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Modeling Bank Panics: Challenges

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Abstract

Our primary finding is that surprisingly small changes in assumptions which determine the amount of net worth available in a bank panic have an important impact on the nature of the equilibria: there may not be a bank panic at all, or there may be several different panics of different severity. The economic reasons for this sensitivity are clarified by transforming the market economy into a game and studying banker best response functions. To establish robustness to model details, we report similar quantitative results across three different model specifications and calibrations. A second, additional result, is displayed in a three-period version of the panic model of Gertler and Kiyotaki (2015). That model naturally suggests the idea that welfare can be improved by imposing a restriction on bank leverage. We compute the Ramsey-optimal leverage restriction, but find that there is an implementation problem: the restriction can be associated with more than one equilibrium, not just the desired one. We discuss one way to address the implementation problem.

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1 Introduction

Beginning in the last half of 2007 there was a sharp rise in the fraction of financial firms having difficulty rolling over their short-term liabilities, especially firms in the sector specializing in the acquisition of mortgage-backed securities.[1] Shortly thereafter, the US economy fell into the Great Recession. Economists have been debating the question, 'what was the cause of these events?'.

There are two types of answer. The *fundamentals hypothesis* posits that fundamentals made the crisis and recession inevitable, though precise details about timing and other things were left to chance.[2] The other hypothesis, (the *panic hypothesis*) is that weakening fundamentals made the US economy vulnerable to a panic that need not have occurred. Under this hypothesis, policy makers noticed the weakening fundamentals but thought that they could minimize the damages if and when a recession occurred. Unfortunately, they were not aware of the increased vulnerability to panic and, alas, a panic then did occur. The resulting damage to the economy was largely a consequence of the panic and not so much a direct consequence of the fundamentals themselves. A prominent example of the latter argument is proposed in Gertler and Kiyotaki (2015) (GK).

We show that the panic hypothesis requires the existence of what seems like implausibly high costs for equity to enter the sector of the financial system experiencing crisis. A more conclusive assessment of whether these costs are too high requires empirical estimates and obtaining these is beyond the scope of this paper. We also show that the number of equilibria and their nature is very sensitive to assumptions made about the entry of equity in a crisis. For example, we find that, generically, there are three equilibria. Moreover, depending on assumptions on the entry of new bankers, it is also possible that there are just one or two equilibria. To establish these qualitative features of equilibria we transform the model into a game and study banker best response functions. As an indicator of the robustness of our conclusions, we show that similar conclusions emerge from a model of bank panics based on very different financial frictions from the ones in GK.

[1] See, for example, Gorton (2010), Bernanke (2010) and Covitz et al. (2013).

[2] Milton Friedman once described this view with a particular analogy. Suppose a determined lumberjack chops down a tree. The precise moment when the tree falls, and the direction of its fall are determined by random, seemingly non-fundamental factors like small changes in wind direction and other such things. Still, the real 'cause' of the ultimate fall of the tree is the exertions of the lumberjack. More discussion of the fundamentals hypothesis appears in footnotes 7 and 10 below.

Our models focus on shadow banks that issue short term liabilities (*deposits*) to buy long-lived assets.[3] Under the 'right' configuration of fundamentals, shadow banks operating outside the protection of a lender of last resort are vulnerable to a panic-induced collapse. In a panic, each creditor believes that the other creditors will refuse to roll over bank deposits, and that this will force the banks into fire sales of assets. Thinking that this will wipe out the net worth of banks, panicky creditors refuse to roll over, and their fear is self-fulfilling. We refer to a scenario in which the fire sales are so severe that the net worth of the banks is wiped out as an *annihilation run*. Our models follow the classic narrative of Diamond and Dybvig (1983), in which agents can differentiate fundamentals from the sunspots that drive the panic component of a crisis.[4] We call a rollover crisis in such a model *a pure panic run*.[5]

We examine models in which rollover crises are pure panic runs. In these models, it is simply assumed that banks do not have the option to issue equity during an annihilation run. We find that the equilibria of these models are surprisingly sensitive to allowing even a small amount of equity to enter the banking system in an annihilation run. To understand the problem it is useful to differentiate between two effects of panic on a shadow bank's debt capacity.[6] First, the fire-sale fall in the price of assets that occurs as financial firms sell assets to pay off liabilities directly reduces bank net worth. The drop in net worth per se reduces debt capacity. But, at the same time the fall in the price of assets raises their rate of return and, other things the same, this raises debt capacity. We find that the latter effect is enormously powerful, so much so that even if only a tiny amount of net worth, or equity, enters the banking system during a run, then this tiny net worth can support a lot of assets via high leverage. Thus, by propping up asset prices, a small amount of net worth can prevent a general panic from occurring in the first place. We quantify the assumption that other financial institutions cannot bring equity into a crisis by computing the entry cost that would

[3]Shadow banks are financial institutions other than commercial banks which are part of the Federal Reserve.

[4]Our models are very different from Diamond and Dybvig (1983) in the sense that we do not have their sequential service constraint. We have not investigated whether the challenges that we discuss in this paper apply to Diamond and Dybvig (1983).

[5]We differentiate a panic run from a 'run'. For example, Gorton (2018) defines a run as "...an information event in which holders of short-term debt no longer want to lend to banks because they receive information leading them to suspect the value of the backing for the debt, so they run." We refer to this kind of run as a 'fundamentals run'. See also Allen and Gale (1998) in which a bank run is fundamental because it occurs when depositors realize the returns on bank investments are low (see also Goldstein and Pauzner (2005)). For additional discussion of such a run see footnote 10.

[6]We interpret the term, debt capacity, as synonymous with leverage.

prevent them from doing so.

We suspect that an economically plausible way to explain why net worth is slow to enter a financial sector in crisis can be found in Chari and Jagannathan (1988). This type of model is in effect a hybrid of the two hypotheses mentioned above. In the model, agents cannot differentiate in real time whether the crisis is is due to a bad shift in fundamentals or to a sunspot (i.e., panic). This sort of confusion has some empirical appeal because even now there is disagreement over the role of fundamentals in the housing market versus financial panic in causing the Great Recession.[7]

To build confidence in the robustness of our results, we consider three models. One is essentially the infinite horizon model in Gertler and Kiyotaki (2015) (GK) and the second is a three-period version of GK. The third model is the one with a very different set of financial frictions, a version of the one proposed in Christiano and Ikeda (2016) (CI). The household side of all three models is the same as in the model in GK. The vision in all three models is that the financial system is segmented into various sub sectors which specialize in different types of assets. The focus is on one particular sub sector henceforth called the *banks*, which for the sake of specificity (but at some risk of oversimplifying) could be thought of as the set of financial institutions that finance the purchase of homes. The remainder of the financial system, as well as the rest of the economy, is handled in a reduced form way by lumping it into a *household*.[8] The people in charge of the banks are uniquely endowed with the ability to make the banks' assets productive. Banks issue short term deposits to finance their assets which are illiquid and long-lived (i.e., housing). When creditors to the banking industry refuse to roll over their bank deposits, the only way banks have to honor their short term liabilities is to sell assets. Because the crisis hits the entire banking sector, this means that bankers must sell their assets to outsiders (in the model, households) who have less expertise in running the assets and so are willing to only pay low prices. The consequence of the resulting fire sales is that bankers' net worth is wiped out as the value of their assets drops below their liabilities. This results in the misallocation of capital away from people (bankers) who know how to make capital productive, and towards other

[7]See, for example, the report of the congressional inquiry into the causes of the financial crisis, (2011). In particular, note the difference between the conclusion of the report and the minority dissent. The latter concluded that most of the troubles were the outcome of government housing policies that, because they were poorly designed, led to an unsustainable rise in residential investment. For additional discussion of other fundamentals interpretations to the panic run hypothesis, see footnote 10.

[8]See Gertler et al. (2020a), which places additional structure on the remainder of the financial system.

people who are less able to do so. The general idea that aggregate fluctuations can be induced by economic shocks which reallocate resources away from a group of economic agents has a long and venerable history in economics.[9]

There are several dimensions of the panic narrative that deserve closer scrutiny.[10] However, our primary focus is on the apparent dependence of the panic scenario on extreme assumptions about the ability to move net worth into a sector of the financial system experiencing a crisis. As noted above, we examine these costs in several models. For example, we find that the present value entry costs to bring \$1 of net worth into a crisis is at least \$378 for the three period version of the GK model, \$151 for the infinite period version of the GK model and \$7.8 dollars for the CI model.[11] These costs all

[9]An influential example is Bernanke (1983)'s reformulation of Fisher (1933)'s debt deflation hypothesis (the idea is further developed in Bernanke and Gertler (1990)). In this view, a fall in the net worth of borrowers (say, because there is deflation and debt is fixed in nominal terms) undercuts the people that drive investment and employment.

[10]There are two challenges to the GK narrative that we do not consider. The first is raised by Acharya et al. (2013) who point to evidence that appears to diminish the role of panic and increases the role of fundamentals in the financial crisis. According to Acharya et al. (2013) and Covitz et al. (2013, p.845) most banks had commercial bank sponsors (in GK, these are part of the household). Sponsors originated and securitized the assets, Asset Backed Securities (ABS), which they then sold to banks, which paid for them by issuing short term Asset Backed Commercial Paper (ABCP). Most of the sale contracts between sponsors and banks contained explicit liquidity guarantees which required the commercial banks to buy back ABS at par from the banks that they sponsored, as long as assets in the ABS had not begun to go into default. According to Acharya et al. (2013, p. 520) the precise definition of 'begun to go into default' implied that if securitized assets began to deteriorate for fundamental reasons, the liquidity guarantee remained in place long enough that ABCP "...almost always matures before the securitized assets are declared in default". In this way, when market participants understood that securitized assets were deteriorating, they decided not to roll over ABCP knowing that sponsoring banks guaranteed the liquidity required to pay off maturing ABCP. Bank creditors guessed that the quality of securitized assets would continue to deteriorate, so that liquidity guarantees would expire on any ABCP that was rolled over, which would then go into default. According to Acharya et al. (2013, p. 520), "In fact, throughout the entire run, no outside investors in ABCP suffered a default under a liquidity guarantee." Thus, a key component of the panic scenario, that individual bank creditors feared a liquidity crisis resulting from a general failure to rollover bank liabilities, is hard to square with the explicit liquidity guarantees from sponsoring banks (many of which had the backing of the FED) to banks. The second challenge to the panic narrative has to do with the assumption used to produce fire sales during a panic. GK assume that households are 'less efficient' than bankers at managing bank capital. Acharya et al. (2013) argue that the relative 'inefficiency' of commercial bank sponsors is that they faced capital requirements while banks did not. Regulators designed the tax implied by capital requirements to get commercial banks to internalize the risks on their portfolios. When the commercial banks moved ABS off their own balance sheets and onto the balance sheets of banks, they avoided the capital requirement tax but held onto the risk ('regulatory arbitrage'). In the GK model a transfer of assets from households to banks is welfare-improving because it puts the capital in more efficient hands. However, under the regulatory arbitrage interpretation this transfer would clearly be inefficient as long as the capital requirements are properly designed.

[11]See sections 2.6.2 and 3.4.2 for the three period GK and CI models, respectively. See the online Appendix, section C.2.3 for the infinite horizon GK model.

seem implausibly high.[12]

We now briefly describe the differences between the GK (*running away*) and CI (*hidden effort*) models, which have to do with the banks. Both models share the property that banks finance long-term assets with short term liabilities. But, that is not enough to guarantee that a panic annihilation run can occur. As noted above, the logic of a panic run is that a belief that aggregate deposits are zero implies that banks must fire-sale assets to pay off past liabilities, and this wipes out banker net worth. To guarantee that zero deposits and bankrupt banks is an equilibrium requires that the individual depositor, contemplating this scenario, best responds by not rolling over her own deposits. So far, we have provided no reason that a depositor would not roll over. The loss to creditors in the fire sale is a sunk cost, and new credit could be used by the bank to acquire new and productive assets.[13] An additional financial friction, beyond maturity mismatch, is required for a panic run to exist in equilibrium.

The financial friction adopted by the running away and hidden effort models differ. In the case of the running away model, it is assumed that bankers have the opportunity to divert (run away with) a fraction of bank assets. When a banker has no net worth (assuming the appropriate conditions are satisfied) then a household knows that the banker would run away with any deposit he gives her. The household best responds by not providing any deposits. The CI model adopts a different financial friction. This model assumes the banker influences the return on her assets by exerting an effort that is not observed. This effort can be interpreted as a combination of good vetting of projects, as well as banker control over projects already underway via the use of income covenants on loan contracts.[14] In the CI model, a run is potentially an equilibrium because a bankrupt banker who has no skin in the game can only commit to a minimal level of effort. As a result, potential depositors turn to other, higher-paying, options and best respond by setting deposits in banks to zero.

Our analysis allows us to make an observation about macro prudential policy which may be of independent interest. A feature of the models we consider is that banks do not internalize the impact of their leverage decision on the probability of a run. This

[12] Jermann and Quadrini (2012) argue that equity issuance costs are very low for non-financial firms, even during the financial crisis. However, these results do not compare directly with ours because, among other things, their results are for non-financial firms.

[13] We follow GK in assuming that if a banker with zero equity issues deposits, these can be applied to the purchase of new bank assets and no part of these deposits is used to pay off 'old' deposits.

[14] See, for example, Greenwald et al. (2019), Lian and Ma (2021) and Chodorow-Reich and Falato (2022).

naturally leads to considering the welfare effects of a regulatory cap on bank leverage. We find that the Ramsey-optimal leverage restriction is associated with two equilibria, the good (Ramsey) equilibrium and a bad equilibrium. This finding reflects the two effects of a credit restriction: the direct effect (*quantity channel*) of a fall in the quantity of assets reduces leverage but the fall in assets simultaneously reduces the price of assets and, other things the same, that raises leverage (*price channel*). If the strength of the price channel is greater for a larger fall in assets, then it is not surprising that two equilibria could be associated with a given leverage constraint: one equilibrium in which quantity and price fall by a small amount and another in which quantity and price fall by a large amount. The first of these is the Ramsey equilibrium and the other is an equilibrium in which welfare is lower than it is in the unregulated economy. Thus, there is a non-trivial implementation problem associated with leverage restrictions. We illustrate one way to resolve the implementation problem.

Section 2 reports our results for the three-period version of the GK model. The infinite horizon version results are qualitative similar and so to save space we report these in section C in the online Appendix. Section 3 reports our analysis of the CI model. Concluding remarks appear in Section 4.

2 Rollover Crisis: Three-Period Version of Running-Away Model

This section first describes a three-period, $t = 0, 1, 2$, version of the infinite-horizon model in GK. We then look at the implications of the model for the design of macro-prudential policy. After that, section 2.6 summarizes our findings for the fragility of equilibria to assumptions about entry. We also explain how the infinite-horizon version of the model, the one studied in GK, implies even greater fragility.[15]

The economy is populated by a representative, competitive household which contains a worker and a unit measure of bankers.[16] The assumption that bankers and workers live in the same household simplifies the welfare analysis we do when we study macroprudential policy. Moreover, the fact that a banker is atomistic relative to the household simplifies the analysis by implying that the banker takes as given the house-

[15]To save space, the details for the infinite horizon model are reported in Section C in the online Appendix.

[16]By a 'competitive' agent we mean one that takes all market prices and rates of return as given.

hold's intertemporal marginal rate of substitution in consumption.[17] In $t = 0$, bankers offer short-term (one-period) deposits and use the combination of the deposits and their own resources to purchase long-term (three-period) assets, capital. Because the assets are long-term and liabilities are short-term, bankers find it convenient to pay off at least some of their short term liabilities in $t = 1$ by rolling over short term debt. This arrangement can work, but it is also vulnerable to systemic collapse in $t = 1$ in the scenario in which depositors refuse to roll over bank liabilities (the *annihilation run* state). There is another state in period $t = 1$ in which banks have positive net worth and are able to rollover their liabilities and we refer to this as the *no run* state. These two states correspond to the two sunspot states analyzed in GK.[18] In Section 2.6.1 below we show that there exists a third period $t = 1$ sunspot state, which we call a *partial run* state. In this section we limit our attention to the annihilation and no run states. All exogenous fundamentals are deterministic.

2.1 Bankers

Bankers are instructed by their households to 'play by the rules' and maximize the present discounted value of profits in period 2. Alternatively, if by running away with assets in periods $t = 0, 1$ the banker can bring home even more profits, the household encourages its banker to do so. We make assumptions which guarantee that in equilibrium, bankers choose to play by the rules. Random matching between depositors and banks ensures that the probability a household places deposits in its own bank is zero.

In the first section we describe notation that is common across states and dates. Then, we discuss the banker's only substantive decisions, which occur in periods $t = 0, 1$. We begin by discussing the banker problem in each of the two possible states in period 1. Then, we consider the problem of the banker in the first period, $t = 0$.

[17]GK assume bankers are risk neutral and that they live separately from households, so we do not study macroprudential policy in that model. However, Section C in the online Appendix shows that our results on the fragility of equilibria are similar in the infinite and finite horizon version of the running away model.

[18]In the analysis of GK the state in any particular period is composed of a sunspot as well as the realization of an exogenous shocks to fundamentals. In this paper, fundamentals are deterministic. Because the agents in GK can differentiate between fundamental and sunspot shocks, our assumption that fundamentals are deterministic simply lets us focus on the pure panic part of a stochastic equilibrium.

2.1.1 Notation and Definitions

In equilibrium, each banker is identical.[19] The representative banker starts each period, $t = 0, 1, 2$, with assets, k_{t-1}^b, and liabilities, $R_{t-1}d_{t-1}$, acquired in the previous period. Here, R_{t-1} denotes the period $t-1$ competitively determined gross return on period $t-1$ bank deposits, d_{t-1}.

A bank's assets inherited from the previous period, k_{t-1}^b, are productive capital with exogenous, deterministic marginal productivity, Z_t, in period t. The model abstracts from frictions between a banker and the firm she funds. For notational convenience we follow GK in assuming that a banker simply operates the capital herself. For the most part, this assumption is benign. However, the assumption complicates the empirical interpretation of 'bank leverage' because here the bank is in effect a combination of a banker and the firms she lends to.

The banker's beginning-of-period t net worth, N_t, is as follows:

$$N_t = \max\left\{0, (Z_t + Q_t)\, k_{t-1}^b - R_{t-1}d_{t-1}\right\}, \tag{1}$$

where Q_t denotes the period t competitive price of capital.[20] The time t value of assets is equal to the time t value of liabilities, including net worth:

$$Q_t k_t^b = N_t + d_t, \tag{2}$$

for $t = 0, 1$. It is convenient to rewrite banker net worth (equation 1) in terms of leverage, ϕ_{t-1}:

$$N_t = \max\left\{0, \left(R_t^k - R_{t-1}\right)\phi_{t-1} + R_{t-1}\right\} N_{t-1}, \tag{3}$$

[19]Actually, our analysis could accommodate the assumption that bankers are heterogeneous if we assumed they all have the same leverage in the pre-period, $t = -1$. In this case we can interpret our analysis as involving the *average* value of deposits, assets and net worth of bankers.

[20]Our equation (1) corresponds to GK(eq 11), though we use slightly different notation. In our notation, R_{t-1} corresponds to $\bar{R}_t$ in GK (eq 5). According to GK, $\bar{R}_t$ denotes the non-contingent promise made in $t-1$ to pay gross rate of return, $\bar{R}_t$ if no bank run occurs in period t. Like our R_{t-1}, $\bar{R}_t$ in GK is not contingent on the period t realization of shocks. The return paid to depositors in GK in case there is a run is $x_t\bar{R}_t$, where x_t has the properties, $(Z_t + Q_t)\, k_{t-1}^b - R_{t-1}x_t d_{t-1} = 0$ and $0 \leq x_t < 1$. The object, R_t in GK (eq. 11) corresponds to our $R_{t-1}x_t$. Because of the property of x_t, the expression for net worth in our equation (1) coincides with GK (eq 11). In our analysis, our representation of N_t is slightly more convenient than the representation in GK (eq 11).

for $t = 0, 1, 2$, where

$$R_t^k \equiv \frac{Z_t + Q_t}{Q_{t-1}}, \phi_t \equiv \frac{Q_t k_t^b}{N_t}. \tag{4}$$

Because capital only generates a payoff in the following period, its price in period 2, Q_2, is zero.

At the start of periods $t = 0, 1$, the banker truthfully announces her deposit and asset decisions, $d_t \geq 0$ and $Q_t k_t^b \geq 0$, subject to equation 2.[21] The banker that chooses to 'play by the rules' seeks to maximize the discounted profits that she transfers to her household in period 2 (in equilibrium, all bankers 'play by the rules'). Such a banker attempts to make good on her obligation to repay $R_t d_t$ to depositors at the start of the next period. A banker also has the option to run away in period t with θ_t of her assets, $Q_t k_t^b$. If the banker runs away, then the remaining $(1 - \theta_t) Q_t k_t^b$ assets are destroyed. A banker who chooses to run away converts $\theta_t Q_t k_t^b$ into period t consumption goods in the goods market and brings the goods home to be consumed by her household. After running away with assets, a banker ceases to be a banker. So, the present discounted value of absconding with assets in period t is $\theta_t Q_t k_t^b$. Because an individual banker is atomistic, she takes as given the household asset pricing kernel when she makes her running-away decision.[22]

When depositors contemplate placing a deposit in a bank, they know the amount of total deposits, d_t, that the bank intends to take. So, the depositor can in effect see which bank intends to run away and which does not. Depositors avoid banks that intend to run away because households receive a zero gross return from such a bank and households have access to a positive return by directly investing in capital (more on this later). Thus, no depositor would place a deposit in a bank if that bank's deposit decision violated the following constraint:

$$\theta_t Q_t k_t^b \leq V_t, \tag{5}$$

where V_t represents the time t value of being a banker. The fact that depositors avoid banks which violate equation (5) implies that a decision by a banker to violate equation

[21] Without the assumption of truthful revelation of d_0 the banker problem would not be well defined. The banker would report a low value of d_0 and then bring in unbounded revenues by setting d_0 to an unboundedly large number. In practice, the assumption of truthful revelation is not without merit because potential creditors do (imperfectly) investigate the balance sheet of potential borrowers before deciding how much to lend.

[22] We assume that the individual banker believes all other bankers play by the rules. This belief is correct in equilibrium.

(5) is tantamount to choosing $d_t = 0$. The banker already has the opportunity to set $d_t = 0$ when it plays by the rules. So, there is no loss in simply assuming that the banker voluntarily self-imposes equation (5), which we refer to as the *incentive constraint*.

2.1.2 Period $t = 1$ Banker Problem: Annihilation Run State

We denote values of variables in the period $t = 1$ annihilation run state by an asterisk, '*'. In this state, the price of capital, Q_1^*, is so low that

$$(Z_1 + Q_1^*)\, k_0^b - R_0 d_0 < 0,$$

so that, according to equation (1), $N_1^* = 0$. In this case, households' period $t = 0$ deposits in effect are converted into equity and the value of assets are distributed to them. That is, depositors receive $x R_0 d_0$, where x denotes the recovery rate on deposits and

$$x = \frac{R_1^{*,k} Q_0 k_0^b}{R_0 d_0}. \tag{6}$$

For model parameterizations that we consider, $x < 1$ in the annihilation run state (see Proposition 1 below).[23]

With $N_1^* = 0$ equation (2) implies that $Q_1^* k_1^{*,b} = d_1^*$. Let $V_1(d_1^*)$ denote the discounted value of net revenues in period 2, conditional on the bank issuing deposits, $d_1^* \geq 0$:

$$V_1(d_1^*) = \beta m_2^* \left[R_2^{*,k} - R_1^* \right] d_1^*.$$

Here, m_2^* denotes $u'(c_2^*)/u'(c_1^*)$ where c_2^* and c_1^* denote consumption of the representative agent in the period $t = 1$ annihilation run state and c_2^* denotes consumption in $t = 2$, after the annihilation run state.[24] Because the banker is atomistic within the household, it takes m_2^* as given. Also, β denotes the representative household's discount factor. Thus, the banker problem can be written as $V_1 = \max_{d_1^*} V_1(d_1^*)$ subject to equation (5), or,

$$\begin{aligned} V_1 &= \max\nolimits_{d_1^* \geq 0} \beta m_2^* \left[R_2^{*,k} - R_1^* \right] d_1^*, \\ \text{s.t.} \quad & \theta_1 d_1^* \leq \beta m_2^* \left[R_2^{*,k} - R_1^* \right] d_1^*. \end{aligned} \tag{7}$$

Consistent with equation (4), $R_2^{*,k} \equiv Z_2/Q_1^*$. The second term in equation (7) is the banker incentive constraint (see equation (5)).

[23]Parameter values are discussed in subsection 2.4 below.

[24]Later (see equation (30) below), we show that $c_2 = c_2^*$ is an equilibrium condition of the model.

The highest level of deposits consistent with the incentive constraint in equation (7) is $d_1^* = 0$ if, and only if, $\theta_1 > \beta m_2^* \left[R_2^{*,k} - R_1^* \right]$, where $\beta m_2^* R_1^* = 1$.[25] Rearranging, we find that the condition which guarantees $d_1^* = 0$ to be the only decision consistent with the incentive constraint is:

$$R_2^{*,k} < (1 + \theta_1) R_1^*. \tag{8}$$

Equation (8) is satisfied for the parameter values we work with in our model.

2.1.3 Period $t = 1$ Banker Problem: No Run State

We now consider the period $t = 1$ no run state in which Q_1 is high enough that $N_1 > 0$ in equation (1). It is convenient to scale variables by $N_1 > 0$. In particular, let ϕ_1 denote leverage, $(N_1 + d_1)/N_1$, and let ψ_1 denote discounted beginning-of-period 2 profits, scaled by N_1. The banker problem in the no run equilibrium is:

$$\begin{gathered} \psi_1 = \max_{\phi_1 \geq 1} \psi_1 (\phi_1) \\ \theta_1 \phi_1 \leq \psi_1 (\phi_1), \end{gathered} \tag{9}$$

where

$$\psi_1 (\phi_1) = \beta m_2 \left[\left(R_2^k - R_1 \right) \phi_1 + R_1 \right]. \tag{10}$$

Here, m_2 denotes $u'(c_2)/u'(c_1)$ where c_2 and c_1 denote consumption of the representative agent in the period $t = 1$ no run state and c_2 denotes period $t = 2$ consumption. The banker takes m_2 as exogenous. To understand equation (9), let $V_1(d_1; N_1)$ denote unscaled discounted profits in the no run period $t = 1$ state. Then,

$$V_1 (d_1; N_1) = \beta m_2 N_1 \psi_1 \left(\frac{N_1 + d_1}{N_1} \right) = \beta m_2 \left[R_2^k (N_1 + d_1) - d_1 R_1 \right].$$

So, the problem in equation (9) is equivalent to the following perhaps more intuitive representation of the banker problem: $\max_{d_1} \beta m_2 \left[R_2^k (N_1 + d_1) - d_1 R_1 \right]$ subject to $\theta_1 Q_1 k_1^b \leq V_1 (d_1; N_1)$.

We consider model parameter values which satisfy the restriction, $R_2^k > R_1$, so the banker finds it optimal to set ϕ_1 to the maximum allowed by the participation

[25] In equilibrium, the volume of deposits traded is zero in the $t = 1$ annihilation run. However, if they were traded, then the return on deposits, R_1^*, would satisfy $\beta m_2^* R_1^* = 1$, where $m_2^* = u'(c_2^*)/u'(c_1^*)$ and c_2^* is period 2 consumption given that an annihilation run occurred in $t = 1$.

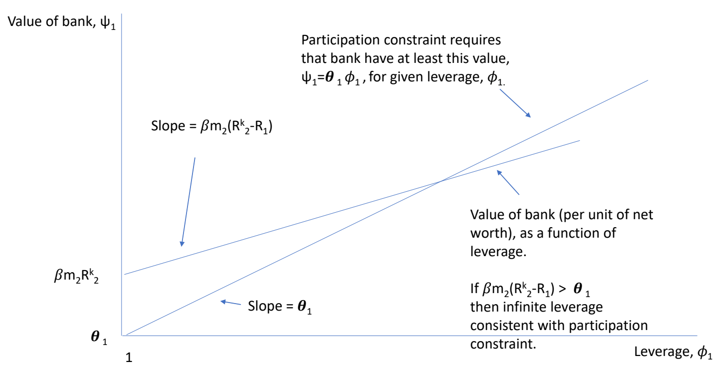
constraint. To ensure that this maximum is finite we also require $\beta m_2 \left(R_2^k - R_1\right) < \theta_1$,which implies that the slope of ψ_1 is not only positive, but flatter than θ_1. If, in addition, the intercept of ψ_1 exceeds the intercept of the incentive function, we know that the maximum allowed by the incentive constraint has the economically interesting property, $1 < \phi_1 < \infty$. To ensure the intercept condition, we require $\beta m_2 R_2^k > \theta_1$. Below, we show that household optimality requires $\beta m_2 = R_1^{-1}$ (see equation (26)), which, given $R_2^k > R_1$, guarantees the intercept condition. We summarize the restrictions that guarantee $1 < \phi_1 < \infty$ as follows:

$$R_1 < R_2^k < (1 + \theta_1) R_1, \tag{11}$$

where the second inequality is a representation of $\beta m_2 \left(R_2^k - R_1\right) < \theta_1$, given the household optimality condition. Under the restrictions in equation (11), we have

$$\phi_1 = \frac{R_1}{(1 + \theta_1) R_1 - R_2^k}, \; 1 < \phi_1 < \infty. \tag{12}$$

Figure 1: Condition for Finite Leverage



Equilibrium scaled profits and leverage are not well defined in the annihilation state considered in the previous subsection because the denominator, net worth, is zero in that state. Still, for the banker's $t = 0$ problem to be well defined, she must be able to contemplate what would happen for off-equilibrium values of ϕ_0. This includes the scenario in which she sets $\phi_0 = 1$. She would want to do that if the reward of having positive net worth in the annihilation run state were large enough, as when P or ψ_1^* are large. When we consider the choice of ϕ_0 in the next subsection, it is convenient to have defined profits per unit of net worth, ψ_1^*, and leverage, ϕ_1^*, in the annihilation state.

We adopt the following assumption:

Assumption 1. *an incumbent banker with positive net worth in an annihilation run state cannot issue deposits, but can invest its net worth with return $R_2^{*,k}$.*

By the modifier, 'incumbent banker', we mean a banker that was operating in the previous period. In the three-period model all bankers in period 1 are incumbent. The modifier matters when we consider the infinite horizon version of this model in the section C of the online Appendix. Under Assumption 1, equation 10 implies

$$\phi_1^* = 1,\ \psi_1^* = \beta m_2^* R_2^{*,k}. \tag{13}$$

One interpretation of Assumption 1 follows GK(page 2024). They suppose that when almost all banks have zero net worth, households have a hard time differentiating any small number of bankers that may have positive net worth from the others.[26] We adopt Assumption 1 because without it, we had difficulty finding an interesting calibration for the model in which P is large. Major financial crises are rare in the US and section 2.4 below explains why we nevertheless think it is interesting to study a model calibrated to have a high value of P.

2.1.4 Period $t = 0$ Banker Problem

We only consider the equilibrium in period $t = 0$ in which $N_0 > 0$. The objective of bankers in period 0 is the present value of period $t = 2$ profits. Expressed as a function

[26]Our assumption here is slightly different from GK(page 2024). As we discuss in section C of the online Appendix, in their infinite horizon economy a small mass of net worth enters the banking system via 'newborn' bankers. GK(page 2024) assume that these newborns cannot issue deposits and *also* do not invest their net worth in an annihilation state. In our three-period model we assume that in the off-equilibrium event that some bankers have net worth in an annihilation run state, they do invest their own net worth. We believe that this difference with GK is inessential.

of period 0 leverage, ϕ_0, and scaled by N_0, the banker's objective is:[27]

$$\psi(\phi_0) = \beta m_1 (1-P)\psi_1 \left[\left(R_1^k - R_0\right)\phi_0 + R_0\right] + \beta m_1^* P \psi_1^* \times \max\left\{0, \left[\left(R_1^{*,k} - R_0\right)\phi_0 + R_0\right]\right\}. \tag{14}$$

Here, ψ_1 and ψ_1^* denote period 1 discounted profits, scaled by period 1 net worth.[28] In equation (14), m_1 and m_1^*, are functions of consumption analogous to the definition of m_2 and m_2^* above. The banker treats these as exogenous. The objects in square brackets in equation (14) are beginning-of-period $t = 1$ net worth, scaled by N_0 in the two $t = 1$ states. Because ψ_1 and ψ_1^* are functions of market prices, the banker views these as independent of her choice of ϕ_0. The function, $\psi : [1, \infty) \rightarrow \mathbb{R}$, is piecewise linear and continuous in ϕ_0 because of the max operator in equation (14) and is represented by the solid line with a kink in Figure 2. The parameter values we choose for our model imply that the equilibrium has the following (economically reasonable) properties:

$$R_1^{*,k} - R_0 < 0,\ R_1^k - R_0 > 0. \tag{15}$$

With these properties the piecewise linear ψ function has a positive slope to the right of the kink, and might have a negative slope to the left of the kink (see Figure 2).

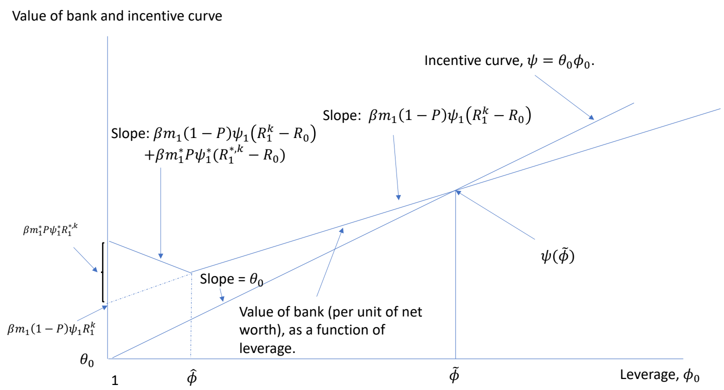
[27] To see how Equation (14) is derived, consider the scaling of the first term after the equality:

$$\beta(1-P)m_1 \frac{V_1}{N_1}\frac{N_1}{N_0} = \beta(1-P)m_1\psi_1\left[\left(R_1^k - R_0\right)\phi_0 + R_0\right],$$

where equation (3) was used (taking into account that $R_1^k > R_0$) and also, $\psi_1 \equiv V_1/N_1$.

[28] The object, ψ_1 is $\psi_1(\phi_1)$, where $\psi_1(\cdot)$ and ϕ_1 are defined in equations 10 and (12), while ψ_1^* is defined in equation (13).

Figure 2: Period $t = 0$



Notes: (i) the piecewise linear curve with kink at $\phi_0 = \hat{\phi}$ represents the value of the bank, ψ, as a function of leverage, $\phi_0 \geq 1$; (ii) the intercept of ψ, $\psi(1)$, lies below the crossing of $\psi(\phi_0)$ with the incentive curve, $\theta_0 \phi_0$, which occurs at $\phi_0 = \tilde{\phi}$; (iii) the object, $\hat{\phi}$, denotes the cutoff value for ϕ_0, below which bank net worth is positive during an annihilation run in period 1 (see equation (16)).

Next, we study the constraint on the banker's leverage choice, $\theta\phi_0 \leq \psi(\phi_0)$ (see the solid straight line, $\theta\phi_0$,in Figure 2, referred to as the *incentive curve*). Let $\hat{\phi}$ denote the level of leverage associated with the kink in Figure 2:[29]

$$\hat{\phi} = \frac{R_0}{R_0 - R_1^{*,k}} > 1. \tag{16}$$

The value of $\hat{\phi}$ is strictly bigger than unity because of the first assumption in equation (15). Evaluating ψ at $\hat{\phi}$, we obtain:

$$\psi\left(\hat{\phi}\right) = \beta m_1 (1-P)\,\psi_1 \left(R_1^k - R_1^{k,*}\right)\hat{\phi}. \tag{17}$$

From equation (17) we infer that $\theta_0\hat{\phi} < \psi\left(\hat{\phi}\right)$ if

$$\beta m_1 (1-P)\,\psi_1 \left(R_1^k - R_1^{k,*}\right) > \theta_0. \tag{18}$$

If the slope of ψ for $\phi_0 > \hat{\phi}$ is less than θ_0, that is,

$$\beta m_1 (1-P)\,\psi_1 \left(R_1^k - R_0\right) < \theta_0, \tag{19}$$

[29]The object, $\hat{\phi}$, is the value of ϕ_0 that solves $\left(R_1^{*,k} - R_0\right)\phi_0 + R_0 = 0$. This object is discussed in GK(Appendix A). Here and throughout we take for granted that gross rates of return are are positive, as is required for equilibrium.

then we know that ψ crosses the incentive curve at a finite point, denoted $\tilde{\phi}$, which is greater than $\hat{\phi}$. We conclude that:

Lemma 1. *Let $\tilde{\phi}$ denote the largest value of ϕ consistent with the banker incentive constraint. If equations (18) and (19), and the first condition in equation (15) hold, then*

$$1 < \tilde{\phi} < \infty.$$

The banker problem in $t = 0$ is:

$$\psi_0 = \max_{\phi_0 \geq 1} \psi(\phi_0), \text{ subject to } \theta\phi_0 \leq \psi(\phi_0). \tag{20}$$

Under the conditions of Lemma 1 this problem is well defined. For the model to be economically interesting, we require that parameters be such that the banker chooses to issue a positive amount of deposits in period $t = 0$, i.e., $\phi_0 > 1$ solves the banker problem. When P or ψ_1^* are large, so that the function, $\psi(\phi_0)$, has a 'V' shape then, $\phi_0 = 1$ might be the best choice of the banker (see equation 14). The restriction on market rates of return that guarantees $\phi_0 = \tilde{\phi}$ is the solution to the banker problem is that the value of ψ at $\phi_0 = 1$ be less than $\psi\left(\tilde{\phi}\right)$:[30]

$$\beta m_1 (1 - P) \psi_1 R_1^k + \beta m_1^* P \psi_1^* R_1^{*,k} < \psi\left(\tilde{\phi}\right). \tag{21}$$

We require that equation (21) be satisfied. Note that $\tilde{\phi} > \hat{\phi}$ implies that the second argument to which the 'max' operator in equations (1) and (3) is applied is strictly negative. It follows that in the annihilation state $x < 1$, where x is defined in equation (6). We summarize our findings in the form of a proposition:[31]

Proposition 1. *Suppose $N_0 > 0$ and that the equilibrium of the model satisfies the conditions of Lemma 1 and equations (8), (11) and (21). Then, $\phi_0 = \tilde{\phi}$ is the unique solution to the banker problem, where $1 < \tilde{\phi} < \infty$. Moreover, $x < 1$ in equation (6).*

[30] GK(Appendix A) confront an analogous issue in the infinite horizon model and they adopt a stronger assumption than we do, to guarantee that the analog of $\tilde{\phi}$ is the optimal decision of the banker. They assume that the slope of ψ is positive for $\phi_0 < \hat{\phi}$. We were not able to find an economically interesting parameterization for our model that satisfies this constraint and implies a value of P that is significantly greater than zero. We return to this observation in subsection 2.4 below. For additional discussion in the infinite horizon version of the running away model, see sections C.1 and C.3 in the online Appendix.

[31] The proposition does not list the second restriction in equation (15) among its assumptions. This is because the second restriction in equation (15) is implied by equation (21).

2.1.5 Sunspot in Period $t = 1$

We have described two period $t = 1$ states. Following GK, we denote the probability of the annihilation run state by P. The latter probability is assumed to be a function of X, the aggregate over all banks' recovery ratios defined in equation (6). Then,

$$P = \max\left[0, 1 - X\right]. \tag{22}$$

One interpretation of equation (22) is that it represents a behavioral assumption about a sunspot which causes agents to coordinate on the no run or annihilation run state. The assumption that a general run on all banks is more likely if the recovery rate in the event of a run is low may have intuitive appeal. The fact that P is a function of the *aggregate* recovery ratio is the source of a key externality in the model. Obviously, X - and, hence, P - is a consequence of each bank's deposit decision, yet each banker rationally treats P as exogenous to her own decision. Another interpretation of equation (22) is that it is a reduced form representation of a global games approach. For a discussion of this interpretation see GK(p. 2031).[32]

2.2 Households

Households live three periods. In periods $t = 0, 1, 2$ they can either make one-period deposits in banks or they can purchase capital. In a first-best scenario, households would not hold capital because they are inefficient at managing it. However, because of the financial frictions in the model, it is possible that households might hold capital anyway. The household's lifetime utility is given by

$$u\left(c_0\right) + \beta E_0\left[u\left(c_1\right) + \beta u\left(c_2\right)\right], \tag{23}$$

where c_t denotes consumption in period $t = 0, 1, 2$, and the only uncertainty is whether or not there is a run in period $t = 1$. The household's budget constraints in periods $t = 0, 1$ are:

$$Q_t\left(k_t^h - k_{t-1}^h\right) + c_t + d_t + f\left(k_t^h\right) \leq x_t R_{t-1} d_{t-1} + Z_t k_{t-1}^h + y_t, \tag{24}$$

[32] See also Ikeda and Matsumoto (2021).

where $x_t = 1$ for $t = 0$ and in the no run state in period $t = 1$. In the annihilation run state in period $t = 1$, x_1 corresponds to x in equation (6). In equation (24) y_t denotes exogenous income in period t, and k_{t-1}^h and d_{t-1} denote beginning-of-period t capital and deposits, respectively, held by the household. Also, $x_t R_{t-1} d_{t-1}$ denote the interest and principal on d_{t-1} paid by banks to the household in period t. Finally, $k_t^h - k_{t-1}^h$ denotes the quantity of capital purchased at the competitive price, Q_t, and $f\left(k_t^h\right)$ denotes increasing, differentiable and convex management costs associated with the household's end-of-period t stock of capital. This management cost is the reason for the fire sale drop in the price of capital in the annihilation run state.[33] The household's budget constraint in the final period is:

$$c_2 \leq R_1 d_1 + Z_2 k_1^h + N_2,$$

where N_2 denotes profits transferred in lump-sum by bankers (see equation (7)).

The household optimality conditions associated with $t = 0$ and the no run state in period 1 are:

$$u'\left(c_0\right) = \beta\left[\left(1-P\right)u'\left(c_1\right) + Pu'\left(c_1^*\right)x\right]R_0 \tag{25}$$

$$u'\left(c_1\right) = \beta u'\left(c_2\right)R_1. \tag{26}$$

Households hold no deposits in the period $t = 1$ annihilation run state. The optimality conditions associated with the capital decision in period 0 is:

$$\begin{gathered} u'\left(c_0\right) = \beta\left(1-P\right)u'\left(c_1\right)\left(\frac{Z_1+Q_1}{Q_0+f'\left(k_0^h\right)}\right) \\ +\beta Pu'\left(c_1^*\right)\left(\frac{Z_1+Q_1^*}{Q_0+f'\left(k_0^h\right)}\right) + \mu u'\left(c_0\right) \\ \mu k_0^h = 0, \quad \mu, k_0^h \geq 0, \end{gathered} \tag{27}$$

where μ denotes the multiplier on the non-negativity constraint on capital. The optimality condition associated with capital in the no run period 1 state is:

$$\begin{gathered} u'\left(c_1\right) = \beta u'\left(c_2\right)\frac{Z_2}{Q_1+f'\left(k_1^h\right)} + \nu \times u'\left(c_1\right) \\ \nu k_1^h = 0, \quad \nu, k_1^h \geq 0 \end{gathered} \tag{28}$$

[33] The function, f, is the reason that households are less efficient at holding capital than banks. For a critical discussion of this assumption, see the 'second challenge' for the GK model in footnote 10 above.

We include multipliers on the non-negativity constraints on capital because these constraints could in principle be binding in this model.

In the period 1 annihilation run state, banks sell all their capital, so that $k_1^h = 1$. For this to be consistent with household optimality, we require

$$u'(c_1^*) = \beta u'(c_2^*) \frac{Z_2}{Q^* + f'(1)}. \tag{29}$$

2.3 Market Clearing and Aggregate Conditions

Total capital in the economy is fixed at unity. Some of the capital is managed by bankers (k_t^b), the rest is managed by the households directly ($k_t^h = 1 - k_t^b$). Total resources available for consumption include exogenous income (y_t), plus the payoff on capital (Z_t), minus the costs of managing capital faced by households ($f(k_t^h)$). The resource constraints are:

$$\begin{aligned} c_t + f\left(k_t^h\right) = Z_t + y_t : \qquad t = 0, 1 \\ c_1^* + f(1) = Z_1 + y_1 \\ c_2 = c_2^* = Z_2 : \qquad t = 2 \end{aligned} \tag{30}$$

The first equation describes the resource constraint in period $t = 0$ and the no run state in $t = 1$. The second is the resource constraint in the annihilation run state in $t = 1$. The third is the resource constraint in period 2. From these equations we see that the cost of an annihilation run is that households hold 'too much' capital in the period of the run, $t = 1$.

2.4 Competitive Equilibrium and Parameter Calibration

We define a competitive equilibrium in our model economy as follows:

Definition 1. A competitive equilibrium is a set of prices and returns,
$\left\{R_0, R_1, R_1^*, Q_0, Q_1, Q_1^*, R_1^k, R_1^{*,k}\right\}$, and quantities,
$\left\{x, P, k_0^b, k_1^{*,b}, k_1^b, c_0, c_1, c_1^*, c_2, c_2^*, d_0, d_1, d_1^*, N_0, N_1, N_1^*, N_2, N_2^*\right\}$ such that
(i) the quantities solve the agents' problems, given the probability, P, in equation (22), as well as prices and returns
(ii) markets clear.

In the above definition, it is understood that $k_t^h = 1 - k_t^b$, for $t = 0, 1$.

We now discuss our parameterization of the model. We set technology, Z_t, equal to Z in periods $t = 0, 1$ and treat Z as a free parameter. Similarly, we treat technology in period $t = 2$, Z_2, as a free parameter. In addition we set the household endowment as follows, $y_0 = y_1 = y$ and treat y as a free parameter, while $y_2 = 0$. We treat the initial assets and liabilities of bankers in period 0, $(R_{-1}d_{-1}, k^b_{-1})$, as free parameters. The other free parameters are those governing utility, the running-away opportunity and management costs, $\beta, \sigma, \theta_0, \theta_1, \alpha$. The utility function and management cost specification are given by:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad f\left(k^h\right) = \frac{\alpha}{2}\left(k^h\right)^2. \tag{31}$$

To ensure that the periods $t = 0, 1$ problems of the banker be well-defined and economically interesting in the baseline equilibrium we choose the free parameters so that equations (8) and (11), as well as the conditions in Proposition 1, hold. In addition, we want the equilibrium to be characterized by a non-trivial positive value of P. In the past century, there have been only about 3 serious financial crises in the US, so the unconditional probability of a crisis is quite low, somewhere in the range of 3 percent. But, consistent with the spirit of GK, we suppose that (for reasons not explained in the model) the fundamentals, such as θ_t, Z_t and y_t, have taken on values that make the economy vulnerable to a crisis.

We choose values for the free parameters (see Table 1) to optimize a loss function defined over all variables and which penalizes violation of the restrictions. The loss function specifies plausible values for the free parameters as well as for the endogenous variables in the model equilibrium. We use the adjective, 'plausible', loosely. It is hard to assess the empirical plausibility of parameter values in a three-period model, so at best the model is useful for its qualitative implications. A list of the model variable values appears in the first column of Table 2 (all numbers have been rounded). The exact loss function is described in section A in the online Appendix and in the code used for the calibration is available in the replication files for this paper.

Consider the parameter values in Table 1, reported after rounding. The discount factor is close to unity. The risk aversion parameter on utility is slightly bigger than log. Our values for θ_0 and θ_1 are somewhat higher than what is reported in the literature.[34] The share of GDP (i.e., consumption) produced by the endowment is

[34] Gertler and Karadi (2011) and GK use values close to 0.381 and 0.19, respectively.

roughly 80 percent in periods $t = 0, 1$.

Table 1: Baseline Parameter Values

β: subjective discount rate	0.91
σ: curvature in utility function	1.37
α: household management cost	0.10
θ_1: bank running away parameter, $t = 1$	0.50
θ_0: bank running away parameter, $t = 0$	0.34
Z: capital productivity, $t = 0, 1$	0.05
Z_2: capital productivity, $t = 2$	0.09
y: labor income, $t = 0, 1$	0.22
k^h_{-1}: household capital, $t = -1$	0.19
$R_{-1} \times d_{-1}$: bank liabilities, $t = -1$	0.24

Note: these are model initial conditions and parameter values chosen by a calibration procedure described in the text.

The endogenous variables of the model are reported in the first column of Table 2 (the other columns are discussed in the next subsection). There are several interesting features to note. First, the impact of the annihilation run on the real economy is substantial. Household consumption in the annihilation run state falls about 17 percent compared to what it is in the no run state.[35] The standard deviation in the log of period 1 consumption, from the perspective of period 0, is about 5.6 percent (see σ_C in the table). Second, the financial consequences of an annihilation run are also substantial. The price of assets in the annihilation run is 44% of its value in the no run state. Also, depositors lose 10% of what they are owed by banks in the annihilation run state, compared to the no run state. For the reasons discussed above, our calibration implies a relatively high value of P, $P = 0.10$. The risk premium in the period $t = 0$ deposit rate is 1.4%. This is the difference between the actual deposit rate, 7.3%, and what the risk free rate of interest would be if there were a market in one-period risk free debt.[36] The gross return on capital in the

[35]Subject to the caveats mentioned above about comparing our model with data, it is interest to compare this drop to the drop in consumption in the 2008 US financial crisis. In particular, from August 2008 to April 2009, US personal consumption expenditures dropped by 4 percent. In this sense the drop in the model is relatively large.

[36]The period 0 risk free interest rate is $100\left[(E_0\beta m_1)^{-1} - 1\right]$. For comparison, consider the TED spread, the difference between what banks pay for loans in the interbank market over the presumably safe rate paid by the US government. Before the 2007-2008 crisis, that rate averaged roughly 0.3 percent (APR), but rose to roughly 4% (APR) at the peak of the crisis in Fall, 2008. Given the relatively large drop in consumption, the relatively small risk premium in the model presumably

annihilation run state is nearly double what it is in the no run state.

Third, the results in the first column of Table 2 provide one way of illustrating the sense in which the existence of an annihilation run state is fragile in the three-period model. Financial frictions underlying the incentive constraint are very weak in such a state. This is because, absent Assumption 1, any banker with non-zero net worth has an enormous debt capacity in an annihilation run. To see why, suppose Assumption 1 applies to all bankers except one, and that that banker finds herself in an annihilation run with positive net worth. Evaluating equation (12) at the values of $R_1^*, R_2^{*,k}$ reported in the first column of Table 2, we find that the exceptional banker's maximal amount of leverage is 756 in the $t = 1$ annihilation run state. Exploiting this extraordinarily high debt capacity, the per-unit-of-net-worth value of banking in the annihilation state for the exceptional banker is equal to 375. This value is two orders of magnitude higher than the value, $\psi_1^* = 1.5$, that she would enjoy if she had net worth, but did not exploit her debt capacity at all by setting deposits to zero in the annihilation run state.[37]

The value of having net worth in the annihilation run state when Assumption 1 is not imposed is so high that the exceptional banker focuses exclusively on that state in period $t = 0$. To maximize her net worth in the annihilation run state she issues zero debt in $t = 0$.[38] The enormous debt capacity this creates in the annihilation state implies that the period 0 value of banking per unit of net worth is 32. This is an order of magnitude greater than $\psi_0 = 1.1612$, the corresponding value of banking when Assumption 1 is imposed (see Table 2).[39] Evidently, if we drop Assumption 1, the numbers in Table 2 do not satisfy the optimality conditions of a banker and so they do not constitute an equilibrium (see Definition 1).[40] The existence of the annihilation run state is fragile in that a small change in the model that is not

reflects the well-known difficulty of models with a utility function like ours to deliver empirically reasonable premia.

[37] The object, ψ_1^*, is defined in equation (13).

[38] An alternative to Assumption 1 is to suppose that a banker with positive net worth in an annihilation run faces convex costs for issuing new deposits in that state. Like Assumption 1, this would reduce the profitability of net worth in an annihilation run. It would therefore also reduce the incentive of a banker to cut back on deposits in $t = 0$ with the objective of preserving net worth in case a run occurs in $t = 1$.

[39] For the definition of $\tilde{\phi}$ recall Lemma 1.

[40] GK(Appendix A) do not adopt Assumption 1 and explain why the annihilation run state is nevertheless part of an equilibrium in their infinite horizon model. Below, we argue that the annihilation run is nevertheless fragile in their model for other reasons.

motivated by the logic of the model itself (i.e., dropping Assumption 1), destroys the equilibrium. We return to this theme in the material below.

Table 2: Baseline and the Alternative Leverage Restriction Equilibrium

	Baseline	Policy (Good)	Change (%)	Policy (Bad)	Change (%)
ϕ_0	3.4116	3.3434	-2.0000	3.3434	-2.0000
ϕ_1	2.2485	2.2540	0.2453	2.5960	15.4580
ψ_0	1.1612	1.1954	2.9412	1.9599	68.7785
ψ_1	1.1157	1.1184	0.2453	1.2881	15.4580
c_0	0.2752	0.2750	-0.0687	0.2643	-3.9652
k_0^b	0.9548	0.9229	-3.3475	0.5227	-45.2538
k_1^b	0.8091	0.8054	-0.4658	0.6204	-23.3287
c_1	0.2735	0.2734	-0.0257	0.2683	-1.9030
c_1^*	0.2269	0.2269	0.0000	0.2269	0.0000
c_2	0.0890	0.0890	0.0000	0.0890	0.0000
Q_0	0.3649	0.3623	-0.7227	0.2958	-18.9405
Q_1	0.3587	0.3582	-0.1386	0.3307	-7.8202
Q^*	0.1953	0.1953	0.0000	0.1953	0.0000
x	0.8982	0.9113	1.4641	1.0680	18.9084
P	0.1018	0.0887	-12.9170	0.0000	-100.0000
R_0	1.0733	1.0746	0.1217	1.1230	4.6344
R_1	0.2360	0.2361	0.0352	0.2423	2.6693
R_1^*	0.3048	0.3048	0.0000	0.3048	0.0000
R_1^k	1.1292	1.1361	0.6064	1.2982	14.9671
$R_1^{*,k}$	0.6815	0.6864	0.7279	0.8407	23.3662
R_2^k	0.2481	0.2484	0.1388	0.2692	8.4836
$R_2^{*,k}$	0.4556	0.4556	0.0000	0.4556	0.0000
d_0	0.2463	0.2343	-4.8580	0.1084	-55.9986
d_1	0.1612	0.1605	-0.4089	0.1261	-21.7453
N_0	0.1021	0.1000	-2.0877	0.0462	-54.7174
N_1	0.1291	0.1280	-0.8470	0.0790	-38.7868
σ_C	5.6445	5.2985	-6.1304	0.0000	-100.0000
Risk Premium	1.3954	1.0631	-23.8131	0.0000	-100.0000
Welfare	-13.8091	-13.8068	0.1417	-13.8743	-3.9311

Notes: see text for discussion of the variables.

2.5 Macroprudential Policy

We impose a leverage restriction in period 0, so that bankers must satisfy:

$$\frac{Q_t k_t^b}{N_t} \leq \kappa.$$

If κ is large enough, then the constraint is not binding because bankers already self-impose a leverage constraint (recall equation (20)). The leverage constraint is motivated by the fact that the probability of the period 1 run state, P, is determined by the aggregate recovery ratio, X. Individual bankers ignore the impact on P of their leverage choice and the purpose of the leverage constraint is to get them to internalize that impact. A high value of P has the undesirable consequence of increasing the likelihood that the run state occurs, resulting in a misallocation of capital between banks and households. We set κ so that the constraint is binding and forces banks to hold 2% less leverage than they do in the baseline equilibrium discussed in Section 2.4. Numerical calculations suggest that this is roughly the Ramsey-optimal leverage restriction.[41]

2.5.1 Equilibrium with Leverage Restriction

We do a systematic search to identify all equilibria associated our leverage restriction. To do so we vary k_0^h on a fine grid, K, of equally-spaced points on the unit interval. The grid includes $k_0^h = 0$. For each value of k_0^h we compute the equilibrium of the economy, ignoring the household Euler equation, equation (27), associated with the k_0^h decision. Figure 3 graphs an error function, $g\left(k_0^h\right)$, associated with equation (27):

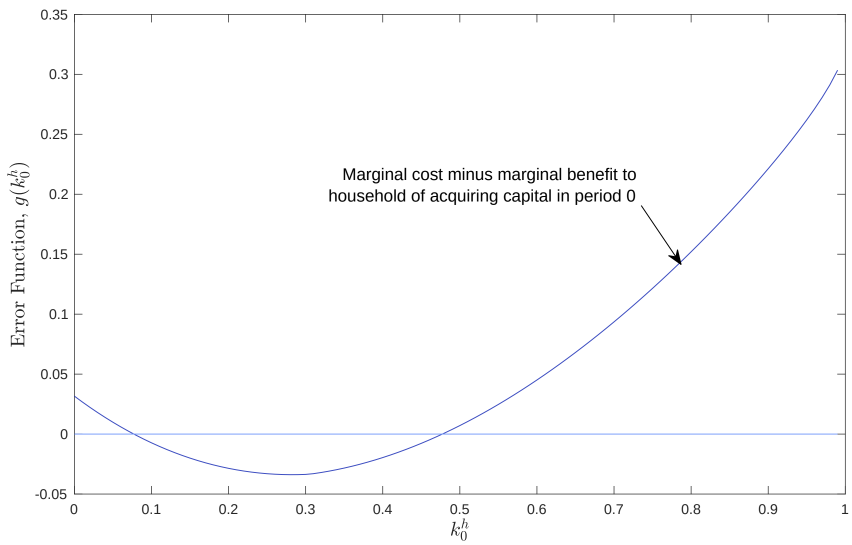
$$g\left(k_0^h\right) = 1 - \left[\beta\left(1-P\right)\frac{u'\left(c_1\right)}{u'\left(c_0\right)}\left(\frac{Z_1+Q_1}{Q_0+f'\left(k_0^h\right)}\right) + \beta P\frac{u'\left(c_1^*\right)}{u'\left(c_0\right)}\left(\frac{Z_1+Q_1^*}{Q_0+f'\left(k_0^h\right)}\right)\right]. \quad (32)$$

For each value of k_0^h in K we use the equilibrium conditions other than equation (27) to solve for $c_0, c_1, c_1^*, Q_0, Q_1, Q_1^*, P$. The first term, unity, on the right of the above expression is the marginal cost (in period 0 goods units) of consumption. The term in square brackets is the corresponding marginal benefit, also measured in period 0 goods units. The function, g, is graphed over its domain, $[0,1]$, in Figure 3. According to equation (27) the multiplier, μ, is zero for $k_0^h \in (0,1]$. At $k_0^h = 0$, the multiplier is $\mu = g(0)$ and the Euler equation, equation (27), holds because $\mu > 0$ and $\mu k_0^h = 0$ (see Figure 3).

So, the household Euler equation holds at three values of k_0^h: $0, 0.08, 0.48$. These points constitute candidate equilibria for the model. However, $k_0^h = 0$ turns out not to be an equilibrium because $k_0^b = 1 - k_0^h = 1$ does not satisfy the bank incentive constraint in equation (20). All banker incentive constraints are satisfied at the other two candidate equilibria and hence these are actual equilibria of the model.

[41] Caveat: we only did a *local* search for the leverage restriction that has the best (i.e., Ramsey) equilibrium associated with it.

Figure 3: Euler Equation Error in $t = 0$, leverage restriction imposed



Note: See equation (32) for a definition of the function, g. We explain that the household Euler equation, equation (27), is satisfied for three values of k_0^h: $0, 0.08, 0.48$. However, only the second two points are consistent with the banker problems being well-defined. $k_0^b = 1 - k_0^h = 1$ violates the bank participation constraint in equation (20). The points, $k_0^h = 0.08$ and $k_0^h = 0.48$ correspond to the equilibrium described in the 'policy' column in Table 2.

Consider the second column of numbers in Table 2. This corresponds to the lower of the two equilibrium values of k_0^h in Figure 3 (see column heading, 'Policy (Good)'). The third column compares the latter equilibrium with the benchmark equilibrium ('Change (%)'). We measure the impact of leverage on welfare as the percent increase in baseline consumption, c_0, required to equate welfare in the baseline equilibrium to welfare in the equilibrium with the leverage constraint.[42] Evidently, the leverage restriction increases welfare by 0.14%. The table indicates that banks respond by reducing deposits by nearly 1 percent (recall, $deposits = (\phi - 1) N$). Bank assets, k_0^b, go down by 3% and their price, Q_0, falls by a small amount, roughly 1 percent. The recovery ratio rises and the probability of crisis falls by one percentage point, from 10% to 9%. The table indicates that there is virtually no impact on consumption of the leverage restriction so that the primary reason that welfare increases is that P falls.

The fact that there is a second equilibrium associated with the leverage restriction illustrates the well-known fact that unique implementation of the Ramsey equilibrium

[42]Let $W(c_0)$ denote discounted utility of the representative household in the baseline equilibrium, where the notation indicates the equilibrium value of c_0 (see equation (23) for the definition of welfare and the first column of Table 2). Let W_p denote welfare in an equilibrium that satisfies the leverage restriction. We define the rise (fall, if negative) in welfare by solving for Δ in the equation, $W(c_0(1 + \Delta)) = W_p$. The table reports 100Δ.

may be a problem. Table 2 shows (see last column) that welfare in the $k_0^h = 0.48$ equilibrium is *lower* than in the baseline equilibrium. Interestingly, in this second equilibrium $P = 0$, so that the financial system is completely stabilized. Because the equilibrium results in lower welfare it also illustrates the 'dark side' of regulation. While regulation might reduce risk, it may simultaneously result in an inefficient allocation of capital.

In sum, it is apparent that there are two ways to reduce leverage by 2%. One increases welfare and is associated with a small reduction in k_0^b and Q_0^b. The other produces large reductions in k_0^b and Q_0^b and reduces welfare. To gain intuition into this result, it is convenient to focus on leverage, ϕ_1, in the no run state in period 1, instead of ϕ_0. This change simplifies the intuition because we hold the period 0 values of variables fixed at their baseline values, thus reducing the number of equilibrium conditions that need to be considered. The period 1 Euler equation of the household (see equation (28)) is:

$$Q_1 = \beta \frac{u'(c_2)}{u'(c_1)} Z_2 - \alpha k_1^h = \beta Z_2^{1-\sigma} (Z_1 + y - \frac{\alpha}{2} \left(k_1^h\right)^2)^\sigma - \alpha k_1^h. \tag{33}$$

The second equality makes use of our specification of utility as well as the period $t = 1, 2$ resource constraints (see equations (30)). The key thing to note about the above expression is that Q_1 is decreasing in k_1^h and, most importantly, it decreases at an increasing rate because of the convexity of management costs.[43] The next important relation is the definition of leverage:

$$\phi_1 = \frac{Q_1(1-k_1^h)}{N_1} = \frac{Q_1(1-k_1^h)}{(Z_1+Q_1)(1-k_0^h) - R_0 d_0} = \frac{1}{1 - k_0^h + \frac{Z_1(1-k_0^h) - R_0 d_0}{Q_1}} (1 - k_1^h). \tag{34}$$

The first equality uses the definition of bank leverage and the fact that $k_1^b = 1 - k_1^h$. The second equality uses the law of motion of period $t = 1$ net worth. The term, $Z_1(1-k_0^h) - R_0 d_0$, is what net worth would be in period $t = 1$ if the value of capital were zero in that period. This term is negative in both the annihilation run and no run states.

Equation (34) shows that an increase in k_1^h (i.e., decrease in k_1^b) has two effects on leverage: a direct effect (the *quantity channel*) holding Q_1 constant and an indirect

[43] Note from equation (33)) that $dQ_1/dk_1^h < 0$ increases in absolute value as k_1^h increases.

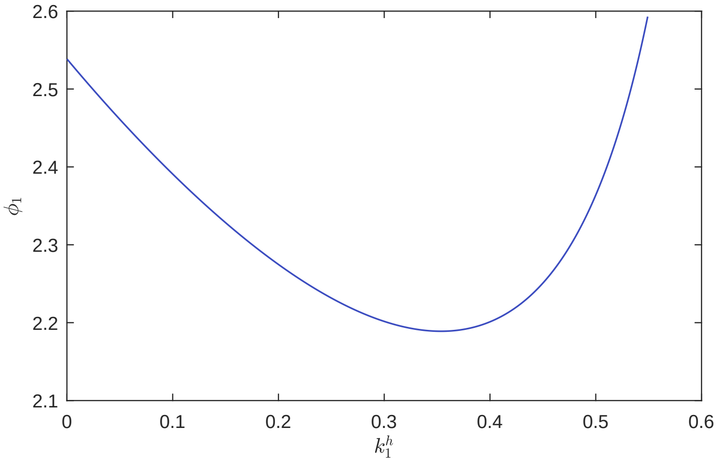
effect (the *price channel*) which operates via the change in Q_1 induced by a change in k_1^h.[44] The quantity channel clearly reduces leverage, ϕ_1. The price channel operates because a rise in k_1^h requires that Q_1 be lower (equation (33)). The price channel implies that, other things the same, a fall in Q_1 drives ϕ_1 up (equation (34)).[45]

The convexity of management costs amplifies the price channel when k_1^h is high and reduces it when k_1^h is small. To see why this nonlinearity can contribute to multiplicity of equilibria under a leverage constraint, consider Figure 4. This displays the values of ϕ_1 corresponding to a range of values of k_1^h. Suppose that in the absence of policy, k_1^h takes on a low value, say 0.05. Suppose a leverage restriction leads to a reduction in bank assets, and an increase in k_1^h. Note that while k_1^h is still small the quantity channel initially dominates the price channel, so that the fall in k_1^b leads to a fall in ϕ_1. As k_1^h increases further, however, the price channel dominates and ϕ_1 begins to rise. Thus, there are two ways that one can obtain a given decline in ϕ_1 starting from $k_1^h = 0.05$. One is via a small rise in k_1^h (i.e., a small fall in k_1^b) and the other is via a large rise in k_1^h (a larger fall in k_1^b). This pattern is similar to what we see in Table 2 which displays the impact on equilibrium of a binding restriction on ϕ_0. In that table there are two equilibria: in one, k_0^b falls a small amount and in the other k_0^b falls a large amount. Consistent with the simple intuition described here, the impact on Q_0 is smaller in the first case than it is in the second.

[44]Note that a change in the value of k_1^h directly affects Q_1 via equation (33). Then, k_1^h and Q_1 jointly affect ϕ_1 via equation (34). Recall that we hold the period 0 variables fixed at their $t = 0$ values in the baseline equilibrium.

[45]Here, we use the fact that $\left(Z_1(1 - k_0^h) - R_0 d_0\right) < 0$ in the third equality in equation (34).

Figure 4: ϕ_1 as a function of k_1^h



Notes: (i) ϕ_1 is determined by solving equations (33) and (34) for a range of values of k_1^h; (ii) see footnote 44 and text for discussion.

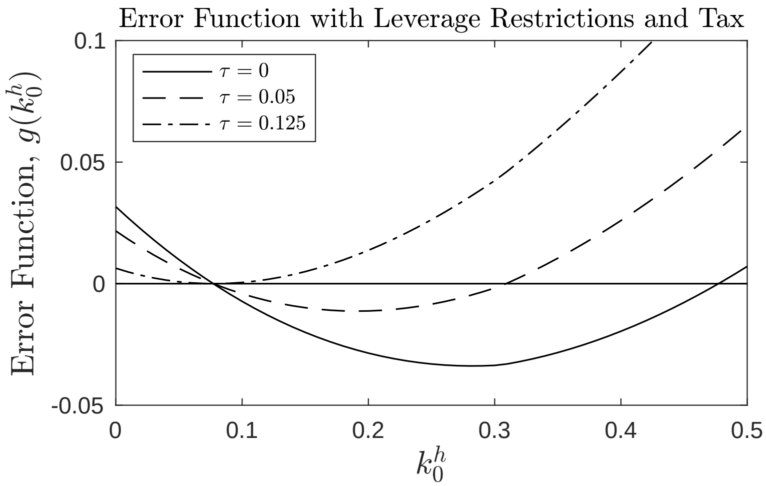
2.5.2 Implementation

In our example, a leverage restriction can induce a bank to internalize the effect of banker leverage on the fragility of the system. In the 'good' equilibrium that satisfies the leverage restriction, deposits are slightly lower and welfare increases. However, the same leverage restriction is also consistent with a 'bad' equilibrium, in which deposits fall significantly and households hold a much higher level of capital, which lowers welfare because of increasing management costs.[46] The objective of good macro prudential policy is not only to identify the Ramsey optimal policy, but also to design policy in a way that ensures that the Ramsey equilibrium is the only equilibrium. In our model this implementation problem can be achieved by coupling the leverage restriction with a particular tax on household capital holdings. Households pay $\tau \times \left(k_0^h - \bar{k}\right)$, where $\bar{k}$ denotes capital holdings in the desired equilibrium. The proceeds of the tax are redistributed back to the household in the form of a lump-sum transfer. Figure 5 plots the Euler equation errors for three levels of tax. A 12.5% tax on household capital holdings achieves a unique good equilibrium. Note that with $\tau = 0$ we reproduce the multiple equilibrium result in Figure 3. As τ increases, the Euler error function rotates and eventually the curve cuts the zero line only once. With this tax, we eliminate the

[46] It would be interesting to systematically explore whether there are other simple policies which encourage banks to internalize externalities and which are not associated with more than one equilibrium.

'bad' equilibrium.[47]

Figure 5: Euler Equation Error in $t = 0$, leverage restriction imposed



Notes: (i) for discussion, see subsection 2.5.2; (ii) the $\tau = 0$ curve coincides with the curve in Figure 3.

2.6 Fragility of Equilibrium to Entry Costs in the Running Away Model

This section reports two results. First, generically there are three states in $t = 1$. In addition to the annihilation and no run states, there is also a partial run state in which the net worth of the banking system is only partially wiped out. The economic intuition for this finding is made transparent by transforming the model into a game. Second, we show that the annihilation run equilibrium is very sensitive to assumptions made about entry. The model assumes that entry is not possible during annihilation run, and we show this requires what appears to be implausibly large entry transactions costs. Thus, in this model the no-entry assumption appears implausible. However, if the no-entry assumption is dropped, then the annihilation run does not occur in equilibrium.

[47]Our policy emphasizes a second potential pitfall associated with implementing a Ramsey equilibrium that was emphasized in Diamond and Dybvig (1983) and generalized by Bassetto (2005) (see also Atkeson et al. (2010)). In particular, the tax policy that the government uses to nudge the economy towards the good equilibrium must be feasible in case the bad equilibrium allocations were to occur. Diamond and Dybvig (1983)'s discussion illustrates this point in a particularly simple way: for deposit insurance to successfully rule out a run equilibrium, the deposit insurance program must have sufficient funds in case a run were to occur. Our policy for uniquely implementing the Ramsey equilibrium is feasible in all states of the world and so does not exhibit the second potential pitfall discussed here.

The findings just summarized apply to the calibrated three-period version of the running away model. Calibration of a three-period model is always problematic, and so it is natural to ask whether similar results apply in the infinite horizon version of the model calibrated in GK. The answer is, 'yes'. In fact, in the infinite horizon model fragility even greater than what we find in the three period model. This is because in the infinite horizon one has to assume that new bankers ('newborns') arrive in each period to replace a small fraction of incumbent bankers who are removed. In GK it is assumed that newborns who arrive when there is an annihilation run, do not enter banking for one period. Consistent with our finding about the importance of entry in the three period model, we show that if the no-entry assumption is dropped and newborns can enter during an annihilation run, then that run cannot be an equilibrium. In addition, given our game-theoretic approach we can examine alternative assumptions about newborns: how many newborns enter and under precisely what circumstances do they enter? We find that, depending on the assumptions made, a large range of equilibria is possible. As in the three-period model one can have three states: an annihilation run state, a partial run state and a no run state. There are other possibilities too. These include that there is no annihilation run, and only partial runs of varying intensity. Since the infinite-horizon model has many details, and the basic findings about fragility are similar to what we find in the three-period model, we save space by reporting the results of the infinite-horizon model in section C in the online Appendix.

2.6.1 Partial Run

An individual banker's period $t = 1$ deposit decision is a function of her net worth, N_1, while the key determinant of N_1 is the price of capital, Q_1.[48] But, Q_1 must clear the capital market and so is determined by the collective deposit decision of the mass of other bankers. Because all bankers make their deposit decision simultaneously and without communication, no individual banker can actually see Q_1. Instead, they must form a belief about Q_1, which is tantamount to forming a belief about the deposit decision of other bankers.[49] In this way, an individual banker's deposit decision, d_1, can be represented as a *best response* to D_1, the decisions of all the other bankers. This

[48]The other variables, $Z_1, k_0^b, R_0 d_0$, required to determine N_1 are predetermined in $t = 1$.

[49]In the market equilibrium of the model in Definition 1 it is simply assumed that agents know objects like Q_1, while sidestepping the question of how agents come to have this knowledge. There is a voluminous literature on this question and our analysis proceeds in the spirit of the strand started in macroeconomics by Guesnerie (1992).

is how a banker making her period $t = 1$ deposit decision before the market for deposits and capital has met and cleared, might find herself in the position of a player in a large, simultaneous-move, non-cooperative game against the mass of other bankers.

We take as given the date $t = 0$ variables. Given a decision, d_1, by an individual banker and the average decision, D_1, by the other bankers, there is a well-defined payoff for the individual banker. This payoff is determined by the values of Q_1, N_1, R_2^k and R_1 in the continuation equilibrium implied by each value D_1.[50] To determine the values of Q_1, N_1, R_2^k, R_1, the individual banker performs computations conditional on the given values of the period $t = 0$ variables, as well as D_1. In particular, the banker computes all aggregate variables that, with one exception, solve the $t = 1, 2$ market equilibrium conditions. The exception is that the list of equilibrium conditions imposed does not include the condition associated with D_1 itself (i.e., the incentive constraint). In effect the individual banker asks a series of 'what if' questions: 'what would my best response be if D_1 were to take on this value or that value?'

In contemplating what her best response to a particular value of D_1 is, the individual banker does not ask why other bankers would choose that value of D_1.[51] Relatedly, we also assume that if the continuation equilibrium given D_1 causes other bankers' incentive constraints to be violated, the other bankers do not run away with their assets. Market rates of return adjust so that the markets clear in the continuation equilibrium and households supply D_1 deposits.

In determining her own best response to D_1 the individual banker asking the 'what if' question carefully pays attention to her own incentive constraint. In a Nash equilibrium (i.e., D_1 such that $d_1(D_1) = D_1$) all bankers satisfy their incentive constraint. Let $d_1(D_1)$ denote the banker's best response to D_1. According to equation (12):

$$d_1(D_1) = \frac{R_2^k(D_1) - \theta R_1(D_1)}{(1+\theta) R_1(D_1) - R_2^k(D_1)} N_1(D_1), \tag{35}$$

where $\zeta(D_1)$ denotes the value of ζ in the continuation equilibrium, for $\zeta = R_2^k, R_1, N_1$. Here, the incentive constraint, equation (11), requires that the numerator and denominator are both positive. Below, we verify that these incentive constraints are satisfied

[50] We make heavy use of the concept of a continuation equilibrium. This concept coincides with the one used in Atkeson et al. (2010)'s definition of concept of a sophisticated equilibrium.

[51] However, with her best response function in hand a banker can potentially use rationality and common knowledge to determine that some values of D_1 can be deleted from further consideration by the individual banker on the grounds of not being 'rationalizable' (see, e.g., Bernheim (1984) and Pearce (1984)).

in our numerical example for all D_1 considered. The Nash equilibria of the game correspond to equilibria of the market economy.[52]

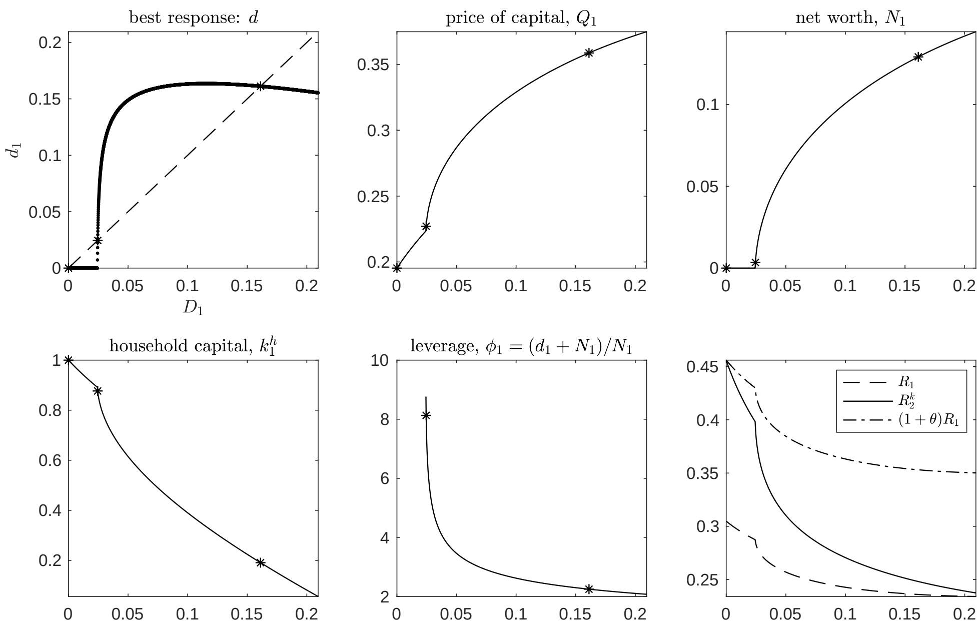
The 1,1 panel of Figure 6 plots the best response function of an individual banker.[53] Best response functions cross the 45-degree line at three distinct points. Two of the points coincide with the time $t = 1$ no run and annihilation run equilibria in the baseline model.[54] The complementarity between bankers' deposit decisions can be seen in the positive slope of the best response function in the 1,1 panel of Figure 6. This complementarity operates via the positive response of Q_1 and N_1 to an increase in D_1 (see panels 1,2 and 1,3).

[52]The reason for this is that the one equilibrium condition that is ignored in the continuation equilibrium associated with D_1 is the first order condition for deposits. For D_1 with the property, $D_1 = d_1(D_1)$, that first order condition is satisfied.

[53]By assumption, each banker has the same amount of net worth, so it makes no difference which banker we look at.

[54]See, e.g., the first column of numbers in Table 2.

Figure 6: Individual Banker Best Response Function



Notes: (i) For discussion, see subsection 2.6.2. (ii) The 1,1 panel describes the best response function, $d_1(D_1)$. The function, $d_1(\cdot)$, is continuous throughout the range of values of D_1 displayed. We document this by displaying dots on a very fine grid of D_1's. Note the small number of visually distinct dots in the neighborhood of the partial run state (the middle crossing), indicating that the curve is continuous there, though very steep. (ii) The 2,2 panel had to be truncated for scale reasons. (iii) The stars indicate the Nash equilibria, which correspond to market equilibria which are, from left to right: the annihilation run, the partial run and the no run states. (iv) Model parameter values are reported in Table 1 and endogenous variables in the annihilation run and no run states are reported in the first column of Table 2.

The third Nash equilibrium in Figure 6 is what we call a *partial run state*, where the price of capital is low, but not so low that banker net worth is wiped out. In the partial run state, bankers cannot pay off all of $R_0 d_0$ by rolling over their deposits so they must also sell some of their capital. The partial run state is part of a general equilibrium of the three-period model in the sense of Definition 1 if we assign it a zero probability from the perspective of period $t = 0$. By continuity, the equilibrium allocations would be similar if the probability of a partial run were positive, but sufficiently small.

Crucial features of the model that account for the third crossing of $d_1(D_1)$ with the 45^o line are (i) the continuity of the best response function, (ii) the fact that in our baseline equilibrium $x < 1$ and (iii) the convexity of the household management costs (see equation (31)). Condition (ii) explains why the best response function is flatter than the 45^o line for D_1 small: a substantial rise in Q_1 is required for bankers to have positive net worth, so that as D_1 increases a small amount above zero, net worth remains zero and the individual banker continues to best respond by setting $d_1 = 0$. Condition (iii) explains why the best response function cuts the 45^o line from above at the no run equilibrium. As households hold a smaller fraction of the stock of capital (panel 2,1), the convexity of management costs imply that those costs exert a vanishingly small effect on the market price, Q_1 (see the concavity in Q_1 in panel 1,2).[55] As a result, further increments in D_1 lead to only a small increase in Q_1 and N_1 and, hence, d_1. Finally, since the best response function lies below the 45^o line for low D_1 and that function crosses from above the 45^o line for high D_1, (i) then implies that there must be a middle crossing, as we see.

The discussion in the previous paragraph provides an indirect explanation for the existence of the third, middle equilibrium. Two features of the model provide a direct explanation for why $d_1(D_1)$ rises so sharply for values of D_1 just barely large enough that banker net worth becomes positive. First, the sharp rise can be explained in part by condition (iii). When D_1 is low and households hold a large proportion of the capital stock the convexity in management costs plays a big role by making Q_1 very sensitive to D_1.[56] The sensitivity of Q_1 to D_1 implies that N_1 is also sensitive to D_1, which in turn helps explain why $d_1(D_1)$ is steeper than the 45^o line in a neighborhood of the partial run equilibrium.

The second model feature that directly explains the existence of the middle equilibrium in the 1,1 panel of Figure 6 is the nonlinearity in the leverage decision of bankers, equation (12). With a fall in D_1, R_2^k rises because Q_1 falls, as noted above and reported in panel 1,2. According to the 2,3 panel in Figure 6, R_2^k rises towards $(1+\theta)R_1$ as D_1 falls, so that leverage is extremely high for small D_1 (see the 2,2 panel).[57] Of course, if net worth is zero this does not translate into a high level of borrowing by banks. However, as soon as N_1 becomes positive, the fact that leverage is so high ensures that bank borrowing rises sharply.

[55]See Footnote 43

[56]See Footnote 43 and recall the concavity of Q_1 in panel 1,2 of Figure 6.

[57]The 2,2 panel had to be truncated for scale reasons, for low values of D_1.

2.6.2 Interpreting the No Entry Assumption in Annihilation Run as an Entry Cost

Recall that we imagine 'banks' in the model are just one sector of the financial system and that 'households' in the model are a reduced form combination of both actual households as well as all the other sectors in the financial system. Suppose the banks experience an annihilation run and consider a financial institution from another sector of the financial system which is well known and easily distinguished from the banks experiencing the rollover crisis. In addition, we might suppose that that financial institution is from a sector of the financial system that is 'close' to the one experiencing the run, in the sense that expertise in one sector of the financial system is reasonably transferrable to the other.[58]

In our model entry into a sector of the financial system experiencing a rollover crisis is ruled out by assumption. In this section we calculate the entry cost which would rationalize our no-entry assumption. We find that the entry cost is very large, possibly orders of magnitude larger than what is empirically plausible.

Recall from the discussion in section 2, that financial frictions are very weak in an annihilation run. In particular, a banker with positive net worth that enters an industry experiencing a rollover can obtain an enormous amount of leverage, $\phi_1^* = 756$.[59] Net worth in the no run equilibrium is $N_1 = 0.129$,[60] and suppose that just 0.3 *percent* of that amount of net worth enters in the annihilation run. That small amount of net worth could support roughly the assets held by banks in the no run equilibrium.[61] In particular,

$$\overbrace{0.129}^{\text{net worth in no run equilibrium}} \times \overbrace{0.003}^{\text{fraction of no run equilibrium net worth entering}} \times \overbrace{756}^{\text{leverage}} = 0.29,$$

which, after rounding, comes close to $d_1 + N_1$ in the no run equilibrium (see Table 2, first column).

That the annihilation run could be destroyed by such a small change in assumption

[58] An example of two sectors of the financial system that may be 'close' may be the sector that deals in car loans and the sector that deals in home mortgages.

[59] To obtain this value of ϕ_1^* we evaluate equation (12), replacing R_1 and R_2^k with $R_1^*, R_2^{*,k}$, respectively, from Table 2 column 1.

[60] See N_1 in the first column of Table 2.

[61] In the model, the net worth could be injected by giving it to one of its bankers.

about entry poses a severe challenge for this environment as a theory of rollover crisis. The return on equity in a period 1 annihilation run is $\psi_1^* = \theta_1 \phi_1^*$ by the incentive constraint evaluated at equality (see equation (13)). Thus, with $\theta_1 = 0.5$, we have that the rate of return per dollar of equity in the annihilation run is extremely high:

$$\psi_1^* = 0.5 \times 756.$$

That is, if a banker enters an annihilation run with \$1 of equity, that dollar generates a return, in present value terms, of $\psi_1^* = \$378$. So, the cost of entry would have to be \$378 or more, per one dollar of equity issued, in order to rationalize the no entry assumption. We suspect that this is implausibly large, by orders of magnitude.[62][63] If the new entering banker had little expertise in banking because they come from a sector not very 'close' to the sector experiencing the annihilation run, that banker's lack of expertise would have to be extreme.

3 Rollover Crisis: Hidden Effort Model

In the model of Section 2, the assumption that the banker can run away with assets limits the amount of credit she receives. This section considers the Christiano and Ikeda (2016) (CI) model in which bankers exert costly effort to improve the performance of their balance sheets. The financial friction stems from the assumption that that effort is unobservable. This friction provides an alternative reason why a banker's net worth limits the amount of credit it receives.

We exploit a feature of the analysis in Section 2 to sharply narrow our focus here. In section 2.5, we studied two *leverage questions*: do leverage restrictions improve welfare? If 'yes', then how do we implement the desired equilibrium with lower leverage? Answering these questions required computing the entire equilibrium for all three periods, because we wanted to study the effects of a leverage restriction put in place in 'normal times' before a rollover crisis has occurred. Section 2.6 studied *rollover crisis questions*: when banks use short term liabilities to finance long term assets, are there multiple equilibria in which lenders supply various amounts of credit and how do these

[62] To obtain the \$1 of equity from a household would require paying the household's opportunity cost, $R_1^* = 0.3048$. So, technically the return on issuing equity is $\$378 - \$0.3048 \simeq 378$, after rounding.

[63] Our analysis is consistent with the findings in Gertler et al. (2020b). They do allow banks to issue equity, but *not* during a run (see their equation 23).

equilibria depend on assumptions about banker entry?

We do not explore the leverage questions in the hidden effort model considered here. In part this is because the desirability of leverage restrictions in the hidden effort model has already been shown in CI.[64] Instead, we explore only the rollover crisis questions. We show that the conclusions of the previous section are robust to the different financial friction that we work with here (unobserved banker effort). In particular we find that: (i) generically, there is an odd number of equilibria in which there is not only an annihilation and a no run equilibrium, but there is also a partial run equilibrium; and (ii) whether there is any equilibrium apart from the no run equilibrium depends delicately on what one assumes about banker entry during an annihilation run.

In the previous section, we found that the essence of (i) and (ii) can be studied simply by considering period 1 before that period's price of capital is determined. We take as given the assets and liabilities of banks inherited from period 0, without discussing how they were determined. In this way, we are able to discuss the rollover crisis arising from a maturity mismatch on bank balance sheets in what is effectively a static model. The essence of the maturity mismatch is captured by the assumption that the value of a bank's assets is determined simultaneously with the bankers' deposit decision.

The following subsection describes the banker problem in the model. Subsection 3.2 briefly reviews the households and market clearing conditions, which are similar to those conditions in the running away model in subsection 2. In subsection 3.3 we provide the definition of equilibrium, discuss our calibration of the model parameters and display the three equilibria of the model: annihilation run, partial run and no-run. Subsection 3.4 discusses the economics of the three equilibria by transforming the market equilibria into the Nash equilibria of a game. That allows us to make points (i) and (ii).

3.1 Bankers

The first subsection describes how the banker selects a credit contract in a competitive market. The second subsection summarizes the properties of the contract chosen by the banker, across a range of values for net worth and other variables outside of the banker's control.

[64]CI do not consider rollover crises, so the case for leverage restrictions when a rollover crisis is presumably greater in their model.

3.1.1 The Banker's Problem

We assume that a unit mass of bankers lives in a representative household. At the start of period $t = 1$, the representative household averages the net worth of all its bankers that were operating in $t = 0$ and distributes the proceeds equally to each banker. Bankers interact in competitive markets with mutual funds, where they receive a loan contract. The period $t = 1$ net worth, N, of a banker is the analog of equation (1):

$$N = \max\left\{0, \left(\hat{Z} + Q\right) k_0^b - L_0\right\}, \tag{36}$$

where L_0 corresponds to the beginning-of-period $t = 1$ liabilities of bankers. Also, $\hat{Z}$ denotes the average, across bankers, of the productivity of $t = 0$ capital, k_0^b. The details of how the liabilities, L_0, break down into deposits and interest are immaterial here. The variables, $\hat{Z}, k_0^b$ and L_0, are state variables at time $t = 1$ and are the same for each banker because of the way net worth is distributed at the start of the period. In this section we avoid notational clutter by deleting the time subscript when $t = 1$.

Bank assets in $t = 1$ can either be highly productive and have gross return, $e^g Z$, or be less productive and have gross return, $e^b Z$, where $g > 0 > b$. By exerting an effort, e, bankers are able to increase the probability, $p\left(e\right)$, that the bank's capital is highly productive, where

$$p(e) = \bar{a} + \bar{b}e,\ 0 \leq e \leq \frac{1 - \bar{a}}{\bar{b}} - \varepsilon. \tag{37}$$

We use lower case p to differentiate the probability in equation (37) (which refers to a random variable that is idiosyncratic to an individual bank) from the probability, P, of a run in the running away model. Because we do the analysis in period $t = 1$, there is no aggregate uncertainty. In equation (37), $1 - \bar{a}, \bar{a}, \bar{b} > 0$ and $\varepsilon > 0$, but small. Although capital is long-lived, effort in a particular period affects the productivity of all the bank's capital in that period, including any part, k_0^b, of the bank's capital that it still owns in period $t = 1$. We interpret e as a combination of skill in discriminating between good and bad entrepreneurs, as well as the ability of the bank to exert on-going influence over the operations of the entrepreneur via various types of covenants. These factors are captured in a reduced-form way in the model which uses the analytically convenient device of supposing that the banker itself operates projects. In the latter respect, the model takes the approach in the running away model in the previous section.

According to equation (37), the banker does not have the ability to set $p = 1$. This ensures that, regardless of the (feasible) level of e chosen, the performance of the banker's project does not reveal e. A banker's disutility of making effort, $(N+d)e^2/2$, is increasing and convex in e, as well as proportional to the amount of assets under management.

The banker receives a contract, $\left(d, R_d^g, R_d^b\right)$, from a mutual fund, where $d \geq 0$. Here, R_d^x denotes the gross interest rate conditional on the realized idiosyncratic state, $x = g, b$. The contract must satisfy three constraints. First, it must be feasible for the bank to repay the mutual fund in each idiosyncratic state:

$$(N+d)R^g \geq R_d^g d \tag{38}$$

$$(N+d)R^b \geq R_d^b d, \tag{39}$$

where

$$R^x = e^x Z/Q, \tag{40}$$

for $x = g, b$ and

$$N + d = Qk^b. \tag{41}$$

Second, the interest rates, R_d^g, R_d^b, must be consistent with (perfectly diversified) competitive mutual funds making zero profits:

$$R = p(e)R_d^g + (1 - p(e))R_d^b, \tag{42}$$

where R is a gross interest rate taken as given by mutual funds. Since equation (42) depends on the value of e, the latter plays a role in the contract. This brings us to the third constraint, the *incentive constraint*. Because e is not observable to creditors, the contract takes for granted that the banker will make the level of effort that is privately optimal conditional on the contract terms, $\left(d, R_d^g, R_d^b\right)$. For a specified value of these terms, the banker's objective is a simple quadratic polynomial in e:

$$\beta m p(e) \left[(N+d)\, R^g - R_d^g d\right] + \beta m \left(1 - p(e)\right) \left[(N+d)\, R^b - R_d^b d\right] - \frac{1}{2}(N+d)e^2, \tag{43}$$

where βm is the rate at which the banker discounts period 2 returns. At an interior optimum, e takes on the value:

$$e = \beta m \bar{b} \left[\left(R^g - R^b \right) - \left(R_d^g - R_d^b \right) \frac{d}{N + d} \right]. \tag{44}$$

It is convenient to substitute out for R_d^b, R_d^g in equation (43) using equation (42). Doing so, the banker objective can be written, after some rearranging, in the following form

$$\beta m N \pi_d (e) + \beta m d \left[\pi_d (e) - R \right], \tag{45}$$

where

$$\pi_d (e) = p(e) R^g + (1 - p(e)) R^b - \frac{1}{2\beta m} e^2. \tag{46}$$

Let $V(N)$ denote the value of equation (45) at the optimizing choice of d, R_d^g, R_d^b, subject to the following constraints: (i) the incentive constraint, equation (44);[65] (ii) the banker zero profit condition, equation (42); and (iii) the cash constraints, equations (38) and (39). The banker has an outside option to simply deposit its net worth in the mutual fund, in which case it earns, in present value terms, $\beta m N R = N$. The latter equality uses the household optimality condition discussed in equation (48) below. The formal definition of the banker problem is:

Definition 2. The *banker's loan contracting problem under unobservable effort is to choose* d, R_d^g, R_d^b *to maximize equation (45) subject to (i)-(iii), and then choose to be a banker if* $V(N) \geq N$. *Otherwise the banker simply deposits* N *in a mutual fund.*

In our computations we have not encountered a case in which the banker would choose to deposit her net worth in a mutual fund. Obviously, this cannot occur in an equilibrium. But, it might in principle have occurred out of equilibrium such as when we display the banker's 'demand for deposits' curve in Subsection 3.1.2 or when we analyze the model in its game representation in Subsection 3.4.

After receiving d from the mutual fund, the banker purchases (or sells) capital so that equation (41) holds.[66] In period $t = 2$ the banker transfers the quantities in square

[65] Equation (44) assumes interiority of the optimal value of e. We make this assumption in the manuscript to avoid complicating the notation, but in the computations we impose the upper and lower bound constraints on e, which are sometimes binding.

[66] Equation (41) is not to be interpreted as suggesting that the firm purchases all its capital, k^b, with $N + d$. The purchases of the bank can be seen by examining its sources and uses of funds in period 1. These are $\hat{Z} k_0^b + d$ and $Q \left(k^b - k_0^b \right) + L_0$, respectively. Equating sources and uses of funds and using equation (36), we obtain equation (41). Note that only $k^b - k_0^b$ units of capital are purchased (or sold, if negative) in period 1.

brackets in equation (43) to the household depending on whether its project turns out good or bad.

3.1.2 The Banker Demand for Deposits

Conditional on values of N, m it is possible to define a *banker's demand for* d, *conditional on* R. In effect, this requires substituting out for R_d^g, R_d^b, e in the solution to the banker's loan contracting problem.

A useful benchmark for understanding the solution to the banker's problem is to compare it to the solution to the contracting problem when effort is observable. In this case, the contract parameters are $\left(e, d, R_d^g, R_d^b\right)$. The banker's problem when e is observable corresponds to the problem in which effort is not observed, minus restriction (i) after equation (46). In this case, R_d^g, R_d^b play no role in the banker's choice of e and d.[67] The banker with observable effort sets e to e^*, the value of e that optimizes $\pi_d\left(e\right)$:

$$e^* = \beta m \bar{b}\left(R^g - R^b\right). \tag{47}$$

This level of effort is optimal independently of $N \geq 0$ and the choice of d. Interestingly, with the level of effort, e^*, the banker's marginal cost of effort is equated to the associated social marginal gain. This gain corresponds to the value of the marginal shift in probability towards the good state. Second, from equation (45) we see that optimization of d implies

$$d = \begin{cases} 0 & R > \pi_d\left(e^*\right) \\ \infty & R < \pi_d\left(e^*\right) \end{cases},$$

with the banker indifferent over $d \geq 0$ when $R = \pi_d\left(e^*\right)$. Thus, when effort is observable, the banker's demand for deposits is infinitely elastic at $R = \pi_d\left(e^*\right)$. In the observable effort case, R_d^g, R_d^b, are simply chosen to satisfy the banker zero profit condition, as well as the good and bad-state cash constraints. In sharp contrast to the important role played by these variables in our model, R_d^g, R_d^b play no allocative role when effort is observable.

To understand the solution to the banker problem, consider Figure 7, which displays

[67] To see this, condition (ii) has already been imposed on the banker objective, equation (45). So, absent condition (i), a banker with observable effort would choose a contract in which e and d optimize equation (45) without any restrictions. After solving for e and d, R_d^g, R_d^b can always be chosen to satisfy condition (iii). This is why condition (iii) is non-binding in the observable effort case.

numerical results based on the model parameterization discussed in subsection 3.3.[68] Consider first the dashed line. For any R this line displays the highest level of deposits, $d(R)$, such that the banker can just barely insulate creditors (the mutual fund) from losses in the bad state. That is, it is possible to set $R_d^g = R_d^b = R$ while satisfying the good and bad state cash constraints. Specifically, $d(R)$ is the value of d that solves $dR = (N+d)R^b$. Points in the figure to the right of the dashed line have the property, $R_d^b < R < R_d^g$. In this case, the loan contract in effect imposes a tax on banker effort, so that $e < e^*$.[69] We see this tax in equation (44) which shows that for given d the tax is smaller the smaller is the spread, $R_d^g - R_d^b$. Also, for a given spread and d, the tax is smaller the larger is net worth, N.

The solid line with the kink is the demand for deposits for the banker with unobserved effort, who has $N > 0$.[70] The horizontal segment of that demand curve overlaps with the demand for deposits when effort is observed. The argument we made before about why a banker with observed effort would set $d = 0$ for $R > \pi_d(e^*)$ applies in the unobserved effort case as well.

We can already see the distortionary effects of the hidden effort assumption when $R = \pi_d(e^*)$. When effort is observable, the banker is indifferent over all $d \geq 0$, but when effort is not observable the banker will not go beyond the kink point. This is because such points lie to the right of the dashed line, which intersects the demand for deposits precisely at the kink. At levels of deposits to the right of the dashed line the banker must share losses with her creditor and this necessarily implies $\pi_d(e) < \pi_d(e^*)$.[71] The banker prefers $d = 0$ over d beyond the kink point because net revenues on deposits are negative for such values of d (see equation (45)).

For $R < \pi_d(e^*)$ the banker can borrow more than $d(R)$ while still making profits on deposits. But, the amount the banker is willing to borrow is limited because moving to the right of the dashed line results in a reduction in $\pi_d(e)$. The solid line indicates how far the banker is willing to go in borrowing for each R. Note that the dashed line is very steep compared to the banker deposit demand curve. This indicates that at low values

[68] Model parameter values appear in Table 3. Variables that are exogenous to the banker but endogenous in the model, such as Q and N, are taken from the first column of Table 4.

[69] It is straightforward to verify that, conditional on N, for d such that $dR > (N+d)R^b$ then the solution to the banker contracting problem has the property that the banker is wiped out in the bad state, i.e., $dR_g^b = (N+d)R^b$. A simple proof-by-contradiction argument establishes this result.

[70] The particular value of N used for deriving the banker deposit demand curve is the one discussed in footnote 68.

[71] It is evident from the definition of e^* and the structure of π_d that π_d is strictly increasing in e for $e < e^*$.

of R the financial friction is more pronounced, and projects are run less efficiently. Again, the force of the hidden effort assumption is on display: for $R < \pi_d(e^*)$ the banker with hidden effort borrows a finite amount whereas if the banker's effort were observable she would want to take an unbounded level of deposits.

It is important to emphasize that the demand curve is drawn by varying the value of R, while holding m and the current and future prices of capital fixed (the current price is Q and the $t = 2$ price is zero). Under these circumstances, a fall in R increases the cross-sectional variance on the performance of banker assets.[72] We know of no empirical evidence that can be used to assess this implication. For example, Gertler and Karadi (2011) argue that interest rate spreads decrease after a reduction in Federal Reserve's policy interest rate. However, Q (and, hence, N, $R^g - R^b$) probably also respond to their monetary shock, and we do not know what our model would predict for cross-sectional banker variation if Q were adjusted simultaneously with a fall in R. Our purpose here is only to shed light on the model properties and the relation of the model to empirical evidence of the type in Gertler and Karadi (2011) is left for future research.

Interestingly, the shape of the banker's demand for deposits in the hidden effort model bears a strong resemblance to the analogous object in period 1 of the running away model. The demand for deposits in that model is also horizontal at a high value of R for low values of d. Like in our model, in that model the horizontal line extends all the way to infinity in the absence of the assumption that the banker can run away with a fraction of the assets. With the running away assumption, the demand for deposits has a kink and becomes downward sloped, as in the hidden effort model. In both models the kink occurs further to the right the larger is banker net worth.[73]

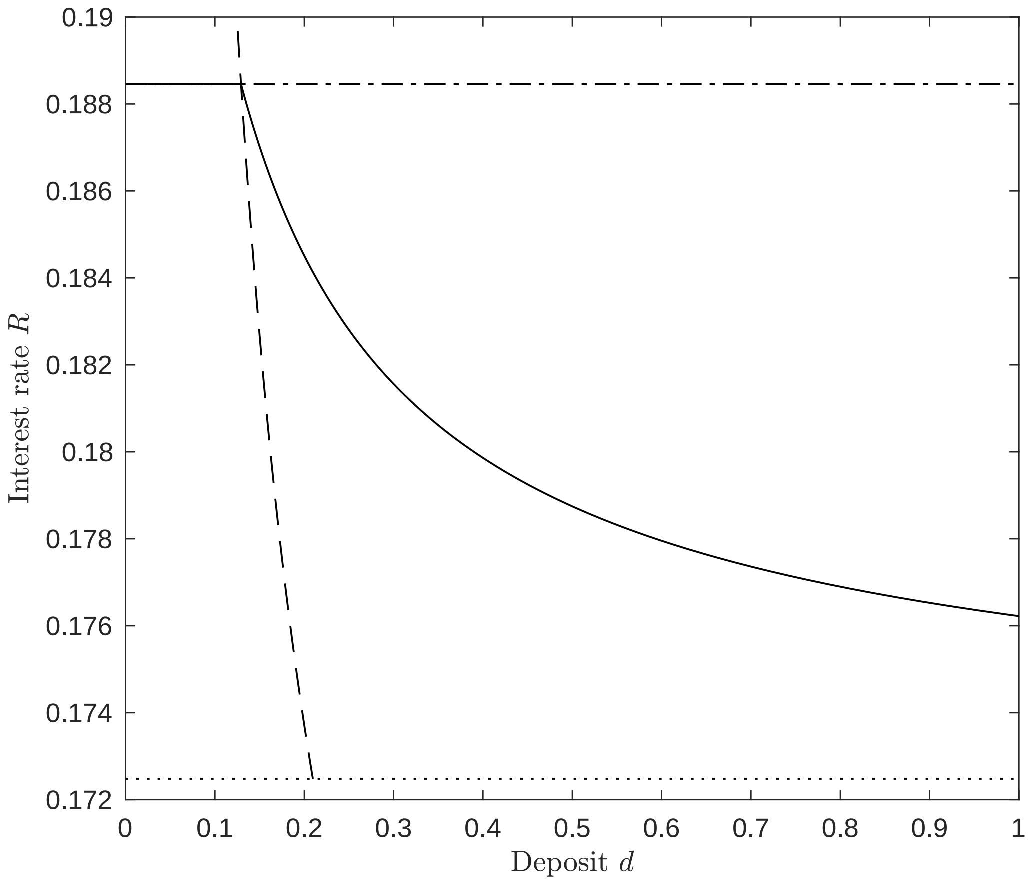
We can also consider the case in which the banker has no net worth, so that $N = 0$. In this case the demand for deposits is infinitely elastic at $R^0 = \pi_d(0)$. For $R > R^0$ the banker sets $d = 0$. For $R = R^0$ the banker sets $e = 0$ and is indifferent over $d \geq 0$. For $R < R^0$ the banker's demand for deposits is unbounded above. This demand curve is displayed as the dotted line in Figure (7). Note that the demand curve for $N > 0$ (solid

[72]The variance of the (undiscounted) rate of return on a unit of bank assets from the perspective of the beginning of $t = 1$ is $p(e)(1 - p(e))(R^g - R^b)^2$. In the text we show that, holding Q fixed, a cut in R reduces $p(e)$ by reducing e. This reduces the above variance when $p(e) > 1/2$, as is the case in our no run equilibrium. If the higher d issued when R is lower resulted in an increase in Q, then this would raise e and reduce $R^g - R^b$ (for the latter, recall equation (40)). So, the total effect on variance of a drop in R is ambiguous. In addition, a cut in R would, in a multi-period context, also be expected to affect future asset prices that too would feed back onto variance.

[73]See Figure 5.2 in Christiano and Ikeda (2013) for a graph of the banker demand for deposits in a static version of the running away model. The derivation can also be found on page 25, here.

line) is declining towards the dotted line. Intuitively, this is not surprising because as d becomes large enough for given N, N becomes a vanishingly small portion of total assets. We computed the demand curve for d up to as far as $d = 30$ and found it to be approaching the dotted line from above. Later, we will see that R^0 is below the household's outside option so that the quantity of deposits demanded is zero when $N = 0$.

Figure 7: Bank Demand for Deposits



Notes: Solid line with kink - banker demand for deposits when effort is not observable; dot-dashed line - banker demand for deposits when effort is observable; dotted line - banker demand for deposits when net worth is zero and effort is not observable; dashed line - values of R and d such that banker cash constraint is satisfied as a strict equality in the bad state with $R_d^b = R$, i.e., $R = (N + d)\, R^b/d$, where R^b is defined in equation (40). Model parameters are reported in Table 3 and the value of Q (which determines R^b) and N are as reported in the first column of Table 4.

3.2 Households, Market Clearing and Aggregate Conditions

The representative household is the same as the one in section 2.2, except that we start the analysis of the household in period $t = 1$, before Q is determined. At that time, the household's state variables are Z, $L_0 = R_0 d_0$ and $k_0^h = 1 - k_0^b$. The household's objective is $u(c) + \beta u(c_2)$, where c, c_2 denote periods $t = 1$ and $t = 2$ consumption, respectively (recall the convention in this section that the time subscript is deleted when $t = 1$). The household's budget constraints for $t = 1$ and $t = 2$ are given by equation (24). Households that hold capital obtain marginal productivity Z in $t = 1$ and Z_2 in period $t = 2$. They do not have the ability to raise the productivity of capital by exerting effort. As in the running away model, the household pays a management cost, $f(k^h)$, for its end-of-period $t = 1$ capital holdings, k^h. We define

$$R = 1/(\beta m), \tag{48}$$

where $m = u'(c_2)/u'(c)$. When deposits are positive, $d > 0$, then equation (48) is the household's optimality condition for deposits. In an annihilation run, deposits are zero and in this case we take equation (48) as the definition of R. In the annihilation run, R coincides with the gross rate of return on capital.[74]

The capital market clearing condition in period $t = 1$ is $k^h + k^b = 1$. The resource constraint in period $t = 1$ is:

$$c + f\left(k^h\right) = Zk_0^h + \hat{Z}k_0^b + y \tag{49}$$

where y denotes an exogenous component of household income in period $t = 1$.[75] The resource constraint in $t = 2$ is[76]

$$c_2 = Z_2 k^h + \hat{Z}_2 k^b, \tag{50}$$

[74] Below, we consider the possibility that a small mass of individual bankers deviate from an annihilation run equilibrium by taking loan contracts from a mutual fund and the R that the mutual fund would have to pay households for those funds is the one given by equation (48).

[75] We can also obtain equation (49) by Walras' law. Here that requires that the sum of purchases by households and banks in period 1 equal the sum of their income. The purchases and income of households in period 1 are given in equation (24). The uses (i.e., expenditures) and sources (i.e., income) of funds for the banks are discussed in Footnote 66.

[76] As usual, this expression can also be derived using Walras' law. In this case, we use equation (40) to substitute out for R^x, $x = g, b$.

where

$$\hat{Z}_2 = \left[p\left(e\right)e^g + \left(1 - p(e)\right)e^b\right] Z_2.$$

3.3 Competitive Equilibrium and Parameter Calibration

We define an equilibrium as follows:

Definition 3. A competitive equilibrium is $\left\{R_d^g, R_d^b, R, d, N, Q, e, k^h, k^b, c, c_2\right\}$ which (i) solve the banker contracting problem (see Definition 2) and the household problem and (ii) satisfy the $t = 1, 2$ resource constraints and market clearing conditions.

The parameters of the model are:

$$\alpha, \beta, \sigma, g, b, \bar{a}, \bar{b}, L_0, k_0^b, \hat{Z}, Z, Z_2, y,$$

where α and σ are the management function and utility function parameters discussed in the previous section (see equation (31)). Similarly, β is the household's discount rate.

Table 3 lists the parameters and exogenous variables in the baseline model. We parameterize the model to guarantee that the annihilation run exists; the financial constraint (equation (39)) binds in the bad state; e is interior and non-negativity constraints on c, c_2, k^b, k^h are non-binding in the no run equilibrium. In selecting parameters we also made sure that standard parameters like the discount rate, did not take on unreasonable values.[77]

[77] We did not use a formal loss function approach in this case, as we did in section 2.4.

Table 3: Baseline Parameters

α: household management cost	0.0610
β: subjective discount rate	0.9700
σ: curvature in utility function	1.4951
g: good project productivity	0.2300
b: bad project productivity	-0.3000
$\bar{a}$: project probability parameter	0.2500
$\bar{b}$: project probability parameter	0.7500
L_0: bank liabilities, $t = 0$	0.4239
k_0^b: bank capital, $t = 0$	0.7300
Z: household productivity, $t = 1$	0.1263
$\hat{Z}$: banker productivity, $t = 1$	0.1263
Z_2: banker productivity, $t = 2$	0.1000
y: household labor income, $t = 1$	0.2000

Notes: parameter values for period $t = 1, 2$ in hidden effort model; the two $t = 0$ variables are state variables in $t = 1$.

As in the running away model, we found three equilibria. This is not surprising since in the hidden effort model it is also true that there is strategic complementarity in the deposit decision of banks, which operates via the market-clearing price, Q, of capital. The three equilibria are reported in Table 4 as the *no run equilibrium*, the *partial run equilibrium* and the *annihilation run equilibrium*. Note that in the last case, $N = d = e = 0$, as expected. As we go from the annihilation equilibrium to the no run equilibrium, net worth increases, banks issue more deposits and buy more capital at a higher price. Note that the efficiency of banking also improves as e increases across the equilibria and the amount of capital held by banks increases. Consumption in period 1 increases while c_2 is essentially constant. So, we see that welfare is increasing as we go from the annihilation run equilibrium to the no run equilibrium.

Table 4: Three Equilibria in Hidden Effort Model

	No-run	Partial run	Annihilation run
k^h	0.5709	0.8884	1.0000
e	0.3590	0.1907	0.0000
$p(e)$	0.5193	0.3930	0.2500
R	0.1854	0.1955	0.2037
Q	0.5046	0.4574	0.4299
N	0.0366	0.0022	0.0000
d	0.1799	0.0488	0.0000
c	0.3164	0.3022	0.2958
c_2	0.1004	0.0994	0.1000
V	0.0399	0.0042	0.0000
V_{Save}	0.0366	0.0022	0.0000

Note: V corresponds to $V(N)$, the present value attained by the banker in the unobserved effort case (see Definition 2). $V_{save} = N$ corresponds to present value of the banker's outside option, which is to deposit its net worth in a mutual fund. Banker optimization requires $V_{save} \leq V(N)$.

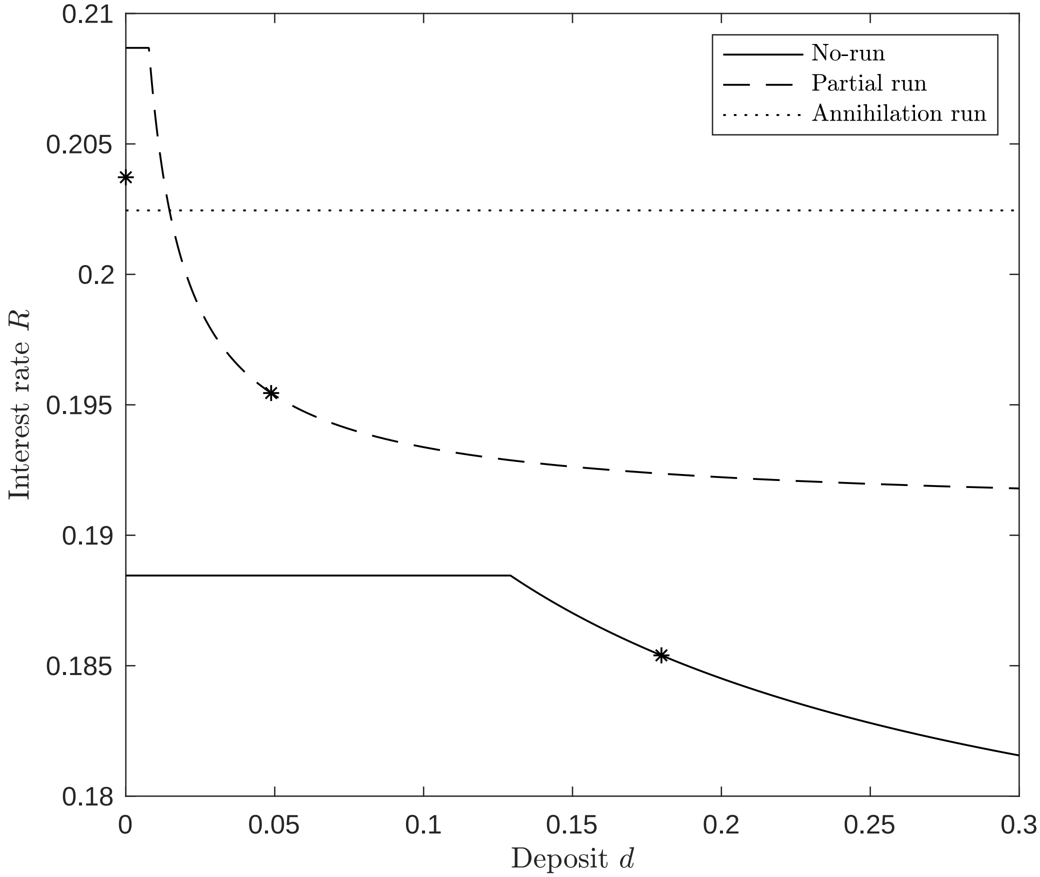
Figure 8 displays the bank demand curves for deposits in each of the three equilibria. The solid line corresponds to the no run equilibrium and coincides with the bank demand curve in Figure 7. The dashed and dotted lines correspond to the demand curves in the partial and annihilation runs, respectively. The stars indicate the location of the supply of deposits by households, i.e., the general equilibrium point.

Consider the case of an annihilation run, in which case market clearing deposits are zero because the marginal value of deposits to households (i.e., their outside option of holding capital directly) lies above the banker demand curve for deposits. Because deposits are zero in this equilibrium, the net worth of banks is zero, $N = 0$. Note that in the partial and no run equilibria, the financial friction is binding because in each case the star lies below the horizontal segment of the corresponding demand curve for deposits (recall the discussion in Subsection 3.1.2). In the annihilation run, the friction is so binding that the banking system shuts down altogether.

Note that the horizontal segment of the demand curve in the partial run equilibrium is much shorter than it is in the no run equilibrium. This is because net worth is relatively small in the former equilibrium. It is also interesting that the demand curve is shifted up in the partial run equilibrium. That reflects the endogeneity of R^g and R^b, which rise with the lower Q in the partial run equilibrium.[78]

[78] Recall the definition of R^x, $x = g, b$ in equation (40). Also, recall the discussion in Subsection 3.1.2, which shows that the horizontal component of the banker demand for deposits corresponds to $R = \pi_d(e^*)$, where $\pi_d(e)$ is defined in equation (46) and e^* is defined in equation (47).

Figure 8: Bank Demand Curves



Notes: The graph contains three banker deposit demand curves for each of the equilibria indicated in the legend. Stars correspond to the general equilibrium points reported in Table 4.

3.4 Fragility of Equilibria to Entry Costs in the Hidden Effort Model

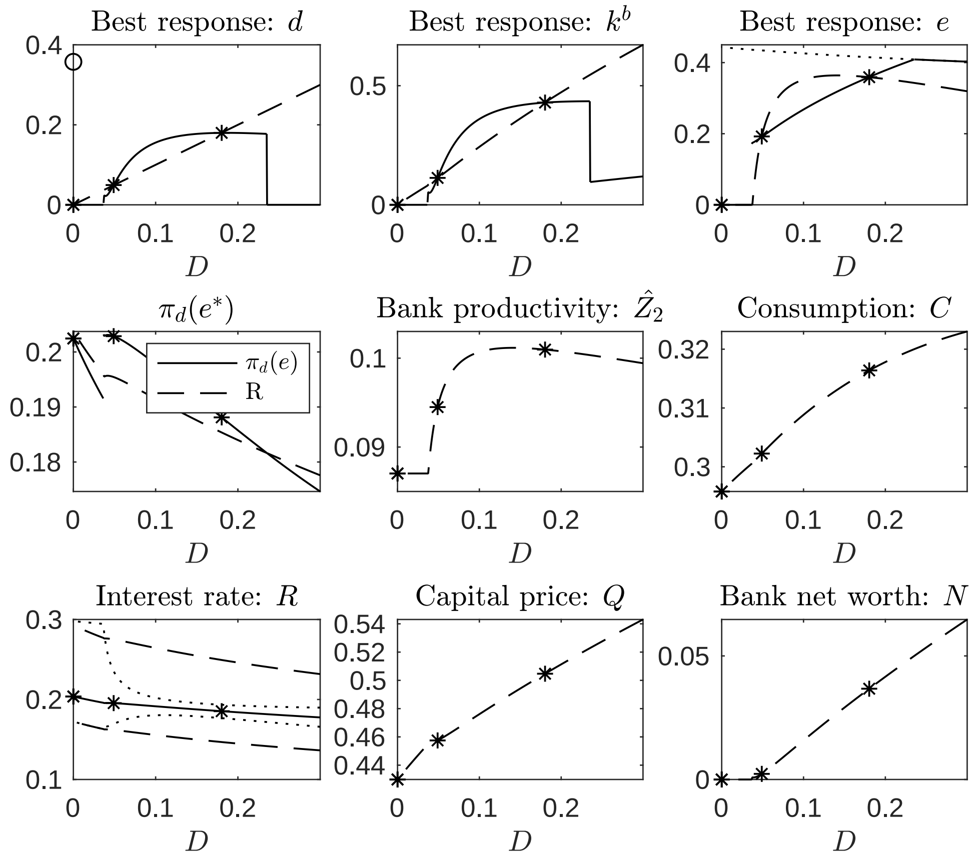
Here, we show that the hidden effort model also displays the fragility of equilibrium reported for the three period running away model in Section 2.6.

3.4.1 Partial Run

To understand the three equilibria identified in Subsection 3.3, it is useful to apply the same device used in Subsection 2.6 for the running away model. In particular, we convert the market equilibrium into a game between a representative individual bank and all the other banks. The individual bank conjectures that the other banks issue D deposits on average and then computes the associated continuation equilibrium. This continuation equilibrium conditional on D is a set of values for the other endogenous variables which solve all of the market equilibrium conditions except one, the bankers' first order condition for deposits.[79] By leaving out the latter equation the individual bank in effect asks 'what if everyone else set deposits on average to D?' without asking whether the value of D is optimal for the other bankers. In the continuation equilibrium conditional on D, variables crucial for solving the individual banker's contracting problem, including d, are determined. The mappings from $D \in [0, .3]$ to a subset of the variables in the continuation equilibrium, as well as to d are displayed in Figure 9. The mapping from D to d is the individual banker's best response function. The three locations marked by a '*' correspond to Nash equilibria. These in turn correspond to the annihilation run, the partial run and the no run market equilibria.

[79] A discussion of how we computed best responses in the Hidden Effort model appears in Section B of the online Appendix. The code for computing the best responses is included in the replication files.

Figure 9: Best response function: Hidden effort model



Note: (1) Apart from the 2,1 and 3,1 panels, the convention is as follows. Solid lines - individual bank best response to aggregate bank deposits, D. A vertical solid line indicates a set of values in the 1,1 and 1,2 panels. At the associated value of D, d and k^b are set-valued (for explanation, see text), so that the best responses for d, k^b, e in panels 1,1, and 1,2, respectively, are set-valued at one value of D. This value of D is the one in which $\pi_d(e^*) = R$ in panel 2,1. Dashed lines - continuation equilibrium for aggregate variables associated with D. (2) In the 3,1 panel: solid line is risk free interest rate, upper dotted line is R_d^g and lower dotted line is R_d^b, while upper and lower dashed lines correspond to R^g and R^b, respectively. All the variables in the 3,1 panel are features of the continuation equilibrium conditional on D. The continuation equilibrium conditional on D is the set of equilibrium variables that solve all equilibrium conditions, except the equilibrium condition for D.

Consider the 1,1 panel in the figure. That displays the individual banker's best response to D (see the solid line). When $D = 0$ banks buy little capital. With the market for capital dominated by households who assign a lower value to it, Q is low. The latter price is a part of the continuation equilibrium associated with D and appears in the 3,2 panel (except in panel 3,1, the dashed line corresponds to the continuation equilibrium). With the low value of Q when $D = 0$, the net worth of bankers is zero (see panel 3,3). Given the relatively high outside option of the household in the annihilation run, the individual banker best responds by setting $d = 0$.[80] As D rises, the price of capital, Q, increases, as well as net worth, N. But, N remains at zero until Q is large enough (see equation (36)). So, d remains at zero too.

Another way to see why the individual bank's best response function lies below the 45^o line for small D is to look at the features of the continuation equilibrium displayed in panel 3,1. In this panel, the dashed line corresponds to R^g, R^d, the dotted line corresponds to R_d^g, R_d^b and the solid line corresponds to R, all in the continuation equilibrium.[81] In panel 3,1 we see that for $D > 0$ but small, $R_d^g > R^g$ and $R_d^b = R^b$.[82] Here, R_d^g, R_d^g, R satisfy the zero profit condition and bad state cash condition of banks given D. Under these circumstances, the pecuniary return on deposits, D, are negative for all e. The individual banker best responds by choosing $d = 0$.

Thus, the banker's best response function is flat compared to the 45^o line (panel 1,1) in a neighborhood of $D = 0$. As D increases, Q eventually rises by enough that net worth becomes positive. Then, the banker best responds with $d > 0$ (see panel 1,1). The individual banker's d rises sharply with D and cuts the 45^o line from below. This crossing corresponds to the partial run equilibrium. Eventually, the best response function flattens out and cuts the 45^o a third time, at the no run equilibrium.

There are several interesting features of the individual bank best response function displayed in the 1,1 panel. First, when D is large enough that $d > 0$ then deposits are strategic complements and the best response function is upward-sloped. This is consistent with the simple intuition that when D is bigger then banks buy more capital, driving Q and, hence, N, up. The individual banker best responds to the rise in N by issuing more d. This pattern continues as D reaches the no run equilibrium. Afterward, the mapping is single-valued over almost all values of D displayed, with one exception. The exception is the value of D when the best response of d is set-valued. This set corresponds to the vertical solid lines in the 1,1 and 1,2 panels of Figure 9. For higher

[80] Recall the discussion in subsection 3.3. The household's outside option is indicated by the star on the vertical axis in Figure 8. The bank's demand curve in the annihilation run is the infinitely elastic horizontal dotted line.

[81] Only the first order condition for D is ignored in these computation of the continuation equilibrium. The other equations associated with the loan contract problem of other bankers are used to determine R_d^g and R_d^b in their loan contracts associated with D. For values of D where the best response function does not intersect the 45^o line, the level of D violates the banker first order condition for deposits.

[82] The bad state cash constraint is binding for all D up to a little above the no run equilibrium. This can be seen from the dashed line in panel 1,3, which shows that $e < e^*$, where e^* is indicated by the dotted line. The appendix shows that when the bad state cash constraint is binding, then $R_d^b = R^b \left(N + D\right)/D$ so that $R_d^b = R^b$ when $N = 0$.

values of D, the individual banker best responds by setting $d = 0$. The reason for these findings can be seen by looking at the 2,1, and 1,3 panels. The first of these panels displays $\pi_d(e^*)$ (solid line) and R (dashed line) as a function of D.[83] Consistent with the discussion in Subsection 3.1.2, panel 1,3 shows that $e = e^*$ for the value of D where $R = \pi_d(e^*)$. So, in the continuation equilibrium associated with this value of D the individual bank is indifferent over values of d in an interval (see, for example, the horizontal portion of the solid line in Figure 7). The vertical line in the 1,1 panel is the corresponding interval of values of d. Associated with those values of d are the quantity of assets, k^b, indicated by the vertical line in the 1,2 panel.[84] The best response of deposits drops to zero when $\pi_d(e^*) < R$ because the banker makes negative revenues on deposits in that case. This is true in general because $\pi_d(e) \leq \pi_d(e^*)$ and because panel 1,3 indicates that $e = e^*$ over this range of D's.

Finally, consider the open circle on the y-axis in panel 1,1, which sheds light on a central theme of the paper. That circle corresponds to the open circle in Figure C1a of the online Appendix for the infinite horizon GK model. We computed ξ, the ratio of the total net worth brought into banking by newborns in the GK model to the net worth of all bankers in the no run Nash equilibrium in Figure C1a. The value of ξ is (not surprisingly) very small, at $\xi = .0384$.[85] We then computed N, the net worth of bankers in the no run Nash equilibrium in the 1,1 panel of Figure 9. The quantity of net worth, ξN, for the hidden effort model is thus comparable to the net worth of newborns in the GK model. The open circle is the amount of deposits that can be supported by the ξN units of net worth in an annihilation run. Because the rate of return on assets is high in that state, leverage is high, at 255. This is a similar order of magnitude for analogous concept in the three-period and infinite period versions of the GK model.[86] Banks with ξN units of net worth can issue an amount of deposits

[83] Recall that $\pi_d(e)$ is the expected revenues on one unit of deposits, net of the utility cost of effort, conditional on e (see equation (45)). Also, e^* is the value of e that maximizes π_d and $\pi_d(e)$ is increasing for all $e < e^*$.

[84] The individual bank's capital, k^b, does not drop to zero because that is funded by the positive and growing bank net worth as D increases.

[85] In section C.2.3 of the online Appendix, we report that the net worth brought in by newborns in the infinite horizon GK model is 0.000676. Also, we report that leverage in the no run state is 21 in a period when the most recent run was 10 periods in the past. Also, the total assets held by the banks in that state is 0.37. So, the net worth of all bankers in the no run state is $0.37/21 = 0.0176$. Then, $0.000676/0.0176 = 0.0384$ after rounding. For more details see section C in the online Appendix.

[86] Recall that the relevant leverage number in the three period model is 756 (see section 2.6.2). The relevant leverage number in the infinite horizon GK model is while it is 782 (see section C.2.3 in the online Appendix).

corresponding to the open circle. Note that this is roughly two times the deposits that are issued in the no run state. This is another example of how a small amount of net worth introduced into an annihilation run can cause that run not to be a Nash equilibrium.

3.4.2 Interpreting the No Entry Assumption in Annihilation Run as an Entry Cost

In this section, we calculate the entry cost that would rationalize the no-entry assumption in an annihilation run. Consistent with our results for the running away model, we find that that entry cost is very high.[87]

To do the entry cost calculation for the hidden effort model, we require the analog of ψ_1^* in the three period running away model (see equation (13)). Recall that the optimized present discounted value of profits (net of borrowing costs) for a banker with net worth, N, is $V(N)$.[88] It is easy to verify that, as in the case of ψ_1^*, $V(N)/N$ well-defined and independent of N for $N > 0$. As it turns out, the only variables required in the annihilation equilibrium are R^g, R^b and R. [89] The values of these market variables

[87]The calculations for the three-period running away model are reported in Section 2.6.2. That section also summarizes the results for the infinite horizon version of the running away model (see section C.2.3 in the online Appendix for details).

[88]See discussion after equation (46).

[89]To see the two results just described and to explain how we compute the findings reported in the text, let $V\left(d, R_d^g, R_d^b; N\right)$ denote the expression in equation (45). Recall that we define, after equation (45), $V(N) \equiv \max_{d,R_d^g,R_d^b} V\left(d, R_d^g, R_d^b; N\right) = N\beta m \max_{e,\phi,R_d^g,R_d^b} v\left(e, \phi; R^g, R^b, R\right)$, where $\beta m = R^{-1}$ by household optimization and R is the cost of funds to the mutual fund. The maximization is subject to the incentive constraint, the cash constraints and the zero profit condition of mutual funds (see Definition 2), and recall that we suppose $N > 0$. The object, ϕ, denotes leverage, $\phi = (N + d)/N$. Also, $v\left(e, \phi; R^g, R^b, R\right) = \pi_d(e) + [(\phi - 1)/\phi][\pi_d(e) - R]$ and π_d is defined in equation (46). Note that in going from V to v, R_d^g, R_d^b have been replaced by R^g, R^b, R, where all three variables are exogenous to the banker. This reflects that we have substituted out the zero profit condition in the banker objective (see equation (42)). We substitute out for R_d^g, R_d^b in the incentive constraint by using the zero profit condition as well as cash constraints. In the case of the cash constraints, we use the implication derived in Footnote 69, that an optimizing banker who chooses to issue so much leverage that she must share risk with creditors, will choose a contract that wipes out her net worth in the bad state. These considerations allow us to substitute out for R_d^g, R_d^b in the banker incentive constraint, which reduces to:

$$e - \beta m \bar{b}\left[\left(R^g - R^b\right) - \frac{\max\left\{0, \frac{\phi-1}{\phi}R - R^b\right\}}{p(e)}\right] = 0.$$

The banker problem is now simply to solve $\max_{\phi,R_d^g,R_d^b} v\left(\phi, R_d^g, R_d^b\right)$ subject to the above constraint. Note that the value of N does not appear in this problem, which coincides with the unscaled version of the problem as long as $N > 0$. This establishes that $V(N)/N$ is independent of N for $N > 0$. We use standard methods to solve a Lagrangian representation of the above banker problem (in practice, can be deduced from the variables reported in the third column of numbers in Table 4. There, we find that $R = 0.2037$, $e = 0.082$, $p(e) = 0.311$, $\phi = 255$. Also, combining the value of Q in that table with b and g in Table 3, we obtain $R^g = 0.293$ and $R^b = 0.172$. These results imply $V(N)/N = 7.8$.

Note that, as in the other two models, leverage is a very high value of 255 for a banker with net worth who enters when there is an annihilation run. Also, that banker earns \$7.8 dollars per \$1 of net worth brought into an annihilation run. This number is two orders of magnitude smaller than the corresponding result for the three period and infinite period running away models, which are \$378 (see section 2.6.2) and \$151 (see online Appendix, section C.2.3), respectively. Still, the notion that it costs \$7.8 to issue \$1 of equity to rationalize the no entry assumption seems implausibly high by an order of magnitude. We suspect that the reason the required entry costs are relatively 'small' in the hidden effort model reflects that high leverage in that model implies low banker effort.

4 Conclusion

We show that a pure banking panic contains within it the seed of its own undoing. In a pure panic, expected returns are very high so that there is a huge incentive for equity to come into the banking system at that time. Equity could be injected by outsiders. Even insiders can preserve equity for a run by not issuing deposits in periods before a run. To preserve the existence of a bank run state, ad hoc assumptions are required to prevent these types of equity injections that would otherwise undo a banking panic.

We also find that the number and nature of equilibria is sensitive to assumptions about equity injections. For example, we find that, generically, there is a second run state in which the net worth of banks is not completely wiped out. We also display an example in the online Appendix (see Figure C1c) in which there are two run states and both have the property that the net worth of banks is not wiped out.

Our paper reports a second result that is of interest. Not surprisingly, we show that a leverage restriction can raise social welfare. But, it can also be associated with a second equilibrium in which welfare is reduced. We display this result in the three-period version of the GK model. The intuition is very simple and so we expect that the result generalizes to other models as well.

we also include Lagrange multipliers to ensure that e satisfies equation (37))).

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Online Appendix

The first section below discusses our loss-function based method for calibrating the parameter values for the three period running away model in Section 2. The second subsection reviews the details of our analysis of the infinite-horizon running away model in GK.

A Calibrating the Values of the Parameters in the Three-Period Running Away Model in Section 2

Here, we flesh out the details of the calibration strategy sketched in subsection 2.4. This calibration turned out to be challenging. First, we are uncertain about the precise meaning of parameter values for a three-period model. Second, it is not straightforward to find model parameter values for which there is an interior equilibrium and the banker problems are interesting in the sense that all banker incentive constraints (i.e., the assumptions of Proposition 1) are satisfied. To address these challenges we develop the quasi-Bayesian strategy described below. All the codes are available in the replication files.

A.1 Strategy

The 10 exogenous parameters of the model are listed in Table 1 and reproduced here for convenience:

$$\sigma,\ \beta,\ Z,\ Z_2,\ \alpha,\ \theta_1,\ k_{-1}^h,\ R_{-1}d_{-1},\ y,\ \theta_0. \tag{A.1}$$

An equilibrium (see Definition 1) corresponds to a list of values for the following 15 variables:

$$Q_0, Q_1, Q_1^*, c_0, c_1, c_1^*/c_1, c_2, P, \psi_1, \psi_1^*, R_0, R_1, R_1^*, k_0^h, k_1^h. \tag{A.2}$$

The actual list endogenous variables in Definition 1 is much longer, but the missing variables can be obtained trivially from the above list. For example, equation (30) implies $c_2^* = c_2$ and similarly, k_t^b can be recovered from the fact, $k_t^b + k_t^h = 1$.

We find it convenient to replace the exogenous variables, θ_1, α, in equation (A.1) with the endogenous variables, $c_1^*/c_1, P$. We denote the resulting list of 'adjusted exogenous variables' by χ. Similarly, the variables $c_1^*/c_1, P$ in equation (A.2) are replaced by θ_1, α and we denote the resulting vector of 'adjusted endogenous variables' by ς. For a given setting for the variables in χ we may be able to compute a corresponding set of variables, ς, that satisfy:

$$\varsigma = g(\chi), \tag{A.3}$$

where g denotes the 15 equilibrium conditions of the model.

Our calibration strategy proceeds in four steps. In the first step we draw n values of χ from a 'prior distribution'. The priors on each variable in χ are independent and reported in Table A1. We then evaluate $\varsigma_i = g(\chi_i), i = 1, ..., n$. For some values of χ, we did not find a satisfactory solution to g. That was either because no ς could be found that satisfies the equilibrium conditions and simple non-negativity constraints, or because the solution violated one of the banker incentive constraints, i.e., the conditions of Proposition 1. In step 2, we delete the unsatisfactory χ_i's and instead continue working with the subset, n_1, of values of χ_i for which we found a satisfactory solution to equation (A.3). With a slight abuse of notation, denote the resulting satisfactory set of $\chi_i's$ by $\chi_1, ..., \chi_{n_1}$. Next, we define a continuous loss function over both ς and χ:

$$\mathcal{L}(\varsigma, \chi).$$

In step 3, for each $i = 1, ..., n_1$ we locally optimize, by choice of χ_i,[90]

$$\mathcal{L}(g(\chi_i), \chi_i).$$

The set of local optimizers is denoted by $\mathcal{L}_i, \hat{\chi}_i, \hat{\varsigma}_i$, $i = 1, ..., n_1$. The calibrated values of χ and ς are denoted by $\hat{\chi}$ and $\hat{\varsigma}$, and are obtained in the final and fourth step:

$$\hat{\chi} = \hat{\chi}_{i'}, \hat{\varsigma} = \hat{\varsigma}_{i'}, \tag{A.4}$$

where i' has the property, $\mathcal{L}_{i'} \geq \mathcal{L}_i$, for $i = 1, ..., n_1$.

[90]We used the optimizer, fminsearch, in MATLAB.

Table A1: Priors for estimation

Variable	Distribution	Mean	β	α
P	Beta	0.15	15	2.65
$\theta_t,\ t \in \{0,1\}$	Beta	0.3	40	17
$\frac{c_1^*}{c_1}$	Beta	0.8	5	20
β	Beta	0.9	9	81
$k_t^h,\ t \in \{-1,0,1\}$	Beta	0.2	60	15
σ	Inverse Gamma	1	1	2
Z	Beta	0.15	10	1.76
Z_2	Beta	0.15	10	1.76
$R_{-1}d_{-1}$	Beta	0.15	15	2.65
y	Beta	0.15	10	1.76

Note: in the case of the Beta distribution, α and β are 'scale' parameters. In the case of Inverse Gamma, α is a 'shape' parameter and β is a 'scale' parameter.

A.2 Constructing the Loss Function

We now describe how we constructed the loss function, $\mathcal{L}$. This loss function is additively separable across the values assigned to a subset of the variables ς and χ and also includes penalties for violating bank incentive constraints. The subset of the variables in ς and χ whose values are directly penalized in $\mathcal{L}$ is $(\sigma, \theta_1, \beta, \frac{c_1^*}{c_1}, P, k_{-1}^h, k_0^h, k_1^h)$. Values of the variables, α, Z, Z_2, which violate their sign constraint are penalized using an indicator function. The other variables are indirectly penalized via their impact on the incentive constraints.

We first discuss the variables, $(\sigma, \theta_1, \beta, \frac{c_1^*}{c_1}, P, k_{-1}^h, k_0^h, k_1^h)$. Any variable in this vector which also appears in Table A1 enters $\mathcal{L}$ via the prior density exhibited in that table. Variables in the previous vector which do not appear in Table A1 are introduced into $\mathcal{L}$ via an inverse Gamma distribution or Beta distribution, depending on whether we want the variable to be positive or inside the $[0,1]$ interval. We use the actual density rather than its log to avoid numerical problems associated with computing the log of zero. This creates a problem when the optimization routine attempts to take a variable outside the support of its distribution. In cases like this, we smoothly extended the density function to apply a substantial (though smooth) penalty for leaving the support. Specifically, we define the following objects:

$$b_{L,\sigma} = \begin{cases} -\sigma 10^3 & \sigma \leq 0 \\ f_{IG}(\sigma, \alpha_\sigma, \beta_\sigma) & \sigma > 0 \end{cases}$$

$$b_{L,\theta_1} = \begin{cases} -\theta_1 10^8 (\theta_1 - 0.5)^2 & \theta \geq 1 \\ f_\beta(\theta_1, \alpha_{\theta_1}, \beta_{\theta_1}) & \theta < 1 \end{cases}$$

$$b_{L,\beta} = \begin{cases} 1 - \beta^{100} & \beta \geq 1 \\ f_\beta(\beta, \alpha_\beta, \beta_\beta) & \beta < 1 \end{cases}$$

$$b_{L,c_1^*/c_1} = f_\beta(\frac{c_1^*}{c_1}, \alpha_{c_1^*/c_1}, \beta_{c_1^*/c_1})$$

$$b_{L,P} = 4 \times f_\beta(P, \alpha_P, \beta_P)$$

$$b_{L,k_t^h} = \begin{cases} f_\beta(0.3, \alpha_k, \beta_k) - 10^4 (k_t^h - 0.3)^2 & k_t^h > 0.3 \\ f_\beta(k_t^h, \alpha_k, \beta_k) & k_t^h \leq 0.3 \end{cases},$$

for $t = -1, 0, 1$. Here, f_ϱ denotes a two-parameter distribution, where $\varrho = IG$ corresponds to the inverse Gamma distribution and $\varrho = \beta$ corresponds to the Beta distribution. Also, α_x, β_x are the parameters of the associated distribution, where, with one exception, x corresponds to the elements of $(\sigma, \theta_1, \beta, \frac{c_1^*}{c_1}, P, k_{-1}^h, k_0^h, k_1^h)$.[91] The exception is that in the case of $x = k$, the same Beta distribution parameter values were used in each of k_{-1}^h, k_0^h, k_1^h.

Now we turn to the incentive constraints. We include a subset of those constraints, namely seven, as follows:

[91] In the case of the Beta distribution, the two parameters are the 'scale' parameters. In the case of the inverse Gamma distribution the two parameters are the shape and scale parameters.

$$b_0 = 10^9 \left(\beta m_1 (1-P)\psi_1 (R_1^k - R_0)/\theta_0 - 1\right)$$
$$b_1 = 10^9 \left(\beta m_1 \left(1-P\right)\psi_1 R_1^k + \beta m_1^* P \psi_1^* R_1^{*,k} - \psi_0\right)$$
$$b_2 = 10^9 \left(R_2^{k,*} - (1+\theta_1)R_1^*\right)$$
$$b_3 = 10^9 \left(R_2^k - (1+\theta_1)R_1\right)$$
$$b_4 = 10^9 \left(R_1 - R_2^k\right)$$
$$b_5 = 10^9 (R_0 - R_1^k)$$
$$b_6 = 0.2 \times 10^3 \left(Z k_{-1}^b - R_{-1} d_{-1}\right).$$

Finally, $\mathcal{L}$ penalizes large deviations between c_0, c_1, and c_2.

We construct the penalty function as follows:

$$\begin{aligned}\mathcal{L} = \; & \textstyle\sum_{x\in\{\sigma,\theta_1,\beta,c_1^*/c_1,P\}} 0.2 b_{L,x} - \sum_{x\in\{\alpha,Z,Z_2\}} 0.2 \mathbb{1}_{(x<0)}|x| + \sum_{t=-1}^{1} b_{L,k_t} \\ & - \textstyle\sum_{j=0}^{6} \mathbb{1}_{(b_j>0)} b_j - 0.2\left((c_0-c_1)^2 + (c_1-c_2)^2\right),\end{aligned} \tag{A.5}$$

where $|x|$ indicates the absolute value of the scalar, x and $\mathbb{1}_{(condition)}$ is the indicator function which is unity if *condition* is satisfied.

The penalty function does not include all the incentive constraints in Proposition 1. At the end of the calibration analysis, we verify that the final parameters in equation (A.4) satisfy all the incentive constraints. So, some constraints play no role in the computation of $\mathcal{L}_i, \hat{\chi}_i, \hat{\varsigma}_i, i = 1, ..., n_1$. However, all the incentive constraints play a role in step 2 of the calibration procedure and in the final results.

B Computing Best Responses in the Hidden Effort Model

We use the best response function method to find all possible equilibria at period 1. Specifically, we compute an individual bank's best response $\left\{k^b, e^*\right\}$ for given the other's choice K^b by the following three steps. For notational simplicity, we use the lower-case to denote the individual banker's choice, while the upper-case to denote aggregate variables.

First, for given K^b, for each possible $e \in [\underline{e}, \overline{e}]$, we compute the other aggregate variables by the following algorithm

$$
\begin{aligned}
K^h\left(K^b, e\right) &= 1 - K^b, \\
C\left(K^b, e\right) &= Z_1 + y_1 - \frac{\alpha}{2}\left(K^h\right)^2, \\
C_2\left(K^b, e\right) &= Z_2 K^h + Z_2\Big[P\left(e\right) e^g + \left(1 - P\left(e\right)\right) e^b\Big] K^b, \\
R\left(K^b, e\right) &= \left(\beta \frac{U'\left(C_2\right)}{U'\left(C\right)}\right)^{-1}, \\
Q\left(K^b, e\right) &= \frac{Z_2}{R} - \alpha K^h, \\
N\left(K^b, e\right) &= \max\left\{\left(Z^b + Q\right) K^b - L_0, 0\right\}, \\
D\left(K^b, e\right) &= Q_1 K_1^b - N,
\end{aligned}
$$

and the banker's incentive constraint error

$$
er\left(K^b, e\right) = e - \frac{\bar{b}}{R}\left(R^g - R^b - \frac{\frac{D}{D+N} R - R^b}{P(e)}\right) = 0.
$$

So the banker's optimal effort $e\left(K^b\right)$ solves $er\left(K^b, e\right) = 0$. We also consider the following scenarios where the banker's optimal effort is a corner solution. If $er\left(K^b, e\right) < 0$ for any possible $e \in [\underline{e}, \overline{e}]$, we choose $e\left(K^b\right) = \overline{e}$. If $er\left(K^b, e\right) > 0$ for any possible $e \in [\underline{e}, \overline{e}]$, we choose $e\left(K^b\right) = \underline{e}$.

Second, the individual banker choose the best response taking the above aggregate variables as given. For given aggregate variables K^b, for each possible banker's capital decision $k^b \in \left[\frac{N}{Q}, 1\right]$, for each possible banker's effort decision $e^* \in [\underline{e}, \overline{e}]$, we compute the following two errors (first order conditions in the individual banker's contracting problem),

$$
\begin{aligned}
d &= Q k^b - N, \\
er_1\left(k^b, e^*\right) &= e^* - \frac{\bar{b}}{R}\left[R^g - R^b - \min\left\{0, \frac{\frac{d}{N+d} R - R^b}{P\left(e^*\right)}\right\}\right], \\
\eta &= \frac{e^* - \frac{\bar{b}}{R}\left(R^g - R^b\right)}{1 - \frac{1}{R}\left(\frac{\bar{b}}{P(e^*)}\right)^2 \left[\frac{d}{N+d} R - R^b\right]}, \\
er_2\left(k^b, e^*\right) &= \frac{1}{R}\Big[P\left(e^*\right) R^g + \left(1 - P\left(e^*\right)\right) R^b - R\Big] - \frac{1}{2}\left(e^*\right)^2 + \eta \frac{\bar{b}}{P\left(e^*\right)} \frac{d}{N+d}.
\end{aligned}
$$

For each possible k^b, we first compute the individual banker's optimal effort $e^*\left(k^b\right)$, which solves $er_1\left(k^b, e^*\right) = 0$. Then k^b is obtained by solving $er_2\left(k^b, e^*\left(k^b\right)\right) = 0$. We also consider the following scenarios where the banker's choice on k^b is a corner solution. If $er_2\left(k_1^b, e_1^*\left(k_1^b\right)\right) > 0$ for any possible $k^b \in \left[\frac{N}{Q}, 1\right]$, we choose $k^b = 1$. If $er_2\left(k^b, e^*\left(k^b\right)\right) < 0$ for any possible k^b, we choose $k^b = \frac{N}{Q}$.
Lastly, we compute the individual banker's net profit in the optimal contract

$$V\left(k^b, e^*\right) = \frac{1}{R}\left[\left(P\left(e^*\right)^2 R^g + \left(1 - P\left(e^*\right)^2\right) R^b\right)(N+d) - Rd\right] - \frac{1}{2}(N+d)\left(e^*\right)^2,$$

and the value of the banker's outside option V_{Save},

$$V_{Save} = \frac{1}{R}\left[P\left(\widetilde{e}^*\right) R^g + \left(1 - P\left(\widetilde{e}^*\right)\right) R^b\right] N - \frac{1}{2} N \widetilde{e}^{*2}.$$

where $\widetilde{e}^*$ is the banker's effort in the outside option, i.e. $\widetilde{e}^* = \min\left\{\frac{\bar{b}}{R}\left(R^g - R^b\right), \bar{e}\right\}$. Therefore, if $V\left(k^b, e^*\right) > V_{Save}$, the banker will choose to invest by borrowing from the mutual funds, and the best response function is $k^b\left(K^b\right) = k^b$, $e^*\left(K^b\right) = e^*$. Otherwise, the banker will only invest with his own net worth N and not borrowing, and the best response function is $k^b\left(K^b\right) = \frac{N\left(K^b\right)}{Q\left(K^b\right)}$, $e^*\left(K^b\right) = \widetilde{e}^*$.

C Analysis of Infinite-Horizon, Running Away Model

Here, we show that the implications of the infinite horizon GK model are qualitatively similar to those of three-period model discussed in subsection 2.6.1. In the first subsection below we sketch the market equilibrium of the model. In the second subsection we transform the model into a game and report our results.

C.1 The Market Equilibrium of the Model

The equilibrium of the GK model can be characterized as a Markov chain in which the states, $X(s)$, are indexed by $s = 1, 2, \ldots$.[92] The elements of $X(s)$ include consumption, bank deposits, banker net worth, the share of capital held by households and banks, the interest rate on bank deposits, the price of capital and the probability that there

[92]Here, the vector, $X(s)$, should not be confused with the aggregate leverage ratio in equation (22).

will be a run in the next period, $P(s)$.[93] In the equilibrium, $s = 1$ indexes the state in which an annihilation run occurs. For $s > 1$, $s - 1$ represents the number of periods since the last annihilation run.

When $s = 1$, the system moves to $s = 2$ with probability 1. It is impossible to have two consecutive annihilation runs because no liabilities are issued in state $s = 1$. For $s \geq 2$ the equilibrium has the property that with probability $P(s)$ the state moves from s to $s = 1$ in the next period and with probability $1 - P(s)$ the state moves from s to $s + 1$ in the next period. The function, $P(s)$ is decreasing in the aggregate loan recovery rate in the event that $s = 1$ in the next period, as in the three-period model (see equation (22)). As in the three-period model, there is no uncertainty in the model other than the possibility of a run.[94]

The sequence of events in a state, $s > 2$, is as follows:

- The only way to be in state $s > 2$ in the current period is to have been in state $s-1$ in the previous period. Among the unit-measure of bankers working in state $s-1$, $1-\sigma$ die at the start of s and σ survive to remain bankers in s.

- Newborn bankers in state s replace the $1-\sigma$ bankers which die in s. The $1-\sigma$ newborns receive a small transfer of net worth denominated in the units of the numeraire good.

- Incumbent bankers pay off liabilities issued in $s-1$. The net worth of all bankers, $N(s)$, working in state s (i.e., the σ surviving incumbent bankers plus the $1-\sigma$ newborns) is determined. Net worth across bankers is heterogeneous because they have different ages.

- State s working bankers make their deposit decision. All bankers then buy capital at the price, $Q(s)$. The $1-\sigma$ bankers that die at the start of state s consume their net worth in s and then exit the economy.

- A random draw from a binary distribution determines whether the system moves from s to $s = 1$ or $s+1$ in the next period.

[93] In practice, $\lim_{s\to\infty} X(s)$ converges in the (small probability) event that $s \to \infty$. GK point out that this allows one to approximate the equilibrium by choosing a large positive integer, S, and setting $X(s) = X(S)$ for all $s > S$. This reduces the number of equilibrium objects that must be computed to a manageable size. For details of what we did, see the Online Appendix. One of the variables in $X(s)$, $P(s)$, is dependent upon the price of capital, $Q(1)$, in the run state. This is an element of $X(1)$.

[94] Gertler and Kiyotaki (2015) allow for a technology shock, but we set that to the unconditional mean implied by their parameterization.

GK assume that in the run state, $s = 1$, newborns do not enter and they instead wait until the next state, $s = 2$. Thus, the number of newborns entering in state $s = 2$ is the sum of the $\sigma(1-\sigma)$ newborns that enter in the run state and survive into the next period, plus the new $(1-\sigma)$ newborns.[95] Apart from the quantity of new entrants in $s = 2$, the sequence of events in $s = 2$ is the same as for $s > 2$. A *Markov chain equilibrium* is a sequence, $X(s)$, for $s = 1, 2, ...$, such that (i) the quantities in this sequence solve agents' problems given the prices and probabilities and (ii) markets clear. This definition of equilibrium is the analog of Definition 1 in our three-period running away model. We find a unique equilibrium that is essentially the same as the one reported in Gertler and Kiyotaki (2015).

Our model and the specified values of the parameters are identical to the specification in GK(table 1), with two exceptions. First, we set the technology shock to a fixed value that corresponds to the unconditional mean of the stochastic technology shock used in GK. With this change, there is an equilibrium in which bankers issue deposits in each $s > 1$. However, the resulting equilibrium also has the property that $P(s)$ is small in the sense that $P(s) \to 0$ as $s \to \infty$. In order to ensure that $P(s) > 0$ for all s, we reduced the size of the endowment given to newborns by a factor of 1.7. But then, we found that the positive deposit decisions of banks, while still locally optimal, were dominated by setting deposits to zero in order to preserve net worth in the case of an annihilation run. This is same phenomenon we found in the three-period model (see section 2.4). As in that discussion, we found that introducing Assumption 1 reduces the attractiveness of hoarding net worth in a non run state. As a result an equilibrium that resembles the one in GK is preserved except that $P(s)$ is positive for all s. This illustrates the robustness of the fragility observation in section 2.4. There, we argued that the existence of the annihilation run state is fragile and depends on exogenous assumptions that directly control the amount of net worth in the hands of bankers in an annihilation run. For a more detailed discussion of the model, see the Online Appendix.[96]

[95] The net worth of bankers born in $s = 1$ and who survive into $s = 2$ is storable from $s = 1$ to $s = 2$. Newborns in $s = 1$ who die in $s = 2$ have zero consumption.

[96] This appendix graphs the equilibrium $X(s)$, $s = 1, ...$ It also reports stochastic simulations of the model to show the frequency of crisis, as well as the average time required to respond to a crisis.

C.2 Representing the Model as a Game

To represent the model as a game we need well-defined best response functions for the players, which we take to be the bankers in a particular state, $\tilde{s}$. We describe the key ingredient of the best response function, a continuation equilibrium, in the first subsection below. We then perform our game-theoretic analysis on the GK model. We show that the annihilation and no run equilibria reported by them correspond to the two Nash equilibria of the game representation of the model. The third subsection uses the game representation of the model to show how sensitive the model properties are to alternative assumptions about entry into banking. These results demonstrate the robustness of the findings for the three-period model in subsection 2.6.1.

C.2.1 The Continuation Markov Chain Equilibrium

Overview

We do the analog of what we did in our three-period model. There, we started the analysis in period $t = 1$ taking what happened in period $t = 0$ as given. Here, we start in a particular state, $\tilde{s} = 10$, and we take as given the liabilities and assets acquired by households and banks in state $\tilde{s} - 1$.[97] In the Markov chain equilibrium, only the no run equilibrium is possible in $\tilde{s}$. For example the two elements, $Q(\tilde{s})$ and $N(\tilde{s})$, of $X(\tilde{s})$ are the given equilibrium prices of capital and value of net worth in state $\tilde{s}$ of the Markov chain equilibrium. As in any market equilibrium concept, in the Markov equilibrium agents do not need to form beliefs about what prices and net worth are. The Markov chain equilibrium concept incorporates the rational expectations assumption that people simply know what those objects are, and sets aside the question of how people arrive at such knowledge. As in section 2.6.1 we take a single step away from the rational expectations assumption. We do so in state $\tilde{s}$ for one period only. In all later dates and states, including times when $\tilde{s}$ is visited again, agents are assumed to have rational expectations.

In state $\tilde{s}$ banks find themselves having to make a guess about Q and N (to simplify notation, we do not include the state index here). The problem of bankers is the same as in subsection 2.6.1 above. In particular, all the bankers make their deposit decisions simultaneously and without coordinating. Because we temporarily drop the rational expectations assumption in state $\tilde{s}$, all the individual banker knows is that the current

[97] These include $k^b(\tilde{s}-1)$, $R(\tilde{s}-1)\, d(\tilde{s}-1)$, which are elements of the vector $X(\tilde{s}-1)$.

price of capital, Q, is determined by market clearing and this depends, in part, on the average deposit decision, D, of the other bankers. Conditional on D, the individual banker can compute the continuation equilibrium. This is a sequence, $X(s;D)$ for $s=\tilde{s},\tilde{s}+1,...$ that satisfies all equilibrium conditions except the one associated with D itself. When we consider values of D different from the one in the Markov chain equilibrium, the sequence, $X(s;D)$ for $s=\tilde{s},\tilde{s}+1,...$ does not coincide with $X(s)$ for $s=\tilde{s},\tilde{s}+1,...$.

The analysis is simplified by the fact that $\lim_{s\rightarrow\infty}X(s;D)$ coincides with analogous limit in the Markov chain equilibrium.[98] Constructed in this way, the system eventually must revert to the Markov chain equilibrium for any D. It either reverts as soon as an annihilation run occurs or in the (zero probability) event that an annihilation run never occurs, $s\rightarrow\infty$.[99] The Markov chain equilibrium is stationary. For D not equal to 0 or $D(\tilde{s})$, the deviation from the Markov equilibrium explored here is not stationary, though eventually it converges back to the Markov chain equilibrium.

Details of the Computations

We compute banker best responses in a way that is analogous to the three-period analysis in subsection 2.6.1. In particular, we choose a specific state, $\bar{s}=11$. That is the analog of period 1 in the three-period model. We then fix the value of D, one of the elements of $X(\bar{s})$. We drop the equation in the state $\bar{s}$ that determines D, so that the state $\bar{s}$ equilibrium conditions are given by:$X(s)$

$$\bar{v}\left(X(\bar{s}-1),\bar{X}_D(\bar{s};D),\bar{X}_D(\bar{s}+1;D);Q(1)\right)=0. \tag{C.1}$$

Here, $\bar{X}(s;D)$ denotes the value $X(s)$ in the continuation equilibrium associated with D occurring in state $\bar{s}$ and $\bar{v}$ replaces v in $\bar{s}$ one time only. In all subsequent periods

[98] To compute the continuation equilibrium, $X(s;D)$ for $s=\tilde{s},\tilde{s}+1,...$, we make use of the same approximation described in footnote 93. For $D>0$, in each state, $s=\tilde{s},\tilde{s}+1,...$ agents believe that in the next state there can only be a run equilibrium, $s=1$, or a no run equilibrium, just like in the Markov Chain equilibrium. So, what happens in a run equilibrium matters for each $X(s;D)$ for $s=\tilde{s},\tilde{s}+1,...$. As noted above, we take $X(s),s=1,...,\tilde{s}-1$ as given. For $D=0$, $X(\tilde{s}+j;D)=X(1+j),j\geq 0$. For $D=D(\tilde{s})$, its value in the Markov Chain equilibrium, $X(\tilde{s}+j;D)=X(\tilde{s}+j)$, $j\geq 0$.

[99] There is a (large) equal number of equations and unknowns to solve for $X(s;D)$ for $s=\tilde{s},\tilde{s}+1,....$ For this, we adapt the same algorithm used in GK. Replication files for the computations are available.

the equilibrium conditions are again given by v, so that

$$v\left(\bar{X}_D\left(s-1\right),\bar{X}_D\left(s\right),\bar{X}_D\left(s+1\right);Q\left(1\right)\right)=0,$$

for $s>\bar{s}$. In those periods, we impose the feature of the baseline equilibrium that the system in state s moves to $s=1$ with probability $P\left(s\right)$ and that s moves to state $s+1$ with probability $1-P\left(s\right)$, where $P(s)$ is the probability of a run assigned by $\bar{X}\left(s;D\right)$. In the (zero probability) event that the system never experiences a run, then,

$$\lim_{s\to\infty}\bar{X}_D\left(s\right)=X_\infty\left(Q\left(1\right)\right). \tag{C.2}$$

Here, $X_\infty\left(Q\left(1\right)\right)$ is the solution to equation (C.5) used in computing the baseline equilibrium and $Q\left(1\right)$ is the baseline equilibrium price of capital in the state $s=1$. For the no run value of D the continuation equilibrium coincides with the baseline equilibrium itself. However, for other values of D the continuation equilibrium returns to the baseline equilibrium as soon as an annihilation run occurs or, if one never occurs, then the system converges according to equation (C.2).

In effect, we study an 'MIT shock' that occurs in a particular date when the state is $\bar{s}$. In the preceding state, $\bar{s}-1$, no one expected any value of D other than its no run value assigned in the baseline equilibrium by $X\left(\bar{s}\right)$. When the system eventually arrives at $s=\bar{s}$ again, then the variables simply take on the value of $X\left(\bar{s}\right)$ assigned in the baseline equilibrium. Thus, the stochastic process eventually 'returns' to where it would have been in the absence of the MIT shock.

C.2.2 Best Response Function in State $\tilde{s}$: the GK Case

Given a belief, D, and with the implied continuation equilibrium, $X\left(s;D\right)$ for $s=\tilde{s},\tilde{s}+1,...$ in hand, the individual banker has the information necessary to solve its deposit decision, d. Our baseline analysis adopts GK's (p. 2024) assumption that if an annihilation run occurs, $D=0$, newborn bankers do not enter because they fear that households will not be able to differentiate them from the σ other banks whose net worth is zero. For positive values of D, newborns do enter.

We seek a single level of deposits, $d\left(D\right)$, as a 'best response' to D. We accommodate the fact that bankers in the model are heterogeneous by computing the average best response deposit decision of bankers, conditional on D :

$$d(D) = (\phi(D) - 1) N(D). \tag{C.3}$$

Here, $N(D)$ denotes average net worth across all operating banks in state $\tilde{s}$, in the continuation equilibrium conditional on D. Also, $\phi(D)$ denotes the best leverage response of a banker in the continuation equilibrium conditional on D.[100] Bankers are not cooperative, so each of the unit measure of bankers underlying equation (C.3) believes that it is atomistic in that it ignores any impact of its deposit decision on prices and rates of return.

The best response function, $d(D)$, is reported in Figure C1a for a range of values of D that includes $D = 0$. The best response function is the solid line, while the dashed line is the 45^o line. By construction of $d(D)$, values of D such that $D = d(D)$ are Nash equilibria and correspond to equilibria of the underlying market economy.[101]

Note that the best response function, $d(D)$, crosses the 45^o line at roughly $D = 0.35$, which corresponds to the no run equilibrium in the Markov chain equilibrium with $s = \tilde{s}$. The best response is generally upward-sloping, reflecting strategic complementarity among bankers. As D increases, the net worth of bankers increases and individual bankers best respond by demanding more deposits. This reflects the fact that with higher D, $Q(D)$ increases and this effect on price operates on banker deposits through two channels. First, net worth is increasing in Q and second, leverage is decreasing in Q as a rise in the price of capital drives down its rate of return. Evidently, the net worth channel dominates the leverage channel on the upward-sloping portion of the best response function.

Note the kink just below $D = 0.05$. That kink is the largest value of D where incumbent bankers' net worth is zero. Those bankers best respond by demanding zero deposits for all D at the kink and smaller. As D falls below the kink point, the only bankers in business are the $1 - \sigma$ newborns who are assumed to enter with a level of

[100] Because of the linearity in the model, ϕ is the same for all bankers with positive net worth. However, for a banker with zero net worth, ϕ is not well-defined. Such a banker sets $d = 0$, a result that we verify when we do the computations. Recall that equation (C.3) is the integral of best response functions, across the unit measure of bankers. So, that integral is correct if we set $\phi = x$ where x is any finite number, for bankers with zero net worth. In practice, we set ϕ in equation (C.3) to its value for bankers with positive net worth.

[101] Recall that for $D = D(\tilde{s})$, where $D(\tilde{s}) > 0$ is the value of deposits in the Markov Chain equilibrium, $D(\tilde{s}) = d(D(\tilde{s}))$ and the continuation equilibrium has the property, $X(s; D(\tilde{s})) = X(s)$, for $s = \tilde{s}, \tilde{s} + 1, ...$. Also, recall that for $D = 0$ the continuation equilibrium is $X(\tilde{s}; D) = X(1)$ and $X(s; D) = X(s)$ for $s \geq 2$.

net worth independent of the value of $D > 0$.[102] It is not surprising that deposits are strategic substitutes among newborns, since for them the net worth channel described in the previous paragraph is shut down. With a reduction in D, the price of capital in the continuation equilibrium is lower, raising the rate of return on capital and thus relaxing financial frictions on newborns. This is why it is that to the left of the kink, the best response function has a negative slope. As D converges to zero, the best response rises sharply and approaches the open circle on the vertical axis. When $D = 0$ the best response discontinuously drops to the origin reflecting that newborns are assumed not to enter in that case.[103] Evidently, there are two Nash equilibria, consistent with the conclusion of the market-based analysis reported in GK.[104]

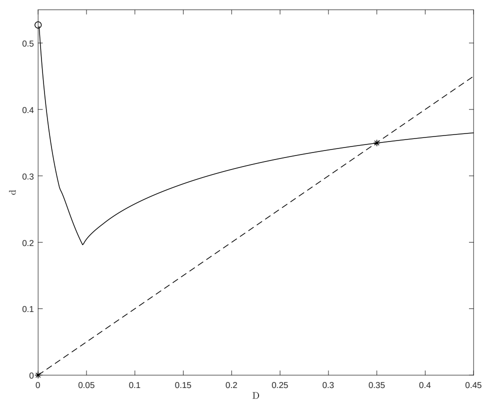
[102] For D to the left of the kink point in Figure C1a, $N(D) = (1-\sigma)\omega^b$, where ω^b denotes the quantity of net worth transferred to each newborn.

[103] Computation of the open circle in Figure C1a is straightforward and closely mirrors the computations surrounding the $t = 0$ banker problem in the three-period running away model (see equation (20)). The problem of a newborn with net worth, $N_1 = \omega^b > 0$, who enters in state $s = 1$ is $V_1 = \max_{d_1} [\beta(1-\sigma) N_2 + \beta\sigma V_2]$ subject to the incentive constraint: $\theta Q_1 k_1^b \leq V_1$. Here, V_s denotes the value of being a banker in state s, $s = 1, 2$. In defining the banker's objective we have taken into account that $P(1) = 0$ and that with probability $1 - \sigma$ the newborn dies and consumes its net worth, N_2, in the next period (an inessential difference with our three-period model is that bankers in this model live separately from the household, are risk neutral and only consume at the date of their death). After scaling by N_1, the problem reduces to $\max_{d_1} \psi_1(\phi_1)$ subject to $\theta\phi_1 \leq \psi_1(\phi_1)$, where $\psi_1(\phi_1) = [\beta(1-\sigma) + \beta\sigma\psi_2] N_2/N_1$, and $N_2/N_1 = \phi_1 \frac{Z+Q_2}{Q_1} - (\phi_1 - 1) R_1$. The latter expression is discussed in footnote 27 and the scaled notation corresponds to that used in subsection 2.1. Also $R_1 = C_2^h / \left(\beta C_1^h\right)$, where C_s^h denotes the consumption of the household in states $s = 1, 2$. It can be verified that the incentive constraint puts an upper bound on ϕ_1 and that that upper bound optimizes the newborn banker's constrained optimization problem. Thus, we solve for the unique value of ϕ_1 that satisfies $\psi_1(\phi_1) = \theta\phi_1$. The empty circle in Figure C1a is $(\phi_1 - 1)(1-\sigma) N_1$. The required numbers are $Q_1 = 0.8870$, $Q_2 = 0.9316$, $R_1 = 1.0550$, $C_1^h = 0.0536$, $C_2^h = 0.0560$, $\psi_2 = 19.7806$, $(1-\sigma)\omega^b = 0.0011487/1.7$, $\sigma = 0.95$, $\theta = 0.1934$, $\beta = 0.99$. These imply $\phi_1 = 781.6647$, so $(1-\sigma)(\phi_1 - 1)\omega^b = 0.53$ as shown in Figure C1a.

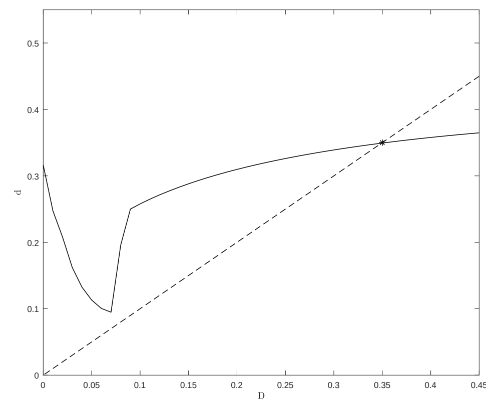
[104] GK do not take a stand on what newborns would do out of equilibrium. So, there another interpretation of their model in which the best response function drops to zero discontinuously at the kink point. This would not change the result that there are only two equilibria. The reason we adopt our particular interpretation of GK is that it is more convenient from the point of view of the subsequent discussion.

Figure C1: Best Response Analysis in the Infinite Period Model

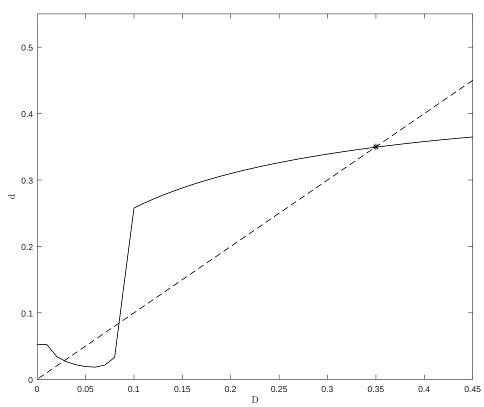
(a) All Newborns Stay Out When $D = 0$



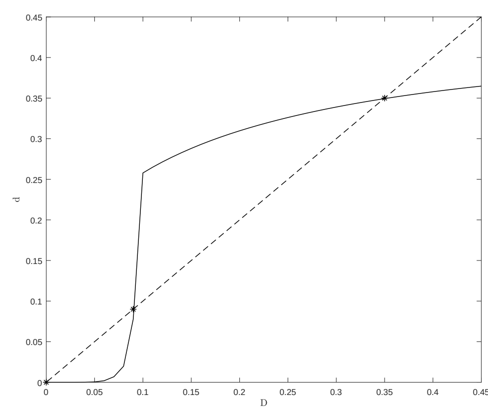
(b) 40% of Newborns Stay Out When $D = 0$



(c) 90% of Newborns Stay Out When $D = 0$



(d) 100% of Newborns Stay Out When $D = 0$



Notes: (i) these figures display the best response, $d(D)$,defined in equation (C.3). (ii) Figure C1a displays a best response that is consistent with the analysis in GK: there are two Nash equilibria with a discontinuity at $D = 0$ because at that point newborns do not enter (the open circle is the limit of the best response function as $D \to 0$). At $D = 0$,the best response is the closed circle at the origin. In terms of footnote 105, $\varphi(0) = 0$ and $\varphi(x) = 1$ for $x > 0$. (iii) Figure C1b displays the best response when $\varphi(x)$ is as defined in footnote 105 with $z = 0.6$, so that 60% of newborns enter when $D = 0$. (iv) Figure C1c displays the best response with $z = 0.1$, so that 10% of newborns enter when $D = 0$. see text for discussion. (v) Figure C1d displays the best response with $z = 0$ so that zero newborns enter during a run. The shape of the best response function for $D \leq D^*$ reflect the gradually rising function, $\varphi(x)$.

C.2.3 Best Response Function in State $\tilde{s}$: Alternative Treatments of Newborns

The panels in Figure C1 make it transparent that the treatment of newborns has a profound impact on the nature of the equilibria. For example, the small open circle on the vertical axis of figure C1a shows how many deposits newborns would issue if they entered when $D = 0$. Given the high returns in an annihilation run, the financial frictions on newborns are very weak and they can obtain leverage roughly equal to 782, almost identical to the leverage of 756 reported for our three-period model in section 2.4. The aggregate net worth of newborns is 0.000676, so if they entered, they would issue 0.53 deposits and hold 0.53 assets, after rounding. To evaluate the magnitude of assets held by newborns that enter when $D = 0$, it is useful to compare the quantity of those assets with the assets held by all banks in the no run Nash equilibrium in state $\tilde{s}$. Leverage in that state is 21 and the total assets held by the banks in that equilibrium is 0.37. Thus, the newborns alone can hold 40 percent more assets if they enter the during the annihilation run state than are held by the entire banking system in the no run equilibrium. Evidently, the rollover crisis property of the model is not at all robust to assumptions about entry.

We can translate the fact that newborns cannot issue equity (net worth) in an annihilation run into a transactions cost they have to pay to enter. If a newborn banker had \$1 of equity during a run, they would earn a present discounted value of $\psi^* = \theta \times 782 = 0.1934 \times 782 = \$151$ (we use the value of θ used in GK, see below). A way to interpret that newborns do not enter in an annihilation costs is that they face a transactions cost slightly higher than \$151 per \$1 of equity issued in order to deploy their equity in a run. As in the three-period model adjustment costs of that magnitude seem implausibly large.

So far we have only considered alternative assumptions about the entry of newborns when $D = 0$. With $D > 0$ we assume all newborns enter. In panels (b)-(d) of Figure C1 we assume that the fraction of newborns that enter, $\varphi(D)$, is continuous in $D \geq 0$. We parameterize this function so that $\varphi(0) = z < 1$ and $\varphi(D) = 1$, for $D \geq D^*$, where D^* corresponds to the kink point in Figure C1a. The function, $\varphi(D)$, is continuous, increasing and convex over the range, $D \in [0, D^*]$.[105]

We just considered case when 0% of newborns stay out of the market when $D = 0$. Using the function φ we can ask what happens if the fraction of newborns that stay out rises above 0% during an annihilation run. The best response function in this case is displayed in Figure C1b, where the fraction of newborn staying out of the market is 40% when $D = 0$ and falls continuously as D increases.[106] In this case too, we see that the annihilation run state is not a Nash equilibrium, so it also is not a market equilibrium. By further increasing the fraction of newborns that stays out of the market we in effect

[105] We use $\varphi(x) = \min\{F(x; \mu, \sigma)/F(.1; \mu, \sigma) + z, 1\}$,where $\mu = 6, \sigma = 1$ and F is the cumulative distribution function of the lognormal distribution. Various values of z are used, as indicated in the panels of Figure C1.

[106] The fraction, $1 - \varphi(D)$, enter in state $s = 2$.

shift the best response function down. The overall shape of the best response function is preserved and it is therefore not surprising that as the best response function shifts down, at some point two new Nash equilibria make an appearance (see Figure C1c). Interestingly, none of these two equilibria are an annihilation run. We continued to reduce $\varphi(D)$ and Figure C1d shows what happens when $\varphi(0) = 0$, so that no newborns enter when $D = 0$ and a very small number enter for $D > 0$. Now, there are the three equilibria we saw in the three-period model in subsection 2.6.1.[107] Overall, Figure C1 shows that the set of equilibria is very sensitive to assumptions about entry.

C.2.4 Computation of Best Response Function

Under our assumption (verified ex post) about the convergence of $X(s)$ in s we have that for a sufficiently large value of S, that the equilibrium is well-approximated as solving

$$v\left(X(s-1), X(s), X(s+1); Q(1)\right) = 0, \tilde{v}\left(X(2), X(3); Q(1)\right) = 0 \qquad \text{(C.4)}$$

for $s = 3, 4, ..., S+1$. In equation (C.4), $X(S+2) \simeq X_\infty = \lim_{s\to\infty} X(s)$. Also, v is a set of equilibrium conditions for the model variables, $X(s)$, for $s > 2$. The equilibrium conditions in state $s = 2$, $\tilde{v}$, are slightly different than those for $s > 2$ in part because the equilibrium conditions do not look back at the elements in $X(1)$, apart from $Q(1)$. The notation makes explicit the dependence of each equilibrium condition on the price of capital in the annihilation run state, $Q(1)$. The price of capital in the annihilation run equilibrium appears in each equilibrium condition because, for each $s \geq 2$, it is possible that the subsequent period is an annihilation run period. In principle, household consumption in $s = 1$ is also required for each s, but in the model equilibrium household consumption in the run state turns out to be a function

[107] In the case of Figures C1c and C1d it is interesting to know whether any of the equilibria could be eliminated on the basis of some kind of stability concept. Angeletos and Sastry (2021) define a rational expectations equilibrium as (locally) 'well behaved' if the associated Nash equilibrium in the game form of the model is locally eductively stable, i.e., the best response function crosses that Nash equilibrium with a slope less than unity in absolute value. The appeal of local eductive stability is that, under certain representations of bounded rationality, if agents believe the economy is in a neighborhood of an equilibrium (and, this belief is common knowledge, CK), then as more mental reasoning is applied, they will coordinate arbitrarily closely on the equilibrium. If a rational expectations equilibrium is ill-behaved, then if boundedly rational agents believe the economy is in a neighborhood of a particular equilibrium (and this is CK), then rationality has no prediction for which belief agents will adopt in that neighborhood, no matter how intense their mental reasoning is. See Evans and Guesnerie (2005) and Evans et al. (2018) for more extended discussions of local eductive stability.

of exogenous parameters. In equation (C.4), $X_\infty(Q(1))$ solves

$$v\left(X_\infty(Q(1)), X_\infty(Q(1)), X_\infty(Q(1)); Q(1)\right) = 0. \tag{C.5}$$

In the run state the household holds all the economy's capital, which (as in GK) we assume to be unity. The household's Euler equation for capital in $s = 1$ is, analogous to our equation (29) and (31),

$$\beta \frac{Q(2) + Z}{Q(1) + \alpha} = \frac{C(2)}{C(1)}. \tag{C.6}$$

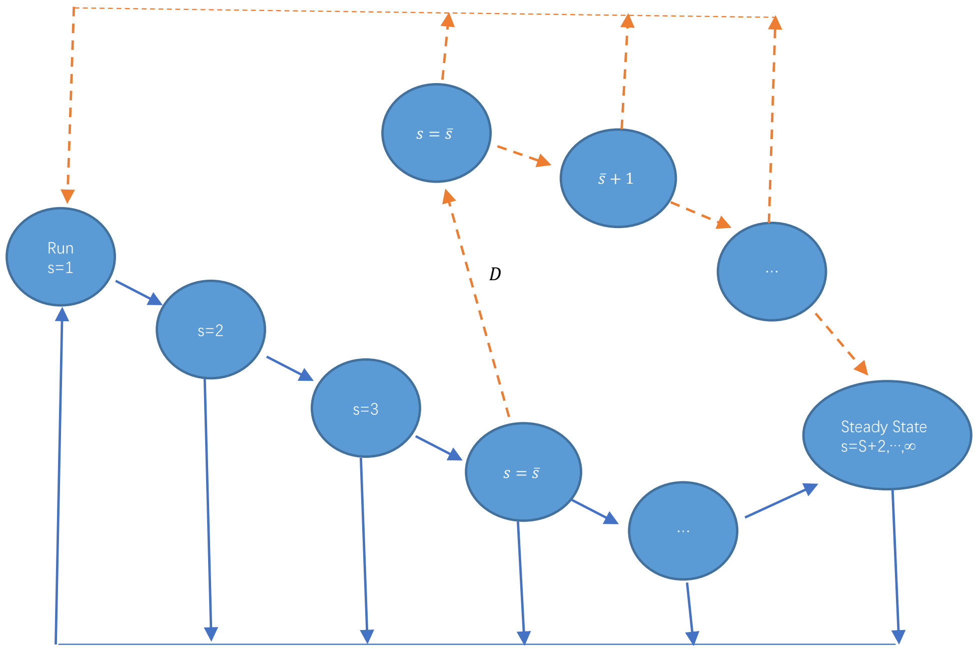
Here, $C(s)$ denotes household consumption in state $s = 1$ and 2, Z denotes the marginal product of capital, α is the household capital management parameter and β its the household's discount rate. The fact that it is consumption growth that is equated to the rate of return on capital (inclusive of marginal management costs) reflects the assumption of log utility in consumption.

We use the numerical algorithm proposed in GK to solve for $X(1), ..., X(S+2)$. In particular, let $Q(1)$ be an initial guess about the price of capital. Then, compute $X_\infty(Q(1))$ to solve equation (C.5). Then, solve for the S objects, $X(2), ..., X(S+1)$, using the S equations in (C.5). After this, use equation (C.6) and $C(2)$ and $Q(2)$ from $X(2)$ to obtain a new value for $Q(1)$ that solves (C.6). (Recall that $C(1)$ is a function of exogenous parameters.) Finally, iterate on this mapping from $Q(1)$ to a new value for the price of capital until convergence, Q_∞. Then, $X_\infty \equiv X_\infty(Q_\infty)$. See subsection C.3 below for more information about this mapping.

Definition 4. A baseline (approximate) equilibrium is a set, $\{X(1), ..., X(S+1), X_\infty\}$,where $X(s)$ solve equations (C.4) and (C.6) for $s = 1, ..., S+1$,and X_∞ solves equation (C.5).

The definition provides an approximate equilibrium because $S < \infty$. We use $S = 120$ and have found that $X(s)$ converges at about $s = 70$. The bottom arc of 'balls' in Figure C2 corresponds to the equilibrium elements of $X(s)$.

Figure C2: Baseline Equilibrium and Continuation Equilibrium, Given D



Note: The bottom arc of balls correspond to the baseline equilibrium, characterized by $s = 1, ..., S + 2$ and discussed in Subsection C.1. The bottom arc is a stationary Markov chain stochastic process: the values of the economic variables are always the same whenever the process visits a given state, s. The upper arc corresponds to the continuation equilibrium associated with a perturbed value of deposits, D, that (potentially) deviates from its baseline value in state $\bar{s}$ one time only. This case is discussed in Subsection C.2.1. In both cases, the state advances from s to $s + 1$ with probability, $P(s)$, and it advances to state $s = 1$ (the annihilation run state) with probability, $1 - P(s)$. The top arc of balls is the response to an 'MIT shock' (completely unanticipated in the previous state) in which D is arbitrary and replaces its value in the baseline equilibrium, $X(\bar{s})$. The upper arc describes a nonstationary Markov chain stochastic process, though for every possible sequence of states, that process converges to the baseline stochastic process. This can be seen in the above diagram, which shows that in the upper arc, the system can jump to $s = 1$,in which case it goes back to the baseline equilibrium Markov chain. In the (zero probability) even that the system never encounters $s = 1$ then it converges to X_∞ where it also rejoins the baseline stochastic process.

In our analysis, we assign the following values to the parameters:

$$\begin{aligned} \alpha &= 0.00797,\ \theta = 0.1934,\ \sigma = 0.95,\ \beta = 0.99, \\ W^h &= 0.045,\ W^b = 0.0011487/1.7, Z = 0.0126. \end{aligned}$$

These are taken from GK (except W^b, which is their number is divided by 1.7 to ensure $P_\infty > 0$ while still ensuring that the banker incentive constraint is compatible only with bankers setting deposits equal to zero in an annihilation run). The properties of X_∞ in this case are:

$$
\begin{aligned}
Q(1) &= 0.89, \quad Q_\infty = 0.98, \quad K^h_\infty = 0.30, \quad D_\infty = 0.62 \\
\Phi_\infty &= 11.1, \quad P_\infty = 0.0031, \\
C^h(1) = .054, C^h_\infty &= 0.055, \quad C^b_\infty = 0.0031, \quad N_\infty = 0.061, \\
R^f_\infty &= 1/\beta - 0.95/10000, \quad \frac{\bar{R}_\infty}{R^f_\infty} - 1 = 0.41/10000.
\end{aligned}
$$

The risk free rate, R^f_∞, is slightly lower than $1/\beta$, reflecting the precautionary saving motive in the presence of a (very small) probability of an annihilation run in $s = \infty$. The upper arc of balls in Figure C2 corresponds to the continuation equilibrium associated with a particular value of D in state $s = \bar{s}$.

C.3 Detailed Parametric Discussion of Infinite Horizon Model

We provide the following discussion in the form of a set of presentation slides. These go into the deepest possible detail about the specification of the baseline model and the various incentive constraints on the banker.