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Partnership Dissolution in a Search Market With On-The-Match Learning

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Abstract

We construct a frictional search-and-matching model with on-the-match learning and rematching. Agents are ex-ante homogeneous, have idiosyncratic preferences, and receive news about the profitability of their current match following a Poisson process. We provide an infinite number of pointwise balance conditions and a finite number of aggregate balance conditions and prove their equivalence. We show that agents follow cutoff strategies in the unique steady-state equilibrium. If the profitability types inside a match have a strong positive (negative) correlation, then a faster learning rate is ex-ante welfare-increasing (decreasing) for the agents.

JEL Codes: C78, D83, J64

Keywords: Search frictions, matching, on-the-job search, learning

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1 Introduction

This paper studies a one-sided matching market with search frictions, on-the-match learning, and on-the match rematching. A continuum of ex-ante homogeneous agents meet each other following a Poisson process and have to decide whether to form a match or not upon meeting. Inside a match, unknown and potentially correlated types are drawn for each agent that specify whether or not that match is beneficial for the corresponding agent. An agent receives an unobserved and constant positive flow payoff if her current match is beneficial for her and a flow payoff of zero otherwise. In particular, a match can be beneficial for one of the two agents and not beneficial for the other one. Agents whose match is not beneficial receive public bad news about their current match according to a Poisson process.

As in the previous paper, a central assumption is that both agents in a match can search for a new partner. This fundamentally shapes the set of possible equilibria. Not only the payoff inside the match but also the endogenous risk of being left by the partner are important factors for the agents' decision-making. The rematching behavior of the partner affects the continuation value of the current match and, as a consequence, affects the own behavior as well. This establishes an endogenous interest of the agents in the match value for the partner.

Possible applications for this analysis are professional relationships between business partners, athletes who search for a duo partner, or scientists searching for a co-author. In these applications it is plausible that a partner does not find her current match valuable anymore and that she tries to find a more fitting partner. One can abstract from the one-sidedness of the search market and obtain the same results for a corresponding two-sided model. In particular, the trade-offs in this paper also apply to applications with two market sides like job markets or marriage markets.

We analyze the agents' rematching behavior, the market structure, and the welfare effect of the speed of learning. First, we provide the existence and uniqueness of a steady-state equilibrium. The equilibrium behavior is as follows: For single agents and agents who received bad news about the profitability for themselves it is a dominant strategy to search for a partner. In matches where both agents have not received bad news, the agents do not search for a new partner as not receiving bad news for a period of time makes their belief about their current match type more optimistic than the belief about a potential new match. There are three counteracting effects that determine the equilibrium behavior of an agent whose partner received bad news: The first effect is that the longer the match persists without receiving bad news herself, the more optimistic is the agent about her own type. The second effect is that the agent takes into account that her partner tries to replace her after receiving bad news. The third effect is that due to the partner's bad news the agent updates her belief about her own type. This third effect can change the belief for better or for worse: Depending on whether there is a positive or negative correlation between the unknown types in a pair, bad news for the partner is bad or good news for oneself. Together, these three effects imply that even if an agent herself has not received bad news, the bad news for the partner can cause the agent to try to replace the partner to avoid the risk of becoming single. As a result, there are three cases of how agents whose partners have received bad news behave in equilibrium. First, an agent searches for a new partner if her current partner received bad news. Second, an agent stays in the current match even if her current partner received bad news. Third, an agent whose partner received bad news follows a cutoff strategy, i.e., she searches for a new partner until the current match persists for a certain time and stops searching afterward.

We use comparative statics to show that a faster learning rate is ex-ante beneficial for the agents entering the market if the agents' goals are aligned, i.e., if there is a positive and sufficiently strong correlation of the unknown types in a pair. If both agents in a match are likely to have the same type, then learning this type benefits both of them. Conversely, if there is a sufficiently strong negative correlation, then a faster learning rate ex-ante hurts the agents. If it is likely that there is exactly one agent who profits from the match, then this agent is worse off by a faster learning rate of the types and this utility loss dominates the utility gain of the partner. In particular, with sufficiently strong negative correlation, agents in a pair would strictly prefer that both of them would commit to never rematch if they could do so, which would correspond to a learning rate of zero, i.e., no learning at all.

This section is concluded with an overview of the related literature. The rest of this paper is organized as follows. Section 2 presents the model. In Section 3, we define the equilibrium concept of a steady-state equilibrium. In Section 4, existence and uniqueness is shown for the subclass of monotone steady-state equilibria. We show in Section 5 that the previous restriction to monotone equilibria is without loss. Section 6 uses comparative statics to analyze the effect of a faster learning rate on the agents' welfare. Section 7 concludes. The proofs can be found in the appendix.

1.1 Related Literature

Our model builds on the search framework developed by Burdett and Coles (1997), Shimer and Smith (2000), Smith (2006), and Kreutzkamp et al. (2021). In these models, having a high type results in a higher flow utility for all potential partners. In this paper, agents are ex-ante homogeneous and draw a new type each time they form a new match. Therefore, here, a high long-term potential does not persist outside of the current match.

The assumption of ex-ante homogeneous agents that has led to this paper was inspired by Smith (1995) who presents a search-and-exchange market for ex-ante homogeneous goods where the valuations for the goods are drawn independently at each meeting. In Smith (1995), agents who meet can exchange goods and separate afterward while in our model matched agents form a pair, and their future utilities also crucially depend on their partners' rematching decisions.

On-the-job search in labor markets has been widely analyzed before. Pissarides (1994) introduces search equilibria with on-the-job search. An important assumption is that only workers can search for new jobs while being employed. For a survey on search models of the labor market, see Rogerson et al. (2005). In contrast to on-the-job search, in our model, both agents in a match can continue searching which is central to our results. Both agents in a pair can rematch and the resulting equilibrium strategies have to be optimal given the partner's future rematching decisions.

Kreutzkamp et al. (2021) features an on-the-match search model similar to this paper. In both papers, the expected continuation payoff inside a match changes over time. In Kreutzkamp et al. (2021), the present value of a match increases as capital is accumulated inside a match. Here, the the absence of bad news over a period of time mathematically has similar effects on the present value of a match as capital accumulation has. The major differences in the models are that Kreutzkamp et al. (2021) assumes a common ranking over heterogeneous agents with constant productivity types and capital accumulation inside the matches, while this paper considers ex-ante homogeneous agents whose productivity types are idiosyncratic, initially unknown to the agents, and have to be learned over time.

A related strand of literature has studied partnership dissolution where two agents jointly own an asset. Cramton et al. (1987) show that an ex-post efficient dissolution is possible if the shares of the asset are sufficiently even. Fieseler et al. (2003) study interdependent valuations and analyze when efficient trade can occur. In recent work, Loertscher and Wasser (2019) study partnership dissolution with interdependent values and derive optimal ownership structures. Van Essen and Wooders (2016) introduce a dynamic auction format to dissolve partnerships. While this strand of literature analyzes how to dissolve a partnership efficiently, we endogenize the question of when to dissolve a partnership by modeling a search market and embedding the partnerships into the market. Also, in our model, there is no jointly owned asset to be divided for the dissolution of a partnership. Fershtman and Szabadi (2020) study a related question and also consider an endogenous partnership dissolution. In contrast to our model, they analyze a single pair of agents with private information about the joint desirability of the partnership who are not ex-ante sure whether or not to dissolve their partnership.

2 The Model

We construct a one-sided search model with continuous time and non-transferable utility where agents learn and search on-the-match. There is a continuum of ex-ante homogeneous agents in the market. Every agent is either single or in a match with another agent. New agents enter the market as singles at a constant rate $\eta > 0$ and agents in the market meet each other following a quadratic meeting technology with parameter $\lambda > 0$, that is, each agent meets an agent from a mass m in the market uniformly at random with Poisson rate λm. When two agents meet, both of them have to simultaneously decide whether to accept or decline forming a new match. If both agents agree, they form a new pair and leave their respective partners (if they are matched) who become singles.

After two agents form a pair, a hidden binary type (h or l) is drawn for each of them that indicates the desirability of the current match. The probabilities for the types are $p_{hh} > 0$ for (h, h), $p_{hl} > 0$ for (l, h) and (h, l), respectively, and $p_{ll} > 0$ for (l, l). This allows the types to be correlated. We call pairs where the types are (l, h) or (h, l) mixed pairs.

An h-agent gains an unobserved constant flow utility of $w > 0$ while being matched with her partner. A single agent or a matched l-agent gains a flow utility of 0. All agents discount future payoffs at rate r. For example, an h-agent whose match is dissolved after time t_0 receives an (unobserved) aggregated payoff of

$$\int_0^{t_0} w \cdot e^{-rt} dt = \frac{w}{r}\left(1 - e^{-rt_0}\right)$$

in that match.

If the hidden type of an agent is l, then the agent will receive bad news about the current match due to a Poisson process at rate γ. The occurrence of bad news is publicly observable by both agents in that match. If the hidden type of an agent is h, then the agent will never receive bad news. Therefore, bad news fully reveal that the type of the corresponding agent is l. For a matched agent, we denote the (public) information about whether or not bad news occured in the current match by $(S, S') \in \{B, U\} \times \{B, U\}$, where $S = B$ if and only if the agent has received bad news herself and $S' = B$ if and only if the partner has received bad news (B standing for "bad news" and U standing for "unknown"). For the remainder of this paper, we use lower-case letters like $i, j \in \{l, h\}$ for hidden types and upper-case letters like $S, S' \in \{U, B\}$ for the public information.

Beliefs If an agent is in a match without receiving bad news for a period of time, she adjusts her belief accordingly. Let $P(ij|SS', t)$ denote the belief that the hidden types are ij given that the information is (S, S') and given that the pair is together for time t. For the information (B, B) we know that the type is ll for sure. The other conditional beliefs of the agents about the hidden types can be calculated by the Bayesian rule. For the information (U, U) we get the beliefs

$$P(hh|UU,t) = \frac{p_{hh}}{p_{hh} + 2p_{hl}e^{-\gamma t} + p_{ll}e^{-2\gamma t}},$$

$$P(hl|UU,t) = P(lh|UU,t) = \frac{p_{hl}e^{-\gamma t}}{p_{hh} + 2p_{hl}e^{-\gamma t} + p_{ll}e^{-2\gamma t}},$$

$$P(ll|UU,t) = \frac{p_{ll}e^{-2\gamma t}}{p_{hh} + 2p_{hl}e^{-\gamma t} + p_{ll}e^{-2\gamma t}},$$

and for the information (U, B) we get the beliefs

$$P(hl|UB,t) = \frac{p_{hl}}{p_{hl} + p_{ll}e^{-\gamma t}},$$

$$P(ll|UB,t) = \frac{p_{ll}e^{-\gamma t}}{p_{hl} + p_{ll}e^{-\gamma t}}.$$

Figure 1 illustrates the change of beliefs over time.

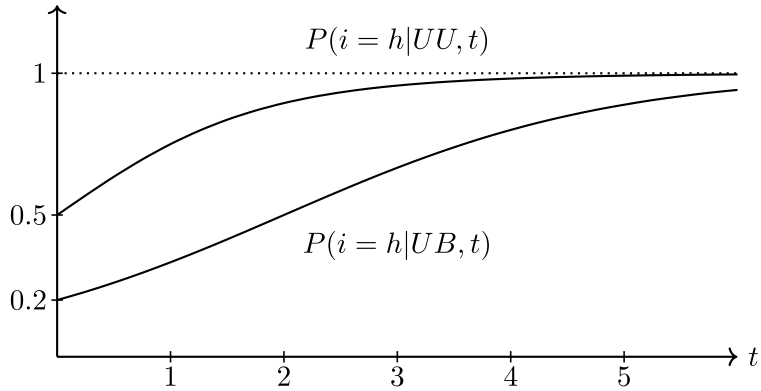

Figure 1: Conditional beliefs in pairs that have not received bad news for time t are strictly increasing. The parameters used for the graphs are $\gamma = \ln(2)$, $p_{hh} = 0.4$, $p_{hl} = 0.1$, and $p_{ll} = 0.4$.

The beliefs for the information (B, U) are analogous to the ones for (U, B). Note that receiving no bad news over a period of time is generally good news for the agents, as the belief of having an h-type is increasing over time. In particular,

$$P(i = h|UU,t) = P(hh|UU,t) + P(hl|UU,t) \quad \text{and}$$
$$P(i = h|UB,t) = P(hl|UB,t)$$

are both strictly increasing in t. Analogously, the belief that the partner has an h-type is also increasing over time. For the remainder of this paper, when we write that the beliefs are increasing over time, we refer to the beliefs $P(i = h|UU,t)$, $P(i = h|UB,t)$, and $P(j = h|UU,t)$ being strictly increasing in t.

Strategies and Masses Note that the belief at time t is independent of the time at which bad news occurred, since bad news fully reveal the state of the corresponding agent and without bad news the conditional probability of having an h-type only depends on the total amount of time without bad news. Therefore, we restrict our attention to symmetric Markov strategies that only condition on the current information type. A Markov strategy for an agent is a measurable function

$$\varphi : \{\emptyset, UU, UB, BU, BB\} \times [0, \infty) \to [0, 1]$$

where $\varphi(SS', t)$ is the probability that an agent who is in a match for exactly time t and for whom the information in the current match is (S, S') accepts to rematch upon meeting another agent. Similarly, $\varphi(\emptyset, t)$ is the probability that an agent who is single for exactly time t agrees to match.

Let $m_{ij,SS'}(t)$ denote mass of agents in pairs which are together for exactly time $t \in [0, \infty)$, who have type i, whose partner has type j, and whose information is SS'. Let Θ denote

$$\{\emptyset, (hh, UU), (hl, UU), (hl, UB), (lh, UU), (lh, BU), (ll, UU), (ll, UB), (ll, BU), (ll, BB)\},$$

i.e., the set of all indices of matches that can occur, where $\emptyset$ denotes singles. Let

$$\mathcal{M} := \big(m_\theta(t)\big)_{t \geq 0, \theta \in \Theta}$$

denote the vector of all masses. For $\theta \in \Theta$ let

$$m_\theta := \int_0^\infty m_\theta(t)dt \in \overline{\mathbb{R}}_+$$

denote the aggregated mass of such agents in the market.[1] The aggregated mass of agents who accept forming a new match is given by

$$m_0 := \int_0^\infty m_\emptyset(t)\varphi(\emptyset, t)\ dt\ + \sum_{(ij,SS') \in \Theta \setminus \{\emptyset\}} \int_0^\infty m_{(ij,SS')}(t)\varphi(SS', t)\ dt.$$

The term is derived by integrating over all masses of agents times their respective probability of accepting. In particular, λm_0 is the rate of the Poisson process with which an individual agent meets accepting agents.

Survival Probabilities For fixed masses and for $\theta \in \Theta$ let $q_\theta(t_0, t_1)$ denote the survival probability from t_0 to t_1. More precisely, $q_{(ij,SS')}(t_0, t_1)$ is the probability that an ij-pair with information (S, S') that is together for exactly time t_0 is still together after time t_1 without changing its information. Similarly, $q_\emptyset(t_0, t_1)$ is the respective probability that a single who is single for time t_0 is single for time t_1.

[1] Note that, in general, the aggregated mass could be infinite.

Formally, the probabilities are

$$q_\emptyset(t_0,t_1) = \exp\Big(-\int_{t_0}^{t_1} \lambda m_0 \varphi(\emptyset,t)\ dt\Big),$$
$$q_{(ij,SS')}(t_0,t_1) = \exp\Big(-\int_{t_0}^{t_1} \lambda m_0\varphi(SS',t) + \lambda m_0\varphi(S'S,t) + \big(\mathbb{1}_{i=l}\mathbb{1}_{S=U} + \mathbb{1}_{j=l}\mathbb{1}_{S'=U}\big)\gamma\ dt\Big),$$

since agents form new matches following an inhomogeneous Poisson process with the corresponding rate $\lambda m_0\varphi(\cdot,t)$. The term $\mathbb{1}_{i=l}\mathbb{1}_{S=U} + \mathbb{1}_{j=l}\mathbb{1}_{S'=U} \in \{0,1,2\}$ denotes the number of agents in the match who can still receive bad news.

Without knowing the hidden types, the expected survival probability of a match with information (S,S') is

$$q_{SS'}(t_0,t_1) = \sum_{i,j\in\{h,l\}} P(ij|SS',t_0)\ q_{(ij,SS')}(t_0,t_1),$$

where $P(ij|SS',t_0)$ denotes the belief of the pair having types ij.

Continuation Payoffs Fix a vector of masses $\mathcal{M}$ and a strategy φ with $m_0 < \infty$. Assume for now that the masses do not change over time. Then, the expected continuation payoffs are well-defined. Let $V(\emptyset,t_0)$ be the expected continuation payoff of an agent who are single for exactly time t_0 and let $V(SS',t_0)$ be the expected continuation payoffs of an agent that is in a match for exactly time t_0 and whose information is (S,S'). In particular, $V(UU,0)$ is equal to the expected utility of forming a new match.

The expected continuation payoff $V(\cdot,t_0)$ can be constructed from the following components: All future flow payoffs in the current match are discounted by

$$q_{SS'}(t_0,t)e^{-r(t-t_0)},$$

i.e., by the survival rate multiplied by the discount factor for time t. The expected flow payoff in the current match is equal to w times the belief of having an h-type:

$$w\big(P(hh|SS',t) + P(hl|SS',t)\big).$$

The rate of accepting a new match multiplied by the corresponding continuation payoff is

$$\lambda m_0\varphi(SS',t)V(UU,0)$$

and the rate of becoming single multiplied by the continuation payoff of being single is

$$\lambda m_0\varphi(S'S,t)V(\emptyset,0).$$

The term

$$\mathbb{1}_{S=U}\big(P(lh|SS',t)+P(ll|SS',t)\big)\gamma V(BS,t)$$

describes the rate at which the agent oneself receives bad news times the continuation payoff after that event. Similarly,

$$\mathbb{1}_{S'=U}\big(P(hl|SS',t)+P(ll|SS',t)\big)\gamma V(SB,t)$$

is the rate at which the partner receives bad news times the continuation payoff.

Now, the expected continuation payoff $V(SS',t_0)$ can be calculated recursively by integrating over the expected future payoffs as follows: For all $t_0 \geq 0$,

$$\begin{aligned}
V(SS',t_0) = \int_{t_0}^{\infty} & q_{SS'}(t_0,t)e^{-r(t-t_0)}\Big(w\big(P(hh|SS',t)+P(hl|SS',t)\big) \\
&+ \lambda m_0 \varphi(SS',t)V(UU,0) + \lambda m_0 \varphi(S'S,t)V(\emptyset,0) \\
&+ \mathbb{1}_{S=U}\big(P(lh|SS',t)+P(ll|SS',t)\big)\gamma V(BS,t) \\
&+ \mathbb{1}_{S'=U}\big(P(hl|SS',t)+P(ll|SS',t)\big)\gamma V(SB,t)\Big)\ dt, \\
V(\emptyset,t_0) = \int_{t_0}^{\infty} & q_{\emptyset}(t_0,t)e^{-r(t-t_0)}\lambda m_0 \varphi(\emptyset,t)V(UU,0)\ dt
\end{aligned}$$

holds.

3 Steady-State Equilibria

We split our equilibrium concept into two parts. The first part is the mutual optimality of the strategies. The second part requires the masses to satisfy certain balance conditions.

We now begin with the first part, the optimality.

Definition 1. The pair $(\mathcal{M},\varphi)$ constitutes a *partial equilibrium* if m_0 is finite and φ is mutually optimal taking the masses as given, i.e., if for all public information $SS' \in \{\emptyset, UU, UB, BU, BB\}$ and $t \geq 0$

$$\begin{aligned}
V(UU,0) < V(SS',t) &\ \Rightarrow\ \varphi(SS',t)=0 \\
V(UU,0) > V(SS',t) &\ \Rightarrow\ \varphi(SS',t)=1
\end{aligned}$$

holds.

Taking the masses and the strategies of the other agents as given and constant over time, as well as the own strategy in the future[2], fixes the continuation payoffs

[2] By the One-Shot Deviation Principle, it is sufficient the require pointwise optimality of the strategies.

$V(UU,0)$ for accepting to form a new match and $V(SS',t)$ of not accepting. If the expected value of a new match is strictly larger than the continuation payoff of the current state, then an agent accepts. Conversely, if the expected value of a new match is strictly smaller, then an agent rejects.

Note that this equilibrium concept includes sequential rationality. In particular, agents are not allowed to play non-optimal even on a measure null set or if the partner would accept them with probability 0. When an agent is indifferent, i.e., at a time t with $V(UU,0) = U(SS',t)$, then she can accept with any probability $q \in [0,1]$.

In any partial equilibrium, singles and agents who received bad news always accept to form a new match. As their current state yields a flow payoff of 0, their only possible payoff comes from forming a new match. Consequently, as the equilibrium strategy of an agent with bad news is constant, the corresponding partner has to follow a monotone equilibrium strategy.

Lemma 1. *In all partial equilibria $\varphi(\emptyset,t) = 1$, $\varphi(BU,t) = 1$, and $\varphi(BB,t) = 1$ hold for all t. Furthermore, agents who have not received bad news, but whose partners have received bad news follow a cutoff strategy, that is, they accept to rematch until some cutoff $t^* \in [0,\infty]$ and they do not accept afterwards.*

For the second part of our equilibrium concept, the masses need to satisfy balance conditions for every state. In short, for each type $\theta \in \Theta$, the masses have to be equal to the inflow times the survival probability, i.e.,

$$m_\theta(t) = \text{Inflow}(\theta) \ \cdot q_\theta(0,t)$$

has to hold with $\text{Inflow}(\theta)$ being the inflow of new agents into state θ due the matching process or new market entries of singles. More precisely:

Definition 2. A partial equilibrium $(\mathcal{M},\varphi)$ is a *steady-state equilibrium* if for all

$t \in [0, \infty)$ the pointwise balance conditions

$$\begin{aligned}
m_\emptyset(t) &= \big(\eta + \lambda m_0(m_0 - m_\emptyset)\big) q_\emptyset(0,t) \\
m_{hh,UU}(t) &= p_{hh}\lambda m_0^2 \; q_{hh,UU}(0,t) \\
m_{hl,UU}(t) &= 2p_{hl}\lambda m_0^2 \; q_{hl,UU}(0,t) \\
m_{ll,UU}(t) &= p_{ll}\lambda m_0^2 \; q_{ll,UU}(0,t) \\
m_{hl,UB}(t) &= \int_0^t \gamma m_{hl,UU}(t') \; q_{hl,UB}(t',t) \; dt' \\
m_{ll,UB}(t) &= \int_0^t 2\gamma m_{ll,UU}(t') \; q_{ll,UB}(t',t) \; dt' \\
m_{ll,BB}(t) &= \int_0^t \gamma m_{ll,UB}(t') \; q_{ll,BB}(t',t) \; dt'
\end{aligned}$$

hold.

The balance conditions state that the masses $\mathcal{M}$ together with the strategy φ and the quadratic meeting technology imply the same masses $\mathcal{M}$ again. This ensures that the masses remain stationary in equilibrium.

4 Characterization of Monotone Equilibria

The belief of having a high type as well as the belief of the partner having a high type both increase over the time in a match. Therefore, agents become more optimistic the longer a match persists without bad news. In the following, we analyze monotone equilibria where agents willingness to accept to rematch decreases as their beliefs increase. Later we will show that there exist in fact no non-monotone steady-state equilibria. Therefore, it is without loss to restrict attention to monotone equilibria.

Definition 3. A partial equilibrium/steady-state equilibrium $(\mathcal{M}, \varphi)$ is called *monotone* if the acceptance probability $\varphi(SS', t)$ is weakly decreasing in t for every information (S, S').

Since single agents and agents who have received bad news always accept to rematch, the only equilibrium behaviors to be specified are the ones for agents with information UU and UB. The next lemma says that in a monotone equilibrium, agents with information UU never accept to match with a new partner.

Lemma 2. *In all monotone partial equilibria $\varphi(UU, t) = 0$ holds for all $t > 0$.*

The reason is that the continuation payoff in a match without bad news is strictly increasing over time as the beliefs get more optimistic and the probability of being left by the partner is non-increasing.

Knowing the agents' equilibrium behavior simplifies the balance conditions. More precisely, for a partial equilibrium to satisfy the infinite set of pointwise balance conditions it is necessary and sufficient to satisfy a finite number of aggregate balance conditions. This reduction of the balance conditions to a finite set of equations is a substantial simplification. In particular, for any given cutoff t^* one can obtain the masses numerically.

Lemma 3. *A monotone steady-state equilibrium satisfies $(\mathcal{M}, \varphi)$ the aggregated balance conditions. For generic parameters*[3] *the aggregated balance conditions are:*

$$\begin{aligned}
m_\emptyset &= \frac{\eta + \lambda m_0^2}{2\lambda m_0} \\
m_{hh,UU} &= \infty \\
m_{hl,UU} &= \frac{2p_{hl}\lambda m_0^2}{\gamma} \\
m_{ll,UU} &= \frac{p_{ll}\lambda m_0^2}{2\gamma} \\
m_{hl,UB,\leq t^*} &= p_{hl}m_0\Big(1 + \frac{2\lambda m_0}{\gamma - 2\lambda m_0}e^{-\gamma t^*} - \frac{\gamma}{\gamma - 2\lambda m_0}e^{-2\lambda m_0 t^*}\Big) \\
m_{hl,UB,>t^*} &= p_{hl}m_0\Big(\frac{2\gamma}{\gamma - 2\lambda m_0}e^{-2\lambda m_0 t^*} - \frac{4\lambda m_0}{\gamma - 2\lambda m_0}e^{-\gamma t^*}\Big) \\
m_{ll,UB,\leq t^*} &= p_{ll}m_0\Big(\frac{2\lambda m_0}{\gamma + 2\lambda m_0} + 2\lambda m_0 e^{-2\gamma t^*} + \frac{4\gamma\lambda m_0}{\gamma + 2\lambda m_0}e^{-(2\lambda m_0 + \gamma)t^*}\Big) \\
m_{ll,UB,>t^*} &= p_{ll}m_0\Big(\frac{4\gamma\lambda m_0}{(\gamma + \lambda m_0)(\gamma - 2\lambda m_0)}e^{-(2\lambda m_0+\gamma)t^*} - \frac{2\lambda m_0(\gamma + 2\lambda m_0)}{(\gamma + \lambda m_0)(\gamma - 2\lambda m_0)}e^{-2\gamma t^*}\Big) \\
m_{ll,BB} &= p_{ll}m_0\Big(\frac{\gamma - 2\lambda m_0}{2(\gamma + 2\lambda m_0)} - \frac{\lambda m_0(\gamma + 2\lambda m_0 - 2\gamma^2 + 2\gamma\lambda m_0 + 4\lambda^2 m_0^2)}{(\gamma + \lambda m_0)(\gamma - 2\lambda m_0)}e^{-2\gamma t^*} \\
&\qquad - \frac{2\gamma\lambda m_0(\gamma + 2\lambda m_0 + 2\gamma^2 - 2\gamma\lambda m_0 - 4\lambda^2 m_0^2)}{(\gamma + 2\lambda m_0)(\gamma + \lambda m_0)(\gamma - 2\lambda m_0)}e^{-(2\lambda m_0+\gamma)t^*}\Big) \\
m_0 &= m_\emptyset + m_{hl,UB,\leq t^*} + \frac{1}{2}m_{hl,UB,>t^*} + m_{hl,UB,\leq t^*} + \frac{1}{2}m_{ll,UB,>t^*} + m_{ll,BB}
\end{aligned}$$

Conversely, if the aggregated masses of a monotone partial equilibrium $(\mathcal{M}, \varphi)$ satisfy the aggregated balance conditions, then there exists a unique monotone steady-state equilibrium $(\mathcal{M}', \varphi)$ that has the same aggregated masses as $(\mathcal{M}, \varphi)$.

These aggregate balance conditions are obtained by integrating the pointwise balance conditions. As a direct consequence, the aggregate balance conditions are necessary for the pointwise balance conditions. The last equation gives the mass of all agents who are willing to accept a new match. The factor $\frac{1}{2}$ of the masses $m_{hl,UB,>t^*}$ and $m_{ll,UB,>t^*}$ accounts for the fact that only half of the agents in such pairs are willing to rematch. The crucial part of Lemma 3 is that the aggregate

[3]With generic, we here mean that $\gamma \neq \lambda m_0$ and $\gamma \neq 2\lambda m_0$ hold. This is without loss as the statement of Lemma 3 also holds for non-generic parameters but with different terms for the masses.

balance conditions are also sufficient. The proof idea is that the aggregate masses determine the mass m_0 of agents who search for a new partner and the mass m_0 determines the pointwise masses.

As a consequence of Lemma 3, we get the following equations which correspond to the more commonly known balance conditions of the form "Inflow equals Outflow".

Corollary 1. *For any $(\mathcal{M}, \varphi)$ that satisfies the aggregated balance conditions, the equations*

$$2p_{hl}\lambda m_0^2 = \lambda m_0(2m_{hl,UB,\leq t^*} + m_{hl,UB,>t^*})$$
$$p_{ll}\lambda m_0^2 = \lambda m_0(2m_{ll,UB,\leq t^*} + m_{ll,UB,>t^*} + 2m_{ll,BB})$$

hold.

Corollary 1 says that the total inflow rate of agents into hl-pairs ($2p_{hl}\lambda m_0^2$) is equal to the total outflow rate ($2\lambda m_0$ times the the number of agents who accept to rematch). The analogue also holds for ll-pairs while Lemma 3 shows the same result for singles.

In a monotone steady-state equilibrium, every pair except for hh-pairs eventually dissolves. This allows us to calculate the mass m_0 of agents who accept forming a new match, without knowing the specific equilibrium cutoff t^*.

Lemma 4. *In every monotone steady-state equilibrium*

$$m_0 = \sqrt{\frac{\eta}{\lambda p_{hh}}}$$

holds.

Lemma 4 uniquely determines the rate λm_0 for a given choice of parameters. Intuitively, the balance conditions imply that the inflow into the market is equal to the rate at which agents enter an absorbing state, i.e., the rate at which hh-pairs meet. Thus, $\eta = p_{hh}\lambda m_0^2$ holds. Formally, adding the aggregated balance conditions yields this result.

Our next lemma shows that for the given m_0 there is a unique cutoff t^* with a corresponding monotone steady-state equilibrium. This gives uniqueness in the class of monotone steady-state equilibria. When we talk about uniqueness, we formally mean that the masses in the steady-state equilibrium are uniquely given and the the strategies are unique up to a (finite) measure zero set of points (more precisely, in equilibrium only the agents' acceptance probabilities $V(UU, 0)$ and $V(UB, t^*)$ upon being indifferent can be arbitrary).

Lemma 5. *There exists a unique monotone steady-state equilibrium.*

This uniqueness result on the class of all monotone steady-state equilibria is in fact without loss as we will show in the next section.

5 Equilibrium Uniqueness

We now analyze the structure of non-monotone partial equilibria and show that those equilibria do not satisfy the balance conditions. Therefore there can only exist monotone steady-state equilibria and by this we get uniqueness for the class of all steady-state equilibria.

The next proposition characterizes the non-monotone partial equilibria.

Proposition 1. *In any non-monotone partial equilibrium there exists a* $t_0 \in [0, \infty)$ *with*

$$V(UU, t) > V(UU, 0) \text{ for all } t \in (0, t_0) \text{ and}$$
$$V(UU, t) = V(UU, 0) \text{ for all } t \geq t_0.$$

Furthermore, $\varphi(UU, t)$ *is strictly increasing after* t_0.

In the first part of the proof it is shown that $V(UU, t)$ cannot go below $V(UU, 0)$. The second part of the proof shows that once $V(UU, t) = V(UU, 0)$ holds for any $t > 0$, then it also holds for all larger t. A key argument for both parts is that agents become more optimistic over time and if all other circumstances are equal for two different points in time, then the later point needs to have a higher continuation payoff. Figure 2 illustrates the continuation payoff $V(UU, t)$ for non-monotone partial equilibria.

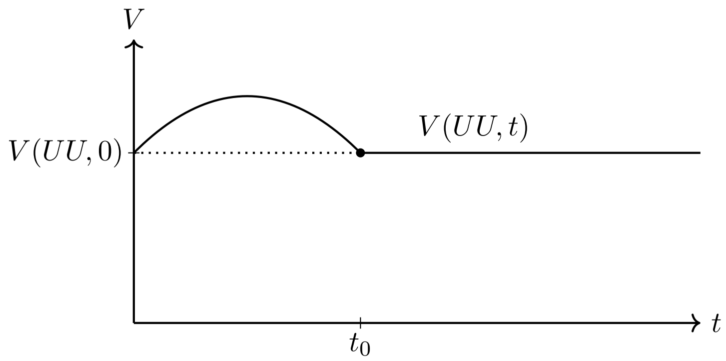

Figure 2: Continuation payoff for non-monotone partial equilibria

As long as $V(UU, t)$ is larger than $V(UU, 0)$, agents with information (U, U) do not accept to rematch. After t_0, the continuation payoff of not accepting is equal to the payoff of accepting. Therefore, agents follow a mixed strategy after t_0 and they mix with strictly increasing probability to keep their partners indifferent. Since both agents in such a match would strictly prefer that both agents do not accept to rematch, this can be interpreted as a coordination failure.

The next theorem states that a non-monotone equilibrium cannot be a steady-state equilibrium.

Theorem 1. *There exists a unique steady-state equilibrium and it is monotone.*

This shows that our restriction to monotone equilibria and our analysis of them are without loss of generality. In particular, in the last section, we have analyzed the equilibrium structure of the unique steady-state equilibrium.

6 The Role of Learning

In this section, we investigate the impact of the learning rate on the agents. We use comparative statics to analyze the welfare effects of a faster (or slower) learning rate γ.

The following lemma considers the case of a strong positive correlation[4]. If p_{hl} is close to 0, then agents have most likely the same type. In particular, bad news for the partner is also bad news for oneself. Therefore, a faster learning rate benefits both partners as both get the opportunity to leave an unprofitable match.

Lemma 6. *Fix $p_{hh} > 0$ and let p_{hl} converge to 0. Then, the expected equilibrium utility $V(\emptyset, 0)$ upon entering the market converges to*

$$V^*(\emptyset, 0) = \frac{\lambda m_0}{r + \lambda m_0} \cdot \frac{1 - p_{ll}}{r(1 - p_{ll}\frac{2\gamma\lambda m_0}{(r+2\gamma)(r+\lambda m_0)})} \cdot w.$$

For p_{hl} sufficiently small, $V(\emptyset, 0)$ is strictly increasing in γ.

In contrast to the previous lemma, now consider the case of a strong negative correlation, i.e., p_{hl} being close to $\frac{1}{2}$. Then, there is most likely one "winner" with an h-type and one "loser" with an l-type in each match. Learning who has a low type in a match allows that agent to find a new match but imposes a negative externality on the partner. We show that the negative externality on an h-agent is larger than the gain of rematching for an l-agent. More precisely, upon forming a match, the two partners would increase their ex-ante expected payoff if they could commit to never leaving. As a consequence, a faster learning rate γ decreases the expected utility in equilibrium, and agents would be better of by learning at a slower pace, or not learning at all.

Lemma 7. *Fix $p_{hh} > 0$ and let p_{ll} converge to 0. Then, the expected equilibrium utility $V(\emptyset, 0)$ upon entering the market converges to*

$$V^{**}(\emptyset, 0) \approx \frac{\lambda m_0}{r + \lambda m_0} \cdot \frac{\frac{p_{hh}}{r} + p_{hl}\frac{r+\lambda m_0+\gamma}{(r+\gamma)(r+\lambda m_0)}}{1 - p_{hl}\frac{\gamma\lambda^2 m_0^2}{(r+\gamma)(r+\lambda m_0)^2} - p_{hl}\frac{\gamma\lambda m_0}{(r+\gamma)(r+\lambda m_0)}} \cdot w.$$

[4]We consider the correlation between the hidden types in a match conditional on being in an non-obsorbing state hl, lh, or ll. In our limit analysis, to prevent the equilibrium mass m_0 of agents who search for a match from diverging to ∞, we fix the probability $p_{hh} > 0$ of entering an absorbing state.

For p_{ll} sufficiently small, $V(\emptyset, 0)$ is strictly decreasing in γ.

As a result, the effect of a faster learning rate is ambiguous. It depends on the correlation, whether faster learning increases or decreases the welfare of agents. If the agents' goals are aligned (strong positive correlation), then faster learning is beneficial. In contrast, with a strong negative correlation, slow learning is more beneficial as the agents prefer not to know who wins and who loses in a match, to prevent the match from being dissolved.

7 Conclusion

In this paper, we analyze a search model with on-the-match search and on-the-match learning. While being matched, agents learn about the idiosyncratic value of the current match. Not only the own valuation but also the partner's valuation of the match are of importance for an agent as the partner's rematching behavior affects the present value of a persisting match. This leads to an endogenous interest in the match being profitable for the partner.

We show the existence and uniqueness of a steady-state equilibrium. In equilibrium, agents follow cutoff strategies. Further, we provide an infinite set of pointwise balance conditions that ensures the stationarity of the masses for each time t that a match persists and we prove the equivalence to a finite set of aggregate balance conditions. For the welfare effects of learning, the correlation between the types in a match is of importance. With a strong positive correlation, faster learning increases the ex-ante expected payoff while with a strong negative correlation, the ex-ante payoff decreases with a faster learning rate. In the latter case, committing together to never dissolve a match is ex-ante preferred by both agents.

An interesting direction of further research would be the extension to other information structures. For instance, if h-agents received good news over time, instead of l-agents receiving bad news, then the beliefs inside a match grow more pessimistic the longer a match persists without news. This would change the rematching behavior of agents in a sense that agents in newly formed matches immediately search for a new partner as even an ε of time without good news decreases the present value of the current match below the value of a newly formed match. For more general information structures, like the occurrence of multiple different types of news, or the beliefs following a Brownian motion, the drift of the belief would be of importance to the agents' equilibrium behavior.

A Proofs

A.1 Proofs for Section 3

Proof of Lemma 1. For single agents and agents with bad news it is a dominant strategy to always accept. An agent in a pair could copy the strategy of a single/agent with bad news and receive a strictly higher payoff.

For the equilibrium behaviour of an agent with information (U, B), note that the partner always accepts to rematch, i.e., her acceptance probability is constant over time. Since the belief $P(hl|UB, t)$ is strictly increasing in t, the continuation payoff $V(UB, t)$ is also strictly increasing in t. Thus, $V(UB, t)$ crosses $V(UU, 0)$ at most once. □

A.2 Proofs for Section 4

Proof of Lemma 2. The equilibrium behavior of agents with information (U, U) follows from the fact that at $t = 0$ an agent is indifferent between accepting to rematch and staying in the current match. By monotonicity, the acceptance probability of the partner is non-increasing. Therefore, the continuation payoff $V(UU, t)$ is strictly increasing over time. Since the continuation payoff at time $t = 0$ is identical to the continuation payoff of accepting, agents with information (U, U) never accept for $t > 0$. □

Proof of Lemma 3. Integrating the pointwise balance equation for singles

$$m_\emptyset(t) = \big(\eta + \lambda m_0(m_0 - m_\emptyset)\big)e^{-\lambda m_0 t}$$

over t yields

$$m_\emptyset = \big(\eta + \lambda m_0(m_0 - m_\emptyset)\big) \cdot \frac{1}{\lambda m_0},$$

which is equivalent to

$$m_\emptyset = \frac{\eta + \lambda m_0^2}{2\lambda m_0}.$$

Integrating the pointwise balance equation for hh-pairs

$$m_{hh,UU}(t) = p_{hh}\lambda m_0^2 \cdot 1$$

over t yields

$$m_{hh,UU} = \infty.$$

Integrating the pointwise balance equation for $m_{hl,UU}$

$$m_{hl,UU}(t) = 2p_{hl}\lambda m_0^2 e^{-\gamma t}$$

over t yields

$$m_{hl,UU} = \frac{2p_{hl}\lambda m_0^2}{\gamma}.$$

Integrating the pointwise balance equation for $m_{ll,UU}$

$$m_{ll,UU}(t) = p_{ll}\lambda m_0^2 e^{-2\gamma t}$$

over t yields

$$m_{ll,UU} = \frac{p_{ll}\lambda m_0^2}{2\gamma}.$$

Integrating the three remaining pointwise balance conditions over t yields the integrals

$$\begin{aligned}
m_{hl,UB,\le t^*} &= \gamma 2p_{hl}\lambda m_0^2 \int_0^{t^*}\int_0^t \exp\big(-\gamma t' - 2\lambda m_0(t-t')\big)\,dt'dt\\
m_{hl,UB,>t^*} &= \gamma 2p_{hl}\lambda m_0^2 \int_{t^*}^{\infty}\int_0^t \exp\big(-\gamma t' - \lambda m_0(t-t') - \lambda m_0 \max(t^*-t',0)\big)\,dt'dt\\
m_{ll,UB,\le t^*} &= 2\gamma p_{ll}\lambda m_0^2 \int_0^{t^*}\int_0^t \exp\big(-2\gamma t' - (2\lambda m_0+\gamma)(t-t')\big)\,dt'dt\\
m_{ll,UB,>t^*} &= 2\gamma p_{ll}\lambda m_0^2 \int_{t^*}^{\infty}\int_0^t \exp\big(-2\gamma t' - (\lambda m_0+\gamma)(t-t') - \lambda m_0 \max(t^*-t',0)\big)\,dt'dt\\
m_{ll,BB} &= 2\gamma^2 p_{ll}\lambda m_0^2 \int_0^{\infty}\int_0^t\int_0^{t'} \exp\big(-2\gamma t'' - (\lambda m_0+\gamma)(t'-t'') - \lambda m_0 \max(\min(t^*,t')-t'',0)\\
&\qquad - 2\lambda m_0(t-t')\big)\,dt''dt'dt,
\end{aligned}$$

where the survival functions $q_{ij,SS'}(t',t)$ are substituted by the corresponding exponential functions given by the equilibrium strategies:

$$\begin{aligned}
q_{hl,UB}(t',t) &= \exp\big(-\lambda m_0(t-t') - \lambda m_0 \max(\min(t^*,t)-t',0)\big)\\
q_{ll,UB}(t',t) &= \exp\big(-(\lambda m_0+\gamma)(t-t') - \lambda m_0 \max(\min(t^*,t)-t',0)\big)\\
q_{ll,BB}(t',t) &= \exp\big(-2\lambda m_0(t-t')\big).
\end{aligned}$$

Solving these integrals gives the aggregate balance conditions

$$m_{hl,UB,\leq t^*} = p_{hl}m_0\Big(1 + \frac{2\lambda m_0}{\gamma - 2\lambda m_0}e^{-\gamma t^*} - \frac{\gamma}{\gamma - 2\lambda m_0}e^{-2\lambda m_0 t^*}\Big)$$

$$m_{hl,UB,>t^*} = p_{hl}m_0\Big(\frac{2\gamma}{\gamma - 2\lambda m_0}e^{-2\lambda m_0 t^*} - \frac{4\lambda m_0}{\gamma - 2\lambda m_0}e^{-\gamma t^*}\Big)$$

$$m_{ll,UB,\leq t^*} = p_{ll}m_0\Big(\frac{2\lambda m_0}{\gamma + 2\lambda m_0} + 2\lambda m_0 e^{-2\gamma t^*} + \frac{4\gamma\lambda m_0}{\gamma + 2\lambda m_0}e^{-(2\lambda m_0+\gamma)t^*}\Big)$$

$$m_{ll,UB,>t^*} = p_{ll}m_0\Big(\frac{4\gamma\lambda m_0}{(\gamma + \lambda m_0)(\gamma - 2\lambda m_0)}e^{-(2\lambda m_0+\gamma)t^*} - \frac{2\lambda m_0(\gamma + 2\lambda m_0)}{(\gamma + \lambda m_0)(\gamma - 2\lambda m_0)}e^{-2\gamma t^*}\Big)$$

$$\begin{aligned} m_{ll,BB} = p_{ll}m_0\Big(&\frac{\gamma - 2\lambda m_0}{2(\gamma + 2\lambda m_0)} - \frac{\lambda m_0(\gamma + 2\lambda m_0 - 2\gamma^2 + 2\gamma\lambda m_0 + 4\lambda^2 m_0^2)}{(\gamma + \lambda m_0)(\gamma - 2\lambda m_0)}e^{-2\gamma t^*} \\ &- \frac{2\gamma\lambda m_0(\gamma + 2\lambda m_0 + 2\gamma^2 - 2\gamma\lambda m_0 - 4\lambda^2 m_0^2)}{(\gamma + 2\lambda m_0)(\gamma + \lambda m_0)(\gamma - 2\lambda m_0)}e^{-(2\lambda m_0+\gamma)t^*}\Big). \end{aligned}$$

Thus, if the pointwise balance conditions are satisfied, so are the aggregated balance conditions.

The final equation

$$m_0 = m_\emptyset + m_{hl,UB,\leq t^*} + \frac{1}{2}m_{hl,UB,>t^*} + m_{hl,UB,\leq t^*} + \frac{1}{2}m_{ll,UB,>t^*} + m_{ll,BB},$$

follows from the fact that the set of all agents who want to rematch consists of the following: All singles, all agents who received bad news, and all agents whose partner has received bad news and who are in match for a time less than t^*. For $t > t^*$, only half of the agents in pairs with information UB are willing to rematch, which implies that only half of the masses $m_{hl,UB,>t^*}$ and $m_{ll,UB,>t^*}$ counts towards m_0.

It remains to show that the aggregate balance conditions are sufficient for the pointwise balance equations. For this, we construct pointwise masses $\mathcal{M}$ as follows: First, the aggregate masses uniquely determine the masses m_0 of agents who search for a new partner. Second, the masses m_0, $m_\emptyset$ and the strategies uniquely determine the pointwise masses

$$\begin{aligned} m_\emptyset(t) &= \big(\eta + \lambda m_0(m_0 - m_\emptyset)\big)q_\emptyset(0,t) \\ m_{hh,UU}(t) &= p_{hh}\lambda m_0^2\; q_{hh,UU}(0,t) \\ m_{hl,UU}(t) &= 2p_{hl}\lambda m_0^2\; q_{hl,UU}(0,t) \\ m_{ll,UU}(t) &= p_{ll}\lambda m_0^2\; q_{ll,UU}(0,t). \end{aligned}$$

Finally, the remaining pointwise masses are uniquely determined by

$$m_{hl,UB}(t) = \int_0^t \gamma m_{hl,UU}(t')\; q_{hl,UB}(t',t)\; dt'$$

$$m_{ll,UB}(t) = \int_0^t 2\gamma m_{ll,UU}(t')\; q_{ll,UB}(t',t)\; dt'$$

$$m_{ll,BB}(t) = \int_0^t \gamma m_{ll,UB}(t')\; q_{ll,BB}(t',t)\; dt'.$$

Therefore, there exists a unique $(\mathcal{M}', \varphi)$ that has the same aggregated masses as $(\mathcal{M}, \varphi)$ and satisfies the pointwise balance conditions. □

Proof of Corollary 1. This corollary follows from summing the aggregate balance conditions together. Adding two times

$$m_{hl,UB,\leq t^*} = p_{hl} m_0 \Big(1 + \frac{2\lambda m_0}{\gamma - 2\lambda m_0} e^{-\gamma t^*} - \frac{\gamma}{\gamma - 2\lambda m_0} e^{-2\lambda m_0 t^*}\Big)$$

plus

$$m_{hl,UB,>t^*} = p_{hl} m_0 \Big(\frac{2\gamma}{\gamma - 2\lambda m_0} e^{-2\lambda m_0 t^*} - \frac{4\lambda m_0}{\gamma - 2\lambda m_0} e^{-\gamma t^*}\Big)$$

yields

$$2m_{hl,UB,\leq t^*} + m_{hl,UB,>t^*} = 2p_{hl} m_0.$$

Multiplying this by λm_0 yields the first equation. Analogously, adding two times

$$m_{ll,UB,\leq t^*} = p_{ll} m_0 \Big(\frac{2\lambda m_0}{\gamma + 2\lambda m_0} + 2\lambda m_0 e^{-2\gamma t^*} + \frac{4\gamma\lambda m_0}{\gamma + 2\lambda m_0} e^{-(2\lambda m_0 + \gamma) t^*}\Big)$$

plus

$$m_{ll,UB,>t^*} = p_{ll} m_0 \Big(\frac{4\gamma\lambda m_0}{(\gamma + \lambda m_0)(\gamma - 2\lambda m_0)} e^{-(2\lambda m_0 + \gamma) t^*} - \frac{2\lambda m_0(\gamma + 2\lambda m_0)}{(\gamma + \lambda m_0)(\gamma - 2\lambda m_0)} e^{-2\gamma t^*}\Big)$$

plus two times

$$\begin{aligned} m_{ll,BB} = p_{ll} m_0 \Big(&\frac{\gamma - 2\lambda m_0}{2(\gamma + 2\lambda m_0)} - \frac{\lambda m_0(\gamma + 2\lambda m_0 - 2\gamma^2 + 2\gamma\lambda m_0 + 4\lambda^2 m_0^2)}{(\gamma + \lambda m_0)(\gamma - 2\lambda m_0)} e^{-2\gamma t^*} \\ &- \frac{2\gamma\lambda m_0(\gamma + 2\lambda m_0 + 2\gamma^2 - 2\gamma\lambda m_0 - 4\lambda^2 m_0^2)}{(\gamma + 2\lambda m_0)(\gamma + \lambda m_0)(\gamma - 2\lambda m_0)} e^{-(2\lambda m_0 + \gamma) t^*}\Big) \end{aligned}$$

yields

$$2m_{ll,UB,\leq t^*} + m_{ll,UB,>t^*} + 2m_{ll,BB} = p_{ll} m_0.$$

Multiplying by λm_0 yields the second equation. $\square$

Proof of Lemma 4. For the proof, we substitute the masses by the aggregate balance conditions in the term that specifies m_0: First, we multiply

$$m_0 = m_\emptyset + m_{hl,UB,\leq t^*} + \frac{1}{2}m_{hl,UB,>t^*} + m_{hl,UB,\leq t^*} + \frac{1}{2}m_{ll,UB,>t^*} + m_{ll,BB}$$

from Lemma 3 by $2\lambda m_0$ to obtain

$$\begin{aligned} 2\lambda m_0^2 = 2\lambda m_0 m_\emptyset &+ \lambda m_0(2m_{hl,UB,\leq t^*} + m_{hl,UB,>t^*}) \\ &+ \lambda m_0(2m_{hl,UB,\leq t^*} + m_{ll,UB,>t^*} + 2m_{ll,BB}). \end{aligned}$$

Now, substituting the aggregate balance conditions

$$\begin{aligned} 2\lambda m_0 m_\emptyset &= \eta + \lambda m_0^2 \\ 2p_{hl}\lambda m_0^2 &= \lambda m_0(2m_{hl,UB,\leq t^*} + m_{hl,UB,>t^*}) \\ p_{ll}\lambda m_0^2 &= \lambda m_0(2m_{ll,UB,\leq t^*} + m_{ll,UB,>t^*} + 2m_{ll,BB}) \end{aligned}$$

from Lemma 3 and Corollary 1 yields

$$2\lambda m_0^2 = \eta + \lambda m_0^2 + 2p_{hl}\lambda m_0^2 + p_{ll}\lambda m_0^2.$$

By $p_{hh} = (1 - 2p_{lh} - p_{ll})$, we get that

$$\eta = p_{hh}\lambda m_0^2$$

holds and therefore, the aggregate balance conditions imply that m_0 has to be

$$m_0 = \sqrt{\frac{\eta}{\lambda p_{hh}}}.$$

$\square$

Proof of Lemma 5. First, we show the existence of a monotone steady-state equilibrium. As shown before, accepting is optimal for singles and agents with bad news. Furthermore, $\varphi(UU, t) = 0$ is not only necessary for all monotone partial equilibrium, but also a best response to itself. For the existence, it remains to show that there exists a cutoff $t^* \in [0, \infty]$ and a corresponding steady-state equilibrium such that it is optimal for agents with information (U, B) to accept to rematch until cutoff t^* and reject to rematch afterwards. Let

$$W(t) := V(UB, t) - V(UU, 0)$$

denote the difference in the expected utility of staying in the current match minus rematching given that all agents follow such a monotone strategy with cutoff t. The difference $W(t)$ is continuous by construction, strictly increasing in t, and bounded. It converges to $W(\infty) := \lim_{t\to\infty} W(t)$. If $W(t) \geq 0$ for all t, then it is never optimal to accept to rematch for an agent with information (U, B) and the cutoff $t^* = 0$ is optimal. If $W(t) \leq 0$ for all t, then it is always optimal to accept to rematch for an agent with information (U, B) and the cutoff $t^* = \infty$ is optimal. If neither of these two cases holds, then there exist $t_0 < t_1$ with $W(t_0) < 0 < W(t_1)$. By the intermediate value theorem, there is an interior cutoff $t^* \in (0, \infty)$ with $W(t^*) = 0$, i.e., agents are indifferent at the cutoff, they strictly prefer to rematch for all $t < t^*$ and strictly prefer to stay in the current match for all $t > t^*$. Thus, there always exists a monotone steady-state equilibrium.

For uniqueness, suppose for contradiction that there are two monotone steady-state equilibria with different cutoffs $t_1^* < t_2^*$. Consider a single agent and take the other agents' strategies as given. In particular, the cutoff t^* of the other agents has no influence on the own continuation payoff, since m_0 is the same in both equilibria and the agents with bad news receive the same continuation payoff as a single agent. The continuation payoff $V(UU, 0)$ of forming a new match cannot be the same in both equilibria. Otherwise, the continuation payoff for all future matches would be equal in both equilibria and since $V(UB, t)$ is strictly increasing in both equilibria, this contradicts $t_1^* \neq t_2^*$. Thus, we get that $V(UU, 0)$ is different in both equilibria and since the cutoff choice t^* of the other agents does not change the own expected payoff, in at least one of the two equilibria the own choice of the cutoff is not optimal. □

A.3 Proofs for Section 5

Proof of Proposition 1. For this proof, we first need the following lemma that shows that the continuation payoff at time t_1 is higher than at time t_0 if the following three conditions are all satisfied: (1) the continuation payoff is higher at $t_1 + \varepsilon$ than at $t_0 + \varepsilon$ for some $\varepsilon > 0$, (2) the partner rematches less often after t_1 than after t_0, and (3) $t_1 > t_0$, i.e., the beliefs are more optimistic at t_1.

Lemma 8. *Fix any partial equilibrium and two points in time $t_0 < t_1$. If there exists an $\varepsilon > 0$ with*

$$V(UU, t_0 + \varepsilon) \leq V(UU, t_1 + \varepsilon)$$

such that for almost all $\xi \in (0, \varepsilon)$

$$\varphi(UU, t_0 + \xi) \geq \varphi(UU, t_1 + \xi)$$

holds, then we get

$$V(UU, t_0) < V(UU, t_1).$$

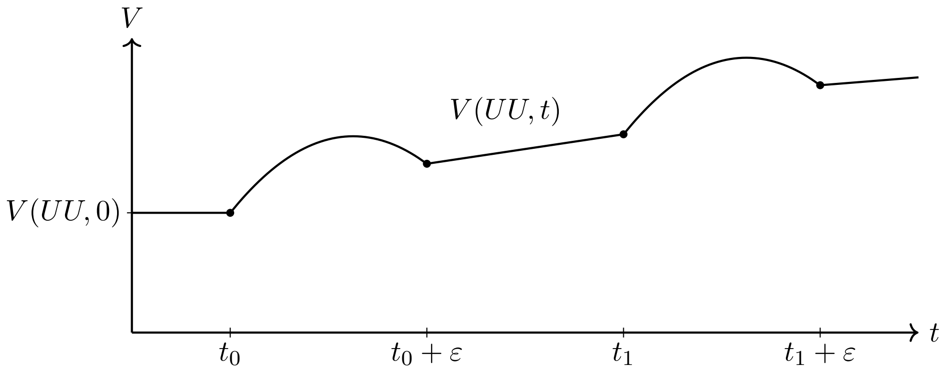

Figure 3: Continuation payoff for Lemma 8

Proof. Fix a partial equilibrium. The continuation payoff at time t can be expressed as a function of the continuation payoff at time $t+\varepsilon$, the own strategy, the partner's strategy, and the belief (which affects the expected rate at which bad news arrive). The continuation payoff is decreasing in the acceptance probability of the partner and it is increasing in the future continuation payoff at time $t+\varepsilon$. Furthermore, the continuation payoff strictly increases as agents become more optimistic over time. Therefore, $V(UU,t_0) < V(UU,t_1)$ holds. $\square$

Now, to prove Proposition 1, fix a non-monotone partial equilibrium. In the following we use Lemma 8 to systematically exclude various cases of how $V(UU,t)$ might behave until only one possible equilibrium type remains. Then, we conclude that all non-monotone partial equilibria must be of the form as described in Proposition 1.

The first claim shows that the continuation payoff cannot go below $V(UU,0)$ without going up again.

Claim 1. There is no t_0 with $V(UU,t) < V(UU,0)$ for all $t > t_0$.

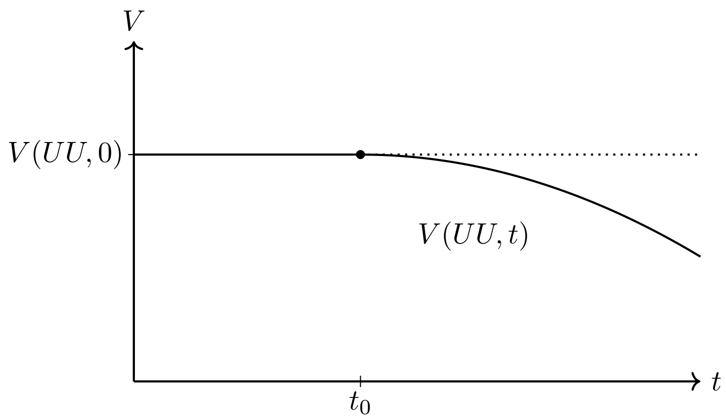

Figure 4: Continuation payoff for Claim 1

Proof. Assume for contradiction that there exists such a t_0. Let

$$t_1 = \inf_t \left\{ \forall t' > t \ : \ V(UU, t') < V(UU, 0) \right\}$$

be the infimum over all such times. After t_1, agents strictly prefer to accept to rematch. Thus, the partner's acceptance probability does not change after t_1. Since agents get more optimistic over time, $V(UU, t')$ is strictly increasing on the interval (t_1, ∞). This is a contradiction, since by construction and continuity $V(UU, t_0) = V(UU, 0)$ holds. □

The next claim shows that the continuation payoff cannot go below $V(UU, 0)$ and up again. Together with the last claim, this implies that $V(UU, t)$ is always at least as large as $V(UU, 0)$.

Claim 2. There are no $t < t'$ with $V(UU, t) < V(UU, t') = V(UU, 0)$.

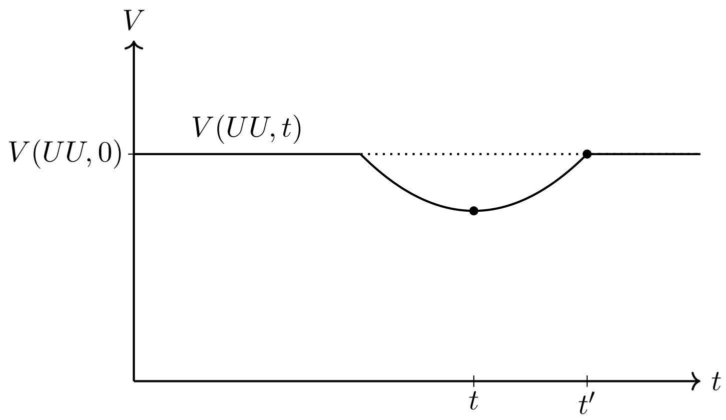

Figure 5: Continuation payoff for Claim 2

Proof. Assume for contradiction that there exist such $t < t'$. Now, we construct an open interval (t_0, t_1) with positive length such that the continuation payoff is smaller than $V(UU, 0)$ at the interval and the interval is maximal under set-inclusion. More precisely, we define

$$t_0 = \inf_{t'} \left\{ \forall t'' \in (t', t) \ : \ V(UU, t'') < V(UU, 0) \right\}$$

and

$$t_1 = \sup_{t'} \left\{ \forall t'' \in (t, t') \ : \ V(UU, t'') < V(UU, 0) \right\}.$$

Then, (t_0, t_1) is such an interval. By continuity the agents are indifferent at the boundary points, i.e., the equality $V(UU, t_0) = V(UU, t_1) = V(UU, 0)$ holds. Now, we compare the continuation payoff at the two times t_0 and $\frac{t_0+t_1}{2}$. Agents are more optimistic at $\frac{t_0+t_1}{2}$, the acceptance probability of the partner is 1 immediately after both times, and the future continuation payoff is higher after $\frac{t_0+t_1}{2}$. We apply Lemma 8 to t_0 and $\frac{t_0+t_1}{2}$ with $\varepsilon = \frac{t_1-t_0}{2}$ and we get $V(UU, t_0) < V(UU, \frac{t_0+t_1}{2})$ which contradicts our construction. $\square$

The next claim shows that after $t = 0$ the continuation payoff cannot go above $V(UU, 0)$ without going down again.

Claim 3. There exists no $t_0 > 0$ with $V(UU, t_0) = V(UU, 0)$ such that for all larger $t > t_0$ $V(UU, t) > V(UU, 0)$ holds.

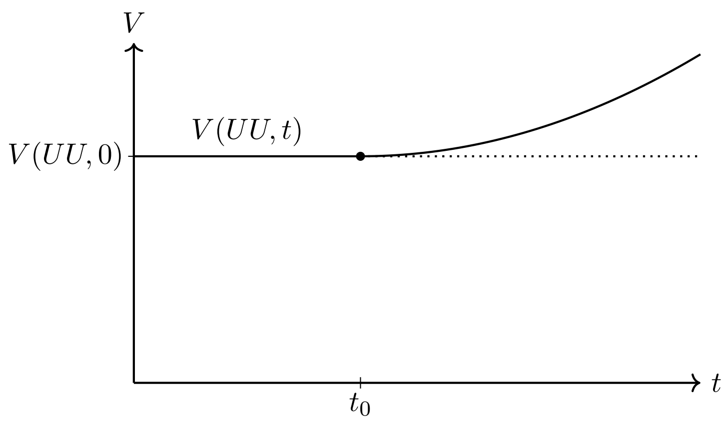

Figure 6: Continuation payoff for Claim 3

Proof. Assume for contradiction that there exists a $t_0 > 0$ such that agents with information (U, U) do not accept to rematch afterwards. Without loss, let t_0 be the minimum of all such times. Then, the agents are more optimistic at t_0 than at $t = 0$ and the partner will always reject to rematch after t_0. We apply Lemma 8 to 0 and t_0 with $\varepsilon = t_0$ and get that the continuation payoff $V(UU, t_0)$ is strictly larger than $V(UU, 0)$. This is a contradiction to the minimality of t_0. $\square$

Finally, the last claim says that after $t = 0$ the continuation payoff cannot go above $V(UU, 0)$ and reach $V(UU, 0)$ again afterward. Together with the previous claim, this implies that if $V(UU, t) = V(UU, 0)$ holds for some $t > 0$, then the same equality also holds for all $t' > t$.

Claim 4. There exist no three points $0 < t_0 < t_1 < t_2$ such that $V(UU, t_0) = V(UU, 0)$, $V(UU, t_1) > V(UU, 0)$, and $V(UU, t_2) = V(UU, 0)$ hold.

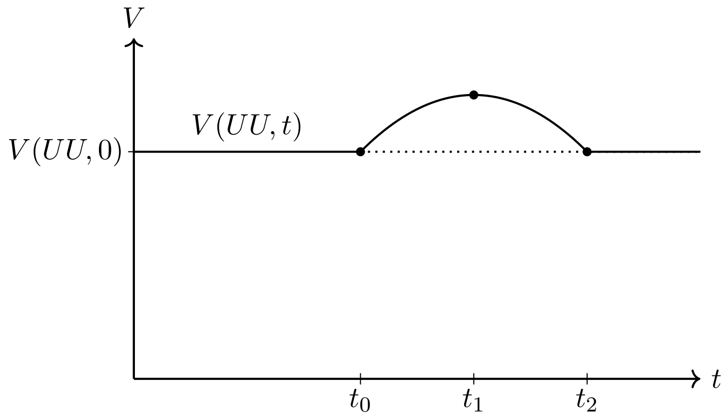

Figure 7: Continuation payoff for Claim 4

Proof. Assume for contradiction that there exist such three points. Without loss let the distance $t_2 - t_0$ be minimal of all such tuples. Then, $V(UU, t) > V(UU, 0)$ holds for all interior points $t \in (t_0, t_2)$. Now, we distinguish two cases.

Case 1: $t_2 - t_0 \geq t_0$, i.e., the length of the interval (t_0, t_2) is larger than the length of the interval $(0, t_0)$. Then, we apply Lemma 8 to 0 and t_0 with $\varepsilon = t_0$. Agents are more optimistic at t_0, agents are never left by their partner in the interval $(t_0, 2t_0)$, and the continuation payoff at $2t_0$ is strictly larger than at t_0. Thus, we get that $V(UU, t_0) \geq V(UU, 0)$ holds which is a contradiction.

Case 2: $t_2 - t_0 < t_0$, i.e., the length of the interval (t_0, t_2) is smaller than the length of the interval $(0, t_0)$. Let $\hat{t} := t_0 - (t_2 - t_0)$. We apply Lemma 8 to $\hat{t}$ and t_0 with $\varepsilon = t_2 - t_0$ to compare the continuation payoffs $V(UU, \hat{t})$ and $V(UU, t_0)$. The beliefs are higher at t_0, the partner does not accept to rematch at the interval (t_0, t_2), and the continuation payoff at the end of the interval (t_0, t_2) is equal to the continuation payoff at the end of the interval $(\hat{t}, t_0)$. Therefore, $V(UU, \hat{t}) < V(UU, t_0) = V(UU, 0)$ holds. This is a contradiction, since we have shown that $V(UU, t) \geq V(UU, 0)$ has to hold for all t. □

Continuation of the proof of Proposition 1. Now, we know that if we have $V(UU, t) = V(UU, 0)$ for any $t > 0$, then this equality also holds for all $t' > t$. If this equality would only hold for $t = 0$, then we would have a monotone equilibrium. Therefore, in any non-monotone partial equilibrium exists a $t \in [0, \infty)$ such that $V(UU, t') = V(UU, 0)$ holds for all $t' > t$. Let

$$t_0 = \inf_t \left\{ \forall t' > t \ : \ V(UU, t') = V(UU, 0) \right\}$$

be the earliest time after which $V(UU, t)$ is constant. By the previous claims, we get $V(UU, t) > V(UU, 0)$ for all $t \in (0, t_0)$. Therefore, the partial equilibrium is exactly as characterized in Proposition 1. □

Proof of Theorem 1. In any non-monotone partial equilibrium, there is a time t_0 after which $\varphi(UU, t)$ is strictly increasing. Thus, the probability that an hh-pair stays together for at least time t converges to 0 as t approaches ∞. Since agents who received bad news accept to rematch with probability 1, there is no absorbing state, i.e., every match is eventually dissolved.

Since the balance conditions imply that the inflow of agents into absorbing states is equal to the inflow into the market, a non-monotone partial equilibrium does not satisfy the balance conditions. Therefore, all steady-state equilibria have to be monotone. Since there exists a unique monotone steady-state equilibrium, we get uniqueness among all steady-state equilibria. □

A.4 Proofs for Section 6

Proof of Lemma 6. First, we show that t^* tends to ∞ as p_{hl} vanishes. Let $\hat{t}$ denote the time at which

$$P(hl|UB, t) = P(i = h|UU, 0) \qquad \Leftrightarrow$$

$$\frac{p_{hl}}{p_{hl} + p_{ll}e^{-\gamma t}} = \frac{p_{hh}}{p_{hh} + 2p_{hl}e^{-\gamma t} + p_{ll}e^{-2\gamma t}} + \frac{p_{hl}e^{-\gamma t}}{p_{hh} + 2p_{hl}e^{-\gamma t} + p_{ll}e^{-2\gamma t}}$$

holds. For agents whose partner received bad news, it is a dominant strategy to search at all times before $\hat{t}$. Thus, for the equilibrium cutoff $t^* > \hat{t}$ holds. Now, consider a sequence where p_{hl} converges to 0. Along this sequence, the time $\hat{t}$ tends to ∞. Therefore, t^* also tends to ∞.

Next, we consider the limit of such a sequence, i.e., let $p_{hl} = 0$ and $t^* = \infty$ hold. Let $V(ij)$ denote the expected continuation payoff of forming a new pair with type i oneself and type j for the partner. Let $V(x)$ denote the expected continuation payoff of forming a new pair before the types are realized. Recall that $V(\emptyset, 0)$ denotes the expected continuation payoff of becoming single. Then, by integrating the expected future utilities, we get for the continuation payoffs:

$$\begin{aligned}
V(x) &= p_{hh}V(hh) + p_{ll}V(ll) \\
V(hh) &= \frac{w}{r} \\
V(ll) &= \frac{2\gamma}{r + 2\gamma}V(\emptyset, 0) \\
V(\emptyset, 0) &= \frac{\lambda m_0}{r + \lambda m_0}V(x).
\end{aligned}$$

Taking these together, we get

$$V(x) = p_{hh}\frac{w}{r} + p_{ll}\frac{2\gamma}{r + 2\gamma}\frac{\lambda m_0}{r + \lambda m_0}V(x)$$

and therefore

$$V^*(\emptyset,0) = \frac{\lambda m_0}{r+\lambda m_0} \cdot \frac{1-p_{ll}}{r(1-p_{ll}\frac{2\gamma\lambda m_0}{(r+2\gamma)(r+\lambda m_0)})} \cdot w$$

holds. Taking the derivative with respect to the learning rate γ shows that $d\frac{V^*(\emptyset,0)}{d\gamma} > 0$ holds.

Now, by the continuity of the payoff functions, the payoff function $V(\emptyset,0)$ converges to $V^*(\emptyset,0)$ as p_{hl} vanishes. As the derivative of $V^*(\emptyset,0)$ is strictly positive, $d\frac{V(\emptyset,0)}{d\gamma} > 0$ is also positive for p_{hl} sufficiently small. □

Proof of Lemma 7. Analogously to the proof of Lemma 6, for p_{ll} sufficiently small, an agent whose partner received bad news stays in the match ($t^* = 0$). Next, consider the limit of such a sequence, i.e., $p_{ll} = 0$. The continuation payoffs are:

$$\begin{aligned}
V(x) &= p_{hh}V(hh) + p_{hl}V(hl) + p_{lh}V(lh) \\
V(hh) &= \frac{w}{r} \\
V(hl) &= \frac{w}{r+\gamma} + \frac{\gamma}{r+\gamma}\Big(\frac{w}{r+\lambda m_0} + \frac{\lambda m_0}{r+\lambda m_0}V(\emptyset,0)\Big) \\
V(lh) &= \frac{\gamma}{r+\gamma}V(\emptyset,0) \\
V(\emptyset,0) &= \frac{\lambda m_0}{r+\lambda m_0}V(x).
\end{aligned}$$

Together, we get

$$V(x) = \frac{p_{hh}\frac{w}{r} + p_{hl}\frac{w}{r+\gamma} + p_{hl}\frac{\gamma}{r+\gamma}\frac{w}{r+\lambda m_0}}{1 - p_{hl}\frac{\gamma}{r+\gamma}\frac{\lambda m_0}{r+\lambda m_0}\frac{\lambda m_0}{r+\lambda m_0} - p_{hl}\frac{\gamma}{r+\gamma}\frac{\lambda m_0}{r+\lambda m_0}}$$

and thus, the expected equilibrium utility upon entering the market is given by

$$V^{**}(\emptyset,0) \approx \frac{\lambda m_0}{r+\lambda m_0} \cdot \frac{\frac{p_{hh}}{r} + p_{hl}\frac{r+\lambda m_0+\gamma}{(r+\gamma)(r+\lambda m_0)}}{1 - p_{hl}\frac{\gamma\lambda^2 m_0^2}{(r+\gamma)(r+\lambda m_0)^2} - p_{hl}\frac{\gamma\lambda m_0}{(r+\gamma)(r+\lambda m_0)}} \cdot w$$

which is strictly decreasing in γ. By the continuity of the payoff functions, the payoff function $V(\emptyset,0)$ converges to $V^{**}(\emptyset,0)$ as p_{ll} vanishes. As the derivative of $V^{**}(\emptyset,0)$ is strictly negative, $d\frac{V(\emptyset,0)}{d\gamma} > 0$ is also negative for p_{ll} sufficiently small. □

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