Discussion Paper Series – CRC TR 224

Discussion Paper No. 315
Project B 03

Consumer Search and Choice Overload

Volker Nocke [1]
Patrick Rey [2]

August 2021

[1] Department of Economics and MaCCI, University of Mannheim, Germany; School of Economics, University of Surrey, UK. Email: volker.nocke@gmail.com
[2] Toulouse School of Economics, University Toulouse Capitole, France. Email: patrick.rey@tse-fr.eu

Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Consumer Search and Choice Overload*

Volker Nocke[†] Patrick Rey[‡]

July 28, 2021

Abstract

We study a model in which a monopoly seller decides which among a set of heterogeneous products to offer, and what prices to charge, and consumers engage in costly (random) sequential search to learn prices and valuations. We show that the equilibrium exhibits *choice overload*: The larger the product line, the fewer consumers start searching. We provide conditions under which the equilibrium size of the product line is socially excessive (or insufficient). We also characterize equilibria when the seller can position products, thereby allowing the possibility of directed search, and disclose product identity. We show that the best equilibrium for the seller may involve randomizing over product positioning and inducing inefficient search. Finally, we extend our analysis to that of a platform choosing which sellers to host.

JEL Classification: L12, L15, D42.

Keywords: Sequential consumer search, product variety, choice overload, multi-product firm, platform.

*We wish to thank Bruno Jullien, Martin Peitz, Andrew Rhodes, Sandro Shelegia, John Thanassoulis, Jidong Zhou, seminar participants at the European University Institute, Bocconi University, Cambridge University, ITAM, London School of Economics, Santa Clara University, Toulouse School of Economics, the University of Surrey, and the Consumer Search Digital Seminar Series as well as participants at the 2015 MaCCI Summer Institute in Competition Policy (Erftstadt), the 2015 MaCCI Workshop on Multi-product Firms (Bad Homburg), the Spring 2021 CRC TR 224 conference. Nocke gratefully acknowledges financial support from the German Research Foundation (DFG) through CRC TR 224 (Project B03). Rey gratefully acknowledges financial support from the European Research Council (ERC) under the European Community's Seventh Framework Programme (FP7/2007-2013) Grant Agreement N 340903 and from the ANR Investissements d'Avenir program under the grant CHESS ANR-17-EURE-0010.

[†]Department of Economics and MaCCI, University of Mannheim, Germany; School of Economics, University of Surrey, UK. Email: volker.nocke@gmail.com

[‡]Toulouse School of Economics, University Toulouse Capitole, France. Email: patrick.rey@tse-fr.eu

1 Introduction

In today's internet age, consumers face a choice among a huge variety of products. However, finding out which product a consumer likes best (and what final price he has to pay) has become a complicated and time-consuming task. Such frictions are at the heart of the literature on consumer search, which has recently undergone a resurgence due to the increasing importance of internet platforms and e-commerce. Experimental evidence also points to adverse reactions by consumers to an increase in product range, a phenomenon dubbed *choice overload*. For example, Iyengar and Lepper (2000) document that an increase in the number of flavors sold at a jam tasting booth in an upscale grocery store induced a significant reduction in the share of customers making a purchase.

In this paper, we develop a sequential search model that is consistent with the choice overload phenomenon, and use it to study the optimal product offering of a monopolistic seller. An increase in product variety has two opposing effects. On the one hand, it induces fewer consumers to search in the first place, thus being consistent with *choice overload*. On the other, it induces a higher probability of a sale for each consumer who chooses to search. Depending on the distribution of search costs in the population of consumers, the seller's optimal product variety may be socially excessive or insufficient. Eliminating choice overload through product positioning or search recommendations (thereby making consumer search more efficient) may, however, not be in the interest of the seller. Instead, even when such possibility is available, the most profitable equilibrium often involves a stochastic strategy by the seller, so as to ensure that active consumers keep searching until finding a match.

In the model, described in Section 2, there is a set of heterogeneous products that differ in their "popularity", i.e., the probability with which any consumer has a "match" with that product. Consumers differ in their match-conditional valuations for products as well as in their search costs. A firm decides which subset of the products to offer and how to price them. Consumers only observe the size of the product line but not the identity of the offered products nor their prices. Upon inspecting a product, a consumer learns whether he has a match with that product and, if so, his match-conditional valuation, which is the same across products. He uses this information to update his beliefs about the not-yet-inspected products. In the baseline setting, products are allocated to indistinguishable "slots" so that (sequential) consumer search is necessarily random.

In Section 3 we show that, for any given size of the product line, the firm chooses the most popular products and offers them at monopoly prices. Consumers with low enough search costs start searching and continue to do so until finding a match (and then either purchase or give up, depending on the realization of their match-conditional valuation). This is because, consumers who did not find a match in previous inspections become increasingly optimistic about finding a match at the next one. Consumer search therefore has the potential to be "addictive:" some consumers unwilling to start searching would, if coerced to make a first inspection, voluntarily keep searching afterwards—others (those with higher search costs) would need to experience more than one inspection before becoming "addicted" to searching.[1]

An increase in the size of the product line has two opposing effects: On the one hand, it offers more opportunities of finding a match; on the other, by reducing the average popularity among the products on offer, it lowers the probability of finding a match on any given inspection. We show that the latter effect always dominates, implying that the number of consumers who choose to search is decreasing in the size of the product line—consistent with the choice overload phenomenon. However, increasing the size of the product line does benefit consumers with sufficiently low search costs as it increases the probability of finding a match.

The seller's optimal size of the product line is socially excessive (resp., insufficient) if, on average, the search cost of infra-marginal consumers increases more (resp., less) than proportionally with the search cost of the marginal consumer.[2] We also show that the seller's optimal size of the product line is decreasing in the elasticity of the search cost distribution, and that it increases (resp., decreases) with consumers' share of the surplus if that elasticity is decreasing (resp., increasing).

In Section 4, we assume that the firm can put products in specific positions (e.,g, because there are distinguishable slots or, alternatively, because the firm can make recommendations about the products), which allows for the possibility of directed search. We first show that the previous outcome can still be sustained by a uniformly random positioning policy: this not only leaves consumers indifferent about their search sequences, but also induces them to keep searching until finding a match—which in turn makes the firm indifferent about positioning.

We then study "positioning" equilibria in which consumer search is directed. [For simplicity, we assume in that part of the analysis that the maximum size of the product line is two.] In particular, we show that there exists a "pure positioning" equilibrium in which the slot allocation or the recommendation policy is deterministic; it is thus perfectly informative, implying that consumers inspect the products in decreasing order of popularity. In this equilibrium, more consumers start searching because they will inspect the most popular product first. However, some consumers then stop searching after the first inspection—even absent finding a match; as a result, this equilibrium may be less profitable than the random positioning equilibrium.

In addition, there exists a continuum of "noisy positioning" equilibria in which the

[1] There is a large literature in psychology and marketing on shopping addiction and compulsive buying disorder (*oniomania*), first described by Kraepelin (1915, p.409).

[2] The seller's optimal size of the product line is socially optimal if the search cost of infra-marginal consumers increases at the same rate as that of the marginal consumer—i.e., if search costs are distributed according to a power law.

slot allocation or the recommendation policy are non-uniformly random, which induces consumers to start with the more promising position (i.e., where they are more likely to encounter the more popular product). In all of these equilibria, consumers who start searching continue to do so until they find a match (and in the best such equilibrium, consumers who are indifferent between starting or not, are also indifferent between continuing or not). For these equilibria to exist, however, the positioning policy has to be sufficiently noisy.

Finally, we allow for the possibility that the firm discloses the identity of a product upon inspection. This prevents the positioning policy from being perfectly informative. By contrast, the noisy positioning equilibria are robust to such disclosure possibility.

In Section 5, we show that the main insights carry over to the case of a monopolistic platform hosting a set of third-party (single-product) sellers. We conclude in Section 6.

Related literature. Our paper is related to several literatures. First, it contributes to the burgeoning literature on consumer search. In the classic Wolinsky (1986) model of sequential search with differentiated products, and the literature that builds upon it, consumers sequentially search for the best "match". Each firm offers a single product, and products are completely symmetric in that a consumer gets a random draw of his valuation for the product from the same distribution. Our paper contributes to that literature in several ways. Most importantly, we introduce product heterogeneity in an analytically tractable way by assuming that products differ in their "popularity" (i.e., the probability with which a consumers likes a product) while maintaining the assumption of consumer heterogeneity.[3] Second, we analyze pricing and product choice of a multiproduct monopolist. The existing literature on consumer search mostly focuses on (competing) single-product firms, and the few papers that do consider multiproduct firms (e.g., Zhou, 2014; Rhodes, Watanabe, and Zhou, 2019) typically assume that, upon visiting a multiproduct firm, a consumer automatically learns his match values (and prices) for all of the products offered by that firm (i.e., there is no within-firm search).[4]

In the second part of our paper, where positioning becomes available, we show that the firm may be worse off when consumer search is directed (in which case consumers first inspect the most promising positions). This is related to some recent papers on consumer obfuscation and search diversion. Petrikaitė (2018) studies a Wolinsky-type model of sequential search where a monopolist sells two (symmetric) products and chooses

[3]In contrast to Wolinsky (1986), consumers not only differ in their valuations for a given product but also in their search costs. Moraga-Gonzalez, Sandor and Wildenbeest (2017) introduce heterogeneous search costs in a version of the Wolinsky (1986) model with an infinite number of firms.

[4]One exception is Rhodes (2015) who studies the pricing and advertising decisions of a multiproduct retailer offering n symmetric products. Contributing to the literature on price (rather than match) search with homogeneous goods, pioneered by Stahl (1989), Hämäläinen (2019) allows not only for inter-firm search but also for intra-firm search.

not only prices but also product-specific search costs.[5] Consumers observe these search costs and then decide which product to inspect first; that is, search is directed. In equilibrium, the seller does obfuscate exactly one product (i.e., sets a positive search cost for that product). By contrast, in our model, product-level search costs are fixed (and identical) but products are heterogeneous, and we compare profits across equilibria (with and without directed search). Hagiu and Jullien (2011) analyze search diversion by a monopolistic intermediary. There are two independent sellers on the platform and the intermediary receives an exogenous seller-specific fee for making a consumer visit a seller. The intermediary observes consumers' types and ex ante commits to seller-specific diversion probabilities (i.e., probabilities with which a consumer has to visit first the less preferred seller). Hagiu and Jullien show that, if prices are exogenous, the intermediary diverts consumers with positive probability toward the firm from which it receives the higher fee.[6,7] By contrast, in our model, the firm is vertically integrated (except in Section 5), does not observe consumers' types, and cannot force consumers to inspect products in a particular order. The second part of our paper is also related to Chen and He (2011), who study consumer search and advertising on an internet platform. In their model, single-product firms that differ in the popularity of their products participate in auctions for distinguishable advertising slots. They show that there exists a separating equilibrium (akin to our pure positioning equilibrium), in which the firm offering the most popular product wins the auction for the advertising slot that consumers visit first, as well as a pooling equilibrium (akin to our random positioning equilibrium), in which firms bid zero and search is random.[8]

Choice overload—the phenomenon that consumers may be less likely to choose a product if they face more options—has been well-documented in several field experiments. As mentioned above, Iyengar and Leppar (2000) find that consumers are less likely to make a choice if they are offered a larger number of jam flavors. Boatwright and Nunes (2001) report an experiment run by an online grocery: in the experimental group in which the product selection was halved, the sales were 11 percent higher than in the control group. Kamenica (2006) has shown that choice overload can be rationalized if consumers make contextual inferences from the number of products on offer. In his model, a firm can choose to offer any subset of N possible flavors of jam (at some exogenous price). Each consumer likes at most one flavor, with flavors differing in the fraction of consumers who like them. There are both informed consumers (who know which

[5] Gamp (2019) analyzes a setup that is, in some ways, more general than Petrikaitė (2018)'s but assumes that prices are observable.

[6] If prices are endogenous, then some consumer types are diverted even if the intermediary receives the same fee from both sellers.

[7] Teh and Wright (forthcoming) study steering of consumers by a monopoly intermediary on a platform where firms pay commission fees to influence the intermediary's ranking of products.

[8] Eliaz and Spiegler (2011) study the pricing of a monopoly platform in a variant of the Che and He (2011) model.

flavor they like) and uniformed consumers (who do not). Informed consumers therefore pick their favorite flavor if it is on offer (and do not choose anything otherwise) whereas uninformed consumers have to randomly pick one flavor, if any. The existence of informed consumers implies that, for a given number of flavors, the firm offers the most popular ones. Offering a larger number of flavors involves a trade-off: on the one hand, more informed consumers are buying (as a larger choice set is more likely to include the favorite flavor); on the other, it makes uninformed consumers less likely to purchase (as they are less likely to find their favorite flavor, given that they have to randomly pick one). By contrast, in our model, all consumers are equally uninformed ex ante but can engage in (costly) sequential search to find a product they like; that is, we endogenize the extent of information that consumers have, and compare random and directed search equilibria. Kuksov and Villas-Boas (2010) also offer a (random) consumer search model to explain choice overload. However, in their model the optimal consumer search rule is quite complicated, which leads them to focus on the case of three or fewer alternatives. The tractability of our model allows us to consider instead arbitrary product ranges, to study pricing and product choice decisions, and to compare random and directed search.

2 Setting

Consider an industry in which there is a countably infinite number of products, indexed by $i = 1, 2, ...$ On the supply side, a monopolist chooses (i) which subset $\mathcal{I}$ of these products to offer, and (ii) the prices $(p_i)_{i\in\mathcal{I}}$ at which it offers them.[9] There is no fixed cost associated with offering a product, and all products involve the same constant unit cost of production, normalized to zero.

On the demand side, a unit mass of consumers have unit demands, and differ in their search costs and valuations. The search cost, c, has a continuous c.d.f. $G(\cdot)$ over the support $\mathbb{R}_+$, satisfying $G(0) = 0$. Consumers value a product only if they have a match with that product, and these matches are independent across consumers and products. As in Chen and He (2011), each product i is characterized by the probability $\mu_i \in (0, 1)$ with which any consumer has a match with that product. We assume that $\mu_i > \mu_{i+1}$ and $\lim_{i\to\infty} \mu_i = 0$; that is, products are labeled in descending order of popularity and there is a finite number of products meeting any given level of popularity. Conditional on having a match, a consumer's valuation, v, is distributed over $[0, \infty)$ with c.d.f. $F(\cdot)$. Consumers' match-conditional valuations are i.i.d. across consumers but, conditional on a match, a consumer's valuation is the same for all products.

Consumer search is random and sequential: consumers first learn their search costs

[9]Throughout the paper, we assume that the price charged by the monopolist for product i, p_i, is the same for all consumers. In particular, the monopolist cannot condition the price on the consumer's search history or other consumer characteristics.

and observe the number of products offered by the firm (but not their identity), and decide whether to start searching or not. If a consumer chooses not to search, his payoff is zero. If he chooses to search, he pays the search cost c and randomly inspects one of the products. Upon inspection, he observes the price of the product and learns whether or not he has a match with that product; if he does, he learns his match-conditional valuation v. He then decides whether or not to purchase the product; if he does, he stops searching. Otherwise, he decides whether to inspect another product, thereby incurring again the search cost c, and so on. Importantly, consumers never learn the identity of inspected products; instead, they update their beliefs on the basis of products' prices, whether or not they had a match and, if so, their match-conditional valuations.

The timing is as follows:

Stage 1 The firm chooses the size n of its product line, which is observed by consumers.

Stage 2 The firm chooses the composition of its product portfolio (i.e., it chooses $\mathcal{I}$ subject to $|\mathcal{I}| = n$) and sets its prices, none of which is observed by consumers.

Stage 3 Consumers sequentially choose whether to inspect (randomly) the offered products.

We will characterize the (pure-strategy) Perfect Bayesian Equilibria (PBE) of this game. For any size of the product line chosen in stage 1, stages 2 and 3 form a proper subgame of incomplete information. Hence, the continuation equilibrium strategies of the firm (product selection and pricing decisions) and of the consumers (search and purchasing decisions) must constitute a PBE of the subgame. In this subgame, consumers never observe the composition of the product portfolio but, upon inspection, observe the prices charged by the firm, whether there is a match, and moreover learn their valuation upon the first match. As long as they observe prices that are consistent with the firm's equilibrium strategy,[10] consumers update their beliefs according to Bayes' rule, using all relevant information. In particular, the occurrence of a match is informative of the popularity of the inspected product. The observed prices may also be informative when equilibrium prices differ across products. By contrast, as a consumer's match valuation is the same for all products, the realization of this valuation does not add any information to that conveyed by the sequence of matches.

When instead consumers observe unexpected prices, Bayes' rule has no bite. However, it is usual to restrict attention to off-equilibrium beliefs that are *consistent* beliefs, that is, based on relevant information;[11] in our setting, this rules out beliefs that would depend on search costs and realized valuations. Still, when encountering an out-of-equilibrium price

[10] As $0 < \mu_i < 1$ for every product i, any observed match sequence is consistent with any strategy of the firm.

[11] See Kreps and Wilson (1982).

in a given subgame, consumers may wonder about the implications of this unexpected deviation in the firm's strategy for the not-yet-inspected products and prices. Common assumptions include *passive*[12] and *wary*[13] beliefs: in our setting, the former means that, when encountering one or more out-of-equilibrium prices during their search process, consumers stick to the belief that the firm has chosen the subgame equilibrium product portfolio and set the subgame equilibrium prices for all the not-yet-inspected products; the latter means instead that consumers expect the firm to have chosen the optimal product portfolio and prices, given the observed out-of-equilibrium price(s).

For tractability, we will use passive beliefs for our baseline analysis. However, when possible we will further focus on equilibria that are robust to alternative (consistent) beliefs:

Definition 1 *A PBE is belief-proof if it remains a PBE when replacing consumers' off-equilibrium beliefs and behavior*[14] *with any other consistent off-equilibrium beliefs and any off-equilibrium behavior induced by these beliefs.*

In other words, an equilibrium is belief-proof if it remains an equilibrium, no matter how consumers interpret unexpected deviations, as long as consumers do not react differently based on their search costs or their realized valuations. By construction, the set of belief-proof equilibrium outcomes is a subset of passive-belief equilibrium outcomes.

Let $\pi(p) \equiv p[1 - F(p)]$ and

$$s(p) \equiv \int_p^\infty (v - p)dF(v)$$

denote the expected profit and consumer surplus, respectively, generated by a match with a product priced at p. We assume that $\pi(p)$ has a unique maximizer, denoted p^m, and let $\pi^m \equiv \pi(p^m)$ and $s^m \equiv s(p^m)$.

3 Equilibrium Analysis

We first show in Section 3.1 that, for any size of the product line chosen in stage 1, there exists a belief-proof continuation equilibrium, in which the firm offers the most popular products at the monopoly price, and any consumer who starts searching keeps doing so until finding a match. We also show that this equilibrium maximizes the profit expected from any active consumer (i.e., any consumer who start inspecting the products) and constitutes the unique belief-proof continuation equilibrium. We note in in Section 3.2

[12] See, e.g., Hart and Tirole (1990) and O'Brien and Shaffer (1992).

[13] See, e.g., McAfee and Schwartz (1994).

[14] That is, purchasing and search decisions subsequent to encountering an unexpected price.

that this equilibrium exhibits "choice overload": an increase in the size of the product line reduces the number of consumers who start searching. We then derive the implications of choice overload for the equilibrium product variety: we provide in Section 3.3 conditions under which the variety offered is socially excessive or insufficient, and analyze in Section 3.4 some of its determinants.

3.1 Pricing and Product Selection

Throughout this section, we fix the size n of the product portfolio chosen by the firm and study the continuation subgame. It is instructive to begin with the case where the firm chose to offer a single product, i.e., $n = 1$. Consider a candidate continuation equilibrium in which the firm offers product i at price p_i. A consumer with search cost c thus inspects the product if and only if $c \leq \mu_i s(p_i)$. Suppose now that the firm deviates by offering product j at price p_j. As consumers cannot observe this deviation before inspecting the product, it does not affect their search behavior, and consumers then buy if they have a match with a valuation exceeding p_j; the profit resulting from this deviation is therefore

$$G\left(\mu_i s(p_i)\right) \mu_j \pi(p_j),$$

where $G\left(\mu_i s(p_i)\right) > 0$ as consumers with sufficiently small search costs find it optimal to inspect the product. This expression is maximized by setting $j = 1$ and $p_j = p^m$. Hence, in the unique equilibrium of this subgame, the firm selects the most popular product, $\mathcal{I} = \{1\}$, and charges the monopoly price p^m.[15] Consumers with search costs $c \leq \mu_1 s^m$ inspect the product (and purchase it if they have a match with valuation $v \geq p^m$), whereas those with higher search costs do not. The firm's equilibrium profit in subgame $n = 1$ is thus equal to $G\left(\mu_1 s^m\right) \mu_1 \pi^m$.

We now show that the logic extends to any larger product line $n \geq 2$. Consider first a candidate equilibrium in which the firm offers the product portfolio $\mathcal{I}$ (such that $|\mathcal{I}| = n$) and prices are *uninformative*: all products are offered at the same price (or price lottery). Let $\alpha_k(\mathcal{I})$ denote the probability of a match at the k^{th} inspection, conditional on not having had any match at the previous $k-1$ inspections, we have:

Lemma 1 (increasing optimism) *For any product portfolio $\mathcal{I}$ of size $n \geq 2$, if prices are uninformative, then $\alpha_k(\mathcal{I})$ is strictly increasing in $k \in \{1, ..., n\}$; as a result, any consumer who starts searching keeps searching until finding a match.*

Proof. See Appendix A. ■

[15] The equilibrium would not necessarily be unique if we had assumed that consumers' search costs are bounded away from zero: in that case, there may be a "coordination failure" equilibrium in which the firm offers a very unpopular product and no consumer engages in search.

As prices are uninformative, consumers update their beliefs only on the basis of their match sequences. That $\alpha_k(\mathcal{I})$ increases with k then follows from the fact that, when an inspected product does not yield a match, consumers put a higher probability on that product being one of the less popular products, and therefore become more optimistic about finding a match at the next inspection. This, in turn, encourages consumers to keep searching until finding a match.

Remark 1 (search addiction) *That consumers become more optimistic in the absence of a match gives rise to a search addiction pattern: consumers who do not want to start searching may choose to keep searching if coerced to do a first search, the result of which turns out to be unsuccessful.*

It follows from Lemma 1 that a consumer who starts searching finds a match with probability

$$M(\mathcal{I}) \equiv 1 - \prod_{i \in \mathcal{I}} (1 - \mu_i) . \tag{1}$$

The next lemma shows further that, to maximize the expected per-consumer profit, the firm cannot do better than charging the monopoly price:

Lemma 2 (monopoly profit) *Fix the size n of the product line. In the continuation subgame, for any given (consistent) consumer beliefs and any optimal search strategy given these beliefs (off as well as on the equilibrium path), the firm cannot obtain an expected per-consumer profit greater than $M(\mathcal{I}_n)\pi^m$.*

Proof. See Appendix B. ■

By construction, by offering any given product portfolio $\mathcal{I}$ at any given constant price p, the firm cannot obtain an expected per-consumer profit greater than $M(\mathcal{I})\pi(p)$, which is maximal for $\mathcal{I} = \mathcal{I}_n$ and $p = p^m$. Lemma 2 shows further that the firm cannot gain from charging different prices across products: although this could enable the firm to discriminate among consumers according to their (costs and) valuations, consumers' self-selection actually works against the interest of the firm, as it is precisely the consumers with higher valuations that are more prone to keep searching, and thus likely to find lower prices. To see this, consider the subgame $n = 2$, a candidate continuation equilibrium in which the firm offers products i and j at prices p_i and $p_j < p_i$, and consider those consumers who learn their valuations on the first inspection. Obviously, consumers who first inspect the lower-priced product, j, stop searching. For those who inspect product i first, the expected benefit of inspecting the other product is equal to $-c + \mu_j [\max\{v - p_j, 0\} - \max\{v - p_i, 0\}]$, which is weakly increasing in v, and strictly so if $v \in (p_i, p_j)$. As a result consumers with higher valuations are more likely to find the lower price p_j.

It follows from Lemma 2 that offering the most popular products at the monopoly price does constitute a belief-proof continuation equilibrium: deviating from this candidate equilibrium cannot affect the number of consumers who start searching, and can only decrease the profit generated by them, regardless of how consumers revise their beliefs when encountering unexpected prices. Conversely, this constitutes the unique belief-proof continuation equilibrium: if consumers' beliefs are "optimistic", in that they induce consumers to keep searching when encountering unexpected prices, then, starting from any other candidate equilibrium, the firm could obtain (arbitrarily close to) $M(\mathcal{I}_n)\,\pi^m$ by offering the most popular products at deviating prices arbitrarily close to p^m. It also follows from Lemma 1 that, in this equilibrium, consumers are indifferent between searching and not when their search cost is equal to

$$\underline{c}_n \equiv \frac{M(\mathcal{I}_n)}{N(\mathcal{I}_n)} s^m, \tag{2}$$

where

$$N(\mathcal{I}) \equiv \frac{1}{|\mathcal{S}(\mathcal{I})|} \sum_{(i_1,\ldots,i_{|\mathcal{I}|})\in\mathcal{S}(\mathcal{I})} \left[1 + (1-\mu_{i_1}) + (1-\mu_{i_1})(1-\mu_{i_2}) + \ldots + \Pi_{k=1}^{|\mathcal{I}|-1}(1-\mu_{i_k})\right] \tag{3}$$

is the expected number of inspections until the first match, with

$$\mathcal{S}(\mathcal{I}) \equiv \{(i_1,\ldots,i_{|\mathcal{I}|}) \in \mathcal{I} \,|i_1 \neq \ldots \neq i_n\}$$

denoting the set of sequences of products in $\mathcal{I}$. Building on these observations leads to:

Proposition 1 (portfolio composition and prices) *Fix the size n of the product line; in the continuation subgame, there exists a PBE with passive beliefs in which:*

- (i) *the firm offers the most popular products at the monopoly price: $\mathcal{I} = \mathcal{I}_n$ and $p_i = p^m$ for every $i \in \mathcal{I}_n$;*
- (ii) *consumers keep searching until finding a match if their search costs is lower than $\underline{c}_n$, and do not search otherwise; upon finding a match, consumers stop searching, and purchase the product if their valuation exceeds the monopoly price.*

Furthermore, this PBE is belief-proof and its outcome is the unique belief-proof PBE outcome.

Proof. See Appendix C. ■

Remark 2 (observability of product portfolio) *In our setting, where consumers only observe the size n of the product line (but not its composition), offering the most popular* *products (i.e., $\mathcal{I} = \mathcal{I}_n$) maximizes the probability of a match, $M(\mathcal{I})$, but does not affect the number of searchers, $G(\underline{c}_n)$. If we had assumed, instead, that consumers observe the composition of the product line as well, then offering the most popular products would also maximize the number of searchers.*[16] *It follows that Proposition 1 would carry over to the case where consumers observe $\mathcal{I}$ before deciding to engage in search.*

Remark 3 (equilibria with passive beliefs) *Given the portfolio size n, Proposition 1 characterizes a PBE with passive beliefs which is also the unique belief-proof PBE of the continuation game. It is easy to see that there is no other PBE with passive beliefs and uniform pricing: with passive beliefs, a deviation from a uniform equilibrium price $p \neq p^m$ to p^m would have no impact on consumers' search behavior (from Lemma 1, active consumers keep searching until finding a match), but would increase the expected profit generated by a match. However, there exist equilibria with passive beliefs in which the firm does not charge the monopoly price on some products, and a deviation to the monopoly price would reduce the likelihood that consumers keep searching. In Online Appendix A, we provide an example of such an equilibrium, in which the firm offers products 1 and 2 at prices $p_1 = p^m$ and $p_2 < p^m$, and consumers who first encounter product 1 stop searching even in the absence of a match. From Proposition 1, this equilibrium is not belief-proof (indeed, the firm would deviate from p_2 to $\tilde{p}_2 \simeq p^m$ if this deviation were to induce consumers to keep searching in the absence of a match) but can be more profitable than the monopoly pricing equilibrium, by encouraging more consumers to start searching: as consumers do not inspect product 2 if they encounter product 1 first, they are more prone to do a first inspection.*

3.2 Choice Overload

We now examine how the size of the product line affects consumers' search behavior and welfare, assuming that the above unique belief-proof continuation equilibrium arises for any size n of the product line. We first note that expanding the product line increases the expected number of searches needed to find a match, as it reduces the average popularity of the selected products, but increases the overall probability of a match:

Lemma 3 (match probability and search intensity) *Expanding the product line increases both the overall match probability and the expected number of searches needed to find a match: $M(\mathcal{I}_n)$ and $N(\mathcal{I}_n)$ are strictly increasing in n.*

Proof. See Appendix D. ■

[16] In both cases, the argument developed in the proof of Proposition 1 can be used to show that the firm has no incentive to deviate from monopoly pricing when deviating on the composition of its product portfolio. Hence, the profit from such a deviation to $\mathcal{I}$ is given by $G(\underline{c}_n)M(\mathcal{I})\pi^m$ when only n is observed, and by $G(M(\mathcal{I})s^m/N(\mathcal{I}))M(\mathcal{I})\pi^m$ when $\mathcal{I}$ is observed; in both cases, this profit is maximal for $\mathcal{I} = \mathcal{I}_n$.

The following proposition shows that, despite this trade-off, expanding the product line reduces the number of consumers who search – to the point that consumer demand would vanish if the firm were to offer all products:

Proposition 2 (choice overload) *Expanding the product line reduces the number of consumers who search: (i)* $\underline{c}_n$ *is strictly decreasing in* n*, and (ii)* $\lim_{n\to\infty} \underline{c}_n = 0$.

Proof. Part *(i)*. The expected surplus of a consumer with search cost c when facing a product line of size n can be expressed as

$$S(c;n) \equiv \frac{\sum_{i=1}^{n} \mu_i}{n} s^m - c + \sum_{i=1}^{n} \left\{ \frac{1-\mu_i}{n} \left[M(\mathcal{I}_n \backslash \{i\}) s^m - N(\mathcal{I}_n \backslash \{i\}) c \right] \right\},$$

where the first two terms correspond to the expected surplus from the first inspection, and each term in curly brackets corresponds to the expected surplus from subsequent searches, conditional on the first product having been inspected being product i.

As $\partial S(c;n)/\partial c < 0$, the marginal type $\underline{c}_n$ is uniquely determined by $S(\underline{c}_n;n) = 0$ and, to prove that $\underline{c}_n > \underline{c}_{n+1}$, we only need to show that $S(\underline{c}_n; n+1) < 0$. We have:

$$\begin{aligned} S(\underline{c}_n; n+1) &= \frac{\sum_{i=1}^{n+1} \mu_i}{n+1} s^m - \underline{c}_n + \frac{1-\mu_{n+1}}{n+1} \left[M(\mathcal{I}_n) s^m - N(\mathcal{I}_n) \underline{c}_n \right] \\ &\quad + \sum_{i=1}^{n} \left\{ \frac{1-\mu_i}{n+1} \left[M(\mathcal{I}_{n+1} \backslash \{i\}) s^m - N(\mathcal{I}_{n+1} \backslash \{i\}) \underline{c}_n \right] \right\} \\ &= \frac{\sum_{i=1}^{n+1} \mu_i}{n+1} s^m - \underline{c}_n + \sum_{i=1}^{n} \left\{ \frac{1-\mu_i}{n+1} \left[M(\mathcal{I}_{n+1} \backslash \{i\}) s^m - N(\mathcal{I}_{n+1} \backslash \{i\}) \underline{c}_n \right] \right\}, \quad (4) \end{aligned}$$

where the last equality follows from the definition of $\underline{c}_n$. To conclude the proof, it suffices to note that, on the RHS of (4): (a) the sum of the first two terms is strictly negative; and (b) each term in curly brackets is strictly negative as well.

To see (a), recall first that $(\sum_{i=1}^{n} \mu_i)/n$ is strictly decreasing in n, implying that

$$\frac{\sum_{i=1}^{n+1} \mu_i}{n+1} s^m - \underline{c}_n < \frac{\sum_{i=1}^{n} \mu_i}{n} s^m - \underline{c}_n.$$

Furthermore, $\underline{c}_n$ is such that

$$[\alpha_1(\mathcal{I}_n) s^m - \underline{c}_n] + [1-\alpha_1(\mathcal{I}_n)][\alpha_2(\mathcal{I}_n) s^m - \underline{c}_n] + ... + \Pi_{i=1}^{n-1}[1-\alpha_i(\mathcal{I}_n)][\alpha_n(\mathcal{I}_n) s^m - \underline{c}_n] = 0,$$

where $\alpha_k(\mathcal{I}_n)$ is the probability of a match at the k^{th} inspection, conditional on no prior match and, from Lemma 1, is strictly increasing in k. Hence, the first term (in brackets) on the LHS of the last equation is non-positive (otherwise, the sum of the terms would be strictly positive). The conclusion then follows from the observation that $\alpha_1(\mathcal{I}_n) = (\sum_{i=1}^{n} \mu_i)/n$.

Finally, (b) follows from the definition of $\underline{c}_n$, in conjunction with the observation that, for any $i \leq n$, $M(\mathcal{I}_{n+1}\backslash\{i\}) < M(\mathcal{I}_n)$ and $N(\mathcal{I}_{n+1}\backslash\{i\}) > N(\mathcal{I}_n)$: this can be seen from (1) and (3), using $\mu_i > \mu_{n+1}$.

Part *(ii)*. See Appendix E. ■

Proposition 2 shows that increasing the size of the product line from n to $n+1$ reduces the number of consumers who search. It implies that consumers with search costs $\underline{c}_{n+1} < c < \underline{c}_n$ suffer from the expansion of the product line: for them, the benefit from the increased probability of a match is outweighed by the increased expected number of searches needed to find a first match. In contrast, consumers with sufficiently low search costs benefit on net as they are hurt less by the increase in the expected number of searches.

Building on these observations leads to:

Corollary 1 (impact of product line on consumers) *For any $n \in \mathbb{N}^*$, there exists $\hat{c}_n \in (0, \underline{c}_{n+1})$ such that increasing the size of the product line from n to $n+1$ does not affect consumers with search costs $c \geq \underline{c}_n$ (as they do not search anyway) but hurts consumers with search costs $c \in (\hat{c}_n, \underline{c}_n)$, while benefiting consumers with search costs $c < \hat{c}_n$.*

Proof. Consider an increase in the size of the product line from n to $n+1$. From Proposition 2, consumers with search costs exceeding $\underline{c}_n$ do not search under either scenario and are therefore unaffected, whereas those with search costs between $\underline{c}_{n+1}$ and $\underline{c}_n$ no longer search and are therefore worse off (by a standard revealed preference argument). For consumers with a search cost lower than $\underline{c}_{n+1}$, the impact of the product line expansion can be expressed as:

$$[M(\mathcal{I}_{n+1}) - M(\mathcal{I}_n)]s^m - [N(\mathcal{I}_{n+1}) - N(\mathcal{I}_n)]c,$$

where both terms in brackets are strictly positive. It follows that consumers with search costs

$$c < \left(\frac{M(\mathcal{I}_{n+1}) - M(\mathcal{I}_n)}{N(\mathcal{I}_{n+1}) - N(\mathcal{I}_n)}\right) s^m \equiv \hat{c}_n$$

are better off, whereas those with $c > \hat{c}_n$ are worse off. ■

3.3 Product Variety

We now turn to the firm's decision about the size of its product line. If all products had the same match probability $\mu > 0$, then the search cost threshold would be independent of n (and given by $\underline{c}_n = \mu s^m$), and the firm would therefore wish to offer as many products as possible. However, under our more plausible assumption that the popularity of additional products tend to vanish as the firm expands its product line, the choice overload effect identified in Proposition 2 implies that the firm chooses to limit the size of its product line, even in the absence of any fixed cost of introducing products:

Proposition 3 (limited product line) *There exists an equilibrium. In equilibrium, the firm offers a finite number of products:* $\mathcal{I}^* = \{1, 2, ..., n^*\}$*, where* $1 \leq n^* < \infty$*.*

Proof. We have already shown that there exists a unique belief-proof continuation equilibrium for any size n of the product line. By assumption, $G(c) > 0$ for any $c > 0$. This implies that, for any size n of the firm's product line, there is a strictly positive mass, $G(\underline{c}_n)$, of consumers who search, generating a strictly positive level of profit, given by

$$\Pi_n \equiv G\left(\underline{c}_n\right) M\left(\mathcal{I}_n\right) \pi^m. \tag{5}$$

However, this profit tends to vanish as n goes to infinity as it is bounded by $G\left(\underline{c}_n\right) \pi^m$, where, from Proposition 2 and the continuity of G at 0, $\lim_{n \to \infty} G(\underline{c}_n) = 0$. Hence, there exists $\bar{n}$ such that $\Pi_n < \Pi_1$ for any $n > \bar{n}$, and in equilibrium the firm never offers more than $\bar{n}$ products. Conversely, for any $n^* \in \arg\max_{n \in \{1,...,\bar{n}\}} \{\Pi_n\}$, there exists an equilibrium in which the firm offers n^* products. ■

To analyze whether the firm will provide too much or too little product variety from consumers' or society's point of view, note that consumer surplus, as a function of the size n of the product line, can be expressed as

$$\begin{aligned} S_n &\equiv \int_0^{\underline{c}_n} \left[M(\mathcal{I}_n)s^m - N(\mathcal{I}_n)c\right] dG(c) \\ &= N(\mathcal{I}_n) \int_0^{\underline{c}_n} \left[\underline{c}_n - c\right] dG(c) \\ &= N(\mathcal{I}_n) \left[\underline{c}_n - c^e(\underline{c}_n)\right] G(\underline{c}_n), \end{aligned} \tag{6}$$

where $c^e(\underline{c}_n) \equiv \int_0^{\underline{c}_n} c dG(c)/G(\underline{c}_n)$ is the average search cost of those consumers who search in equilibrium. Let $n^S \equiv \arg\max_n S_n$ and $n^W \equiv \arg\max_n W_n$ denote the size of the product line that maximizes consumer surplus and social welfare, respectively, where $W_n \equiv \Pi_n + S_n$ denotes social welfare when the firm offers n products. The same argument as in the proof of Proposition 3 implies that both n^S and n^W are finite.[17]

We have:

Proposition 4 (private vs. social provision of product variety) *If the average cost of inframarginal consumers,* $c^e(c)$*, is proportional to the cost of the marginal consumer,*

[17] For the sake of exposition, we assume here that n^S, n^W and n^* are all unique; in (non-generic) cases of multiple optima, Proposition 4 applies to every solution in $\arg\max_n S_n$, $\arg\max_n W_n$ and $\arg\max_n W_n$.

c, then the equilibrium size of the product line maximizes consumer surplus and social welfare.

If instead $c^e(c)$ increases less (resp., more) than proportionally with c, then the firm tends to offer more (resp., fewer) products than what would maximize consumer surplus or social welfare: $n^* \geq n^S, n^W$ (resp., $n^* \leq n^S, n^W$).

Proof. When the firm offers n products, its profit is given by (5) which, using $M(\mathcal{I}_n)\, s^m = N(\mathcal{I}_n)_n \underline{c}_n$, can be expressed as

$$\Pi_n = N(\mathcal{I}_n)\underline{c}_n G(\underline{c}_n)\frac{\pi^m}{s^m}.$$

From (6), consumer surplus can be written as

$$S_n = [1 - \rho(\underline{c}_n)]\, N(\mathcal{I}_n)\, \underline{c}_n G(\underline{c}_n),$$

where

$$\rho(c) \equiv \frac{c^e(c)}{c} < 1.$$

It follows that if $c^e(\underline{c}_n)$ is proportional to $\underline{c}_n$ (that is, if $\rho(\cdot)$ is constant), then S_n, Π_n and W_n are all maximal for $n^* = \arg\max_n N(\mathcal{I}_n)\, \underline{c}_n G(\underline{c}_n)$.

More generally, using a revealed preference argument, we have:

$$\begin{aligned} S_{n^S} &= [1 - \rho(\underline{c}_{n^S})]\, N(\mathcal{I}_{n^S})\, \underline{c}_{n^S} G(\underline{c}_{n^S}) \\ &\geq S_{n^*} = [1 - \rho(\underline{c}_{n^*})]\, N(\mathcal{I}_{n^*})\, \underline{c}_{n^*} G(\underline{c}_{n^*}) \\ &\geq [1 - \rho(\underline{c}_{n^*})]\, N(\mathcal{I}_{n^S})\, \underline{c}_{n^S} G(\underline{c}_{n^S}), \end{aligned}$$

where the last inequality follows from $\Pi_{n^*} \geq \Pi_{n^S}$, and thus

$$\rho(\underline{c}_{n^*}) \geq \rho(\underline{c}_{n^S}). \tag{7}$$

From (7), it follows that if $\rho(\cdot)$ is decreasing, then $\underline{c}_{n^S} \geq \underline{c}_{n^*}$. To show that this implies $n^S \leq n^*$, suppose otherwise that $n^S > n^*$. From Lemma 3, we would have

$$\Pi_{n^S} = N(\mathcal{I}_{n^S})\underline{c}_{n^S} G(\underline{c}_{n^S})\frac{\pi^m}{s^m} > N(\mathcal{I}_{n^*})\underline{c}_{n^*} G(\underline{c}_{n^*})\frac{\pi^m}{s^m} = \Pi_{n^*},$$

a contradiction.

If instead $\rho(\cdot)$ is increasing, then, from (7), $\underline{c}_{n^S} \leq \underline{c}_{n^*}$. To show that this implies $n^S \geq n^*$, suppose otherwise that $n^S < n^*$. Noting that, integrating by parts, consumer surplus can be expressed as

$$S_n = N(\mathcal{I}_n)\int_0^{\underline{c}_n} (\underline{c}_n - c)\, dG(c) = N(\mathcal{I}_n)\int_0^{\underline{c}_n} G(c),$$

Lemma 3 would then imply

$$S_{n^S} = N(\mathcal{I}_{n^S}) \int_0^{\underline{c}_{n^S}} G(c) < N(\mathcal{I}_{n^*}) \int_0^{\underline{c}_{n^*}} G(c) = S_{n^*},$$

a contradiction.

A similar reasoning applies to social welfare. Using a revealed preference argument, we have:

$$\begin{aligned} W_{n^W} &= \left\{1 + [1 - \rho(\underline{c}_{n^W})] \frac{s^m}{\pi^m}\right\} \Pi_{n^W} \\ &\geq W_{n^*} = \left\{1 + [1 - \rho(\underline{c}_{n^*})] \frac{s^m}{\pi^m}\right\} \Pi_{n^*} \\ &\geq \left\{1 + [1 - \rho(\underline{c}_{n^*})] \frac{s^m}{\pi^m}\right\} \Pi_{n^W}, \end{aligned}$$

which in turn implies

$$\rho(\underline{c}_{n^W}) \leq \rho(\underline{c}_{n^*}).$$

The same reasoning as above then shows that $n^W \leq n^*$ (resp., $n^W \geq n^*$) if $\rho(\cdot)$ is decreasing (resp., increasing). ■

We show in Online Appendix B that the average inframarginal cost $c^e(c)$ is proportional to c if and only if search costs are distributed according to a power law, that is, c is distributed over $[0, \bar{c}]$ according to $G(c) = (c/\bar{c})^\alpha$ for some $\alpha > 0$ – this holds, for instance, when the distribution is uniform: $G(c) = c/\bar{c}$. If instead the density of search costs is of the form $g(c) = 1 + \alpha(c - 1/2)$, where $|\alpha| \leq 2$, then $c^e(c)$ increases less (resp., more) than proportionally with c if $\alpha > 0$ (resp., $\alpha < 0$).

The intuition underlying Proposition 4 is reminiscent of Spence's (1975) price versus quality trade-off. In Spence's analysis, the cost of increasing quality is borne by the firm and is thus the same from the private and social standpoints, whereas the private and social benefits of such an increase may differ: the firm focuses on the impact of quality on marginal consumers, as this is what drives demand, whereas consumer surplus and social welfare account for the impact of quality on infra-marginal consumers as well. In our model, expanding the product line increases the probability of a match, $M(\mathcal{I}_n)$, which affects the firm and consumers in the same way, but it also increases the expected number of inspections, $N(\mathcal{I}_n)$, which may affect differently marginal and infra-marginal consumers. This explains why the firm tends to provide too much (resp., too little) product variety if the average search cost of the inframarginal consumers increases less (resp., more) than proportionally with the search cost of the marginal consumer.[18]

[18]Because n is necessarily an integer, however, the firm may provide the socially optimal product variety even when its incentives are biased.

3.4 Comparative Statics

We provide here some insights on the determinants of the size of the product line. We start with a brief discussion of the impact of relative popularity of the products, before relating the optimal size of the product line to key elasticities.

To study the impact of relative popularity of the products, we focus on the choice between one versus two products:

Lemma 4 (impact of relative product popularity) *Suppose that there are two products available, of popularity μ_1 and μ_2 respectively, and that the cost distribution has a continuous density $g(\cdot)$ satisfying $g(0) > 0$. There exists a continuous function $\hat{\eta}(\mu_1) \in [0, 1]$, satisfying $\hat{\eta}(0) \simeq 0.585$ and $\hat{\eta}(1) = 0$, such that it is optimal for the firm to offer both products if and only if*

$$\mu_2 > [1 - \hat{\eta}(\mu_1)] \mu_1.$$

Proof. See Online Appendix C. ■

As is intuitive, it is optimal to introduce a second product if it is sufficiently popular. Interestingly, the acceptable (percentage) reduction of popularity, $\hat{\eta}(\mu_1)$, is lower when the first product is already popular. In particular, if the first product is highly popular (i.e., μ_1 close to 1), the second product must be almost as popular to be added (i.e., $\hat{\eta}(\mu_1)$ is close to 1). This can be contrasted with what we might expect when the size of the product line is driven by cost considerations. For example, if adding another product involves a fixed cost, we would expect narrower product lines when products are not too popular. By contrast, when the size is driven by choice overload concerns, the optimal size of the product line may be greater if both products are not too popular than if they are both popular. For example, assuming that the percentage reduction $(\mu_1 - \mu_2)/\mu_1$ remains constant and lower than $\bar{\eta}$, $n = 2$ dominates $n = 1$ when μ_1 (and thus μ_2) is close to 0, whereas the opposite holds when μ_1 (and thus μ_2) is close to 1.

We now revert to arbitrary sets of available products and relate the optimal size of the product line to key elasticities. For the sake of exposition, we ignore integer constraints and, further, assume here that the private and social optima are uniquely defined; this, in turn, enables us to rely on the first-order approach.[19] We have:

[19] Recall from Proposition 2 that the privately optimal number of products, n^*, is an interior solution; the same applies to n^S and n^W. Furthermore, note that if the firm's profit were to achieve its maximum for multiple sizes of the product line, almost any small perturbation would break the firm's indifference and yield uniqueness. Hence, the comparative statics provided by the following propositions hold generically for small changes in the parameters of interest.

Lemma 5 (optimality condition for product variety) *Ignoring integer constraints, the first-order condition characterizing the equilibrium size of the product line, n^*, can be expressed as*

$$\varepsilon_M(n^*) = \varepsilon_G(\underline{c}_{n^*})\,\varepsilon_{M/N}(n^*)\,, \tag{8}$$

where

$$\varepsilon_G(c) \equiv \frac{cg(c)}{G(c)}$$

denotes the elasticity of the search cost distribution $G(\cdot)$, and

$$\varepsilon_M(n) \equiv \frac{n}{M(\mathcal{I}_n)}\frac{d}{dn}(M(\mathcal{I}_n)) \text{ and } \varepsilon_{M/N}(n) \equiv -\frac{nN(\mathcal{I}_n)}{M(\mathcal{I}_n)}\frac{d}{dn}\left(\frac{M(\mathcal{I}_n)}{N(\mathcal{I}_n)}\right)$$

respectively denote the elasticities of $M(\mathcal{I}_n)$ and $M(\mathcal{I}_n)/N(\mathcal{I}_n)$.[20]

Proof. The firm's equilibrium profit as a function of n can be expressed as

$$\Pi_n = M(\mathcal{I}_n)\,G\left(\frac{M(\mathcal{I}_n)\,s^m}{N(\mathcal{I}_n)}\right)\pi^m,$$

and thus

$$\begin{aligned}\frac{d\Pi_n}{dn} &= \frac{d}{dn}(M(\mathcal{I}_n))\,G(\underline{c}_n)\pi^m + M(\mathcal{I}_n)\,s^m\frac{d}{dn}\left(\frac{M(\mathcal{I}_n)}{N(\mathcal{I}_n)}\right)g(\underline{c}_n)\,\pi^m \\ &= \frac{\Pi_n}{n}\left[\varepsilon_M(n) - \varepsilon_G(\underline{c}_n)\varepsilon_{M/N}(n)\right]. \end{aligned} \tag{9}$$

Hence, evaluated at n^*, the term in brackets (which is equal to the elasticity of the firm's profit with respect to the size of its product line) is equal to zero, yielding (8). ■

Condition (8) has a simple interpretation: Expanding the product line by one percent increases the probability of a match by ε_M percent, but decreases the marginal consumer's inspection cost threshold by $\varepsilon_{M/N}$ percent, translating into a $\varepsilon_G\varepsilon_{M/N}$ percent decrease in the population of consumers who search.

Building on this insight, allows us to provide comparative statics. The following proposition shows that the equilibrium size of the product line is independent of π^m and, depending on the search cost distribution, can either increase or decrease with s^m:

Proposition 5 (division of surplus) *The equilibrium size of the product line, n^*, is independent of π^m. By contrast, ignoring integer constraints, n^* strictly increases (resp., strictly decreases) with s^m if*

$$\frac{d\varepsilon_G}{dc}(\underline{c}_{n^*}) < 0 \text{ (resp., } > 0\text{).}$$

[20] By construction, $\varepsilon_G(c)$ and $\varepsilon_M(n)$ are both positive, for any c and n; furthermore, from Proposition 2, $\underline{c}_n$ is decreasing in n and thus $\varepsilon_{M/N}(n) > 0$.

Proof. From the proof of Lemma 5, the elasticity of the firm's profit with respect to the size of its product line is given by

$$\frac{d\Pi_n}{dn}\frac{n}{\Pi_n} = \varphi(n; s^m) \equiv \varepsilon_M(n) - \varepsilon_G\left(\frac{M(\mathcal{I}_n)}{N(\mathcal{I}_n)}s^m\right)\varepsilon_{M/N}(n).$$

As n^* is the profit-maximizing number of products (which we assume to be unique), it follows that $\varphi(n; s^m)$ is strictly decreasing in n at $n = n^*$. Furthermore,

$$\frac{\partial\varphi}{\partial s^m}(n^*; s^m) = -\varepsilon_G'(\underline{c}_{n^*})\frac{M(\mathcal{I}_{n^*})}{N(\mathcal{I}_{n^*})}\varepsilon_{M/N}(n^*),$$

and is thus strictly positive (resp., strictly negative) if $d\varepsilon_G(\underline{c}_{n^*})/dc < 0$ (resp., $d\varepsilon_G(\underline{c}_{n^*})/dc > 0$). The assertion follows from the implicit function theorem. ■

The next proposition shows that the equilibrium product line shrinks when the distribution of search costs becomes more elastic:

Proposition 6 (elasticity of search cost distribution) *Suppose that the search cost distribution can be indexed in such a way that its elasticity, $\varepsilon_G(\cdot)$, strictly increases with some parameter γ. Then, ignoring integer constraints, n^* decreases with γ.*

Proof. Following the same steps as in the proof of Proposition 5, it suffices to note that the elasticity of the firm's profit with respect to the size of its product line can be expressed as

$$\frac{d\Pi_n}{dn}\frac{n}{\Pi_n} = \varphi(n; \gamma) \equiv \varepsilon_M(n) - \varepsilon_G(\underline{c}_n; \gamma)\varepsilon_{M/N}(n).$$

The assertion then follows from the assumption that $\varepsilon_G(\cdot; \gamma)$ strictly increases with γ. ■

4 Positioning and Disclosure

So far, we have assumed that, prior to inspection, consumers could not distinguish products, neither directly nor indirectly; as a result, search is necessarily random. In some environments, however, a "positioning technology" may be available and potentially allow for directed search. For example, supermarkets place their products in "distinguishable slots" such as aisles and shelves and online platforms can make recommendations. In this section, we therefore extend our baseline setting by assuming that the firm can use such a technology to position its offerings.

We first show that a "no-positioning" equilibrium still exists, in which the firm adopts a completely random positioning policy (e.g., it randomly allocates products to slots, or makes completely uninformative recommendations); consumers are thus indifferent about the search sequence, and the outcome is as characterized by Proposition 1. Next, we show that there also exist "positioning" equilibria, in which consumer search is perfectly directed. We characterize these equilibria for the two-product case. One such equilibrium exhibits "pure positioning": the firm adopts a deterministic positioning policy, which is therefore perfectly informative; as a result, consumers inspect the more popular product first, but some of them then stop searching even in the absence of a match. Other equilibria rely instead on "noisy" positioning: the firm adopts a positioning policy which is random but not uniformly so; this suffices to direct consumer search, as consumers inspect first the more promising position, but maintains enough uncertainty to induce all active consumers to keep searching until finding a match. These equilibria are Pareto ranked: they only differ in the probability of placing the more popular product in the first position, which determines the informativeness of the positioning policy; the best equilibrium (for consumers as well as for the firm) is the most informative one, in which, in the absence of a match, consumers are indifferent between continuing searching or not.

For the sake of exposition, we assume that, for any size n of the product line, the firm places the products in n distinguishable positions, $j = 1, ..., n$, in such a way that there is one position per product, and one product per position. The timing becomes:

Stage 1 The firm publicly chooses the size n of its product line, which now also determines the number of positions.

Stage 2 The firm now privately chooses not only its product portfolio and its prices, but also the positioning of the n selected products among the n positions.

Stage 3 Consumers sequentially decide which positions to inspect.

We extend the notion of passive beliefs by assuming that, when encountering one or more out-of-equilibrium prices, consumers not only stick to the belief that the firm has chosen the equilibrium product portfolio and the equilibrium prices for the products in the not-yet-inspected positions but they also maintain the belief that the firm has chosen the equilibrium positioning policy:

4.1 Random positioning

Our first result shows that the "no-positioning" equilibrium characterized in Proposition 1 can still be sustained by completely randomizing over the slot allocation or the recommendations.

Proposition 7 (random positioning - portfolio composition and prices) *Fix the size n of the a product line; in the continuation subgame, there exist infinitely many Perfect Bayesian Equilibria with passive beliefs in which:*

(i) *the firm offers the most popular products at the monopoly price:* $\mathcal{I} = \mathcal{I}_n$ *and* $p_i = p^m$ *for every* $i \in \mathcal{I}_n$*;*

(ii) *the firm randomizes uniformly over its positioning policy;*

(iii) *consumers start searching if and only if their search cost is lower than* $\underline{c}_n$*, in which case they keep searching until finding a match; upon finding a match, consumers stop searching and purchase the product if their valuation exceeds the monopoly price.*

These equilibria all give the firm the same expected profit, equal to

$$\Pi_n = G\left(\underline{c}_n\right) M\left(\mathcal{I}_n\right) \pi^m.$$

They only differ in the order in which consumers inspect the positions: any (deterministic or random) distribution of search sequences for each of the active consumers can be supported in equilibrium.

Proof. The proof follows closely that of Proposition 1. To see that the firm has no incentive to deviate from the random positioning policy, note first that such a deviation would not be observed by consumers; hence, it would not affect consumer participation. Second, note that all active consumers keep searching until finding a match. As a result, the firm is indifferent about its positioning policy. Conversely, given that the firm uniformly randomizes over its positioning policy, consumers are indifferent as to the order in which they inspect the positions – even if they were to observe out-of-equilibrium prices, due to passive beliefs. Hence, any arbitrary search sequence can be supported in equilibrium, and different consumers can pick different sequences. ■

This equilibrium corresponds to the case in which the firm adopts a completely uninformative positioning policy; for example, it uniformly randomizes over the assignment of products to slots, or the order in which it recommends consumers to inspect products. In equilibrium, consumers are then indifferent as to whether or not to follow the recommendations (and may or may not do so).

If this continuation equilibrium is played for every size of the product line, then the overall equilibrium is the same as that characterized by Proposition 3:

Corollary 2 (random positioning - product line) *There exists an equilibrium in which the firm chooses a product line of size* $n^* \in \arg\max_n \Pi_n$*, and offers products in* $\mathcal{I} = \{1, 2, ..., n^*\}$ *at the monopoly price, and randomizes uniformly over its positioning policy.*

The insights from our previous analysis still apply to that equilibrium. However, as shown below, there now also exist equilibria featuring some positioning.

4.2 Positioning

We now turn our attention to equilibria in which the firm adopts an informative positioning policy. For the sake of exposition, throughout the rest of this section we assume that the firm can at most offer two products: $n \in \{1, 2\}$.

Obviously, positioning is a moot issue if the firm chooses to offer a single product. In that case, there is a unique continuation equilibrium, in which the firm offers the most popular product at the monopoly price (i.e., $p_1 = p^m$) and consumers inspect the product if their search cost is lower than $\underline{c}_1$, in which case they buy if they have a match with a valuation exceeding p^m; the resulting profit is thus $\Pi_1 = G(\underline{c}_1)\,\mu_1\pi^m$.

We now study the subgame when the firm chooses to offer two products. For the sake of exposition, we focus on equilibria in which the firm still charges monopoly prices; it readily follows that it selects the two most popular products (that is, $\mathcal{I} = \{1, 2\}$)[21] and that, upon finding a match, consumers stop searching (and then purchase the product if their valuation exceeds the monopoly price).

We first note that positioning induces a directed search behavior:

Lemma 6 (directed search) *Fix the size of the product line to $n = 2$, and consider a continuation equilibrium in which the firm offers the two most popular products at monopoly prices: $\mathcal{I} = \{1, 2\}$ and $p_1 = p_2 = p^m$; unless the firm randomizes uniformly over its positioning policy, active consumers inspect first the position that is more likely to correspond to the more popular product.*

Proof. See Appendix F. ■

The intuition is simple: in any equilibrium with informative positioning, in the sense that the firm is more likely to place the more popular product in a given position, consumers prefer to inspect that position first, as this increases the probability of a match if they stop searching afterwards, and otherwise decreases the probability of having to incur the cost of a second inspection. In what follows, whenever an equilibrium exhibits directed search, we refer to first inspected position as the "first position" and to the other one as the "second position".

[21] Starting from a candidate equilibrium in which the firm selects product $j > 2$ rather than product $i \in \{1, 2\}$, replacing j with i would increase its expected profit. This is obvious for those consumers who then inspect product i on a second inspection: they would do so only if they did not have a match on the first inspection, and increasing the probability of a match on that second inspection would therefore benefit both the firm and consumers. But this is also the case for consumers who inspect product i on a first inspection, as the expected profit conditional on a match is then π^m, and is lower in the absence of a match, as the consumer may stop searching, and a match on the second inspection is not certain anyway.

4.2.1 Pure positioning

The next proposition characterizes the (essentially unique) equilibrium featuring "pure positioning", that is, in which the positioning policy is deterministic and thus perfectly informative:

Proposition 8 (pure positioning) *Suppose that the firm has chosen to offer two products: $n = 2$; in the continuation subgame, there exists a Perfect Bayesian Equilibrium with passive beliefs in which:*

(i) *the firm offers the most popular products at the monopoly price: $\mathcal{I} = \{1, 2\}$ and $p_1 = p_2 = p^m$;*

(ii) *the firm places product 1 in first position, and product 2 in second position;*

(iii) *consumers start searching if and only if their search cost is lower than $\underline{c}_1 = \mu_1 s^m$, in which case they start with the first position and, in the absence of a match, inspect the second position if and only if their search cost is lower than $\mu_2 s^m$.*

The firm's expected profit is equal to

$$\Pi^P \equiv G(\mu_1 s^m)\, \mu_1 \pi^m + G(\mu_2 s^m)\,(1 - \mu_1)\mu_2 \pi^m.$$

Finally, there is no other equilibrium with passive beliefs, monopoly pricing and a deterministic positioning policy.

Proof. See Appendix G. ■

A deterministic (and, thus, perfectly informative) positioning policy encourages more consumers to start searching, as they expect to find the most popular product on their first inspection. However, fewer consumers keep searching in the absence of a match: without positioning, all consumers with a cost lower than $\underline{c}_2$ not only start searching, but keep doing so until finding a match. By contrast, with positioning, only those with a cost lower than $\mu_2 s^m$ ($< \underline{c}_2$) are willing to make a second inspection. [22]

If the positioning equilibrium is played when the firm chooses to offer both products, expanding the product line does not discourage consumers from searching: expecting to find the most popular product on their first inspection, all consumers with search cost $c < \underline{c}_1 = \mu_1 s^m$ inspect it, as is the case when the firm chooses to offer only that product. It

[22] To see that $\underline{c}_2 > \mu_2 s^m$, note that:

$$\underline{c}_2 - \mu_2 s^m = \left[\frac{\mu_1 + \mu_2 - \mu_1 \mu_2}{2 - \frac{\mu_1 + \mu_2}{2}} - \mu_2 \right] s^m = \frac{(2 - \mu_2)(\mu_1 - \mu_2)}{4 - \mu_2 - \mu_1} s^m > 0.$$

follows that expanding the product line is always profitable, as some consumers (namely, those with cost $c < \mu_2 s^m$) inspect the second product as well:

Corollary 3 (pure positioning - product line) *Suppose that the firm can offer at most two products; there exists an equilibrium in which the firm offers the two most popular products at the monopoly price, and adopts a pure positioning policy.*

Interestingly, despite inducing more consumers to start searching, positioning may *decrease* the profitability of the firm, as it discourages a fraction of them from making a second inspection. Indeed, the firm is better off in the random positioning equilibrium whenever $\Pi_2 \geq \Pi^P$,[23] which amounts to

$$G\left(\underline{c}_2\right) > \frac{\mu_1 G\left(\mu_1 s^m\right) + \left(1-\mu_1\right)\mu_2 G\left(\mu_2 s^m\right)}{\mu_1 + \left(1-\mu_1\right)\mu_2},$$

where

$$\underline{c}_2 = \frac{\mu_1 + \mu_2 - \mu_1\mu_2}{2 - \frac{\mu_1+\mu_2}{2}} s^m \in (\mu_2 s^m, \mu_1 s^m).$$

Holding μ_1 and μ_2 fixed, this condition holds if the distribution of search costs is such that $G(\mu_1 s^m) - G(\underline{c}_2)$ is small relative to $G(\underline{c}_2) - G(\mu_2 s^m)$. By contrast, the positioning equilibrium is more profitable for instance when, holding the distribution $G(\cdot)$ of search costs fixed, μ_2 is sufficiently small (close to zero) or sufficiently large (close to μ_1);[24] this is actually the case for any $\mu_2 < \mu_1$ when G is uniform.

4.2.2 Noisy positioning

The equilibria characterized by Propositions 7 and 8 feature two alternative, arguably extreme ways of resolving the trade-off between encouraging more consumers to participate (the extensive margin) on the one hand, and fostering the search intensity of these consumers (the intensive margin) on the other: the pure positioning equilibrium maximizes consumer participation, whereas the random positioning equilibrium ensures that all participating consumers keep searching until finding a match. Potentially, there is a variety of intermediate solutions, in which the more popular product is more likely to be placed in first position (thus inducing consumers to start with that position, and also encouraging more of them to do so), but is also placed in second position with some probability (so as to encourage consumers to keep searching). The next proposition shows that among these intermediate solutions, those that maximize search intensity, and only those, can indeed be supported in equilibrium:

[23] Positioning is irrelevant when it chooses to offer a single product, and the positioning equilibrium is more profitable when the firm chooses to offer two products than one; hence, $\Pi^P > \Pi_1$.

[24] As $\mu_2 \to 0$, the LHS converges to $G(\mu_1 s^m/(2-\mu_1/2))$, whereas the RHS converges to $G(\mu_1 s^m) > G(\mu_1 s^m/(2-\mu_1/2))$. As $\mu_2 \to \mu_1$, the LHS converges to the RHS from below.

Proposition 9 (noisy positioning) *Suppose that the firm has chosen to offer two products: $n = 2$. There exists an $\bar{r} < 1$, which is decreasing in μ_2 and increases from $1/2$ to 1 as μ_1 increases from μ_2 to 1, such that, for any $r \in [1/2, \bar{r}]$, in the continuation subgame there exists a Perfect Bayesian Equilibrium with passive beliefs in which:*

(*i*) *the firm offers the most popular products at the monopoly price: $\mathcal{I} = \{1, 2\}$ and $p_1 = p_2 = p^m$;*

(*ii*) *with probability r, the firm places product 1 in first position and product 2 in second position; with complementary probability $1 - r$, it does the reverse;*

(*iii*) *consumers start searching if and only if their search cost is lower than some threshold $\underline{c}_2^N(r)$, in which case they start with the first position and keep searching in the absence of a match; their total probability of having a match is therefore maximal and equal to:*

$$M_2 \equiv \mu_1 + \mu_2 - \mu_1\mu_2.$$

These equilibria differ in the probability r of placing the more popular product in first position; the firm's expected profit, given by

$$\Pi^N(r) \equiv G\left(\underline{c}_2^N(r)\right) M_2 \pi^m,$$

coincides with that obtained in the random positioning equilibrium for $r = 1/2$, and increases with r.

Finally, together with the equilibria characterized by Propositions 7 and 8, these equilibria are the only ones with passive beliefs and monopoly pricing.

Proof. See Appendix H. ■

This proposition shows that there is no equilibrium with noisy positioning in which a consumer stops searching when a first inspection did not produce a match. The intuition is as follows. On the one hand, positioning, that is, placing the more popular product (i.e., product 1) with greater probability in first position, induces all active consumers to start with that position. On the other hand, to be willing to maintain some noise, the firm must be indifferent about which product to place in which position. But if all active consumers start with the first position and some of them stop searching after that, even if they do not have a match, then the firm strictly prefers to place product 1 in first position, a contradiction. Hence, to sustain noisy positioning, all active consumers must keep searching in the absence of a match; this, in turn, imposes an upper bound on the probability r that product 1 is placed in first position – this upper bound is such that consumers who are indifferent between inspecting or not the first position are also indifferent between inspecting or not the second one if they do not obtain a match on the first inspection.

The limit case $r = 1/2$ constitutes one of the random positioning equilibria characterized by Proposition 7. [25] Increasing r above $1/2$ reduces the noise, which benefits consumers: a match on the first inspection becomes more likely, which reduces the likelihood of having to do a second inspection. This, in turn, induces more consumers to participate, which increases the expected profit of the firm, all the more so as the probability of finding the more popular product on the first inspection further increases. This is why these equilibria can be ranked, and the most profitable equilibrium corresponds to $r = \bar{r}$. The scope for noisy positioning moreover expands with the popularity gap between the two products: the best equilibrium boils down to random positioning when the products are similar, and instead approaches pure positioning when μ_1 tends to 1.

The equilibria with less noisy positioning are thus more profitable than the random positioning equilibrium – by the same token, making positioning less noisy may encourage the firm to offer more products: positioning is irrelevant when the firm chooses to offer a single product, but adopting a less noisy positioning policy enhances profit when the firm chooses to offer an additional product; hence, it tilts the balance in favor of the latter option – however, the firm may still prefer to offer a single product. [26] Maintaining some noise can however be optimal for the firm: indeed, noisy positioning is also more profitable than pure positioning whenever

$$G\left(\underline{c}_2^N(r)\right) > \frac{\mu_1 G(\mu_1 s^m) + (1-\mu_1)\mu_2 G(\mu_2 s^m)}{\mu_1 + (1-\mu_1)\mu_2}.$$

4.3 Disclosure

We have so far assumed that an inspection reveals product characteristics such as prices and match valuations, but does not convey any additional information about the identity of the inspected products – and, thus, of the uninspected ones. Suppose now that the firm has a "disclosure technology" at its disposal, which enables it to communicate, upon inspection, the identity of the product. Of course, the availability of such a technology does not matter if the firm offers a single product. We therefore confine attention to the subgame $n = 2$. Moreover, for simplicity, we assume that only two products, 1 and 2, are available.[27] We first note that the possibility of disclosure eliminates some of the

[25] Specifically, it corresponds to the equilibrium in which active consumers start with the first position but, anticipating a random allocation, always inspect the second position in the absence of a match.

[26] This is the case when $\Pi^R(\bar{r}) \equiv G\left(\underline{c}_2^N(\bar{r})\right) M_2 \pi^m < \Pi_1 = G(\mu_1 s^m)\mu_1 \pi^m$, which holds for μ_2 small enough, as $\lim_{\mu_2 \to 0} M_2 = \mu_1$ and $\lim_{\mu_2 \to 0} \underline{c}_2^N(\bar{r}) < \mu_1 s^m$ (see Appendix H).

[27] Otherwise we would have to specify additional out-of-equilibrium beliefs if the firm were to deviate by offering a product that it does not sell in equilibrium *and* disclosing its identity. When a consumer then inspects that product first, passive beliefs no longer pin down the consumer's beliefs about the not-yet-inspected product.

equilibria previously identified. Conversely, additional equilibria arise.

In any of the previous equilibria in which consumers keep searching until finding a match, the firm has no incentive to deviate and disclose the identity of inspected products, as this could only induce some consumers to stop searching – namely, upon learning that they already inspected the most popular product(s). It follows that the possibility of disclosure does not disrupt the monopoly pricing equilibrium characterized by Proposition 1 for the baseline setting, nor the random and noisy positioning equilibria characterized by Propositions 7 and 9 for the extended setting with positioning. These equilibria remain supported by the firm never disclosing any product, or disclosing the identity of the less popular product(s) with sufficiently small probability, so as to ensure that *not disclosing* does not constitute an excessively bad signal.

By contrast, the non-monopoly pricing equilibria discussed in Remark 3, in which prices reveal the identity of the products, disappear when disclosure becomes possible. These equilibria have the feature that popular products are offered at the monopoly price, whereas less popular products are offered at sub-optimal prices; yet, the firm is prevented from deviating to the monopoly price, as this would "signal" a popular product and prompt some consumers to stop searching. Thanks to the disclosure technology, the firm can instead deviate to the monopoly price *and* reassure consumers that they inspected a not-so-popular product, thus encouraging consumers to keep searching in the absence of a match.

Likewise, the possibility of disclosure destroys the pure positioning equilibrium characterized by Proposition 8, in which some consumers only inspect the most promising position, even if they have no match with the product in that position. The firm would therefore have an incentive to swap the positions of the two products, so as to induce consumers to inspect first the less popular product, and inform them afterwards, so as to encourage them to inspect the other product as well.

Conversely, the possibility of disclosure gives rise to additional equilibria, in which the firm selectively discloses (upon inspection) the identity of the less popular products. For the sake of exposition, in the remainder of this section we assume again that only two products are available, and focus on the case where the firm chooses to offer both of them at the monopoly price.[28] We start with the case without positioning:

Proposition 10 (disclosure without positioning) *Suppose that the disclosure technology is available (but not the positioning technology) and the firm has chosen to offer two products (i.e., $n = 2$); the continuation Perfect Bayesian Equilibria with passive*

[28] When more than two products are available, an additional equilibrium condition must be met to ensure that the firm would not have an incentive to replace product 1 with a slightly less popular product and disclose the identity of that product, so as to encourage additional consumers to keep searching. See the remark at the end of Appendix I.

beliefs in which the firm offers the most popular products at the monopoly price (i.e., $\mathcal{I} = \{1, 2\}$ and $p_1 = p_2 = p^m$) are:

1. *The equilibrium characterized by Proposition 1 (together with not disclosing the identity of product 1 and disclosing the identity of product 2 with zero or small enough probability, so as to induce consumers to keep searching in the absence of a match and of disclosure).*

2. *A "disclosure" equilibrium in which:*

   (i) *upon inspection, the firm discloses the identity of product 2 with probability one, and the identity of product 1 with probability strictly less than one (possibly zero);*

   (ii) *consumers start searching if and only if their search cost is lower than*

   $$\underline{c}_2^D \equiv \frac{\mu_1 + \mu_2 + (1 - \mu_2)\,\mu_1}{3 - \mu_2} s^m \,(> \underline{c}_2)\,,$$

   in which case they keep searching until finding a match unless they learn (through disclosure, or the absence of it) that they already inspected product 1 and their search cost exceeds $\mu_2 s^m$.

   This equilibrium gives the firm an expected profit equal to

   $$\Pi^D \equiv G\left(\underline{c}_2^D\right)\left(\mu_1 + (1 - \mu_1)\frac{\mu_2}{2}\right)\pi^m + G\left(\mu_2 s^m\right)\frac{(1 - \mu_1)\,\mu_2}{2}\pi^m.$$

Proof. See Appendix I. ■

Proposition 10 confirms that, besides the original equilibrium, a new equilibrium exists, which provides another solution to the above-mentioned trade-off between consumer participation and search intensity. On the one hand, as the identity of only one product is formally disclosed, consumers either learn or infer which product they inspected first; hence, if it was the more popular product, then they stop searching if they face a high search cost, namely, if $c > \mu_2 s^m$. On the other hand, the prospect of avoiding a second costly inspection encourages more consumers to start searching: $\underline{c}_2^D > \underline{c}_2$. Depending on the match probabilities and on the distribution of search costs, either this new equilibrium or the original equilibrium characterized by Proposition 1 generates more profit. For example, keeping μ_1 and μ_2 fixed, the disclosure equilibrium increases profit (i.e., $\Pi^D > \Pi_2$) if and only if:

$$G\left(\underline{c}_2\right) < \frac{\left[2\mu_1 + (1 - \mu_1)\,\mu_2\right] G\left(\underline{c}_2^D\right) + (1 - \mu_1)\,\mu_2 G\left(\mu_2 s^m\right)}{2\mu_1 + 2\,(1 - \mu_1)\,\mu_2}.$$

The following proposition characterizes the equilibria that arise when disclosure and positioning are both available:

Proposition 11 (positioning and disclosure) *Suppose that the positioning and disclosure technologies are both available and that the firm has chosen to offer two products, $n = 2$. Then, in the continuation subgame, the only Perfect Bayesian Equilibria with passive beliefs in which the firm offers the most popular products at the monopoly price (i.e., $\mathcal{I} = \{1, 2\}$ and $p_1 = p_2 = p^m$) are the following:*

1. *The random positioning equilibrium of Proposition 7 and the noisy positioning equilibria of Proposition 9 (together with not disclosing the identity of product* 1 *– except possibly when placed in second position in the noisy positioning equilibria – and disclosing the identity of product* 2 *with zero or small enough probability, so as to induce consumers to keep searching in the absence of a match and of disclosure).*

2. *A disclosure equilibrium similar to that of Proposition 10, sustained thanks to random positioning; specifically:*

   (i) *the firm randomizes uniformly over its positioning policy and uses the same disclosure policy as in Proposition 10;*

   (ii) *consumers adopt the same search behavior as in Proposition 10.*

Proof. See Appendix J. ■

Proposition 11 first confirms that the possibility of disclosure does not disrupt the random and noisy positioning equilibria characterized by Propositions 7 and 9 (in which all active consumers search until finding a match), but eliminates the pure positioning equilibrium identified by Proposition 8 (in which some active consumers only inspect the first position). It also shows that random positioning can support the "disclosure" equilibrium characterized by Proposition 10. The intuition is simple and mirrors that for the no-positioning equilibrium: if the firm uniformly randomizes over its positioning policy, consumers are indifferent about which position to inspect first; conversely, given this consumer behavior, the firm is indeed indifferent about its positioning policy.

Thus, when positioning and disclosure are both available, Proposition 11 provides a third solution, in addition to those already identified by Propositions 8 and 9, to the trade-off between consumer participation and search intensity. Among these three solutions, the pure positioning equilibrium maximizes the number of consumers who inspect the more popular product, but minimizes the number of consumers who inspect both products. The noisy positioning equilibria ensure instead that all active consumers inspect the second position in the absence of a match with the product in the first, but induces fewer consumers to participate; among these equilibria, participation is maximized for $r = \bar{r}$, where marginal consumers are not only indifferent between participating or not, but also between continuing searching or not. Finally, the "disclosure with no positioning"equilibrium provides an intermediate solution, both in terms of participation and search intensity. It can be checked that, depending on the distribution of search costs, any of these three monopoly pricing equilibria can be the most profitable for the firm.[29]

4.4 Welfare analysis

The most profitable type of equilibrium is also the one that maximizes total welfare if the monopoly price generates relatively more profit than consumer surplus, that is, if the ratio π^m/s^m is sufficiently large. If instead this ratio is small, or if the regulatory objective focuses on consumer surplus rather than total welfare, then the welfare analysis will be primarily driven by consumers' interests, which calls for favoring the provision of information.

Specifically, let $S^N(r)$ denote the consumer surplus generated by the random (for $r = 1/2$) and noisy positioning equilibria identified by Propositions 7, 9 and 11, S^P denote the consumer surplus generated by the pure positioning equilibrium identified by Proposition 8, and S^D the one generated by the disclosure equilibria identified by Propositions 10 and 11. We have:

Proposition 12 (consumers prefer having more information) *$S^N(r)$ increases with r in the relevant range $r \in [0, \bar{r}]$, and $S^P > \max\left\{S^N(\bar{r}), S^D\right\}$.*

Proof. See Online Appendix E.1. ■

Among the noisy positioning equilibria, the least noisy one thus Pareto dominates: providing more accurate information improves search efficiency, which benefits consumers but also enhances profit, by inducing more consumers to search. However, consumers' interest may conflict with the profitability of the firm when considering alternative types of equilibria. Indeed, consumers always prefer the pure positioning equilibrium: it provides full information about product popularity, which allows consumers to inspect each product i if, and only if, its expected value, $\mu_i s^m$, exceeds their search costs. By contrast, as already noted, the noisy positioning and disclosure equilibria may generate more profit, by inducing more consumers to keep searching in the absence of a match.

When pure positioning is not an option (e.g., because the firm cannot commit not to use the disclosure technology, which destroys this equilibrium), consumers' preferences over disclosure and noisy positioning depend on their search costs. Consumers with low

[29] See Online Appendix D for an illustration.

search costs—who will inspect both products anyway—favor positioning, which reduces their expected number of searches (the more so, the higher μ_1). By contrast, consumers with higher search costs—who would rather not inspect product 2—may favor disclosure (particularly if μ_1 is close to μ_2, so that positioning has little impact on the expected number of searches). Indeed, we have:

Proposition 13 (disclosure versus positioning) *For any $\mu_2 \in (0,1)$, there exists $\mu_1^D(\mu_2)$ and $\mu_1^N(\mu_2)$ satisfying $\mu_2 < \mu_1^D(\mu_2) \leq \mu_1^N(\mu_2) < 1$ such that $S^D > S^N(\bar{r})$ for $\mu_1 < \mu_1^D(\mu_2)$ and $S^N(\bar{r}) > S^D$ for $\mu_1 > \mu_1^N(\mu_2)$.*

Furthermore, if $c^e(\underline{c}) = \int_0^{\underline{c}} cdG(c)/G(\underline{c})$ is proportional to $\underline{c}$, then consumers' and the firm's interests are perfectly aligned among noisy positioning and disclosure equilibria. Otherwise, their interests may conflict in either direction (i.e., $S^N(\bar{r}) > S^D$ but $\Pi^N(\bar{r}) < \Pi^D$, or $S^N(\bar{r}) < S^D$ but $\Pi^N(\bar{r}) > \Pi^D$).

Proof. See Online Appendix E.2. ■

If μ_1 is large enough, noisy positioning—which maximizes the probability of a match, as all active consumers inspect both products—can reduce the number of expected searches to a *larger* extent than disclosure (even for consumers who do not inspect product 2 under disclosure). In that case, *all* consumers prefer noisy positioning to disclosure. For lower values of μ_1, however, positioning has less impact on the expected number of searches. Some consumers may then favor disclosure to avoid inspecting product 2. When this is the case, disclosure fosters consumer participation (i.e., $\underline{c}_2^D > \underline{c}_2^N(\bar{r})$); whether this also yields higher total consumer surplus depends on product characteristics (e.g., disclosure dominates if μ_1 is close to μ_2), but also on the distribution of search costs (e.g., how many consumers have a search cost lower than $\mu_2 s^m$, rather than between $\underline{c}_2^N(\bar{r})$ and $\underline{c}_2^D$).

Disclosure is clearly unattractive if all consumers prefer noisy positioning: disclosure then discourages consumer participation and thus also generates less profit than noisy positioning (under which active consumers keep searching until finding a match). More generally, if the distribution of search costs is such that the average cost $c^e(\underline{c})$ of active consumers is proportional to the cost $\underline{c}$ of the marginal one, then—as in Proposition 4—the most profitable equilibrium (among the noisy positioning and disclosure ones) is the one that maximizes consumer surplus (and, thus, total welfare). Otherwise, a bias may arise, in favor of either disclosure or positioning.

5 Platform

In this section, we study product variety and pricing by a hosting platform. More specifically, we depart from the baseline setting of Section 2 in that each product i is now produced by a single-product seller i, who sets the price p_i for its product if present on the platform, and the firm is a monopoly platform that charges fixed fees in return for hosting the sellers. As in the baseline setting, the (platform) slots are indistinguishable to consumers, and so search is necessarily random. The timing is as follows:

Stage 1 The platform publicly sets its size n.

Stage 2 The platform chooses the set $\mathcal{I}$ of sellers to approach, where $|\mathcal{I}| \leq n$; this set is observed by the sellers but not by the consumers.

Stage 3 The platform privately makes take-it-or-leave-it offers $(\phi_i)_{i\in\mathcal{I}}$, which the sellers then privately accept or reject;[30] the set $\hat{\mathcal{I}} \subseteq \mathcal{I}$ of sellers eventually present on the platform is not observed, but the number of these sellers, $\hat{n} = \left|\hat{\mathcal{I}}\right| \leq n$, is publicly observed.

Stage 4 Each seller $i \in \hat{\mathcal{I}}$ privately sets its price p_i.

Stage 5 Having observed n and $\hat{n}$, consumers sequentially choose whether to inspect (randomly) the products offered on the platform.

As in the baseline setting, for any size of the platform set in stage 1, the remaining stages form a proper subgame of incomplete information. We assume that sellers have passive beliefs when encountering out-of-equilibrium offers at stage 2. Similarly, we assume that consumers have passive beliefs when encountering out-of-equilibrium prices at the final stage. We do not restrict sellers' and consumers' beliefs about the identities of the sellers on the platform in the out-of-equilibrium event in which $\hat{n} < n$.

The following proposition is the analog of Proposition 1:

Proposition 14 (platform composition and prices) *Fix the size n of the platform. In the continuation subgame, there exists a PBE with passive beliefs that yields the same outcome (in terms of product portfolio and prices, consumer surplus and total profit) as the equilibrium characterized by Proposition 1:*

(*i*) *the platform hosts the n most popular products, each of which is offered at the monopoly price: $\hat{\mathcal{I}} = \mathcal{I} = \mathcal{I}_n$ and $p_i = p^m$ for every $i \in \mathcal{I}_n$;*

(*ii*) *consumers keep searching until finding a match if their search cost is lower than $\underline{c}_n$, and do not search otherwise;*

(*iii*) *the platform appropriates all the profit through the fees charged to the sellers.*

Proof. See Appendix K. ■

[30] The equilibrium derived below would be unaffected if the offers and acceptance decisions were observed by the sellers.

For any given size n adopted by the platform, the equilibrium characterized by Proposition 14 gives the platform exactly the same profit as the firm's equilibrium profit in Proposition 1. It follows that the analysis of choice overload and product variety conducted in Section 3 for the baseline setting carries over as well.

Remark 4 (bargaining power) *The analysis of choice overload and product variety still carries over when sellers have some bargaining power in their negotiations with the platform. Suppose for example that, in stage 3, the fees are determined through "Nash-in-Nash" bilateral bargaining; that is, the platform and every seller $i \in \mathcal{I}$ engage in Nash bargaining, holding fixed the equilibrium agreements between the platform and the sellers in $\mathcal{I} \setminus \{i\}$. As sellers obtain zero profit in the absence of an agreement and the platform derives its profit solely through the fixed fees, it follows that the platform obtains a fraction $\omega \in (0, 1]$, reflecting its bargaining power in bilateral negotiations, of the profit generated by each seller on the platform.*[31] *Hence, the equilibrium profit of the platform is exactly a fraction ω of the equilibrium profit obtained under take-it-or-leave-it offers, and the implications for the equilibrium product variety are therefore unchanged.*

6 Conclusion

We analyze pricing and product choice by a firm in a setting where products differ in their ex ante appeal ("popularity") and initially uninformed consumers can sequentially search through the offered products. We first consider the case of random (i.e., non-directed) search. Interestingly, as in the case of (ex ante) symmetric products, only those consumers with low enough search costs become active, but all those who start searching become seemingly addicted to it and keep searching until finding a match. Building on this insight, we show that this search pattern generates choice overload: increasing the number of products offered—which reduces their popularity on average—induces marginal consumers to stop searching. As a result, even if additional products can be offered at no cost, and despite the fact that expanding the product range increases the probability of a purchase from each active consumer, the firm chooses to offer a limited number of products. Compared with the socially optimal product portfolio, the firm offers too much (resp., too little) variety when the average search cost of active consumers is increasing (resp., decreasing) in the search cost of the marginal consumer.

We then explore—for a product range limited to two products—the firm's incentive to direct consumer search by assigning products to distinguishable "slots" or, alternatively,

[31] Let π_i denote the profit obtained by seller $i \in \mathcal{I}$ if it joins the platform. If the negotiation breaks down, seller $i \in \mathcal{I}$ obtains $\underline{\pi}_i = 0$ and the platform obtains $\underline{\Pi} = \sum_{j \in (\mathcal{I} \setminus \{i\})} \phi_j$; if instead the negotiation is successful, their profits are respectively $\hat{\pi}_i = \pi_i - \phi_i$ and $\hat{\Pi} = \underline{\Pi} + \phi_i$. The Nash bargaining rule then yields $\phi_i = \hat{\Pi} - \underline{\Pi} = \omega \left(\hat{\Pi} + \hat{\pi} - \underline{\Pi} - \underline{\pi} \right) = \omega \pi_i$.

by making informative product recommendations. Compared with pure random search, directing consumer search increases the number of active consumers—who can then first inspect the most popular product. But, unless there is sufficient randomness in the firm's positioning or recommendation strategies, directing consumer search also induces some consumers to stop searching even in the absence of a match. As a result, the firm may therefore want to adopt strategies that create enough uncertainty in consumers' minds—for instance, by making noisy recommendations. Finally, we show how our insights carry over to the case of a platform hosting third-party sellers.

Extending the analysis of firms' communication strategies and their impact on consumer search behavior to arbitrarily large product portfolios would be an interesting avenue for future research, particularly in the light of the diversity of practices adopted by firms in different industries and the policy concerns that they sometimes generate. Further, while we have focused on the case of a monopolist, another important avenue would be to study the implications for competition of firms' and platforms' influence over consumers' information on product popularity.

References

Chen, Yongmin and Chuan He (2011). Paid Placement: Advertising and Search on the Internet, *Economic Journal*, vol. 121: F309-F328.

Eliaz, Kfir and Ran Spiegler (2011). A Simple Model of Search Engine Pricing, *Economic Journal*, vol. 121: F329-F339.

Gamp, Tobias (2019). Guided Search. *Mimeo.*

Hagiu, Andrei and Bruno Jullien (2011). Why Do Intermediaries Divert Search?, *RAND Journal of Economics*, vol. 42(2): 337-362.

Hämäläinen, Saara (2019). Intrafirm Comparison Shopping, *Mimeo.*

Hart, O., and J. Tirole (1990). Vertical Integration and Market Foreclosure, *Brookings Papers on Economic Activity. Microeconomics*, 1990: 205-276.

Iyengar, Sheena S. and Mark R. Lepper (2000). When Choice is Demotivating: Can One Desire Too Much of a Good Thing?, *Journal of Personality and Sociological Psychology*, vol. 79(6), 995-1006.

Kamenica, Emir (2008). Contextual Inference in Markets: On the Informational Content of Product Lines, *American Economic Review*, vol. 98(5): 2127-2149.

Kraepelin, Emil (1915). *Psychiatrie: Ein Lehrbuch für Studierende und Ärzte.* 8th edition. Verlag von Johann Ambrosius Barth: Leipzig.

Kreps, David M., and Robert Wilson (1982). Sequential Equilibria, *Econometrica*, vol. 50(4): 863-894.

Kuksov, Dmitri and J. Miguel Villas-Boas (2010). When More Alternatives Lead to Less Choice, *Marketing Science*, vol 29(3): 507-524.

McAfee, R. Preston, and Marius Schwartz (1994). Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity, *American Economic Review*, 84(1): 210-230.

Moraga-González, José Luis, Zsolt Sándor, and Matthijs R. Wildenbeest (2017). Prices and Heterogeneous Search Costs, *RAND Journal of Economics*, 48(1): 125-146.

O'Brien, Dan P., and Greg Shaffer (1992). Vertical Control with Bilateral Contracts, *RAND Journal of Economics*, 23(3): 299-308.

Petrikaitė, Vaiva (2018). Consumer Obfuscation by a Multiproduct Firm, *RAND Journal of Economics*, vol. 49(1): 206-223.

Rhodes, Andrew (2015). Multiproduct Retailing, *Review of Economic Studies*, vol. 82(1): 360-390.

Rhodes, Andrew, Makoto Watanabe, and Jidong Zhou (2019). Multiproduct Intermediaries, *Mimeo.*

Spence, A. Michael (1975). Monopoly, Quality, and Regulation, *Bell Journal of Economics*, vol. 6(2): 417-429.

Stahl, Dale O. (1989). Oligopolistic Pricing with Sequential Consumer Search, *American Economic Review*, vol. 79(4): 700-712.

Teh and Julian Wright (forthcoming). Intermediation and Steering: Competition in Prices and Commissions, *American Economic Journal: Microeconomics.*

Wolinsky, Asher (1986). True Monopolistic Competition as a Result of Imperfect Information, *Quarterly Journal of Economics*, vol. 101(3): 493-512.

Zhou, Jidong (2014). Multiproduct Search and the Joint Search Effect, *American Economic Review*, vol. 104(9): 2918-2939

Appendix

A Proof of Lemma 1

Fix a product portfolio, $\mathcal{I}$ of size n and, for $k \in \{1, ..., n\}$, let $m_k \in \{0, 1\}$ denote whether a match occurs ($m_k = 1$) or not ($m_k = 0$) at the k^{th} inspection, $M_k \equiv \Sigma_{h=1}^{h=k} m_h$ denote the number of matches at the first k inspections (with the convention $M_0 = 0$) and

$$\alpha_k \equiv \Pr\left[m_k = 1 \mid M_{k-1} = 0\right]$$

denote the probability of a match at the k^{th} inspection, conditional on not having had any match at the previous $k-1$ inspections. Furthermore, for $k \in \{2, ..., n\}$, let

$$\hat{\alpha}_k \equiv \Pr\left[m_k = 1 \mid M_{k-2} = 0 \text{ and } m_{k-1} = 1\right]$$

denote the probability of a match at the k^{th} inspection, conditional on not having had any match at the previous $k-2$ inspections and having had a match at the $(k-1)^{\text{th}}$ inspection.

We first show that observing a match makes consumers more pessimistic about future matches:

Lemma 7 (a match brings bad news about future matches) *For $k \in \{2, ..., n\}$, $\alpha_k > \hat{\alpha}_k$.*

Proof. For $k \in \{1, ..., n\}$, let P_k denote the sub-portfolio of products visited at the first k inspections, and

$$\mathcal{P}_k \equiv \{\mathcal{J} \subseteq \mathcal{I} \mid |\mathcal{J}| = k\}$$

denote the set of such sub-portfolios (with the convention $P_0 = \mathcal{P}_0 = \varnothing$). For $k \in \{2, ..., n\}$, we have:

$$\alpha_k = \sum_{\mathcal{J}_{k-2} \in \mathcal{P}_{k-2}} \Pr\left[P_{k-2} = \mathcal{J}_{k-2}\right] \beta_k\left(\mathcal{J}_{k-2}\right),$$
$$\hat{\alpha}_k = \sum_{\mathcal{J}_{k-2} \in \mathcal{P}_{k-2}} \Pr\left[P_{k-2} = \mathcal{J}_{k-2}\right] \hat{\beta}_k\left(\mathcal{J}_{k-2}\right),$$

where

$$\begin{aligned} \beta_k\left(\mathcal{J}_{k-2}\right) &= \Pr\left[m_k = 1 \mid M_{k-1} = 0 \text{ and } P_{k-2} = \mathcal{J}_{k-2}\right], \\ \hat{\beta}_k\left(\mathcal{J}_{k-2}\right) &= \Pr\left[m_k = 1 \mid M_{k-2} = 0,\ m_{k-1} = 1 \text{ and } P_{k-2} = \mathcal{J}_{k-2}\right]. \end{aligned}$$

Fix $\mathcal{J}_{k-2}$, and let $\hat{n} \equiv \left|\mathcal{I} \diagdown \mathcal{J}_{k-2}\right| (= n + 2 - k)$ denote the size of the complement sub-portfolio $\mathcal{I} \diagdown \mathcal{J}_{k-2}$, and, for $i \in I_{\hat{n}} \equiv \{1, ..., \hat{n}\}$, let $\hat{\mu}_i$ denote the popularity of its i^{th} most popular product; we thus have: and $\hat{\mu}_1 > ... > \hat{\mu}_{\hat{n}}$. $\beta_k(\mathcal{J}_{k-2})$ can then be expressed as:

$$
\begin{aligned}
\beta_k(\mathcal{J}_{k-2}) &= \Pr[m_k = 1 \mid M_{k-2} = 0, m_{k-1} = 0 \text{ and } P_{k-2} = \mathcal{J}_{k-2}] \\
&= \frac{\Pr[m_{k-1} = 0 \text{ and } m_k = 1 \mid P_{k-2} = \mathcal{J}_{k-2} \text{ and } M_{k-2} = 0]}{\Pr[m_{k-1} = 0 \mid P_{k-2} = \mathcal{J}_{k-2} \text{ and } M_{k-2} = 0]} \\
&= \frac{\sum_{i \in I_{\hat{n}}} \sum_{j \in I_{\hat{n}} \setminus \{i\}} (1 - \hat{\mu}_j) \hat{\mu}_i}{\sum_{h \in I_{\hat{n}}} \sum_{j \in I_{\hat{n}} \setminus \{h\}} (1 - \hat{\mu}_j)} \\
&= \sum_{i \in I_{\hat{n}}} w_i \hat{\mu}_i
\end{aligned}
$$

where

$$
\begin{aligned}
w_i &\equiv \frac{\sum_{j \in I_{\hat{n}} \setminus \{i\}} (1 - \hat{\mu}_j)}{\sum_{h \in I_{\hat{n}}} \sum_{j \in I_{\hat{n}} \setminus \{h\}} (1 - \hat{\mu}_j)} \\
&= \frac{[\sum_{j \in I_{\hat{n}}} (1 - \hat{\mu}_j)] - (1 - \hat{\mu}_i)}{\sum_{h \in I_{\hat{n}}} \{[\sum_{j \in I_{\hat{n}}} (1 - \hat{\mu}_j)] - (1 - \hat{\mu}_h)\}} \\
&= \frac{\hat{n}(1 - \hat{\mu}) - (1 - \hat{\mu}_i)}{\hat{n}^2 (1 - \hat{\mu}) - \sum_{h \in I_{\hat{n}}} (1 - \hat{\mu}_h)} \\
&= \frac{\hat{n}(1 - \hat{\mu}) - (1 - \hat{\mu}_i)}{\hat{n}^2 (1 - \hat{\mu}) - \hat{n}(1 - \hat{\mu})} \\
&= \frac{\hat{n}(1 - \hat{\mu}) - (1 - \hat{\mu}_i)}{\hat{n}(\hat{n} - 1)(1 - \hat{\mu})},
\end{aligned}
$$

where

$$
\hat{\mu} \equiv \frac{\sum_{h \in I_{\hat{n}}} \hat{\mu}_h}{\hat{n}}
$$

denotes the expected popularity of the products in the complement sub-portfolio $\mathcal{I} \diagdown \mathcal{J}_{k-2}$.

Likewise, $\hat{\beta}_k(\mathcal{J}_{k-2})$ can be expressed as:

$$\begin{aligned}
\hat{\beta}_k(\mathcal{J}_{k-2}) &= \Pr[m_k = 1 \mid M_{k-2} = 0, m_{k-1} = 1 \text{ and } P_{k-2} = \mathcal{J}_{k-2}] \\
&= \frac{\Pr[m_{k-1} = 1 \text{ and } m_k = 1 \mid P_{k-2} = \mathcal{J}_{k-2} \text{ and } M_{k-2} = 0]}{\Pr[m_{k-1} = 1 \mid P_{k-2} = \mathcal{J}_{k-2} \text{ and } M_{k-2} = 0]} \\
&= \frac{\sum\limits_{i \in I_{\hat{n}}} \sum\limits_{j \in I_{\hat{n}} \setminus \{i\}} \hat{\mu}_j \hat{\mu}_i}{\sum\limits_{h \in I_{\hat{n}}} \sum\limits_{j \in I_{\hat{n}} \setminus \{h\}} \hat{\mu}_j} \\
&= \sum_{i \in I_{\hat{n}}} \hat{w}_i \hat{\mu}_i
\end{aligned}$$

where

$$\begin{aligned}
\hat{w}_i &\equiv \frac{\sum\limits_{j \in I_{\hat{n}} \setminus \{i\}} \hat{\mu}_j}{\sum\limits_{h \in I_{\hat{n}}} \sum\limits_{j \in I_{\hat{n}} \setminus \{h\}} \hat{\mu}_h} \\
&= \frac{\left[\sum\limits_{j \in I_{\hat{n}}} \hat{\mu}_j\right] - \hat{\mu}_i}{\sum\limits_{h \in I_{\hat{n}}} \left\{\left[\sum\limits_{j \in I_{\hat{n}}} \hat{\mu}_j\right] - \hat{\mu}_h\right\}} \\
&= \frac{\hat{n}\hat{\mu} - \hat{\mu}_i}{\hat{n} \sum\limits_{j \in I_{\hat{n}}} \hat{\mu}_j - \sum\limits_{h \in I_{\hat{n}}} \hat{\mu}_h} \\
&= \frac{\hat{n}\hat{\mu} - \hat{\mu}_i}{\hat{n}(\hat{n} - 1)\hat{\mu}}.
\end{aligned}$$

The weight distribution used for $\hat{\beta}_k(\mathcal{J}_{k-2})$, $\{\hat{w}_i\}_{i \in \{1,\dots,\hat{n}\}}$, dominates (in the sense of first-order stochastic dominance) the weight distribution used for $\beta_k(\mathcal{J}_{k-2})$, $\{w_i\}_{i \in \{1,\dots,\hat{n}\}}$: for any $i \in \{1, \dots, \hat{n}\}$, we have:

$$\begin{aligned}
\sum_{j=1}^{i} (w_j - \hat{w}_j) &= \sum_{j=1}^{i} \left\{ \frac{\hat{n}(1 - \hat{\mu}) - (1 - \hat{\mu}_j)}{\hat{n}(\hat{n} - 1)(1 - \hat{\mu})} - \frac{\hat{n}\hat{\mu} - \hat{\mu}_j}{\hat{n}(\hat{n} - 1)\hat{\mu}} \right\} \\
&= \sum_{j=1}^{i} \frac{\hat{n}(1 - \hat{\mu})\hat{\mu} - (1 - \hat{\mu}_j)\hat{\mu} - \hat{n}\hat{\mu}(1 - \hat{\mu}) + \hat{\mu}_j(1 - \hat{\mu})}{\hat{n}(\hat{n} - 1)(1 - \hat{\mu})\hat{\mu}} \\
&= \frac{\sum\limits_{j=1}^{i} (\hat{\mu}_j - \hat{\mu})}{\hat{n}(\hat{n} - 1)(1 - \hat{\mu})\hat{\mu}} \\
&\geq 0,
\end{aligned}$$

where the inequality is strict for $i < n$ and follows from $\hat{\mu}_1 > \dots > \hat{\mu}_{\hat{n}}$ and the definition of $\hat{\mu}$.

As $\hat{\mu}_1 > ... > \hat{\mu}_{\hat{n}}$, it follows that $\beta_k(\mathcal{J}_{k-2}) > \hat{\beta}_k(\mathcal{J}_{k-2})$ for any $\mathcal{J}_{k-2} \in \mathcal{P}_{k-2}$, which in turn implies $\alpha_k > \hat{\alpha}_k$. ■

Given the information available after the first k inspection, the expected probability of having a match is the same for all future inspections. Hence, we have:

$$\begin{aligned}
\alpha_k &= \Pr[m_k = 1 \mid M_{k-1} = 0] \\
&= \Pr[m_{k+1} = 1 \mid M_{k-1} = 0] \\
&= \Pr[m_k = 1 \mid M_{k-1} = 0] \Pr[m_{k+1} = 1 \mid M_{k-1} = 0 \text{ and } m_k = 1] \\
&\quad + \Pr[m_k = 0 \mid M_{k-1} = 0] \Pr[m_{k+1} = 1 \mid M_{k-1} = 0 \text{ and } m_k = 0] \\
&= \alpha_k \hat{\alpha}_{k+1} + (1 - \alpha_k)\alpha_{k+1} \\
&= \alpha_{k+1} + \alpha_k(\hat{\alpha}_{k+1} - \alpha_{k+1}) \\
&< \alpha_{k+1},
\end{aligned}$$

where the inequality follows from Lemma 7.

It follows that if a consumer finds it optimal to make the first inspection, he will optimally continue to inspect all products until he has found a match (and learned his match-conditional valuation). To see this, suppose by way of contradiction, that the consumer finds it optimal to stop searching after $k < n$ inspections. This requires $\alpha_{k+1}s^m \leq c$ as, otherwise, the consumer would have an incentive to search at least one more time. But the consumer must also find it optimal to conduct the k^{th} inspection, *knowing* that it will be the last one, which in turn requires $\alpha_k s^m \geq c$. We thus have $\alpha_k s^m \geq c \geq \alpha_{k+1}s^m$, a contradiction.

B Proof of Lemma 2

Fix the size n of the product line and consider the associated continuation subgame. Belief consistency requires consumers to update their beliefs about product characteristics and prices on the basis of observed matches and prices. Thus, for any given number of inspected slots, $k \in \{1, ..., n-1\}$, let $\nu(\mathbf{m}^k; \mathbf{p}^k)$ denote consumers' updated beliefs about the firm's product portfolio and prices in the remaining $n - k$ slots, given the observed sequence of matches $\mathbf{m}^k = (m_1, ..., m_k)$ and the observed sequence of prices $\mathbf{p}^k = (p_1, ..., p_k)$.[32]

We first show that consumers with higher realized valuations are more likely to keep searching:

[32] For simplicity of notation, we do not introduce k as an argument of the function ν; however, it does depend on k through the length of the sequences $\mathbf{m}^k$ and $\mathbf{p}^k$. A similar comment applies to the other functions introduced below.

Lemma 8 (consumer search) *Suppose that consumers' beliefs are consistent and that their search strategy is optimal given these beliefs. For any* $k \in \{1, ..., n-1\}$*, any* $\mathbf{m}^k$ *satisfying* $\sum_{i=1}^{k} m_i \geq 1$ *and any* $\mathbf{p}^k = (p_1, ..., p_k)$*, let* $\sigma\left(\mathbf{m}^k; \mathbf{p}^k \mid v, c\right)$ *denote a consumer's probability of inspecting one more slot when facing a search cost* c *and having observed the match and price sequences* $\mathbf{m}^k$ *and* $\mathbf{p}^k$*, and a valuation* v *on the first* k *inspected slots; we have:*

$$\sigma\left(\mathbf{m}^k; \mathbf{p}^k \mid v, c\right) > 0 \Longrightarrow \sigma\left(\mathbf{m}^k; \mathbf{p}^k \mid v', c'\right) = 1 \text{ for any } v' \geq v \text{ and } c' < c.$$

Proof. For any $k \in \{1, ..., n-1\}$, $\mathbf{m}^k$ satisfying $\sum_{i=1}^{k} m_i \geq 1$ and $\mathbf{p}^k = (p_1, ..., p_k)$, let $\underline{p}\left(\mathbf{m}^k; \mathbf{p}^k\right) \equiv \min\left\{p_h \in \mathbf{p}^k \mid m_h = 1\right\}$ denote the lowest price for the products which produced a match, and let $\tilde{\nu}_i\left(\mathbf{m}^k, \mathbf{p}^k\right)$ and $\hat{p}_i\left(\mathbf{m}^k, \mathbf{p}^k\right)$ denote consumers' beliefs about, respectively, the probability that an additional inspection would expose them to product $i = 1, 2, ...$, and the price of this product. Finally, let $\psi\left(\mathbf{p}^k; \mathbf{m}^k \mid v, c\right)$ denote the expected benefit (gross of the search cost) of an $(k+1)^{\text{th}}$ inspection, assuming optimal search behavior thereafter (and netting out search costs of any subsequent inspections).

We start by showing that the following property $(\mathcal{P}_k)$ holds for every $k \in \{1, ..., n-1\}$: for any $\mathbf{m}^k = (m_1, ..., m_k)$ satisfying $\sum_{i=1}^{k} m_i \geq 1$ and any $\mathbf{p}^k = (p_1, ..., p_k)$, $\psi\left(\mathbf{p}^k; \mathbf{m}^k \mid v, c\right)$ is weakly increasing in v, and weakly decreasing in c.

We first establish that $(\mathcal{P}_{n-1})$ holds: for any $\mathbf{m}^{n-1}$ satisfying $\sum_{h=1}^{n-1} m_h \geq 1$ and any $\mathbf{p}^{n-1}$, we have

$$\psi\left(\mathbf{m}^{n-1}; \mathbf{p}^{n-1} \mid v, c\right) = \sum_i \tilde{\nu}_i\left(\mathbf{m}^{n-1}; \mathbf{p}^{n-1}\right) \mu_i b\left(\underline{p}\left(\mathbf{m}^{n-1}; \mathbf{p}^{n-1}\right); \hat{p}_i\left(\mathbf{m}^{n-1}; \mathbf{p}^{n-1}\right) \mid v\right),$$

where

$$\begin{aligned} b\left(\underline{p}; p \mid v\right) &\equiv \max\left\{v - \min\left\{\underline{p}, p\right\}, 0\right\} - \max\left\{v - \underline{p}, 0\right\} \\ &= \max\left\{\min\left\{v, \underline{p}\right\} - p, 0\right\}. \end{aligned}$$

The conclusion follows from the fact that $b\left(\underline{p}; p \mid v\right)$ is weakly increasing in v and independent from c.

Reasoning by induction, suppose now that $(\mathcal{P}_{k+1})$ holds. For any $\mathbf{m}^k = (m_1, ..., m_k)$ satisfying $\sum_{i=1}^{k} m_i \geq 1$ and any $\mathbf{p}^k = (p_1, ..., p_k)$, any v and any c, we have:

$$\psi\left(\mathbf{m}^k; \mathbf{p}^k \mid v, c\right) = \sum_i \tilde{\nu}_i\left(\mathbf{m}^k; \mathbf{p}^k\right) \hat{\psi}_i\left(\mathbf{m}^k; \mathbf{p}^k, \hat{p}_i\left(\mathbf{m}^k, \mathbf{p}^k\right) \mid v, c\right),$$

where

$$\begin{aligned}\hat{\psi}_i\left(\mathbf{m}^k;\mathbf{p}^k,\hat{p}\mid v,c\right) &\equiv (1-\mu_i)\max\left\{\psi\left(\mathbf{m}^k,0;\mathbf{p}^k,\hat{p}\mid v,c\right)-c,0\right\}\\ &\quad +\mu_i\left[\begin{array}{c} b\left(\underline{p}\left(\mathbf{m}^k,\mathbf{p}^k\right);\hat{p}\mid v\right)\\ +\max\left\{\psi\left(\mathbf{m}^k,1;\mathbf{p}^k,\hat{p}\mid v,c\right)-c,0\right\}\end{array}\right].\end{aligned}$$

The conclusion then follows from the observation that every term in $\hat{\psi}_i$ is weakly increasing in v and weakly decreasing in c.

By construction, a consumer having observed the match and price sequences $\mathbf{m}^k$ and $\mathbf{p}^k$ and a valuation v on the first k inspected slots is willing to search an additional slot if $\psi\left(\mathbf{m}^k;\mathbf{p}^k\mid v,c\right)\geq c$. It then follows from $(\mathcal{P}_k)$ that $\psi\left(\mathbf{m}^k;\mathbf{p}^k\mid v',c'\right)>c'$ for any $v'\geq v$ and $c'<c$; hence, any consumer with such valuation v' and cost c' strictly prefers to inspect an additional slot. ■

We now prove Lemma 2. Fix the size n of the product line, the composition $\mathcal{I}$ of the product portfolio and the prices of these products, and consider a consumer with search cost c. If the consumer starts searching, the expected profit that he generates can be expressed as

$$E\left[\Pi\mid c\right]=\frac{1}{\left|\mathcal{S}\left(\mathcal{I}\right)\right|}\sum_{\mathbf{s}\in\mathcal{S}(\mathcal{I})}\sum_{\mathbf{m}\in\{0,1\}^n}\rho\left(\mathbf{m}\mid\mathbf{s}\right)E\left[\Pi\mid c,\mathbf{s},\mathbf{m}\right]$$

where $\mathbf{s}\in\mathcal{I}^n$ is an arbitrary search sequence[33] and $\mathcal{S}\left(\mathcal{I}\right)$ is the set of such sequences, $\mathbf{m}\in\{0,1\}^n$ is an arbitrary sequence of matches,

$$\rho\left(\mathbf{m}\mid\mathbf{s}\right)\equiv\Pi_{i=1}^n\mu_{s_i}^{m_i}\left(1-\mu_{s_i}\right)^{1-m_i}$$

denotes the probability of match sequence $\mathbf{m}$, conditional on $\mathbf{s}$, and $E\left[\Pi\mid c,\mathbf{s},\mathbf{m}\right]$ represents the expected profit (with respect to the realized valuation v) obtained from consumers facing a search cost c, taking as a given the search sequence $\mathbf{s}=(s_1,...,s_n)\in\mathcal{S}\left(\mathcal{I}\right)$ and the match sequence $\mathbf{m}=(m_1,...,m_n)$ that they would face if they were to inspect all products, but assuming that these consumers search optimally given their (consistent) beliefs (and may thus stop searching before reaching the end of the sequence).

We now consider a given $(c,\mathbf{s},\mathbf{m})$ and provide an upper bound for $E\left[\Pi\mid c,\mathbf{s},\mathbf{m}\right]$. If $\sum_{k=1}^n m_k=0$, the firm obtains zero profit. Consider now the case where $\sum_{k=1}^n m_k\geq 1$. From Lemma 8, a consumer who buys at price p when drawing a valuation v buys at a weakly lower price $p'\leq p$ whenever drawing a higher valuation $v'>v$, as the higher valuation induces the consumer to inspect at least as many products. Let $\hat{v}$ denote the infimum of the valuations for which a consumer buys,[34] and $p(v)$ denote the price at which a consumer with valuation $v>\hat{v}$ buys. As noted above, $p(v)$ is weakly decreasing;

[33] That is, $s_k\in\mathcal{I}$ denotes the index of the k^{th} product according to the search sequence $\mathbf{s}$.

[34] $\hat{v}$ depends on $(c,\mathbf{s},\mathbf{m})$; we drop the argument to simplify the exposition.

hence, $\hat{p} = \lim_{v \longrightarrow \hat{v}^+} \{p(v)\}$ exists and satisfies, for every $v \geq \hat{v}$:

$$p(v) \leq \hat{p} \leq \hat{v},$$

where the first inequality stems from the monotonicity of $p(v)$, and the second inequality follows from the fact that, by construction, $p(v) \leq v$ for any v (above but) arbitrarily close to $\hat{v}$. Hence, we have:

$$E[\Pi \mid c, \mathbf{s}, \mathbf{m}] = \int_{\hat{v}}^{+\infty} p(v)\, dF(v) \leq \int_{\hat{v}}^{+\infty} \hat{v} dF(v) = [1 - F(\hat{v})]\, \hat{v} \leq \pi^m,$$

where the last inequality follows from $\pi^m = \max\{[1 - F(p)]\, p\}$.

As

$$\sum_{\mathbf{m} \in \{0,1\}^n} \rho(\mathbf{m} \mid \mathbf{s}) \equiv 1 - \Pi_{i=1}^n (1 - \mu_i) = M(\mathcal{I}),$$

aggregating over all possible product and match sequences yields:

$$E[\Pi \mid c] \leq \frac{1}{|\mathcal{S}(\mathcal{I})|} \sum_{\mathbf{s} \in \mathcal{S}(\mathcal{I})} \sum_{\mathbf{m} \in \{0,1\}^n} \rho(\mathbf{m} \mid \mathbf{s})\, \pi^m = M(\mathcal{I})\, \pi^m \leq M(\mathcal{I}_n)\, \pi^m. \tag{10}$$

C Proof of Proposition 1

Fix the portfolio size chosen by the firm in the first stage, n, and consider the continuation subgame.

Part (ii). Suppose that the firm offers the most popular products at the monopoly price: $\mathcal{I} = \mathcal{I}_n$ and $p_i = p^m$ for every $i \in \mathcal{I}_n$. A consumer then stops searching upon the first match: he purchases the product if his match-conditional valuation exceeds the charged price, and gives up without any purchase otherwise. From Lemma 1, any consumer who starts searching keeps searching until finding a match. A consumer will therefore engage in search if the expected probability of a match times the expected surplus conditional on a match, $M(\mathcal{I}_n)s^m$, exceeds the expected search cost, which is equal to the expected number of searches times the consumer's cost per search, $N(\mathcal{I}_n)c$, i.e., if $c < \underline{c}_n$, and will not engage in search if $c > \underline{c}_n$.

Part (i). We first show that offering the most popular products at the monopoly prices constitutes a PBE with passive beliefs. Suppose that the firm deviates and offers the product portfolio $\mathcal{I}^d$ at prices $(p_i^d)_{i \in \mathcal{I}^d}$. Consumers cannot observe this deviation before their initial search decision. Hence, they start searching if $c < \underline{c}_n$, and do not do so if $c > \underline{c}_n$. As passive beliefs are consistent, Lemma 2 implies that the deviation cannot generate an expected profit exceeding $G(\underline{c}_n)\, M(\mathcal{I}_n)\, \pi^m$, which is the equilibrium expected profit. Hence, the deviation is not profitable.

To conclude the proof, we first show that this PBE is belief-proof, before establishing uniqueness. Consider any other alternative consumers' consistent off-equilibrium beliefs and induced off-equilibrium behavior. By construction, along the equilibrium path consumers' beliefs are pinned down by Bayes' rule, and their behavior constitutes a best-response to the firm's equilibrium strategy, given these beliefs. It remains to check that the equilibrium strategy of the firm constitutes a best-response to these consumer strategies; this follows again from Lemma 2.

Turning to uniqueness, we first note that, in any PBE in which the firm offers the most popular products at the monopoly price (i.e., $\mathcal{I} = \mathcal{I}_n$ and $p_i = p^m$ for every $i \in \mathcal{I}_n$), consumers' equilibrium behavior is given by part (ii), which leads to the same equilibrium outcome as above. Consider now any other candidate belief-proof PBE outcome, in which the firm chooses a different product portfolio and/or different prices. The firm then obtains an expected profit of the form $G(\underline{c})\,\Pi$, where $\underline{c}$ denotes the cost threshold determining consumers' decision to start searching[35] and Π denotes the firm's expected per-consumer profit. From (10) in the proof of Lemma 2, we then have $\Pi < M(\mathcal{I}_n)\,\pi^m$. To see this, suppose first that the firm chooses $\mathcal{I} \neq \mathcal{I}_n$. We then have $\Pi \leq M(\mathcal{I})\,\pi^m < M(\mathcal{I}_n)\,\pi^m$. If the firm choose $\mathcal{I} = \mathcal{I}_n$ and a price $p_i \neq p^m$ for at least one product $i \in \mathcal{I}_n$, then Π satisfies:

$$\Pi \leq [1 - M(\mathcal{I}_n)] \times 0 + M_i \times \pi(p_i) + [M(\mathcal{I}_n) - M_i] \times \pi^m,$$

where:

$$M_i = \mu_i \prod_{j \in \mathcal{I}_n \setminus \{i\}} (1 - \mu_j)$$

denotes the probability of having a match with product i only. Using $\pi(p_i) < \pi^m$ yields $\Pi < M(\mathcal{I}_n)\,\pi^m$.

Suppose now that the firm deviates and: (i) selects the most popular products (i.e., $\mathcal{I} = \mathcal{I}_n$) and (ii) prices them at the same price, which is chosen arbitrarily close to p^m among the out-of-equilibrium prices. Suppose further that consumers have "optimistic beliefs" and interpret this unexpected price as signalling that the firm stuck to the product portfolio $\mathcal{I}$ but offers all uninspected products at largely subsidized prices (zero price, say). As long as consumers do not have a match, these beliefs induce consumers to be more optimistic about the probability of a match at next inspections (from Lemma 1) and about the prices found at these inspections. Hence, consumers who started searching keep searching until finding a match; whether or not they keep doing so afterwards, they then end up buying if the realized valuation exceeds p, and not buying otherwise. Hence, the deviation gives the firm an per-consumer expected profit arbitrarily close to

[35] Whenever consumers' beliefs are consistent, and thus independent of their search cost, there indeed exists such a cost threshold, equal to consumers' expected utility from starting inspecting the products (gross of the cost of the first inspection, but net of the costs of any additional inspections), below which consumers starts searching and above which they do not do so.

$M(\mathcal{I}_n)\,\pi^m > \Pi$, making the deviation profitable.

D Proof of Lemma 3

Let $\lambda_i \equiv 1-\mu_i$ denote the probability of not having a match with product i. From (1), increasing the size of the product line from n to $n+1$, changes the expected probability of a match by

$$M(\mathcal{I}_{n+1}) - M(\mathcal{I}_n) = \lambda_1 ... \lambda_n (1-\lambda_{n+1}) > 0,$$

where the inequality follows from $\lambda_i \in (0,1)$ for any $i \in \mathcal{I}_{n+1}$.

For any $n \in \mathbb{N}^*$ and any $s \in \mathcal{S}(\mathcal{I}_n)$, let

$$\varphi_n(s) \equiv 1 + \sum_{k=1}^{n-1} \Pi_{i=1}^{k} \lambda_{s_i}$$

denote the expected number of searches needed to find a match in $\mathcal{I}_n$ given the search sequence s. The expected number of searches needed to find a match in $\mathcal{I}_{n+1}$ can then be written as

$$N(\mathcal{I}_{n+1}) = \frac{1}{(n+1)!} \sum_{s\in\mathcal{S}(\mathcal{I}_{n+1})} \varphi_{n+1}(s).$$

Regrouping the terms in which product $n+1$ appears in the i^{th} position yields:

$$N(\mathcal{I}_{n+1}) = \frac{1}{(n+1)!} \sum_{i=1}^{n+1} \left\{ \sum_{s\in\mathcal{S}(\mathcal{I}_n)} \varphi_{n+1}(\sigma(s,i)) \right\},$$

where, for any $s \in \mathcal{S}(\mathcal{I}_n)$ and any $i \in \mathcal{I}_{n+1}$,

$$\sigma(s,i) = (\sigma_j(s,i))_{j\in\mathcal{I}_{n+1}} \in \mathcal{S}(\mathcal{I}_{n+1})$$

is defined by:

$$\sigma_j(s,i) \equiv \begin{cases} s_j & \text{for } j<i, \\ n+1 & \text{for } j=i, \\ s_{j-1} & \text{for } j>i. \end{cases}$$

Furthermore, we have

$$\varphi_{n+1}(\sigma(s,n+1)) - \varphi_n(s) = \lambda_{s_1} ... \lambda_{s_n} > 0,$$

and, for $i \in \mathcal{I}_n$,

$$\varphi_{n+1}(\sigma(s,i)) - \varphi_n(s) = \sum_{k=i}^{n-1} \left\{ (\lambda_{n+1} - \lambda_{s_k}) \lambda_{s_1} ... \lambda_{s_{k-1}} \right\} + \lambda_{s_1} ... \lambda_{s_{n-1}} \lambda_{n+1} > 0,$$

where the inequalities follow from $\lambda_{n+1} > \lambda_k > 0$ for any $k \in \mathcal{I}_n$. Hence,

$$\begin{aligned} N(\mathcal{I}_{n+1}) &> \frac{1}{(n+1)!} \sum_{i=1}^{n+1} \left\{ \sum_{s \in \mathcal{S}(\mathcal{I}_n)} \varphi_n(s) \right\} \\ &= \frac{1}{n!} \sum_{s \in \mathcal{S}(\mathcal{I}_n)} \varphi_n(s) \\ &= N(\mathcal{I}_n). \end{aligned}$$

E Proof of Part (ii) of Proposition 2

Fix n and $(\mu_1, ..., \mu_n)$, and let $\lambda_i \equiv 1 - \mu_i$ denote the probability of not having a match with product i, and

$$\hat{\lambda}_n \equiv (\lambda_1 ... \lambda_n)^{\frac{1}{n}}$$

denote the geometric mean of these probabilities. Note that, by assumption, $\lambda_1 < ... < \lambda_n$.

We first establish:

Lemma 9 $\underline{c}_n \leq (1 - \hat{\lambda}_n) s^m$.

Proof. From (1) and the definition of $\hat{\lambda}_n$, we have:

$$M(\mathcal{I}_n) = 1 - \hat{\lambda}_n^n.$$

As $\lambda_1 < \lambda_n$, there exist two products i and j such that $\lambda_i < \hat{\lambda}_n < \lambda_j$. Consider the following geometric-mean-preserving contraction, which consists in replacing λ_i and λ_j by, respectively, $\tilde{\lambda}_i$ and $\tilde{\lambda}_j$ such that: (i) $\tilde{\lambda}_i \tilde{\lambda}_j = \lambda_i \lambda_j$; and (ii) $(\tilde{\lambda}_i - \lambda_i)(\tilde{\lambda}_j - \lambda_j) = 0$. That is, the transformation preserves the geometric mean and moves both probabilities toward the geometric mean, up to the point that at least one of them is equal to it. For the sake of exposition, let $\tilde{\mathcal{I}}_n$ denote the set of products with no-match probabilities $(\tilde{\lambda}_1, ..., \tilde{\lambda}_n)$, where $\tilde{\lambda}_k = \lambda_k$ if $k \notin \{i, j\}$.

From (3), we have

$$\begin{aligned} N(\mathcal{I}_n) &= \frac{1}{n!} \sum_{s \in \mathcal{S}(\mathcal{I}_n)} \left[1 + \lambda_{s_1} + \lambda_{s_1} \lambda_{s_2} + ... + \lambda_{s_1} ... \lambda_{s_{n-1}} \right] \\ &= 1 + \sum_{k=1}^{n-1} T_k(\mathcal{I}_n), \end{aligned}$$

where

$$T_k(\mathcal{I}_n) \equiv \frac{1}{n!} \sum_{s \in \mathcal{S}(\mathcal{I}_n)} \lambda_{s_1} ... \lambda_{s_k} = \frac{(n-k)!}{n!} \sum_{(s_1,...,s_k) \in \mathcal{S}^k(\mathcal{I}_n)} \lambda_{s_1} ... \lambda_{s_k},$$

where

$$S^k(\mathcal{I}_n) \equiv \{(s_1, ..., s_k) \in (\mathcal{I}_n)^k | s_1 \neq ... \neq s_k\}.$$

We now show that replacing $\mathcal{I}_n$ with $\tilde{\mathcal{I}_n}$ decreases the expected number of searches. To see this, note first that this transformation does not affect the terms in T_k in which either both λ_i and λ_j appear or neither of them appears. Hence, for every $k \in \mathcal{I}_{n-1}$, we have:

$$T_k(\mathcal{I}_n) - T_k(\tilde{\mathcal{I}_n}) = \frac{(n-k)!}{n!} \sum_{(s_1,...,s_{k-1}) \in \tilde{\mathcal{S}}^{k-1}(\mathcal{I}_n)} (\lambda_i - \tilde{\lambda}_i + \lambda_j - \tilde{\lambda}_j) \lambda_{s_1} ... \lambda_{s_{k-1}},$$

where

$$\tilde{\mathcal{S}}^{k-1}(\mathcal{I}_n) \equiv \{(s_1, ..., s_{k-1}) \in (\mathcal{I}_n \setminus \{i, j\})^{k-1} | s_1 \neq ... \neq s_{k-1})\},$$

and, using the fact that $(\tilde{\lambda}_i, \tilde{\lambda}_j)$ is a geometric-mean-preserving contraction of (λ_i, λ_j),

$$\lambda_i + \lambda_j > \tilde{\lambda}_i + \tilde{\lambda}_j.$$

To see the last inequality, note that the transformation is the integral over infinitesimal changes $d\lambda_i$ and $d\lambda_j$ such that $d(\lambda_i \lambda_j) = \lambda_j d\lambda_i + \lambda_i d\lambda_j = 0$ and $d\lambda_i > 0$. As $\lambda_i < \lambda_j$, the induced infinitesimal change in $\lambda_i + \lambda_j$ is given by

$$d(\lambda_i + \lambda_j) = \left(1 - \frac{\lambda_j}{\lambda_i}\right) d\lambda_i < 0.$$

It follows that, for every $k \in \mathcal{I}_{n-1}$, $T_k(\mathcal{I}_n) > T_k(\tilde{\mathcal{I}_n})$. Hence, the transformation (i) reduces the expected number of searches needed to find a first match, and (ii) reduces (by at least one) the number of no-match probabilities that are different from the geometric mean $\hat{\lambda}_n$. Re-iterating the process at most n times results in replacing $\mathcal{I}_n$ with $\hat{\mathcal{I}}_n$, where $\hat{\mathcal{I}}_n$ is the set of products with no-match probabilities $(\hat{\lambda}_n, ..., \hat{\lambda})$, and yields:

$$N(\mathcal{I}_n) > N(\hat{\mathcal{I}}_n) = 1 + \hat{\lambda}_n + (\hat{\lambda}_n)^2 + ... + (\hat{\lambda}_n)^{n-1} = \frac{1 - (\hat{\lambda}_n)^n}{1 - \hat{\lambda}_n}.$$

It follows that

$$\underline{c}_n = \frac{M(\mathcal{I}_n)}{N(\mathcal{I}_n)} s^m < \frac{M(\mathcal{I}_n)}{N(\hat{\mathcal{I}}_n)} s^m = (1 - \hat{\lambda}_n) s^m.$$

■

To conclude the proof, it suffices to note that $\lim_{n \to \infty} \lambda_n = 1$ (i.e., $\lim_{n \to \infty} \mu_n = 0$) implies $\lim_{n \to \infty} \hat{\lambda}_n = 1$. As $\log \lambda_n$ is strictly increasing in n and $\lim_{n \to \infty} \log \lambda_n = 0$, for any $\epsilon > 0$, there exists $\hat{n}$ such that

$$\log \lambda_n \geq -\frac{\epsilon}{2}$$

for any $n > \hat{n}$. Therefore (using $\log \lambda_1 < \log \lambda_k$ for $k > 1$):

$$\begin{aligned}
\log \hat{\lambda}_n &= \frac{1}{n} \sum_{k=1}^{n} \log \lambda_k \\
&< \frac{1}{n} \left[\hat{n} \log \lambda_1 + (n - \hat{n}) \left(-\frac{\epsilon}{2} \right) \right] \\
&= - \left[\frac{\hat{n}}{n} |\log \lambda_1| + \left(1 - \frac{\hat{n}}{n} \right) \frac{\epsilon}{2} \right],
\end{aligned}$$

where the last expression lies between $-\epsilon$ and 0 for n large enough. Hence, as n goes to infinity, $\log \hat{\lambda}_n$ tends to 0, and thus $\hat{\lambda}_n$ tends to 1.

F Proof of Lemma 6

Suppose without loss of generality that the firm allocates product 1 to slot 1 with probability $r \in (1/2, 1]$, and let μ^i denote the probability that slot $i \in \{1, 2\}$ generates a match on a first visit, and $\bar{\mu}^i$ denote the probability that slot $i \in \{1, 2\}$ generates a match on a second visit, conditional on the other slot not having generated a match on the first visit. We have:

$$\begin{aligned}
\mu^1 &\equiv r\mu_1 + (1 - r)\, \mu_2, \\
\mu^2 &\equiv (1 - r)\, \mu_1 + r\mu_2,
\end{aligned}$$

and

$$\bar{\mu}^1 \equiv \frac{r\,(1 - \mu_2)\, \mu_1 + (1 - r)\,(1 - \mu_1)\, \mu_2}{r\,(1 - \mu_2) + (1 - r)\,(1 - \mu_1)} = \frac{\mu^1 - \mu_1\mu_2}{1 - \mu^2}, \tag{11}$$

$$\bar{\mu}^2 \equiv \frac{r\,(1 - \mu_1)\, \mu_2 + (1 - r)\,(1 - \mu_2)\, \mu_1}{r\,(1 - \mu_1) + (1 - r)\,(1 - \mu_2)} = \frac{\mu^2 - \mu_1\mu_2}{1 - \mu^1}, \tag{12}$$

As $r > 1/2$, $\mu^1 > \mu^2$ and $\bar{\mu}^1 > \bar{\mu}^2$; the first inequality is obvious and the second one follows from:

$$\begin{aligned}
\bar{\mu}^1 - \bar{\mu}^2 &= \frac{\mu^1 - \mu^2}{(1 - \mu^1)\,(1 - \mu^2)} \left[1 + \mu_1\mu_2 - \left(\mu^1 + \mu^2 \right) \right] \\
&= \frac{(1 - \mu_1)\,(1 - \mu_2)}{(1 - \mu^1)\,(1 - \mu^2)} \left(\mu^1 - \mu^2 \right) \\
&> 0,
\end{aligned}$$

where the second equality stems from $\mu^1 + \mu^2 = \mu_1 + \mu_2$.

When a consumer starts visiting slot i, his expected utility is given by, for $j \neq i \in \{1, 2\}$:

$$V^i \equiv -c + \mu^i s^m + \left(1 - \mu^i\right) \max\left\{0, \bar{\mu}^j s^m - c\right\}.$$

If $c \geq \bar{\mu}^1 s^m \ (> \bar{\mu}^2 s^m)$, then we have:

$$V^1 - V^2 = \left(\mu^1 - \mu^2\right) s^m > 0,$$

where the strict inequality follows from $\mu^1 > \mu^2$. If instead $c < \bar{\mu}^1 s^m$, then we have:

$$\begin{aligned} V^1 - V^2 &= \left(\mu^1 - \mu^2\right) s^m + \left(1 - \mu^1\right) \max\left\{0, \bar{\mu}^2 s^m - c\right\} - \left(1 - \mu^2\right)\left(\bar{\mu}^1 s^m - c\right) \\ &\geq \left(\mu^1 - \mu^2\right) s^m + \left(1 - \mu^1\right)\left(\bar{\mu}^2 s^m - c\right) - \left(1 - \mu^2\right)\left(\bar{\mu}^1 s^m - c\right) \\ &= \left(\mu^1 - \mu^2\right) c \\ &> 0, \end{aligned}$$

where the equality relies on (11) and (12), and the strict inequality follows again from $\mu^1 > \mu^2$.

G Proof of Proposition 8

Consider a candidate continuation equilibrium in which the firm offers the two most popular products at the monopoly price (i.e., $\mathcal{I} = \{1, 2\}$ and $p_1 = p_2 = p^m$) and places product 1 in first position, and product 2 in second position. From Lemma 6, active consumers then start with the first position. Obviously, consumers with search cost $c \leq \mu_1 s^m$ have an incentive to do so, and will stop searching in case of a match; in the absence of a march, consumers have an incentive to inspect the other position only if $c \leq \mu_2 s^m \ (< \mu_1 s^m)$. Conversely, consumers with search cost exceeding $\mu_1 s^m$ will not search, and those with a search cost $c \in (\mu_2 s^m, \mu_1 s^m)$ will only inspect the first position, even in the absence of a match.

This candidate equilibrium thus yields an expected profit equal to

$$\Pi^* = \left[G\left(\mu_1 s^m\right)\mu_1 + G\left(\mu_2 s^m\right)(1 - \mu_1)\mu_2\right]\pi^m.$$

We first check that these strategies constitute an equilibrium.[36] Suppose that the firm deviates and offers product i at price p^1 in first position, and product j at price p^2

[36] It suffices to check that there is no profitable deviation in pure strategies (over positions and prices); this implies that there is no profitable deviation in mixed strategies either, as the expected profit from a probability distribution over positions and prices is equal to the same probability distribution over profits from a realized vector of positions and prices.

in second position (we use superscripts to refer to positions, as opposed to products). As this deviation is not observed by consumers, it does not affect their decision about whether to start searching. Hence, consumers with search cost $c > \mu_1 s^m$ do not search, whereas those with $c < \mu_1 s^m$ inspect the first position. Furthermore, with passive beliefs:

- in the absence of a match, observing an unexpected price does not affect active consumers' search behavior: they inspect the second position if $c < \mu_2 s^m$, and stop searching if $c > \mu_2 s^m$;
- in case of a match, they inspect the second position if $c < \mu_2 (\max\{v, p^1\} - p^m)$, and stop searching if $c > \mu_2 (\max\{v, p^1\} - p^m)$.

Following an approach similar to that of the proof of Lemma 2, let $E[\Pi|c, \mathbf{m}]$ denote the expected profit (with respect to the realized valuation v) obtained from active consumers with cost c, given the search behavior just described, and taking as a given the match sequence $\mathbf{m} = (m^1, m^2)$ that they would face if they were to visit both positions. The insight from Lemma 2 carries over: conditional on having a match, a consumer cannot generate a higher expected profit than $\max_p \pi(p) = \pi^m$. Indeed, we have:

- in the absence of any match, consumers generate zero profit: $E[\Pi|c, (0,0)] = 0$;
- if there would be a match only with the product placed in i^{th} position, consumers generate an expected profit $\pi(p^i)$ if $c < \mu_i s^m$, and no profit otherwise:

$$E[\Pi|c, (1,0)] = \pi\left(p^1\right) \le \pi^m \text{ and } E[\Pi|c, (0,1)] = \begin{cases} \pi(p^2) \le \pi^m & \text{if } c < \mu_2 s^m, \\ 0 & \text{if } c > \mu_2 s^m. \end{cases}$$

- if instead they would have a match with both products, the following possible cases arise:
  - If $p^1 \le p^2$ or $c > \mu_2 (p^1 - p^m)$, then consumers buy at p^1 if $v \ge p^1$, and do not buy otherwise: in the first case, p^1 is obviously the relevant price, regardless of whether consumers inspect the second position; and in the second case, p^1 remains the relevant price as consumers never inspect the second position, as they believe that this would bring an expected benefit equal to $\max\{\mu_2 (p^1 - p^m), 0\}$ if $v \ge p^1$, and equal to $\max\{\mu_2 (v - p^m), 0\} \le \max\{\mu_2 (p^1 - p^m), 0\}$ if $v < p^1$. We thus have:

$$E[\Pi|c, (1,1), \mathbf{p}] = \pi\left(p^1\right) \le \pi^m.$$

  - If $p^1 > p^2$ and $c < \mu_2 (p^1 - p^m)$, then consumers never buy at p^1: consumers with a match on the first inspection and $v \ge p^1$ continue to search, and thus end-up buying at p^2. It follows that consumers who buy do so at p^2; as consumers would not buy if their valuations lie below p^2, we have:

$$E[\Pi|c,(1,1)] \leq \left[1 - F\left(p^2\right)\right] p^2 = \pi\left(p^2\right) \leq \pi^m.$$

It follows that the expected profit satisfies:

$$\begin{aligned} E\left[\Pi|c,(1,1)\right] &\leq \begin{cases} \pi\left(p^1\right) & \text{if } p^1 \leq p^2 \text{ or } c > \mu_2\left(p^1 - p^m\right), \\ \pi\left(p^2\right) & \text{if } p^1 > p^2 \text{ and } c < \mu_2\left(p^1 - p^m\right), \end{cases} \\ &\leq \pi^m. \end{aligned}$$

Hence, following the deviation, the expected profit obtained from active consumers (i.e., those with cost $c < \mu_1 s^m$) is given by:

$$\begin{aligned} & \int_0^{\mu_1 s^m} \left\{\mu_i\left(1-\mu_j\right) E\left[\Pi|c,(1,0)\right] + \left(1-\mu_i\right)\mu_j E\left[\Pi|c,(0,1)\right] + \mu_i\mu_j E\left[\Pi|c,(1,1)\right]\right\} dG(c) \\ \leq\ & \mu_i\left(1-\mu_j\right) G\left(\mu_1 s^m\right)\pi^m + \left(1-\mu_i\right)\mu_j G\left(\mu_2 s^m\right)\pi^m + \mu_i\mu_j G\left(\mu_1 s^m\right)\pi^m \\ =\ & \left[1-\left(1-\mu_i\right)\left(1-\mu_j\right)\right] G\left(\mu_2 s^m\right)\pi^m + \mu_i\left[G\left(\mu_1 s^m\right) - G\left(\mu_2 s^m\right)\right]\pi^m. \end{aligned}$$

The last expression is maximal when the firm selects the two most popular products (i.e., $\mathcal{I} = \{1,2\}$), so as to maximize the first term, and moreover places the most popular product (product 1) in first position, so as to maximize the second term; as the resulting profit then corresponds to the candidate equilibrium profit Π^*, the deviation is therefore unprofitable.

To conclude the proof, consider an alternative "pure positioning" equilibrium in which the firm offers products i in first position and j in second position, both at monopoly prices. From Lemma 6, consumers then start searching by inspecting the position hosting the more popular product, say position 1; it follows that consumers with cost $c < \mu_i s^m$ inspect position i, and among these, those with $c < \mu_j s^m$ also inspect the other position in the absence of a match on the first inspection. The firm's expected profit is therefore:

$$\begin{aligned} \Pi &= \mu_i G\left(\mu_i s^m\right)\pi^m + \left(1-\mu^i\right)\mu_j G\left(\mu_j s^m\right)\pi^m \\ &= \left[1-\left(1-\mu_i\right)\left(1-\mu_j\right)\right] G\left(\mu_j s^m\right)\pi^m + \mu_i\left[G\left(\mu^i s^m\right) - G\left(\mu_j s^m\right)\right]\pi^m \\ &\leq \left[1-\left(1-\mu_1\right)\left(1-\mu_2\right)\right] G\left(\mu_2 s^m\right)\pi^m + \mu_1\left[G\left(\mu_1 s^m\right) - G\left(\mu_2 s^m\right)\right]\pi^m. \end{aligned}$$

As altering the product portfolio and the positioning policy cannot be detected by consumers, who would therefore stick to the same search behavior, the last expression corresponds to the profit that the firm could obtain by offering instead product 1 in first position and product 2 in second position.

H Proof of Proposition 9

Consider a candidate continuation equilibrium in which the firm offers the two most popular products at the monopoly price (i.e., $\mathcal{I} = \{1, 2\}$ and $p_1 = p_2 = p^m$) and: (i) with probability $r \in (1/2, 1)$, it places product 1 in first position, and product 2 in second position; (ii) with complementary probability, it does the opposite. From Lemma 6, active consumers start with the first position, where they are more likely to find the more popular product.

We first show that active consumers must continue to search in the absence of a match. Using again superscripts to refer to positions, the probability of a match on the first inspection is

$$\mu^1(r) = r\mu_1 + (1 - r)\mu_2,$$

which decreases from $(\mu_1 + \mu_2)/2$ to μ_2 as r increases from $1/2$ to 1. If a fraction $\xi > 0$ of active consumers do not inspect the second position in the absence of a match on their first inspection, then the average probability of a match is

$$\xi\mu^1(r) + (1 - \xi) M_2,$$

where

$$M_2 \equiv \mu_1 + \mu_2 - \mu_1\mu_2$$

denotes the total probability of having a match when inspecting both products. It would therefore be profitable for the firm to deviate and place product 1 in first position with probability 1, so as to increase this probability of a match to

$$\xi\mu_1 + (1 - \xi) M_2.$$

Hence, active consumers must keep searching in the absence of a match. Their overall probability of a match is thus given by M_2, and the expected number of searches is equal to

$$\mu^1(r) \times 1 + \left[1 - \mu^1(r)\right] \times 2 = 2 - \mu^1(r).$$

Consumers therefore choose to start searching if their search cost c satisfies

$$c \leq \underline{c}_2^N(r) \equiv \frac{M_2 s^m}{2 - \mu^1(r)}.$$

As $\mu^1(r)$ increases with r, so does $\underline{c}_2^N(r)$.

When they do not have a match, consumers' expected probability of a match with the other product becomes:

$$\bar{\mu}^2(r) \equiv \frac{r(1-\mu_1)\mu_2 + (1-r)(1-\mu_2)\mu_1}{r(1-\mu_1) + (1-r)(1-\mu_2)},$$

which decreases[37] from μ_1 to μ_2 as r increases from 0 to 1 and, for $r = 1/2$, coincides with the updated probability in the absence of positioning, namely,

$$\mu^2 \equiv \frac{\mu_1 + \mu_2 - 2\mu_1\mu_2}{2 - \mu_1 - \mu_2}.$$

All active consumers will indeed choose to inspect the second position if the first inspection does not produce a match if $\underline{c}_2^N(r) \leq \bar{\mu}^2(r)\, s^m$. We know that this holds in the absence of positioning, that is, for $r = 1/2$. Conversely, for $r = 1$ we have:[38]

$$\underline{c}_2^N(1) = \frac{\mu_1 + \mu_2 - \mu_1\mu_2}{2 - \mu_1} s^m > \mu_2 s^m = \bar{\mu}^2(1)\, s^m.$$

As $\underline{c}_2^N(r)$ increases with r whereas $\bar{\mu}^2(r)$ is a decreasing function of r, it follows that there exists $\bar{r} \in (1/2, 1)$ such that all consumers who start searching keep searching until finding a match as long as $r \leq \bar{r}$.

In this candidate equilibrium, each match generates an expected profit π^m and consumer surplus s^m; as there are $G\left(\underline{c}_2^N(r)\right)$ consumers who start searching, and each one obtains a match with probability M_2, the total expected profit and expected surplus are respectively given by

$$\Pi^N(r) \equiv G\left(\underline{c}_2^N(r)\right) M_2 \pi^m \text{ and } S^N(r) \equiv G\left(\underline{c}_2^N(r)\right) M_2 s^m.$$

As $\underline{c}_2^N(r)$ increases with r, so do the expected profit and consumer surplus; furthermore, they coincide with the no positioning equilibrium values when $r = 1/2$. The threshold $\bar{r}$ is such that $\underline{c}_2^N(r) = \bar{\mu}^2(r)\, s^m$, which yields:

$$\bar{r} = \frac{\sqrt{1-\mu_2}}{\sqrt{1-\mu_1} + \sqrt{1-\mu_2}}.$$

It is therefore decreasing in μ_2, and increases from 1/2 to 1 as μ_1 increases from μ_2 to 1. Hence, the set of noisy positioning equilibria expands with the (un-)popularity ratio $\frac{1-\mu_2}{1-\mu_1}$: the best (i.e., least noisy) one boils down to random positioning when the two products

[37] To see this, note that $\hat{\mu}^2(r)$ is a weighted average of μ_1 and $\mu_2 < \mu_1$, with a relative weight on μ_2 that increases with r.

[38] The inequality amounts to

$$\mu_1 + \mu_2 - \mu_1\mu_2 > 2\mu_2 - \mu_1\mu_2 \iff \mu_1 > \mu_2.$$

have a similar popularity (i.e., $\mu_1 = \mu_2$), and converges to pure positioning as μ_1 tends to 1.

We now check that the firm has no incentive to deviate.[39] To see this, suppose that it does, and offers products i at price p^1 in slot 1, and product j at price p^2 in slot 2. As before, the deviation does not affect consumers' decision about whether to start searching; hence, consumers with search cost $c > \underline{c}_2^N(r)$ do not search, whereas those with $c < \underline{c}_2^N(r)$ start searching by inspecting slot 1. Furthermore, with passive beliefs:

- in the absence of a match, observing an unexpected price does not affect active consumers' search behavior, who thus keep searching;
- in case of a match, they inspect slot 2 if $c < \underline{\mu}^2(r)(\max\{v, p^1\} - p^m)$, and stop searching if $c > \underline{\mu}^2(r)(\max\{v, p^1\} - p^m)$, where

$$\underline{\mu}^2(r) \equiv \frac{r\mu_1\mu_2 + (1-r)\mu_2\mu_1}{r\mu_1 + (1-r)\mu_2},$$

Following again a similar approach as in the proof of Lemma 2, let $E[\Pi|c, \mathbf{m}]$ denote the expected profit (with respect to the realized valuation v) obtained from active consumers with cost c, given the search behavior just described, and taking as a given the match sequence $\mathbf{m} = (m_1, m_2)$ that they would face if they were to visit both slots. As all active consumers keep searching until finding a match, consumers who would have a match only with the product allocated to slot i now generate an expected profit $\pi(p^i) \leq \pi^m$. If instead they would have a match with both products, then, following the same steps as in Appendix G for the proof of Proposition 8 yields:

$$\begin{aligned} E[\Pi|c,(1,1)] &\leq \begin{cases} \pi(p^1) & \text{if } p^1 \leq p^2 \text{ or } c > \underline{\mu}^2(r)(p^1 - p^m), \\ \pi(p^2) & \text{if } p^1 > p^2 \text{ and } c < \underline{\mu}^2(r)(p^1 - p^m), \end{cases} \\ &\leq \pi^m. \end{aligned}$$

Hence, following the deviation, the expected profit obtained from active consumers (i.e., those with cost $c < \underline{c}_2^N(r)$) is given by:

$$\int_0^{\underline{c}_2^N(r)} \left\{\mu_i\left(1-\mu_j\right)E[\Pi|c,(1,0)] + (1-\mu_i)\mu_j E[\Pi|c,(0,1)] + \mu_i\mu_j E[\Pi|c,(1,1)]\right\} dG(c)$$
$$\leq G\left(\underline{c}_2^N(r)\right)\left[1-(1-\mu_i)\left(1-\mu_j\right)\right]\pi^m.$$

where the last expression is maximal when the firm selects the two most popular products (i.e., $\mathcal{I} = \{1,2\}$), and then corresponds to the candidate equilibrium profit $\Pi^R(r)$; the deviation is therefore unprofitable.

[39] By the argument in footnote 36 it suffices to check for deviations in pure strategies.

To conclude the proof, the same reasoning as at the end of the proof of Proposition 8 can be used to show that it is not profitable to offer other products than the two most popular ones. To see this, consider a candidate equilibrium in which the firm offers at monopoly prices a set of products with random positioning, in such a way that if offers some product $k \notin \{1,2\}$ with positive probability ϵ_k and does not offer product $i \in \{1,2\}$ with positive probability ϵ_i. Replacing product k with product j with probability $\min\{\epsilon_i, \epsilon_k\}$ would be undetected by consumers, who would therefore stick to the same search behavior, and would increase the expected profit of the firm by increasing the overall probability of a match.

Summing-up, we have:

- Proposition 7 characterizes all the equilibria with passive beliefs and monopoly pricing in which the products are uniformly randomly assigned to slots (i.e., $r = 1/2$);
- Proposition 8 characterizes instead all the equilibria with passive beliefs and monopoly pricing in which the products are deterministically assigned to slots (i.e., $r = 1$); and
- the above analysis characterizes all the equilibria with passive beliefs and monopoly pricing in which the allocation of products to slots is a non-degenerate lottery that is not uniform (i.e., $1/2 < r < 1$).

I Proof of Proposition 10

We suppose here that slots are indistinguishable (i.e., the positioning technology is not available) and the firm has chosen to offer the two products (i.e., $n = 2$). As mentioned in the text, to simplify the exposition and avoid discussing out-of-equilibrium beliefs in case the firm were to deviate and disclose the identity of unexpected products, we further assume that only two products are available; it is therefore common knowledge that the firm offers the two most popular products (i.e., $\mathcal{I} = \{1,2\}$, and we study the continuation equilibria in which the firm charges monopoly prices (i.e., $p_1 = p_2 = p^m$). We denote by δ_i the probability with which the firm discloses the identity of product $i = 1,2$ upon inspection.

Consider first a candidate equilibrium such that $\delta_1 < \delta_2 = 1$. Observing that the identity of the first inspected product is undisclosed then reveals that it was product 1; hence, in any such equilibrium, active consumers fully learn the identity of the first inspected product. Intuitively, marginal consumers (i.e., those consumers indifferent between starting a search or not) will be willing to keep searching, in the absence of a match on the first inspection, only when becoming more optimistic about the product assigned to the remaining slot, that is, when encountering the less popular product on their first inspection. Hence, the marginal cost threshold satisfies:

$$\begin{aligned} c &= \frac{1}{2}\mu_1 s^m + \frac{1}{2}\left[\mu_2 s^m + (1-\mu_2)(\mu_1 s^m - c)\right] \\ \Leftrightarrow c &= \underline{c}_2^D \equiv \frac{\mu_1 + \mu_2 + (1-\mu_2)\mu_1}{3-\mu_2} s^m. \end{aligned}$$

As $\underline{c}_2^D$ is strictly increasing in μ_1 and μ_2, we have:

$$\mu_2 s^m = \underline{c}_2^D\Big|_{\mu_1=\mu_2} < \underline{c}_2^D < \underline{c}_2^D\Big|_{\mu_2=\mu_1} = \mu_1 s^m,$$

thus validating the working assumption that marginal active consumers keep searching only when expecting to find product 1 in the remaining slot. It can moreover be checked that, compared with the equilibrium that arises in the absence of disclosure, more consumers now start searching:

$$\begin{aligned} \underline{c}_2^D - \underline{c}_2 &= \frac{\mu_1 + \mu_2 + (1-\mu_2)\mu_1}{3-\mu_2} s^m - \frac{\mu_1 + \mu_2 - \mu_1\mu_2}{2 - \frac{\mu_1+\mu_2}{2}} s^m \\ &= \frac{(2-\mu_2)(1-\mu_1)}{(3-\mu_2)(4-\mu_1-\mu_2)} (\mu_1 - \mu_2) s^m > 0. \end{aligned}$$

This candidate equilibrium thus yields an expected profit equal to

$$\begin{aligned} \Pi^D &\equiv G\left(\underline{c}_2^D\right)\left\{\frac{1}{2}\mu_1 \pi^m + \frac{1}{2}\left[\mu_2 \pi^m + (1-\mu_2)\mu_1 \pi^m\right]\right\}\pi^m + \frac{G(\mu_2 s^m)}{2}(1-\mu_1)\mu_2 \pi^m \\ &= G\left(\underline{c}_2^D\right)\left(\mu_1 + (1-\mu_1)\frac{\mu_2}{2}\right)\pi^m + G(\mu_2 s^m)\frac{(1-\mu_1)\mu_2}{2}\pi^m. \end{aligned}$$

We now check that the firm has no incentive to deviate. To see this, suppose that it does and offers products 1 and 2 at prices p_1 and p_2. As the deviation does not affect consumers' decisions about whether to start searching, consumers with search cost $c > \underline{c}_2^D$ do not search, whereas those with $c < \underline{c}_2^D$ start searching and inspect either product with equal probability. Furthermore, with passive beliefs, consumers' decision to make a second inspection depends on the disclosure outcome of their first inspection only through their expectation about the identity of the remaining product; specifically, if the first product was disclosed to be product 2, then they expect to encounter product 1, offered at the monopoly price; otherwise (i.e., in the absence of disclosure, or if disclosure reveals that they first inspected product 1), consumers expect to encounter product 2 offered at the monopoly price. Therefore, after the first inspection:

- In the absence of a match:
  - if disclosure reveals that the first inspected product was product 2, then con-

sumers expect to encounter product 1 offered at the monopoly price and, as $\underline{c}_2^D < \mu_1 s^m$, all active consumers inspect the remaining product;

  - otherwise, they expect to encounter product 1 offered at the monopoly price and thus inspect the remaining product only if their cost lies below $\mu_2 s^m \left(< \underline{c}_2^D\right)$.

- In case of a match at the expected monopoly price, consumers stop searching and buy if their valuation exceeds the monopoly price.

- In case of a match at an unexpected price p^1, consumers inspect the remaining slot only if their cost lies below $c < \mu^2 (\max\{v, p^1\} - p^m)$, where $\mu^2 = \mu_1$ if the inspected product was disclosed to be product 2, and $\mu^2 = \mu_2$ otherwise.

Adapting the approach of used in the proof of Lemma 2, let $E[\Pi \mid c, \mathbf{s}, \mathbf{m}, \delta]$ denote the expected profit (with respect to the realized valuation v) obtained from active consumers with cost c, given the search behavior just described, the product sequence $\mathbf{s} = (s_1, s_2) \in \{(1,2), (2,1)\}$ and the match sequence $\mathbf{m} = (m_1, m_2) \in \{0,1\}^2$ that they would face if they were to visit both slots, and the disclosure outcome $\delta \in \{0,1,2\}$. Consumers who have a match only in their first inspection (i.e., $\mathbf{m} = (1,0)$) at price $p^1 \in \{p_1, p_2\}$ generate an expected profit $\pi(p^1) \leq \pi^m$. Those who would have a match only in their second inspection (i.e., $\mathbf{m} = (0,1)$) at price $p^2 \in \{p_1, p_2\}$ generate an expected profit $\pi(p^2) \leq \pi^m$ if product 2 is disclosed in the first inspection (i.e., $\delta = 2$), otherwise (i.e., $\delta \in \{0,1\}$) they generate that expected profit only if they have a cost $c < \mu_2 s^m$. For consumers that would have matches on both inspections (i.e., $\mathbf{m} = (1,1)$), the following possible cases arise:

- If they expect to encounter product 2 on the second inspection and $c > \mu_2 s^m$: $E[\Pi | c, \mathbf{s}, (1,1), \delta] = \pi(p_1) \leq \pi^m$.

- Otherwise, letting $(p^1, p^2) \in \{\{p_1, p_2\}, \{p_2, p_1\}\}$ denote the price encountered on the first and second inspections, and letting μ^2 denote consumers' updated beliefs about the probability of a match with the remaining product:

  - If $p^1 \leq p^2$ or $c > \mu^2 (p^1 - p^m)$, then consumers buy at p^1 if $v \geq p^1$, and do not buy otherwise; we thus have:

$$E[\Pi | c, \mathbf{s}, (1,1), \delta] = \pi\left(p^1\right).$$

  - If instead $p^1 > p^2$ and $c < \mu^2 (p^1 - p^m)$, then consumers never buy at p^1: consumers with a match on the first slot and $v \geq p^1$ continue to search, and thus end-up buying at p^2. It follows that consumers who buy do so at p_2; as consumers would not buy if their valuations lie below p^2, we then have:

$$E\left[\Pi|c, \mathbf{s}, (1,1), \delta\right] = \pi\left(p^2\right).$$

Hence, we have $E\left[\Pi|c, \mathbf{s}, (1,1), \delta\right] \leq \pi^m$.

To check that the deviation cannot be profitable, it suffices to note that:

- The deviation does not affect the number of active consumers and, from the above, cannot increase the profit generated by consumers that would have matches on both inspections, or a match on the first inspection only. As for consumers that would have a match on the second inspection only:
  - deviating on prices cannot increase the profit generated by those who make that second inspection;
  - deviating on disclosure cannot increase profit either: disclosing the identity of product 1 would have no impact (consumers would anyway expect to find product 2 in the next inspection), and not disclosing the identity of product 2 would reduce profit, by making consumers more pessimistic about the identity of the other product, thus inducing consumers with cost $c > \mu_2 s^m$ to stop searching.

We now turn to alternative candidate continuation equilibria. Consider first a candidate equilibrium in which $\delta_2 < \delta_1 = 1$. After a first, unsuccessful inspection, if the identity of the product is undisclosed then consumers infer that they inspected product 2 and stop searching if their cost exceeds $\mu_2 s^m$. But then, the firm would better not disclose the identity of product 1, so as make consumers believe that they inspected product 2 and induce them to keep searching as long as their cost does not exceed $\mu_1 s^m > \mu_2 s^m$. A similar argument applies to to candidate equilibria in which $\delta_1 = \delta_2 = 1$. By deviating and not disclosing the identity of product 1, consumers' passive beliefs would lead them to become more optimistic and keep searching more often. Specifically, consumers who inspect product 1 first would then expect the remaining product to deliver a match with higher probability (namely, $(\mu_1 + \mu_2)/2 > \mu_2$), which in turn raises the cost threshold under which they keep searching in the absence of a match.

Consider now a candidate equilibrium such that $\delta_1 = \delta_2 = 0$. As product identities are never disclosed, consumers' behavior is then the same as in our baseline setting. In particular, consumers who start searching keep searching. It follows that, indeed, the firm does not have an incentive to deviate and disclose the identity of the products: by construction, this cannot affect consumers' decision about whether to start searching, and may only induce some of them to stop searching in the absence of a first match.

Finally, consider a candidate equilibrium such that $\delta_1 = 0 < \delta_2 < 1$. It must be the case that all active consumers keep searching in the absence of a match with the first inspected product. Otherwise, the firm would have an incentive to disclose the identity of product 2 with probability 1, so as to make consumers more optimistic. But then, any such equilibrium induces the same consumer behavior as the "no disclosure" equilibrium $\delta_1 = \delta_2 = 0$. Conversely, to induce consumers to keep searching, the probability of disclosing product 2 should not be too large, so as to ensure that consumers remain sufficiently optimistic in the absence of disclosure. Specifically, in the absence of a match on the first inspection the posterior belief, which we will denote by $\hat{\mu}_2(\delta_2)$, should be sufficiently optimistic that marginal consumers (i.e., those with seach cost $c = \underline{c}_2$) are willing to keep searching. The posterior belief if given by:

$$\hat{\mu}_2(\delta_2) = \frac{\frac{1}{2}(1-\mu_1)\mu_2 + \frac{1}{2}(1-\delta_2)(1-\mu_2)\mu_1}{\frac{1}{2}(1-\mu_1) + \frac{1}{2}(1-\mu_2)(1-\delta_2)} = \frac{\mu_1 + \mu_2 - 2\mu_1\mu_2 - \mu_1(1-\mu_2)\delta_2}{1-\mu_1+(1-\mu_2)(1-\delta_2)},$$

which is decreasing in δ_2:

$$\frac{d}{d\delta_2} = -\frac{(1-\mu_1)(1-\mu_2)(\mu_1-\mu_2)}{(1-\mu_1+(1-\mu_2)(1-\delta_2))^2} < 0.$$

Hence, as δ_2 increases, $\hat{\mu}_2(\delta_2)$ decreases from

$$\hat{\mu}_2(0) = \frac{\mu_1+\mu_2-2\mu_1\mu_2}{2-\mu_2-\mu_1}$$

to $\hat{\mu}_2(1) = \mu_2$. From Proposition 1, we know that $\underline{c}_2 < \hat{\mu}_2(0)s^m$, as consumers who start searching then keep searching; conversely, it is straightforward to check that $\underline{c}_2 > \hat{\mu}_2(1)s^m = \mu_2 s^m$:

$$\frac{\underline{c}_2}{s^m} - \mu_2 = \frac{\mu_1+\mu_2-\mu_1\mu_2}{2-\frac{\mu_1+\mu_2}{2}} - \mu_2 = \frac{(\mu_1-\mu_2)(2-\mu_2)}{4-\mu_2-\mu_1} > 0.$$

It follows that there exists $\hat{\delta}_2 \in (0,1)$ such that, as long as $\delta_2 < \hat{\delta}_2$, consumers who start searching keep searching in the absence of a match.

J Proof of Proposition 11

Establishing existence is straightforward given our earlier results; we therefore provide only a brief sketch of the proof here. First, consumers have no incentive to deviate. In the equilibria in which the firm uniformly randomizes over the slot allocation, consumers are indifferent as to which slot to inspect first. Moreover, in the equilibria described in part 1 of the proposition, the probability of disclosure of product 2 is "sufficiently small" that—in the absence of disclosure and of a match on the first inspection—each consumer still puts a high enough probability on the event that the inspected product in slot 1 is the less popular product, which ensures that he is willing to inspect slot 2.[40] The firm has no incentive to deviate either. As before, a deviation on the slot allocation would not be observed by consumers and thus would have no effect on consumer participation. In part 1 of the proposition, active consumers keep searching until finding a match; hence, the firm cannot improve upon the outcome by changing the slot allocation or the disclosure policy. In part 2, active consumers randomize over which slot to visit first; hence, the firm is indifferent as to which product to allocate to which slot. Moreover, the firm cannot benefit by changing its disclosure policy: disclosing or not product 1 does not make a difference to consumer behavior, and disclosing product 2 is strictly optimal as consumers without a match upon the first inspection would otherwise stop searching.

We now show that there do not exist other equilibria. W.l.o.g., we confine attention to $r \geq 1/2$. Note first that any (monopoly-pricing) equilibrium must have the property that there exists a cutoff type $\underline{c} \in (\mu_2 s^m, \mu_1 s^m]$ such that consumers with search costs $c < \underline{c}$ inspect at least one slot whereas those with search costs $c > \underline{c}$ do not inspect any slot. Consumers with search costs $c < \mu_2 s^m$ will continue searching until finding a match, no matter what the disclosure policy (on or off the equilibrium path); the order of their search depends only on the equilibrium value of r: If $r > 1/2$, all of these consumers necessarily start by inspecting slot 1 first; if $r = 1/2$, they are indifferent.

Let δ_{ik} the probability that the firm discloses the identity of product i when allocated to slot k.

We start by noting that there cannot be an equilibrium with $r = 1$. In such an equilibrium, all active consumers with search costs $c > \mu_2 s^m$ would stop searching after inspecting slot 1. The firm could thus profitably deviate by setting $r = 0$ (i.e., allocating product 2 to slot 1) and disclosing the identity of product 2: all consumers who do not have a match with the product in slot 1 would then choose to inspect slot 2, knowing that product 1 is allocated to that slot.

It follows that, in equilibrium, each product is allocated to both slots with positive probability. Next, we claim that any equilibrium must have the following properties. First, any slot j is inspected first by some consumers with search costs $c > \mu_2 s^m$, then either $\delta_{2j} = 1$, in which case consumers fully learn the identity of the product and consumers with search costs $c > \mu_2 s^m$ stop searching in the absence of a match, or else $\delta_{1j} = 0$ and δ_{2j} is small enough(possibly zero) to ensure that all consumers continue searching in the absence of a match. Second, if both slots are visited first by some consumers with search costs $c > \mu_2 s^m$, then either $\delta_{21} = \delta_{22} = 1$ or else $\delta_{11} = \delta_{12} = 0$ and δ_{21} and δ_{22} are both sufficiently small that consumers continue searching in the absence

[40] Recall that if the probability of disclosure is zero, then for any $r < \overline{r}$ any active consumer who does not have a match with the product offered in slot 1 has a strict incentive to continue searching.

of a match, *regardless of which slot they inspected first.*

To this end, suppose first that there exists a slot j that is inspected first by some of the consumers with search costs $c > \mu_2 s^m$ and that, with positive probability, some of those consumers stop searching in the absence of a match. Then, we must have $\delta_{2j} = 1$. Suppose otherwise that $\delta_{2j} < 1$. If some of these consumers were to stop searching in the absence of disclosure, then the firm could profitably deviate by increasing δ_{2j} so as to induce consumers who inspected product 2 to keep searching. Therefore, all consumers must continue searching in the absence of disclosure. It is then optimal for the firm to set $\delta_{1j} = 0$, so as to ensure that all these consumers keep searching. We thus have a contradiction: no matter which product is allocated to slot j, all consumers continue searching in the absence of a match.

Suppose now that there exists a slot j that is inspected first by some of the consumers with search costs $c > \mu_2 s^m$ and that all these consumers continue searching in the absence of a match. Then, $\delta_{1j} = 0$ as otherwise all consumers with search costs $c > \mu_2$ would stop searching after product 1 is revealed to be in that slot; moreover, δ_{2j} has to be sufficiently small so that all of those consumers without a match are still willing to continue searching when nothing is disclosed.

Second, suppose that slots 1 and 2 are both visited first by some consumers with search costs $c > \mu_2 s^m$. If $\delta_{2j} = 1$ for $j = 1$ or $j = 2$, then $\delta_{21} = \delta_{22} = 1$. To see this, suppose otherwise that $\delta_{2j} = 1$ but $\delta_{2k} < 1$. By the first property just established, this means that in the absence of a match on the first inspection: (i) consumers with search costs $c > \mu_2 s^m$ who visit slot j first stop searching whenever product 1 is allocated to that slot; by contrast, those who visit slot k first all continue searching. But then the firm could profitably deviate by allocating product 1 with probability 1 to slot k without disclosing its identity, together with disclosing the identity of product 2 whenever consumers inspect slot j. But this contradicts our earlier finding that there does not does exist a pure positioning equilibrium (i.e., we cannot have $r = 0$ nor $r = 1$).

Note that this implies that if slots 1 and 2 are both visited first by some consumers with search costs $c > \mu_2 s^m$ and $\delta_{21} = \delta_{22} = 1$, then we must have $r = 1/2$ as otherwise, if $r > 1/2$, all active consumers would inspect slot 1 first.

Finally, note that this also implies that if $\delta_{2j} = 1$ for only one slot j (i.e., $\delta_{2k} < 1$), then slot j must be the slot that is visited second by all consumers (i.e., given the convention that $r \geq 1/2$, it must be slot 2), and the disclosure policy in the slot that is visited first (i.e., slot 1) is non-revealing, so that all active consumers continue searching in the absence of a match.

K Proof of Proposition 14

Suppose that the platform chose a size $n \geq 1$ in stage 1. We show that, in the proper continuation subgame that starts in stage 2, the following beliefs and strategies, in which $\mathcal{I}^e(n, \hat{n})$ denotes an arbitrary set of sellers of size $\hat{n}$ and $\hat{\mathcal{I}}_i(\mathcal{I})$ denotes an arbitrary constitute a PBE with passive beliefs:

- In stage 5, for any number $\hat{n}$ of sellers present on the platform ("active sellers" hereafter), consumers:
  - expect the set of active sellers to be $\mathcal{I}_n$ if $\hat{n} = n$ and $\mathcal{I}^e(n, \hat{n})$ if $\hat{n} < n$, and expect all products to be offered at the monopoly price p^m (regardless of the prices they actually encounter);
  - start searching if and only if their cost is lower than $\underline{c}(\mathcal{I}^e(n, \hat{n}))$, where

    $$\underline{c}(\mathcal{I}) \equiv \frac{M(\mathcal{I})}{N(\mathcal{I})} s^m,$$

    where $M(\mathcal{I})$ and $N(\mathcal{I})$ are respectively given by (1) and (3) – in particular, $\underline{c}(\mathcal{I}_n) = \underline{c}_n$–, in which case they keep searching until finding a match (again, regardless of the prices they encounter);
  - upon finding a match, stop searching if they the associated price does not exceed the monopoly price, in which case they buy if their valuation exceeds the associated price (if the associated price exceeds the monopoly price, they keep searching if their search cost is low enough).

- In stage 4, for any set $\mathcal{I}$ of sellers approached in stage 2 and any number $\hat{n}$ of active sellers, each of these sellers:
  - expects charges the monopoly price p^m and expects the others to do the same.

- In stage 3, for any $\mathcal{I}$:
  - the platform offers each seller $i \in \mathcal{I}$ to host it for a fee $\hat{\phi}_i(\mathcal{I}) \equiv G(\underline{c}_n)\hat{\alpha}^i(\mathcal{I})\pi^m$, where

    $$\hat{\alpha}^i(\mathcal{I}) \equiv \frac{1}{n}\left[1 + \sum_{k=1}^{n-1} \frac{1}{|\mathcal{P}_k^i(\mathcal{I})|} \sum_{\mathcal{J} \in \mathcal{P}_k^i(\mathcal{I})} \Pi_{j \in \mathcal{J}}(1 - \mu_j)\right] \mu_i$$

    denotes the probability that a consumer has his first match with product $i \in \mathcal{I}$, and $\mathcal{P}_k^i(\mathcal{I})$ denotes the set of subsets of $\mathcal{I}$ that are of size $k \leq n$ and do not include product i;

– each seller $i \in \mathcal{I}$ expects all other sellers in $\mathcal{I} \backslash \{i\}$ to join the platform, is willing to pay up to $\hat{\phi}_i(\mathcal{I})$ for joining the platform, and accepts the platform's offer.

- In stage 2, the plaftorm approaches the sellers of the n most popular products: $\mathcal{I} = \mathcal{I}_n$.

We start with the last stage. For any $\hat{n} \leq n$, as consumers expect a uniform price and have passive beliefs, after k unsuccesful inspections their updated match probability is given by $\alpha_k(\mathcal{I}^e(n, \hat{n}))$, which, from Lemma 1 increases with k; hence, if they found it desirable to start searching, they are willing to keep searching – see the proof of Proposition 1. Upon finding a match, however, they stop searching if the associated price does not exceed the monopoly price, as they do not expect to find a better price; by contrast, if the associated price exceeds the monopoly price, the prospect of finding a lower price induces consumers with low enough search costs to inspect additional products. Hence, consumers who start searching expect to find a match with probability $M(\mathcal{I}^e(n, \hat{n}))$ and anticipate an expected number of inspections given by $N(\mathcal{I}^e(n, \hat{n}))$; it follows that consumers are willing to start searching if and only if $c \leq \underline{c}(\mathcal{I}^e(n, \hat{n}))$.

Moving to stage 4, it is straightforward to check that, for any $\mathcal{I}$ and any $\hat{n} \leq n$, no active seller has an incentive to deviate and charge $p \neq p^m$: this would have no impact on the number of visits (active consumers visit the seller if and only if they do not have a prior match with another seller, regardless of the price actually charged by the seller) and would strictly decrease the expected profit per visit: as a consumer would not buy if there is no match or his valuation is lower than p, this expected profit per visit cannot exceed $\mu_i p[1 - F(p)] = \mu_i \pi(p) < \mu_i \pi(p^m)$ (in addition, raising the price above p^m may induce some consumers with $v > p$ to inspect additional products and eventually buy from another seller).

We now turn to stage 3. As seller $i \in \mathcal{I}$ expects all other sellers in $\mathcal{I}$ to join the platform, if it also join the platform it expects consumers to observe $\hat{n} = n$ and thus to become active if their search costs lies below $\underline{c}(\mathcal{I}_n) = \underline{c}_n$; hence, its expected profit from joining the platform is given by $\hat{\phi}_i(\mathcal{I}) = G(\underline{c}_n)\hat{\alpha}^i(\mathcal{I})\pi^m$, and it is thus willing to pay up to $\hat{\phi}_i(\mathcal{I})$; conversely, as acceptance decisions are private, the platform finds it profitable to make an acceptable offer to every seller $i \in \mathcal{I}$, and the most profitable of these offers is precisely given by $\hat{\phi}_i(\mathcal{I})$.

Finally, consider stage 2. Given the above continuation strategies, approaching a set $\mathcal{I}$ of sellers gives the platform an expected profit equal to obtains

$$\sum_{i \in \mathcal{I}} \hat{\phi}_i(\mathcal{I}) = G(\underline{c}_n) \sum_{i \in \mathcal{I}} \alpha^i(\mathcal{I}) \pi^m = G(\underline{c}_n) M(\mathcal{I}) \pi^m.$$

The conclusion then follows from the fact that $M(\mathcal{I})$ is maximal for $\mathcal{I} = \mathcal{I}_n$.

Online Appendix for
Consumer Search and Choice Overload

A On the Multiplicity of Equilibria with Passive Beliefs

In this section, we construct an equilibrium for the case where $n = 2$ in which the firm offers its products at different prices. For simplicity, we assume that all consumers who start searching face the same search cost, c. All other assumptions are as in the baseline setting of Section 2.

A.1 Candidate Equilibrium

We consider a candidate equilibrium in which the firm offers products 1 at the monopoly price ($p_1^* = p^m$) and product 2 at a moderately lower price ($p_2^* < p^m$), in such a way that product 1 still generates the greater expected surplus: $\mu_1 s^m > \mu_2 s_2^*$, where $s_2^* \equiv s\left(p_2^*\right)$. In what follows, we fix ($F\left(\cdot\right)$, μ_1, and) the equilibrium price p_2^*, and characterize the relevant range of c and μ_2 for which there exists such an equilibrium; the above condition amounts to:

$$\mu_1 s^m > \mu_2 s_2^* \Longleftrightarrow \mu_2 < \bar{\mu}_2 \equiv \mu_1 \frac{s^m}{s_2^*}. \tag{13}$$

We further focus on an equilibrium with passive beliefs in which consumers make a first inspection, and then inspect the remaining product only if they observe p_2^* and have no match. To ensure that consumers start searching, we must have:

$$\begin{aligned} c \quad &< \quad \frac{1}{2}\mu_1 s^m + \frac{1}{2}\left[\mu_2 s_2^* + \left(1-\mu_2\right)\left(\mu_1 s^m - c\right)\right] \\ &\Longleftrightarrow \quad c < \bar{c}\left(\mu_2\right) \equiv \frac{\left(2-\mu_2\right)\mu_1 s^m + \mu_2 s_2^*}{3-\mu_2}. \end{aligned} \tag{14}$$

For the second inspection decision we must have:

- In the absence of a match at the first inspection, upon observing an unexpected price $p \notin \{p^m, p_2^*\}$, consumers stop searching:

$$c > \hat{c}\left(\mu_2\right) \equiv \frac{\left(1-\mu_2\right)\mu_1 s^m + \left(1-\mu_1\right)\mu_2 s_2^*}{2-\mu_1-\mu_2}. \tag{15}$$

To ensure that this condition is compatible with (14), we will assume that[1]

$$\mu_1 < \frac{1}{2}.$$

Altogether, the above conditions yield:

$$\mu_2 s^m < \hat{c}(\mu_2) < \bar{c}(\mu_2) < \mu_1 s^m.$$

It follows that consumers stop searching if they observe $p = p^m$ (as $\mu_1 s^m > (\bar{c}(\mu_2) >) \, c$), and keep searching if instead they observe $p = p_2^*$ (as $\mu_2 s_2^* < (\hat{c}(\mu_2) <) \, c$).

- After a first match, consumers stop searching. This is obviously the case if they observed $p = p_2^* < p^m$; if instead they observed $p \notin \{p^m, p_2^*\}$, this holds if they stop searching even when p is prohibitive (i.e., such that $s(p) = 0$), which amounts to:

$$c > \frac{\mu_2 \mu_1 s^m + \mu_1 \mu_2 s_2^*}{\mu_1 + \mu_2},$$

and is implied by (15).[2] Finally, if they observed $p = p^m$, this requires:

$$c > \tilde{c}(\mu_2) \equiv \mu_2 (p^m - p_2^*).$$

It follows that all the above conditions are satisfied if $\mu_2 < \bar{\mu}_2$ and $c \in (\underline{c}(\mu_2), \bar{c}(\mu_2))$, where:

$$\underline{c}(\mu_2) \equiv \max\{\tilde{c}(\mu_2), \hat{c}(\mu_2)\}.$$

As μ_2 goes to zero, $\underline{c}(\mu_2)$ tends to $\hat{c}(0) = \mu_1 s^m / (2 - \mu_1)$, whereas $\bar{c}(\mu_2)$ tends to $2\mu_1 s^m / 3$, which exceeds $\hat{c}(0)$ for $\mu_1 < 1/2$; hence, for any given $\mu_1 < 1/2$, there exists a non-empty search cost range for μ_2 small enough.

A.2 Possible Deviations

In the above candidate equilibrium, the firm obtains an expected profit equal to:

$$\begin{aligned} \Pi^* &\equiv \frac{1}{2}\mu_1 \pi^m + \frac{1}{2}\left[\mu_2 \pi_2^* + (1 - \mu_2) \mu_1 \pi^m\right] \\ &= \frac{(2 - \mu_2) \mu_1 \pi^m + \mu_2 \pi_2^*}{2}, \end{aligned}$$

[1] We have:

$$\bar{c}(\mu_2) - \hat{c}(\mu_2) = \frac{1 - 2\mu_1 + \mu_1 \mu_2}{(3 - \mu_2)(2 - \mu_1 - \mu_2)} (\mu_1 s^m - \mu_2 s_2^*) > 0,$$

where the inequality follows from (13) and $\mu_1 < 1/2$.

[2] We have:

$$\hat{c}(\mu_2) - \frac{\mu_2 \mu_1 s^m + \mu_1 \mu_2 s_2^*}{\mu_1 + \mu_2} = \frac{(\mu_1 - \mu_2)(\mu_1 s^m - \mu_2 s_2^*)}{(2 - \mu_1 - \mu_2)(\mu_1 + \mu_2)} > 0.$$

where $\pi_2^* \equiv \pi(p_2^*)$. As μ_2 goes to zero, this expected profit tends to

$$\Pi_0^* \equiv \mu_1 \pi^m.$$

We can distinguish three types of deviation, depending on which prices are affected.

A.2.1 Single deviation on p_1

A single deviation on the price of the product 1 from p^m to $p_1 \notin \{p^m, p_2\}$ does not affect consumers' search behavior: they stop searching after the first inspection unless they encountered p_2 and had no match, as along the equilibrium path. It follows that such deviation cannot be profitable, as it simply replaces the monopoly profit π^m with a lower profit $\pi(p_1) < \pi^m$ in case of a match with the product 1.

A single deviation from $p_1 = p^m$ to $p_1 = p_2^*$ induces instead consumers to keep searching in the absence of a match (and stop searching otherwise); hence, it yields:

$$\begin{aligned} \hat{\Pi} &\equiv \frac{1}{2}\left[\mu_1 \pi_2^* + (1-\mu_1)\mu_2 \pi_2^*\right] + \frac{1}{2}\left[\mu_2 \pi_2^* + (1-\mu_2)\mu_1 \pi_2^*\right] \\ &= \left[\mu_1 + \mu_2 - \mu_1 \mu_2\right] \pi_2^*. \end{aligned}$$

We have:

$$\begin{aligned} \Pi^* - \hat{\Pi} &= \frac{1}{2}\mu_1 \pi^m + \frac{1}{2}\left[\mu_2 \pi_2^* + (1-\mu_2)\mu_1 \pi^m\right] \\ &\quad -\frac{1}{2}\left[\mu_1 \pi_2^* + (1-\mu_1)\mu_2 \pi_2^*\right] - \frac{1}{2}\left[\mu_2 \pi_2^* + (1-\mu_2)\mu_1 \pi_2^*\right] \\ &= \frac{(2-\mu_2)\mu_1(\pi^m - \pi_2^*) - (1-\mu_1)\mu_2 \pi_2^*}{2}. \end{aligned}$$

This deviation is therefore unprofitable as long as:

$$(2-\mu_2)\mu_1(\pi^m - \pi_2^*) > (1-\mu_1)\mu_2 \pi_2^*. \tag{16}$$

As μ_2 goes to zero, the left-hand tends to $2\mu_1(\pi^m - \pi_2^*) > 0$ whereas the right-hand side tends to 0. Hence, there exists $\hat{\mu}_2$ such that this deviation is unprofitable as long as $\mu_2 < \hat{\mu}_2$.

A.2.2 Other deviations

Deviating on the price of the second product – in isolation or combined with deviating on the price of the first product – induces all consumers to stop searching after the first inspection, regardless of whether there is a match; hence, a deviation to $(\tilde{p}_1, \tilde{p}_2)$ (where $\tilde{p}_1 = p^m$ in case of an isolated deviation on p_2, and $\tilde{p}_1 \neq p^m$ in case of a simultaneous deviation on both prices) yields:

$$\tilde{\Pi} \equiv \frac{1}{2}\mu_1 \pi\left(\tilde{p}_1\right) + \frac{1}{2}\mu_2 \pi\left(\tilde{p}_2\right).$$

Using $\pi\left(\tilde{p}_i\right) \leq \pi^m$ and $\pi_2^* \geq 0$, we have:

$$\begin{aligned} \Pi^* - \tilde{\Pi} &\geq \frac{1}{2}\mu_1 \pi^m + \frac{1}{2}\left(1-\mu_2\right)\mu_1 \pi^m - \frac{1}{2}\mu_1 \pi^m - \frac{1}{2}\mu_2 \pi^m \\ &= \left(\mu_1 - \left(1+\mu_1\right)\mu_2\right)\frac{\pi^m}{2}. \end{aligned}$$

It follows that this deviation is unprofitable whenever

$$\mu_2 < \tilde{\mu}_2 \equiv \frac{\mu_1}{1+\mu_1}.$$

A.3 Existence

For any $\mu_1 \in (0, 1/2)$ and any $p_2^* < p^m$, the above strategies constitute an equilibrium for any $\mu_2 \in (0, \min\{\bar{\mu}_2, \hat{\mu}_2, \tilde{\mu}_2\})$ and any $c \in \left(\underline{c}\left(\mu_2\right), \bar{c}\left(\mu_2\right)\right)$.

Illustration: Suppose that match valuations are uniformly distributed over $[0, 1]$; we have:

$$\pi\left(p\right) = p\left(1-p\right) \text{ and } s\left(p\right) = \frac{\left(1-p\right)^2}{2},$$

and thus:

$$p^m = \frac{1}{2}, s^m = \frac{1}{8} \text{ and } \pi^m = \frac{1}{4}.$$

Fix $p_2^* = p^m/2 = 1/4$; we have

$$\begin{aligned} s_2^* &= \left[\frac{\left(1-p\right)^2}{2}\right]_{p=\frac{1}{4}} = \frac{9}{32}, \\ \pi_2^* &= \left[p\left(1-p\right)\right]_{p=\frac{1}{4}} = \frac{3}{16}, \end{aligned}$$

and:

$$\bar{\mu}_2 = \mu_1 \frac{\frac{1}{8}}{\frac{9}{32}} = \frac{4\mu_1}{9} < \tilde{\mu}_2 \equiv \frac{\mu_1}{1+\mu_1}.$$

Condition (16) amounts to:

$$\begin{aligned} 0 &< \left[\left(2-\mu_2\right)\mu_1\left(\pi^m - \pi_2^*\right) - \left(1-\mu_1\right)\mu_2 \pi_2^*\right]_{\pi^m=\frac{1}{4}, \pi_2^*=\frac{3}{16}} \\ &= \frac{1}{8}\mu_1 - \frac{3}{16}\mu_2 + \frac{1}{8}\mu_1\mu_2 \\ &\Leftrightarrow \mu_2 < \hat{\mu}_2 = \frac{2\mu_1}{3-2\mu_1}, \end{aligned}$$

where $\hat{\mu}_2$ also exceeds $\bar{\mu}_2$:

$$\hat{\mu}_2 - \bar{\mu}_2 = \frac{2\mu_1}{3-2\mu_1} - \frac{4\mu_1}{9} = \frac{2\mu_1(3+4\mu_1)}{9(3-2\mu_1)} > 0.$$

Finally, we have:

$$\begin{aligned}
\hat{c}(\mu_2) &= \left[\frac{(1-\mu_2)\mu_1 s^m + (1-\mu_1)\mu_2 s_2^*}{2-\mu_1-\mu_2}\right]_{s^m=\frac{1}{8}, s_2^*=\frac{9}{32}} = \frac{4\mu_1+9\mu_2-13\mu_1\mu_2}{32(2-\mu_1-\mu_2)}, \\
\tilde{c}(\mu_2) &= [\mu_2(p^m - p_2^*)]_{p^m=\frac{1}{2}, p_2^*=\frac{1}{4}} = \frac{\mu_2}{4}, \\
\bar{c}(\mu_2) &= \left[\frac{(2-\mu_2)\mu_1 s^m + \mu_2 s_2^*}{3-\mu_2}\right]_{s^m=\frac{1}{8}, s_2^*=\frac{9}{32}} = \frac{8\mu_1+9\mu_2-4\mu_1\mu_2}{32(3-\mu_2)}
\end{aligned}$$

As expected, $\bar{c}(\mu_2) > \hat{c}(\mu_2)$:

$$\frac{8\mu_1+9\mu_2-4\mu_1\mu_2}{32(3-\mu_2)} - \frac{4\mu_1+9\mu_2-13\mu_1\mu_2}{32(2-\mu_1-\mu_2)} = \frac{(4\mu_1-9\mu_2)(1-2\mu_1+\mu_1\mu_2)}{32(2-\mu_1-\mu_2)(3-\mu_2)},$$

where the right-hand side is positive for $\mu_1 \in (0,1/2)$ and $\mu_2 \in (0, 4\mu_1/9)$. In addition, $\bar{c}(\mu_2) > \tilde{c}(\mu_2)$ as long as:

$$\begin{aligned}
0 &> \frac{8\mu_1+9\mu_2-4\mu_1\mu_2}{32(3-\mu_2)} - \frac{\mu_2}{4} = \frac{8\mu_1-15\mu_2+8\mu_2^2-4\mu_1\mu_2}{32(3-\mu_2)} \\
&\Leftrightarrow \mu_2 < \check{\mu}_2 \equiv \frac{15+4\mu_1-\sqrt{225-136\mu_1+16\mu_1^2}}{16}
\end{aligned}$$

where the threshold $\check{\mu}_2$ exceeds $\bar{\mu}_2$:

$$\begin{aligned}
\check{\mu}_2 - \bar{\mu}_2 &= \frac{15+4\mu_1-\sqrt{225-136\mu_1+16\mu_1^2}}{16} - \frac{4\mu_1}{9} \\
&= \frac{15-\frac{28}{9}\mu_1-\sqrt{225-136\mu_1+16\mu_1^2}}{16} \\
&= \frac{\left(15-\frac{28}{9}\mu_1\right)^2-(225-136\mu_1+16\mu_1^2)}{16\left(15-\frac{28}{9}\mu_1+\sqrt{225-136\mu_1+16\mu_1^2}\right)} \\
&= \frac{128}{1296}\frac{\mu_1(27-4\mu_1)}{15-\frac{28}{9}\mu_1+\sqrt{225-136\mu_1+16\mu_1^2}} \\
&> 0,
\end{aligned}$$

where the inequality stems from the fact that the numerator and the denominator of the last expression are both positive for any $\mu_1 \in (0,1)$.

It follows that, for $p_2^* = 1/4$, there exists an equilibrium such as described above for any $\mu_1 \in (0,1/2]$, any $\mu_2 \in (0, 4\mu_1/9)$ and any $c \in (\underline{c}(\mu_2), \bar{c}(\mu_2))$, where $\underline{c}(\mu_2) =$

$\max\{\tilde{c}(\mu_2), \hat{c}(\mu_2)\}$.

A.4 Profitability

The above equilibrium can be more profitable than the monopoly price equilibrium, by encouraging more consumers to search. The upper bound on the search cost, given by (14), is indeed lower than that for monopoly pricing, which is given by:

$$\underline{c}_2 = \frac{M_2}{N_2} s^m,$$

where

$$M_2 = \mu_1 + \mu_2 - \mu_1 \mu_2 \text{ and } N_2 = 2 - \frac{\mu_1 + \mu_2}{2}.$$

Indeed, we have:

$$\begin{aligned}
\bar{c}(\mu_2) - \underline{c}_2 &= \frac{(2-\mu_2)\mu_1 s^m + \mu_2 s_2^*}{3-\mu_2} - \frac{\mu_1 + \mu_2 - \mu_1\mu_2}{1 + \frac{1-\mu_1}{2} + \frac{1-\mu_2}{2}} s^m \\
&> \frac{(2-\mu_2)\mu_1 + \mu_2}{3-\mu_2} s^m - \frac{\mu_1 + \mu_2 - \mu_1\mu_2}{1 + \frac{1-\mu_1}{2} + \frac{1-\mu_2}{2}} s^m \\
&= (1-\mu_1) \frac{2-\mu_2}{3-\mu_2} \frac{\mu_1 - \mu_2}{4 - \mu_1 - \mu_2} s^m \\
&> 0.
\end{aligned}$$

It follows that, in the above illustration, for any $c \in (\max\{\underline{c}(\mu_2), \underline{c}_2\}, \bar{c}(\mu_2))$:

- the monopoly pricing equilibrium would generate no search, and therefore yield zero profit.[3]
- by contrast, the above equilibrium induces all consumers to search, and yields a positive profit.

B Properties of the Average Inframarginal Cost

We show here that the average cost of inframarginal consumers, $c^e(c)$, is proportional to the cost of the marginal consumer, c, if and only if $G(c) = (c/\bar{c})^\alpha$ for some $\alpha > 0$. Using integration by parts, we have:

$$\frac{c^e(c)}{c} = \frac{\int_0^c x g(x) dx}{cG(c)} = \frac{cG(c) - \Gamma(c)}{cG(c)} = 1 - \frac{\Gamma(c)}{cG(c)},$$

[3] Although the firm is there indifferent about the product portfolio and its prices, introducing an infinitesimal number of consumers with low enough search costs would not materially affect the analysis but would eliminate this indifference.

where $\Gamma(c) = \int_0^c G(x)dx$ denotes the primitive of $G(c)$. For $c^e(c)$ to be proportional to c, the ratio $c^e(c)/c$ must remain constant; the above equality therefore implies that the ratio $\Gamma(c)/cG(c)$, too, is constant. Hence, there exists α such that

$$\frac{G(c)}{\Gamma(c)} = \frac{1+\alpha}{c}.$$

Solving this differential equation yields:

$$\log\Gamma(c) = (1+a)\log c + \log b,$$

for some constant b, or:

$$\Gamma(c) = bc^{1+\alpha}$$

and thus:

$$G(c) = \Gamma'(c) = (1+\alpha)\,bc^{\alpha}.$$

The function $G(c)$ must be increasing in c, which requires $\alpha > 0$, and it must be equal to 1 for $c = \bar{c}$, which determines the constant b, and leads to $G(c) = (c/\bar{c})^{\alpha}$.

C Proof of Lemma 4

Fix $\mu_1 \in (0,1)$ and denote by η the percentage reduction in the probability of a match for the second product; that is:

$$\eta \equiv \frac{\mu_1 - \mu_2}{\mu_1} \iff \mu_2 = (1-\eta)\,\mu_1.$$

The profit achieved with n products is given by

$$\Pi_n = G(\underline{c}_n)\,M(\mathcal{I}_n)\,\pi^m.$$

Therefore:

- for $n = 1$, we have $M_1 = \mu_1$, $N_1 = 1$, $\underline{c}_1 = \mu_1 s^m$, and thus

$$\Pi_1 = G(\mu_1 s^m)\,\mu_1\pi^m.$$

- for $n = 2$, we have:

$$
\begin{aligned}
M_2 &= 1-(1-\mu_1)(1-\mu_2) = (2-\mu_1-(1-\mu_1)\eta)\,\mu_1, \\
N_2 &= 1+\frac{(1-\mu_1)+(1-\mu_2)}{2} = 2-\mu_1+\frac{\mu_1}{2}\eta, \\
\underline{c}_2 &= \frac{2-\mu_1-(1-\mu_1)\eta}{2-\mu_1+\frac{\mu_1}{2}\eta}\mu_1 s^m = \frac{1-\frac{1-\mu_1}{2-\mu_1}\eta}{1+\frac{\mu_1}{2(2-\mu_1)}\eta}\mu_1 s^m,
\end{aligned}
$$

leading to:

$$
\Pi_2 = G\left(\frac{1-\frac{1-\mu_1}{2-\mu_1}\eta}{1+\frac{\mu_1}{2(2-\mu_1)}\eta}\mu_1 s^m\right)(2-\mu_1-(1-\mu_1)\eta)\,\mu_1\pi^m.
$$

Hence, $n=2$ dominates $n=1$ (i.e., $\Pi_2 > \Pi_1$) if and only if:

$$
\phi(\eta;\mu_1) \equiv G\left(\frac{1-\frac{1-\mu_1}{2-\mu_1}\eta}{1+\frac{\mu_1}{2(2-\mu_1)}\eta}\mu_1 s^m\right)(2-\mu_1-(1-\mu_1)\eta) - G(\mu_1 s^m) > 0.
$$

The function $\phi(\eta;\mu_1)$ satisfies:

$$
\begin{aligned}
\phi(0;\mu_1) &= (1-\mu_1)\,G(\mu_1 s^m) > 0, \\
\phi(1;\mu_1) &= G\left(\frac{1-\frac{1-\mu_1}{2-\mu_1}}{1+\frac{\mu_1}{2(2-\mu_1)}}\mu_1 s^m\right) - G(\mu_1 s^m) = G\left(\frac{2\mu_1 s^m}{4-\mu_1}\right) - G(\mu_1 s^m) < 0,
\end{aligned}
$$

where the first inequality stems from $\mu_1 \in (0,1)$ and the last inequality follows from $2/(4-\mu_1) < 2/3 < 1$ for $\mu_1 \in (0,1)$. The function $\phi(\eta;\mu_1)$ is moreover decreasing in η in the range $\eta \in [0,1]$:

$$
\begin{aligned}
\frac{\partial\phi}{\partial\eta}(\eta;\mu_1) &= -g\left(\frac{1-\frac{1-\mu_1}{2-\mu_1}\eta}{1+\frac{\mu_1}{2(2-\mu_1)}\eta}\mu_1 s^m\right)\frac{2(2-\mu_1)^2\mu_1 s^m}{(4-2\mu_1+\eta\mu_1)^2}(2-\mu_1-(1-\mu_1)\eta) \\
&\quad -(1-\mu_1)\,G\left(\frac{1-\frac{1-\mu_1}{2-\mu_1}\eta}{1+\frac{\mu_1}{2(2-\mu_1)}\eta}\mu_1 s^m\right) \\
&< 0.
\end{aligned}
$$

It follows that, for any $\mu_1 \in (0,1)$, there exists a unique $\hat{\eta}(\mu_1)$ such that $\phi(\hat{\eta}(\mu_1);\mu_1)$; it is then strictly better to offer two products rather than one if $\eta < \hat{\eta}(\mu_1)$, and strictly better to offer one product rather than two if $\eta > \hat{\eta}(\mu_1)$. The continuity of the density $g(\cdot)$ ensures that $\hat{\eta}(\mu_1)$ varies continuously with μ_1. Furthermore:

- for $\mu_1 = 1$, we have:

$$
\phi(\eta;1) = G\left(\frac{2}{\eta+2}s^m\right) - G(s^m),
$$

implying that $\hat{\eta}(1) = 0$;

- for $\mu_1 = 0$, $\phi(\cdot; 0) = 0$; hence, as μ_1 tends to 0, the first-order approximation of $\phi(\eta; \mu_1)$ is given by:

$$\phi(\eta; \mu_1) \simeq \frac{\partial \phi}{\partial \mu_1}(\eta; 0)\, \mu_1 = \left[\frac{(2-\eta)^2}{2} - 1\right] g(0)\, \mu_1 s^m.$$

It follows that, as μ_1 tends to 0, $\hat{\eta}(\mu_1)$ tends to the solution to:

$$\frac{(2-\eta)^2}{2} - 1 = 0,$$

that is:[4]

$$\lim_{\mu_1 \longrightarrow 0} \hat{\eta}(\mu_1) = 2 - \sqrt{2} \simeq 0.585$$

D Profitability of Monopoly Pricing Equilibria

We consider here the case where positioning and/or disclosure are available, and provide a comparison of the profits obtained by the firm in the various monopoly pricing equilibria that may arise. As the random positioning equilibrium characterized by Proposition 7 is a particular case of a noisy positioning equilibrium, we focus on the *pure positioning* equilibrium characterized by Proposition 8, the best *noisy equilibrium* among those identified by Proposition 9, and the *disclosure equilibrium* (with either no or random positioning) characterized by Propositions 10 and 11. The pure positioning equilibrium generates an expected profit equal to

$$\Pi^P \equiv G(c_1)\, \mu_1 \pi^m + G(c_2)\,(1-\mu_1)\mu_2 \pi^m,$$

where

$$c_1 = \mu_1 s^m \text{ and } c_2 \equiv \mu_2 s^m.$$

By contrast, the disclosure equilibrium generates an expected profit given by

$$\Pi^D = G\left(\underline{c}_2^D\right)\left(\mu_1 + (1-\mu_1)\frac{\mu_2}{2}\right)\pi^m + G(c_2)\,\frac{(1-\mu_1)\,\mu_2}{2}\pi^m,$$

where

$$\underline{c}_2^D = \frac{\mu_1 + \mu_2 + (1-\mu_2)\,\mu_1}{3-\mu_2} s^m \in (c_2, c_1).$$

[4]We focus here on the relevant solution: the other solution is equal to $\sqrt{2} + 2$ and thus exceeds 1.

Finally, the best noisy positioning equilibrium generates a profit equal to

$$\Pi^R(\bar{r}) = G\left(\underline{c}_2^N(\bar{r})\right) M_2 \pi^m,$$

where

$$M_2 = \mu_1 + \mu_2 - \mu_1\mu_2,$$

is the overall probability of a match and

$$\underline{c}_2^N(\bar{r}) = \frac{M_2}{2 - \mu^1(\bar{r})} s^m$$

is the cost of the marginal searcher, where

$$\mu^1(\bar{r}) = \bar{r}\mu_1 + (1 - \bar{r})\mu_2$$

denotes the expected probability of a match on the first position.

As $r = \bar{r}$ is such that a consumer with search cost $\underline{c}_2^N(\bar{r})$ is not only indifferent between participating or not, but also between inspecting or not the second position if the first one does not produce a match, we have:

$$\underline{c}_2^N(\bar{r}) = \frac{M_2}{2 - \mu^1(\bar{r})} s^m = \mu^1(\bar{r}) s^m,$$

leading to:

$$\underline{c}_2^N(\bar{r}) = (\mu^1(\bar{r}) s^m =)(1 - \sqrt{1 - M_2}) s^m.$$

Note that

$$\underline{c}_2^D = \frac{(2 - \mu_2) c_1 + c_2}{3 - \mu_2}$$

can be expressed as a weighted average of c_1 and c_2, with positive weights on both thresholds. The same applies to $\underline{c}_2^N(\bar{r}) = \mu^1(\bar{r}) s^m$. Therefore

$$c_2 < \min\left\{\underline{c}_2^D, \underline{c}_2^N(\bar{r})\right\} \leq \max\left\{\underline{c}_2^D, \underline{c}_2^N(\bar{r})\right\} < c_1.$$

The last inequality implies that the pure positioning equilibrium generates the greatest participation. Among the other two equilibria, either one can generate greater participation:

Lemma D.1 *We have:*

$$\underline{c}_2^N(\bar{r}) \gtrless \underline{c}_2^D \iff \mu_1 \gtrless \phi(\mu_2) \equiv \frac{3 - 3\mu_2 + \mu_2^2}{(2 - \mu_2)^2},$$

where $\phi(\mu_2)$ increases from $3/4$ to 1 as μ_2 varies from 0 to 1

Proof. We have:

$$\begin{aligned}
\underline{c}_2^D \quad > \quad & \underline{c}_2^N(\bar{r}) \Longleftrightarrow \frac{\mu_1 + \mu_2 + (1-\mu_2)\mu_1}{3-\mu_2} > 1 - \sqrt{1-M_2} \\
\Longleftrightarrow \quad & (3-\mu_2)\sqrt{1-M_2} > 3 - \mu_2 - [(\mu_1 + \mu_2 + (1-\mu_2)\mu_1)] = 3 + \mu_1\mu_2 - 2\mu_2 - 2\mu_1 \,(>0) \\
\Longleftrightarrow \quad & (3-\mu_2)^2 (1 - \mu_1 - \mu_2 + \mu_2\mu_1) > (3 + \mu_1\mu_2 - 2\mu_2 - 2\mu_1)^2 \\
\Longleftrightarrow \quad & (\mu_1 - \mu_2)\left(\mu_2^2 - 3\mu_2 - 4\mu_1 + 4\mu_1\mu_2 - \mu_1\mu_2^2 + 3\right) > 0 \\
\Longleftrightarrow \quad & \mu_1 < \frac{3 - 3\mu_2 + \mu_2^2}{(2-\mu_2)^2} = \phi(\mu_2).
\end{aligned}$$

Conversely, $\underline{c}_2^D < \underline{c}_2^N(\bar{r})$ if and only $\mu_1 > \phi(\mu_2)$. ■

Building on this, the next proposition shows that any of the three types of equilibria can be the most profitable:

Proposition D.1 (most profitable equilibrium) *Suppose that positioning and/or disclosure are available, that only two products are available, and consider the pure positioning equilibrium characterized by Proposition 8, the best noisy equilibrium among those identified by Proposition 9, and the disclosure equilibrium (with either no or random positioning) characterized by Propositions 10 and 11. For each of these equilibria, there exist match probabilities, μ_1 and μ_2, and distributions of search costs, $G(s)$, such that this equilibrium is the most profitable one.*

Proof. Recall that the pure positioning equilibrium is more profitable when the firm offers both products than when it offers a single product, and that in the latter case, positioning and disclosure plays no role. Hence, without loss of generality, we can focus on the case where the firm offers both products.

We first note that pure positioning is the most profitable equilibrium when μ_2 is small enough, as it generates the greatest participation; indeed, in the limit case where μ_2 tends to vanish, we have:

$$\lim_{\mu_2 \to 0} \Pi^P = G(c_1)\mu_1\pi^m > \max\left\{\lim_{\mu_2 \to 0} \Pi^D, \lim_{\mu_2 \to 0} \Pi^R\right\} = \max\left\{G\left(\underline{c}_2^D\right), G\left(\underline{c}_2^N(\bar{r})\right)\right\}\mu_1\pi^m.$$

By contrast, for any given probabilities μ_1 and μ_2, the noisy positioning equilibrium is the most profitable one if there are few consumers with search costs between $\underline{c} \equiv \min\left\{\underline{c}_2^N(\bar{r}), \underline{c}_2^D\right\}$ and c_1; indeed, in the limit case where no consumer has a search cost between $\underline{c}$ and c_1, using $G\left(\underline{c}_2^D\right) = G\left(\underline{c}_2^N(\bar{r})\right) = G(c_1)$ yields:

$$\Pi^D - \Pi^P = \Pi^R - \Pi^D = [G(c_1) - G(c_2)](1-\mu_1)\frac{\mu_2}{2}\pi^m.$$

Hence, $\Pi^R > \Pi^D > \Pi^P$ whenever some consumers have a search cost between c_2 and $\underline{c}$ (so that $G(c_1) > G(c_2)$).

Finally, for any given probabilities μ_2 and $\mu_1 < \phi(\mu_2)$, implying $\underline{c}_2^N(\bar{r}) < \underline{c}_2^D$, the disclosure equilibrium is the most profitable one if there are few consumers with search costs between $\underline{c}_2^D$ and c_1, as well as between c_2 and $\underline{c}_2^N(\bar{r})$; indeed, in the limit case where no consumer has a search cost in these intervals, using $G\left(\underline{c}_2^D\right) = G(c_1)$ and $G\left(\underline{c}_2^N(\bar{r})\right) = G(c_2)$ yields:

$$\begin{aligned}\Pi^D - \Pi^P &= [G(c_1) - G(c_2)](1-\mu_1)\frac{\mu_2}{2}\pi^m,\\ \Pi^P - \Pi^R &= [G(c_1) - G(c_2)]\mu_1\pi^m.\end{aligned}$$

Hence, $\Pi^D > \Pi^P > \Pi^R$ whenever some consumers have a search cost between $\underline{c}_2^N(\bar{r})$ and $\underline{c}_2^D$ (so that $G(c_1) > G(c_2)$). ■

E Consumer Surplus under Positioning and Disclosure

E.1 Proof of Proposition 12

We compare here the consumer surplus generated by the noisy positioning equilibria identified by Propositions 7, 9 and 11, the pure positioning equilibrium identified by Proposition 8, and the disclosure equilibria identified by Propositions 10 and 11.

• $S^N(r)$ *increases with* r. We first show that, among the random ($r = 1/2$) and noisy positioning ($1/2 < r \leq \bar{r}$) equilibria, consumers prefer the least noisy one ($r = \bar{r}$). The noisy positioning equilibrium associated with a given $r \in [1/2, 1]$ gives a consumer with search cost c an expected surplus equal to

$$\hat{S}^N(c;r) \equiv \begin{cases} M_2 s^m - N_2^N(r)\,c & \text{if } c \leq \underline{c}_2^N(r), \\ 0 & \text{otherwise,} \end{cases} \tag{17}$$

where $M_2 = \mu_1 + \mu_2 - \mu_1\mu_2$ and

$$N_2^N(r) = 2 - [r\mu_1 + (1-r)\mu_2]. \tag{18}$$

As $N_2^N(r)$ is decreasing in r, and $\hat{S}^N(c;r) > 0$ for c sufficiently small, it follows that $S^N(r) = \int_0^{+\infty} \hat{S}^N(c;r)\,dG(c)$ increases with r.

• $S^P > \max\left\{S^N(\bar{r}), S^D\right\}$. Next, we show that the pure positioning equilibrium yields higher consumer surplus than any noisy positioning and disclosure equilibrium.

Pure positioning gives a consumer with search cost c an expected surplus given by

$$\hat{S}^P(c) \equiv \begin{cases} 0 & \text{if} \quad c \geq \mu_1 s^m \\ \hat{S}_1^P(c) \equiv \mu_1 s^m - c & \text{if} \quad \mu_1 s^m \geq c \geq \mu_2 s^m, \\ \hat{S}_2^P(c) \equiv \hat{S}_1^P(c) + (1-\mu_1)(\mu_2 s^m - c) & \text{if} \quad c \leq \mu_2 s^m. \end{cases}$$

Eliminating the noise in positioning enhances consumer surplus: for any $r \leq \bar{r} < 1$,

$$\hat{S}^P(c) - \hat{S}^N(c;r) = \begin{cases} \mu_1 s^m - c > 0 & \text{if } \mu_1 s^m > c \geq \underline{c}_2^N(r), \\ (1-r)(\mu_1-\mu_2)c - (1-\mu_1)(\mu_2 s^m - c) > 0 & \text{if } \underline{c}_2^N(r) \geq c \geq \mu_2 s^m, \\ (1-r)(\mu_1-\mu_2)c & \text{if } c \leq \mu_2 s^m. \end{cases}$$

It readily follows that $S^P = \int_0^{+\infty} \hat{S}^P(c) dG(c) > S^N(r)$ for any $r \leq \bar{r}$.

The disclosure equilibrium gives a consumer with search cost c an expected surplus, $\hat{S}^D(c)$, equal to $\hat{S}^N\left(c; \frac{1}{2}\right) \left(< S_2^P(c)\right)$ for $c \leq \mu_2 s^m$ and, for $\underline{c}_2^D \geq c > \mu_2 s^m$, given by

$$\begin{aligned} & \frac{1}{2}(\mu_1 s^m - c) + \frac{1}{2}\left[\mu_2 s^m - c + (1-\mu_2)(\mu_1 s^m - c)\right] \\ & < \frac{1}{2}(\mu_1 s^m - c) + \frac{1}{2}(1-\mu_2)(\mu_1 s^m - c) \\ & < \mu_1 s^m - c \\ & = \hat{S}_1^P(c), \end{aligned}$$

where the first inequality stems from $c > \mu_2 s^m$ and the second one from $\mu_2 > 0$ and $\underline{c}_2^D < \mu_1 s^m$. Hence, $S^P > S^D = \int_0^{+\infty} \hat{S}^D(c)\, dG(c)$.

E.2 Proof of Proposition 13

The least noisy positioning equilibrium corresponds to $r = \bar{r}$, which, from the proof of Proposition 9, is given by:

$$\bar{r} = \frac{\sqrt{1-\mu_2}}{\sqrt{1-\mu_1} + \sqrt{1-\mu_2}}.$$

The search cost of the marginal consumer is then equal to:

$$\underline{c}_2^N \equiv \underline{c}_2^N(\bar{r}) = \left. \frac{\mu_1 + \mu_2 - \mu_1\mu_2}{2 - [r\mu_1 + (1-r)\mu_2]} s^m \right|_{r = \frac{\sqrt{1-\mu_2}}{\sqrt{1-\mu_1}+\sqrt{1-\mu_2}}} = \left[1 - \sqrt{(1-\mu_1)(1-\mu_2)}\right] s^m. \tag{19}$$

Any consumer with search cost $c < \underline{c}_2^N$ obtains a positive expected surplus, which can be expressed as

$$\hat{S}(c; \bar{r}) = \underline{N}_2^N \left(\underline{c}_2^N - c\right),$$

where

$$\underline{N}_2^N \equiv N_2^N(\bar{r}) = 1 + \sqrt{1-\mu_1}\sqrt{1-\mu_2}. \tag{20}$$

Integrating by parts, expected consumer surplus is thus given by

$$\bar{S}^N \equiv \int_0^{\underline{c}_2^N} \hat{S}(c;\bar{r})\,dG(c) = \underline{N}_2^N \int_0^{\underline{c}_2^N} \left(\underline{c}_2^N - c\right) dG(c) = \underline{N}_2^N \Gamma\left(\underline{c}_2^N\right),$$

where

$$\Gamma(\underline{c}) \equiv \int_0^{\underline{c}} dG(c).$$

The disclosure equilibria generate an expected consumer surplus equal to:

$$S^D = \int_0^{\mu_2 s^m} (M_2 s^m - N_2 c)\,dG(c) + \int_{\mu_2 s^m}^{\underline{c}_2^D} \left(M_2^D s^m - N_2^D c\right) dG(c),$$

where

$$M_2^D \equiv \frac{2\mu_1 + \mu_2 - \mu_1\mu_2}{2} \text{ and } N_2^D \equiv \frac{3-\mu_2}{2}. \tag{21}$$

Re-arranging yields:

$$\begin{aligned} S^D &= \int_0^{\mu_2 s^m} \left[\left(M_2 - M_2^D\right) s^m - \left(N_2 - N_2^D\right) c\right] dG(c) + \int_0^{\underline{c}_2^D} \left(M_2^D s^m - N_2^D c\right) dG(c) \\ &= \left(N_2 - N_2^D\right) \int_0^{\mu_2 s^m} (\mu_2 s^m - c)\,dG(c) + N_2^D \int_0^{\underline{c}_2^D} \left(\underline{c}_2^D - c\right) dG(c) \\ &= \left(N_2 - N_2^D\right) \Gamma(\mu_2 s^m) + N_2^D \Gamma\left(\underline{c}_2^D\right), \end{aligned}$$

where the second equality follows from $M_2 s^m - N_2 c = M_2^D s^m - N_2^D c$ for $c = \mu_2 s^m$ and the last one stems from integration by parts.

Consumers with search cost $c < \mu_2 s^m$ keep searching until finding a match; they thus favor positioning (even if noisy), which reduces their expected number of inspections by increasing the probability of a match on the first inspection: $N_2^N(r) < N_2 \left(= N_2^N(1/2)\right)$. Conversely, consumers with search cost $c > \mu_2 s^m$ may favor disclosure, which allows them to avoid inspecting product 2. However, as $M_2^D < M_2$, this happens only if:

- Disclosure allows them to reduce their own expected number of searches: $N_2^D < \underline{N}_2^N$; otherwise, $\hat{S}^D(c) = M_2^D s^m - N_2^D c$ is obviously lower than $\hat{S}(c;\bar{r}) = M_2 s^m - N_2^N c$.
- It does so to an extent sufficient to offset the reduced probability of a match (from M_2 to M_2^D); this is the case if and only if disclosure expands the number of active

consumers:[5] $\underline{c}_2^D > \underline{c}_2^N$, which amounts to[6]

$$\mu_1 < \mu_1^N(\mu_2) \equiv \frac{3 - 3\mu_2 + \mu_2^2}{(2-\mu_2)^2}.$$

It follows that, if $\mu_1 > \mu_1^N(\mu_2)$, then *all* consumers prefer noisy positioning to disclosure.

Conversely, suppose now that $\mu_1 = \mu_2 + \varepsilon$, where ε is positive but small. We have:

- For noisy positioning:

$$\begin{aligned}
\underline{c}_2^N(\varepsilon) &= \left[1 - \sqrt{(1-\mu_2-\varepsilon)(1-\mu_2)}\right] s^m \\
&= \left[1 - (1-\mu_2)\sqrt{1 - \frac{\varepsilon}{1-\mu_2}}\right] s^m \\
&\simeq \left[1 - (1-\mu_2)\left[1 - \frac{\varepsilon}{2(1-\mu_2)} - \frac{\varepsilon^2}{8(1-\mu_2)^2}\right]\right] s^m \\
&= \left[\mu_2 + \frac{\varepsilon}{2} + \frac{\varepsilon^2}{8(1-\mu_2)}\right] s^m.
\end{aligned}$$

Similarly:

$$\begin{aligned}
\underline{N}_2^N(\varepsilon) &= 1 + \sqrt{(1-\mu_2-\varepsilon)(1-\mu_2)} \\
&\simeq 1 + (1-\mu_2)\left[1 - \frac{\varepsilon}{2(1-\mu_2)} - \frac{\varepsilon^2}{8(1-\mu_2)^2}\right] \\
&= 2 - \mu_2 - \frac{\varepsilon}{2} - \frac{\varepsilon^2}{8(1-\mu_2)}.
\end{aligned}$$

[5] For consumers with search cost $c > \mu_2 s^m$, the generated surplus can be expressed as $\hat{S}^N(c,\bar{r}) = N_2^N(\bar{r})\left(\underline{c}_2^N - c\right)$ and $\hat{S}^D(c) = N_2^D\left(\underline{c}_2^D - c\right)$, where $N_2^D < N_2^N$. Hence, $\underline{c}_2^D < \underline{c}_2^N$ implies $\hat{S}^D(c) < \hat{S}^N(c,\bar{r})$; conversely, if $\underline{c}_2^D > \underline{c}_2^N$, then any consumer with search cost $c \in \left(\underline{c}_2^N, \underline{c}_2^D\right)$ obtains a positive surplus with disclosure, and no surplus with noisy positioning.

[6] See Lemma D.1 in Online appendix D.

Therefore

$$\begin{aligned}
\bar{S}^N &= \underline{N}_2^N \Gamma\left(\underline{c}_2^N\right) \\
&\simeq \left[2-\mu_2-\frac{\varepsilon}{2}-\frac{\varepsilon^2}{8\left(1-\mu_2\right)}\right] \Gamma\left(\left[\mu_2+\frac{\varepsilon}{2}+\frac{\varepsilon^2}{8\left(1-\mu_2\right)}\right] s^m\right) \\
&\simeq \left[2-\mu_2-\frac{\varepsilon}{2}-\frac{\varepsilon^2}{8\left(1-\mu_2\right)}\right] \\
&\quad \times\left\{\Gamma\left(\mu_2 s^m\right)+G\left(\mu_2 s^m\right)\left[\frac{\varepsilon}{2}+\frac{\varepsilon^2}{8\left(1-\mu_2\right)}\right] s^m+\frac{1}{2} g\left(\mu_2 s^m\right)\left(\frac{\varepsilon s^m}{2}\right)^2\right\} \\
&\simeq \Gamma\left(\mu_2 s^m\right)\left(2-\mu_2\right)-\left[\Gamma\left(\mu_2 s^m\right)-G\left(\mu_2 s^m\right)\left(2-\mu_2\right) s^m\right] \frac{\varepsilon}{2} \\
&\quad -\left[\frac{\Gamma\left(\mu_2 s^m\right)}{2\left(1-\mu_2\right)}-G\left(\mu_2 s^m\right) \frac{\mu_2 s^m}{2\left(1-\mu_2\right)}-g\left(\mu_2 s^m\right) \frac{2-\mu_2}{2}\left(s^m\right)^2\right] \frac{\varepsilon^2}{4} .
\end{aligned}$$

- For disclosure:

$$\begin{aligned}
\underline{c}_2^D &= \frac{\mu_2+\varepsilon+\mu_2+\left(1-\mu_2\right)\left(\mu_2+\varepsilon\right)}{3-\mu_2} s^m=\mu_2 s^m+\frac{2-\mu_2}{3-\mu_2} \varepsilon s^m, \\
N_2 &= 2-\frac{\mu_2+\mu_2+\varepsilon}{2}=2-\mu_2-\frac{\varepsilon}{2}, \\
N_2^D &= \frac{3-\mu_2}{2} .
\end{aligned}$$

Therefore:

$$\begin{aligned}
S^D &= \left(N_2-N_2^D\right) \Gamma\left(\mu_2 s^m\right)+N_2^D \Gamma\left(\underline{c}_2^D\right) \\
&= \left(2-\mu_2-\frac{\varepsilon}{2}-\frac{3-\mu_2}{2}\right) \Gamma\left(\mu_2 s^m\right)+\frac{3-\mu_2}{2} \Gamma\left(\mu_2 s^m+\frac{2-\mu_2}{3-\mu_2} \varepsilon s^m\right) \\
&\simeq \Gamma\left(\mu_2 s^m\right) \frac{1-\mu_2}{2}-\Gamma\left(\mu_2 s^m\right) \frac{\varepsilon}{2} \\
&\quad +\left\{\Gamma\left(\mu_2 s^m\right)+G\left(\mu_2 s^m\right) \frac{2-\mu_2}{3-\mu_2} \varepsilon s^m+\frac{1}{2} g\left(\mu_2 s^m\right)\left(\frac{2-\mu_2}{3-\mu_2} \varepsilon s^m\right)^2\right\} \frac{3-\mu_2}{2} \\
&= \Gamma\left(\mu_2 s^m\right) \frac{1-\mu_2}{2}-\Gamma\left(\mu_2 s^m\right) \frac{\varepsilon}{2}+\Gamma\left(\mu_2 s^m\right) \frac{3-\mu_2}{2}+G\left(\mu_2 s^m\right) \frac{2-\mu_2}{3-\mu_2} \varepsilon s^m \frac{3-\mu_2}{2} \\
&\quad +\frac{1}{2} g\left(\mu_2 s^m\right)\left(\frac{2-\mu_2}{3-\mu_2} \varepsilon s^m\right)^2 \frac{3-\mu_2}{2} \\
&= \Gamma\left(\mu_2 s^m\right)\left(\frac{1-\mu_2}{2}+\frac{3-\mu_2}{2}\right)-\Gamma\left(\mu_2 s^m\right) \frac{\varepsilon}{2}+G\left(\mu_2 s^m\right)\left(2-\mu_2\right) s^m \frac{\varepsilon}{2} \\
&\quad +g\left(\mu_2 s^m\right) \frac{\left(2-\mu_2\right)^2}{3-\mu_2}\left(s^m\right)^2 \frac{\varepsilon^2}{4} \\
&= \Gamma\left(\mu_2 s^m\right)\left(2-\mu_2\right)-\left[\Gamma\left(\mu_2 s^m\right)-G\left(\mu_2 s^m\right)\left(2-\mu_2\right) s^m\right] \frac{\varepsilon}{2} \\
&\quad +g\left(\mu_2 s^m\right) \frac{\left(2-\mu_2\right)^2}{3-\mu_2}\left(s^m\right)^2 \frac{\varepsilon^2}{4}
\end{aligned}$$

It follows that, for ε small enough and $\mu_1 < \mu_2 + \varepsilon$, disclosure yields greater consumer surplus:

$$\begin{aligned}
S^D - S^N &\simeq \Gamma\left(\mu_2 s^m\right)\left(2-\mu_2\right) - \left[\Gamma\left(\mu_2 s^m\right) - G\left(\mu_2 s^m\right)\left(2-\mu_2\right) s^m\right] \frac{\varepsilon}{2} \\
&\quad + g\left(\mu_2 s^m\right) \frac{\left(2-\mu_2\right)^2}{3-\mu_2} \left(s^m\right)^2 \frac{\varepsilon^2}{4} \\
&\quad - \left\{ \begin{array}{c} \Gamma\left(\mu_2 s^m\right)\left(2-\mu_2\right) - \left[\Gamma\left(\mu_2 s^m\right) - G\left(\mu_2 s^m\right)\left(2-\mu_2\right) s^m\right] \frac{\varepsilon}{2} \\ - \left[\frac{\Gamma(\mu_2 s^m)}{2(1-\mu_2)} - G\left(\mu_2 s^m\right) \frac{\mu_2 s^m}{2(1-\mu_2)} - g\left(\mu_2 s^m\right) \frac{2-\mu_2}{2} \left(s^m\right)^2\right] \frac{\varepsilon^2}{4} \end{array} \right\} \\
&= \left\{ \frac{G\left(\mu_2 s^m\right) \mu_2 s^m - \Gamma\left(\mu_2 s^m\right)}{2\left(1-\mu_2\right)} + \frac{7-3\mu_2}{2\left(3-\mu_2\right)} \left(2-\mu_2\right) g\left(\mu_2 s^m\right) \left(s^m\right)^2 \right\} \frac{\varepsilon^2}{4} \\
&> 0,
\end{aligned}$$

where the inequality stems from $\Gamma'(\cdot) = G(\cdot)$ and $\Gamma''(\cdot) > 0$, which implies $G\left(\mu_2 s^m\right) \mu_2 s^m > \Gamma\left(\mu_2 s^m\right)$.[7]

We now turn to the second part of Proposition 13. The expected consumer surplus generated by the least noisy positioning equilibrium can be expressed as:

$$\begin{aligned}
\bar{S}^N &= \int_0^{\underline{c}_2^N} \left(M_2 s^m - \underline{N}_2^N c\right) dG(c) \\
&= M_2 s^m \int_0^{\underline{c}_2^N} \left(1 - \frac{c}{\underline{c}_2^N}\right) dG(c) \\
&= M_2 s^m \left[1 - \frac{c^e\left(\underline{c}_2^N\right)}{\underline{c}_2^N}\right] G\left(\underline{c}_2^N\right),
\end{aligned}$$

where

$$c^e\left(\underline{c}\right) \equiv \frac{\int_0^{\underline{c}} c\, dG(c)}{\int_0^{\underline{c}} dG(c)}.$$

As the equilibrium expected profit is given by $\bar{\Pi}^N = M_2 \pi^m G\left(\underline{c}_2^N\right)$, if the distribution of search costs is such that $c^e\left(\underline{c}\right) = \rho \underline{c}$,[8] for some $\rho > 0$, we thus have:

$$\bar{S}^N = \frac{s^m}{\pi^m} (1-\rho) \bar{\Pi}^N.$$

Likewise, using $\left(M_2 - M_2^D\right) s^m = \left(N_2 - N_2^D\right) \mu_2 s^m$ and $M_2^D s^m = N_2^D \underline{c}_2^D$, the expected

[7] We have:

$$G\left(\mu_2 s^m\right) \mu_2 s^m - \Gamma\left(\mu_2 s^m\right) = \int_0^{\mu_2 s^m} G\left(\mu_2 s^m\right) - G(c)\, dc > 0.$$

[8] Recall that this corresponds to power distributions of the form $G(c) = (c/\bar{c})^\alpha$ over $[0, \bar{c}]$, for some $\alpha > 0$.

consumer surplus generated by the disclosure equilibrium can be expressed as:

$$\begin{aligned} S^D &= N_2^D \int_0^{\underline{c}_2^D} \left(\underline{c}_2^D - c\right) dG(c) + \left(N_2 - N_2^D\right) \int_0^{\mu_2 s^m} \left(\mu_2 s^m - c\right) dG(c) \\ &= M_2^D s^m \int_0^{\underline{c}_2^D} \left(1 - \frac{c}{\underline{c}_2^D}\right) dG(c) + \left(M_2 - M_2^D\right) s^m \int_0^{\mu_2 s^m} \left(1 - \frac{c}{\mu_2 s^m}\right) dG(c) \\ &= M_2^D s^m (1-\rho) G\left(\underline{c}_2^D\right) + \left(M_2 - M_2^D\right) s^m (1-\rho) G\left(\mu_2 s^m\right) \\ &= \frac{s^m}{\pi^m} (1-\rho) \bar{\Pi}^D, \end{aligned}$$

where $\bar{\Pi}^D = M_2 \pi^m G(\mu_2 s^m) + M_2^D \pi^m \left[G\left(\underline{c}_2^D\right) - G(\mu_2 s^m)\right]$ is the equilibrium expected profit. It follows that, among the least noisy positioning and disclosure equilibria, the profit-maximizing one also maximizes consumer surplus.

To show that a bias can arise with alternative distributions of search costs, suppose that $\mu_1 < \mu_1^N(\mu_2)$ (otherwise, as already noted, disclosure is unattractive for the firm and all consumers), and consider the following examples:[9]

• *Example 1:* consumers have the same search cost $\hat{c}$, which is slightly below $\underline{c}_2^N$; we then have

$$\bar{\Pi}^N = M_2 \pi^m > \Pi^D = M_2^D \pi^m,$$

and[10]

$$S^D \simeq M_2^D s^m \left(1 - \frac{\underline{c}_2^N}{\underline{c}_2^D}\right) > \bar{S}^N \simeq 0.$$

• *Example 2:* a mass $1-m$ of consumers have the same search cost $\hat{c}$, which is slightly below $\mu_2 s^m$, whereas the remaining mass $m \in (0,1)$ of consumers have a search cost slightly below $\underline{c}_2^D$; we then have

$$\Pi^D = (1-m) M_2 \pi^m + m M_2^D \pi^m > \bar{\Pi}^N = (1-m) M_2 \pi^m,$$

and

$$\begin{aligned} \bar{S}^N &\simeq (1-m) M_2 s^m \left[1 - (1-m) \frac{\mu_2 s^m}{\underline{c}_2^N}\right], \\ S^D &\simeq M_2^D s^m \left(1 - \frac{(1-m)\mu_2 s^m + m \underline{c}_2^D}{\underline{c}_2^D}\right) = (1-m) M_2^D s^m \left(1 - \frac{\mu_2 s^m}{\underline{c}_2^D}\right). \end{aligned}$$

Therefore:

$$\bar{S}^N - S^D \simeq (1-m) s^m \left\{ M_2 \left[1 - (1-m) \frac{\mu_2 s^m}{\underline{c}_2^N}\right] - M_2^D \left(1 - \frac{\mu_2 s^m}{\underline{c}_2^D}\right) \right\},$$

[9] For simplicity, these examples assume that consumers' search costs take at most two values; introducing an infinitesimal mass of costs distributed over $[0, +\infty)$ would not affect the insights.

[10] The approximation stems from $\hat{c}$ being slightly below $\underline{c}_2^N$.

where, as $M_2 > M_2^D$, the right-hand side is positive whenever $m > 1 - \underline{c}_2^N / \underline{c}_2^D$.