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The Payday Loan Puzzle: A Credit Scoring Explanation

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Abstract

Many credit cardholders in the U.S. turn to expensive payday loans, even though they have not yet exhausted their credit lines. This results in significant monetary costs and has been coined the "Payday Loan Puzzle." We propose the novel explanation that households use payday loans to protect their credit scores since payday lenders do not report to credit bureaus. To quantitatively examine this hypothesis, we build a two-asset Huggett-type model with two default options as well as hidden information and actions. Using our calibrated model, we can account for 40% of the empirically identified payday loan borrowers with liquidity left on their credit cards. We can also match the magnitude of monetary costs due to this seeming pecuniary mistake. To inform the policy debate over payday lending, we assess the welfare implications of several policy counterfactuals. We find that either banning payday loans or increasing their default costs results in aggregate welfare losses.

Keywords: Consumer Credit, Bankruptcy, Default, Payday Loan, Financial Regulation, Type Score, Asymmetric Information, Hidden Action, Cross-Subsidization

JEL Classifications: D82, E21, E49, G18, G51, K35

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1 Introduction

Agarwal, Skiba, and Tobacman (2009) observe that two-thirds of individuals who use both credit cards and payday loans have at least $1,000 of credit card liquidity left when taking out a payday loan.[1] This behavior is seemingly puzzling as payday loans carry very high interest rates corresponding to annualized percentage rates of several hundred percent, compared to 10 to 30 percent on credit cards. The authors calculate that this seeming pecuniary mistake is very costly: these people could have saved on average $200 over a year by borrowing up to their credit card limits before taking out payday loans. This phenomenon has been termed the "Payday Loan Puzzle."

Why do households take out expensive payday loans when they have far cheaper credit options available? Various behavioral explanations, such as self-control problems and financial illiteracy, have been put forward. In this paper, we propose a novel rational explanation for the payday loan puzzle, inspired by the following interview of an actual payday lender:

> *"Why are people taking out [payday] loans instead of using their cards?" Ranney told me, "This guy was implying that these people weren't smart enough to make the 'right' decision. I laughed in his face.* ***'They're protecting the card!'*** *I told him. [...]"* ***Whereas failure to repay a payday loan won't affect a consumer's credit score, failure to repay a credit card will.***
>
> — Lisa Servon (2017): *The Unbanking of America*[2]

Our proposed "reputation protection" hypothesis is that people do not exhaust their credit card limits because they want to protect their credit scores. A credit score is a statistic computed by credit bureaus to access a person's default risk.[3] Borrowing or defaulting on credit cards will affect one's credit score, while payday lenders in the U.S. usually do not report to credit bureaus (Consumer Financial Protection Bureau, 2017).[4] People care about

[1] A payday loan is a short-term unsecured loan with a duration of a few weeks for a typically small amount of around $300. In the SCF 2010, around 5% of households used payday loans in the previous year. About 60% of payday loan borrowers possess credit cards. See, for example, Elliehausen and Lawrence (2001).

[2] Servon was interviewing Tim Ranney, a payday lender, and Ranny was sharing a conversation he had with a risk manager at one of the largest credit card issuers in the U.S.

[3] The most well-known credit score in the U.S. is the FICO score, 35% of which is determined by the payment history and 30% by the debt burden.

[4] In line with our hypothesis, Bhutta, Skiba, and Tobacman (2015) empirically document that payday borrowings has no impact on credit scores.

their credit scores as they influence credit access, credit costs, mortgage terms, and even job application prospects in the future. Therefore, using payday loans to protect one's credit score leads to dynamic reputational benefits at the static cost of higher interest fees.

To better understand the reasons behind the payday loan puzzle and to formalize the above hypothesis, we extend the type scoring framework of Chatterjee, Corbae, Dempsey, and Ríos-Rull (2020). The authors study a Huggett-type model with consumer default and asymmetric information. Households differ in their degrees of patience measured by discount factors (called their "types"). These factors influence their default behavior and thus their riskiness as borrowers. However, banks are unable to observe household types directly. As a result, banks resort to using "type scores" to infer the probability of each individual being patient with a high discount factor (the good type). A type score thus represents an individual's reputation in the credit markets and is analogous to a credit score in practice.[5]

We extend their framework by adding a second debt option (payday loans) and a second default option on only payday loans. Thus, in addition to bank loans, households in our model can also borrow using payday loans offered by the second type of financial intermediary called payday lenders. Households can default in two ways: (1) "formal default" where households default on both bank and payday loans;[6] and (2) "payday default" where households default selectively only on their payday loans. Default costs include filing fees, utility loss (stigma), and temporary exclusion from the respective asset markets. In equilibrium, payday loans have higher interest rates compared to bank loans because of higher default premia and operating costs. Crucially, banks cannot observe the payday loan choices of households. Payday loans thus introduce hidden actions into the price setting and type score updating problem of banks. To our knowledge, we are the first to explicitly model payday loans using a two-asset structure and two default options.

In our model, a dynamic trade-off emerges between the short-run costs of payday loans and the long-run reputational credit score gains. Households trade off between the marginal benefit of maintaining one's type scores versus the marginal cost of borrowing on more expensive payday loans. The intuition behind the type score protection is as follows. Banks cannot observe a household's type and its payday loan usage. If a household

[5] Chatterjee et al. (2020) show that there exists a mapping from type scoring economy to credit scoring economy under some sufficient conditions.

[6] This is modeled in line with Chapter 7 bankruptcy in the U.S. which entails the liquidation of non-exempt assets in return for debt dischargement.

is hit by a low income shock and borrows using bank loans to smooth consumption, banks regard this as being indicative of impatience and thus downgrade the type score. Taking up payday loans instead helps protect against being misclassified in the current period. Moreover, it also lowers the probability of a type score downgrade due to default on bank loans in the future in case of sufficiently low future income shocks. We are the first to formally examine the reputation protection explanation for the payday loan puzzle in a theoretical model.

Limited information of banks regarding households' types and payday loan choices gives rise to cross-subsidization in the bank loan market. Conditional on the same level of bank borrowing, impatient households or payday loan borrowers are more likely to default. However, banks cannot observe either a household's type or payday loan usage. This imperfect information restricts banks from designing contracts conditioned on these two characteristics. Both impatient households and payday loan borrowers thus face cheaper borrowing rates than the actuarially fair rates when banks have full information. As a result, impatient households (payday loan borrowers) are subsidized by patient households (non-payday loan borrowers) in the bank lending market.

To understand the payday loan puzzle documented in Agarwal et al. (2009), we calibrate our model to the U.S. households in 2004. Most parameters are exogenously determined by direct empirical evidence or estimates from the literature. We internally calibrate the stigma costs of defaults to match default rates in the bank and payday markets. Our calibrated model can account for various untargeted moments, such as the fraction of payday loan borrowers and the average interest rate on payday loans.

Our calibrated model endogenously gives rise to the reputation protection channel: households invest in their type scores by paying higher interest costs on payday loans. We can quantitatively account for 40% of the empirically identified payday loan borrowers who have not exhausted their credit cards yet. We can also match the magnitude of the monetary costs. Neither of these moments was targeted in the calibration.[7] In particular, the model predicts average annual monetary costs of $230, which is similar to its empirical counterpart of $200 as calculated by Agarwal et al. (2009). Using our calibrated model, we are the first to generate and quantitatively match the empirically identified payday loan puzzle.

[7] As mentioned previously, the unaccounted 60% of the puzzle occurrence could be potentially explained by other behavioral explanations.

Payday loans have been a controversial subject of debate in the U.S. in recent years. Critics of payday loans have focused on the high costs of these loans and have argued for outright payday loan bans.[8] However, we show that payday loans serve an essential insurance purpose even in the presence of these high costs. We are the first to inform the payday loan policy debate in a structural framework by conducting a series of counterfactual policy experiments.

First, we investigate the effects of limiting the maximum payday loan size, a quantity cap, and an outright ban of payday loans. We find that a quantity cap decreases overall welfare. However, there is heterogeneity across households: impatient households lose while patient ones gain. Impatient households are more likely to borrow larger payday loans and are thus more heavily affected by the quantity cap. In addition, the quantity cap imposes less unobservable options on payday loans. This reduction in hidden actions enables banks to better infer payday loan usage of households, thus reducing the amount of information asymmetry in the bank loan market. As a result, banks can better identify households' discount factors, leading to a decline in cross-subsidization of impatient by patient households. In contrast to the quantity cap, a full ban on payday loans is welfare-reducing for both types of households. The reason for the welfare loss is the reduction in available insurance. Both impatient and patient households use payday loans to smooth idiosyncratic shocks without harming their type scores. With a full ban, the insurance loss outweighs the gains from reduced cross-subsidization for patient households. These results imply that current regulatory efforts in certain U.S. states to ban payday loans may be misguided in the sense that they end up hurting all households.[9]

Second, we examine the implications of increasing either the formal or payday default cost. The increase in default costs is calibrated to reflect the increase in Chapter 7 filing costs after the 2005 Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) in the U.S.[10] We find that increasing formal default costs leads to a welfare gain, whereas increasing payday default costs leads to a welfare loss for both types of households. Higher default costs make it harder to *smooth consumption across states* by defaulting, but easier to *smooth consumption over time* by borrowing through lower default premia

[8] For example, 16 states and the District of Columbia in the U.S. either prohibit payday loans or impose limits, while 23 states allow payday lending (Consumer Federation of America, 2021).

[9] For example, Morse (2011) uses natural disasters to identify a causal, positive relationship between welfare and access to payday loans. In other words, banning payday loans results in a welfare loss.

[10] The 2005 BAPCPA was the most significant reform of bankruptcy law in recent years. Among other changes, it significantly increased the total out-of-pocket filing costs. See also Albanesi and Nosal (2020).

(Zame, 1993). In equilibrium, households prefer smoothing across states by defaulting on payday loans while smoothing over time by borrowing bank loans for three reasons: (1) defaulting on payday loans does not directly affect a household's type score, whereas formally defaulting on a bank loan does; (2) interest rates for bank loans are much lower than payday loans; and (3) payday default costs are lower than formal default costs. Higher formal (payday) default costs exactly help (hamper) households in achieving smoothing over time (across states).

The rest of the paper is organized as follows. Section 2 gives an overview of the related literature. Section 3 details the model framework. Section 4 presents the calibration of the model. Section 5 illustrates the fundamental mechanism of pooling and cross-subsidization in our framework. In Section 6, we discuss in detail the payday loan puzzle and the reputation protection channel in our model. Section 7 presents the policy experiments and Section 8 concludes with some potential extensions.

2 Related Literature

In this section, we discuss the literature related to our paper. The consumer finance literature (both empirical and theoretical) is extensive; thus, we will only focus on the papers most directly related to our own. We start by discussing papers that we build on in terms of the underlying methodology and then briefly summarize the literature on payday loans.

Our theoretical framework is based on the type scoring framework developed by Chatterjee et al. (2020). In their paper, they build on the consumer default workhorse models developed by Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Livshits, MacGee, and Tertilt (2007) in which households are allowed to default on their loans as insurance against idiosyncratic risk.[11] Both Chatterjee et al. (2007) and Livshits et al. (2007) assume that lenders are fully informed about all household characteristics that affect repayment in the next period. Chatterjee et al. (2020) depart from this assumption and introduce heterogeneity across households in the form of different discount factors, which are unob-

[11] Some papers extend the standard consumer default framework by incorporating behavioral components. For example, Nakajima (2017) considers households with temptation and analyzes the welfare implications of the 2005 BAPCPA. Exler, Livshits, MacGee, and Tertilt (2020) introduce over-optimism of households about future income. See also Exler and Tertilt (2020) for a complete survey.

servable by banks. As the patience of households affects their loan repayment probability, banks try to infer households' types by computing an individual-specific type score. This score denotes the Bayesian assessment by banks of individual type based on observable household behavior. Our paper extends this model by introducing a second asset and an additional default option. In addition, banks cannot observe payday loans and default and thus face hidden actions.

Our paper is also closely related to the empirical literature on the seeming pecuniary mistakes in using payday loans. Using matched credit card and payday loan data, Agarwal et al. (2009) document that many borrowers use payday loans when they still have sufficient credit left on their credit cards, even though payday loans carry much higher interest rates. They compute that this behavior is very costly and leads to monetary costs of several hundred U.S. dollars over one year. They coin this finding the "Payday Loan Puzzle." Furthermore, Carter, Skiba, and Tobacman (2011) look at a dataset of credit union members and their payday loan borrowing behavior. They also find a pecuniary loss due to the usage of payday loans instead of cheaper alternatives similar to the previous paper. We contribute to this literature by generating the payday loan puzzle in a theoretical model and offering a rational explanation for part of its occurrence.

Payday loans and their effects on consumers are a hotly debated regulatory topic in the U.S. The literature on the effects of payday loans on consumers is in disagreement about its sign. Using household panel survey data, Zinman (2010) finds that restricting access to payday loans leads consumers to shift to bank overdrafts and late payments. The result is a decline in the financial health of affected households and an overall harmful effect of restricting payday loans. Similarly, Morse (2011) uses natural disasters and estimates that access to payday lenders increases welfare. Morgan, Strain, and Seblani (2012) find that the banning of payday lending leads to an increase in bounced checks and overdraft fees. Bhutta, Goldin, and Homonoff (2016) find that consumers switch to other high-cost alternatives in response to payday loan bans. These authors stress that payday loans are instrumental for households to mitigate the negative effects of transitory income or expenditure shocks, especially when access to the mainstream financial system is impaired.

On the other hand, many authors point out that using payday loans can further worsen households' financial situation. Skiba and Tobacman (2019) estimate that using payday loans significantly increases bankruptcy rates by depressing the cash flow of households.

Melzer (2011) finds that access to payday loans worsens the ability of households to pay mortgages, rent, and utility bills. Carrell and Zinman (2014) use exogenous variation in payday loan access for military personnel to estimate that usage of payday loans decreases job performance, retention, and readiness. Campbell, Martínez-Jerez, and Tufano (2012) find that access to payday lending increases rates of involuntary bank account closures. We contribute to this literature by offering a theoretical framework in which we jointly model mainstream financial and payday loans as well as their interaction with credit scores. We then use our framework to conduct counterfactual policy exercises, such as banning payday loans, and investigate the resulting welfare implications for households.

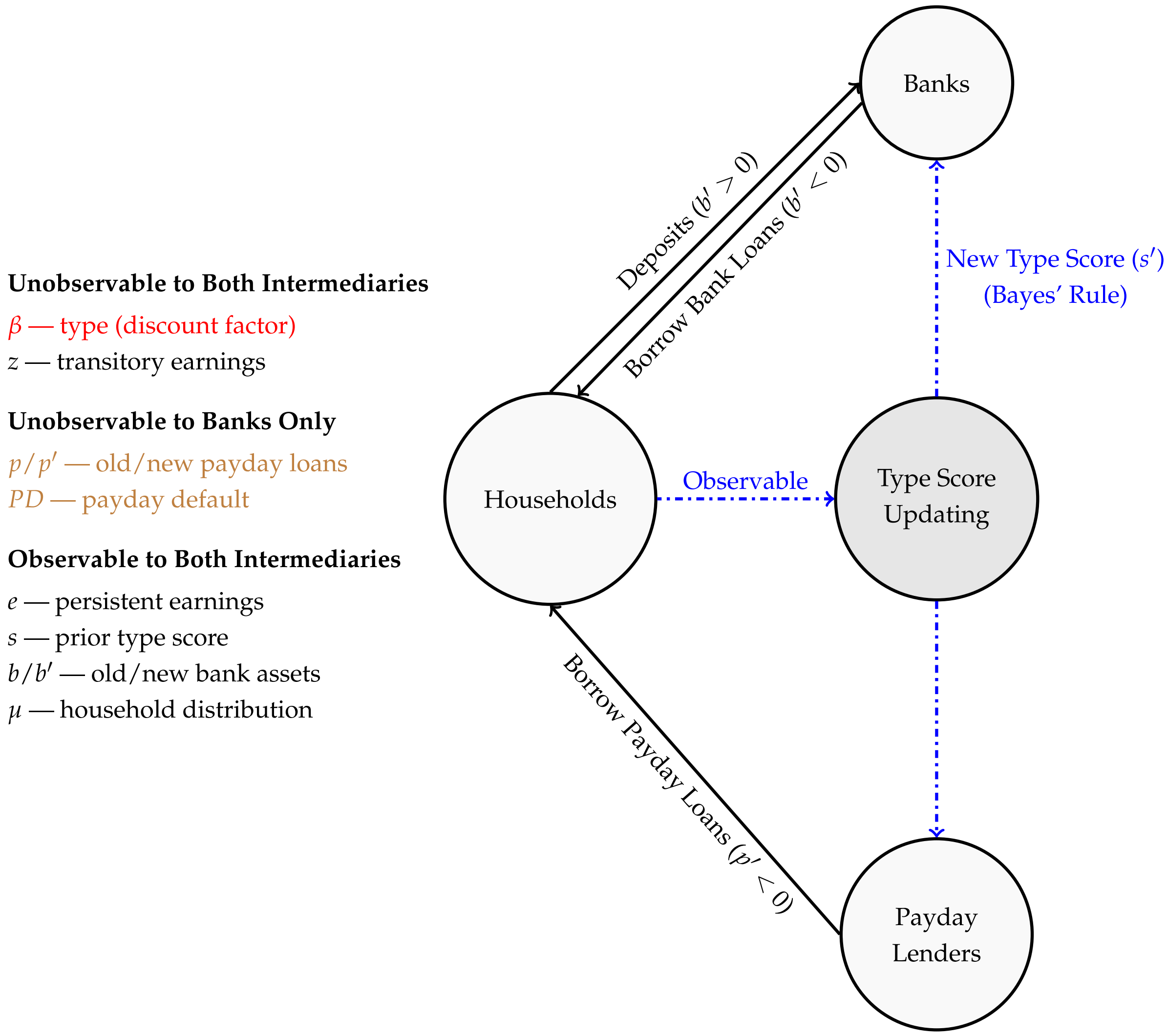
Our paper is also related to Exler (2020). He examines the welfare impact of different policy alternatives to regulate small-dollar loans. He builds and calibrates a quantitative model of unsecured lending where individuals can declare bankruptcy or become delinquent. His findings suggest welfare improving changes to the legislation proposed by the Consumer Financial Protection Bureau (CFPB). In contrast to our approach, he considers only one asset and does not model credit scores. Saldain (2021) considers a model of only payday loans with behavioral households and studies policy regulations on payday lending.

3 The Model

Time is discrete and infinite. We follow the convention of dynamic programming that the time subscript is removed, and the next-period variable is expressed with prime $'$. The market is incomplete. There is a measure one of rational households populating the economy. In addition, there exist two financial intermediaries, banks and payday lenders, which operate in perfectly competitive markets. Both offer lending services in one-period unsecured loans. Banks also provide saving services. The layout of the economy is illustrated in Figure 1.

In every period, households survive at a rate ρ, and those who die are replaced by newborns. Households receive persistent earnings e following a stationary finite-state Markov process $Q^e(e'|e)$ and transitory earnings z determined by an i.i.d. process $Q^z(z)$. All income realizations are independent across individuals. There are two types of households: impatient households with a low discount factor β_L and patient households with a high

Figure 1: Layout of the Economy and Information Structure



discount factor β_H. A household's discount factor follows a stationary two-state Markov process $Q^{\beta}(\beta'|\beta)$ and evolves independently across individuals. We call a household's discount factor her type.

Households derive utility from consumption c. They can either borrow or save an amount b' at the discount price q_b with banking institutions. Furthermore, they may also take out payday loans p' at the discount price q_p. These actions are illustrated with the solid arrows in Figure 1. At the beginning of each period, if a household has any kind of debt, she can choose to repay ($d = R$) or default. There are two default options available: formal default ($d = FD$) and payday default ($d = PD$). Formal default discharges all debts (including potential payday loans) but incurs the out-of-pocket bankruptcy costs κ_{FD} (e.g., attorney fees) and stigma (utility) costs ξ_{FD}. In addition, no saving or borrowing is possible in the filing period. Alternatively, she may choose payday default to selectively discharge her payday loan only at the cost of filing fees κ_{PD} and stigma costs ξ_{PD}.

Compared to formal default, she becomes excluded only from the payday lending market, and potential bank loans still need to be repaid, but she retains access to the bank asset market.[12]

Banks can observe households' persistent earnings e, bank asset position b, bank asset choice b', formal default FD, and household distribution μ. On the contrary, they cannot observe households' transitory earnings z, payday loan position p, payday loan choice p', payday default $(d = PD)$, and discount factors β. We denote (e, b, s) as the bank-observable state ω_b. This information structure is summarized on the left-hand side in Figure 1. As all unobservable variables are relevant for the repayment probability of loans in the next period, banks would like to infer them. While banks cannot infer transitory earnings z as they are i.i.d. across time and households, the other variables can be.

For a household's payday loan position p, we assume that banks are not able to track it at an individual level, but banks know the aggregate distribution of payday loans in the population (rational expectations). As a result, banks exploit the cross-sectional distribution of households to form their expectation about a household's payday loan position.[13] Banks then handle unobservable payday loan choices p' by summing them out. In addition, banks cannot observe whether payday loans are repaid. Hence, they cannot distinguish between full repayment or payday default by households. These two choices are accordingly subsumed under non-formal default ($\tilde{d} = NFD \equiv R \vee PD$).

Households' discount factors are unobservable to financial intermediaries. Banks infer these factors using type scores s, which denote the probability of being patient. Past actions are informative about a household's discount factor as it follows a persistent process. The prior assessment of a household being patient at the beginning of a period is denoted as $s \equiv \mathbb{P}(\beta_i = \beta_H)$. Given bank-observable states ω_b and choices $(\tilde{d}, b')$, banks will update a household's type score s using Bayes' rule each period. The posterior type score is denoted as $s' = \psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b)$ where superscripts denote actions and variables in parentheses denote states. As the updated type score may not lie on the type score grid, it is assigned to the nearest grid points using the function $Q^s(s'|\psi)$.[14] The type score up-

[12] Note that, compared to most papers in the consumer finance literature, there is no long-term exogenous exclusion imposed in our model.

[13] In principle, it is also possible to assume that banks form a joint score over type and payday loan choices $s(\beta, p)$ for each household.

[14] To be precise, s' will be randomly assigned to one of the two nearest points between which s' lies, with probabilities inversely proportional to the relative distance of s' to the respective grid points. This assignment is captured by the function $Q^s(s'|\psi)$.

dating process is indicated by the dashed arrows in Figure 1. Thus, the bank loan pricing function $q_b^{(NFD,b')}(\omega_b)$ will be affected by an individual's observable choices and characteristics, including type scores.

Payday lenders are assumed to be more informed than banks. In addition to what banks can observe, payday lenders can certainly tell payday loan decisions. This information structure is also summarized on the left-hand side in Figure 1. For simplicity, we assume that payday lenders use the identical type scores as banks.[15]

The rest of the section is structured as follows. Section 3.1 summarizes the timing in each period. Section 3.2 details the household's maximization problem. Section 3.3 presents the problems of both financial intermediaries. In particular, type score updating is discussed in Section 3.3.1. Section 3.4 shows the evolution of the cross-sectional distribution of households. In Section 3.5, we close the section by defining the equilibrium.

3.1 Timing

The timing in every period is summarized as follows:

1. Households begin each period with state (β, z, ω_b, p).
2. Given bank prices $q_b^{(NFD,b')}(\omega_b)$ and payday prices $q_p^{(R,b',p')}(\omega_b)$, households choose to either repay all debt $d = R$, default on the payday loan only $d = PD$, or formally default on both loans $d = FD$.
   - If $d = R$, they also choose b' and p' and consume $c^{(R,b',p')}$.
   - If $d = PD$, they also choose b' and $p' = 0$ and consume $c^{(PD,b',0)}$.
   - If $d = FD$, they consume the leftover earnings $c^{(FD,0,0)}$.
3. Based on bank-observable states ω_b and choices $(\tilde{d}, b')$, banks update their type scores from prior s to posterior $\psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b)$.
4. β', z', e', and s' are drawn from $Q^\beta(\beta'|\beta)$, $Q^z(z')$, $Q^e(e'|e)$, and $Q^s(s'|\psi)$. Newborn households begin with discount factor β' drawn from the initial distribution G_β,

[15] In principle, payday lenders can form another "type score" using their richer information set compared to banks. This simplifying assumption is meant to keep computation numerically tractable. Nonetheless, payday lenders can still better predict the repayment probability than banks in our economy.

transitory earnings z' from G_z, persistent earnings e' from G_e, no bank or payday loan assets $(b', p') = (0, 0)$, and a type score s' consistent with G_β.

3.2 Households

Households take as given the bank and payday loan pricing functions $q_b^{(NFD,b')}(\omega_b)$ and $q_p^{(R,b',p')}(\omega_b)$ as well as the type scoring function $\psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b)$. Households can choose between repayment ($d = R$), defaulting on payday loans only ($d = PD$), or formally defaulting on both bank and payday loans ($d = FD$).

Following Chatterjee et al. (2020), we introduce the action-specific utility shocks. These shocks are i.i.d. across time and households. For each action (d, b', p') and household, an unobservable additive utility shock $\epsilon^{(d,b',p')}$ is drawn from an extreme value distribution. These shocks capture other unobservable heterogeneity that is not explicitly modeled in a reduced but tractable way. Policy functions also become probabilistic with these shocks. Without such randomness, households' actions are perfectly informative about their true types.

The value function is thus given by:

$$V(\epsilon, \beta, z, \omega_b, p) = \max_{(d,b',p')} v^{(d,b',p')}(\beta, z, \omega_b, p) + \epsilon^{(d,b',p')}, \quad (1)$$

where $\epsilon^{(d,b',p')}$ is drawn from the following extreme value distribution $EV(\epsilon)$:

$$EV(\epsilon) = \exp\left\{ -\exp\left(-\frac{\epsilon - \mu_\epsilon}{\alpha} \right) \right\}, \quad (2)$$

where $\alpha > 0$ determines the variance of the shock and $\mu_\epsilon = -\alpha\gamma_E$ makes the shock mean zero and γ_E is the Euler's constant.[16]

The conditional value function is given by:

$$\begin{aligned} v^{(d,b',p')}(\beta, z, \omega_b, p) = {} & u\left(c^{(d,b',p')}(z, \omega_b, p)\right) - \xi_{PD} \cdot \mathbb{I}_{[d=PD]} - \xi_{FD} \cdot \mathbb{I}_{[d=FD]} \\ & + \beta\rho \cdot \sum_{(\beta',z',e',s')} Q^\beta(\beta'|\beta) \cdot Q^z(z') \cdot Q^e(e'|e) \cdot Q^s(s'|\psi) \cdot W(\beta', z', \omega'_b, p'), \end{aligned} \quad (3)$$

[16] Note that the noise of extreme value shocks is not the reason why our model is able to generate the payday loan puzzle. In fact, we control for it while identifying the puzzle. Refer to Section 6.1 for details.

where the utility function defined on consumption $u(c)$ is additively separable over time, continuous, increasing, and concave; ξ_{PD} and ξ_{FD} represents the stigma costs for payday and formal default; $\mathbb{I}$ denotes the indicator function equal to one if the condition in the squared parentheses is true; W is the unconditional value function which will be defined below; and consumption $c^{(d,b',p')}(z,\omega_b,p)$ is defined as:

$$\begin{cases} e\cdot z+b+p-q_b^{(NFD,b')}(\omega_b)\cdot b'-q_p^{(R,b',p')}(\omega_b)\cdot p', & \text{if } (d,b',p')=(R,b',p') \\ e\cdot z-\kappa_{PD}+b-q_b^{(NFD,b')}(\omega_b)\cdot b', & \text{if } (d,b',p')=(PD,b',0)\,, \\ e\cdot z-\kappa_{FD}, & \text{if } (d,b',p')=(FD,0,0) \end{cases} \tag{4}$$

where κ_{PD} and κ_{FD} denote the out-of-pocket bankruptcy costs for payday and formal default.[17]

Let the set of feasible actions be defined as:

$$\mathcal{F}(z,\omega_b,p)=\left\{(d,b',p')|c^{(d,b',p')}(z,\omega_b,p)>0\right\}. \tag{5}$$

Under the distributional assumption on the utility shocks in Equation (2), the choice probabilities take the following form:[18]

$$\sigma^{(d,b',p')}(\beta,z,\omega_b,p)=\begin{cases} \frac{\exp\left\{v^{(d,b',p')}(\beta,z,\omega_b,p)/\alpha\right\}}{\sum_{(\hat{d},\hat{b}',\hat{p}')\in\mathcal{F}}\exp\left\{v^{(\hat{d},\hat{b}',\hat{p}')}(\beta,z,\omega_b,p)/\alpha\right\}} & \text{if } (d,b',p')\in\mathcal{F}(z,\omega_b,p) \\ 0 & \text{otherwise} \end{cases}. \tag{6}$$

The unconditional value function is then given by:

$$\begin{aligned} W(\beta,z,\omega_b,p) &= \mathbb{E}_\epsilon V(\epsilon,\beta,z,\omega_b,p) \\ &= \alpha\cdot\ln\left(\sum_{(d,b',p')\in\mathcal{F}(\beta,z,\omega_b,p)}\exp\left\{\frac{v^{(d,b',p')}(\beta,z,\omega_b,p)}{\alpha}\right\}\right). \end{aligned} \tag{7}$$

We use $\mu(\beta,z,\omega_b,p)$ to denote the cross-sectional distribution of households.

[17] There are two technical assumptions. First, we assume for computational reasons that households can only take out payday loans if they also borrow in the banking sector. Second, we assume that default is restricted to households who have debts larger than the respective monetary bankruptcy costs. For example, formal default is feasible only if $b+p<-\kappa_{FD}$.

[18] See, for example, Rust (1987).

3.3 Financial Intermediaries

In this section, we detail the financial intermediaries. Section 3.3.1 presents the banking sector and Section 3.3.2 outlines the payday lenders.

3.3.1 Banks

Banks can borrow from the international credit market at risk-free interest rate r_f. The bank's profit $\pi_b^{(NFD,b')}(\omega_b)$ for a contract (NFD, b') is given by:

$$\pi_b^{(NFD,b')}(\omega_b) = \begin{cases} \rho \cdot \frac{\mathbb{P}_b^{(NFD,b')}(\omega_b)\cdot(-b')}{1+r_f} - q_b^{(NFD,b')}(\omega_b)\cdot(-b') & \text{if } b' < 0 \\ q_b^{(NFD,b')}(\omega_b)\cdot b' - \rho \cdot \frac{b'}{1+r_f} & \text{if } b' \geq 0 \end{cases}, \tag{8}$$

where ρ is the survival probability and $\mathbb{P}_b^{(NFD,b')}(\omega_b)$ denotes the repayment probability of a contract (NFD, b') conditional on bank-observable states ω_b. Given perfect competition, the zero-profit condition implies for each contract that:

$$q_b^{(NFD,b')}(\omega_b) = \begin{cases} \rho \cdot \frac{\mathbb{P}_b^{(NFD,b')}(\omega_b)}{1+r_f} & \text{if } b' < 0 \\ \frac{\rho}{1+r_f} & \text{if } b' \geq 0 \end{cases}. \tag{9}$$

Recall that banks cannot observe discount factors β, transitory earnings z, payday loan holdings and choices (p, p'), as well as the exact choice of repayment or payday default $(d = PD \vee R)$. To determine the repayment probability $\mathbb{P}_b^{(NFD,b')}(\omega_b)$, banks solve an inference problem over these unobservables in three steps.

1. Filter out unobservable states and actions (p, p', R, PD) to obtain the choice probabilities of bank-observable actions $\tilde{\sigma}_b^{(\tilde{d},b')}(\beta, z, \omega_b)$.
2. Assess the probability that an individual is patient tomorrow β' given bank-observable state ω_b and choices $(\tilde{d}, b')$, i.e., the posterior type score $s' = \psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b)$.
3. Compute the individual's repayment probability given transition over ω_b for each possible β'. Then, use the weighted sum over β' to compute $\mathbb{P}_b^{(NFD,b')}(\omega_b)$.

In the first step, banks filter out payday loan holdings p using the household distribution μ and sum out payday loan choices p' as follows:

$$\sigma_b^{(d,b')}(\beta, z, \omega_b) = \sum_{p'} \sum_{p} \sigma^{(d,b',p')}(\beta, z, \omega_b, p) \cdot \frac{\mu(\beta, z, \omega_b, p)}{\sum_{\hat{p}} \mu(\beta, z, \omega_b, \hat{p})}, \tag{10}$$

where the last fraction denotes the marginal distribution of p conditional on (β, z, ω_b). The idea is straightforward: since banks have rational expectations, they deal with the unobservables by weighting them with the distribution of unobservables conditional on the observables. Banks then form the probability of formal default ($\tilde{d} = FD$) versus non-formal default ($\tilde{d} = NFD \equiv R \vee PD$) to obtain the choice probabilities of bank-observable actions as follows:

$$\tilde{\sigma}_b^{(\tilde{d},b')}(\beta, z, \omega_b) = \begin{cases} \sigma_b^{(d,b')}(\beta, z, \omega_b) & \text{if } \tilde{d} = FD \\ \sum_{d \in \{R,PD\}} \sigma_b^{(d,b')}(\beta, z, \omega_b) & \text{if } \tilde{d} = NFD \end{cases}. \tag{11}$$

Accordingly, the feasible set from the bank's perspective is defined as:

$$\tilde{\mathcal{F}}_b(\beta, z, \omega_b) = \left\{ (\tilde{d}, b') \,\middle|\, \tilde{\sigma}_b^{(\tilde{d},b')}(\beta, z, \omega_b) > 0 \right\}. \tag{12}$$

In the second step, an individual's type score update is computed using Bayes' rule:[19]

$$\psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b) = \begin{cases} \sum_z Q^z(z) \cdot \sum_\beta Q^\beta(\beta'|\beta) \cdot \frac{\tilde{\sigma}_b^{(\tilde{d},b')}(\beta,z,\omega_b) \cdot s(\beta)}{\sum_{\hat{\beta}} \tilde{\sigma}_b^{(\tilde{d},b')}(\hat{\beta},z,\omega_b) \cdot s(\hat{\beta})} & \text{for } (\tilde{d}, b') \in \tilde{\mathcal{F}}_b \\ \sum_\beta Q^\beta(\beta'|\beta) \cdot s(\beta) & \text{for } (\tilde{d}, b') \notin \tilde{\mathcal{F}}_b \end{cases}, \tag{13}$$

where $s(\beta_L) \equiv 1 - s(\beta_H)$ by abuse of notation. For completeness, the second case in Equation (13) handles the score updating for an infeasible action. The updating process is intuitive: banks' prior belief s is updated with the relative choice likelihood of observable actions across types $\left(\tilde{\sigma}_b^{(\tilde{d},b')} \Big/ \sum_\beta \tilde{\sigma}_b^{(\tilde{d},b')} \cdot s \right)$, and with the exogenous transition of discount factors Q^β and transitory earnings Q^z. The posterior type score s' is denoted by $\psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b)$. There are two observations: (1) rebuilding type scores is costly due to priors; and (2) the updating process is dominated by priors when banks are certain about households' types. As s' may not lie on the score grid, we randomly assigned it to one of the two nearest points. This assignment is characterized by the function $Q^s(s'|\psi)$. Refer to Appendix A

[19]Note that $\psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b) \in [0, 1]$ and its value is bounded by the transition probability of becoming patient for all ω_b and $(\tilde{d}, b')$.

for details.

In the final step, the next-period repayment probability of a contract (NFD, b') for banks is computed as:

$$\mathbb{P}_b^{(NFD,b')}(\omega_b) = \sum_{(\beta',z',e',s')} s'(\beta') \cdot Q^z(z') \cdot Q^e(e'|e) \cdot Q^s\left(s'(\beta')\,\middle|\,\psi_{\beta'}^{(NFD,b')}(\omega_b)\right)$$
$$\Bigg[\mathcal{W}_{PD}^{b'}(\omega_b) \cdot \left(1 - \sigma^{(FD,0,0)}(\beta', z', \omega_b', p' = 0)\right) +$$
$$\left(1 - \mathcal{W}_{PD}^{b'}(\omega_b)\right) \cdot \sum_{p'} \mathcal{W}_{p'}^{(R,b')}(\omega_b) \cdot \left(1 - \sigma^{(FD,0,0)}(\beta', z', \omega_b', p')\right)\Bigg], \quad (14)$$

where the weighting factor $\mathcal{W}_{PD}^{b'}(\omega_b)$ denotes the probability that a household with bank-observable states ω_b and bank loan choice b' chooses payday default $d = PD$ between full repayment and payday default in the current period. It is given by:

$$\mathcal{W}_{PD}^{b'}(\omega_b) = \sum_z Q^z(z) \cdot \frac{\sum_\beta s(\beta) \cdot \sigma_b^{(PD,b')}(\beta, z, \omega_b)}{\sum_{\hat{d}\in\{PD,R\}} \sum_\beta s(\beta) \cdot \sigma_b^{(\hat{d},b')}(\beta, z, \omega_b)}. \quad (15)$$

In this case, provided that an individual has chosen to default on her payday loan in the current period, the bank realizes that the only possible payday loan choice in the next period is zero $p' = 0$.

Analogously, $1 - \mathcal{W}_{PD}^{b'}(\omega_b)$ gives the probability of choosing full repayment $d = R$. As banks do not observe p', they must form an expectation over the individual's payday loan choice. Conditional on full repayment, $\mathcal{W}_{p'}^{(R,b')}(\omega_b)$ denotes the probability of a household choosing a certain payday loan p' and is given by:

$$\mathcal{W}_{p'}^{(R,b')}(\omega_b) = \sum_z Q^z(z) \cdot \frac{\sum_\beta s(\beta) \cdot \hat{\sigma}_b^{(R,b',p')}(\beta, z, \omega_b)}{\sum_{\hat{p}'} \sum_\beta s(\beta) \cdot \hat{\sigma}_b^{(R,b',\hat{p}')}(\beta, z, \omega_b)}. \quad (16)$$

3.3.2 Payday Lenders

The payday loan pricing schedule is also endogenously determined by the zero-profit condition due to the assumption of perfect competition.[20] For computational tractability,

[20] This assumption can be justified by: (1) there are more payday loan storefronts than McDonald's and Starbucks combined in the U.S (Karger, 2005); (2) Flannery and Samolyk (2005) find that the annual interest rates of payday loans can be accounted by significant fixed operating costs and higher default premia.

we assume payday lenders use the same type score as banks to infer a household's hidden type.[21] The repayment probability of a contract (R, b', p') for bank-observable states ω_b is thus given by:

$$\mathbb{P}_p^{(R,b',p')}(\omega_b) = \sum_{(\beta',z',e',s')} s(\beta') \cdot Q^z(z') \cdot Q^e(e'|e) \cdot Q^s\left(s'(\beta')|\psi_{\beta'}^{(NFD,b')}(\omega_b)\right) \left(1 - \sum_{d' \in \{FD,PD\}} \sum_{b''<0} \sigma^{(d',b'',0)}(\beta', z', \omega_b', p')\right). \tag{17}$$

Note that payday lenders have to take into account both formal default FD and payday default PD because payday loans can be discharged in both cases. Moreover, a payday loan can be taken only if she does not save at banks $b'' < 0$. The payday loan pricing function is thus given by:

$$q_p^{(R,b',p')}(\omega_b) = \rho \cdot \frac{\mathbb{P}_p^{(R,b',p')}(\omega_b)}{1 + r_p}, \tag{18}$$

where r_p denotes the operating costs in the payday lending industry.

3.4 Evolution of the Household Distribution

The probability for an individual to move from state (β, z, ω_b, p) to $(\beta', z', \omega_b', p')$ is governed by the following mapping:

$$\begin{aligned} & T^*(\beta', z', \omega_b', p'|\beta, z, \omega_b, p) \\ & = \rho \cdot Q^\beta(\beta'|\beta) \cdot Q^z(z') \cdot Q^e(e'|e) \cdot \sigma^{(d,b',p')}(\beta, z, \omega_b, p) \cdot Q^s\left(s'(\beta')|\psi_{\beta_H'}^{(\tilde{d},b')}(\omega_b)\right) \\ & + (1-\rho) \cdot G_\beta(\beta') \cdot G_z(z') \cdot G_e(e') \cdot \mathbb{I}_{[b'=0]} \cdot \mathbb{I}_{[s'=G_\beta]} \cdot \mathbb{I}_{[p'=0]}. \end{aligned} \tag{19}$$

The second line describes the transition of surviving households. The third line describes the birth of newborn households. Therefore, the cross-sectional distribution of households μ evolves according to:

$$\mu'(\beta', z', \omega_b', p') = \sum_{(\beta,z,\omega_b,p)} T^*(\beta', z', \omega_b', p'|\beta, z, \omega_b, p) \cdot \mu(\beta, z, \omega_b, p). \tag{20}$$

[21] One possible justification is that developing a separate type score technology is too expensive for payday lenders.

3.5 Equilibrium

A stationary Recursive Competitive Equilibrium (RCE) is a set of (un)conditional value functions v^* and W^*, bank loan pricing functions q_b^* and repayment probability $\mathbb{P}_b^*$, payday loan pricing functions q_p^* and repayment probability $\mathbb{P}_p^*$, a type scoring function ψ^*, choice probability functions σ^* and $\tilde{\sigma}_b^*$, and a distribution $\overline{\mu}^*$ such that:

1. Household Optimality: $v^{*(d,b',p')}(\beta, z, \omega_b, p)$, $\sigma^{*(d,b',p')}(\beta, z, \omega_b, p)$, and $W^*(\beta, z, \omega_b, p)$ satisfy Equation (3), (6), and (7) for all (β, z, ω_b, p), respectively.

2. Type Score Updating: $\tilde{\sigma}_b^{*(\tilde{d},b')}(\beta, z, \omega_b)$ and $\psi_{\beta'_H}^{*(\tilde{d},b')}(\omega_b)$ satisfy Equation (11) and (13) for all (β, z, ω_b), respectively.

3. Zero Profits for Banks: $q_b^{*(NFD,b')}(\omega_b)$ and $\mathbb{P}_b^{*(NFD,b')}(\omega_b)$ satisfy Equation (9) and (14) for all ω_b, respectively.

4. Zero Profits for Payday Lenders: $q_p^{*(R,b',p')}(\omega_b)$ and $\mathbb{P}_p^{*(R,b',p')}(\omega_b)$ satisfy Equation (18) and (17) for all ω_b, respectively.

5. Stationary Distribution: $\overline{\mu}^*(\beta, z, \omega_b, p)$ solves Equation (20).

Note that the banking problem requires the knowledge of the cross-sectional distribution of households μ. As a result, all equilibrium objects depend on the distribution, and solving the model numerically becomes a daunting task. To accelerate the computation, we implement the one-loop algorithm where value functions, the type scoring function, pricing schedules, and the distribution are updated simultaneously in each iteration until convergence.[22] Refer to Appendix B for computational details.

4 Calibration

The goal of the paper is to explore to what extent the reputation protection channel can explain the payday loan puzzle documented in Agarwal et al. (2009). Given they used a payday loan dataset collected from 2000 to 2004 and to circumvent the effects of the 2005 Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA), we set the baseline calibration year to 2004. The model period is one year. We calibrate the model to

[22] A similar algorithm is implemented by Hatchondo, Martinez, and Sapriza (2010).

the whole U.S. households. Median earnings are set to \$33,176 in 2004 from the Current Population Survey (CPS).[23] Our calibration strategy is threefold: (1) standard parameters are taken from the literature; (2) parameters with a direct empirical counterpart are exogenously calibrated; and (3) the rest are internally calibrated to match targeted data moments.

The persistent and transitory earnings processes are taken from Floden and Lindé (2001). We use their process because they estimated it using wage earnings in the U.S. for the same time period considered in our paper and without life-cycle components. We assume newborn households are endowed with the lowest persistent earnings realization and with transitory earnings drawn randomly from the estimated process. These assumptions imply that newborn households start with low earnings. Following Chatterjee et al. (2020),[24] we set discount factors to 0.886 and 0.915, respectively. The turn-over rates for discount factors are $Q^{\beta}(\beta_H|\beta_L) = 0.013$ and $Q^{\beta}(\beta_L|\beta_H) = 0.011$. These rates imply that households change their types on average every 77 to 91 years. The share of impatient households among newborns is set to 72%. This is consistent with the upward moving of credit ranking along ages observed in data.[25] All are summarized in Table 1.

We set the CRRA parameter of the utility function to 2, the standard value in the macro literature. The survival probability of households every period is set to 0.975, implying an average working life span of 40 years. The risk-free rate r_f is set to 1.4% and implies an effective interest rate of 4%, consistent with the literature. According to calculations in Albanesi and Nosal (2020), the out-of-pocket filing costs for Chapter 7 before the 2005 bankruptcy reform amounted to approximately \$697, implying $\kappa_{FD} = 0.02$. As Montezemolo and Wolff (2015) pointed out that payday defaults in practice involve two bounced checked fees (one by banks and the other by payday lenders, \$35 each), we set the out-of-pocket filing costs for payday defaults κ_{PD} to 0.002. According to Flannery and Samolyk (2005), the average operating costs (without default losses) per two-week payday loan of size \$230 is around \$19, thus implying the annualized operating cost for payday lenders r_p is 1.925. The dispersion parameter of the extreme value distribution is set to 0.005.[26]

[23] \$638 earnings per week $\times$ 52 weeks $=$ \$33,176.

[24] To determine discount factors, Chatterjee et al. (2020) use an affine approximation using the model-generated data to match the means and standard deviations of credit rankings across ages. Our calibrated model can match these moments fairly well.

[25] μ_H denote the share of patient households. Solving $\mu_H = \rho\left[(1 - Q^{\beta}(\beta_L|\beta_H))\mu_H + Q^{\beta}(\beta_H|\beta_L)(1-\mu_H)\right] + (1-\rho)G_{\beta_H}$ yields that there are 41% of patient and 59% of impatient households in equilibrium.

[26] This value is comparable the those used in Chatterjee et al. (2020). To rule out the contribution of ex-

Parameter		Value	Target / Source
Persistence of persistent earnings	ρ_e	0.9136	Floden and Lindé (2001)
S.D. to persistent earnings	σ_e^2	0.0426	Floden and Lindé (2001)
S.D. to transitory earnings	σ_z^2	0.0421	Floden and Lindé (2001)
Persistent earnings at birth	G_e	(1,0,0)	Upward earnings profile
Transitory Earnings at birth	G_z	(1/3,1/3,1/3)	Upward earnings profile
Low discount factor	β_L	0.886	Chatterjee et al. (2020)
High discount factor	β_H	0.915	Chatterjee et al. (2020)
Transition from low to high	$Q^\beta(\beta_H\|\beta_L)$	0.013	Chatterjee et al. (2020)
Transition from high to low	$Q^\beta(\beta_L\|\beta_H)$	0.011	Chatterjee et al. (2020)
Discount factor at birth	G_β	(0.72,0.28)	Chatterjee et al. (2020)
CRRA	γ	2	Standard
Survival probability	ρ	0.975	40 years
Risk-free rate	r_f	0.014	Effective interest rate = 4%
Formal default cost	κ_{FD}	0.02	Albanesi and Nosal (2020)
Payday default cost	κ_{PD}	0.002	Montezemolo and Wolff (2015)
Operating cost for payday lenders	r_p	1.925	Flannery and Samolyk (2005)
S.D. of extreme value shocks	α	0.005	$\approx$ Chatterjee et al. (2020)

Table 1: Exogenously Chosen Parameters

Table 1 provides a summary.

We internally calibrate the stigma costs for formal default κ_{FD} and for payday default κ_{PD} jointly by matching the formal default rate and the conditional payday default rate. The conditional payday default rate refers to the write-off rate among payday loan borrowers in the year after they took out their first payday loans. Results are summarized in Table 2. The formal default rate in the data is computed as the total number of non-business Chapter 7 filings from American Bankruptcy Institute (ABI) normalized by the total number of U.S. households in 2004. The conditional payday default rate is taken from Skiba and Tobacman (2018) where they used the same payday loan data as in Agarwal et al. (2009). The formal and payday stigma costs are accordingly set to 0.02235 and 0.00702, respectively.[27]

We also evaluate our model fit on a set of untargeted moments standard in the consumer finance literature. The data and model moments are summarized in Table 3.[28] For

treme value shocks to the payday loan puzzle, we check whether households are making such a seeming pecuniary mistake with higher values. See Section 6.1.

[27] The values for formal and payday stigma costs correspond to 2.18% and 0.7% of consumption loss on average.

[28] Note that for all SCF-related data moments, we restrict the sample to households with household heads

Parameter		Value	Target	Data	Model
Formal stigma cost	ξ_{FD}	0.02235	Formal default rate	0.99%	0.99%
Payday stigma cost	ξ_{PD}	0.00702	Payday default rate (cond.)	29.7%	29.7%

Table 2: Internally Calibrated Parameters

Moment (in %)	Data	Model
Households in Debt		
Fraction of bank loan borrowers	20.9	24.26
Fraction of payday loan borrowers	5.61	9.46
Bank debt-to-earnings (cond.)	11.75	6.48
Interest Rate		
Avg. interest rate for bank loans	9.26	8.56
Avg. interest rate for payday loans	447.88	410.85

Table 3: Untargeted Moments: Data v.s. Model

the fraction of bank loan borrowers in the data, we use the 2004 Survey of Consumer Finances (SCF) and construct a measure of liquid net worth.[29] We then compute the fraction of households with negative liquid net worth. The fraction of payday loan borrowers is computed with the 2010 SCF since information on payday loans was first collected in the 2010 wave. We also use the 2004 SCF to compute the bank debt-to-earnings ratio conditional on borrowing bank loans.[30] Bank debt is measured using the same liquid net worth definition as above. Earnings is computed as wage income measured in the 2004 SCF.

The average interest rate for bank loans is computed as the average credit card interest rate among those having a positive credit card balance in the 2004 SCF, net of the one-year ahead CPI inflation of all urban consumers from the U.S. Bureau of Labor Statistics. We use the payday loan statistics reported in Skiba and Tobacman (2018) to calculate the average interest rate for payday loans, net of the one-year ahead CPI inflation.[31]

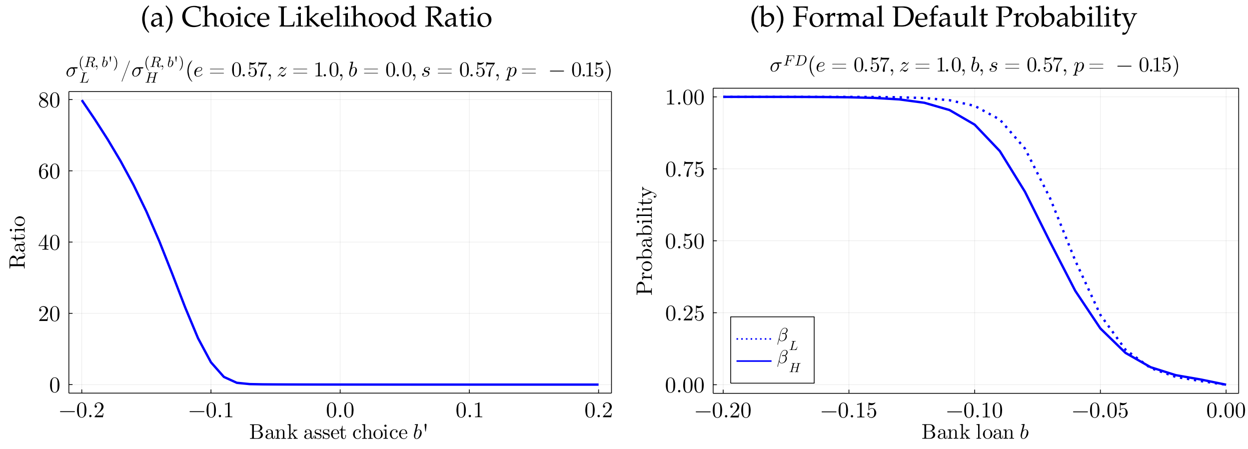
aged between 20 and 60. We do this since our model does not account for retirement or childhood.

[29] We follow Herkenhoff (2019) in constructing this measure of liquid net worth. It is calculated as the difference between a household's liquid assets, such as checking and savings accounts, and credit card debt. We prefer this measure of net worth as we do not explicitly model illiquid assets such as housing in our framework.

[30] We compute the ratio of average debt to average earnings conditional on having bank debts.

[31] The average bi-weekly payday loan size is \$317.55 with an average interest payment of \$56.4. It implies that $\frac{56.4}{317.55} \times \frac{365}{14} \times \frac{1}{1.03388} \times 100 = 447.88\%$.

Figure 2: Borrowing and Default Behavior across Types



Notes: Left figure: The choice likelihood ratio denotes the probability of an impatient household making a certain choice relative to a patient one. A high value for a certain choice b' implies that an impatient household is much more likely to make this choice compared to a patient one. Right figure: The solid line denotes the probability of formal default for a patient household across bank loans b. The dashed line denotes the same probability for an impatient household.

5 Pooling and Cross-Subsidization

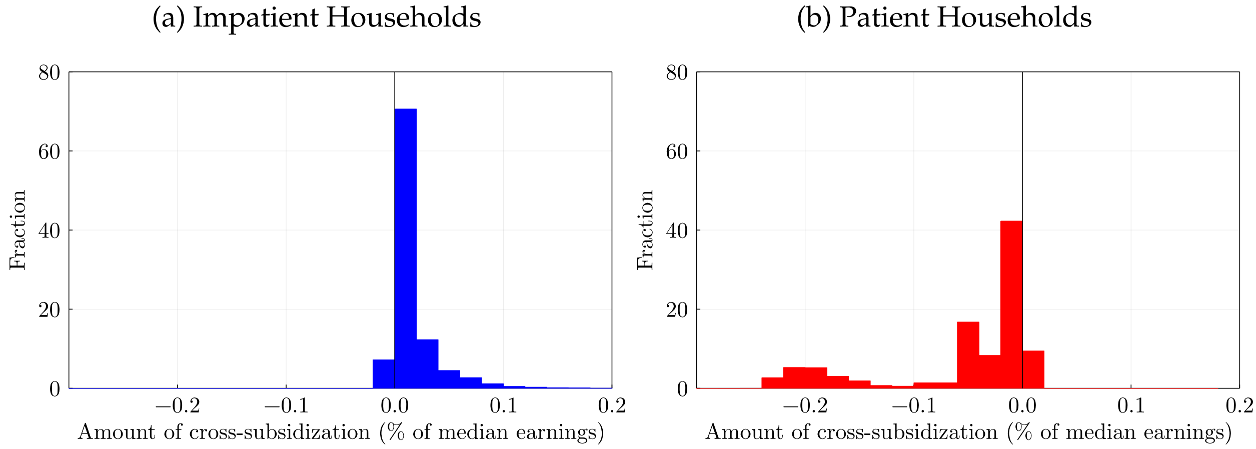
In our economy, there is hidden information about a household's type in addition to hidden actions (a household's payday loan choice is unobservable to banks). Because banks cannot observe household types and payday loan choices, they cannot directly design contracts conditioned on these variables.[32] As a result, this limited information structure leads to two-dimensional pooling across household types and payday loans when banks price their loans.[33]

We first illustrate the heterogeneity in behavior and the resulting cross-subsidization of bank loans across types. Figure 2 illustrates differences in borrowing and default behavior across impatient and patient households. Figure 2a plots the choice likelihood ratio across different bank asset choices b' conditional on a certain state. The choice likelihood ratio denotes the probability of an impatient household saving or borrowing a certain amount relative to a patient one. A high value for the ratio implies that a certain choice is more likely to be taken by an impatient household than a patient one. We can see that impatient households are much more likely to borrow and to borrow more relative to patient

[32] As we discussed in Section 3.3.1, banks will instead use type scores and the conditional distribution of payday loans given observed variables.

[33] There is only pooling across types for payday lenders since they can observe a household's payday loan choice. In this section, we will focus on pooling and cross-subsidization in the bank lending market.

Figure 3: Cross-Subsidization of Bank Loans across Types



Notes: Cross-subsidization is computed as the difference between actuarially fair interest payments when banks can observe household type and actual interest payments in equilibrium.

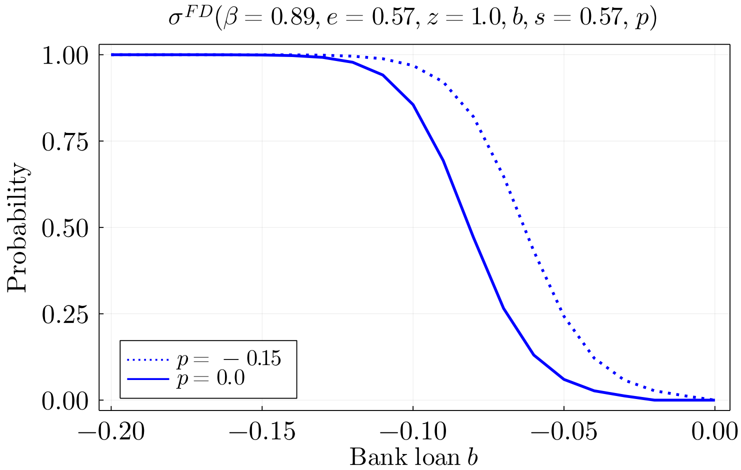
households. This is intuitive as households with a lower discount factor value consumption today more and will therefore tend to borrow more. Figure 2b illustrates how the formal default probability varies across levels of bank debt b. The solid line presents the formal default probability for a patient household, while the dashed line shows the probability for an impatient one. It can be seen that the impatient households are more likely to formally default than patient ones across most bank loan positions b. As a consequence, conditional on the same state (and in particular, the same bank loan size), impatient households are riskier borrowers for banks.

Since banks cannot perfectly infer a household's type, this imperfect distinction across types results in the cross-subsidization of bank loans across types. In Figure 3, we plot the distribution of cross-subsidization amounts in the percentage of median earnings for impatient and patient households. Such an amount denotes the extra interest payments that households face in the counterfactual when banks were able to see their types compared to the benchmark, computed as:

$$\left(q^{(NFD,b')} - q^{(NFD,b')}_{fair}(\beta)\right) \cdot b' \times 100, \tag{21}$$

where $q^{(NFD,b')}_{fair}(\beta)$ represents the actuarially bank loan price schedule as if banks knew household types. As shown in Figure 3, it is mostly impatient households who are cross-subsidized by patient households. This is due to the fact that the impatient tend to be riskier borrowers as they are more likely to default. In other words, conditional on the

Figure 4: Formal Default Probability across Payday Loans



Notes: The solid line depicts the probability for a household with no payday loans to formally default. The dashed line shows the same probability for a household with a payday loan size of 0.15.

same level of bank borrowing, impatient households face lower interest rates on bank loans than actuarially fair rates in our economy.

Moreover, there are also differences in default behavior across payday loan borrowers. Figure 4 shows how the formal default probability varies across different levels of bank debt b and households with extra payday debt $p = -0.15$ (dashed line) or not $p = 0$ (solid line). Conditional on the same bank loan position, households with additional payday loan positions are more likely to formally default on both loans. This is straightforward as households with more payday loans have a higher total debt burden and are thus more likely to default. As a result, bank loan borrowers who take out extra payday loans are riskier for banks.

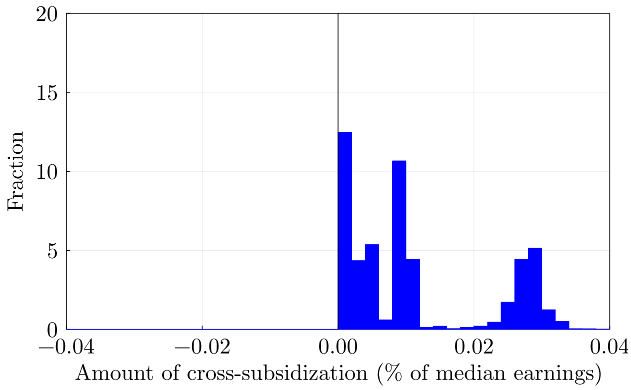
These differences in default behavior lead to cross-subsidization of bank loans across payday and non-payday loan borrowers. Because banks cannot observe payday loan usage by households, borrowers with extra payday loans face the same bank loan pricing schedule as borrowers who do not have payday loans. Conditional on the same level of bank loan, payday loan borrowers tend to have a higher default probability as they have more debt in total. As a result, payday (non-payday) loan borrowers pay lower (higher) rates on bank loans than actuarially fair rates. Figure 5 plots the distribution of the cross-subsidization amounts across payday and non-payday loan borrowers. In this case, the amount of cross-subsidization is computed as below.

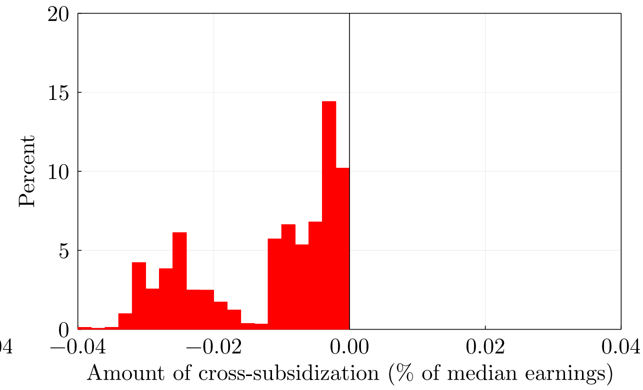
$$\left(q^{(NFD,b')} - q_{fair}^{(R\vee PD,b',p')}\right) \cdot b' \times 100, \tag{22}$$

Figure 5: Cross-Subsidization of Bank Loans across (Non-)Payday Loan Borrowers

(a) Payday Loan Borrowers

(b) Non-Payday Loan Borrowers





Notes: Cross-subsidization is computed as the difference between actuarially fair interest payments when banks can observe payday loan usage and actual interest payments in equilibrium.

where $q_{fair}^{(R \vee PD, b', p')}$ represents the actuarially bank loan price schedule as if banks were able to observe payday loan default and choices.

Table 4 summarizes the main equilibrium outcomes across types. Compared to patient households, impatient households are more likely to default and borrow, and hold larger debts for both bank and payday loans. This leads to overall higher borrowing costs for the impatient even though they are partially cross-subsidized by patient households as shown in Figure 3.

6 The Payday Loan Puzzle

In this section, we first illustrate how we identify the payday loan puzzle in our model. Then, we examine to what extent our model can account for the puzzle in the data. In addition, we quantify the type score gains and interest costs from using payday loans and investigate under what circumstances households use payday loans to protect their type scores in our model.

6.1 Identification of the Payday Loan Puzzle

In our model, we identify the households who make seeming pecuniary mistakes that are consistent with the payday loan puzzle in the following way: for each possible state (β, z, ω_b, p), we identify those feasible borrowing choices with repayment $(R, b' < 0, p' <$

Moment (in %)	Aggregate	Impatient	Patient
Default			
Formal default rate	0.99	1.27	0.57
Payday default rate (cond.)	29.7	30.6	27.9
Households in debt			
Fraction of bank loan borrowers	24.26	27.5	19.55
Fraction of payday loan borrowers	9.46	10.7	7.65
Fraction of both loan borrowers	8.42	9.54	6.77
Bank debt-to-earnings (cond.)	6.48	6.54	6.36
Payday debt-to-earnings (cond.)	1.91	2.00	1.73
Interest rate			
Avg. interest rate for bank loans	8.56	8.79	8.06
Avg. interest rate for payday loans	410.85	433.89	362.74

Table 4: Equilibrium across Types

Notes: The payday default rate and the payday debt-to-earnings ratio are conditional on having any payday loans. The bank debt-to-earnings ratio is conditional on having any bank loans.

$0) \in \mathcal{F}(z, \omega_b, p)$ that involve a payday loan where the same total amount of borrowing $\hat{b}' = b' + p'$ could have been achieved at lower borrowing costs using bank loans only. That is:

$$\left| q_b^{(NFD,b')}(\omega_b) \cdot b' + q_p^{(R,b',p')}(\omega_b) \cdot p' \right| < \left| q_b^{(NFD,\hat{b}')}(\omega_b) \cdot \hat{b}' \right|. \tag{23}$$

The borrowing choices that fulfil the above condition are the choices that we classify as the payday loan puzzle. Let the set of these choices be called $\mathcal{P}(\beta, z, \omega_b, p)$.[34]

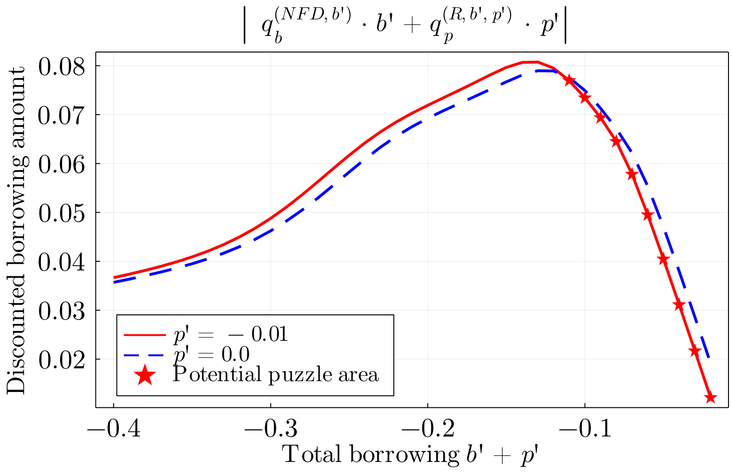
To illustrate where the region with payday loan puzzle can happen, Condition (23) is visualized in Figure 6 where we plot the discounted borrowing amounts across total borrowing conditional on a certain state. The solid line denotes the discounted borrowing amounts involving a given payday loan $p' = -0.01$ and the dashed line denotes the

[34] Recall that, in a model with utility shocks, any feasible action will be chosen with positive probability (not just the choice with the highest value). As a result, households might take up payday loans because mainly of such shocks. To control for this nuisance, we additionally check whether households are conscious of making this decision with higher values. To be specific, for each state (β, z, ω_b, p), the feasible borrowing choices with repayment $(R, b' < 0, p' < 0) \in \mathcal{F}(z, \omega_b, p)$ where the value of borrowing a certain amount is higher when using payday loans compared to only using bank loans. That is:

$$v^{(R,b',p')}(\beta, z, \omega_b, p) > v^{(R,\hat{b}',p=0)}(\beta, z, \omega_b, p). \tag{24}$$

Hence, there exists the general dependency of $\mathcal{P}(\cdot)$ on β. In fact, Condition (24) is pretty weak as almost all borrowing choices using both loans are fulfilled.

Figure 6: Identification of the Payday Loan Puzzle



Notes: The discounted borrowing amount is computed as the borrowing amount multiplied by the associated discount borrowing price.

discounted borrowing amounts without any payday loan $p' = 0$. The region of choices satisfying the condition is marked by asterisks and labeled as "Potential Puzzle Area."

Recall that Agarwal et al. (2009) use a matched dataset of credit cards and payday loans to identify the payday loan puzzle. We accordingly define the rate of puzzle occurrence as the fraction of households that make a choice which would be classified as the payday loan puzzle relative to all households that borrow using both loans. More specifically, the rate of puzzle occurrence in the model is calculated as follows:

$$\frac{\sum_{\beta,z,\omega_b,p} \mu(\beta,z,\omega_b,p) \cdot \sum_{(d,b',p')\in\mathcal{P}(\beta,z,\omega_b,p)} \sigma^{(d,b',p')}(\beta,z,\omega_b,p)}{\sum_{\beta,z,\omega_b,p} \mu(\beta,z,\omega_b,p) \cdot \sum_{(d,b',p')\in\mathcal{F}_{both}(z,\omega_b,p)} \sigma^{(d,b',p')}(\beta,z,\omega_b,p)}, \tag{25}$$

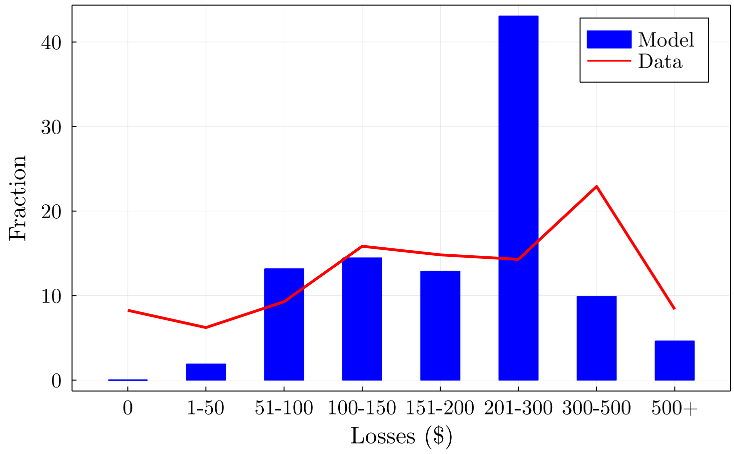
where the numerator represents the unconditional fraction of households making the puzzling behavior; the denominator denotes the fraction of households borrowing using both types of loans; and the feasible set of borrowings choices using both loans $\mathcal{F}_{both}(z,\omega_b,p)$ is defined as:

$$\mathcal{F}_{both}(z,\omega_b,p) \equiv \left\{(d,b',p') | (d=R, b'<0, p'<0) \in \mathcal{F}(z,\omega_b,p)\right\}. \tag{26}$$

Our model can account for a significant fraction of the puzzling households who take out expensive payday loans with cheaper borrowing alternatives available, identified in the data. In the model, the rate of puzzle occurrence is around 26.44%.[35] Agarwal et al.

[35] The rate of puzzle occurrence among impatient households is 25.55% and among patient ones is 28.31%.

Figure 7: Histogram of Monetary Costs of Payday Loan Puzzle



Notes: The data series is from Agarwal et al. (2009). The monetary costs are the amounts which households could have saved if they first exhausted their credit cards before taking out payday loans over one year.

(2009) empirically identify a rate of around two-thirds using a matched dataset. Thus, our model can account for around 40% of the payday loan puzzle found in the data.[36]

Our model can also match the magnitude of monetary costs from the payday loan puzzle. Recall that these costs denote the amounts which the puzzling payday loan borrowers could have saved if first exhausting their credit cards. Figure 7 shows the distribution of the corresponding annual monetary costs per household in both data (solid line) and our calibrated model (bar chart). We can see that in our model most monetary costs have the same magnitude ranging from \$0 to \$500 as in the data.[37] Moreover, our calibrated model predicts average annual monetary costs of \$230, which is aligned with the average amount of around \$200 reported in Agarwal et al. (2009). Essentially, these costs represent the value of reputation protection in our model.

6.2 The Reputation Protection Channel

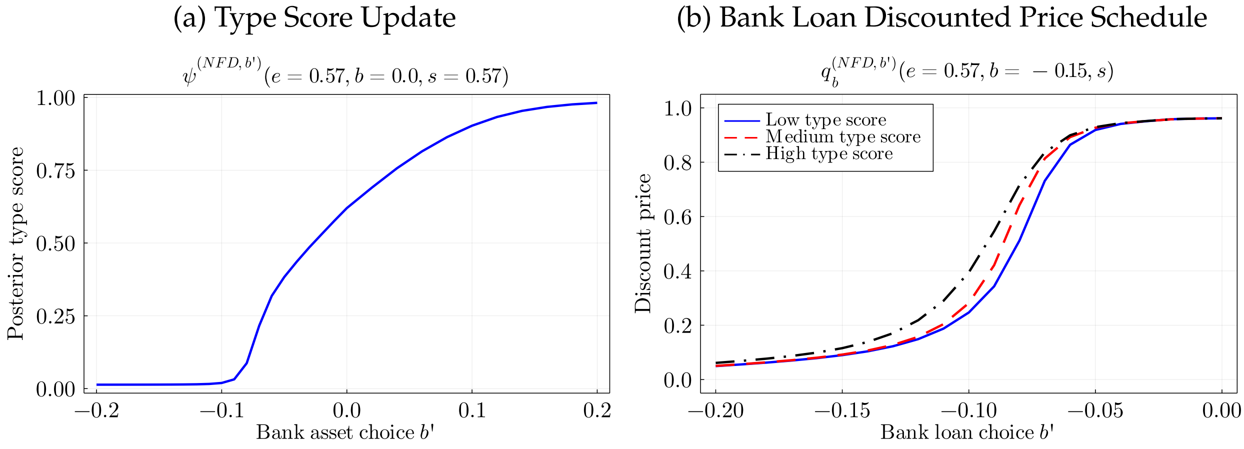
We now explore the reputation protection hypothesis quantitatively in our model. In our model, borrowing larger bank loans leads to a lower type score. In addition, households with lower type scores face higher bank interest rates. Hence, households have an in-

The unconditional fraction of puzzling households is 2.28% in aggregate, 1.5% among impatient households, and 0.78% among patient ones.

[36]Note that cheaper costs for payday default than formal default are not the main factor with which our calibrated model can generate the payday loan puzzle. Refer to Appendix C for details.

[37]We can even match the distribution of these costs rather well, apart from the bins of \$201-\$300 and \$300-\$500.

Figure 8: Reputation Protection Incentive



Notes: Left figure: The type score update is plotted across different bank asset choices b' conditional on a certain state (e, b, s). A new type score of 1.0 means that a household is assessed to be patient with probability one. Right figure: The discounted price schedule for bank loans is shown across different bank loan choices b' conditional on a certain state (e, b, s). The discount price is inversely related to the interest rate. The solid/dashed/dash-dotted lines denote the schedules offered to households with low/medium/high type scores.

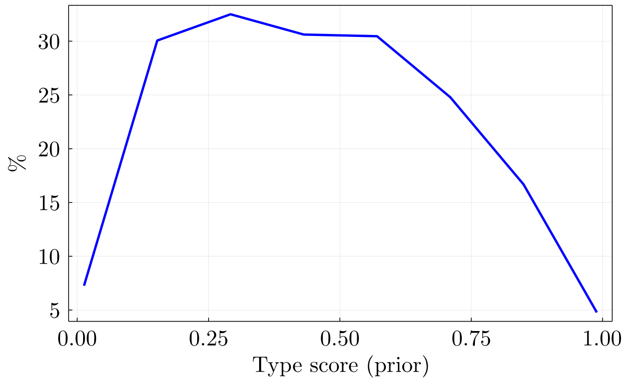
centive to borrow using payday loans instead of bank loans in order to avoid a negative impact on their type scores, thus giving them access to cheaper bank credit in the future.

Figures 2 and 8 illustrate how this mechanism works. Figures 2a and 8a show the effects of bank loan choices on type scores. In Figure 2a, we can see how impatient households are more likely to borrow and to borrow more relative to patient households. Figure 8a shows the type score updating function and depicts how a household's type score is updated conditional on different bank asset choices b'. We can see that taking out a larger bank loan (or saving less) leads to a worse type score update because banks realize that the impatient are more likely to borrow larger amounts. Figures 2b and 8b show how a lower type score leads to higher interest rates. Figure 2b illustrated how impatient households are more likely to formally default than patient ones across different levels of debt. Figure 8b illustrates the bank loan discounted price schedules for households with low (solid line), medium (dashed line), and high type scores (dash-dotted line). Banks will charge households with lower type scores lower discounted prices (higher interest rates) in order to be compensated for the additional default risk.

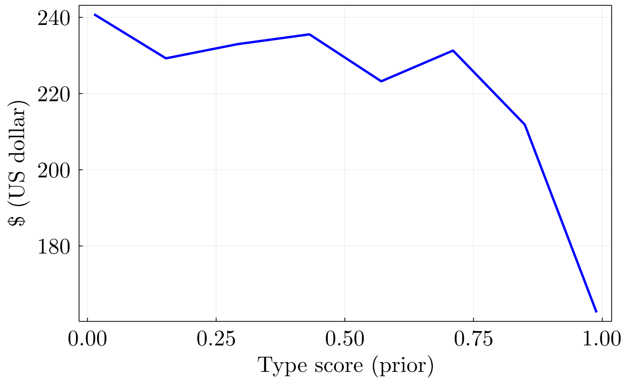
Figure 9 looks at the trade-off between type score protection and monetary costs for using payday loans among the payday loan borrowers with cheaper credit available. Figure 9a illustrates the relative gain in posterior type scores from using payday loans compared

Figure 9: Cost-Benefit Analysis among Seemingly Puzzling Households

(a) Posterior Type Score Gain



(b) Monetary Costs



Notes: Left figure: The type score gain is computed by comparing the posterior type score of using payday loans relative to using only banks loans for the same borrowing amount, conditional on a prior type score, and expressed in percentage points. Right figure: The monetary costs denote the extra interest payments incurred using payday loans compared to using bank loans for the same borrowing amount across prior type scores.

to borrowing the same amount using only bank loans across different prior type scores.[38] There exists significant prior-dependent heterogeneity.[39] In particular, the gain is over 30% for those who have lower medium prior type scores. Figure 9b calculates the monetary costs in U.S. dollars across prior type scores.[40] These costs refer to the extra interest expenses incurred by using payday loans compared to using bank loans for the same borrowing amount. Such pecuniary costs are significant and vary across prior type scores. For example, households with the lowest possible type score are willing to pay an additional $240 in payday loan interest fees to achieve higher type scores. On average, these puzzling households, i.e., taking out payday loans while having cheaper borrowing alternatives available, are willing to pay an additional $230 in interest payments on payday loans for an increase in type scores by 23%. On average, an 1% increase in type scores, in turn, leads to a lower borrowing interest rate by 16% in the future bank lending market.[41]

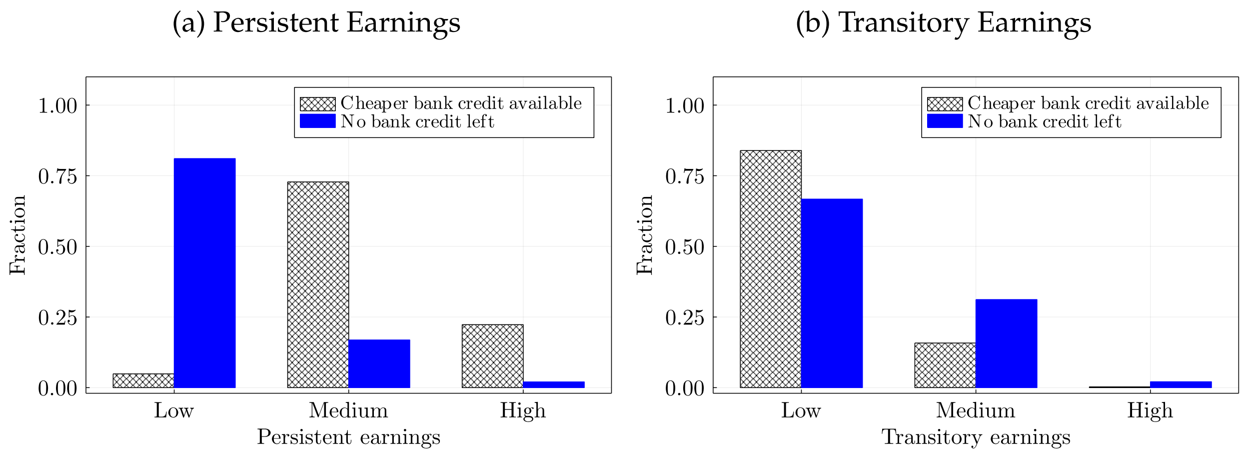
[38] To be precise, the relative gain in posteriors for given bank-observable states ω_b is computed as: $\left(\psi^{(NFD,b')}_{\beta'_H}(\omega_b) - \psi^{(NFD,\hat{b}')}_{\beta'_H}(\omega_b)\right) \Big/ \psi^{(NFD,\hat{b}')}_{\beta'_H}(\omega_b) \times 100$ where $\psi^{(NFD,b')}_{\beta'_H}(\omega_b)$ and $\psi^{(NFD,\hat{b}')}_{\beta'_H}$ denote the updated type scores for borrowing a bank loan of b' and for borrowing a mixture of bank and payday loans $\hat{b}' = b' + p'$.

[39] The hump shape results from the fact that prior dominates in the type score updating at both ends (i.e., when banks believe a household to be a certain type).

[40] If we express these monetary costs in percentage points relative to the counterfactual, the resulting plot also exhibits a hump-shaped pattern.

[41] See Appendix D for more general results.

Figure 10: Earnings Distribution among Both Loan Borrowers



Notes: These figures show the distribution of payday loan borrowers who have exhausted their cheaper bank loans or not across persistent (left figure) and transitory (right figure) earnings. ”Cheaper bank credit available” refers to the households who borrow using both loans even though they have not exhausted cheaper bank credit (see conditions 23 and 24). ””No bank credit left” refers to the households who borrow using both loans but have exhausted cheaper bank credit.

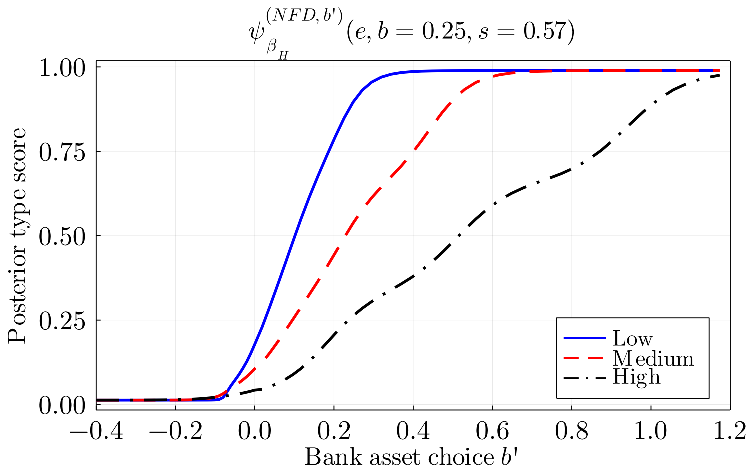
6.3 Profile of Puzzling Households

In the previous subsection, we illustrated how using payday instead of bank loans can lead to significant type score gains at the cost of substantially higher interest costs in the short run. Better type scores thus lead to better access to credit markets in the long run. In this subsection, we further investigate when households engage in this seemingly puzzling behavior in our calibrated model.

Figure 10 plots the distribution of both loan borrowers across persistent earnings (Figure 10a) and transitory earnings (Figure 10b), conditional on whether the cheaper bank credit has been exhausted or not yet. We can see that, compared to the borrowers who have exhausted their cheaper bank credit (solid bar chart), borrowers who have not exhausted their cheaper bank credit yet (argyle bar chart) tend to have higher persistent but lower transitory earnings. In particular, households take out payday loans before exhausting cheaper bank loans when they have medium to high persistent earnings but low transitory earnings in our model. This observation indicates that these puzzling households use payday loans to smooth out the shortfall in transitory earnings without significantly damaging their type scores (such a trade-off has been explained in Figure 9).

However, why are the households with this earnings profile especially incentivized to borrow using payday loans instead of cheaper bank loans? Recall that banks can ob-

Figure 11: Type Score Updating across Persistent Earnings



Notes: This figure plots the updated type score for different bank asset choices b' across persistent earnings for a certain state. The solid/dashed/dash-dotted lines denote the type score updating function with low/medium/high persistent earnings.

serve persistent earnings but not transitory earnings. Therefore, taking out bank loans to smooth out a negative transitory earnings shock while having high persistent earnings will lead to a downgraded type score. This explanation is illustrated in Figure 11 which shows the type score updating across bank asset choices for different persistent earnings. Conditional on the same bank asset choice b', a household with low persistent earnings (solid line) will receive a higher type score update than a household with medium (dashed line) or high (dash-dotted line) persistent earnings. The intuition is as follows. Borrowing a larger bank loan is more indicative of impatience (low discount factor) when having high compared to low persistent earnings because banks think those with higher persistent earnings are not supposed to borrow that much. Instead, by complementing bank loans with payday loans, which are unobservable to banks, households can reduce the negative impact on their type scores while still being able to smooth out transitory earnings shocks.

7 Policy Experiments

In this section, we consider two different policy experiments that are highly relevant in the consumer credit market: policies curtailing (or outright banning) payday loans and bankruptcy law regulation.

7.1 Payday Loan Regulation

Payday loans have been a subject of intensive public debate. Opponents of payday loans have long argued that payday lenders prey on poor households and should be banned. Advocates emphasized the role of payday loans in smoothing consumption.

We contribute to this debate by investigating the welfare implications of limiting access to payday loans through quantity caps or an outright ban on payday loans in our model. Table 5 summarizes the key results of these policy counterfactuals where we report the key moments and welfare outcomes measured in consumption equivalent variation (CEV) units relative to the benchmark in percentage points.[42] The column "Benchmark" describes the calibrated model as presented in the previous sections. The column "Quantity Cap" denotes the counterfactual where the possible payday loan choices are limited to a size of \$300 which is the smallest possible payday loan in the benchmark economy.[43] The column "Full Ban" describes the counterfactual where payday loans become unavailable in the economy.

Compared to the benchmark, a quantity cap leads to fewer payday loan borrowers as there are less payday loan choices available. Conditional on borrowing payday loans, payday debt-to-earnings ratio also drops. It then leads to a decrease in the (unconditional) payday default rate to 2.2% since it is less advantageous to default on smaller payday loans.[44] The unconditional payday default rate also drops mechanically as there are less payday loan borrowers. In addition, the conditional effective default rate on payday loans, which is defined as the fraction of households defaulting on payday loans through either formal or payday default conditional on have any payday loans, also decreases from around 34.68% in the benchmark to 31.24%. Accordingly, the average payday interest rate decreases. The formal default rate also decreases slightly and as such there is no substitution from payday default to formal default as a consequence of the payday loan cap. This in turn gives rise to a mild decrease in average bank interest rate. Surprisingly, the *extensive margin* of bank loan borrowing also decreases: the fraction of bank loan borrowers drops slightly. The lack of an increase in the extensive margin of bank loan borrowers is explained by the fact that most payday loan borrowers were already borrowing bank

[42] Note that households barely change their types even though types are assumed to be stochastic for the technical reason. Given our calibration, the average life expectancy of 40 years is two times smaller than the average type-switching period of around 80 years. Refer to Section 4 for details.

[43] \$300 is the average payday loan size in the data.

[44] The monetary filing cost stays the same as in the benchmark economy.

Variables (in %)	Benchmark	Quantity Cap	Full Ban
Default			
Formal default rate	0.99	0.96	0.89
Payday default rate	2.81	2.19	–
Eff. payday default rate (cond.)	34.68	31.24	–
Households in debt			
Fraction of bank loan borrowers	24.26	24.06	23.15
Fraction of payday loan borrowers	9.46	8.22	–
Fraction of both loan borrowers	8.42	7.36	–
Bank debt-to-earnings (cond.)	6.48	6.61	6.84
Payday debt-to-earnings (cond.)	1.91	1.40	–
Interest rate			
Avg. interest rate for bank loans	8.56	8.53	8.46
Avg. interest rate for payday loans	410.85	341.88	–
Welfare			
Welfare – aggregate	–	-0.0012	-0.0291
Welfare – impatient households	–	-0.0029	-0.0331
Welfare – patient households	–	0.0013	-0.0233
Cross-Subsidization of bank loans			
Avg. cross-sub. across types	–	-10.5	-15.1
Avg. cross-sub. across payday loan borrowers	–	-21.0	-100.0

Table 5: Policy Counterfactual: Restricting Payday Loan Size

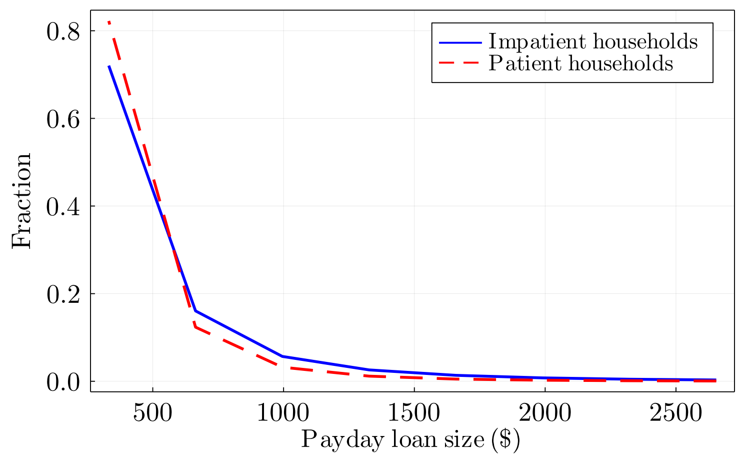
Notes: The conditional effective payday default rate is defined as the fraction of households choosing to default on payday loans through either formal or payday default, conditional on having any payday loans. The bank debt-to-earnings ratio is conditional on having any bank loans. The payday debt-to-earnings ratio is conditional on having any payday loans. Welfare is measured in CEV units relative to the benchmark in percentage points. The average cross-subsidization amount of bank loans is computed as in Section 5 but expressed in percentage changes relative to the benchmark.

loans in the benchmark economy. Instead, limiting the size of payday loans leads to an increase in the *intensive margin* of bank loan usage: conditional on borrowing, bank debt-to-earnings ratio rises. This is because borrowers now partially substitute bank loans for payday loans. In the full ban counterfactual, all of these changes are magnified.

The overall welfare effects of both policy counterfactuals are negative.[45] More interestingly, the welfare implications of experiments are heterogenous across household types. Impatient households lose in terms of welfare whenever the payday loan market becomes

[45] Note that our framework measures the lower bound of the welfare effects of type scores since, in practice, individuals with higher credit scores have better mortgage terms and labor market outcomes, both of which are not considered in our model.

Figure 12: Distribution of Payday Loan Size across Types



Notes: This figure illustrates the distribution of payday loan borrowers across different payday loan amounts for impatient households (solid line) and patient households (dashed line).

more constrained. In contrast, patient households have higher welfare in the quantity cap counterfactual but lower welfare in the full ban counterfactual compared to the benchmark economy. The reasons for the declines in welfare for impatient households are intuitive. First, impatient households are more likely to borrow larger payday loans in the benchmark economy and are thus more affected by the quantity cap or ban, as shown in Figure 12 of the distribution of payday loan size conditional payday loan borrowers across types in the benchmark. Second, imposing a payday loan quantity cap or banning payday loans also reduces the informational asymmetry regarding payday loan usage in the bank market. In turn, this reduction allows banks to better assess a household's type and reduces pooling across types in the bank loan market. As a result, there is less cross-subsidization of impatient by patient households as we can see in Table 5. This decrease in cross-subsidization explains the increase in welfare for patient households but the decrease in welfare for impatient households in the quantity cap counterfactual.

So what explains the decrease in welfare for patient households when payday loans are fully banned? The answer is that there is a second factor at play apart from cross-subsidization: insurance. Constraining payday loan choices makes it harder for everyone in the economy, including patient households, to insure against idiosyncratic shocks. When payday loans are quantity capped but still available in the economy, the reduction in cross-subsidization outweighs this reduced insurance for patient households. But patient households do depend on payday loans to smooth shocks, for example in order to reduce the negative effect on type scores of a transitory earnings shock as discussed in

Section 6. In the full ban economy, this loss of insurance outweighs the gain from reduced cross-subsidization for patient households. This result implies that in our model fully banning payday loans makes both types of households worse off.

7.2 Bankruptcy Regulation

Another approach to regulation in the consumer finance market taken by policy makers is through bankruptcy laws. The most notable overhaul of bankruptcy regulation in recent years is the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) in 2005. Among other changes, this legislation increased the total out-of-pocket filing cost for Chapter 7 filings by around 35% (Albanesi and Nosal, 2020). To examine the effect of such an increase in monetary filing cost in our model, we simulate a counterfactual where the formal filing cost is increased by 35% ($1.35 \times \kappa_{FD}$). In addition, we also consider the policy counterfactual where the payday filing cost rises by the same magnitude ($1.35 \times \kappa_{PD}$) to assess the implication of stricter regulation on payday lending. The key results of these policy counterfactuals are summarized in Table 6. The column "$1.35 \times \kappa_{FD}$" denotes the counterfactual where the formal filing cost is increased by 35%. The column "$1.35 \times \kappa_{PD}$" describes the counterfactual where the payday filing cost is increased by 35%.

Focusing first on the case where the formal filing cost is increased, we can observe that this change leads to a significant decrease in the formal default rate. This is caused by substitution from formal default to payday default as the (unconditional) payday default rate rises. The drop in the formal default rate leads to a decrease in the average bank interest rate as banks require a lower default premium on their loans. This, in turn, makes borrowing using bank loans cheaper and increases bank loan borrowing both in terms of the extensive (fraction of loan borrowers) and intensive (debt-to-earnings) margins. Interestingly, the increase in bank loan borrowing is not accompanied by a decrease in payday loan borrowing. Rather payday loan usage also increases, leading to an overall higher level of debt in the economy. This is because the conditional effective default rate on payday loans actually drops from 34.68% in the benchmark to 33.59%, thus implying cheaper borrowing costs for payday loans.

Continuing to the case where the filing cost for payday default is increased, the payday default rate drops mechanically as it becomes more expensive to default on payday loans. This is associated with a lower average payday loan interest rate. We can also see that the

Variables (in %)	Benchmark	$1.35 \times \kappa_{FD}$	$1.35 \times \kappa_{PD}$
Default			
Formal default rate	0.99	0.84	0.99
Payday default rate	2.81	3.03	2.60
Eff. payday default rate (cond.)	34.68	33.59	33.78
Households in debt			
Fraction of bank loan borrowers	24.26	26.35	24.21
Fraction of payday loan borrowers	9.46	10.11	9.07
Bank debt-to-earnings (cond.)	6.48	7.56	6.48
Interest rate			
Avg. interest rate for bank loans	8.56	7.51	8.56
Avg. interest rate for payday loans	410.85	395.01	398.23
Welfare			
Welfare – aggregate	–	0.1236	-0.0032
Welfare – impatient households	–	0.1404	-0.0036
Welfare – patient households	–	0.0991	-0.0026

Table 6: Policy Counterfactual: Higher Filing Costs

Notes: The conditional effective payday default rate is defined as the fraction of households choosing to default on payday loans through either formal or payday default, conditional on having any payday loans. The bank debt-to-earnings ratio is conditional on having any bank loans. The payday debt-to-earnings ratio is conditional on having any payday loans. Welfare is measured in CEV units relative to the benchmark in percentage points.

fraction of payday loan borrowers drops even though payday interest rates have fallen. The reason is that in our economy households often default on payday loans. The utility of payday loan borrowers decreases as the increase in payday default costs outweighs the lower payday interest costs. All bank-related variables remain roughly unchanged.

The welfare implications of increasing the filing costs for either formal or payday default are the opposite: an increase in formal default costs leads to a welfare gain for both types of households, whereas an increase in payday default costs leads to a welfare loss. On the one hand, a stricter bankruptcy regime through higher default costs leads to lower interest rates, making borrowing cheaper. On the other hand, a stricter regime makes it more costly to default in response to bad shocks.[46] In our model, it is cheaper to borrow using bank loans compared to payday loans. At the same time, it is less costly to default on payday than bank loans as both the reputational and monetary filing costs are lower. Thus, households prefer to borrow using bank loans and to default on their payday loans

[46]This explanation refers to the insurance-efficiency trade-off of a bankruptcy regime between *smoothing over time* and *smoothing across states* (Zame, 1993).

first.[47] Increased formal default costs exactly allow households to take out bank loans at even lower interest rates, which explains the welfare gain in this counterfactual. In contrast, increased payday default costs make it harder for households to default on their payday loans, which explains the welfare loss in this case.

8 Conclusion

One puzzle in the consumer finance literature is the so-called ”Payday Loan Puzzle”: households use expensive payday loans even when they still have cheaper alternatives, such as credit cards. We propose a new rational explanation of this behavior: these households use payday loans to protect their credit scores since payday lenders do not report to credit bureaus. To investigate this hypothesis, we build a two-asset Huggett-type model with two types of consumer default as well as asymmetric information and hidden actions. Households can be of one of two types: patient with a high discount factor or impatient with a low discount factor. This household type is unobservable to lenders. In order to form an expectation of a household's type, lenders compute an individual-specific type score based on one's credit history. In addition, a household's payday loan choice is also not observable to banks. This information structure then endogenously creates an incentive for households to use payday loans instead of cheaper bank loans to protect their type scores.

Our model can successfully replicate the payday loan puzzle by matching both the fraction of households that show behavior consistent with the payday loan puzzle as well as the magnitude of the monetary costs. Furthermore, we illustrate how the reputation protection channel leads to the emergence of the payday loan puzzle in our framework. We then conduct a series of policy experiments. We show that restricting the size of payday loans benefits patient households at the expense of impatient ones, while a full ban on payday loans results in a welfare loss for both types of households. In addition, we also show that increasing the costs of defaulting on payday loans is welfare-reducing, whereas increasing the costs of formal default is beneficial in terms of welfare. These results imply that current regulatory efforts in the U.S. to curtail or even ban the payday loan sectors may potentially be harmful to households.

[47] This argument is also valid across types. As shown in Table 4, the average payday interest rates are far higher than the ones for bank loans for both types.

In the future, estimating the model using the simulated method of moments could make the policy conclusions more robust. However, such an estimation is often constrained by the availability of payday loan data at the individual level. In addition, we are planning to consider a case where banks can observe payday loan usage by requiring payday lenders to report. This alternative specification would allow us to more cleanly separate the effect on policy outcomes of pooling across types versus pooling across payday loan borrowers, thus guiding the regulation of the payday lending industry.

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A Assignment of Posterior Type Score

As the updated type score ψ may not lie on the original type score grid, it is randomly assigned to one of the two nearest grid points $s'_i(\beta')$ and $s'_j(\beta')$ for all β' with $s'_i(\beta') \leq \psi^{(\tilde{d},b')}_{\beta'_H} \leq s'_j(\beta')$, and assign probability $\chi(\beta'|\psi)$ to $s'_i(\beta')$ and $1-\chi(\beta'|\psi)$ to $s'_j(\beta')$, where

$$\chi(\beta'|\psi) = \frac{s'_j(\beta') - \psi^{(\tilde{d},b')}_{\beta'_H}(\omega_b)}{s'_j(\beta') - s'_i(\beta')}, \quad \forall \beta'. \tag{27}$$

For all s' such that $s'(\beta') \in \{s'_i(\beta'), s'_j(\beta')\}$ for all β', the probability of receiving score s' in the next period is thus equal to

$$Q^s(s'|\psi) = \prod_{s'(\beta')=s'_i(\beta')} \chi(\beta'|\psi) \cdot \prod_{s'(\beta')=s'_j(\beta')} (1-\chi(\beta'|\psi)). \tag{28}$$

For all other s', $Q^s(s'|\psi) = 0$.

B Computation

B.1 Grid Specifications

Variable	Symbol	# Points	Range
Persistent earnings	e	3	$\{0.57, 1.00, 1.74\}$
Transitory earnings	z	3	$\{0.78, 1.00, 1.29\}$
Bank assets	b	191	$[-0.40, 15.00]$
Payday loans	p	16	$[-0.15, 0.00]$
Type scores	s	8	$[0.013, 0.989]$

Table 7: Grids Used for Model Computation

We discretize the persistent and transitory earnings processes, each with three points, using Adda and Cooper (2003) and uniform distribution, respectively. We choose the lower bounds for bank and payday loans to ensure that the endogenous borrowing limits are included. Check Appendix D for the pricing schedules in equilibrium. We then consider an equally-spaced grid of 40 points for bank loans and an exponentially-spaced grid of 150 points for bank savings. More importantly, the grid for payday loans is designed with the same spacing as bank loans to properly compare the borrowing choices between bank and payday loans when identifying the payday loan puzzle.

B.2 One-Loop Algorithm

1. Set parameters and tolerances for convergence.

2. Create grids for (β, z, ω_b, p) with lengths $(n_\beta, n_z, n_\omega, n_p)$ where $n_\omega = n_e \times n_b \times n_s$.

3. Initialize algorithm with starting guesses:

   (a) $W(:,:,:,:,s,:) = W^{FI}$ for all s where W^{FI} denotes the unconditional value function under full information.

   (b) $\psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b) = s \cdot Q^\beta(\beta_H|\beta_H) + (1-s) \cdot Q^\beta(\beta_H|\beta_L)$ for all ω_b and $(\tilde{d}, b')$.

      i. $s'_i = \max\left\{s \in \mathcal{S} | s \leq \psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b)\right\}$ and $s'_j = \min\left\{s \in \mathcal{S} | s \geq \psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b)\right\}$.

      ii. $Q^s\left(s'_i(\beta_H)|\psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b)\right) = \frac{s'_j - \psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b)}{s'_j - s'_i}$ and $Q^s\left(s'_j(\beta_H)|\psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b)\right) = \frac{\psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b) - s'_i}{s'_j - s'_i}$.

   (c) $q_b^{(NFD,b')}(:,b,s) = q_b^{FI}$ for all b, s where q_b^{FI} denotes the bank loan price function under full information.

   (d) $q_p^{(R,b',p')}(:,b,s) = q_p^{FI}$ for all b, s where q_p^{FI} denotes the payday loan price function under full information.

   (e) $\mu(:,:,:,:,s,:) = \frac{1}{n_s} \times \mu^{FI}$ for all s where μ^{FI} denotes the cross-sectional distribution of households under full information.

4. Begin the one-loop algorithm:

   (a) Solve for new W_1 taking as given W_0.

      i. Find set of feasible actions (d, b', p') using (4).

      ii. For each (β, z, ω_b, p), compute the value $v^{(d,b',p')}(\beta, z, \omega_b, p)$ for each feasible action (d, b', p') according to (3).

      iii. Compute new W_1 using (7).

   (b) Compute $\sigma^{(d,b',p')}(\beta, z, \omega_b, p)$ according to (6).

   (c) Compute new equilibrium functions.

      i. On bank side:

A. Compute $\tilde{\sigma}_b^{(\tilde{d},b')}(\beta, z, \omega_b)$ using (10) and (11).

B. Then $\psi_{\beta'_H}^{(\tilde{d},b')}(\omega_b)$ using (13).

C. Then $\chi(\beta'|\psi)$ using (27) for all ψ from previous step.

D. Then $Q^s(s'|\psi)$ using (28) for all ψ from previously.

E. Then $\mathbb{P}_b^{(NFD,b')}(\omega_b)$ using (14).

F. Finally $q_b^{(NFD,b')}(\omega_b)$ using (9).

ii. On payday lender side:

A. Compute $\mathbb{P}_p^{(R,b',p')}(\omega_b)$ using (17).

B. Then $q_p^{(R,b',p')}(\omega_b)$ using (18).

(d) Compute stationary distribution μ_1 using (20).

(e) Assess convergence of W, ψ, q_b, q_p, and μ.

i. If achieved, continue to the next step.

ii. Otherwise, update the initialization of the targeted objects with relaxation and return to step (a).

5. Compute moments.

C Robustness Check: Same Default Costs

Given that payday default costs are lower than those for formal default, households might take out payday loans because of the better across-state insurance through defaulting on payday loans at lower costs. To argue that this filing channel is not the primary driver for our calibrated framework to generate the payday loan puzzle, we consider a counterfactual where we set the filing and stigma costs for formal default to those for payday default. That is, defaulting on bank loans is as cheap as on payday loans, either pecuniarily or mentally. Important moments and the rate of payday loan puzzle occurrence are reported in Table 8, along with the benchmark results.

We can see that, compared to the benchmark, households substitute formal default for payday default as it becomes cheaper to execute formal default. A higher formal default rate increases the interest costs for bank and payday loans since households can discharge both loans with formal default. Higher borrowing costs result in drops in the fractions

Moment (in %)	Benchmark	Same Default Costs
Default		
Formal default rate	0.99	5.21
Payday default rate (cond.)	29.7	22.0
Households in debt		
Fraction of bank loan borrowers	24.26	16.40
Fraction of payday loan borrowers	9.46	9.24
Fraction of both loan borrowers	8.42	8.42
Interest rate		
Ave. interest rate for bank loans	8.56	59.38
Ave. interest rate for payday loans	410.85	1435.12
Payday loan puzzle		
Rate of puzzle occurrence	26.44	51.38

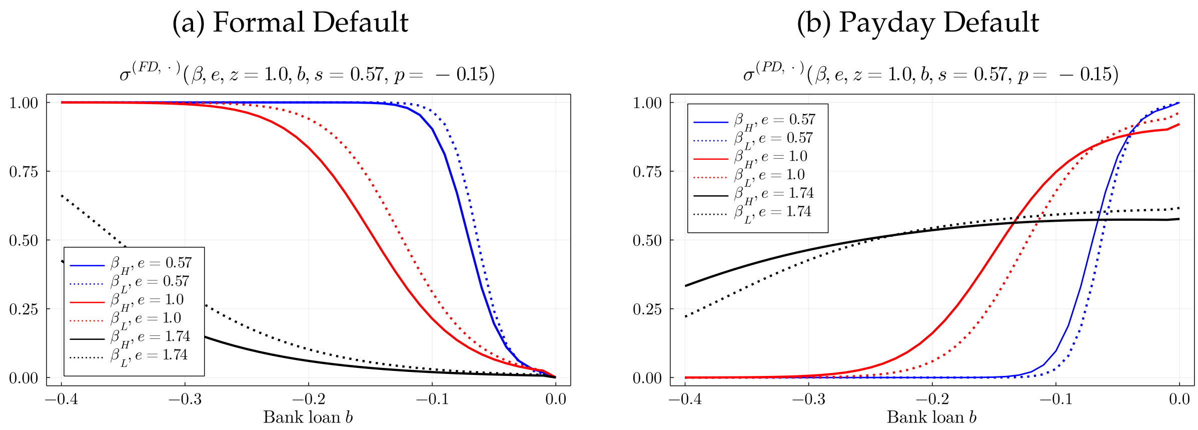
Table 8: Counterfactual: Same Default Costs

of either loan borrowers at the extensive margin. More importantly, the rate of puzzle occurrence is almost two times larger than the one in the benchmark. The increase can be explained by the fact that payday loans are very costly in the counterfactual. As a result, Condition (23) is much more likely to be satisfied, conditional on borrowing using both loans. This result suggests that cheaper costs for payday default than formal default are not the main driving force for our calibrated model to generate the payday loan puzzle.

D General Results

Figure 13 depicts how default probabilities vary across (persistent) earnings e and types β. The left-hand side shows how the probability of a household choosing formal default increases as its debt burden grows (b becomes more negative). Households with lower earnings start to formally default at lower debt burdens compared to households with higher earnings. Furthermore, more impatient households (β_L) also start to formally default at smaller debt levels. In contrast, as can be seen on the right-hand side the probability of payday default decreases as the debt burden grows. This is due to the switching from payday to formal default: As bank loans increase households switch from payday defaulting on their payday loans only to formally defaulting on all debt in order to discharge their larger bank loans. We can see in Figure 13b that this switching starts earlier

Figure 13: Default Probabilities

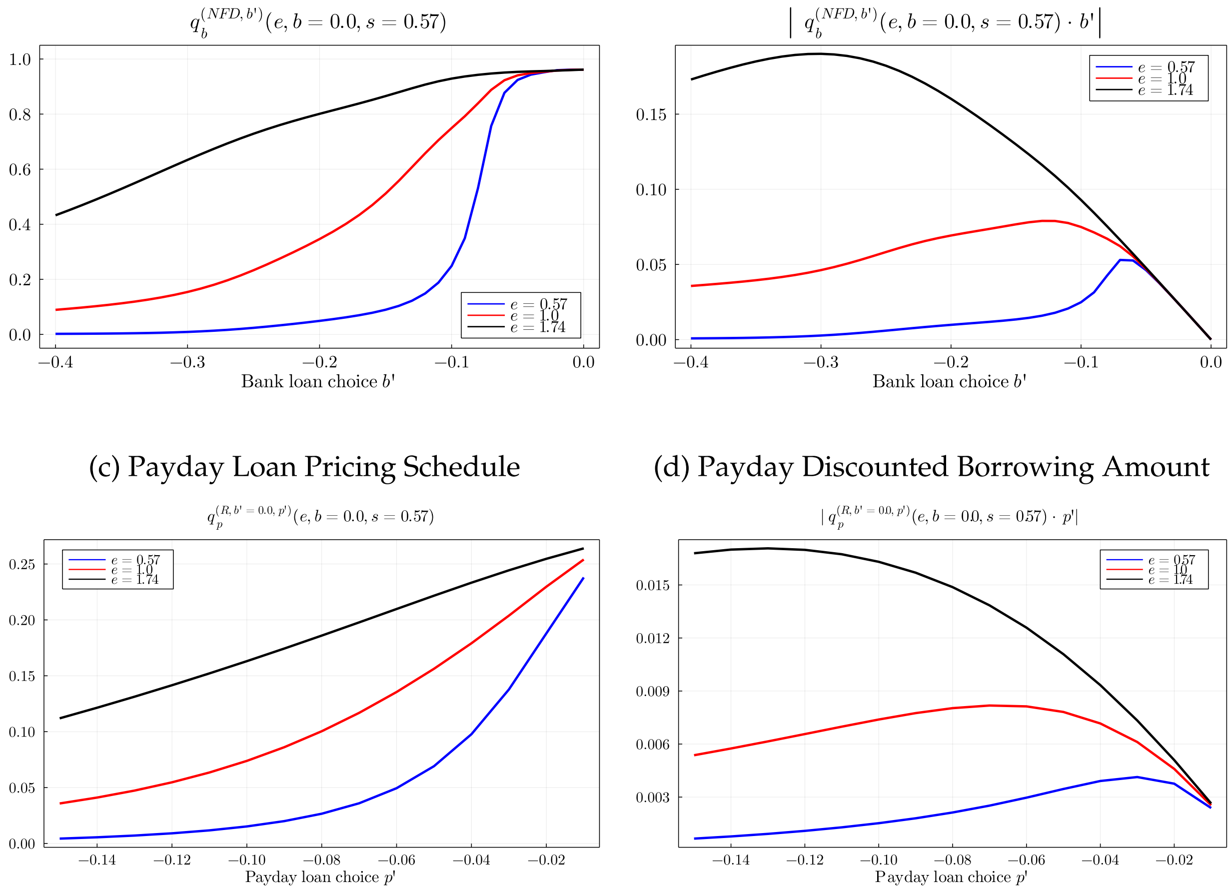


at lower debt levels for households with less income (black line starts dropping at lower b) and for households that are more impatient (dashed lines drop more quickly than solid lines). This happens because low types are less concerned about the long-term reputational damage from formal default.

The pricing schedules and the risky borrowing limits of bank and payday loans across earnings in the model are depicted in Figure 14. These results are quite standard in consumer default models. The intuition is clear: On the one hand, borrowing more this period will lead to a higher default probability next period c.p. as the gain from defaulting is larger. As a result we can see in Figure 14a that borrowing more (more negative b') leads to lower prices/higher interest rates. Furthermore, an individual with lower persistent earnings e will face lower prices compared to one with higher e c.p. due to the difference in default probability in the following period. Similarly, the payday loan pricing schedules and the risky borrowing limits across earnings in the model are in the bottom panel. These results are similar to those of bank loans. The significant disparity in levels across bank and payday loans results from the fact that payday lenders have higher operating costs than banks (i,e, higher lending costs).

Figure 15 illustrates what kind of household in our economy saves or borrows. On the left, Figure 15a shows the distribution of savers and borrowers across *persistent* income. Unsurprisingly, savers in our economy tend to have higher (persistent) income compared to borrowers. We can also see that households who use bank loans (either only bank loans or together with payday loans) are overwhelmingly poor (the red bars). Perhaps more interestingly, payday loan borrowers, while still being poor compared to savers, tend to have higher persistent income than bank loan borrowers. On the right, Figure

Figure 14: Pricing Schedule and Discounted Borrowing Amount



(a) Bank Loan Pricing Schedule

(b) Bank Discounted Borrowing Amount

(c) Payday Loan Pricing Schedule

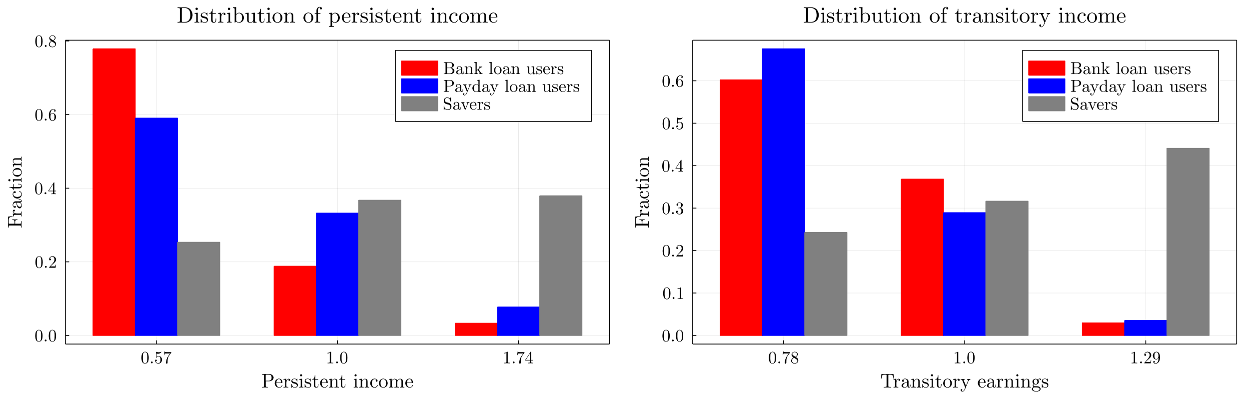
(d) Payday Discounted Borrowing Amount

15b shows the distribution of households across *transitory* income. Compared to Figure 15a it can be seen that payday loan borrowers tend to have lower transitory income than bank loan borrowers. These two figures suggest that the two types of loans are used to smooth different types of income shocks in our model: households use bank loans to smooth persistent income shocks whereas payday loans are used to smooth transitory shocks. This makes sense: Payday loans are more expensive than bank loans and are much more costly to smooth a persistent negative income shock. On the other hand, using payday loans does not (directly) affect your type score. As a result, it can make sense to smooth transitory income shocks using payday loans in order avoid long-term reputational damage to a household.

Figure 16a plots the type score distributions among borrowers and savers. We can see that savers in our economy tend to have higher type scores compared to either bank or payday loan borrowers. Interestingly, payday loan borrowers have slightly lower type

Figure 15: Earnings Distribution among Borrowers and Savers

(a) Persistent Earnings (b) Transitory Earnings



scores compared to bank loan borrowers. Figure 16b instead depicts the type score distribution among puzzle and non-puzzle users. We can see that the prior type score distributions of both users are skewed to the right. More importantly, puzzle borrowers, those who take out payday loans before exhausting cheaper bank credit, tend to have lower prior type scores in contrast to non-puzzle borrowers, those who take out payday loans without cheaper bank credit available. This is because the reputation gain (the interest costs) are higher (lower) for households with lower type scores (see Figure 9).

Figure 16: Type Score Distribution

(a) Persistent Earnings (b) Transitory Earnings

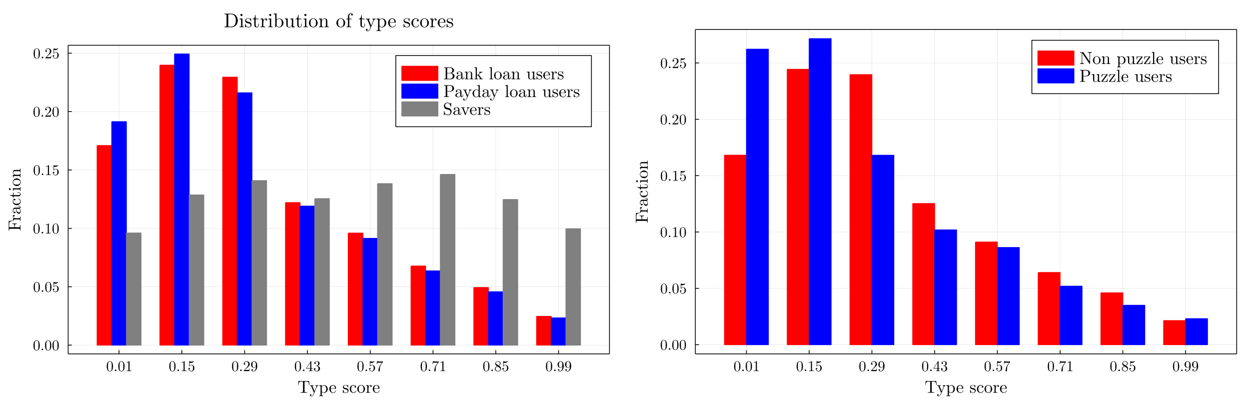
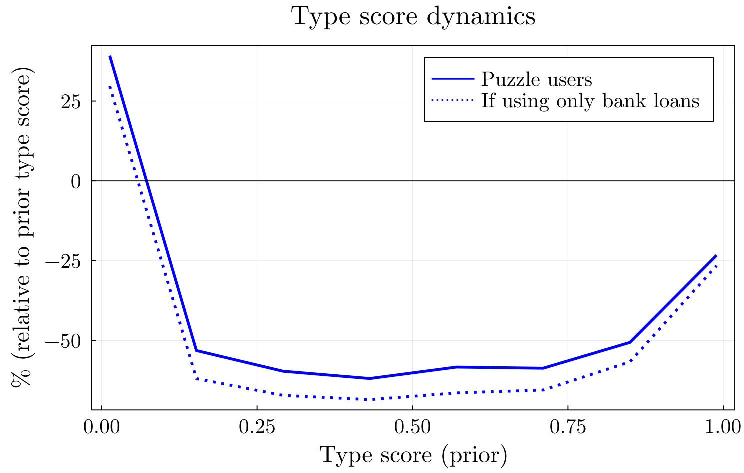


Figure 17 plots the variation in updated type scores relative to priors among puzzle users in percentage (solid line) compared to the counterfactual when they were to borrow the same amount using only bank loans (dotted line). Borrowing only banks loans results in overall lower posterior type scores across all priors, compared to borrowing a mixture of bank and payday loans. This is intuitive as banks can observe only bank loans.

Figure 17: Posterior Type Score Dynamic

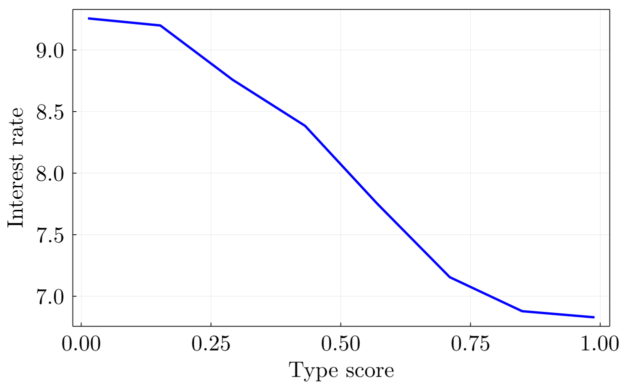


Borrowing more bank loans thus indicates more impatience.

Figure 18 plots the average interest rates for bank loans (Figure 18a) and payday loans (Figure 18b) across type scores. We can see that higher type scores lead to lower interest rates in both bank and payday lending markets. In particular, the difference in bank loan interest rates between households with the lowest and highest type scores is over 2%. On the other hand, the interest rate difference in the payday lending market can be up to 90%.

Figure 18: Average Interest Rates for Bank and Payday Loans across Type Scores

(a) Bank Loans



(b) Payday Loans