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Information Transmission in Voluntary Disclosure Games

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Abstract

Does a better-informed sender transmit more accurate information in equilibrium? We show that, in a general class of voluntary disclosure games, unlike other strategic communication environments, the answer is positive. If the sender's evidence is more Blackwell informative, then the receiver's equilibrium utility increases. We apply our main result to show that an uninformed sender who chooses a test from a Blackwell-ordered set does so efficiently.

Keywords: Evidence, Informativeness.

JEL Classification: D82, D83, L15.

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1 Introduction

Voluntary disclosure plays a central role in many markets with information asymmetry. Even when agents' interests are not aligned, if an informed agent holds hard evidence, she can disclose pieces of it to promote her interests. The ubiquitousness of voluntary disclosure in communication environments, e.g., of annual reports by public companies, verifiable curricula vitae by job candidates, etc., has inspired a vast body of literature in economics, finance, and accounting.

A question that has not been answered yet by this literature is whether a sender with better information transmits more of it in equilibrium. In cheap-talk models, for example, the answer is trivially negative.[1] Recent literature addressing other communication models, such as costly disclosure and signaling,[2] also points to a non-monotone link between the sender's access to information and equilibrium communication. Nevertheless, we show that in voluntary disclosure environments, better-informed agents transmit more accurate information *in equilibrium*.

We study a general model of voluntary disclosure: in each state of the world, nature performs $n+1$ conditionally independent lotteries. The first n lotteries determine the realizations of a given set of n signals (Blackwell experiments), and the $n+1$ lottery determines which subset of those signals is in the sender's possession.[3] The sender decides which of the signals in her possession she should disclose, and the receiver chooses an action (a real number). The two players' interests regarding this action are not aligned. Whereas the receiver's goal is to coordinate his action with the state, the sender aims to maximize the receiver's action. Our main result states that, in (a truth-leaning) equilibrium,[4] whenever the signals are more Blackwell informative the receiver's expected utility increases.

When analyzing the equilibrium effect of a change in the sender's informativeness, the

[1] In simple examples of Crawford and Sobel's (1982) model, limiting the sender's information can ease her incentive compatibility constraints and allow for a more refined equilibrium partition; see Fischer and Stocken (2004).

[2] For examples of each, see Harbaugh and Rasmusen (2018) and Ball (2020), respectively.

[3] In Section 6 we show that several general models of voluntary disclosure, previously discussed in the literature, fit in this framework.

[4] For the exact definition of the truth-leaning refinement, see Section 3.2.

first obstacle one faces is the characterization of the equilibrium. In general voluntary disclosure games, the equilibrium can be quite involved and usually includes mixing. Therefore, we apply a recent result by Hart et al. (2017) to show that the equilibrium question can be reduced to a mechanism design question. In this way, we can prove that the receiver's utility is increasing in the signals' informativeness, without providing a characterization of the equilibrium.

Hart et al. (2017) show that commitment power does not help the receiver obtain higher utility. That is, the receiver's utility under the optimal (deterministic) mechanism is the same as his utility in (a truth-leaning) equilibrium. To prove our general claim, we construct a direct (potentially) random mechanism that generates the exact same joint distribution of the state of the world and the receiver's action as the optimal mechanism of any given less informative evidence structure. Subsequently, we construct a deterministic incentive-compatible mechanism that gives the receiver a higher utility than the mimicking mechanism. Therefore, we deduce that the mimicking mechanism gives the receiver a lower utility than the optimal deterministic mechanism, and the reduction implies that the receiver's *equilibrium* utility is increasing in the sender's informativeness. In Section 2, we exemplify this proof method in a simple evidence structure, namely, the Dye (1985) model.

We apply our main result to DeMarzo et al.'s (2019) model of an endogenous Dye (1985) evidence structure, where an uninformed sender chooses a test (Blackwell experiment) in private and then decides whether she should disclose its result. They show that, in equilibrium, the sender's choice minimizes the disclosure threshold. Therefore, our main result implies that when the sender faces a Blackwell-ordered set her choice is efficient. Since the disclosure threshold is decreasing in the test's informativeness (Jung and Kwon, 1988), the sender chooses the most informative test which, according to our result, maximizes the receiver's expected utility.

To provide the reader with some intuition for our main result, we discuss a simple example in Section 6.2. Intuitively speaking, making the sender's information more accurate has two effects. First, when presented to the receiver in full, the sender's evidence is more informative about the state of the world, which is clearly beneficial to the receiver. Second, the informational shift also has a strategic effect. In Example 2, by presenting this shift in a model that keeps the set of types fixed, we show that this shift can be interpreted as a reduction in the sender's set of "lies." In addition, we employ Example 2 to explain why changing the probability that the sender obtains evidence in a way that seemingly makes her more informed can decrease equilibrium communication.[5]

Related Literature This paper contributes to the literature on strategic disclosure. Starting with Grossman (1981) and Milgrom (1981), the economic literature discusses environments in which communication is verifiable. Though the early models indicate information unraveling, Dye (1985) and Jung and Kwon (1988) establish an equilibrium with partial disclosure by allowing for uncertainty regarding the sender's informativeness. Verrecchia (1983) shows that such an equilibrium can be obtained if disclosure is costly.

Our model generalizes other voluntary disclosure models previously discussed in the literature. In Section 6.1, we briefly discuss a few other structural models, such as Shin's (1994) evidence model and Hart et al.'s (2016) partition model, in both of which the evidence formation process is defined explicitly, and we show that they can be accommodated in our model. Moreover, we show that even reduced-form models, such as Hart et al. (2017) and Ben-Porath et al. (2019), in which the evidence formation process is not defined, can be accommodated in our model. Even though our framework is structural, it is rich and flexible, and we can specify the primitives in a way consistent with those reduced-form models. Finally, note that, with regards to Hart et al.'s (2017) model, the converse is also true. That is, for every instance of our model, we can define the primitives of Hart et al.'s (2017) model in a way that will induce the same game.[6] Thus, for our main result, we can use Hart et al.'s (2017) equivalence result.

Our proof applies findings in the disclosure literature on the value of commitment power. Glazer and Rubinstein (2004, 2006) study verifiable communication models in which the receiver's action is binary, and they show that the receiver does not gain from commitment

[5]Proposition 3 deals with the special case where the probability that the sender obtains each signal is independent of the state and whether she obtains other signals. We apply the same proof method as in our main result to show that increasing those probabilities makes the receiver better off.

[6]See the proof of Proposition 1.

power. This result is extended to multi-action environments by Sher (2011) and is further generalized by Hart et al. (2017). Ben-Porath et al. (2019) prove that the "no value for commitment" result can be extended to a multi-sender environment, in which some senders wish to maximize the receiver's action and some wish to minimize it. A similar result is shown to hold in a special case by Bhattacharya and Mukherjee (2013). To the best of our knowledge, our paper is the first to show that these equivalency results can constitute a tool for answering natural questions in the disclosure literature.

The closest paper to ours in the disclosure literature is Rappoport (2020) who shows that a more informed sender encounters more pessimism on the receiver's part. Unlike our model, Rappoport's (2020) does not distinguish between the distribution of the underlying state and the sender's ability to provide evidence. He studies a reduced-form evidence model in the spirit of Hart et al. (2017) where the sender's type is defined as a pair: the first dimension corresponds to the receiver's utility, and the second dimension specifies the set of types the sender can mimic. In this model, Rappoport (2020) defines the evidence structure as more informative if, for every two types of the sender such that one type can mimic the other, the relative probability of the mimicking type is higher. He shows that if the evidence structure is more informative according to this definition then the receiver's equilibrium action is lower for each report of the sender.

Our model differs from the reduced-form model in a way that allows us to compare the receiver's utility across different evidence structures. In principle, one can interpret the reduced-form model as if the sender's type corresponds to an actual state of the world or a distribution over a state space. However, under such interpretation, Rappoport's (2020) definition of a more informed sender would typically imply a change in the prior distribution of the state and thus does not allow for a comparison of the receiver's utility in the two evidence structures. Moreover, in Hart et al.'s (2017) model, an alternative definition of an increase in the informativeness of an evidence structure based on the Blackwell order (similar to the one in our model), is also not applicable for such a comparison. In Section 6 (Example 2), we show why such a definition does not capture a change in the informativeness of the evidence structure in a clean way. Since the sender's type also determines which types she can mimic, a garbling of the types' distribution changes not only the quality of the information but also the strategic environment. As a result, a garbling of the types in this example can *increase* the receiver's utility. Therefore, to study the effect of an informativeness change in the evidence structure on the receiver's utility, we define an explicit model of evidence that disentangles the quality of the sender's information from her available strategies.

Our paper also connects to the literature that studies the relation between the sender's informativeness and equilibrium performance. Harbaugh and Rasmusen (2018) study a model of voluntary certification with a disclosure cost in the spirit of Verrecchia (1983), and they show that grade-coarsening is optimal. Due to higher participation, the receiver may observe more information in equilibrium if the quality of the sender's information worsens. Similarly, Bertomeu et al. (2021) show, in a simple evidence structure à la Dye (1985), that if sending a message is costly, the receiver can be better off when the sender's information is obscured. They also show that if there is no cost, a better-informed sender transmits more information in equilibrium. Our main result extends the latter result to a general voluntary disclosure environment. Ball (2020) studies a communication model with costly distortion, in which an intermediary observes the sender's message and aggregates it into a score. He shows that a partly informative score can be optimal for the receiver. In a related framework, Whitmeyer (2019) shows how a receiver can profit from garbling the sender's message.[7]

The rest of the paper is organized as follows. In Section 2 we present an example. In Section 3 we discuss our model. In Section 4 we study the effect of a change in the informativeness of the evidence structure. In Section 5 we present an application. In Section 6 we discuss our results and conclude.

[7]See also Frankel and Kartik (2022) who show that, in a costly distortion environment, commitment can help the receiver achieve better results.

2 Example 1

Before we present the general model, we start with an analysis of a simple evidence structure, the Dye (1985) model. With probability q, the sender (she) obtains a verifiable realization of a signal σ containing information about the state of the world ω, and is otherwise uninformed. An informed sender can either disclose her only piece of evidence truthfully or pretend to be uninformed. Then, the receiver (he) observes the sender's disclosure (or lack thereof) and chooses an action $a \in \mathbb{R}$. In contrast to the sender who wishes to maximize action a regardless of the state, the receiver wishes to coordinate action a with ω. Specifically, assume that the receiver's utility takes the quadratic form;[8] i.e., for every action a and state ω, $u_R(a, \omega) = -(a - \omega)^2$.

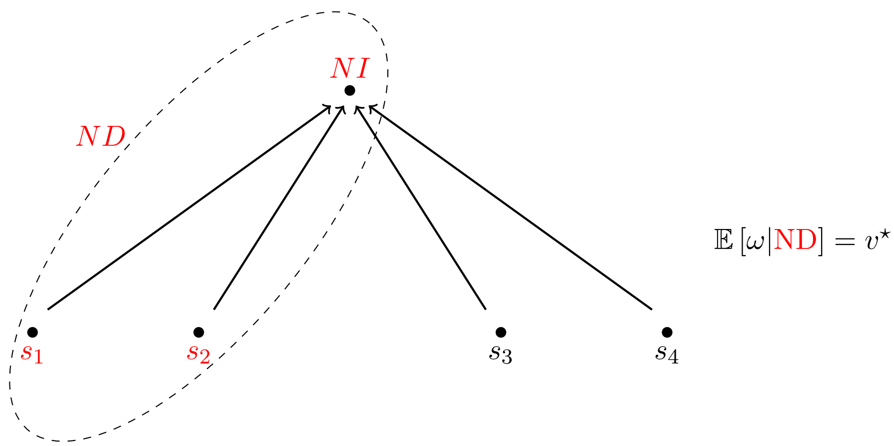

Figure 1: Dye (1985) Model

The no-information type (NI) has only one available strategy – nondisclosure (ND). Type s_i can either send message s_i or mimic NI.

For every realization of σ, s_i, let v_i denote the implied posterior mean, and let H denote the distribution (CDF) of the posterior mean. Dye (1985) and Jung and Kwon (1988) show that the equilibrium in this game is defined by a threshold $v^\star$. Informed types disclose their evidence if and only if their implied posterior mean is above the no-disclosure action $v^\star$, which is defined by the (unique; see Acharya et al., 2011) solution of

[8]Recall that quadratic preferences imply that the receiver's optimal action is the posterior mean given his belief.

$$v^{\star} = \frac{qH(v^{\star})\,\mathbb{E}[\omega|v \leq v^{\star}] + (1-q)\,\mathbb{E}[\omega]}{qH(v^{\star}) + (1-q)}. \tag{1}$$

Assume now that $v_2 < v^{\star} < v_3$ as depicted in Figure 1, and consider the following informational improvement in signal σ: realization s_3 is replaced with two more accurate pieces of evidence, $\underline{s_3}$ and $\overline{s_3}$, where $\underline{v_3} < v_3 < \overline{v_3}$, and further assume that $\underline{v_3} < v^{\star}$. Jung and Kwon (1988) show that any mean-preserving spread decreases the disclosure threshold in the equilibrium of the new information structure. As Bertomeu et al. (2021) show, this decrease implies that the receiver's utility in equilibrium increases. However, their argument is based on a characterization of the equilibrium strategies, and since the equilibrium of a general evidence model might be quite involved, we cannot generalize the direct argument. Therefore, we take an indirect approach.

An Indirect Approach There is another way to show why the receiver is better off if the sender's signal is more informative. Consider a receiver who, for some reason, can commit to the action he would take after the disclosure of each piece of evidence. In the more informative evidence structure, such a receiver can commit to the following direct mechanism:

$$\psi(x) = \begin{cases} v^{\star}, & x \in \{s_1, s_2\} \cup \emptyset, \\ v_3, & x \in \left\{\underline{s_3}, \overline{s_3}\right\}, \\ v_4, & x = s_4. \end{cases} \tag{2}$$

Mechanism ψ mimics the sender's equilibrium payoffs of the less informative structure. Both $\underline{s_3}$ and $\overline{s_3}$ are treated as the corresponding piece of evidence under the less informative signal, s_3. If the sender does not disclose any evidence ($x = \emptyset$), or if she discloses evidence with a posterior mean below $v^{\star}$ (other than $\underline{s_3}$), the receiver chooses the corresponding no-disclosure action $v^{\star}$. And, if the sender discloses evidence with a posterior mean above $v^{\star}$ (other than $\overline{s_3}$), the receiver chooses the corresponding optimal action.

In this simple example, it is easy to see that each informed type obtains a weakly higher payoff when disclosing truthfully, and therefore mechanism ψ is incentive compatible. In addition, mechanism ψ mimics the equilibrium of the less informative signal state by state, and therefore it guarantees the receiver the same utility. Now, we can apply Hart et al.'s (2017) result on the equivalency between the optimal mechanism and equilibrium. They show that, in voluntary disclosure games, commitment power does not help the receiver obtain a higher payoff. The optimal (deterministic) mechanism in the game with the more informative signal, which is better than ψ by definition, provides the receiver with the same utility that he achieves in equilibrium. Therefore, we can deduce that the receiver's *equilibrium* utility is higher in the game with the more informative signal.

Unlike the direct approach, the indirect one applies to a general class of voluntary disclosure games. We show that, even without a characterization of the equilibrium, we can construct a mechanism that mimics the joint distribution of the receiver's action and the state induced by any less informative evidence structure. The general "mimicking" mechanism is more complex and involves randomization. Therefore, in addition to the above argument, we still need to show that the incentive compatibility constraints hold in general and that we can apply Hart et al.'s (2017) result.

3 Model and Preliminary Analysis

A voluntary disclosure game is a communication game between an informed player (the sender, or she) and a decision-maker (the receiver, or he). First, the sender decides which evidence to disclose. Then, the receiver chooses an action $a \in \mathbb{R}$. In the next section, we present our general voluntary disclosure model. In Section 6, we show that our structural model, which specifies the evidence formation process explicitly, can accommodate several models previously discussed in the literature, including the reduced-form models of Hart et al. (2017) and Ben-Porath et al. (2019).

3.1 Model

State of the World and Preferences The state of the world is $\omega \in \Omega$, where Ω is a finite set and $f \in \Delta\Omega$ is its prior distribution. We assume that the sender's preferences do not depend on the state of the world and that she wishes to maximize the receiver's action; i.e., $u_S(a)$ is a strictly increasing function. The receiver's utility, by contrast, depends on the state. Specifically, we assume that, for every state ω, $u_R(a,\omega)$ is differentiable, single-peaked,[9] and concave.

Evidence There is a set of n conditionally independent signals $\Sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_n\}$. For each signal $\sigma_i \in \Sigma$, we denote by S_i its finite set of possible realizations, and by s_i a generic realization. A signal σ_i is a mapping $\sigma_i : \Omega \longrightarrow \Delta S_i$, where $\sigma_i(s_i|\omega)$ denotes the probability of a realization $s_i \in S_i$ given a state $\omega \in \Omega$. In addition, there is a mapping $Q : \Omega \to \Delta 2^N$, where $N = \{1, 2, \ldots, n\}$, that determines which signals the sender obtains given a state ω. That is, for every $A \subseteq N$, $Q(A|\omega)$ denotes the conditional probability that the sender observes the realizations of the subset of signals $\{\sigma_i \in \Sigma | i \in A\}$.

The set of signals Σ and the mapping Q induce a conditional probability distribution $G(\cdot|\omega)$ over the set of possible profiles of evidence $\mathcal{E} := \times_{i=1}^{n}(S_i \cup \{\emptyset\})$, with a generic element E. The interpretation is as follows: the sender can obtain at most one realization of each signal, and if she does not obtain any realization of signal σ_i then the i-th coordinate of the profile $E \in \mathcal{E}$ is $\emptyset$, i.e., $E_i = \emptyset$. We denote the set of signals for which $E_i \neq \emptyset$ by $A_E \subseteq N$. Consider a profile of evidence $E \in \mathcal{E}$ such that for every $i \in A_E$ we have $E_i = s_i \in S_i$. The probability that the sender obtains evidence profile E in state $\omega \in \Omega$ is given by

$$G(E \mid \omega) := Q(A_E|\omega) \cdot \Pi_{i \in A_E} \sigma_i(s_i|\omega). \tag{3}$$

As can be seen from (3), the distribution of the profile of evidence conditional on the state, $G(\cdot|\cdot)$, is pinned down by the set of signals Σ and the mapping Q. We call this distribution an *evidence structure* and denote it by $G(\Sigma, Q)$.

Strategies The set of strategies available to the sender depends on her type, that is, the profile of evidence in her possession. Intuitively, we assume that the sender must disclose

[9] For every ω, there exists a' such that $\frac{\partial}{\partial a} u_R(a', \omega) = 0$, $\frac{\partial}{\partial a} u_R(a, \omega) > 0$ for $a < a'$, and $\frac{\partial}{\partial a} u_R(a, \omega) < 0$ for $a > a'$.

the truth but not necessarily the whole truth. Formally, the set of pure strategies of a sender who possesses evidence profile $E \in \mathcal{E}$, denoted by $\Gamma_E \subseteq \mathcal{E}$, is defined as follows.

Definition 1. *$E' \in \mathcal{E}$ is a pure strategy that is available for a sender of type $E \in \mathcal{E}$ if and only if the following conditions hold:*

- $A_{E'} \subseteq A_E$.
- $\forall i \in A_{E'}$, $E'_i = E_i$.

Thus, type E's set of available strategies is $\Delta\Gamma_E$, where γ_E denotes a generic element of this set, and $\gamma_E(E')$ denotes the probability that the strategy assigns to message $E' \in \Gamma_E$. The set of available strategies for the sender is $\Gamma := \times_{E\in\mathcal{E}}\Delta\Gamma_E$, with a generic element γ.

The set of available strategies for the receiver, Π, is the set of all mappings[10] $\pi : \mathcal{E} \to \mathbb{R}$. Following the disclosure decision of the sender, the receiver forms a belief about the state, $\mu : \mathcal{E} \to \Delta\Omega$. We denote by $\mu_E(\omega)$ the probability that the belief μ assigns to state $\omega \in \Omega$ after the sender has disclosed the profile $E \in \mathcal{E}$.

3.2 Preliminary Analysis

Equilibria An equilibrium is defined as a pair, $(\gamma^\star, \pi^\star)$, together with a belief $\mu^\star$.

The strategy of the sender, $\gamma^\star$, satisfies

$$\gamma^\star_E(E') > 0 \implies E' \in \underset{E''\in\Gamma_E}{\arg\max}\, u_S\left(\pi^\star(E'')\right). \tag{4}$$

The receiver's strategy, $\pi^\star$, satisfies

$$\pi^\star(E) = \arg\max_{a\in\mathbb{R}} \mathbb{E}_{\mu^\star_E}\left[u_R(a,\omega)\right], \tag{5}$$

where $\mu^\star$ is consistent with $\gamma^\star$ along the equilibrium path. That is, for every $E \in \mathcal{E}$ that is played with positive probability,

[10] The concavity of u_R implies that we can assume without loss of generality that the receiver can play only pure strategies.

$$\mu_E^{\star}(\omega) = \frac{f(\omega) \sum\limits_{E' \in \{E'' \mid E \in \Gamma_{E''}\}} G(E' \mid \omega)\, \gamma_{E'}^{\star}(E)}{\sum\limits_{\omega' \in \Omega} f(\omega') \sum\limits_{E' \in \{E'' \mid E \in \Gamma_{E''}\}} G(E' \mid \omega')\, \gamma_{E'}^{\star}(E)}. \tag{6}$$

Equilibrium Selection As is well known, communication games admit a plethora of equilibria. Our selection criterion is the truth-leaning refinement defined by Hart et al. (2017). This refinement selects the unique equilibrium (in payoff terms) where the following two properties hold:

1. If $E \in \arg\max\limits_{E' \in \Gamma_E} u_S(\pi^{\star}(E'))$ then $\gamma_E^{\star}(E) = 1$.
2. If $\gamma_{E'}(E) = 0$ for every $E' \in \mathcal{E}$ then $\mu_E^{\star}(\omega) = \frac{f(\omega) G(E|\omega)}{\sum\limits_{\omega' \in \Omega} f(\omega') G(E|\omega')}$.

The first condition states that if the sender cannot strictly gain by concealing a subset of her evidence, then she discloses all evidence in her possession. The second condition deals with off-path beliefs. Assume that the receiver sees an unexpected message, i.e., a message E such that $\gamma_{E'}(E) = 0$ for every $E' \in \mathcal{E}$. In a truth-leaning equilibrium, the receiver believes that the sender does not hold additional evidence.

As Hart et al. (2017) show, a truth-leaning equilibrium is a limit of an equilibrium sequence in games where truthful disclosure yields a small "bonus" for the sender. For every $\epsilon > 0$, let $G_\epsilon(\Sigma, Q)$ be a game with evidence structure $G(\Sigma, Q)$ in which the sender obtains an additional payoff ϵ for a truthful disclosure. The utility of type E when sending message E', $u_{S\epsilon}(a, E, E')$, is equal to $u_S(a) + \epsilon \cdot \mathbb{1}_{E'=E}$. Consider now a sequence of $\{\epsilon_n\}_{n=1}^{\infty}$ where $\epsilon_n \to 0$. The games sequence $\{G_{\epsilon_n}(\Sigma, Q)\}_{n=1}^{\infty}$ converges to $G(\Sigma, Q)$. Hart et al. (2017) show that if an equilibrium of the game $G(\Sigma, Q)$ is a limit of an equilibrium sequence of the games $\{G_{\epsilon_n}(\Sigma, Q)\}_{n=1}^{\infty}$ then it is a truth-leaning equilibrium. They also show that, in any truth-leaning equilibrium, every type of sender (evidence profile) induces the same action. Finally, truth-leaning equilibria are receiver-optimal equilibria and are the focus of recent literature on disclosure games; see, for example, Jiang (2019), and Rappoport (2020). From now on, an equilibrium signifies a truth-leaning equilibrium; the receiver's utility in this equilibrium is denoted by $\widetilde{U}_R(G)$.

4 Informativeness

We now turn to study the effect of informational changes in the evidence structure. We show that an increase in the Blackwell informativity of the sender's signals implies an increase in the receiver's utility. Here, we focus on the proof technique. In Section 6.2 (Example 2), after providing a construction that fits Hart et al.'s (2017) model in our framework, we give some intuition for our main result by using this construction.

Proposition 1. *Let $Q : \Omega \to \Delta 2^N$ and let $\Sigma = \{\sigma_1, ..., \sigma_n\}$ and $\widehat{\Sigma} = \{\widehat{\sigma}_1, ..., \widehat{\sigma}_n\}$ be two sets of signals. If, for every i, σ_i is more Blackwell informative than $\widehat{\sigma}_i$, then $\widetilde{U}_R\left(G(\Sigma, Q)\right) \geq \widetilde{U}_R\left(G(\widehat{\Sigma}, Q)\right)$.*

We defer the proof of Proposition 1 to the Appendix. Here, we provide a sketch of it. First, using a standard transitivity argument, it is sufficient to prove the claim for the case where there exists $i \in N$ such that σ_i is more Blackwell informative than $\widehat{\sigma}_i$ and, for every $j \neq i$, we have that $\sigma_j = \widehat{\sigma}_j$. We prove Proposition 1 for this case using Hart et al.'s (2017) equivalence between the optimal *deterministic* mechanism and the equilibrium.[11] That is, instead of comparing the receiver's equilibrium payoff under both evidence structures, we compare his payoffs under the optimal *deterministic* mechanisms, where he can commit in advance to an action given any report of the sender.

Let $\psi^\star(G)$ denote the optimal deterministic mechanism for evidence structure G. Since σ_i is more Blackwell informative than $\widehat{\sigma}_i$, evidence structure $G(\widehat{\Sigma}, Q)$ can be obtained by a garbling of $G(\Sigma, Q)$. We construct a direct mechanism for evidence structure $G(\Sigma, Q)$ that mimics this garbling. For every report of the sender that includes an element of S_i, the mimicking mechanism performs a lottery that "garbles" S_i into $\widehat{S}_i$, and, for each realization, the mimicking mechanism commits the action that corresponds to the resulting profile of evidence under $\psi^\star\left(G\left(\widehat{\Sigma}, Q\right)\right)$. Otherwise, if the sender's report does not include an element of S_i, i.e., $E_i = \emptyset$, the mimicking mechanism chooses the same action as in $\psi^\star\left(G\left(\widehat{\Sigma}, Q\right)\right)$. This construction shows that, as long as the mimicking mechanism is incentive compatible,

[11] In the proof we show that our model fits in Hart et al.'s (2017) framework and thus we can apply their result.

a receiver with commitment power can obtain, under evidence structure $G(\Sigma, Q)$, the same payoff as in $\psi^\star\left(G\left(\widehat{\Sigma}, Q\right)\right)$.

However, the mimicking mechanism is potentially random. Therefore, we can not apply Hart et al.'s (2017) equivalence result directly, since, in general, the receiver's utility from a random mechanism might be strictly higher than in equilibrium.[12] To get around this problem, we construct a *deterministic* mechanism that improves upon the mimicking mechanism. For each report of the sender, the deterministic mechanism executes an action equal to the expectation of the actions induced by the corresponding lottery under the mimicking mechanism. Assuming it is incentive compatible, the deterministic mechanism improves upon the mimicking mechanism since the receiver's preferences are concave. It follows that the receiver's payoff in $\psi^\star\left(G\left(\widehat{\Sigma}, Q\right)\right)$ is (weakly) lower than his payoff in the constructed deterministic mechanism, which is (weakly) lower than the receiver's payoff in $\psi^\star(G(\Sigma, Q))$ (and in the equilibrium of $G(\Sigma, Q)$). Therefore, it is left to show that both mechanisms are indeed incentive compatible.[13]

In the proof, we go over all possible sender's types and show that the incentive constraints hold. Here, we concentrate on the most challenging case in which a type $E \in \mathcal{E}$ where $E_i \neq \emptyset$ is contemplating whether she should report $E' \in \Gamma_E$ where $E'_i \neq \emptyset$. Since the garbling that generates $\widehat{\sigma}_i$ from σ_i is independent of the state, it is also independent of coordinates different from i. Therefore, reporting E' induces the same lottery as E but with lower action for each lottery successive realization. Fix such a realization; for both E and E', the mimicking mechanism "replaces" the i-th coordinate in the sender's report with the same piece of evidence, and commits the action that corresponds to the resulting profile under $\psi^\star\left(G\left(\widehat{\Sigma}, Q\right)\right)$. In other words, reporting E induces an action that corresponds to a truthful disclosure of some type under $\psi^\star\left(G\left(\widehat{\Sigma}, Q\right)\right)$, whereas reporting E' induces an action that corresponds to a possible deviation of the same type. Since $\psi^\star\left(G\left(\widehat{\Sigma}, Q\right)\right)$ is

[12] In different environments, Hart et al. (2016) and Sher (2011) prove an equivalence between the receiver-optimal equilibrium and the optimal random mechanism. However, they impose additional assumptions that do not hold in our model. In particular, we require that the sender's preferences be monotone in the receiver's action and that the receiver's preferences be concave without any additional restriction, whereas they require a "relative" concavity condition between the receiver's and the sender's preferences.

[13] In fact, it is enough to show that the constructed deterministic mechanism is incentive compatible. However, our proof uses the incentive compatibility of the (potentially) random mechanism.

incentive compatible, we know that, for each realization of the lottery, reporting E induces a (weakly) higher action than reporting E', and thus this deviation is not profitable. Using similar arguments, we show that no type of sender has a profitable deviation from truthful disclosure. Moreover, the action distribution induced by truthful disclosure dominates the action distribution induced by a deviation. Therefore, in the *deterministic* mechanism, which replaces each lottery with its expectation, truthful disclosure implies a (weakly) higher action than any other report. Applying Hart et al.'s (2017) equivalence result, we show that Proposition 1 follows.

5 Application: Evidence Gathering

In this section, we apply our main result to discuss test choice efficiency. We show that, in DeMarzo et al.'s (2019) endogenous evidence model, the sender's choice is efficient. A sender who chooses a test from a Blackwell-ordered set chooses the test that maximizes the receiver's utility.[14]

In DeMarzo et al.'s (2019) model of evidence gathering, a sender chooses a test in private from a given set $\mathcal{T} = \{T_1, T_2, \ldots, T_m\}$, where each test induces a Dye (1985) disclosure game in which the sender obtains evidence with probability q. As discussed in Section 2, if it is known that the sender's choice was T_i, then the equilibrium of the induced subgame is defined by the unique solution to[15]

$$v_i^\star = \frac{qH_i\left(v_i^\star\right)\mathbb{E}\left[\omega|v_i\left(s_i\right) \leq v_i^\star\right] + \left(1-q\right)\mathbb{E}\left[\omega\right]}{qH_i\left(v_i^\star\right) + 1 - q}, \tag{7}$$

where $v_i\left(s_i\right)$ denotes the posterior mean of ω implied by realization s_i in test T_i, and H_i denotes its CDF.

For a sender whose preferences are linear, e.g., $u\left(a\right) = a$, her equilibrium choice can be characterized by what DeMarzo et al. (2019) call the "minimum principle." That is, the sender chooses a test $\widetilde{T}$ that minimizes the equilibrium action for nondisclosure, i.e.,

[14] We adopt here DeMarzo et al.'s (2019) terminology of "tests," but these are equivalent to the signals discussed above.

[15] As in Section 2, we assume that the receiver's utility is quadratic; hence, his optimal action is the posterior mean given his beliefs.

$\widetilde{T} \in \arg\min_i v_i^{\star}$. This characterization allows us to show the following result concerning their framework. Denote by $\widetilde{U}_R(T_i)$ the expected utility of the receiver in the subgame induced by an *observed* choice of T_i, and denote by $\widehat{U}_R$ his expected utility in the equilibrium of the evidence-gathering game.

Proposition 2. *Assume that the set of available tests, $\mathcal{T}$, is Blackwell-ordered. In the equilibrium of the evidence-gathering game, $\widehat{U}_R = \arg\max_{T_i \in \mathcal{T}} \widetilde{U}_R(T_i)$; i.e., the sender chooses the most informative test and maximizes the receiver's utility.*

Proof. By Jung and Kwon's (1988) result discussed in Section 2, a more Blackwell informative test implies a lower disclosure threshold. Therefore, because $\mathcal{T}$ is Blackwell-ordered, DeMarzo et al.'s (2019) "minimum principle" implies that the sender chooses the most informative test that minimizes the no-disclosure action. By Proposition 1, we know that the most informative test maximizes the receiver's utility among all possible tests in $\mathcal{T}$. Therefore, we deduce that the sender's choice maximizes the receiver's utility. ■

Note that DeMarzo et al. (2019) already characterize choice efficiency in the sense that the sender chooses the most informative test. However, without our Proposition 1, this result does not imply that the receiver's utility is maximized.[16]

6 Discussion and Conclusion

Before we conclude, we further discuss our model and results. First, we discuss the generality of our model. Then, we give some intuition for our main result and discuss changes in the probability of the sender observing information.

6.1 Discussion of the Model

Next, we discuss the generality of our model. We show that our model generalizes several structural models previously analyzed in the literature. First, our model generalizes models

[16]See also Ben-Porath et al. (2018) who study a project choice in a disclosure environment and, among other results, characterize the sender's choice between projects that have the same expectation and are ranked according to second-order stochastic dominance. This choice is closely related to a choice between information structures that are Blackwell-ordered.

such as multidimensional Dye (1985) and Shin (1994), where the sender can obtain multiple signals about the state of the world. Second, our model generalizes partition models in which a type of the sender is a subset of states, and she can disclose different supersets of those. An example of such a model can be found in Hart et al. (2016, Appendix C.3). In addition, we show that our model can accommodate Ben-Porath et al.'s (2019) model, where the primitives are a set of types and the evidence in their possession, and the reduced-form model of Hart et al. (2017), where the primitives are a set of types and a disclosure order.

6.1.1 Multidimensional Dye (1985) Model

Hart et al.'s (2017) model generalizes other structural models, such as Shin's (1994) evidence model and Hart et al.'s (2016) partition model. However, by fitting, for example, Shin's (1994) model in Hart et al.'s (2017) model, we are no longer able to retrieve the original state space. Therefore, we begin with fitting a few structural models in our framework in a way that preserves their original properties.

In a multidimensional Dye (1985) model, the state space is $\Omega \subset \mathbb{R}^N$ with a prior distribution $p \in \Delta\Omega$. The receiver wants his action $a \in \mathbb{R}$ to match the sum of the state coordinates. For example, the utility of the receiver given the state $\omega \in \Omega$ and an action $a \in \mathbb{R}$ is $-\left(a - \sum_{i=1}^{n} \omega_i\right)^2$. The evidence structure is as follows: given a state $\omega \in \Omega$, the sender possesses, with probability q_i, a piece of evidence that certifies that the i-th coordinate of the state is ω_i, where q_i is independent of the state. To fit this model in our model, the state space, the prior distribution, and the payoff functions do not need to change. The only thing we need to work out is the evidence structure. First, for every $i \in N$, define σ_i to be a signal that perfectly reveals the i-th coordinate of the state, ω_i. In addition, for every $A \subseteq N$ and for every $\omega \in \Omega$, let $Q\left(A|\omega\right) = \prod_{i \in A} q_i \prod_{i \notin A} \left(1 - q_i\right)$. Note that our model allows for a more general relation between the state and the signals. Our model also allows for a more general dependence between the state and the probability of the sender observing certain evidence profiles.

6.1.2 Shin (1994)

In Shin (1994), the game is supposed to capture a specific economic scenario involving a firm and its shareholders. Therefore, the game description includes specific details related to this scenario. However, the underlying structure is strategically equivalent to a standard disclosure game. The description of the strategic environment in Shin's (1994) model is as follows. The state space is $\Omega = \{\omega_1, \omega_2, ..., \omega_n\} \subseteq \mathbb{R}$ where $\omega_i > \omega_j$ if and only if $i > j$, and the prior distribution is $p \in \Delta\Omega$. Given a state $\omega_i \in \Omega$, nature draws $N - 1 \equiv \{1, 2, ..., n-1\}$ binary signals $\sigma = \{\sigma_i\}_{i \in N-1}$. Each signal σ_i certifies whether the state is above or below ω_i. The probability that the sender observes a result of the signal σ_i is q_i and is independent of the state. This description is a special case of our evidence structure where the set of signals is the same and $Q(A|\omega) = \prod\limits_{i \in A} q_i \prod\limits_{i \notin A} (1 - q_i)$ for any $A \subseteq N - 1$. As stated above, our model allows for a more general relation between the state and the signal and a more general dependence between the state and the probability of the sender observing certain evidence profiles.

6.1.3 Partitions

Our formulation can also accommodate a partition model, such as the one in Hart et al. (2016). In this model, the primitives are a state of the world $\omega \in \Omega$ and an increasing sequence of partitions $\{S_i\}_{i=1}^n$ such that S_{i+1} is a refinement of S_i. The evidence structure is defined as follows. First, nature draws a state $\omega \in \Omega$ according to a prior distribution $\hat{f}$. Then, nature draws an index $i \in N$ according to a distribution $\hat{Q}$ that may depend on the state. If a state ω and an index i are drawn, the sender's type is the element s_i^l in partition S_i such that $\omega \in s_i^l$. A sender of type s_i^l can report any s_j^k such that $j < i$ and $\omega \in s_j^k$. In order to fit this model in our model we need to define the signals in the following way: $\sigma_i\left(s_i^k|\omega\right) = 1 \Leftrightarrow \omega \in s_i^k$. In addition, we need to define $Q(A|\omega) > 0$ only if $A \subset N$ has the property $i \in A \Rightarrow j \in A\ \forall j < i$ and $Q(\{1, \ldots, i\}|\omega) = \hat{Q}(i|\omega)$ for every $\omega \in \Omega$. Note that our model allows for a more general signal structure. For example, partition S_i need not be a refinement of partition S_j for every $j < i$.

6.1.4 Ben-Porath et al. (2019)

The model in Ben-Porath et al. (2019) is a multi-agent model that allows for some type dependency in the utility of the agents. Again, our focus is on the evidence structure, and thus we present it for the single-agent case. There is a set of types T with a full support prior distribution ρ. There is a mapping $\hat{\varepsilon} : T \to 2^{2^T}$ that represents the set of events that each type $t \in T$ can prove. That is, if $e \in \hat{\varepsilon}(t)$ then type t can prove the event $e \subseteq T$. Ben-Porath et al. (2019) impose the following conditions. First, for every $t \in T$ and $e \in \hat{\varepsilon}(t)$, it must be that $t \in e$. Second, for any $e \in \bigcup_{s \in T} \hat{\varepsilon}(s)$, we have that $t \in e$ if and only if $e \in \hat{\varepsilon}(t)$. Lastly, they assume the normality condition (Lipman and Seppi, 1995): for every $t \in T$, $\left(\bigcap_{e \in \hat{\varepsilon}(t)} e\right) \in \hat{\varepsilon}(t)$.

To fit this evidence structure in our framework, we first define $\Omega = T$ and $f = \rho$. Second, for every $e \in \bigcup_{s \in T} \hat{\varepsilon}(s)$, we define a degenerate signal σ_e that gives the same result regardless of the realized state, i.e., $\Sigma = \{\sigma_e\}_{e \in \bigcup_{s \in T} \hat{\varepsilon}(s)}$. Finally, for every $A \subseteq \Sigma$ and every state $\omega = t$, we define $Q(A|\omega = t) = 1$ if and only if[17] $A = \{e \in \bigcup_{s \in T} \hat{\varepsilon}(s) \,|t \in e\}$. This specification of the primitives of our model accommodates Ben-Porath et al.'s (2019) evidence structure.[18]

6.1.5 Hart et al. (2017)

In Hart et al.'s (2017) model, there is a set of sender's types T, with a prior distribution denoted by $p \in \Delta(T)$. Each type $t \in T$ has two characteristics: first, a set of types she can mimic, $L(t) \subseteq T$; second, a utility function of the receiver $h_t(\cdot)$ that defines his payoff for every action $a \in \mathbb{R}$. They assume that the sets $\{L_t\}_{t \in T}$ have the following two properties:

- $t \in L(t)$ for every $t \in T$.
- If $s \in L(t)$ and $r \in L(s)$ then $r \in L(t)$.

[17] There is a slight abuse of notation here. In the main model, the function Q gets as input subsets of the index set $N = \{1, 2, ..., n\}$ where $n = |\Sigma|$ whereas here Q gets as input subsets of Σ.

[18] Another way to model evidence structures is by mapping every type $t \in T$ to a set of abstract feasible messages $M(t)$ instead of to a set of events $\hat{\varepsilon}(t) \in 2^{2^T}$; see Green and Laffont (1986) and Bertomeu and Cianciaruso (2018). Similar to Ben-Porath et al. (2019), these models can be accommodated in our model by defining a degenerate signal for every message.

That is, L defines a partial order on T.

To fit Hart et al.'s (2017) model in our model, we map each type $t \in T$ to an evidence profile E_t in a way that will reproduce $\{L(t)\}_{t \in T}$ in the sense that $L(t) = \{s \in T | \Gamma_{E_s} \subseteq \Gamma_{E_t}\}$. Note that there can be multiple ways to formulate a given instance of Hart et al.'s (2017) model in our model. However, we present a simple way to define the state space and the signals that maps each instance of Hart et al.'s (2017) model to ours. Intuitively, we map each type t in Hart et al.'s (2017) to a distinct state of nature ω and a distinct degenerate signal σ. The signals Σ are degenerate and give the same realization in every state, but they store information through the mapping Q. That is, in state ω, the sender can present evidence profiles that include σ' if and only if the corresponding type in the original formulation $t(\omega)$ can mimic the type that corresponds to σ', $t(\omega')$.

Formally, we map Hart et al.'s (2017) to our model in the following way. First, we arbitrarily index the finite set of types T. We map every type $t \in T$ to a natural number between 1 and $|T|$ in a one-to-one way, and we denote this mapping by $m(\cdot)$. We define the state space to be the type space with the given prior distribution and with the given utility functions; i.e., $T = \Omega$, $f = p$, and, for every $t \in T$, $u_R(a, \omega = t) = h_t(a)$. Next, we define $|T| = |\Omega|$ degenerate signals, i.e., $\Sigma = \{\sigma_{m(t)}\}_{t \in T}$, such that $\sigma_{m(t)}$ is independent of the state for every $t \in T$. Without loss of generality, we can assume that each signal has only one possible result; i.e., for every $t \in T$ we have $S_{m(t)} = \{s_{m(t)}\}$, and every signal $\sigma_{m(t)}$ corresponds to a type $t \in T$. Finally, for every $A \subseteq \{1, 2, \ldots, |T|\}$ and for every state $\omega \in \Omega = T$, we define $Q(A|\omega = t) = 1$ if and only if $A = \{m(t)|t \in L(t)\}$. It follows that, given any state $\omega = t$, the sender observes exactly the result of the degenerate signals $\{\sigma_{m(t)}\}_{t \in L(t)}$.

By construction, the set of types in Hart et al.'s (2017) model T is mapped in a one-to-one way to a set of evidence profiles $\mathcal{E} = \times_{i=1}^{|T|}(s_i \cup \{\emptyset\})$ such that a type $t \in T$ is mapped to the vector E_t whose i-th coordinate is given by

$$E_{ti} = \begin{cases} s_i, & \text{if } i \in \{m(r)|r \in L(t)\}, \\ \emptyset, & \text{otherwise.} \end{cases} \tag{8}$$

It follows that, for every $t \in T$, we have $p(t) = G(E_t)$, and that for any two types $t, t' \in T$ we have $E_{t'} \in \Gamma_{E_t}$ if and only if $t' \in L(t)$. Therefore, we have accommodated Hart et al.'s (2017) model in our model.[19]

6.2 Example 2

Next, we present an example of Hart et al.'s (2017) model. First, we use the example to convey some intuition for our main result. Second, we use it to explain why we study the role of informativeness in a framework that models evidence explicitly. Finally, using this example, we discuss the strategic consequences of changes of Q in our model.

Let $T = \{0, 6, 7\}$, $\forall t \in T\ f(t) = \frac{1}{3}$ with $h_t(a) = -(a-t)^2$ and the following disclosure order: $L(0) = \{0, 6\}$, $L(6) = \{6\}$, $L(7) = \{6, 7\}$; see Figure 2. In the unique equilibrium of this example, types 6 and 7 reveal themselves, while type 0 mimics type 6.

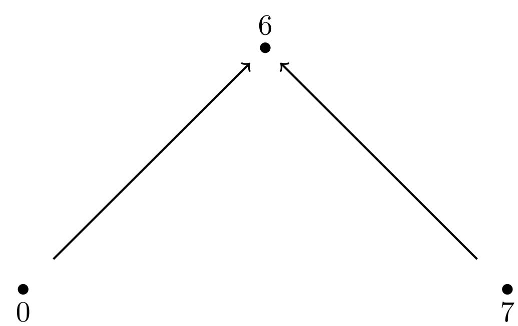

Figure 2: Example 2

Types 0 and 7 can mimic type 6.

6.2.1 Intuition for the Main Result

As discussed in Section 6.1.5, to account for this example with our model, we define the set of states by $\Omega = T$. We define the set of degenerate signals by $\{\sigma_i\}_{i\in\Omega}$ where every $S_i = \{s_i\}$ and $\sigma_i(s_i|\omega) = 1$. Finally, we define $Q(A|\omega) = 1 \Leftrightarrow A = \{\omega' | \omega' \in L(\omega)\}$, and $U_R(a, \omega) = -(a-\omega)^2$.

[19]Notice that our procedure (weakly) expands the strategy space of each type. Specifically, a type E_t in our model can report some profile of evidence $E \in \Gamma_{E_t}$ such that $E \neq E_{t'}$ for all $t' \in T$. Hart et al. (2017) show that this kind of change does not entail a change in the equilibrium; see Proposition 7 in their Online Appendix C.4.

Consider now the following informational change by which we make σ_6 informative about the state. Specifically, we add another result to S_6, r_6, where $\sigma_6(r_6|\omega \in \{6,7\}) = 1$ and $\sigma_6(r_6|\omega = 0) = 0$. That is, signal σ_6 reveals whether the state is $\omega = 0$ or not. By Proposition 1, in the equilibrium of the new information structure, the utility of the receiver increases.

When we examine how this informational shift changes the structure of the original formulation of the example, we can obtain some intuition for our general result. In the new evidence structure, we delete an edge from the original graph (see Figure 3), thus allowing type 6 to separate herself.[20] Making the sender's evidence more informative generates two effects. First, the evidence contains more information about the relevant state. Since the receiver faces a decision problem, as long as the sender discloses truthfully, this effect makes the receiver better off. Second, the strategy set of the sender also changes, which, in principle, can make the receiver worse off. However, the nature of equilibria in voluntary disclosure games implies that such a change is good for the receiver. In (a truth-leaning) equilibrium, if a type mimics another type, she induces a higher than the receiver's optimal action, given all the information she possesses. Therefore, in the new evidence structure, where the sender cannot play this strategy, she induces a (weakly) lower action closer to the receiver's optimal action.[21]

6.2.2 Informativeness in an Abstract Model

Using Example 2, we also demonstrate the difficulties that arise when we try to define an informativeness change in an abstract model in the spirit of Hart et al. (2017). Hart et al. (2017) define only a distribution p over the set of types T and a disclosure order. By contrast, our model defines the state space and the evidence structure (which can then be reduced to Hart et al.'s (2017) model).

[20] In general, an informativeness shift of the signal does not necessarily correspond to a deletion of an edge since it can also induce a change in the set of types. However, this simple example still uncovers some intuition for our main result.

[21] Hart et al.'s (2017) equivalency result implies that, in their model, omitting an edge is always beneficial for the receiver. In the mechanism design problem, each edge is represented by an IC constraint. That is, the equilibrium of the game where we omit an edge is equivalent to the solution of an optimization problem where we omit an IC constraint.

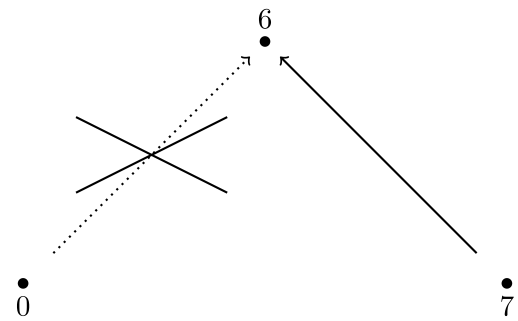

Figure 3: The New Information Structure

Type 0 can no longer mimic type 6.

However, we can think of Hart et al.'s (2017) model *as if* there exists a state space Ω and the sender's type is sampled according to $p : \Omega \to \Delta T$. A naïve way to define worse information, under such an interpretation of Hart et al.'s (2017) model, is by using a garbling of the distribution p. In Example 2, we can see why such garbling is not equivalent to information coarsening. Assume that $T = \Omega$ and $p(t|\omega = t) = 1$, and consider the following garbling of p. In the case where the state is 6, with probability $\frac{1}{2}$ the sender obtains evidence as above, and with probability $\frac{1}{2}$ she obtains evidence of type 7. In the garbled evidence structure, each type of sender plays the same strategy as in the source distribution. Type 7 still finds it optimal to disclose truthfully, and type 0 still wants to mimic type 6. Admittedly, the new type 7 corresponds to a mixture between states 6 and 7, which implies a loss for the receiver. However, the receiver's expected utility increases. When the sender reports message 6, the receiver attributes a higher probability to type 0 and makes a considerably smaller mistake. Thus, the expected quadratic distance between the receiver's action and the state of the world decreases.[22]

Example 2 shows why a garbling of p does not capture cleanly an informational coarsening. Though the sender's evidence is less correlated with the state, the strategy space also changes, and type 0 finds it harder to mimic others. Obviously, our proof method fails if we try to employ it in this example. Yet, it is worthwhile to trace the source of this failure. First, note that we can define a mechanism $\widetilde{\psi}$ for the ungarbled evidence structure that mimics the joint distribution of the types (states) and the receiver's actions in the

[22]The quadratic penalty decreases from $\frac{2}{3} \cdot 3^2$ to $\frac{1}{6} \cdot 4^2 + \frac{1}{3} \cdot 2^2 + \frac{1}{6} \cdot \left(\frac{2}{3}\right)^2 + \frac{1}{3} \cdot \left(\frac{1}{3}\right)^2$.

equilibrium of the garbled evidence structure:

$$\widetilde{\psi}(t) = \begin{cases} 2, \ t = 0, \\ \left(\frac{20}{3}, \frac{1}{2}\right) \bigoplus \left(2, \frac{1}{2}\right), \ t = 6, \\ \frac{20}{3}, \ t = 7, \end{cases} \tag{9}$$

where $\left(\frac{20}{3}, \frac{1}{2}\right) \bigoplus \left(2, \frac{1}{2}\right)$ denotes the following lottery. With probability $\frac{1}{2}$ the action $\frac{20}{3}$ is played and with probability $\frac{1}{2}$ the action 2 is played. The picture that arises from the definition of $\widetilde{\psi}$ is clear. This mechanism is not incentive compatible, and therefore our proof method fails. In the ungarbled evidence structure, type 0 can mimic type 6, and type 6 gets a lottery that dominates the deterministic action of type 0. Thus, type 0 would find it optimal to report non-truthfully.

6.2.3 Changes in the Probability of Obtaining Evidence

Example 2 also demonstrates why a change in Q in our model does not capture cleanly an informativeness change. Changes of Q affect the strategic environment, and their effect on the receiver's utility can go either way. Recall that Q is a stochastic mapping from the state space Ω to subsets of signals. Two natural comparative static exercises over Q come to mind. First, we can ignore its strategic aspect and treat Q as a standard signal. That is, we can say that Q is more informative than Q' if Q' is obtained by a garbling of Q. To see why the receiver can be better off under Q', consider the change discussed in Section 6.2.2. The original stochastic mapping Q is fully informative. Q is deterministic, and the sender observes a distinct set of signals in each state. After the change, Q' is no longer fully informative as in state 6, with probability $1/2$, the sender observes the same signals as in state 7. However, as we already showed, the receiver prefers the new evidence structure over the original one.[23]

The second comparative static we consider is a (weak) increase of the (marginal) prob-

[23] One can also see this insight directly from a simple generalization of Dye (1985). Suppose $\Omega = [0, 1]$, $\Sigma = \{\sigma_1\}$. If Q is not informative, i.e., $Q(\{1\} \mid \omega) = 1$ for every $\omega \in \Omega$, we have unraveling as the unique equilibrium. If, for example, we consider $Q'(\{1\} \mid \omega) = \omega$ we have only partial disclosure in equilibrium. That is, Q' is obtained by a garbling of Q, yet the receiver prefers Q'.

ability that the sender observes each signal in every state. This change can also make the receiver worse off since it (weakly) enlarges the sender's strategy set. To see this, consider the change discussed in Section 6.2.1. There we show that a more informative signal in our model could be manifested by omitting a link in Hart et al. (2017). We can take one more step and map the new evidence structure, without the link from type 0 to type 6, back into our model according to the procedure described in Section 6.1.5. A comparison of the resulting evidence structure and the original one, as formalized in our model, shows that the only difference is that the sender's probability of observing σ_6 in state $\omega = 0$ is zero instead of one. That is, in this example, the receiver is better off when the sender obtains less evidence.[24]

Nevertheless, if the probability that the sender obtains each signal is independent, as in the models discussed in Sections 6.1.1 and 6.1.2, the receiver always prefers a sender with more evidence. Applying a variant of the proof method we use in Proposition 1, we show that, in this special case, an increase of each such probability benefits the receiver.

Definition 2. *$Q : \Omega \longrightarrow \Delta 2^N$ represents an independent process of evidence gathering if there exists a vector $q = (q_1, q_2, \dots q_n)$, such that, for every $A \subseteq N$ and for every $\omega \in \Omega$, we have $Q(A|\omega) = \prod_{i \in A} q_i \prod_{i \notin A} (1 - q_i)$.*

Proposition 3. *Let $G(\Sigma, Q)$ and $G\left(\Sigma, \widehat{Q}\right)$ be two evidence structures, where both Q and $\widehat{Q}$ represent independent processes of evidence gathering. If, for every i, $q_i \geq \widehat{q}_i$, then $\tilde{U}_R(G(\Sigma, Q)) \geq \tilde{U}_R\left(G\left(\Sigma, \widehat{Q}\right)\right)$.*

6.3 Conclusion

In this paper, we have asked whether a better-informed sender communicates more information in equilibrium in voluntary disclosure games. Applying recent results in the disclosure literature, we have shown that this question can be reduced to a mechanism design problem and proven that a better-informed sender communicates information more effectively. If the sender's evidence is more Blackwell informative, then the receiver's expected utility in

[24] We wish to thank an anonymous referee for suggesting this exercise.

equilibrium increases. We have also applied our findings to discuss choice efficiency in a model with an endogenous evidence structure.

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Appendix

Proof of Proposition 1

As mentioned in the main text, it is enough to prove the proposition for Σ and $\widehat{\Sigma}$, which differ only in the first coordinate; i.e., σ_1 is more Blackwell informative than $\widehat{\sigma}_1$. We prove Proposition 1 in four steps: we reduce the equilibrium question to a mechanism design question, construct a (potentially) random mimicking mechanism, construct a deterministic mechanism that improves upon the mimicking mechanism, and show that both mechanisms are incentive compatible.

Reduction To confirm that our model is a special case of Hart et al.'s (2017) reduced-form model, one only needs to check the transitivity property of the disclosure order that Hart et al. (2017) impose in their model. Consider $E, E', E'' \in \mathcal{E}$ such that $E \in \Gamma_{E'}$ and $E' \in \Gamma_{E''}$. We need to show that $E \in \Gamma_{E''}$. If $E \in \Gamma_{E'}$ then $A_E \subseteq A_{E'}$, and if $E' \in \Gamma_{E''}$ then $A_{E'} \subseteq A_{E''}$. It follows that $A_E \subseteq A_{E''}$. Additionally, for every $i \in A_E$ it holds that $E_i = E'_i$ and for every $i \in A_{E'}$ it holds that $E'_i = E''_i$. Because $A_E \subseteq A_{E'}$ it follows that for every $i \in A_E$ $E_i = E''_i$. We showed that $A_E \subseteq A_{E''}$ and that for every $i \in A_E$ we have that $E_i = E''_i$; it follows that $E \in \Gamma_{E''}$. Therefore, we can use Hart et al.'s (2017) result. That is, consider a receiver with commitment power who states in advance which action would follow each disclosure of the sender. A deterministic mechanism is defined by a function $\psi : \mathcal{E} \to \mathbb{R}$, where $\psi(E)$ is the action the receiver takes in the case where the sender discloses E. The optimal mechanism is given by

$$\psi^\star := \underset{\psi:\mathcal{E}\to\mathbb{R}}{\arg\max}\,\mathbb{E}\left[u_R\left(\psi\left(E\right),\omega\right)\right], \tag{10}$$

$$s.t.$$

$$(IC): \ \forall E \in \mathcal{E}, \forall E' \in \Gamma_E, \psi\left(E\right) \geq \psi\left(E'\right). \tag{11}$$

Hart et al. (2017) show that in the optimal deterministic mechanism $\psi^\star$, the receiver chooses the same actions as in the unique (in payoff terms) truth-leaning equilibrium, and thus the expected payoff of the receiver is the same in both cases.

The Mimicking Mechanism Next, we construct a (potentially) random mechanism for evidence structure $G(\Sigma, Q)$ that mimics the joint distribution of the state and actions that is induced by the optimal deterministic mechanism of $G(\widehat{\Sigma}, Q)$. The signal σ_1 is more Blackwell informative than $\widehat{\sigma}_1$. That is, there exists a $|S_1|$ by $|\widehat{S}_1|$ "garbling" matrix L where $l^{k,j}$ denotes the probability that a realization $s_k^1 \in S_1$ is "garbled" to a realization $\widehat{s}_j^1 \in \widehat{S}_1$. Specifically, for every $\widehat{s}_j^1 \in \widehat{S}_1$ and a state of the world $\omega \in \Omega$ it holds that

$$\widehat{\sigma}_1(\widehat{s}_j^1 \mid \omega) = \sum_{s_k^1 \in S_1} \sigma_1(s_k^1 \mid \omega) \cdot l^{k,j}. \tag{12}$$

Let $\psi^\star(G(\widehat{\Sigma}, Q)) : \widehat{\mathcal{E}} \to \mathbb{R}$ be the optimal deterministic mechanism of the less informative evidence structure $G(\widehat{\Sigma}, Q)$, and define the (potentially) random mimicking mechanism $\widetilde{\psi} : \mathcal{E} \to \Delta\mathbb{R}$ for the more informative evidence structure $G(\Sigma, Q)$ as follows. Let $E = (E_1, E_2, ..., E_n) \in \mathcal{E}$. If $E_1 = \phi$, i.e., $E \in \widehat{\mathcal{E}}$, then $\widetilde{\psi}(E) = \psi^\star(G(\widehat{\Sigma}, Q))(E)$. If this is not the case, that is, if there exists $s_k^1 \in S_1$ such that $E_1 = s_k^1$, then the mechanism runs a lottery and with probability $l^{k,j}$ the receiver's action is $\psi^\star(G(\widehat{\Sigma}, Q))(\widehat{s}_j^1, E_2, ..., E_n)$.

The Deterministic Mechanism Hart et al.'s (2017) result establishes an equivalence between the optimal *deterministic* mechanism and the truth-leaning equilibrium. That is, to use this result, we need to find a deterministic mechanism that is incentive compatible and improves upon the mimicking mechanism. Such a mechanism, by definition, gives the receiver a (weakly) lower expected payoff than the optimal deterministic mechanism. Thus, we can conclude that the expected payoff of the receiver under the (potentially) random mimicking mechanism is (weakly) lower than under the optimal *deterministic* mechanism. We define this deterministic mechanism in the following way. For every $E \in \mathcal{E}$, the action of the receiver given the report E is the expectation of the (potentially degenerate) lottery

$\widetilde{\psi}(E)$. If this mechanism is incentive compatible, it (weakly) improves the expected payoff of the receiver relative to the mimicking mechanism since his preferences are concave.

Incentive Compatibility It is left to show that both mechanisms we have constructed are incentive compatible. First consider $E = (E_1, E_2, ..., E_n) \in \mathcal{E}$ where $1 \notin A_E$, i.e., $E \in \widehat{\mathcal{E}}$. For every such profile of evidence and for every available strategy to a sender of type E, the mechanisms $\widetilde{\psi}$ and $\psi^{\star}(G(\widehat{\Sigma}, Q))$ coincide. It follows that such a type of sender would find it optimal to report truthfully because the mechanism $\psi^{\star}(G(\widehat{\Sigma}, Q))$ is incentive compatible. Note also that given a report of such an evidence profile the action of the receiver is deterministic also in $\widetilde{\psi}$ as it coincides with $\psi^{\star}(G(\widehat{\Sigma}, Q))$. Thus, given such a report the deterministic mechanism we have constructed coincides with $\widetilde{\psi}$, and thus such a type of sender would find it optimal to report truthfully also in the deterministic mechanism. Now consider a sender of type $E = (E_1, E_2, ..., E_n) \in \mathcal{E}$ where $1 \in A_E$. First, it is not profitable to omit the realizations of any $B \subset A_E$ with $1 \in B$. By the definition of mechanism $\widetilde{\psi}$, if the sender chooses to omit the realizations of such $B \subset A_E$ and to report some $E'' \in \Gamma_E$, where $E''_1 = \emptyset$, her payoff is $\psi^{\star}(G(\widehat{\Sigma}, Q))(E'')$ whereas if she reports truthfully she gets a lottery. Each realization of this lottery yields a payment that a type who can report E'' receives under the mechanism $\psi^{\star}(G(\widehat{\Sigma}, Q))$. Since this is a lottery over payments that are weakly larger than $\psi^{\star}(G(\widehat{\Sigma}, Q))\,(E'')$, it follows that omitting the realizations of such $B \subset A_E$ is not profitable both in $\widetilde{\psi}$ and in the constructed deterministic mechanism. It is left to show that it is not profitable to omit any $B \subset A_E$ where $1 \notin B$. If the sender reports such $E' \in \Gamma_E$ and if she reports E then, by the definition of the mechanism $\widetilde{\psi}$, she gets the same lottery in terms of the probability of each result, but the action given each result is different. If the sender reports E' she gets the lottery[25]

$$\bigoplus_{\widehat{s}^1_j \in \widehat{S}_1} (\psi^{\star}(G(\widehat{\Sigma}, Q))(\widehat{s}^1_j, E'_2, ..., E'_N), l^{k,j}), \tag{13}$$

[25] We denote by $\bigoplus_{i \in N}(x_i, p_i)$ the lottery in which for every $i \in N$ the probability to get the prize x_i is p_i.

and if the sender reports truthfully she gets the lottery

$$\bigoplus_{\widehat{s}^1_j \in \widehat{S}_1} (\psi^\star(G(\widehat{\Sigma}, Q))(\widehat{s}^1_j, E_2, ..., E_N), l^{k,j}). \tag{14}$$

Again, because $(\widehat{s}^1_j, E'_2, ..., E'_N) \in \Gamma_{(\widehat{s}^1_j, E_2, ..., E_N)}$ and because the mechanism $\psi^\star(G(\widehat{\Sigma}, Q))$ is incentive compatible we have that, for every j,

$$\psi^\star(G(\widehat{\Sigma}, Q))(\widehat{s}^1_j, E_2, ..., E_N) \geq \psi^\star(G(\widehat{\Sigma}, Q))(\widehat{s}^1_j, E'_2, ..., E'_N). \tag{15}$$

It follows that the lottery that the sender gets if she reports truthfully dominates the lottery she gets if she reports E', and thus such a deviation from truthful disclosure is not profitable both under $\widetilde{\psi}$ and under the deterministic mechanism. Since we have covered every possible deviation from truthful disclosure, we can conclude that both mechanisms are indeed incentive compatible. It follows that $\psi^\star(G(\Sigma, Q))$ is at least as good for the receiver as $\psi^\star(G(\widehat{\Sigma}, Q))$. This completes the proof.

Proof of Proposition 3

We have already established that we can reduce the problem to a comparison between the optimal mechanisms under both evidence structures. Similarly to Proposition 1, it is enough to consider the case where $q_1 \geq \widehat{q}_1$ and, for every $i > 1$, $q_i = \widehat{q}_i$. We construct a mechanism for evidence structure $G(\Sigma, Q)$ that mimics the joint distribution of the state and actions induced by the optimal deterministic mechanism of $G(\Sigma, \widehat{Q})$. If the sender discloses a profile of evidence $E \in \mathcal{E}$ such that $E_1 = \emptyset$, then the mimicking mechanism $\widetilde{\psi}$ operates in the same way as the optimal mechanism for $G\left(\Sigma, \widehat{Q}\right)$, i.e., $\widetilde{\psi}(E) = \psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)(E)$. If the sender discloses a profile of evidence $E \in \mathcal{E}$ such that $E_1 \neq \emptyset$, then the mimicking mechanism runs a lottery: with probability $\frac{\widehat{q}_1}{q_1}$, the receiver's action is $\psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)(E)$, and with probability $1-\frac{\widehat{q}_1}{q_1}$ the receiver's action is $\psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)(E')$, where $\forall i \in \{2, 3, ..., n\}$ $E'_i = E_i$ and $E'_1 = \emptyset$.

Next, we show that mechanism $\widetilde{\psi}$ mimics the joint distribution of the receiver's action and the state of the world, induced by mechanism $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)$. Consider a profile of evidence $E\in\mathcal{E}$ such that $E_1\neq\emptyset$ and consider another profile of evidence $E'\in\mathcal{E}$ such that $\forall i\in\{2,3,...,n\}$ $E'_i=E_i$ and $E'_1=\emptyset$. For every $i\in A_E$ assume $E_i=s_i$ for some $s_i\in S_i$. To show that mechanism $\widetilde{\psi}$ mimics the joint distribution of the state and the receiver's actions induced by the mechanism $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)$, it is sufficient to show that, given some state of the world $\omega\in\Omega$, the probability that actions $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E), \psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E')$ are played is the same under both mechanisms. Given a state $\omega\in\Omega$, the probability that the sender possesses the profile of evidence E under $G(\Sigma,Q)$ is:

$$\left(\Pi_{\{i\in A_E,i\neq 1\}}q_i\sigma_i(s_i\mid\omega)\right)\left(\Pi_{i\notin A_E}(1-q_i)\right)q_1\sigma_1(s_1\mid\omega) \tag{16}$$

The probability that the sender possesses the profile of evidence E under $G(\Sigma,\widehat{Q})$ is:

$$\left(\Pi_{\{i\in A_E,i\neq 1\}}q_i\sigma_i(s_i\mid\omega)\right)\left(\Pi_{i\notin A_E}(1-q_i)\right)\widehat{q}_1\sigma_1(s_1\mid\omega) \tag{17}$$

By the definition of mechanism $\widetilde{\psi}$, the receiver takes action $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E)$ with probability:

$$\left(\Pi_{\{i\in A_E,i\neq 1\}}q_i\sigma_i(s_i\mid\omega)\right)\left(\Pi_{i\notin A_E}(1-q_i)\right)q_1\sigma_1(s_1\mid\omega)\frac{\widehat{q}_1}{q_1} \tag{18}$$

$$=\left(\Pi_{\{i\in A_E,i\neq 1\}}q_i\sigma_i(s_i\mid\omega)\right)\left(\Pi_{i\notin A_E}(1-q_i)\right)\widehat{q}_1\sigma_1(s_1\mid\omega)$$

It follows that given state $\omega\in\Omega$, the probability that the receiver executes action $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E)$ is the same under both mechanisms. Consider now action $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E')$. The probability that the receiver executes this action given state $\omega\in\Omega$ in mechanism $\widetilde{\psi}$ is:

$$\sum_{s'_1\in S_1}\left(\Pi_{\{i\in A_E,i\neq 1\}}q_i\sigma_i(s_i\mid\omega)\right)\left(\Pi_{i\notin A_E}(1-q_i)\right)q_1\sigma_1(s'_1\mid\omega)(1-\frac{\widehat{q}_1}{q_1}) \tag{19}$$

$$+\left(\Pi_{i\in A_{E'}}q_i\sigma_i(s_i\mid\omega)\right)\left(\Pi_{i\notin A_{E'}}(1-q_i)\right)$$

We can rewrite (19) as

$$\left(\Pi_{\{i\in A_E, i\neq 1\}} q_i \sigma_i(s_i \mid \omega)\right) \left(\Pi_{i\notin A_E}(1-q_i)\right) q_1 (1-\frac{\widehat{q}_1}{q_1}) \tag{20}$$

$$+\left(\Pi_{i\in A_{E'}} q_i \sigma_i(s_i \mid \omega)\right) \left(\Pi_{\{i\notin A_{E'}, i\neq 1\}}(1-q_i)\right)(1-q_1).$$

Rewriting again we get:

$$\left(\Pi_{i\in A_{E'}} q_i \sigma_i(s_i \mid \omega)\right) \left(\Pi_{i\notin A_E}(1-q_i)\right)(q_1-\widehat{q}_1+1-q_1), \tag{21}$$

which is equal to:

$$\left(\Pi_{i\in A_{E'}} q_i \sigma_i(s_i \mid \omega)\right) \left(\Pi_{i\notin A_E}(1-q_i)\right)(1-\widehat{q}_1). \tag{22}$$

The expression in (22) is the probability that the sender possesses the profile of evidence $E' \in \mathcal{E}$ given the state ω under $G(\Sigma, \widehat{Q})$. In addition, it is equal to the probability that action $\psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)(E')$ is executed given the state ω under $G\left(\Sigma, \widehat{Q}\right)$. It follows that mechanism $\widetilde{\psi}$, which corresponds to the evidence structure $G(\Sigma, Q)$, mimics the joint distribution of the receiver's actions and the state of the world under mechanism $\psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)$ which corresponds to $G(\Sigma, \widehat{Q})$.

The next step of the proof is to show that $\widetilde{\psi}$ is incentive compatible. First, because mechanism $\psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)$ is IC, we have that every type with a profile of evidence E such that $E_1 = \emptyset$ would find it optimal to report truthfully. This is true because mechanism $\widetilde{\psi}$ treats E and every $E' \in \Gamma_E$ exactly the same as $\psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)$. Second, let E be a profile of evidence such that $E_1 = s_1$ for some $s_1 \in S_1$, and consider deviations to $E' \in \Gamma_E$ such that $E'_1 = \emptyset$. We have that $\widetilde{\psi}(E)$ is a lottery between $\psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)(E)$ and $\psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)(E'')$, where $\forall i \in \{2, 3, ..., n\}$, $E''_i = E_i$ and $E''_1 = \emptyset$. Additionally, we have that $\widetilde{\psi}(E') = \psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)(E')$ and that $E' \in \Gamma_{E''}$. Since mechanism $\psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)$ is IC, it follows that $\psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)(E) \geq \psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)(E'') \geq \psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)(E')$. Clearly, a lottery between $\psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)(E)$ and $\psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)(E'')$ is preferred to getting $\psi^\star\left(G\left(\Sigma, \widehat{Q}\right)\right)(E')$ for sure; i.e., a deviation to reporting E' is not profitable. Lastly, we need to consider deviations to $E' \in \Gamma_E$ such that $E_1 = s_1$. In this case we have that $\widetilde{\psi}(E')$

is a lottery between $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E')$ and $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E''')$, where $\forall i\in\{2,3,...,n\}$ we have that $E_i'''=E_i'$ and $E_1'''=\emptyset$. We have that $E'\in\Gamma_E$ and $E'''\in\Gamma_{E''}$, and since mechanism $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)$ is IC we know $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E)\geq\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E')$ and $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E'')\geq\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E''')$. Additionally, according to the definition of mechanism $\widetilde{\psi}$, if the sender reports E she induces action $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E)$ with probability $\frac{\widehat{q}_1}{q_1}$, and action $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E'')$ with probability $\frac{\widehat{q}_1}{q_1}$. However, if she reports E' she induces action $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E')$ with probability $\frac{\widehat{q}_1}{q_1}$ and action $\psi^\star\left(G\left(\Sigma,\widehat{Q}\right)\right)(E''')$ with probability $\frac{\widehat{q}_1}{q_1}$. Therefore, the lottery given a truthful report is preferred; i.e., this kind of deviation is also not profitable. It follows that mechanism $\widetilde{\psi}$ is incentive compatible.

Finally, as in our proof of Proposition 1, we can construct a better (for the receiver) deterministic mechanism by replacing each lottery with its expectation. Since truthful disclosure dominates all other strategies, we know that the deterministic mechanism is also incentive compatible. Therefore, we can apply Hart et al.'s (2017) equivalence result, and the proof is completed.