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# Currency Competition With Firms

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# Currency Competition with Firms*

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**Abstract**

This paper analyses the consequences for monetary policy if firms issue money which generates seignorage revenues and information on consumers. I present a benchmark economy with a unique monetary equilibrium in which firms form digital currency areas if information rents are large. The central bank loses its autonomy and is forced to implement deflationary monetary policy. I extend the benchmark to show that the central bank may regain policy autonomy when firms form currency consortia with decision powers and claims on seignorage concentrated in the hands of one firm.

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*"What would set Facebook's Libra apart, if it were to proceed, is the combination of an active-user network representing more than a third of the global population with the issuance of a private digital currency opaquely tied to a basket of sovereign currencies. [...]*

*A significant concern regarding Facebook's Libra project is the potential for a payment system to be adopted globally in a short time period and to establish itself as a potentially new unit of account."*

— **Lael Brainard**, Board of Governors of the Federal Reserve System (Dec 18, 2019)

*"I sincerely hope that the people of Europe will not be tempted to leave behind the safety and soundness of established payment solutions and channels in favour of the beguiling but treacherous promises of Facebook's siren call."*

— **Yves Mersch**, Executive Board of the ECB (Sep 2, 2019)

# 1 Introduction

The last decade has seen large-scale innovation in the realm of digital currencies. The most prominent example is Bitcoin, a private, decentralised digital currency for which transactions are verified using cryptographic technologies, and which has been issued in large amounts. Not yet in existence in advanced economies but already the subject of research and discussion are central bank digital currencies (CBDC), to be issued by monetary authorities complementing banknotes, coins and reserves. This paper discusses a third type of digital currency: private, centralised digital currencies issued and operationally managed by firms or groups of firms that produce consumption goods. In June 2020, a consortium of firms assembled by Facebook shook the policy world when they announced their digital currency project Libra, later renamed Diem and now folded under regulatory pressure. Evidently, central banks perceive such currencies as serious rivals in the future.

Unlike decentralised, private digital currencies, the centralised counterparts do not offer anonymity. The owner of the technology knows the identity of the consumer and verifies transactions centrally. The collected transaction data have great value in understanding consumer tastes, raising the profits of the seller. Therefore, introducing a currency brings at least one benefit: it generates information rents. Unlike CBDC, private, centralised money generates income that stays in private hands and is not rebated to the fiscal authorities. Issuing currency backed by interest-bearing assets, firms obtain a second benefit: seignorage revenues. This paper studies how these two benefits affect the issuance and acceptance of private and public money, and characterises the consequences for monetary policy.

To this end, I develop a benchmark general equilibrium framework of money as medium of exchange, imperfect competition and information frictions. Consumers value consumption heterogeneously, but their types and purchases are unobservable initially. Firms—best thought of as vertically-integrated platforms, conglomerates or firm consortia which supply the entire range of consumption goods—can introduce a payment technology which, when being used, helps identify consumer types. The model thus contains a notion of information based on past purchasing behaviour. Furthermore, firms have market power and charge prices above marginal costs. Information is useful to both retain a firm's own most profitable customers as well as to attract their competitors' ones. While firms already generate data at a large scale, the unobservability assumption captures the idea that firms can still learn *more* about their and especially other firms' customers using transaction data.[1]

Given the Diem consortium's plans to issue a currency, the payment technology is modelled as money. In particular, consumers face a cash-in-advance constraint following Lucas and Stokey (1987), linking their currency holdings and consumption expenditure. Money does not pay interest and thus incurs an opportunity cost which consumers look to minimise when choosing a currency. I assume that firms back their currencies using assets of the same denomination. It follows that the opportunity cost faced by the consumer is the firm's seignorage income. Given the link between money and consumption, seignorage acts as a tax on consumption. The nominal interest rate of the economy can be then interpreted as a tax rate and consumption expenditure as the tax base.

Firms optimally choose not to accept competitor currencies due to the information rents. The model's prediction of this limited interoperability is consistent with empirical observations from the world of digital platforms and payment technologies. In China, Alipay and WeChat Pay dominate the market for digital payments with more than one billion active users each. However, customers of Alibaba cannot use WeChat Pay to purchase goods. Similarly, Amazon does not accept Apple Pay

[1]Transaction data could also complement other data collected by digital platforms, allowing for deeper insights into consumer tastes and behavioural patterns. Furthermore, data may be lost to producers when goods are sold through retailers. As an example, Google uses Mastercard credit card data from purchases in brick-and-mortar stores to improve their digital advertising algorithms (Source: Bloomberg, August 30, 2018). Last, producing firms may not have the capacities to collect and match consumer data. Lewis and Rao (2015) show that measuring the returns to advertising is very challenging given the large number of transaction data needed and the noise in individual purchase decisions. Shopify, an e-commerce platform and payment provider for small companies and brands, was one of the first members of the Diem consortium.

and Google Pay. This limited acceptance of firm currency breaks the money demand indeterminacy discovered in Kareken and Wallace (1981) and present in recent models of currency competition (Schilling and Uhlig (2019), Fernández-Villaverde and Sanches (2019) and Benigno et al. (2022)). Here, demand for one firm's private money is bound from above by their consumption good sales. Information limits private currency holdings—and thus the seignorage tax base—to consumption expenditure with the issuing firm. The paper's first main result follows: optimally, firms do not levy a seignorage tax on top of their profit-maximising price.

Consumers choose the currency associated with the lowest seignorage tax. The second main result is that the central bank is forced to follow suit and set its interest rate to zero. Firms therefore restrict the central bank's policy autonomy. This privately-enforced Friedman rule improves welfare. However, while zero interest rates are desirable in a model of money as medium of exchange, they are associated with deflation—an outcome that may be undesirable for reasons outside of this model.[2] The paper's third main result is that information rents determine whether the public currency loses its role as medium of exchange. For low levels of central bank seignorage taxes, it is only profitable to introduce a currency if transaction data generate large benefits. In this case, firms form digital currency areas as introduced by Brunnermeier et al. (2019): all firms issue and accept only their own currencies.

I further derive two sufficient conditions for private money to be valued, i.e. for equilibrium uniqueness. First, firms promise to accept their private currency at a strictly positive price. Second, the nominal private money supply is sufficiently low. Then consumers bid up the market price of private money into the strictly positive domain. In other words, as product producers and thus a party directly involved in the transaction, firms have a unique ability to select the monetary equilibrium.

Inspired by the institutional set-up of a currency consortium, I extend the benchmark framework: firms form currency consortia but decision powers and seignorage dividend claims are concentrated in the hands of one firm. Inflationary pressures arise. In such a scenario, the consortium leader optimally levies a seignorage tax on the consortium member firms; if the consortium is sufficiently big, it is the central bank that restricts private monetary policy. Importantly, this extension captures

[2]One example includes economies with nominal wage rigidities. The central bank may want to use inflation as a tool to reduce real wages in response to a negative productivity shock. See i.e. Uhlig and Xie (2021) for a model with nominal rigidities and multiple currencies.

an economy in which a company such as Facebook has little transactions of their own and generates information from other firms' transactions.

In the final part of the paper, I analyse government policies. First, I consider interest-bearing CBDC, and find that the central bank indeed regains full autonomy as long as CBDC does not generate any seignorage income. Second, policies which require firms to back their money using assets denominated in the public currency induce commitment problems on the firms' side. In the benchmark, firms cannot achieve capital gains: both assets and liabilities are of the same denomination and thus decrease equally in value with surprise inflation. This implies the ability to commit to a particular pre-announced monetary policy ex-post. With an asset portfolio denominated in the public currency, surprise inflation does lead to capital gains due to a separation of the medium of exchange and the unit of account. While such a policy can indeed regain central bank policy autonomy given the resulting price instability of private money, it harms consumers in the process.

**Contribution to the literature.** This paper's contribution is to analyse currency competition with profit-maximising producers of consumption goods, which also use private money to generate information. On the one hand, the existing literature on currency competition does not consider such competitors. Schilling and Uhlig (2019) and Fernández-Villaverde and Sanches (2019) discuss competition among cryptocurrencies and a public currency, rediscovering the famous portfolio indeterminacy result of Kareken and Wallace (1981). In this paper, information breaks the portfolio indeterminacy and allows me to set-up the joint product profit and seignorage maximisation problem. In Benigno et al. (2022), the provider of some global currency pays interest on money in a futile attempt to increase its market share. While the monetary policy consequences resonate with this paper's results for the benchmark economy, here they are an outcome of the goods producers' profit-maximisation problem. Skeie (2019) discusses digital currency runs when a decentralised currency competes with public currency experiencing high inflation rates. Cong and Mayer (2021) analyse competition among traditional fiat currencies, cryptocurrencies, and CBDC.[3]

On the other hand, the literature on tokens issued by firms does not consider currency competition. In Chiu and Wong (2022), a digital platform faces the choice between introducing private token money and accepting government currency. However, the platform's size is fixed and currency

[3]For papers discussing the pricing of cryptocurrencies, see, among others, Athey et al. (2016), Chiu and Koeppl (2022), Biais et al. (2022), Prat and Walter (2021), Sockin and Xiong (2020) and Choi and Rocheteau (2021). See John et al. (2021) for a survey.

competition thus muted. Brunnermeier et al. (2019) describe, albeit without a model, how the introduction of firms' digital currency areas may help promote platform cohesion and information collection, and thus might threaten central bank policy autonomy. In Brunnermeier and Payne (2022), information-generating platforms may limit interoperability in order to lock in consumers. The optimal CBDC design trades off allowing for beneficial information generation against preventing suck lock-in effects. Similarly, Gans and Halaburda (2015) discuss private money as a customer retention device in a partial equilibrium setting. Mayer (2021) analyses the interaction between a platform issuing tokens, speculators, and users. Li and Mann (2018), Catalini and Gans (2018), Garratt and Van Oordt (2022), Prat et al. (2019), Rogoff and You (2020), Cong et al. (2022) and Gryglewicz et al. (2021) analyse private tokens with a focus on optimal financing strategies.

My paper also relates to existing work on payment intermediaries' market power and information generation. Garratt and van Oordt (2021) and Garratt and Lee (2021) discuss whether and how payment data collection leads to welfare losses due to price discrimination and monopoly formation. CBDC preserves privacy in a Kahn et al. (2005) sense. In Lagos and Zhang (2019, 2022), sellers are subject to payment intermediary market power. Monetary policy restricts market power and remains effective through a medium-of-exchange channel even in pure credit economies. Huberman et al. (2021) discuss how the Bitcoin payment system can reduce payment intermediary costs. In Ahnert et al. (2022), merchants trade off transaction efficiency against information generation by intermediaries.[4] In this paper, product producers take on the role of intermediaries themselves when introducing private money; information allows me to characterise equilibrium demand for a given private currency. Firms' optimal private monetary policy, i.e. their preferred cost of payment intermediation, then restricts the central bank's policy space.[5]

**Organisation of the paper.** Section 2 presents the benchmark model. Optimal private monetary policy and the resulting consequences for public monetary public are characterised in Section 3. Section 4 extends the framework to break the benchmark model's results. Section 5 discusses policy and Section 6 concludes.

[4]Keister and Monnet (2022) discuss how CBDC can generate information on the quality of banks' balance sheets.

[5]See Goldfarb and Tucker (2019) and Bergemann and Bonatti (2019) for extensive surveys on information and privacy in the digital economy, as well as Acquisti and Varian (2005) and Fudenberg and Villas-Boas (2012) for surveys on customer recognition by imperfectly competing firms.

# 2 Model framework

## 2.1 Environment

### 2.1.1 Households

The model features overlapping generations (OLG) of consumers who live for three periods and discount the future at rate $\beta$. At each point in time, three cohorts of consumers—the young, the adolescent, and the old—co-exist. Each generation consists of a continuum of consumers on the unit interval. They consume a credit good $X$ and a money good $C$, and supply labour $N$. Their age is denoted by $A \in \{y, a, o\}$. Period utility for consumer $j$ is separable and quasi-linear:

$$U_{A,j} \;=\; U(X) \;+\; \theta_{A,j}^{1-\alpha} \left(C\right)^{\alpha} \;-\; N \tag{1}$$

The credit good serves as the model's numeraire. I assume that $U(X)$ satisfies the Inada conditions, implying the existence of a consumption level $X^*$ for which $U'(X^*) = 1$. The consumer's period utility thus corresponds to a buyer's period utility in Lagos and Wright (2005). As in their seminal work, this specification ensures model tractability. With constant disutility from supplying labour, any labour supply effects on consumption—either direct or operating indirectly in general equilibrium via the real wage—are muted. Furthermore, consumers can perfectly smooth consumption by adjusting their labour supply. Thus, this specification allows me to focus on the direct effect of monetary policy on consumption given the need for money as medium of exchange.

Turning to the money good, consumers value consumption heterogeneously according to their type $\theta_j$. I assume that this type is private information. At the beginning of their life, consumers draw their type from a publicly known, common, binary distribution: $\theta_j \in \{\theta_L, \theta_H\}$, $P[\theta_j = \theta_H] = q$, with $\theta_H \geq \theta_L \geq 0$. Consumers do not value consumption of the money good in the final period of their life:

$$\left(\theta_{y,j},\, \theta_{a,j},\, \theta_{o,j}\right) \;=\; \left(\theta_j,\, \theta_j,\, 0\right) \tag{2}$$

Two firms $i \in \{f, g\}$ supply the money good and charge a price $p_{i,t}$. Each period, consumers must choose a firm from which they purchase the money good without knowledge of their prices. Firms cannot transmit any information about their price to consumers.

Consumers form portfolios consisting of money and nominal bonds which are denoted in the public currency or—if private currencies have been introduced—in private currencies. Let the public currency be denoted by $M^{\$}$, firm $f$'s private currency by $M^{\approxeq}$, and firm $g$' currency by $M^{\mathbb{G}}$. Going forward, I refer to the public currency as the *Dollar* and to firm $f's$ private currency as *Diem*. Let $\phi_t^z$ denote the value of currency $z \in Z = \{\$, \approxeq, \mathbb{G}\}$ in terms of the credit good. Consumers are subject to short-sale constraints for each currency.

Let $Q_t^z$ denote the price of a nominal bond denoted in currency $z$, paying one nominal unit in the following period. As usual, bond prices are inversely related to the interest rate prevailing in the respective currencies: $Q_t^z = \frac{1}{1+i_t^z}$. In sum, consumer $j$'s budget constraint is given by:

$$X_{j,t} \ + \ p_{i,t}C_{j,t} \ + \sum_{z \in Z} \phi_t^z \left[M_{j,t}^z + Q_t^z B_{j,t}^z\right] \ \leq \ w_t N_{j,t} \ + \sum_{z \in Z} \phi_t^z \left[M_{j,t-1}^z + B_{j,t-1}^z\right] \ + \ T_{j,t} \tag{3}$$

where $T_{j,t}$ denotes the real lump-sum transfer from firms and government to consumer $j$.[6]

### 2.1.2 Firms

There are two sectors, the credit good and the money good sector. All firms maximise lifetime profits, discounting future profits with discount factor $\beta$.[7] The firms two firms $i \in \{f, g\}$ in the money good sector supply a homogeneous non-durable good and choose a price $p_{i,t}$.

Each firm in each sector produces given production functions that are linear in the only input factor labour:

$$Y_t^X \ = \ N_t^X \qquad\qquad Y_t^C \ = \ N_t^C \tag{4}$$

I assume perfect competition in the credit good market. Given the linearity of the production function, the real wage and thus real marginal costs for all firms in this economy are given by $w_t = 1$ for all $t$.

[6] I assume that firms and government transfer all proceeds to the young in equal proportions. As long as $X^*$ is sufficiently high, this assumption ensures that each consumer's labour supply is strictly positive, without affecting any other equilibrium outcomes.

[7] In equilibrium, given the quasi-linearity of the period utility function, consumer discount factors are equalised at $\beta$. Firms thus use the representative discount factor.

### 2.1.3 Money

Consumers need to hold currency in order to facilitate transactions of the money consumption good. In particular, they face a cash-in-advance (CIA) constraint as in Lucas and Stokey (1987). At the beginning of the game, firms may pay a fixed cost to introduce a private currency which reveals otherwise unobservable purchases of the young. Denote firm $i$'s decision whether to accept currency $z$ by $\gamma_{i,t}^{z} \in \{0, 1\}$. The CIA constraint faced by consumer $j$ at firm $i$ is therefore given by

$$p_{i,t} C_{j,t} \;\leq\; \sum_{z \in Z} \gamma_{i,t}^{z} \phi_{t}^{z} M_{j,t}^{z} \tag{5}$$

Equation (5) implies that consumers require one unit of real money that is accepted by firm $i$ for each unit of real expenditure, all measured in units of the credit good. For simplicity, I assume that consumers that are indifferent between using multiple currencies, use only one of these currencies. I also assume that consumers perfectly anticipate which currencies each firm accepts. This assumption rules out network effects and ensures that firms do not forgo any revenues by not accepting a particular currency. Since firm currencies are newly being introduced, this assumption initially boils down to the public currency being a widely used medium of exchange.

The OLG structure addresses a feature of the CIA model: consumers need to hold real money balances proportional to their consumption expenditure but there is no direct exchange of money and goods. Consumers enter the following period with exactly the same amount of nominal money even if they have consumed. Including a third period in which money holdings unravel is useful since adolescent consumers would otherwise strategically reduce their money good consumption. Furthermore, the model requires an infinite horizon for money to be valued in equilibrium.

### 2.1.4 Seignorage

Besides producing consumption goods using labour, firms may issue money and purchase bonds.

**Assumption 1.** *Firms back their currencies with bonds issued by the household and denominated in said currency:* $M_t^{z,S} = B_{i,t}^{z}$.

If firm $i$ issues private currency $z$, their budget constraint is given by

$$p_{i,t}Y_{i,t} \; - \; w_t N_{i,t} \; + \; \phi_t^z \left( M_t^{z,S} - M_{t-1}^{z,S} \right) \; = \; \sum_{z \in Z} \phi_t^z \left( Q_t B_{i,t} - B_{i,t-1} \right) \; + \; T_{i,t} \tag{6}$$

Assumption 1 yields a very natural notion of seignorage. Define $\tau_t^z = 1 - Q_t^z$ as the opportunity cost of holding money: both money and bonds pay one nominal unit in the following time period but bonds cost $Q_t^z \leq 1$. Using this definition, seignorage revenues at time-$t$ are given by $s_t^z = \tau_t^z \phi_t^z M_t^{z,S}$.[8]

Assumption 1 also gives firms the ability to commit to a particular private monetary policy. Firms hold assets corresponding in size and denomination to their money liabilities; both depreciate equally in value with private currency inflation. Thus, inducing surprise inflation cannot raise profits. Section 5.2 discusses optimal commitment when firms can only purchase Dollar-denominated bonds.

#### *2.1.5 Information and choice of firm*

I assume that private currency usage generates information on consumers. The firm's prior belief that $\theta_j = \theta^H$ is given by the true population share of high types. Whenever a transaction between consumer $j$ and firm $i$ is settled using private currency $z$, the issuing firm observes the transaction quantity and price and updates their belief about consumer $j$'s type using Bayes' rule. Public currency transactions do not generate any information.

Let $\psi_{j,A,t}$ and $\mu_{j,A,t}$ denote consumer $j$'s firm choice and their beliefs about firms' product prices, respectively. Clearly, $\psi_{j,a,t} = i$ if the adolescent consumer $j$ expects a lower total cost of purchasing consumption goods from firm $i$ than from firm $-i$. I assume that adolescent consumers randomise among firms whenever they expect both firms to charge the same total price; *information* affects the *probability* of choosing a given firm. In particular, adolescent consumers, which have not been identified as high types, choose each firm with equal probability of 1/2. Adolescent consumers, which have been identified as high types by one firm, choose this firm with probability one.[9,10] Young consumers randomise, choosing each firm with equal probability of 1/2, if this does not

[8] Households are perfectly happy supplying bonds in exchange for money as long as the real bond return does not exceed $1 + r_t = \beta^{-1}$. Since $w_t = 1$, every unit of interest payments will have to be made up by supplying one unit of labour in the future, but the disutility from supplying labour is discounted by the rate of time preference $\beta$.

[9] This assumption can be microfounded in a model of advertising in which firms face convex cost to advertising or have to allocate scarce advertising capacity across multiple consumption goods for each consumer.

[10] Randomising adolescent consumers are unable to punish firms for the prices charged to their younger selves. In this sense, I am limiting the consumers' strategy space.

leave them strictly worse off. The immediate consequence of this assumption is that, if the two firms charge the same total prices in equilibrium, then one firm's information strictly increases their profits and strictly decreases their competitor's profits. For simplicity, I assume that firms are endowed with the equilibrium steady state level of information generated by their private currency upon its introduction.[11]

The OLG structure is useful to avoid Folk theorem-type results: it cuts off the purchase history and thus limits the degree of learning to the first period of consumer lives; it also allows the game among firms and consumers to be solved backwards from the final period in which consumers derive utility from money good consumption. The OLG structure does not make money essential as a store of value, as in Wallace (1980).

## 2.2 *Efficiency & Equilibrium definition*

The social planner maximises the sum of consumer utilities subject to the economy's resource constraints, $X_t = N_t^X$ and $C_t = N_t^C$. Given quasi-linear utility and linear production functions, the efficient levels of credit and money good consumption equate marginal utilities to one. These allocations can be implemented in the competitive monetary equilibrium if $p_{i,t} = 1$ and $\tau_t^z = 0$ for all $i$, $z$ and $t$. That is, the equilibrium is efficient if firms set prices equal to marginal costs and there is no opportunity cost of money.

The competitive monetary equilibrium is defined as follows:

**Equilibrium definition.** *The competitive monetary equilibrium of this economy is given by the*

1. *Set of initial currency introduction decisions*
2. *Set of currency acceptance decisions,* $\left\{\gamma_{i,t}^z\right\}_{i\in\{f,g\},\, z\in\{\$,≋,\mathbb{G}\},\, t\geq 0}$
3. *Set of firm strategies that solve the profit maximisation problems:*

$$\left\{p_{i,t}, M_t^{z,S}\right\}_{i\in\{f,g\},\, z\in\{≋,\mathbb{G}\},\, t\geq 0}$$

[11] In equilibrium, all firms charge the same total price and thus young consumers choose each firm with probability 1/2. Hence, firms are endowed with information on a measure p/2 of young high valuation consumers.

*4. Set of consumer strategies that solve the utility maximisation problem, given beliefs $\mu_{j,t}$:*

$$\left\{\psi_{j,A,t}, C_{j,A,t}, X_{j,A,t}, M^z_{j,A,t}, B^z_{j,A,t}, N_{j,A,t}\right\}_{j\in[0,1],\ A\in\{y,a,o\},\ z\in\{\$,\approx,\mathbb{G}\},\ t\geq 0}$$

*5. Set of prices* $\left\{w_t, \phi^z_t, Q^z_t\right\}_{z\in\{\$,\approx,\mathbb{G}\},\ t\geq 0}$

*such that the markets for labour, the credit good, the money good, bonds and money clear. Beliefs are formed rationally and are updated according to Bayes' Law.*

# 3 Equilibrium

This section characterises the competitive monetary equilibrium. If introduced, I postulate that private money is valued in equilibrium, $\phi^z_t > 0$, and that the desired monetary policy is indeed implementable by controlling the money supply. I verify this postulate in Section 3.7. I also focus on equilibria in which the government ensures that public money is valued, and characterise the necessary conditions.

## *3.1 Money balances and demand schedules of the adolescent*

I begin by providing a summary and intuitive discussion of the consumer optimality conditions. A formal, step-by-step solution to the consumer problem is presented in Appendix A.

First, credit good consumption is equal for all consumers in all time periods $t$:

$$X_{j,A,t} \ = \ X^* \tag{7}$$

Intuitively, given the period utility function and the unit real wage, consumers can always purchase one more unit of the credit good by supplying an additional unit of labour at constant disutility of one. The real interest rate of the economy is then pinned down by the time rate of preference: $1+r_t = \beta^{-1}$. For all currencies $z \in \{\$, \approx, \mathbb{G}\}$, define the gross inflation rate as $(1+\pi^z_{t+1}) = \phi^z_t/\phi^z_{t+1}$. The first order conditions for bonds for all consumers, regardless of their type, simplify to the Fisher equation:

$$Q_t^z \;=\; \beta(1+\pi_{t+1}^z)^{-1} \tag{8}$$

The price of bonds needs to compensate for the fact that consumers discount the future and that nominal bonds lose real value over time, captured by the inflation rate.

Second, adolescent consumers only hold money in order to enable consumption purchases. Money is dominated by bonds in terms of returns whenever $\tau_t^z > 0$. If this is true for all currencies $z$, then consumers do not hold real money balances in excess of their real money good expenditure. Furthermore, consumers do not hold currencies that are not accepted by their chosen firm $i$. If multiple currencies are accepted, they hold the currency with the lowest opportunity cost, denoted by $\tau_{i,t}^z$. Whenever $\tau_{i,t}^z > 0$, consumer $j$'s CIA constraint is binding:

$$p_{i,t}\, C_{j,t} \;=\; \phi_t^z M_{j,a,t}^z \tag{9}$$

If $\tau_{i,t}^z = 0$, there is no opportunity cost of holding money and the CIA constraint is slack.[12]

Third, the adolescent consumer's demand schedule for the money good is given by

$$C_{j,t} \;=\; C\big(\theta_j, p_{i,t}(1+\tau_{i,t}^z)\big) \;=\; \theta_j \left[\frac{\alpha}{p_{i,t}(1+\tau_{i,t}^z)}\right]^{\frac{1}{1-\alpha}} \tag{10}$$

The demand schedule is a function of the *seignorage-adjusted price*: firms charge a price $p_{i,t}$ which is scaled up by the opportunity cost of money. If $\tau_{i,t}^z = 0$, bonds and money have the same return. There is no opportunity cost of money and consumers pay the real price only once. If $\tau_{i,t}^z > 0$, consumers pay the full price once to firms, and another $(\tau_{i,t}^z)$-times to the issuer of currency.

## *3.2 Characterising a monopolist's seignorage-adjusted prices*

As an intermediate step, I characterise the prices that a monopolist would set. First, suppose money is not necessary in order to purchase consumption goods. This is equivalent to a world without an

[12] The solution also implies a zero lower bound on nominal interest rates for all currencies $z \in \{\$, \approx, \mathbb{G}\}$: $Q_t^z \leq 1 \Leftrightarrow i_t^z \geq 0$. For negative interest rates, markets for bonds and money do not clear: consumers want to borrow infinite amounts at negative rates to purchase money which pays zero interest.

opportunity cost of holding money. The consumption demand schedule simplifies to

$$C(\theta_j, p) \;=\; \theta_j \left[\frac{\alpha}{p}\right]^{\frac{1}{1-\alpha}} \tag{11}$$

The profits from selling to consumer $j$ are then given by $\Pi(p, \theta_j) = (p-1)C(\theta_j, p)$.

**Lemma 1.** *The profit function is continuous in $p$ and has a unique maximum $\tilde{p} = \frac{1}{\alpha}$.*

The profit-maximising price $\tilde{p}$ is a constant mark-up over marginal costs. It is independent of the consumer's type $\theta_j$ for two reasons. First, the type does not affect the price elasticity of consumption. Second, marginal costs are constant. It follows that monopoly profits are linear in the consumer type, $\Pi(\theta_j) = \kappa\theta_j$, with $\kappa = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} > 0$.

Consider next a monopoly economy in which money is required to purchase consumption. A firm without private currency sets the price $p$ and the central bank implements the seignorage tax rate $\tau_t^{\$}$. Crucially, a firm that issues private currency chooses both.

**Lemma 2 (Monopoly prices).** *Consider an economy in which the only money in circulation is public currency. In this economy, the monopolist charges a seignorage-adjusted price given by $\tilde{p}(1+\tau_t^{\$}) \;>\; \tilde{p}$ whenever $\tau_t^{\$} > 0$.*

*Consider next an economy in which the only money in circulation is issued by the monopoly firm. This firm charges a seignorage-adjusted price $p_t(1+\tau_t^{\approx}) = \tilde{p}$. In other words, the firm fully removes the seignorage tax either by pursuing a private monetary policy of $\tau_t^{\approx} = 0$ or by providing compensating product discounts.*

*Proof.* Consider a monopolist that transacts in the public currency and does not obtain seignorage revenues on a given transaction with consumer $j$. The firm's corresponding profits are given by $\Pi_t \;=\; (p_t - 1)\; C\big(\theta_j, p_t(1+\tau_t^{\$})\big)$. The profit maximising price is again given by $\tilde{p}$. Note that the seignorage tax directly reduces per-transaction profits:

$$\Pi\big(\theta_j, \tau_t^{\$}\big) \;=\; \kappa\theta_j\big(1+\tau_t^{\$}\big)^{\frac{1}{\alpha-1}} \tag{12}$$

Next consider a monopolist which also issues $M^{\approx}$, the only currency in circulation. I have established that each consumer's CIA constraint binds whenever seignorage revenues are strictly positive. Thus, seignorage revenues are given by

$$s_t^{\approx} \;=\; \sum_{A\in\{y,a\}} \int_0^1 \tau_t^{\approx} p_t\, C\big(\theta_j, p_t(1+\tau_t^{\approx})\big)\, dj \tag{13}$$

Aggregating the per-consumer product profits and total seignorage revenues, the profit-maximisation problem is then given by

$$\max_{p_t(1+\tau_t^{\approx})} \quad \sum_{A\in\{y,a\}} \int_0^1 \Big(p_t(1+\tau_t^{\approx})-1\Big)\, \theta_j \, \left[\frac{\alpha}{p_t(1+\tau_t^{\approx})}\right]^{\frac{1}{1-\alpha}} dj \tag{14}$$

The seignorage-adjusted price of Lemma 2 is the solution to this problem. □

Intuitively, a firm optimally does not levy a tax on top of their profit-maximising price. Since the firms controls both product price and seignorage tax rate, the firm implements a private currency variant of the Friedman rule. Firms obtain a degree of freedom: they can implement any private monetary policy and then set prices with compensating discounts accordingly.

## *3.3 Firms issuing private currency fully remove the seignorage tax*

In order to solve for the equilibrium private money introduction decisions, I first characterise the firms' profits for a one-sided introduction. To this end, suppose that firm $f$ has introduced its currency Diem, the only private currency in circulation. The model then contains two types of money good producers: one that issues private currency, and one that doesn't. For this and the next subsection, I postulate that Diem demand is bound from above by the transactions with firm $f$ using Diem. Proposition 3 verifies this postulate (subsection 3.5). Given this bound, the paper's first proposition jointly characterises optimal product pricing and profit-maximising, private monetary policy for the full monetary framework:

**Proposition 1.** *Both firms charge the monopoly prices as characterised by Lemma 2 to all consumers in all time periods. It follows that, in equilibrium, firm g's transaction are subject to the government's seignorage tax rate. Firm f optimally removes the seignorage tax rate on its private currency transactions and perfectly identifies all of its high valuation customers using private currency.*

*Proof.* Adolescent consumers choose the firm which they expect to charge the lowest seignorage-adjusted price. When consumers expect both firms to charge the same seignorage-adjusted price, they randomise. Once a choice is made, it cannot be reverted. Hence, firms charge monopoly prices to adolescent consumers. Anticipating these future monopoly prices independent of their type, young high valuation consumers have no incentive to hide their type. Hence the young consumers' optimality conditions perfectly mirror those of the adolescent consumers: the demand schedule is given by Equation (10) and they visit the firm which they expect to charge the lowest seignorage-adjusted price when young; they randomise for equal expected seignorage-adjusted prices. Hence, firms again charge their monopoly prices of Lemma 2. Finally, since firm $f$ charges the same price to both high and low valuation consumers using Diem, consumers of different types demand different quantities of the consumption good. Hence, firm $f$ receives a perfectly informative signal to update its prior beliefs using Bayes' rule. □

The Starbucks rewards system is a real world example of such seignorage discounts. When customers pay using preloaded credit in their Starbucks app, they double their rewards, which can be used to purchase other Starbucks products. Similarly, automotive producer banks give credit at discounted rates when purchasing new cars. Both of these are examples of bundling product and payment, with a cross-subsidisation from payments to products.

## *3.4 The central bank loses its policy autonomy*

Having characterised the seignorage-adjusted prices that firms charge in different currencies, I am now ready to determine the consequences for private and public monetary policy.

**Lemma 3 (Choice of firm and currency).** *Suppose $\tau_t^{\$} > 0$. Consumers only choose firms which have introduced their own private currencies. Whenever these firms accept the Dollar and their private currency, consumers prefer to transact using the private currency.*

**Proposition 2 (Central bank loses policy autonomy).** *Suppose the central bank supplies a strictly positive amount of money, $M_t^{\$,S} > 0$. Then government fiat money is only valued, $\phi_t^{\$} > 0$, if $\tau_t^{\$} = 0$. This policy is associated with deflation: $\pi_{t+1}^{\$} = \beta - 1 < 0$.*

Lemma 3 follows from Proposition 1. Proposition 2 then follows from Lemma 3. Consumers rationally form beliefs about firms' prices. By Proposition 1, and unless $\tau_t^{\$} = 0$, purchasing at firm

$f$ using Diem is less costly than a) using the Dollar with firm $f$, and b) purchasing from firm $g$. Suppose $\tau_t^{\$} > 0$. No consumer visits firm $g$'s store, and thus aggregate consumption of the money good provided by firm $g$ is zero. Since no consumer uses the Dollar at firm $f$, the non-negativity constraint binds, and $\phi_t^{\$} M_t^{\$} = 0$. Therefore government fiat money is not valued if the central bank supplies a positive Dollar supply, $M_t^{\$,S} > 0$, unless $\tau_t^{\$} = 0$. Deflation is necessary as it ensures a strictly positive real return on money, compensating consumers for time-discounting.

One private currency is sufficient to impose the Friedman rule on the central bank and fully remove its policy autonomy. Consumer welfare unambiguously increases. Money serves the vital role of facilitating transactions, and levying a seignorage tax reduces consumption and thus consumer utility. However, the allocations are not efficient due to monopoly pricing.

## 3.5 Firms do not accept competitor currencies

This section verifies the postulate that private currency demand is bound from above by consumption good purchases using private currency from the issuing firm.

**Proposition 3 (No interoperability).** *Firm $g$ does not accept Diem:*

$$\gamma_{g,t}^{\approx} \;=\; 0 \quad \forall\, t \tag{15}$$

*It follows that Diem holdings are indeed bound from above by consumption good purchases using private currency from the issuing firm.*

*Proof.* See Appendix B.1.

The intuition is simple. Information is used to improve each firm's customer base at the expense of their competitor. Thus, firms strictly prefer not to accept competitor currencies in order to generate less information. The model therefore predicts that digital giants such as Amazon would not have accepted Diem. This prediction is consistent with observations from the real world. In China, the market for digital payments is dominated by Alipay and WeChat Pay. The social media platform WeChat resembles Facebook, and its payment technology is not accepted by Alibaba, the owner of Alipay. Similarly, Amazon does not accept Apple Pay and Google Pay.

## 3.6 Firms form digital currency areas for large information rents

So far, I have characterised the equilibrium outcome for a one-sided private currency introduction. Does firm $g$ want to counter-innovate by also introducing a private currency?

Let $\Delta\Big(\{\tau^{\$}_{t+s}\}_{s\geq 0}\Big)$ denote the lifetime seignorage gains due to the privately-enforced Friedman rule in an economy without information rents. Similarly, let $\Delta^I$ denote the lifetime information rents for an economy without seignorage taxes.

**Proposition 4 (Digital currency areas).** *If the fixed cost of introducing the currency is smaller than the information rents, $k \leq \Delta^I$, both firms introduce private digital currencies. Firms neither accept the Dollar nor their competitor's currency. Each private money's real balances are given by the issuing firm's product revenues. The Dollar loses its role as medium of exchange.*

*For an intermediate cost $k$, with $\Delta^I < k \leq \Delta^I + \Delta\Big(\{\tau^{\$}_{t+s}\}_{s\geq 0}\Big)$, only one firm introduces a private currency and only accepts this currency. The competitor firm only accepts the Dollar. Private and public real money balances are given by the firms' respective product revenues.*

*Proof.* See Appendix B.2.

The first mover gains can be neatly decomposed into lifetime information rents and lifetime seignorage gains. Prior to the introduction of a private currency, firms split the market equally. The Dollar seignorage tax lowers profits for both firms in every time period. Proposition 1 showed that this tax is fully removed for all currencies upon the introduction of one private currency. Both firms benefit equally and achieve the lifetime seignorage gains for an economy without information. In this sense, the seignorage gains are a *positive externality* on the competitor firm.

The lifetime information rents for an economy without seignorage taxes are a *negative externality* on the competitor firm. Information allows the currency issuing firm to improve its customer base at the expense of the competitor firm. To maximise information collection, currency-issuing firms optimally do not accept the public currency.[13] It follows that the first mover trades off the total gains from introducing a private currency against a fixed cost of doing so. The second mover only trades off the information rents against the fixed cost.

[13] If the government forces acceptance of public currency by making it legal tender, then firms could charge a high mark-up in public currency transactions to encourage private currency usage.

Information rents matter for equilibrium outcomes. First, private currency provisions in a low inflation economy is only profitable with information generation. Second, whether the public currency loses its role as medium of exchange depends on the size of the economy's information rents. For sufficiently large information rents, all firms form digital currency areas as suggested by Brunnermeier et al. (2019): although transactions take place within one economy, they are settled using different currencies in different marketplaces. However, given the privately-enforced Friedman rule result, payment technologies introduced by second movers need not take the form of private currencies.

### *3.7 Uniqueness & Implementation of private monetary policy*

This section verifies my initial postulate that, in equilibrium, private money is valued and optimal private monetary policy can indeed be implemented. To this end, let $\tilde{m}^z$ denote the equilibrium real money balances conditional on private money being valued, as characterised by Proposition 4.

**Proposition 5 (Uniqueness and implementation).** *Suppose firm $i$ accepts its currency $z$ at some minimal price $\phi_t^{z,min}$ and sets the profit-maximising seignorage-adjusted price with a product price $p_{i,t} < \tilde{p}$. Suppose further that the supply of currency $z$ is bound from above by $\phi_t^{z,min} M_t^{z,S} \leq \tilde{m}^z$. It follows that its equilibrium price is unique and strictly positive.*

*Given equilibrium uniqueness and the results of Propositions 1 - 4, firm $i$ can implement its desired private monetary policy $\tau_t^z$ at time $t+1$ by setting*

$$M_{t+1}^{z,S} = \frac{\beta}{1-\tau_t^z} \frac{p_{i,t+1}}{p_{i,t}} M_t^{z,S} \qquad \text{and} \qquad M_0^{z,S} = M > 0 \tag{16}$$

*Proof.* See Appendix B.3.

Intuitively, firm $f$ can induce demand for its currency by accepting it at some minimal price of money. Consumers are then willing to purchase private currency in centralised markets for a price that is weakly larger than the firm's minimum acceptance price of currencies as long as the aggregate nominal token supply is sufficiently small. This uniqueness argument resonates with Obstfeld and Rogoff (1983, 2021). The authors show that the price of money is bound away from zero if the government uses its ability to tax to credibly promise to repurchase money at a strictly positive price. Here the price of money is bound away from zero since firms can promise to accept their private currency at a strictly positive price in exchange for consumption goods. Comparing this result to the literature on the strategic complementarities of token and payment technology adoption, i.e. Crouzet et al. (2019), Cong et al. (2021) and Alvarez et al. (2022), the firm considered here has a unique ability to internalise such strategic complementarities as both the recipient of payments and issuer of money. Finally, implementability is included for completeness.

### *3.8 Discussion*

*Economy without information frictions.* In existing models of currency competition, the portfolio breakdown among competing currencies is indeterminate.[14] With perfect substitutability, consumers are only willing to hold all competing currencies if the relative prices of currencies are constant. As a consequence, there are not enough equilibrium conditions to pin down individual currency balances. Furthermore, issuers of private money do not internalise the equilibrium effect of private money issuance.

In this paper, information crucially limits the seignorage tax base. The benchmark results rely on the idea that firm $g$ does not accept Diem, ensuring that firm $f$'s seignorage revenues correspond exactly to their product sales in Diem. Thus, the equilibrium Diem demand curve is determined to a degree that allows me to formulate the profit-maximisation problem of firm $f$. Without information frictions, the Diem money demand curve is again indeterminate. Since the opportunity cost of money is directly linked to profits, firms perfectly internalise the equilibrium effect of private monetary policy.

*Heterogeneous information.* The model can easily accommodate heterogeneity in firms' ability to learn about consumer types even without a private currency. The benchmark results require that firms do not accept competitor currencies, i.e. Amazon does not accept currency issued by Facebook. Equilibrium introduction decisions are of course affected by the degree to which firms already obtain transaction data: the more data firms generate without a private currency, the lower are the incentives to introduce such a currency.

*Price discrimination.* Since equilibrium prices are independent of consumer types, firms do not price discriminate. The advantage is that it allows me to combine notions of imperfect competition,

[14] This result was initially obtained by Kareken and Wallace (1981) and and features in Schilling and Uhlig (2019), Fernández-Villaverde and Sanches (2019) and Benigno et al. (2022).

information based on based purchase behaviour, and money as medium of exchange into a tractable general equilibrium framework. Information is useful because it gives firms the means to attract the most profitable consumers even if all firms charge the same seignorage-adjusted price in equilibrium.

The disadvantage is that the model is silent on strategic consumer behaviour: consumers are indifferent with regards to information provision. However, even if consumers were experiencing price discrimination, existing research suggests that consumers struggle to protect their privacy in digital environments. Bian et al. (2022) highlight that consumers lack awareness of firms' data collection practices. Chen et al. (2021) document a data privacy paradox: evaluating survey and Alipay data, they find no relationship between stated privacy preferences and privacy-preserving actions.[15] Consumers may also be willing to share transaction data in full knowledge of future price discrimination if a) the convenience of data-generating payments is sufficiently large relative to the privacy-preserving alternative, i.e. cash, or b) data provision imposes an externality on other consumers rather the transacting consumers themselves, as in Garratt and van Oordt (2021).

*CBDC and disintermediation of banks.* The findings in this paper also speak to the literature on disintermediation of banks due to CBDC (Andolfatto (2021), Chiu et al. (2022), Keister and Sanches (2022), Piazzesi and Schneider (2020) and Williamson (2022)). The 'no seignorage' result, here derived for public money, also applies to bank deposits and credit card issuers charging transaction fees if they compete with private money.

# 4 Breaking the benchmark

Inspired by the previously proposed institutional set-up of Diem as a currency consortium consisting of multiple firms and initiated by Facebook, I adjust the benchmark environment. In particular, I study the effect of a concentration of private monetary policy decisions powers and seignorage dividend claims on optimal private monetary policy.

[15] See also Liu et al. (2021) who analyse behavioural vulnerabilities, in particular limited resistance to purchases of consumption goods that do not increase utility, in the context of data privacy regulation.

## 4.1 Environment

In this section, consumers derive utility from *two* money goods according to their type $\left(\theta_{j,A}, \hat{\theta}_{j,A}\right)$:

$$U_{j,A} \;=\; U(X) \;+\; \theta_{j,A}^{1-\alpha} C^{\alpha} \;+\; \left(\omega \hat{\theta}_{j,A}\right)^{1-\alpha} \hat{C}^{\alpha} \;-\; N \tag{17}$$

The second money good type $\hat{\theta}_j$ is also drawn from a binary distribution which is common knowledge, similar to the first money good type $\theta_j$. I allow for any correlation structure of types across goods. As before, firms $f$ and $g$ supply the first money good, $C$. The parameter $\omega$ captures the relative size of the second market. Two firms $(\hat{f}, \hat{g})$ produce the second money good, $\hat{C}$. The market structure mirrors that of the first money good, and so do the consumers' optimality conditions. Consider the scenario in which firms $\left(f, \hat{f}\right)$ have formed a currency consortium that issues Diem, the only private money in the economy. I assume that firm $f$ is the *consortium leader*, deciding on Diem monetary policy and thus on the corresponding seignorage tax rate. I refer to firm $\hat{f}$ as the *consortium member*. I further assume that consortium leader and member share the total Diem seignorage revenues equally.[16] Importantly, by Proposition 3, competitor firms $g$ and $\hat{g}$ do not accept Diem given the consortium's information rents; consortium firms do not accept public currency by Proposition 4.

Given the results of Section 3, I assume that private money is valued and its price unique conditional on monetary policy. Real Diem balances are given by the consumption expenditure with the consortium firms. Let the (constant) equilibrium customer bases of the consortium firms be denoted by $\Theta$ and $\hat{\Theta}$. The leader's total profits are then given by:

$$\Pi_{f,t} \;=\; \left[p_{f,t}\left(1 + \frac{\tau_t^{\approx}}{2}\right) - 1\right] \; C\big(p_{f,t}(1+\tau_t^{\approx}), \Theta\big) \;+\; \frac{\tau_t^{\approx}}{2}\, p_{\hat{f},t}\, \hat{C}\big(p_{\hat{f},t}(1+\tau_t^{\approx}), \omega\hat{\Theta}\big) \tag{18}$$

The first term captures firm $f$'s product profits and half of the corresponding seignorage dividends. The second term captures the seignorage dividends corresponding to firm $\hat{f}$'s transactions that firm $f$ receives. Appendix C.1 provides the consumer demand functions, the corresponding profit function

[16] One interpretation is that the leading firm determines the initial private currency set-up, including private monetary policy, and the second firm joins the currency consortium afterwards taking this set-up as given. All firms pay a fixed fee to join the consortium and receive equal dividend shares. The Diem consortium's proposal to issue many public currency-denominated stablecoins before all of the proposed 100 members have joined for a fee of USD 10m is one such scenario.

for firm $\hat{f}$ and derives equilibrium pricing strategies for both firms. Taking pricing strategies as given, the consortium leader maximises profits with respect to the Diem seignorage tax rate $\tau_t^{\approx}$, subject to an upper bound on $\tau_t^{\approx}$ (derived in Appendix C.2):

$$\tau_t^{\approx} \ \leq \ \bar{\tau}\left(\tau_t^{\$}\right) \tag{19}$$

Consumers only demand the consortium firms' goods if they charge a weakly lower seignorage-adjusted price than their competitors. If the desired Diem seignorage tax rate is too high, consumers do not purchase consumption goods using Diem. It follows that the Diem seignorage tax rate is bounded from above.

In equilibrium, the consortium leader's Diem transaction share for a one-sided private currency introduction is given by $\sigma = \Theta/(\Theta + \omega\hat{\Theta})$. I restrict the parameter space such that $\sigma \leq \frac{1}{2}$.

**Definition.** *Ownership is concentrated whenever* $\sigma < \frac{1}{2}$.

Figure 1 illustrates Equations (18) and (19) for $(\sigma, \tau^{\$}, \alpha) = (\frac{1}{3}, 0.06, 0.9)$.

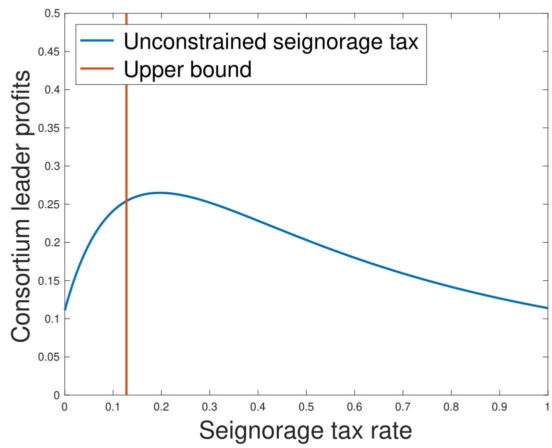


Figure 1: Consortium leader profit function

## *4.2 Concentrated ownership induces inflationary pressures*

The following proposition characterises optimal private monetary policy in this new environment:

**Proposition 6 (Concentrated ownership induces inflationary pressures).** *If* $\sigma = 1/2$*, then* $\tau_t^{\approx} = 0$*. If ownership is concentrated, the consortium leader implements a strictly positive seignorage tax rate, inducing inflationary pressures. If the optimal seignorage tax rate is unbounded,* $\tau_t^{\approx} < min\{\bar{\tau}(\tau_t^{\$}), 1\}$*, it is strictly decreasing in the leader's transaction share* $\sigma$*.*

*Proof.* See Appendix C.3.

**Corollary 1 (Public monetary policy disciplines private monetary policy).** *The upper bound on the consortium's seignorage tax rate binds for a sufficiently small* $\sigma$*.*

The consortium leader trades off maximising its own product profits against levying a seignorage tax on the consortium member. The former is maximised as $\tau_t^{\approx} = 0$ but this sets the latter to zero. As ownership becomes concentrated, the tax base available to the consortium leader grows relative to its own product profits, tipping the trade-off in the favour of the latter. One interpretation of Proposition 6 is that inflationary pressures arise as the private currency becomes more commonly used in the economy (i.e. $\omega$ increases). Another interpretation is that information gains, which are larger for the consortium member than the consortium leader (i.e. $\hat{\Theta} > \Theta$), are inflationary.

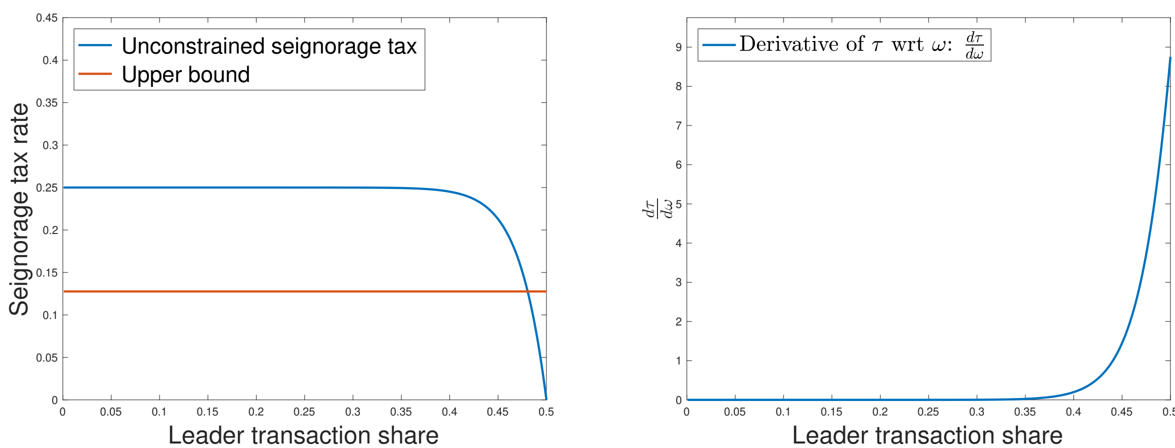


Figure 2: Unconstrained profit-maximising Diem seignorage tax rate and its upper bound

The optimal Diem seignorage tax rate—if the consortium leader were unconstrained by central bank policy—cannot be fully characterised analytically for all parameter combinations. Figure 2 plots the numerical solution as a function of the consortium leader's transaction share $\sigma$. It demonstrates that the profit-maximising, unconstrained seignorage tax rate is decreasing in the leader's transaction share.

**Corollary 2 (Equilibrium introduction decisions).** *Relative to Proposition 4, the second mover's currency introduction incentives increase by* $\Delta\left(\left\{\tau^{\approx}_{t+s}\right\}_{s\geq 0}\right) > 0$.

In Section 3, a one-sided introduction fully disciplines the central bank and both firms equally benefit from seignorage gains. In this section's partial equilibrium of a one-sided introduction, firm $f$ optimally implements strictly positive seignorage tax rates. This reinstates central bank policy autonomy, at least in part, but also increases the second mover benefits by the implied seignorage gains. The general equilibrium threat to public money as medium of exchange thus increases.

# 5 Policy

Given the stark consequences for monetary policy outlined above, it is natural to ask whether the government can regain policy autonomy via certain policies. I consider two policies that were discussed upon the announcement of Diem: introduction of CBDC and regulation of the asset portfolio used to back private money.

## 5.1 Interest-bearing CBDC can restore policy autonomy

Suppose the central bank introduces central bank digital currency (CBDC). Given its digital nature, it is technologically feasible to pay interest on CBDC.

**Proposition 7.** *The central bank can escape the benchmark equilibrium outcome by issuing interest-bearing CBDC as long as the interest rate on CBDC matches the interest rate of bonds.*

*Proof.* See Appendix D.1.

In the benchmark, the central bank is forced to remove its seignorage tax. Instead of setting interest rates on bond to zero, the central bank can also remove the seignorage tax by paying equivalent interest on money.

## 5.2 Forcing firms to hold Dollar bonds induces commitment issues

Suppose the regulator requires the issuer of private currency to fully back its money using bonds denominated in the public currency, $\phi^z_t M^{z,S}_t = \phi^\$_t B^\$_{i,t}$. Importantly, holding Dollar-denominated assets induces commitment issues. In particular, firms may be tempted to inflate away their liabilities denominated in private currencies ex-post to achieve capital gains on their Dollar-denominated asset holdings. I make one additional assumption for this economy with commitment issues:

**Assumption 2.** *Firms lose the ability to issue money after surprise ex-post inflation.*

I consider both the benchmark economy of Section 3 and the currency consortium economy of Section 4. In the benchmark economy, the firm's budget constraint is now given by

$$p_{i,t}C_{i,t} \; - \; w_t N_{i,t} \; + \; \phi_t^z \left( M_t^{z,S} - M_{t-1}^{z,S} \right) \; = \; \phi_t^\$ \left( Q_t^\$ B_{i,t}^\$ - B_{i,t-1}^\$ \right) \; + \; T_{i,t} \tag{20}$$

where $C_{i,t}$ denotes the total consumption purchases from firm $i$. Using the definitions of the opportunity cost of money and inflation as well as market clearing conditions, it becomes

$$(p_{i,t} - 1)C_{i,t} \; + \; \underbrace{\tau_t^\$ m_t^z}_{A} + \underbrace{\left[ \frac{1}{1+\pi_t^\$} - \frac{1}{1+\pi_t^z} \right] m_{t-1}^z}_{B} \; = \; T_{i,t} \tag{21}$$

where $m_t^z = p_{i,t}C_{i,t}$. Total seignorage revenues in period $t$ now consist of two objects: immediate seignorage revenues due to the purchase of Dollar-denominated bonds (A); and delayed capital gains due to the appreciation of the Dollar-denominated bonds vis-à-vis Diem-denominated liabilities (B). In the economy with a currency consortium, the leader's rearranged budget constraint reads

$$(p_{f,t} - 1)C_{f,t} \; + \; \frac{1}{2} \left( \tau_t^\$ m_t^{≋} + \left[ \frac{1}{1+\pi_t^\$} - \frac{1}{1+\pi_t^{≋}} \right] m_{t-1}^{≋} \right) \; = \; T_{f,t} \tag{22}$$

where $m_t^{≋} = p_{f,t}C_{f,t} + p_{\hat{f},t}C_{\hat{f},t}$.

**Lemma 4.** *Optimal private monetary policy with commitment is unchanged vis-à-vis Sections 3 and 4 and characterised by Propositions 1 and 6, respectively.*

*Proof.* The lemma is shown by evaluating the respective firms' life-time payoffs, given by the discounted sum of the LHS of Equation (21). Given equilibrium bond pricing, express the term $B$ as

$\beta^{-1}(\tau^z_{t-1} - \tau^\$_{t-1})m^z_{t-1}$. Consider then the benchmark firm's profits due to time-$t$ transactions:

$$\begin{aligned}\Pi_{i,t} &= (p_{i,t} - 1)C_{i,t} + \tau^\$_t m^z_t + \beta\frac{\tau^z_t - \tau^\$_t}{\beta}m^z_t \\ &= (p_{i,t} - 1)C_{i,t} + \tau^z_t p_{i,t}C_{i,t} \end{aligned} \tag{23}$$

Setting a seignorage-adjusted price $\tilde{p}$ maximises this expression, and hence setting it in every period maximises the benchmark firm's life-time payoffs. The proof for the currency consortium leading firm employs the exact same reasoning. □

Without the ability to commit, delayed capital gains open the door for discretionary, inflationary private monetary policy. Once consumers have formed money and bond portfolios based on their expectations of private monetary policy, the issuer is tempted to surprise consumers and inflate away their currency liabilities. To illustrate, suppose a firm sets $p_{i,t-1} = \tilde{p}$ and announces a private monetary policy of $\tau^z_{t-1} = 0$. Consumers act expecting a low level of $\tau^z_{t-1}$. However, ex-post, the firm implements $\tau^z_{t-1} = 1$ by issuing large amounts of private money at time-$t$. This firm generates the benchmark profits plus the capital gains.

**Proposition 8.** *Suppose firms cannot commit their ex-post private monetary policy. Suppose that private money is valued at time $t$, $\phi^z_t > 0$. In the benchmark economy, the firm can implement the optimal private monetary policy under commitment ex-ante by setting $p_{i,t} = \tilde{p}/2$ and $\tau^z_t = 1$. In the economy with a currency consortium, the consortium leader is unable to implement the commitment solution ex-ante. Ex-post, they optimally do so if and only if they are sufficiently patient, $\beta \geq \underline{\beta}$.*

*Proof.* See Appendix D.2.

Intuitively, the benchmark firm can give sizeable product discounts such that consumers must expect very high inflation rate even with commitment. This ability to implement the commitment solution is unique to the benchmark firm which perfectly internalises the seignorage tax-effect on its profits. One interpretation of such an arrangement is that money corresponds to expiring vouchers which consumers replenish every period to purchase consumption goods.

However, the currency consortium leader only partially internalises the seignorage tax ex-ante, since half of the tax revenues accrue to the consortium member. Hence, they only implement the commitment solution under discretion if it is *ex-post optimal* to do so. To illustrate, consider the economy in the steady state with constant public monetary policy $\pi^{\$}$. It is optimal not to induce surprise inflation and thus maintain the ability to issue money at any time $t$ if

$$\frac{1}{2}\frac{m^{\approx}}{1+\pi^{\$}} \;<\; \frac{1}{1-\beta}\left[\delta^{I} \;+\; \delta\big(\tau^{\$},\tau^{\approx}\big) \;+\; \hat{\delta}\big(\tau^{\approx}\big)\right] \tag{24}$$

The LHS captures the leader's one-off capital gains. The RHS captures the leader's life-time benefits from implementing the commitment solution. These benefits consist of the discounted sum of the period information gains, $\delta^{I} > 0$, the period seignorage revenues associated with the consortium leader's own transactions, $\delta\big(\tau^{\$},\tau^{\approx}\big) \geq 0$, as well as the period seignorage tax on the consortium member, $\hat{\delta}\big(\tau^{\approx}\big) \geq 0$. The sum of period benefits is strictly positive. Thus, as $\beta \nearrow 1$, the RHS tends to infinity. It follows that—as long as real private money balances in the economy are finite—a sufficiently patient consortium leader prefers to implement the commitment solution in any time period.

The commitment issue is a result of separate denominations of the consortium's bond portfolio and currency liabilities. Thus, thus section highlights the complementarities between the unit of account and the medium of exchange. Commitment issues do not arise when asset-backed private money combines both roles. Policymakers, that require firms to back their currencies using assets denominated in the public currency, may regain policy autonomy if firms are tempted by discretionary, inflationary polices and lose the ability to issue money. However, such a policy harms consumers with private money holdings.

# 6 Conclusion

This paper is the first work to formally analyse currency competition between the central bank and firms. Importantly, information ensures that money demand is determined in equilibrium. The monetary policy consequences differ widely depending on the structure of private currency markets. In the benchmark, firms fully remove the central bank's policy autonomy unless the central introduces interest-bearing CBDC. The central bank regains policy autonomy for a one-sided currency introduction of a currency consortium if decision powers and seignorage dividend claims are concentrated in the hands of one firm. However, public currency may be under greater threat to lose its role as medium of exchange altogether. The model also highlights commitment issues on the firms' side for a separation of the unit of account and the medium of exchange.

Applying the model to real world examples, these results suggest that the Diem currency consortium assembled by the social network company Facebook, with little consumer sales of their own, would have restricted central bank policy autonomy less than a private currency issued by a product-selling platform such as Amazon.

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# Appendices

# A Appendix to Section 3

## *A.1 The adolescent consumer's maximisation problem*

Consider the second period of consumer $j$'s life, born at time $t-1$, having visited firm $i$'s shop and learnt their price $p_{i,t}$. The consumer maximises remaining lifetime utility subject to budget constraints when adolescent and old (Equation 3) and the CIA constraint when adolescent (Equation 5). For ease of exposition, I drop all $j$-subscripts. Formally, the non-negativity constraints on their real money holdings reads $\phi_t^z M_{a,t}^z \geq 0$ for all $z \in Z$. The full utility maximisation problem is given by

$$\max_{\substack{\{X_{a,t}, X_{o,t+1}, C_{a,t}, \\ N_{o,t+1}, N_{a,t}, B^z_{a,t}, M^z_{a,t}\}_{\{z\in Z\}}}} \quad U(X_{a,t}) \;+\; \theta^{1-\alpha}(C_{a,t})^{\alpha} \;-\; N_{a,t} \;+\; \beta\Big[U(X_{o,t+1}) - N_{o,t+1}\Big]$$

$$\begin{aligned}
\text{s.t.} \quad X_{a,t} + p_{i,t}C_{a,t} + \sum_{z\in Z} \phi^z_t \left[M^z_{a,t} + Q^z_t B^z_{a,t}\right] \;&\leq\; N_{a,t} + \sum_{z\in Z} \phi^z_t \left[M^z_{y,t-1} + B^z_{y,t-1}\right] \\
X_{o,t+1} \;&\leq\; N_{o,t+1} + \sum_{z\in Z} \phi^z_{t+1} \left[M^z_{a,t} + B^z_{a,t}\right] \\
p_{i,t}C_{a,t} \;&\leq\; \sum_{z\in Z} \gamma^z_{i,t}\phi^z_t M^z_{a,t} \\
\phi^z_t M^z_{a,t} \;&\geq\; 0 \quad \text{for each } z \in Z
\end{aligned} \tag{IA.1}$$

The first order conditions (FOCs) are then given by

$$X_{a,t}: \quad U'(X_{a,t}) \;=\; \lambda_{a,t} \tag{IA.2}$$

$$X_{o,t+1}: \quad U'(X_{o,t+1}) \;=\; \lambda_{a,t+1} \tag{IA.3}$$

$$C_{a,t}: \quad \alpha\theta_j^{1-\alpha}(C_{a,t})^{\alpha-1} \;=\; p_{i,t}(\lambda_{a,t} + \nu_{a,t}) \tag{IA.4}$$

$$N_{a,t}: \quad 1 \;=\; \lambda_{a,t} \tag{IA.5}$$

$$N_{o,t+1}: \quad 1 \;=\; \lambda_{o,t+1} \tag{IA.6}$$

$$B^z_{a,t}: \quad Q^z_t \;=\; \beta\,\frac{\lambda_{o,t+1}}{\lambda_{a,t}}\,\frac{\phi^z_{t+1}}{\phi^z_t} \tag{IA.7}$$

$$M^z_{a,t}: \quad 1 \;=\; \beta\frac{\lambda_{o,t+1}}{\lambda_{a,t}}\frac{\phi^z_{t+1}}{\phi^z_t} \;+\; \gamma^z_{i,t}\nu_{a,t} \;+\; \rho^z_{a,t} \tag{IA.8}$$

for all currencies $z \in Z$. The Lagrange and Kuhn-Tucker multipliers of the budget and CIA constraints are denoted by $\lambda_{a,t}$ and $\nu_{a,t}$. The Kuhn-Tucker conditions for the CIA and the non-negativity constraints are given by

$$\nu_{a,t}\left(-p_{i,t}C_{j,t} + \sum_{z\in Z}\gamma^z_{i,t}\phi^z_t M^z_{a,t}\right) \;=\; 0 \qquad \text{and} \qquad \nu_{a,t} \;\geq\; 0 \tag{IA.9}$$

$$\rho^z_{a,t}\,\phi^z_t M^z_{a,t} \;=\; 0 \qquad \text{and} \qquad \rho^z_{a,t} \;\geq\; 0 \tag{IA.10}$$

for all currencies $z \in Z$. Combining FOCs for consumption of the credit good and labour supply immediately yields that $X_{a,t} = X_{o,t+1} = X^*$. Because consumption of the credit good is equal across time, the real interest rate of the economy is pinned down by the discount factor $\beta$. The FOCs for bonds in all currencies simplify to

$$Q_t^z = \beta(1 + \pi_{t+1}^z)^{-1} \tag{IA.11}$$

Using this expression for bond prices, money FOCs become

$$\gamma_{i,t}^z \nu_{a,t} = 1 - Q_t^z - \rho_{a,t}^z \tag{IA.12}$$

Consumers only hold currencies accepted by firms, and only hold the one with the lower inflation rate (higher bond price) if multiple currencies are accepted. Consider first the case where firm $i$ only accepts one currency: $(\gamma_{i,t}^\$, \gamma_{i,t}^{\approx}, \gamma_{i,t}^{\mathbb{G}}) = (0, 1, 0)$. The LHS of the above is zero, requiring $(\rho_t^\$, \rho_t^{\mathbb{G}}) > 0$ whenever $(Q_t^\$, Q_t^{\mathbb{G}}) < 1$; it follows that $M_{a,t}^\$ = M_{a,t}^{\mathbb{G}} = 0$. Consider next a firm that accepts multiple currencies, i.e. the Dollar and Diem. Combining the two FOCs for $M^\$$ and $M^{\approx}$ shows that whenever $Q_t^\$ < Q_t^{\approx}$, it must be that $\rho_{a,t}^\$ > \rho_{a,t}^{\approx}$; since $\rho_{a,t}^{\approx} \geq 0$, this requires $\rho_{a,t}^\$ > 0$, yielding $M_{a,t}^\$ = M_{a,t}^{\mathbb{G}} = 0$.

Denote the lowest seignorage tax rate with firm $i$ by $\tau_{i,t}^z$. The FOC for this money then reads

$$\nu_{a,t} = \tau_{i,t}^z$$

which implies that $\nu_{a,t} > 0$ whenever $\tau_{i,t} > 0$. Then the CIA constraint holds with equality:

$$m_{a,t} = p_{i,t} C_{a,t}(p_{i,t}, \tau_{i,t})$$

where $m_{a,t}$ denote real money balances of currencies held by the consumer. Turning to the FOC for money good consumption, consumer $j$'s demand schedule is given by

$$C_{j,a,t}(p_{i,t}, \tau_{i,t}) = \theta_j \left[ \frac{\alpha}{p_{i,t}(1 + \tau_{i,t})} \right]^{\frac{1}{1-\alpha}} \tag{IA.13}$$

The value function of a middle aged consumer at time-$t$ having chosen firm $i$ is given by

$$V_{j,a,t} = U(X^*) \;+\; \theta_j^{1-\alpha}(C_{j,a,t})^\alpha \;-\; N_{j,a,t} \;+\; \beta\big[U(X^*) - N_{j,o,t+1}\big] \tag{IA.14}$$

where

$$N_{j,a,t} = X^* + p_{i,t}C_{j,a,t} + \sum_{z\in Z}\phi_t^z\left[M_{j,a,t}^z + Q_t^z B_{j,a,t}^z\right] - \sum_{z\in Z}\phi_t^z\left[M_{j,y,t-1}^z + B_{j,y,t-1}^z\right] \tag{IA.15}$$

$$N_{j,o,t+1} = X^* - \sum_{z\in Z}\phi_{t+1}^z\left[M_{j,a,t}^z + B_{j,a,t}^z\right] \tag{IA.16}$$

$$C_{j,a,t} = \theta\left[\frac{\alpha}{p_{i,t}(1+\tau_{i,t})}\right]^{\frac{1}{1-\alpha}} \tag{IA.17}$$

Combining and using the equilibrium bond pricing equations for all denominations $z \in Z$, the expression becomes

$$V_{j,a,t} \;= \bar{V}_{j,a} \;+\; \theta_j\left[\frac{\alpha}{p_{i,t}(1+\tau_{i,t})}\right]^{\frac{\alpha}{1-\alpha}} \;-\; p_{i,t}\theta_j\left[\frac{\alpha}{p_{i,t}(1+\tau_{i,t})}\right]^{\frac{1}{1-\alpha}} \tag{IA.18}$$

$$-\; \sum_{z\in Z}\phi_t^z(1-Q_t^z)M_{j,a,t}^z \;+\; \sum_{z\in Z}\phi_t^z\left[M_{j,y,t-1}^z + B_{j,y,t-1}^z\right] \tag{IA.19}$$

Optimally consumers only hold the currency with the lowest inflation rate among those accepted by firm $i$ and the cash-in-advance constraint holds with equality. It follows that

$$\tau_{i,t}^z p_{i,t}C_{j,a,t} = \sum_{z\in Z}\phi_t^z(1-Q_t^z)M_{j,a,t}^z \tag{IA.20}$$

and the expression for the value function becomes

$$V_{j,a,t} \;=\; \bar{V}_{j,a} \;+\; \theta_j\left[\frac{\alpha}{p_{i,t}(1+\tau_{i,t})}\right]^{\frac{\alpha}{1-\alpha}} \;-\; p_{i,t}(1+\tau_{i,t})\theta_j\left[\frac{\alpha}{p_{i,t}(1+\tau_{i,t})}\right]^{\frac{1}{1-\alpha}}$$

$$+\; \sum_{z\in Z}\phi_t^z\left[M_{j,y,t-1}^z + B_{j,y,t-1}^z\right] \tag{IA.21}$$

Collecting terms, the adolescent consumer $j$'s value function at time-$t$ is given by

$$V_{j,a,t}\left(\{M^z_{j,y,t}, B^z_{j,y,t}\}_{\{z\in Z\}}\right) \;=\; \bar{V}_{j,a} \;+\; \sum_{z\in Z}\phi^z_t\left[M^z_{j,y,t-1} + B^z_{j,y,t-1}\right] \;+\; \theta_j\tilde{\kappa}\Big[p_{i,t}(1+\tau_{i,t})\Big]^{\frac{\alpha}{\alpha-1}} \tag{IA.22}$$

where $\tilde{\kappa} = 1-\alpha > 0$. Optimally, consumers indeed visit the firm which they expect to charge the lowest seignorage-adjusted real price.

## *A.2 The young consumer's maximisation problem*

The adolescent consumer's value function is independent of any consumer's decisions taken when young, apart from asset holdings. Consider a consumer born at time $t$. Since consumers face monopoly prices independent of their types when adolescent, they can only affect future utility through asset holdings. Hence I proceed using the following adolescent value function:

$$V_{j,a,t+1}\left(\{M^z_{j,y,t}, B^z_{j,y,t}\}_{\{z\in Z\}}\right) \;=\; \tilde{V}_{j,a} + \sum_{z\in Z}\phi^z_{t+1}\left(M^z_{j,y,t} + B^z_{j,y,t}\right) \tag{IA.23}$$

Having visited firm $i$'s shop and learnt their price $p_{i,t}$, the young consumer's utility maximisation problem is given by (again omitting $j$-subscripts for ease of exposition):

$$\begin{aligned}
\max_{\{X_{y,t},C_{y,t},N_{y,t},B^z_{y,t},M^z_{y,t}\}_{\{z\in Z\}}} \quad & U(X_{y,t}) \;+\; \theta^{1-\alpha}(C_{y,t})^{\alpha} \;-\; N_{y,t} \;+\; \beta\, V_{a,t}\left(\{M^z_{y,t}, B^z_{y,t}\}_{\{z\in Z\}}\right) \\
\text{s.t.} \quad X_{y,t} + p_{i,t}C_{y,t} + \sum_{z\in Z}\phi^z_t\left[M^z_{y,t} + Q^z_t B^z_{y,t}\right] \;&\leq\; N_{y,t} \;+\; T_{y,t} \\
p_{i,t}C_{y,t} \;&\leq\; \sum_{z\in Z}\gamma^z_{i,t}\phi^z_t M^z_{y,t} \\
\phi^z_t M^z_{y,t} \;&\geq\; 0 \quad \text{for each } z\in Z
\end{aligned} \tag{IA.24}$$

The resulting equilibrium conditions below, together with the budget and CIA constraint holding with equality, mirror those for the adolescent consumer:

$$\begin{aligned} X_{y,t} &= X^* \\ C(\theta_j, p_{i,t}, \tau_{i,t}) &= \theta\left[\frac{\alpha}{p_{i,t}(1+\tau_{i,t})}\right]^{\frac{1}{1-\alpha}} \end{aligned} \quad \text{(IA.25)}$$

Plugging in yields

$$V_{y,t} = U(X^*) + \theta\left[\frac{\alpha}{p_{i,t}(1+\tau_{i,t})}\right]^{\frac{\alpha}{1-\alpha}} - N_{y,t} + \beta\left[\tilde{V}_{j,a} + \sum_{z\in Z}\phi^z_{t+1}\left[M^z_{y,t} + B^z_{y,t}\right]\right]$$

$$N_{y,t} = X^* + p_{i,t}\theta\left[\frac{\alpha}{p_{i,t}(1+\tau_{i,t})}\right]^{\frac{1}{1-\alpha}} + \sum_{z\in Z}\phi^z_t\left[M^z_{y,t} + Q^z_t B^z_{y,t}\right] - T_{y,t} \quad \text{(IA.26)}$$

Combine the two expressions and use the equilibrium bond pricing condition, the optimally binding CIA constraint as well as the definition of $\tau_{i,t}$ to find:

$$V_{j,y,t} = \bar{V}_y + T_{y,t} + \theta_j\tilde{\kappa}\left[p_{i,t}(1+\tau_{i,t})\right]^{\frac{\alpha}{\alpha-1}} + \beta\,\tilde{V}_{j,a} \quad \text{(IA.27)}$$

Young consumers thus also optimally choose the firm with the lowest seignorage-adjusted price.

# B Proofs

## *B.1 Proof of Proposition 3*

Suppose firm $g$ does accept Diem: $\gamma^{\approx}_{g,t} = 1$ for all $t$. Let $\eta_t$ be the measure of young consumers using Diem with firm $g$ at time $t$. If $\eta_t = 0$ for all $t$, the claim follows. Suppose that the probability of $\eta_t > 0$ is strictly greater than zero for at least one $t$. Since the measure of consumers using Diem at firm $g$ is strictly positive, it follows that $\tau^{\$}_t \geq \tau^{\approx}_t$. The government ensures that the Dollar is valued, and thus $\tau^{\$}_t \leq \tau^{\approx}_t$. Hence it must be that $\tau^{\$}_t = \tau^{\approx}_t$. Note that profits, for any level of the seignorage rate $\tau_t$, are strictly decreasing in the amount of information that the competitor firm has generated. Then, if the probability of the measure of young consumers using Diem with firm $g$ is strictly positive, firm $g$ prefers not to accept Diem. Thus, $\gamma^{\approx}_{g,t} = 1$ for all $t$ in which $\eta_t > 0$, and the claim follows. □

## B.2 Proof of Proposition 4

Suppose firm $i$ has introduced the private currency and, as by Proposition 2, all seignorage taxes have been removed. If a strictly positive measure of consumers shops at firm $i$ using public currency, then by not accepting public currency, $\gamma^{\$}_{i,t} = 0$, the firm can induce larger holdings of private currency without losing any customers–who are indifferent between using private and public currency–and thus generate more information. Not accepting public currency strictly increase profits.

Let the time-$t$ profits of firm $f$ when both firms accept the Dollar and neither firm has introduced their private currency be denoted by $\Pi^{\$,\$}_{f,t}$. Similarly, let $\Pi^{\approx,\mathbb{G}}_{f,t}$ denote the time-$t$ profits if both firms have introduced and only accept private currency. Finally, let $\Pi^{\approx,\$}_{f,t}$ denote the profits for a one-sided introduction with acceptance of currencies as in the proposition. These period profits are given by

$$\Pi^{\$,\$}_{f,t} \;=\; \mathbb{E}[\theta]\,\kappa\,(1+\tau^{\$}_t)^{\frac{1}{\alpha-1}} \tag{IB.1}$$

$$\Pi^{\approx,\mathbb{G}}_{f,t} \;=\; \mathbb{E}[\theta]\,\kappa \tag{IB.2}$$

$$\Pi^{\approx,\$}_{f,t} \;=\; \Theta_t\,\kappa \tag{IB.3}$$

where $\Theta_t$ denotes firm $f$'s customer base with information. The first mover gains, $\Pi^{\approx,\$}_{f,t} - \Pi^{\$,\$}_{f,t}$, can be rearranged to read

$$\Pi^{\approx,\$}_{f,t} \;-\; \Pi^{\$,\$}_{f,t} \;=\; \underbrace{\Big[\Theta_t \;-\; \mathbb{E}[\theta]\Big]\,\kappa}_{A} \;+\; \underbrace{\mathbb{E}[\theta]\,\kappa\,\Big[1 \;-\; (1+\tau^{\$}_t)^{\frac{1}{\alpha-1}}\Big]}_{B} \tag{IB.4}$$

where term $A$ captures the per-period information rents in a no-seignorage tax economy and term $B$ captures the per-period seignorage gains in a no-information economy. The terms $\Delta^I$ and $\Delta\Big(\big\{\tau^{\$}_{t+s}\big\}_{s\geq 0}\Big)$ are the discounted sums of all per-period information rents and seignorage gains, respectively. The second mover's period gains are also given by term A, and thus life-time gains are given by the life-time information rents. □

## B.3 Proof of Proposition 5

To show uniqueness, consider the following two equilibrium candidates:

1. $\phi_t^z > 0$ and $m_t^z = \tilde{m}^z$ for all $t$

2. $\phi_t^z = 0$ for at least one $t$

The first equilibrium candidate is characterised by Propositions 1 and 4 for which I assumed that $\phi_t^z > 0$. Consider the second equilibrium candidate. Suppose that firm $i$ accepts payment using its currency $M^z$ at $\phi_t^{z,min}$. That is, whenever $\phi_t^z < \phi_t^{z,min}$, the cash-in-advance constraint faced by consumer $j$ for purchases with firm $i$ is given by

$$p_{i,t}C_{j,t} \;\leq\; \phi_t^{z,min} M_{j,t}^z \tag{IB.5}$$

The utility maximisation problem of adolescent consumer $j$ at firm $i$ is given by Equation (IA.1) with the CIA constraint replaced with Equation (IB.5). The FOC with respect to $M_{j,t}^z$ if $M_{j,t}^z > 0$ reads

$$-\phi_t^z \lambda_t + \phi_t^{z,min}\nu_t + \beta\phi_{t+1}^z\lambda_{t+1} \;=\; 0 \tag{IB.6}$$

where $\lambda_t$ is the time-$t$ Lagrange multiplier of the budget constraint and $\nu_t$ is the Lagrange multiplier of the CIA constraint. Suppose that $\phi_t^z = 0 < \phi_t^{z,min}$. First note that it cannot be that $\phi_t^z = 0$ and $\phi_{t+1}^z > 0$ since $\nu_t \geq 0$ and $\beta\lambda_{t+1} > 0$. Next, consider two time periods for which $\phi_t^z = \phi_{t+1}^z = 0$. Holding private money does not incur an opportunity cost when purchasing it at a zero price, and hence $\nu_t = 0$ and $\tau_t^z = 0$. It follows

$$C_{a,t}(p_{i,t}) \;=\; \theta_j \left[\frac{\alpha}{p_{i,t}}\right]^{\frac{1}{1-\alpha}} \;>\; \theta_j \left[\frac{\alpha}{\tilde{p}}\right]^{\frac{1}{1-\alpha}} \tag{IB.7}$$

since $p_{i,t} < \tilde{p}$. Thus, every consumer expects lower seignorage-adjusted prices with firm $i$ (and thus higher consumption levels) than with competitor firm $-i$, regardless of the opportunity cost of money accepted by firm $-i$. This creates excess demand for currency $z$ since $\phi_t^{z,min} M_t^{z,S} \leq \tilde{m}^z$, and hence money markets cannot clear at $\phi_t^z = \phi_{t+1}^z = 0$ for any $t$ on the equilibrium path. Since both price sequences $\phi_t^z = \phi_{t+1}^z = 0$ as well as $\phi_t^z = 0$ with $\phi_{t+1}^z > 0$ are ruled out on the equilibrium path for any $t$, it cannot occur that $\phi_t^z > 0$ but $\phi_{t+1}^z = 0$: the money market at time $t+1$ does not clear for any $\phi_{t+2}^z \geq 0$. Thus, it must be that $\phi_t^z > 0$ for all $t$, and uniqueness follows.

Turning to implementation, note that by definition $(1+\pi^z_{t+1})m^z_{t+1}/m^z_t = M^z_{t+1}/M^z_t$ and $m^z_t = p_{i,t}C_{i,t}$ given Propositions 1 - 4. Using the definition of $\tau^z_t$, equilibrium bond pricing (Equation 8) and the money market clearing condition yields

$$M^{z,S}_{t+1} \;=\; \frac{\beta}{1-\tau^z_t}\frac{m^z_{t+1}}{m^z_t}M^{z,S}_t \qquad \text{and} \qquad M^{z,S}_0 \;=\; M \;>\; 0 \tag{IB.8}$$

With equilibrium uniqueness and private real money balances given by consumption expenses with firm $i$, the claim follows. □

# C Appendix to Section 4

## C.1 *Optimal pricing strategies*

Consumer $j$'s demand schedules, conditional on a purchase using Diem with one of the consortium firms, resemble the demand schedules of the previous sections:

$$C_{j,t}(p_{f,t},\tau^{\approx}_t) \;=\; \theta_j\left[\frac{\alpha}{p_{f,t}(1+\tau^{\approx}_t)}\right]^{\frac{1}{1-\alpha}} \tag{IC.1}$$

$$C_{j,t}(p_{\hat{f},t},\tau^{\approx}_t) \;=\; \omega\hat{\theta}_j\left[\frac{\alpha}{p_{\hat{f},t}(1+\tau^{\approx}_t)}\right]^{\frac{1}{1-\alpha}} \tag{IC.2}$$

The consortium firms' customer bases are given by:

$$\Theta_t = \sum_{A\in\{y,a\}}\int_0^1 1_{\{\psi_{j,A,t}=f\}}\theta_j dj \tag{IC.3}$$

$$\hat{\Theta}_t = \sum_{A\in\{y,a\}}\int_0^1 1_{\{\hat{\psi}_{j,A,t}=\hat{f}\}}\hat{\theta}_j dj \tag{IC.4}$$

where $\hat{\psi}$ denotes the firm choice in the second money good market. Since Diem holdings correspond exactly to consumption purchases from consortium firms, the two firms' profit functions are given by:

$$\Pi_{f,t} \;=\; \Big(p_{f,t}\Big(1+\frac{\tau_t^{\approx}}{2}\Big)-1\Big)\,\Theta_t\Big[\frac{\alpha}{p_{f,t}(1+\tau_t^{\approx})}\Big]^{\frac{1}{1-\alpha}} \;+\; \frac{\tau_t^{\approx}}{2}p_{\hat{f},t}\omega\hat{\Theta}_t\Big[\frac{\alpha}{p_{\hat{f},t}(1+\tau_t^{\approx})}\Big]^{\frac{1}{1-\alpha}}$$

$$\Pi_{\hat{f},t} \;=\; \Big(p_{\hat{f},t}\Big(1+\frac{\tau_t^{\approx}}{2}\Big)-1\Big)\,\omega\hat{\Theta}_t\Big[\frac{\alpha}{p_{\hat{f},t}(1+\tau_t^{\approx})}\Big]^{\frac{1}{1-\alpha}} \;+\; \frac{\tau_t^{\approx}}{2}p_{f,t}\Theta_t\Big[\frac{\alpha}{p_{f,t}(1+\tau_t^{\approx})}\Big]^{\frac{1}{1-\alpha}} \qquad \text{(IC.5)}$$

The firms' first order conditions then reveal that firms charge the following prices in Diem taking $\tau_t^{\approx}$ as given:

$$p_{f,t}(\tau_t^{\approx}) = \frac{\tilde{p}}{1+\frac{\tau_t^{\approx}}{2}} \qquad p_t^{\hat{f}}(\tau_t^{\approx}) = \frac{\tilde{p}}{1+\frac{\tau_t^{\approx}}{2}} \qquad \text{(IC.6)}$$

The equilibrium profit function of the consortium-leading firm $f$ is then given by:

$$\Pi_{f,t}\big(\tau_t^{\approx}\big) = (\tilde{p}-1)\Theta_t\Bigg[\frac{1+\frac{\tau_t^{\approx}}{2}}{1+\tau_t^{\approx}}\Bigg]^{\frac{1}{1-\alpha}} \;+\; \frac{\tau_t^{\approx}}{2}\,\omega\hat{\Theta}_t\,\tilde{p}\Bigg[\frac{\big[1+\frac{\tau_t^{\approx}}{2}\big]^{\alpha}}{1+\tau_t^{\approx}}\Bigg]^{\frac{1}{1-\alpha}} \qquad \text{(IC.7)}$$

where the first term captures firm $f$'s product profits and seignorage revenues due to its own transactions; the second term captures the seignorage revenues generated by firm $\hat{f}$ which accrue to firm $f$.

## *C.2 Upper bound on the Diem seignorage tax rate*

To derive the lower bound on the Diem seignorage tax rate, begin by noting that firm $\hat{f}$ only wants to remain part of the consortium if they pay weakly lower seignorage-adjusted prices than their competitor—otherwise they do not sell any goods and prefer to only accept the Dollar.

$$p_t^{f}(1+\tau_t^{\approx}) \;\leq\; \tilde{p}(1+\tau_t^{\$}) \qquad \text{(IC.8)}$$

where the above expression imposed that the non-consortium firms charge a real product price of $\tilde{p}$ as by Proposition 1. Given the consortium firms' pricing strategies as above, rearrange to find

$$\tau_t^{\approx} \;\leq\; \frac{\tau_t^{\$}}{1-\frac{1+\tau_t^{\$}}{2}} \qquad \text{(IC.9)}$$

whenever this expression's numerator is strictly greater than zero. If the numerator is weakly less than zero, Diem inflation is unconstrained by central bank policy.

## C.3 Proof of Proposition 6

For ease of exposition, I ignore the time-subscripts in this proof. I begin by showing that $\tau^{\approx} = 0$ is indeed a maximum if $\sigma = 1/2$. Note that $\sigma = 1/2$ implies that $\Theta = \omega\hat{\Theta}$. Also note that both firms charge the same price and internalise the seignorage tax effect to an equal degree. It follows that both firms' revenues and profits are exactly equal, and so do the respective seignorage revenues generated by each firm. Hence, Equation (IC.5) becomes

$$\Pi_f \;=\; \Big(p_f\Big(1+\frac{\tau^{\approx}}{2}\Big)-1\Big)\,\Theta\left[\frac{\alpha}{p_f(1+\tau^{\approx})}\right]^{\frac{1}{1-\alpha}} \;+\; \frac{\tau_t^{\approx}}{2}p_f\Theta\left[\frac{\alpha}{p_f(1+\tau^{\approx})}\right]^{\frac{1}{1-\alpha}} \tag{IC.10}$$

which corresponds to the profit function of Section 3 maximised at $\tau_t^{\approx} = 0$.

Second, I show that the optimal seignorage tax rate for concentrated ownership is strictly positive. Let $W$ denote the derivative of profits with respect to the Diem seignorage tax rate: $W = \frac{\partial \Pi_f}{\partial \tau^{\approx}}$. Evaluating this derivative at zero shows that it is strictly positive whenever ownership is concentrated:

$$\sigma \;<\; \frac{1}{2} \qquad \Rightarrow \qquad W|_{\tau^{\approx}=0} \;>\; 0 \tag{IC.11}$$

This implies that there is a $\tau^{\approx} > 0$ in the neighbourhood of zero associated with larger profits. Hence it must be that the maximum of the profit function over the permissible domain of $\tau^{\approx} \in [0,1]$ is strictly positive.

Next, if the maximum is not bounded, $\tau^{\approx} < min\big\{\bar{\tau}\big(\tau^{\$}\big),1\big\}$, it is characterised by the first order condition $W = 0$. The change in this maximum in $\sigma$ is then given by

$$\frac{d\tau^{\approx}}{d\sigma} \;=\; -\frac{\partial\Theta}{\partial\sigma}\frac{\frac{\partial W}{\partial\Theta}}{\frac{\partial W}{\partial\tau^{\approx}}} \tag{IC.12}$$

Since $\tau^{\approx}$ is a max point, by the second order condition it must be that $\frac{\partial W}{\partial \tau^{\approx}} < 0$ when evaluated at the max point. By definition of $\sigma$ it must be that $\frac{\partial\Theta}{\partial\sigma} > 0$. It follows that $d\tau^{\approx}/d\sigma < 0$ if $\frac{\partial W}{\partial\Theta} < 0$.

$$\frac{\partial W}{\partial\Theta} \;=\; (\tilde{p}-1)\frac{1}{1-\alpha}\frac{1}{2}\big(1+\tau^{\approx}/2\big)^{\frac{1}{1-\alpha}-1}\big(1+\tau^{\approx}\big)^{\frac{1}{\alpha-1}} \;+\; (\tilde{p}-1)\big(1+\tau^{\approx}/2\big)^{\frac{1}{1-\alpha}}\frac{1}{\alpha-1}\big(1+\tau^{\approx}\big)^{\frac{1}{\alpha-1}-1} \tag{IC.13}$$

Rearranging and dividing by strictly positive terms, it follows that

$$sign\Big(\frac{\partial W}{\partial\Theta}\Big) \;=\; sign\Big(\frac{1}{2+\tau^{\approx}} - \frac{1}{1+\tau^{\approx}}\Big) \tag{IC.14}$$

Hence $\frac{\partial W}{\partial\Theta} < 0$ and the claim follows. □

# D Appendix to Section 5

## D.1 Proof of Proposition 7: CBDC

Suppose the central bank pays an interest rate $i_t^{M\$}$ on CBDC. Equivalently, the price of CBDC can be expressed as $\phi_t^{\$}Q_t^{M\$}$, where $Q_t^{M\$} = 1/(1+i_t^{M\$})$. Note that $Q_t^{M\$} = 1$ if $i_t^{M\$} = 0$ as in the benchmark model. The consumer budget constraint is thus given by

$$X_{j,t} + p_{i,t}C_{j,t} + \sum_{z\in Z}\phi_t^z Q_t^z B_{j,t}^z + \phi_t^{\$}Q_t^{M\$}M_{j,t}^{\$} + \sum_{z\in\{\approx,\mathbb{G}\}}\phi_t^z M_t^z \;\leq\; w_t N_{j,t} + \sum_{z\in Z}\phi_t^z\left[B_{j,t-1}^z + M_{j,t-1}^z\right] \tag{ID.1}$$

Since consumers issue bonds to purchase money of the same denomination, $B_{j,t}^z + M_{j,t}^z = 0$, it follows that the opportunity cost of Dollar holdings is given by $\tau_t^{\$} = Q_t^{M\$} - Q_t^{\$}$. If $i_t^{M\$} = i_t^{\$}$, it follows that $Q_t^{M\$} = Q_t^{\$}$ and hence $\tau_t^{\$} = 0$. Thus, the central bank can implement $i_t^{\$} > 0$ as long as equivalent interest is paid on CBDC. □

## D.2 Proof of Proposition 8

Let the one-off surprise inflation gains at time-$t$ be denoted by $W_{f,t}$ and given by

$$W_{f,t} \;=\; \frac{1}{2}\left[\frac{1}{1+\pi_t^{\$}} - \frac{1}{1+\pi_t^{\approx}}\right]m_{t-1}^{\approx} \;\rightarrow\; \frac{m_{t-1}^{\approx}}{2(1+\pi_t^{\$})} \tag{ID.2}$$

as $\pi_t^{\approx} \nearrow \infty$. With a binding CIA constraint, the one-off surprise inflation gains are given by

$$W_{f,t} \;=\; \frac{1}{2(1+\pi_t^{\$})}\;\tilde{p}\;\Big[\Theta + \omega\hat{\Theta}\Big]\left[\frac{(1+\tilde{\tau}_{t-1}/2)^{\alpha}}{1+\tilde{\tau}_{t-1}}\right]^{\frac{1}{1-\alpha}} \tag{ID.3}$$

where $\tilde{\tau}_{t-1}$ captures the private monetary policy that was *expected* at time $t-1$. $W_{f,t}$ is independent of $\beta$. Let the benefit from implementing the commitment solution at time-$t$ be given by

$$V_{f,t} \;=\; \delta^{I}\big(\tau_t^{\approx}\big) \;+\; \delta\big(\tau_t^{\$},\tau_t^{\approx}\big) \;+\; \hat{\delta}\big(\tau_t^{\approx}\big) \;+\; \beta\;\max\left\{W_{f,t+1}, V_{f,t+1}\right\} \tag{ID.4}$$

where $\delta^{I}\big(\tau_t^{\approx}\big)$ denotes the period information rents given optimal private monetary policy as characterised by Proposition 6, $\delta\big(\tau_t^{\$},\tau_t^{\approx}\big)$ denotes the period seignorage gains associated with own transactions, and $\hat{\delta}\big(\tau_t^{\approx}\big)$ denotes the period seignorage tax revenues from the consortium member, respectively given by

$$\delta^{I}(\tau_t^{\approx}) \;=\; \big[\Theta - \mathbb{E}[\theta]\big]\,\kappa\left(\frac{1+\tau_t^{\approx}/2}{1+\tau_t^{\approx}}\right)^{\frac{1}{1-\alpha}} \tag{ID.5}$$

$$\delta(\tau_t^{\$},\tau_t^{\approx}) \;=\; \mathbb{E}[\theta]\kappa\left[\left(\frac{1+\tau_t^{\approx}/2}{1+\tau_t^{\approx}}\right)^{\frac{1}{1-\alpha}} - \left(\frac{1}{1+\tau_t^{\$}}\right)^{\frac{1}{1-\alpha}}\right] \tag{ID.6}$$

$$\hat{\delta}(\tau_t^{\approx}) \;=\; \frac{\tau_t^{\approx}}{2}\omega\hat{\Theta}\tilde{p}\left[\frac{(1+\tau_t^{\approx}/2)^{\alpha}}{1+\tau_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \tag{ID.7}$$

Equation (ID.4) takes into account that the consortium leader again faces the decision to deviate from the commitment monetary policy in the following period. I postulate that $W_{f,t+s} \leq V_{f,t+s}$ for all $s \geq 1$, verified below. Equation (ID.4) becomes

$$V_{f,t} \;=\; \sum_{s=0}^{\infty}\beta^{s}\Big[\delta^{I}\big(\tau_{t+s}^{\approx}\big) \;+\; \delta\big(\tau_{t+s}^{\$},\tau_{t+s}^{\approx}\big) \;+\; \hat{\delta}\big(\tau_{t+s}^{\approx}\big)\Big] \tag{ID.8}$$

Note that $\delta^{I}\big(\tau_t^{\approx}\big) > 0$ for all $\tau_t^{\approx}$. Further note that $\delta(\tau_t^{\$},\tau_t^{\approx}) \geq 0$ and $\hat{\delta}(\tau_t^{\approx}) \geq 0$. Thus, Equation (ID.8) is continuous and strictly increasing in $\beta$.

First, I show sufficiency. Let the highest level of the private seignorage tax rate on the equilibrium path under commitment be denoted by $\bar{\tau}^{\approx}$. It follows that

$$V_{f,t} \;\geq\; \sum_{s=0}^{\infty} \beta^s \, \delta^I\big(\tau^{\approx}_{t+s}\big) \;\geq\; \sum_{s=0}^{\infty} \beta^s \, \delta^I\big(\bar{\tau}^{\approx}\big) \;=\; \frac{\delta^I\big(\bar{\tau}^{\approx}\big)}{1-\beta} \tag{ID.9}$$

Note that $V_{f,t} \nearrow \infty$ as $\beta \nearrow 1$. Since $W_{f,t}$ is finite, there exists a threshold level $\beta_t = \beta_t\left(\pi^{\$}_t, \tilde{\tau}_{t-1}, \{\tau^{\approx}_{t+s}, \tau^{\$}_{t+s}\}_{s\geq 0}\right) < 1$ such that $V_{f,t} \geq W_{f,t}$ if $\beta \geq \beta_t$. Let $\underline{\beta} = \max\{\beta_t\}$. Then $V_{f,t} \geq W_{f,t}$ for all $t$ if $\beta \geq \underline{\beta}$. This verifies above postulate and sufficiency follows.

Necessity follows from the definition of $\underline{\beta}$. Consider $\beta < \underline{\beta}$. Then there exists a time period $t$ in which $\beta < \beta_t\left(\pi^{\$}_t, \tilde{\tau}_{t-1}, \{\tau^{\approx}_{t+s}, \tau^{\$}_{t+s}\}_{s\geq 0}\right)$, and $V_{f,t} < W_{f,t}$. Note that

$$V_{f,t}(\beta \searrow 0) \;=\; \delta^I\big(\tau^{\approx}_t\big) \;+\; \delta\big(\tau^{\$}_t, \tau^{\approx}_t\big) \;+\; \hat{\delta}\big(\tau^{\approx}_t\big) \tag{ID.10}$$

If $V_{f,t}(\beta \searrow 0) < W_{f,t}$ for at least one $t$, then $\underline{\beta} > 0$. Otherwise, $\underline{\beta} = 0$ and the consortium leader always implements the commitment solution ex-post, regardless of the discount factor $\beta$.