

Discussion Paper Series – CRC TR 224

Discussion Paper No. 415
Project C 03

Inefficient Bank Recapitalization, Bailout and Post-Crisis Recoveries

Andrea Modena ¹

April 2023

¹ University of Mannheim, Email: andrea.modena@uni-mannheim.de

Support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation)
through CRC TR 224 is gratefully acknowledged.

Inefficient Bank Recapitalization, Bailout, and Post-Crisis Recoveries*

Andrea Modena[†]

This Version: 25th November 2022

Abstract

We study bailouts in a macroeconomic model where banks provide services that facilitate firms' investments but limit their own leverage to prevent costly recapitalizations. This precautionary motive can generate financial crises, in which banks' limited intermediation capacity discourages investments and dampens growth. Bank recapitalizations are constrained-inefficient because they do not internalize that, in the aggregate, higher equity buffers allow for more intermediation, favouring investments and accelerating recoveries. System-wide bailouts can mitigate this inefficiency and improve long-run welfare as long as their positive effect on banks' equity value outweighs their negative impact on risk-taking incentives. (JEL D51, G21)

Keywords: bailout; efficiency; financial crisis, general equilibrium; recovery; welfare

*Previously circulated as "Recapitalization, Bailout, and Long-run Welfare in a Dynamic Model of Banking". My gratitude goes to Markus Brunnermeier, Stefano Colonnello, Husnu Dalgic, Pietro Dindo, Piotr Dybka, Giorgio Ferrari, Christian Kubitzka, Marcella Lucchetta, Paul Mayer, Elisabetta Marzano, Aakriti Mathur, Sebastian Merkel, Christian Mücke, Giovanna Nicodano, Hendrik Hakenes, Roberto Savona, Isabel Schnabel, Paolo Pasquariello, Loriana Pelizzon, Luca Regis, Jean-Charles Rochet, Josef Schroth, Farzad Saidi, Paul Schempp, Christian Schlag, Javier Suarez, Jan Pieter Krahenen, Ernst-Ludwig von Thadden, and Raphael Repullo as well as to the participants to the VERA-Ca' Foscari Workshop in Macro-Finance (Venice), the Workshop in Stochastics for Economics and Finance (Siena), the University of Bonn, the SAEe 2019 (Alicante), the EWMES 2019 (Rotterdam), the QFW 2020 (Naples), the student research seminars at Princeton University, the ISCEF 2020 (Paris), the ASSET 2020 (Padova), the CEF 2021 (Tokyo), the 37th GdRE Symposium in Money, Banking, and Finance (Paris), the LTI@Unito (Turin), the WEHIA 2021 (Milan), the SasCa PhD workshop (Alghero), the DGF 2021 (Innsbruck), and the University of Mannheim for the helpful conversations and suggestions. I gratefully acknowledge support from the Leibniz Institute for Financial Research SAFE, the Collegio Carlo Alberto, and the German Research Foundation (DFG) through CRC TR 224 (Project C03). All remaining remaining errors are my own.

[†]University of Mannheim. Email: andrea.modena@uni-mannheim.de

1 Introduction

In the wake of the 2007-2008 Financial Crisis, more than 100 EU banks benefited from some form of public support. In this regard, the European Commission (2019) reported that its member states provided \$430 bn, about 3 per cent of their aggregate GDP, as new capital to ailing distressed financial institutions. Similarly, on the other side of the Atlantic, the US treasury acquired \$235 bn of 707 bank stocks and equity warrants as a part of its \$700 bn Troubled Asset Relief Program.

Adopting these measures generated widespread discontent about the privatization of banks' profits and the socialization of their losses. The argument was that bailouts would stimulate excessive risk-taking across the targeted institutions (Farhi and Tirole, 2012), whose downside risks would be carried mainly by the taxpayers. Conversely, bailouts were justified by their role in sustaining financial intermediation during the crisis, thereby preventing economic stagnation. (Caballero et al., 2008).

While the moral-hazard problem of bank bailouts has been extensively investigated, both theoretically (e.g. Repullo, 2000; Gorton and Huang, 2004; Keister, 2016) and empirically (e.g. Dam and Koetter, 2012; Hryckiewicz, 2014), features of the trade-off between their fiscal costs and their macroeconomic benefits are not yet fully understood. In this paper, we tackle this issue through the lens of a macroeconomic model in which banks can intermediate capital to improve firms' productivity but limit their leverage to avoid costly equity issuance.

In this context, we describe the mechanisms by which banks' precautionary motive can generate capital misallocation, thereby depressing investments and channelling persistent financial crises. Then, we show that bank recapitalization strategies are constrained-inefficient because they do not internalize the positive effect of their sector's *aggregate* capitalization on firms' investment decisions, which determines the dynamics of post-crisis recoveries. Finally, we study the role of bank bailouts in undoing this inefficiency and characterize the trade-off between their (short-run) fiscal costs and their (long-run) macroeconomic benefits. We find that *system-wide* bailouts (i.e., contingent on the realization of a systemic crisis) can improve social welfare by accelerating post-crisis recoveries under the condition that their positive impact on banks' equity value is more significant than the negative one on their risk-taking incentives.

In consonance with the work of Reinhart and Rogoff (2014), our framework outlines a

precise mechanism by which the benefits of bank bailouts are not only to avoid crises but also to avoid long recoveries. Doing so offers guidance to policymakers and empirical researchers who want to measure and disentangle these two effects. In this regard, our findings square nicely with studies showing that bank recapitalizations positively affect the development of financially-dependent firms (Laeven and Valencia, 2013), that bailouts can curb systemic risk by providing additional equity buffers (Homar and van Wijnbergen, 2017; Kapan and Minoiu, 2018; Berger et al., 2020), and that they can speed up economic recoveries (Tsionas et al., 2015; Dinger et al., 2021). Moreover, we are in line with the empirical evidence in Veronesi and Zingales (2010), who estimate that the 2008 “Paulson’s” plan generated an increment of \$130 bn in banks’ valuation at a taxpayers’ expense of about \$35 bn.

The model builds on Løkka and Zervos (2008), Brunnermeier and Sannikov (2014), and Klimenko et al. (2016). The economy features a continuum of identical producers (firms and their managers), households, and banks. Firms produce output using capital supplied by households and banks in exchange for risky claims written on firms’ profits. Investing in firms requires some management fees, which can be high or low depending on the ability of each firm’s managers; all securities are evenly exposed to the same source of systematic risk. Banks are owned and managed by households, who receive their dividends and finance their (costly) recapitalizations. Therefore, bank-level decisions are also optimal for their (individual) shareholders.¹ Banks are valuable because they tell the low- from the high-fee managers. By doing that, they can intermediate capital to increase aggregate productivity and thus foster investments. However, banks limit their activity (i.e., retain precautionary equity buffers) to prevent costly recapitalizations.

We solve the model for its competitive equilibrium and characterize its transition dynamics between normal and crisis states, depending on how banks’ intermediation capacity affects capital allocation between high- and low-fee firms. In normal conditions, when the banking sector’s equity is significant (and cheap), banks have no precautionary motive and

¹Due to this assumption, we substantially abstract from household-bank agency issues that are typical in the traditional bailout literature. In a complementary fashion, we focus on the role of banks in mitigating information frictions between households and firms. In the same spirit of Sandri and Valencia (2013), this choice is motivated by the following observations. First, moral hazard is likely to be small if bailouts are only adopted in response to systemic crises, as it will always be the case in this paper. Second, quantifying the extent to which agency problems can jeopardize bailout efficacy is immaterial unless we first identify the mechanisms by which their macroeconomic benefits can be substantial. Third, from the empirical standpoint, there is little evidence that public guarantees increase protected banks’ risk-taking, except for those that have outright public ownership (Gropp et al., 2010).

pay dividends regularly. Coherently, they are willing to finance the entire universe of firms at low management fees, which fosters productivity and promotes further investments. Conversely, following adverse shocks, the economy may enter a crisis. Bank equity is scarce (and costly) when that happens, and recapitalizations frequently occur. To avoid recapitalization costs, banks find it optimal to reduce their intermediation activity and delay dividends, which forces households to finance firms directly at a high management fee. This choice diminishes the productivity of capital and discourages investments, which decelerates the build-up of bank equity and extends the crisis duration.

Mindful of this mechanism, the second part of the paper shows that, even though bank recapitalizations are optimal at the individual level, they are constrained-inefficient in the aggregate. This happens for the following three reasons. First, banks are evenly exposed to systematic shocks, meaning that their recapitalizations are always contemporaneous (i.e., crises are always systematic). Second, banks are competitive and thus fail to internalize the positive impact of their synchronous decisions on the market value of their equity, which determines their intermediation capacity. Third, banks ignore that this capacity affects the share of capital allocated to low-fee managers, which accelerates post-crisis recoveries by increasing productivity and encouraging investments. This pattern provides scope for government intervention.

Therefore, the third part of the paper investigates the government's role in providing banks with tax-financed equity transfers (bailouts) in the context of crises. In the same spirit of Bianchi (2016), we label the policy as “systemic” and implement it conditional on the entire banking sector's capitalization being smaller than a certain threshold. Similarly to Cordella and Levy-Yeyati (2003), we find that bailouts affect banks' valuations in two opposite ways. Positively, by raising future dividend expectations (“value” effect); negatively, by fostering their risk-taking incentives (“risk-taking” effect).

We complement their study by showing how these effects interact with the equilibrium allocation of capital, which determines the likelihood of experiencing crises and the speed of subsequent recoveries. Moreover, we show that bailouts can be welfare-enhancing, provided that the policy implementation threshold is sufficiently low (and transfers are not too large). When that is the case, the value effect of the policy outweighs its negative consequences on banks' risk-taking. Therefore, banks find it optimal to postpone their dividend payments and accumulate additional equity buffers. This reaction induces financial stability by reducing

the sensitivity of bank balance sheets to systematic shocks. At the same time, larger equity buffers stimulate banks' intermediation capacity, channelling higher investments across the business cycle. This response generates additional equity buffers in a positive feedback loop that accelerates post-crisis recoveries and enhances welfare in the long run.

Conversely, when the bailout threshold and magnitude conditions are not met, the risk-taking effect takes over. Accordingly, banks find it optimal to anticipate dividend payouts and thus reduce their equity buffers. This behaviour exacerbates allocative inefficiency by reversing the abovementioned forces, which causes more violent state transitions and persistent crises.

The rest of the paper is organized as follows. Section 2 presents the model and derives its competitive equilibrium. Section 3 presents a normative analysis of its inefficiency. Section 3 analyses bailout interventions and discusses the results in light of the empirical literature. Section 5 concludes.

1.1 Related literature

From a broad perspective, we contribute to the vast macro-finance literature in which financial frictions generate inefficiency and amplification of shocks.² As in He and Krishnamurthy (2011, 2013), the banking sector's capitalization in our model plays a key role in generating highly non-linear dynamics. Close to Klimenko et al. (2016), we incorporate a banking sector with equity issuance costs into a general equilibrium framework.³ From the foundational work of Brunnermeier and Sannikov (2014), we borrow the modelling apparatus that generates endogenous shifts between good and crisis regimes. We complement this literature by investigating the role of bank bailouts in shaping the macroeconomics of post-crisis recoveries and how that affects social welfare in the long run.

By investigating bailout costs and benefits, we connect to several studies showing that un-proper bank regulation can aggravate liquidity and debt overhang problems (Gorton and Huang, 2004; Acharya and Yorulmazer, 2007a; Philippon and Schnabl, 2013), encourage herding behaviours (Acharya and Yorulmazer, 2007b), generate allocative inefficiency

²A comprehensive overview of this literature appears in Quadrini (2011) and Brunnermeier et al. (2012). On the relationship between financial frictions and externalities, we refer to Dávila and Korinek (2018).

³In this regard, we also connect to the dynamic corporate finance literature investigating optimal equity issuance, default, and liquidity such as Løkka and Zervos (2008), Hilscher and Raviv (2014), Hugonnier and Morellec (2017), and Berger et al. (2020), among others.

(Gersbach and Rochet, 2012), and foster financial fragility (Keister, 2016; Mitkov, 2020). In particular, we relate to the work of Cordella and Levy-Yeyati (2003) and Lambrecht and Tse (2021), showing that increments in bank valuations following bailouts may dominate the policy’s moral-hazard-related costs. We differentiate from these papers by studying the long-run effects of bank bailouts in a fully-dynamic general equilibrium model in which crises are endogenous in both their likelihood and duration.

Focusing on the long-run effects of bailouts, we also relate to the literature exploring how bank regulation affects the real economy in a dynamic environment. De Nicoló et al. (2014), for example, study the impact of micro-prudential regulation on bank lending and find a U-shaped relationship between welfare and capital requirements. However, their model does not feature financial crises. Mendicino et al. (2019) show that capital requirements make banks safer by addressing long-run stability risks but negatively impact aggregate demand by imposing short-run costs. Schroth (2020) demonstrates that bailouts generate long-term costs associated with adverse wealth redistribution. Unlike these papers, we show that, under suitable implementation rules, bailouts can generate long-run benefits by stabilizing business cycles and accelerating post-crisis recoveries.

Finally, we relate to several papers investigating bank recapitalizations and regulation in the context of crises. Noticeable contributions include Gertler et al. (2012), who find that banks’ risk-taking incentives can reduce the benefits of counter-cyclical credit-support policies during crises, and Sandri and Valencia (2013), who show that state-contingent bank recapitalizations can improve welfare by smoothing output fluctuation in response to significant losses. Along the same lines, Rampini and Viswanathan (2019) develop a model in which capital accumulates slower across banks than producers, which impairs efficient reallocations following adverse shocks. Similarly, Gersbach et al. (2021) show that bailouts can stabilize capital accumulation by efficiently re-allocating resources after slumps in economic fundamentals.

Unlike the current study, none of these papers allows for endogenous bank equity issuance. Moreover, from a more technical standpoint, they evaluate crises based on an impulse-response analysis, which linearizes their competitive equilibriums around a deterministic steady-state. Conversely, we characterize the relationship between bailouts and endogenous crisis dynamics using a fully-fledged dynamic system. A few relevant exceptions are Phelan (2016) and Schroth (2021). Phelan (2016) shows that bank leverage constraints can stabilize

business cycles and prevent crises. However, it does not consider private or public bank recapitalizations. Schroth (2021) studies bank dividends in a model economy where banks do not internalize that higher payout commitments during crises can increase shareholder value and accelerate recoveries.

Another closely related paper is Bianchi (2016), showing that bailouts can release balance sheet constraints and mitigate crisis-related recessions. Although very similar in spirit, our paper sets itself apart in at least three dimensions. First, whereas Bianchi (2016) studies the bailout of productive firms, we focus on banks. Second, the allocative inefficiency in our model comes from the combined effect of banks' intermediation services (management fees) and costly equity issuance. Conversely, in Bianchi (2016), they are generated by the interplay of firms' labour demand and leverage constraints. Third, the crisis probability in Bianchi (2016) is exogenous, while in our model is endogenous.

2 Model

This section provides a general model overview and discusses its main assumptions. Then, it characterizes the model's competitive equilibrium.

2.1 Environment

Time is continuous and indexed as $t \in [0, \infty]$. The economy features two goods, physical capital (or simply "capital") and perishable consumer goods. Three actors populate it: production entities (firms and their managers, henceforth simply "firms"), households, and banks. At each time, the economy's net worth equals the aggregate stock of capital K_t , which is evaluated at the competitive price q_t ; output acts as the numéraire.

Firms are coupled with managers to form production entities. Managers raise capital from households and banks to finance firms at each instant. In exchange, they issue risky claims that promise to repay their holders with consumption goods ("dividends") plus newly-generated capital. To remunerate their service, managers charge a fee, which can be high or low, depending on their ability.

Households are infinitely lived and equipped with an initial endowment of capital, a part of which is exogenously designated to constitute banks. At each instant of time, they receive

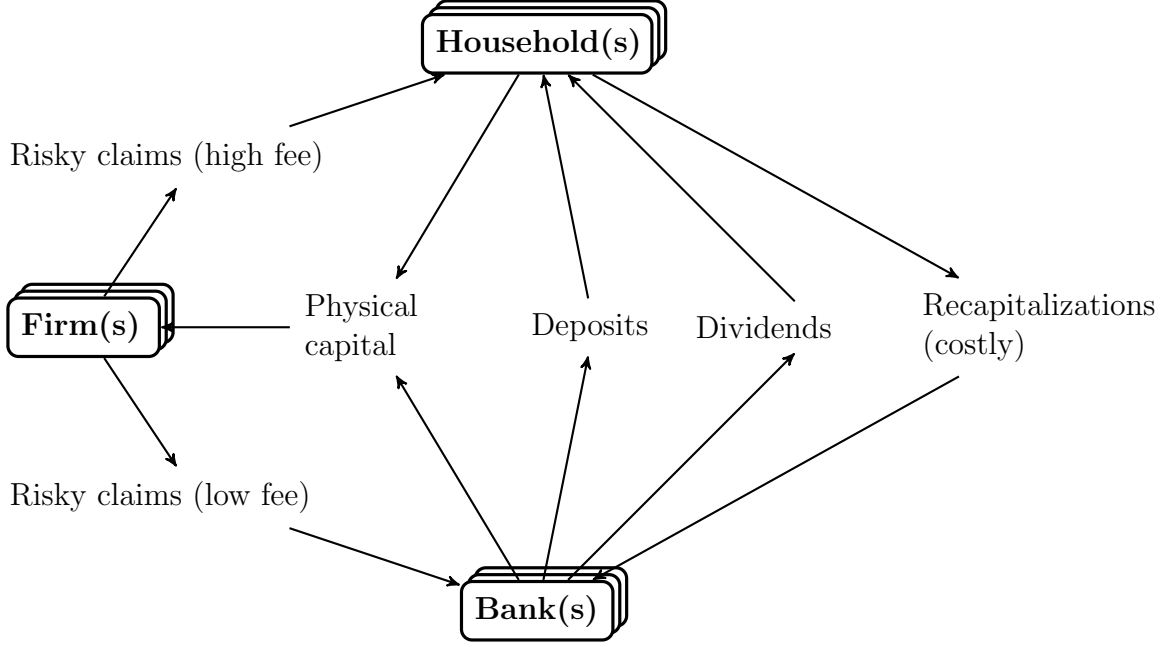


Figure 1: Structure of the model.

bank dividends, pay for their recapitalizations (“equity issuance”), and consume; then, they allocate their net worth between firm risky claims and bank liabilities (“deposits”).

Banks are valuable because they can costlessly identify low-fee firms. However, their intermediation capacity is limited because issuing bank equity is costly. Given these features, banks adjust their intermediated capital share by issuing deposits, paying dividends, and issuing equity to maximize their market value.

Figure 1 provides a snapshot of the model’s structure.

2.2 Production entities and technologies

The unit mass of firms is denoted by $i \in \mathbb{I}$. Similarly to Dindo et al. (2022), they are randomly coupled with an equivalent mass of managers to form a continuum of production entities. Each entity uses capital to produce perishable consumption goods by using the following technology:

$$y_t^i = Ak_t^i, \quad (1)$$

in which A is a positive constant parametrising total factor productivity.

Firms are also equipped with a technology that regenerates capital by using consumption

goods as inputs. Let Z_t denote a standard Brownian motion defined on the filtered probability space $(\Omega, \mathcal{H}, \mathbb{P})$ with natural filtration $\{\mathcal{H}_t, t > 0\}$. Then, the capital stock managed by firm i follows the uni-variate Itô's process

$$dk_t^i = k_t^i[\Phi(\iota_t^i)dt + \sigma dZ_t], \text{ with } \Phi(\iota) = \ln(1 + \theta\iota)^{\frac{1}{\theta}} - \delta, \quad (2)$$

in which θ , δ , and σ are three positive constant. Adopting the concave function $\Phi(\bullet)$ is equivalent to assuming a standard investment technology with (convex) adjustment costs parametrized by θ (on this point, see Brunnermeier and Sannikov, 2016). Note that Z_t is the unique source of (systematic) risk within the economy and is common across firms, meaning that the returns on risky claims are perfectly correlated. This is important because, as we will discuss in Section 3.2.2, it implies that when individual banks recapitalize in response to a negative shock, they always do it simultaneously.

Besides their technologies, firms have no endowment on their own. Accordingly, they are constituted by their managers who raise capital from households and banks in exchange for risky claims with stochastic returns R_t^i written on their firms' profits. To remunerate their service, managers charge a proportional fee η^i , which can be either positive or zero, depending on their ability.⁴

To characterize the return on risky claims net of management fees, we follow Brunnermeier and Sannikov (2014) and conjecture that capital prices evolve as

$$dq_t = q_t(\mu_t^q dt + \sigma_t^q dZ_t), \quad (3)$$

whose drift and diffusion terms are \mathcal{H} -adapted processes to be determined in equilibrium. By Itô's lemma, given the production function in Eq. (1), the investment technology in Eq. (2), and the conjectured stochastic processes in Eq. (3), investing in firms yields

$$dR_t^i = \underbrace{\frac{A - \iota_t^i}{q_t} dt}_{\text{Dividend yield}} + \underbrace{[\Phi(\iota_t^i) + \mu_t^q + \sigma_t^q \sigma]}_{\text{Capital gain}} dt + \underbrace{(\sigma + \sigma_t^q)}_{:=\sigma_t} dZ_t. \quad (4)$$

$\underbrace{\hspace{15em}}_{:=\mu_t^i}$

To prevent arbitrage opportunities, a risky claim written on one unit of capital must be

⁴A simple micro-foundation of the management fee appears in Online Appendix B.1.

valued at q_t ; the optimal investment rate is the one that maximizes Eq. (4), which satisfies Tobin's q relationship

$$\frac{\partial \Phi(\iota_t^i)}{\partial \iota_t^i} = \frac{1}{q_t} \Rightarrow \iota_t^i = \nu_t, \forall i \in \mathbb{I}, \quad (5)$$

which implies that $dR_t^i = dR_t$ and $\mu_t^i = \mu_t, \forall i \in \mathbb{I}$.

2.3 Households

The unit mass of households is indexed $h \in \mathbb{H}$. They are infinitely lived, risk-neutral, and discount future utility at the constant rate ρ . They are born with a time-zero net worth endowment e_0 , a share of which is exogenously designated to constitute bank b 's equity e_0^b . Household h acts as a manager and shareholder of bank b . Therefore, at each instant, she receives her dividends and can subscribe to new equity issuances at the market price ν_t .

Households enjoy utility from consumption flows c_t^h and liquidity/payment services, which are provided by bank deposits d_t^h and parametrized by Γ .⁵ Accordingly, they allocate their disposable net worth $e_t^h := e_t - \nu_t e_t^b$ between capital k_t^h and deposits. As we have explained above, the former asset yields returns as specified in Eq. (4) and requires the payment of management fees; since households have no information to select low-fee managers, they rebate their investments equally across the universe of firms and pay the average fee $\eta = \int_{\mathbb{I}} \eta^i di$ per unit of capital. Bonds are remunerated at the endogenous rate r_t .

Formally, households solve the unconstrained problem

$$H_0^h := \max_{\{c_t^h, d_t^h, k_t^h\}} \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} (c_t^h + \Gamma d_t^h) dt \right], \quad (6)$$

subject to

$$de_t = r_t d_t^h dt + q_t k_t^h dR_t - \eta k_t^h dt - c_t^h + d(\nu_t e_t^b). \quad (7)$$

As we show in Appendix A.1, households' optimal strategies satisfy

$$d_t^h : r_t = \rho - \Gamma, \quad (8)$$

$$k_t^h : \mu_t - \frac{\eta}{q_t} \leq \rho. \quad (9)$$

⁵A similarly parsimonious way of modelling deposits demand can be found in Stein (2012), Klimenko et al. (2016), and Homar and van Wijnbergen (2017), among others.

Their optimal consumption level c_t^h is indeterminate; it will be identified in equilibrium by that of aggregate output.

Eq. (8) tells us that, to be attractive, deposits must pay the inter-temporal discount rate ρ minus the marginal utility of liquidity services Γ . Similarly, Eq. (9) says that households are unwilling to hold capital unless its expected return net of management fees matches ρ . When the inequality holds slack, households mandate all risky investments to banks (i.e., $k_t^h = 0$ and $e_t^h = d_t^h$); else, they directly constitute a share of firms consistently with banks' leverage decisions.

2.4 Banks

A unit mass of banks is indexed $b \in \mathbb{B}$. Each bank is owned and managed by one household, which constitutes her initial equity e_0^b using a time-zero transfer of capital. For this reason, the model does not feature moral hazard problems between individual banks and their shareholders since the former always take actions that the latter find desirable. Importantly, this assumption does not prevent banks from being opportunistic when they can be bailed out by the government, as we will discuss in Section 4.3.

Banks matter because, differently from households, they can screen managers and choose the ones charging the low fee (without loss of generality, we henceforth set its level to zero). With this advantage, banks leverage their balance sheet by issuing deposits d_t^b and allocate their assets in risky claims, valued $q_t k_t^b$. Moreover, they adjust their capital structure by paying dividends $e_t^b \delta_t^b$, issuing equity $e_t^b \xi_t^b$, or choosing to default.

As in Løkka and Zervos (2008), recapitalizations entail the payment of additional consumption flows $\lambda e_t^b d\xi_t^b$, in which λ parametrizes illiquidity, organizational, and administrative costs.⁶ The presence of these costs motivates banks to build precautionary equity buffers against systematic shocks, thereby limiting their leverage capacity in certain states of the world.

By imposing the constraint that $d_t^b + e_t^b = q_t k_t^b$, banks solve the problem

$$J_0^b = \nu_0 e_0^b := \max_{\{k_t^b, \delta_t^b, \xi_t^b\}} \mathbb{E}_0 \left[\limsup_{S \rightarrow \infty} \sup_{\tau} \int_0^{\tau \wedge S} e^{-\rho t} e_t^b (d\delta_t^b - (1 + \lambda)d\xi_t^b) \right], \quad (10)$$

⁶Evidence on the negative relationship between market liquidity and equity issuance costs is documented in Butler et al. (2005)

subject to

$$de_t^b = e_t^b r_t dt + q_t k_t^b (dR_t - r_t dt) + e_t^b (d\xi_t^b - d\delta_t^b), \quad (11)$$

in which $\tau := \inf \{t \in [0, \infty) : e_t^b \leq 0\}$ denotes the optimal default time. Similarly to Brunnermeier and Sannikov (2014), the endogenous term $\nu \geq 1$ can be interpreted as the market price of a bank with book value e^b ; that is, the maximal expected value that she can attain per unit of equity endowment.

As we show in Appendix A.2, banks find optimal to postpone recapitalizations unless their book value approaches zero. When that happens, their shareholders are always willing to subscribe new equity issuances at the market price ν_t , which implies that $\tau = \infty$. Put differently, bank recapitalizations take place every time their equity buffers are not sufficient to remunerate deposits; by subscribing new equity, shareholders back up the shortfall by using $e_t^b \xi_t^b (1 + \lambda)$ units of their net worth. Importantly, this keeps banks solvent and guarantees the absolute safety of their deposits.

As we show in Appendix A.3, banks' dividends and recapitalizations depend on the level of ν_t , which evolves over the interval $(1, 1 + \lambda)$ with dynamics⁷

$$d\nu_t = \nu_t \left(\Gamma dt - \frac{\mu_t - r}{\sigma_t} dZ_t \right). \quad (12)$$

In particular, they satisfy

$$\delta_t^b > 0 \text{ if } \nu_t = 1, \quad \delta_t^b = 0 \text{ else}; \quad (13)$$

$$\xi_t^b > 0 \text{ if } \nu_t = 1 + \lambda, \quad \xi_t^b = 0 \text{ else}. \quad (14)$$

Eqs. (13) and (14) relate the *reflecting barriers* of the process in Eq. (12) to the optimal strategies of the banks. According to the first equation, the dividend yield $d\delta_t^b$ is positive only when banks' market value matches their book value ($\nu_t = 1$). This condition pins down the “upper” reflecting barrier at which banks pay out dividends instantaneously to guarantee a minimum valuation of e_t^b . Symmetrically, banks issue equity at a the rate $d\xi_t^b$ so that their marginal market valuation does not exceed that of their book plus the recapitalization cost λ . The state $\nu_t = 1 + \lambda$ identifies the “lower” reflecting barrier. In all intermediate states in which $\nu_t \in (1, 1 + \lambda)$ banks neither pay dividends nor issue equity, thereby cumulating

⁷Notice that, being banks the marginal investors in the economy, the drift and diffusion of their equity price dynamics are proportional to both the marginal utility of deposits and the Sharpe ratio.

precautionary buffers to avoid costly recapitalizations; within this regions, their leverage satisfy

$$k_t^b : \mu_t - r \leq -\frac{1}{dt} \text{Cov}_t \left(\frac{d\nu_t}{\nu_t}, dR_t \right). \quad (15)$$

When Eq. (15) holds with equality, which is always the case in equilibrium, banks are the marginal investors in the economy and act *as if* they were risk averse. Accordingly, they limit their leverage (i.e., hold precautionary equity buffers) and set the risk premium depending on the covariance between the market price of their equity and the return on investing in firms (on this point, see also Brunnermeier and Sannikov, 2014).⁸

2.5 Equilibrium and aggregation

Definition 1. (*Competitive equilibrium*) A competitive equilibrium Ω is a map from histories of systematic shocks $\{W_t\}$ to prices $\{q_t, \nu_t\}$, returns $\{R_t\}$, risk-free rates $\{r_t\}$, investments $\{\iota_t\}$, consumption $\{c_t^h\}$, capital allocations $\{k_t^j, d_t^j, j \in \{h, b\}\}$, dividends and recapitalizations $\{\delta_t^b, \xi_t^b\}$ so that firms maximize the return on their risky claims, households maximize their inter-temporal utility, banks maximize their market value, and all markets clear. A fully-fledged definition of the equilibrium appears in Online Appendix B.2.

To summarize the main features of the equilibrium, Figure 2 provides a snapshot of the relationship between households' and banks' the balance sheets at each instant of time.

2.5.1 Characterizing the equilibrium

To characterize the equilibrium, we conjecture that its dynamics depend on the level of the state variable ψ , defined as the ratio between the book value of aggregate bank equity $E_t^b = \int_{\mathbb{B}} e_t^b db$ and that of the whole economy $q_t K_t$. In particular, we look for a real-valued function $\Omega : \psi \rightarrow \mathbb{R}^n$ that is consistent with Definition 1, which depends only on the current value of ψ but not on its history. Therefore, we drop all time subscripts t .

We identify the equilibrium as a solution to a boundary value problem for the value of $\Omega(\psi) := \{q(\psi), \nu(\psi)\}$ over the closed interval Ψ . The result is summarized in the following.

⁸In the same spirit of He and Krishnamurthy (2013), this is an intermediary asset pricing model because bank capital has a central role in determining the price of both real and financial assets. Evidence that the marginal value of intermediaries' net worth provides relevant information for asset pricing can be found in Adrian et al. (2014).

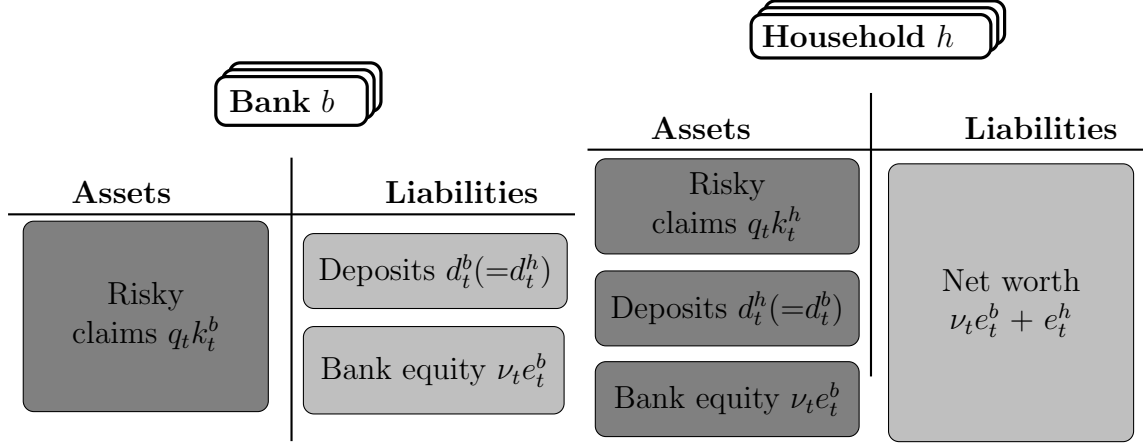


Figure 2: Banks' and households' balance sheet in equilibrium.

Proposition 1. (Equilibrium dynamics) *The state variable ψ has law of motion*

$$\begin{aligned}
 d\psi = & \underbrace{\left[\frac{\psi}{\theta} \left(\frac{1 + \theta A}{q(\psi)} - 1 \right) + (\sigma + \sigma^q(\psi)) ((\kappa(\psi) - \psi) \sigma^\nu(\psi) + \psi (\sigma + \sigma^q(\psi))) \right]}_{:= \psi \mu^\psi(\psi)} dt + \\
 & + \underbrace{(\kappa(\psi) - \psi) (\sigma + \sigma^q(\psi))}_{:= \psi \sigma^\psi(\psi)} dW + d\chi, \quad (16)
 \end{aligned}$$

in which $\kappa(\psi) = K^b/K$ and the level of ψ is regulated within the interval $\Psi := (0, \bar{\psi} \leq 1]$ by the singular control $d\chi = \int_{\mathbb{B}} [d\delta^b - d\xi^b] db$. The unknown functions in $\Omega(\psi)$ solve the following system of ODEs:

$$\begin{cases}
 q(\psi) \mu^q(\psi) = \mathcal{A}q(\psi), \\
 q(\psi) \sigma^q(\psi) = \frac{\partial q(\psi)}{\partial \psi} \psi \sigma^\psi(\psi), \\
 \nu(\psi) \Gamma = \mathcal{A}\nu(\psi), \\
 \nu(\psi) \sigma^\nu(\psi) = -\frac{\partial \nu(\psi)}{\partial \psi} \psi \sigma^\psi(\psi),
 \end{cases} \quad (17)$$

in which \mathcal{A} denotes the Dynkin's operator.⁹

Proof. See Appendix A.4. □

Similarly to the dynamics of ν in Eq. (12), which is bounded over $[1, \lambda]$, the banking sector's relative size fluctuates within Ψ . Given individual banks' optimal strategies in Eqs.

⁹Given a generic diffusion process $dx = x(\mu_x dt + \sigma_x dW)$, the Dynkin's operator \mathcal{A} of a continuous function $h(x)$ is: $\mathcal{A}f(x) = \frac{\partial f}{\partial x} x \mu_x + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} x^2 \sigma_x^2$. Technical details can be found in Øksendal (2003).

(13) and (14), in the interior of Ψ , banks neither pay dividends nor issue equity, and the control term $d\chi$ equals zero. Conversely, when ψ reaches either of its boundaries, $d\chi$ acts as an impulse that regulates the motion of Eq. (16) so that it remains in Ψ .¹⁰

3 Normative analysis

This section shows that the competitive equilibrium and the associated bank recapitalization strategies are neither efficient nor constrained-efficient. Then, it explains inefficiencies as pecuniary externalities of the banking sector's capitalization on individual banks' risk-taking capacity and firms' investments. Finally, it relates the effect of these externalities on the economy's transition between normal and crisis states.

3.1 Efficiency and welfare

To begin the analysis, we describe the equilibrium in which banks have no equity issuance costs ($\lambda = 0$) and show that the resulting allocation is efficient. We henceforth label as "inefficient" any deviation from this benchmark case.

Proposition 2. (First-best equilibrium) *When $\lambda = 0$, the aggregate stock of capital reaches its first-best allocation. In this equilibrium, the market price of bank equity is constant and equals $\nu^{Ib} = 1$, capital prices and firms' investments are constant and satisfy $q^{Ib} = \max_{\iota^{Ib}} \frac{A - \iota^{Ib}}{\rho - \Phi(\iota^{Ib}) - \Gamma} \geq q(\psi), \forall \psi \in \Psi$, households' aggregate value is constant and is proportional to the capital price level $\int_{\mathbb{H}} H^{h,Ib} dh \propto h^{Ib} = q^{Ib}$, aggregate consumption is proportional to aggregate capital; that is, $\int_{\mathbb{H}} c^{h,Ib} dh = C^{Ib} = K[A - \iota^{Ib}]$. Moreover, all macroeconomic aggregates have the law of motion in Eq. (2) with drift $\Phi(\iota^{Ib}) \geq \Phi(\iota(\psi)), \forall \psi \in \Psi$.*

Proof. See Appendix A.5. □

In this equilibrium, banks can offset their equity shortfalls at no cost. Therefore, their optimal strategy consists of continuously paying out dividends and issuing equity to maintain their net worth at zero. Moreover, the equilibrium is efficient because banks' risk-taking capacity is unconstrained; thus, they can entrust the whole capital stock with the low-fee

¹⁰Note that adjustment takes place either when banks pay out dividends or when they issue new equity, but never contemporaneously. There is no issuance friction in the limit case where $\lambda = 0$ and banks pay dividends and issue equity instantaneously to hold $\nu = 1$ at each time t .

managers. As a result, capital prices and investments remain at their maximum levels, and the economy grows at the highest possible rate.

3.1.1 Inefficiency

We now investigate the equilibrium with positive equity issuance costs ($\lambda > 0$) and show that the resulting allocation is inefficient. To start the analysis, we define social welfare in the short run (or “ex-post”); that is, conditional on the economy being in state ψ_0 (analytical derivations appear in Appendix A.7).

Definition 1. (*Short-run welfare*) *Contingent on an initial stock of capital K_0 and an initial level of bank capitalization ψ_0 , the economy’s social welfare is*

$$\int_{\mathbb{H}} H^h(\psi_0, K_0) dh = \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \Theta(\psi_t) K_t dt \right] = h(\psi_0) K_0, \quad (18)$$

in which

$$\Theta(\psi) =: \underbrace{A - \iota(q(\psi))}_{\text{Consumption}} + \underbrace{(\kappa(\psi) - 1)\eta}_{\text{Management fees}} + \underbrace{\Gamma(\kappa(\psi) - \psi)q(\psi)}_{\text{Deposits}} \quad (19)$$

where $\kappa(\psi) := \omega^b(\psi)\psi \in [0, 1]$ denotes the share of bank-intermediated capital, $\{K, \psi\}$ evolve according to Eqs. (2) and (16), and the unknown function $h(\bullet)$ satisfies the ODE

$$[\rho - \Phi(\iota(q(\psi)))] h(\psi) = \Theta(\psi) + \mathcal{A}h(\psi) + \frac{\partial h(\psi)}{\partial \psi} \psi \sigma^\psi(\psi) \sigma, \quad (20)$$

subject to the boundary conditions $\lim_{\psi \rightarrow 0} h(\psi) = \frac{A - \iota(q(\psi)) - \eta}{\rho - \Phi(\iota(q(\psi))) - \Gamma}$ and $\lim_{\psi \rightarrow 0} \frac{\partial h(\psi)}{\partial \psi} = 0$.

Eq. (18) tells us that, for a given stock of aggregate capital K_0 , social welfare does not depend on the economy’s absolute size but on how capital is allocated between households and banks. For this reason, we set $K_0 = 1$ and measure short-run welfare simply as $h(\psi_0)$.

When comparing the boundary condition of Eq. (20) with the first-best welfare level h^{Ib} in Proposition 2, and using that the share of bank-intermediated capital $\kappa(\psi_0) \leq 1$, it is easy to see that $h(\psi_0) \leq h^{Ib}$ for all levels of ψ_0 , meaning that the equilibrium with $\lambda > 0$ is not efficient. As we will see next, inefficiency arises because households cannot insure against the risk of facing costly recapitalizations, low deposit supply, and high management fees in states when the banking sector’s capitalization (and thus leverage capacity) is scarce.

3.1.2 Constrained inefficiency

To show that the competitive equilibrium is not only inefficient but also constrained-inefficient, let $\mathcal{L}(\kappa, \psi)$ denote the right-hand side of Eq. (20) as a function of the bank-intermediated capital share κ and the banking sector's relative capitalization ψ . By taking the partial derivative with respect to κ , we can write the utility flow of increasing bank-intermediated capital share above its competitive level as

$$\frac{\partial \mathcal{L}}{\partial \kappa} = \eta + \Gamma q + \frac{\partial h}{\partial \psi} \frac{\partial(\psi \mu^\psi)}{\partial \kappa} + \frac{\partial(\psi \sigma^\psi)}{\partial \kappa} \left[\sigma \frac{\partial h}{\partial \psi} + \frac{\partial^2 h}{\partial \psi^2} \psi \sigma^\psi \right] \stackrel{\leq}{\geq} 0. \quad (21)$$

The first two terms of Eq. (21), which are always positive, capture the direct effect of having lower management fees and a higher supply of liquid deposits. The second and third terms, either positive or negative, capture the effect of ψ being stochastic. Since Eq. (21) does not generally hold with equality for any $\psi \in \Psi$, the competitive equilibrium with $\lambda > 0$ is not only inefficient but also constrained-inefficient. The constraint-efficient allocation $\{\kappa^{ce}, \psi^{ce}\}$ such that $\partial \mathcal{L}(\kappa^{ce}, \psi^{ce}) / \partial \kappa = 0$ could be implemented by the government if it were able to offset any variation of ψ redistributing capital between households and banks at no cost. In the rest of the paper, we refer to “constrained-inefficient” as any allocation that does not satisfy this condition.

Equipped with Eq. (21), we can immediately verify that individual bank recapitalizations are also constrained-inefficient.

Proposition 3. (*Inefficient recapitalization*) *Provided that management fees η and the liquidity benefits of bank deposits Γ are large enough, an additional increment in bank-intermediated capital above the recapitalization threshold $\psi \rightarrow 0$ is always welfare improving.*

Proof. See Appendix A.6. □

Why are bank recapitalizations constraint-inefficient? The answer is two-folded. In the first place, inefficiencies occur because banks do not internalize the effect of their *synchronous* recapitalizations on the market price of equity, which determines their leverage capacity. At the same time, banks do not take into account how their leverage affects the share of capital handled by low-fee managers, which, in turn, stimulates aggregate productivity and promotes investments. The following section explores each of these aspects in greater detail.

Parameter	Meaning	Value
σ	Systematic volatility	0.06
θ	Technological friction	3
δ	Capital depreciation	0.065
λ	Equity issuance cost	0.15
Γ	Utility for deposits (liquidity)	0.01
ρ	Subjective discount rate	0.05
η	Management fee	0.023
A	Total productivity	0.15

Table 1: Calibrated parameters.

3.2 Pecuniary externalities and crises: a numerical example

In this section, we show that recapitalization inefficiencies materialize as two different (but strictly related) pecuniary externalities of the banking sector’s capitalization on banks’ intermediation capacity and firms’ investments. Moreover, we illustrate how these externalities generate endogenous transitions between normal and crisis states.

3.2.1 Calibration

The model’s parameters are set as in Table 1; they are determined as follows. Systematic volatility is in line with the estimates of Campi and Duenas (2020), who find that global GDP volatility ranges between two and ten per cent in the long run. The values of adjustment costs, capital depreciation, and the subjective discount rate are standard in the macro-finance literature (see e.g., He and Krishnamurthy, 2011; Di Tella, 2017). The liquidity of deposits parameter is set to obtain a risk-free rate of 4 per cent. Consistently with the estimates of annual intermediation costs in Philippon (2015), the management fee is 2 per cent. The productivity parameter is set so that bank leverage at the dividend payout threshold $\bar{\psi}$ (i.e., the “good” steady state) is about 10. This is the mean value across US banks reported in Dindo et al. (2022). The equity issuance cost is the same as in Klimenko et al. (2016).

3.2.2 Discussion

Fig. 3 reports the numerical solution of the model (details of the solution method appear in Online Appendix B.3). The solid blue lines report different macroeconomic aggregates as functions of the state variable ψ . The shaded silhouettes depict the equilibrium’s stationary

density π , which we obtain by solving the associated Fokker-Plank equation.¹¹ The blue stars denote the threshold level ψ^* below which banks become constrained, and the economy enters a crisis state.

The first inefficiency materializes because equity issuance costs generate a (pecuniary) externality through which the banking sector's relative size affects the banks' risk-taking behaviour, depending on the state of the economy. The underlying mechanism is the following. When the banking sector is large ($\psi > \psi^*$) and its equity is abundant and thus cheap (see Panel (b)), aggregate fluctuations and risk premiums are low (Panels (c) and (f)). Accordingly, banks are willing to bear the entire risk of the economy and dividends are paid out frequently (Panels (d) and (e)). Quite the opposite, when the banking sector grows smaller, its equity becomes increasingly scarce and costly (Panel (b)). In particular, when ψ approaches ψ^* risk premiums skyrocket, and banks expand their equity buffers to prevent costly recapitalizations (Panel (d)), depending on the covariance between the price of their equity and the returns on risky investments (see Eq. (15)). Inefficiency occurs because banks do not internalize that their individual (but synchronous) strategies affect their equity prices through the dynamics of ψ , which determines the economy's risk-bearing capacity and, in turn, the speed of transition between normal and crisis states.

The second inefficiency relates to the fact that, in the context of crises, banks' precautionary motive restrains their capacity to finance low-management-fee firms. As a result, lower levels of ψ generate lower aggregate productivity, capital prices and, in turn, investments (Panel (a)). Ignoring that a better-capitalized banking sector allocates resources more efficiently, banks do not internalize that their (simultaneous) recapitalizations foster investments through to Tobin's q relationship in Eq. (5), thereby accelerating the post-crisis recovery process.

The interaction between these two pecuniary externalities generates *stationary instability* á la Brunnermeier and Sannikov (2014); that is, the economy transitions endogenously between normal and crisis states. In particular, its stationary density is bi-modal and exhibits two stochastic steady states corresponding to the upper and lower boundaries (reflecting barriers) of the state space Ψ . The associated transition dynamics take place as follows.

For every ψ , the equilibrium tends to drift towards the upper boundary $\bar{\psi}$ (Panel (e)). Therefore, the economy spends most of its time in ordinary (i.e., unconstrained) states, where

¹¹Details of the derivation of the stationary density of an Itô's process can be found in Risken (1996).

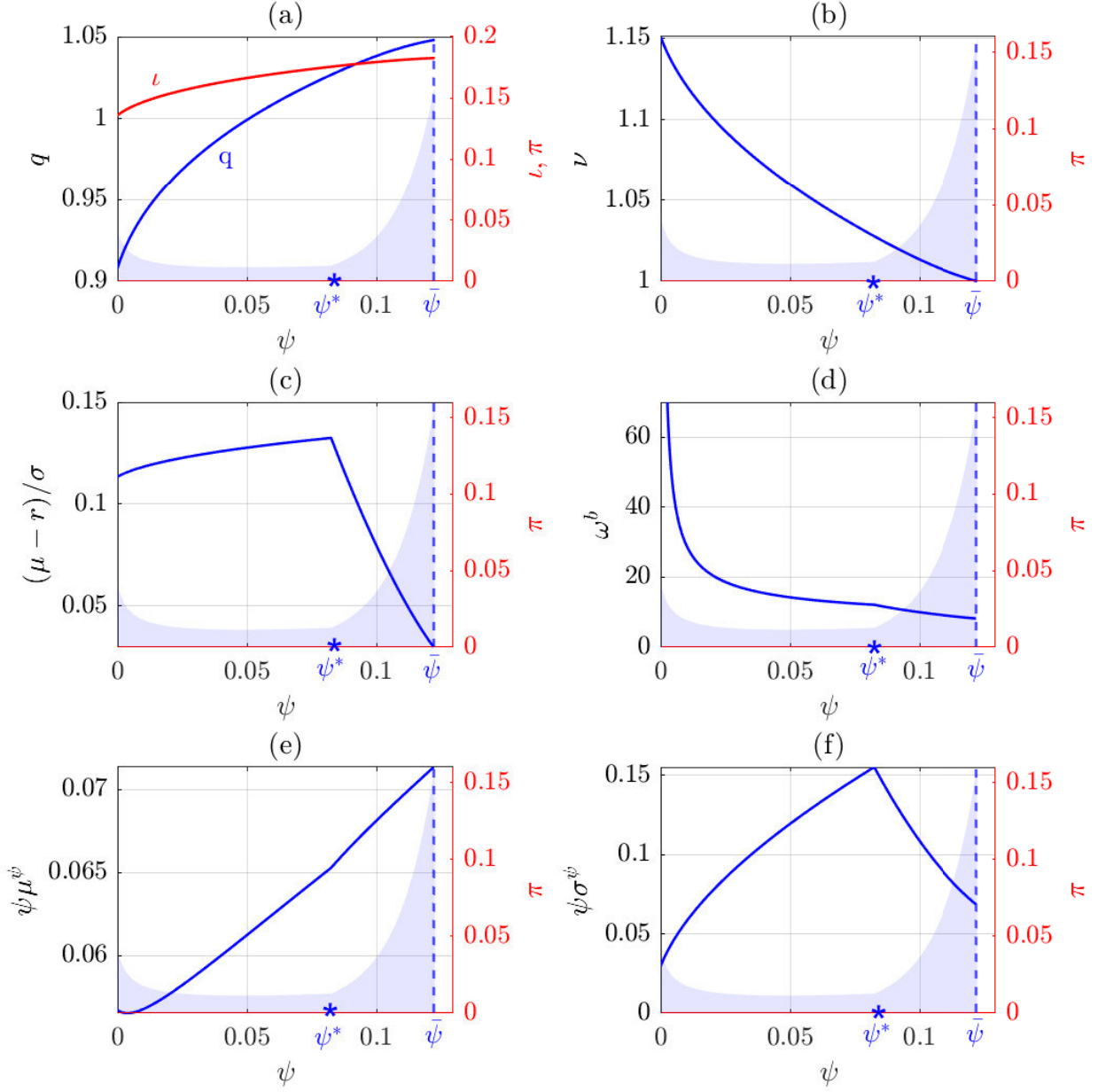


Figure 3: Numerical solution of the model. Parameters are as in Table 1.

capital is allocated efficiently, investments are high, and dividends are paid out frequently. As long as ψ remains sufficiently high, transitions from good to crisis states are unlikely because the state's drift (diffusion) is large (low) (Panel (f)). Following a stream of systematic adverse shocks that drive ψ in the neighbourhood of ψ^* (where the state's volatility is maximal), however, the economy can face an abrupt shift in the constrained region of the state space $\psi \leq \psi^*$. When that happens, the economy experiences persistently high risk premiums and management fees, which reduce capital prices and hinder investments. At the height of the crisis, when $\psi \rightarrow 0$, banks recapitalize frequently but inefficiently due to the pecuniary externalities described above. As a result, allocative inefficiency persists for a protracted period, slowing the recovery process towards the unconstrained region.

In summary, competitive recapitalizations are inefficient and generate persistent financial crises because banks do not internalize the pecuniary externalities of their individual (but synchronous) decisions. More specifically, banks do not consider that, despite their recapitalization costs, additional equity buffers at the aggregate level relieve their precautionary motif and low-fee investment capacity, fostering economic growth and promoting financial stability. This mechanism provides scope for state-contingent bailout interventions, which can improve welfare during crises. We discuss the implementation and implications of such a policy in the next section.

4 Bailout regulation

We have examined the laissez-faire economy in which banks' individually optimal strategies give bank recapitalizations. In this context, we have shown that the competitive equilibrium's outcome is inefficient and constrained-inefficient.

Mindful of this result, we now extend the baseline model by introducing a benevolent government that, by internalizing the externalities described in Section 3.2.2, can decide to bail out the banking sector, depending on the state of the economy. Then, we solve the model for its regulated equilibrium and show that government interventions can be welfare-enhancing. Finally, we discuss our results in light of the empirical literature on the macroeconomic effects of bank bailouts.

4.1 Policy implementation

Similarly to Bianchi (2016), the government cannot use taxes to implement the constrained-efficient allocation. In other words, it is not feasible to make Eq. (21) hold with equality in all states. To capture bailout policies, we assume that the government can enforce systemic tax transfers from households' net worth to banks' equity, but only contingent on the banking sector being excessively small. We label the policy as systemic because the transfers are evenly rebated across all banks rather than targeted to any specific institution.¹²

Formally, let $d\Xi^{b,G}$ denote bank-wise bailout flows, and let them be financed using a linear tax rate T on each household's disposable (or "bail-out-able") net worths e^h . Similarly to private recapitalizations, each unit of bailout capital is costly and entails the payment of $d\Xi^{b,G}\lambda^G$ additional consumption units. The parameter $\lambda^G \geq \lambda$ captures the government's operative and administrative costs in reallocating capital within the economy.

Bailout transfers are evenly distributed across banks when the sector's relative size shrinks below the exogenous threshold ψ^G , which can be interpreted as the minimum size the regulator retains essential for the sector to operate. Due to this policy, households' and banks' budget constraints in Eqs. (7) and (11) modify as

$$de^G = de - Te^h \mathbb{I}_{\psi^G} dt, \quad (22)$$

$$de^{b,G} = de^b + d\Xi^{b,G} \mathbb{I}_{\psi^G}, \quad (23)$$

\mathbb{I}_{ψ^G} being an indicator function that takes value one when $\psi \leq \psi^G$. Implementing the policy, the government always breaks even so that, in the aggregate,

$$\underbrace{(1 + \lambda^G) \left(\int_{\mathbb{B}} d\Xi^{b,G} db \right)}_{\text{Total bailout expenditure}} = \underbrace{T \left(\int_{\mathbb{H}} e^h dh \right)}_{\text{Total tax revenues}} dt, \quad \forall \psi \leq \psi^G. \quad (24)$$

The government evaluate bailouts from a long-run perspective. Therefore, its objective function weights the short-run welfare measure in Definition 1 by using the state's stationary density π .

¹²Note that, due to the assumption that households own and manage banks, the latter always take decisions that the former also find individually desirable. As a result, we fundamentally abstract from agency-based issues such as risk-shifting. As we will see, however, this will not prevent banks from behaving opportunistically and improving their risk-taking under the bailout regime.

Definition 2. (Long-run welfare) For an arbitrary level of aggregate capital stock K , the long-run social welfare W equals the K -rescaled welfare in Eq. (18) integrated over Ψ and weighted by the stationary density $\pi(\psi)$. Formally:

$$W := \int_{\Psi} h(x)\pi(x)d\psi. \quad (25)$$

Eq. (25) can be interpreted as an “ex-ante” measure of welfare; that is, unconditional on the level of ψ . It represents households’ value when $t \rightarrow \infty$ and ψ visits every state in Ψ with density π .

4.2 Regulated equilibrium

Definition 2. (Regulated equilibrium) A regulated equilibrium is a map from histories of systematic shocks to macroeconomic aggregates so that firms, households, and banks solve their problems, the government chooses a tax rate T to maximize the long-run social welfare in Eq. (25) subject to Eq. (24), and all markets clear.

As we show in Appendix A.8, the banking sector’s relative size in the regulated equilibrium has law of motion

$$d\psi^G = d\psi + \mathbb{I}_{\psi \leq \psi^G} T (1 - \psi) (1 + \psi \lambda^G) dt, \quad (26)$$

in which $d\psi$ is the process in Eq. (16).

4.3 Policy evaluation

We explore the effect of bank bailouts in two steps. First, we numerically solve the regulated equilibrium for the arbitrary policy $\{T, \psi^G\}$ and compare the outcome to that of the laissez-faire economy explored in Section 3.2.¹³ Second, we numerically compute the optimal couple $\{T^*, \psi^{G,*}\}$ that maximizes Eq. (25) subject to Eq. (24). In doing that, we discuss the trade-off between the policy’s short-run costs and its long-run benefits. Both experiments adopt the parameters specified in Table 1.

¹³For the same of completeness, a similar comparative static analysis with respect to T appears in Online Appendix B.4.

Figure 1 reports the outcome of the first exercise, in which $T = 0.08$, $\lambda^G = 0.2$ ($> \lambda = 0.15$), and ψ^G is either 2 (red) or 3 per cent (purple). What first stands out is that, as intuition suggests, bailouts accelerate the speed of post-crisis recoveries; that is, they foster the transition from bad to normal states (Panel (e)). However, they do not necessarily make crises less likely unless their implementation does not encourage excessive risk-taking. According to our simulations, this happens when the implementation threshold ψ^G is sufficiently small. The intuition is the following.

Reminiscent of the results in Cordella and Levy-Yeyati (2003), receiving bailouts affects the market price of bank equity in two opposite ways. Positively, by raising future dividend expectations (“value” effect); negatively, by making banks riskier (“risk-taking” effect). When ψ^G is small enough (see red lines in all panels), the value effect dominates (Panel (b)). Accordingly, banks find it optimal to postpone dividend payments and accumulate additional equity buffers. In equilibrium, this choice shrinks the length of the constrained region (see blue and red stars in all panels), which associates with lower (state-contingent) risk premiums (Panel (c)) and lower aggregate uncertainty (Panel (f)).

The risk-taking effect can be visualized by noticing that banks increase their leverage in the constrained region, which associates with higher risk premiums and more significant state volatility (Panels (c) and (f)). In turn, highly volatile crisis states generate lower capital prices (and thus investments) for each level of ψ (i.e., in the short run) (Panel (a)).¹⁴ Despite this unfavourable reaction, the benefits of the value effect dominate and the policy improves financial stability by relocating mass to the right-hand side of the stationary density π . In other words, bailouts generate a positive feedback loop that reduces the likelihood of crises while fostering the *average* price of capital and, in turn, the economy’s long-run investments.

Quite the opposite happens when the bailout implementation threshold ψ^G is too large (see purple lines in all panels). Then, the risk-taking effect dominates, and the policy has the unintended consequence of depressing bank equity prices in every possible state (Panel (b)). Banks find it optimal to anticipate their dividend payouts when that happens, thereby reducing their equity buffers and increasing leverage (Panel (d)). Accordingly, the policy exacerbates allocative inefficiency, which depresses capital prices and, in turn, investments (Panel (a)). All in all, this generates more violent transitions between good and bad states

¹⁴Remember that banks are more productive relative than households due to low monitoring fees and that investments relate to capital prices by Tobin’s q relationship in Eq. (5).

while lengthening the duration of each crisis.

Concerning the risk-taking effect, it is relevant to stress that the model abstracts from agency problems between the banks and their shareholders (on this point, see also Section 2.4. Accordingly, the choices of the former are also (individually) optimal for the latter, and there is no risk-shifting externality.¹⁵ In this respect, we fundamentally differentiate from Cordella and Levy-Yeyati (2003) since, in our model, inefficiency occurs through pecuniary externalities, which affect the economy’s allocation of capital and thus its transition between good and crisis states.

Another observation is that, in the short run (i.e., conditional on ψ), the sign of the relationship between bailouts and bank equity prices ν does not trivially transmit the banking sector’s franchise value, which is defined as $\psi\nu q$. In particular, our numerical results suggest that when the value effect is stronger (ψ^G is small), then q and ν respond to the bailout policy by moving in opposite directions. Conversely, they both decrease when the risk-taking effect prevails (see Panels (a) and (b)).

The third aspect relevant to stress is that welfare-enhancing bailouts entail a trade-off between short-run costs and long-run benefits. This result can be visualized by looking at the households’ short-run welfare, denoted as h , before and after the policy implementation (blue and red dotted lines in Figure 4, Panel (a)) and the associated stationary density π in Panels (a)-(f). On the one hand, our simulation shows that the policy generates a (mild) welfare loss contingent on each level of the state ψ ; that is, in the short run. This is due to the drop in capital prices and dead loss in households’ net worth following each bailout. On the other hand, however, the policy makes good (crisis) states more (less) likely and persistent. Accordingly, it improves the average price of capital and, thus, firms’ investments across the business cycle (i.e., in the long run). In this regard, the trade-off between bailout costs and benefits can also be understood as the “dynamic complementarity” that there is between investing in banks during crises and its positive externalities on the economy’s conditions at consequent stages.

¹⁵Even though this may be considered a strong assumption, it is reasonable to say that moral hazard is likely to be small when bailouts are exclusively used in response to a systemic crisis, which is the case in our model (on this point, see also Sandri and Valencia, 2013). Moreover, while accounting for the channel through which agency issues may lessen the gain from public bank recapitalizations is essential, its role is not compelling without first establishing the mechanisms by which those policies may be beneficial in a long-run perspective.

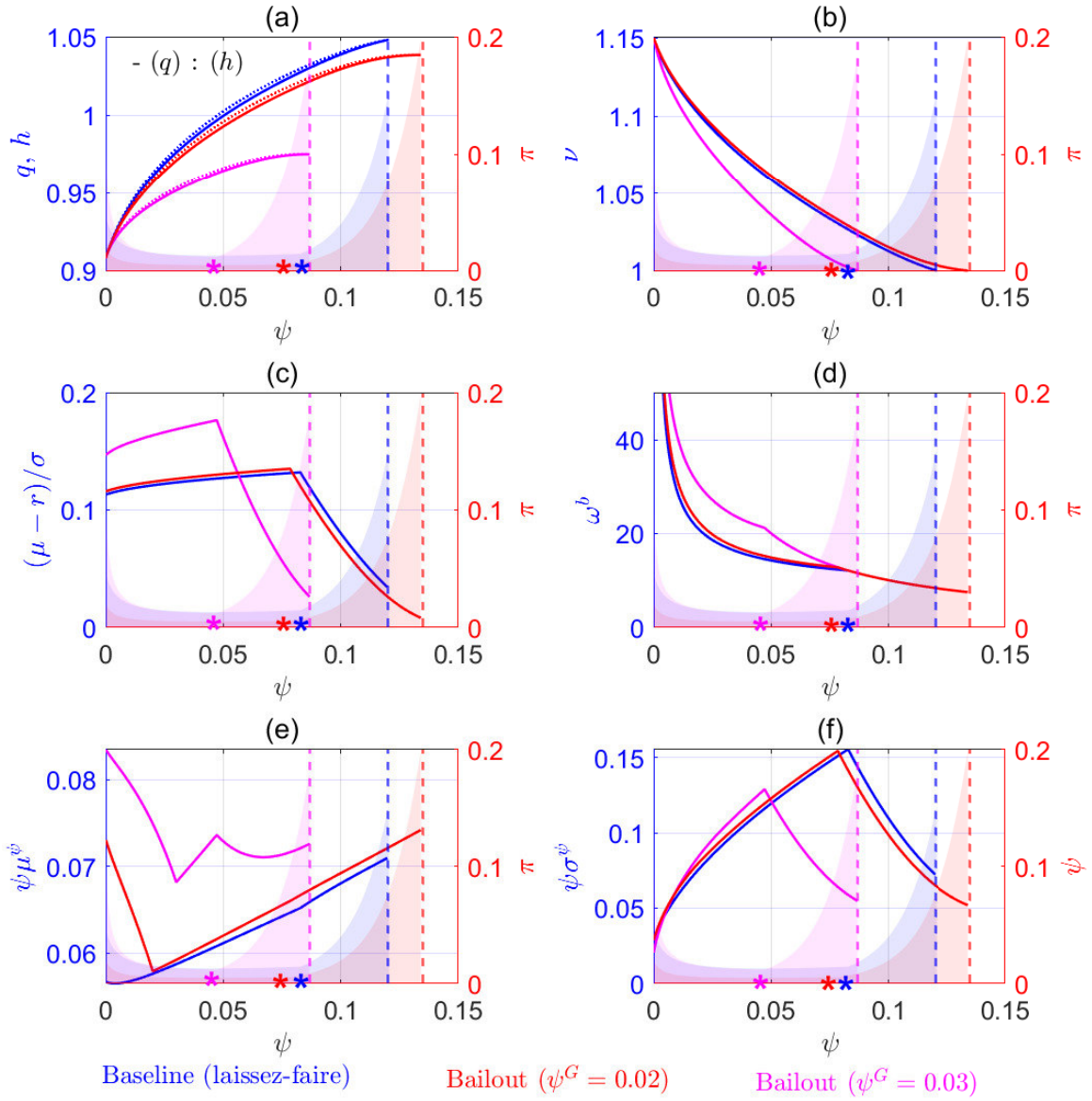


Figure 4: Numerical solution of the model: baseline (laissez-faire) vs regulated (bailout) equilibrium. Parameters are as in Table 1, with $T = 6\%$, $\lambda^G = 0.2$, and $\psi^G = 0.02$ (red) or 0.03 (violet).

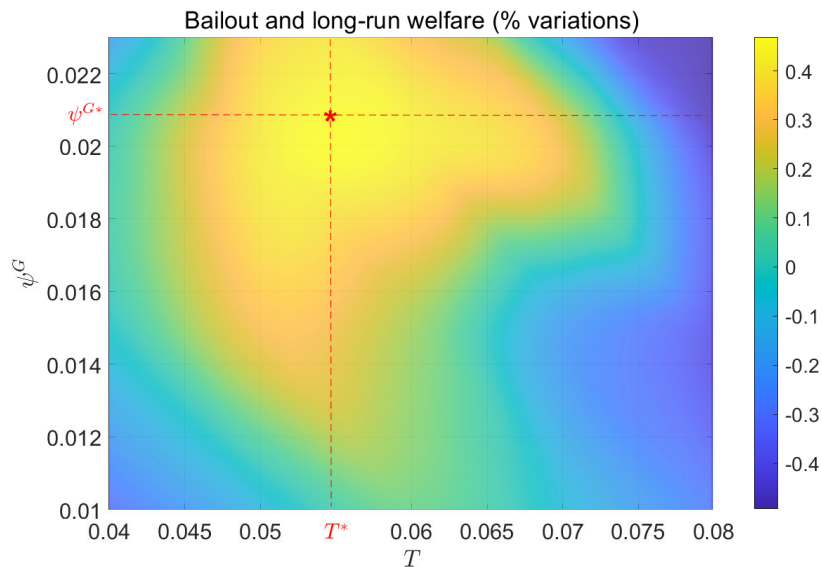


Figure 5: Long-run welfare as a function of bailout threshold ψ^G and the tax rate T .

4.3.1 Optimal policy

To complete the bailout policy analysis, we evaluate numerically the optimal level of long-run welfare as a function of the policy instruments ψ^G and T . The heat map in Figure 5 displays the result.

According to our simulations, the combination of instruments $\{\psi^{G,*}, T^*\}$ increases households' welfare by about half a percentage point with respect to the baseline (laissez-faire) economy. Coherently with the results of Section 4.3, bailouts improve long-run welfare when both their implementation threshold and magnitude are not too large so that the positive effect on banks' valuation overtakes the negative one of their excessive risk-taking incentives.

Note that, due to the highly stylized nature of our model, its quantitative implications are not to be taken by the book. However, going beyond the figures, our findings provide a few pertinent intuitions concerning the mechanism that connects bank recapitalization regimes to their aggregate outcomes in a fully-fledged general equilibrium dynamic environment.

4.3.2 Alternative policy: dividend tax

This section uses our framework to investigate an alternative macro-prudential policy in the form of dividend taxation. In the same spirit of Schroth (2021), the policy aims to incentivise banks to retain additional equity buffers, thereby mitigating the probability of

experiencing a crisis.

To conduct this exercise, we generalize banks' preferences in Eq. (10) as

$$\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} e_t^b (d\delta_t^b(1 - \varepsilon) - (1 + \lambda)d\xi_t^b) \right], \quad (27)$$

in which the new term ε parameterizes a constant tax rate on dividends. As a result, banks' optimal policy in Eq. (13) modifies as:

$$\delta_t^b > 0, \text{ if } \nu = 1 - \varepsilon, \delta_t^b = 0, \text{ else.}$$

Following the same steps of Section 4.3, we solve the model numerically for ε equal to 1 or 2 per cent and compare the outcomes. Figure 6 displays the results.

As it is for the bailout policy, the result of taxing bank dividends on the macroeconomic dynamics is ambiguous. A moderate dividend tax (green) can improve banks' capital buffers by making them postpone their shareholder payments. However, unlike the bailout policy, this decision decreases the value of bank equity for each level of ψ (Panel (b)). Similarly to bailouts, the approach mildly increases risk-taking in the constrained region but decreases it in the unconstrained one (Panels (c) and (d)). Overall, the benefit of having higher capital buffers overtakes the risk-taking by reducing systematic volatility and thus the possibility (and duration) of crises (Panels (e) and (f)).

Conversely, an excessive taxation level can generate highly different results. When paying out dividends becomes too expensive, banks find it convenient to do it earlier; that is when the book value of their assets is small. Accordingly, the value of their equity and overall market evaluation falls, whereas their leverage increases. This behaviour depresses the price of physical capital and fosters risk premiums over a more extensive, constrained region. As a result of these forces, the state's drift flattens, meaning that the economy has a higher probability of switching regimes between good and crisis states (Panel (e)).

4.4 Empirical evidence and model predictions

Before we conclude, we briefly discuss our results in light of the empirical literature on bank bailouts. We develop our considerations along the following three dimensions: (i) the role of bailouts in releasing financial constraints; (ii) their positive effect on bank valuation,

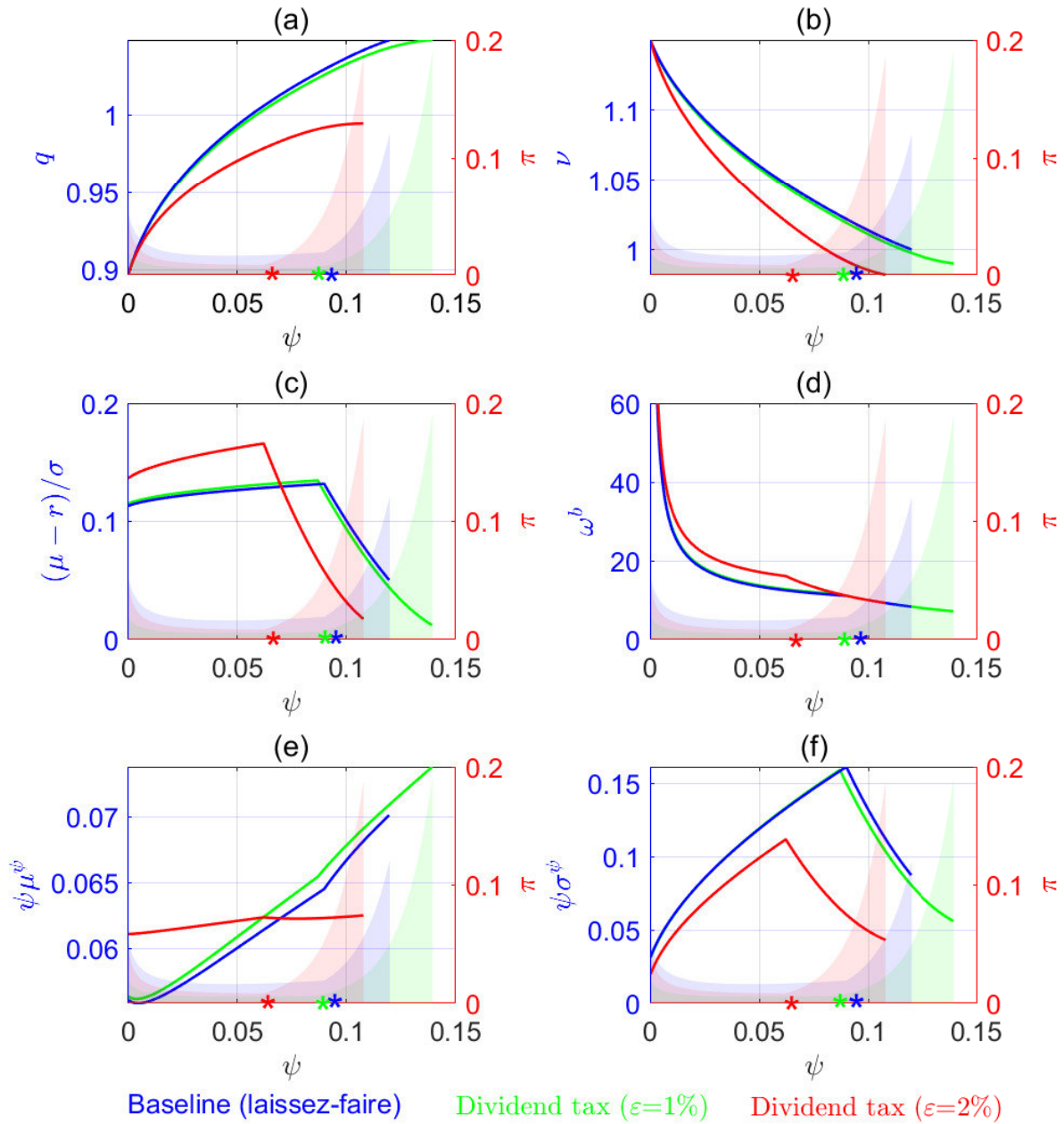


Figure 6: Numerical solution of the model: baseline (laissez-faire) vs regulated (dividend tax) equilibrium. Parameters are as in Table 1, with $\varepsilon = 0.01$ (green) and $\varepsilon = 0.02$ (red).

firm investments, and growth; (iii) their relationship with post-crisis recoveries and financial stability.

Concerning Points (i) and (ii), there is well-established literature testing the so-called “bank recovery” hypothesis. According to this hypothesis, government interventions in the aftermath of crises can sustain banks’ credit by ameliorating their balance sheet conditions, critical determinants of bank-financed firms’ investments.

Two seminal contributions on the topic are Giannetti and Simonov (2013) and Laeven and Valencia (2013). The former paper investigates the role of public support policies during the Japanese financial crisis of the 1990s and documents their fundamental role in supporting bank lending to creditworthy borrowers, which responded by increasing their investments. The latter paper corroborates these results by looking at a large cross-section of countries and showing that an additional one percentage point of GDP expenditure in stimulus can enhance investment growth by 1.3 percentage points. Along the same lines, Barucci et al. (2019) find that, over the period 2008-2012, a 2 per cent increment of cumulated state aids over the financial sector’s total assets prompted investments and GDP growth by 25 and 11 basis points, respectively. Interestingly, they also show that bank recapitalizations did not foster moral hazard but that, on the contrary, they associated with an overall de-leveraging across EU financial institutions.¹⁶

Coherently with this evidence, our paper features banks whose risk-bearing capacity during crises is bounded by their precautionary motif. Consequently, due to their lower management fees, banks’ equity shortage reflects lower productivity and thus depresses firms’ investments. Since individual banks do not internalize the externality of their sector’s aggregate capitalization, public interventions alleviate their precautionary motif in the context of crisis. Higher bank equity buffers are associated with a greater intermediated capital share, thereby improving productivity, supporting investments and, thus, fostering growth in subsequent stages.¹⁷

¹⁶An important difference between Barucci et al. (2019) and previous studies is that it supports the so-called “spare tire” hypothesis. This alternative hypothesis asserts that public interventions affected the market not only via intermediaries’ balance sheets but also by restoring trust in financial markets (i.e., firms can directly issue shares and bonds in substitution for bank loans). Our model does not capture this particular aspect of bank bailouts due to our simplifying assumptions on firms’ capital structure.

¹⁷Evidence endorsing the idea that financial intermediaries’ balance sheets (and in particular their liabilities) were a key driver in explaining the economic recovery after the 2008 crisis appears in Kapan and Minoiu (2018), showing that banks which relied more on market-based funding were particularly vulnerable. Our model parsimoniously captures this aspect by considering financial frictions in the form of equity issuance costs.

Also, in connection with Point ii), one aspect of our results relevant to stress concerns the effect of bailouts on bank equity value. Our model predicts that public aid can create value by redistributing net worth while alleviating allocative inefficiencies. By doing so, our paper characterizes the mechanism behind this value creation as a trade-off between bailouts' short-run costs (losses in households' current net worth) and long-run benefits (gains in the NPV of future bank dividends). This idea is supported by the seminal work of Veronesi and Zingales (2010) who assess the costs and benefits of the Emergency Economic Stabilization Act, which was adopted in response to the 2007-2008 Financial Crisis.¹⁸ Consistently with our results, they estimate that public interventions increased the value of banks' claims by about \$130 bn at a taxpayers' cost of approximately \$35 bn.

Additional evidence supporting our model's predictions appears in Tsionas et al. (2015), which studies EU banks' efficiency following the 2008 financial crisis. Consistently with the mechanism of this paper, they document a substantial difference between banks' performance in the short and the long run.

Concerning the relationship between bailouts, post-crisis recovery, and systemic risk (Point iii)), recent work by Dinger et al. (2021) examines the effect of bailouts on a sample of 72 banking crises in 63 countries over the period 1970-2017. Their findings suggest that bailouts generate a substantial long-run positive effect on growth, which can be detected on average until six years after each intervention. Moreover, they indicate that bailouts are the most effective when targeted to the whole banking system due to their role in preventing allocative inefficiencies and moral hazards. In a likely manner, Homar and van Wijnbergen (2017) investigate the lifespan of recessions after 69 systemic banking crises over the period 1980-2014 and find that public interventions substantially diminish the average crisis' duration. In the same spirit, Homar (2016) and Homar and van Wijnbergen (2017) show that bailouts have an essential role in mitigating systemic risk through what they call a "capital cushion channel", which alleviates the macroeconomic consequences of crises.

More recently, Berger et al. (2020) investigate the Troubled Asset Relief Program's role in reducing its recipient banks' systemic risk contributions and found that, according to indicators such as ΔCoVaR (Brunnermeier and Adrian, 2016) and SRISK (Brownlees and Engle, 2017), it was remarkably effective in doing so. Along the same lines, Kubitza (2021)

¹⁸The "Emergency Economic Stabilization Act", also known as the "Paulson's" plan, created the \$700 bn TARP aimed at purchasing toxic assets from banks and involved a \$125 bn equity infusion in the nine largest US banks after the 2008 financial crisis.

highlights the critical role of financial spillover persistence when measuring systemic risk build-up before crises.

In accordance with these studies, in our model, bailout recapitalization can be advantageous during systemic crises when the banking sector’s intermediation capacity is structurally constrained, investments are persistently low, and the economy is trapped in a long-lasting, slow-growth steady state. This squares nicely with the results of Beck et al. (2020), which show that systematic risk increases more for economies whose banks are regulated by a more conservative resolution regime after adverse system-wide shocks.

5 Conclusions

This paper develops a macro-finance model in which banks’ intermediation ameliorates capital allocations, but their limited risk-bearing capacity can generate occasional and persistent financial crises. In this framework, we show that bank recapitalisations are constrained-inefficient in the context of crises. This happens because banks do not internalize the positive effect of their aggregate capitalization on firms’ investment decisions and how those accelerate post-crisis recoveries. We then investigate the role of system-wide bailouts in mitigating this inefficiency. We find that as long as the banking sector’s capitalization level that regulates the policy’s implementation is small enough, bailouts can improve long-run welfare by fostering financial stability and accelerating economic recoveries.

Our analysis complements the macro-finance literature by developing a model in which crises are endogenous, and their duration depends on public and private bank recapitalizations. Unlike previous studies, our results highlight the dynamic aspects of bank bailouts and, more specifically, the inter-temporal trade-off between their short-run fiscal costs and long-run macroeconomic benefits.

Focusing on systematic bank bailouts, our paper abstracts from at least two significant aspects. First, we do not consider the role of “within” households heterogeneity. A natural extension of the model would include two households that differentiate depending on whether they are bank shareholders. Another interesting refinement would have a more central role for firms by including an entrepreneur sector whose capitalization also affects equilibrium outcomes. Second, we do not consider the bank-level idiosyncratic risk or asymmetric information between managers and shareholders. Including them in the analysis

could generate non-trivial results through different channels. For example, idiosyncratic risks would generate moral hazard if banks were saved after non-systemic defaults. At the same time, they could be an additional instrument to regulate banks' risk-taking by designing macro-prudential policies that distribute losses unevenly across different stakeholders (e.g., depositors and equity holders). Interestingly, this generalization would allow exploring the interplay between bail-in and bailout regulations. We leave these exciting topics for future work.

A Appendix - proofs and derivations

A.1 Households

Households' value is linear in their individual net worth and satisfies the following HJBE (we omit time sub-scripts and household up-scripts for the sake of clear notation):

$$0 = \sup_d \{d^h(\Gamma + r - \rho)\} + \sup_{k^h} \{k^h((\mu - \eta/q) - \rho)\},$$

which is linear in both controls. The FOCs imply that

$$k^h : \mu - \frac{\eta}{q} \leq \rho, \tag{28}$$

which holds with equality when $k^h > 0$, and

$$b^h : r = \rho - \Gamma, \tag{29}$$

Households' consumption level is indeterminate; it will be determined in equilibrium by the associated market clearing condition.

A.2 Optimal stopping time

To find the stopping time τ that solves the problem in Eq. (10), we must set a suitable boundary condition to banks' value $J(e^b)$ (see Stokey, 2009, Chapter 6). This can be done by noticing that, since the process in Eq. (2) is continuous, bank b does never expire her equity over the interval dt . Therefore, due to the time value of money, it is always optimal

to delay recapitalization as long as $e^b > 0$. It follows that there exists some arbitrarily small but positive constant ϵ such that it is optimal to issue equity before the process in Eq. (11) reaches $(-\infty, 0)$. Given the recursive structure of banks' problem, the optimal strategy that defines τ either lets the equity process hit the boundary $(-\infty, 0]$ (absorbing barrier) or it prevents from reaching it (reflecting barrier). Accordingly, the problem in Eq. (10) shall be complemented with the so-called smooth pasting condition, such that¹⁹

$$\max \left\{ -J^*(0), \frac{\partial J^*}{\partial e^b}(0) - (1 + \lambda) \right\} = 0, \quad (30)$$

in which

$$J^*(e^b) = \max_{\{d^b, k^b, \delta^b, \xi^b, e^b\}} J(d^b, k^b, \delta^b, \xi^b, e^b). \quad (31)$$

Equipped with Eqs. (30) and (31), one can show that as long as the equity issuance cost λ is finite, banks' default time is $\tau = \infty$. Therefore, banks find it always optimal to issue equity when $e^b \rightarrow 0$.

Proof. Let bank b 's initial equity be \bar{e} ; let τ be the first time that e^b reaches $(-\infty, 0)$, when the bank chooses whether issue equity or default. Moreover, let J solve the problem in Eq. (10) with the complementary condition in Eq. (30). Then, by definition, bank's value at \bar{e} satisfies

$$J(\bar{e}) = \int_0^{\tau \wedge t} e^{-\rho t} e_t^b (d\delta_t - (1 + \lambda)d\xi_t) + \underbrace{e^{-\rho(t \wedge \tau)} J(e_{t \wedge \tau}^b)}_{\text{Continuation value}}. \quad (32)$$

By taking the differential $d(Je^{-\rho t})$, applying Itô's lemma, and integrating over $(0, t \wedge \tau)$, the following relationship between the motion of e^b and the bank's continuation value holds:

$$\begin{aligned} e^{-\rho(t \wedge \tau)} J(e_{t \wedge \tau}^b) = J(\bar{e}) + \int_0^{t \wedge \tau} e^{-\rho s} \left[-\rho J_s + \mu_s \frac{\partial J_s}{\partial e_s^b} + \frac{1}{2} \sigma_s^2 \frac{\partial^2 J_s}{\partial (e_s^b)^2} \right] ds + \\ + \int_0^{t \wedge \tau} e^{-\rho s} \sigma_s \frac{\partial J_s}{\partial e_s^b} dZ_s + \int_0^{t \wedge \tau} \frac{\partial J_s}{\partial e_s^b} e^{-\rho s} e_s^b (d\xi_s^b - d\delta_s^b), \quad (33) \end{aligned}$$

in which μ_s and σ_s are the drift and diffusion of Eq. (11).

¹⁹A formal statement of a more general problem and proof of uniqueness can be found in Løkka and Zervos (2008). A model in which recapitalization takes place in presence of a positive capital buffer can be obtained by considering a discontinuous noise process (e.g., a Poisson "jump") or by assuming that banks face some when banks collect new capital on the markets. An example of this approach appears in Peura and Keppo (2006).

By matching Eqs. (32) and (33), we obtain the following inequality between banks' market value J and the book value of their equity e^b :

$$\begin{aligned}
\int_0^{\tau \wedge t} e^{-\rho s} [e_s^b(d\delta_s - (1 + \lambda)d\xi_s)] ds &\leq -e^{-\rho(t \wedge \tau)} J(e_{t \wedge \tau}^b) + \\
&+ \int_0^{\tau \wedge t} e^{-\rho s} \underbrace{\left[1 - \frac{\partial J_s}{\partial e_s^b}\right]}_{\leq 0} e_s^b d\delta_s^b + \int_0^{\tau \wedge t} e^{-\rho s} \underbrace{\left[\frac{\partial J_s}{\partial e_s^b} - (1 + \lambda)\right]}_{\leq 0} e_s^b d\xi_s^b + J(\bar{e}) + \\
&+ \int_0^{t \wedge \tau} e^{-\rho s} \underbrace{\left[-\rho J_s + \mu_s \frac{\partial J_s}{\partial e_s^b} + \frac{1}{2} \sigma_s^2 \frac{\partial^2 J_s}{\partial e_s^{b,2}}\right]}_{=0} ds + \int_0^{t \wedge \tau} e^{-\rho s} \sigma_s \frac{\partial J_s}{\partial e_s^b} J_s dZ_s. \quad (34)
\end{aligned}$$

As long as $\lambda < \infty$, the bank's value is maximal if $d\delta^b > 0$ when $\partial J / \partial e^b = 1$ (and $e^b = e^{\max}$ and $d\xi^b > 0$ when $\partial J / \partial e^b < 1$ (and $e^b = 0$). Accordingly, Eq. (34) holds with equality $\forall e^b \in [0, e^{\max}]$ and the process de^b is reflected at 0 and e^{\max} . As a result, $\tau = \infty$ and

$$\lim_{t \rightarrow \infty} \int_0^t e^{-\rho s} (e_t^b(d\delta_t^b - (1 + \lambda)d\xi_t^b)) ds = -\lim_{t \rightarrow \infty} e^{-\rho t} J_t + \lim_{t \rightarrow \infty} \int_0^t e^{-\rho s} \sigma_s \frac{\partial J_s}{\partial e_s^b} dZ_s + J(\bar{e}).$$

By taking conditional expectations at time zero and imposing the transversality condition that $\lim_{t \rightarrow \infty} \mathbb{E}_0 [e^{-\rho t} J_t] = 0$, it holds that

$$J_0 = \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} e_t^b ((d\delta_t^b - (1 + \lambda)d\xi_t^b)) dt \right]. \quad (35)$$

□

A.3 Banks

Given Eq. (35), banks' value J satisfies the following HJBE:

$$\rho J dt = \sup_{\{d^b, k^b, \delta^b, \xi^b\}} \left\{ e^b (d\delta^b - (1 + \lambda)d\xi^b) + \frac{1}{dt} \mathbb{E}[dJ] \right\}. \quad (36)$$

To solve this problem, we postulate that $J = \nu e^b$ and that the unknown function ν evolves with dynamics $d\nu = \nu(\mu^\nu dt + \sigma^\nu dZ)$. By applying Itô's Lemma, using that $\nu > 0$,

and imposing the balance sheet constraint that $e^b = k^b q + b^b$, we can rewrite Eq. (36) as

$$(\rho - r)dt = \mu^\nu dt + \sup_{\delta^b} \{d\delta^b/\nu - d\delta^b\} + \sup_{\xi^b} \{d\xi^b - (1 + \lambda)d\xi^b/\nu\} + \sup_{\omega^b} \{\omega^b((\mu - r) - \sigma^\nu \sigma)\} dt,$$

in which $\omega^b = qk^b/e^b$. By taking the FOCs, the optimal policies in Eqs. (13)-(15) follow suit. Moreover, under the optimal strategy $\{d^b, k^b, \delta^b, \xi^b\}$, it holds that

$$\mu^\nu = \rho - r = \Gamma. \quad (37)$$

A.4 Proof of Proposition 1

To compute the dynamics of $\psi = E^b/(Kq)$, we can apply Itô's Lemma to get

$$d\psi/\psi = dE^b/E^b - d(Kq)/Kq + d(Kq)^2/(Kq)^2. \quad (38)$$

The first term of Eq. (38) can be obtained by integrating Eq. (11) over \mathbb{B} , which yields

$$dE^b = E^b r dt + E^b \omega^b (dR - r dt) - E^b \int_{\mathbb{B}} (d\delta^b - d\xi^b) db. \quad (39)$$

The remaining terms are found by applying Itô's Lemma to Eqs. (2) and (3), which gives

$$d(Kq)/(Kq) = (\Phi(\iota) + \mu^q + \sigma^q \sigma) dt + (\sigma + \sigma^q) dZ. \quad (40)$$

To conclude the proof, one can apply once again Itô's Lemma to compute the motion of $\Omega(\psi) := \{q(\psi), \nu(\psi)\}$ as $dq = \frac{\partial q}{\partial \psi} \psi \mu^\psi + \frac{1}{2} \frac{\partial^2 q}{\partial \psi^2} (\psi \sigma^\psi)^2$ and $d\nu = \frac{\partial \nu}{\partial \psi} \psi \mu^\psi + \frac{1}{2} \frac{\partial^2 \nu}{\partial \psi^2} (\psi \sigma^\psi)^2$. By matching the drift and diffusions of these equations with those of the guessed process in Eqs. (3) and using Eqs. (15) and (37), the system in Eq. (17) follows suit.

The “value matching” condition for capital prices is $\lim_{\psi \rightarrow 0} q(\psi) = \frac{A - \eta - \iota(q(0))}{\rho - \Phi(\iota(q(0))) - \Gamma}$; the economy's reflecting barrier at $\psi = \bar{\psi}$ requires that $\frac{\partial q(\bar{\psi})}{\partial \psi} = 0$. Banks' optimal strategies in Eq. (13) require that $\lim_{\psi \rightarrow 0} \nu(\psi) = 1 + \lambda$ and $\nu(\bar{\psi}) = 1$.

A.5 Proof of Proposition 2

When $\lambda = 0$, banks can instantaneously and costlessly pay dividends (issue equity) in response to positive (negative) systematic shocks. Therefore, the banking sector's capitalization is immaterial and the market value of its equity is constant and equal to 1. The equilibrium's allocation is always efficient because banks are free to intermediate the aggregate capital stock K . The economy's productivity is maximal because no output is spent to remunerate the firms' management cost.

The market clearing condition for consumption implies that $C^{h,IB} = K (A - \iota^{Ib})$, while households' deposits are $D^{Ib} = E^h = q^{Ib}K$. Accordingly, for an initial stock of capital \bar{K} households' welfare satisfies

$$H^h = \bar{K}q = \mathbb{E}_0 \left[\int_0^\infty \bar{K} e^{-(\rho - \Phi(\iota^{Ib}) + 0.5\sigma^2)t + \sigma Z_t} (A - \iota^{Ib} + \Gamma q^{Ib}) \right],$$

which simplifies as

$$q^{Ib} = \max_{\iota^{Ib}} \frac{A - \iota^{Ib}}{\rho - \Phi(\iota^{Ib}) - \Gamma} := h^{Ib} \quad (41)$$

Given that all macroeconomic aggregates are proportional to K , they evolve with the dynamics in Eq. (2), in which $\iota = \iota^{Ib}$ is the maximizer of Eq. (41).

A.6 Proof of Proposition 3

By taking the limit of Eq. (21) for $\psi \rightarrow 0$ and considering the "smooth pasting" condition in Eq. (19), one obtains that

$$\lim_{\psi \rightarrow 0} \frac{\partial \mathcal{L}(\kappa, \psi)}{\partial \kappa} = \underbrace{\eta + \Gamma \lim_{\psi \rightarrow 0} q(\psi)}_{>0} + \underbrace{\lim_{\psi \rightarrow 0} \frac{\partial^2 h(\psi)}{\partial \psi^2} [\psi \sigma^{\kappa, \psi}(\psi)] \frac{\partial \sigma^\psi(\kappa, \psi)}{\partial \kappa}}_{\lesssim 0}. \quad (42)$$

By using the boundary condition following Eq. (20), it is easy to see that the first term of Eq. (42) is always positive. The second term, instead, is negative and approaches zero for $\psi \rightarrow 0$. This can be seen by noticing that $\lim_{\psi \rightarrow 0} \psi \sigma^\psi(\psi) = \lim_{\psi \rightarrow 0} (\kappa(\psi) - \psi)(\sigma^q(\psi) + \sigma) \approx 0$; $\lim_{\psi \rightarrow 0} \partial [\psi \sigma^\psi(\kappa, \psi)] / \partial \kappa = \lim_{\psi \rightarrow 0} (\sigma^q(\psi) + \sigma) = \sigma > 0$; and $\lim_{\psi \rightarrow 0} \frac{\partial^2 h(\psi)}{\partial \psi^2} = \lim_{\psi \rightarrow 0} \frac{\partial^2 q(\psi)}{\partial \psi^2} + 2 \lim_{\psi \rightarrow 0} \frac{\partial q(\psi)}{\partial \psi} \lambda + \lim_{\psi \rightarrow 0} q(\psi) (2 \lim_{\psi \rightarrow 0} \frac{\partial \nu(\psi)}{\partial \psi} - 1) < \infty$; and that, moreover, as we will also see in our numerical simulations, the K -rescaled short-run welfare function h is concave.

A.7 Short-run welfare

Under the optimal strategy $\{C^h, D^h\}$, households' welfare is

$$H_0 = \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} (C^h + \Gamma D^h) dt \right]. \quad (43)$$

By using the market clearing condition for consumption $C^h = K((A - \iota) - (1 - \psi\omega^b)\eta)$ and that aggregate deposits must be $\Gamma D^h = \psi(\omega^b - 1)Kq$, Eq. (43) can be rewritten as

$$H = \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} K_t \Theta(\psi_t) \right] dt. \quad (44)$$

in which $\Theta(\psi) = (A - \iota(q(\psi))) - (1 - \omega^b(\psi)\psi)\eta + \Gamma\psi(\omega^b(\psi) - 1)q(\psi)$. Eq. (44) is the Feynman-Kac representation of the following PDE:

$$\rho H = K\Theta(\psi) + \mathcal{A}H(K, \psi). \quad (45)$$

By guessing (and verifying) that $H(K, \psi) = Kh(1, \psi)$, Eq. (45) simplifies as

$$[\rho - \Phi(\iota(q(\psi)))]h = \Theta(\psi) + \frac{\partial h}{\partial \psi} \psi(\mu^\psi + \sigma^\psi \sigma) + \frac{1}{2} \frac{\partial^2 h}{\partial \psi^2} (\psi \sigma^\psi)^2,$$

with boundary conditions $h(0) = \frac{A - \eta - \iota(0)}{\rho - \Phi(\iota(0)) - \Gamma}$ and $\partial h(0)/\partial \psi = 0$.

A.8 Regulated equilibrium dynamics

In the regulated economy, the aggregate capital stock's value $d(Kq)^G$ has dynamics

$$\frac{d(Kq)^G}{(Kq)^G} = \frac{d(Kq)}{(Kq)} - \mathbb{I}_{\psi^G} \frac{TE^{h,G}}{(Kq)^G} \lambda^G,$$

in which $d(Kq)/(Kq)$ denotes its motion in the unregulated equilibrium. Similarly, banking sector's equity evolves as

$$\frac{dE^{b,G}}{E^{b,G}} = \frac{dE^b}{E^b} + \mathbb{I}_{\psi^G} \frac{TE^{h,G}}{E^{b,G}}.$$

Therefore, by applying Itô's Lemma, the dynamics of relative banking sector's capitalization in the regulated economy is

$$\frac{d\psi^G}{\psi^G} = \frac{d\psi}{\psi} + \mathbb{I}_{\psi^G} T \left(\frac{E^{h,G}}{E^{b,G}} + \lambda^G \frac{E^G}{(Kq)^G} \right) dt.$$

References

- Acharya, V. and T. Yorulmazer (2007a). Cash-in-the-market pricing and optimal resolution of bank failures. *The Review of Financial Studies* 21(6), 2705–2742.
- Acharya, V. and T. Yorulmazer (2007b). Too many to fail-An analysis of time-inconsistency in bank closure policies. *Journal of Financial Intermediation* 16(1).
- Adrian, T., E. Etula, and T. Muir (2014). Financial intermediaries and the cross-section of asset returns. *The Journal of Finance* 69(6), 2557–2596.
- Barucci, E., T. Colozza, and C. Milani (2019). The effect of bank bail-outs in the EU. *Journal of International Money and Finance* 95, 14–26.
- Beck, T., D. Radev, and I. Schnabel (2020). Bank resolution regimes and systemic risk. *WP 05/20, Centre for Banking Research, University of London*.
- Berger, A., N. Himmelberg, C. P. Roman, and S. Tsyplakov (2020). Bank bailouts, bail-ins, or no regulatory intervention? A dynamic model and empirical tests of optimal regulation. *Working paper*.
- Berger, A. N., R. Roman, and J. Sedunov (2020). Did tarp reduce or increase systemic risk? The effects of government aid on financial system stability. *Journal of Financial Intermediation* 43.
- Bianchi, J. (2016). Efficient bailouts? *American Economic Review* 106(12), 3607–59.
- Brownlees, C. and R. F. Engle (2017). SRISK: A conditional capital shortfall measure of systemic risk. *The Review of Financial Studies* 30(1), 48–79.
- Brunnermeier, M. and T. Adrian (2016). Covar. *American Economic Review* 106(7), 1705–41.

- Brunnermeier, M., T. Eisenbach, and Y. Sannikov (2012). Macroeconomics with financial frictions: A survey. *National Bureau of Economic Research* (w18102).
- Brunnermeier, M. and Y. Sannikov (2014). A macroeconomic model with a financial sector. *The American Economic Review* 104(2), 379–421.
- Brunnermeier, M. and Y. Sannikov (2016). The I theory of money. *NBER WP w22533*.
- Butler, A., G. Grullon, and J. Weston (2005). Stock market liquidity and the cost of issuing equity. *Journal of Financial and Quantitative Analysis* 40(2), 331–348.
- Caballero, R., T. Hoshi, and A. Kashyap (2008). Zombie lending and depressed restructuring in japan. *American economic review* 98(5), 1943–77.
- Campi, M. and M. Duenas (2020). Volatility and economic growth in the twentieth century. *Structural Change and Economic Dynamics* 53, 330–343.
- Commission, E. (2019). Supplementay table for reporting government intervention to support financial institutions. Technical report, Eurostat. Can be found at <https://ec.europa.eu/eurostat/documents/1015035/9720710/Background-note-on-gov-interventions-Apr-2019.pdf/3c91ed77-504b-46e3-8ecb-de1bed673c6d>.
- Cordella, T. and E. Levy-Yeyati (2003). Bank bailouts: moral hazard vs. value effect. *Journal of Financial Intermediation* 12(4), 300–330.
- Dam, L. and M. Koetter (2012). Bank bailouts and moral hazard: Evidence from germany. *The Review of Financial Studies* 25(8), 2343–2380.
- Dávila, E. and A. Korinek (2018). Pecuniary externalities in economies with financial frictions. *The Review of Economic Studies* 85(1), 352–395.
- De Nicoló, G., A. Gamba, and M. Lucchetta (2014). Microprudential regulation in a dynamic model of banking. *The Review of Financial Studies* 27(7), 2097–2138.
- Di Tella, S. (2017). Uncertainty shocks and balance sheet recessions. *Journal of Political Economy* 125(6), 2038–2081.
- Dindo, P., A. Modena, and L. Pelizzon (2022). Risk pooling, intermediation efficiency, and the business cycle. *Journal of Economic Dynamics and Control* 144.

- Dinger, V., L. Erman, and D. te Kaat (2021). Bank bailouts and economic growth: Evidence from cross-country, cross-industry data. *Working paper*.
- Farhi, E. and J. Tirole (2012). Collective moral hazard, maturity mismatch, and systemic bailouts. *American Economic Review* 102(1), 60–93.
- Gersbach, H. and J. Rochet (2012). Aggregate investment externalities and macroprudential regulation. *Journal of Money, Credit and Banking* 44, 73–109.
- Gersbach, H., J.-C. Rochet, and M. Scheffel (2021). Financial intermediation, capital accumulation, and recovery. *Review of Finance (Forthcoming)*.
- Gertler, M., N. Kiyotaki, and A. Queralto (2012). Financial crises, bank risk exposure and government financial policy. *Journal of Monetary Economics* 59, 17–34.
- Giannetti, M. and A. Simonov (2013). On the real effects of bank bailouts: micro evidence from japan. *American Economic Journal: Macroeconomics* 5(1), 135–67.
- Gorton, G. and L. Huang (2004). Liquidity, efficiency, and bank bailouts. *American Economic Review* 94(3), 455–483.
- Gropp, H., H. Hakenes, and I. Schnabel (2010). Competition, risk-shifting, and public bailout policies. *The Review of Financial Studies* 24(6), 2084–2120.
- He, Z. and A. Krishnamurthy (2011). A model of capital and crises. *The Review of Economic Studies* 79(2), 735–777.
- He, Z. and A. Krishnamurthy (2013). Intermediary asset pricing. *The American Economic Review* 103(2), 732–770.
- Hilscher, J. and A. Raviv (2014). Bank stability and market discipline: The effect of contingent capital on risk taking and default probability. *Journal of Corporate Finance* 29, 542–560.
- Homar, T. (2016). Bank recapitalizations and lending: A little is not enough. *ESRB Working Paper Series* 16.
- Homar, T. and S. van Wijnbergen (2017). Bank recapitalization and economic recovery after financial crises. *Journal of Financial Intermediation* 32, 16–28.

- Hryckiewicz, A. (2014). What do we know about the impact of government interventions in the banking sector? An assessment of various bailout programs on bank behavior. *Journal of Banking & Finance* 46, 246–265.
- Hugonnier, J. and E. Morellec (2017). Bank capital, liquid reserves, and insolvency risk. *Journal of Financial Economics* 125(2), 266–285.
- Kapan, T. and C. Minoiu (2018). Balance sheet strength and bank lending: Evidence from the global financial crisis. *Journal of Banking & Finance* 92, 35–50.
- Keister, T. (2016). Bailouts and financial fragility. *The Review of Economic Studies* 83(2), 704–736.
- Klimenko, N., S. Pfeil, J.-C. Rochet, and G. D. Nicolo (2016). Aggregate bank capital and credit dynamics. *Swiss Finance Institute Research Paper*, 19–42.
- Kubitza, C. (2021). Tackling the volatility paradox: Spillover persistence and systemic risk. *Working paper*.
- Laeven, L. and F. Valencia (2013). The real effects of financial sector interventions during crises. *Journal of Money, Credit and Banking* 45(1), 147–177.
- Lambrecht, B. and A. Tse (2021). Liquidation, bailout, and bail-in: Insolvency resolution mechanisms and bank lending. *Journal of Financial and Quantitative Analysis Forthcoming*.
- Løkka, A. and M. Zervos (2008). Optimal dividend and issuance of equity policies in the presence of proportional costs. *Insurance: Mathematics and Economics* 42(3), 954–961.
- Mendicino, C., K. Nikolov, J. Suarez, and D. Supera (2019). Bank capital in the short and in the long run. *Journal of Monetary Economics* 112, 64–79.
- Mitkov, Y. (2020). Inequality and financial fragility. *Journal of Monetary Economics* 115, 233–248.
- Øksendal, B. (2003). *Stochastic differential equations*. Springer, Berlin.
- Peura, S. and J. Keppo (2006). Optimal bank capital with costly recapitalization. *The Journal of Business* 79(4), 2163–2201.

- Phelan, G. (2016). Financial intermediation, leverage, and macroeconomic instability. *American Economic Journal: Macroeconomics* 8(4), 199–224.
- Philippon, T. (2015). Has the US finance industry become less efficient? On the theory and measurement of financial intermediation. *American Economic Review* 105(4), 1408–38.
- Philippon, T. and P. Schnabl (2013). Efficient recapitalization. *The Journal of Finance* 68(1), 1–42.
- Quadrini, V. (2011). Financial frictions in macroeconomic fluctuations. *FRB Richmond Economic Quarterly* 97(3), 209–254.
- Rampini, A. A. and S. Viswanathan (2019). Financial intermediary capital. *The Review of Economic Studies* 86(1), 413–455.
- Reinhart, C. M. and K. S. Rogoff (2014). Recovery from financial crises: Evidence from 100 episodes. *American Economic Review* 104(5), 50–55.
- Repullo, R. (2000). Who should act as lender of last resort? An incomplete contracts model. *Journal of Money, Credit and Banking*, 580–605.
- Risken, H. (1996). *The Fokker-planck Equation*, Volume 63-95. Springer, Berlin.
- Sandri, D. and F. Valencia (2013). Financial crises and recapitalizations. *Journal of Money, Credit and Banking* 45(2), 59–86.
- Schroth, J. (2020). On the distributional effects of bank bailouts. *Review of Economic Dynamics*.
- Schroth, J. (2021). Macroprudential policy with capital buffers. *Journal of Monetary Economics* 118, 296–311.
- Stein, J. (2012). Monetary policy as financial stability regulation. *The Quarterly Journal of Economics* 127(1), 57–95.
- Stokey, N. L. (2009). *The Economics of Inaction: Stochastic Control models with fixed costs*. Princeton University Press.

- Tsionas, E. G., G. A. Assaf, and R. Matousek (2015). Dynamic technical and allocative efficiencies in european banking. *Journal of Banking & Finance* 52, 130–139.
- Van Der Ghote, A. (2020). Interactions and coordination between monetary and macro-prudential policies. *American Economic Journal: Macroeconomics* 13(1), 1–34.
- Veronesi, P. and L. Zingales (2010). Paulson’s gift. *Journal of Financial Economics* 97(3), 339–368.

B Online appendix – supplementary material

B.1 Management fees – micro-foundation

This appendix provides a simple micro-foundation for the management fee η described in Section 2.2. As in the baseline model, firms are run by a unit mass of managers. In addition, however, they are now heterogeneous in their ability. In particular, a constant share $f \in [0, 1]$ of them is highly skilled; the remaining $1 - f$ is low-skilled.

As in the baseline model, managers collect physical capital from the firms’ shareholders (households and banks) and use it to produce output by using the (stochastic) technology $y_t = a_t k_t$. The TFP level a_t takes value 1 with probability p or zero; its realizations are i.i.d. over time and across firms.

Similarly to Van Der Ghote (2020) (Appendix B), managers can exert effort to improve the probability that their own firms’ productivity equals one from p to $p^\epsilon > p$. Exerting effort entails a forgone private benefit of zero or ϵ per unit of capital, depending on whether managers are skilled. The firms’ shareholders cannot observe their decisions. Thus, under the parametric restriction that $p^\epsilon - \epsilon \geq p$, moral hazard arises. Importantly, we assume that banks can tell high from low-skill managers, while households are not.

The moral-hazard problem can be addressed by writing an incentive-compatible contract. The contract is crafted so that shareholders commit to paying a premium (“management fee”) η per unit of capital, which remunerates the managers’ effort. It is straightforward to see that the optimal fee is either $\eta = \epsilon / (p^\epsilon - p)$ or zero, depending on the managers’ ability.

Since households cannot tell high-skill from low-skill managers, their best choice is to rebate their investments across the universe of firms and remunerate all managers with η , independently of their skill. In the baseline model’s notation, this implies that $A = p^\epsilon$. Conversely, banks can identify highly-skill managers. Therefore, they only rebate capital across them and earn A by paying no fee.

B.2 Equilibrium – definition

An equilibrium is an \mathcal{H} -adapted stochastic process $\Omega : [0, 1] \rightarrow \mathbb{R}^n$ that maps histories of systematic shocks $\{W_t\}$ to prices $\{q_t, \nu_t\}$, risky returns $\{R_t\}$, deposit rates $\{r_t\}$, production $\{K_t, \iota_t\}$ and consumption choices $\{c_t^h : h \in \mathbb{H}\}$, allocations $\{d_t^i, k_t^i : i \in \{h, b\}\}$, dividends and

recapitalizations $\{\delta_t^b, \xi_t^b : b \in \mathbb{B}\}$ such that:

1. Firms maximize their profits subject the dynamic bc constraint in Eq. (4):

$$\{k_t^i, \iota_t^i\} = \arg \max_{k_t^i, \iota_t^i} \left\{ \mathbb{E}_t \left[k_{t+dt}^i q_{t+dt}^i \exp \left\{ \int_t^{t+dt} r_s ds \right\} - k_t q_t \right] \right\}, \forall i \in \mathbb{I}. \quad (46)$$

2. Household maximize their inter-temporal utility subject to the dynamic bc in Eq. (7):

$$\{c_t^h, d_t^h, k_t^h\} \in \arg \sup_{\{c_t^h, d_t^h, k_t^h\}} \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} (c_t^h + \Gamma d_t^h) \right] dt, \forall h \in \mathbb{H}. \quad (47)$$

3. Banks maximize their market value subject to the dynamic bc in Eq. (11):

$$\{d_t^b, k_t^b, \delta_t^b, \xi_t^b\} \in \arg \sup_{\{d_t^b, k_t^b, \delta_t^b, \xi_t^b\}} \mathbb{E}_0 \left[\lim_{S \rightarrow \infty} \sup_{\tau} \int_0^{\tau \wedge S} e^{-\rho t} e_t^b (d\delta_t^b - (1 + \lambda)d\xi_t^b) dt \right], \forall b \in \mathbb{B}. \quad (48)$$

4. All markets clear:

- (a) Deposits

$$\int_{\mathbb{H}} d_t^h dh + \int_{\mathbb{B}} d_t^b db = 0; \quad (49)$$

- (b) Consumption

$$\int_{\mathbb{H}} (A - \iota_t - \eta) k_t^h dh + \int_{\mathbb{B}} (A - \iota_t) k_t^b db = C_t^h; \quad (50)$$

- (c) Capital

$$\int_{\mathbb{H}} k_t^h dh + \int_{\mathbb{B}} k_t^b db = K_t. \quad (51)$$

B.3 Numerical solution algorithm

To compute the model's competitive equilibrium, we must solve the ODE system in Eq. (17) numerically. For this purpose, we follow Brunnermeier and Sannikov (2014). First, we subtract the optimality conditions in Eqs. (9) from (15), which gives

$$\eta/q(\psi) + \rho - r \leq \sigma^\nu(\psi) (\sigma + \sigma^q(\psi)). \quad (52)$$

Second, we transform the system by using the auxiliary variable $\kappa(\psi) = \psi\omega^b(\psi)$ and guess an initial value for $\partial\nu(0)/\partial\psi \in (0, -\infty)$. Third, we iterate the following steps until convergence.

1. Guess $\kappa \in (\psi, \psi + q/(\partial q/\partial\psi))$ so that Eq. (52) holds with equality, σ^q and σ^ν satisfy Eq. (17), and σ^ψ is the diffusion of Eq. (16).
2. If $\kappa(\psi) \geq 1$, set $\kappa(\psi) = 1$ and recompute Eq. (52). Else, proceed with Step 3.
3. Solve recursively the ODE system in Eq. (17) by using that, by Eq. (28), $\mu^q = \rho + \eta/q - \sigma^q\sigma - \Phi(\iota(q))$. Stop when either $\partial q(\bar{\psi})/\partial\psi$ or $\partial\nu(\bar{\psi})/\partial\psi$ equal zero. We implement this step numerically by using Matlab *ODE45* solver.
4. Rescale ν so that $\nu(\bar{\psi}) = 1$ and compute $\omega^b = \kappa/\psi$.
5. Check whether $|\nu(0) - (1 + \lambda)|$ is below the arbitrary tolerance threshold ϵ (i.e., the boundary condition is met). If yes, stop. Else, update the initial guess for $\partial\nu(0)/\partial\psi$ with a bisection method and repeat from Step 1.

B.4 Comparative static analysis wrt T

Fig. 7 depicts the comparative analysis of the bailout policy with respect to changes in the tax rate parameter T . The results are similar to those of the comparative statics analysis with respect to ψ^G . Thus, we refer to Section 3.2.2 in the main body of the paper for a comprehensive discussion of the underlying mechanisms.

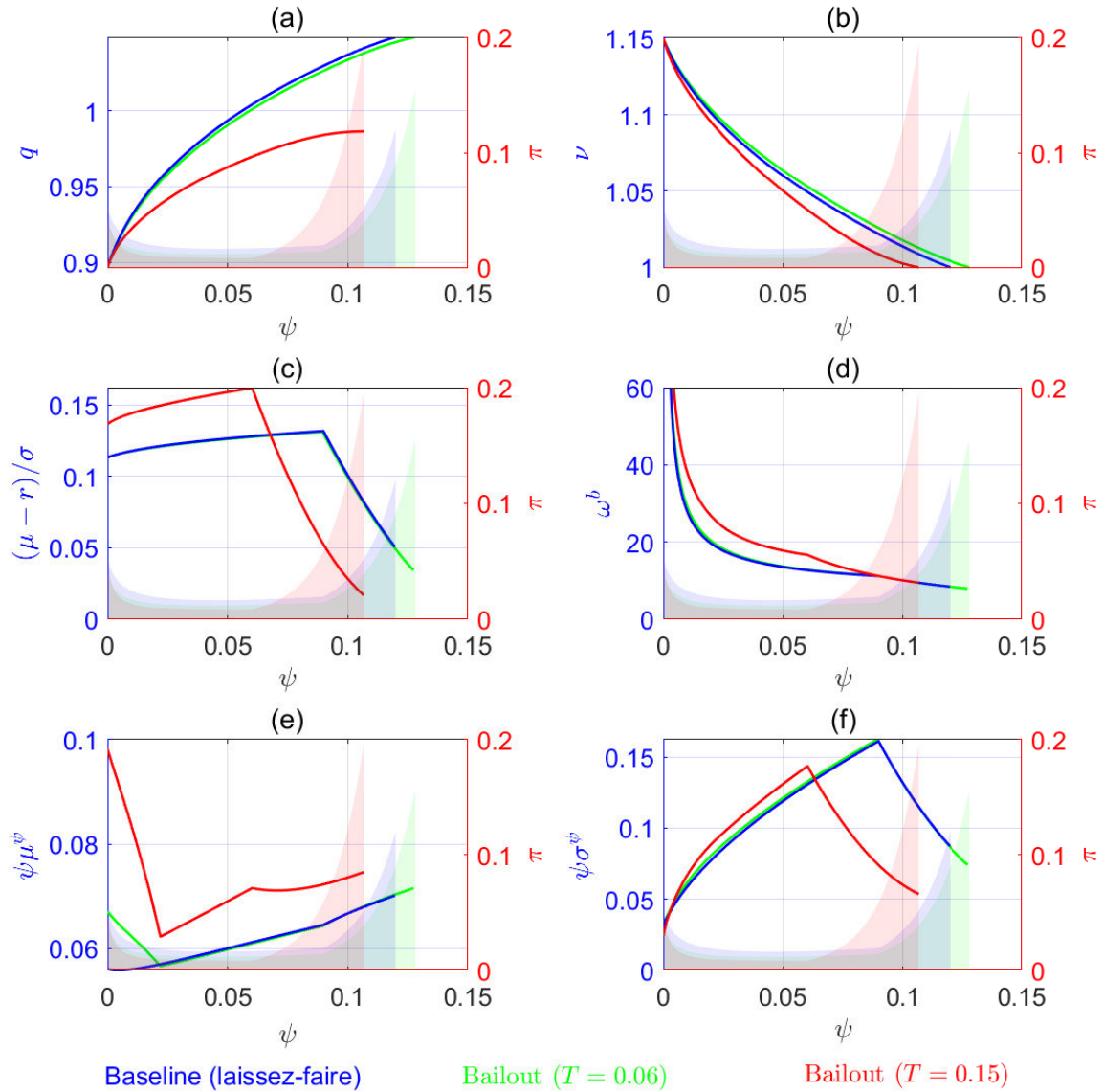


Figure 7: Numerical solution of the model: baseline (laissez-faire) vs regulated (bailout) equilibrium. Parameters are as in Table 1, with $\psi^G = 0.02$, $\lambda^G = 0.2$, and $T = 6\%$ (green) or 15% (red).