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Search Disclosure

Carl-Christian Groh [1]
Marcel Preuss [2]

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[1] Department of Economics, University of Mannheim, Email: cgroh@mail.uni-mannheim.de
[2] Department of Economics, University of Mannheim, Email: preuss@cornell.edu

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Carl-Christian Groh† Marcel Preuss‡

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Abstract

We study information sharing between competing sellers in markets where consumers sample sellers sequentially. Sellers can disclose to their rival when they encounter a buyer. Providing this information, which we call *search disclosure*, can enable all forms of search history-based price discrimination. Yet, firms only conduct search disclosure in equilibrium if search costs are low or price revisions are infeasible. The kind of search disclosure that can emerge in equilibrium leads to price discrimination that reduces consumer surplus and total welfare. However, if firms were mandated to use search disclosure at all times, consumer surplus would be higher.

Keywords: Search, Information Exchange, Antitrust, Price Discrimination

JEL Codes: D18, D83, L13, L86

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†University of Mannheim carl-christian.groh@uni-mannheim.de

‡Cornell University preuss@cornell.edu

1 Introduction

We investigate the exchange of user data between firms in search markets. Advances in tracking technologies have made it ever easier for online sellers to collect and share data about consumers. In fact, almost every website on the internet relies on such technologies to recognize the same user over time (Englehardt and Narayanan, 2016). By exchanging consumer identifiers with rivals, firms have a means to inform their rivals that a certain buyer has obtained an offer from them.[1] This form of information sharing may be collusive and harm consumers if it enables firms to coordinate prices. In addition, it provides firms with accurate information about a consumer's shopping history and thereby facilitates price discrimination, which regulatory bodies around the world are increasingly worried about.[2]

In this article, we analyze the effects of such information sharing, which we call search disclosure. Specifically, we ask when search disclosure occurs in equilibrium and whether search disclosure is anti-competitive and harms consumers. We show that search disclosure prevails in equilibrium only if search costs are sufficiently small or if firms cannot adjust a price offer made to a given consumer. Otherwise, firms do not share said information even though industry profits are higher if firms use search disclosure. Evidence of search history-based price discrimination is indeed very limited despite its technical feasibility.[3] Our analysis, which shows that firms share the necessary information only under limited circumstances, thus provides an explanation for this phenomenon.

Formally, we consider the possibility of search disclosure within the sequential search framework by Wolinsky (1986), in which consumers engage in costly sequential search to discover the prices of goods and how much they value them. Specifically, consumers randomly pick which firm to visit first and, based on the firm's price offer and their willingness-to-pay for that firm's good, decide whether to visit a second firm or not. In the model, firms can disclose a consumer's visit to their rival — this is possible when they have received search disclosure regarding the same consumer before as well as when they have not. Search disclosure thus endogenizes the sellers' beliefs about the search history of consumers and can give rise to rich forms of search history-dependent pricing, which we allow for. This includes the possible revision of prices for consumers who continue to search after getting an initial price quote.

[1] These identifiers can be obtained via cookies, tracking pixels, digital fingerprinting and consumer sign-in.

[2] In 2016, the OECD's competition committee recognized that "there are particular reasons to worry that price discrimination in digital markets will be harmful" (OECD Secretariat, 2016). The EU has recently adopted new compliance rules for firms engaging in online price discrimination (European Commission, 2019).

[3] We refer to price discrimination based on which products a consumer has inspected during a sequential search process as search history-based price discrimination.

In equilibrium, firms never disclose a consumer's visit to their rival after having received disclosure for the consumer. This is because any such consumer must have visited the rival first and continued to search. By conducting search disclosure in this situation, a firm would inform its rival about this fact. Since consumers only continue searching if they have a low willingness-to-pay for the initially inspected product, this induces the rival to revise its price downward. Such downward price revisions harm the disclosing seller, which is why there exists no equilibrium in which firms use search disclosure after having received it before.

Consequently, firms will only ever disclose a buyer's visit if they have not received search disclosure regarding the same buyer before – we call this *partial disclosure*. If firms use partial disclosure in equilibrium, they do not know when a consumer continues to search after visiting them. However, a firm that encounters a buyer without having received disclosure about this buyer before will believe to be visited first. In addition, a firm that encounters a buyer after having received disclosure about this buyer knows that it is visited second. That is, partial disclosure enables sellers to price discriminate based on the consumer's search order, which leads to higher industry profits but lower consumer welfare.

However, even though industry profits are higher under partial disclosure, the latter does not arise in equilibrium unless search costs are small. If a firm uses partial disclosure, its rival will quote a higher price to a consumer when being visited second, which benefits the disclosing firm.[4] However, deviating by withholding search disclosure has a surprising benefit as well. Without receiving search disclosure, the rival will always believe that it is visited first and, thus, use search disclosure even if it is actually visited second. Consequently, by withholding search disclosure, a firm will be informed (by its rival) if the buyer continues to search. This allows the deviating seller to screen its buyers and to set a lower price for buyers who continue to search. Since buyers do not expect any price revisions, there are no Coasian dynamics as in Gul et al. (1986). The ability to screen buyers in said way will thus grant a firm strictly higher profits, creating strong incentives to withhold search disclosure.

If search costs are sufficiently large, partial disclosure is not an equilibrium outcome because being able to screen buyers is more valuable than inducing the rival to charge a higher price. If search costs are high, buyers only continue to search if the net utility from buying the first product they sampled is close to or less than zero. Thus, almost no consumer who continues to search would eventually buy the first product at the initially offered price, regardless of the other firm's price. The cost of deviating to not disclosing, which is that a firm's rival will charge a lower price when being visited second, is therefore negligible when search costs are high. By contrast, being able to screen buyers is very profitable in this case

[4] If the rival knows that it is visited second, it understands that the consumer had a low valuation for the disclosing firm's product. This puts the rival in a favorable position, inducing it to set a high price.

because setting a lower price for consumers who continue searching is the only way to earn any profits from them.

If an equilibrium with partial disclosure does not exist, we find that firms use no disclosure in equilibrium, which implies they cannot price discriminate. Firms do not use search disclosure because of the risk of triggering a downward price revision by their rival, which will happen if the consumer has sampled the rival first. In particular, this detrimental effect of deviating to disclosure dominates the beneficial effect of inducing the rival to set a higher price when it is visited second.

The interaction of search costs and search disclosure has major implications for welfare and consumer surplus because both are lower with partial disclosure than without any search disclosure. Thus, a reduction in search costs can lower both total welfare and consumer surplus if it induces sellers to conduct search disclosure in equilibrium. Future advances in technology will likely lower search costs (think of augmented reality) while also enhancing the feasibility of search disclosure. Our research shows that the combination of both can have adverse surplus effects by facilitating a collusive information exchange.

Interestingly, the equilibrium without disclosure is not the best possible outcome from a consumer surplus and welfare perspective. We numerically show that total and consumer surplus is highest if firms always disclose. The intuition for this result is that firms will revise prices downward for any buyer who searches. This encourages search, which additionally raises buyer welfare by improving the average match quality. Because this outcome is never reached if the decision to conduct search disclosure is left in the hands of the firms, regulation that mandates the full provision of search history information might improve outcomes.

Another important result is that the feasibility of price revisions weakly raises total and consumer surplus. This is because what discourages search disclosure is the possibility that the rival might revise its price downward. We show that if price revisions are impossible, firms always conduct search disclosure in equilibrium, leading to price discrimination that reduces consumer surplus and total welfare. In practice, making revised offers to consumers often requires re-targeting. Our analysis thus implies that privacy regulation which limits the ability of firms to conduct re-targeting can potentially lead to more data sharing, with adverse effects on consumer surplus and welfare.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model and Section 4 contains the equilibrium analysis as well as the comparative statics. In Section 5, we discuss the policy implications of our work. Section 6 concludes.

2 Literature review

We make two main contributions to the literature. First, we are the first who analyze the possibility that firms can inform their rivals about the visit of a given buyer. Second, the information that sellers may obtain in our framework gives rise to rich forms of price discrimination, some of which have not been studied before.

We thus contribute to the consumer search literature, in particular to the work on sequential consumer search for differentiated products, which builds on the workhorse model by Wolinsky (1986) and Anderson and Renault (1999). Since information sharing can inform rival sellers about an arriving buyer's search path, our analysis is related to Armstrong et al. (2009) and Zhou (2011), who study prominence and ordered search, respectively. We add to this literature by showing when an outcome comparable to ordered search can emerge endogenously as a result of sellers' information sharing choices.[5]

In addition, search disclosure in our model may enable sellers to revise prices for consumers who continue to search other sellers before making a purchase decision. The idea of discriminating against such consumers is reminiscent of Armstrong and Zhou (2016). The authors explore the phenomenon of search deterrence, i.e., when sellers commit to higher prices for returning consumers.[6] The key differences to that paper are that we 1) allow for discrimination not only against returning consumers but also against consumers who visit the rival first, 2) study discrimination that is based on endogenously provided information, and 3) do not allow firms to commit to future prices.

Search disclosure allows individual firms to price discriminate based on the inferred search history of the buyer. A handful of other recent papers study price discrimination in search markets. Fabra and Reguant (2020) study a simultaneous search model in which firms price discriminate based on perfect information about the quantity that consumers demand. Preuss (2022) studies price discrimination based on the search behavior of consumers, like this paper. Mauring (2021) considers firms which can discriminate against consumers using information about whether a given consumer is a shopper or a non-shopper.[7] In Bergemann et al. (2021), competing firms receive noisy signals about the size of the consumers' choice sets

[5]Ordered search or, similarly, search with prominence, are also studied by Armstrong (2017), Moraga-González and Petrikaitė (2013), and Haan and Moraga-González (2011).

[6]Pan and Zhao (2022) experimentally investigate the role of commitment power for search deterrence. Other related work is by Zhu (2012), who study a sequential bargaining framework with repeat contacts in a market for over-the-counter financial securities.

[7]While not directly addressing price discrimination, De Corniere (2016) studies a model in which consumers differ based on their search query, providing sellers information they use when setting prices. Similarly, consumers in Yang (2013) differ ex ante and thus search within different pools of firms, again giving firms information relevant to their pricing decision.

and the consumers' search costs.[8] None of these papers consider the endogenous exchange of information about consumers' search histories.

This paper also contributes to the literature on information exchange between competitors. The question when rivals benefit from sharing their information with one another was first addressed by Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984), and Gal-Or (1985), who studied the effects of agreements to exchange private information about demand conditions, as well as by Shapiro (1986) and Gal-Or (1986), who consider firms that can share information about private costs. Focusing on information about individual consumers, Chen et al. (2001) study settings where firms receive imperfect information about buyers' consideration sets that they can share (more recent work on the strategic sharing of consumer data includes Kim and Choi, 2010; Zhao, 2012; Choe, Cong, and Wang, 2022). These papers, however, neither consider the exchange of endogenously collected information like this article, nor do they study environments in which buyers sample offers sequentially.

The first work on the topic of sharing endogenously collected consumer information is by Taylor (2004), who studies a multi-period model in which sellers can sell their customer lists to one another.[9] Relatedly, Liu and Serfes (2006) study a two-period Hotelling model in which firms can share preference information they have acquired for all buyers that initially purchase at their firm.[10] In an online advertising context, Johnson et al. (2022) study the conditions under which online sellers agree to share unique identifiers of their websites' visitors with ad exchanges to facilitate re-targeting. In contrast to the above papers, we focus on the sharing of search-related information in a sequential search model.

3 Setting

In this section, we introduce a model of sequential search and information sharing, which is based on Wolinsky (1986). Two firms indexed $j \in \{A, B\}$ each produce a horizontally differentiated and indivisible good at constant marginal cost, which is normalized to zero. A representative consumer wants to buy at most one unit of the good.[11] The consumption utility this consumer attains when buying firm j's good is given by the match value u_j, which is uniformly and independently drawn from the unit interval. This distribution of match values, which we denote by F, is common knowledge.

[8] Garrett et al. (2019) consider a model of second-degree price discrimination in which consumers differ in their choice sets, but firms do not have information about consumers.

[9] De Nijs (2017) considers a related model of a three-firm oligopoly.

[10] Extensions are studied by Choe et al. (2020) and Lin et al. (2021), among others.

[11] Our results continue to hold for a unit mass of consumers because we allow sellers to price discriminate. Laying out the model for a representative consumer is merely for conciseness.

The consumer does not know the realizations $\{u_j\}_{j=A,B}$ at the beginning of the game and has to discover these match values as well as prices via sequential search. When visiting any firm for the first time, she incurs a search cost $s > 0$ to inspect the firm's product, which means to discover the product's match value and price. The consumer has free recall, i.e., she can costlessly return to purchase at a firm that she has previously visited. Without loss of generality, the order in which the consumer visits firms is random.[12]

In addition to this relatively standard set-up, a firm can, upon being visited by the consumer, disclose to its rival that the consumer has inspected its product. We call the sharing of this information, which enables discriminatory pricing, search disclosure. We assume that search disclosure must be truthful so that firms cannot misreport a buyer's visit.[13] Firms do not observe the prices set by the rival firm, nor any match values. In the absence of search disclosure, firms do not know anything about the consumer's search path. Consumers do not observe the firms' disclosure choices. The exact timing is explained below.

The game begins whenever the consumer starts searching. Without loss of generality, let firm A be the first seller the consumer samples. Upon sampling firm A, firm A sets its price p_A and the consumer observes p_A together with her match value u_A. At this stage, firm A also decides whether to disclose the buyer's visit to its rival or to withhold this information. This choice is captured by the variable $d_A \in \{D, ND\}$, where $d_A = D$ indicates that firm A has disclosed and $d_A = ND$ that it has not. The effect of disclosure is that, if the consumer continues to sample firm B, then firm B knows that the consumer visited firm A before.[14] Without observing firm A's disclosure decision, the consumer decides whether to buy (and receive net utility $u_A - p_A$), to continue searching, or to stop searching without a purchase.

If she continues and samples firm B, firm B quotes a price p_B, which the consumer then observes together with her match value u_B. At the same time, firm B decides whether to use search disclosure itself or not ($d_B \in \{D, ND\}$). Notably, firm A knows for sure that the consumer continued to sample firm B if and only if $d_B = D$.

Next, the consumer can, without incurring additional cost, again check firm A, and firm A can make a new price quote p'_A, which may differ from the original price p_A. Whether firm A wants to revise its price depends on whether or not it has learned new information about the consumer. Afterwards, the consumer makes her decision immediately. Specifically, she chooses firm A if $u_a - p'_A > \max(0, u_B - p_B)$, firm B if $u_B - p_B > \max(0, u_A - p'_A)$ and makes

[12]This holds because firms are ex-ante identical and we restrict attention to symmetric equilibria.

[13]This is a plausible assumption in our context since search disclosure is about sharing some unique identifiers, which a firm would have to guess correctly if it wanted to fake search disclosure.

[14]For example, firm A might install a cookie on the consumer's browser that is readable to firm B. Or, it collects identifiable information and shares these with firm B.

no purchase otherwise.[15]

Given this description of the game, we can formalize the information sets in which a firm can be called to act using the following notation:

- $\mathcal{H}(j) = R$ if firm j has received disclosure and has not met the buyer before.
- $\mathcal{H}(j) = NR$ if firm j has not received disclosure and has not met the buyer before.
- $\mathcal{H}(j) = D \times p_j \times R$ if firm j has received disclosure and has met the buyer before, at which point firm j disclosed the visit to its rival (D) and offered the price p_j.
- $\mathcal{H}(j) = ND \times p_j \times R$ if firm j has received disclosure and has met the buyer before, at which point firm j did not disclose the visit to its rival (ND) and offered the price p_j.

A firm's strategy thus has to define (i) what prices to offer if in the information sets $\mathcal{H}(j) = NR$ and $\mathcal{H}(j) = R$, (ii) whether or not to disclose if in the information sets $\mathcal{H}(j) = NR$ and $\mathcal{H}(j) = R$, and (iii) what revision price to set in any information set $\mathcal{H}(j) = D \times p_j \times R$ and $\mathcal{H}(j) = ND \times p_j \times R$.

Characterizing all relevant information sets as above makes clear that we abstract from the passing of time. This reflects the notion that making inferences about the consumer's preferences based on the passage of time alone is challenging for sellers because consumers differ greatly in how long it takes them before they continue their search. As Ursu et al. (2021) document, consumers often take (quite long) breaks in the search process. As a result, merely observing that a buyer has not bought after some time has passed since she received the offer is not very informative about whether she has indeed continued to search. By contrast, receiving search disclosure resolves any uncertainty about whether the consumer has continued to search and is thus more informative than the passing of time could be. Consequently, neglecting the possibility of learning from the passage of time does not affect the predictions of our model qualitatively, but greatly simplifies the analysis.

Moreover, we note that returning to a previously visited firm before making a purchase decision weakly dominates not returning. This is because free recall implies that returning has either no value (if the consumer does not buy) or a positive value (if the consumer buys). Moreover, there are information sets in which the revised price may be lower than

[15] That is, we implicitly rule out the possibility that the second visited firm, firm B in this example, can revise its price as well. The rationale for this assumption is that the consumer has perfect information about all match values at this point and thus makes a decision relatively quickly (she does not even need to leave B's website in order to see updated prices from A). In addition, the analysis would be equivalent if we assumed that firm A never discloses to firm B if the consumer returns. The intuition we build throughout the forthcoming analysis strongly suggests that firm A would indeed never disclose to firm B in this case.

the original price, making returning a weakly dominant strategy even for consumers who would not buy at the original price. We therefore specify that the buyer will always return to any previously visited firm before making a purchase here and throughout the main analysis.[16] We also study a model in which some consumers incur a cost to return to a previously visited firm (and thus do not always return) in Appendix Section B.

As a solution concept, we use perfect Bayesian equilibrium (PBE). In a PBE, the buyer's and firms' beliefs must be consistent with Bayes' rule. In information sets that are off the equilibrium path, however, Bayes' rule does not apply. To discipline beliefs, we impose the following standard assumptions on off-equilibrium beliefs: Firstly, the buyer's beliefs are passive — whenever the buyer is offered an off-equilibrium price, her beliefs and expectations about future prices remain unchanged. Secondly, we assume that firms hold passive beliefs about their rivals' prices as well. That is, a firm that unexpectedly receives disclosure continues to believe that the other firm follows the equilibrium pricing strategy.

Thirdly, we need to make assumptions about the beliefs that a firm forms about the possible match values of the buyer it faces in the information sets $\mathcal{H}(j) = D \times p_1 \times R$ and $\mathcal{H}(j) = ND \times p_1 \times R$, if any such information set is off path. Then, the assumption of passive beliefs is not sufficient to pin down beliefs. We specify that these beliefs must be consistent as well. Consistency requires that (1) the firm believes that the buyer it faces has searched according to her equilibrium search strategy and (2) that the firm takes into account what it believes (or knows) about the prices the consumer has received along the search path.

4 Equilibrium analysis

There are three candidates for a symmetric pure-strategy PBE, namely (1) an equilibrium in which firms never disclose to their rivals, (2) an equilibrium in which firms disclose to their competitors if and only if they have not previously received disclosure, and (3) an equilibrium in which firms always disclose to their competitors.[17] We refer to these equilibrium candidates as (1) the *no disclosure equilibria*, (2) the *partial disclosure equilibria*, and (3) the *full disclosure equilibria*, respectively. We distinguish novel equilibrium objects in these different equilibrium candidates via superscripts, namely "n" (no disclosure), "d" (partial

[16] When searching for products online, it takes just one click to return to a previously visited seller to check their offer again. Returning to a previously visited seller is thus different from sampling a new one, which requires finding the seller and inspecting the good. Compared to the search cost associated with the latter two actions, a click is essentially free. Moreover, in online markets free recall is facilitated by re-targeting, which provides consumers with the opportunity to easily return to a previously visited website.

[17] An equilibrium in which firms only disclose after having received disclosure, but not when receiving no previous disclosure would be outcome-equivalent to equilibrium candidate (1).

disclosure), and “f” (full disclosure). In the analysis that follows, we will characterize these three equilibrium candidates and determine when they exist.

Throughout the analysis, we restrict attention to equilibria with active search, which requires that search costs be not too large. Specifically, we restrict attention to search costs for which the buyer would participate in the market if there is no search disclosure, i.e., we assume that $s \leq 1/8$.

4.1 No disclosure equilibria

In a *no disclosure equilibrium*, only one information set is on the equilibrium path, namely $\mathcal{H}(j) = NR$, implying that firms never receive disclosure before meeting a buyer. Consequently, firms charge a uniform price p^* in any symmetric equilibrium. In particular, firms do not discriminate against consumers who continue to search since they cannot observe search behavior.

Consumers anticipate that firms do not price discriminate in equilibrium. Thus, their optimal search rule is given by a simple cutoff strategy: continue searching if and only if $u_j < w^n(p_j)$ (if j is the first seller sampled). Note that consumers do not stop searching without a purchase after sampling the first seller because our assumption of active search ($s \leq 1/8$) implies that $w^n(p_j) \geq p_j$ in equilibrium. To derive $w^n(p_j)$, suppose a consumer enjoys utility r if she stops searching. In this case, she is indifferent between receiving the incremental utility from sampling another seller $-j$ at cost $s > 0$ and consuming r if r satisfies

$$\mathbb{E}_{u_{-j}}[\max\{u_{-j} - p^e_{-j} - r,\ 0\}] = s, \tag{1}$$

where p^e_{-j} denotes the anticipated price at seller $-j$. Solving for r yields $r = w^* - p^e_{-j}$, where $w^* = 1 - \sqrt{2s}$, because match values are drawn from a uniform distribution on $[0, 1]$. To obtain the cutoff $w^n(p_j)$, note that the consumer buys from firm j if $u_j - p_j \geq r = w^* - p^*$, where we used that $p^e_{-j} = p^*$ in equilibrium. Thus, the critical match value satisfies

$$w^n(p_j) = w^* - p^* + p_j. \tag{2}$$

The ensuing pricing game is, of course, equivalent to the problem sellers face in the original Wolinsky (1986) model. The unique equilibrium price p^* thus solves

$$p^* = \frac{1 - (p^*)^2}{1 + w^*}. \tag{3}$$

To support this equilibrium, firms must not have an incentive to disclose when being visited by a buyer. The effect of deviating to disclosure depends on whether a buyer visits the deviating firm, say firm A, first or second. If the buyer visits firm A first and continues to search, then firm A's deviation to disclosure makes firm B reach the off-path information set $\mathcal{H}(B) = R$ (it receives disclosure for an unknown buyer). Consequently, disclosure informs firm B that it is visited second. Thus, B learns that the buyer's match at the rival firm, given by u_A, lies below $w^n(p_A)$, as she would not continue to search otherwise. The profit function firm B maximizes in this case, which we denote by $\Pi^2(p_B)$, is therefore given by

$$\Pi^2(p_B) = p_B\left[\frac{1}{2}F(w^*)\left[1 - F(w^n(p_B))\right] + \frac{1}{2}\int_{p_B}^{w^n(p_B)} F(p^* + u_B - p_B)du_B\right], \tag{4}$$

where we account for firm B's passive beliefs that the consumer received the price p^* at firm A, which implies that $w^n(p_A) = w^*$, i.e., the search cutoff consumers use in equilibrium.

By contrast, if the buyer visits firm A second, then firm B reaches the off-path information set $\mathcal{H}(B) = ND \times p^* \times R$ (it has received disclosure for a buyer to whom it offered the equilibrium price p^* before and whose visit it did not disclose to A). That is, disclosure by firm A informs firm B that the buyer has continued to search after sampling B, allowing firm B to infer that the buyer's match value u_B satisfies $u_B < w^n(p^*) = w^*$. Firm B's expected profit function in this information set, which we denote by $\Pi^3(p_B)$, is therefore given by

$$\Pi^3(p_B) = \frac{1}{2}p_B \int_{p_B}^{w^*} F(u_B - p_B + p^*)du_B. \tag{5}$$

Search disclosure by A thus endows firm B with valuable information to price discriminate against buyers with low match values for B. As it turns out, the ensuring price discrimination is detrimental to A's profits. We learn this from Lemma 1 which characterizes the prices p_2^n and p_3^n that maximize Π^2 and Π^3, respectively.

Lemma 1 *The optimal prices p_2^n and p_3^n following a rival's deviation to disclosure are*

$$p_2^n = \frac{1}{2}\left(1 - \left(w^* - p^*\right)\right) + \frac{1}{4}\left((w^*) - \frac{(p^*)^2}{w^*}\right) \tag{6}$$

$$p_3^n = \frac{2}{3}(w^* + p^*) - \frac{1}{3}\sqrt{(w^*)^2 + 2w^*p^* + 4(p^*)^2} \tag{7}$$

These prices are uniquely determined and satisfy the ordering $p_3^n < p^* < p_2^n$.

The price p_2^n is strictly above p^* because receiving disclosure for a previously unknown buyer lets firm B know that this buyer has visited A first. Thus, if the buyer shows up at firm B, this indicates that she did not obtain a good match at firm A (formally, that $u_A < w^*$). This puts firm B in a competitively favorable position in which it can charge a higher price. The effect is reminiscent of environments in which consumers search firms in a certain and known order as in Zhou (2011).[18]

By contrast, the price p_3^n is below p^*. This is because A's search disclosure informs B that the buyer has continued to search, which B would otherwise not be able to observe. More precisely, B originally sets a uniform price p^* to maximize the joint profits from (i) buyers who arrive at B first and buy immediately, (ii) buyers who arrive at B first and return later, and (iii) buyers who sample A first. Upon receiving (unexpected) search disclosure about a buyer it met before, however, B knows that it faces a buyer from group (ii). Buyers in this group must have a low match value at firm B as they would not have continued to search otherwise. This induces firm B to revise its price downward. Notably, the ordering of the revised and the equilibrium price is different from the one found in Armstrong and Zhou (2016). This is because Armstrong and Zhou (2016) consider a setting in which firms can commit to future prices prices for buyers who continue to search.[19]

Thus, a deviation to search disclosure is beneficial for the deviating firm if the buyer visits this firm first and detrimental if the buyer visits the deviating firm second.[20] To evaluate this trade-off, consider the profit function of firm A if it deviates. Because the buyer neither anticipates nor observes any disclosure, she expects firm B to charge p^* so that her search rule after arriving at firm A is still characterized by the function $w^n(p_A)$. Thus, the profit function of firm A if it deviates to disclosure, which we denote by Π^1, is given by:

$$\Pi^1(p_A) = \underbrace{\frac{1}{2}p_A\left[\big[1 - F(w^n(p_A))\big] + \int_{p_A}^{w^n(p_A)} F(p_2^n + u_A - p_A)du_A\right]}_{\text{expected profits from a consumer who samples } A \text{ first}} +$$

$$\underbrace{\frac{1}{2}p_A\left[F(w^*)\big(1 - F(w^* - p_3^n + p_A)\big) + \int_{p_A}^{w^*-p_3^n+p_A} F(p_3^n + u_A - p_A)du_A\right]}_{\text{expected profits from a consumer who samples } B \text{ first}} \tag{8}$$

[18]Nonetheless, there is a difference between the derivation of p_2^n in our analysis and the derivation of the price the firm visited second would charge in Zhou (2011). This is because there is no price discrimination in equilibrium here, implying that firms' and consumers' expectations, which determine p_2^n, differ.

[19]In their analysis of the monopoly case, the authors include an example without commitment. In this example, the return price is actually higher than the initial price as well. Their result obtains because of the strong asymmetry between the monopolist's offer and the outside option. Specifically, the distribution of the outside option is significantly more attractive than the distribution of the seller's net utility.

[20]Note also that firm B does not disclose "back" to firm A in the event that A was visited first because this would induce A to revise its price downward. We show this formally in the proof of Proposition 1.

To understand the expected profits from a consumer who starts at firm B, notice that $u_B < w^*$ must hold if the buyer shows up at firm A. Thus, such a buyer will surely consume at firm A if $u_A - p_A > w^* - p_3^n$, as reflected in the first term. If the buyer's net surplus at firm A is below $w^* - p_3^n$, she still consumes at firm A if $u_A - p_A$ is greater than zero and greater than $u_B - p_3^n$, which holds with probability $F(p_3^n + u_A - p_A)$. If this event holds for a buyer with $u_A < w^* - p_3^n + p_A$, the buyer will surely sample firm A.[21] We find that the adverse effect of disclosure strictly dominates for any $s > 0$.

Proposition 1 *There always exists a unique no disclosure equilibrium in which sellers charge p^*. In this equilibrium, deviating by non-disclosure is strictly unprofitable.*

There are two reasons why an equilibrium with no disclosure can be sustained for any level of search costs. The first is that the rival's price reduction $(p^* - p_3^n)$ for buyers that sampled the rival first exceeds the rival's price increase $(p_2^n - p^*)$ for buyers who sampled the disclosing seller first. Figure 1 visualizes this fact, which holds by the following logic: When firm B receives search disclosure about a buyer it has not seen before, the only inference this firm can make when the buyer arrives is that $u_A < w^*$, which concerns the rival's product. By contrast, when B receives search disclosure about a buyer it has seen before, it learns that $u_B < w^*$, which concerns its own product. Because search disclosure is more informative about the buyer's demand for the own product in the latter case, the subsequent price reduction is greater in magnitude than the price increase in the former.

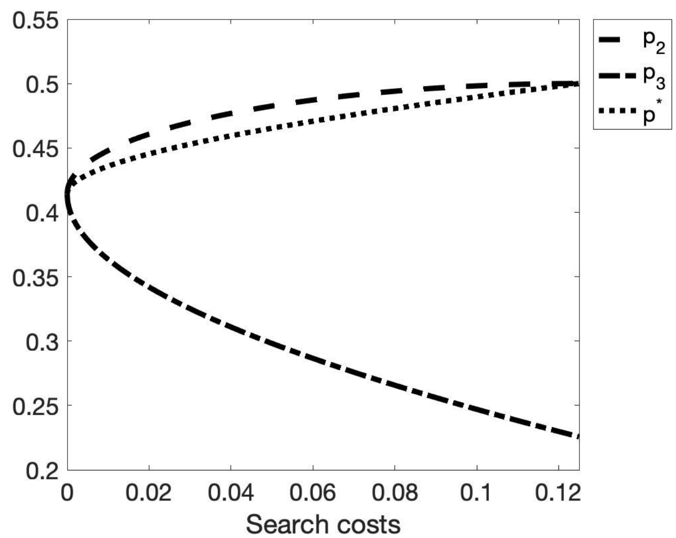

Figure 1: Equilibrium (Wolinsky) price and off-path prices

[21]This is because $u_B < p_3^n + u_A - p_A < p_3^n + (w^* - p_3^n + p_A) - p_A = w^*$, where the latter equals $w^n(p^*)$.

The second effect underlying Proposition 1 is that disclosure by firm A will reduce demand from a consumer who sampled B first more than it increases demand from a consumer who samples A first, even if the changes in p_2^n and p_3^n were equal in magnitude. This holds by the following logic: Suppose $p_2^n = p^* + \delta$ and $p_3^n = p^* - \delta$. If the buyer starts at firm B (the non-deviating firm), B's revised price of $p^* - \delta$ instead of p^* means that she will be $1/2\delta$ less likely to choose firm A for sure. By contrast, if the buyer starts at firm A, a price of $p^* + \delta$ instead of p^* at firm B has no comparable effect. This is because the probability that such a buyer surely buys at firm A, namely $1 - F(w^* - p^* + p_A)$, is unaffected by search disclosure, given that buyers continue to expect the price p^* at firm B. Thus, buyers are more likely (by a difference of $1/2\delta$) to fall into the category in which their demand decreases.

4.2 Partial disclosure equilibria

We now consider equilibria in which firms disclose to their competitors if and only if they have not received disclosure beforehand. This disclosure strategy implies that firms are certain about whether they are visited first or second in the two information sets that are on path in such an equilibrium. If a firm faces an unknown buyer for whom no disclosure was received (i.e. $\mathcal{H}(j) = NR$), this firm knows that it is being visited first. If a firm faces an unknown buyer for whom disclosure was received (i.e. $\mathcal{H}(j) = R$), this firm knows that it is being visited second.

Consequently, the symmetric pure strategy equilibrium features two on-path prices p_1^* (set by the seller sampled first) and p_2^* (set by the seller sampled second). Consumers anticipate that sellers do not know whether they continue to search or not and, thus, do not expect prices to be revised in equilibrium. The optimal search rule thus still uses a simple cutoff value. Using previous notation, this cutoff value is given by

$$w^d(p_j) = w^* - p_2^* + p_j \tag{9}$$

so that a consumer buys from firm j (when j is visited first) and without sampling $-j$ if and only if $u_j \geq w^d(p_j)$.

This set-up is, of course, comparable to a model of ordered search. We can therefore invoke existing results from Armstrong et al. (2009) to characterize the partial disclosure equilibrium prices p_1^* and p_2^*. Importantly, Armstrong et al. (2009) show that $p_2^* > p_1^*$. Moreover, the cutoff for an active search market to exist is still $s \leq 1/8$.[22]

[22]The reason is that as $s \to 1/8$, p_1^* and p_2^* converge to p^* so that the buyer expects the same prices (and thus the same surplus from search) with and without disclosure.

There are two information sets in which each seller $j \in \{A, B\}$ can deviate from the partial disclosure equilibrium strategy. Without loss of generality, suppose the buyer samples firm A first. That is, firm A will be in the information set $\mathcal{H}(A) = NR$ when the game begins as the buyer samples firm A. In this information set, firm A must not want to deviate to non-disclosure, or there is no partial disclosure equilibrium. In addition, if the consumer continues to sample firm B and firm A sticks to its equilibrium strategy, then B is in the information set $\mathcal{H}(B) = R$ as it will have received search disclosure from A. In this information set, the partial disclosure strategy dictates that firm B must not disclose back to firm A when the buyer arrives. Notably, if $\mathcal{H}(B) = R$, deviating to disclosure informs A that the buyer has continued to search. As argued before, this will lead seller A to revise its price downward, making such a deviation generally unprofitable.

The effects of a deviation to non-disclosure in the first event when no disclosure was received previously are more complex. Suppose that firm A does not disclose a buyer's visit when $\mathcal{H}(A) = NR$. If the buyer continues to search, then firm B faces a buyer for whom it has not received disclosure before. Firm B will thus incorrectly believe that it is the first firm the buyer visits, and offer p_1^* instead of p_2^*. In addition, firm A's deviation has a second effect on A's profits via the influence on B's subsequent disclosure decisions. If the buyer continues to search firm B, this firm is now in the information set $\mathcal{H}(B) = NR$ (instead of $\mathcal{H}(B) = R$ if A followed the equilibrium strategy). As a result, firm B, following the equilibrium play, will use search disclosure itself. This, in turn, will let firm A know if a consumer has continued to search. Since only buyers with match values below $w^d(p_A)$ continue to search, deviating to non-disclosure practically allows A to sequentially screen its buyers.

In sum, deviating to not disclosing when a consumer arrives at firm A first (firm A knows this in the partial disclosure equilibrium) has two opposing effects. On the one hand, it makes firm B charge a lower price, which lowers firm A's profits. On the other hand, it allows A to sequentially screen its buyers and to revise the price for buyers that continued to search. Notice that buyers do not expect any price revisions in a partial disclosure equilibrium and, thus, would never sample the other seller (firm B) only to get a lower price at firm A. Thus, even though the buyer's optimal search rule leads to negative selection, there are no Coasian dynamics as in Gul, Sonnenschein, and Wilson (1986). Revising the price for consumers who continue searching is therefore strictly profitable.

To evaluate which effect dominates, we first characterize how firms revise their initial prices when deviating to non-disclosure. To that end, we must derive the price a firm offers in each information set $\mathcal{H}(j) = ND \times p_1 \times R$, which depends on the firm's initial price p_1. If firm A charges an initial price p_1 when deviating to non-disclosure, the buyer continues to search firm B if and only if $u_A < w^d(p_1) = w^* - p_2^* + p_1$. Additionally, firm A knows that firm B sets the equilibrium price p_1^* after the deviation to non-disclosure (if the buyer continues to search). Thus, the revised price p_3 maximizes $\Pi^{3,d}(p_3|p_1)$:

$$\Pi^{3,d}(p_3|p_1) = p_3 \int_{p_3}^{w^d(p_1)} \frac{1}{2} F(u_A - p_3 + p_1^*) du_A. \tag{10}$$

The optimal price in $\mathcal{H}(j) = ND \times p_1 \times R$, which we denote by $p_3^d(p_1)$, is a function of p_1. Next, we analyze the profit function that A maximizes when choosing an initial price p_1 if it deviates to non-disclosure, knowing that it will be able to screen buyers based on whether they continue to search or not. This profit function, which we call $\Pi^{1,d}(p_1)$, is given by:

$$\Pi^{1,d}(p_1) = p_1 \frac{1}{2}\big[1 - F(w^d(p_1))\big] + p_3^d(p_1) \int_{p_3^d(p_1)}^{w^d(p_1)} \frac{1}{2} F(p_1^* + u_A - p_3^d(p_1)) du_A \tag{11}$$

Let p_1^d maximize $\Pi^{1,d}(p_1)$. By analyzing the first-order conditions that $p_3^d(p_1)$ and p_1^d must satisfy in the subgame following a deviation to non-disclosure, we obtain the following result.

Lemma 2 *Suppose firm j deviates at $\mathcal{H}(j) = NR$ to $d_j = ND$. Then, the optimal initial price p_1^d and the optimal revision price function $p_3^d(p_1)$ satisfy*

$$p_3^d(p_1) = (2/3)(w^d(p_1) + p_1^*) - (1/3)\sqrt{(w^d(p_1))^2 + 2w^d(p_1)p_1^* + 4(p_1^*)^2} \tag{12}$$

$$p_1^d = 1 - w^d(p_1^d) - \big(p_3(p_1^d)\big)^2 + p_3(p_1^d)\big(w^d(p_1^d) + p_1^*\big). \tag{13}$$

Both p_1^d and $p_3^d(p_1)$ are uniquely determined.

The uniqueness of p_1^d and $p_3^d(p_1)$ is important because it implies that the partial disclosure equilibrium exists if and only if the deviation profits given p_1^d and $p_3^d(p_1^d)$ do not exceed the equilibrium profits given p_1^* and p_2^*. We plot the four prices $(p_1^*, p_2^*, p_1^d, p_3^d(p_1^d))$ for different values of search costs in Figure 2.

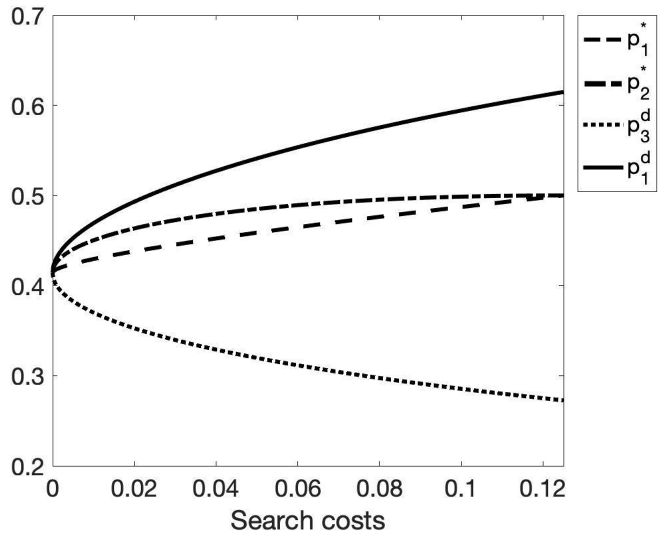

Figure 2: Equilibrium and deviation prices

With the system of equations for p_1^d and $p_3^d(p_1)$ as well as for p_1^* and p_2^*, we can characterize analytically when the partial disclosure equilibrium exists.

Proposition 2 *There exists a threshold $\bar{s} > 0$ such that the partial disclosure equilibrium exists if $s \leq \bar{s}$. In this equilibrium, prices are uniquely determined. There also exists a $\bar{s}' > \bar{s}$ such that a partial disclosure equilibrium does not exist if $s > \bar{s}'$.*

Our numerical analysis complements the proposition by showing that $\bar{s} = \bar{s}'$. That is, the partial disclosure equilibrium exists up to a unique cutoff value $\bar{s} \approx 0.01$.

To understand why the partial disclosure equilibrium cannot be sustained if search costs are high, consider the case that search is very costly. In particular, suppose it is so costly that a buyer would search after sampling the first firm, say firm A, only if u_A is approximately equal to or smaller than p_A. Then, any buyer who continues to search is very unlikely to buy from firm A after sampling firm B, at least at A's original price. This mitigates the detrimental effect of deviating to non-disclosure because buyers who search further will most likely not buy from firm A regardless of whether B charges p_1^* or p_2^*. By contrast, learning that a buyer continued to search is very profitable for A if search costs are high. This is because a revised price of p_3^d can lead to significant demand from a buyer who continued to search, which is almost zero if A does not revise its price. Consequently, the beneficial effect of a deviation by non-disclosure dominates when search costs are high.

To understand why the partial disclosure equilibrium does exist if search costs are small, however, examine Figure 2 and note how the prices compare to each other. The gap between

p_2^* and p_1^* increases the fastest in s when s is small. As argued before, the larger this difference, the greater is the cost of deviating from partial disclosure. Intuitively, the ratio of the $p_2^* - p_1^*$ gap and the $p_1^d - p_3^d(p_1^d)$ gap is largest when search costs are small, which means that the detrimental effect of a deviation by non-disclosure dominates when search costs are small.

4.3 Full disclosure equilibria

Consider the third possible equilibrium candidate, in which firms disclose to their competitors regardless of whether they have received disclosure before or not, i.e. $d_j = D$ if $\mathcal{H}(j) = NR$ or $\mathcal{H}(j) = R$. If both firms stick to this disclosure strategy, a firm can reach any of the following information sets: $\{NR\}$, $\{R\}$, and $\{D \times p_1 \times R\}$, where p_1 is the firm's arbitrary initial price. If $\mathcal{H}(j) = NR$, firm j believes it is visited first. We denote the equilibrium price firm j would set in this information set by p_1^f, where the superscript f refers to "full" disclosure. If $\mathcal{H}(j) = R$, firm j believes it is visited second and sets the price p_2^f in equilibrium. Lastly, firm j knows that it was visited first but that the buyer has also sampled firm $-j$ if $\mathcal{H}(j) = D \times p_1 \times R$, in which case firm j will offer a revised price given by $p_3^f(p_1)$.

To characterize the equilibrium prices, we must first solve for the consumers' optimal search rule. If firms follow the full disclosure strategy, prices will be revised on the equilibrium path. As a result, a consumer who has sampled one firm already anticipates that if she samples the other firm as well, then the price at the initially visited firm will change from p_1 to $p_3^f(p_1)$. The optimal search rule in this case is thus non-standard and must first be derived.

Lemma 3 *For any initial price p_1, there exists a $w^f(p_1)$ such that consumers will continue searching if and only if their initial match value is below $w^f(p_1)$. In equilibrium, $p_3^f(p_1) < w^f(p_1)$. If this cutoff is interior (strictly below 1), it solves:*

$$w^f(p_1) - p_1 = \underbrace{\int_0^{w^f(p_1) - p_3^f(p_1) + p_2^f} (w^f(p_1) - p_3^f(p_1))du + \int_{w^f(p_1) - p_3^f(p_1) + p_2^f}^1 (u - p_2^f)du - s}_{\text{Expected utility conditional on searching}} \quad (14)$$

Suppose that the consumer visits firm A first. The first integral on the right-hand side of (14) captures the expected value of being able to buy from firm A at the anticipated revised price $p_3^f(p_1)$ while the second integral captures the expected value of being able to buy from firm B at the anticipated price p_2^f. Given a value $w^f(p_1)$ as determined by (14), the consumer will, in equilibrium, continue to search if and only if her match value at the first firm is below $w^f(p_1)$.[23] For future reference, we define the equilibrium value of this cutoff as $w^f := w^f(p_1^f)$.

[23]We show in the appendix that the search cutoff must be interior in an equilibrium.

Taking note of the consumer's optimal search rule, we are ready to derive the equilibrium prices. If firm A receives disclosure for a buyer whom it quoted the price p_1 before, A knows that $u_A < w^f(p_1)$ because the buyer continued to search. In the information set $\mathcal{H}(A) = D \times p_1 \times R$, firm A maximizes the following profit function through choice of p_3:

$$\Pi^{3,f}(p_3|p_1) = \frac{1}{2}p_3 \int_{p_3}^{w^f(p_1)} F(u_A - p_3 + p_2^f)du_A. \tag{15}$$

We have already defined the solution to this as $p_3^f(p_1)$. Now consider the situation of a firm, say B, which receives disclosure for a previously unknown buyer ($\mathcal{H}(B) = R$). When this consumer shows up at firm B, it believes that the match value of the consumer at firm A satisfies $u_A < w^f$. Additionally, firm B expects that by following the equilibrium strategy of disclosing back to firm A, the price at which the consumer can buy from firm A is the revised equilibrium price p_3^f, where $p_3^f := p_3^f(p_1^f)$. Thus, firm B sets p_2 to maximize:

$$\Pi^{2,f}(p_2) = \frac{1}{2}p_2F(w^f)\big[1 - F(w^f - p_3^f + p_2)\big] + \frac{1}{2}p_2 \int_{p_2}^{w^f - p_3^f + p_2} F(p_3^f + u_B - p_2)du_B, \tag{16}$$

the solution to which we have already defined as p_2^f.[24]

Finally, consider a firm, say A, which is visited by a buyer about whom no disclosure was received yet so that A's information set is $\mathcal{H}(A) = NR$. In a full disclosure equilibrium, this information set is only reached if the buyer did not visit another firm before. Thus, firm A knows that it is the first firm this buyer visits. When setting its price p_1, firm A takes into account how p_1 affects the consumer's search decision (captured by the function $w^f(p_1)$) as well as the price $p_3^f(p_1)$, which A will revise its original price to if it subsequently receives disclosure from B. Formally, firm A's optimal price in this case (given by p_1^f) maximizes:

$$\Pi^{1,f}(p_1) = \frac{1}{2}\big[1 - F(w^f(p_1))\big]p_1 + \underbrace{\frac{1}{2}p_3^f(p_1) \int_{p_3^f(p_1)}^{w^f(p_1)} F(p_2^f + u_A - p_3^f(p_1))du_A}_{=\Pi^{3,f}(p_3|p_1)} \tag{17}$$

Pinning down the prices that maximize (15) – (17) allows us to describe necessary conditions that the prices in a full disclosure equilibrium must satisfy.

Lemma 4 *Consider a full disclosure equilibrium. Given w^f, the equilibrium prices (p_1^f, p_2^f, p_3^f)*

[24]The function $\Pi^{2,f}(p_2)$ correctly depicts the profits of firm B in the information set $\mathcal{H}(B) = R$ if $w^f - p_3^f + p_2 \le 1$. We verify that this condition must hold true for prices p_2 in an open ball around p_2^f.

must jointly solve the following system of equations:

$$p_3^f = \frac{2}{3}(w^f + p_2^f) - \frac{1}{3}\sqrt{\left(w^f + p_2^f\right)^2 + 3\left(p_2^f\right)^2} \tag{18}$$

$$p_2^f = \frac{1}{2}\left(1 - w^f + p_3^f\right) + \frac{1}{4}w^f - \frac{1}{4}w^f\left(p_3^f\right)^2 \tag{19}$$

$$p_1^f = (1 - w^f)\left(\frac{\partial w^f(p_1^f)}{\partial p_1}\right)^{-1} + p_3^f\left(p_2^f + w^f - p_3^f\right) \tag{20}$$

We show that, for any $w^f \in [0,1]$, there exists a unique vector of prices that jointly solves these three equations. This allows us to establish that full disclosure cannot constitute an equilibrium.

Proposition 3 *A full disclosure equilibrium does not exist.*

Intuitively, a full disclosure equilibrium requires that $p_1^f \leq p_3^f$. To see this, consider the decision situation of a firm who faces a buyer it has already received disclosure about so that this firm knows that it is visited second. In a full disclosure equilibrium, this firm is supposed to disclose the buyer's visit to its competitor. However, doing so is strictly unprofitable if the price revision induced by this disclosure decision is downward, i.e., it is unprofitable if $p_3^f < p_1^f$. This is because a lower price at the rival firm entails a reduction in the demand of the disclosing firm. However, the negative selection effect implies that the optimal prices satisfy $p_3^f < p_1^f$.

Formally, we show that the unique joint solution to the equations (18) - (20) has the property $p_3^f < p_1^f$ for any $w^f < 1$, which thus cannot support a full disclosure equilibrium. In addition, if $w^f = 1$, all prices would be exactly equal because a search cutoff of $w^f = 1$ means that buyers sample both sellers before making a purchase decision regardless of their match values, rendering any information obtained from search disclosure irrelevant. However, if all prices are the same, then consumers with sufficiently high match values at the first firm they visit must not find it profitable to continue to search since there are strictly positive search costs. In other words, consumers with initial match values below 1 would buy immediately, contradicting the specification that $w^f = 1$.

4.4 Comparative statics

We here seek to understand the relationship between the incidence of search disclosure and search costs. To deal with equilibrium multiplicity, we make the following assumption.

Assumption 1 *If multiple equilibria exist, we remove any equilibrium which is strictly Pareto dominated by another equilibrium, in terms of the payoffs of the firms.*

Assumption 1 seems justified in our context because tracking a consumer is a game that is usually repeated several thousand times per day. As a result, coordinating on their preferred equilibrium should be feasible for sellers. Moreover, our model describes an environment in which consumers do not arrive all at once but over time. While we do not model this explicitly, we imagine that if firms play the equilibrium strategy of their preferred equilibrium, consumers will quickly notice and adjust equilibrium beliefs accordingly. Since each consumers' individual mass is zero, ignoring the beliefs of a few initial consumers and simply playing the equilibrium strategy of their preferred equilibrium is essentially costless for firms.

The previous analysis has shown that we can restrict attention to the partial and the no disclosure equilibrium. Using Assumption 1 requires knowing which equilibrium firms prefer if both exist. To this end, recall that profits in the no disclosure equilibrium equal profits if search is random while profits in the partial disclosure equilibrium equal average profits if search is ordered. Thus, we know from Armstrong et al. (2009) that the partial disclosure equilibrium is preferred from the firms' perspective if $s \leq 0.021$. Because the partial disclosure equilibrium exists when search costs are sufficiently small by Proposition 2, while the no disclosure equilibrium exists for all search costs by Proposition 1 (and full disclosure never prevails), the next result follows immediately from the preceding discussion.

Proposition 4 *There is search disclosure in equilibrium if s is sufficiently small and no search disclosure if s is sufficiently high.*

Based on previous calculations that show that the partial disclosure equilibrium exists when search costs are below $\bar{s} \approx 0.01$, we can in fact conclude that there is a unique cutoff value $\bar{s} \approx 0.01$ which determines whether there is search disclosure in equilibrium or not.

Recall that the partial disclosure equilibrium achieves the same welfare and consumer surplus as the equilibrium of an ordered search model whereas the no disclosure equilibrium is outcome-equivalent to the standard random search equilibrium. We can thus leverage results from Armstrong et al. (2009) once more, who show that prominence (ordered search) lowers total surplus and consumer surplus compared to random search. Thus, the next result is an immediate corollary to Proposition 4.

Corollary 1 *A marginal reduction in search costs that triggers a shift to the partial disclosure equilibrium will lower consumer surplus and total welfare.*

5 Policy implications

In this section, we study the equilibrium outcomes that emerge when modifying the framework we outlined and solved previously. We consider the equilibrium outcomes when firms cannot revise prices (Section 5.1) and when firms exogenously receive full search history information (Section 5.2). This analysis shows that a policymaker interested in maximizing consumer welfare should ensure that price revisions by firms are feasible and consumers can easily observe revised prices. In addition, the sharing of search history related information should not be left up to firms. This is because consumer welfare is maximal when firms exogenously receive full search history information (in the model with price revisions), an outcome which is impossible under voluntary search disclosure by the preceding analysis.

5.1 Banning price revisions

In this section, we solve the aforementioned model when firms cannot revise prices. Under this specification, there are just two relevant candidates for a symmetric pure-strategy PBE, namely (1) the no disclosure equilibrium and (2) the partial disclosure equilibrium. The full disclosure and the partial disclosure equilibrium are outcome-equivalent because a firm that receives disclosure for a known buyer has no more choices to make.

In a nutshell, the benefits of non-disclosure in the main analysis emerged from the possibility of price revisions, either by the rival firm or by the firm itself. When this possibility is eliminated, disclosing becomes strictly profitable regardless of search costs. Thus, when price revisions are impossible, the equilibrium result from the previous section basically flips.

Proposition 5 *If firms cannot revise prices, the no disclosure equilibrium does not exist, while the partial disclosure equilibrium always exists.*

To see why the no disclosure equilibrium never exists when price revisions are impossible, recall the trade-off firms face in the no disclosure equilibrium of the baseline model. If a firm discloses and that firm is the first firm the consumer visits, its rival will set a higher price (compared to the price the rival sets if the firm does not disclose). If a firm discloses and that firm is the second firm the consumer visits, the rival learns that it should revise its price downward. In the baseline model, the detrimental effect of potentially triggering a downward price revision dominates the benefit that accrues if a consumer visits the disclosing firm first. If price revisions are impossible, however, the second channel is shut down, and only the beneficial effect of disclosure remains. This implies that firms always deviate from the no disclosure strategy.

By contrast, the partial disclosure equilibrium always exists when price revisions are impossible. The two equilibrium prices, namely p_1^* and p_2^*, are equal to the equilibrium prices from Armstrong et al. (2009). As before, a firm that deviates by non-disclosure when being visited first will make its rival wrongly believe to be visited first. The non-disclosure deviation thus reduces the rival's price from p_2^* to p_1^*. In the baseline model, such a deviation also offered the benefit of being notified by the rival if a consumer continued to search, which allowed the deviating firm to screen consumers. When price revisions are impossible, however, screening buyer types with different prices is not feasible. Thus, only the detrimental effect of the deviation to non-disclosure remains, rendering partial disclosure an equilibrium strategy for any level of search costs.

Consequently, if price revisions are impossible, there is search history-based price discrimination in equilibrium. Consumers will be charged different prices at either firm, depending on whether they visit this firm first or second. By previous arguments, consumer welfare in such an equilibrium is lower than in the no disclosure uniform price equilibrium.

The policy implications of this section are thus twofold: Firstly, the feasibility of price revisions weakly raises consumer welfare. Secondly, note that we modeled the effects of price revisions if firms have no commitment power. Thus, the feasibility of price revisions has to go hand in hand with provisions ensuring that firms cannot discourage search by committing to high prices for consumers who continue to search as in Armstrong and Zhou (2016).

5.2 Exogenous search disclosure

In this section, we suppose that a third party guarantees that each firm is informed about a buyer's entire search history, i.e., about all search decisions. This could be achieved, for example, by regulation that mandates search disclosure at all times. Alternatively, an online platform on which firms sell their products and consumers search, or more generally any large data intermediary, could make such search history information available to sellers.

As in Section 4.3, there are three prices a consumer can be offered on the equilibrium path, which we call $p_1^{f,*}, p_2^{f,*}$, and $p_3^{f,*}$. The prices $p_1^{f,*}$ and $p_2^{f,*}$ will be offered by a firm when a consumer visits this firm first and second, respectively. The price $p_3^{f,*}$ is offered by the first visited firm after the consumer samples the second firm as well. By Lemma 3, the

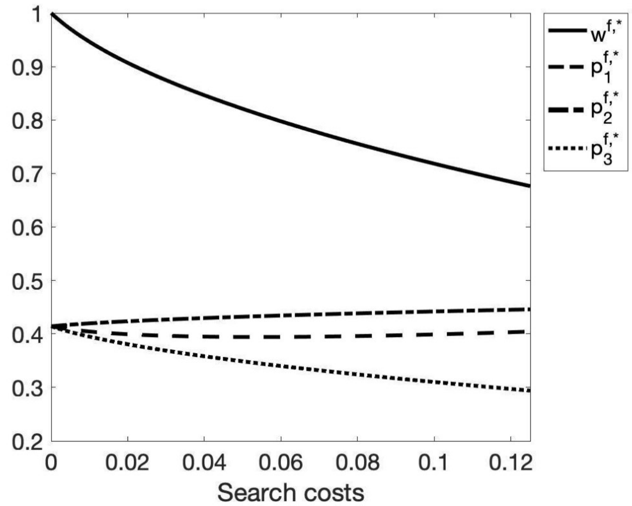

Figure 3: Equilibrium with Exogenous Information

consumers' optimal search behavior can be characterized by a cutoff rule $w^f(p_1)$ as defined in (14), where p_1 is an arbitrary initial price offered by the first visited seller.

It remains to derive the equilibrium objects for different levels of search costs. In equilibrium, there must be a search cutoff $w^{f,*}$ such that consumers find it optimal to continue searching if and only if the first seller's match value is below $w^{f,*}$, as well as a vector of prices $(p_1^{f,*}, p_2^{f,*}, p_3^{f,*})$ that jointly solve (18), (19), and (20), given $w^{f,*} = w^f(p_1^{f,*})$. Using numerical methods, we are able to compute the joint solution for any $s \in [0, 1/8]$. Moreover, we verify that this combination of prices, together with the search cutoff $w^f(p_1)$ as defined in equation (14) and the optimal revision price function $p_3^f(p_1)$ for an arbitrary initial price p_1, satisfy the sufficient conditions for an equilibrium. We visualize the on-path objects $(p_1^{f,*}, p_2^{f,*}, p_3^{f,*}, w_f^*)$ for any $s \in [0, 1/8]$ in Figure 3.

The Figure shows that $p_2^{f,*} > p_1^{f,*} > p_3^{f,*}$ for all $s > 0$, which aligns with the intuition we have developed in the previous sections. Contrary to some of the previous results, however, the prices $p_2^{f,*}$ and $p_1^{f,*}$ do not approach the monopoly price of 1/2 as search frictions grow large. This is because consumers now anticipate that continuing to search leads the initially visited firm to lower its price to $p_3^{f,*}$, which creates additional incentives to search and, thus, more competition. This notion is also reflected by the fact that the search cutoff w_f^* lies firmly above $p_1^{f,*}$ even if search costs are large. That is, while in the baseline model consumers would always buy immediately from the first visited firm (without continuing to search) at any price below their match value if $s \to 1/8$, they only buy immediately in this extension if the offered price is sufficiently smaller than their match value.

With all these results in hand, we now discuss how buyer surplus and profits are affected by the exogenous provision of search history information. Specifically, we are interested in how the outcome with exogenous search history information compares with the outcome of the game with endogenous search disclosure.

The exogenous provision of search history information increases consumer surplus. We visualize this in Figure 4, in which we compare the surplus in the no disclosure equilibrium to the consumer surplus under exogenous provision of search history information. In the no disclosure equilibrium, firms set the uniform price p^*, which means that consumer surplus in this equilibrium is equal to consumer surplus in the Wolinsky (1986) equilibrium.

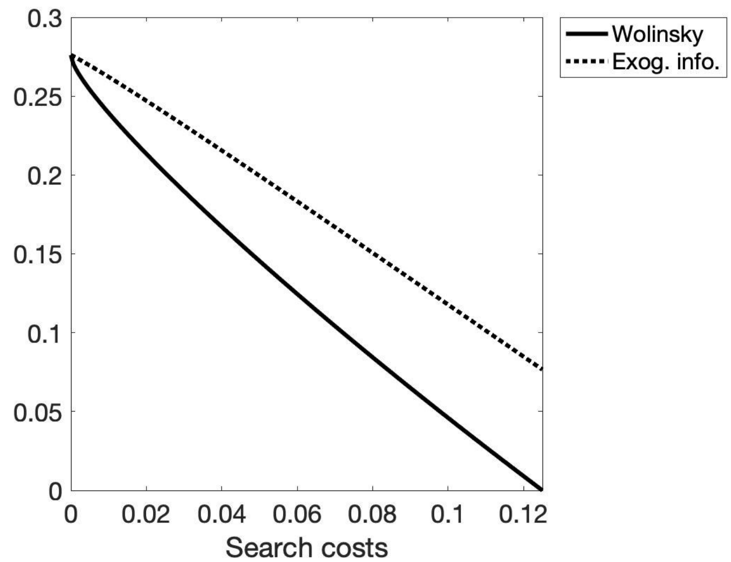

Figure 4: Consumer Surplus with Exogenous Information

Consumer surplus is higher under the exogenous provision of search history information than in the Wolinsky (1986) equilibrium. This is because the revision prices are comparatively low, which is favorable for consumers in general and also implies that even consumers with low match values make a purchase eventually. Moreover, low revision prices encourage search, which improves the average match quality of the purchased good. Both of these effects raises consumer surplus. In addition, these effects also imply that total welfare is higher when firms have exogenous access to full search history information.

Finally, recall that the no disclosure equilibrium, in which consumer welfare is the same as in the Wolinsky (1986) equilibrium, exists for all search costs under endogenous search disclosure (if price revisions are feasible). However, if search costs are sufficiently small or price revisions impossible, the partial disclosure equilibrium exists and will be selected because it is preferred by firms. Notably, previous arguments have established that buyer surplus in the partial disclosure equilibrium is even lower than in the Wolinsky (1986) equilibrium. Thus, exogenous information provision makes buyers weakly better off compared to the outcome under endogenous search disclosure, regardless of our equilibrium selection criterion.

6 Conclusion

We have studied the incentives of firms to exchange information about consumers in a sequential search framework. When being visited by the buyer, a firm can notify its rival — we refer to this as *search disclosure*. Search disclosure benefits the disclosing firm if it is visited first by the buyer, but is detrimental if the buyer visits the disclosing firm second and firms are able to adjust the price they offer to a given consumer.

The possibility of price revisions is thus of central importance for the incidence of search disclosure. If revising prices is not feasible, firms will always disclose to their competitors in equilibrium. This prediction is reversed if revising prices is feasible. Then, we show that an equilibrium without disclosure is the unique pure-strategy equilibrium for a large range of search costs. Firms will only conduct search disclosure in equilibrium when search costs are sufficiently small, even though industry profits can be raised by sharing said information. Thus, the possibility of price revisions prevents price discrimination and thus weakly raises consumer welfare, even though prices are never revised in equilibrium.

An important implication of our work is that policymakers should codify an explicit right for price revisions in the markets we study and make the possibility of them common knowledge. The importance of price revisions suggests a critical benefit of firms being able to re-target visitors because re-targeting is a means to inform consumers about revised prices. Moreover, firms must not be able to commit to future prices as in Armstrong and Zhou (2016). Arguably, ensuring and announcing such a right may be easier than prohibiting communication between firms.

Another obstacle to firms being able to offer revised prices might arise if some consumers can only see the revised price at a cost. We therefore study a scenario in which a positive (and possibly large) fraction of consumers face recall costs in Appendix Section B. We find that the no disclosure equilibrium continues to exist for a wide range of search costs, except if search costs are small. The reason it does not exist for small search costs is that the downside of disclosing, which is the potential downward price revision by a rival, weighs less when some consumers do not see revised offers due to costly recall. By the same token, we argue that the partial disclosure equilibrium continues to exist for small search costs (with a widening range as more consumers face costly recall).[25]

Finally, we note that third parties in the search models we study could ensure access to detailed search history information for the participating firms, offering a substitute for voluntary search disclosure by firms. We show that buyers would benefit from the exogenous availability of this information if prices are revisable, and are indifferent otherwise.

[25]We limit ourselves to offering a verbal discussion and conjectures regarding the partial and full disclosure equilibrium because the analysis is not very tractable.

A Mathematical Appendix

Proof of Lemma 1: This proof consists of four parts. In the first part, we derive p_2^n and in the second part, we show that $p_2^n > p^*$. In part three, we derive p_3^n and part four verifies that $p_3^n < p^*$.

Part 1: Deriving p_2^n. In $\mathcal{H}(B) = R$, the perceived profit function of firm B is:

$$\Pi_2(p_B) = p_B \underbrace{\left\{\frac{1}{2}F(w^*)\big[1 - F(w^n(p_B))\big] + \frac{1}{2}\int_{p_B}^{w^n(p_B)} F(u_B + p^* - p_B)du_B\right\}}_{:=D^2(p_B)},$$

where $w^n(p_B) = w^* - p^* + p_B$ as defined in the main text. The derivative of $D^2(p_B)$ with respect to p_B reads:

$$\frac{\partial D^2(p_B)}{\partial p_B} = -\frac{1}{2}F(w^*)f(w^n(p_B)) + \frac{1}{2}\left[F(w^*) - F(p^*) - \int_{p_B}^{w^n(p_B)} f(u_B + p^* - p_B)du_B\right] = -\frac{1}{2}w^*$$

Similarly, demand simplifies to:

$$D^2(p_B) = \frac{1}{2}w^*\big[1 - \big(w^* - p^* + p_B\big)\big] + \frac{1}{2}\left[\frac{1}{2}(w^*)^2 - \frac{1}{2}(p^*)^2\right]$$

The price p_2^n must solve the first-order condition $p_B\frac{\partial D^2(p_B)}{\partial p_B} + D^2(p_B) = 0$. Plugging in the above results yields:

$$p_2^n = \frac{1}{2}\left(1 - (w^* - p^*)\right) + \frac{1}{4}\left(w^* - \frac{(p^*)^2}{w^*}\right) \tag{21}$$

Part 2: Showing that $p_2^n - p^* > 0$. Using (21), $p_2^n - p^* > 0$ if and only if

$$2 > w^* + 2p^* + \frac{p^{*2}}{w^*} \Leftrightarrow \sqrt{2w^*} > w^* + p^* \tag{22}$$

Substituting the equilibrium expression for p^* given by

$$p^* = -\frac{1}{2}(1 + w^*)\left(1 - \sqrt{1 + \frac{4}{(1 + w^*)^2}}\right) \tag{23}$$

which can be obtained from solving equation (3) explicitly, allows us to rewrite (22) as

$$\sqrt{2w^*} > -\frac{1}{2}(1-w^*)+\frac{1}{2}(1+w^*)\sqrt{1+\frac{4}{(1+w^*)^2}} \tag{24}$$

$$\Leftrightarrow \sqrt{2w^*}+\frac{1}{2}(1-w^*) >\sqrt{\frac{1}{4}(1+w^*)^2+1} \tag{25}$$

$$\Leftrightarrow \frac{1}{4}(1-w^*)^2+(1-w^*)\sqrt{2w^*}+2w^* >\frac{1}{4}(1+w^*)^2+1 \tag{26}$$

$$\Leftrightarrow (1-w^*)\sqrt{2w^*} >1-w^* \tag{27}$$

The last inequality holds if $1 > w^* > 1/2$. Since $w^* = 1-\sqrt{2s}$ (which follows from the standard Wolinsky analysis), $w^* < 1$ holds for any positive search costs. Moreover, $w^* > 1/2$ for all $s < 1/8$, which is exactly the threshold above no equilibrium with active search exists ($w^* \geq p^*$ if and only if $s \leq 1/8$). This proves that $p_2^n - p^* > 0$ if $w^* \geq p^*$.

Part 3: Derivation of p_3^n. If the consumer visited firm B before, disclosure by firm A leads to $\mathcal{H}(B) = ND \times p^* \times R$. Accordingly, B's profits are given by

$$\Pi^3(p_B) = p_B\int_{p_B}^{w^*}\frac{1}{2}F(u_B-p_B+p^*)du_B \tag{28}$$

as already derived in the main text (see equation (5)). This is the correct profit function when restricting attention to prices p_B that satisfy: $w^*-p_B+p^* < 1$. Suppose for now that the optimal p_B falls in this interval (which we verify later). Then, p_B must solve the following first-order condition:

$$\int_{p_B}^{w^*}\frac{1}{2}(u_B-p_B+p^*)du_B + p_B\Big[-\frac{1}{2}F(p^*)+\int_{p_B}^{w^*}\frac{1}{2}(-1)du_B\Big]=0$$

$$\Longleftrightarrow$$

$$1.5(p_B)^2+p_B\big[-w^*-2p^*-w^*\big]+\big[\frac{1}{2}(w^*)^2+w^*p^*\big]=0$$

$$\Longleftrightarrow$$

$$3(p_B)^2-p_B\big[(4)(w^*+p^*)\big]+\big[(w^*)(w^*+2p^*)\big]$$

Denote the solution to this equation by p_3^n. Then,

$$p_3^n = (2/3)(w^*+p^*)-(1/3)\sqrt{(w^*)^2+2w^*p^*+4(p^*)^2} \tag{29}$$

where we have ignored the positive root because calculations show that the negative one is the appropriate one. Moreover, this price will always be in the region that we have restricted our attention to, namely $p_3^n > w^*+p^*-1$. For prices p_B at which $p_B \leq w^*+p^*-1$, true profits are below the profit function laid out above - because there is no deviation into this region under this optimistic formulation of profits, the optimal price must satisfy $p_3^n > w^* + p^* - 1$.

Part 4: Verifying the ordering $p_3^n < p^*$. Using (29), $p^* - p_3^n > 0$ holds if and only if

$$\frac{1}{3}\sqrt{w^{*2} + 2w^*p^* + 4p^{*2}} > \frac{2}{3}w^* - \frac{1}{3}p^* \Leftrightarrow \sqrt{w^{*2} + 2w^*p^* + 4p^{*2}} > 2w^* - p^* \tag{30}$$

$$\Leftrightarrow w^{*2} + 2w^*p^* + 4p^{*2} > 4w^{*2} - 4w^*p^* + p^{*2} \tag{31}$$

$$\Leftrightarrow p^{*2} + 2w^*p^* > w^{*2} \tag{32}$$

The equilibrium condition pinning down p^* (from Wolinsky, 1986) tells us that

$$p^* = \frac{1 - p^{*2}}{1 + w^*} \Leftrightarrow w^* = \frac{1 - p^* - p^{*2}}{p^*}.$$

Substituting the expression for w^* into inequality (32) above yields

$$2 - 2p^* - p^{*2} > \left(\frac{1 - p^* - p^{*2}}{p^*}\right)^2 \Leftrightarrow 2p^* - 2p^{*4} + 3p^{*2} - 4p^{*3} > 1. \tag{33}$$

We know that $p^* \in (\sqrt{2} - 1, 1/2]$ in any equilibrium with active search. This follows from the necessary condition that $w^* \geq p^*$ and $w^* = 1 - \sqrt{2s}$. It can be verified that $2p^* - 2p^{*4} + 3p^{*2} - 4p^{*3} = 1$ for $p^* = \sqrt{2} - 1$. Thus, inequality (33) holds if

$$\partial_{p^*}\left(2p^* - 2p^{*4} + 3p^{*2} - 4p^{*3}\right) > 0 \text{ for all } p^* \in [\sqrt{2} - 1, 1/2] \tag{34}$$

$$\Leftrightarrow (2 - 8p^{*3}) + (6p^* - 12p^{*2}) > 0 \text{ for all } p^* \in [\sqrt{2} - 1, 1/2] \tag{35}$$

$$\Leftrightarrow (1 - 4p^{*3}) + 3p^*(1 - 2p^*) > 0 \text{ for all } p^* \in [\sqrt{2} - 1, 1/2] \tag{36}$$

Since $p^* \leq 1/2$, it is easy to verify that $1 - 4p^{*3} > 0$ and $3p^*(1 - 2p^*) \geq 0$, implying that the above inequality always holds. This completes the proof. ■

Proof of Proposition 1: The proof has two parts. We first show that the profit function depicted in (8) is the correct one. This follows from the discussion in the main text subject to one additional observation: Firm B, after reaching $\mathcal{H}(B) = R$ due to firm A's deviation, must not use search disclosure itself. Search disclosure by firm B would inform firm A that the buyer continued to search. Then, firm A would reach the information set $\mathcal{H}(A) = D \times p_1 \times R$, in which it can revise its price for return consumers, which expression (8) does not allow for.

After establishing that $\mathcal{H}(A) = D \times p_1 \times R$ will not be reached, we show that disclosure in $\mathcal{H}(A) = NR$, through its effect on (8) via p_2 and p_3, is not profitable.

Part 1: $\mathcal{H}(A) = D \times p_1 \times R$ will not be reached (even) if $d_A = D$, because B would not find it profitable to disclose in $\mathcal{H}(B) = R$.

Recall that firm B believes that firm A offered the consumer the price p^*. Thus, it believes that $d_B = D$ (disclosing back) leads to $\mathcal{H}(A) = D \times p^* \times R$. In this information set, B anticipates that A expects B's price to equal p_2^n, since this is part of B's equilibrium strategy. Consequently, firm B believes that firm A would revise its price p_A to maximize the following profit function:

$$\Pi^{3,dd}(p_A) = p_A \int_{p_A}^{w^*} \frac{1}{2} F(u_A - p_A + p_2^n) du_A, \tag{37}$$

where we have accounted for the fact that, in the information set $\mathcal{H}(A) = D \times p^* \times R$, firm A must believe that the consumers initial match value satisfied $u_j \in [0, w^*]$. The optimal price in this information set, which we denote by p_3^{dd} to account for the "double" deviation, is given by:

$$p_3^{dd} = (2/3)(w^* + p_2) - (1/3)\sqrt{(w^*)^2 + 2w^* p_2 + 4(p_2)^2}, \tag{38}$$

where we have again ignored the positive root because calculations show that the negative root is the appropriate one. If $p_3^{dd} < p^*$, firm B would expect firm A to revise its price downward and firm B has no incentive to disclose "back" if $\mathcal{H}(B) = R$. At $w^* = 1$ or, equivalently, at $s = 0$, $p_3^{dd} = p^*$. To show that $p_3^{dd} < p^*$, it is thus sufficient to show that the derivatives with respect to w^* have opposite signs. We begin with p^*. By taking the derivative of p^* given in (23), we obtain

$$\frac{\partial p^*}{\partial w^*} = \frac{1 - \sqrt{1 + \frac{4}{(1+w^*)^2}}}{2\sqrt{1 + \frac{4}{(1+w^*)^2}}}, \tag{39}$$

which makes it easy to verify that $\partial p^* / \partial w^* \in (-1/2, 0)$. The derivative of p_3^{dd} is

$$\frac{\partial p_3^{dd}}{\partial w^*} = \frac{2}{3} - \frac{w^* + p_2}{3\sqrt{(w^* + p_2)^2 + 3p_2^2}} + \left(\frac{2}{3} - \frac{w^* + 4p_2}{3\sqrt{(w^* + p_2)^2 + 3p_2^2}} \right) \frac{\partial p_2}{\partial w^*} \tag{40}$$

$$= \left(\frac{2}{3} - \frac{w^* + p_2}{3\sqrt{(w^* + p_2)^2 + 3p_2^2}} \right) \left(1 + \frac{\partial p_2}{\partial w^*} \right) - \frac{p_2}{\sqrt{(w^* + p_2)^2 + 3p_2^2}} \frac{\partial p_2}{\partial w^*}. \tag{41}$$

It can be verified that the first term above is greater than 1/3. Thus, $\partial p_3^{dd} / \partial w^* > 0$ follows if $\partial p_2 / \partial w^* \in (-1, 0)$. Taking the derivative of p_2 (see Lemma 1 for the expression) w.r.t p^* yields:

$$\frac{\partial p_2}{\partial w^*} = \frac{1}{2}\left(1 - \frac{p^*}{w^*}\right)\frac{\partial p^*}{\partial w^*} - \frac{1}{4}\left(1 - \left(\frac{p^*}{w^*}\right)^2\right) = \frac{1}{2}\left(1 - \frac{p^*}{w^*}\right)\left(\frac{\partial p^*}{\partial w^*} - \frac{1}{2}\left(1 + \frac{p^*}{w^*}\right)\right). \tag{42}$$

Since $\partial p^* / \partial w^* < 0$ as shown above, $\partial p_2^n / \partial w^* < 0$ holds because $p^* < w^*$. To bound $\partial p_2^n / \partial w^*$ from below, observe that $\partial p^*/\partial w^* > -1/2$ implies that

$$\frac{\partial p_2^n}{\partial w^*} > \frac{1}{2}\left(1 - \frac{p^*}{w^*}\right)\left(-\frac{1}{2} - \frac{1}{2}\left(1 + \frac{p^*}{w^*}\right)\right) = -\frac{1}{2}\left(1 - \left(\frac{p^*}{w^*}\right)^2\right) > -\frac{1}{2}, \tag{43}$$

which is sufficient to prove that $\partial p_2^n/\partial w^* \in (-1, 0)$. Thus, $\partial p_3^{dd}/\partial w^* > 0$ whereas $\partial p^*/\partial w^* < 0$, which proves $p_3^{dd} < p^*$. The firm that receives disclosure will thus not disclose back, and the deviating firm has no chance to revise its price later. Expression (8) thus correctly represents the deviating firm's profits.

Part 2: A deviation to disclosure is strictly unprofitable at $\mathcal{H}(A) = NR$.

Knowing that (8) correctly represents the profits of the deviating firm, we next focus on analyzing the average effect of changing the rival's price from p^* to either p_3^n or p_2^n. Let $D(p_A, p^*)$ represent firm A's demand if it does not disclose and charges a price p_A. Also, let $D^d(p_A, p^*, p_2^n, p_3^n)$ denote firm A's demand after disclosure (deviation), where p_2^n is the price firm B sets if $\mathcal{H}(B) = R$ and p_3^n the revised price if $\mathcal{H}(B) = ND \times p^* \times R$. Because firm B will not disclose back to firm A, disclosure by firm A never leads to $H(A) = D \times p_A \times R$ so that firm A will never revise its price. Accordingly, total profits are equal to $\max_{p_A} p_A D^d(p_A, p^*, p_2^n, p_3^n)$ if A deviates, while they are equal to $\max_{p_A} p_A D(p_A, p^*)$ in the no disclosure equilibrium. Thus, a deviation is strictly unprofitable if

$$D(p_A, p^*) < D^d(p_A, p^*, p_2^n, p_3^n) \text{ for all } p_A. \tag{44}$$

Let $p_2^n = p^* + \delta$ and $p_3^n = p^* - \delta - \varepsilon$. By Lemma 1, $\delta > 0$ holds but the sign of ε is unknown. Then, $D^d(p_A, p^*, p_2^n, p_3^n) = D^d(p_A, p^*, p^* + \delta, p^* - \delta - \varepsilon)$. It is easy to verify that $\partial_\varepsilon D^d < 0$. Thus, (44) holds if (i) $\varepsilon \geq 0$ and (ii) $\partial_\delta D^d < 0$ because $D(p_A, p^*) = D^d(p_A, p^*, p^*, p^*)$.

To see that (i) is true, notice that:

$$\begin{aligned} \varepsilon =& (p^* - p_3^n) - \delta = (p^* - p_3^n) - (p_2^n - p^*) && (45)\\ =& \frac{10}{12}p^* - \frac{5}{12}w^* - \frac{6}{12} + \frac{3}{12}\frac{(p^*)^2}{w^*} + \frac{4}{12}\sqrt{(w^*)^2 + 2w^*p^* + 4(p^*)^2} && (46) \end{aligned}$$

Substituting $w^* = (1 - p^* - p^*)/p^*$, which follows from equation (3), into the expression above yields that $\varepsilon \geq 0$ if and only if

$$\begin{aligned} &\frac{1}{12}\left(-1 - \frac{5}{p^*} + 15p^* - \frac{3(p^*)^3}{-1 + p^* + (p^*)^2} + 4\sqrt{1 + \frac{1}{(p^*)^2} - \frac{2}{p^*} + 3(p^*)^2}\right) \geq 0 && (47)\\ \Leftrightarrow &\left(4\sqrt{1 + \frac{1}{(p^*)^2} - \frac{2}{p^*} + 3(p^*)^2}\right)^2 \geq \left(1 + \frac{5}{p^*} - 15p^* + \frac{3(p^*)^3}{-1 + p^* + (p^*)^2}\right)^2 && (48)\\ \Leftrightarrow\ &16\left(1 + \frac{1}{(p^*)^2} - \frac{2}{p^*} + 3(p^*)^2\right) \geq \frac{(5 - 4p^* - 21(p^*)^2 + 14(p^*)^3 + 12(p^*)^4)^2}{(p^*)^2(-1 + p^* + (p^*)^2)^2} && (49) \end{aligned}$$

Additional steps show that inequality (49) holds if and only if

$$\begin{aligned}
&16 - 64p^* + 64(p^*)^2 + 32(p^*)^3 - 16(p^*)^4 - 96(p^*)^5 - 32(p^*)^6 + 96(p^*)^7 + 48(p^*)^8 \geq \\
&25 - 40p^* - 194(p^*)^2 + 308(p^*)^3 + 449(p^*)^4 - 684(p^*)^5 - 308(p^*)^6 + 336(p^*)^7 + 144(p^*)^8 \\
\Leftrightarrow\ &-3(3 + 8p^* - 86(p^*)^2 + 92(p^*)^3 + 155(p^*)^4 - 196(p^*)^5 - 92(p^*)^6 + 80(p^*)^7 + 32(p^*)^8) \geq 0
\end{aligned}$$

One can easily verify that the left-hand side equals 0 if $p^* = \sqrt{2} - 1$ and $3/8$ if $p^* = 1/2$. Thus, showing that the left-hand side of this expression is concave over $[\sqrt{2} - 1, 1/2]$ is sufficient to show that inequality (49) is true (by concavity, the boundary conditions imply that the derivative of the left-hand side is positive on this interval). The second derivative of the left-hand side is given by

$$-12\big(-43 + 138p^* + 465(p^*)^2 - 980(p^*)^3 - 690(p^*)^4 + 840(p^*)^5 + 448(p^*)^6\big) \tag{50}$$

To show that (50) is negative, we show separately that (1) $465(p^*)^2 - 980(p^*)^3 + 448(p^*)^6$ and (2) $-43 + 138p^* - 690(p^*)^4 + 840(p^*)^5$ are both non-negative for all $p^* \in [\sqrt{2} - 1, 1/2]$. Consider term (1) first, which we show satisfies:

$$465(p^*)^2 - 980(p^*)^3 + 448(p^*)^6 \geq 0 \Leftrightarrow 465 \geq 980p^* - 448(p^*)^4 \tag{51}$$

It can be verified that inequality (51) holds at $p^* = 1/2$. Moreover, $980p^* - 448(p^*)^4$ increases in p^* for all $p^* \leq (35/64)^{1/3}$ (note $(35/64)^{1/3} > 1/2$), implying that (51) holds for all $p^* \in [\sqrt{2} - 1, 1/2]$. Consider term (2) next, which we show satisfies:

$$-43 + 138p^* - 690(p^*)^4 + 840(p^*)^5 \geq 0 \tag{52}$$

Again, it can be verified that inequality (52) holds (strictly) at $p^* = \sqrt{2} - 1$. Thus, it is sufficient to show that the left-hand side of (52) increases in p^*. By taking the derivative, we see that this conditions holds if and only if

$$6(23 - 460(p^*)^3 + 700(p^*)^4) \geq 0 \Leftrightarrow 23 \geq 460(p^*)^3 - 700(p^*)^4 \tag{53}$$

One can check that the function $460(p^*)^3 - 700(p^*)^4$ obtains its maximum at $p^* = 69/140$. Since $23 > 460(69/140)^3 - 700(69/140)^4$, we know that (53) holds for all $p^* \in [\sqrt{2} - 1, 1/2]$. This completes the proof of subpart (i).

We next prove (ii): $\partial_\delta D^d < 0$. Evaluating $D^d(p_A, p^*, p^* + \delta, p^* - \delta - \varepsilon)$, we obtain

$$\frac{1}{2}\Bigg(\underbrace{w^*(1 - w^* + p^* - \delta - \varepsilon - p_A) + \int_{p_A}^{w^* - p^* + \delta + \varepsilon + p_A} (p^* - \delta - \varepsilon + u_A - p_A)\mathrm{d}u_A}_{\text{Modified searcher profits}}\Bigg)$$
$$+\frac{1}{2}\Bigg(\underbrace{(1 - w^* + p^* - p_A) + \int_{p_A}^{w^* - p^* + p_A} (p^* + \delta + u_A - p_A)\mathrm{d}u_A}_{\text{Modified first arriver profits}}\Bigg).$$

The derivative of this demand function with respect to δ satisfies

$$\frac{1}{2}\left((-1)w^* + (1)w^* + \int_{p_A}^{w^* - p^* + \delta + \varepsilon + p_A} (-1)\mathrm{d}u_A\right) + \frac{1}{2}\int_{p_A}^{w^* - p^* + p_A} (1)\mathrm{d}u_A = -\frac{\delta + \varepsilon}{2} < 0,$$

which shows that demand falls in δ for any $\delta > 0$.

Part 3: Establishing uniqueness of the candidate equilibrium.

On the equilibrium path, the uniform price p^* will be set, which is uniquely determined. All off-path prices, namely p_2^n, p_3^n and the revision price function, are also uniquely determined by Lemma 1, implying the desired result. ■

Proof of Lemma 2: We derive $p_3^d(p_1)$ first. As shown in the main text, the profit function of the deviating firm A in the information set $\mathcal{H}(A) = D \times p_1 \times R$ is given by (10):

$$\Pi^{3,d}(p_3|p_1) = p_3 \int_{p_3}^{w^d(p_1)} \frac{1}{2}F(u_A - p_3 + p_1^*)du_A = p_3 \int_{p_3}^{w^d(p_1)} \frac{1}{2}(u_A - p_3 + p_1^*)du_A$$

Consider a generic initial price p_1. We restrict attention to prices p_3 which satisfy $w^d(p_1) - p_3 + p_1^* < 1$ (and verify that the optimal revision price will satisfy this). This allows us to rewrite the above profit function with the second equality. The optimal price $p_3^d(p_1)$ needs to solve the following first-order condition:

$$\int_{p_3^d}^{w^d(p_1)} \frac{1}{2}(u_A - p_3^d + p_1^*)du_j - p_3^d\frac{1}{2}(p_1^*) + p_3^d \int_{p_3^d}^{w^d(p_1)} \frac{1}{2}(-1)du_A = 0$$

$$\Longleftrightarrow$$

$$\frac{1}{2}(w^d(p_1) - p_3^d + p_1^*)^2 - \frac{1}{2}(p_1^*)^2 - p_3^d p_1^* - p_3^d\big(w^d(p_1) - p_3^d\big) = 0 \tag{54}$$

The unique solution to this equation that satisfies $p_3^d(p_1) \in [0,1]$ is given by

$$p_3^d(p_1) = (2/3)(w^d(p_1) + p_1^*) - (1/3)\sqrt{(w^d(p_1))^2 + 2w^d(p_1)p_1^* + 4(p_1^*)^2}. \tag{55}$$

We derive p_1^d next. Firm A's profit function if it does not disclose when $\mathcal{H}(A) = NR$ is shown in (11), which we repeat here for convenience.

$$\Pi^{1,d}(p_1) = p_1 \frac{1}{2}\big[1 - F(w^d(p_1))\big] + p_3^d(p_1) \int_{p_3^d(p_j)}^{w^d(p_1)} \frac{1}{2} F(p_1^* + u_j - p_3^d(p_1)) du_A$$

Note that this expression is valid only if $p_3^d(p_1) < w^d(p_1)$, which must hold in a PBE (else, no profits are made when setting $p_3^d(p_1)$). The derivative of $\Pi^{1,d}(p_1)$ with respect to p_1 is:

$$\frac{\partial \Pi^{1,d}(p_1)}{\partial p_1} = p_1 \frac{1}{2}[-1] + \frac{1}{2}\big[1 - F(w^d(p_1))\big] + p_3^d(p_1)\Bigg[\frac{1}{2}(p_1^* + w^d(p_1) - p_3^d(p_1))$$

$$-\frac{1}{2}(p_1^*)\frac{\partial p_3^d(p_1)}{\partial p_1} - \int_{p_3^d(p_1)}^{w^d(p_1)} \frac{1}{2}\frac{\partial p_3^d(p_1)}{\partial p_1} du_A\Bigg] + \frac{\partial p_3^d(p_1)}{\partial p_1}\Bigg[\int_{p_3^d(p_1)}^{w^d(p_1)} \frac{1}{2}(p_1^* + u_A - p_3^d(p_1)) du_A\Bigg],$$

from which we obtain the following first-order condition for the optimal deviation price p_1^d:

$$\big[1 - p_1^d - w^d(p_1^d)\big] + p_3^d(p_1^d)(p_1^* + w^d(p_1^d) - p_3^d(p_1^d)) + \frac{\partial \Pi^{3,d}(p_3^d|p_1)}{\partial p_3}\frac{\partial p_3^d(p_1^d)}{\partial p_1} = 0 \tag{56}$$

By the Envelope theorem, $\partial \Pi^{3,d}(p_3^d|p_1)/\partial p_3 = 0$ so that (56) simplifies to

$$p_1^d = 1 - w^d(p_1^d) - \big(p_3^d(p_1^d)\big)^2 + p_3^d(p_1^d)\big(w^d(p_1^d) + p_1^*\big), \tag{57}$$

which equals the expression provided in the lemma. The solution to (57) is unique if the right-hand side of (57) decreases in p_1^d everywhere. Its derivative shows that this is true if and only if

$$1 > \frac{\partial p_3^d(p_1^d)}{\partial p_1}\left(w^d(p_1^d) + p_1^*\right) + p_3^d(p_1^d)\left(1 - 2\frac{\partial p_3^d(p_1^d)}{\partial p_1}\right). \tag{58}$$

We first show that this inequality holds if $\partial p_3^d/\partial p_1 \in (\frac{1}{3}, \frac{1}{2})$. Second, we verify that $\partial p_3^d/\partial p_1 \in (\frac{1}{3}, \frac{1}{2})$. If $\partial p_3^d/\partial p_1 \in (\frac{1}{3}, \frac{1}{2})$, the right-hand side of (58) satisfies

$$\frac{\partial p_3^d(p_1^d)}{\partial p_1}\left(w^d(p_1^d) + p_1^*\right) + p_3^d(p_1^d)\left(1 - 2\frac{\partial p_3^d(p_1^d)}{\partial p_1}\right) < \frac{1}{2}\left(w^d(p_1^d) + p_1^*\right) + \frac{1}{3}p_3^d(p_1^d). \tag{59}$$

Additionally, we know that $w^d(p_1^d) < 1$, $p_1^* < 1/2$ as well as $p_3^d(p_1) < 1/2$. The latter follows from Remark 1 (presented at the end of this proof) upon substituting $w^d(p_1^d)$ with w and p_1^* with p. Thus, $\frac{1}{2}\big(w^d(p_1^d) + p_1^*\big) + \frac{1}{3}p_3^d(p_1^d) < \frac{3}{4} + \frac{1}{6} < 1$ if $\partial p_3^d/\partial p_1 \in (\frac{1}{3}, \frac{1}{2})$, which proves the first claim.

To verify that $\partial p_3^d/\partial p_1 \in (\frac{1}{3}, \frac{1}{2})$, we first calculate the derivative of p_3^d and find that

$$\frac{\partial p_3^d(p_1^d)}{\partial p_1} = \frac{1}{3}\left(2 - \frac{w^d(p_1^d) + p_1^*}{\sqrt{\left(w^d(p_1^d) + p_1^*\right)^2 + 3\left(p_1^*\right)^2}}\right) > \frac{1}{3}\left(2 - \frac{w^d(p_1^d) + p_1^*}{\sqrt{\left(w^d(p_1^d) + p_1^*\right)^2}}\right) = \frac{1}{3}. \tag{60}$$

Additionally, $\partial p_3^d/\partial p_1 < 1/2$ if and only if

$$\frac{w^d(p_1^d) + p_1^*}{\sqrt{\left(w^d(p_1^d) + p_1^*\right)^2 + 3\left(p_1^*\right)^2}} > \frac{1}{2} \iff \left(w^d(p_1^d) + p_1^*\right)^2 > \frac{1}{4}\left(w^d(p_1^d) + p_1^*\right)^2 + \frac{3}{4}\left(p_1^*\right)^2 \tag{61}$$

$$\iff \left(w^d(p_1^d) + p_1^*\right)^2 > \left(p_1^*\right)^2 \tag{62}$$

where the last inequality holds always. This proves the second claim that $\partial p_3^d/\partial p_1 < 1/2 \in (\frac{1}{3}, \frac{1}{2})$ and, thus, guarantees uniqueness of the optimal deviation price p_1^d. ■

Remark 1 *Let*

$$p_3(w,p) = \frac{2}{3}(w+p) - \frac{1}{3}\sqrt{(w+p)^2 + 3p^2}, \tag{63}$$

$w \leq 1$ *and* $p \leq 1/2$. *Then* $p_3(w,p) \leq 1 - \frac{\sqrt{3}}{3} < 1/2$.

Proof of Remark 1: Taking the partial derivatives, it is easy to see that p_3 increases in w and in p for all $p \in (\sqrt{2}-1, 1/2)$ and $w \in [1/2, 1)$:

$$\frac{p_3(w,p)}{\partial w} = \frac{2}{3} - \frac{p+w}{3\sqrt{(w+p)^2 + 3p^2}} > \frac{2}{3} - \frac{p+w}{3\sqrt{(w+p)^2}} = \frac{1}{3} > 0. \tag{64}$$

In addition,

$$\frac{p_3(w,p)}{\partial p} = \frac{2}{3} - \frac{4p+w}{3\sqrt{3p^2 + (w+p)^2}} > 0 \tag{65}$$

$$\Leftrightarrow 2\sqrt{3p^2 + (w+p)^2} > 4p + w \quad \Leftrightarrow \quad 3w^2 > 0, \tag{66}$$

which holds always if $w > 0$. Thus, p_3 is bounded from above by $p_3(1, 1/2) = 2/3(3/2) - 1/3\sqrt{3}$, which proves the claim. ■

Proof of Proposition 2: The Proposition has two parts. First, we show that there exists a threshold $\bar{s}$ such that the partial disclosure equilibrium exists for all search costs below this threshold. Thereafter, we show that there exists another threshold $\bar{s}'$ such that the partial disclosure equilibrium does not exist for search costs above this threshold.

Part 1: There is a value $\bar{s}$ such that partial disclosure is an equilibrium if $s \leq \bar{s}$.

In a partial disclosure equilibrium, $d_j = D$ if $\mathcal{H}(j) = NR$ and $d_j = ND$ if $\mathcal{H}(j) = R$. To prove that the partial disclosure equilibrium exists, we show that deviating from these disclosure strategies is not profitable if s is sufficiently small.

Claim 1: Deviating to non-disclosure in the information set $\mathcal{H}(j) = NR$ is not profitable if $s < \hat{s}$ ($\hat{s} > 0$).

If firm j encounters a buyer while $\mathcal{H}(j) = NR$, then the buyer must have started her search at j. To evaluate the effect of deviating to $d_j = ND$ when $\mathcal{H}(j) = NR$, we can thus restrict attention to buyers who search order in this order. In equilibrium, total profits from such buyers are given by

$$\Pi^{EQ} = \frac{1}{2}p_1^*\big[1 - F(w^d(p_1^*))\big] + \frac{1}{2}p_1^* \int_{p_1^*}^{w^d(p_1^*)} F(p_2^* + u_j - p_1^*)\mathrm{d}u_j, \tag{67}$$

where $w^d(p_1^*) = w^* - p_2^* + p_1^*$ is the equilibrium search cutoff. As argued in the main text, the equilibrium prices p_1^* and p_2^* can be obtained from Armstrong et al. (2009), who provide the following implicit equations for these prices when there are two firms:

$$p_1^* = \frac{1}{2}\big(1 - (w^* - p_2^*)\big) + \frac{1}{4}(w^*)^2 - \frac{1}{4}(p_2^*)^2 \tag{68}$$

$$p_2^* = \frac{1}{2}\big(1 - (w^* - p_2^*)\big) + \frac{1}{4}(w^* - p_2^* + p_1^*) - \frac{1}{4}\frac{(p_1^*)^2}{w^* - p_2^* + p_1^*}. \tag{69}$$

Armstrong et al. (2009) show that these prices are unique and satisfy $p_2^* \geq p_1^*$. In comparison, profits after deviating to non-disclosure are given by:

$$\Pi^{DEV}(p_1) = \frac{1}{2}p_1\big[1 - F(w^d(p_1))\big] + \frac{1}{2}p_3^d(p_1) \int_{p_3^d(p_1)}^{w^d(p_1)} F(p_1^* + u_j - p_3^d(p_1))\mathrm{d}u_j, \tag{70}$$

Lemma 2 provides implicit equations for the optimal prices $p_3^d(p_1)$ and p_1^d and guarantees that p_1^d and $p_3^d(p_1^d)$ are uniquely determined.

One can verify that $p_1^d = p_3^d(p_1) = p_1^* = p_2^* = \sqrt{2} - 1$ if $w^* = 1$ or, equivalently, $s = 0$. That is, $\Pi^{DEV} = \Pi^{EQ}$ if $s = 0$, where we define Π^{DEV} as the deviation profits for the optimal deviation price, i.e., $\Pi^{DEV} := \Pi^{DEV}(p_1^d)$. To show that a threshold $\hat{s} > 0$ exists such that $\Pi^{DEV} \leq \Pi^{EQ}$ for all $s \leq \hat{s}$, it is thus sufficient to show that

$$\left.\frac{\partial \Pi^{EQ}}{\partial s}\right|_{s=0} > \left.\frac{\partial \Pi^{DEV}}{\partial s}\right|_{s=0} \quad \Longleftrightarrow \quad \left.\frac{\partial \Pi^{EQ}}{\partial w^*}\right|_{w^*=1} < \left.\frac{\partial \Pi^{DEV}}{\partial w^*}\right|_{w^*=1}. \tag{71}$$

To prove the validity of the second inequality in (71), take the derivative of Π^{EQ}:

$$\frac{\partial \Pi^{EQ}}{\partial w^*} = \left(\frac{p_1^*}{2} - \frac{p_1^* w^*}{2} + \frac{p_1^*(w^* - p_2^*)}{2}\right)\frac{\partial p_2^*}{\partial w^*} + \left(-\frac{p_1^*}{2} + \frac{p_1^* w^*}{2}\right), \tag{72}$$

where we have already used that $\frac{\partial \Pi^{EQ}}{\partial p_1^*}\frac{\partial p_1^*}{\partial w^*} = 0$ due to the Envelope Theorem. Since $p_1^* = p_2^* = \sqrt{2} - 1$ at $w^* = 1$, the derivative further simplifies to

$$\left.\frac{\partial \Pi^{EQ}}{\partial w^*}\right|_{w^*=1} = \frac{\sqrt{2}-1}{2}\left(2-\sqrt{2}\right)\frac{\partial p_2^*}{\partial w^*} \tag{73}$$

The derivative of Π^{DEV} is given by

$$\begin{aligned}\frac{\partial \Pi^{DEV}}{\partial w^*} &= \left(\frac{p_1^d}{2} - \frac{p_3(w^* + p_1^* - p_2^* + p_1^d - p_3^d(p_1^d))}{2}\right)\frac{\partial p_2^*}{\partial w^*} \\ &+ \left(\frac{p_3^d(p_1^d)(w^* - p_2^* + p_1^d - p_3^d(p_1^d))}{2}\right)\frac{\partial p_1^*}{\partial w^*} + \left(-\frac{p_1^d}{2} + \frac{p_3(w^* + p_1^* - p_2^* + p_1^d - p_3^d(p_1^d))}{2}\right)\end{aligned} \tag{74}$$

where we have again used the Envelope Theorem, which implies $\frac{\partial \Pi^{DEV}}{\partial p_1} = 0$ at p_1^d. At $w^* = 1$, the solution is again $p_1^d = p_1^* = p_2^* = p_3^d(p_1^d) = \sqrt{2} - 1$. Thus, the derivative further simplifies to

$$\left.\frac{\partial \Pi^{DEV}}{\partial w^*}\right|_{w^*=1} = \frac{\sqrt{2}-1}{2}\left(2-\sqrt{2}\right)\frac{\partial p_1^*}{\partial w^*}. \tag{75}$$

Consequently, (71) holds if and only if $\partial p_2^*/\partial w^* < \partial p_1^*/\partial w^*$. We prove this inequality using the equations determining p_1^* and p_2^* given by (68) and (69) and the multi-variate version of the Implicit Function Theorem. Rewriting (68) and (69) implies the following system of implicit equations:

$$0 = -\,p_1^* + \frac{1}{2}(1 - w^* + p_2^*) + \frac{1}{4}\left(w^*\right)^2 - \frac{1}{4}\left(p_2\right)^* \tag{76}$$

$$0 = \frac{1}{2} + \frac{1}{4}(p_1^* - w^*) - \frac{3}{4}p_2^* + \frac{1}{4}\frac{\left(p_1^*\right)^2}{p_2^* - p_1^* - w^*} \tag{77}$$

By differentiating everything with respect to w^* and collecting terms, we get

$$(-1)\frac{\partial p_1^*}{\partial w^*} + \frac{1}{2}(1 - p_2^*)\frac{\partial p_2^*}{\partial w^*} = \frac{1}{2}(1 - w^*) \text{ as well as} \tag{78}$$

$$\begin{aligned}\frac{1}{4}\left(1 + \frac{2p_1^*(p_2^* - p_1^* - w^*) + \left(p_1^*\right)^2}{(p_2^* - p_1^* - w^*)^2}\right)\frac{\partial p_1^*}{\partial w^*} - \left(\frac{3}{4} + \frac{1}{4}\frac{\left(p_1^*\right)^2}{(p_2^* - p_1^* - w^*)^2}\right)\frac{\partial p_2^*}{\partial w^*} \\ = \frac{1}{4}\left(1 - \frac{\left(p_1^*\right)^2}{(p_2^* - p_1^* - w^*)^2}\right).\end{aligned} \tag{79}$$

At $(p_1^*, p_2^*, w^*) = (\sqrt{2}-1, \sqrt{2}-1, 1)$, this reduces to

$$\begin{pmatrix} -1 & \frac{1}{2}(2-\sqrt{2}) \\ \frac{1}{2}(3-2\sqrt{2}) & -\frac{1}{2}(3-\sqrt{2}) \end{pmatrix} \begin{pmatrix} \frac{\partial p_1^*}{\partial w^*} \\ \frac{\partial p_2^*}{\partial w^*} \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{1}{2}(1-\sqrt{2}) \end{pmatrix} \tag{80}$$

It can also be verified that the derivatives exist by checking that the determinant of the first matrix above is non-zero. Solving the system of linear equations yields

$$\begin{pmatrix} \frac{\partial p_1^*}{\partial w^*} \\ \frac{\partial p_2^*}{\partial w^*} \end{pmatrix} = \begin{pmatrix} \frac{1}{17}(4\sqrt{2}-7) \\ \frac{1}{17}(\sqrt{2}-6) \end{pmatrix} \approx \begin{pmatrix} -0.08 \\ -0.27 \end{pmatrix}, \tag{81}$$

which shows that $\partial p_2^*/\partial w^* < \partial p_1^*/\partial w^*$ at $w^* = 1$, completing the proof of (71) and Claim 1.

Claim 2: deviating to disclosure when in the information set $\{R\}$ is not profitable.

Consider a firm $-j$ that is at the information set $\mathcal{H}(-j) = R$. Since $\mathcal{H}(-j) = R$ is on-path, firm $-j$ believes that firm j, which must have been visited before, set the price p_1^* and that the buyer continued to search if $u_j < w^d(p_1^*) = w^* - p_2^* + p_1^*$. If firm $-j$ follows the equilibrium strategy and does not disclose, firm j's price remains at p_1^*. But if firm $-j$ deviates to $d_{-j} = D$, then $\mathcal{H}(j) = D \times p_1^* \times R$, which leads firm j to revise its price to maximize the following profit function:

$$p_j \int_{p_j}^{w^d(p_1^*)} \frac{1}{2} F(u_j - p_j + p_2^*) du_j \tag{82}$$

Let p_3^{dr} denote the price that maximizes (82), where the superscripts reflect that this stage is preceded by j both using and receiving search disclosure itself (i.e., $\mathcal{H}(j) = D \times p_1^* \times R$). Clearly, $d_{-j} = D$ is not a profitable deviation for firm $-j$ in the information set $H(-j) = R$ if $p_3^{dr} \leq p_1^*$.

We seek to show that there is a value $\tilde{s} > 0$ such that $p_3^{dr} \leq p_1^*$ if $s \leq \tilde{s}$.[26] By taking the derivative of (82) with respect to p_j, we obtain the first-order condition that p_3^{dr} needs to satisfy. Similarly to $p_3^d(p_1)$ that we derived before, solving for p_3^d yields:

$$p_3^{dr} = \frac{2}{3}(w^d(p_1^*) + p_2^*) - \frac{1}{3}\sqrt{(w^d(p_1^*))^2 + 2w^d(p_1^*)p_2^* + 4(p_2^*)^2} \tag{83}$$

$$= \frac{2}{3}(w^* + p_1^*) - \frac{1}{3}\sqrt{(w^* + p_1^*)^2 + 3\,(p_2^*)^2} \tag{84}$$

To show that $p_3^{dr} < p_1^*$ when s is sufficiently small, observe that $p_3^{dr} = p_1^* = p_2^* = \sqrt{2}-1$ at $s = 0$ or equivalently, at $w^* = 1$. Thus, the claim is true if

$$\left.\frac{\partial p_1^*}{\partial w^*}\right|_{w^*=1} < \left.\frac{\partial p_3^{dr}}{\partial w^*}\right|_{w^*=1}. \tag{85}$$

[26]Calculations show that $p_3^{dr} < p_1^*$ for all $s > 0$ but restricting attention to small s suffices here.

Since $\partial p_1^*/\partial w^*$ at $w^* = 1$ is already known from before, it only remains to calculate $\partial p_3^{dr}/\partial w^*$. Taking the derivative of (84) with respect to w^* while taking into account that p_1^* and p_2^* are functions of w^* as well yields:

$$\frac{\partial p_3^{dr}}{\partial w^*} = \left(\frac{2}{3} - \frac{1}{3} \frac{w^* + p_1^*}{\sqrt{(w^* + p_1^*)^2 + 3\left(p_2^*\right)^2}} \right) \left(1 + \frac{\partial p_1^*}{\partial w^*} \right) - \frac{p_2^*}{\sqrt{(w^* + p_1^*)^2 + 3\left(p_2^*\right)^2}} \frac{\partial p_2^*}{\partial w^*}. \tag{86}$$

To evaluate this expression at $w^* = 1$, recall that $p_1^* = p_2^* = \sqrt{2} - 1$ and $\partial p_1^*/\partial w^* = -1/17(4\sqrt{2} - 7)$ and $\partial p_2^*/\partial w^* = -1/17(\sqrt{2} - 6)$ (see (81) above) at $w^* = 1$. Thus

$$\frac{\partial p_3^{dr}}{\partial w^*} = 1/119(22 + 19\sqrt{2}) > \partial p_1^*/\partial w^*,$$

which completes the proof that there is a value $\tilde{s} > 0$ such that $p_3^{dr} \leq p_1^*$ if $s \leq \tilde{s}$. Consequently, deviating to $d_{-j} = D$ is not profitable when $\mathcal{H}(-j) = R$.

In sum, let $\bar{s} = \min(\tilde{s}, \hat{s})$. Then, $s \leq \bar{s}$ implies that deviating from the equilibrium disclosure strategy in either information set is not profitable. This establishes the existence of a partial disclosure equilibrium for all $s \leq \bar{s}$ and completes the proof of the first part.

Part 2: There is a value $\bar{s}' > 0$ such that partial disclosure is not an equilibrium if $s > \bar{s}'$.

In the partial disclosure equilibrium, we know that $p_1^* \leq p^* \leq p_2^*$, with $p_1^* \to p^*$ and $p_2^* \to p^*$ as $s \to 1/8$. The latter follows from the fact that the prices p_1^* and p_2^* are continuous in s and $p_1^* = p_2^* = p^*$ when $s = 1/8$. Recall also that the search cutoff $w^d(p_1)$ satisfies: $w^d(p_1) = w^* - p_2^* + p_1$. Thus, $p_2^* \to p^*$ as $s \to 1/8$, which implies that $w^d(p_1) - p_1 \to 0$ as $s \to 1/8$.

Consider the equilibrium profits of firm j when it sets the price p_1^* and discloses in $\mathcal{H}(j) = NR$:

$$\Pi^{1,*}(p_1^*) = p_1^* \frac{1}{2} \left[1 - F(w^d(p_1^*)) \right] + p_1^* \int_{p_1^*}^{w^d(p_1^*)} \frac{1}{2} F(p_2^* + u_j - p_1^*) du_j$$

By continuity of the expression above and because $w^d(p_1) \to p_1$ as $s \to 1/8$, it follows that:

$$\lim_{s \to 1/8} \Pi^{1,*}(p_1^*) = \frac{1}{2} p_1^* [1 - p_1^*] \tag{87}$$

Suppose that firm j deviates to non-disclosure when $\mathcal{H}(j) = NR$, in which case it gets a chance to screen its buyers and receives profits as defined in equation (11). To establish that deviating to non-disclosure is profitable if $s \to 1/8$, it is sufficient to show that the deviation is profitable if firm j charges p_1^*. For $p_1 = p_1^*$, the profits a firm makes as $s \to 1/8$ (for which $w^d(p_1^*) \to p_1^*$) converge to

$$\lim_{s \to 1/8} \Pi^{1,d}(p_1^*) = p_1^* \frac{1}{2} \left[1 - p_1^* \right] + p_3^d(p_1^*) \int_{p_3^d(p_1^*)}^{p_1^*} \frac{1}{2} F(p_1^* + u_j - p_3^d(p_1^*)) du_j, \tag{88}$$

where we have simplified notation by neglecting the limit of p_1^* as $s \to 1/8$. It is easy to see that (88) is strictly greater than (87) as $s \to 1/8$ if $p_1^* > p_3^d(p_1^*)$ in the limit since this implies that the second term in (88) is strictly positive. To calculate the limit of $p_1^* - p_3^d(p_1^*)$, note that:

$$\lim_{s\to 1/8}\left(p_3^d(p_1^*)\right) = \frac{4}{3}\lim_{s\to 1/8}(p_1^*) - \frac{1}{3}\sqrt{7\left(\lim_{s\to 1/8}(p_1^*)\right)^2} = \frac{4-\sqrt{7}}{3}\lim_{s\to 1/8}(p_1^*)$$

where we again used that $w^d(p_1^*) \to p_1^*$ as $s \to 1/8$. It thus follows that

$$\lim_{s\to 1/8}\left(p_3^d(p_1^*) - p_1^*\right) = \frac{1-\sqrt{7}}{3}\lim_{s\to 1/8}(p_1^*) < 0,$$

which shows that deviating to non-disclosure is strictly profitable. This completes the proof. ■

Proof of Lemma 3: Consider the search decision of a consumer who has visited firm j first. Note that consumers have passive beliefs. When receiving the (potentially off-equilibrium) price p_1, a consumer with match value u_j will thus search if and only if:

$$\int_0^{u_j - p_3^f(p_1)+p_2^f} \max\{u_j - p_3^f(p_1), 0\} f(u_{-j}) du_{-j} + \int_{u_j - p_3^f(p_1)+p_2^f}^1 \max\{u_{-j} - p_2^f, 0\} f(u_{-j}) du_{-j} - s >$$

$$\max\{u_j - p_1, 0\}, \tag{89}$$

where the left-hand side depicts the value of continuing to search and the right-hand side the value of not doing so. The optimal search strategy is a cutoff rule with cutoff $w^f(p_1)$. This holds by the following logic: Because we restrict attention to search costs which admit on-path search, a consumer always strictly prefers to continue search (and does not buy immediately) if $u_j \leq p_1$. Moreover, if $u_j > p_1$, a consumer's gains from search (the difference between the left-hand side and the right-hand side of equation (89) is strictly falling in u_j. To see this, note that the derivative of the gains of search for $u_j \geq p_1$ is bounded from above by

$$F\left(u_j - p_3^f(p_1) + p_2^f\right) - 1. \tag{90}$$

Thus, there must be a unique cutoff $w^f(p_1)$ so that consumers continue searching if and only if their initial match value is below $w^f(p_1)$.

To show that $w^f(p_1)$ is given by the expression presented in the lemma, observe that the maximum functions on both sides of (89) vanish if $u_j = w^f(p_1)$. This follows from the fact that (i) $w^f(p_1) > p_1$ and (ii) $w^f(p_1) > p_3^f(p_1)$ must hold.

While (i) holds by previous arguments, (ii) is due to the following logic: In a PBE, $w^f(p_1) > p_3^f(p_1)$ must hold for any initial price p_1. To see this, suppose toward a contradiction that $w^f(p_1) \leq$

$p_3^f(p_1)$. Since firms must have consistent beliefs in any PBE, firm j in the information set $\mathcal{H}(j) = D \times p_1 \times R$ will believe that all returning consumers have $u_j \leq w^f(p_1)$. When setting a price $p_3^f(p_1)$ above $w^f(p_1)$, the firm's profits in this information set would be zero. By contrast, it is easy to see that j could earn strictly positive profits in the information set $\mathcal{H}(j) = D \times p_1 \times R$ by charging a price of $p_3^f(p_1) < w^f(p_1)$, contradicting that $w^f(p_1) \leq p_3^f(p_1)$ holds in equilibrium.

Point (ii) also implies that, for any $u_{-j} > w^f(p_1) - p_3^f(p_1) + p_2^f$, we have that $\max\{u_{-j} - p_2^f, 0\} = u_{-j} - p_2^f$. Thus, regardless of whether $p_1 < p_3^f(p_1)$ or not, the cutoff $w^f(p_1)$, if it lies strictly below 1, sets the following equation equal to 0:

$$T(w^f, p_1) := (w^f - p_3^f(p_1) + p_2^f)(w^f - p_3^f(p_1)) + \int_{w^f - p_3^f(p_1) + p_2^f}^{1} (u_{-j} - p_2^f) du_{-j} - s - (w^f - p_1) = 0 \tag{91}$$

which completes the proof of the lemma.

For future reference, we use $T(w^f, p_1)$ to find the derivative of $w^f(p_1)$ w.r.t. p_1:

$$\frac{\partial T}{\partial p_1} = 1 - (w^f(p_1) - p_3^f(p_1) + p_2^f)\frac{\partial p_3^f(p_1)}{\partial p_1} \quad ; \quad \frac{\partial T}{\partial w^f} = (w^f(p_1) - p_3^f(p_1) + p_2^f) - 1 \tag{92}$$

This establishes that, so long as $w^f(p_1)$ is interior:

$$\frac{\partial w^f(p_1)}{\partial p_1} = \frac{1 - (w^f(p_1) - p_3^f(p_1) + p_2^f)\frac{\partial p_3^f(p_1)}{\partial p_1}}{1 - (w^f(p_1) - p_3^f(p_1) + p_2^f)}. \tag{93}$$

■

Proof of Lemma 4: There are two cases, namely $p_1^f > p_3^f$ and $p_1^f \leq p_3^f$. The first case cannot be an equilibrium because no firm j would disclose in the information set $\mathcal{H}(j) = R$ if $p_1^f > p_3^f$. This is because $d_j = D$ when $\mathcal{H}(j) = R$ reduces the competitor's price (since $p_1^f > p_3^f$) and intensifies competition. It is therefore sufficient to study the case that $p_1^f \leq p_3^f$.

Part 1: Derivation of p_3^f and p_2^f.

To derive the optimal price $p_3^f(p_1)$ firm j offers in the information set $\mathcal{H}(j) = D \times p_1 \times R$, we compute the derivative of $\Pi^{3,f}(p_3|p_1)$ as defined in equation (15). Given that $w^f(p_1^f) - p_3^f + p_2^f < 1$ must hold, the equilibrium price $p_3^f(p_1) < w^f(p_1)$ must satisfy the following first-order condition for p_1 close to p_1^f:

$$\int_{p_3}^{w^f(p_1)} \frac{1}{2}\left(u_j - p_3 + p_2^f\right) du_j - \frac{1}{2} p_3 \left(w^f(p_1) - p_3 + p_2^f\right) = 0 \tag{94}$$

Thus, the price $p_3^f(p_1)$ is given by:

$$p_3^f(p_1) = \frac{2}{3}(w^f(p_1) + p_2^f) - \frac{1}{3}\sqrt{\left(w^f(p_1) + p_2^f\right)^2 + 3\left(p_2^f\right)^2} \tag{95}$$

Consider p_2^f next. We showed in the main text that the profit function of firm B in the information set $\mathcal{H}(B) = R$ for prices in an open ball around the equilibrium p_2^f is

$$\Pi^{2,f}(p_2) = p_2 \frac{1}{2} F(w^f)\big[1 - F(w^f - p_3^f + p_2)\big] + p_2 \int_{p_2}^{w^f - p_3^f + p_2} \frac{1}{2} F(p_3^f + u_B - p_2) du_B \tag{96}$$

The corresponding first-order condition is:

$$w^f\left[1 - (w^f - p_3^f + p_2)\right] - w^f p_2 + \int_{p^2}^{w^f - p_3^f + p_2} (p_3^f + u_B - p_2) du_B = 0 \tag{97}$$

Thus, for a fixed w^f, the equilibrium price p_2^f must solve:

$$p_2^f = (1/2)\big[1 - w^f + p_3^f\big] + \frac{1}{4}(w^f) - \frac{1}{4}\frac{(p_3^f)^2}{(w^f)} \tag{98}$$

Part 2: For any w^f, the equilibrium prices p_2^f and $p_3^f := p_3^f(p_1^f)$ are uniquely determined.

Using (95), the revision price for a fixed search cutoff w^f is given by

$$p_3^f = \frac{2}{3}\left(w^f + p_2^f\right) - \frac{1}{3}\sqrt{\left(w^f + p_2^f\right)^2 + 3\left(p_2^f\right)^2} \tag{99}$$

For a fixed search cutoff w^f, the derivative of p_3^f with respect to p_2^f is:

$$\frac{\partial p_3^f}{\partial p_2^f} = (2/3) - (1/3)(1/2)\frac{2\big(w^f + p_2^f\big) + 6p_2^f}{\sqrt{\big(w^f + p_2^f\big)^2 + 3\big(p_2^f\big)^2}} < 2/3 \tag{100}$$

Moreover, for a fixed w^f, we have:

$$\frac{\partial p_2^f}{\partial p_3^f} = (1/2) - (1/2)\underbrace{(p_3^f/w^f)}_{<1} \in (0, 1/2) \tag{101}$$

We can define the solution price p_2^f using a fixed-point expression:

$$T^2(p_2^f) := p_2^f - p_2\big(p_3(p_2^f)\big) = 0 \implies \frac{\partial T^2}{\partial p_2^f} = 1 - \frac{\partial p_2}{\partial p_3}\frac{\partial p_3}{\partial p_2} \tag{102}$$

Note that $\frac{\partial p_3}{\partial p_2} < 2/3$ and $\frac{\partial p_2}{\partial p_3} \in \big(0, 1/2\big)$, which implies that $\frac{\partial p_2}{\partial p_3}\frac{\partial p_3}{\partial p_2} < (1/3)$. Thus, T^2 is strictly rising in p_2. Consequently, there is a unique solution for p_2^f, and by extension, for p_3^f, for any given w^f.

Part 3: Initial equilibrium characterization: $w^f < 1$ must hold.

Suppose toward a contradiction that $w^f = 1$ for some level of search costs $s > 0$ and recall that $p_1^f \leq p_3^f$ must hold in a full disclosure equilibrium. If $w^f = 1$, it is easy to verify that the unique solutions for p_2^f and p_3^f are given by $p_2^f = p_3^f = 0.4142$. In addition, $p_2^f = p_3^f$ combined with $p_1^f \leq p_3^f$ and $s > 0$ implies that consumers with a match value close to 1 at the first firm would not continue to search, contradicting that $w^f = 1$. To see this, note that the gains of search are continuous in the initial match value and that a consumer with initial match value of 1 would strictly prefer to not continue searching.

Part 4: Derivation of p_1^f.

p_1^f must maximize $\Pi^{1,f}(p_1)$ as defined in equation (17). Using the Envelope theorem, which implies that $\partial\Pi^{3,f}(p_3|p_1)/\partial p_3 = 0$, the first-order condition that p_1^f must solve can be written as:

$$\frac{1}{2}(1 - w^f(p_1)) - \frac{1}{2}\frac{\partial w^f(p_1)}{\partial p_1}p_1 +$$

$$\frac{1}{2}p_3(p_2(w^f), w^f(p_1))\frac{\partial w^f(p_1)}{\partial p_1}\big(p_2(w^f) + w^f(p_1) - p_3(p_2(w^f), w^f(p_1))\big) = 0 \tag{103}$$

In an equilibrium, $w^f(p_1^f) = w^f$ by definition, which implies that p_1^f must solve:

$$p_1^f = (1 - w^f)\left(\frac{\partial w^f(p_1^f)}{\partial p_1}\right)^{-1} + p_3^f\big(p_2^f + w^f - p_3^f\big) \tag{104}$$

■

Proof of Proposition 3: We seek to show that $p_3^f < p_1^f$ in any full disclosure equilibrium, implying a profitable deviation when $\mathcal{H}(j) = R$. We prove this in three parts.

Part 1: A solution for $\frac{\partial w^f(p_1)}{\partial p^1}$ at $p_1 = p_1^f$.

By optimal consumer search, it was established that the derivative of $w^f(p_1)$ w.r.t p_1 depended on $\frac{\partial p_3^f(p_1^f)}{\partial p_1}$. To pin down $\frac{\partial w^f(p_1^f)}{\partial p_1}$, recall that the deviation $p_3^f(p_1)$ solves — for a given w^f and for p_1 around p_1^f — the following function:

$$p_3^f(p_1) = \frac{2}{3}\left(w^f(p^1) + p_2^f\right) - \frac{1}{3}\sqrt{\left(w^f(p^1) + p_2^f\right)^2 + 3\left(p_2^f\right)^2} \tag{105}$$

At $p_1 = p_1^f$, we have $w^f(p_1^f) = w^f$, which implies that:

$$\frac{\partial p_3^f(p_1^f)}{\partial p_1} = \frac{\partial w^f(p_1)}{\partial p_1}\left[\frac{2}{3} - \frac{1}{3}\frac{1}{2}\frac{2\big(w^f + p_2^f\big)}{\sqrt{\big(w^f + p_2^f\big)^2 + 3\big(p_2^f\big)^2}}\right] \tag{106}$$

Plugging this back into the expression for $\frac{\partial w^f(p_1)}{\partial p_1}$ given in equation (93), which was derived from consumers' optimal search behaviour, implies that, at $p_1 = p_1^f$, expression (106) becomes:

$$\frac{\partial w^f(p_1^f)}{\partial p_1}\left[1 - (w^f - p_3^f + p_2^f) + (w^f - p_3^f + p_2^f)\left(\frac{2}{3} - \frac{1}{3}\frac{\big(w^f + p_2^f\big)}{\sqrt{\big(w^f + p_2^f\big)^2 + 3\big(p_2^f\big)^2}}\right)\right] = 1 \tag{107}$$

The term in brackets is strictly positive, because $(w^f - p_3^f + p_2^f) < 1$ must hold in equilibrium. Thus, the derivative of $w_1^f(p_1)$ w.r.t. p_1 (evaluated at the equilibrium value p_1^f) is independent of the exact value of p_1^f. Moreover, it is strictly positive.

Part 2: Uniqueness of p_1^f.

The results from Part 1 imply that $\frac{\partial w^f(p_1^f)}{\partial p_1}$ will be strictly positive and independent of the exact value of p_1^f. This establishes that p_1^f is uniquely pinned down and given by:

$$p_1^f = (1 - w^f)\left(\frac{\partial w^f(p_1^f)}{\partial p_1}\right)^{-1} + p_3^f\big(p_2^f + w^f - p_3^f\big) \tag{108}$$

Part 3: There exists no full disclosure equilibrium.

Previous arguments have established that $w^f < 1$ must hold in equilibrium. However, examining the unique joint solutions (p_1^f, p_2^f, p_3^f) establishes that, for any $w^f < 1$, the ordering $p_3^f < p_1^f$ will hold. This, however, is a contradiction. Under this ordering of equilibrium prices, firms would prefer to deviate and not disclose after receiving disclosure.

■

Proof of Proposition 4: The result directly follows from the discussion on page 20. ■

Proof of Proposition 5: The result directly follows from the discussion in the two paragraphs after the Proposition on pages 21 and 22. ■

Consumer surplus calculations: We define consumer surplus as the ex-ante expected utility of the buyer that we rely on throughout the paper, in particular in Section 5.2. To calculate consumer surplus, define $u^s(u_j)$ and $u^{ns}(u_j)$ as the expected utilities of searching and not searching, respectively, for a buyer that draws an initial match value u_j.

Consider the following general formulation where we define p_1^f as the price the buyer would receive at the initial firm she visits and p_2^f and p_3^f as the other prices she could receive second (or when returning to the initially visited firm) on the search path. Note that this formulation nests all our equilibria. Recall further that there was always a unique search cutoff w^f such that buyers search (in equilibrium) if and only if their initial match value is below this cutoff. Noting this, the ex-ante utility (BS) of the buyer is:

$$BS = \int_0^{w^f} u^s(u_j)du_j + \int_{w^f}^1 u^{ns}(u_j)du_j, \tag{109}$$

where $u^{ns}(u_j) = \max\{u_j - p_1^f, 0\} - s$ and $u^s(u_j)$ is given by

$$u^s(u_j) = \int_0^{\min\{\max\{u_j - p_3^f, 0\}+p_2^f, 1\}} \max\{u_j - p_3^f, 0\}du_{-j} + \int_{\min\{\max\{u_j - p_3^f, 0\}+p_2^f, 1\}}^1 (u_{-j} - p_2^f)du_{-j} - 2s.$$

We use (109) to calculate buyer surplus for different search costs in Figure 4.

B Extension: Search with Costly Recall

A key comparative static result of our analysis is that sellers do not use search disclosure in equilibrium if search costs are too large. Crucially, this prediction hinges on another result, namely that the no disclosure equilibrium exists when the partial disclosure equilibrium does not (the former always exists in the base model). The reason why no disclosure is an equilibrium in the baseline model regardless of search costs was that deviating to disclosure can induce the rival to revise its price downward, the negative effect of which outweighed any benefits from disclosing. The dominance of this negative effect on the deviating firm's profits is directly related to the free recall assumption. This is because free recall guarantees that every consumer learns about the revised price of the firm they visited first before they make a purchase decision, which is to the detriment of the disclosing firm.

In this extension, we therefore study the case in which a positive mass of consumers face strictly positive recall costs. While positive recall costs mitigate the negative effects of deviating to search disclosure, we document that our results are robust in the sense that no disclosure remains an equilibrium for a large share of parameter combinations, and in particular when search costs are not too low. This is exactly what was to be expected, given that the costs of triggering a price revision by one's rival are smallest for low search costs. This is because price revisions are small in magnitude when search costs are small, given that the decision to search is not very informative about a consumer's match value. We also argue why partial disclosure does not emerge as an equilibrium if search costs are too large.

To model costly recall, we extend the framework of Janssen and Parakhonyak (2014), who consider costly recall for all consumers. In our analysis, we assume that a share $1 - \rho < 1$ of all consumers face recall costs. Precisely, $1 - \rho$ consumers must incur the cost $b > 0$ if they want to return to the seller they visited first after having continued to search. The remaining $\rho > 0$ share of consumers have free recall as in the baseline model. Everything else is identical to the baseline model as well.

We focus on the no disclosure equilibrium in this framework with costly recall. In this equilibrium, there is just one equilibrium price, which we call p^c, given that firms know nothing about the consumers' search histories in equilibrium. That is, consumers expect to receive the equilibrium price p^c at any firm and expect no revisions of prices. Thus, consumers with free recall will search if their match value is below $w^0(p_j;p^c) = \left(1 - \sqrt{2s}\right) - p^c + p_j$, which is similar to the baseline model. The optimal cutoff for consumers with recall costs, however, is different. Consumers with costly recall search if their match value is below

$$w^b(p_j;p^c) = \left(1 + b - \sqrt{2s + 2b}\right) - p^c + p_j. \tag{110}$$

As before, we restrict attention to equilibria with active search. As in Janssen and Parakhonyak (2014), we restrict attention to search costs under which there also is positive return demand by consumers with recall costs in equilibrium, i.e. $w^b(p^c;p^c) > p^c + b$[27].

In equilibrium, the firms obtain profits from both consumers with free recall and consumers with costly recall. The demand from the former group, which we denote by $D^0(p_j)$ here, has the same structure as defined in equation (8). Equilibrium demand from consumers with recall costs, which we here denote by $D^b(p_j)$, is novel and given by:

$$D^b(p_j) = \underbrace{\int_{w^b(p_j;p^c)}^{1} (1/2)du_j + \int_{p_j+b}^{w^b(p_j;p^c)} (1/2)F(u - p_j - b + p^c)du_j}_{\text{first arriver demand}} +$$

$$\underbrace{w^b(p^c;p^c) \int_{w^b(p_j;p^c)-b}^{1} (1/2)du_j + \int_{p_j}^{w^b(p_j;p^c)-b} (1/2)F(u_j - p_j + b + p^c)du_j}_{\text{searcher demand}} \tag{111}$$

To understand this expression, consider first the demand from consumers who arrive at firm j first. Because $w^b(p_j;p^c) > p_j$, any consumer who arrives at firm j first and obtains a match value above $w^b(p_j;p^c)$ will directly buy. Any consumer with $u_j < w^b(p_j;p^c)$ will search and ultimately buy at firm j if and only if it is worthwhile to return at all (i.e. $u_j > p_j + b$) *and* it is better to return than to purchase at the other firm (i.e. $u_j - p_j - b > u_{-j} - p^c$).

[27] The equivalent condition on b and s when there are no consumers with free recall can be found in proposition 6 in Janssen and Parakhonyak (2014).

Next, consider consumers who visit firm $-j$ first. These consumers are expected to sample firm j only if $u_{-j} < w^b(p^c; p^c)$, given that they are expected to receive the price p^c at firm $-j$. If their match at firm j is above $w^b(p_j; p^c) - b$, any such consumer will surely buy at firm j. If their match value u_j is below this cutoff, they will buy at firm j if this match value exceeds p_j and it is better to purchase at firm j than to return to firm $-j$, i.e., $u_j - p_j > u_{-j} - p^c - b$.

We define the equilibrium profit function of the firm as $\Pi(p_j; p^c)$, which is given by:

$$\Pi(p_j; p^c) = p_j \left[\rho D^0(p_j) + (1-\rho) D^b(p_j)\right] \tag{112}$$

The equilibrium price p^c in the no disclosure equilibrium must thus solve:

$$p^c = arg \max_{p_j} \Pi(p_j; p^c) \tag{113}$$

The no disclosure equilibrium exists if it is not worthwhile for a firm to deviate from the equilibrium by disclosing when an unknown buyer arrives at the firm. As in the baseline framework, such a deviation by firm j will have different effects, depending on whether the buyer has visited firm $-j$ before or not. If a buyer who arrives at firm j first (without firm j knowing) and firm j discloses, then firm $-j$ will offer a price p_2^c to the consumer. As before, this price p_2^c is above the equilibrium price p^c.

By contrast, if a buyer arrives at firm j after having visited firm $-j$ before, then search disclosure leads the rival firm $-j$ to revise its price. We define the price that a firm would choose in this information set as p_3^c. Recall that the revised price was always below the equilibrium price in the baseline framework. Due to presence of consumers with costly recall this is no longer true in general. This is because consumers with recall costs who return to the firm they initially visited generate inelastic demand around the original price.[28]

The profit function of a firm that receives disclosure for a known buyer is non-differentiable at the price $p^c + b$. This holds because, given the firm's beliefs, it anticipates that all consumers with recall costs who return to the firm surely buy when offered a return price below $p^c + b$, which is not true when the return price is above $p^c + b$.

For any price $p_j \leq p^c + b$, a firm j who receives disclosure for a known buyer believes it will make obtain the following profits:

$$\Pi^R(p_j; p^c) = p_j \Bigg[\underbrace{(1-\rho) \int_{p^c+b}^{w^b(p^c;p^c)} (u_j - b) du_j}_{\text{Consumers with recall costs}} + \underbrace{\rho \int_{p_j}^{w^0(p^c;p^c)} (u_j - p_j + p^c) du_j}_{\text{Consumers with free recall}} \Bigg] \tag{114}$$

[28]Note that we still assume passive beliefs here. This implies that a firm who receives disclosure still believes that its rival set the equilibrium price p^c, and that consumers searched according to the equilibrium search rules.

To understand this expression, note the following. From firm j's points of view, all consumers with recall costs who return to firm j must (i) have a match value above $p^c + b$ and (ii) must have a difference in match values, namely $u_j - u_{-j}$, that is greater than b. Result (ii) holds because firm j believes that the consumer received the price p^c at its rival. In that situation, it would only be worthwhile for a consumer with recall costs to return to firm j instead of buying at the rival if $u_j - b - p^c > u_{-j} - p^c$, i.e. if and only if $u_j - u_{-j} > b$.

These results imply that the demand from returning consumers with costly recall is fully inelastic for $p_j \leq p^c + b$. When offering a price $p_j \leq p^c + b$, the match value of any any such consumer who returns will exceed the price, by result (i). Moreover, firm j would also expect any such consumer to buy at firm j rather than at firm $-j$. This is because, by result (ii), the consumption utility at firm j (namely $u_j - p_j$) remains above the consumption utility at firm $-j$ (namely $u_j - p^c$) for any $p_j \leq p^c + b$.

At any price $p_j > p^c + b$, by contrast, the demand generated by returning consumers with recall costs is elastic. Then, the objective function $\Pi^R(p_j; p^c)$ is:

$$\Pi^R(p_j; p^c) = p_j \Bigg[\underbrace{(1-\rho) \int_{p_j}^{w^b(p^c;p^c)} (u_j - p_j + p^c) du_j}_{\text{Consumers with recall costs}} + \underbrace{\rho \int_{p_j}^{w^0(p^c;p^c)} (u_j - p_j + p^c) du_j}_{\text{Consumers with free recall}} \Bigg] \tag{115}$$

The two different consumer groups hence affect the optimal return price in different ways: Consumers with recall costs push up the optimal revision price p_3^c, while consumers with free recall exert downward pressure on this price. As a result, the relationship between p_3^c and s is non-monotonic. Using numerical methods, we compute p_3^c (and all other relevant equilibrium objects) for different parameter combinations. The results are visualized by the following graphs, which plot the relationship between p_3^c and s for different levels of ρ and b:

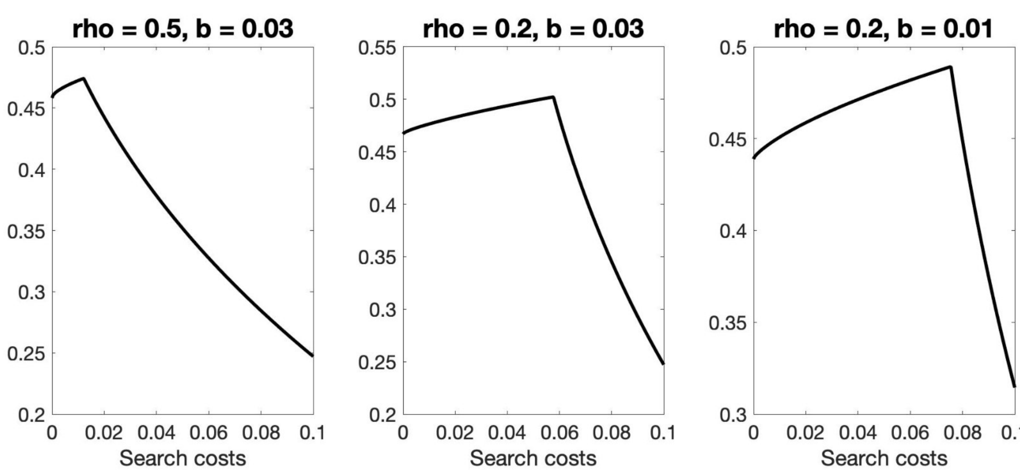

Figure 5: Costly recall - return prices

For low levels of s, the price p_3^c equals $p^c + b$ and thus lies strictly above p^c. To see why this is optimal, recall that consumers with recall costs generate inelastic demand for prices $p_j \leq p^c + b$. These consumers thus push the optimal revision price of the firm up towards $p^c + b$, but no further than that, because the associated profits have a kink at this price. When search costs are low, the measure of consumers with costly recall who arrive at a firm is relatively high, which implies that the optimal revision price equals $p^c + b$. The optimal revision price p_3^c will fall below p^c only for sufficiently high search costs, at which the weight of returning consumers with free recall, which push the price below p^c, becomes high enough.

For low values of s, there is hence no detrimental effect of disclosure because disclosure always leads to upward changes in the rival's price. Thus, the no disclosure equilibrium does not exist for low search costs in this setup. Only when p_3^c falls sufficiently far below p^c and the detrimental effect of disclosure is sufficiently strong, no disclosure becomes an equilibrium. This is visualized by the following figure. In each graph, we compare the equilibrium profits $\Pi(p^c; p^c)$ (dotted line) to the profits that are attainable via a deviation to disclosure (solid line) for different combinations of ρ and b. If the equilibrium profits are above the deviation profits, no disclosure is an equilibrium.[29]

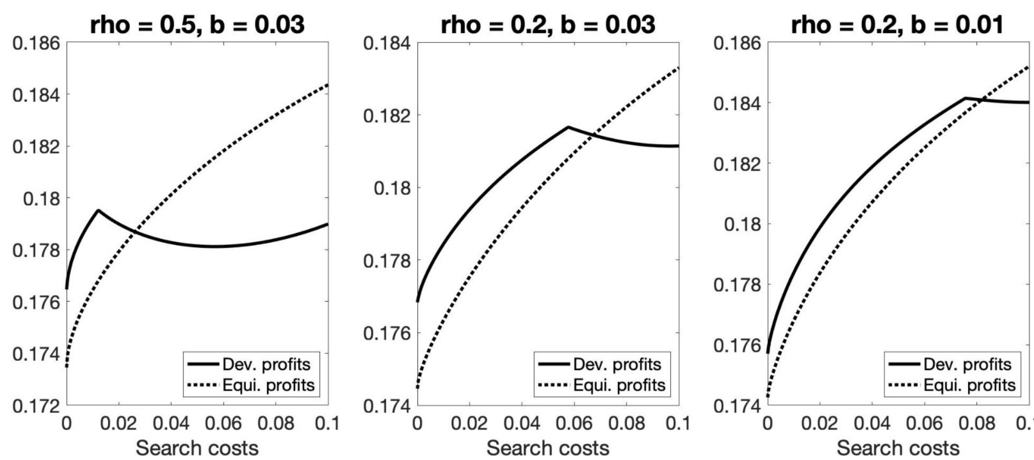

Figure 6: Costly recall - deviation incentives

There are three main takeaways: First, the no disclosure equilibrium continues to exist for a wide range of search costs even when 50% of consumer have substantial return costs ($b = 0.03$).

Second, the presence of return costs does strongly counteract the existence of this equilibrium if search costs are too small, which has implications for the optimal regulation of the markets we describe. For instance, enabling price retargeting of consumer through tracking (i.e. raising ρ) may be quite beneficial, because it counteracts the incidence of search history based price discrimination.

[29]The validity of this claim hinges on one additional observation. No firm must find it optimal to deviate by disclosing after having received disclosure. We numerically verified that this is the case if search costs are low enough such that $\Pi(p^c; p^c)$ is greater than the depicted deviation profits.

Third, the analysis has reaffirmed that the incentives to conduct search disclosure are reduced when search costs increase. Intuitively, this notion would also carry over when analyzing the partial disclosure equilibrium in the extension with costly recall. We conjecture that the partial disclosure equilibrium will only exist when search costs are sufficiently small. When they are high, consumers who leave a firm to search will never return (this effect is only reinforced by recall costs). Thus, the benefits of search disclosure (i.e. increasing the price the rival would set upon being visited second) will be negligible even under costly recall. By contrast, the benefits of withholding disclosure are still large even if search costs are high, because the resulting selection will be particularly pronounced. By similar arguments, the full disclosure equilibrium may exist if recall is costly, but only if search costs are very small.

B.1 Missing Proofs

Part 1: Equilibrium characterization

Janssen and Parakhonyak (2014) show that the reservation utility of a consumer with recall costs in a uniform price equilibrium (with equilibrium price p^c) is:

$$w^b(p_j) = \left(1 + b - \sqrt{2b + 2s}\right) - p^c + p_j \tag{116}$$

In equilibrium, consumers with free and costly recall will search if and only if their initial valuation is below $w^{*,0}$ and $w^{*,b}$, which are respectively defined as follows:

$$w^{*,0} = w^0(p^c) := 1 - \sqrt{2s} \quad ; \quad w^{*,b} = w^b(p^c) := 1 + b - \sqrt{2b + 2s} \tag{117}$$

Consider consumers for whom recall is free and who arrive at firm j:

- First arrivers with $u_j > w^0(p_j)$ will buy at firm j.
- First arrivers with $u_j \in [p_j, w^0(p_j)]$ buy at firm j iff $u_{-j} < u_j - p_j + p^c$.
- Second arrivers with $u_j > w^0(p_j)$ buy at firm j if and only if $u_{-j} < w^{*,0}$.
- Second arrivers with $u_j \in [p_j, w^0(p_j)]$ buy at firm j if and only if $u_{-j} < u_j - p_j + p^c$.

Now consider consumers for whom recall is costly.

- First arrivers with $u_j > w^b(p_j)$ will buy at firm j.
- First arrivers with $u_j \in [p_j, w^b(p_j)]$ buy at firm j iff $u_{-j} < u_j - p_j - b + p^c$ and $u_j - p_j - b > 0$.
- Second arrivers with $u_j > w^b(p_j)$ buy at firm j if and only if $u_{-j} < w^{b,*}$.
- Second arrivers with $u_j \in [p_j, w^b(p_j)]$ buy at firm j iff $u_{-j} < w^{b,*}$, and $u_{-j} < u_j - p_j + b + p^c$.

Firstly, consider the components of demand from consumers with strictly positive return costs $b > 0$. Consider first arrivers with $u_j > w^b(p_j)$. Demand from these consumers is:

$$D^1(p_j) = \int_{w^b(p_j)}^{1} (1/2) du_j \tag{118}$$

Consider second arrivers with $u_j \in [p_j, w^b(p_j) - b]$. The condition that $u_{-j} < u_j - p_j + b + p^c$ implies that they have searched (i.e. $u_{-j} < w^{b,*}$), because:

$$u_{-j} < w^b(p_j) - b - p_j + b + p^c = w^{b,*}$$

Now consider second arrivers with $u_j \in [w^b(p_j) - b, 1]$. For these consumers, search guarantees consumption. Search requires that $u_{-j} < w^{b,*}$. Consumption occurs at j if $u_j - p_j > u_{-j} - p^c - b \iff u_j - p_j + b > u_{-j} - p^c$. If these consumers searched, we have:

$$u_{-j} - p^c < w^{b,*} - p^c = w^b(p_j) - p_j \leq u_j - p_j + b$$

Thus, demand implied by second arrivers is:

$$D^S(p_j) = w^{b,*} \int_{w^b(p_j)-b}^{1} (1/2) du_j + \int_{p_j}^{w^b(p_j)-b} (1/2)(u_j - p_j + b + p^c) du_j \tag{119}$$

Thirdly, consider the demand that comes from agents with return costs that visit firm j first, search, and then return. This demand is:

$$D^R(p_j) = \int_{p_j+b}^{w^b(p_j)} (1/2)(u_j - p_j - b + p^c) du_j \tag{120}$$

One can show that demand from consumers with return cost b is thus:

$$D(p_j; b) = (1/2)\big(2p^c - 2p_j + 1 - (p^c)^2 - p_j b - p^c\sqrt{2b+2s} + p_j\sqrt{2b+2s}\big) \tag{121}$$

Thus, total demand is:

$$D(p_j) = \frac{\big(2p^c - 2p_j + 1 - (p^c)^2 - p_j(1-\rho)b\big) - p^c\big(\rho\sqrt{2s} + (1-\rho)\sqrt{2b+2s}\big) + p_j\big(\rho\sqrt{2s} + (1-\rho)\sqrt{2b+2s}\big)}{2} \tag{122}$$

Thus, the equilibrium price will need to solve:

$$p_j \frac{\partial D(p_j)}{\partial p_j} + D(p_j) = 0 \iff$$

$$\big(2p^c - 2p_j + 1 - (p^c)^2 - p_j(1-\rho)b\big) - p^c\big(\rho\sqrt{2s} + (1-\rho)\sqrt{2b+2s}\big) + p_j\big(\rho\sqrt{2s} + (1-\rho)\sqrt{2b+2s}\big)$$

$$+p_j\bigg(- 2 - (1-\rho)b + \big(\rho\sqrt{2s} + (1-\rho)\sqrt{2b+2s}\big)\bigg) = 0 \tag{123}$$

The equilibrium price is thus:

$$p^* = \frac{-d^* - \sqrt{(d^*)^2 - 4(a^*)(c^*)}}{2a^*} \tag{124}$$

We have defined $a^* = -1$, $c^* = 1$, and:

$$d^* = -2 - 2(1-\rho)b + \rho\sqrt{2s} + (1-\rho)\sqrt{2b+2s}$$

Naturally, this is only the correct equilibrium price as long as $p^c + b < w^{b,*} := w^b(p^c)$.

Part 2: Optimal second arriver pricing:

Suppose a firm j receives search disclosure for a previously unknown buyer, which implies that this consumer must have arrived second and must have $u_{-j} < w^{*,b}$ or $u_{-j} < w^{*,0}$, respectively. For second arrivers with return costs b, demand is:

$$D^S(p_j; b) = w^{b,*}\int_{w^b(p_j)-b}^{1} (1/2)du_j + \int_{p_j}^{w^b(p_j)-b} (1/2)(u_j - p_j + b + p^c)du_j \tag{125}$$

For second arrivers with free recall (share ρ), demand is:

$$D^S(p_j; 0) = w^{0,*}\int_{w^0(p_j)}^{1} (1/2)du_j + \int_{p_j}^{w^0(p_j)} (1/2)(u_j - p_j + p^c)du_j \tag{126}$$

Thus, a firm who receives disclosure for a previously unknown buyer will maximize the following through choice of p_2:

$$p_2\big[\rho D^S(p_j; 0) + (1-\rho)D^S(p_j; b)\big] \tag{127}$$

Part 3: Optimal revision of prices

Now consider the optimal price that a firm j would set when receiving disclosure for a consumer it has seen before. Any such consumer with return costs must have $u_j < w^{*,b}$ and must find it optimal to return (expecting to receive p^c at the initial firm they visit). This requires that (i) $u_j > p^c + b$ and (ii) $u_j - p^c - b > u_{-j} - p^c \iff u_{-j} < u_j - b$.

Consider prices $p_j \in [p^c, p^c + b]$. For any such price, a consumer who returns can buy (by condition (i)) and will buy, given that they still prefer to buy at firm j (by condition (ii)). Thus, demand (from consumers with return costs) for prices $p_j \in [p^c, p^c + b]$ is:

$$D^R(p_j, b) = \int_{p^c+b}^{w^{*,b}} \int_0^{u_j - b} (1/2) du_{-j} du_j \tag{128}$$

Moreover, demand (from consumers with return costs) for prices $p_j \in [p^c + b, 1]$ is:

$$D^R(p_j, b) = \int_{p_j}^{w^{*,b}} \int_0^{u_j - p_j + p^*} (1/2) du_{-j} du_j = \int_{p_j}^{w^{*,b}} (1/2)(u_j - p_j + p^c) du_j \tag{129}$$

Both these probabilities will be strictly interior.

Recalling that a share ρ of agents have no recall costs, total demand from returning consumers for prices $p_j \leq p^c + b$ is:

$$D^R(p_j) = \rho \int_{p_j}^{w^{*,0}} (1/2)(u_j - p_j + p^*) du_j + (1 - \rho) \int_{p^*+b}^{w^{*,b}} (1/2)(u_j - b) du_j \tag{130}$$

Return demand can be similarly computed for $p_j > p^* + b$, noting that the component derived from consumers without return costs stays the same.

Part 4: The effects of a deviation by disclosure

Now suppose that firm j, who initially receives no disclosure, deviates by disclosing.

Consider any second arriver with return costs and $u_j \in [w^b(p_j) - b, 1]$. By previous arguments, any such consumer will arrive at firm j and not return to firm $-j$, because she does not anticipate a revision of the original price at firm $-j$. Thus, conducting search disclosure will not harm the disclosing firm when facing such a consumer.

Now consider any second arriver with return costs and $u_j \in [p_j, w^b(p_j) - b]$ and $u_{-j} \in [0, u_j - p_j + b + p^c]$. In the perception of any such consumer, it is not optimal to return to firm $-j$. Thus, conducting search disclosure will not harm the disclosing firm when facing such a consumer.

Any consumer with $u_j \in [p_j, w^b(p_j) - b]$ and $u_{-j} \in [u_j - p_j + b + p^c, 1]$ would in fact return to firm $-j$ (anticipating to receive the price p^c there). Any such consumer would thus buy at firm j iff and only if $u_j - p_j > u_{-j} - p_3$. When $p_3 < p^c$, no such consumer would ever buy at firm j. However, if $p_3 \geq p^c$, there is a chance that any such consumer buys at firm j. This occurs if $u_j - p_j > u_{-j} - p_3$. Thus, any such consumer will buy at firm j if $u_{-j} \in [u_j - p_j + p^c + b, u_j - p_j + p_3]$.

Thus, demand from second arrivers with return costs is:

$$D^S(p_j) = (1/2)w^{b,*}[1-(w^b(p_j)-b)] + \int_{p_j}^{w^b(p_j)-b} (1/2)(u_j - p_j + b + p^c)du_j +$$

$$\int_{p_j}^{w^b(p_j)-b} \int_{u_j-p_j+p^c+b}^{u_j-p_j+p_3} (1/2)du_{-j}du_j \tag{131}$$

Now consider first arrivers with return costs. Consumers with $u_j > w^b(p_j)$ will never search, so demand from them is still given by:

$$D^1(p_j) = \int_{w^b(p_j)}^{1} (1/2)du_j \tag{132}$$

This is because $w^b(p_j) > p_j$. Thirdly, consider consumers with return costs $b > 0$ who arrive at firm j first, search, and return. Given the timing of search disclosure, they have received the price $p^{2,d}$. Because the rival firm will not engage in search disclosure, the demand implied by these consumers is:

$$D^R(p_j) = \int_{p_j+b}^{w^b(p_j)} (1/2)(u_j - p_j - b + p^2)du_j \tag{133}$$

Deviation demands from consumers without return costs have been derived previously — this is given in equation (11). We can use all these notions to compute the total demand (and thus the optimal price) set by a firm after deviating by non-disclosure. This profit is:

$$\Pi^{1,b}(p_j) = p_j(1-\rho)\frac{1}{2}\Bigg[\int_{w^b(p_j)}^{1}(1)du_j + \int_{p_j+b}^{w^b(p_j)}(u_j - p_j - b + p^2)du_j +$$

$$w^{b,*}[1-(w^b(p_j)-b)] + \int_{p_j}^{w^b(p_j)-b}(u_j - p_j + b + p^c)du_j + \int_{p_j}^{w^b(p_j)-b}\int_{u_j-p_j+p^c+b}^{u_j-p_j+p_3}(1)du_{-j}du_j\Bigg] +$$

$$p_j\frac{\rho}{2}\Bigg[\big[1-w^0(p_j)\big] + \int_{p_j}^{w^0(p_j)}(p_2 + u_j - p_j)du_j + w^{*,0}\big(1-(w^{*,0} - p_3 + p_j)\big) + \int_{p_j}^{w^{*,0}-p_3+p_j}(p_3 + u_j - p_j)du_j\Bigg] \tag{134}$$

Part 5: Deviations to disclosure after receiving disclosure.

Suppose a firm j receives disclosure for a buyer that it has not seen before. By the passive beliefs assumption, it believes that it's rival $-j$ offered the price p^* and that consumers who visited the rival first continued searching if and only if their match there was below $w^{,0}$ and $w^{*,b}$, respectively.

By not disclosing, the rival's price would remain unchanged. By disclosing, firm j gives it's rival the chance to revise it's price. From firm j's point of view, the rival will then maximize the following profit function:

$$\Pi^{dd}(p_{-j}) = \rho \int_0^{w^{*,0}} (u_{-j} - p_{-j} + p_2)\mathbb{1}[p_{-j} \leq u_{-j}]du_{-j}+$$

$$(1-\rho) \int_{p^*+b}^{w^{*,b}} \int_0^{u_{-j}-p^*-b+p_2} \mathbb{1}[u_{-j} - p_{-j} \geq 0]\mathbb{1}[u_{-j} - p_{-j} \geq u_j - p_2]du_j du_{-j} \tag{135}$$

The rival's $(-j)$ beliefs are passive and the rival knows that it initially disclosed. Receiving disclosure lets the rival know that it was visited first. The rival thus knows that firm j was in the information set $\mathcal{H}(j) = R$ after receiving disclosure by $-j$. By the assumption of passive beliefs, the rival firm $-j$ must believe that firm j offered the price p_2.

Upon offering a price $p_{-j} \leq p^c + b$, profits are thus:

$$\Pi^{dd}(p_{-j}) = \rho \int_0^{w^{*,0}} (u_{-j} - p_{-j} + p_2)\mathbb{1}[p_{-j} \leq u_{-j}]du_{-j} + (1-\rho) \int_{p^*+b}^{w^{*,b}} (u_{-j} - p^* - b + p_2)du_{-j} \tag{136}$$

This is because:

$$u_{-j} - p^c - b + p_2 \leq u_{-j} - p_{-j} + p_2$$

By contrast, if $p_{-j} > p^c + b$, profits are:

$$\Pi^{dd}(p_{-j}) = \rho \int_0^{w^{*,0}} (u_{-j} - p_{-j} + p_2)\mathbb{1}[p_{-j} \leq u_{-j}]du_{-j} + (1-\rho) \int_{p_{-j}}^{w^{*,b}} (u_{-j} - p_{-j} + p_2)du_{-j} \tag{137}$$

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