Discussion Paper Series – CRC TR 224

Discussion Paper No. 354
Project B 01

Voting to Persuade

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June 2022

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Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Voting to Persuade*

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May 30, 2022

Abstract

We consider a model of collective persuasion, in which members of an advisory committee receive private continuous signals and then vote on a policy change. A decision maker then decides whether to adopt the change upon observing each vote. Information transmission between the committee and the decision maker is possible if and only if there exists an informative equilibrium with the unanimity rule. When the decision maker is more conservative, a higher level of consensus is needed to persuade her to abandon the status quo in equilibrium. Our result thus provide a rationale for the use of the unanimity rule, despite its poor performance in information aggregation (Feddersen and Pesendorfer 1998). Furthermore, the continuous-signal model considered in this paper produces results that contrast the discrete-

*We would like to thank Sourav Bhattacharya, Georg Nöldeke, Xianwen Shi, and Thomas Tröger for their advice and encouragement, Tilman Börgers for suggesting us to study the model with continuous signals, Hülya Eraslan, Timothy Feddersen, Ronen Gradwohl, Stephan Lauermann, Nicola Persico and participants of the 38th Australasian Economic Theory Workshop, the 6th World Congress of the Game Theory Society, University of Technology Sydney and Xiamen University for comments and suggestions. Wong gratefully acknowledges financial support from the Swiss National Science Foundation (SNF) through project grant 172680. Yang gratefully acknowledges funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 (Project B01). All remaining errors are our own.

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signal model considered in the literature (Battaglini 2017; Gradwohl and Feddersen 2018) and we discuss how the results depend on the coarseness of the signal structure.

1 Introduction

A manager is considering whether to hire a candidate or leave the post open. To evaluate the candidate, she consults a committee of experts who have private information about the competence of the candidate. However, committee members often have their own interest in the decision. For example, the committee may be more eager to have a new hire, while the manager is more concerned with the salary involved. A common practice is for each committee member to make a binary recommendation, either for the candidate or against the candidate, to the manager, and the manager then makes the final decision.

In this case, the group of experts serves as an advisory committee and provides decision-relevant information to the manager through voting. This paper studies information transmission between the committee and the manager when there is a conflict of interest between these two parties. Our model applies to a lot of real-world situations. Examples include the Federal Advisory Council, which advises the Federal Reserve Board; the Investor Advisory Committee, which advises the US Securities and Exchange Commission; advisory committees for the US Food and Drug Administration; and expert panels appointed in WTO dispute settlement proceedings. Despite their importance in real-world decision making, such committees, usually called advisory committees, are understudied in the literature (Gradwohl and Feddersen 2018).[1]

We study the behavior of such an advisory committee with a standard voting framework. There is an unknown state of the world, and the decision maker needs to choose between a policy change and the status quo. Each member of the committee receives a continuous signal about the state, and each casts a vote on whether to adopt the policy change. The decision maker is informed of each vote, and then chooses between the policy change and the status

[1] There are also committees that are usually regarded as decision-making committees but face a decision-making problem similar to the one we describe. For example, the Federal Open Market Committee independently determines monetary policy, but the economic consequences of the policy depend on the market response. In the United States, the Congress collectively determines whether to pass a bill, but the Article I of the Constitution requires that every bill passed by the Congress must be presented to the president for approval, and the president can veto the passed bill within 10 days. Presidential vetos do happen sometimes.

quo. All committee members share the same preference. Their preference is completely aligned with the decision maker's preference when the state is known. But with incomplete information, there is a conflict of interest: the decision maker is more biased toward the status quo; she can be persuaded to switch her decision only if the committee provides sufficiently convincing information supporting the policy change. Thus, there is meaningful information transmission only if the decision maker can be persuaded to adopt the policy change in equilibrium.

Recent work on advisory committees (Levit and Malenko 2011; Battaglini 2017; Gradwohl and Feddersen 2018) shows that information transmission between the committee and the decision maker is impossible when the conflict of interest is too large. This is because an advisory committee differs from a decision-making committee in one key aspect. In a decision-making committee, votes are aggregated by an exogenous voting rule that selects one of the two options. In an advisory committee, the decision rule is endogenous and chosen by the decision maker. In equilibrium, the decision maker adopts a certain decision (voting) rule and the committee members cast their votes in anticipation of the equilibrium decision rule and the other committee members' equilibrium voting behavior. As a result, a given voting rule may not be supported in equilibrium if the decision maker is not willing to follow it. When the conflict of interest is large enough, the decision maker cannot be persuaded to adopt the policy change under any voting rule, and the committee's information is never utilized.

We first characterize the necessary and sufficient condition for information transmission. The characterization also provides a tractable algorithm to verify whether information transmission is possible in equilibrium. In Section 4, we have the following main result of the paper:

Proposition *Information transmission is possible if and only if the decision maker can be persuaded in an equilibrium where she adopts the unanimity rule.*

This result implies that to determine whether information transmission is possible, it is sufficient to verify the existence of an equilibrium in which the decision rule is the unanimity rule. If such an equilibrium does not exist, then information transmission fails in any equilibrium.

Ever since Feddersen and Pesendorfer (1998) pointed out the inferiority of the unanimity rule in a jury voting model, it remains puzzling why the unanimity rule is so prevailing in many decision-making processes despite its poor performance in information aggregation from a theoretical point of view. Moreover, the unanimity rule is also the uniquely bad voting rule if deliberation is allowed (Austen-Smith and Feddersen 2006; Gerardi and Yariv 2007). Our result provides a rationale for the use of the unanimity rule. When the conflict of interest is large, the unanimity rule could be the only decision rule that arises in equilibrium and makes information transmission possible. Although information transmission may also be possible under other voting rules when the conflict of interest is moderate, our result shows that whenever some other voting rules can sustain communication, so does the unanimity rule, but not vice versa. This means that the unanimity rule can be adopted to transmit information for a wider range of scenarios than all other voting rules.

More generally, we can compare two decision rules and ask which of them is more likely to survive when the conflict of interest between the committee and the decision maker gets larger. More precisely, which voting rule can sustain information transmission for a wider range of scenarios? The following result in Section 5 provides a partial answer by comparing all k-rules. Under a k-rule, the decision maker chooses the policy change when there are no less than k votes supporting the change.

Proposition *If a decision maker can be persuaded in an equilibrium where she adopts k-rule, then she can be persuaded in an equilibrium where she adopts $(k+1)$-rule.*

Thus, we expect a higher k-rule to arise in equilibrium when the decision maker gets more conservative towards the policy change. This aspect has not been considered in the literature, which mainly focuses on information aggregation in a decision-making committee.[2] For an advisory committee, voting behavior depends not only on the preference of the committee, but also on the degree of the conflict of interest between the decision maker and the committee. When the decision maker is more conservative and reluctant to adopt the policy change, the committee members may have to aim for a higher consensus level, a higher k-rule, for example, to persuade her.

Our two baseline results above thus confirm the intuition that more affirmative votes make the case for a policy change more convincing. Therefore, a higher k-rule could allow information

[2] For example, Persico (2004) shows that the optimal voting rule for a committee is the statistical rule, which depends on the preference of the committee.

transmission to a decision maker when a lower k-rule cannot. Such reasoning, however, ignores the fact that the committee members adjust their votes strategically and vote more aggressively for a policy change when facing a higher k-rule (Austen-Smith and Banks 1996). While we show that such naïve reasoning leads to the right conclusion when signals are continuous under fairly general conditions, in Section 5, we consider a version of our model with binary signals (Gradwohl and Feddersen 2018) and show that this often-used specification leads to drastically different results. This suggests that the assumption of binary signals is not innocuous in studying advisory committees. To investigate the reasons underlying these contrasting results, we further consider a general discrete-signal model, in which the signal of each committee member can take more than two values. The analysis of the general model suggests that if the signals are fine enough, then the results will be similar to the continuous-signal case; if the signals are very coarse, the results will be similar to the binary-signal case.

2 Literature review

Our model builds on the voting model pioneered by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997). These two seminal papers model the voting of a decision-making committee as non-cooperative strategic games. Austen-Smith and Banks (1996) show that truthful voting is generally not an equilibrium. Feddersen and Pesendorfer (1997) show that in spite of that, private information held by the committee members can be fully aggregated in a large election. Thereafter, the literature has been mainly focusing on studying the performance of decision-making committees in information aggregation, including the effect of voting rules adopted by the committees on information aggregation (Feddersen and Pesendorfer 1998; Duggan and Martinelli 2001; Li et al. 2001; Martinelli 2002), the efficiency of information aggregation when pre-voting communication is allowed (Coughlan 2000; Austen-Smith and Feddersen 2006; Gerardi and Yariv 2007), and costly information acquisition (Li 2001; Persico 2004; Martinelli 2006).[3] The committees in this literature are all decision-making committees, but the committee we consider in the current paper is an advisory committee. Thus, the voting rules in these papers are exogenously given while the voting rule in our paper is endogenous.

[3] Li and Suen (2009) and Gerling et al. (2005) provide excellent surveys of the earlier works in this literature.

This paper is also related to the cheap-talk literature initiated by Crawford and Sobel (1982). In the cheap-talk literature, the papers closest to the current paper are the ones with multiple senders (Krishna and Morgan 2001; Battaglini 2002; Ambrus and Takahashi 2008). Different from the current paper, these papers mostly focus on whether full revelation is obtainable. In this paper, we restrict our attention to binary state and action spaces, and focus on the feasibility of information transmission rather than the possibility of full revelation.

The papers most closely related to this paper are Austen-Smith (1993), Wolinsky (2002), Levit and Malenko (2011), Battaglini (2017), Gradwohl and Feddersen (2018) and Battaglini, Morton and Patacchini (2020). Austen-Smith (1993) considers a heterogeneous two-expert committee and a decision maker who does not commit to a decision rule. He compares simultaneous voting and sequential voting, and examines the informational properties of these two mechanisms. Levit and Malenko (2011), Battaglini (2017), and Gradwohl and Feddersen (2018) consider simultaneous voting models similar to ours but with discrete signals. They find that information transmission is impossible if the conflict of interest between the committee and the decision maker is large enough, regardless of the size of the committee. Wolinsky (2002) also reaches a similar impossibility result in a model with verifiable information but with a different information and payoff structure. Ekmekci and Lauermann (2022) introduce costly participation and noise to Battaglini (2017). In contrast to the previous literature, they show that if there are only costs and no noise, information is fully aggregated when the size of the committee is large enough. Battaglini, Morton and Patacchini (2020) test the predictions of Battaglini (2017) experimentally. We go beyond these papers by not only deriving the necessary and sufficient conditions for information transmission but also studying the corresponding condition for a given voting rule in detail.

Pei and Strulovici (2021) study a related voting problem where the voting rule is endogenous. In their model, the voters have endogenous and corrected signals and the underlying state of the world is also endogenous. But different from the current paper, they study the attainability of the full-commitment outcome when the decision maker cannot commit.

The rest of the paper is organized as follows. Section 3 introduces the model. Section 4 characterizes the equilibria and presents the main results of the paper. Section 5 discusses the model with discrete signals. Section 6 concludes. Most of the proofs are relegated to the

Appendix.

3 Model

3.1 Setup

There is a committee of $\mathcal{N}$ homogeneous members. Each member i receives information about the state $\theta \in \{y, n\}$, and then votes simultaneously on two options, status quo N (or nay) and alternative Y (or yay), $v_i \in \{Y, N\}$. The decision maker (DM) is first informed of each vote, and then makes a final decision $D \in \{Y, N\}$ between status quo N and alternative Y. The common prior probability that the state is y is $p \in (0, 1)$.

Payoffs. The payoffs of the committee members and the DM, denoted by $u_C(D, \theta)$ and $u_{DM}(D, \theta)$, respectively, depend on DM's choice D and the state θ. The payoffs of status quo N are normalized to zero in both states for both parties, i.e., $u_C(N, \theta) = u_{DM}(N, \theta) = 0$ for $\theta \in \{y, n\}$. The payoffs of alternative Y depend on the state: For the committee members, $u_C(Y, n) = -1/2$, $u_C(Y, y) = 1/2$; for the DM, $u_{DM}(Y, n) = -\alpha$, and $u_{DM}(Y, y) = 1 - \alpha$. The parameter $\alpha \in (1/2, 1)$ measures the conflict of interest between the DM and the committee members. Under perfect information, all players have the same preference, i.e., all players strictly prefer alternative Y in state y and status quo N in state n. For every interior belief about the state, the DM's expected payoff of adopting alternative Y is lower than that of the committee members. Moreover, we assume that the optimal uninformed decision for the DM is status quo N, i.e., $\alpha > p$.[4]

	$\theta = n$	$\theta = y$
$D = N$	0	0
$D = Y$	$-\alpha$	$1 - \alpha$

	$\theta = n$	$\theta = y$
$D = N$	0	0
$D = Y$	$-\frac{1}{2}$	$\frac{1}{2}$

Table 1: The DM's (left) and committee member's (right) payoffs.

Information. Before voting, each member i receives a private signal s_i regarding the state θ.

[4] This assumption is non-essential for our results. If $\alpha < p$, then the DM will choose alternative Y instead of status quo N in an uninformative equilibrium. Our characterizations for informative equilibria remain valid.

Signal s_i is distributed on (a,b) according to distribution function $F(.)$ if the state is y and $G(.)$ if the state is n, where $a,b \in \mathbb{R} \cup \{-\infty,+\infty\}$. The distributions $F(.)$ and $G(.)$ admit continuous density functions $f(.)$ and $g(.)$, respectively, on (a,b). All signals are independently distributed conditional on the true state.

Strategies. A *voting strategy* of a committee member i is a (measurable) function, $m_i : (a,b) \rightarrow [0,1]$, that maps his signal s_i into the probability of voting for alternative Y. A voting strategy m_i is a *partisan* strategy if committee member i votes for the same option regardless of his signal. A voting strategy m_i is *increasing* if $m_i(s_i) \geq m_i(s_i')$ for all $s_i \geq s_i'$. A voting strategy m_i is a *cutoff strategy* if there exists an $s_i^* \in (a,b)$ such that $m_i(s_i) = I$ for all $s_i > s_i^*$ and $m_i(s_i) = J$ for all $s_i < s_i^*$, where $I, J \in \{0,1\}$ and $I \neq J$. We use $m := (m_1, m_2, \ldots, m_{\mathcal{N}})$ to denote the voting strategy profile of the committee members.

A *decision rule* of the DM is a function $d : \{Y,N\}^{\mathcal{N}} \rightarrow \{Y,N\}$ that maps a vote profile $v \in \{Y,N\}^{\mathcal{N}}$ into one of the two options. We effectively assume that the DM uses a pure strategy. In the online appendix, we show that allowing mixed strategies for the DM does not affect any of our main results. Below we introduce two types of decision rules that will be important for our analysis. Denote the number of yay votes in a vote profile v by $|v|$. A decision rule d is a k-rule if there exists a threshold $k \in \{1,2,...,\mathcal{N}\}$ such that, for all $v \in \{Y,N\}^{\mathcal{N}}$, $d(v) = Y$ if $|v| \geq k$, and $d(v) = N$ if $|v| < k$. Each k-rule is uniquely characterized by the corresponding threshold k. A decision rule d is a *weighted voting rule* if there exists a weight profile $w = (w_1, w_2, ...w_{\mathcal{N}}) \in \mathbb{R}_+^{\mathcal{N}}$ and a quota $Q \in \mathbb{R}_+$ such that $d(v) = Y$ if $\sum_{i=1}^{\mathcal{N}} w_i \mathbf{1}_{\{v_i=Y\}} \geq Q$ and $d(v) = N$ if $\sum_{i=1}^{\mathcal{N}} w_i \mathbf{1}_{\{v_i=Y\}} < Q$, where $\mathbf{1}$ is the indicator function.[5] A k-rule, for example, corresponds to a weighted voting rule in which $w_1 = ... = w_{\mathcal{N}} = 1$ and $Q = k$.

Equilibrium. We use perfect Bayesian equilibrium (PBE) as the solution concept. A PBE consists of a voting strategy profile m, a decision rule d, and a system of belief $\mu : \{Y,N\}^{\mathcal{N}} \rightarrow [0,1]$ that specifies the posterior belief of the DM about the state for each vote profile. Each committee member i's voting strategy m_i is optimal given the other committee members' voting strategies m_{-i} and the decision rule d; the decision rule d is optimal given the voting strategy profile m and the belief system μ; and μ is updated according to Bayes' rule whenever possible. Finally,

[5] See Shapley and Shubik (1954) and Felsenthal and Machover (1998), for example, for the study of these voting rules in cooperative game theory.

we assume that the DM always chooses alternative Y when indifferent.

An equilibrium is *informative* if the DM chooses both options with positive probability in equilibrium. We say that a DM, characterized by her preference parameter α, *can be persuaded* if there exists an informative equilibrium. An equilibrium is *symmetric* if all committee members use the same voting strategy, and *asymmetric* otherwise. Finally, two equilibria with strategy profiles (m, d) and (m', d') are *outcome-equivalent* if for all signal profiles $(s_1, s_2, .., s_{\mathcal{N}}) \in (a, b)^{\mathcal{N}}$, $\Pr(d(v) = Y|\,(s_1, s_2, .., s_{\mathcal{N}})\,; m) = \Pr(d'(v) = Y|\,(s_1, s_2, .., s_{\mathcal{N}})\,; m')$.

3.2 Assumptions on information structure

Let $h_F(s) := \frac{f(s)}{1-F(s)}$ and $h_G(s) := \frac{g(s)}{1-G(s)}$ be the hazard functions for distributions $F(.)$ and $G(.)$, respectively. Define the hazard ratio at signal s as the ratio of the hazard functions at signal s, i.e., $h_F(s)/h_G(s)$. We impose the following three assumptions on $F(.)$ and $G(.)$.

Assumption 1 (MLRP) *$F(.)$ and $G(.)$ satisfy the strict monotone likelihood ratio property (MLRP), i.e., $f(s)/g(s)$ is strictly increasing in s.*

Assumption 2 (Unbounded likelihood ratio) *As s approaches a, $f(s)/g(s)$ approaches zero. As s approaches b, $f(s)/g(s)$ approaches positive infinity.*

Assumption 3 (IHRP) *$F(.)$ and $G(.)$ satisfy the strict increasing hazard ratio property (IHRP), i.e., $h_F(s)/h_G(s)$ is strictly increasing in s.*

Assumption 1 is a standard assumption in the literature, which guarantees that a higher signal is more indicative of the state being y. We summarize some useful properties of distributions satisfying MLRP in Lemma 4 in the Appendix.

Assumption 2 implies unbounded posterior odds, which means that signals can be arbitrarily precise. This assumption guarantees that for any fixed k-rule, an informative equilibrium exists in the corresponding decision-making committee.[6] Moreover, under this assumption, a sufficiently

[6] A weaker assumption that guarantees this is $f(a)/g(a) < (1-p)/p < f(b)/g(b)$. This assumption is employed by Duggan and Martinelli (2001). It makes sure that a committee member who behaves "naively" (i.e., as if his vote alone determines the outcome) will vote for status quo N (alternative Y) after receiving a signal that is low (high) enough.

large committee can persuade any conservative DM (see Corollary 2) and persuade the DM to choose the correct option in both states (see Proposition 5).

Assumption 3 is a regularity assumption that has been studied by Duggan and Martinelli (2001) in the context of a decision-making committee. Kalashnikov and Rachev (1986) first introduced this property in the statistical literature.[7] It was also shown to be an important condition for the absence of information cascades in the observational learning models (Herrera and Hörner 2011, 2013). Most but not all distributions commonly used in economics and political science satisfy IHRP.[8] For example, if both $F(.)$ and $G(.)$ are normal distributions that satisfy MLRP, then they satisfy IHRP.[9]

To provide some intuition for Assumption 3, consider two signals s and s' such that $s > s'$. IHRP implies that

$$\frac{f(s)/g(s)}{f(s')/g(s')} > \frac{(1-F(s))/(1-G(s))}{(1-F(s'))/(1-G(s'))}.$$

Given signal s, $f(s)/g(s)$ is the likelihood ratio of signal s, $(1-F(s))/(1-G(s))$ is the likelihood ratio of a truncation above s. Then IHRP implies that the likelihood ratio of a signal increases faster than that of the upper truncation. The interpretation of Assumption 3 will be more transparent once we discuss the equilibrium voting strategy.

4 Equilibrium analysis

In this section, we study the properties of informative equilibria and establish the necessary and sufficient conditions for the existence of an informative equilibrium. We first consider symmetric voting in Section 4.1. We show that when voting is symmetric, the DM can be persuaded if and only if there exists an informative equilibrium with the unanimity rule. Then in Section 4.2, we extend the results to asymmetric voting. We show that focusing on symmetric equilibrium is without loss in investigating the possibility of information transmission.

[7] In survival analysis, IHRP is referred to as the "ageing faster property." This is because the hazard function represents the instantaneous probability of death. Thus, the probability of death for an agent whose lifetime distribution function is given by $F(.)$ increases faster as s increases than the probability of death for an agent whose lifetime distribution function is given by $G(.)$.

[8] See Herrera and Hörner (2011) for a discussion and a list of distributions that satisfy IHRP. A notable case that fails IHRP is the exponential distribution, whose hazard ratio is a constant (Duggan and Martinelli 2001; Herrera and Hörner 2011). It is thus a knife edge case.

[9] See Lemma 5 in the Appendix.

The equilibrium strategies of the committee members and the DM can be arbitrarily complex. However, the following proposition indicates that since we are only interested in the equilibrium outcome, it is without loss to restrict attention to a specific class of equilibria, in which the committee members use either cutoff strategies or partisan strategies and the DM uses a weighted voting rule.

Proposition 1 *For any equilibrium, there exists an outcome-equivalent equilibrium with strategy profile* (m, d) *such that*

1. *each member* i*'s voting strategy* m_i *is either an increasing cutoff strategy or a partisan strategy;*
2. *the DM's decision rule* d *is a weighted voting rule.*

In the following analysis, we further restrict attention to equilibria in which all committee members use increasing cutoff strategies. When a committee member adopts a partisan strategy, his vote does not depend on the signal received. For other committee members who use increasing cutoff strategies, they decide how to vote as if they ignore how partisan members vote. To see why this is the case, note that committee members who use increasing cutoff strategies decide how to vote conditional on their own signal as well as being pivotal.[10] The posterior belief conditional on being pivotal does not depend on the vote cast by a partisan voter, because a partisan member's vote is independent of the state. Therefore, these members behave as if the partisan voters are absent. By the same logic, for every vote profile that occurs with positive probability in equilibrium, the DM decides between the two options as if she ignores how partisan members vote as well. Hence, dropping the partisan members out of the committee affects neither how the other members vote, nor how the DM decides between alternative Y and status quo N, and generates the same equilibrium outcome. Conversely, given an equilibrium in which all members of a smaller committee use increasing cutoff strategies, there is always an equilibrium that generates the same outcome in a larger committee where the extra members use partisan strategies.

[10] A member is pivotal when his vote can affect the final outcome. For example, when the decision rule is a k-rule, then a member is pivotal when there are exactly $k-1$ yay votes from all other members.

This means that the existence of partisan committee members effectively reduces the committee size. Since our focus is on the existence of an informative equilibrium, for a given committee size, it is without loss to focus only on equilibria in which all members vote informatively. In these equilibria, the cutoff s_i^* fully characterizes member i's equilibrium voting strategy, so we also use m_i and s_i^* interchangeably.

4.1 Symmetric voting

We first consider symmetric informative equilibria in which all committee members use the same cutoff strategy. By Proposition 1, every yay vote in such an equilibrium will be assigned the same weight, because the voter identity provides no extra information to the DM in addition to the number of yay votes $|v|$. Thus, the equilibrium decision rule must be a k-rule.

We now characterize the equilibrium cutoff strategy of the committee members. Note that a committee member with the cutoff signal is indifferent between voting for alternative Y and status quo N conditional on being pivotal, and member i is pivotal when there are exactly $k-1$ yay votes among all other members, i.e., $|v_{-i}| = k-1$. So the cutoff signal s^* solves the following equation,

$$\frac{p}{1-p}\left(\frac{1-F(s)}{1-G(s)}\right)^{k-1}\left(\frac{F(s)}{G(s)}\right)^{\mathcal{N}-k}\frac{f(s)}{g(s)}=1. \tag{1}$$

To understand this equilibrium condition, note that $p/(1-p)$ is the prior likelihood ratio of the state (state y versus state n), $(1-F(s))/(1-G(s))$ is the likelihood ratio of a yay vote, $F(s)/G(s)$ is the likelihood ratio of a nay vote, and $f(s)/g(s)$ is the likelihood ratio of signal s. By Bayes' rule, the product of these terms on the left-hand side of (1) is the posterior likelihood ratio of the state conditional on a committee member receiving a signal s, knowing that $k-1$ other members cast yay votes and $\mathcal{N}-k$ cast nay votes. For a member with the cutoff signal s^*, this posterior likelihood ratio must be 1, which means that he is indifferent between status quo N and alternative Y.

By MLRP, $f(s)/g(s)$ is strictly increasing in s, and both $(1-F(s))/(1-G(s))$ and $F(s)/G(s)$ are strictly increasing in s. Therefore, the left-hand side of (1) is strictly increasing in s. Combined with Assumption 2, this implies that (1) has a unique solution. Denote the unique solution to (1) by $s(k,\mathcal{N})$. We have,

Lemma 1 *The voting cutoff* $s(k, \mathcal{N})$ *is strictly decreasing in* k.

Intuitively, when the DM asks for more yay votes to choose alternative Y, the members cast yay votes more often in equilibrium. To see why this must be the case, note that when the decision rule is k-rule, member i is pivotal when $|v_{-i}| = k - 1$. By the definition of $s(k, \mathcal{N})$, a member with signal $s(k, \mathcal{N})$ is indifferent between alternative Y and status quo N conditional on being pivotal. When the decision rule is $(k+1)$-rule, member i is pivotal when $|v_{-i}| = k$. If $s(k+1, \mathcal{N})$ is higher than $s(k, \mathcal{N})$, then under $(k+1)$-rule, a member with signal $s(k+1, \mathcal{N})$ would never be indifferent between alternative Y and status quo N conditional on pivotal, because he is conditioning on more yay votes and each yay vote is more indicative of the state being y than under k-rule. Thus, $s(k+1, \mathcal{N})$ must be strictly lower than $s(k, \mathcal{N})$.

The DM decides between alternative Y and status quo N after observing the vote profile v. Given that each member uses an increasing cutoff strategy featuring cutoff $s(k, N)$, we can therefore find the condition for the existence of a symmetric informative equilibrium with the corresponding k-rule. The following lemma provides the necessary and sufficient condition for the existence of a symmetric informative equilibrium with k-rule.

Lemma 2 *For each* $k \in \{1, 2, ..., \mathcal{N}\}$, *a symmetric informative equilibrium with* k*-rule exists if and only if* $\alpha \leq \boldsymbol{\alpha}(k, \mathcal{N})$, *where* $\boldsymbol{\alpha}(k, \mathcal{N})$ *is the unique solution to*

$$\frac{\alpha}{1-\alpha} = \frac{h_G(s(k, \mathcal{N}))}{h_F(s(k, \mathcal{N}))}.$$

Note that in a symmetric equilibrium with k-rule, the DM optimally chooses alternative Y if $|v| \geq k$, and status quo N if $|v| < k$. Given $s^* = s(k, \mathcal{N})$, this means that the posterior likelihood ratio of the state that makes the DM indifferent between Y and N, $\alpha/(1-\alpha)$, satisfies

$$\frac{p}{1-p}\left(\frac{1-F(s^*)}{1-G(s^*)}\right)^{k-1}\left(\frac{F(s^*)}{G(s^*)}\right)^{\mathcal{N}-k+1} < \frac{\alpha}{1-\alpha} \leq \frac{p}{1-p}\left(\frac{1-F(s^*)}{1-G(s^*)}\right)^{k}\left(\frac{F(s^*)}{G(s^*)}\right)^{\mathcal{N}-k}, \quad (2)$$

where the lower bound is the posterior likelihood ratio when $|v| = k - 1$, and the upper bound is the posterior likelihood ratio when $|v| = k$. According to condition (1), when observing the signal $s^* = s(k, \mathcal{N})$ and $|v_{-i}| = k - 1$, one believes that states y and n are equally likely. Thus, the posterior likelihood ratio when observing $|v_{-i}| = k$ can be replaced by the inverse of the likelihood ratio of signal $s^* = s(k, \mathcal{N})$. As a result, condition (2) can be reformulated as

$$\frac{F(s^*)}{G(s^*)}\frac{g(s^*)}{f(s^*)} < \frac{\alpha}{1-\alpha} \leq \frac{(1-F(s^*))}{(1-G(s^*))}\frac{g(s^*)}{f(s^*)}. \tag{3}$$

By MLRP, $F(s^*)/G(s^*) < f(s^*)/g(s^*)$. That is, a nay vote is less indicative of the state being y than the signal s^*. Thus the posterior likelihood ratio when $|v| = k-1$ is less than 1, and the left inequality of (3) always holds for $\alpha > 1/2$. Therefore, for a given k, the right inequality in (3) is a necessary and sufficient condition for the existence of a symmetric informative equilibrium with the corresponding k-rule. The value $\boldsymbol{\alpha}(k, \mathcal{N})$ represents the most conservative DM that may adopt k-rule in a symmetric informative equilibrium when the committee size is $\mathcal{N}$.

We are now ready to state our first main result.

Proposition 2 (Symmetric voting) *There exists a symmetric informative equilibrium if and only if*

$$\alpha \leq \boldsymbol{\alpha}(\mathcal{N}, \mathcal{N}).$$

Furthermore, for all $k' > k$, there exists a symmetric informative equilibrium with k'-rule if there exists a symmetric informative equilibrium with k-rule, but the converse is in general not true.

The first part of the result establishes a necessary and sufficient condition for the existence of a symmetric informative equilibrium. It also provides a simple algorithm to check whether information transmission is possible when the committee members adopt symmetric voting strategies.

The second part of the result implies that a higher k-rule can sustain information transmission for a larger set of α. In other words, a higher k-rule is more likely to arise endogenously in equilibrium when the DM becomes more conservative. It also provides a rationale for the pressure for a higher level of consensus in many institutions.

We start with the second part of the result to explain the intuition. When increasing the threshold of the decision rule, say from k to $k+1$, there are two opposite effects. One is a direct *consensus* effect. The increase in the threshold means that the DM asks for a higher consensus level among the members to choose alternative Y. A higher consensus level is more indicative of the state being y. The other is an indirect *strategic* effect. As shown in Lemma 1, an increase in the threshold leads to the committee members casting yay votes less conservatively, that is, $s(k+1,\mathcal{N}) < s(k,\mathcal{N})$. It makes each single yay vote less indicative of the state being y. By IHRP, the consensus effect dominates the strategic effect, so we have $\boldsymbol{\alpha}(k+1,\mathcal{N}) > \boldsymbol{\alpha}(k,\mathcal{N})$. Combined with Lemma 2, we obtain the second part of the proposition. The first part of this proposition naturally follows.

In Figure 1, we illustrate how $\boldsymbol{\alpha}(k,\mathcal{N})$ increases with k when the signals are normally distributed in both states. We show in Lemma 5 in the Appendix that if two normal distributions $F(.)$ and $G(.)$ satisfy MLRP, they satisfy IHRP. The distributions illustrated in the figure, $F(s) = \Phi(s-1)$ and $G(s) = \Phi(s+1)$,[11] satisfy MLRP, thus also satisfy the other assumptions of the model.

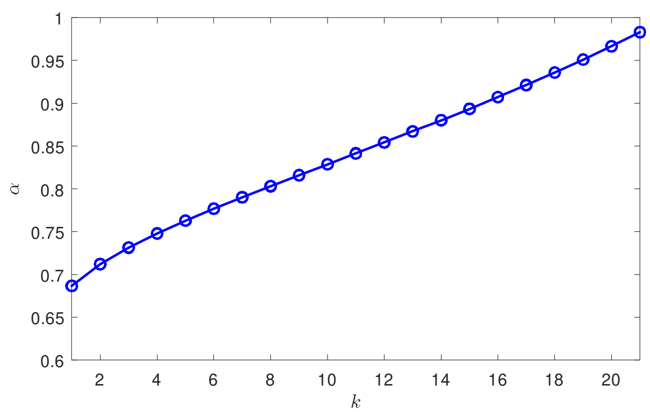

Figure 1: The function $\boldsymbol{\alpha}(k,\mathcal{N})$.

Parameters: $p = 1/2$, $\mathcal{N} = 21$, $F(s) = \Phi(s-1)$, $G(s) = \Phi(s+1)$.

According to Proposition 2, the existence of a symmetric informative equilibrium depends on the size of the committee. Our next result further explores the role of committee size and shows that, under symmetric voting, if there is informative transmission with a smaller committee, then there is informative transmission with a larger committee, but not vice versa.[12]

[11] $\Phi(s) := \left(1/\sqrt{2\pi}\right)\int_{-\infty}^{s} e^{-t^2/2}dt$ is the cumulative distribution function of the standard normal distribution.

[12] This is not necessarily the case, when the signals are binary. See the discussion in Section 5.

Corollary 1 *For all $\mathcal{N}' > \mathcal{N}$, if there exists a symmetric informative equilibrium when the committee size is $\mathcal{N}$, then there exists a symmetric informative equilibrium when the committee size is $\mathcal{N}'$, but the converse is in general not true.*

This corollary is a direct implication of Proposition 2 and the fact that $\boldsymbol{\alpha}(\mathcal{N},\mathcal{N})$ is strictly increasing in $\mathcal{N}$. To understand why $\boldsymbol{\alpha}(\mathcal{N},\mathcal{N})$ is strictly increasing in $\mathcal{N}$, note that $s(\mathcal{N},\mathcal{N})$ is the unique solution to

$$\frac{p}{1-p}\left(\frac{1-F(s)}{1-G(s)}\right)^{\mathcal{N}-1}\frac{f(s)}{g(s)}=1. \tag{4}$$

By MLRP, $f(s)/g(s)$ is strictly increasing in s, and $((1-F(s))/(1-G(s)))^{\mathcal{N}-1}$ is strictly increasing in s and $\mathcal{N}$. Therefore, the left-hand side of (4) is strictly increasing in both s and $\mathcal{N}$. Thus, $s(\mathcal{N},\mathcal{N})$ must be strictly decreasing in $\mathcal{N}$. This is because under the unanimity rule, there are more yay votes in a larger committee when a member is pivotal, so the members vote less conservatively. By IHRP, this implies that $\boldsymbol{\alpha}(\mathcal{N},\mathcal{N})$ is strictly increasing in $\mathcal{N}$.

Corollary 1 mainly concerns with the existence of an informative equilibrium when the committee is of different sizes. However, if the unanimity rule is not used in equilibrium, a larger committee may fail to transmit information when a smaller committee voting under the unanimity rule is able to. We illustrate this point through the following example.

Example 1 *Suppose voting is symmetric, and $F(.)$ and $G(.)$ are two normal distributions satisfying MLRP. When $p > 1/2$, $\boldsymbol{\alpha}(1,1) > \boldsymbol{\alpha}(\kappa+1, 2\kappa+1)$ for all $\kappa \geq 1$.*

In this example, for a DM with α such that $\boldsymbol{\alpha}(\kappa+1,2\kappa+1) < \alpha < \boldsymbol{\alpha}(1,1)$, there is a symmetric informative equilibrium when there is a single expert, but no symmetric informative equilibrium with the simple majority rule, even when the committee is arbitrarily large. This example shows that with regard to information transmission, not only the committee size matters, the equilibrium decision rule is also crucial.

4.2 Asymmetric voting

In this section, we generalize the equilibrium analysis by allowing asymmetric voting. With fewer constraints on the voting strategies of the committee members, one natural question is: Can a committee persuade a more conservative DM under asymmetric voting than under symmetric voting? Proposition 3 says the answer is no.

Proposition 3 *There exists an informative equilibrium if and only if*

$$\alpha \leq \boldsymbol{\alpha}(\mathcal{N}, \mathcal{N}).$$

Proposition 3 generalizes the first part of Proposition 2 to asymmetric voting. There exists an informative equilibrium if and only if there exists a symmetric informative equilibrium with the unanimity rule. Therefore, allowing asymmetric voting does not make a difference in terms of the possibility of information transmission. Proposition 3 also provides a tight upper bound for the degree of conflict of interest between the DM and the committee members that allows information transmission. This upper bound can and can only be achieved by the unanimity rule. Gradwohl and Feddersen (2018) also derive an upper bound in a binary-signal model, but their upper bound is not achievable by the unanimity rule nor by symmetric voting. In a more general discrete-signal model, Battaglini (2017) derives an upper bound that is not tight. In Section 5, we will discuss the differences between the continuous-signal model and the discrete-signal model, with a special focus on the binary-signal case.

To see why asymmetric voting would not change the existence condition for an informative equilibrium, consider a two-member committee. By Lemma 1, there exists a symmetric informative equilibrium with the unanimity rule for all $\alpha \leq \boldsymbol{\alpha}(2, 2)$. Consider an asymmetric equilibrium. There are three possibilities:

Case 1: The DM chooses alternative Y if and only if there are two yay votes. In this case, the decision rule is the unanimity rule. As shown by Duggan and Martinelli (2001), under IHRP, voting under the unanimity rule is always symmetric. Therefore, this case is irrelevant.

Case 2: The DM chooses alternative Y if and only if there is at least one yay vote. In this case, the decision rule is a k-rule and $k = 1$. Without loss of generality, assume that member 1 has the higher cutoff. It must be the case that $s_1^* > s(1, 2) > s_2^*$, otherwise either member 1 with signal s_1^* or member 2 with signal s_2^* is not indifferent between alternative Y and status quo N conditional on being pivotal. Moreover, member 1's pivotal consideration implies that s_2^*

must be low enough. IHRP makes sure that the effect of the decrease in s_2^* relative to $s(1,2)$ dominates the effect of the increase in s_1^* relative to $s(1,2)$, so that a yay vote from member 1 and a nay vote from member 2 would not be more indicative of the state being y than one yay vote and one nay vote under symmetric voting.[13] Thus, DM cannot be more conservative than $\boldsymbol{\alpha}(1,2)$. Therefore, such an equilibrium does not exist when $\alpha \geq \boldsymbol{\alpha}(2,2)$.

Case 3: The DM chooses alternative Y if and only if a specific committee member, say member 1, casts a yay vote. In this case, the decision rule means that how member 2 votes is irrelevant, so member 2 is never pivotal. Effectively, the committee is a one-man committee. By Corollary 1, $\boldsymbol{\alpha}(1,1) < \boldsymbol{\alpha}(2,2)$. Therefore, such an equilibrium does not exist when $\alpha \geq \boldsymbol{\alpha}(2,2)$.

These three cases together imply that there does not exist an asymmetric informative equilibrium when $\alpha \geq \boldsymbol{\alpha}(2,2)$. More generally, for a committee of arbitrary size, the basic idea underlying this example still applies. That is, asymmetric voting implies that the DM must be willing to choose alternative Y given some vote profiles other than unanimity. Some of these vote profiles are less indicative of the state being y than the others. IHRP ensures that we can find a vote profile that leads to a lower posterior belief than unanimity.

When we focus on the class of equilibria in which the equilibrium decision rule is a k-rule, we can also show that asymmetric voting does not help a committee to persuade a more conservative DM, when a particular k-rule is adopted in equilibrium. This immediately implies that the second part of Proposition 2 still holds when asymmetric voting is allowed.

Proposition 4 (k-rules) *For all $k \in \{1,2,...,\mathcal{N}\}$, there exists an informative equilibrium with k-rule if and only if there exists a symmetric informative equilibrium with k-rule.*

The proof of Proposition 4 generalizes the argument in Case 2 of the two-member committee example. We only need to show that asymmetric voting under k-rule would not enable the

[13] Formally, member 1's indifference condition is given by

$$\frac{p}{1-p}\frac{F(s_2^*)}{G(s_2^*)}\frac{f(s_1^*)}{g(s_1^*)} = 1.$$

Thus, the posterior likelihood ratio of the state given a yay vote from member 1 and a nay vote from member 2 is:

$$\frac{p}{1-p}\frac{1-F(s_1^*)}{1-G(s_1^*)}\frac{F(s_2^*)}{G(s_2^*)} = \frac{g(s_1^*)}{f(s_1^*)}\frac{1-F(s_1^*)}{1-G(s_1^*)} = \frac{h_G(s_1^*)}{h_F(s_1^*)}.$$

By IHRP, this ratio is strictly smaller than $h_G(s(1,2))/h_F(s(1,2))$.

committee to persuade a more conservative DM. In an asymmetric informative equilibrium with k-rule, among all vote profiles with k yay votes, some induce a lower posterior belief of the state being y than others. IHRP makes sure that at least one of these vote profiles is less indicative of the state being y than k votes under symmetric voting.

We now examine the effect of the committee size. Combined with the fact that $\boldsymbol{\alpha}(\mathcal{N}, \mathcal{N})$ is increasing in $\mathcal{N}$, Proposition 3 naturally implies that Corollary 1 can be generalized to any informative equilibrium, that is, if there is an informative equilibrium with a smaller committee, there must exist an informative equilibrium with a larger one. This justifies our focus on the case in which all committee members vote informatively, when considering the possibility of information transmission. It is because, as discussed at the beginning of this section, the partisan members essentially only play the role of reducing the committee size.

Given that a more conservative DM can be persuaded by a larger committee, one may ask whether any DM, no matter how conservative she is, can be persuaded by a committee that is sufficiently large. The answer is yes. We state this result formally in the following corollary.

Corollary 2 *For any $\alpha \in (1/2, 1)$, there exists an $\mathbf{N}(\alpha) < \infty$, such that an informative equilibrium exists for all committees larger than $\mathbf{N}(\alpha)$.*

The above corollary implies that, for any DM, there always exists an informative equilibrium when the committee is large enough. This result relies on Assumption 2, which implies $\boldsymbol{\alpha}(\mathcal{N}, \mathcal{N})$ is increasing in $\mathcal{N}$ without bound.

Though a sufficiently large committee can persuade any DM to choose alternative Y some of the time, it could still be that the probability that the DM chooses alternative Y is very small. If this is the case, then at a practical level, the insights obtained above are not very useful. However, we show below that this is not the case. When the committee size goes to infinity, the DM chooses the correct option in both states almost surely. Therefore, both information aggregation and transmission can be efficient asymptotically.

Proposition 5 (Duggan and Martinelli (2001)) *There is a sequence of informative equilibria with strategy profiles $\left(m^{\mathcal{N}}, d^{\mathcal{N}}\right)_{\mathcal{N}=1}^{\infty}$ such that the probabilities that the DM chooses alternative*

Y in state y and chooses status quo N in state n converge to 1, that is,

$$\lim_{\mathcal{N}\to\infty} \Pr\left(d^{\mathcal{N}} = Y|\theta = y\right) = \lim_{\mathcal{N}\to\infty} \Pr\left(d^{\mathcal{N}} = N|\theta = n\right) = 1.$$

Duggan and Martinelli (2001) and Martinelli (2002) proved this result for a decision-making committee voting under the unanimity rule. Proposition 5 follows immediately from their result and Corollary 2. Intuitively, when the unanimity rule is used, alternative Y is rejected only if one of the committee members casts a nay vote. Thus, the DM effectively delegates the decision to the committee member who has received the most negative information against alternative Y. Under Assumption 2, when the committee size goes to infinity, the cutoff signal becomes arbitrarily low, then a nay vote becomes arbitrarily indicative of the state being n. Therefore, the DM chooses the correct option in each state with probability one in the limit.

5 Discrete signals

In the baseline model, we assume that each committee member receives a continuous signal. In this section, we consider the situation in which each member receives a discrete signal. We first focus on the binary-signal case and then discuss briefly the general case with more than two signals. The results we have for the binary-signal case are very different from those in the baseline model, and the discussion on the general discrete-signal case explains how and why our results depend on the coarseness of the signal structure.

5.1 Binary signals

The binary-signal model considered in this section has been studied by Gradwohl and Feddersen (2018). The assumption of binary signal is common in the collective decision-making literature, as it is often thought to be an innocuous assumption that provides a good first approximation of models with more general information structures. As in the previous sections, we first analyze the existence conditions for the symmetric informative equilibrium with a particular k-rule, which are not discussed in Gradwohl and Feddersen (2018), then we discuss whether the existence conditions change when asymmetric voting is allowed. One will see that the results we obtain strikingly contrast with those in the continuous-signal model.

Suppose each committee member i privately receives a binary signal $s_i \in \{Y, N\}$ about the state θ. The signals are symmetrically informative about the state, i.e.,

$$\Pr\left(s_i = Y|\theta = y\right) = \Pr\left(s_i = N|\theta = n\right) = q,$$

where $q \in (1/2, 1)$. For simplicity, we assume that the prior of the state being y is $1/2$. The setting is so far identical to the one considered in Gradwohl and Feddersen (2018).

A voting strategy of committee member i is a tuple $\left(\rho_N^i, \rho_Y^i\right)$, where ρ_N^i and ρ_Y^i are the probabilities of voting for alternative Y after receiving an N-signal and a Y-signal, respectively. Voting is *truthful* if one votes according to his signal, i.e., $\left(\rho_N^i, \rho_Y^i\right) = (0, 1)$; and voting is *informative* if voting is responsive to the signal, i.e., $\rho_Y^i \neq \rho_N^i$.

First, we consider symmetric voting, where $\left(\rho_N^i, \rho_Y^i\right) = (\rho_N, \rho_Y)$ for every member $i = 1, 2, \ldots, \mathcal{N}$. For simplicity, we assume that $\mathcal{N}$ is *odd*. In an informative equilibrium, if voting is symmetric and increasing, the equilibrium decision rule must be a k-rule. When the decision rule is the simple majority rule, i.e., $k = (\mathcal{N}+1)/2$, voting is truthful, i.e., $\rho_N = 0$ and $\rho_Y = 1$. When the decision rule is a minority rule, i.e., $k < (\mathcal{N}+1)/2$, the committee members vote for each option with positive probability after receiving a Y-signal, i.e., $\rho_N = 0$ and $\rho_Y \in (0, 1)$. When the decision rule is a supermajority rule, i.e., $k > (\mathcal{N}+1)/2$, the committee members vote for each option with positive probability after receiving an N-signal, i.e., $\rho_N \in (0, 1)$ and $\rho_Y = 1$.

Consider the supermajority rules. For all $k > (\mathcal{N}+1)/2$, the indifferent signal is the N-signal. Therefore, conditional on being pivotal, a committee member with an N-signal is indifferent between voting for alternative Y and status quo N, so ρ_N solves the following equation,

$$\left(\frac{q + (1-q)\,\rho_N}{q\rho_N + (1-q)}\right)^{k-1} \left(\frac{1-q}{q}\right)^{\mathcal{N}-k} \left(\frac{1-q}{q}\right) = 1. \tag{5}$$

Note that $\left(q + (1-q)\,\rho_N\right) / \left(q\rho_N + (1-q)\right)$ is the likelihood ratio of a yay vote, and $(1-q)/q$ is the likelihood ratio of a nay vote. Therefore, on the left-hand side of (5), the first term is the likelihood ratio of $k-1$ yay votes, the second term is the likelihood ratio of $\mathcal{N}-k$ nay votes, and the third term is the likelihood ratio of an N-signal. The left-hand side of (5) is thus the posterior likelihood ratio of the state conditional on being pivotal and receiving an N-signal.[14] The posterior likelihood ratio must be 1 to make a member with an N-signal indifferent between alternative Y and status quo N. Note that the likelihood ratio is strictly larger than 1 when $\rho_N = 0$, strictly smaller than 1 when $\rho_N = 1$, and strictly decreasing in ρ_N. Therefore, (5) has a unique solution in $(0, 1)$. Denote the unique solution by $\rho_N (k, \mathcal{N})$, for $k > (\mathcal{N} + 1)/2$.

It is worth mentioning that for $k > (\mathcal{N} + 1)/2$, $\rho_N (k, \mathcal{N})$ is increasing in k, that is, when the DM asks for more yay votes to choose alternative Y, the members cast yay vote more often after receiving the indifferent signal, N-signal. The reason is very similar to that of Lemma 1. If $\rho_N (k + 1, \mathcal{N})$ is smaller than $\rho_N (k, \mathcal{N})$, then under $(k + 1)$-rule, a member with an N-signal cannot be indifferent between alternative Y and status quo N conditional on being pivotal, because compared with k-rule, there are more yay votes when the member is pivotal and each yay vote is more indicative of the state being y. Thus $\rho_N (k + 1, \mathcal{N})$ must be strictly larger than $\rho_N (k, \mathcal{N})$.

For a k-rule to be supported in an informative equilibrium, it must be the case that the DM finds it optimal to follow the given k-rule, that is,

$$\frac{\Pr (|v| = k - 1|\theta = y)}{\Pr (|v| = k - 1|\theta = n)} < \frac{\alpha}{1 - \alpha} \leq \frac{\Pr (|v| = k|\theta = y)}{\Pr (|v| = k|\theta = n)}. \tag{6}$$

The left-hand side of the first inequality is the posterior likelihood ratio when $|v| = k - 1$, and the right-hand side of the second inequality is the posterior likelihood ratio when $|v| = k$. Similar to $\boldsymbol{\alpha} (k, \mathcal{N})$ in the continuous-signal model, define $\boldsymbol{\alpha}_2 (k, \mathcal{N})$ in the following way: for $k < (\mathcal{N} + 1)/2$, let $\boldsymbol{\alpha}_2 (k, \mathcal{N}) := 1/2$; for $k = (\mathcal{N} + 1)/2$, let $\boldsymbol{\alpha}_2 (k, \mathcal{N}) := q$; for $k > (\mathcal{N} + 1)/2$, let $\boldsymbol{\alpha}_2 (k, \mathcal{N})$ be the unique solution to

$$\frac{\alpha}{1 - \alpha} = \frac{q}{1 - q} \frac{q + (1 - q) \rho_N (k, \mathcal{N})}{q \rho_N (k, \mathcal{N}) + (1 - q)}. \tag{7}$$

The right-hand side of (7) is the likelihood ratio of a Y-signal and a yay vote, given the voting strategy $(\rho_N (k, \mathcal{N}), 1)$. Since $\rho_N (k, \mathcal{N}) \in (0, 1)$, the right-hand side of (7) is strictly larger than $q/(1 - q)$ and strictly smaller than $q^2/(1 - q)^2$. Moreover, the right-hand side of (7) is strictly decreasing in $\rho_N (k, \mathcal{N})$. Hence, $\boldsymbol{\alpha}_2 (k, \mathcal{N})$ is strictly decreasing in k given that $\rho_N (k, \mathcal{N})$

[14] Since $p = 1/2$, the likelihood ratio of the state is 1, so the parameter p does not appear in the expression of the posterior likelihood ratio.

is strictly increasing in k.

The following lemma characterizes the necessary and sufficient conditions for the existence of symmetric informative equilibria with k-rule.

Lemma 3 *When voting is symmetric,*

1. *there exists no informative equilibrium with k-rule where $k < (\mathcal{N}+1)/2$;*

2. *there exists an informative equilibrium with k-rule where $k \geq (\mathcal{N}+1)/2$ if and only if $\alpha \leq \boldsymbol{\alpha}_2(k, \mathcal{N})$.*

To understand this lemma, we consider three cases, $k = (\mathcal{N}+1)/2$, $k > (\mathcal{N}+1)/2$, and $k < (\mathcal{N}+1)/2$, separately. As discussed above, when $k = (\mathcal{N}+1)/2$, voting is truthful. Thus, observing $|v| = (\mathcal{N}+1)/2$ is informationally equivalent to observing $(\mathcal{N}+1)/2$ Y-signals and $(\mathcal{N}-1)/2$ N-signals, which is informationally equivalent to observing a single Y-signal, given that the signals are symmetrically informative. Similarly, observing $|v| = (\mathcal{N}-1)/2$ is informationally equivalent to observing a single N-signal. Therefore, to have the simple majority rule supported in a symmetric informative equilibrium, it must be the case that $1-q < \alpha \leq q$, according to (6). Since $\alpha > 1/2$, there exists a symmetric informative equilibrium with the simple majority rule if and only if $\alpha \leq q$.

When $k > (\mathcal{N}+1)/2$, the indifferent signal is the N-signal, which implies that conditional on the pivotal event v_{-i} such that $|v_{-i}| = k-1$ is equivalent to observing a Y-signal. Thus, for the DM, observing a vote profile $v = (v_i, v_{-i})$ with $v_i = Y$ and $|v_{-i}| = k-1$ is equivalent to observing a Y-signal and a yay vote. The corresponding posterior likelihood ratio is

$$\frac{q}{1-q}\frac{q+(1-q)\,\rho_N(k,\mathcal{N})}{q\rho_N(k,\mathcal{N})+(1-q)},$$

which is the right-hand side of (7). Similarly, observing $|v| = k-1$ is equivalent to observing a Y-signal and a nay vote, which is equivalent to receiving no information at all. Therefore, there exists a symmetric informative equilibrium with $k > (\mathcal{N}+1)/2$ if and only if $\alpha \leq \boldsymbol{\alpha}_2(k, \mathcal{N})$.

When $k < (\mathcal{N}+1)/2$, committee member i is indifferent between alternative Y and status quo N conditional on being pivotal after receiving a Y-signal. Similar to the logic in the previous case, for the DM, observing $|v| = k$ is equivalent to observing an N-signal and a yay vote. Since a member casts a yay vote only after receiving a Y-signal, the DM's posterior belief after observing an N-signal and a yay vote is equal to that after observing a Y-signal and an N-signal. Thus, the DM's posterior is exactly 1/2 after observing a vote profile v with $|v| = k$. Since $\alpha > 1/2$, she is never willing to choose alternative Y when $|v| = k$. Therefore, there is no symmetric informative equilibrium with $k < (\mathcal{N} + 1)/2$.

Based on the above discussion, we illustrate in Figure 2 how $\boldsymbol{\alpha}_2(k, \mathcal{N})$ changes with $k = 1, 2, \ldots, \mathcal{N}$, using an example with $\mathcal{N} = 21$ and $q = 0.7$.

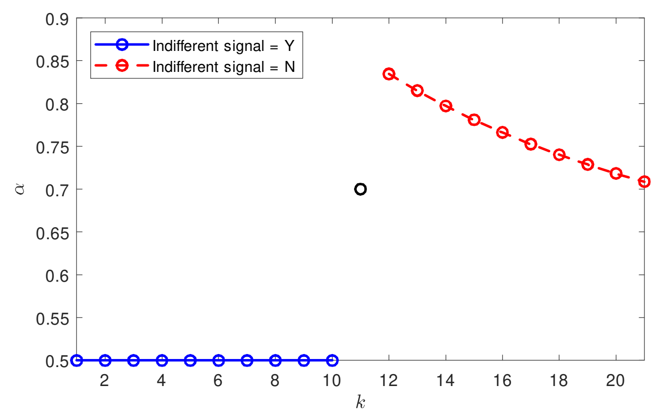

Figure 2: The function $\boldsymbol{\alpha}_2(k, \mathcal{N})$.

Parameters: $p = 1/2$, $\mathcal{N} = 21$, $q = 0.7$.

In Figure 2, note that $\boldsymbol{\alpha}_2((\mathcal{N} + 1)/2, \mathcal{N})$ is lower than $\boldsymbol{\alpha}_2(k, \mathcal{N})$ for all $k > (\mathcal{N} + 1)/2$, so according to Lemma 3, the existence of an informative equilibrium with the simple majority rule implies the existence of an informative equilibrium with a supermajority rule. Furthermore, for all supermajority rules, $\boldsymbol{\alpha}_2(k, \mathcal{N})$ is strictly decreasing in k, so the existence of an informative equilibrium with a higher k-rule implies the existence of an informative equilibrium with a lower k-rule. We summarize these observations formally in the following proposition.

Proposition 6 *When voting is symmetric,*

1. *there exists an informative equilibrium with a supermajority rule if there exists an informative equilibrium with the simple majority rule, but the converse is in general not true;*

2. *for all $k > (\mathcal{N}+1)/2$ and $k' > k$, there exists an informative equilibrium with k-rule if there exists an informative equilibrium with k'-rule, but the converse is in general not true.*

The second part of the proposition is the opposite of what we obtained in the continuous-signal model (See Proposition 2). Moreover, it implies that the existence of an informative equilibrium with the unanimity rule is no longer the necessary and sufficient condition for information transmission. The stark difference is because $\boldsymbol{\alpha}(k, \mathcal{N})$ in the continuous-signal model is increasing in k but $\boldsymbol{\alpha}_2(k, \mathcal{N})$ in the binary-signal model is decreasing in k for $k > (\mathcal{N}+1)/2$.

We now explain how the two information structures generate the differences. Consider first the continuous-signal model. To be consistent with the binary-signal model under consideration, we assume $p = 1/2$. Suppose that the voting cutoff in a symmetric equilibrium with k-rule is s, then as shown in (3), the likelihood ratio when $|v| = k$ is

$$\frac{\Pr(|v| = k|\theta = y)}{\Pr(|v| = k|\theta = n)} = \underbrace{\frac{g(s)}{f(s)}}_{\text{a signal } s^-} \times \underbrace{\frac{1 - F(s)}{1 - G(s)}}_{\text{a yay vote}}. \tag{8}$$

For a given signal s, consider a hypothetical signal s^-, such that $f(s^-)/g(s^-) = g(s)/f(s)$. A signal s^- cancels a signal s exactly, that is, the posterior belief conditional on a signal s^- and a signal s is exactly the prior. We call s^- the *anti-signal* of signal s. Thus, the likelihood ratio when $|v| = k$ is equal to that of an anti-signal s^- and a yay vote. By MLRP, when s decreases, the likelihood ratio of the anti-signal s^- increases, while that of a yay vote decreases. By IHRP, the likelihood ratio of the anti-signal s^- increases "faster" than the likelihood ratio of a yay vote decreases. Therefore, the likelihood ratio when $|v| = k$ decreases with s, which implies that $\boldsymbol{\alpha}(k, \mathcal{N})$ increases with k.

Consider next the binary-signal model. For $k > (\mathcal{N}+1)/2$, in a symmetric equilibrium with k-rule, the indifferent signal is the N-signal, and the likelihood ratio when $|v| = k$ is equal to the likelihood ratio of a Y-signal (the *anti-signal* of an N-signal) and a yay vote, i.e.,

$$\frac{\Pr(|v| = k|\theta = y)}{\Pr(|v| = k|\theta = n)} = \underbrace{\frac{q}{1-q}}_{\text{a } Y\text{-signal}} \times \underbrace{\frac{q + (1-q)\rho_N(k, \mathcal{N})}{q\rho_N(k, \mathcal{N}) + (1-q)}}_{\text{a yay vote}}. \tag{9}$$

The likelihood ratio of a Y-signal is constant, and the likelihood ratio of a yay vote depends on how often the committee members vote for alternative Y after receiving an N-signal. When k increases, the committee members vote for alternative Y more often after receiving an N-signal, which means that the likelihood ratio of a yay vote gets lower. Unlike in the continuous-signal model, the decrease in the likelihood ratio of the yay vote is not compensated by a corresponding increase in the likelihood ratio of the anti-signal, because the likelihood ratio of a Y-signal is constant. As a result, for all $k > (\mathcal{N}+1)/2$, the likelihood ratio of $|v| = k$ decreases with k, which implies that $\boldsymbol{\alpha}_2(k, \mathcal{N})$ decreases with k.

Now we will discuss asymmetric voting, and illustrate another difference between the binary-signal model and the continuous-signal model. In the continuous-signal model, allowing asymmetric voting has no effect on the existence of an informative equilibrium. But this is not the case in the binary-signal model, as shown in the following proposition.

Proposition 7 (Gradwohl and Feddersen (2018)) *Suppose $\mathcal{N} \in \{3, 5, 7, \dots\}$. There exists an informative equilibrium if and only if $\alpha \leq \overline{\alpha}$, where $\overline{\alpha}$ is the unique solution to*

$$\frac{\alpha}{1-\alpha} = \left(\frac{q}{1-q}\right)^2.$$

Moreover, there exists an asymmetric informative equilibrium if there exists a symmetric informative equilibrium, but the converse is in general not true.

Gradwohl and Feddersen (2018) derived this result.[15] We include it here to highlight the difference between the binary-signal model and the continuous-signal model with regard to asymmetric voting. Note that $\overline{\alpha}$ represents the most conservative DM, who is only willing to choose alternative Y after observing two Y-signals. Thus, the first part of Proposition 7 implies that if a DM strictly prefers status quo N given two Y-signals, then information transmission is never possible. Different from the continuous-signal model, the upper bound $\overline{\alpha}$ cannot be achieved by symmetric voting. The reason is that according to Lemma 3, under symmetric voting, the existence of an informative equilibrium depends on the posterior belief of the DM upon observing

[15] Gradwohl and Feddersen (2018) state the "only if" part of our Proposition 7 as a separate lemma (Lemma 1), while constructing an asymmetric informative equilibrium for any DM with $\alpha \leq \overline{\alpha}$.

a Y-signal and a yay vote. A yay vote, however, is always strictly less indicative of the state being y than a Y-signal under symmetric voting. Allowing asymmetric voting can change the situation. For example, consider a three-member committee. When $\alpha = \overline{\alpha}$, there exists an asymmetric informative equilibrium in which member 1 always votes for alternative Y, the other two members vote truthfully, and the DM chooses alternative Y if and only if both members 2 and 3 vote for alternative Y. But there does not exists a symmetric informative equilibrium.

To summarize, there are four main differences between the binary-signal model and the continuous-signal model. First, when the signals are continuous, a necessary and sufficient condition for information transmission is the existence of an informative equilibrium with the unanimity rule (Proposition 3), but it is not the case when the signals are binary (Proposition 6). Second, restricting to supermajority rules, when the signals are continuous, there exists a symmetric informative equilibrium with a lower k-rule only if there exists a symmetric informative equilibrium with a higher k-rule (Proposition 4), but it is just the opposite when the signals are binary (Proposition 6). Third, when the signals are binary, allowing asymmetric voting could make information transmission possible with a more conservative DM (Proposition 7), but it is not the case when the signals are continuous (Proposition 3). Finally, information transmission is always possible with a committee large enough in the continuous-signal model (Corollary 1), but not in the binary-signal model (Proposition 7).

The first three differences arise because of the assumption of discrete signals. We have deliberately chosen the binary-signal structure and a set of parameters that make the comparison easiest and sharpest. When we relax the assumptions on the prior probability p (which could be nonuniform), the signal structure (which could be asymmetric) or the committee size (which could be even), the results may change. However, the point remains that the binary-signal model generates results that are very different from those of the continuous-signal model. The last difference comes from Assumption 2, and has little to do with whether the signals are continuous or not. If Assumption 2 is relaxed, the last difference disappears, and we get a result similar to both Battaglini (2017) and Gradwohl and Feddersen (2018) in the continuous-signal model.

5.2 General discrete signals

In this section, we consider the case where the committee members receive discrete signals that can take more than two values and try to reconcile the seemingly conflicting findings from the continuous-signal model and the binary-signal model. Battaglini (2017) adopts a very similar information structure in a Poisson voting game.[16] However, he does not investigate how the existence condition of the symmetric informative equilibrium with k-rule changes with k.

Suppose each member i receives a private signal $s_i \in \{t_1, t_2, ...t_M\}$, where $M \geq 2$ is the number of possible signal realizations. We continue to assume $p = 1/2$ as in the last section, even though it is inessential for our discussion here. Let $q_F(t_m)$ and $q_G(t_m)$ be the probabilities that $s_i = t_m$ when the state is y and n, respectively. Define the hazard functions in the current information structure as $h_F(t_m) := q_F(t_m) / \sum_{l=m}^{M} q_F(t_l)$ and $h_G(t_m) := q_G(t_m) / \sum_{l=m}^{M} q_G(t_l)$. Correspondingly, MLRP and IHRP become:

Assumption 4 (MLRP) *$F(.)$ and $G(.)$ satisfy the strict monotone likelihood ratio property (MLRP), i.e., $q_F(t_m)/q_G(t_m)$ is strictly increasing in m.*

Assumption 5 (IHRP) *$F(.)$ and $G(.)$ satisfy the strict increasing hazard ratio property (IHRP), i.e., $h_F(t_m)/h_G(t_m)$ is strictly increasing in m.*

Note that when the signals are discrete, there is no loss to assume that MLRP holds. This is because we can simply reorder the signals and combine signals that have the same likelihood ratio.

We focus on symmetric voting. Thus, as in the previous sections, the equilibrium decision rule in an informative equilibrium must be a k-rule. Similar to $\boldsymbol{\alpha}(k, \mathcal{N})$ and $\boldsymbol{\alpha}_2(k, \mathcal{N})$, define $\boldsymbol{\alpha}_M(k, \mathcal{N})$ as the unique solution to

$$\frac{\alpha}{1-\alpha} = \frac{\Pr(|v| = k | \theta = y)}{\Pr(|v| = k | \theta = n)}.$$

Thus, $\boldsymbol{\alpha}_M(k, \mathcal{N})$ is the upper bound of α such that a symmetric informative equilibrium with k-rule exists. We will show that the pattern of how $\boldsymbol{\alpha}_M(k, \mathcal{N})$ changes with k combines features of $\boldsymbol{\alpha}(k, \mathcal{N})$ and $\boldsymbol{\alpha}_2(k, \mathcal{N})$.

[16] Different from the current paper, Battaglini (2017) models the voting problem using the Poisson game approach introduced by Myerson (1998a, 1998b, 2000). Therefore, his model is, strictly speaking, different from ours. However, the two approaches often produce similar results.

To see that, consider a symmetric informative equilibrium in which the decision rule is k-rule and the indifferent signal is t_m, that is, the committee members are indifferent between the two options after receiving signal t_m conditional on being pivotal. Then, the posterior likelihood ratio of the vote profile v with $|v| = k$ is given by

$$\frac{\Pr(|v| = k|\theta = y)}{\Pr(|v| = k|\theta = n)} = \underbrace{\frac{q_G(t_m)}{q_F(t_m)}}_{\text{anti-signal } t_m^-} \times \underbrace{\frac{q_F(t_m)\,\rho_m(k,\mathcal{N}) + \sum_{l=m+1}^{M} q_F(t_l)}{q_G(t_m)\,\rho_m(k,\mathcal{N}) + \sum_{l=m+1}^{M} q_G(t_l)}}_{\text{a yay vote}}, \tag{10}$$

where $\rho_m(k,\mathcal{N})$ is the probability that a committee member votes for alternative Y after receiving the indifferent signal t_m. As indicated in (10), observing a vote profile v with $|v| = k$ is informationally equivalent to observing the anti-signal t_m^- of the indifferent signal t_m and a yay vote, similar to (9).

As k increases, intuitively the committee members vote for alternative Y more often in equilibrium. The increase in the probability of a yay vote could be associated with an increase in $\rho_m(k,\mathcal{N})$ with the same indifferent signal or a lower indifferent signal.[17] These two changes potentially have opposing effects on the likelihood ratio when $|v| = k$, $\Pr(|v| = k|\theta = y) / \Pr(|v| = k|\theta = n)$. When the indifferent signal t_m is unchanged as k increases, then $\rho_m(k,\mathcal{N})$ increases. By MLRP, the likelihood ratio of a yay vote in (10) decreases (Lemma 7 in the Appendix), so the value of $\boldsymbol{\alpha}_M(k,\mathcal{N})$ decreases as in the binary-signal case with supermajority rules. If the indifferent signal t_m decreases, the likelihood ratio of the anti-signal t_m^- in (10) increases according to MLRP. Although the likelihood ratio of a yay vote also decreases when t_m decreases, the overall effect on $\boldsymbol{\alpha}_M(k,\mathcal{N})$ is not determined by IHRP. When the increase in the likelihood ratio of the anti-signal t_m^- dominates, $\boldsymbol{\alpha}_M(k,\mathcal{N})$ increases.

Given the two effects discussed above, the relationship between $\boldsymbol{\alpha}_M(k,\mathcal{N})$ and k may not

[17] For illustrative purposes, we focus on situations where the committee members are indifferent between alternative Y and status quo N after receiving some signal. When the committee members are never indifferent, (10) does not apply and the behavior of $\boldsymbol{\alpha}_M(k,\mathcal{N})$ is less regular. However, suppose the committee members strictly prefer to vote for alternative Y after receiving signal t_m but strictly prefer to vote for status quo N after receiving signal t_{m-1}. Then, the posterior likelihood ratio is bounded below by the inverse of the hazard ratio, i.e.,

$$\frac{\Pr(|v| = k|\theta = y)}{\Pr(|v| = k|\theta = n)} \geq \frac{q_G(t_m)}{q_F(t_m)} \frac{\sum_{l=m}^{M} q_F(t_l)}{\sum_{l=m}^{M} q_G(t_l)} = \frac{h_G(t_m)}{h_F(t_m)}.$$

By IHRP, $h_G(t_m)/h_F(t_m)$ is strictly decreasing in m. This suggests that, as k increases and the cutoff signal t_m decreases, this lower bound rises and $\boldsymbol{\alpha}_M(k,\mathcal{N})$ could exhibit an upward trend as in the continuous-signal case, even when the committee members are not indifferent at any signal (see Figure 3).

exhibit a clear monotone pattern as $\boldsymbol{\alpha}(k, \mathcal{N})$. Note that in the binary-signal case, for all supermajority rules, only the effect of the increasing $\rho_m(k, \mathcal{N})$ is present, so $\boldsymbol{\alpha}_2(k, \mathcal{N})$ is decreasing in k for all $k >(\mathcal{N}+1)/2$, while in the continuous-signal model, the effect of the decreasing indifferent signal dominates, so $\boldsymbol{\alpha}(k, \mathcal{N})$ is increasing in k. With both effects present in the general discrete-signal model, the pattern of $\boldsymbol{\alpha}_M(k, \mathcal{N})$ has the features of both $\boldsymbol{\alpha}(k, \mathcal{N})$ and $\boldsymbol{\alpha}_2(k, \mathcal{N})$. When the increase in k does not change the indifferent signal t_m, $\boldsymbol{\alpha}_M(k, \mathcal{N})$ decreases with k as $\boldsymbol{\alpha}_2(k, \mathcal{N})$ under supermajority rules. When the increase in k induces a decrease in the indifferent signal, $\boldsymbol{\alpha}_M(k, \mathcal{N})$ may decrease or increase with k. If the effect of the decreasing indifferent signal t_m dominates, then $\boldsymbol{\alpha}_M(k, \mathcal{N})$ increases with k as $\boldsymbol{\alpha}(k, \mathcal{N})$.

For a concrete illustration of the above two effects, we show in Figure 3 how $\boldsymbol{\alpha}_M(k, \mathcal{N})$ changes with k using an example with $\mathcal{N}=21$ and $M=4$.

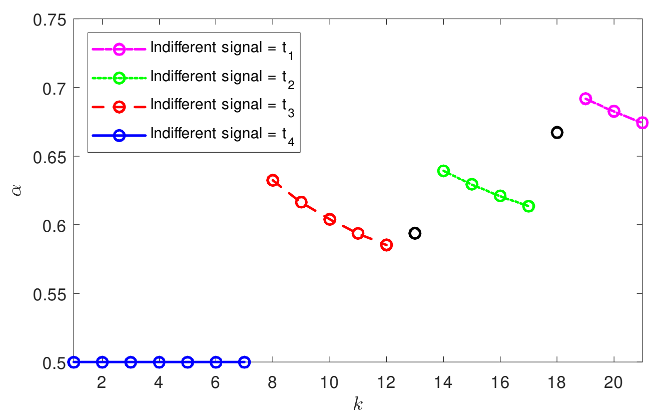

Figure 3: The function $\boldsymbol{\alpha}_M(k, \mathcal{N})$.

Parameters: $p=1/2$, $\mathcal{N}=21$, $q_F=(1/8, 3/16, 1/4, 7/16)$, $q_F=(1/4, 1/4, 1/4, 1/4)$.

When $k \leq 7$, the committee members vote for each option with a positive probability at signal t_4. Since t_4 is the highest possible signal, we have

$$\frac{\Pr(|v|=k|\theta=y)}{\Pr(|v|=k|\theta=n)} = \frac{q_G(t_4)}{q_F(t_4)} \frac{q_F(t_4)\,\rho_4(k, \mathcal{N})}{q_G(t_4)\,\rho_4(k, \mathcal{N})} = 1.$$

This means that the committee cannot persuade any DM with $\alpha > 1/2$. When $8 \leq k \leq 12$, the committee members mix at signal t_3, and $\boldsymbol{\alpha}_M(k, 21)$ first jumps above $1/2$ and then decreases with k. The same is true for $14 \leq k \leq 17$ and $19 \leq k \leq 21$, where the committee members mix at signals t_2 and t_1, respectively. Within these ranges, the committee members are mixing at the same signal. The effect of the decreasing $\rho_m(k, \mathcal{N})$ is the only effect present, and $\boldsymbol{\alpha}_m(k, \mathcal{N})$ decreases with k. However, if we compare $k = 12$, $k = 17$, and $k = 21$, we have $\boldsymbol{\alpha}_M(12, 21) < \boldsymbol{\alpha}_M(17, 21) < \boldsymbol{\alpha}_M(21, 21)$. When the committee members are mixing at different signals, both effects are present. In this comparison, the effect of the decreasing cutoff signal dominates, and $\boldsymbol{\alpha}_M(k, \mathcal{N})$ increases with k.

It is clear that the function $\boldsymbol{\alpha}_M(k, \mathcal{N})$ exhibits patterns that are both similar to and different from the two extreme cases, $\boldsymbol{\alpha}(k, \mathcal{N})$ and $\boldsymbol{\alpha}_2(k, \mathcal{N})$. Thus, the general discrete-signal model forms a bridge between the continuous-signal model and the binary-signal model. For the analysis of symmetric voting equilibria, what is important is not whether the signal space is continuous, but whether the signals are "coarse" or "fine". When the signals are fine enough, the results will be similar to the continuous-signal case, and $\boldsymbol{\alpha}_M(k, \mathcal{N})$ is mostly increasing in k. When the signals are coarse enough, the results will be similar to the binary-signal case. If we consider a sequence of signal structures converging to a continuous signal structure that satisfies Assumptions 1–3, $\boldsymbol{\alpha}_M(k, \mathcal{N})$ would be increasing in k in the limit.

In general, since $\boldsymbol{\alpha}_M(k, \mathcal{N})$ is not monotone in k, we are unable to obtain a tight upper bound on α that allows information transmission. However, similar to Lemma 1 in Battaglini (2017), we can obtain a loose upper bound by bounding the two terms in (10) individually: choosing t_1 for the anti-signal and t_M for the yay vote. Define $\overline{\alpha}_M$ as the unique solution to

$$\frac{\alpha}{1-\alpha} = \frac{q_G(t_1)}{q_F(t_1)} \frac{q_F(t_M)}{q_G(t_M)},$$

If $\alpha > \overline{\alpha}_M$, there does not exist any informative equilibrium. In the example presented in Figure 3, $\overline{\alpha}_M = 7/9 \approx 0.778$.

6 Conclusion

We consider a voting model in which members of an advisory committee receive private continuous signals and vote on a policy change before a DM makes the final decision. Our approach differs from previous works in that the voting rule is endogenous in our model. We show that information transmission between the committee and the decision maker is possible if and only if there exists an informative equilibrium with the unanimity rule. Our results offer a new perspective on an old debate between Feddersen and Pesendorfer (1998) and Coughlan (2000). In our setting, the committee must first be able to persuade the DM before information could be put into use. We suggest that a more stringent majority requirement could be adopted to make information transmission possible, even though it may result in suboptimal information aggregation.

With respect to modeling choice, our comparison between the binary-signal model and continuous-signal model shows that the binary-signal simplification is not innocuous. When the signals are coarse, it significantly alters most of the results. Modelers, therefore, should be heedful of such a possibility when applying voting models. Discrete-signal models should be used when it better approximates the reality, but not as an approximation of the continuous-signal model, especially in an advisory committee.

Finally, we have made several assumptions on the environment: 1) the committee is homogenous, 2) the DM observes each vote, 3) there is no pre-voting communication, 4) information is exogenous. Relaxing any of these assumptions could lead to a better understanding of how advisory committees work.

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Appendix

The following lemma summarizes some basic properties of distributions that satisfy MLRP. These properties will be useful later in the proofs.

Lemma 4 *Suppose $F(.)$ and $G(.)$ satisfy MLRP. Then,*

1. *For all $s \in (a, b)$,*
$$\frac{f(s)}{1 - F(s)} < \frac{g(s)}{1 - G(s)} \text{ and } \frac{f(s)}{F(s)} > \frac{g(s)}{G(s)};$$

2. *For all $s \in (a, b)$,*
$$F(s) < G(s);$$

3. *$\frac{1-F(s)}{1-G(s)}$ and $\frac{F(s)}{G(s)}$ are strictly increasing in s;*

Proof of Lemma 4. Part 1 follows from the proofs that likelihood ratio dominance implies hazard rate dominance and reverse hazard rate dominance (See Shaked and Shanthikumar (2007)). It is enough to note that the proofs apply when the weak inequalities are replaced by strict inequalities.

Part 2 follows from the proof that likelihood ratio dominance implies first-order stochastic dominance. Again, we note that the proof goes through when the weak inequality are replaced by strict inequality.

Part 3:
$$\frac{\partial}{\partial s}\left(\frac{1 - F(s)}{1 - G(s)}\right) = \frac{1 - F(s)}{1 - G(s)}\left(\frac{g(s)}{1 - G(s)} - \frac{f(s)}{1 - F(s)}\right) > 0,$$

where the inequality follows from Part 1.

$$\frac{\partial}{\partial s}\left(\frac{F(s)}{G(s)}\right) = \frac{F(s)}{G(s)}\left(\frac{f(s)}{F(s)} - \frac{g(s)}{G(s)}\right) > 0,$$

where the inequality follows from Part 1. ■

The following lemma shows that two normal distributions satisfying MLRP also satisfy IHRP.

Lemma 5 (Normal distribution) *Suppose $F(.)$ and $G(.)$ are two normal distributions satisfying MLRP. Then,* *1. $F(.)$ and $G(.)$ have the same variance, that is, $f(s) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(s-\mu_F)^2}{2\sigma^2}}$, and $g(s) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(s-\mu_G)^2}{2\sigma^2}}$ for some $\mu_F > \mu_G$ and $\sigma > 0$;*

2. $F(.)$ and $G(.)$ satisfy IHRP, that is, $h_F(s)/h_G(s)$ is strictly increasing in s.

Proof of Lemma 5. Suppose $g(.) = \frac{1}{\sigma_G\sqrt{2\pi}}e^{-\frac{(s-\mu_G)^2}{2\sigma_G^2}}$ and $f(.) = \frac{1}{\sigma_F\sqrt{2\pi}}e^{-\frac{(s-\mu_F)^2}{2\sigma_F^2}}$.

1. $f(s)/g(s) = \frac{\sigma_G}{\sigma_F}\exp\left(\frac{(s-\mu_G)^2}{2\sigma_G^2} - \frac{(s-\mu_F)^2}{2\sigma_F^2}\right)$. Consider $\ln(f(s)/g(s)) = \ln f(s) - \ln g(s)$.

$$\frac{\partial}{\partial s}(\ln f(s) - \ln g(s)) = s\left(\frac{1}{\sigma_G^2} - \frac{1}{\sigma_F^2}\right) + \left(\frac{\mu_F}{\sigma_F^2} - \frac{\mu_G}{\sigma_G^2}\right).$$

By MLRP, $\frac{\partial}{\partial s}(\ln f(s) - \ln g(s)) > 0$ for all s. However, if $\sigma_G^2 < \sigma_F^2$, the above expression goes to negative infinity when s goes to negative infinity. If $\sigma_G^2 > \sigma_F^2$, it goes to negative infinity when s goes to positive infinity. Hence, we must have $\sigma_G^2 = \sigma_F^2 = \sigma^2$. Thus,

$$\frac{\partial}{\partial s}(\ln f(s) - \ln g(s)) = \frac{1}{\sigma^2}(\mu_F - \mu_G).$$

MLRP then implies that $\mu_F > \mu_G$.

2. Taking derivative of $h_F(.)/h_G(.)$, we have

$$\begin{aligned}
\frac{\partial}{\partial s}\left(\frac{h_F(s)}{h_G(s)}\right) &= \frac{\partial}{\partial s}\left(\frac{f(s)(1-G(s))}{g(s)(1-F(s))}\right) \\
&= \frac{1}{\sigma^2}(\mu_F - \mu_G)\frac{f(s)}{g(s)}\frac{1-G(s)}{1-F(s)} + \frac{f(s)}{g(s)}\frac{(1-G(s))f(s) - g(s)(1-F(s))}{(1-F(s))^2} \\
&= \frac{f(s)(1-G(s))}{g(s)(1-F(s))}\left(\frac{f(s)}{1-F(s)} - \frac{g(s)}{1-G(s)} + \frac{1}{\sigma^2}(\mu_F - \mu_G)\right) \\
&= \frac{h_F(s)}{h_G(s)}\left(h_F(s) - h_G(s) + \frac{1}{\sigma^2}(\mu_F - \mu_G)\right) \\
&= \frac{h_F(s)}{h_G(s)}\left(h_F(s) - h_F(s + (\mu_F - \mu_G)) + \frac{1}{\sigma^2}(\mu_F - \mu_G)\right),
\end{aligned}$$

where the last equality follows from the observation that $h_G(s) = h_F(s + (\mu_F - \mu_G))$. Thus, $\frac{\partial}{\partial s}\left(\frac{h_F(s)}{h_G(s)}\right)$ has the same sign as $h_F(s) - h_F(s + (\mu_F - \mu_G)) + \frac{1}{\sigma^2}(\mu_F - \mu_G)$.

By the mean value theorem, there exists an $s' \in (s, s + (\mu_F - \mu_G))$ such that

$$h_F(s) - h_F(s + (\mu_F - \mu_G)) = -h'_F(s')(\mu_F - \mu_G).$$

Note that $h_{N(\mu,\sigma^2)}(s) = \frac{1}{\sigma} h_{N(0,1)}\left(\frac{s-\mu}{\sigma}\right)$. Thus, $h'_{N(\mu,\sigma^2)}(s) = \frac{1}{\sigma^2} h'_{N(0,1)}\left(\frac{s-\mu}{\sigma}\right)$. Sampford (1953) shows that $0 < h'_{N(0,1)}(.) < 1$. Hence, $0 < h'_G(s') < \frac{1}{\sigma^2}$. Therefore,

$$h'_F(s')(\mu_F - \mu_G) < \frac{1}{\sigma^2}(\mu_F - \mu_G),$$

which implies that $\frac{\partial}{\partial s}\left(\frac{h_F(s)}{h_G(s)}\right) > 0$.

■

Before proving Proposition 1, we prove the following lemma first.

Lemma 6 *Suppose in equilibrium member i uses a nonpartisan strategy and is pivotal with positive probability. Then, member i uses a cutoff strategy.*

Proof of Lemma 6. Suppose in equilibrium member i uses a nonpartisan strategy and is pivotal with positive probability. Denote the set of vote profiles of the other committee members, given which member i is pivotal, by $\boldsymbol{piv}_i$. Since $\Pr(\boldsymbol{piv}_i) > 0$, there exists $v_{-i} \in \boldsymbol{piv}_i$ such that $\Pr(v_{-i}) > 0$. Since the committee member i uses a nonpartisan strategy in equilibrium, for all $v_{-i} \in \boldsymbol{piv}_i$, $\Pr(v_{-i}) > 0$ implies that $\Pr(v = (Y, v_{-i})) > 0$ and $\Pr(v = (N, v_{-i})) > 0$. This means that the DM can update her belief using Bayes' rule after seeing the vote profiles (Y, v_{-i}) or (N, v_{-i}). Denote the likelihood ratio of a yay vote from member i by $L_i^+ := \frac{\Pr(v_i = Y|\theta = y)}{\Pr(v_i = Y|\theta = n)}$. We must have either $L_i^+ > 1$ or $L_i^+ < 1$. Otherwise, member i's vote is uninformative. Since the DM always chooses Y in the case of indifference by assumption, this means that for all $v_{-i} \in \boldsymbol{piv}_i$ such that $\Pr(v_{-i}) > 0$, the DM takes the same action after seeing (Y, v_{-i}) or (N, v_{-i}). This contradicts the assumption that member i is pivotal with positive probability.

If $L_i^+ > 1$, casting a yay vote can only change the final outcome from N to Y. Given the others' voting strategies m_{-i} and the DM's decision rule d, ignoring non-pivotal events, member

i with signal s_i voting for alternative Y gets

$$\frac{1}{2}\frac{pf(s_i)}{pf(s_i)+(1-p)g(s_i)}\Pr(\boldsymbol{piv}_i|\theta=y)-\frac{1}{2}\frac{(1-p)g(s_i)}{pf(s_i)+(1-p)g(s_i)}\Pr(\boldsymbol{piv}_i|\theta=n).$$

If he votes for status quo N, he gets 0. When member i with signal s_i is indifferent between voting for alternative Y and status quo N, we have

$$\frac{pf(s_i)}{(1-p)g(s_i)}=\frac{\Pr(\boldsymbol{piv}_i|\theta=n)}{\Pr(\boldsymbol{piv}_i|\theta=y)}, \tag{11}$$

where the left-hand side is strictly increasing in s_i by MLRP and the right-hand side is strictly positive and independent of s_i. Therefore, by Assumption 2, a solution to (11) exists and is unique. Member i must use an increasing cutoff strategy in equilibrium. Similarly, when $L_i^+ < 1$, member i must use a *decreasing* cutoff strategy in equilibrium. ■

Proof of Proposition 1. Consider an arbitrary equilibrium (m, d, μ) of our model. In equilibrium, the committee members can be classified into three types. Type I members use partisan strategies. Type II members use nonpartisan strategies, but are never pivotal. Type III members use nonpartisan strategies and are pivotal with positive probability. We construct the desired equilibrium $(\hat{m}, \hat{d}, \hat{\mu})$ by going through the committee members one by one. Let $(m^0, d^0, \mu^0) = (m, d, \mu)$ and re-order the committee members according to their types so that type I members appear first, type II second, and type III last. For each step i, we modify committee member i's strategy, the DM's decision rule and belief and show that $\left(m^i, d^i, \mu^i\right)$ remains an equilibrium and it is outcome-equivalent to equilibrium $\left(m^{i-1}, d^{i-1}, \mu^{i-1}\right)$. The final product $\left(m^{\mathcal{N}}, d^{\mathcal{N}}, \mu^{\mathcal{N}}\right)$ will satisfy the desired properties. Consider step i. There are three cases.

1. Suppose in equilibrium $\left(m^{i-1}, d^{i-1}, \mu^{i-1}\right)$ member i is a type I member. We consider only the case where member i always votes for Y. The other case is similar. We modify the decision rule such that, for all $v_{-i} \in \{Y, N\}^{\mathcal{N}-1}$, $d^i(N, v_{-i}) = d^i(Y, v_{-i}) = d^{i-1}(Y, v_{-i})$, and leave the strategy profile unchanged, that is, $m^i = m^{i-1}$. After the modification, d^i is constant in m_i^i. Since d^{i-1} is an equilibrium decision rule given m^{i-1}, for all $v_{-i} \in \{Y, N\}^{\mathcal{N}-1}$, $d^i(Y, v_{-i}) = d^{i-1}(Y, v_{-i})$ is optimal. For all $v_{-i} \in \{Y, N\}^{\mathcal{N}-1}$, (N, v_{-i}) does not

occur under m^i, so $d^i(N, v_{-i}) = d^i(Y, v_{-i})$ is also optimal, if we simply set $\mu^i(N, v_{-i}) = \mu^i(Y, v_{-i}) = \mu^{i-1}(Y, v_{-i})$. Under the new decision rule d^i, member i is never pivotal. Thus, the partisan strategy is optimal for member i. For member $j \neq i$, d^i assigns the same outcome to any vote profile that occurs with positive probability as d^{i-1}. Thus, m^i_j is optimal for member j. Hence, (m^i, d^i, μ^i) remains an equilibrium. Finally, since d^i assigns the same outcome to any vote profile that occurs with positive probability as d^{i-1}, equilibrium (m^i, d^i, μ^i) is clearly outcome-equivalent to equilibrium $(m^{i-1}, d^{i-1}, \mu^{i-1})$.

2. Suppose in equilibrium $(m^{i-1}, d^{i-1}, \mu^{i-1})$ member i is a type II member. We modify the strategy profile such that $m^i_i(.) = 1$, and the decision rule such that, for all $v_{-i} \in \{Y, N\}^{\mathcal{N}-1}$, $d^i(Y, v_{-i}) = d^i(N, v_{-i}) = d^{i-1}(Y, v_{-i})$. After the modification, member i uses a partisan strategy and the decision rule d^i is constant in m^i_i. For all $v_{-i} \in \{Y, N\}^{\mathcal{N}-1}$ such that $\Pr(v_{-i}) > 0$ given m^{i-1}, since member i is never pivotal, we must have $d^{i-1}(Y, v_{-i}) = d^{i-1}(N, v_{-i})$. Thus, d^i assigns the same outcomes to all these profiles v_{-i} as d^{i-1}. Therefore, d^i is optimal. For all $v_{-i} \in \{Y, N\}^{\mathcal{N}-1}$ such that $\Pr(v_{-i}) = 0$, d^i is also optimal, if we assign the DM the proper off-the-equilibrium-path belief. Under the new decision rule d^i, member i is never pivotal. Thus, the partisan strategy is optimal for him. For member $j \neq i$, if m^{i-1}_j is partisan, then m^i_j is still optimal because, by Part 1 and Part 2 of this proof, the decision rule d^i is constant in m^i_j; if m^{i-1}_j is nonpartisan, m^i_j is still optimal because, under (m^{i-1}, d^{i-1}), the probability that Y is chosen given member j's vote and any signal profile for members $-j$ is the same as before. This also means that equilibrium (m^i, d^i, μ^i) is outcome-equivalent to equilibrium $(m^{i-1}, d^{i-1}, \mu^{i-1})$.

3. Suppose in equilibrium $(m^{i-1}, d^{i-1}, \mu^{i-1})$ member i is a type III member. By Lemma 6, m^{i-1}_i is a cutoff strategy, which can be either increasing or decreasing. If m^{i-1}_i is increasing, we leave the strategy profile and the decision rule unchanged, i.e., $(m^i, d^i) = (m^{i-1}, d^{i-1})$. If m^{i-1}_i is decreasing, we modify the strategy profile such that $m^i_i(.) = 1 - m^{i-1}_i(.)$, and the decision rule such that for all $v_{-i} \in \{Y, N\}^{\mathcal{N}-1}$, $d^i(Y, v_{-i}) = d^{i-1}(N, v_{-i})$ and $d^i(N, v_{-i}) = d^{i-1}(Y, v_{-i})$. This in effect relabels member i's a yay vote as a nay vote and a nay vote as a yay vote. We leave the DM's belief unchanged, that is, $\mu^i = \mu^{i-1}$. Obviously, d^i is

optimal for the DM and m_j^i is optimal for member j. The profile $\left(m^i, d^i, \mu^i\right)$ remains an equilibrium and it is outcome-equivalent to equilibrium $\left(m^{i-1}, d^{i-1}, \mu^{i-1}\right)$.

It remains to show that $\hat{d} = d^{\mathcal{N}}$ is a weighted voting rule. We would like to find a pair of w and Q that represents $\hat{d}$. For player i who uses a cutoff strategy s_i^*, denote the likelihood ratio of a yay vote from player i by L_i^+, i.e., $L_i^+ = \frac{1-F\left(s_i^*\right)}{1-G\left(s_i^*\right)}$, and the likelihood ratio of a nay vote from player i by L_i^-, i.e., $L_i^- = \frac{F\left(s_i^*\right)}{G\left(s_i^*\right)}$. Let the weight of a yay vote from member i be $\ln L_i^+ - \ln L_i^-$, i.e.,

$$w_i := \ln L_i^+ - \ln L_i^-.$$

For player i who uses a partisan strategy, set $w_i := 0$.

Denote the total weight of vote profile v by $W(v) := \sum_{i=1}^{\mathcal{N}} w_i \mathbf{1}_{\{v_i = Y\}}$. Now we show that for all vote profiles v and v' that occur with positive probability in equilibrium, the total weight of v is larger than the total weight of v' if and only if $\Pr(\theta = y|v) \geq \Pr(\theta = y|v')$.

Let C be the set of committee members who use cutoff strategies. Consider a vote profile v, the posterior likelihood ratio of the state conditional on v is

$$\begin{aligned}
&\prod_{i \in C \cap \{i : v_i = Y\}} \frac{1 - F\left(s_i^*\right)}{1 - G\left(s_i^*\right)} \prod_{i \in C \cap \{i : v_i = N\}} \frac{F\left(s_i^*\right)}{G\left(s_i^*\right)} \\
&= \exp\left(\sum_{i \in C \cap \{i : v_i = Y\}} \ln L_i^+ + \sum_{i \in C \cap \{i : v_i = N\}} \ln L_i^- \right) \\
&= \exp\left(\sum_{i \in C \cap \{i : v_i = Y\}} \ln L_i^+ - \sum_{i \in C \cap \{i : v_i = Y\}} \ln L_i^- + \sum_{i \in C} \ln L_i^- \right) \\
&= \exp\left(W(v) + c\right),
\end{aligned}$$

where c is a constant. This implies that the total weight is strictly increasing in $\Pr(\theta = y|v)$.

Now we find the quota Q. Given the equilibrium decision rule $\hat{d}$ and voting strategy $\hat{m}$, consider the corresponding V^+. There exists a $\underline{v} \in V^+$ that such that $\Pr(\underline{v}) > 0$ and for all $v \in V^+$ that occur with positive probability in equilibrium, we have

$$\Pr(\theta = y|\underline{v}) \leq \Pr(\theta = y|v).$$

Set Q to be the total weight of vote profile $\underline{v}$, that is,

$$Q := W\left(\underline{v}\right).$$

This proves that $\hat{d}$ is a weighted voting rule. ■

Proof of Lemma 1. For all $s \in (a, b)$, we have

$$\begin{aligned}\frac{\left(1-F\left(s\right)\right)^{k-1}F\left(s\right)^{\mathcal{N}-k}}{\left(1-G\left(s\right)\right)^{k-1}G\left(s\right)^{\mathcal{N}-k}} &= \frac{\left(1-F\left(s\right)\right)^{k}F\left(s\right)^{\mathcal{N}-(k+1)}}{\left(1-G\left(s\right)\right)^{k}G\left(s\right)^{\mathcal{N}-(k+1)}}\frac{\left(1-G\left(s\right)\right)F\left(s\right)}{\left(1-F\left(s\right)\right)G\left(s\right)}\\ &< \frac{\left(1-F\left(s\right)\right)^{k}F\left(s\right)^{\mathcal{N}-(k+1)}}{\left(1-G\left(s\right)\right)^{k}G\left(s\right)^{\mathcal{N}-(k+1)}},\end{aligned}$$

where the inequality follows from MLRP and Part 2 of Lemma 4. Thus, the left-hand side of (1) is strictly increasing in k. Moreover, by Part 3 of Lemma 4, the left-hand side of (1) is strictly increasing in s. Thus, to satisfy (1), it must be the case that $s\left(k+1,\mathcal{N}\right) < s\left(k,\mathcal{N}\right)$. ■

Proof of Lemma 2. In the main text. ■

Proof of Proposition 2. The proposition follows immediately from IHRP and Lemmas 1 and 2. ■

Proof of Corollary 1. In the main text. ■

Proof of Example 1. Suppose both $G\left(.\right)$ and $F\left(.\right)$ are normal distributions satisfying MLRP. Then, by Part 1 of Lemma 5, $G = N\left(\mu_G, \sigma^2\right)$ and $F = N\left(\mu_F, \sigma^2\right)$, where $\mu_F > \mu_G$. For a single expert, $s\left(1,1\right)$ is defined by

$$\frac{p}{1-p}\frac{f\left(s\left(1,1\right)\right)}{g\left(s\left(1,1\right)\right)} = 1.$$

Since $f\left(\frac{\mu_G+\mu_F}{2}\right) = g\left(\frac{\mu_G+\mu_F}{2}\right)$, $p > \frac{1}{2}$ implies that $s\left(1,1\right) < \frac{\mu_g+\mu_f}{2}$. Note that, when $s = \frac{\mu_G+\mu_F}{2}$, $\frac{(1-F(s))F(s)}{(1-G(s))G(s)} = 1$. Thus, by Part 3 of Lemma 4, $s\left(1,1\right) < \frac{\mu_g+\mu_f}{2}$ implies that

$$\left(\frac{\left(1-F\left(s\left(1,1\right)\right)\right)F\left(s\left(1,1\right)\right)}{\left(1-G\left(s\left(1,1\right)\right)\right)G\left(s\left(1,1\right)\right)}\right)^{\kappa} < 1 = \frac{\left(1-p\right)g\left(s\left(1,1\right)\right)}{pf\left(s\left(1,1\right)\right)}.$$

At $s^* = s(\kappa+1, 2\kappa+1)$,

$$\left(\frac{(1-F(s^*))\,F(s^*)}{(1-G(s^*))\,G(s^*)}\right)^{\kappa} = \frac{(1-p)\,g(s^*)}{pf(s^*)}. \tag{12}$$

By MLRP, the right-hand side of (12) is strictly decreasing in s. By Part 3 of Lemma 4, the left-hand side of (12) is strictly increasing in s. Hence, $s(1,1) < s(\kappa+1, 2\kappa+1)$. By the definition of $\boldsymbol{\alpha}(k,\mathcal{N})$ and IHRP, $\boldsymbol{\alpha}(1,1) > \boldsymbol{\alpha}(\kappa+1, 2\kappa+1)$ for all κ. ■

Proof of Proposition 3. To prove this proposition, we only need to show that the existence of an asymmetric informative equilibrium implies the existence of an informative symmetric equilibrium with the unanimity rule. Consider an asymmetric informative equilibrium. By Proposition 1, it is without loss to assume that every committee member uses either an increasing cutoff strategy or a partisan strategy. Suppose further that no committee member uses a partisan strategy. Since all possible vote profiles appear with positive probability in equilibrium, the equilibrium in this case is characterized by the pair (s^*, d) alone, where $s^* \in (a,b)^{\mathcal{N}}$ is a cutoff profile and d is a weighted voting rule.

In the asymmetric equilibrium (s^*, d), there must be a committee member i such that $s_i^* > s(\mathcal{N},\mathcal{N})$. Otherwise, for all $j \in \{1, 2, .., \mathcal{N}\}$, $s_j^* \leq s(\mathcal{N},\mathcal{N})$. By Parts 2 and 3 of Lemma 4, this means that for all vote profile $v \in \{Y, N\}^{\mathcal{N}}$, we have

$$\frac{\Pr(v|\theta = y)}{\Pr(v|\theta = n)} \leq \left(\frac{1-F(s(\mathcal{N},\mathcal{N}))}{1-G(s(\mathcal{N},\mathcal{N}))}\right)^{\mathcal{N}}.$$

Thus, if the asymmetric equilibrium (s^*, d) is informative, then an informative symmetric equilibrium with the unanimity rule exists.

Suppose $s_i^* > s(\mathcal{N},\mathcal{N})$. In equilibrium, since for every $v_{-i} \in \boldsymbol{piv}_i$, the profile (Y, v_{-i}) induces the DM to choose alternative Y, we have, for all $v_{-i} \in \boldsymbol{piv}_i$,

$$\frac{\alpha}{1-\alpha} \leq \frac{p}{1-p}\frac{\Pr(v_{-i}|\theta = y)\,(1-F(s_i^*))}{\Pr(v_{-i}|\theta = n)\,(1-G(s_i^*))}.$$

The optimality of the cutoff s_i^* implies that $\exists\ v_{-i} \in \boldsymbol{piv}_i$,

$$\frac{p}{1-p}\frac{\Pr\left(v_{-i}|\theta=y\right)}{\Pr\left(v_{-i}|\theta=n\right)} \leq \frac{g(s_i^*)}{f(s_i^*)}.$$

Therefore,

$$\frac{\alpha}{1-\alpha} \leq \frac{g(s_i^*)(1-F(s_i^*))}{f(s_i^*)(1-G(s_i^*))} < \frac{g\left(s\left(\mathcal{N},\mathcal{N}\right)\right)\left(1-F(s\left(\mathcal{N},\mathcal{N}\right))\right)}{f\left(s\left(\mathcal{N},\mathcal{N}\right)\right)\left(1-G\left(s\left(\mathcal{N},\mathcal{N}\right)\right)\right)},$$

where the last inequality follows from IHRP and the fact that $s_i^* > s\left(\mathcal{N},\mathcal{N}\right)$. This implies that an informative symmetric equilibrium with the unanimity rule exists.

Finally, suppose in equilibrium some committee members use partisan strategies. As noted in the main text, in this case the committee in effect becomes a smaller committee. By Corollary 1 and the first part of this proof, such an informative equilibrium exists only if there exists a symmetric informative equilibrium with the unanimity rule with the original committee. ■

Proof of Proposition 4. We first show that in an informative (symmetric or asymmetric) equilibrium in which the equilibrium decision rule is a k-rule, all committee members must use cutoff strategies. Since the equilibrium is informative, we have $\Pr\left(|v| \geq k\right) > 0$ and $\Pr\left(|v| < k\right) > 0$. Independence of committee members' signals then implies that all committee members are pivotal with positive probability. Given that the decision rule is k-rule and that the realized signals of the committee members can be arbitrarily precise, they cannot use partisan strategies in equilibrium. By Lemma 6, all committee members must use cutoff strategies.

Consider an asymmetric informative equilibrium (s^*, d) in which the equilibrium decision rule d is a k-rule. There must be a committee member i such that $s_i^* > s\left(k,\mathcal{N}\right)$. Otherwise, for all $j \in \{1,2,..,\mathcal{N}\}$, $s_j^* \leq s\left(k,\mathcal{N}\right)$. By Part 3 of Lemma 4, this means that for all vote profile v such that $|v| = k$,

$$\frac{\alpha}{1-\alpha} \leq \frac{\Pr\left(v|\theta=y\right)}{\Pr\left(v|\theta=n\right)} \leq \left(\frac{1-F\left(s\left(k,\mathcal{N}\right)\right)}{1-G\left(s\left(k,\mathcal{N}\right)\right)}\right)^k\left(\frac{F\left(s\left(k,\mathcal{N}\right)\right)}{G\left(s\left(k,\mathcal{N}\right)\right)}\right)^{\mathcal{N}-k}.$$

Thus, an informative symmetric equilibrium with k-rule exists.

Suppose $s_i^* > s\left(k,\mathcal{N}\right)$. Since for every $v_{-i} \in \{Y,N\}^{\mathcal{N}-1}$ such that $|v_{-i}| = k-1$, the profile (Y, v_{-i}) induces the DM to choose alternative Y in equilibrium, we have, for all $v_{-i} \in \{Y,N\}^{\mathcal{N}-1}$

such that $|v_{-i}| = k - 1$,

$$\frac{\alpha}{1-\alpha} \leq \frac{p}{1-p} \frac{\Pr\left(v_{-i}|\theta = y\right)\left(1 - F(s_i^*)\right)}{\Pr\left(v_{-i}|\theta = n\right)\left(1 - G(s_i^*)\right)}.$$

The optimality of the cutoff s_i^* implies that $\exists\ v_{-i} \in \{Y, N\}^{\mathcal{N}-1}$ such that $|v_{-i}| = k - 1$ and

$$\frac{p}{1-p} \frac{\Pr\left(v_{-i}|\theta = y\right)}{\Pr\left(v_{-i}|\theta = n\right)} \leq \frac{g(s_i^*)}{f(s_i^*)}.$$

Therefore,

$$\frac{\alpha}{1-\alpha} \leq \frac{g(s_i^*)(1 - F(s_i^*))}{f(s_i^*)(1 - G(s_i^*))} < \frac{g\left(s\left(k, \mathcal{N}\right)\right)\left(1 - F(s\left(k, \mathcal{N}\right))\right)}{f\left(s\left(k, \mathcal{N}\right)\right)\left(1 - G\left(s\left(k, \mathcal{N}\right)\right)\right)},$$

where the last inequality follows from IHRP and the fact that $s_i^* > s\left(k, \mathcal{N}\right)$. This implies that an informative symmetric equilibrium with k-rule exists. ■

Proof of Corollary 2. By Proposition 3, there exists an informative equilibrium with the unanimity rule if and only if $\alpha \leq \boldsymbol{\alpha}\left(\mathcal{N}, \mathcal{N}\right)$. We complete the proof by showing that $\boldsymbol{\alpha}\left(\mathcal{N}, \mathcal{N}\right)$ goes to 1 as $\mathcal{N}$ goes to infinity. Consider the optimality condition for the committee members in the informative equilibrium with the unanimity rule, we have

$$\frac{p}{1-p}\left(\frac{1 - F\left(s\left(\mathcal{N}, \mathcal{N}\right)\right)}{1 - G\left(s\left(\mathcal{N}, \mathcal{N}\right)\right)}\right)^{\mathcal{N}-1} = \frac{g\left(s\left(\mathcal{N}, \mathcal{N}\right)\right)}{f\left(s\left(\mathcal{N}, \mathcal{N}\right)\right)}. \tag{13}$$

Suppose $\lim_{\mathcal{N}\to\infty} s\left(\mathcal{N}, \mathcal{N}\right) = \underline{s} > a$. Then $\lim_{\mathcal{N}\to\infty} \frac{g(s(\mathcal{N},\mathcal{N}))}{f(s(\mathcal{N},\mathcal{N}))} = \frac{g(\underline{s})}{f(\underline{s})} < \infty$, and

$$\lim_{\mathcal{N}\to\infty}\left(\frac{1 - F\left(s\left(\mathcal{N}, \mathcal{N}\right)\right)}{1 - G\left(s\left(\mathcal{N}, \mathcal{N}\right)\right)}\right)^{\mathcal{N}-1} = \lim_{\mathcal{N}\to\infty}\left(\frac{1 - F\left(\underline{s}\right)}{1 - G\left(\underline{s}\right)}\right)^{\mathcal{N}-1} = \infty,$$

which violate (13). Therefore, $\lim_{\mathcal{N}\to\infty} s\left(\mathcal{N}, \mathcal{N}\right) = a$. Then, we have,

$$\lim_{\mathcal{N}\to\infty} \frac{h_G\left(s\left(\mathcal{N}, \mathcal{N}\right)\right)}{h_F\left(s\left(\mathcal{N}, \mathcal{N}\right)\right)} = \lim_{s\to a} \frac{h_G\left(s\right)}{h_F\left(s\right)} = \lim_{s\to a} \frac{g\left(s\right)}{f\left(s\right)} \frac{1 - F\left(s\right)}{1 - G\left(s\right)} = \lim_{s\to a} \frac{g\left(s\right)}{f\left(s\right)} = \infty,$$

where the last equality follows from Assumption 2. This implies $\boldsymbol{\alpha}\left(\mathcal{N}, \mathcal{N}\right) \to 1$ as $\mathcal{N} \to \infty$. ■

Proof of Proposition 5. See Duggan and Martinelli (2001), Theorem 4. ■

Proof of Lemma 3. In any symmetric informative equilibrium, the DM finds it optimal to follow a given k-rule if and only if (6) holds. There are three cases.

1. Suppose $k < \frac{\mathcal{N}+1}{2}$. If a symmetric informative equilibrium with k-rule exists, the voting strategy (ρ_N, ρ_Y) must satisfy $\rho_N = 0$ and $\rho_Y \in (0,1)$, which implies that member i must be indifferent between alternative Y and status quo N conditional on being pivotal after receiving a Y-signal, i.e.,

$$\frac{\Pr\left(\boldsymbol{piv}_i|\theta = y\right)}{\Pr\left(\boldsymbol{piv}_i|\theta = n\right)} = \frac{\Pr\left(|v_{-i}| = k-1|\theta = y\right)}{\Pr\left(|v_{-i}| = k-1|\theta = n\right)} = \frac{1-q}{q}.$$

Given the voting strategy, we have

$$\frac{\Pr\left(|v| = k-1|\theta = y\right)}{\Pr\left(|v| = k-1|\theta = n\right)} = \frac{1-q}{q}\frac{q\left(1-\rho_Y\right)+\left(1-q\right)}{\left(1-q\right)\left(1-\rho_Y\right)+q} < 1,$$

and

$$\frac{\Pr\left(|v| = k|\theta = y\right)}{\Pr\left(|v| = k|\theta = n\right)} = \frac{1-q}{q}\frac{q}{1-q} = 1 < \frac{\alpha}{1-\alpha},$$

as $\alpha > \frac{1}{2}$. Thus, (6) can never be satisfied. Therefore, there does not exist a symmetric informative equilibrium in which the equilibrium decision rule is a minority rule.

2. Suppose $k = \frac{\mathcal{N}+1}{2}$. If a symmetric informative equilibrium with k-rule exists, voting is truthful. We have

$$\frac{\Pr\left(\boldsymbol{piv}_i|\theta = y\right)}{\Pr\left(\boldsymbol{piv}_i|\theta = n\right)} = \frac{\Pr\left(|v_{-i}| = k-1|\theta = y\right)}{\Pr\left(|v_{-i}| = k-1|\theta = n\right)} = \frac{q^{\frac{\mathcal{N}-1}{2}}\left(1-q\right)^{\frac{\mathcal{N}-1}{2}}}{\left(1-q\right)^{\frac{\mathcal{N}-1}{2}}q^{\frac{\mathcal{N}-1}{2}}} = 1.$$

Given truthful voting, we have

$$\frac{\Pr\left(|v| = k-1|\theta = y\right)}{\Pr\left(|v| = k-1|\theta = n\right)} = \frac{q^{\frac{\mathcal{N}-1}{2}}\left(1-q\right)^{\frac{\mathcal{N}-1}{2}}}{\left(1-q\right)^{\frac{\mathcal{N}-1}{2}}q^{\frac{\mathcal{N}-1}{2}}}\frac{1-q}{q} = \frac{1-q}{q} < 1,$$

and

$$\frac{\Pr\left(|v| = k|\theta = y\right)}{\Pr\left(|v| = k|\theta = n\right)} = \frac{q^{\frac{\mathcal{N}-1}{2}}\left(1-q\right)^{\frac{\mathcal{N}-1}{2}}}{\left(1-q\right)^{\frac{\mathcal{N}-1}{2}}q^{\frac{\mathcal{N}-1}{2}}}\frac{q}{1-q} = \frac{q}{1-q}.$$

Therefore, (6) is equivalent to $\alpha \leq q = \boldsymbol{\alpha}_2\left(\frac{\mathcal{N}+1}{2}, \mathcal{N}\right)$.

3. Suppose $k > \frac{\mathcal{N}+1}{2}$. If a symmetric informative equilibrium with k-rule exists, the voting strategy (ρ_N, ρ_Y) must satisfy $\rho_N \in (0,1)$ and $\rho_Y = 1$, which implies that member i is indifferent between alternative Y and status quo N conditional on being pivotal after receiving an N-signal, i.e.,

$$\frac{\Pr(\boldsymbol{piv}_i|\theta = y)}{\Pr(\boldsymbol{piv}_i|\theta = n)} = \frac{\Pr(|v_{-i}| = k-1|\theta = y)}{\Pr(|v_{-i}| = k-1|\theta = n)} = \frac{q}{1-q}.$$

Given the voting strategy, we have

$$\frac{\Pr(|v| = k-1|\theta = y)}{\Pr(|v| = k-1|\theta = n)} = \frac{q}{1-q}\frac{1-q}{q} = 1,$$

and

$$\frac{\Pr(|v| = k|\theta = y)}{\Pr(|v| = k|\theta = n)} = \frac{q}{1-q}\frac{q+(1-q)\rho_N}{q\rho_N + (1-q)} = \frac{\boldsymbol{\alpha}_2(k,\mathcal{N})}{1-\boldsymbol{\alpha}_2(k,\mathcal{N})}.$$

Therefore, (6) is equivalent to $\alpha \leq \boldsymbol{\alpha}_2(k,\mathcal{N})$.

■

Proof of Proposition 6. Follows immediately from Lemma 3. ■

Proof of Proposition 7. Consider an informative equilibrium. In equilibrium, there must be at least one committee member who uses a nonpartisan strategy and is pivotal with positive probability. Let member i be that committee member. Moreover, it is without loss to assume that the equilibrium decision rule is increasing.

From pivotal consideration, we have

$$\frac{1-q}{q} \leq \frac{\Pr(\boldsymbol{piv}_i|\theta = y)}{\Pr(\boldsymbol{piv}_i|\theta = n)} \leq \frac{q}{1-q},$$

which implies

$$\min_{\substack{v_{-i}\in\boldsymbol{piv}_i \\ \Pr(v_{-i})>0}} \frac{\Pr(v_{-i}|\theta = y)}{\Pr(v_{-i}|\theta = n)} \leq \frac{q}{1-q}.$$

Therefore,

$$\min_{\substack{v\in V^+ \\ \Pr(v)>0}} \frac{\Pr(v|\theta=y)}{\Pr(v|\theta=n)} \leq \min_{\substack{v_{-i}\in \boldsymbol{piv}_i \\ \Pr(v_{-i})>0}} \frac{\Pr(v_{-i}|\theta=y)}{\Pr(v_{-i}|\theta=n)} \frac{q}{1-q} \leq \left(\frac{q}{1-q}\right)^2 .$$

Thus, we must have $\alpha \leq \overline{\alpha}$ in an informative equilibrium.

Next, we consider $\alpha = \overline{\alpha}$ and construct an informative equilibrium which involves the committee members voting asymmetrically. Consider the following asymmetric voting strategy profile and decision rule: 1) committee member 1 always votes for status quo N, and 2) committee member $i \in \{2,..,\mathcal{N}\}$ votes truthfully; the DM chooses alternative Y when there are at least $\frac{\mathcal{N}+1}{2}$ votes for alternative Y from committee members $\{2,..,\mathcal{N}\}$ and status quo N otherwise.

To check that this is an equilibrium, consider committee member 1. Since he is never pivotal, it is optimal for him to vote for status quo N.

Next, consider committee member $i \in \{2,..,\mathcal{N}\}$. Since

$$\frac{q}{1-q} \frac{\Pr\left(|v_{-i}| = \frac{\mathcal{N}-1}{2}|\theta=y\right)}{\Pr\left(|v_{-i}| = \frac{\mathcal{N}-1}{2}|\theta=n\right)} = \frac{q^{\frac{\mathcal{N}+1}{2}}(1-q)^{\frac{\mathcal{N}-3}{2}}}{(1-q)^{\frac{\mathcal{N}+1}{2}} q^{\frac{\mathcal{N}-3}{2}}} = \left(\frac{q}{1-q}\right)^2 > 1,$$

he strictly prefers voting for alternative Y after receiving a Y-signal. Moreover, he is indifferent between the two options after receiving an N-signal, since

$$\frac{1-q}{q} \frac{\Pr\left(|v_{-i}| = \frac{\mathcal{N}-1}{2}|\theta=y\right)}{\Pr\left(|v_{-i}| = \frac{\mathcal{N}-1}{2}|\theta=n\right)} = \frac{q^{\frac{\mathcal{N}-1}{2}}(1-q)^{\frac{\mathcal{N}-1}{2}}}{(1-q)^{\frac{\mathcal{N}-1}{2}} q^{\frac{\mathcal{N}-1}{2}}} = 1.$$

Finally, since

$$1 < \frac{\overline{\alpha}}{1-\overline{\alpha}} = \left(\frac{q}{1-q}\right)^2,$$

the DM also finds it optimal to follow the decision rule. ■

The following lemma implies that MLRP implies that the likelihood ratio of a yay vote decreases as k increases in equilibrium in the discrete-signal model.

Lemma 7 *Suppose the signals are discrete. If $F(.)$ and $G(.)$ satisfy MLRP, then, for $m \in \{1,2,..,M-1\}$, the function*

$$H(\rho) := \frac{q_F(t_m)\rho + \sum_{l=m+1}^{M} q_F(t_l)}{q_G(t_m)\rho + \sum_{l=m+1}^{M} q_G(t_l)}$$

is strictly decreasing.

Proof of Lemma 7. Differentiating $H(.)$, we get

$$H'(\rho) = \frac{q_F(t_m)\left(\sum_{l=m+1}^{M} q_G(t_l)\right) - q_G(t_m)\left(\sum_{l=m+1}^{M} q_F(t_l)\right)}{\left(q_G(t_m)\rho + \sum_{l=m+1}^{M} q_G(t_l)\right)^2} < 0.$$

To see why the inequality holds, note that by MLRP, for all $l \in \{m+1, .., M\}$,

$$\frac{q_F(t_m)}{q_G(t_m)} q_G(t_l) < q_F(t_l),$$

which means

$$\frac{q_F(t_m)}{q_G(t_m)}\left(\sum_{l=m+1}^{M} q_G(t_l)\right) < \sum_{l=m+1}^{M} q_F(t_l).$$

■