proposition 3 *Consider the baseline setting. The price tuple $(p^{L,M}, p^{H,M})$ can be supported in perfect Bayesian equilibrium if and only if:*

$$0.5 \leq \frac{s}{\sigma} + p^{L,M} = \frac{s}{\sigma} + \frac{1}{4\sigma} \tag{15}$$

By contrast, the price tuple $(p^{L,D}, p^{H,D}) = (p^{L,M}, s/\sigma + p^{L,M})$ can be supported in perfect

[2] Consumers with $\nu < s/\sigma + p^L$ do not fulfil the necessary conditions for search.

Bayesian equilibrium if and only if the following two conditions jointly hold:

$$0.5 > (1-\sigma)p^{L,D} + \sigma p^{H,D} + s \tag{16}$$

$$\Pi^C((1-\sigma)p^{L,D} + \sigma p^{H,D} + s; \tilde{\nu}^H) \leq \Pi^M(p^{H,D}; \tilde{\nu}^H) \tag{17}$$

The condition laid out in equation (15) is both necessary and sufficient for existence of the monopoly equilibrium. This inequality is equivalent to the property that the cutoff price $\hat{p}(\nu)$ of any consumer is above $p^{H,M} = 0.5$. If this holds true, there will be no search on the equilibrium path and monopoly profits are an upper envelope for the profits that can be attained in this equilibrium. Since both prices maximize monopoly profits after the respective signal, there will be no deviations. Condition (15) is also necessary for existence of the monopoly equilibrium. If this condition is violated, some consumers would search at these prices, which would create incentives for an upward deviation from the equilibrium low price.

As discussed previously, there will not be on-path search in the search deterrence equilibrium. Moreover, this equilibrium must satisfy:

$$p^{H,D} = \frac{s}{\sigma} + p^{L,D} < 0.5 \tag{18}$$

A high signal price above 0.5 can not be supported in an equilibrium without search, since there would be a downward deviation to the monopoly price $p^{H,M} = 0.5$. That inequality (18) holds is guaranteed by condition (16). Furthermore, the search deterrence equilibrium can only be supported when all agents with $\nu > 0.5$ would search upon being offered the monopoly high signal price $p^{H,M} = 0.5$. Condition (16) is also sufficient for this to hold true. Suppose, for a contradiction, that consumers with $\nu > 0.5$ would not search when being offered the monopoly high signal price in the search deterrence equilibrium. Then, this deviation would grant the firm monopoly profits. Because there is no on-path search in this equilibrium and monopoly high signal profits have a strict optimum at $p^{H,M}$, deviating to the monopoly high signal price would be profitable, breaking the initial equilibrium.

It remains to check other profitable deviations from the equilibrium prices. There exist no profitable deviations from the low signal price, since there is no on-path search and $p^{L,D}$ maximizes monopoly profits. By an analogous logic, there exist no profitable downward deviations from $p^{H,D}$ since $p^{H,D} < p^{H,M}$. Now consider deviation prices $p_j > p^{H,D}$. Setting such a price triggers search by all consumers with $\nu \in (s/\sigma + p^{L,D}, 0.5)$. In addition, consumers with $\nu > 0.5$ would move on to search if and only if:

$$p_j > (1-\sigma)p^{L,D} + \sigma p^{H,D} + s \tag{19}$$

The term $(1-\sigma)p^{L,D} + \sigma p^{H,D} + s$ is the search price cutoff $\hat{p}(\nu)$ of consumers with $\nu > 0.5$. Note that condition (16) implies that this cutoff price is strictly below 0.5, which is necessary for existence of the search deterrence equilibrium. Note further that any consumer who leaves firm j to search will never return. Thus, setting a price above $(1-\sigma)p^{L,D} + \sigma p^{H,D} + s$ will yield zero profits. In addition, any price in the interval $(p^{H,D}, (1-\sigma)p^{L,D} + \sigma p^{H,D} + s)$ is dominated by setting the price $p_j = (1-\sigma)p^{L,D} + \sigma p^{H,D} + s$. At any such price, consumers with $\nu \leq 0.5$ will surely not buy at the firm while the sale is surely made to all consumers with $\nu > 0.5$. A deviation to the price $p_j = (1-\sigma)p^{L,D} + \sigma p^{H,D} + s$ is not profitable if and only if condition 17 holds true.

This completes the characterization of the conditions which guarantee existence of the equilibria without search on the equilibrium path. I thus turn my attention to the equilibrium candidate which features search on the equilibrium path. In this equilibrium, the mass of agents that search on the equilibrium path and the equilibrium prices are determined jointly. Firstly, I pin down the sequentially rational search behaviour in such an equilibrium in the following lemma:

Lemma 2 *Consider the baseline setting. A symmetric pure-strategy equilibrium (p^L, p^H) with search on the equilibrium path must satisfy:*

$$\frac{s}{\sigma} + p^L < 0.5 \leq \frac{s}{1-\sigma} + p^L \tag{20}$$

In such an equilibrium, the sequentially rational consumer search behaviour is given by:

$$\hat{p}(\nu) = \begin{cases} \infty & \nu \in [0, s/\sigma + p^L] \\ s/\sigma + p^L & \nu \in (s/\sigma + p^L, 0.5] \\ \infty & \nu \in (0.5, \sigma p^H + (1-\sigma)p^L + s] \\ \sigma p^H + (1-\sigma)p^L + s & \nu \in (\sigma p^H + (1-\sigma)p^L + s, 1] \end{cases} \tag{21}$$

It was previously shown that any equilibrium with on-path search must satisfy $p^L < p^H = 0.5$. Moreover, agents with $\nu > 0.5 = p^H$ cannot search on the equilibrium path in a symmetric pure-strategy equilibrium of the above form. The inequality $0.5 \leq s/(1-\sigma) + p^L$ is equivalent to this property. Intuitively, this holds true when $1-\sigma$ is comparatively small, which means that agents with $\nu > 0.5$ have a relatively small chance of generating the low signal and being offered the low signal price. Moreover, an equilibrium with search on path must have a property which guarantees that on-path search is optimal for some agents. As indicated above, consumers with $\nu \in (s/\sigma + p^L, 0.5)$ will have a cutoff price equal to:

$$\hat{p}(\nu) = p^L + \frac{s}{\sigma} \tag{22}$$

The inequality $s/\sigma + p^L < 0.5$ thus guarantees that there exists an interval of consumers with $\nu \in (s/\sigma + p^L, 0.5)$ who will search when being offered the price $p^H = 0.5$.

Having established the properties of the sequentially rational search behaviour in an equilibrium with on-path search, I now move on to characterize profits. This is necessary to pin down a closed form solution for the low signal price in such an equilibrium. Note that firms form beliefs over three consumer characteristics: (i) the true valuation of the consumer, (ii) the consumer's search queue, and (iii) the price the consumer has received from the other firm (if any). In the search equilibrium, the profit functions of a firm (after any signal) can be split into three distinct segments. Before describing these, I define the density of consumers with valuation ν that arrive at a firm in position $k \in \{1, 2\}$ as $f^k(\nu)$. Because the first search is random and valuations are drawn from the uniform distribution, $f^k(\nu) = 0.5$ holds true for both $k \in \{1, 2\}$.

Firstly, suppose that a firm j sets a price $p_j \in (0, s/\sigma + p^L)$. By lemma (2), no consumer that arrives at firm 1 first would move on to search when receiving such a price. Thus, a consumer that arrives at firm j first buys at firm j if and only if $\nu \geq p_j$. By lemma (2), consumers that arrive at firm j second must have generated the high signal (and thus received the high price $p^H = 0.5$) at firm $-j$ and must have $\nu \in (s/\sigma + p^L, 0.5]$.

Recall further that $s/\sigma + p^L < 0.5 = p^H$ must hold in an equilibrium with search. Thus, when firm j offers consumers who arrive at this firm second a price $p_j \in (0, s/\sigma + p^L)$, any such consumer would thus surely buy the good at firm j. In this price interval, the demand implied by these consumers is thus fully inelastic. For prices $p_j \in (0, s/\sigma + p^L)$, the competitive profit functions $\Pi^C(p_j; \tilde{\nu}^k)$ are given by:

$$\Pi^C(p_j; \tilde{\nu}^k) = \underbrace{p_j \int_{p_j}^{1} f^1(\nu) Pr(\tilde{\nu}^k|\nu) d\nu}_{\Pi^M(p_j; \tilde{\nu}^k)} + \underbrace{p_j \int_{s/\sigma + p^L}^{0.5} f^2(\nu) Pr(\tilde{\nu}^k|\nu) Pr(\tilde{\nu}^H|\nu) d\nu}_{M^k(\sigma, s; p^L)} \tag{23}$$

Note that the first term on the right-hand side is just equal to the monopoly profits at this price (for the given signal). The second term is the mass of people that arrive at firm j after searching and generate the signal $\tilde{\nu}$, multiplied by the price. Observe that I have defined:

$$M^L(\sigma, s; p^L) = \int_{s/\sigma+p^L}^{0.5} f^2(\nu)Pr(\tilde{\nu}^L|\nu)Pr(\tilde{\nu}^H|\nu)d\nu \tag{24}$$

Profits in the price interval $p_j \in (s/\sigma + p^L, 0.5)$ are not of fundamental importance and are only relevant when checking for potential deviations.

Now consider prices in the interval $p_j \in [0.5, \sigma p^H + (1-\sigma)p^L + s]$. Any consumer that arrives at firm j after searching would not buy the good at firm j when being offered such a price. However, no consumer that arrives at firm j first and has $\nu > 0.5$ would move on to search at these prices. Thus, profits are equal to monopoly profits, i.e:

$$\Pi^C(p_j; \tilde{\nu}) = p_j \int_{p_j}^{1} f^1(\nu)Pr(\tilde{\nu}|\nu)d\nu \tag{25}$$

A natural candidate for an equilibrium high signal price would be the global maximizer of this function, namely the monopoly high signal price $p^{H,M}$. By proposition 2, we know that this price is the unique high signal price that can be supported in a perfect Bayesian equilibrium with on-path search. In the price interval $p_j \in [\sigma p^H + (1-\sigma)p^L + s, 1]$, profits will be zero, because all consumers leave to search and never return.

Given that the equilibrium low signal price must satisfy $p^L \in [0, s/\sigma + p^L]$ and competitive profits are continuously differentiable in this interval, the lower price p^L must be a fixed point of the following first-order condition:

$$\left.\frac{\partial \Pi^M(p_j; \tilde{\nu}^L)}{\partial p_j}\right|_{p_j=p^L} + M^L(\sigma, s; p^L) = 0 \tag{26}$$

Strict concavity of the monopoly low signal profit function $\Pi^M(p_j; \tilde{\nu}^L)$ in the interval $[0, 0.5]$ implies that there is a unique solution to this equation (if a solution that satisfies the properties laid out in lemma 2 exists), which is given by:

$$p^{L,S} = \frac{1 + (1+2s)\sigma - \sigma^2 - 2s}{4(1.5\sigma - 0.5\sigma^2)} \tag{27}$$

Evaluating this first-order condition shows that the price p^L must solve the following:

$$p^L = p^{L,M} + \frac{M(\sigma, s; p^L)}{\sigma} \tag{28}$$

Thus, the equilibrium low signal price will be *higher* in a competitive equilibrium with on-path search than the monopoly low signal price $p^{L,M}$. We also know that the high signal price p^H must equal the monopoly high signal price $p^{H,M} = 0.5$ in an equilibrium with search. Moreover, the previous insights imply that equilibrium profits in the search equilibrium are pinned down by the following expressions.

$$\Pi^C(p^L; \tilde{\nu}^L) = \Pi^M(p^L; \tilde{\nu}^L) + p^L M^L(\sigma, s; p^L) \tag{29}$$

$$\Pi^C(p^H; \tilde{\nu}^H) = \Pi^M(p^H; \tilde{\nu}^H) \tag{30}$$

The following proposition defines when the search equilibrium can be supported as a perfect Bayesian equilibrium:

Proposition 4 *There is a unique pure-strategy equilibrium candidate $(p^{L,S}, p^{H,S})$ with search on the equilibrium path, where $p^{L,S}$ is given by equation (27) and $p^{H,S} = p^{H,M} = 0.5$.*

This price tuple constitutes a perfect Bayesian equilibrium (together with the sequentially rational search behaviour given in lemma 2) if the following conditions both hold:

$$\frac{3\sigma - 1 - 2\sigma^2}{10\sigma - 2 - 4\sigma^2} < s \leq 0.5\sigma - 0.25 \tag{31}$$

$$\Pi^C(s/\sigma + p^L; \tilde{\nu}^H) \leq \Pi^M(0.5; \tilde{\nu}^H) \tag{32}$$

Moreover, firm profits are always stricly higher in the search equilibrium than in the monopoly equilibrium and in the search deterrence equilibrium.

Uniqueness of the equilibrium follows from previous arguments. Equation (31) is obtained when plugging the equilibrium prices into equation (20). In addition, it needs to be ensured that there are no profitable deviations from equilibrium prices. In the baseline model, monopoly profits for either signal are an upper envelope for competitive profits in the price interval $p_j \in (s/\sigma + p^L, 0.5)$. There is not necessarily an analogous property in the generalized settings I study later. Moreover, monopoly profits for either signal are an upper envelope for the respective competitive profits in the price interval $p_j \in (0.5, 1)$. Now consider possible deviations from the low signal price $p^{L,S}$. Recall that the equilibrium low signal price in the search equilibrium is higher than the monopoly low signal price. Note further that low signal profits in the price interval $p_j \in (0, p^L + s/\sigma]$ were given by the following expression:

$$\Pi^C(p_j; \tilde{\nu}^L) = \Pi^M(p_j; \tilde{\nu}^L) + p_j M^L(\sigma, s; p^L) \tag{33}$$

Because $p^{L,M} < p^{L,S}$, equation (33) implies that competitive profits when setting $p^{L,M}$ are higher than the monopoly profits when setting this price. Moreover, recall that $p^{L,S}$ must maximize $\Pi^C(p_j; \tilde{\nu})$ in the interval $p_j \in [0, s/\sigma + p^{L,S}]$. These two notions imply that the profits a firm obtains from consumers that generate the low signal are higher in the search equilibrium than in the monopoly outcome.

This result is already noteworthy as such - it means that firms obtain higher profits in an equilibrium where consumers have more firms in their choice set. Moreover, it is very useful when checking deviations. Because monopoly low signal profits are an upper envelope for competitive low signal profits at all prices $p_j \in (s/\sigma + p^L, 1)$ and $\Pi^C(p^{L,S}; \tilde{\nu}^L) > \Pi^M(p^{L,M}; \tilde{\nu}^L)$ no profitable deviations from the competitive low price exist. By an analogous logic, one can show that the most profitable deviation from the high signal price would be to the price $p_j = s/\sigma + p^L$. Equation 32 formalizes the notion that this deviation is not profitable.

This result that firm profits are highest in the search equilibrium has important consequences. It breaks the positive link between the size of consumers' choice sets and the degree of competition. As a corollary, it suggests that observing a large volume of search on a market may not be beneficial to consumers. Instead, equilibrium search may just act as an imperfect screening device that additionally allows firms multiple chances to make a sale to a given consumer.

I visualize the existence results pinned down in propositions (3) and (4) in the following graph, where I plot realizations of signal precision (σ) on the x-axis and realizations of search costs (s) on the y-axis. Blue dots indicate existence of the monopoly equilibrium, yellow dots indicate existence of the search equilibrium, and the dark green dots indicate existence of the search deterrence equilibrium. Light green dots indicate that both the search and the search deterrence equilibrium exist. In the red-shaded areas, no equilibrium exists.

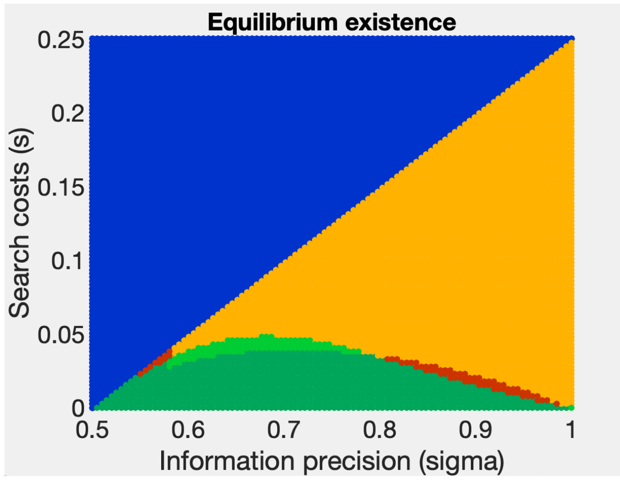

Figure 1: Equilibrium existence in the baseline model

The relationship between parameters and the existence of equilibria is based on the following ideas: The monopoly equilibrium exists if and only if no consumers search at the price tuple $(p^{L,M}, p^{H,M})$. This property holds true if and only if consumers with $\nu \leq 0.5$, who have the highest probability to generate the low signal, have no incentives to search at these prices. The incentives to search for these agents are falling in search costs and rising in information precision. Thus, this equilibrium exists when search costs are relatively high or information precision is relatively low.

The search equilibrium requires that some consumers with $\nu < 0.5$ would search when offered the price $p^{H,M} = 0.5$, but consumers with $\nu \geq 0.5$ would not search at this price. Such an outcome is possible because incentives to search at equilibrium prices are lower for consumers with $\nu \geq 0.5$ than for consumers with $\nu \in (s/\sigma + p^L, 0.5)$. In order for said outcome to be an equilibrium, search costs must be in an intermediate range. Then, search costs are low enough such that consumers with $\nu \in (s/\sigma + p^L, 0.5)$ would optimally search at the price $p^{H,M}$, but also high enough to ensure that consumers with $\nu \geq 0.5$ do not search on path. The possible range of search costs that can support this equilibrium shrinks as information precision falls. This is because the difference inbetween the search calculus of consumers with $\nu < 0.5$ and those with $\nu \geq 0.5$ becomes smaller as signal precision falls.

Now consider the search deterrence equilibrium. Recall that the equilibrium high price was based on making consumers with $\nu \in (s/\sigma + p^{L,D}, 0.5)$ exactly indifferent between searching and not searching. By contrast, existence of this equilibrium is determined by the search incentives of agents with $\nu > 0.5$. Existence of this equilibrium requires that consumers with

$\nu > 0.5$ can credibly promise to search when being offered the monopoly high signal price. Moreover, existence of the equilibrium requires that the cutoff price of these consumers is sufficiently far away from 0.5. This is necessary to ensure that a deviation from the high signal price to this cutoff price is not profitable.

The interval of search costs under which this equilibrium can be supported is greatest for intermediate levels of information precision. This is because the effect of information precision on search incentives is non-linear. Fixing equilibrium prices, higher levels of σ reduce the search incentives of agents with $\nu > 0.5$. This is because greater levels of information precision imply that these agents are more likely to generate the high signal and receive the high signal price. On the other hand, higher levels of σ reduce the equilibrium prices $p^{L,D}$ and $p^{H,D}$, which raises incentives to search.

Finally, it remains to explain the regions where both the search deterrence and the search equilibrium exist and the regions where no equilibrium exists. In the region where no equilibrium exists for low levels of information precision, search costs are too high to support the search deterrence equilibrium. Moreover, the search equilibrium cannot be supported since there exist profitable deviations from the high signal price. In the light green region in the middle of the graph, both the search deterrence and the search equilibrium exist. This follows from the fact that equilibrium prices are lower in the search deterrence equilibrium, which means that incentives to search are higher in this equilibrium. This implies that consumers with $\nu > 0.5$ can effectively constrain prices with the threat of searching in this equilibrium, but would not search at $p_j = 0.5$ when the search equilibrium is played.

Next to this, there exists a region of parameter combinations where no equilibrium exists. In this region, consumers with $\nu > 0.5$ would search at $p_j = 0.5$, no matter whether they would expect the prices $(p^{L,D}, p^{H,D})$ or the prices $(p^{L,S}, p^{H,S})$ at the firm they visit second. This means that the search equilibrium cannot exist. However, the cutoff price of these consumers, given the potential equilibrium price vector $(p^{L,D}, p^{H,D})$, is quite close to $p^{H,M} = 0.5$. Setting a price equal to the cutoff price of consumers with $\nu > 0.5$ when observing the high signal thus yields deviation profits that are approximately equal to monopoly high signal profits. Because these are higher than equilibrium high signal profits in the search deterrence equilibrium, said deviation is profitable.

4.4 Comparative statics

Now I investigate the comparative statics of prices and profits in any three equilibria. The above considerations already indicate that the effects of changes in the parameters are non-monotonous and fundamentally depend on the equilibrium that is being played.

Corollary 1 *Consider the impacts of a rise in s (search costs) for fixed σ (signal precision) and vice versa. If the search equilibrium $\left(p^{L,S}(\sigma,s),p^{H,S}(\sigma,s)\right)$ is played for the parameter vectors under consideration, then*

1. *The price $p^{H,S}(\sigma,s)$ is independent of s and σ, while $p^{L,S}(\sigma,s)$ is falling in s and in σ.*
2. *Firm profits are falling in the level of search costs.*

Standard intuition regarding the effect of search costs on prices suggests the following: An increase of search costs should reduce the number of firms an average consumer has in her choice set, thus reducing competition, and hence raising prices and firm profits. In the search equilibrium, both the sign of the comparative statics result and the working channel behind it are diametrically opposed to the standard intuition.

The low signal price will fall in response to an increase of search costs. Recall that any consumer that arrives at a firm after searching generates fully inelastic demand for this firm at prices $p_j \in (0, s/\sigma + p^{L,S})$. By contrast, the demand created by consumers that arrive at firm j first is sensitive to the price p_j offered by this firm. When search costs increase, less consumers search on the equilibrium path. Thus, the weight of consumers that arrive after searching and generate locally inelastic demand in the low signal profit function falls. As a consequence, the optimal low signal price falls.

The profits a firm makes from consumers that generate the low signal fall when search costs rise. Lower search costs imply that less agents arrive at a firm after searching, which reduces profits because these agents generate locally price-inelastic demand. The profits a firm makes from consumers that generate the high signal are unaffected by search costs.

Now consider the search deterrence equilibrium:

Corollary 2 *Consider the impacts of a rise in s (search costs) for fixed σ (signal precision) and vice versa. If the search deterrence equilibrium $\left(p^{L,D}(\sigma,s),p^{H,D}(\sigma,s)\right)$ is played for the parameter vectors under consideration, then*

1. *The price $p^{L,D}(\sigma,s)$ is independent of s and is falling in σ. The price $p^{H,D}(\sigma,s)$ is rising in search costs and is falling in σ.*

2. *Firm profits are rising in the level of search costs.*

Consider first the prices in the search deterrence equilibrium. The low signal equilibrium price is equal to the monopoly low signal price, which falls in σ by previous arguments and is independent of s. By contrast, the high price $p^{H,D}$ is rising in search costs. Recall that the high price is set in a way that makes consumers with $\nu \in (s/\sigma + p^{L,D}, 0.5)$ exactly indifferent between searching and not searching. As s rises, their gains of search at any given price offer fall. This implies that the price achieving indifference rises. This comparative statics result can also be explained by market forces. When s rises from s' to s'', competitive profits are now equal to monopoly profits in the interval $(s'/\sigma + p^{L,D}, s''/\sigma + p^{L,D})$. Because the old high signal price had to be strictly below 0.5 and monopoly profits are thus increasing at the initial high signal price, firms will raise their high signal price in response to the rise of s.

In the search deterrence equilibrium, lower prices are not generated by an expanded choice set of consumers. Instead, the mere threat of searching is sufficient to generate downward pressure on prices. A similar reasoning lies behind the effect of an increase in information precision σ. As σ rises, the incentives to search will increase for consumers with $\nu \in (s/\sigma + p^{L,D}, 0.5)$. To re-achieve indifference, the high signal price must fall. This new equilibrium is reached via the following transitional dynamics: An increase of σ implies that consumers with $\nu \in (s/\sigma + p^{L,D}, 0.5)$ will now search on-path at the old price tuple. This creates undercutting incentives for firms, who will successively reduce their high signal price until the new equilibrium is reached.

This concludes the study of the effects of parameter changes on equilibrium prices in a given equilibrium. Next, I study the how equilibrium prices respond to larger parameter changes that may switch the equilibrium that is being played. When parameter changes induce shifts in the nature of the equilibrium that is played, this has a considerable effect on prices. In the following, I fix two levels of σ at 0.65 and 0.75 and study the equilibrium prices under different search costs. Equilibrium prices are plotted on the y-axis while search costs are plotted on the x-axis. The vertical green lines and the green text specify which equilibrium is being played at a given (s, σ) combination.

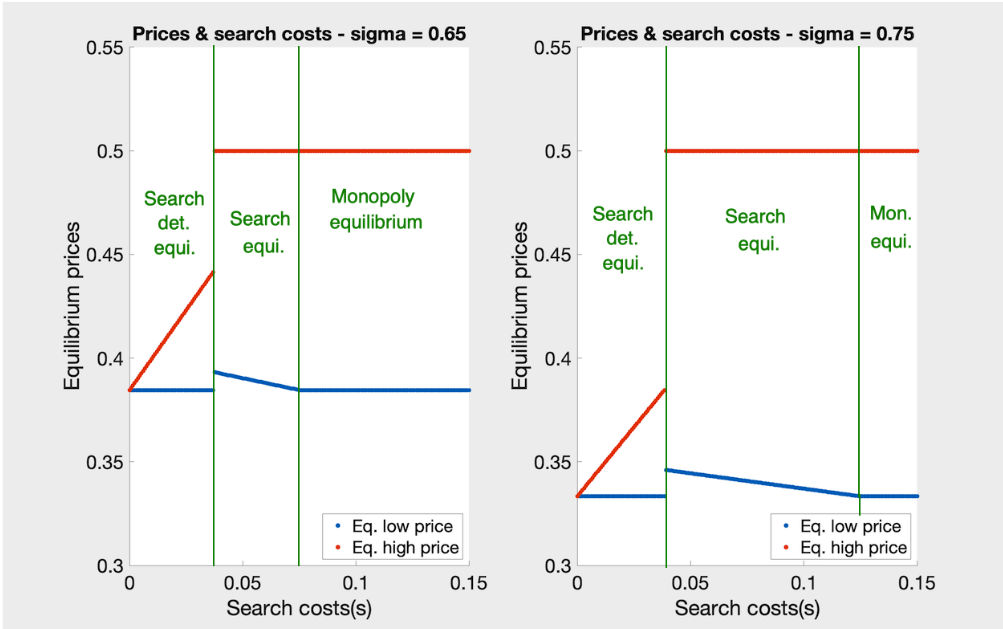

Figure 2: Search costs and prices in the baseline model

Recall the existence results laid out in the previous section. Fixing a given level of information precision (σ), the search deterrence equilibrium will be played for low search costs s. The search equilibrium and the monopoly equilibrium will be played for intermediate and high levels of search costs, respectively.

The effect of search costs on prices is non-monotonic. Within the search deterrence equilibrium, higher search costs raise the equilibrium high price. When search costs are such that a marginal rise of search costs ensures existence of the search equilibrium, this equilibrium will be played, which implies a stark upward jump of prices. As is standard in many models, an increase of search costs thus raises prices in this interval. However, the forces that drive this result in my model are fundamentally different to the logic underlying the standard explanation of the positive relationship between search costs and prices.

In my model, an increase of search costs goes along with a discontinuous *increase* of the extent of equilibrium search when the market shifts from the search deterrence equilibrium to the search equilibrium. Equilibrium search is induced because consumers with $\nu > 0.5$ loose their ability to credibly promise to search when being offered the monopoly high signal price in the search equilibrium. This leads firms to offer the monopoly high signal price to all consumers that generate the high signal, which induces low valuation consumers to search on-path. As discussed before, a further increase of search costs has a favorable effect on the lower equilibrium price within the search equilibrium.

I move on to study the effect of signal precision on prices. In the following, I fix two levels of s at 0.02 and 0.04 and study the equilibrium prices under different levels of signal precision. Importantly, the nature of the equilibria that exist is fundamentally different for the two levels of search costs. For low search costs ($s = 0.02$), all three equilibria exist and increases of σ carry the market from the monopoly equilibrium into the search deterrence equilibrium and finally into the search equilibrium. For higher search costs ($s = 0.04$), the search deterrence equilibrium is never the unique equilibrium. Equilibrium prices are plotted on the y-axis while signal precision is plotted on the x-axis. As before, the green lines and text indicate which equilibrium is being played at a given parameter combination.

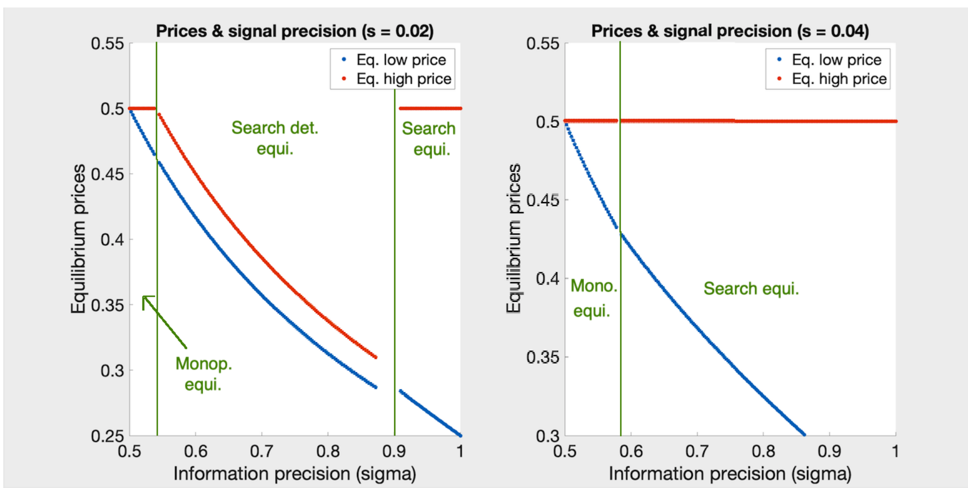

Figure 3: Signal precision and prices in the baseline model

In general, increases of information precision uniformly reduce the equilibrium low signal price. By contrast, the effect of information precision on the equilibrium high signal price is non-monotonic. When $s = 0.02$, an increase of information precision reduces both prices for a large interval of information precision because the search deterrence equilibrium is constantly played. However, when a threshold level of information precision is surpassed, the economy reverts back to the search equilibrium which is extremely unfavorable to consumers. These results add nuance to the intuition that more precise price targeting must be harmful to consumers.

5 Generalized signal distribution

5.1 Setup and initial remarks

This section is devoted to emphasizing that the key insights of the previous simplified framework also hold in more general contexts. The main points stressed by the previous model were the following: Firstly, the presence of an imperfect, but informative signal about consumer valuations is sufficient to sustain an equilibrium with search on the equilibrium path in certain parameter conditions. Secondly, the effect of search costs and information precision on prices is non-monotonic. Thirdly, the search equilibrium exists for intermediate levels of search costs and greater information precision shrinks the interval of search costs for which the search equilibrium exists. By contrast, the search deterrence equilibrium requires low search costs and the interval of search costs for which this equilibrium exists is largest at intermediate levels of information precision. Thus, there is a non-monotonic relationship between search costs and the degree of equilibrium search.

In the following, I show under which assumptions these results hold for generalized signal distributions. For now, I only assume that $Pr(\tilde{\nu}^H|\nu)$ is weakly increasing in ν.

5.2 Equilibrium analysis

Proposition 1 still holds in any generalized setting of the model framework I consider. In a pure-strategy equilibrium $p := (p^L, p^H)$ with $p^L < p^H$, all consumers that arrive at a firm after searching can not buy at the high price p^H.

Recall that the previous discussion of optimal search first pinned down a necessary condition for search in terms of a consumer's valuation and then moved on to define sufficient conditions. Mirroring these insights, I define the set $\hat{V}(p^L)$, which pins down what consumers *can* search on the equilibrium path. This set is defined as follows:

$$\hat{V}(p^L) = \left\{ \nu \in [0,1] : Pr(\tilde{\nu}^L|\nu) \max\{\nu - p^L, 0\} > s \right\} \tag{34}$$

A consumer can search on the equilibrium path if and only if her valuation ν is in the set $\hat{V}(p^L)$. Define further that $S(p^L, p^H)$ is the set of consumer valuations that search on the equilibrium path, with $\underline{\nu} := \inf S(p^L, p^H)$, and $\bar{\nu} = \sup S(p^L, p^H)$. The properties of these sets are pinned down by the following lemma:

Lemma 3 *Consider a pure-strategy equilibrium (p^L, p^H). The following must hold:*

- *$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s = 0$ must hold when $\nu = \inf \hat{V}(p^L)$, which implies that $\inf \hat{V}(p^L) > p^L$. Moreover, the function $Pr(\tilde{\nu}^L|\nu)(\nu - p^L)$ must be continuous at $\nu = \inf \hat{V}(p^L)$.*

- *The set of consumer valuations that search on the equilibrium path (ignoring measure zero sets) is $\hat{V}(p^L) \cap [p^L, p^H]$. Thus, $\underline{\nu} = \inf \hat{V}(p^L)$.*

The first result has mostly technical relevance. Intuitively, it holds because any jumps in $Pr(\tilde{\nu}^L|\nu)$ at points of discontinuities must always be downwards by the assumption that $Pr(\tilde{\nu}^H|\nu)$ is weakly increasing in ν. The second result is useful to pin down the measure of people that search on the equilibrium path. The following lemma establishes useful connections between the optimal search and consumption choices:

Lemma 4 *Consider a pure-strategy equilibrium (p^L, p^H). The following must hold:*

- *All consumers that arrive after search would buy at firm j when this firm offers them a price $p_j \leq \underline{\nu}$.*

- *No consumer would search after receiving a price $p_j \leq \underline{\nu}$.*

By lemma 3, all consumers that arrive after searching must have a valuation above $\underline{\nu}$. The fact that any consumer arrives after searching requires that the set $S(p^L, p^H)$ is not empty, which in turn requires that $\underline{\nu} < p^H$. Together, these notions imply the first result. To understand the second result, recall the definition of the price cutoff function $\hat{p}(\nu)$: A consumer with valuation ν will search if and only if she receives an initial price offer $p_j > \hat{p}(\nu)$. Note that any consumer with $\nu \in S(p^L, p^H)$ has a cutoff price $\hat{p}(\nu) > \nu$. Together with the fact that $\inf S(p^L, p^H) = \underline{\nu}$, this notion implies the second result.

These two lemmas imply that the structure of profits around any equilibrium low signal price mirrors the structure of this function in the baseline model, even for general settings. Consider an equilibrium with on-path search. The demand that is implied by searchers is also fully inelastic around the equilibrium low price in these generalized settings. In other words, there must exist an interval of prices $p_j \in [0, \underline{\nu}]$ that includes p^L in its interior where the profit functions satisfy the following stucture:

$$\Pi^C(p_j; \tilde{\nu}^k) = p_j \int_{p_j}^{1} f^1(\nu) Pr(\tilde{\nu}^k|\nu) d\nu + p_j M^k(p) \tag{35}$$

Note that $M^k(p)$ is defined as follows:

$$M^k(p) = \int_{\nu \in S(p^L, p^H)} Pr(\tilde{\nu}^H|\nu) Pr(\tilde{\nu}^k|\nu) f^2(\nu) d\nu \tag{36}$$

The term $M^L(p)$ thus represents the mass of consumers that arrive at a firm after searching and generate the low signal. Before moving forward, recall that the set of valuations that searched on the equilibrium path in the baseline model was $S(p^L, p^H) = [s/\sigma + p^L, 0.5]$ and $\inf \hat{V}(p^L) = s/\sigma + p^L$. Thus, the high price in the search deterrence equilibrium satisfied $p^H = \inf \hat{V}(p^L)$. The set of possible equilibria that can exist in generalized settings is also very similar to the set of equilibrium candidates in the baseline model under weak assumptions. This is formalized in the following result:

Proposition 5 *Consider the set of symmetric pure strategy equilibria. Suppose that the probability function $Pr(\tilde{\nu}^H|\nu)$ is continuous on $\nu \in [0,1]$. The lower equilibrium price must be characterized by the following expression:*

$$\left.\frac{\partial \Pi^M(p_j; \tilde{\nu}^L)}{\partial p_j}\right|_{p_j = p^L} + M^L(p) = 0 \tag{37}$$

The equilibrium high price must satisfy one of the following expressions:

$$\left.\frac{\partial \Pi^M(p_j; \tilde{\nu}^H)}{\partial p_j}\right|_{p_j = p^H} = 0 \tag{38}$$

$$Pr(\tilde{\nu}^L|p^H)(p^H - p^L) - s = 0 \tag{39}$$

If the function $Pr(\tilde{\nu}^H|\nu)$ is once continuously differentiable, there is a unique solution to equation (38). Suppose further that:

(i) $g(\nu) := Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s$ is a quasiconcave function in the interval $[0, p^{H,M}]$ for any $p^L \in [0, p^{H,M}]$.

(ii) $Pr(\tilde{\nu}^H|0.5) = 0.5$ holds true.

(iii) In any equilibrium, $g(\nu) = 0 \implies g'(\nu) \neq 0$.

Then, there are at most two potential equilibrium high signal prices that solve equation (39) but not equation (38), namely $p^H = \inf \hat{V}(p^L)$ and $p^H = \sup \hat{V}(p^L) \cap [0, p^{H,M})$.

This proposition characterizes the set of potential symmetric pure-strategy equilibria. The equilibrium low price must satisfy expression (37) because the profit function in the interval of prices $p_j \in [0, \underline{\nu}]$, which includes p^L, is differentiable and satisfies the structure laid out in equation (35). If the first order condition given in (37) is violated, there would surely exist a profitable deviation to another price within the interval $[0, \underline{\nu}]$.

Given that competitive profits are equal to monopoly profits at p^H, a natural candidate for the equilibrium high signal price is the monopoly high signal price. Now consider an equilibrium candidate that does not satisfy this first-order condition. Suppose that $Pr(\tilde{\nu}^L|p^H)(p^H - p^L) - s > 0$. By continuity of $Pr(\tilde{\nu}^L|\nu)$, there would exist an open interval of valuations above p^H for which a consumer would search at p^H, which yields a contradiction. Suppose alternatively that $Pr(\tilde{\nu}^L|p^H)(p^H - p^L) - s < 0$. Then, competitive profits are equal to monopoly profits in an open ball around p^H. If the price p^H does not satisfy the first-order condition, it cannot be an equilibrium. Hence, the only possible candidate high signal price that does not satisfy the monopoly FOC must satisfy equation (39).

Assume that the regularity conditions laid out in proposition 5 hold. Then, the following equilibria exist:

- The search equilibrium, where p^L satisfies the FOC laid out in equation 37 and $p^H = arg\max \Pi^M(p_j; \tilde{\nu}^H)$.
- The monopoly equilibrium, where $p^L = arg\max \Pi^M(p_j; \tilde{\nu}^L)$ and $p^H = arg\max \Pi^M(p_j; \tilde{\nu}^H)$.
- The search deterrence equilibrium, where $p^L = arg\max \Pi^M(p_j; \tilde{\nu}^L)$ and $p^H = \inf \hat{V}(p^L)$.
- An equilibrium where p^L satisfies the FOC laid out in equation 37 and $p^H = \sup \hat{V}(p^L) \cap [0, p^{H,M})$

The only new equilibrium candidate is the fourth one, which I will refer to as the *constrained search equilibrium*. The numerical simulations that follow highlight that this equilibrium candidate is extremely unlikely to actually exist, because two conflicting forces are necessary to support this as a perfect Bayesian equilibrium. First, no consumer with a valuation above p^H can search on the equilibrium path. However, any equilibrium must satisfy $p^H < p^{H,M}$ under plausible assumptions on the signal distribution such as $Pr(\tilde{\nu}^H|0.5) = 0.5$. To avoid the existence of a profitable upward deviation towards $p^{H,M}$, enough agents with valuation in the interval $\nu \in (p^H, p^{H,M})$ need to search for prices slightly above p^H. However, these consumers cannot search at price p^H. Given the continuity of the signal probability distribution, these forces fundamentally conflict.

5.3 Numerical results

In the following numerical simulations, I will consider signal distributions that take the following form:

$$Pr(\tilde{\nu}^L|\nu) = \sigma\left(1 - \frac{1}{1 + e^{k(0.5-\nu)}}\right) + (1-\sigma)\left(\frac{1}{1 + e^{k(0.5-\nu)}}\right) \tag{40}$$

For $k \to \infty$, this distribution is for all intents and purposes equal to the one used in the baseline setting. Lower values of k amount to making the signal distribution more linear, while the parameter σ governs the upper and lower bounds of the probability distribution. In the following, I plot this distribution for different values of σ and k.

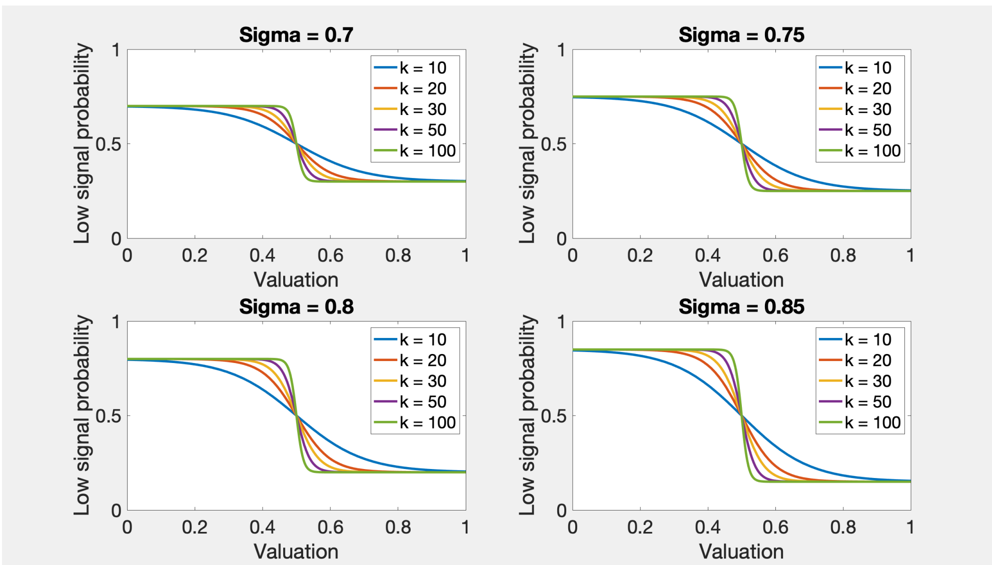

Figure 4: General signal distributions

The precision of a binary signal can be seen as the probability that the signal correctly identifies whether a consumer is in the upper or the lower part of the valuation interval. Under this interpretation, signal precision is rising both in k and σ.

In the following, I study different parameter combinations and show when the aforementioned equilibria exist. As of now, I have not found any parameters for which the constrained search equilibrium actually exists. A given graph always corresponds to a fixed level of σ. I consider two values of σ, namely $\sigma \in \{0.75, 0.85\}$, in the following analysis. On the x-axis, different levels of k are being plotted. On the y-axis, different levels of s are plotted. As before, the different colors indicate existence of a given equilibrium. The search deterrence equilibrium exists at green points, the search equilibrium exists at yellow points, and the monopoly equilibrium exists at blue points. Red points indicate that none of these equilibria exist and a yellow dot with a green border indicates that both the search and the search deterrence equilibrium exist.

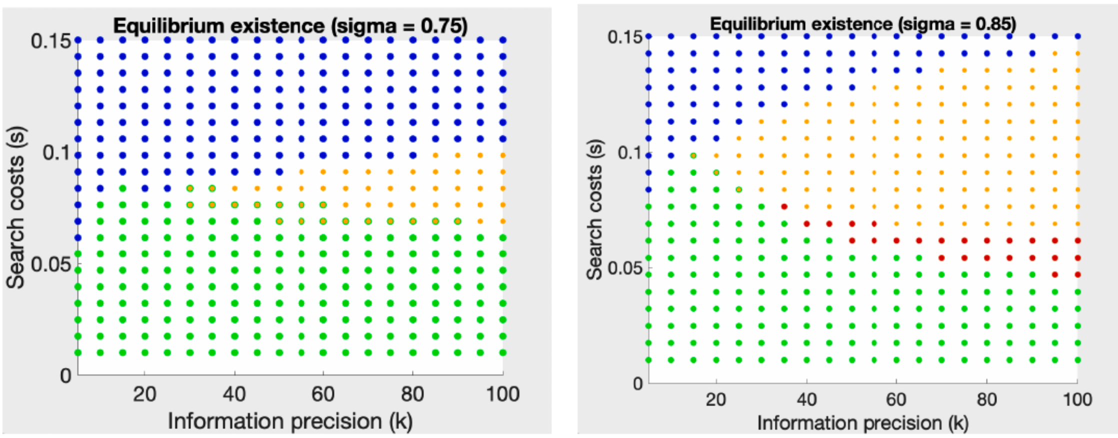

Figure 5: Existence for general signal distributions

The general trends outlined previously are being confirmed. The search deterrence equilibrium exists for low search costs, the search equilibrium exists for intermediate search costs, and the monopoly equilibrium exists for high search costs. Both the search equilibrium and the search deterrence equilibrium exist for substantial parameter ranges even when k is in the region $[25, 40]$, at which the signal distribution is relatively linear.

High levels of information precision facilitate existence of the search equilibrium. Shifting σ from 0.85 to 0.75 and decreasing k shrinks the interval of search costs for which the search equilibrium exists. There is a potential issue of equilibrium non-existence when search costs are too high to support the search deterrence equilibrium but not high enough to support the search equilibrium. This issue is particularly relevant at high values of k and disappears as the signal probability distribution becomes more linear in the valuation.

Finally, I plot the comparative statics effects of an increase of search costs, fixing levels of k and σ. I consider $\sigma = \{0.75, 0.85\}$ and $k \in \{30, 50\}$. A given figure corresponds to a given level of σ, and a given plot within a figure corresponds to a fixed level of k. On the x-axis, different search costs are plotted, while equilibrium prices are plotted on the y-axis. Consider first $\sigma = 0.75$.

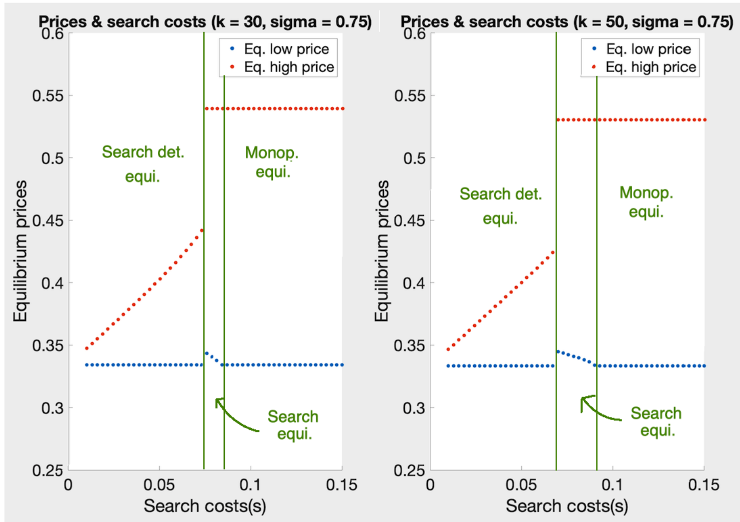

Figure 6: Prices and search costs in general settings - I

Now consider $\sigma = 0.85$:

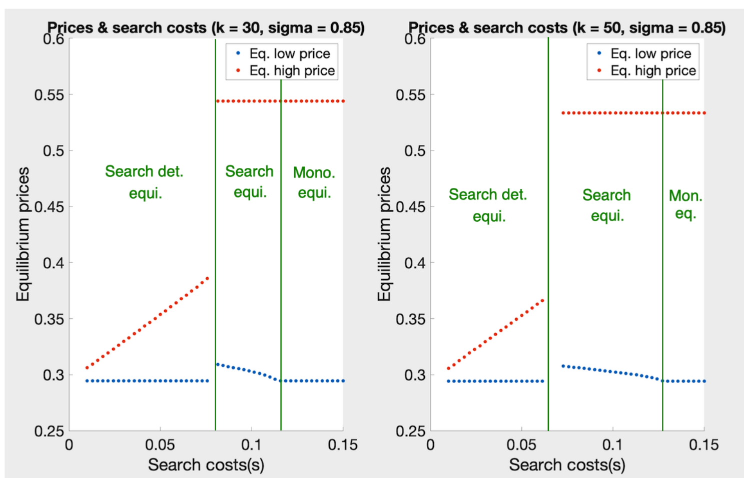

Figure 7: Prices and search costs in general settings - II

The comparative statics effects of an increase in search costs mirror those in the baseline setting.

6 Conclusion

I have studied price discrimination based on imperfect information in markets where consumers engage in sequential search to obtain price offers. Whenever a consumer visits a firm, this firm receives a binary and informative signal about the consumer's valuation. A firm observes nothing else for any consumer and offers a consumer a price purely based on the signal the consumer has generated. I have first considered a simplified version of this framework where the distribution of the signal is a step-function. I have highlighted that different parameter combinations give rise to fundamentally different equilibria. This implies that the role of search costs and information precision in such markets is highly nuanced.

Importantly, my results offer a strong rebuttal of the idea that observing a high level of equilibrium search is equivalent to high levels of competitive pressure. It crucially matters what kinds of consumers choose to search. When only a select group of consumers search, the presence of equilibrium search may simply offer firms an effective screening tool which generates higher equilibrium prices and firm profits. In the unique equilibrium with search on the equilibrium path, profits and prices are higher than in any other equilibrium, including the monopoly equilibrium that is attained when search is prohibitively costly.

The effect of search costs on equilibrium prices is non-monotonic. When search costs are small, increases of search costs lead to increased prices, akin to standard results. When search costs cross a certain threshold, the equilibrium that is being played switches. This leads to an upward jump in prices. Afterwards, a further increase of search costs reduces prices. The effect of information precision is similarly nuanced. On the one hand, high levels of information precision facilitate existence of the search equilibrium in which prices are highest. However, increased levels of information precision reduce both prices when search costs are low and reduce the lower equilibrium price in any equilibrium.

A Proofs:

A.1 Proof of proposition 1

The proof of this result mirrors the discussion in the text.

A.2 Proof of lemma 1

Suppose that $s \to \infty$, which de facto makes both firms into monopolists. Since the first search is random, the profit functions are:

$$\Pi^M(p_j; \tilde{\nu}^k) = p_j \int_{p_j}^{1} f(\nu_i = \nu, r^*_{i,j} = 1, \tilde{\nu}_{i,j} = \tilde{\nu}^k) d\nu = 0.5 p_j \int_{p_j}^{1} f(\nu_i = \nu, \tilde{\nu}_{i,j} = \tilde{\nu}^k) d\nu$$

. Note that:

$$f(\nu_i = x, \tilde{\nu}_{i,j} = \tilde{\nu}^L) = \begin{cases} (1-\sigma) f_\nu(x) & \text{, if } x \geq 0.5 \\ (\sigma) f_\nu(x) & \text{, if } x < 0.5 \end{cases}$$

$$f(\nu_i = x, \tilde{\nu}_{i,j} = \tilde{\nu}^H) = \begin{cases} (\sigma) f_\nu(x) & \text{, if } x \geq 0.5 \\ (1-\sigma) f_\nu(x) & \text{, if } x < 0.5 \end{cases}$$

<u>Evaluation of $\Pi^L(p_j)$:</u>

Plugging all this back into the function $\Pi^L(p_j)$ yields the following with $p_j < 0.5$:

$$\Pi^L(p_j) = 0.5 p_j \left[\sigma \int_{p_j}^{0.5} f_\nu(x) dx + (1-\sigma) \int_{0.5}^{1} f_\nu(x) dx \right] = 0.5 p_j \left[0.5 - \sigma p_j \right]$$

The derivative of this w.r.t p_j, when $p_j < 0.5$, is:

$$\left. \frac{\partial \Pi^L(p_j)}{\partial p_j} \right|_{p_j < 0.5} = 0.5[0.5 - \sigma p_j] + 0.5 p_j[-\sigma] = 0.25 - 0.5\sigma p_j - 0.5\sigma p_j = 0.25 - \sigma p_j$$

Let's evaluate the sign of this derivative:

$$\left. \frac{\partial \Pi^L(p_j)}{\partial p_j} \right|_{p_j < 0.5} < 0 \iff 1 - 4\sigma p_j < 0 \iff p_j > \frac{1}{4\sigma}$$

$$\left. \frac{\partial \Pi^L(p_j)}{\partial p_j} \right|_{p_j < 0.5} = 0 \iff 1 - 4\sigma p_j = 0 \iff p_j = \frac{1}{4\sigma}$$

This is the optimum of this function in the interval $[0, 0.5)$ when $\sigma > 0.5$. Now, we need to examine this function at $p_j \geq 0.5$:

$$\Pi^L(p_j) = 0.5p_j \left[(1-\sigma) \int_{p_j}^{1} f_\nu(x)dx \right] = 0.5(1-\sigma)p_j[1-p_j]$$

The derivative of this w.r.t p_j at prices weakly above 0.5 is:

$$\left. \frac{\partial \Pi^L(p_j)}{\partial p_j} \right|_{p_j \geq 0.5} = 0.5(1-\sigma)(1-2p_j) < 0 \iff p_j > 0.5$$

Hence, this function is a parabola that has a maximum at $p^* = 1/4\sigma$ and is decreasing in all directions as you move away from there.

<u>Evaluation of $\Pi^H(p_j)$:</u>

Now, let's repeat this for the other function:

$$\Pi^H(p_j) = p_j \int_{p_j}^{0.5} 0.5(1-\sigma) f_\nu(x)dx + p_j \int_{0.5}^{1} 0.5(\sigma) f_\nu(x)dx$$

If $p_j < 0.5$, the value of this function is:

$$\Pi^H(p_j) = 0.5p_j(1-\sigma)[0.5-p_j] + 0.5p_j(\sigma)[0.5]$$

Let's take the derivative of this w.r.t p_j:

$$\left. \frac{\partial \Pi(p_j)}{\partial p_j} \right|_{p_j < 0.5} = 0.5(1-\sigma)[0.5-2p_j] + 0.5(\sigma)[0.5] = 0.25(1-\sigma) - (1-\sigma)p_j + 0.25\sigma =$$

$$0.25 - (1-\sigma)p_j > 0 \iff p_j < \frac{0.25}{1-\sigma}$$

Note that:

$$1-\sigma \leq 0.5 \iff 2 \leq \frac{1}{1-\sigma} \iff \frac{0.25}{1-\sigma} \geq 2(0.25) = 0.5$$

This implies that this function is rising as long as $p_j < 0.5$ if $\sigma > 0.5$, so there cannot be a maximum here. Thus, I consider $\Pi^H(p_j)$ on the interval where $p_j \geq 0.5$:

$$\Pi^H(p_j) = 0.5(\sigma)[p_j - p_j^2]$$

The derivative of this w.r.t. p_j is:

$$0.5(\sigma)[1 - 2p_j] < 0 \iff p_j > 0.5$$

This means that the maximum of this function is at $p_j = 0.5$.

A.3 Proof of proposition 2

Possible equilibrium category 1: $p = p^L = p^H$.

In such an equilibrium, there is no search on path because there are zero incentives to search but positive costs of doing so. Thus, no consumer would arrive at any firm j after search.

Since search costs are strictly positive, there exists an interval of prices $p_j \in [p, p + \epsilon]$ where consumers would not search. Thus, no equilibrium with $p < 0.5$ can be supported, since an upward deviation would be optimal when observing the high signal.

Consider instead a price $p \geq 0.5$. Since $p^{L,M} < 0.5$ holds true, there would be a profitable downward deviation. No consumer would move on to search.

Possible equilibrium category 2: $p^H < p^L$.

A consumer with $\nu > p^H$ will search after an initial price $p_j \in [p^H, p^L]$ if and only if:

$$Pr(\tilde{\nu}^H|\nu)(\nu - p^H) + Pr(\tilde{\nu}^L|\nu)\max\{\nu - p_j, 0\} - s > \max\{\nu - p_j, 0\}$$

Moreover, search after an initial price $p_j > p^L$ is optimal if and only if:

$$Pr(\tilde{\nu}^H|\nu)(\nu - p^H) + Pr(\tilde{\nu}^L|\nu)\max\{\nu - p^L, 0\} - s > \max\{\nu - p_j, 0\}$$

Note that $Pr(\tilde{\nu}^H|\nu) = 1 - \sigma$ and later $Pr(\tilde{\nu}^H|\nu) = \sigma$, i.e. for $\nu > 0.5$.

(i) $p^L > 0.5$

Consider the search calculus of an agent with $\nu \in [0.5, p^L]$ that receives a price $p_j \leq p^L$.

$$\sigma(\nu - p^H) + (1 - \sigma)\max\{\nu - p_j, 0\} - s > \max\{\nu - p_j, 0\} \iff \sigma(\nu - p^H) - s > \sigma \max\{\nu - p_j, 0\}$$

An agent with $\nu > p^L$ that receives a price p^L will search iff:

$$\sigma(\nu - p^H) + (1-\sigma)(\nu - p^L) - s > (\nu - p^L) \iff \sigma(\nu - p^H) - s > \sigma(\nu - p^L)$$

$$\iff$$

$$p^L > p^H + \frac{s}{\sigma} \iff \sigma(p^L - p^H) > s$$

Suppose:

$$\sigma(p^L - p^H) - s > 0$$

This yields a direct contradiction since agents with $\nu > p^L$ will search, which breaks the equilibrium by proposition 1. Suppose alternatively that:

$$\sigma(p^L - p^H) \leq s$$

Thus, for all agents with $\nu \in [0.5, p^L]$, we have that:

$$\sigma(\nu - p^H) < \sigma(p^L - p^H) \leq s$$

This means that all these consumers cannot search at prices $p_j \in [0.5, p^L]$. Moreover, consumers with $\nu > p^L$ would also not search.

In this price interval, profits would thus equal monopoly profits. No other consumer would search at equilibrium prices, since:

$$(1-\sigma)(p^L - p^H) < \sigma(p^L - p^H)$$

Thus, there is a profitable downward deviation.

(ii) $p^H < p^L \leq 0.5$

Assume that the following condition holds:

$$(1-\sigma)(p^L - p^H) - s > 0 \implies \sigma(p^L - p^H) - s > 0$$

Consider agents with $\nu > 0.5 > p^L$, who will search at price $p_j = p^L$ if and only if:

$$(\sigma)(\nu - p^H) + (1-\sigma)(\nu - p^L) - s > (\nu - p^L) \iff \sigma(p^L - p^H) > s$$

Thus, these consumers will search at $p_j = p^L$, which breaks the equilibrium. Thus, it must hold that:

$$(1-\sigma)(p^L - p^H) - s \leq 0 \implies p^L \leq p^H + \frac{s}{1-\sigma}$$

Now consider agents with $\nu \in [p^H, 0.5]$, who search for prices $p_j \leq p^L$ if and only if:

$$(1-\sigma)(\nu - p^H) - s > (1-\sigma)\max\{\nu - p_j, 0\}$$

Agents with $\nu \in [p^H, p^L]$ have a negative LHS, i.e. won't search. For the rest, the cutoff price is interior and above p^L, (this means search won't occur at this price), i.e.:

$$\hat{p}(\nu) = p^H + \frac{s}{1-\sigma} \geq p^L$$

Moreover, no consumer with $\nu > 0.5$ can search when being offered a price $p_j = p^L$. Thus, no consumers search when being offered the price p^L.

A deviation to this price is profitable after the high signal since $p^L \leq 0.5$ and monopoly profits after the high signal are rising in this price interval.

Possible equilibria category 3: $p^L < p^H$

The fact that $\sigma > 0.5$ implies that:

$$\frac{s}{\sigma} + p^L < \frac{s}{1-\sigma} + p^L$$

Case 1: Suppose that $s/\sigma + p^L < 0.5$.

(i) $p^H < s/\sigma + p^L$.

Let's define optimal consumer search behaviour:

Consider consumers with $\nu \in [p^H, 0.5]$:

At prices $p_j \geq p^H$, search occurs iff:

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) + Pr(\tilde{\nu}^H|\nu)\max\{\nu - p^H, \nu - p_j, 0\} - s > \max\{\nu - p_j, 0\}$$

$$\Longleftrightarrow$$

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) + Pr(\tilde{\nu}^H|\nu)(\nu - p^H) - s > \max\{\nu - p_j, 0\}$$

$$\Longleftrightarrow$$

$$\nu - \sigma p^L - (1-\sigma)p^H - s > \max\{\nu - p_j, 0\}$$

Solving for the cutoff price in this interval of valuations yields that:

$$\hat{p}(\nu) = \sigma p^L + (1-\sigma)p^H + s$$

Let's investigate whether this is above p^H:

$$\sigma p^L + (1-\sigma)p^H + s > p^H \iff \sigma(p^L - p^H) + s > 0 \iff p^L + \frac{s}{\sigma} > p^H$$

In the price interval $p_j \in [p^H, \sigma p^L + (1-\sigma)p^H + s]$, no consumer would search. For consumers with $\nu > 0.5 > p^H$, the price cutoff will also surely be above p^H. Why? Consider the search calculus of these agents for prices $p_j \in [p^L, p^H]$:

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) + Pr(\tilde{\nu}^H|\nu)\max\{\nu - p_j, 0\} - s > \max\{\nu - p_j, 0\} \iff$$

$$(1-\sigma)(\nu - p^L) - s > (1-\sigma)(\nu - p_j)$$

The potential cutoff price would be:

$$\hat{p}(\nu) = p^L + \frac{s}{1-\sigma} > p^L + \frac{s}{\sigma} > p^H$$

Thus, their cutoff price would be equal to:

$$\hat{p}(\nu) = (1-\sigma)p^L + \sigma p^H + s > (\sigma)p^L + (1-\sigma)p^H + s > p^H$$

Thus, profits are equal to monopoly profits in this price interval interval.

Because $p^H < s/\sigma + p^L < 0.5$, monopoly profits are increasing in this price interval. Thus, an upward deviation is profitable.

(ii) $p^H = s/\sigma + p^L$: Could potentially be an equilibrium, deviations will be checked later.

(iii) $p^H \in (s/\sigma + p^L, 0.5)$.

Consider a consumer with $\nu \in (p^H, 0.5)$. Such a consumer would search at price p^H if and only if:

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) + Pr(\tilde{\nu}^H|\nu)(\nu - p^H) - s > (\nu - p^H)$$

$$\iff$$

$$\sigma(-p^L) - s > \sigma(-p^H) \iff p^H > p^L + \frac{s}{\sigma}$$

This holds true for all such consumers. Thus, all these consumers would search at p^H, which breaks the equilibrium.

(iv) $p^H > 0.5$:

Consider a price $p^H > 0.5$. Note that all consumers with $\nu < 0.5$ would have a cutoff price equal to:

$$\hat{p}(\nu) = p^L + \frac{s}{\sigma} < 0.5 < p^H$$

Note that all consumers with $\nu \in (0.5, p^H)$ will have the following price cutoff, supposing that this price cutoff is weakly below p^H.

$$\hat{p}(\nu) = p^L + \frac{s}{1-\sigma}$$

Suppose that $p^H \leq p^L + \frac{s}{1-\sigma}$. Then, the price cutoff of consumers with $\nu > 0.5$ is:

$$\hat{p}(\nu) = (1-\sigma)p^H + \sigma p^L + s \geq p^H$$

Then, a downward deviation would be immidiately optimal since consumers with $\nu \in (0.5, 1)$ would not move on to search for prices below p^H.

Thus, suppose that $p^H > p^L + \frac{s}{1-\sigma}$ and now consider consumers with $\nu \in (p^H, 1)$. These agents will find it strictly optimal to search when receiving the price p^H if and only if:

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) + Pr(\tilde{\nu}^H|\nu)\max\{\nu - p^H, 0\} - s > \max\{\nu - p^H, 0\} \iff$$

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s > Pr(\tilde{\nu}^L|\nu)(\nu - p^H) \iff p^H > p^L + \frac{s}{1-\sigma}$$

This implies that these agents will search on the equilibrium path, which breaks the equilibrium.

Case 2: Suppose that $s/\sigma + p^L > 0.5$. This implies:

$$\sigma(0.5 - p^L) < s \implies (1-\sigma)(0.5 - p^L) < s$$

(i) $p^H < 0.5$.

Note firstly that this implies that $p^H < s/\sigma + p^L$

Consider any consumer with $\nu \in (p^H, 0.5)$. If the cutoff price of these consumers is below p^H, it equals:

$$\frac{s}{\sigma} + p^L$$

We know this cannot be true by assumption. The resulting cutoff price for these consumers would be:

$$\hat{p}(\nu) = \sigma p^L + (1-\sigma)p^H + s > p^H \iff \sigma p^L + s > \sigma p^H \iff p^L + \frac{s}{\sigma} > p^H$$

This holds true by assumption. Moreover, the cutoff price for $\nu > 0.5$, which has to be above p^H as well, will be even higher. Thus, you would have an optimal marginal upward deviation since profits are monopoly profits up until this interval.

(ii) $p^H \in (0.5, 1]$

For these prices, only the search behaviour of agents with $\nu > 0.5$ is relevant, since all other agents cannot buy.

Suppose:

$$0.5 < p^H \leq p^L + \frac{s}{1-\sigma}$$

Now consider consumers with $\nu \in [0.5, 1]$. For prices $p_j = 0.5$ which is a better deal than p^H, search occurs iff:

$$(1-\sigma)(\nu - p^L) - s > (1-\sigma)(\nu - 0.5) \iff 0.5 > p^L + \frac{s}{1-\sigma}$$

Thus, none of these consumers will search when being offered $p_j = 0.5$. A downward deviation to $p_j = 0.5$ is profitable.

Suppose instead that:

$$p^H > p^L + \frac{s}{1-\sigma}$$

Consider consumers with $\nu > p^H$. Suppose $p_j = p^H$. These consumers will search because:

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s > Pr(\tilde{\nu}^L|\nu)(\nu - p^H) \iff p^H > p^L + \frac{s}{1-\sigma}$$

This breaks the equilibrium.

<u>Possible equilibria</u>

Sofar, we have obtained that an equilibrium must either satisfy $p^H = p^{H,M}$ or $p^H = p^L + s/\sigma$. In the second equilibrium, there will be no on-path search.

Why? Suppose that $p^L = 1/4\sigma$ and $p^H = s/\sigma + p^L$.

It must hold that

$$p^L + \frac{s}{\sigma} < 0.5$$

Since $p^H = p^L + \frac{s}{\sigma}$, consumers with $\nu \leq 0.5$ will not search at $p^H = s/\sigma + p^L$. Instead, consumers with $\nu \in (s/\sigma + p^L, 0.5)$ will search for prices above p^H.

Note that consumers with $\nu > 0.5$ will not search at p^H by the following logic:

$$(1-\sigma)(\nu - p^L) - s < (1-\sigma)(\nu - p^H) \iff p^H < \frac{s}{1-\sigma} + p^L$$

If there is no on-path search, the only optimal solution for the low price is $p^L = p^{L,M}$. Setting $p^L > p^{L,M}$ is not optimal, since a downward deviation to $p^{L,M}$ is optimal.

Setting $p^L < p^{L,M}$ is also not optimal since sequentially rational search implies that marginally raising the price above p^L will not trigger search. Strict concavity of the monopoly profit functions then implies the result.

Thus, there are two equilibria without search on path.

An equilibrium with search must thus satisfy $p^H = p^{H,M}$ and $p^L < p^H$ by initial arguments.

A.4 Proof of proposition 3

(i) Monopoly equilibrium

In the monopoly equilibrium, $p^L = 1/4\sigma$ and $p^H = 0.5$. At p^H, no agent may search, i.e. all agents must have a cutoff price above 0.5. This requires:

$$0.5 \leq \frac{s}{\sigma} + \frac{1}{4\sigma} < \frac{s}{1-\sigma} + \frac{1}{4\sigma}$$

Consider any consumer with $\nu \leq 0.5$. This consumer will search at $p^H = 0.5$ if and only if:

$$\sigma(\nu - p^L) - s > \sigma \max\{\nu - 0.5, 0\}$$

Search requires that the LHS is positive, i.e.

$$\nu > \frac{s}{\sigma} + p^L$$

This cannot be true. Note that if the above condition is violated, you will be able to find agents that will search, which makes p^L not optimal. Now examine consumers with $\nu > 0.5$. These agents will not search at p^H iff:

$$(1-\sigma)(\nu - p^L) - s \leq (1-\sigma)\{\nu - 0.5, 0\} \iff 0.5 \leq \frac{s}{1-\sigma} + p^L$$

This holds by assumption. There are no deviations in terms of prices, as competitive profits are below monopoly profits everywhere.

(ii) Search deterrence equilibrium:

In this equilibrium, $p^L = 1/4\sigma$ and $p^H = s/\sigma + p^L$.

We have already proven that there will not be search on-path. Now let's check possible deviations. There are no deviations from the low price. There is no search on-path, which means that monopoly profits are an upper envelope for competitive profits.

Consider deviations from p^H. There will not be any profitable deviations to $p_j < p^H$, since profits are equal to monopoly profits here. Monopoly profits are rising in this price interval since $p^H = s/\sigma + p^L < 0.5$ holds true.

First consider a deviation to $p_j = 0.5$. To ensure that this is not profitable requires that all consumers with $\nu \geq 0.5$ will move on to search at this price. Suppose that they do not search - then, profits at $p_j = 0.5$ will equal monopoly profits. Since equilibrium prices are also monopoly profits, the deviation is profitable. Thus, all consumers with $\nu \geq 0.5$ must search at $p_j = 0.5$.

Thus, the following must hold for all $\nu > 0.5$:

$$(1-\sigma)(\nu - p^L) + \sigma(\nu - p^H) - s > (\nu - 0.5) \iff 0.5 > (1-\sigma)p^L + \sigma p^H + s$$

Note that this condition is independent of ν, so long as $\nu > 0.5$.

Moreover, the above condition implies that the initial condition that $p^H < 0.5$ holds true.

We can repeat this argument for generic prices $p_j > p^H$ to pin down the cutoff price of these consumers as:

$$\hat{p}(\nu) = (1-\sigma)p^L + \sigma p^H + s \in (p^H, 0.5)$$

By a similar logic, no deviations to prices $p_j \in ((1-\sigma)p^L + \sigma p^H + s, 1)$ will be profitable, since profits will be zero as all consumers move on to search and never return.

Now consider price deviations in the interval $p_j \in (p^H, (1-\sigma)p^L + \sigma p^H + s)$. This interval is non-degenerate.

In this interval of prices, all consumers with $\nu \leq 0.5$ will search and will not return. All consumers with $\nu \geq 0.5$ do not search at these prices and buy directly. Thus, the best deviation price is $p_j = (1-\sigma)p^L + \sigma p^H + s$, since profits are rising at these prices.

When setting this price, the firm will only make the sale to consumers with $\nu \geq 0.5$ - but the sale will be made to all these consumers. Thus, profits from the deviation are:

$$\Pi^C((1-\sigma)p^L + \sigma p^H + s; \tilde{\nu}^H) =$$

$$\big((1-\sigma)p^L + \sigma p^H + s\big) \int_{0.5}^{1} Pr(\tilde{\nu}^H|\nu)(1)(0.5)d\nu = 0.25\sigma\big((1-\sigma)p^L + \sigma p^H + s\big)$$

By contrast, equilibrium profits are:

$$\Pi^{C,H}(p_j = s/\sigma + p^L) = \left(\frac{s}{\sigma} + p^L\right)\left[\int_{\frac{s}{\sigma}+p^L}^{0.5} Pr(\tilde{\nu}^H|\nu)(1)(0.5)d\nu + \int_{0.5}^{1} Pr(\tilde{\nu}^H|\nu)(1)(0.5)d\nu\right] =$$

$$\left(\frac{s}{\sigma} + p^L\right)\left[0.5(1-\sigma)\left(0.5 - \frac{s}{\sigma} - p^L\right) + 0.25\sigma\right]$$

Thus, a necessary condition for equilibrium existence (which is then also sufficient given that said cutoff price is below 0.5) is:

$$\left(\frac{s}{\sigma} + p^L\right)\left[0.5(1-\sigma)\left(0.5 - \frac{s}{\sigma} - p^L\right) + 0.25\sigma\right] \geq 0.25\sigma\big((1-\sigma)p^L + \sigma p^H + s\big)$$

A.5 Proof of lemma 2

We are studying an equilibrium where $p^L < p^H = 0.5$.

Ordering:

(i) Suppose:

$$p^H = 0.5 \leq \frac{s}{\sigma} + p^L < \frac{s}{1-\sigma} + p^L$$

Then, no consumer will search on-path. Consider consumers with $\nu \leq 0.5$, who search at $p_j = p^H$ iff:

$$\sigma(\nu - p^L) + (1-\sigma)(0) - s > (0) \iff \nu > \frac{s}{\sigma} + p^L$$

Such consumers don't exist under the above assumption.

Consider consumers with $\nu > 0.5$, who search at $p_j = p^H$ iff:

$$(1-\sigma)(\nu - p^L) + (\sigma)(\nu - 0.5) - s > (\nu - 0.5) \iff 0.5 > \frac{s}{1-\sigma} + p^L$$

(ii) Suppose instead that:

$$0.5 > \frac{s}{1-\sigma} + p^L$$

Then, consumers with $\nu > 0.5$ will search on path by the above logic, which breaks the equilibrium. This pins down the ordering such a PSE needs to satisfy.

Calculating the search process

I will pin down the optimal search process for the relevant intervals of valuations seperately.

(i) $\nu \leq p^L$

There is no price after which such a consumer would search. This implies that $\hat{p}(\nu) = \infty$ for these consumers.

(ii): $\nu \in (p^L, 0.5)$:

Search is strictly optimal for prices $p_j \in [p^L, p^H]$ if and only if:

$$\mathbb{E}[u^{gs}(p_j; \nu \in (p^L, 0.5))] = \sigma[\nu - p^L] + (1-\sigma)\max\{\nu - p_j, 0\} - s > \max\{\nu - p_j, 0\}$$

$$\iff$$

$$\sigma[\nu - p^L] - s > \underbrace{\sigma \max\{\nu - p_j, 0\}}_{\leq 0}$$

Since the RHS is weakly positive, a necessary condition for search to occur at these prices is $\sigma[\nu - p^L] - s > 0$. If this is true, all prices $p_j \geq \nu$ will induce search and the indifference price is pinned down by:

$$\sigma(\nu - p^L) - s = \sigma(\nu - p_j) \iff \hat{p}(\nu) = p^L + s/\sigma$$

Note that our assumption implies that there exist consumers with $\nu \in [p^L + s/\sigma, 0.5]$ for which this holds. For all other consumers, search is never optimal and $\hat{p}(\nu) = \infty$.

(iii) $\nu > p^H$: Such a consumer will find it strictly optimal to search for prices $p_j > p^L$ if and only if:

$$\sigma \max\{\nu - p^H, \nu - p_j\} + (1-\sigma)(\nu - p^L) - s > \max\{v - p_j, 0\}$$

If $p^L < p_j \leq p^H$, this inequality becomes:

$$(1-\sigma)(\nu - p^L) + \sigma(\nu - p_j) - s > (v - p_j)$$

The necessary condition we need to examine now is: $(1-\sigma)(\nu - p^L) - s > 0 \iff \nu > s/(1-\sigma) + p^L$. If this fails, search is not optimal. If this is strictly positive, we can calculate the search cutoff in this interval of prices as:

$$\hat{p}(\nu) = \frac{s}{1-\sigma} + p^L$$

In order to be the correct search cutoff, this equation must be a price in $[p^L, p^H]$.

Our assumption was that $0.5 = p^H < \frac{s}{1-\sigma} + p^L$, which means that this cannot be the correct search cutoff. In other words, consumers will never search at these prices.

If $p_j > p^H$, the search inequality becomes:

$$\sigma(\nu - p^H) + (1-\sigma)(\nu - p^L) - s > \max\{v - p_j, 0\}$$

The relevant condition to check now is $\sigma(\nu - p^H) + (1-\sigma)(\nu - p^L) - s > 0 \iff \nu > s + \sigma p^H + (1-\sigma)p^L$. If this is weakly negative, you won't search.

If this is strictly positive, you will have an interior solution for your search cutoff and:

$$\hat{p}(\nu) = \sigma p^H + (1-\sigma)p^L + s$$

We have to verify that this is above p^H:

$$\hat{p}(\nu) = \sigma p^H + (1-\sigma)p^L + s > p^H \iff p^H < p^L + \frac{s}{1-\sigma}$$

This pins down search behaviour.

A.6 Proof of proposition 4

Closed-form solution for p^L:

A word on notation: $f^p(\nu, \tilde{\nu}^k, \tilde{\nu}_{i,-j} = \tilde{\nu}^z)$ indicates the density of consumers with valuation ν that arrive at firm j at $p-th$ order, generate the signal $\tilde{\nu}^k$ at k and $\tilde{\nu}^z$ at the other firm.

I first focus on pinning down the optimal low price p^L. Until a price $p_j \in (0, s/\sigma + p^L]$, the objective function after the low signal is:

$$\Pi^C(p_j;\tilde{\nu}^L) = p_j \int_{p_j}^{1} f^1(\nu,\tilde{\nu}^L)d\nu + p_j \int_{s/\sigma+p^L}^{0.5} f^2(\nu,\tilde{\nu}^L,\tilde{\nu}_{i,-j}=\tilde{\nu}^H)d\nu$$

To evaluate this, note the following:

$$\int_{s/\sigma+p^L}^{0.5} f^2(\nu,\tilde{\nu}^L,\tilde{\nu}_{i,-j}=\tilde{\nu}^H)d\nu = \int_{s/\sigma+p^L}^{0.5} 0.5\sigma(1-\sigma)f_v(\nu)d\nu =$$

$$\Big[0.5\sigma(1-\sigma)\Big]\big[(0.5-p^L)-s/\sigma\big] = 0.5\Big[(0.5-p^L)(\sigma-\sigma^2)-s(1-\sigma)\Big]$$

$$=$$

$$0.5\Big[(0.5-p^L+s)(\sigma)-(0.5-p^L)\sigma^2-s\Big]$$

Define

$$M(\sigma,p^L) = 0.5\Big[(0.5-p^L+s)(\sigma)-(0.5-p^L)\sigma^2-s\Big]$$

Now, let us plug this function $M(.)$ into the objective for the prices $p_j \in (0, s/\sigma+p^L]$:

$$\Pi^C(p_j;\tilde{\nu}^L) = p_j \int_{p_j}^{0.5} \sigma 0.5 f_\nu(x)dx + p_j \int_{0.5}^{1} (1-\sigma)0.5 f_\nu(x)dx + p_j M(\sigma,p^L)$$

Integrating up, the objective function becomes:

$$\Pi^C(p_j;\tilde{\nu}^L) = p_j 0.5\sigma[0.5-p_j] + p_j 0.5(1-\sigma)[0.5] + p_j M(\sigma,p^L) = 0.25p_j - 0.5\sigma(p_j)^2 + p_j M(\sigma,p^L)$$

Setting the derivative equal to zero and assuming that such a solution exists in the price interval we are examining yields:

$$\frac{\partial \Pi^C(p_j;\tilde{\nu}^L)}{\partial p_j} = 0 \iff 0.5 - 2\sigma p_j + 2M(\sigma,p^L) = 0 \iff p_j = \frac{1}{4\sigma} + \frac{M(\sigma,p^L)}{\sigma}$$

Now let's obtain a closed-form expression for p^L in symmetric equilibrium:

$$p^L = \frac{1}{4\sigma} + \left[\frac{0.5-p^L+s}{2} - \frac{0.5-p^L}{2}\sigma - \frac{s}{2\sigma}\right] = \frac{1}{4\sigma} + p^L\left[\frac{\sigma}{2}-\frac{1}{2}\right] + \left[\frac{0.5+s-0.5\sigma}{2} - \frac{s}{2\sigma}\right]$$

$$\iff$$

$$p^L = \frac{1}{4\sigma} + p^L[0.5(\sigma-1)] + \left[\frac{(1-\sigma)}{4} + \frac{s\sigma}{2\sigma} - \frac{s}{2\sigma}\right] = \frac{1}{4\sigma} - 0.5p^L(1-\sigma) + \frac{(1-\sigma)\sigma}{4\sigma} + \frac{2(\sigma-1)s}{4\sigma}$$

$$\Longleftrightarrow$$

$$p^L + 0.5p^L(1-\sigma) = p^L(1.5-0.5\sigma) = \frac{1+\sigma-\sigma^2-2(1-\sigma)s}{4\sigma}$$

$$\Longleftrightarrow$$

$$p^L = \frac{1+\sigma-\sigma^2-2(1-\sigma)s}{4\sigma(1.5-0.5\sigma)} = \frac{1+(1+2s)\sigma-\sigma^2-2s}{4(1.5\sigma-0.5\sigma^2)}$$

Note uniqueness of this solution.

Checking deviations from p^L:

Consider first prices $p_j \in [0, s/\sigma + p^L]$. Note that the objective function is strictly concave in this price range. Thus, there will be no deviations in this interval.

Consider secondly prices $p_j \in (s/\sigma + p^L, 1]$. I will show now that the monopoly profit functions are an upper envelope for the competitive profits function for both signals.

Consider first competitve profits made from consumers that generate the signal $\tilde{\nu}^K$ at firm j for prices $p_j \in (s/\sigma + p^L, 0.5]$:

$$\Pi^C(p_j;\tilde{\nu}^k) < \Pi^M(p_j;\tilde{\nu}^k)$$

$$\Longleftrightarrow$$

$$p_j \int_{0.5}^{1} f^1(\nu_i=\nu,\tilde{\nu}^k)d\nu + p_j \int_{p_j}^{0.5} f^1(\nu_i=\nu,\tilde{\nu}_{i,-j}=\tilde{\nu}^H,\tilde{\nu}^k)d\nu + p_j \int_{p_j}^{0.5} f^2(\nu_i=\nu,\tilde{\nu}_{i,-j}=\tilde{\nu}^H,\tilde{\nu}^k)d\nu$$

$$<$$

$$p_j \int_{p_j}^{0.5} f^1(\nu_i=\nu,\tilde{\nu}^k)d\nu + \int_{0.5}^{1} f^1(\nu_i=\nu,\tilde{\nu}^k)d\nu$$

$$\Longleftrightarrow$$

$$p_j \int_{p_j}^{0.5} f^2(\nu_i=\nu,\tilde{\nu}_{i,-j}=\tilde{\nu}^H,\tilde{\nu}^k)d\nu < p_j \int_{pj}^{0.5} f^1(\nu_i=\nu,\tilde{\nu}_{i,-j}=\tilde{\nu}^L,\tilde{\nu}^k)d\nu$$

$$\Longleftrightarrow$$

$$\int_{p_j}^{0.5} \left[0.5Pr(\tilde{\nu}^k|\nu)(1-\sigma)f_\nu(\nu)\right]d\nu < \int_{p_j}^{0.5} 0.5Pr(\tilde{\nu}^k|\nu)\sigma f_\nu(\nu)d\nu$$

This equality holds since $\sigma > 0.5$.

Now consider prices $p_j \geq 0.5$. At these prices, the sale will not be made to any searchers. As long as prices are below $\sigma p^H + (1-\sigma)p^L + s$, profits are exactly equal to monopoly profits. For prices above this, profits are zero.

Thus, we have the following for all prices $p_j > s/\sigma + p^L$:

$$\Pi^C(p_j; \tilde{\nu}^L) < \Pi^M(p_j; \tilde{\nu}^L)$$

By constrast, note the following: For all prices $p_j \leq s/\sigma + p^L$:

$$\Pi^C(p_j; \tilde{\nu}^L) > \Pi^M(p_j; \tilde{\nu}^L)$$

Moreover, one can show that the following regarding the relationship between the search equilibrium low price and the monopoly low price.

$$p^L = \frac{1}{4\sigma} + \frac{M(\sigma, p^L)}{\sigma} > \frac{1}{4\sigma} = p^{L,M}$$

Thus, $p^{L,M} < s/\sigma + p^L$ holds. Taking note of this and the fact that p^L maximizes $\Pi^C(p_j)$ on $p_j \in (0, s/\sigma + p^L]$ then yields:

$$\Pi^C(p^L; \tilde{\nu}^L) \geq \Pi^C(p^{L,M}; \tilde{\nu}^L) > \Pi^M(p^{L,M}; \tilde{\nu}^L)$$

Since $p^{L,M}$ maximizes $\Pi^M(p^{L,M})$ over the entire domain, we can also say the following for prices $p_j \in (s/\sigma + p^L, 1)$:

$$\Pi^M(p^{L,M}; \tilde{\nu}^L) \geq \Pi^M(p_j; \tilde{\nu}^L)$$

These arguments together imply the following chain of inequalities for the prices $p_j \in (s/\sigma + p^L, 1)$:

$$\Pi^C(p^L; \tilde{\nu}^L) > \Pi^M(p^{L,M}; \tilde{\nu}^L) \geq \Pi^M(p_j; \tilde{\nu}^L) \geq \Pi^C(p_j; \tilde{\nu}^L)$$

This completes the proof.

Checking deviations from $p^H = 0.5$

For the interval $(\sigma p^H + (1-\sigma)p^L + s, 1]$, the firm's profits will be zero.

In the interval $[0.5, \sigma p^H + (1-\sigma)p^L + s)$, the firm's profits are equal to the monopoly profits. We know there will be no deviations into this region.

As argued before, the firm's profits are bounded from above by $\Pi^M(p_j)$ in the price interval $p_j \in (s/\sigma + p^L, 0.5)$. Since $p^{H,S}$ maximizes monopoly profits, there won't be any deviations into this region.

Finally, I need to show that there is no deviation to a price in the interval $p_j \in (0, s/\sigma + p^L]$. Competitive profits in such a region are:

$$\Pi^C(p_j; \tilde{\nu}^H) = \Pi^M(p_j; \tilde{\nu}^H) + p_j \int_{pj}^{0.5} f^2(\nu_i = \nu, \tilde{\nu}_{i,j} = \tilde{\nu}^H, \tilde{\nu}^H) d\nu$$

To evaluate this, recall that $\Pi^M(p_j; \tilde{\nu}^H)$ is rising for all prices $p_j \leq 0.5$. Thus, the maximum of the above will be at $p_j = s/\sigma + p^L$. To evaluate this, recall that monopoly profits can be expressed as follows that

$$\Pi^M(p_j; \tilde{\nu}^H) = 0.25 p_j - 0.5(1-\sigma)p_j^2$$

Moreover, note that:

$$(s/\sigma + p^L) \int_{s/\sigma + p^L}^{0.5} f^2(\nu_i = \nu, \tilde{\nu}_{i,j} = \tilde{\nu}^H, \tilde{\nu}^H) d\nu =$$

$$(s/\sigma + p^L) \int_{s/\sigma + p^L}^{0.5} 0.5(1-\sigma)(1-\sigma) d\nu = 0.5(1-\sigma)^2 (s/\sigma + p^L)[0.5 - (s/\sigma + p^L)] =$$

$$0.25(1-\sigma)^2(s/\sigma + p^L) - 0.5(1-\sigma)^2(s/\sigma + p^L)^2$$

Combining these yields that:

$$\Pi^C(s/\sigma + p^L; \tilde{\nu}^H) = 0.25\big[1 + (1-\sigma)^2\big](s/\sigma + p^L) - 0.5(1-\sigma)\big[1 + (1-\sigma)\big](s/\sigma + p^L)^2$$

By contrast, monopoly profits at $p^H = 0.5$ are:

$$\Pi^M(0.5; \tilde{\nu}^H) = (0.5) \int_{0.5}^{1} f^1(\nu_i = \nu; \tilde{\nu}^H) d\nu \approx (0.5)(0.5)(0.5)\sigma = (1/8)\sigma$$

In other words, the following inequality has to hold in order to ensure that there are no deviations:

$$(1/8)\sigma - 0.25\big[1 + (1-\sigma)^2\big](s/\sigma + p^L) + 0.5(1-\sigma)\big[1 + (1-\sigma)\big](s/\sigma + p^L)^2 > 0$$

Checking the fundamental premise of the equilibrium:

I now characterize how the two fundamental inequalities underlying this equilibrium behave when plugging in equilibrium prices:

$$p^L(\sigma, s) + s/\sigma < 0.5$$

$$\iff$$

$$\frac{\sigma(1 + (1 + 2s)\sigma - \sigma^2 - 2s)}{\sigma(6\sigma - 2\sigma^2)} + \frac{s(6\sigma - 2\sigma^2)}{\sigma(6\sigma - 2\sigma^2)} < 0.5$$

$$\iff$$

$$\frac{(\sigma + \sigma^2 + 2s\sigma^2 - \sigma^3 - 2s\sigma) + 6s\sigma - 2s\sigma^2}{\sigma(6\sigma - 2\sigma^2)} < 0.5$$

$$\iff$$

$$\frac{\sigma + \sigma^2 - \sigma^3 + 4s\sigma}{(6\sigma^2 - 2\sigma^3)} < 0.5$$

$$\iff$$

$$\sigma + \sigma^2 - \sigma^3 + 4s\sigma < 3\sigma^2 - \sigma^3 \iff \sigma - 2\sigma^2 + 4s\sigma < 0$$

$$\iff$$

$$4s < 2\sigma - 1$$

$$\iff$$

$$\sigma > 2s + 0.5$$

Analogously, we can check validity of the second condition:

$$0.5 < p^L(s, \sigma) + s/(1 - \sigma)$$

$$\iff$$

$$\frac{1 + (1 + 2s)\sigma - \sigma^2 - 2s}{(6\sigma - 2\sigma^2)} + \frac{s}{(1 - \sigma)} > 0.5$$

$$\iff$$

$$\frac{(1 + (1 + 2s)\sigma - \sigma^2 - 2s)(1 - \sigma)}{(6\sigma - 2\sigma^2)(1 - \sigma)} + \frac{s(6\sigma - 2\sigma^2)}{(1 - \sigma)(6\sigma - 2\sigma^2)} > 0.5$$

$$\iff$$

$$\frac{(1 + \sigma + 2s\sigma - \sigma^2 - 2s) + (-\sigma - \sigma^2 - 2s\sigma^2 + \sigma^3 + 2s\sigma)}{(6\sigma - 2\sigma^2) - 6\sigma^2 + 2\sigma^3} + \frac{6\sigma s - 2s\sigma^2}{(6\sigma - 2\sigma^2) - 6\sigma^2 + 2\sigma^3} > 0.5$$

$$\iff$$

$$\frac{(1 + 4s\sigma - 2\sigma^2 - 2s - 2s\sigma^2 + \sigma^3) + 6\sigma s - 2s\sigma^2}{6\sigma - 8\sigma^2 + 2\sigma^3} > 0.5$$

$$\iff$$

$$1 + 10s\sigma - 2\sigma^2 - 2s - 4s\sigma^2 + \sigma^3 > 3\sigma - 4\sigma^2 + \sigma^3$$

$$\iff$$

$$1 + 10s\sigma + 2\sigma^2 - 2s - 4s\sigma^2 - 3\sigma > 0 \iff s(10\sigma - 2 - 4\sigma^2) > 3\sigma - 1 - 2\sigma^2$$

$$\iff$$

$$s > \frac{3\sigma - 1 - 2\sigma^2}{10\sigma - 2 - 4\sigma^2}$$

Profits in the search deterrence equilibrium vs monopoly profits.

There is no search on-path and the low prices are the same in the monopoly setting and in the search deterrence equilibrium. Thus, low signal profits are the same.

In the search deterrence equilibrium, the firm makes the high signal profits it would make in the monopoly setting when offering the price $p_j = s/\sigma + p^L < 0$. We know this must be below the monopoly price and that monopoly profits are strictly concave in this price domain.

Thus, the monopoly price is a strict optimum and high signal profits are higher in monopoly than in the search deterrence equilibrium.

A.7 Proof of corollary 1

Part 1: High price comparative statics:

The equilibrium high price is unaffected by search costs and signal precision.

Part 2: Low price comparative statics:

We know that that monopoly profits are weakly concave and differentiable on $(0, 0.5)$ and $(0.5, 1)$.

By differentiability of monopoly profits, note that the equilibrium low price has to satisfy the following FOC:

$$\left.\frac{\partial \Pi^M(p_j;\tilde{\nu}^L)}{\partial p_j}\right|_{p_j=p^L} + M^L(p) = 0$$

Note that:

$$M^L(p_j) = \int_{p^L+s/\sigma}^{0.5} 0.5Pr(\tilde{\nu}^H|\nu_i)Pr(\tilde{\nu}^L|\nu)d\nu = 0.5\int_{p^L+s/\sigma}^{0.5} \sigma(1-\sigma)d\nu$$

Thus, it holds that:

$$\frac{\partial M^L(p_j)}{\partial s} < 0$$

Using the implicit function theorem and weak concavity of the monopoly profit function then yields the desired result regarding the effect of search costs.

Now regarding the effect of a rise in information precision. Recall that the equilibrium low price can be characterized as follows

$$T(p^L,\sigma) = p^L - \frac{1}{4\sigma} - \frac{M(\sigma,p^L)}{\sigma} = 0 \iff T(p^L,\sigma) = p^L - \frac{1}{4\sigma} - 0.5\int_{p^L+s/\sigma}^{0.5}(1-\sigma)d\nu$$

Note that:

$$\frac{\partial T}{\partial p^L} = 1-(-1)0.5(1-\sigma) > 0$$

$$\frac{\partial T}{\partial \sigma} = -\frac{-1}{4\sigma^2} - 0.5\int_{p^L+s/\sigma}^{0.5}(-1)d\nu - (-1)(-s/\sigma^2)0.5(1-\sigma) = 0.5\int_{p^L+s/\sigma}^{0.5}(1)d\nu + \frac{0.25}{\sigma^2} - \frac{0.5(1-\sigma)s}{\sigma^2} > 0$$

The last inequality holds because:

$$0.5(1-\sigma)s < 0.5(1-\sigma) < 0.25$$

The result follows from application of the implicit function theorem.

Part 3: Firm profits and search costs

Competitive high signal profits are independent of s and so is the high price in the search equilibrium. Thus, high signal profits are not affected by an increase in s.

Consider the search equilibrium. We know $p^L < s/\sigma + p^L < 0.5$ must hold, which implies that the competitive objective function is continuously differentiable around p^L. Thus, the following FOC must hold:

$$\left.\frac{\partial \Pi^C(p_j;\tilde{\nu}^L)}{\partial p_j}\right|_{p_j=p^L} = 0$$

This result directly allows for the application of the envelope theorem. The effect of a rise in search costs on low signal profits runs only through the direct impact on M, which is negative.

A.8 Proof of corollary 2

Prices and comparative statics in the search deterrence equilibrium:

In the search deterrence equilibrium, it holds that $p^{L,D} = \frac{1}{4\sigma}$, $p^{H,D} = s/\sigma + p^{L,D}$.

The equilibrium low price is uanffected by search costs and falls in σ.

The equilibrium high price rises in s and falls in σ.

Firm profits in the search deterrence equilibrium.

There is no search on-path and a rise in s does not affect $p^{L,D}$. Thus, this rise in search costs has no effect on the low signal profits.

For a higher s, $p^{H,D}$ will be higher. We know monopoly high signal profits are rising in prices in this interval. Since high signal profits in the search deterrence equilibrium are equal to monopoly high signal profits, profits must be higher.

A.9 Proof of lemma 3:

Part 1: Define the following:

$$g(\nu) := Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s$$

The first part requires that $lim_{\nu\uparrow x} g(\nu) \geq g(x) \geq lim_{\nu\downarrow x} g(\nu)$. By the limit rule for products, we have:

$$\lim_{\nu\uparrow x} Pr(\tilde{\nu}^L|\nu)(\nu - p^L) = \lim_{\nu\uparrow x} Pr(\tilde{\nu}^L|\nu) \lim_{\nu\uparrow x}(\nu - p^L) = \lim_{\nu\uparrow x} Pr(\tilde{\nu}^L|\nu)(x - p^L) \geq$$

$$Pr(\tilde{\nu}^L|x)(x - p^L) \geq$$

$$\lim_{\nu \downarrow x} Pr(\tilde{\nu}^L|\nu)(x - p^L) = \lim_{\nu \downarrow x} \left[Pr(\tilde{\nu}^L|\nu)(\nu - p^L)\right]$$

This implies the desired result.

Now I need to show that $Pr(\tilde{\nu}^L|\underline{\nu})(\underline{\nu} - p^L) - s = 0$. Define $\underline{\nu} = \inf \hat{V}(p^L)$, and note that:

$$\hat{V}(p^L) = \left\{ Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s > 0 \right\}$$

Suppose that $g(\underline{\nu}) = Pr(\tilde{\nu}^L|\underline{\nu})(\underline{\nu} - p^L) - s > 0$. We know that $lim_{\nu \uparrow x} g(\nu) \geq g(\underline{\nu})$. By definition of limits, $\underline{\nu}$ cannot be the infimum then, since it would not constitute a lower bound.

Now suppose that $Pr(\tilde{\nu}^L|\underline{\nu})(\underline{\nu} - p^L) - s < 0$. We know that $lim_{\nu \downarrow x} g(\nu) \leq g(\underline{\nu}) < 0$. By definition of limits, any point just above $\underline{\nu}$ would also be a lower bound, which means we cannot have an infimum at $\underline{\nu}$.

Next, I need to show continuity. Assume, for a contradiction, that $lim_{\nu \uparrow \underline{\nu}} g(\nu) > g(\underline{\nu}) = 0$. This is a contradiction to the fact that $\underline{\nu}$ is an infimum.

Assume, for a contradiction, that $g(\underline{\nu}) > lim_{\nu \downarrow \underline{\nu}} g(\nu)$. We can show that this would be a contradiction to the infimum definition as well, since a point just above $\underline{\nu}$ would also be a infimum. This prooves continuity.

Part 2:

Search on the equilibrium path is only possible when receving the price offer $p_j = p^H$.

We know consumers with $\nu \neq \hat{V}(p^L)$ cannot search on path. Similarly, consumers with $\nu < p^L$ cannot search on path. Moreover, only sets of consumers with $\nu > p^H$ that have zero measure can search on-path.

The set of consumers that searchers on path (ignoring measure zero sets) is thus a subset of $\hat{V}(p^L) \cap [p^L, p^H]$. Now I show that any consumer in this set will search when being offered the price p^H. Such a consumer will search then if and only if:

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) + Pr(\tilde{\nu}^H|\nu) \max\{\nu - p^H, 0\} - s > \max\{\nu - p^H, 0\} \iff$$

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s > 0$$

This holds true by construction.

A.10 Proof of lemma 4:

Part 1: Consider any consumer that has arrived after search. This requires that $\underline{\nu} < p^H$, which means the firm beats the price of the other firm for such consumers. In an equilibrium, this consumer must have $\nu \in \hat{V}(p^L) \cap [p^L, p^H]$. We know that the infimum of this set is $\underline{\nu}$, so consumption is possible for all these agents. This implies the result.

Part 2:

I need to show that no consumer would move on to search when receiving a price $p_j \leq \nu$. We know that no consumer with $\nu \notin \hat{V}(p^L) \cap [p^L, p^H]$ will ever search.

Now consider a consumer with $\nu \in \hat{V}(p^L) \cap [p^L, p^H]$. Recall that all these consumers have a price cutoff $\hat{p}(\nu)$ equal to:

$$\hat{p}(\nu) = p^L + \frac{s}{Pr(\tilde{\nu}^L|\nu)}$$

It was shown previously that:

$$Pr(\tilde{\nu}^L|\underline{\nu})(\underline{\nu} - p^L) - s = 0$$

This implies that $\hat{p}(\underline{\nu}) = \underline{\nu}$. Note that $\hat{p}'(\nu) > 0$, which means that for all $\nu \in \hat{V}(p^L) \cap [p^L, p^H]$:

$$\hat{p}(\nu) \geq \hat{p}(\underline{\nu}) = \underline{\nu}$$

This means that none of these consumers will search after being offered the price $p_j = \underline{\nu}$.

Finally, consider any consumer with $\nu \in \hat{V}(p^L) \cap [p^H, 1]$. I have shown that any such consumer must have $\hat{p}(\nu) \geq p^H$ (ignoring measure zero sets).

A.11 Proof of proposition 5

Part 1:

Given that the signal probability functions are continuous, the monopoly low signal profit function is differentiable. Since competitive low signal profits take the given form in the price interval $p_j \in [0, \underline{\nu}]$, we know that this FOC must hold.

Part 2: I have to show the following: When the probability function is continuous, then an equilibrium high price must satisfy on of the following equations:

$$\left.\frac{\partial \Pi_j^M(p_j; \tilde{\nu}^H)}{\partial p_j}\right|_{p_j = p^H} = 0$$

$$Pr(\tilde{\nu}^L | p^H)(p^H - p^L) - s = 0$$

(i) $Pr(\tilde{\nu}^L | p^H)(p^H - p^L) - s > 0$ cannot be an equilibrium.

Suppose, for a contradiction, that $Pr(\tilde{\nu}^L | p^H)(p^H - p^L) - s > 0$. If this holds true, $p^H \in \hat{V}(p^L)$. Consider the search decision of an agent with $\nu > p^H$. This consumer searches at p^H if and only if:

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) + Pr(\tilde{\nu}^H|\nu)(\nu - p^H) - s > (\nu - p^H) \iff$$

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s > Pr(\tilde{\nu}^L|\nu)(\nu - p^H)$$

Both these objects are continuous in ν and we know that this inequality is satisfied at $\nu = p^H$. This shows that a strictly positive measure of consumers with $\nu > p^H$ search, which breaks the equilibrium.

(ii) If $Pr(\tilde{\nu}^L | p^H)(p^H - p^L) - s < 0$, the equilibrium must satisfy:

$$\frac{\partial \Pi^M(p_j; \tilde{\nu}^H)}{\partial p_j} = 0$$

Consider any consumer with $\nu \in [p^H, 1]$. This consumer would search when offered the price $p_j > p^H$ if and only if:

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) + Pr(\tilde{\nu}^H|\nu)(\nu - p^H) - s > (\nu - p^H)$$

In order for a consumer to search at a price $p_j > p^H$, the LHS of this expression needs to be strictly positive. Define the following set::

$$\hat{V}^H(p) = \left\{\nu - p^H + Pr(\tilde{\nu}^L|\nu)(p^H - p^L) - s > 0\right\}$$

Further recall that the cutoff price (if it is above p^H) of agents with a valuation in the above set is:

$$\hat{p}^H(\nu) = Pr(\tilde{\nu}^L|\nu)\big(p^L - p^H\big) + p^H + s$$

By continuity of $Pr(\tilde{\nu}^L|\nu)$, it holds that:

$$\lim_{\nu \downarrow p^H} Pr(\tilde{\nu}^L|\nu) = Pr(\tilde{\nu}^L|p^H)$$

This implies that:

$$\lim_{\nu \downarrow p^H} \Big[\nu - p^H + Pr(\tilde{\nu}^L|\nu)(p^H - p^L) - s\Big] = 0 + Pr(\tilde{\nu}^L|p^H)(p^H - p^L) - s < 0$$

Continuity implies that you can find an open interval $[p^H - \delta, p^H + \delta]$ such that any $\nu' \in [p^H - \delta, p^H + \delta]$ will satisfy:

$$\nu' \notin \hat{V}^H(p)$$

$$\nu' \notin \hat{V}^L(p)$$

Thus, for this price interval, all consumers will have $\hat{p}(\nu') = \infty$ by construction.

Find the first valuation ν' above p^H that solves:

$$\nu' - p^H + Pr(\tilde{\nu}^L|\nu')(p^H - p^L) - s = 0$$

By continuity arguments, $\hat{p}(\nu) = \infty$ must hold for all consumers with $\nu \in [p^H, \nu')$.

Note that $\hat{p}(\nu') = \nu'$. All consumers with $\nu > \nu'$ will have a cutoff price above this. Thus, all consumers with $\nu > p^H$ will not search for prices below ν'.

This means that profits are equal to monopoly profits in the interval $[p^H, \nu']$. No consumer with a valuation above p^H can arrive at firm j after searching. No consumer with $\nu > p^H$ that arrives at firm j first will search for these prices.

Consider prices just below p^H. We know that consumers with $[p^H - \delta, p^H]$ must have $\nu \notin \hat{V}(p^L)$, which is the necessary condition for search at prices $p_j \leq p^H$, i.e. for equilibrium search.

Thus, no consumers with $\nu > p^H - \delta$ will arrive at firm j after search. All consumers that arrive first and have $\nu > p^H - \delta$ won't move on to search at these prices.

In the price interval $[p^H - \delta, p^H]$, profits will thus also be monopoly profits. This implies that the high signal price must satisfy said FOC.

Part 3

I have to show that the monopoly high signal price is unique when the signal probability function is once continuously differentiable. Note that monopoly high signal profits are given by the following:

$$\Pi^M(p_j; \tilde{\nu}^H) = p_j \int_{p_j}^{1} 0.5 Pr(\tilde{\nu}^H|\nu) d\nu$$

Taking the derivative of this w.r.t prices yields:

$$\frac{\partial \Pi^M(p_j; \tilde{\nu}^H)}{\partial p_j} = \int_{p_j}^{1} 0.5 Pr(\tilde{\nu}^H|\nu) d\nu - 0.5 p_j Pr(\tilde{\nu}^H|p_j)$$

Note that this derivative is continuous, since the signal probability function is continuous.

Now let's evaluate the second derivative of profits, which is:

$$\frac{\partial^2 \Pi^M(p_j; \tilde{\nu}^H)}{\partial p_j^2} = -0.5 Pr(\tilde{\nu}^H|p_j) - 0.5 Pr(\tilde{\nu}^H|p_j) - 0.5 p_j \frac{\partial Pr(\tilde{\nu}^H|p_j)}{\partial p_j}$$

This implies that monopoly high signal profits are strictly concave, which implies uniqueness.

Part 4

I have to show that the given regularity conditions are sufficient to ensure that there are only two potential high signal equilibrium prices that are unequal to $p^{H,M}$ and set $g(p^H) = 0$.

The proof of this proceeds in several steps.

(i) The monopoly high signal price must be weakly above 0.5.

Suppose, for a contradiction, that $p^{H,M} < 0.5$.

Recall firstly that the monopoly high signal price must solve:

$$\int_{p_j}^{1} 0.5 Pr(\tilde{\nu}^H|\nu)d\nu - 0.5 p_j Pr(\tilde{\nu}^H|p_j) = 0 \iff p^{H,M} Pr(\tilde{\nu}^H|p^{H,M}) = \int_{p^{H,M}}^{1} Pr(\tilde{\nu}^H|\nu)d\nu$$

In general, we have:

$$\int_{p^{H,M}}^{1} Pr(\tilde{\nu}^H|\nu)d\nu > \int_{p^{H,M}}^{1} Pr(\tilde{\nu}^H|p^{H,M})d\nu = [1 - p^{H,M}] Pr(\tilde{\nu}^H|p^{H,M})$$

Because $p^{H,M} < 0.5$, we have:

$$[1 - p^{H,M}] > p^{H,M}$$

This implies a contradiction to the first-order condition.

(ii) It must hold that $p^H \leq p^{H,M}$ in any equilibrium.

Suppose, for a contradiction, that $p^H > p^{H,M}$ in some equilibrium.

We know that monopoly high signal profits are strictly concave. We also know that any firm can only make monopoly profits when setting the high signal price.

Consider a given equilibrium without search on the equilibrium path. Then, there will also not be search at prices below p^H, which directly implies that a deviation to $p^{H,M}$ is profitable.

Now consider an equilibrium with on-path search and consider a deviation price $p_j \leq p^H$. The necessary condition for search is then given by membership in the set $\hat{V}(p^L)$.

We know $g(\nu) = Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s$ must satisfy $g(p^H) = 0$ and that this function cannot be flat here. Further, we have assumed that the signal probability is differentiable, which then implies that $g(\nu)$ is differentiable.

Assume, for a contradiction, that $g'(p^H) > 0$. By the definition of a derivative, we must have that:

$$\lim_{v' \to p^H} \frac{g(v') - g(p^H)}{v' - p^H} = x > 0$$

Thus, we can find a $\delta > 0$ such that, for all $\nu \in [p^H - \delta, p^H)$, the following term must be arbitrarily close to the positive number x:

$$\frac{g(v') - g(p^H)}{v' - p^H}$$

Setting $\epsilon = x/2$, there would exist such a delta where:

$$\left| \frac{g(v') - g(p^H)}{v' - p^H} - x \right| < \frac{x}{2}$$

It is easy to show that the following must then hold true:

$$\frac{g(v') - g(p^H)}{v' - p^H} > 0$$

Since the denominator is negative, the numerator must also be negative and we have $g(v') < g(p^H) = 0$ for all these ν'.

But then none of the agents with $\nu \in [p^H - \delta, p^H]$ would search at prices $p_j \in [p^H - \delta, p^H]$, and neither could agents with $\nu > p^H$. This implies that no agents with $\nu > p^H - \delta$ arrive after search and no agents with $\nu > p^H - \delta$ that arrive at j first would move on to search at these prices.

This implies that profits at these prices equal monopoly profits and there is a downward deviation by strict concavity of high signal monopoly profits.

Thus, it must hold that $g'(p^H) < 0$. By the definition of a derivative, we must have that:

$$\lim_{v' \to p^H} \frac{g(v') - g(p^H)}{v' - p^H} = x < 0$$

Setting $\epsilon = -x/2$, there must exist a delta where all $\nu' \in [p^H - \delta, p^H)$ satisfy:

$$\left| \frac{g(v') - g(p^H)}{v' - p^H} - x \right| < \frac{-x}{2}$$

It follows directly that the following must hold for all such ν':

$$\frac{g(v') - g(p^H)}{v' - p^H} < 0$$

Since the denominator is negative, the numerator must be positive, i.e. $g(v') > g(p^H) = 0$. Thus, for all these $\nu \in [p^H - \delta, p^H]$, $\nu \in \hat{V}(p^L)$ and these agents must have a cutoff price given by:

$$\hat{p}(\nu) = \frac{s}{Pr(\tilde{\nu}^L|\nu)} + p^L$$

We also know that this cutoff must be interior. Moreover, we know that this cutoff must be weakly below p^H, because for all agents with $\nu \in [p^H - \delta, p^H)$, it must hold that:

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s > 0 \implies$$

$$Pr(\tilde{\nu}^L|\nu)(p^H - p^L) - s > Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s > 0 \implies$$

$$p^H > p^L + \frac{s}{Pr(\tilde{\nu}^L|\nu)}$$

This means that there must exist agents with $\nu \in [p^H - \delta, p^H]$ that arrive at firm j after searching.

In the following, I define a valuation cutoff function:

$$\nu > \hat{\nu}(p_j) \iff \hat{p}(\nu) > p_j$$

In words, consumers with $\nu \in [p^H - \delta, p^H]$ will search if and only if $\nu < \hat{\nu}(p_j)$.

Thus, consider competitive profits in the price interval $p_j \in [p^H - \delta, p^H]$. Here, profits will be:

$$\Pi^C(p_j; \tilde{\nu}^k) = p_j \int_{p^H}^{1} (0.5) Pr(\tilde{\nu}^k|\nu) d\nu + p_j \int_{\hat{\nu}(p_j)}^{p^H} (0.5) Pr(\tilde{\nu}^k|\nu) d\nu +$$

$$p_j \int_{p_j}^{\hat{\nu}(p_j)} (0.5) Pr(\tilde{\nu}^k|\nu) Pr(\tilde{\nu}^H|\nu) d\nu + p_j \int_{p_j}^{p^H} (0.5) Pr(\tilde{\nu}^k|\nu) Pr(\tilde{\nu}^H|\nu) d\nu$$

Let's take the derivative of this w.r.t the price p_j. This is:

$$\frac{\partial \Pi^C(p_j; \tilde{\nu}^k)}{\partial p_j} = \int_{p^H}^{1} (0.5) Pr(\tilde{\nu}^k|\nu) d\nu + \int_{\hat{\nu}(p_j)}^{p^H} (0.5) Pr(\tilde{\nu}^k|\nu) d\nu +$$

$$\int_{p_j}^{\hat{\nu}(p_j)} (0.5) Pr(\tilde{\nu}^k|\nu) Pr(\tilde{\nu}^H|\nu) d\nu + \int_{p_j}^{p^H} (0.5) Pr(\tilde{\nu}^k|\nu) Pr(\tilde{\nu}^H|\nu) d\nu +$$

$$p_j \Big(- \frac{\partial \hat{\nu}(p_j)}{\partial p_j} \Big) \big(0.5 Pr(\tilde{\nu}^k|\hat{\nu}(p_j)) \big) + p_j \Big(\frac{\partial \hat{\nu}(p_j)}{\partial p_j} \Big) \big(0.5 Pr(\tilde{\nu}^k|\hat{\nu}(p_j)) Pr(\tilde{\nu}^H|\hat{\nu}(p_j)) \big)$$

$$-p_j(0.5)Pr(\tilde{\nu}^k|p_j)Pr(\tilde{\nu}^H|p_j) - p_j\big(0.5Pr(\tilde{\nu}^k|p_j)Pr(\tilde{\nu}^H|p_j)\big)$$

Note that:

$$p_j\bigg(-\frac{\partial\hat{\nu}(p_j)}{\partial p_j}\bigg)\big(0.5Pr(\tilde{\nu}^k|\hat{\nu}(p_j))\big) + p_j\bigg(\frac{\partial\hat{\nu}(p_j)}{\partial p_j}\bigg)\big(0.5Pr(\tilde{\nu}^k|\hat{\nu}(p_j))Pr(\tilde{\nu}^H|\hat{\nu}(p_j))\big) =$$

$$0.5p_j\bigg(\frac{\partial\hat{\nu}(p_j)}{\partial p_j}\bigg)\bigg(Pr(\tilde{\nu}^k|\hat{\nu}(p_j))Pr(\tilde{\nu}^H|\hat{\nu}(p_j)) - Pr(\tilde{\nu}^k|\hat{\nu}(p_j))\bigg) < 0$$

This inequality holds since $Pr(\tilde{\nu}^k|\hat{\nu}(p_j))Pr(\tilde{\nu}^H|\hat{\nu}(p_j)) < Pr(\tilde{\nu}^k|\hat{\nu}(p_j))$ and $\frac{\partial\hat{\nu}(p_j)}{\partial p_j} > 0$.

Thus, the derivative satisfies:

$$\frac{\partial\Pi^C(p_j;\tilde{\nu}^k)}{\partial p_j} < \int_{p^H}^{1}(0.5)Pr(\tilde{\nu}^k|\nu)d\nu + \int_{\hat{\nu}(p_j)}^{p^H}(0.5)Pr(\tilde{\nu}^k|\nu)d\nu +$$

$$\int_{p_j}^{\hat{\nu}(p_j)}(0.5)Pr(\tilde{\nu}^k|\nu)Pr(\tilde{\nu}^H|\nu)d\nu + \int_{p_j}^{p^H}(0.5)Pr(\tilde{\nu}^k|\nu)Pr(\tilde{\nu}^H|\nu)d\nu - 2p_j\big(0.5Pr(\tilde{\nu}^k|p_j)Pr(\tilde{\nu}^H|p_j)\big)$$

Evaluating this at $p_j = p^H$ yields that:

$$\frac{\partial\Pi^C(p_j;\tilde{\nu}^k)}{\partial p_j}\bigg|_{p_j=p^H} < \int_{p^H}^{1}(0.5)Pr(\tilde{\nu}^k|\nu)d\nu + \int_{p^H}^{p^H}(0.5)Pr(\tilde{\nu}^k|\nu)d\nu - 2p^H(0.5)Pr(\tilde{\nu}^k|p^H)Pr(\tilde{\nu}^H|p^H) +$$

$$\int_{p^H}^{p^H}(0.5)Pr(\tilde{\nu}^k|\nu)Pr(\tilde{\nu}^H|\nu)d\nu + \int_{p^H}^{p^H}(0.5)Pr(\tilde{\nu}^k|\nu)Pr(\tilde{\nu}^H|\nu)d\nu =$$

$$\int_{p^H}^{1}(0.5)Pr(\tilde{\nu}^k|\nu)d\nu - 2p^H(0.5)Pr(\tilde{\nu}^k|p^H)Pr(\tilde{\nu}^H|p^H)$$

Note that $p^H > p^{H,M} \geq 0.5$. Note further that $Pr(\tilde{\nu}^H|\nu)$ is rising in ν and $Pr(\tilde{\nu}^H|0.5) = 0.5$.

This implies that $Pr(\tilde{\nu}^H|p^H) \geq 0.5$. Then, it holds that $-2Pr(\tilde{\nu}^H|p^H) \leq -1$, which implies that :

$$\frac{\partial\Pi^C(p_j;\tilde{\nu}^k)}{\partial p_j}\bigg|_{p_j=p^H} < \int_{p^H}^{1}(0.5)Pr(\tilde{\nu}^k|\nu)d\nu - p_j(0.5)Pr(\tilde{\nu}^k|p^H) = \frac{\partial\Pi^M(p_j;\tilde{\nu}^k)}{\partial p_j}\bigg|_{p_j=p^H}$$

If this is true, we cannot have an equilibrium - there would be a downward deviation. This follows from strict concavity of the high signal monopoly profit function.

(iii) We have proven that $p^H \leq p^{H,M}$ must hold.

Quasiconcavity of the function $Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s$ on $[0, p^{H,M}]$ implies that the following sets are convex:

$$\left\{\nu \in [0, p^{H,M}] : Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s > 0\right\}$$

$$\left\{\nu \in [0, p^{H,M}] : Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s \geq 0\right\}$$

An equilibrium high signal price that does not equal $p^{H,M}$ must be strictly below $p^{H,M}$ by previous arguments.

There can be at most two solutions of the following equation in the interval $[0, p^{H,M})$ that can be supported as an equilibrium high signal price:

$$Pr(\tilde{\nu}^L|\nu)(\nu - p^L) - s = 0$$

Suppose, for a contradiction, that there are three solutions to the above equation in the interval $[0, p^{H,M})$ that could be supported as an equilibrium high signal price. By the regularity condition, $g'(p^H) \neq 0$ must hold. Then, it could be that two solutions feature a strictly rising g. Alternatively, there could be two solutions with a strictly falling g. By differentiability, there will be a violation of convexity of the above sets.

These potential solutions are $\inf \hat{V}(p^L)$ and $\sup \hat{V}(p^L) \cap [0, p^{H,M})$.

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