Discussion Paper Series – CRC TR 224

Discussion Paper No. 475
Project B 06

Union and Firm Labor Market Power

Miren Azkarate-Askasua[1]
Miguel Zerecero[2]

November 2023

[1] University of Mannheim, Email: azkarate-askasua@uni-mannheim.de
[2] University of California, Email: mzerecer@uci.edu

Support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Collaborative Research Center Transregio 224 - www.crctr224.de
Rheinische Friedrich-Wilhelms-Universität Bonn - Universität Mannheim

Union and Firm Labor Market Power

Miren Azkarate-Askasua Miguel Zerecero*

October 9, 2023

Abstract

Can union and firm market power counteract each other? What are the output and welfare effects of employer and union labor market power? Using data from French manufacturing firms, we leverage mass layoff shocks to competitors to identify a negative effect of employment concentration on wages. In line with the reduced form evidence and the French institutional setting, we develop and estimate a multi-sector bargaining model that incorporates employer market power. We find that in the absence of unions output decreases by 0.48 percent because they partially counteract distortions coming from oligopsony power. Furthermore, eliminating employer and union labor market power increases output by 1.6 percent and the labor share by 21 percentage points. Workers' geographic mobility is key to realizing the output gains.

JEL Codes: J2, J42, J51

Keywords: Labor markets, Wage setting, Misallocation, Monopsony, Unions

*Azkarate-Askasua: University of Mannheim (azkarate-askasua@uni-mannheim.de); Zerecero: University of California, Irvine (mzerecer@uci.edu). This paper was previously circulated as *The Aggregate Effects of Labor Market Concentration*. We are indebted to our advisor Christian Hellwig for his invaluable guidance throughout this project. We are grateful to Fabrice Collard and Patrick Fève for their extensive suggestions. We also thank Tiziana Assenza, Thomas Chaney, Jan Eeckhout, Sergio Feijoo, Simon Fuchs, Elia Lapenta, Salvatore Lo Bello, Ramon Marimon, Nour Meddahi, Federico Mendelman, Elias Papaioannou, François Poinas, Tommaso Porzio, Jean-Marc Robin, Antonio Rodríguez-López, Sébastien Roux, Uta Schönberg, Robert Ulbricht and Nicolas Werquin for their comments and suggestions. Finally, we thank the participants at the TSE Macro workshop and researchers at the Federal Reserve Bank of Atlanta, Universitat de Girona, ECB, Tilburg, ITAM Business, Stockholm University, EUI, CEMFI, Banco de España, CREi, UPF, Mannheim, Goethe, CEPR ESSIM 2021, Barcelona Summer Forum in Market Power and the Labor Market, EEA-ESEM 2023, and SED in Minneapolis for useful comments. We gratefully acknowledge financial support from TSE to get access to the administrative data provided by CASD. This work is supported by a public grant overseen by the French National Research Agency (ANR) as part of the 'Investissements d'avenir' program (reference: ANR-10-EQPX-17 – Centre d'accès sécurisé aux données – CASD). Zerecero acknowledges the financial support from CONACYT (reference: 329103/383874). Azkarate-Askasua acknowledges the support from the German Research Foundation (through the CRC-TR-224 project B06). First version: October 1, 2018. All errors are our own.

1 Introduction

There is growing evidence, especially for the United States, that links lower wages to labor market concentration.[1] Indeed, if this concentration reflects monopsony power in the labor market, standard theory predicts that establishments *mark down* wages by paying workers less than their marginal revenue product of labor. On the other hand, if labor market institutions enable workers to organize and have a say over the wage setting process, bargaining can mitigate, or even reverse, the effect of establishments' market power on wages.

In this paper, we study the interaction between union and firm labor market power and quantify their effects on productivity and welfare in the French manufacturing sector. The French case stands out over other developed countries, especially with respect to the U.S., for having regulations that significantly empower workers over employers. We therefore provide a theoretical framework that incorporates both, employer and union labor market power.

Our main result is that unions mitigate the negative efficiency effects of employer market power. We find that in the absence of unions and holding the total labor supply constant, output decreases by 0.48%. While the effect on output is small, unions have a meaningful distributional role. Without unions, the labor share would be 12 percentage points smaller and average wages would be 24% smaller. This translates into median welfare losses for the workers of more than 26%. We also find that in a counterfactual with monopsonistic competition—a limit case where establishments become atomistic due to entry of a continuum of competitors—both output and workers' welfare increase compared to a scenario with oligopsonistic competition. The output effect of changing from oligopsonistic to monopsonistic competition greater than that of adding unions to an oligopsonistic environment, but the effect on workers' welfare is smaller. These results imply that unions can serve as a partial alternative to more firms competing in the labor market. Similar to the effects of increasing the number of competitors, unions boost output and workers' welfare compared to a situation with purely oligopsonistic competition, albeit not reaching the same output gains of the monopsonistic competition counterfactual.

We proceed in three steps. First, we establish empirically that, within the same firm, establishments with higher local employment shares pay lower wages for the same occupations. We identify this effect by using a competitor's national mass layoff shock as an external source of variation to an establishment's local employment share. Second, in line with the previous empirical result and the French labor institutional setting, we build and estimate a model where labor market power arises from (i) employers that face upward-sloping labor supplies, and (ii) workers that bargain over wages. Third, we use the model to quantify the output and welfare consequences of employers and workers' labor market power.

We start by documenting the link between employer's market power and wages. We use data on French manufacturing firms from the years 1994 to 2007. To say something about employer labor market power, we first need to specify the relevant labor markets. We define a local labor market as a combination of a commuting zone, industry, and occupation. At the establishment-occupation level, our proxy for the strength of labor market power is the employment share within the local labor market. To explore a link between labor market power and wages, we need to overcome the

[1] See for example Benmelech, Bergman, and Kim (2018), Azar, Marinescu, Steinbaum, and Taska (2020b), Berger, Herkenhoff, and Mongey (2022), Jarosch, Nimczik, and Sorkin (2019), Benmelech et al. (2018) among others.

endogeneity of the employment shares and the wages. We propose a novel identification strategy where we instrument employment shares with negative employment shocks—or mass layoffs—to competitors. Identification comes from residual within firm-occupation-year variation across establishments located in different local labor markets. Depending on the specification, the estimated semi-elasticity ranges from -0.16 to -0.04. That is, a 1 percentage point increase of employment share lowers the establishment wage by up to 0.16 percent.[2]

After presenting the reduced form evidence, we build a general equilibrium model that incorporates two elements: employer and union labor market power. Our framework without bargaining is similar to the one in Berger, Herkenhoff, and Mongey (2022) (BHM) under Bertrand competition. First, we borrow from the trade and urban economics literature (e.g. Eaton and Kortum, 2002; Ahlfeldt, Redding, Sturm, and Wolf, 2015) and assume workers have stochastic preferences to work at different places, as in Card, Cardoso, Heining, and Kline (2018). Heterogeneity of workers' tastes implies that individual establishment-occupations face an upward sloping labor supply curve, which potentially gives rise to employer labor market power. In the absence of bargaining, as there is a discrete set of establishment-occupations per local labor market, employers act strategically and compete for workers in an oligopsonistic fashion. Wages are therefore paid with a markdown, which is a function of the labor supply elasticity. Similarly to Atkeson and Burstein (2008), this elasticity depends on the employment share within the local labor market. The second important element of the model is collective wage bargaining. We assume wages are set at the establishment-occupation level between establishments and unions acting symmetrically. Both sides internalize how rents are generated and bargain with zero as the outside option.

This wage-setting process leads to a distortion that is reflected in a wedge between the equilibrium negotiated wage and the marginal revenue product of labor. This wedge summarizes both sides of market power as it is a combination of both, a markdown due to oligopsony power, and a markup due to wage bargaining. The model clearly captures that union and firm labor market power constitute countervailing forces. The smaller this wedge is, the larger the market power of employers relative to workers and vice-versa. Heterogeneity of the labor wedge across establishments distorts relative wages and potentially generates misallocation of resources that decrease aggregate output. Heterogeneity comes from two sources: (i) the dependence of the markdown on sector specific labor supply elasticities and employment shares; and (ii) the across sector differences in the markup due to the different bargaining powers. Our model nests as special cases both, a full bargaining setting or a model with oligopsonistic competition only.

The framework features a large number of different establishment-occupation wages plus the product prices. We show that the model is block recursive and how to solve for the general equilibrium in two steps. We solve first for the employment shares within each local labor market ignoring aggregates. Second, we show how to build sector level fundamentals and solve for aggregate prices. This two-step procedure eases the solution because the model can be rewritten at the sector level.[3] We provide an analytical characterization of the equilibrium at the sector level and along the way prove the existence and uniqueness of the equilibrium.

[2] This corresponds to a reduction of roughly 1000 euros (at 2015 prices) per year if we pass from the first to the third quartile of the employment share distribution.

[3] The intuition behind this is that after solving for the employment shares while ignoring sector and economy-wide constants, we can fully characterize the allocation of labor and capital *within* each sector. This fact, combined with the information about the establishment-level fundamentals, allows us to aggregate the model at the sector level with corresponding sector-level fundamentals.

After the model set-up, we discuss how to identify and estimate the model parameters. We have two types of parameters: the ones related to the labor supply and bargaining, and the ones related to production. Regarding the labor supply, we assume that workers face a sequential decision: in a first stage, they observe their preferences for different local labor markets and choose the one that maximizes their expected utility; in a second stage, they observe their preferences to work for different employers and choose the establishment. Therefore, these labor supplies depend on two key parameters related to the heterogeneity of workers' preferences. These parameters are the *local* and *across-market* elasticities of substitution. The local elasticity measures how strongly workers substitute across establishments within a local labor market, while the across-market elasticity measures how strongly workers substitute across local labor markets in the economy. These two elasticities jointly determine the magnitude of employers' labor market power.

The main challenge is to separately identify the union bargaining powers from the local and across-market elasticities of substitution. We propose a strategy to estimate the elasticities of substitution that is independent from the underlying wage setting process. Therefore, our identification strategy is readily applicable to set-ups with or without bargaining.

We first estimate the across-market elasticity of substitution. We use the insight that this is the only relevant elasticity for the establishments that are alone in their local labor markets, which we refer to as full monopsonists. Their local labor market equilibrium boils down to a standard system representing the labor supply and demand equations. Ordinary least squares estimates present the traditional problem of other price-quantity systems as the estimated elasticities are biased towards zero. Rather than instrumenting to get exogenous variation in the labor supply and demand, we identify the across local labor market elasticities and the inverse labor demand elasticity adapting the identification through heteroskedasticity of Rigobon (2003). This identification strategy allows to recover the structural parameters by assuming heteroskedasticity across sub-samples. We adapt the method by imposing the heteroskedasticity restriction on the variance-covariance of structural shocks across occupations.[4]

In the second step of our estimation strategy, we estimate the local elasticities of substitution using within-market variation in wages and employment levels. To instrument for wages, we rely on firm-level revenue productivities. Even when strategic interactions are present, our approach avoids violating the stable unit of treatment value assumption (SUTVA) that leads to biased estimates as highlighted by Berger, Herkenhoff, and Mongey (2022). BHM show that *within-establishment, across-time* variation cannot identify the labor supply elasticity because non-atomistic establishments' strategic interactions can affect the overall equilibrium, resulting in a SUTVA violation. Instead, we condition on an equilibrium allocation and use *across-establishment, within-market* variation to identify the local elasticity of substitution, which is related to the labor supply elasticity. We expand on BHM's argument in three ways. First, we clarify the general relationship between the elasticity of substitution and the labor supply elasticity and explain the scenarios where they are equivalent. Second, we generally establish the bias between the labor supply elasticity and a reduced form estimate. Third, we show that within equilibrium variation can identify the local elas-

[4]To see the notion behind Identification through Heteroskedasticity, consider the following system: $y = \alpha x + u$ and $x = \beta y + v$, with $var(\epsilon) \equiv \sigma_\epsilon$ and $cov(u, v) = 0$. The system is under-identified as the covariance matrix of (x, y) (which can be directly estimated) yields three moments (σ_x, σ_y and $cov(x, y)$) while we have to solve for four unknowns: $(\alpha, \beta, \sigma_u, \sigma_v)$. Suppose we can split the data into two sub-samples with the same parameters (α, β) but different variances. Now the two sub-samples give us 3+3=6 data moments with only six unknowns: the two parameters (α, β) and the four variances of structural errors.

ticity of substitution. After estimating both elasticities of substitution, we estimate the bargaining powers to match the sector labor shares.

Even when our model has some elements like bargaining and oligopsony that depart from more traditional environments in the trade and urban literatures, we show that the general equilibrium counterfactual can be computed using only observed wages and employment levels in the data. We do that by writing the model in terms of relative changes with respect to the current equilibrium. This approach, borrowed from the trade literature, allows us to solve for changes of equilibrium variables relative to a baseline scenario.[5] We are able to do that because the observed wages and employment levels are sufficient statistics of the fundamentals of the model, in this case the establishments' productivities and amenities.

We quantify the output losses of employers' and workers' labor market power by removing those distortions in a counterfactual economy while keeping workers' preferences fixed. This is a counterfactual scenario where employers are competitive and workers have no bargaining power leading to wages that are equal to the marginal revenue product of labor. We find that output increases by 1.6 percent while the labor share rises by 21 percentage points. This increased labor share goes together with wage gains that in turn translate into 42 percent median expected welfare gains for workers. Removing the heterogeneity of wedges improves the allocation of labor by increasing the employment of more productive establishments. The counterfactual gains in the labor share suggest that employer labor market power is stronger than that of the unions. This is a consequence of the estimated low elasticities of substitution that are in the range but a bit lower than the estimates of Berger, Herkenhoff, and Mongey (2022) for the U.S. Interestingly, we find that removing the bargaining would slightly reduce output compared to the baseline. Thus, given the presence of employers' labor market power, unions help to counteract distortions coming from oligopsony power.

Additionally, we find that geographic mobility is the key margin of adjustment to achieve the baseline counterfactual productivity gains, rather than within local labor market or within sector mobility. The intuition behind this is that there are a handful of concentrated and productive firms in rural areas and removing labor market power increases their wage and employment more relative to urban areas. We find that labor market distortions account for 13 percentage points—about a third—of the urban/rural wage gap. Consequently, in the counterfactual with no distortions, the total employment decreases in urban areas relative to the baseline, which changes the geographical composition of manufacturing employment in France.

Finally, we incorporate two extensions to the model. First, we introduce an endogenous labor force participation decision by assuming that workers may voluntarily stay out of the labor force. This additional margin of adjustment amplifies the output responses in the counterfactuals. For example, if aggregate productivity and wages go up, more workers would enter the labor force, further increasing output. In a second extension, we allow for agglomeration forces within the local labor market that also amplify the output responses compared to the baseline counterfactual.

Literature. This paper speaks to several strands of the literature. First, and most closely related, is the literature on employer labor market power. Several empirical papers have documented the

[5] Costinot and Rodríguez-Clare (2014) refer to this method as "exact hat algebra". They use this approach to compute welfare effects of trade liberalizations using easily accessible macroeconomic data.

importance of labor market concentration on wages, employment and vacancies.[6] The concentration relates critically to the definition of a *local* labor market which most of the papers consider as rigid entities based on combinations of location-industry or location-geography identifiers.[7] Most empirical papers focus on aggregate measures of concentration, as the Herfindahl-Hirschman Index, as a proxy for employer labor market power. Our contribution to this empirical literature is to focus on market power at the establishment level and propose a novel identification strategy using exogenous variation from *competitors' mass layoff shocks*.

This paper also contributes to structural work on employer labor market power. We depart from the traditional monopsony power framework (e.g. Manning, 2011; Card et al., 2018) and recent papers with monopsony power by having heterogeneous markdowns arising from market structure and by extending it to allow for wage bargaining.[8] The paper is complementary to Jarosch et al. (2019) as they consider employer labor market power in a search framework. Recently Jäger, Roth, Roussille, and Schoefer (2022) show that workers do not hold correct beliefs about their outside options which allows firms to mark down wages if there are search costs that are sufficiently high. Bachmann, Bayer, Stüber, and Wellschmied (2022), MacKenzie (2021) and Trottner (2022) have focused on misallocation effects of monopsony power. We contribute to this literature by including unions and studying their counterbalancing effect to the labor market power of firms.

In recent important work, Berger, Herkenhoff, and Mongey (2022) build a structural model with oligopsonistic competition within local labor markets. We share the objective of measuring the output effects of labor market distortions and reach similar quantitative conclusions when taking together union and firm labor market power. However, our contribution differs from theirs in several dimensions: (i) our framework nests theirs as an special case without bargaining; (ii) we incorporate occupations and use them for the identification of the structural parameters; (iii) we allow for differences in structural parameters across industries. In particular, we allow for different local elasticities of substitution and bargaining powers across industries. Importantly, this adds heterogeneity to the labor wedges and employment misallocation; (iv) we show that counterfactuals can be computed without the need to back out underlying productivities and we perform the counterfactuals using actual establishment data; and (v) we evaluate the role of unions as an alternative to labor market competition.

We also contribute to the literature studying the role of unions. Some papers have focused on the impact of unions on reducing wage inequality (DiNardo, Fortin, and Lemieux, 1995; Farber, Herbst, Kuziemko, and Naidu, 2021). On the contrary, evidence using quasi-experimental variation has found insignificant effects of unionization on wages (Freeman and Kleiner, 1990; Lee and Mas, 2012; Frandsen, 2021). Our paper is related to Lagos (2020) that studies worker amenity and wage compensation under bargaining in Brazil and how they depart from monopsony compensation. We contribute to that paper in studying aggregate effects of firm and union labor market power. There

[6]Among them there are the papers by Azar, Marinescu, and Steinbaum (2020a); Benmelech et al. (2018); Azar et al. (2020b); Schubert, Stansbury, and Taska (2020); Dodini, Lovenheim, Salvanes, and Willén (2020); Marinescu, Ouss, and Pape (2021)

[7]There have been some advances in considering flexible local labor markets either based on labor flows (Nimczik, 2018), commuting patterns (Manning and Petrongolo, 2017), skill composition (Macaluso, 2017; Dodini et al., 2020), or broadly on workers' outside options (Schubert et al., 2020) inferred from labor flows. We take a more traditional approach and define them based on location-industry-occupation identifiers.

[8]Recent papers studying the effects of monopsony power include Lamadon, Mogstad, and Setzler (2022), Deb, Eeckhout, Patel, Warren et al. (2022b), Deb, Eeckhout, Patel, and Warren (2022a), Amodio and De Roux (2021), Amodio, Medina, and Morlacco (2022), Datta (2021) and Felix (2021) among others.

is growing empirical evidence of the ability of unions on weakening the effects of labor market concentration. Marinescu et al. (2021) find negative effects of local labor market concentration on wages for new hires in France that are less pronounced in more unionized industries. Similar findings have been reported by Benmelech et al. (2018) in the U.S. and Dodini, Salvanes, and Willén (2021) in Norway. These findings are in line with our structural model and we find that allowing for collective bargaining is key to matching certain empirical regularities. We furthermore provide a framework to rationalize those findings and evaluate their aggregate implications.

The paper relates to the literature on imperfect competition in general. Our approach is similar to Edmond, Midrigan, and Xu (2021) and Morlacco (2018) in trying to quantify the effect of heterogeneous market power on aggregate output. They study output and intermediate input market powers respectively while we focus on the effects of labor market power. Recently Hershbein, Macaluso, and Yeh (2020) and Wong (2019) disentangle output and labor market power using, respectively, a production function approach for the U.S. and France. They both find the presence of employer labor market power even when controlling for production function heterogeneity and output market power.

Contrary to the evidence on output market power, other studies suggest that employer labor market power is not the driver behind the decreasing trends of the U.S. labor share (e.g. Lipsius, 2018; Berger et al., 2022) with the exception of Hershbein et al. (2020).[9] The focus of this paper is not on labor share trends but on the effects of employer and union labor market power in a given cross section of firms, markets and industries.

We estimate local and across-market elasticities of substitution, which bound the elasticity of the labor supply.[10] Therefore, our paper contributes to micro estimates of firm labor supply elasticities. Staiger, Spetz, and Phibbs (2010), Falch (2010), Berger et al. (2022) and Datta (2021) use quasi-experimental variation on wages to estimate the firm labor supply elasticities that go from 0.1 (Staiger et al., 2010) to 10.8 (Berger et al., 2022).[11] Both our local and across market elasticities of substitution lie in that range. Dube, Jacobs, Naidu, and Suri (2020) and Datta (2021) estimate a labor supply elasticity to firm-level wage policies ranging between 3 and 5, which is close to our local elasticity of substitution. Azar, Berry, and Marinescu (2022) estimate market elasticities of 0.5 and firm elasticities of 5 which are very close to our estimated elasticities of subsitution. Lastly, the median estimate in the meta-analysis of Sokolova and Sorensen (2021) and the estimates in Webber (2015) are close to 1, which is in between our estimates for the across-market and local elasticities of substitution.

The rest of the paper is organized as follows. Section 2 introduces the data. Section 3 shows the stylized facts and our empirical strategy. Section 4 presents the model. Section 5 discusses details about identification and estimation of the model. Section 6 shows the results from counterfactual exercises. Section 7 presents extensions of the model and Section 8 concludes.

[9] Karabarbounis and Neiman (2013) documented the falling trend of the labor share and Barkai (2020) and Gutiérrez and Philippon (2016) the rising trend of the profit share for different countries. Output market power has been pointed out as an explanation for the decline of the labor share (e.g. De Loecker, Eeckhout, and Unger, 2020; De Loecker and Eeckhout, 2021).

[10] In particular, the local elasticity of substitution bounds from above the supply elasticity while the across-market elasticity bounds it from below.

[11] Berger et al. (2022) estimate elasticities of substitution. Here we report their local elasticity of substitution which is an upper bound for the labor supply elasticity.

2 Data

Most of our analysis relies on *FICUS* for the years 1994-2007 with firm-level fiscal records consisting of balance sheet information including wage bill, capital stock, number of employees and value added. A break in the industry classification series prevents us from extending the time span of the sample.[12] This dataset includes all French firms, except for the smallest ones declaring at the micro-BIC regime and some agricultural firms. We also use *CASD Postes*, an employer-employee dataset with the universe of salaried employees with firm and establishment identifiers. We recover the location, occupation classification, wages and employment that are necessary to distinguish across different establishment-occupations of a given firm. Additionally we use data relating the city codes to commuting zones and Consumer Price Index data to deflate nominal variables.[13]

We define four broad categories of occupations: top management, supervisor, clerical and operational.[14] We define a local labor market as the combination of commuting zone, 3-digit industry, and occupation. On average, there are 57,900 local labor markets per year.[15]

Our sample consists of approximately 4 million establishment-occupation-year observations that belong to around 1.25 million firms. Details about sample selection are in Online Appendix G.2.

2.1 Summary statistics

Table 1 presents summary statistics at the establishment-occupation level for the final sample. The median occupation at a given establishment has 2 employees and pays 27,439 euros per worker. Certain firms have the same occupation in different locations, which we denote as multilocation firm-occupations. The micro evidence in the next section focuses on multilocation firm-occupations.[16] Panels (a) and (b) of Table 1 have the summary statistics of occupations belonging to monolocation and multilocation firms. The majority of observations, roughly 80%, belong to monolocation firm-occupations. Occupations in firms with establishments at multiple locations are larger on average with 27 employees versus 7 for occupation-firms at a single location. In both groups, the distribution of employment is concentrated in few large employers, as both medians are smaller than the means. Firms with multilocation occupations pay wages per capita that are 15% higher than the monolocation ones.

We categorize manufacturing firms into 97 different 3-digit industries, or sub-industries, which are distributed across 364 different commuting zones. We denote the 3-digit industries as h and the commuting zones as n. The average 3-digit industry labor share is 52% and the share of capital is 26%.[17] Taking those averages, the profit share would be around 22%. We refer to the interested reader to consult the tables on Online Appendix H, which present summary statistics on the commuting zones, local labor markets and sub-industries.

[12] Before 1994 the wage data was imputed and after 2007 the industry classification (APE) is not consistent with previous versions. On the contrary, the classification change between the 1993 and 2003 codes is consistent at the 3-digit level.

[13] The sources are https://www.insee.fr/fr/information/2114596 and https://www.insee.fr/fr/statistiques/serie/001643154 respectively.

[14] The classification is very similar to the one in Caliendo, Monte, and Rossi-Hansberg (2015). We group together their first two categories (firm owners receiving a wage and top management positions) into top management because the distinction between the two was not stable in 2002.

[15] We use interchangeably 3-digit industry or sub-industry throughout the text.

[16] The multilocation definition is occupation specific. A firm can have both monolocation and multilocation occupations.

[17] We follow Barkai (2020) to compute the capital share.

Table 1: Establishment-Occupation Summary Statistics

	Mean	Pctl(25)	Median	Pctl(75)	St. Dev.
All sample					
L_{iot}	11.1	1.1	2.3	6.2	59.5
$w_{iot}L_{iot}$	367.2	31.6	71.8	196.6	2,379.5
w_{iot}	34.0	20.9	27.4	39.5	117.1
$s_{io\|m}$	0.20	0.01	0.05	0.24	0.30
(a) Monolocation					
L_{iot}	7.4	1.0	2.1	5.1	29.7
$w_{iot}L_{iot}$	216.7	29.7	64.5	159.6	925.2
w_{iot}	32.8	20.3	26.6	38.5	35.5
$s_{io\|m}$	0.18	0.01	0.04	0.19	0.29
(b) Multilocation					
L_{iot}	26.6	1.3	4.1	15.1	120.3
$w_{iot}L_{iot}$	1,004.7	45.7	139.3	533.0	5,052.4
w_{iot}	39.0	23.6	30.7	43.7	257.7
$s_{io\|m}$	0.29	0.02	0.11	0.48	0.35

Notes: The top panel shows summary statistics for the whole sample. Panels (a) and (b) present respectively summary statistics of monolocation and multilocation firm-occupations. Number of observations for All Sample is 4,151,892. The Monolocation sample is 3,359,236; and the Multilocation sample is 792,656. L_{iot} is full time equivalent employment at the establishment-occupation *io*, $w_{iot}L_{iot}$ is the wage bill, w_{iot} is establishment-occupation wage or wage per FTE, $s_{io|m}$ is the employment share out of the local labor market. All the nominal variables are in thousands of constant 2015 euros.

The local labor market, denoted by m, is a combination of commuting zone n, 3-digit industry h and occupation o. We take the standard approach of defining local labor markets based on these administrative classifications.[18] The median local labor market is small and has only 2 establishments and 10 employees. This is a consequence of the handful of manufacturing firms that are present in the countryside demanding certain occupations. Blue collar workers working in the food industry in Lourdes, close to the Pyrenees, are one example of a local labor market. The median local labor market is concentrated with a Herfindahl-Hirschman Index (HHI henceforth) of 0.68.[19] High median local labor market concentrations do not imply that most of the workers are in highly concentrated environments but rather that there are few local labor markets with low concentration levels and high employment.

3 Empirical evidence

This section provides suggestive evidence of employer labor market power in France and presents the French institutional setting. We start by presenting evidence of a negative relation between employers' local labor market power, proxied by their employment share, and wages. We later explain the institutional framework of the French labor market and the importance of wage bargaining.

[18]We abstract from defining flexible local labor markets as in Nimczik (2018) for Austria, or how easy it is to change to similar occupations, as considered by Macaluso (2017) or by Schubert et al. (2020) using rich mobility data coming from resumes in the U.S.

[19]The HHI of local labor market m ranges from the inverse of the number of competitors ($1/N_m$), if all the establishments have the same shares, to 1. A local labor market can have a HHI of almost one if one establishment has virtually all the employment. The median HHI is very similar (0.69) if we consider wage bill shares $s^w_{io|m}$ instead of employment shares $s_{io|m}$.

3.1 Labor market power and wages

This section explores the relationship between employer labor market power and wages at the establishment level. The challenge is finding a source of exogenous variation for our proxy of local labor market power, the employment share $s_{io|m}$, that will allow to estimate the effect of employer market power on wages. In what follows, we focus on multilocation firm-occupations where the effects are estimated using residual variation across local labor markets within a firm-occupation-year.[20]

The baseline specification is:

$$\log(w_{io,t}) = \beta\, s_{io|m,t} + \psi_{\mathbf{J}(i),o,t} + \delta_{\mathbf{N}(i),t} + \epsilon_{io,t}\,, \tag{1}$$

where $\log(w_{io,t})$ is the log average wage at plant i of firm j and occupation o at local labor market m in year t, $s_{io|m,t}$ is the employment share of the plant out of the market m, $\psi_{\mathbf{J}(i),o,t}$ is a firm-occupation-year fixed effect, $\delta_{\mathbf{N}(i),t}$ is a commuting zone-year fixed effect and $\epsilon_{io,t}$ is an error term. Our parameter of interest is β.

The specification controls for sector labor demand differences with firm-occupation-year fixed effects $\psi_{\mathbf{J}(i),o,t}$. These include, for example, differences in the usage of capital for a given sector or a firm. Occupation labor demand shocks that are firm specific are also captured by the fixed effects $\psi_{\mathbf{J}(i),o,t}$. Lastly, the commuting zone times year fixed effects $\delta_{\mathbf{N}(i),t}$ control for permanent differences across locations and also for potential geographical spillovers of mass layoff shocks as stressed by Gathmann, Helm, and Schönberg (2017).

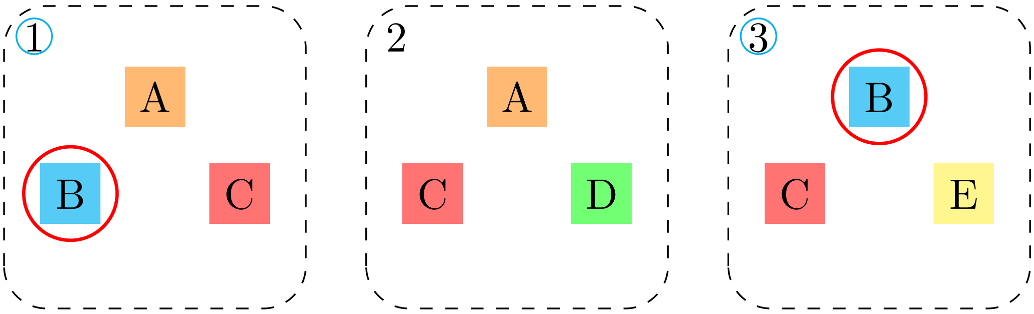
The establishment's employment share, $s_{io|m,t}$, is likely to be endogenous to the wages. On the one hand, everything else equal, higher wages attract more workers and therefore increase the employment share. On the other hand, if there is labor market power on the employer side, we expect two establishments with the same fundamentals to pay differently depending on their local labor market power. That is, everything else equal, we expect a plant with higher employment share to pay relatively less than the one in a more competitive local labor market. Given these endogeneity issues, we propose a novel identification strategy based on mass layoff shocks to competitors.

Our approach uses idiosyncratic shocks to a given firm and instruments the employment shares by using quasi-experimental variation coming from mass layoffs of competitors. We want to have an instrument that induces variation on an establishment's employment dominance in a local labor market that is unrelated to its idiosyncratic characteristics, which an exogenous shock to an establishment's competitor should satisfy. The instrument is built by the presence of a firm having a *national* mass layoff in the same local labor market as non affected establishments. We expect that a national level shock to a competitor is exogenous to the residual within firm-occupation variation across local labor markets that identifies the effect. The main specification is an instrumental variable regression where we compare establishment-occupations of firms that had exogenous increases in concentration due to the competitors' shock against establishment-occupations that were not exposed to the competitors' shock. Online Appendix I.1 discusses the intuition of the instrument within the context of our structural model in a simplified scenario with two establishments.

Figure 1 illustrates how the mass-layoff instrument is implemented. We show an economy with

[20]Recall that a multilocation occupation of a firm is an occupation that is present in several establishments across the geography.

Figure 1: Mass Layoff Instrument



Notes: This figure illustrates how we construct the instrument. Firm B suffers a *national* mass layof shock which reduces employment in all the local labor markets where it is present. This idiosyncratic shock changes the labor market power of non-affected establishments in markets 1 and 3.

three local labor markets and five firms from A to E. We abstract from different occupations for simplicity. The sample we use in the regression analysis excludes firms that only have establishments in a single local labor market. So in the example portrayed in Figure 1, we would exclude firms D and E from the sample.

In the example, firm B suffers an idiosyncratic shock that leads to a *national* mass layoff (employment decreases in all the local labor markets where it is present) which would change the labor market power of its competitors' establishments in those markets where firm B has an establishment. These are markets 1 and 3. More precisely, the presence of firm B's establishments in the different markets would indicate the "treatment" status of its competitors establishments. Thus, firm A's establishment in market 1 would have an exogenous increase in the local employment share but not firm A's establishment in market 2. The underlying identification assumption is that the national mass layoff shock to firm A is independent of its competitors establishments' locations.

As we use a firm fixed effect, our regression would compare the outcomes between firm A's establishments across markets. In other words, we use within-firm, across-establishments variation to identify the reduced-form effect. We restrict our sample to multi-location firms that did not suffer a mass layoff shock and have establishments in local labor markets affected by a mass layoff shock to a competitor (markets 1 and 3) and establishments in non-affected local labor markets (market 2). In the example of Figure 1, our sample would be the establishments of firms A and C.

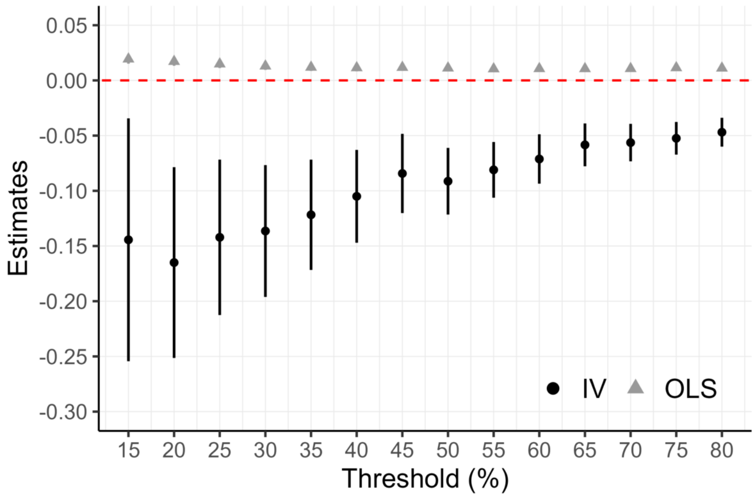
To construct our instrument in the data, we first need to identify the firms suffering a mass layoff.[21] We classify a firm-occupation as having a mass layoff if the establishment-occupation employment at t is less than a threshold κ% of the employment last year for all the firm establishments. Ideally, we would like to identify firms that went bankrupt ($\kappa = 0$). Unfortunately, we cannot externally identify if a firm disappears because it went bankrupt or changes firm identifiers keeping the number of competitors at the local market constant.

The choice of κ presents a trade-off as a lower threshold leads to considering stronger negative shocks and the generated instrument will more likely capture purely idiosyncratic firm shocks. But at the same time, a lower threshold reduces the number of events considered potentially leading to a higher variance of the estimates. This creates a bias-variance trade-off in the selection of the threshold. Lacking a clear candidate for κ, we try different cut-off values.[22]

[21] We give more details on the construction of the instrument in Online Appendix I.1.

[22] A standard value in the literature is κ =70% (e.g. Hellerstein, Kutzbach, and Neumark, 2019; Dodini et al., 2020). That is a 30%

Figure 2: Impact of Employment Share on Wages



Notes: This figure presents the point estimates and 95% confidence bands of the OLS and IV exercises on the y-axis. The x-axis presents different thresholds κ that define a mass layoff shock. The instrument is the presence of a firm with a mass layoff shock in the local labor market. We focus on non-affected competitors (not suffering a mass layoff shock). Equation (1) shows the specification.

We present the OLS and IV point estimates and their 95% confidence intervals in Figure 2.[23] As the employment share is endogenous, the OLS estimates are biased towards zero. On the contrary, the IV estimates are negative and statistically significant, regardless of the choice of the cutoff κ. Moreover, the figure shows clearly the trade-off in the selection of κ. The lower the threshold, the stronger the impact but the higher the variance of the estimated effect. We estimate a semi-elasticity of -0.14 with $\kappa = 15\%$ (i.e. an 85% employment loss). This estimate implies that one p.p. increase in the employment share causes a 0.16% decrease of the establishment wage. This translates into a wage loss of roughly 900 euros per year when passing from the first to the third quartile of employment shares.[24] For the more standard threshold of $\kappa = 70\%$ (a 30% employment reduction) the estimate is halved to -0.06. As we increase the threshold κ the estimated coefficient converges to the OLS estimate and the variance is reduced. In Online Appendix I.1.2 we perform several robustness checks by changing the instrument, the fixed effects, and the definition of local labor market. We also include the logarithm of establishment employment as an additional control to take into account changes along the labor demand curve. We find that our main result—that the wages are negatively related to employment shares—is robust to these different specifications.

The empirical evidence up to now focused on establishing the presence of employer labor market power of French manufacturing firms. We find that firms pay lower wages in local labor markets where they have relatively higher labor market power. We now explain the institutional setting and the importance of unions in France.

employment loss.

[23]We are restricting to firm-occupations classified as not having a mass layoff. The regression sample therefore changes depending on κ which is why the OLS estimates change slightly with κ.

[24]This computation is done taking the employment share differences between the percentile 75 and 25 from Table 1 for the median wage. The analogous computation with the average wage gives a wage reduction of roughly 1100 euros.

Table 2: Union Density and Collective Bargaining Coverage

Country	Union Density	Coverage	Country	Union Density	Coverage
Western Europe			**Southern Europe**		
Austria	27.7	98.0	Italy	36.4	80.0
France	9.0	98.5	Spain	16.8	80.2
Germany	17.7	57.8	**Americas**		
Netherlands	18.1	85.9	Canada	29.3	30.4
Switzerland	16.1	49.2	Chile	15.3	19.3
Northern Europe			United States	10.7	12.3
Finland	67.6	89.3	**Asia & Oceania**		
Ireland	26.3	33.5	Australia	15.1	59.9
Norway	49.7	67.0	Japan	17.5	16.9
United Kingdom	25.0	27.5	Korea	10.0	11.9

Notes: Year 2014. All the variables are in percents. *Union Density* is the unionization rate which is unionized workers relative to total employment. *Coverage* is the collective agreement coverage; the ratio of employees covered by collective agreements divided by all wage earners with the right to bargain. The data comes from the OECD and the sources are administrative data except for Australia, Ireland and the United States which are based on survey data. The regions are defined according to the United Nations M49 area codes.

3.2 Unions

The institutional framework of the French labor market is characterized by legal requirements that give unions an important role even in medium sized firms. The French labor market is known to be one where unions are relevant players, despite the fact that trade union affiliation in France is among the lowest of all the OECD countries.[25] According to administrative data, the unionization rate in France was 9% in 2014, which is slightly below the one in the U.S. (10.7%) and well below the ones in Germany (17.7%) or Norway (49.7%).[26]

Low affiliation rates do not translate into low collective bargaining coverage for the French case. Collective bargaining agreements extend almost automatically to all the workers, unionized or not. That is, if an agreement is reached in a particular sector, all the workers within the sector are covered. Table 2 presents the unionization and collective bargaining coverage rates for several countries. This institutional framework implies that coverage of collective agreements in 2014 was as high as 98.5% in France despite the low union affiliation rates.[27] This is in stark contrast to the U.S. collective bargaining agreements that only apply to union members and therefore coverage is very similar to the unionization rate.

Collective bargaining can happen at different levels. Firms and unions can negotiate at some aggregate level (e.g. industry, occupation, region) and also at economic units such as the group, firm or plant.[28] When wage bargaining happens at the firm level it affects all the workers. Most firms that explicitly bargain over wages do so at the firm level (rather than at the plant or occupation level). In 2010, 92% of mono-establishment firms that had a specific collective bargaining agreement negotiated it at the firm level. Of the multi-establishment firms with specific agreements, 45% negotiated at least partially at the establishment level (Naouas and Romans, 2014).[29]

Legal requirements regarding union representation depend on firm or plant size. The first re-

[25] Article in The Economist 'Why French unions are so strong'.
[26] Source: OECD data https://stats.oecd.org/Index.aspx?DataSetCode=TUD. Unionization rate is also denoted as union density.
[27] The data source of collective bargaining agreements is the OECD as for unionization rates.
[28] Several collective agreements can coexist at a given establishment.
[29] Source https://dares.travail-emploi.gouv.fr/dares-etudes-et-statistiques/etudes-et-syntheses/dares-analyses-dares-indicateurs-dares-resultats/article/la-negociation-salariale-d-entreprise-de-2004-a-2010, page 7.

quirements start when the establishment reaches 10 employees and there is an important tightening of duties when reaching the threshold of 50 employees.[30] As a consequence, firm level wage bargaining is common even at relatively small establishments. In fact, 52% (51%) of establishments with at least 20 employees bargained over wages in 2010 (in 2004) (See Table 1 of Naouas and Romans, 2014).[31]

Theoretically, workers organize into unions to extract rents from the firm through bargaining. Bargaining can happen at different levels in France and here we want to inform the modeling decisions in the next section by quantifying bargaining differences depending on industries or occupations. We build a proxy of rents at the firm level and then compare how the correlation of wages with rents is differentiated depending on the industries and occupations. In particular we compute rents at the firm level $y_{\mathbf{J}(i),t}$ by computing value added minus capital expenditures per worker. The reduced form model is the following:

$$\ln w_{io,t} = \gamma_k \ln y_{\mathbf{J}(i),t} + \epsilon_{io,t},$$

where γ_k is the elasticity of wages with respect to rents and k denotes either 2-digit sector b or occupation o, $y_{\mathbf{J}(i),t}$ is the proxy of rents at the firm level and $\epsilon_{io,t}$ is the error term.

Results in Online Appendix I.3 show that the elasticities at the sector level range from 0.14 for *Metallurgy* to 0.4 for *Food*. On the contrary, when running the same regressions per occupation the elasticities range from 0.27 for *Supervisor* to 0.38 for *Top management*. Given the higher dispersion of the elasticities at the sector level, we will assume differentiated bargaining powers depending on the sector later on in the model.

Having established the existence of employer labor market power and the importance of unions, the next section lays out a model in line with the stylized facts and the French labor market institutions.

4 Model

The economy consists of discrete sets of establishments $\mathcal{I} = \{1, ..., I\}$, locations $\mathcal{N} = \{1, ..., N\}$ and sectors $\mathcal{B} = \{1, ..., B\}$. Each establishment can have several occupations $o \in \mathcal{O} = \{1, ..., O\}$. Each establishment i is located in a specific location n and belongs to sub-industry h in a particular sector b. We define a local labor market m as the combination between location n, 3-digit industry h and occupation o, i.e. m will be combinations of $n \times h \times o$.

We denote the set of establishments that are in local labor market m as $\mathcal{I}_m$ with cardinality N_m. We define the set of all local labor markets as $\mathcal{M}$ and the set of all markets within sector b (within sub-industry h) as $\mathcal{M}_b$ ($\mathcal{M}_h$).[32] The distribution of establishments across local labor markets is determined exogenously. Every establishment can belong only to one location and one sub-sector but can have several occupations and therefore belong to different local labor markets. We define the set of local labor markets that have at least one establishment of sector b as $\mathcal{N}_b$.

The economy is populated by an exogenous measure L of workers who are homogeneous in abil-

[30]The Appendix of Caliendo et al. (2015) provides a comprehensive summary of size related legal requirements in France.
[31]The prevalence of wage bargaining was 44% for establishments with 11 employees or more.
[32]More formally this means $\mathcal{M} = \bigcup_{b\in\mathcal{B}} \mathcal{M}_b$ and $\mathcal{M}_b = \bigcup_{h\in b} \mathcal{M}_h$.

ity but heterogeneous in tastes for different workplaces. They decide their workplace (establishment-occupation) in two steps without any restriction on mobility. First, workers choose in which local labor market m they would like to be employed, and second, they choose in which establishment i of that local labor market they will work. Workers do not save so they do not own any capital.

Capital and output markets are competitive. Establishments are owned by absent entrepreneurs who rent the capital and collect the profits. We assume the economy is a small open economy and capital is specific for each sector. Thus, the sector specific rental rates of capital R_b are exogenous.

Firms and workers bargain over wages at the establishment-occupation io level. The equilibrium bargained wage is the solution to a reduced form Nash bargaining problem where establishments and unions are symmetric. Both have zero threat points and internalize how the marginal cost changes when moving along the labor supply curve. The assumption of null outside options for workers is in line with new evidence of insensitivity of wages to outside options such as the value of nonemployment (Jäger, Schoefer, Young, and Zweimüller, 2020).[33]

Having a discrete set of establishments per local labor market means that when bargaining, both parties internalize the effect of their wages on the labor supply of their most immediate competitors. This reflects the idea that competition for labor is mostly local. Geography in our model is only important to define local labor markets.

In the following we first set up the production side of the economy and workers' labor supply decisions. Second we present equilibrium wages in the oligopsonistic competition case (in the absence of bargaining) and finally we incorporate bargaining into the model.

Production

The final good Y is produced by a representative firm with an aggregate Cobb-Douglas production function using as inputs a composite good Y_b for each sector b:

$$Y = \prod_{b \in \mathcal{B}} Y_b^{\theta_b}, \tag{2}$$

where θ_b is the elasticity of the intermediate good produced by firms in sector b and $\sum_b \theta_b = 1$. Profit maximization implies that the representative firm spends a fixed proportion θ_b on the sector composite Y_b:

$$P_b Y_b = \theta_b P Y. \tag{3}$$

The final good price, which we choose as the numeraire, is equal to:

$$P = 1 = \prod_{b \in \mathcal{B}} \left(\frac{P_b}{\theta_b} \right)^{\theta_b}.$$

Firms produce in a perfectly competitive goods market. P_b is the price of the homogeneous good produced by every firm in sector b, Y_b is their production and P is the price of the final good. Y_b is

[33] We show in the Appendix that the same equilibrium wages arise with a different bargaining protocol where employer labor market power is incorporated through workers' outside options.

the aggregate of output of all the firms in that sector:

$$Y_b = \sum_{i \in \mathcal{I}_b} y_i, \tag{4}$$

where $\mathcal{I}_b$ is the set of establishments that belong to sector b. The establishment production function y_i is an aggregate of occupation productions. Establishment i produces using occupation o specific inputs, labor L_{io} and capital K_{io}, with a decreasing returns to scale technology. Output elasticity with respect to labor β_b and capital α_b are sector specific and establishment-occupations are heterogeneous in their total factor productivity. We assume that occupations are perfect substitutes and their output is aggregated linearly. That is, total establishment output y_i is the sum of occupation specific outputs y_{io}. Decreasing returns to scale in the occupation output y_{io} generate an incentive to produce using several occupations.

Establishment i's output, y_i, is defined as:

$$y_i = \sum_{o=1}^{O} y_{io} = \sum_{o=1}^{O} \widetilde{A}_{io} K_{io}^{\alpha_b} L_{io}^{\beta_b}. \tag{5}$$

The choice of this particular production function is motivated by tractability and empirical reasons. The linearity of the aggregation within establishments allows for the separability of different local labor markets which has an important computational advantage as it allows us to solve each labor market independently. The second reason is data motivated. With our specification, the absence of a particular occupation in an establishment can be rationalized by having null productivity in that occupation. An alternative specification, where labor is a Cobb-Douglas composite of occupations, is at odds with the pervasive prevalence of missing at least one occupation category. The median establishment lacks at least one occupation and those establishments would not be able to produce if labor is a Cobb-Douglas composite of occupations, unless we were to assume establishment-specific output elasticities.

Substituting the demand for capital, the establishment-occupation production is:

$$y_{io} = P_b^{\frac{\alpha_b}{1-\alpha_b}} A_{io} L_{io}^{\frac{\beta_b}{1-\alpha_b}}, \quad A_{io} \equiv \widetilde{A}_{io}^{\frac{1}{1-\alpha_b}} \left(\frac{\alpha_b}{R_b}\right)^{\frac{\alpha_b}{1-\alpha_b}}, \tag{6}$$

where A_{io} is a transformed productivity of io that incorporates elements coming from the demand of capital. Details on these derivations and the model under an alternative production function—where all occupations within an establishments are aggregated within a Cobb-Douglas specification—are in Online Appendix C.4. From now on we work with the production function after substituting out the optimal choice for capital.

Labor supply

We now present the worker preferences that give rise to upward sloping establishment-occupation specific labor supplies. A worker k derives utility by consuming the final good and by the product of two idiosyncratic utility shocks: one establishment-occupation specific preference shifter z_{kio} and another one common for all establishments in local labor market m, u_{km}. The utility of a worker k

working for establishment i at occupation o in local labor market m is:

$$\mathcal{U}_{kio} = c_k z_{kio} u_{km}. \tag{7}$$

Following Eaton and Kortum (2002) in the trade literature and Redding (2016) and Ahlfeldt et al. (2015) in urban economics literature we assume that the idiosyncratic utility shocks are drawn from two independent Fréchet distributions:

$$P(z) = e^{-T_{io}z^{-\varepsilon_b}}, \quad T_{io} > 0, \varepsilon_b > 1 \tag{8}$$

$$P(u) = e^{-u^{-\eta}}, \quad \eta > 1, \tag{9}$$

where the parameter T_{io} determines the average utility derived from working in establishment i and occupation o. In contrast, we normalize these parameters to one for the sub-market specific shock u. The shape parameters ε_b and η control the dispersion of the idiosyncratic utility. They are inversely related to the variance of the taste shocks. We name the parameters ε_b and η as the *local* and *across* labor market elasticities of substitution. If both elasticities have high values, workers have similar tastes for different local labor markets and establishment-occupations. This in turn implies that workers can substitute more easily different jobs and their labor supply is more elastic.

The labor supply elasticities in this framework are different from the Frisch elasticity studied by public economists. Our baseline model features a constant level of aggregate employment and workers do not decide the *amount* of hours to work but rather the *workplace* to which they want to supply their labor. The Frisch elasticity of labor supply is zero in our baseline environment but workers do not supply their labor inelastically to any establishment.

We assume that establishments cannot discriminate against workers based on their taste shocks. This implies that establishment i for occupation o pays the same wage w_{io} to all its employees, leaving the marginal worker indifferent between working in io or somewhere else. Small wage reductions induce the movement of the marginal worker but infra-marginal workers stay.[34]

The only source of worker income are wages, therefore the indirect utility of worker k is:

$$\mathcal{U}_{kio} = w_{io} z_{kio} u_{km}. \tag{10}$$

A worker chooses where to work in two steps: first, they choose their local labor market after observing local labor market shocks u_{km}. After picking a local labor market, the worker then observes the establishment idiosyncratic shocks and chooses the establishment that maximizes expected utility. Following the usual derivations as in Eaton and Kortum (2002), the probability of a worker choosing establishment i and occupation o is a product of two terms: the employment share of the establishment-occupation within the local labor market $s_{io|m}$ and the employment share of the local labor market itself s_m. The probability $\Pi_{io} = s_{io|m} \times s_m$ writes as:

$$\Pi_{io} = \frac{T_{io} w_{io}^{\varepsilon_b}}{\Phi_m} \times \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^{\eta}}{\Phi}, \tag{11}$$

where $\Phi_m \equiv \sum_{i'} T_{i'o} w_{i'o}^{\varepsilon_b}$ is a local labor market aggregate, and the economy-wide constant is

[34] One can view these taste shocks as mobility costs in a static model that could be present when changing jobs across the geography, industry and occupations.

$\Phi \equiv \sum_{b' \in \mathcal{B}} \Phi_{b'} \Gamma_{b'}^{\eta}$, where $\Phi_{b'} \equiv \sum_{m' \in \mathcal{M}_{b'}} \Phi_{m'}^{\eta/\varepsilon_{b'}}$. The Γ_b terms are sector-specific constants. In equilibrium, the first fraction is equal to $s_{io|m}$ and the second term in (11) is s_m.

Integrating over the continuous measure of workers L, the labor supply L_{io} for establishment and occupation o is:

$$L_{io}(w_{io}) = \frac{T_{io} w_{io}^{\varepsilon_b}}{\Phi_m} \times \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^{\eta}}{\Phi} L. \tag{12}$$

The inverse labor supply is upward sloping as long as the local and across labor market elasticities of substitution are finite. In the limit where both tend to infinity, workers are indifferent across workplaces and the inverse labor supply becomes flat.

4.1 Absence of bargaining

To ease the exposition of our baseline model, in this section we characterize equilibrium wages in the absence of bargaining. Given the labor supply curves with finite elasticities, establishments post wages taking into account the labor supply curves (12) they face. This monopsony power translates into a markdown between the wages and the marginal revenue products of labor. When the establishments solve their wage posting problem they act strategically. They look at probability Π_{io} and take into account the effect of wages on the establishment-occupation term $T_{io} w_{io}^{\varepsilon_b}$ and also on the local labor market aggregate Φ_m. However, they take as given economy-wide aggregates (Φ and L).[35] The finite set of establishments per local labor market generates strategic interactions among the competitors. The strategic interactions within a local labor market induces oligopsonistic competition that features a heterogeneous markdown.

The first order condition for the establishment-occupation wage io under oligopsonistic competition is:

$$w_{io}^{MP} = \frac{e_{io}(s_{io|m})}{e_{io}(s_{io|m}) + 1} \beta_b A_{io} L_{io}^{\frac{\beta_b}{1-\alpha_b} - 1} P_b^{\frac{1}{1-\alpha_b}}, \tag{13}$$

where $e_{io}(s_{io|m}) = \varepsilon_b (1 - s_{io|m}) + \eta\, s_{io|m}$ is the labor supply elasticity.[36] This expression is similar to Card et al. (2018) with the difference that we have variable perceived elasticities that arise from the strategic interaction between establishments. The fraction $\frac{e_{io}(s_{io|m})}{e_{io}(s_{io|m})+1}$ in equation (13) is the markdown and it is defined as:

$$\mu(s_{io|m}) \equiv \frac{\varepsilon_b (1 - s_{io|m}) + \eta\, s_{io|m}}{\varepsilon_b (1 - s_{io|m}) + \eta\, s_{io|m} + 1}. \tag{14}$$

In the absence of bargaining, the wedge between the marginal revenue product of labor and the

[35] Similar to Atkeson and Burstein (2008), this type of behavior could be rationalized either by assuming a myopic behavior of the establishment or by having a continuum of local labor markets.

[36] On the contrary, the labor supply elasticity in Berger et al. (2022) is related to payroll shares. This difference comes from the fact that agents in their model make an intensive labor supply decision (equation (B3) in their online appendix) while in ours they do not (which would be equivalent to labor supply being their equation on top of (B3) times employment). Under Bertrand competition, the labor supply elasticity in their model is: $\frac{\partial \log(L_{io})}{\partial \log(w_{io})} = \eta + (\theta - \eta) \frac{\partial \log(W_j)}{\partial \log(w_{ij})}$. The latter partial derivative in their framework is $(\frac{w_{ij}}{W_j})^{1+\eta}$ which is the payroll share of the establishments. Note that if one was abstracting from the intensive labor supply margin, that wage ratio would be equal to the employment share as it can be seen in the equation on top of (B3) in their online appendix. Similarly, taking their notation and writing our framework without amenities and $W_j \equiv (\sum_{i \in \mathcal{I}_m} w_{io}^{\eta})^{1/\eta}$, the latter partial derivative would be the employment share.

wages boils down to a markdown (14). We denote this object in short notation as μ_{io}.

As long as workers find different local labor markets to be less substitutable than establishments within a local labor market (i.e. as long as $\eta < \varepsilon_b$), the markdown (14) is a decreasing function of the employment share $s_{io|m}$. Once an establishment is big with respect to the nearby competitors, it internalizes that it is facing a more inelastic labor supply of workers willing to stay and applies a markdown further away from 1. In the limit where ε_b or η tend to infinity, establishments face an infinitely elastic labor supply and the labor market would be perfectly competitive with a markdown $\mu(s_{io|m}) = 1$.

Heterogeneous markdowns distort relative wages across establishment-occupations and therefore the labor allocations. Heterogeneous markdowns generate misallocation of resources and potentially reduce aggregate output even in the case where total employment is fixed, and the lower wages do not discourage workers from participating in the labor force. We formalize the source of misallocation in Section 4.4.[37]

Abstracting from capital,when the markdowns are constant and total labor supply fixed, labor market power has no effect on output and only affects the division of output into the labor and profit shares. This is no longer true if we were to allow an endogenous leisure or labor force participation decision. Counterfactually increasing wages would increase total labor supply L and therefore total output.[38]

4.2 Bargaining

We now introduce bargaining between employers and unions.[39] We assume that bargaining happens at the establishment-occupation level and involves only wages rather than indirect utilities because workers do not know each others' taste shocks. Given the perfect substitutability of occupations in the production function, bargaining at the establishment-occupation level is equivalent to bargaining at the establishment level, except that there are distinct wage agreements per occupation.

We assume that workers and establishments are symmetric in the bargaining protocol: first, both parties enter the bargaining with a null outside option and, second, internalize how they generate rents as they move along the labor supple curve. The former implies that if bargaining were to fail, workers could not earn any income and establishments could not produce. The zero outside option for the workers is in line with recent evidence of a lack of response of wages to changes in outside options such as unemployment benefits (Jäger et al., 2020). The second assumption, implies that unions bargain to extract part of the generated rents by internalizing how the marginal cost changes when introducing an additional worker. In section 6.1 we explain with more detail how unions redistribute the rent.

The bargained equilibrium wage is the solution to a reduced form Nash bargaining where the union's bargaining power is φ_b and that of the establishment is $1 - \varphi_b$. On the Appendix, we give

[37] Online Appendix D provides an illustration of the distributional and output consequences.

[38] The constant $\mu = \frac{\eta}{\eta+1}$ drives down the wages. If total labor supply were endogenous, workers' decision between consumption c and leisure l would be distorted. Denote by w the wage under monopsonistic competition and by $\tilde{w}$ the wage under a competitive labor market. Worker's maximization under endogenous labor supply leads the marginal rate of substitution to be equal to the wage rate. $w < \tilde{w}$ and therefore $-\frac{U_l}{U_c} = w < \tilde{w}$. Meaning that workers would supply less labor than in the perfectly competitive case.

[39] We use the terms workers and unions interchangeably.

more detail on the bargaining set up and discuss alternative bargaining arrangements that lead to the same negotiated equilibrium wages.

The equilibrium bargained wage is:

$$w_{io} = \underbrace{\left[(1 - \varphi_b)\, \mu_{io} + \varphi_b \, \frac{1-\alpha_b}{\beta_b} \right]}_{\text{Wedge } \lambda(\mu_{io}, \varphi_b)} \times \underbrace{\beta_b A_{io} L_{io}^{\frac{\beta_b}{1-\alpha_b}-1} P_b^{\frac{1}{1-\alpha_b}}}_{\text{MRPL}} . \tag{15}$$

The wedge between equilibrium wages and the marginal revenue product of labor, $\lambda(\mu_{io}, \varphi_b) \equiv (1-\varphi_b)\mu_{io} + \varphi_b \frac{1-\alpha_b}{\beta_b}$, is a combination of two parts. First, a markdown μ_{io} that would be present under oligopsonistic competition in the absence of bargaining, and second, a markup $\frac{1-\alpha_b}{\beta_b}$. When there are decreasing returns to scale, $\frac{1-\alpha_b}{\beta_b} > 1$, workers can extract some quasi-rents through the bargaining process. Bargained wages will be above or below the marginal revenue product depending on the union's bargaining power φ_b and the relative strength of markdowns and markups as the labor wedge is a convex combination between $\mu_{io} < 1$ and $\frac{1-\alpha_b}{\beta_b} \geq 1$.

If the within local labor market elasticity is greater than the across one, i.e. $\varepsilon_b > \eta$, then the perceived labor supply elasticity e_{io} is decreasing in the local labor market employment share. Hence, even if unions bargain over wages, one would observe a negative relationship between employment shares $s_{io|m}$ and wages w_{io} as long as they don't extract all the quasi-rents i.e. as long as $\varphi_b < 1$.

A desirable feature of the model is that it nests both the oligopsonistic competition and bargaining only settings as special cases. The former is equivalent to the limit case where the union's bargaining power φ_b is equal to zero. Equilibrium wages would be equal to a markdown times the marginal revenue product of labor $w^{MP} = \mu_{io} \times MRPL$. A traditional bargaining model on a perfect competition setting—where the outside option for workers is the competitive wage—would yield the same result as in our specification when $e_{io} \longrightarrow \infty$. In such case, if there are decreasing returns to scale, bargained wages incorporate a markup over the marginal product and become $w^B = (1 + \varphi_b \frac{1-\alpha_b-\beta_b}{\beta_b}) \times MRPL$.

4.3 Equilibrium

For given sector rental rates of capital $\{R_b\}_{b=1}^{B}$, the general equilibrium of this economy is a set of wages $\{w_{io}\}_{io=1}^{IO}$, output prices $\{P_b\}_{b=1}^{B}$, a measure of labor supplies to every establishment and occupation $\{L_{io}\}_{io=1}^{IO}$, capital $\{K_{io}\}_{io=1}^{IO}$ and output $\{y_{io}\}_{io=1}^{IO}$, sector $\{Y_b\}_{b=1}^{B}$ and economy-wide output Y, such that equations (2)-(12) and (15) are satisfied $\forall\ io \in \mathcal{I}_m$, $m \in \mathcal{M}$ and $b \in \mathcal{B}$.

4.4 Characterization of the equilibrium

Solving the model amounts to finding establishment wages, sector prices and allocations. The perfect substitutability assumption of the production function implies the block recursivity of the model where local labor market equilibria are independent from aggregates. To further simplify the aggregation and the solution of the model, we restrict the labor demand elasticity to be the same across sectors. That is, we assume the output elasticities to satisfy $\frac{\beta_b}{1-\alpha_b} = 1 - \delta$, where $\delta \in [0, 1]$. Block recursivity allows to split the solution of the model in two, while the parametric restriction on the output elasticities allow us to solve for the aggregate prices in closed form. First, we solve for local employment shares, which are independent of aggregate variables. We show that there is always a unique equilibrium of employment shares in each local labor market. Second, with the solution for the employment shares, we aggregate the local labor markets and show that the model can be rewritten at the sector b level. This last aggregate model is, in turn, enough to solve for sector prices in closed-form.

We first establish the fact that the model is block recursive where we can solve the equilibrium of each local labor market separately without taking into account aggregate variables.

Proposition 1 (Block Recursivity)**.** *Each local labor market equilibrium is independent of aggregate variables and is given by the following N_m systems:*

$$s_{io|m} = \frac{\left(T_{io}^{\frac{1}{\varepsilon_b}} \lambda_{io} A_{io}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}}{\sum_{j \in \mathcal{I}_m} \left(T_{jo}^{\frac{1}{\varepsilon_b}} \lambda_{jo} A_{jo}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}}, \tag{16}$$

$$\lambda_{io} = (1 - \varphi_b) \frac{e_{io}}{e_{io} + 1} + \varphi_b \frac{1}{1 - \delta}, \tag{17}$$

$$e_{io} = \varepsilon_b (1 - s_{io|m}) + \eta \, s_{io|m}. \tag{18}$$

The proof of Proposition 1 is in Online Appendix B. The employment shares are not affected by the aggregate variables for two reasons: (i) the relative wages within a local labor market do not change with aggregate variables; and (ii) employment shares are homogeneous of degree zero to labor-market changes in productivities, amenities, or wedges. Given the block recursivity of the local labor market equilibria, we can now establish the existence and uniqueness of the local labor market equilibrium:

Proposition 2 (Existence and Uniqueness of Local Equilibrium)**.** *If $\eta < \varepsilon_b \, \forall \, b \in \mathcal{B}$, then there exist unique vectors of employment shares $\{s_{io|m}\}_{io \in \mathcal{I}_m}$, wedges $\{\lambda_{io}\}_{io \in \mathcal{I}_m}$, and elasticities $\{e_{io}\}_{io \in \mathcal{I}_m}$ for every local labor market m that solve the system formed by equations* (16)-(18)*.*

Proposition 2 tells us that the characterization of the local labor market is uniquely pinned down, so we can use the employment shares and wedges as inputs when aggregating the model. For the proof see Online Appendix B. Our proof of existence and uniqueness can be applied easily to the local labor market equilibrium presented in Berger et al. (2022). As far as we know, this result has not been previously demonstrated in their paper, so we also derive it in the Online Appendix.

Before turning to the characterization of the general equilibrium of the model, the following proposition measures the aggregate misallocation effects of the heterogeneous labor wedges $\lambda(\mu_{io}, \varphi_b)$.

Proposition 3 (Aggregation at the Sector Level)**.** *Give each local labor market equilibrium $\{s_{io|m}\}_{io \in \mathcal{I}_m}$ we can characterize the output and labor supply at the sector level as functions of sectoral measures of productivities, labor wedges and misallocation, as well as the vector of sector prices $\{P_b\}_{b \in \mathcal{B}}$ as follows:*

Productivities:

$$A_m = \sum_{i \in \mathcal{I}_m} A_{io} \tilde{s}_{io|m}^{1-\delta}, \quad A_b = \sum_{m \in \mathcal{M}_b} A_m \tilde{s}_{m|b}^{1-\delta},$$

where $\tilde{s}_{io|m}$ and $\tilde{s}_{m|b}$ are the establishment and local labor market employment shares that would arise if all establishments had a constant labor wedge λ.

Labor wedges:

$$\lambda_m = \sum_{j \in \mathcal{I}_m} \lambda_{jo} \frac{A_{jo}}{A_m \Omega_m} s_{io|m}^{1-\delta},$$
$$\lambda_b = \sum_{m \in \mathcal{M}_b} \lambda_m \frac{A_m \Omega_m}{A_b \Omega_b} s_{m|b}^{1-\delta},$$

Misallocation:

$$\Omega_m = \sum_{i \in \mathcal{I}_m} \frac{A_{io}}{A_m} s_{io|m}^{1-\delta}, \quad \Omega_b = \sum_{m \in \mathcal{M}_b} \Omega_m \frac{A_m}{A_b} s_{m|b}^{1-\delta}.$$

Let $\mathbf{s}_b \equiv \{s_{io|m}\}_{io \in \mathcal{I}_b}$ be the vector containing all the employment shares of all the establishment-occupations in sector b. Then, sector level measures A_b, λ_b and Ω_b and the vector of sector prices $\{P_b\}_{b \in \mathcal{B}}$ are enough to characterize employment and output at the sector level:

$$L_b = \frac{\Phi_b (P_b, \mathbf{s}_b) \Gamma_b^{\eta}}{\sum_{b' \in \mathcal{B}} \Phi_{b'} (P_{b'}, \mathbf{s}_{b'}) \Gamma_{b'}^{\eta}} L$$
$$Y_b = P_b^{\frac{\alpha_b}{1-\alpha_b}} \Omega_b A_b L_b^{1-\delta}$$

Online Appendix B contains the aggregation of the model to the sector level and the characterization of sector employments as functions of prices and the vector of employment shares $\mathbf{s}_b$.

We now turn to the second step of the model solution. The block recursive nature of the local labor market equilibria allow to characterize the employment shares within the sector without the knowledge of prices $\{P_b\}_{b \in \mathcal{B}}$. That was the key part of the first step. In the second step we take as given these employment shares and solve for the sector prices. Let $\mathbf{s} = \{\mathbf{s}_b\}_{b \in \mathcal{B}}$ be the vector of all employment shares obtained in the first step. Also, let $\mathbf{P} = \{P_b\}_{b \in \mathcal{B}}$. Then, as shown in Proposition 3, the sector labor supply L_b will depend on both $\mathbf{s}$ and $\mathbf{P}$, and the sector level output can be written as:

$$Y_b = P_b^{\frac{\alpha_b}{1-\alpha_b}} \Omega_b A_b L_b(\mathbf{P}, \mathbf{s})^{1-\delta}. \tag{19}$$

Solving the model now amounts to finding the sector prices that clear the markets for intermediate goods. Using the final good production function (2), the intermediate good demand (3), and sector output (19) we obtain:

$$P_b^{\frac{1}{1-\alpha_b}} A_b \Omega_b L_b(\mathbf{P}, \mathbf{s})^{1-\delta} = \theta_b \prod_{b' \in \mathcal{B}} P_{b'}^{\frac{\alpha_{b'}}{1-\alpha_{b'}}} A_{b'} \Omega_b L_{b'}(\mathbf{P}, \mathbf{s})^{1-\delta}. \tag{20}$$

Online Appendix B explains in detail how to get to this expression. Collecting all these expressions for the different sectors forms a system of B equations with B unknowns. Given the employment shares obtained in the first step of the solution, the following proposition establishes the existence and uniqueness of the general equilibrium.

Proposition 4 (Existence and Uniqueness of General Equilibrium)**.** *Given a vector of employment shares* $\mathbf{s}$*, and* $\frac{\beta_b}{1-\alpha_b} = 1 - \delta \quad \forall\, b$*, then there exists a unique vector of prices* $\mathbf{P}$ *such that solves the system formed by* (20) *and it has a closed-form expression.*

On top of showing existence and uniqueness, Proposition 4 shows that there is an analytical solution for the sector prices. For the proof and the closed-form solution for the prices, see Online Appendix B. Taking together Propositions 2 and 4 we can conclude that there exists a unique solution to the model for any set of valid parameters and vectors of productivities and amenities.

5 Identification and estimation

We follow a sequential identification and estimation strategy of the parameters consisting of three steps. First, we identify the parameters that are constant across markets: the inverse elasticity of labor demand δ and the across-market elasticity of substitution η. To do so, we adapt the identification-through-heteroskedasticity proposed by Rigobon (2003). Second, we identify the local elasticities of substitution $\{\varepsilon_b\}_{b=1}^{B}$ by instrumenting for wages in the labor supply equation. In the third step, we calibrate the remaining parameters—the output elasticities $\{\alpha_b\}_{b=1}^{B}$, union bargaining powers $\{\varphi_b\}_{b=1}^{B}$, and final good elasticities $\{\theta_b\}_{b=1}^{B}$—to match their respective industry-specific capital, labor, and expenditure shares. After showing how to estimate the parameters, we explain how to use the observed data on employment and wages to identify the amenities and revenue productivities. Finally we present the estimation results and the estimation fit.

5.1 Common parameters η and δ

We identify the common parameters η and δ by focusing on establishment-occupations that are the sole employer in their local labor markets (i.e., $s_{io|m} = 1$), which we refer to as full monopsonists. These establishments only compete for workers across local labor markets, making the across-market elasticity of substitution η the only relevant parameter for their labor supply.

The labor demand of full monopsonists can be expressed in logs as:

$$\ln w_{io} = \mathcal{C}_b^D - \delta \ln L_{io} + \ln A_{io}, \tag{21}$$

where $\mathcal{C}_b^D$ is an industry-specific demand constant. Their labor supply in logs is:

$$\ln L_{io} = \mathcal{C}_b^S + \eta \ln w_{io} + \ln \widetilde{T}_{io}, \tag{22}$$

where $\widetilde{T}_{io} = T_{io}^{\eta/\varepsilon_b}$ and $\mathcal{C}_b^S$ is an industry-specific supply constant.[40] These equations form a standard price-quantity linear system, which suffers from a textbook case of simultaneity bias.

The standard approach to obtain consistent estimates for, say, the inverse demand elasticity δ is to find an instrument that shifts the supply curve. However, finding such instruments can be context-specific and not portable to other scenarios. Thus, we propose an alternative method to obtain consistent estimates without relying on specific demand and supply shifters.

We identify the across-market elasticity of substitution η and the inverse elasticity of labor demand δ using the identification through heteroskedasticity method proposed by Rigobon (2003). This approach imposes restrictions on the covariance matrix of the structural shocks—the productivities and amenities—across different subsets of the data.[41]

To gain intuition on the method, we follow Rigobon's example of the simplest demand and supply system, where the shocks are independent. Split the sample in two and assume that the supply shocks have a larger variance in the second subsample than in the first subsample, while the demand shocks have a constant variance. As the variance of the supply shocks increases, the cloud of price and quantity realizations spreads across the demand curve. This can be visualized as an ellipse that tilts towards the demand curve. When the variance of the supply shocks approaches infinity, the ellipse converges to the demand curve, and the slope of the demand can be estimated using OLS.

Rigobon's method extends this idea when the form of heteroskedasticity is unknown, showing that the relative change in variances across subsamples identifies the system. In the example above, we get three moments per subsample from the covariance matrix of prices and quantities. With two subsamples, we get six moments to identify the six unknowns of the system: the slopes of the demand and supply curve, and the variances of the demand and supply shocks in each subsample.

Let us return to the labor demand and supply equations for the full monopsonists as described by (21) and (22). After subtracting the sector b average and rearranging we get:

$$\begin{pmatrix} \overline{\ln(L_{io})} \\ \\ \overline{\ln(w_{io})} \end{pmatrix} = \frac{1}{1+\eta\delta} \begin{pmatrix} 1 & -\eta \\ \\ \delta & 1 \end{pmatrix} \begin{pmatrix} \ln(\widetilde{T}_{io}) \\ \\ \ln(A_{io}) \end{pmatrix},$$

where $\overline{\ln(L_{iot})}$ and $\overline{\ln(w_{iot})}$ are the demeaned logarithms of employment and wages respectively.

To apply Rigobon's method, we split the data using the four different occupations, resulting in twelve moments from the covariance matrix of employment and wages per occupation. However, this also means that the system has fourteen unknowns: η, δ, and twelve unknowns from the four covariance matrices of the structural shocks. Therefore, we need to impose at least two restrictions.

We impose the necessary restrictions by first grouping the four occupations into two categories: white-collar (top management and clerical) and blue-collar (supervisor and operational). Our identification assumption is that the covariance between productivities and amenities is constant across occupations within each category. This assumption reflects the idea that amenities such as working

[40]More precisely, $\mathcal{C}_b^D = \ln\left(\left[(1-\varphi_b)\frac{\eta}{\eta+1} + \varphi_b\frac{1}{1-\delta}\right]\beta_b P_b^{\frac{1}{1-\alpha_b}}\right)$, where the markdown is equal to $\mu(s=1) = \frac{\eta}{\eta+1}$, and $\mathcal{C}_b^S = \ln(L/\Phi) + \eta\ln(\Gamma_b)$.

[41]Rigobon and Sack (2004) and Nakamura and Steinsson (2018) also use heteroskedasticity-based estimates to quantify the impact of monetary policy on asset prices and, on real interest rates and inflation respectively.

hours, repetitiveness of the tasks or more general working environments are similarly related to productivity within our two categories. With these restrictions, we end up with a system with twelve unknowns, allowing us to identify δ and η. See Online Appendix E for details.

5.2 Local elasticities of substitution ε_b

We identify local elasticities of substitution ε_b using variation within a local labor market and an instrumental variables approach. The establishment-occupation labor supply (12) in logs is:

$$\ln(L_{io}) = \varepsilon_b \ln(w_{io}) + f_m + \ln(T_{io}), \tag{23}$$

where f_m is a local labor market fixed-effect which absorbs all the endogenous intercepts within each local labor market.

We instrument for wages using a proxy $\widehat{Z}_j$ of firm revenue productivity:

$$\widehat{Z}_j = \frac{P_b Y_j}{\sum_{\mathbf{J}(i)} \sum_o L_{io}^{1-\delta}},$$

where $P_b Y_j$ is value added at the firm j and $\mathbf{J}(i)$ denotes the set of establishments belonging to firm j. We use the estimate for δ from our first estimation step to build the instrument.[42]

In the first estimation step, we allow for the possibility that the structural shocks T_{iot} and A_{iot} are correlated *across* local labor markets. The validity of our instrument in this step is not compromised by the possible across local labor market correlation of amenities and productivities as they may remain uncorrelated *within* each local labor market. The local labor market fixed effect f_m can account for any such cross-market correlation. Nonetheless, we use a lagged instrument instead of a contemporaneous one to minimize potential endogeneity concerns.

It is important to note that our identification strategy so far does not rely on any assumptions about the wage-setting process. This makes our approach easy to adapt and use in different contexts, for example, in settings with no bargaining.

Elasticity of substitution and labor supply elasticity with strategic interactions. Our method avoids the identification issues raised by Berger et al. (2022) (BHM) for identifying supply or demand elasticities under strategic interactions. Paraphrasing BHM, the labor supply elasticity asks the following question: *how much would employment change within a firm after increasing its wage by one percent and holding the other firms' response constant?* Thus, the supply elasticity is a partial equilibrium object: $\left.\frac{d\ln L_{io}}{d\ln w_{io}}\right|_{w_{-io}}$.

BHM argue that even when there is a well-identified idiosyncratic demand shock and no labor supply shifters, we cannot identify the firm's labor supply elasticity. This is because the strategic interactions of other market participants will change the labor supply curve that the firm faces after the shock has occurred. This change in the equilibrium allocation violates the stable unit treatment value assumption (SUTVA), meaning that when we use within-firm across-equilibrium variation in a reduce-form exercise, we are measuring $\frac{d\ln L_{io}}{d\ln w_{io}}$ rather than $\left.\frac{d\ln L_{io}}{d\ln w_{io}}\right|_{w_{-io}}$.

[42]Ideally, we would like to have the average of the revenue productivities, $\frac{1}{N_{\mathbf{J}}}\sum_{\mathbf{J}(i)}\sum_o \frac{P_b Y_{io}}{L_{io}^{1-\delta}}$, but as we only observe value added at the firm level we use the ratio estimator instead $\frac{\sum_{\mathbf{J}(i)}\sum_o P_b Y_{io}}{\sum_{\mathbf{J}(i)}\sum_o L_{io}^{1-\delta}} = \frac{P_b Y_j}{\sum_{\mathbf{J}(i)}\sum_o L_{io}^{1-\delta}}$.

More precisely, consider the following decomposition of the reduced-form estimate:[43]

$$\frac{d\ln L_{io}}{d\ln w_{io}} = \frac{d\ln\left(L_{io}/L_{jo}\right)}{d\ln\left(w_{io}/w_{jo}\right)}\left(1-\frac{d\ln w_{jo}}{d\ln w_{io}}\right)+\frac{d\ln L_{jo}}{d\ln w_{io}}, \tag{24}$$

where L_{jo} and w_{jo} are the employment and wages for any other establishment jo within the local labor market of establishment io. In our setup, the elasticity of substitution $\frac{d\ln(L_{io}/L_{jo})}{d\ln(w_{io}/w_{jo})}$ is constant within a sector and equal to ε_b. In contrast to the reduced form response, the structural labor supply elasticity is equal to

$$\left.\frac{d\ln L_{io}}{d\ln w_{io}}\right|_{w_{-io}} = \varepsilon_b + \underbrace{\left.\frac{d\ln L_{jo}}{d\ln w_{io}}\right|_{w_{-io}}}_{\text{Cross-elasticity}},$$

where the cross-elasticity is equal to $-\varepsilon_b s_{io} + \eta s_{io}$ given our Bertrand competition environment. Thus, we get the expression for the labor supply elasticity, $\varepsilon_b(1-s_{io})+\eta s_{io}$.

The relation between the reduced form estimate and the labor supply elasticity is:

$$\underbrace{\frac{d\ln L_{io}}{d\ln w_{io}}}_{\text{Reduced-form}} = \underbrace{\left.\frac{d\ln L_{io}}{d\ln w_{io}}\right|_{w_{-io}}}_{\text{Supply elasticity}} + \underbrace{\left(\frac{d\ln L_{jo}}{d\ln w_{io}} - \left.\frac{d\ln L_{jo}}{d\ln w_{io}}\right|_{w_{-io}}\right) - \varepsilon_b\frac{d\ln w_{jo}}{d\ln w_{io}}}_{\text{Bias}}.$$

The reduced-form estimate is equal to the labor supply elasticity in two cases. First, when the establishment is atomistic i.e. $s_{io}\to 0$. In that case, the other firms in the market do not respond, so $\frac{d\ln w_{jo}}{d\ln w_{io}}=0$, and $\frac{d\ln L_{jo}}{d\ln w_{io}} = \left.\frac{d\ln L_{jo}}{d\ln w_{io}}\right|_{w_{-io}}$. The second case is when the local and across-market elasticity of substitution are the same. In such case, the employment loss of the competitors is completely offset by the increase in employment to the local labor market. Then, the cross-elasticity is zero, and $\frac{d\ln L_{jo}}{d\ln w_{io}} - \varepsilon_b\frac{d\ln w_{jo}}{d\ln w_{io}} = 0$. In both cases, the labor supply elasticity is equal to the local elasticity of substitution.[44]

BHM suggests using the relation between the reduced-form estimate and the structural elasticity to indirectly infer the structural parameters ε_b and η. However, this requires to assume a wage-setting process which will pin down the form of the structural elasticity. In contrast, our method identifies both parameters without making any assumptions about the wage-setting process.

Consider Figure 3 panel A to illustrate the argument. Assume there is no bargaining. The structural labor supply elasticity measures the employment response to a wage increase along the same labor supply curve LS_i. But if the firm is not atomistic, the wage increase after a positive idiosyncratic productivity shock will affect the other firms in the market, leading to a shift in establishment i's labor supply curve. Under Bertrand competition, wages are strategic complements, resulting in an upward shift in the labor supply, and the reduced-form elasticity will be smaller than the structural elasticity. Under Cournot competition, the employment levels are strategic substitutes, resulting in a downward shift in the labor supply, and the reduced-form elasticity will be greater than the structural elasticity. Therefore, different assumptions about the type of competition

[43]Derivation: $\frac{d\ln L_{io}}{d\ln w_{io}} = \frac{d\ln L_{io}}{d\ln(w_{io}/w_{jo})}\frac{d\ln(w_{io}/w_{jo})}{d\ln w_{io}} = \frac{d(\ln L_{io}-\ln L_{jo}+\ln L_{jo})}{d\ln(w_{io}/w_{jo})}\frac{d(\ln w_{io}-\ln w_{jo})}{d\ln w_{io}} = \frac{d\ln(L_{io}/L_{jo})}{d\ln(w_{io}/w_{jo})}\left(1-\frac{d\ln w_{jo}}{d\ln w_{io}}\right)+\frac{d\ln L_{jo}}{d\ln w_{io}}$.

[44]There is a third trivial case where the reduced-form estimate is equal to the labor supply elasticity. This is when the firm is the only one in the market. Then the supply elasticity is equal to the across-market elasticity of substitution η.

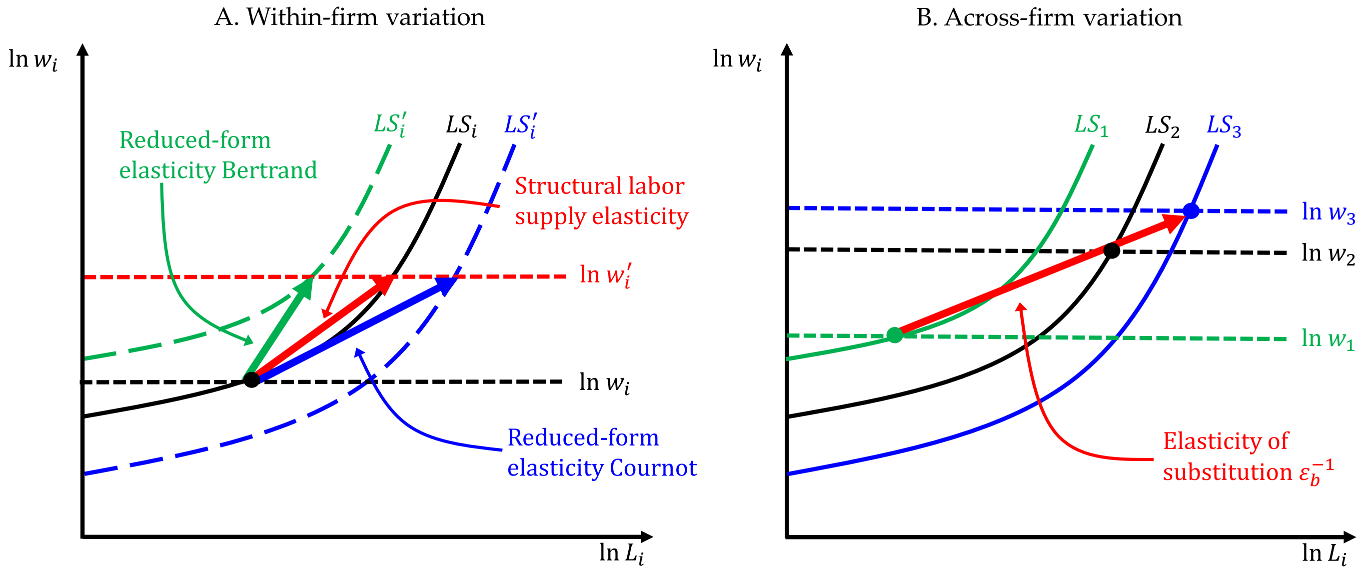


Figure 3: Within-firm, across-equilibria variation vs Across-firm, within-equilibrium variation

Notes: **Panel A** shows how the relationship between reduced-form and structural labor supply elasticities following an idiosyncratic shock depend on the nature of the market competition. **Panel B** illustrates that variation across employment and wages within a local labor market identify the elasticity of substitution ε_b.

can lead to different estimates for ε_b and η.[45]

Now we explain why a regression of log employment on log wages within a local labor market *conditional* on an equilibrium allocation identifies ε_b. To do so, consider Figure 3 panel B. We have three different establishments that only differ in their productivities. As they are not atomistic, the labor supply intercepts—which are a function of the competitors wages—are different.[46] For any two establishments i, j, the slope of the straight line connecting two points in the log wage and employment plane is $\frac{\ln w_i - \ln w_j}{\ln L_i - \ln L_j} = \varepsilon_b^{-1}$. Since the equilibrium allocation remains the same, SUTVA is not violated in this setting. This slope is constant for any pair of points, so the slope estimate of regressing log employment on log wages must be equal to ε_b.[47] This regression therefore does not estimate the labor supply elasticity but rather the local elasticity of substitution.

In conclusion, the source of BHM's identification problem comes from using variation from different equilibrium allocations and not controlling for the equilibrium changes. However, this problem is not present when using within-equilibrium variation. In Online Appendix E.2 we do a simulation exercise to show that even in a set-up like ours, with bargaining and supply shifters, we can recover the elasticity of substitution ε_b using our instrumental variable approach. [48]

[45]The labor supply elasticity in the Cournot competition case is given by $\left(\frac{1}{\varepsilon_b}(1-s_i)+\frac{1}{\eta}s_i\right)^{-1}$.

[46]The log inverse labor supply is equal to $\log w_{io} = \frac{1}{\varepsilon}\log\left(\frac{L_{io}}{L_m - L_{io}}\right) + \frac{1}{\varepsilon}\log\left(\sum_{j\neq i} w_{jo}\right)$.

[47]More explicitly, consider a regression model of the form $\ln L_i = b_0 + b_1 \ln w_i$, where we assume there are no supply side shifters so we do not include an error term. We can first demean both $\ln L_i$ and $\ln w_i$ and then regress those demeaned variables without a constant term to get the estimate of b_1. Thus, the estimate for b_1 is equal to $\frac{d\left[\ln(L_i)-\frac{1}{N}\sum_j \ln(L_j)\right]}{d\left[\ln(w_i)-\frac{1}{N}\sum_j \ln(w_j)\right]} = \frac{d\ln\left(L_i/\left(\prod_j L_j\right)^{1/N}\right)}{d\ln\left(w_i/\left(\prod_j w_j\right)^{1/N}\right)} = \varepsilon_b$.

[48]The argument extends even when using observations from different equilibria, provided we control for the equilibrium changes. We can use the differentiated wage and employment responses across time to labor demand shocks within a labor market. In such setting, one could run the regression in time differences and condition on a labor market fixed effect that controls for equilibrium changes. This variation identifies the local elasticity of substitution. To see this, let Δx be the change across time of variable x. Then, without supply shifters, $\frac{\Delta \ln w_i - \Delta \ln w_j}{\Delta \ln L_i - \Delta \ln L_j} = \varepsilon_b^{-1}$. This is useful because, in a setting with labor supply shifters, we would need instruments to estimate the elasticity of substitution. We instrument by previously identified revenue productivities, but in other applications it can be hard to find cross-sectional instruments that are time specific. Instead, the identification strategy would also work with instruments that generate differentiated labor demand responses within a local labor market. One such instrument is using changes in tax codes.

5.3 Bargaining power φ_b and output elasticities α_b, θ_b

We follow Barkai (2020) to construct the sector rental rates per year $\{R_{bt}\}_{b=1}^{B}$. Using these rates, we can get capital shares of output from the data. From the first order condition for capital, the industry b capital share is:

$$\frac{R_{bt}K_{bt}}{P_{bt}Y_{bt}} = \alpha_b.$$

We estimate α_b such that $\mathbb{E}_t\left[\frac{R_{bt}K_{bt}}{P_{bt}Y_{bt}}\middle| b\right] = \alpha_b$. We use the restriction of constant inverse labor demand elasticity $\frac{\beta_b}{1-\alpha_b} = 1-\delta$, to back out the output elasticities with respect to labor.

We pin down the union bargaining powers with the sector labor shares. Given data on wages and employment, the sector labor share is equal to:

$$LS_b(\varphi_b) = \frac{\beta_b \sum_{io\in\mathcal{I}_b} w_{io}L_{io}}{\sum_{io\in\mathcal{I}_b} w_{io}L_{io}/\lambda(\mu_{io},\varphi_b)},$$

where φ_b is the only parameter left to estimate. We set φ_b such that we match the average sector labor shares across years in the data with the model counterpart.

5.4 Amenities and Revenue Productivities

Amenities and revenue productivities are identified to match wages and labor allocations in equilibrium. We recover establishment-occupation revenue productivities (TFPR) using the wage first order conditions. We observe employment and nominal wages at the establishment-occupation level from the data. Equation (15) in nominal terms is:

$$P_t w_{iot} = \beta_b \lambda(\mu_{iot},\varphi_b)\; P_t P_{bt}^{\frac{1}{1-\alpha_b}} A_{iot} L_{iot}^{-\delta}, \tag{25}$$

where $P_t w_{iot}$ and L_{iot} are observed and $\beta_b\lambda(\mu_{iot},\varphi_b)$ depends on the estimated parameters and observed employment shares. Equation (25) makes clear that given the observed nominal wages and employment, one can only back out transformed TFPRs, $Z_{iot} = P_t P_{bt}^{\frac{1}{1-\alpha_b}} A_{iot}$, which are a function of the establishment-occupation physical productivity A_{iot} and prices $P_t P_{bt}^{\frac{1}{1-\alpha_b}}$.[49] Online Appendix E.4 contains details on how we back out amenities T_{iot} to ensure that we match employment. The next section presents the main estimation results and how well our model fits the data.

5.5 Estimation results

Table 3 shows the estimation results of the main parameters. The most important parameters of the estimation are the elasticities of substitution and the union bargaining powers.

The estimated across local labor market elasticity is $\widehat{\eta} = 0.42$ and the sector specific local labor market labor supply elasticities $\widehat{\varepsilon}_b$ range from 1.22 to 4.05. The across local labor market elasticity being lower than the within ones ($\widehat{\eta} < \widehat{\varepsilon}_b \quad \forall b$), workers are more elastic within than across local

[49] Normally in the literature, revenue productivities are defined as $P_t P_{bt} A_{iot}$. Instead, we define the revenue total factor productivities $P_t P_{bt}^{\frac{1}{1-\alpha_b}} A_{iot}$. Given that one cannot observe sector prices P_{bt}, backing out productivities A_{iot} from the data would require carrying out some normalizations to get rid of sector prices and be able to compute counterfactuals.

Table 3: Main Estimates

Param.	Name	Estimate	Identification
η	Across labor market elasticity	0.42	Heteroskedasticity
δ	1 - Returns to scale	0.04	Heteroskedasticity
$\{\varepsilon_b\}$	Within labor market elasticity	1.22 - 4.05	Labor supply
$\{\beta_b\}$	Output elasticity labor	0.57 - 0.85	Capital share and δ
$\{\varphi_b\}$	Union bargaining	0.06 - 0.73	sector LS

labor markets. This implies that the structural labor wedge $\lambda(\mu_{io}, \varphi_b)$ of our calibrated model is decreasing in employment shares $s_{io|m}$, in line with the empirical evidence from Section 3.

Our across local labor market labor supply elasticity is the same as a recent estimate by Berger et al. (2022) for the U.S.[50] On the contrary, all of our sector specific within local labor market elasticities lie below their estimate of 10.85. This might be a consequence of the lower job-to-job transition rates that characterize the French labor market.[51]

According to our estimates, (i) the employment weighted average bargaining power of French manufacturing is 0.37[52]; and (ii) there is important heterogeneity of bargaining power across industries ranging from 0.06 for *Chemical* to 0.73 for *Telecommunications*.

Our estimates for manufacturing bargaining power in France are consistent with previous studies. In particular, Cahuc, Postel-Vinay, and Robin (2006) estimate a bargaining power of 0.35 for top management workers, which is similar to our estimate, in a framework with search frictions and on-the-job search. Additionally, recent estimates for different manufacturing industries in France by Mengano (2022) are also in line with the middle range of our estimates, although his estimate is lower and less variable across industries (0.24).[53] Finally, we find our bargaining power estimates to be reasonable as there is a positive correlation of 0.33 between establishment size and union bargaining power, in line with the more restrictive legal duties regarding union representation for larger establishments in the French law.

The estimate of the inverse labor demand elasticity, δ, is $\widehat{\delta} = 0.04$. This parameter is also related to the average returns to scale of the production function which are about 0.97. The combination of δ and the estimated capital elasticities per sector $\{\alpha_b\}_{b\in\mathcal{B}}$ allow us to recover the values for the output elasticities with respect to labor, $\{\beta_b\}_{b\in\mathcal{B}}$, as $\beta_b = (1-\alpha_b)(1-\delta)$. These elasticities go from 0.56 for *Transport* to the 0.85 for *Shoe and Leather Production*.

5.6 Estimation fit

We validate the model by replicating the empirical evidence of Section 3 linking micro-level concentration to wages and sector concentration to the sector labor shares. We then compare the model's predicted reduced-form relationship between labor shares and labor market concentration at the industry level. In the Online Appendix, we also show that the model fits well non-targeted labor shares at the sub-industry level and the evolution of total value added.[54]

[50] See Table 3 in Berger et al. (2022).
[51] See Jolivet, Postel-Vinay, and Robin (2006) for a comparison of French mobility against the U.S.
[52] The simple average of sector bargaining powers is 0.41.
[53] See Tables A.2. and A.3. in the Appendix of his paper and Table E1 in our Online Appendix.
[54] See Figure E2.

Micro evidence

In the empirical evidence from Section 3 we measure the effects of concentration on wages by using shocks to local competitors. These shocks capture an exogenous change in the relative position within the local labor market of an establishment. Thus, our aim is to induce such exogenous changes to the local labor market of establishments and test if the quantitative response in the simulated model is similar to the reduced form regression. The firm and commuting zone fixed effects in the regression absorb the general equilibrium effects of the shocks. Therefore, we exploit the model variation in wages without taking into account changes in sector prices or employment levels.

Using wage and employment data, we can identify amenities T_{io} and revenue productivities (TFPRs) $Z_{io} = PP_b^{\frac{1}{1-\alpha_b}} A_{io}$. The TFPRs are multiplied by the same sector-wide constant, $P_b^{\frac{1}{1-\alpha_b}}$. As implied by Proposition 1, the local labor market equilibrium is invariant to any rescaling of the productivities. This means we can completely characterize the employment shares $s_{io|m}$ in the local labor market equilibrium using only the identified TFPRs and amenities. Using the estimated amenities and TFPRs for 2007, we simulate shocks in establishment-occupations' productivities and solve again for the equilibrium employment shares $s^S_{io|m}$ within each local labor market. With this employment shares, we compute wages that ignore any local market, sector, or economy-wide constants, meaning, the wages we use in this simulation are only a function of TFPRs, amenities, and wedges, which are a function of local market employment shares. More precisely, we use $w^S_{io} = \left(T_{io}^{\frac{1}{\varepsilon_b}} \lambda_{io} Z^S_{io} \right)^{\frac{1}{1+\varepsilon_b \delta}}$, where Z^S_{io} is the simulated TFPR. With these simulated data, we explore the link of employment shares to log wages according to the following linear model:

$$\log(w^S_{io}) = f_b + \beta\, s^S_{io|m} + u_{io},$$

where $\log(w^S_{io})$ is the logarithm of simulated wages, f_b is a sector fixed effect, $s^S_{io|m}$ is the equilibrium employment share of establishment-occupation io in the local labor market m, and u_{io} is an error term. We include the sector fixed effect f_b to capture any remaining sector price differences in the revenue productivities.

To replicate the exogenous change in establishment's employment share $s^S_{io|m}$, we use as an instrument the weighted average of the productivity changes of each establishment-occupation's competitors, where the weights are the employment shares in the baseline scenario. More clearly, the instrument for $s^S_{io|m}$, is:

$$\sum_{jo \in \{m \setminus io\}} \frac{Z^S_{jo}}{Z_{jo}} \frac{L_{jo}}{\sum_{ko \in \{m \setminus io\}} L_{ko}},$$

where L_{jo} is the observed employment for establishment-occupation jo in the baseline year, 2007.

The estimated coefficient is -0.203 with a standard error of 0.035. The point estimate is a little below the minimum one presented on Figure 2 but still within the confidence intervals. We take this as evidence that the model is able to replicate the strength of the relationship between employment shares and wages.[55]

[55] The first stage of the instrumental variable is highly significant (p-value<0.001) and the point estimate is negative. This is an expected

Table 4: Concentration and Labor Share: Data vs. Model

	Data: $\log(LS^{D}_{h,t})$		Oligopsony: $\log(LS^{M,MP}_{h,t})$		Model: $\log(LS^{M}_{h,t})$	
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(\overline{HHI}_{h,t})$	-0.054^{***}	-0.056^{***}	-0.388^{***}	-0.416^{***}	-0.175^{***}	-0.161^{***}
	(0.013)	(0.013)	(0.009)	(0.003)	(0.007)	(0.005)
Sector FE	Y	N	Y	N	Y	N
Sector-Year FE	N	Y	N	Y	N	Y
R^2	0.29	0.343	0.901	0.903	0.946	0.909

Notes: The number of observations is 1357. The dependent variable of the first two columns is the logarithm of 3-digit industry labor share at year t, $\log(LS^{D}_{h,t})$ from the data. Next two columns present the model generated log labor shares $\log(LS^{M,MP}_{h,t})$ when the model does not incorporate wage bargaining. This is a framework where the labor wedge λ boils down to $\lambda(\mu_{io}, 0) = \mu_{io}$. Last two columns present the analogous regressions with our framework where bargaining is incorporated $\log(LS^{M}_{h,t})$. Throughout the different frameworks column 1 presents estimates with sector fixed effects and column 2 results with sector-year fixed effects. $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Macro evidence

We now turn to aggregate empirical evidence relating labor market concentration to the labor share, while underscoring the importance of unions to match the aggregate empirical evidence in France.

We run regressions of 3-digit industry labor shares from the data and model generated labor shares on the average industry Herfindahl-Hirschman Indices and compare their estimates. We also compare the results when using a model without unions. Table 4 presents the results. The first 2 columns show the estimates using the labor shares from the data, while the rest correspond to the two alternative models with and without unions.[56] The negative relationship between labor share and concentration in the model with only oligopsonistic competition (columns 3 and 4) is about 8 times higher than in the data. Looking at the last two columns that correspond to our model, we find that the negative relationship is half as much as the model without unions.[57] These results highlight the importance of including employer market power in the model to explain the negative correlation between labor shares and concentration measures, as well as the need for unions to moderate this relationship.

6 Counterfactuals

In this section we evaluate the output and welfare effects of the labor wedges coming from labor market power and we quantify how firm and union labor market power counteract each other. As mentioned in section 5.6, observed employment and wage levels are sufficient to identify amenities and revenue productivities in the model. Also, the characterization of the local labor market equilibrium is invariant to local market-wide constants that are multiplicative of productivities. Since the revenue productivities Z_{io} are a product of the sector price and the productivities A_{io}, we can fully characterize the counterfactual local labor market equilibrium. With the counterfactual employment shares and wedges, we can rewrite the rest of the model in relative changes with respect

result, as for a larger average productivity of the competitors the employment share should diminish. The OLS estimate is 0.851 with standard error 0.005. The positive sign is expected as all the variation comes from productivity changes shifting establishment's demand.

[56] The empirical evidence is complemented in Table I2 of the Online Appendix, where we show alternative regressions of labor share on concentration measures using different fixed effects. The results do not change significantly.

[57] On the contrary, models with bargaining only and with employer labor market power without strategic interactions would not match the data as the effect of concentration on the labor shares would be null. These results support the mechanism of our structural model where union bargaining power and employer labor market power are relevant.

to the baseline equilibrium. This allows us to use "exact hat algebra" techniques, as outlined by Costinot and Rodríguez-Clare (2014), to calculate the counterfactual equilibrium using only the observed levels of wages and employment and without the need to estimate or parameterize a distribution of productivities and amenities. Appendix C.1 explains the details.

In our main analysis we start by eliminating unions and computing the output and welfare changes when workers have free mobility and total labor supply is fixed. We then evaluate if unions can be substitutes of increased competition by removing the structural labor wedges and characterizing the perfect competition equilibrium. Finally, we evaluate how firm and union power neutralize each other by turning off firm labor market power and by characterizing a counterfactual with infinitesimal firms. That is, we first characterize the oligopsony-only counterfactual where the labor wedge is purely a markdown. Then, we characterize the competitive equilibrium allocation where wages are equal to the marginal revenue product, and also the bargaining-only and monopsony-only equilibria to disentangle the relative distortions coming from each side of labor market power. We perform counterfactuals for 2007, the last year of our sample.

We then evaluate how important is labor mobility to obtain output gains by progressively restricting worker's adjustment. We first allow workers to reallocate within an sector. Then, we only allow workers to move across local labor markets that have the same sector-occupation combination. In this case, workers can move across commuting zones and (3-digit) sub-industries within their (2-digit) sector and occupation. Finally, in the most restricted case, workers can only reallocate in establishments within their local labor markets.

6.1 The effects of two-sided labor market power

Table 5 presents the results of various counterfactual scenarios assuming free mobility of workers. The first column displays the labor shares in the baseline and each counterfactual scenario. The subsequent columns show the percentage gains of each scenario compared to the baseline, with column 2 representing output gains. Eliminating unions in the oligopsonistic competition counterfactual reduces output by 0.48%, implying that union bargaining power attenuates labor market distortions in the calibrated model. The slightly more heterogeneous labor wedges in this scenario, lead to increased distortions and a reduction in output.

The second counterfactual, which eliminates labor wedges as in perfect competition, increases aggregate output by 1.62%. The third counterfactual which assumes unions retain their labor market power while employer power is eliminated, achieves output gains almost comparable to eliminating both distortions with no wedges.[58] This is because the labor wedges become $\lambda(1, \varphi_b) = 1 + \varphi_b \frac{\delta}{1-\delta}$ and the only heterogeneity comes from the sector-specific bargaining powers, resulting in small distortionary effects. Lastly, we explore the role of unions as substitutes of competition in the *Monopsony* counterfactual which is a limiting case when employment shares tend to zero, and there is no bargaining. Surprisingly, this counterfactual yields the highest output gains of 1.82, but the workers' welfare falls. Similarly to the *Bargain* counterfactual, the labor wedge is a sector constant (below 1 in this case) and the higher output gains come from employment reallocation across sectors, as it will be obvious in Table 6.

[58] The *Bargain* counterfactual is a situation where none of the sides would internalize movements along the labor supply but bargain over wages.

Table 5: Counterfactuals: Efficiency and Distribution

	LS (%)	Gains (%)		
		ΔY	Δ Wage	Δ Welfare (L)
Baseline $\lambda(\mu, \varphi_b)$	50.62	-	-	-
Counterfactuals				
Oliposony $\lambda(\mu, 0) = \mu_{io}$	38.57	-0.48	-24.18	-26.17
No wedges $\lambda(1, 0) = 1$	72.26	1.62	45.06	42.07
Bargain $\lambda(1, \varphi_b) = 1 + \varphi_b \frac{\delta}{1-\delta}$	73.38	1.60	47.27	44.34
Monopsony $\lambda(\mu, 0) = \frac{\varepsilon_b}{\varepsilon_b+1}$	48.95	1.82	-1.55	-4.92

Notes: First column presents the aggregate labor share (in percent) for the baseline and the different counterfactuals. The last three columns are changes with respect to the baseline in percentages. ΔY is the change of aggregate output, Δ *Wage* is the change in aggregate wage. Aggregate wage is an employment weighted average of establishment-occupation wages. Δ *Welfare (L)* is the change of the median expected welfare of the workers. The main counterfactual is the oligopsonistic competition one with $\lambda = \mu_{io}$. The second counterfactual *Bargain* is the standad bargaining framework where the workers' outside options are the competitive wages and they do not internalize movements along the labor supply. *No wedges* is the counterfactual where the wedge is equal to one and *Monopsony* is a counterfactual where there is no bargaining but firms compete monopsonistically. Counterfactuals are performed for the year 2007.

With respect to the distributional effects we look at the aggregate labor share, which is a value added weighted sum of sector labor shares, $\beta_b \lambda_b$. Using the demand of the final good producer (3), the aggregate labor share is:[59]

$$LS = \sum_{b \in \mathcal{B}} \beta_b \lambda_b \theta_b. \tag{26}$$

Column 1 of Table 5 compares the aggregate labor shares of the baseline and the different counterfactuals. Removing unions decreases the labor share by 12 percentage points, from 50.62% in the baseline to 38.57% in the counterfactual. The labor share rises to 73.38% in the counterfactual where employer labor market power is eliminated which is slightly above to the labor share in the counterfactual without labor wedges (72.26%). Finally, the labor share is slightly reduced from the baseline to 48.95% in the counterfactual where we increase competition and firms compete monopsonistically. There are two reasons why the labor share changes are large relative to output gains: (i) baseline labor wedges have limited heterogeneity, so there is limited room for output gains; and (ii) the calibrated baseline model features high aggregate profit shares, so there is a high potential for labor share changes. As δ is close to zero, there are almost no quasi-rents coming from the decreasing returns to scale, which, combined with the low capital shares, implies high profit shares in the baseline. Removing the labor wedges shifts the profit previously earned by the firm owners to the workers.

Reductions in the aggregate wage do not imply that wage inequality is increased. Due to the idiosyncratic preferences of workers for different establishments, they face a labor supply with finite elasticity. Combined with differences in productivities and amenities, this set-up still yields different wages, even if wedges are equalized.[60]

A lower labor share implies lower wages and lower welfare for the workers. To measure the change in workers' welfare, we calculate their median welfare.[61] Columns 3 and 4 of Table 5

[59] All the counterfactuals leading to the same sector wedge, λ_b, lead to the same aggregate labor share. That is the case when we restrict employment at sectoral or lower levels.

[60] In the limit where workers would be indifferent across different workplaces, there would remain a unique equilibrium wage.

[61] As the across local labor market elasticity η is smaller than 1, the expected value of the Fréchet distribution is not defined. We therefore can only compute the median and the mode of the workers' welfare which is proportional to the aggregation of outside options $\Phi^{\frac{1}{\eta}}$.

show the relative change in average wages and median welfare with respect to the baseline. In the oligopsonistic case, average wages and median welfare are reduced respectively by 24% and 26%. In contrast, in the case with no labor wedges, average wages and median welfare increase by 45% and 42%. The welfare losses exceed the output losses because workers not only experience a decline in productivity but also suffer from the redistribution of pure rents to the owners. In the *Monopsony* counterfactual workers benefit from higher output but receive a lower labor share and lower wages, which implies that their welfare slightly decreases compared to the baseline.

The main takeaways of the counterfactuals are: (i) unions not only redistribute a significant portion of total output towards workers, but also increase the economy's overall productivity compared to the case with only oligopsony; and (ii) unions can be an alternative to increased labor market competition in improving productivity, but fall short.[62] Focusing on the first, while the redistribution effect of unions is expected, the productivity gains are not so obvious. In the model, an increase in the bargaining powers reduces the dispersion of wedges across firms. To see this, note that:

$$\frac{\partial^2 \lambda_{io}}{\partial \varphi_b \partial s_{io}} = -\frac{\partial \mu_{io}}{\partial s_{io}} > 0. \tag{27}$$

As a result, unions lead to a greater increase in the labor wedge in larger establishments compared to smaller ones. This, in turn, reduces the overall variance of wedges and the potential misallocation of labor within a local labor market. It is easier to understand expression (27) by considering the redistribution of a establishment-occupation's output to the labor share: $\frac{w_{io}L_{io}}{P_b Y_{io}} = \beta_b \lambda_{io}$, and realizing that the second order derivative of the labor share is (27) multiplied by β_b. We can decompose the labor share into:

$$\frac{w_{io}L_{io}}{P_b Y_{io}} = \beta_b \lambda_{io} = \underbrace{\beta_b}_{\text{Perf. Comp.}} - \underbrace{\beta_b(1-\mu_{io})}_{\text{Oligopsonistic Rents}} + \underbrace{\varphi_b \overbrace{\left[\underbrace{(1-\alpha_b-\beta_b)}_{\text{DRS Rents}} + \underbrace{\beta_b\,(1-\mu_{io})}_{\text{Oligopsonistic Rents}}\right]}^{\text{Total Rents}}}_{\text{Bargaining gains}}.$$

Under perfect competition and no bargaining, the labor share is equal to the output elasticity β_b. The term $\beta_b(1-\mu_{io})$ captures the oligopsonistic rents that the firm would obtain in the absence of bargaining under the current equilibrium allocation. The third term, involving φ_b, represents the bargaining gains and reflects the redistribution of rents from the establishment towards the workers based on their bargaining power. These bargaining gains consist of the total rents, which are a combination of oligopsonistic rents resulting from labor market power and rents arising from the decreasing returns to scale technology.

Bargaining allows for a redistribution of rents, leading to an increase in the firm's labor share. Abstracting from changes in employment shares, the redistribution narrows the gap between the labor share of large establishments –with low markdowns and high rents– and small establishments –with high markdowns and low rents–, reflecting a lower dispersion of wedges across establishments and lower misallocation. Importantly, this argument holds even in cases with constant re-

[62]In the extreme case where workers have all of the bargaining power, i.e. $\varphi_b = 1$ for all b, the allocation would be the same as the one with perfect competition and the productivity gains would be the same.

turns to scale, where $1 - \alpha_b - \beta_b = 0$. In cases with decreasing returns to scale ($1 - \alpha_b - \beta_b > 0$), the heterogeneity of wedges is further reduced, as the ratio of wedges across establishments approaches one, resulting in reduced misallocation.

The introduction of unions, while decreasing the heterogeneity of wedges within a labor market, can still distort employment across sectors when bargaining power differs across sectors. This can be seen from the *Bargain* counterfactual with slightly lower output gains than the one without wedges. Furthermore, the counterfactuals show that unions might not be the best tool to fight the misallocation effects from heterogeneous firm market power and increase aggregate output. Compared to the *Oligopsony* counterfactual, the output gains from the *Monopsony* counterfactual—an scenario without unions but with infinitesimal firms—suggests that increasing competitors in an oligopsonistic labor market might be a more effective way of increasing output than incorporating unions, but the welfare gains to workers are smaller.

6.2 The importance of labor mobility

We check three additional cases to locate the output changes in an environment with mobility costs. These cases differ in their mobility restrictions, where we allow mobility to happen only within sector, sector-occupation and local labor market. Table 6 compares the free mobility case with the restricted mobility cases for different counterfactuals. Comparing the output changes in column 3 across the different scenarios, we find that restricting mobility reduces the output gains from removing the labor wedges. In the *Oligopsony* counterfactual, when labor is constrained to remain in the local labor market, output does not change as much as in the free mobility case. However, in the other two cases, *Fixed sector* and *Fixed sector-occupation*, the output losses are greater. Comparing the free mobility counterfactuals to the ones with restricted labor mobility we see that the key margin of adjustment is geographical mobility.

Fixing employment at the sector-occupation level accounts for 82% of the gains of the free mobility case without labor wedges. Restricting workers to stay in their particular local labor market in the counterfactual without labor wedges output gains are 0.49%, which constitute only 30% of the gains under free mobility without wedges. In the oligopsonistic competition counterfactual, fixing employment across sectors but allowing for geographical mobility exacerbates the output losses. Restricting employment to move only within a local labor market would contain output losses as productivity losses are reduced by more than 60%.

These results underscore the importance of free mobility of labor to counteract the output losses from the misallocation coming from heterogeneous wedges. The left panel of Figure 4 shows the percentage change of manufacturing employment in the free mobility case in the oligopsonistic competition counterfactual. Each block is the aggregation of local labor markets to the commuting zone.[63] The main conclusion from the counterfactual analysis is that, in the absence of unions, manufacturing employment in in the rural areas would be reduced. The counterfactual reveals that there are a handful of rural productive establishments in concentrated local labor markets. In the baseline these have lower wage markdowns and lower employment that are further dampened without unions. Moving to the counterfactual, those are the ones with the biggest relative wage and

[63] A commuting zone can host several local labor markets across industries and occupations.

Table 6: Counterfactuals: Limited Mobility

		LS (%)	ΔY (%)	Δ Prod (%)	Contribution ΔY (%) GE	Productivity	Labor
Oligopsony	Free mobility	38.57	-0.48	-0.95	-20	200	-80
	Fixed sector	38.57	-0.94	-0.95	-1	101	0
	Fixed sector-occ	38.56	-0.94	-0.97	-2	102	0
	Fixed local market	38.05	-0.37	-0.38	-3	103	0
No wedge	Free mobility	72.26	1.62	1.33	9	83	8
	Fixed sector	72.26	1.32	1.33	-1	101	0
	Fixed sector-occ	72.26	1.33	1.35	-2	102	0
	Fixed local market	72.26	0.49	0.49	-2	102	0
Monopsony	Free mobility	48.95	1.82	1.33	-7	74	33

Notes: Results are in percentages. LS is the labor share. ΔY is the change of aggregate output with respect to the baseline and Δ *Prod* is the change in aggregate productivity from decomposition (28). Last three columns present the contribution of each of the elements of the decomposition (28) to output gains. The first column is the counterfactual type. *Oligopsony* presents the main counterfactual without unions and under oligopsonistic competition, *No wedges* is the counterfactual without labor wedges that corresponds to the perfect competition case, and *Monopsony* is the counterfactual under monopsonistic competition. The second column is the mobility type of the counterfactual with four potential cases. One without mobility restrictions (*Free mobility*) and three restricting mobility: *Fixed sector* allows for mobility only within sector, *Fixed sector-occ* fixes employment at the sector-occupation and allows for mobility across locations and across 3-digit sub-industries, and *Fixed local market* allows for mobility only across establishments within local labor markets. Counterfactuals are performed for the year 2007.

employment losses.[64] The right panel of Figure 4 shows a positive relationship between the logarithm of baseline employment at the commuting zone and employment gains in the oligopsonistic competition counterfactual without unions.[65] Rural areas or commuting zones with low employment levels in the baseline are the ones that benefit the most from the presence of unions. They would experience greater wage reductions and job losses in the counterfactual without unions.

Turning now to the source of the output gains, we can use the aggregate production function and the relative sector output and decompose the logarithm of the relative final output into three terms:[66]

$$\ln \widehat{Y} = \underbrace{\sum_{b \in \mathcal{B}} \theta_b \ln \widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}_{\Delta \text{ GE}} + \underbrace{\sum_{b \in \mathcal{B}} \theta_b \ln \widehat{\Psi}_b}_{\Delta \text{ Productivity}} + \underbrace{\sum_{b \in \mathcal{B}} \theta_b \ln \widehat{L}_b^{1-\delta}}_{\Delta \text{ Labor}} . \tag{28}$$

The first term on the right hand side corresponds to the capital effects or general equilibrium effects of capital flowing to different sectors as a response to changes in relative prices. The second term, arguably the most important, represents total productivity gains or losses from less or more misallocation, respectively. This term suffers the most from labor market concentration as big productive firms are shrinking their relative participation, therefore reducing overall productivity. The third term corresponds to how labor is allocated across sectors.

Columns 3 to 5 of Table 6 show the decomposition of relative changes of output.[67] The main source of output changes come from productivity because sector productivity is an employment

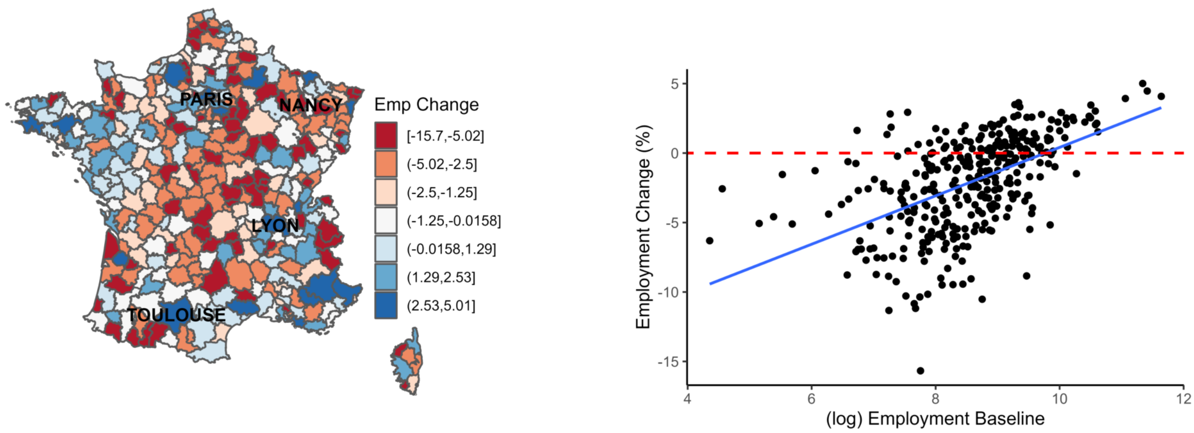
[64] Another potential reason is the differential in the amenities. The reduction of manufacturing labor in the rural areas could be magnified if they have in general worse amenities.

[65] In Online Appendix F.1 we show the employment impact across the French geography of the counterfactual without labor wedges. The general idea remains the same: in a counterfactual with perfect competition rural areas increase their wages and employment the most as they are the most concentrated markets in the baseline scenario.

[66] See Online Appendix A for the detailed derivations.

[67] Note that $\Delta Y = \widehat{Y} - 1 \approx \ln \widehat{Y}$. The decomposition is with respect to $\ln \widehat{Y}$. The share of the gains that come from *Productivity* is $\frac{\sum_{b \in \mathcal{B}} \theta_b \ln \widehat{\Psi}_b}{\ln \widehat{Y}}$. Each row from columns 3 to 5 sums up to 100.

Figure 4: Employment Change (%) in the Counterfactual: Oligopsonistic Competition



Notes: The map presents employment changes with respect to the baseline economy in percentages within commuting zones. The oligopsonistic competition counterfactual is performed for the year 2007. The figure in the right plots the employment change in the counterfactual versus the log of employment in the baseline. The blue line is a fitted line from an OLS regression.

weighted sum of establishment-occupation productivities (which are unchanged). The source of aggregate productivity and output losses without unions is therefore the reallocation of workers towards less productive establishments. Sectoral mobility dampens the negative productivity effects of removing unions under free mobility. When restricting mobility by keeping employment constant at the local labor market level, the misallocation effects are curbed and output changes to -0.37%. On the contrary, productivity gains in the free mobility *Monopsony* counterfactual are identical to the ones in the *No wedge* counterfactual but output gains are amplified in the former due to employment reallocation across sectors. The labor shares in the *No wedge* counterfactual are identical in all cases from (26) as the sector wedge is a constant equal to one.

We show in Online Appendix F.1 that most significant productivity losses in the *Oligopsony* counterfactual happen outside urban areas. As a result, the largest losses relative to the baseline in wages and employment are in commuting zones that do not include big cities.

6.3 Additional exercises

In Online Appendix F.2 we explore how differences in labor market power affect the population composition and wage gap between rural and urban locations. We find that the importance of cities in manufacturing would have declined more slowly in absence of labor market power coming from firms and unions. A potential reason is that the closure of manufacturing establishments in cities would increase the labor concentration of urban areas, making small labor markets relatively more attractive.

We also find that the urban-rural wage gap is affected by the presence of union and firm labor market power. In the counterfactual without labor wedges, the gap is reduced from 36 to 23 percent. This implies that labor market power can explain more than a third of the observed urban-rural wage gap. On the contrary, the urban-rural wage gap is amplified to 50 percent in the oligopsonistic competition counterfactual. Unions therefore disproportionately boost rural wages and help close the urban-rural wage gap.

7 Extensions

The main counterfactual had some strong assumptions. In particular, the total labor supply was fixed and there were no agglomeration externalities. In this section, we propose extensions to relax these assumptions. First, we allow for an endogenous labor participation decision. Second, we introduce agglomeration forces in the local labor markets. All the details are left for the Online Appendix C.

7.1 Endogenous labor force participation

We briefly present the extension with endogenous labor force participation decisions. We assume workers can decide between working and staying at home. In the latter case, they earn wages related to home production. Staying at home is now an endogenous choice, as workers compare the utility from working and staying at home when making their labor decision.

We incorporate the option of being out-of-the-labor-force (from now on OTLF) by defining a new (3-digit) sub-industry for each (2-digit) sector. These new sub-industries have only one 'establishment', indexed by u, per commuting zone. Each of these establishments 'employ' different occupations paying them a home production wage w_{uo}. The establishment-occupations define a new set of local labor markets $\mathcal{U}$ that correspond to combinations of commuting zones, occupations, and the new sub-industries.

Similar to the baseline model, we assume that workers face idiosyncratic shocks that are labor market specific and have the same Fréchet distribution. Thus, the number of workers OTLF in a particular commuting zone-sector u and occupation o is:

$$L_{uo} = \frac{(T_{uo} w_{uo}^{\varepsilon_b})^{\eta/\varepsilon_b} \Gamma_b^{\eta}}{\Phi} L, \quad \Phi \equiv \Phi_e + \Phi_u.$$

L is the total labor supply of both, the employed and the OTLF workers. Φ is the aggregate outside option that now formed of two components. Φ_e is the part that comes from the outside options of the employed workers and Φ_u collects the outside options that correspond to all workers out of the labor force.

We lack detailed data on the geographical distribution of out-of-the-labor-force status as labor force surveys provide information only at the more aggregated regional level. Basing our counterfactuals in those surveys would require the assumption of constant rates of labor participation for entire regions. Instead, we use commuting zone level unemployment rates as proxies for OTLF rates.

To map the model with the data, we assume that the OTLF rate is the same across industries and occupations in each commuting zone and define the proportion of workers OTLF in each local labor market uo accordingly. Similar to how we identify amenities in the baseline model, the proportion of OTLF workers in each local market identifies the home production amenity and income $T_{uo} w_{uo}^{\varepsilon_b}$. We assume that the home production incomes are fixed in the counterfactuals.

Table 7 shows the results of the counterfactuals with endogenous labor force participation. Introducing the endogenous labor participation margin induces higher output losses than in the baseline (*Fixed L*). The counterfactual output change without unions is -1.42% as the total labor supply de-

Table 7: Counterfactual: Endogenous Participation

	ΔY (%)	Δ Prod (%)	Δ L (%)	Contribution ΔY (%)		
				GE	Prod	Labor
Fixed L	-0.48	-0.95		-20	200	-80
Endogenous participation (EP)						
Oligopsony $\lambda(\mu, 0) = \mu_{io}$	-1.42	-0.95	-0.98	-7	67	40
No wedges $\lambda(1, 0) = 1$	2.61	1.33	1.01	6	52	42
Bargain $\lambda(1, \varphi_b) = 1 + \varphi_b \frac{\delta}{1-\delta}$	2.63	1.33	1.06	6	51	43
EP Mobility within sector						
Oligopsony $\lambda(\mu, 0) = \mu_{io}$	-1.86	-0.95	-1.02	-1	51	50

Notes: Results are in percentages. First column ΔY is the change of aggregate output with respect to the baseline, Δ *Prod* is the change in aggregate productivity from decomposition (28) and ΔL is the counterfactual change in total employment. Last three columns present the contribution of each of the elements of the decomposition (28) to output gains. *Fixed L* is the main counterfactual with oligopsonistic competition ($\lambda = \mu_{io}$), under free mobility of labor and fixed total labor supply. All the other counterfactuals in this table allow for endogenous labor force participation. *Oligopsony* is analogous to the main counterfactual. The first instance allows for free mobility of workers within and across sectors while the second one keeps sector workers (employed and unemployed) constant. *No wedges* is the counterfactual without wedges. *Bargain* is the standard bargaining framework where the workers' outside options are the competitive wages and they don't internalize movements along the labor supply.

creases by 0.98%. In contrast to the output gain decomposition in Table 6, 40% of the losses come from the decreased total employment. This extensive margin of adjustment in the total labor supply amplifies the original differences in output gains across counterfactuals. In particular, output gains without labor wedges are as high as 2.61% because total labor force participation is increased by 1%. Despite featuring high wage changes, the differences in total employment are minor in the counterfactuals because we assume that workers have idiosyncratic shocks to stay out of the labor force.

Table 7 shows that sector reallocation contributes to the negative output effects of oligopsonistic competition as it constitutes 40% of the output loss. Fixing total sectoral labor force –employed and unemployed– output changes by -1.86% which is roughly 30 percent higher than the free mobility counterfactual. When fixing the sector labor force, the *Labor* component also contributes to output changes as we allow for changes in sector employment/unemployment.

7.2 Agglomeration

In this section we present an extension of the model that includes agglomeration forces at the local labor market level. To keep the model tractable, we assume that the productivity is: $\widehat{A}_{io} = \widetilde{A}_{io} L_m^{\gamma(1-\alpha_b)}$. The agglomeration effect is a local labor market externality with elasticity $\gamma(1-\alpha_b)$. The wage first order condition is:

$$Pw_{io} = \beta_b \lambda(\mu_{io}, \varphi_b) Z_{io} L_{io}^{-\delta} L_m^{\gamma}. \tag{29}$$

Similarly to the baseline counterfactual, we back out the transformed TFPRs, Z_{io}, to match observed establishment-occupation wages, w_{io}, under the assumption of agglomeration externalities. In the case where employment for a given local labor market is high, the backed out productivity of the establishments in that market m is lower than for the main counterfactual.

Table 8 summarizes the counterfactual results for different values of γ. All the counterfactuals in Table 8 also assume the absence of unions and free mobility but introduce agglomeration forces

Table 8: Counterfactuals: Agglomeration. Oligopsony

	ΔY (%)	Δ Prod (%)	Contribution ΔY (%)		
			GE	Productivity	Labor
No Agglomeration	-0.48	-0.95	-20	200	-80
Agglomeration					
$\gamma = 0.05$	-0.47	-0.99	-16	209	-92
$\gamma = 0.1$	-0.46	-1.02	-13	219	-106
$\gamma = 0.2$	-0.45	-1.08	-4	240	-136
$\gamma = 0.25$	-0.44	-1.11	1	252	-154
$\gamma = 0.3$	-0.43	-1.13	8	265	-172

Notes: Results are in percentages. First column ΔY is the change of aggregate output with respect to the baseline, Δ *Prod* is the change in aggregate productivity from decomposition (28). Last three columns present the contribution of each of the elements of the decomposition (28) to output gains. *No Agglomeration* is the main counterfactual under oligopsonistic competition, under free mobility of labor, fixed total labor supply and no agglomeration forces. All the other counterfactuals in this table allow for agglomeration within the local labor market. Similarly to the main counterfactual, workers are freely mobile and total employment is fixed. We present different counterfactuals depending on the agglomeration parameter γ.

in local labor markets. As γ becomes higher, the more important are the agglomeration forces and the more contained are the output losses. The reason behind this result is that increasing γ the local labor market employment L_m becomes more important in (29). Consequently, differences in baseline employment levels across local labor markets amplify their productivity differences. The movements out of small local labor markets towards middle sized ones are therefore bigger than in the main counterfactual (*No Agglomeration*) and amplify productivity losses but are counteracted by labor reallocation across sectors. Output losses reduce monotonically with the importance of agglomeration externalities. Online Appendix F presents the agglomeration counterfactuals without labor wedges where baseline output gains are amplified.

8 Conclusion

We evaluate the output and welfare costs of employer and union labor market power and how those forces counteract each other. We find that unions countervail distortions generated from firm labor market power as output would be reduced by 0.48% in their absence. Removing structural labor wedges increases output by 1.62% and the gains are amplified up to 2.61% when we allow for an endogenous labor force participation margin. Likewise, the output losses from removing unions and competing under oligopsonistic competition are greater with a participation margin, reaching -1.42%. The main mechanism behind the output losses (gains) is the reallocation of resources towards less (more) productive establishments. Removing unions also leads to significant labor share and wage losses. In conclusion, our findings suggest that employer labor market power is more important than that of unions in determining the extent of labor market power in the French manufacturing sector.

There are some potential insights for policy. The counterfactual without unions suggests that they sustain employment in French rural areas. Our estimated model shows how unions counteract employer labor market power and homogenize the wedge across establishments within a sector. Meaning, as long as establishments within a sector face a similar union institutional setting, workers' bargaining power will dampen the heterogeneity of labor wedges that stem from differentiated employer labor market power and increase allocation efficiency.

However, empowering unions can not completely correct for the inefficiencies caused by the employers' market power and could lead to some unmodeled adverse effects like reducing the entry of new establishments. Counterfactuals also show that increasing labor market competition in the absence of unions would generate output gains. Unions can therefore be thought as substitutes to increased competition in the labor market as both help improve the productivity of the economy. Increasing labor market competition is quantitatively stronger as it generates the highest output gains but brings smaller welfare gains to workers.

The efficient allocation can be implemented by hiring subsidies that would be establishment-occupation specific and depend on the establishments' employment share in equilibrium. Those subsidies could be financed by uniform taxes on profits or payroll. Compared to the efficient allocation, large employers restrict their hiring more than small ones. Then, the hiring subsidies would be greater for large employers. The implementation of such a tax policy would be cumbersome and perhaps not realistic. So what could be a second-best alternative? Our paper offers a partial answer to this question: unions. By redistributing rents that come from employer labor market power, unions can improve the labor allocation and the aggregate productivity in the economy.

References

Ahlfeldt, G. M., S. J. Redding, D. M. Sturm, and N. Wolf (2015): "The Economics of Density: Evidence from the Berlin Wall," *Econometrica*, 83, 2127–2189.

Amodio, F. and N. De Roux (2021): "Labor Market Power in Developing Countries: Evidence from Colombian Plants," .

Amodio, F., P. Medina, and M. Morlacco (2022): "Labor market power, self-employment, and development," .

Atkeson, A. and A. Burstein (2008): "Pricing-to-Market, Trade Costs, and International Relative Prices," *American Economic Review*, 98, 1998–2031.

Azar, J., I. Marinescu, and M. Steinbaum (2020a): "Labor Market Concentration," *Journal of Human Resources*, 1218.

Azar, J., I. Marinescu, M. Steinbaum, and B. Taska (2020b): "Concentration in US Labor Markets: Evidence from Online Vacancy Data," *Labour Economics*, 66, 101886.

Azar, J. A., S. T. Berry, and I. Marinescu (2022): "Estimating labor market power," Tech. rep., National Bureau of Economic Research.

Bachmann, R., C. Bayer, H. Stüber, and F. Wellschmied (2022): "Monopsony Makes Firms not only Small but also Unproductive: Why East Germany has not Converged," .

Barkai, S. (2020): "Declining Labor and Capital Shares," *The Journal of Finance*, 75, 2421–2463, _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/jofi.12909.

Benmelech, E., N. Bergman, and H. Kim (2018): "Strong Employers and Weak Employees: How Does Employer Concentration Affect Wages?" Tech. rep., National Bureau of Economic Research.

Berger, D., K. Herkenhoff, and S. Mongey (2022): "Labor Market Power," *American Economic Review*, 112, 1147–93.

Cahuc, P., F. Postel-Vinay, and J.-M. Robin (2006): "Wage Bargaining with On-the-Job Search: Theory and Evidence," *Econometrica*, 74, 323–364.

Caliendo, L., F. Monte, and E. Rossi-Hansberg (2015): "The Anatomy of French Production Hierarchies," *Journal of Political Economy*, 123, 809–852.

Card, D., A. R. Cardoso, J. Heining, and P. Kline (2018): "Firms and Labor Market Inequality: Evidence and Some Theory," *Journal of Labor Economics*, 36, S13–S70.

Centre d'accès sécurisé aux données (CASD) (2020a): "BTS-Postes 1994-2007 (previously DADS Postes)," Dataset.

——— (2020b): "FICUS 1994-2007," Dataset.

Costinot, A. and A. Rodríguez-Clare (2014): "Trade Theory with Numbers: Quantifying the Consequences of Globalization," in *Handbook of International Economics*, Elsevier, vol. 4, 197–261.

Datta, N. (2021): "Local Monopsony Power," Tech. rep., University College London.

De Loecker, J. and J. Eeckhout (2021): "Global Market Power," *Unpublished manuscript*.

De Loecker, J., J. Eeckhout, and G. Unger (2020): "The Rise of Market Power and the Macroeconomic Implications," *The Quarterly Journal of Economics*, 135, 561–644.

Deb, S., J. Eeckhout, A. Patel, and L. Warren (2022a): "What drives Wage Stagnation: Monopoly or Monopsony?" Tech. rep., UPF mimeo.

Deb, S., J. Eeckhout, A. Patel, L. Warren, et al. (2022b): "Market Power and Wage Inequality," Tech. rep.

DiNardo, J., N. Fortin, and T. Lemieux (1995): "Labor market institutions and the distribution of wages, 1973-1992: A semiparametric approach," .

Dodini, S., M. Lovenheim, K. G. Salvanes, and A. Willén (2020): "Monopsony, Skills, and Labor Market Concentration," *Unpublished manuscript*.

Dodini, S., K. G. Salvanes, and A. Willén (2021): "The Dynamics of Power in Labor Markets: Monopolistic Unions versus Monopsonistic Employers," .

Dube, A., J. Jacobs, S. Naidu, and S. Suri (2020): "Monopsony in Online Labor Markets," *American Economic Review: Insights*, 2, 33–46.

Eaton, J. and S. Kortum (2002): "Technology, Geography, and Trade," *Econometrica*, 70, 1741–1779.

Edmond, C., V. Midrigan, and D. Y. Xu (2021): "How Costly Are Markups?" *Unpublished manuscript*.

Falch, T. (2010): "The Elasticity of Labor Supply at the Establishment Level," *Journal of Labor Economics*, 28, 237–266.

Farber, H. S., D. Herbst, I. Kuziemko, and S. Naidu (2021): "Unions and inequality over the twentieth century: New evidence from survey data," *The Quarterly Journal of Economics*, 136, 1325–1385.

Felix, M. (2021): "Trade, Labor Market Concentration, and Wages," *Job Market Paper*.

Frandsen, B. R. (2021): "The surprising impacts of unionization: Evidence from matched employer-employee data," *Journal of Labor Economics*, 39, 861–894.

Freeman, R. B. and M. M. Kleiner (1990): "The impact of new unionization on wages and working conditions," *Journal of Labor Economics*, 8, S8–S25.

Gathmann, C., I. Helm, and U. Schönberg (2017): "Spillover Effects of Mass Layoffs," *Journal of the European Economic Association*.

Gutiérrez, G. and T. Philippon (2016): "Investment-Less Growth: An Empirical Investigation," Tech. rep., National Bureau of Economic Research.

Hellerstein, J. K., M. J. Kutzbach, and D. Neumark (2019): "Labor Market Networks and Recovery from Mass Layoffs: Evidence from the Great Recession Period," *Journal of Urban Economics*, 113, 103192.

Hershbein, B., C. Macaluso, and C. Yeh (2020): "Monopsony in the US Labor Market," Tech. rep., Working Paper.

Jäger, S., C. Roth, N. Roussille, and B. Schoefer (2022): "Worker beliefs about outside options," Tech. rep., National Bureau of Economic Research.

Jäger, S., B. Schoefer, S. Young, and J. Zweimüller (2020): "Wages and the Value of Nonemployment," *The Quarterly Journal of Economics*, 135, 1905–1963.

Jarosch, G., J. S. Nimczik, and I. Sorkin (2019): "Granular search, market structure, and wages," Tech. rep., National Bureau of Economic Research.

Jolivet, G., F. Postel-Vinay, and J.-M. Robin (2006): "The Empirical Content of the Job Search Model: Labor Mobility and Wage Distributions in Europe and the US," *Contributions to Economic Analysis*, 275, 269–308.

Karabarbounis, L. and B. Neiman (2013): "The Global Decline of the Labor Share," *The Quarterly Journal of Economics*, 129, 61–103.

Lagos, L. (2020): "Labor market institutions and the composition of firm compensation: Evidence from brazilian collective bargaining," .

Lamadon, T., M. Mogstad, and B. Setzler (2022): "Imperfect Competition, Compensating Differentials, and Rent Sharing in the US Labor Market," *American Economic Review*, 112, 169–212.

Lee, D. S. and A. Mas (2012): "Long-run impacts of unions on firms: New evidence from financial markets, 1961–1999," *The Quarterly Journal of Economics*, 127, 333–378.

Lipsius, B. (2018): "Labor Market Concentration Does Not Explain the Falling Labor Share," *Available at SSRN 3279007*.

Macaluso, C. (2017): "Skill Remoteness and Post-Layoff Labor Market Outcomes," in *2017 Meeting Papers*, Society for Economic Dynamics, 569.

MacKenzie, G. (2021): "Trade and market power in product and labor markets," Tech. rep., Bank of Canada Staff Working Paper.

Manning, A. (2011): "Imperfect Competition in the Labor Market," in *Handbook of Labor Economics*, Elsevier, vol. 4, 973–1041.

Manning, A. and B. Petrongolo (2017): "How Local Are Labor Markets? Evidence from a Spatial Job Search Model," *American Economic Review*, 107, 2877–2907.

Marinescu, I., I. Ouss, and L.-D. Pape (2021): "Wages, Hires, and Labor Market Concentration," *Journal of Economic Behavior & Organization*, 184, 506–605.

Mengano, P. (2022): "Trends in Worker Bargaining Power," Tech. rep., University of Zurich.

Morlacco, M. (2018): "Market Power in Input Markets: Theory and Evidence from French Manufacturing," *Unpublished manuscript*.

Nakamura, E. and J. Steinsson (2018): "High-frequency identification of monetary non-neutrality: the information effect," *The Quarterly Journal of Economics*, 133, 1283–1330.

Naouas, A. and F. Romans (2014): "La Négociation Salariale d'entreprise de 2004 à 2010. Entre Renforcement de l'obligation de Négocier et Baisse de l'implantation Des Délégués Syndicaux," Tech. Rep. 067, Dares.

Nimczik, J. S. (2018): "Job Mobility Networks and Endogenous Labor Markets," *Unpublished manuscript*.

Redding, S. J. (2016): "Goods Trade, Factor Mobility and Welfare," *Journal of International Economics*, 101, 148–167.

Rigobon, R. (2003): "Identification through Heteroskedasticity," *Review of Economics and Statistics*, 85, 777–792.

Rigobon, R. and B. Sack (2004): "The impact of monetary policy on asset prices," *Journal of monetary economics*, 51, 1553–1575.

Schubert, G., A. Stansbury, and B. Taska (2020): "Employer Concentration and Outside Options," *Unpublished manuscript*.

Sokolova, A. and T. Sorensen (2021): "Monopsony in Labor Markets: A Meta-Analysis," *ILR Review*, 74, 27–55.

Staiger, D. O., J. Spetz, and C. S. Phibbs (2010): "Is There Monopsony in the Labor Market? Evidence from a Natural Experiment," *Journal of Labor Economics*, 28, 211–236.

Trottner, F. (2022): "Misallocations in Monopsonistic Labor Markets," Tech. rep., Working Paper.

Webber, D. A. (2015): "Firm Market Power and the Earnings Distribution," *Labour Economics*, 35, 123–134.

Wong, H. C. (2019): "Wage Dispersion Across Firms," *Unpublished manuscript*.

Appendix for Publication

Bargaining details

We provide derivations under the baseline bargaining protocol where employers and unions have zero outside options and internalize movements along the labor supply curve. An alternative bargaining protocol leading to the same equilibrium condition for the wages is at the end.

Each establishment has different occupation profit functions $(1-\alpha_b)P_bF(L_{io}) - w^u_{io}L_{io}$, where the optimal capital decision has been taken. We assume that workers and establishments are symmetric both having null threat points and internalizing the generation of rents as they move along the labor supply curve. During the bargaining establishments and unions choose wages to maximize:

$$\max_{w^u_{io}} \left[w^u_{io}L_{io}(w^u_{io})\right]^{\varphi_b} \left[(1-\alpha_b)P_bF(L_{io}(w^u_{io})) - w^u_{io}L_{io}(w^u_{io})\right]^{1-\varphi_b},$$

where we made explicit the fact that both parties internalize how labor supply is a function of equilibrium wages. φ_b is the union's bargaining power, w^u_{io} the wage bargained with the unions at establishment-occupation io, L_{io} the number of workers employed at establishment-occupation io in equilibrium, $(1-\alpha_b)F(L_{io})$ is the output of the establishment-occupation after substituting for the optimal decision of capital. The first order condition of the above maximization problem are:

$$\varphi_b \frac{(1-\alpha_b)P_bF(L_{io}) - w^u_{io}L_{io}}{w^u_{io}L_{io}} \left[L_{io} + w^u_{io}\frac{\partial L_{io}}{\partial w^u_{io}}\right] + (1-\varphi_b)\left[(1-\alpha_b)P_b\frac{\partial F(L_{io})}{\partial L_{io}}\frac{\partial L_{io}}{\partial w^u_{io}} - L_{io} - w^u_{io}\frac{\partial L_{io}}{\partial w^u_{io}}\right] = 0.$$

Using the definition of the perceived labor supply elasticity $e_{io} = \dfrac{\partial L_{io}}{\partial w_{io}}\dfrac{w_{io}}{L_{io}}$ and rearranging the first order condition:

$$w^u_{io} = \varphi_b(1-\alpha_b)P_b\frac{F(L_{io})}{L_{io}} + (1-\varphi_b)(1-\alpha_b)P_b\frac{\partial F(L_{io})}{\partial L_{io}}\frac{e_{io}}{e_{io}+1},$$

where $\mu(s_{io}) \equiv \dfrac{e_{io}}{e_{io}+1}$ is the markdown that establishments would set under oligopsonistic competition.

After substituting the optimal decision for capital, the output elasticity with respect to labor of $F(L_{io})$ is $1-\delta$. Then, from the definition of the output elasticity we have that $\frac{1}{1-\delta}\frac{\partial F(L_{io})}{\partial L_{io}} = \frac{F(L_{io})}{L_{io}}$. Thus, the bargained wage becomes:

$$w^u_{io} = \underbrace{(1-\alpha_b)P_b\frac{\partial F(L_{io})}{\partial L_{io}}}_{MRPL_{io}}\left[(1-\varphi_b)\frac{e_{io}}{e_{io}+1} + \varphi_b\frac{1}{1-\delta}\right],$$

where we recovered the expression from the main text.

Alternative bargaining protocol. The alternative bargaining assumption leading to the same equilibrium wages is that employers and unions bargain over wages without internalizing movements along the labor supply and workers' outside options are the oligopsonistic competition wages w^M_{io} under the allocation with the given equilibrium wages. This alternative protocol can be rationalized by a set up where firms have to pay workers before production would start. They would pay the workers the oligopsonistic wage. Then, workers would force a negotiation where they would split the prospective remaining rents after payments to capital. The bargaining problem would be:

$$\max_{w^u_{io}} \left[w^u_{io}L_{io} - w^M_{io}L_{io}\right]^{\varphi_b} \left[(1-\alpha_b)P_bF(L_{io}) - w^u_{io}L_{io}\right]^{1-\varphi_b}.$$

Online Appendix

Union and Firm Labor Market Power

Miren Azkarate-Askasua and Miguel Zerecero

This Appendix is organized as follows. Section A presents the model derivations for the baseline equilibrium. Section B shows the proofs of the propositions in the main text. Section C presents additional derivations, which includes the derivation of the counterfactual economy relative to the baseline, as well as the different extensions of the model. Section D illustrates the distributional and productivity consequences of labor market power. Section E presents the details of our identification strategy and additional estimation results. Section F provides additional details about the counterfactual exercises. Section G gives details about sample selection and variable construction. Section H presents some additional summary statistics. Section I provides details on our reduced form exercise and the unions. It also provides a link between the reduced-form exercise and our model.

A Derivations

In this section we provide the derivations of the model that are not presented in the main text. First, we show how to obtain the establishment labor supplies by solving the workers establishment choice problem. Later, we show how we obtain the markdown function from the establishments optimality conditions. We then show how to get a closed-form solution for the prices given the solution for the normalized wages.

A.1 Establishment-occupation labor supply

To simplify the notation, we get rid of the occupation subscript o in this subsection. The indirect utility of a worker k that is employed in establishment i in sub-market m is:

$$u_{kim} = w_i z^1_{i|m} z^2_m,$$

where $z^1_{i|m}$ and z^2_m are independent utility shocks. They are both distributed Fréchet with shape and scale parameters ε_b and T_i for $z^1_{i|m}$, and η and 1 for z^2_m.

Workers first see the realizations of the shocks z^2_m for all local labor markets. After choosing their labor market, the workers then observe the establishment specific shocks. Therefore, there is a two stage decision: first, the worker chooses the local labor market that maximizes her expected utility, and subsequently she chooses the establishment that maximizes her utility conditional on the chosen sub-market.

The goal is to compute the unconditional probability of a worker going to establishment i in sub-market m. This probability is equal to:

$$\Pi_i = P\left(w_i z^1_{i|m} \geq \max_{i' \neq i} w_{i'} z^1_{i'|m}\right) P\left(\mathbb{E}_m(\max_i w_i z^1_{i|m}) z^2_m \geq \max_{m' \neq m} \mathbb{E}_{m'}(\max_i w_i z^1_{i|m'}) z^2_{m'}\right)$$

We first solve for the left term. Let's define the following distribution function:

$$G_i(v) = P\left(w_i z^1_{i|m} < v\right) = P\left(z^1_{i|m} < v/w_i\right) = e^{-T_i w_i^{\varepsilon_b} v^{-\varepsilon_b}}.$$

To ease notation, define *conditional* utility $v_i = w_i z^1_{i|m}$ for all i, i'. We need to solve for $P\left(v_i \geq \max_{i' \neq i} v_{i'}\right)$. Fix $v_i = v$. Then we have:

$$P\left(v \geq \max_{i' \neq i} v_{i'}\right) = \bigcap_{i' \neq i} P\left(v_{i'} < v\right) = \prod_{i' \neq i} G_{i'}(v) = e^{-\Phi_m^{-i} v^{-\varepsilon_b}} = G_m^{-i}(v),$$

where $\Phi_m^{-i} \equiv \sum_{i' \neq i} T_{i'} w_{i'}^{\varepsilon_b}$. Similarly, the probability of having at most conditional utility v is equal to:

$$G_m(v) = P\left(v \geq \max_{i'} v_{i'}\right) = e^{-\Phi_m v^{-\varepsilon_b}},$$

where $\Phi_m \equiv \sum_{i'} T_{i'} w_{i'}^{\varepsilon_b}$. Integrating $G_m^{-i}(v)$ over all possible values of v, we get:

$$\begin{aligned}
P\left(v_i \geq \max_{i' \neq i} v_{i'}\right) &= \int_0^\infty e^{-\Phi_m^{-i} v^{-\varepsilon_b}} dG_i(v) \\
&= \int_0^\infty \varepsilon_b T_i w_i^{\varepsilon_b} v^{-\varepsilon_b - 1} e^{-\Phi_m v^{-\varepsilon_b}} dv \\
&= \frac{T_i w_i^{\varepsilon_b}}{\Phi_m} \int_0^\infty \varepsilon_b \Phi_m v^{-\varepsilon_b - 1} e^{-\Phi_m v^{-\varepsilon_b}} dv \\
&= \frac{T_i w_i^{\varepsilon_b}}{\Phi_m} \int_0^\infty dG_m(v) = \frac{T_i w_i^{\varepsilon_b}}{\Phi_m}.
\end{aligned}$$

Now we need to find $P\left(\mathbb{E}_m(\max_i w_i z^1_{i|m}) z^2_m \geq \max_{m' \neq m} \mathbb{E}_{m'}(\max_i w_i z^1_{i|m'}) z^2_{m'}\right)$. So, the expected utility of working in sub-market m is:

$$\mathbb{E}_m(\max_i w_i z^1_{i|m}) = \int_0^\infty v_i dG_m(v) = \int_0^\infty \varepsilon_b \Phi_m v^{-\varepsilon_b} e^{-\Phi_m v^{-\varepsilon_b}} dv.$$

We define this new variable:

$$x = \Phi_m v^{-\varepsilon_b} \qquad dx = -\varepsilon_b \Phi_m v^{-(\varepsilon_b + 1)} dv.$$

Now we can change variable in the previous integral and obtain:

$$\int_0^\infty x^{-1/\varepsilon_b} \Phi_m^{1/\varepsilon_b} e^{-x} dx = \Gamma\left(\frac{\varepsilon_b - 1}{\varepsilon_b}\right) \Phi_m^{1/\varepsilon_b},$$

where $\Gamma(\cdot)$ is the Gamma function. Defining $\Gamma_b \equiv \Gamma\left(\frac{\varepsilon_b - 1}{\varepsilon_b}\right)$, we can rewrite:

$$P\left(\mathbb{E}_m(\max_i w_i z^1_{i|m}) z^2_m \geq \max_{m' \neq m} \mathbb{E}_{m'}(\max_i w_i z^1_{i|m'}) z^2_{m'}\right) = P\left(\Phi_m^{1/\varepsilon_b} \Gamma_b z^2_m \geq \max_{m' \neq m} \Phi_{m'}^{1/\varepsilon_{b'}} \Gamma_{b'} z^2_{m'}\right).$$

Following similar arguments as above, this probability is equal to:

$$P\left(\Phi_m^{1/\varepsilon_b} \Gamma_b z^2_m \geq \max_{m' \neq m} \Phi_{m'}^{1/\varepsilon_b} \Gamma_{b'} z^2_{m'}\right) = \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^\eta}{\Phi},$$

where $\Phi \equiv \sum_{b' \in \mathcal{B}} \sum_{m' \in \mathcal{M}_{b'}} \Phi_{m'}^{\eta/\varepsilon_{b'}} \Gamma_{b'}^{\eta}$.

Finally, combining the two probabilities we obtain:

$$\Pi_i = \frac{T_i w_i^{\varepsilon_b}}{\Phi_m} \times \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^{\eta}}{\Phi}.$$

By integrating Π_i over the whole measure of workers L, we can obtain the labor supply for each establishment:

$$L_i = \frac{T_i w_i^{\varepsilon_b}}{\Phi_m} \times \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^{\eta}}{\Phi} L.$$

Workers' welfare. An obvious way to measure workers welfare would be to compute the average utility for workers. However this is not possible as the estimated shape parameter η is smaller than 1.[1] This implies that the mean for the Fréchet distributed utilities is not defined. Instead, we compute the median utility agents expect to receive in each local labor market. This is equal to:

$$\text{Median}\left[\max_m \mathbb{E}_m (\max_i w_i z_{i|m}^1) z_m^2\right] \propto \Phi^{1/\eta}.$$

A.2 Establishment decision

In the absence of bargaining, the profit maximization problem of establishment i is:

$$\max_{w_{io}, K_{io}} P_b \widetilde{A}_{io} K_{io}^{\alpha_b} L_{io}^{\beta_b} - w_{io} L_{io}(w_{io}) - R_b K_{io}, \tag{A1}$$

where $L_{io}(w_{io})$ is the labor supply (12) where they take Φ and L as given but internalize their effect on Φ_m. P_b and R_b are respectively the sector price and rental rate of capital.[2]. The first order conditions of this problem are:

$$\begin{aligned} w_{io} &= \beta_b \frac{e_{io}}{e_{io}+1} P_b \widetilde{A}_{io} K_{io}^{\alpha_b} L_{io}^{\beta_b - 1}, \\ R_b &= \alpha_b P_b \widetilde{A}_{io} K_{io}^{\alpha_b - 1} L_{io}^{\beta_b}. \end{aligned} \tag{A2}$$

$e_{io} = \varepsilon_b \left(1 - s_{io|m}\right) + \eta \, s_{io|m}$ is the perceived elasticity of supply for establishment i in occupation o.

We can use the first order conditions of capital to substitute it into the establishment's production function and obtain an expression that depends only in labor:

$$y_{io} = \left(\frac{\alpha_b}{R_b}\right)^{\frac{\alpha_b}{1-\alpha_b}} \widetilde{A}_{io}^{\frac{1}{1-\alpha_b}} L_{io}^{\frac{\beta_b}{1-\alpha_b}} P_b^{\frac{\alpha_b}{1-\alpha_b}}. \tag{A3}$$

In order to gain tractability in the solution of the model, we restrict the output elasticity with respect to capital, such that $1 - \frac{\beta_b}{1-\alpha_b} = \delta$, where $\delta \in [0, 1]$ is a constant across sectors. This specification would nest a constant returns to scale technology when $\delta = 0$. As long as $0 < \delta < 1$ the establishment faces decreasing returns to scale within occupations. Define a transformed

[1] See Section 5 in the main text.

[2] The construction details of the rental rate of capital are in Section G.3 of the Online Appendix

productivity $A_{io} \equiv \widetilde{A}_{io}^{\frac{1}{1-\alpha_b}} \left(\frac{\alpha_b}{R_b}\right)^{\frac{\alpha_b}{1-\alpha_b}}$. The establishment-occupation production is:

$$y_{io} = P_b^{\frac{\alpha_b}{1-\alpha_b}} A_{io} L_{io}^{1-\delta}. \tag{A4}$$

Maximization (A1) is therefore equivalent to:

$$\max_{w_{io}} (1-\alpha_b) P_b^{\frac{1}{1-\alpha_b}} A_{io} L_{io}^{1-\delta} - w_{io} L_{io}(w_{io}), \tag{A5}$$

A.3 Markdown function

We obtain the markdown function from the establishment's optimality condition with respect to wages abstracting from wage bargaining. For the full derivation of the wage with also with bargaining, see the main Appendix. Establishments post a wage and choose capital quantity in order to maximize profits subject to their individual labor supply. Establishments only take into account the effect on their local labor market. As explained in the main text, this can happen because of a myopic behavior from the establishments or if there is a continuum of local labor markets. The establishment problem is:

$$\max_{w_{io}, K_{io}} P_b \sum_{o=1}^{O} \widetilde{A}_{io} K_{io}^{\alpha_b} L_{io}^{\beta_b} - \sum_{o=1}^{O} w_{io} L_{io}(w_{io}) - R_b \sum_{o=1}^{O} K_{io},$$

The first order condition with respect to the wage is:

$$P_b \frac{\partial F}{\partial L_{io}} \frac{\partial L_{io}}{\partial w_{io}} = L_{io}(w_{io}) + w_{io} \frac{\partial L_{io}}{\partial w_{io}},$$

where the derivative of the labor supply L_{io} with respect to the establishment-occupation wage w_{io} is:

$$\begin{aligned}
\frac{\partial L_{io}}{\partial w_{io}} &= \frac{L\Gamma_b^{\eta}}{\Phi} \left(\left[\frac{\varepsilon_b T_{io} w_{io}^{\varepsilon_b - 1} \Phi_m - T_{io} w_{io}^{\varepsilon_b} \varepsilon_b T_{io} w_{io}^{\varepsilon_b - 1}}{\Phi_m^2} \right] \Phi_m^{\eta/\varepsilon_b} + \eta \frac{T_{io} w_{io}^{\varepsilon_b}}{\Phi_m} \Phi_m^{\eta/\varepsilon_b - 1} T_{io} w_{io}^{\varepsilon_b - 1} \right) \\
&= \frac{\varepsilon_b T_{io} w_{io}^{\varepsilon_b - 1}}{\Phi_m} \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^{\eta}}{\Phi} L - \frac{\varepsilon_b T_{io} w_{io}^{\varepsilon_b - 1} \Phi_m^{\eta/\varepsilon_b} \Gamma_b^{\eta}}{\Phi_m \Phi} L \frac{T_{io} w_{io}^{\varepsilon_b}}{\Phi_m} + \eta \frac{T_{io} w_{io}^{\varepsilon_b}}{\Phi_m} \frac{T_{io} w_{io}^{\varepsilon_b - 1}}{\Phi_m} \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^{\eta}}{\Phi} L \\
&= \varepsilon_b \frac{L_{io}}{w_{io}} - \varepsilon_b \frac{L_{io}}{w_{io}} \frac{L_{io}}{L_m} + \eta \frac{L_{io}}{w_{io}} \frac{L_{io}}{L_m} \\
&= \frac{L_{io}}{w_{io}} \left(\varepsilon_b (1 - s_{io|m}) + \eta s_{io|m} \right).
\end{aligned}$$

Substituting this last derivative into the first order condition we get:

$$\begin{aligned}
& L_{io} + L_{io} \left(\varepsilon_b (1 - s_{io|m}) + \eta s_{io|m} \right) = P_b \frac{\partial F}{\partial L_{io}} \frac{L_{io}}{w_{io}} \left(\varepsilon_b (1 - s_{io|m}) + \eta s_{io|m} \right) \\
\Rightarrow \quad & w_{io} = \frac{\varepsilon_b (1 - s_{io|m}) + \eta s_{io|m}}{\varepsilon_b (1 - s_{io|m}) + \eta s_{io|m} + 1} P_b \frac{\partial F}{\partial L_{io}} \\
& w_{io} = \mu(s_{io|m}) P_b \frac{\partial F}{\partial L_{io}}.
\end{aligned}$$

B Proofs

Proof of Proposition 1. Substituting the labor supply (12) into (15) and using the restriction $\frac{\beta_b}{1-\alpha_b} = 1-\delta \in [0,1]$, we obtain:

$$w_{io} = \left(\lambda(\mu_{io}, \varphi_b)\beta_b \frac{A_{io}}{(T_{io}\Gamma_b^{\eta})^{\delta}}\right)^{\frac{1}{1+\varepsilon_b\delta}} \Phi_m^{\frac{\delta(1-\eta/\varepsilon_b)}{1+\varepsilon_b\delta}} P_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b\delta)}} \left(\frac{\Phi}{L}\right)^{\frac{\delta}{1+\varepsilon_b\delta}}. \tag{B1}$$

Equation (11) implies that in equilibrium the employment share of the establishment-occupation is:

$$\begin{aligned}
s_{io|m} &= \frac{T_{io}w_{io}^{\varepsilon_b}}{\Phi_m} \\
&= \frac{T_{io}w_{io}^{\varepsilon_b}}{\sum_{j\in\mathcal{I}_m} T_{jo}w_{jo}^{\varepsilon_b}} \\
&= \frac{T_{io}\left(\beta_b\lambda(\mu_{io},\varphi_b)\frac{A_{io}}{(T_{io}\Gamma_b^{\eta})^{\delta}}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}{\sum_{j\in\mathcal{I}_m} T_{jo}\left(\beta_b\lambda(\mu_{jo},\varphi_b)\frac{A_{jo}}{(T_{jo}\Gamma_b^{\eta})^{\delta}}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}} \\
&= \frac{T_{io}^{\frac{1}{1+\varepsilon_b\delta}}\lambda_{io}^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}A_{io}^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}{\sum_{j\in\mathcal{I}_m} T_{jo}^{\frac{1}{1+\varepsilon_b\delta}}\lambda_{io}^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}A_{jo}^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}},
\end{aligned}$$

where we used equation (B1) in the second step and simplified terms in the last one. The solutions of the labor wedge $\lambda_{io}(\mu_{io}, \varphi_b)$ and the markdown come respectively from equations (15) and (14). □

Proof of Proposition 2.

Existence. We follow closely the proof by Kucheryavyy (2012). Define the right hand side of (B1) as:

$$\begin{aligned}
f_{io}(\mathbf{w}) &= [\lambda(\mu_{io}(\mathbf{w}), \varphi_b)]^{\frac{1}{1+\varepsilon_b\delta}} c_{io}, \\
f_{io}(\mathbf{w}) &= [\lambda(\mu(s(\mathbf{w})))]^{\frac{1}{1+\varepsilon_b\delta}} c_{io},
\end{aligned}$$

where $\mathbf{w}$ denotes the vector formed by $\{w_{io}\}$, we simplified the notation of the wedge $\lambda(\mu_{io}, \varphi_b)$ from the main text by getting rid of the second argument. $c_{io} = \left(\beta_b \frac{A_{io}}{(T_{io}\Gamma_b^{\eta})^{\delta}}\right)^{\frac{1}{1+\varepsilon_b\delta}} \Phi_m^{(1-\eta/\varepsilon_b)\frac{\delta}{1+\varepsilon_b\delta}}$ $P_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b\delta)}} \left(\frac{\Phi}{L}\right)^{\frac{\delta}{1+\varepsilon_b\delta}}$ is an establishment-occupation specific parameter. This means we consider Φ_m and Φ as constants and not as functions of w_{io}.

Under the assumption $0 < \eta < \varepsilon_b$, the function $\mu(s) = \frac{\varepsilon_b(1-s)+\eta s}{\varepsilon_b(1-s)+\eta s+1}$ is decreasing in s, the employment share out of the local labor market. Therefore, we can conclude that the wedge $\lambda(\mu(s)) = (1-\varphi_b)\mu(s) + \varphi_b\frac{1}{1-\delta}$ is also decreasing in s. The employment share has bounds $0 \le s \le 1$, which implies $(1-\varphi_b)\frac{\eta}{\eta+1} + \varphi_b\frac{1}{1-\delta} \le \lambda(\mu(s)) \le (1-\varphi_b)\frac{\varepsilon_b}{\varepsilon_b+1} + \varphi_b\frac{1}{1-\delta}$. Also, $1+\varepsilon_b\delta > 0$.

Therefore, it follows that $f_{io}(\mathbf{w})$ is bounded:

$$\left((1-\varphi_b)\frac{\eta}{\eta+1}+\varphi_b\frac{1}{1-\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} c_{io} \leq f_i(\mathbf{w}) \leq \left((1-\varphi_b)\frac{\varepsilon_b}{\varepsilon_b+1}+\varphi_b\frac{1}{1-\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} c_{io}.$$

If the number of participants in sub-market m is $N_m > 0$, we can define the compact set S where $f_{io}(\mathbf{w})$ maps into itself as:

$$\begin{aligned} S = & \left[\left((1-\varphi_b)\frac{\eta}{\eta+1}+\varphi_b\frac{1}{1-\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} c_1, \left((1-\varphi_b)\frac{\varepsilon_b}{\varepsilon_b+1}+\varphi_b\frac{1}{1-\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} c_1\right] \times ... \\ & \times \left[\left((1-\varphi_b)\frac{\eta}{\eta+1}+\varphi_b\frac{1}{1-\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} c_{N_m}, \left((1-\varphi_b)\frac{\varepsilon_b}{\varepsilon_b+1}+\varphi_b\frac{1}{1-\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} c_{N_m}\right]. \end{aligned}$$

The function $f_{io}(\mathbf{w})$ is continuous in wages on S. We can therefore apply Brouwer's fixed point theorem and claim that at least one solution exists for the system of equations formed by (B4). □

Uniqueness. First we introduce the following Theorem and Corollary that we will use later to establish uniqueness in our proofs. These are taken from Allen, Arkolakis, and Li (2016) as they are not present any more in the current version of their paper Allen, Arkolakis, and Li (2021). Of course, any error should be attributed to us.

Theorem 1. *Consider $g : \mathbb{R}^n_{++} \times \mathbb{R}^m_{++}$ for some $n \in \{1, ..., N\}$ and $m \in \{1, ..., M\}$ such that:*

(i) homogeneity of any degree: $g(tx, ty) = t^k g(x, y)$, $t \in \mathbb{R}_{++}$ and $k \in \mathbb{R}$,

(ii) gross-substitution property: $\frac{\partial g_i}{\partial x_j} > 0$ for all $i \neq j$,

(iii) monotonicity with respect to the joint variable: $\frac{\partial g_i}{\partial y_k} \geq 0$, for all i, k.

Then, for any given $y^0 \in \mathbb{R}^M_{++}$ there exists at most one solution satisfying $g(x, y^0) = 0$.

Proof. We proceed by contradiction. Suppose there are two different up-to-scale, solutions, x^1, x^2, such that $f(x^1) = f(x^2) = 0$ i.e. $g(x^1, y^0) = g(x^2, y^0) = 0$. Without loss of generality, suppose there exists some $t > 1$ such that $tx^1_j \geq x^2_j$ for all $j \in \{1, ..., n\}$ and the equality holds for at least one $j = \bar{j}$. Then the inequality must strictly hold since x^1 and x^2 are different up-to-scale. Condition (iii) $\frac{\partial g_i}{\partial y_k} \geq 0$, for all i, k implies that $g(tx^1, y^0) \leq g(tx^1, ty^0) = 0$ where $g(tx^1, ty^0) = 0$ is from condition (i) (and also $g(tx^2, ty^0) = 0$ because x^1 and x^2 are solutions). However, condition (ii) implies $g_j(tx^1, y^0) > g_j(x^2, y^0) = 0$, thus a contradiction. □

Corollary 1. *Assume (i) $f(x)$ satisfies gross-substitution and (ii) $f(x)$ can be decomposed as $f(x) = \sum_{j=1}^{\nu_f} g^j(x) - \sum_{k=1}^{\nu_g} h^k(x)$, where $g^j(x), h^k(x)$ are non-negative vector functions and, respectively, homogeneous of degree α_j and β_k, with $\bar{\alpha} = \max \alpha_j \leq \min \beta_k$.*

1. *Then there is at most one up-to-scale solution of $f(x) = 0$.*

2. *In particular, if for some j, k $\alpha_j \neq \beta_k$, then there is at most one solution.*

Proof. Define $m(x,y)$ as a vector function where $m_i(x,y) = \sum_{j=1}^{\nu_f} y^{\bar{\alpha}-\alpha_j} g_i^j(x) - \sum_{k=1}^{\nu_g} y^{\bar{\alpha}-\beta_k} h_i^k(x)$. Obviously, $m(x,y)$ is of homogenous degree $\bar{\alpha}$ and $\frac{\partial m_i}{\partial y} \geq 0$. Also we have $f(x) = m(x,y^0)$ where $y^0 = 1$, thus the above theorem applies.

Furthermore, if $f_i(x)$ is not homogeneous of some degree because $\alpha_j \neq \beta_k$, there is at most one solution. Suppose not, if tx^1 and x^1 are the solutions, then $f_i(x^1) > t^{-min(\beta_k)} f_i(tx^1) = 0$, also a contradiction. $\square$

In order to prove uniqueness we use Theorem 1 and Corollary 1 stated above.

Define the function $g : \mathbb{R}^n_{++} \to \mathbb{R}^n$ for some $n \in \{1, ..., N\}$ as:

$$g_{io}(\mathbf{w}) = f_{io}(\mathbf{w}) - w_{io}, \quad \forall i \in \{1, .., N_m\}.$$

We want to prove that the solution satisfying $g(\mathbf{w}) = 0$ is unique. In order to do so, we first need to show that $g(\mathbf{w})$ satisfies the gross substitution property ($\frac{\partial g_{io}}{\partial w_{jo}} > 0$ for any $j \neq i$).

Taking the partial derivative of g_{io} with respect to w_{jo} for any $j \neq i$:

$$\frac{\partial g_{io}}{\partial w_{jo}} = \frac{\partial f_{io}(\mathbf{w})}{\partial \lambda(\mu(s(\mathbf{w}))} \times \frac{\partial \lambda(\mu(s_{io|m}))}{\partial \mu(s_{io|m})} \times \frac{\partial \mu(s_{io|m})}{\partial s_{io|m}} \times \frac{\partial s_{io|m}}{\partial w_{jo}},$$

where $\frac{\partial f_{io}(\mathbf{w})}{\partial \lambda(\mu(s(\mathbf{w}))} = \frac{1}{1+\varepsilon_b \delta} \frac{f_{io}(\mathbf{w})}{\lambda(\mu(s(\mathbf{w}))} > 0$. We have that $\frac{\partial \lambda(\mu(s_{io|m}))}{\partial \mu(s_{io|m})} > 0$. Furthermore, we previously established that $\frac{\partial \mu(s_{io|m})}{\partial s_{io|m}} < 0$ under the assumption that $0 < \eta < \varepsilon_b$. The share of an establishment i with occupation o in sub-market m is defined as:

$$s_{io|m} = \frac{T_{io} w_{io}^{\varepsilon_b}}{\sum_{j \in \mathcal{I}_m} T_{jo} w_{jo}^{\varepsilon_b}}.$$

Clearly, $\frac{\partial s_{io|m}}{\partial w_{jo}} < 0$ for any $i \neq j$. Therefore $\frac{\partial g_{io}}{\partial w_{jo}} > 0$ for any $i \neq j$ and g satisfies the gross-substitution property.

The remaining condition to use Corollary 1 is simply that $f_{io}(\mathbf{w})$ is homogeneous of a degree smaller than 1.[3] Clearly, $f_{io}(\mathbf{w})$ is homogeneous of degree 0 as a consequence that the markdown function itself $\mu(s_{io|m})$ is homogeneous of degree 0. Therefore, the function g satisfies the conditions of Corollary 1, and we can conclude that there exists at most one solution satisfying $g(\mathbf{w}) = 0$. $\square$

Existence and uniqueness of local market equilibrium in Berger, Herkenhoff, and Mongey (2022). Our proof extends easily to the case consider by Berger et al. (2022), where instead of using shares of employment $s_{io|m}$, they use wage bill shares $s^w_{io|m} = \frac{w_{io} L_{io}}{\sum_{j \in \mathcal{I}_m} w_{jo} Ljo}$, and no bargaining power. i.e. $\varphi_b = 0$. The existence proof is exactly the same. For uniqueness and to establish gross-substitution of a similar function $g_{io}(\mathbf{w})$, we can follow all the steps of the previous proof and note that:

$$s^w_{io|m} = \frac{T_{io} w_{io}^{1+\varepsilon_b}}{\sum_{j \in \mathcal{I}_m} T_{jo} w_{jo}^{1+\varepsilon_b}}.$$

[3] The degree of homogeneity of $h_{io}(\mathbf{w}) = w_{io}$ is 1.

Thus, clearly, $\frac{\partial s_{io|m}}{\partial w_{jo}} < 0$ for any $i \neq j$ and $g_{io}(\mathbf{w})$ also satisfies the gross-substitution property. Then we can conclude that the local labor market equilibrium of Berger et al. (2022) also exists and is unique.

Proof of Proposition 3. Aggregating establishment-occupation output (6)and using the restriction $\frac{\beta_b}{1-\alpha_b} = 1 - \delta \in [0, 1]$, the local labor market output is:

$$
\begin{aligned}
Y_m &= \sum_{i \in \mathcal{I}_m} y_{io} \\
&= P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{i \in \mathcal{I}_m} A_{io} L_{io}^{1-\delta} \\
&= P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{i \in \mathcal{I}_m} A_{io} s_{io|m}^{1-\delta} L_m^{1-\delta} \\
&= P_b^{\frac{\alpha_b}{1-\alpha_b}} \Omega_m A_m L_m^{1-\delta},
\end{aligned}
$$

where the local labor market productivity and misallocation are measured as:

$$
\begin{aligned}
\Omega_m &\equiv \sum_{i \in \mathcal{I}_m} \frac{A_{io}}{A_m} s_{io|m}^{1-\delta} \\
A_m &\equiv \sum_{i \in \mathcal{I}_m} A_{io} \tilde{s}_{io|m}^{1-\delta} \\
\tilde{s}_{io|m} &= \frac{\left(T_{io}^{1/\varepsilon_b} A_{io}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}}{\sum_{j \in \mathcal{I}_m} \left(T_{jo}^{1/\varepsilon_b} A_{jo}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}}.
\end{aligned}
$$

The definition of $\tilde{s}_{io|m}$ comes from Proposition 1 with constant labor wedges.

Further aggregating to sector level according to (4):

$$
\begin{aligned}
Y_b = \sum_{m \in \mathcal{M}_b} Y_m &= P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{m \in \mathcal{M}_b} \Omega_m A_m L_m^{1-\delta} \\
&= P_b^{\frac{\alpha_b}{1-\alpha_b}} \Omega_b A_b L_b^{1-\delta}.
\end{aligned} \tag{B2}
$$

The sector level measures of productivity and misallocation are:

$$\begin{aligned}
\Omega_b &\equiv \sum_{m\in\mathcal{M}_b} \Omega_m \frac{A_m}{A_b} s_{m|b}^{1-\delta} \\
&= \sum_{m\in\mathcal{M}_b} \sum_{io\in\mathcal{I}_m} \frac{A_{io}}{A_b} s_{io|m}^{1-\delta} s_{m|b}^{1-\delta}, \\
A_b &\equiv \sum_{m\in\mathcal{M}_b} A_m \tilde{s}_{m|b}^{1-\delta} \\
&= \sum_{m\in\mathcal{M}_b} \sum_{io\in\mathcal{I}_m} A_{io} \tilde{s}_{io|m}^{1-\delta} \tilde{s}_{m|b}^{1-\delta}, \\
\tilde{s}_{m|b} &= \frac{\left[\sum_{j\in\mathcal{I}_m} \left(T_{jo}^{1/\varepsilon_b} A_{jo}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}\right]^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta)}}}{\sum_{m'\in\mathcal{M}_b} \left[\sum_{j'\in\mathcal{I}_{m'}} \left(T_{j'o}^{1/\varepsilon_b} A_{j'o}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}\right]^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta)}}}.
\end{aligned}$$

A_b is an employment weighted industry productivity with the employment shares that would arise with constant labor wedges. Similary, Ω_b is an employment weighted sum of productivities where employment shares incorporate the labor wedge normalized by A_b. The covariance between productivities and employment shares is key in order to determine sector productivity. As long as market power distorts the employment distribution making more productive firms to constrain their size, the covariance between productivity and employment is lower than in the case with constant wedges.

Turning to wages, from (15), the establishment wage bill is:

$$\begin{aligned}
w_{io} L_{io} &= \beta_b P_b^{\frac{1}{1-\alpha_b}} \lambda_{io} A_{io} L_{io}^{1-\delta} \\
&= \beta_b \lambda_{io} P_b y_{io},
\end{aligned}$$

where we used the production function (6). The local labor market wage bill is,

$$\begin{aligned}
\sum_{i\in\mathcal{I}_m} w_{io} L_{io} &= \beta_b \sum_{i\in\mathcal{I}_m} \lambda_{io} P_b y_{io} \\
&= \beta_b \sum_{i\in\mathcal{I}_m} \lambda_{io} \frac{P_b y_{io}}{P_b Y_m} P_b Y_m \\
&= \beta_b \lambda_m P_b Y_m, \\
\lambda_m &\equiv \sum_{i\in\mathcal{I}_m} \lambda_{io} \frac{A_{io}}{\Omega_m A_m} s_{io|m}^{1-\delta},
\end{aligned}$$

where λ_m is a value added weighted sum of establishment labor wedges. Aggregating to the sector,

$$\begin{aligned}
\sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} w_{io}L_{io} &= \beta_b \sum_{m\in\mathcal{M}_b} \lambda_m \frac{P_b Y_m}{P_b Y_b} P_b Y_b \\
&= \beta_b \lambda_b P_b Y_b, \\
\lambda_b &\equiv \sum_{m\in\mathcal{M}_b} \lambda_m \frac{A_m \Omega_m}{\Omega_b A_b} s_{m|b}^{1-\delta} \\
&= \sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} \lambda_{io} \frac{A_{io}}{\Omega_b A_b} s_{io|m}^{1-\delta} s_{m|b}^{1-\delta}.
\end{aligned}$$

Using the sectoral production function (B2) and the final good production function (2) we have that:

$$\begin{aligned}
Y &= \prod_{b\in\mathcal{B}} \left(P_b^{\frac{\alpha_b}{1-\alpha_b}} A_b \Omega_b L_b^{1-\delta} \right)^{\theta_b} \\
&= \prod_{b\in\mathcal{B}} P_b^{\frac{\alpha_b\theta_b}{1-\alpha_b}} \prod_{b\in\mathcal{B}} \left(A_b \Omega_b s_b^{1-\delta} \right)^{\theta_b} L^{1-\delta} \\
&= \prod_{b\in\mathcal{B}} \bar{P}_b^{\frac{\alpha_b\theta_b}{1-\alpha_b}} \prod_{b\in\mathcal{B}} \left[\Omega_b \frac{A_b}{A} s_b^{1-\delta} \left(\frac{P_b}{\bar{P}_b} \right)^{\frac{\alpha_b}{1-\alpha_b}} \right]^{\theta_b} A L^{1-\delta} \\
&= \bar{P} \Omega A L^{1-\delta},
\end{aligned}$$

where:

$$\begin{aligned}
\Omega &\equiv \prod_{b\in\mathcal{B}} \left[\Omega_b \frac{A_b}{A} s_b^{1-\delta} \left(\frac{P_b}{\bar{P}_b} \right)^{\frac{\alpha_b}{1-\alpha_b}} \right]^{\theta_b} \\
A &\equiv \sum_{b\in\mathcal{B}} \sum_{m\in\mathcal{M}_b} \sum_{io\in\mathcal{I}_m} A_{io} \tilde{s}_{io|m}^{1-\delta} \tilde{s}_{m|b}^{1-\delta} \tilde{s}_b^{1-\delta} \\
&= \sum_{b\in\mathcal{B}} A_b \tilde{s}_b^{1-\delta} \\
\tilde{s}_b &= \frac{\sum_{m\in\mathcal{M}_b} \left[\sum_{j\in\mathcal{I}_m} \left(T_{jo}^{1/\varepsilon_b} A_{jo} \right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} \right]^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta)}}}{\sum_{b'\in\mathcal{B}} \sum_{m'\in\mathcal{M}_{b'}} \left[\sum_{j'\in\mathcal{I}_{m'}} \left(T_{j'o}^{1/\varepsilon_{b'}} A_{j'o} \right)^{\frac{\varepsilon_{b'}}{1+\varepsilon_{b'}\delta}} \right]^{\frac{\eta(1+\varepsilon_{b'}\delta)}{\varepsilon_{b'}(1+\eta)}}}.
\end{aligned}$$

Ω represents an aggregate misallocation measure taking into account general equilibrium effects, $\bar{P}_b$ is the price of sector b good if all the labor wedges in the economy where constant and A is a measure of undistorted productivity.

Aggregate labor share. From the above, the sector labor share is,

$$LS_b = \beta_b \lambda_b, \quad LS = \frac{\sum_{b\in\mathcal{B}} \sum_{m\in\mathcal{M}_b} \sum_{i\in\mathcal{I}_m} w_{io} L_{io}}{\sum_{b\in\mathcal{B}} \sum_{m\in\mathcal{M}_b} \sum_{i\in\mathcal{I}_m} P_b Y_{io}}. \tag{B3}$$

Realizing that industry b expenditure share is equal to θ_b,

$$LS = \sum_{b \in \mathcal{B}} \beta_b \lambda_b \theta_b.$$

For given parameters, knowing the industry wedges $\{\lambda_b\}_{b=1}^{B}$ is enough to compute the aggregate labor share.

□

Proof of Proposition 4.

Equation (B1) can be separated into two terms. First, a local labor market m constant. Second, an establishment-occupation specific component which is enough to characterize the local equilibrium as shown in Proposition 1. We denote this second term as:

$$\widetilde{w}_{io} = \left(\beta_b \lambda(\mu_{io}, \varphi_b) \frac{A_{io}}{\left(T_{io}\Gamma_b^{\eta}\right)^{\delta}} \right)^{\frac{1}{1+\varepsilon_b \delta}}, \tag{B4}$$

where $\widetilde{w}_{io}$ is a function of the employment shares of all the establishment-occupations in the local labor market equilibrium. The real wage w_{io} is,

$$w_{io} = \widetilde{w}_{io} \Phi_m^{(1-\eta/\varepsilon_b)\frac{\delta}{1+\varepsilon_b\delta}} P_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b\delta)}} \left(\frac{\Phi}{L}\right)^{\frac{\delta}{1+\varepsilon_b\delta}}.$$

We can use the definition of $\Phi_m \equiv \sum_{io \in \mathcal{I}_m} T_{io} w_{io}^{\varepsilon_b}$ to find:

$$\Phi_m = \widetilde{\Phi}_m^{\frac{1+\varepsilon_b\delta}{1+\eta\delta}} P_b^{\frac{\varepsilon_b}{(1-\alpha_b)(1+\eta\delta)}} \left(\frac{\Phi}{L}\right)^{\frac{\varepsilon_b\delta}{1+\eta\delta}}, \quad \widetilde{\Phi}_m \equiv \sum_{io \in \mathcal{I}_m} T_{io} \widetilde{w}_{io}^{\varepsilon_b}, \tag{B5}$$

where, as we mentioned before, $\widetilde{w}_{io}$ is a function of the employment share of io and $\widetilde{\Phi}_m$ is a function of the local labor market equilibrium $\{s_{io|m}\}_{io \in \mathcal{I}_m}$ that can be solved separated from aggregates as shown in Proposition 1.

Plugging the expression of Φ_m into the wage,

$$w_{io} = \widetilde{w}_{io} \widetilde{\Phi}_m^{\frac{(\varepsilon_b-\eta)\delta}{\varepsilon_b(1+\eta\delta)}} P_b^{\frac{1}{(1-\alpha_b)(1+\eta\delta)}} \left(\frac{\Phi}{L}\right)^{\frac{\delta}{1+\eta\delta}}. \tag{B6}$$

The establishment-occupation labor supply L_{io} can be written as $L_{io} = s_{io|m} s_{m|b} L_b$. Given the solution of normalized wages per sub-market $\widetilde{w}_{io}$, we can actually compute the employment share out of the local labor market $s_{io|m}$:

$$s_{io|m} = \frac{T_{io} w_{io}^{\varepsilon_b}}{\Phi_m} = \frac{T_{io} \widetilde{w}_{io}^{\varepsilon_b}}{\widetilde{\Phi}_m}, \quad \widetilde{\Phi}_m \equiv \sum_{i \in \mathcal{I}_m} T_{io} \widetilde{w}_{io}^{\varepsilon_b}.$$

We can also compute the employment share of the local labor market out of the industry $s_{m|b}$. Using the definition of $\Phi_b \equiv \sum_{m\in\mathcal{M}_b} \Phi_m^{\eta/\varepsilon_b}$ and (B5),

$$s_{m|b} = \frac{\Phi_m^{\eta/\varepsilon_b}}{\Phi_b} = \frac{\widetilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta\delta)}}}{\widetilde{\Phi}_b}, \quad \widetilde{\Phi}_b \equiv \sum_{m\in\mathcal{M}_b} \widetilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta\delta)}}.$$

where $\mathcal{M}_b$ is the set of all local labor markets that belong to industry b. This just formalizes the notion that, as long as we know the relative wages within an industry, we can compute the measure of workers that go to each establishment, conditioning on industry employment.

Using (B5), sector labor supply can be written as function of aggregators of 'tilde' variables that are functions of the local employment shares $\widetilde{\Phi}_b(\mathbf{s}_b)$, where $\mathbf{s}_b \equiv \{s_{io|m}\}_{io\in\mathcal{I}_b}$, and prices:

$$L_b = \frac{\Phi_b\Gamma_b^\eta}{\sum_{b'\in\mathcal{B}} \Phi_{b'}\Gamma_{b'}^\eta} L = \frac{P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \widetilde{\Phi}_b(\mathbf{s}_b)\Gamma_b^\eta}{\widetilde{\Phi}} L, \quad \widetilde{\Phi} \equiv \sum_{b'\in\mathcal{B}} P_{b'}^{\frac{\eta}{(1-\alpha_{b'})(1+\eta\delta)}} \widetilde{\Phi}_{b'}(\mathbf{s}_{b'})\Gamma_{b'}^\eta. \tag{B7}$$

This is where the simplifying assumption on the labor demand elasticity $\delta \equiv 1 - \frac{\beta_b}{1-\alpha_b}$ being constant across industries buys us tractability. We can factor out the economy wide constant from (B5) and leave everything in terms of normalized wages and transformed prices.

In order to find equilibrium allocations, we need to solve for the transformed prices $\mathbf{P} = \{P_b\}_{b=1}^{\mathcal{B}}$. Using the intermediate input demand from the final good producer (3) and the above expression for industry labor supply L_b we get:

$$P_b^{\frac{1+\eta}{(1-\alpha_b)(1+\eta\delta)}} A_b\Omega_b \left(\widetilde{\Phi}_b\Gamma_b^\eta\right)^{1-\delta} = \theta_b \prod_{b'\in\mathcal{B}} \left(A_{b'}\Omega_{b'}\left(\widetilde{\Phi}_{b'}\Gamma_{b'}^\eta\right)^{1-\delta}\right)^{\theta_{b'}} \prod_{b'\in\mathcal{B}} \left(P_{b'}^{\frac{\alpha_{b'}(1+\eta\delta)+\eta(1-\delta)}{(1-\alpha_{b'})(1+\eta\delta)}}\right)^{\theta_{b'}}. \tag{B8}$$

Define $f_b \equiv \frac{1}{1-\alpha_b}\log(P_b)$ and $\mathbf{f}$ as a $B\times 1$ vector whose element b' is $f_{b'}$. Then, taking logs and rearranging the previous expressions for all $b \in \mathcal{B}$ we obtain:

$$\mathbf{f} = \mathbf{C} + \mathbf{D}\mathbf{f}, \tag{B9}$$

where $\mathbf{C}$ is a $B\times 1$ vector whose b element is

$$(\mathbf{C})_b = \frac{1+\eta\delta}{1+\eta}\left[\log\left(\frac{\theta_b}{A_b\Omega_b}\right) - (1-\delta)\log\left(\widetilde{\Phi}_b\Gamma_b^\eta\right) + \sum_{b'\in\mathcal{B}}\theta_{b'}\left(\log(A_{b'}\Omega_{b'}) + (1-\delta)\log(\widetilde{\Phi}_{b'}\Gamma_{b'}^\eta)\right)\right],$$

and $\mathbf{D}$ is a $B\times B$ matrix whose b row b' column element is:

$$(\mathbf{D})_{bb'} = \frac{(\alpha_{b'}(1+\eta\delta)+\eta(1-\delta))\,\theta_{b'}}{1+\eta}.$$

A solution to the system (B9) exists and is unique if the matrix $\mathbf{I}-\mathbf{D}$ is invertible. This matrix has an eigenvalue of zero, and therefore is not invertible, if and only if $\mathbf{D}$ has an eigenvalue equal to one.[4] The matrix $\mathbf{D}$ has an eigenvalue equal to one if and only if the sum of the elements of the rows in matrix $\mathbf{D}$ are equal to 1. To see this, let $\mathbf{v}$ be the eigenvector associated with the unit eigenvalue of $\mathbf{D}$, i.e. $\mathbf{D}\mathbf{v} = \mathbf{v}$. If $\mathbf{v} = \mathbf{1}$, then, by the Perron-Frobenius theorem, it is the only eigenvector (up-to-

[4]Proof: If 1 is an eigenvalue of $\mathbf{D}$, then $\mathbf{D}\mathbf{v} = \mathbf{v}$ for a nonzero vector $\mathbf{v}$. Then $(\mathbf{I}-\mathbf{D})\mathbf{v} = 0$, so 0 is an eigenvalue of $\mathbf{I}-\mathbf{D}$ with the associated eigenvector $\mathbf{v}$. Conversely, if 0 is an eigenvalue of $\mathbf{I}-\mathbf{D}$, then $\mathbf{D}\mathbf{v} = \mathbf{v}$ and 1 is an eigenvalue of $\mathbf{D}$.

scale) associated with the unit eigenvalue. Furthermore, if $\mathbf{v} = \mathbf{1}$, then $\sum_{b'} (D)_{bb'} = 1$ for all $b \in \mathcal{B}$. Conversely, if $\sum_{b'} (D)_{bb'} = 1$ for all $b \in \mathcal{B}$, then $\mathbf{v} = \mathbf{1}$ is a solution for the eigensystem $\mathbf{Dv} = \mathbf{v}$. But, by the Perron-Frobenius theorem, $\mathbf{v} = \mathbf{1}$ is the unique (up-to-scale) eigenvector associated with the unit eigenvalue. Therefore, the matrix $\mathbf{I} - \mathbf{D}$ is not invertible if and only if the sum of the elements of the rows in matrix $\mathbf{D}$ are equal to 1.

This sum is equal to 1 if and only if $\sum_b \alpha_b \theta_b = 1$ as:

$$\begin{aligned}\sum_{b'} (\mathbf{D})_{bb'} = 1 \; &\Leftrightarrow \sum_{b'} \left(\alpha_{b'}(1+\eta\delta) + \eta(1-\delta)\right)\theta_{b'} = 1+\eta \\ &\Leftrightarrow \sum_{b'} \alpha_{b'}\theta_{b'}(1+\eta\delta) = 1+\eta-\eta(1-\delta) \\ &\Leftrightarrow \sum_{b'} \alpha_{b'}\theta_{b'} = \frac{1+\eta-\eta(1-\delta)}{1+\eta\delta} \;\Leftrightarrow \sum_{b} \alpha_{b'}\theta'_{b} = 1.\end{aligned}$$

Therefore we can conclude that whenever $\sum_b \alpha_b \theta_b \neq 1$, $\mathbf{f}$ has a unique solution. Also, if $\alpha_b \neq 1$ for all $b \in \mathcal{B}$, then the vector of prices $[P_b]_{b\in\mathcal{B}}$ has a unique solution as well.

Solving for P_b in (B8) we get:

$$P_b = X_b X^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}}, \tag{B10}$$

$$X_b = \left(\frac{\theta_b}{A_b\Omega_b(\widetilde{\Phi}_b\Gamma_b^{\eta})^{(1-\delta)}}\right)^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}}, \quad X = \left(\prod_{b'\in\mathcal{B}}\left(\frac{\theta_{b'}}{X_{b'}}\right)^{\theta_{b'}}\right)^{\frac{1+\eta}{(1+\eta\delta)\sum_{b'\in\mathcal{B}}\theta_{b'}(1-\alpha_{b'})}},$$

for all $b \in \mathcal{B}$ where we used the aggregate price index $1 = \prod_{b\in\mathcal{B}} \left(\frac{P_b}{\theta_b}\right)^{\theta_b}$ to find the economy wide constant X. The above is the closed-form solution of prices in Proposition 4.

The sector price P_b depends positively on the final good elasticity θ_b, reflecting that a higher demand for goods of sector b will increase its price. It also negatively depends on the product of productivity and misallocation $A_b\Omega_b$ and the labor supply shifter for sector b, Γ_b. An increase in any of both terms translates into more supply of sector b goods, either by being more productive or by increasing the labor employed in sector b. This in turn would reduce its price.

□

C Additional derivations

C.1 Hat algebra

This section shows that it is possible to compute the counterfactuals in general equilibrium by using revenue productivities (TFPRs), which are a function of prices determined in general equilibrium, and not just the underlying physical productivities. A priori, the issue is that counterfactually changing the labor wedge changes equilibrium prices and therefore the 'fundamental' TFPRs.

The literature on misallocation has used the TFPRs, together with a modeling assumption on the sector price, to compute the normalized within sector productivity distribution. This has prevented performing general equilibrium counterfactuals that also take into account productivity differences across industries.[5] We show that we can: (i) carry out counterfactuals in general equilibrium by writing the model in relative terms from a baseline scenario; and (ii) compute the movement of production factors across industries.

Our approach is to write counterfactual sector prices relative to the baseline and to fix the transformed revenue productivities Z_{io}.[6] Using the definition for $Z_{io} = PP_b^{\frac{1}{1-\alpha_b}} A_{io}$ and equation (25), nominal wages are equal to:

$$Pw_{io} = \beta_b \lambda(\mu_{io}, \varphi_b) Z_{io} L_{io}^{-\delta}.$$

We denote with a prime the variables in the counterfactual (e.g. P_b') and with a hat the relative variables (e.g. $\widehat{P}_b = \frac{P_b'}{P_b}$). Writing the model as deviations from a baseline scenario has been dubbed 'exact-hat-algebra' by Costinot and Rodríguez-Clare (2014). We can then rewrite the revenue productivity in a counterfactual in hat terms as:

$$Z_{io}' = P' P_b'^{\frac{1}{1-\alpha_b}} A_{io} = \widehat{P} \widehat{P}_b^{\frac{1}{1-\alpha_b}} Z_{io}.$$

The counterfactual revenue productivity is a function of the relative price $\widehat{P}_b$ and the observed revenue productivity Z_{io}. Denoting by λ_{io}' the counterfactual wedge, the counterfactual real wages are:

$$\begin{aligned} w_{io}' &= \beta_b \lambda_{io}' Z_{io}' L_{io}'^{-\delta} \frac{1}{P'} \\ &= \beta_b \lambda_{io}' Z_{io} \frac{\widehat{P}_b^{\frac{1}{1-\alpha_b}}}{P} L_{io}'^{-\delta}, \end{aligned} \tag{C1}$$

where in the last step we used the definition of the transformed TFPRs. In the counterfactuals Z_{io} is taken as a fixed fundamental and we have to solve for sector prices relative to the baseline $\widehat{P}_b$.

[5]For example, Hsieh and Klenow (2009) conduct a counterfactual where they remove distortions at the firm level and compute the productivity gains at the *sector* level. The productivity gains are a result of factors of production reallocating to more productive firms *within* each sector. This allows them to compute a *partial* equilibrium effect on total factor productivity, i.e. keeping the production factors constant *across* industries. A general equilibrium effect on total factor productivity takes into account, not only the reallocation of inputs within, but also across industries. They cannot do this as they can identify only relative productivity differences within each sector while normalizing average differences across industries. For more details, see equation (19) and the discussion below in their paper.

[6]Solving the counterfactuals in levels as stated in Section 4 would require to back out the productivities. It would be possible to do so by making some additional normalizations per sector. For example, one could assume that the minimum physical productivity (or Total Factor Productivity, TFP) is constant across industries and get rid of sector relative prices by normalizing the minimum TFP per sector.

The system (B1) in the counterfactual writes as:

$$w'_{io} = \omega_{io} \left(\frac{\widehat{P}_b^{\frac{1}{1-\alpha_b}}}{P} \right)^{\frac{1}{1+\varepsilon_b\delta}} \Phi'^{\frac{\delta(\varepsilon_b-\eta)}{\varepsilon_b(1+\varepsilon_b\delta)}}_m \left(\frac{\Phi'}{L} \right)^{\frac{\delta}{1+\varepsilon_b\delta}}, \tag{C2}$$

where the establishment-occupation component in the counterfactual, ω_{io}, is:

$$\omega_{io} \equiv \left(\beta_b \lambda'_{io} \frac{Z_{io}}{(T_{io}\Gamma_b^{\eta})^{\delta}} \right)^{\frac{1}{1+\varepsilon_b\delta}}.$$

Finally, the counterfactual establishment-occupation components ω_{io} are enough to compute the employment shares at the local labor market level, $s'_{io|m}$, and at the sector level, $s'_{m|b}$, as shown in Propositions 1 and 3.

To see why the employment shares and wages in the baseline are sufficient statistics for the fundamentals, we can rewrite equation (16) in Proposition 1 with revenue productivities instead of physical productivities for the counterfactual:

$$s'_{io|m} = \frac{\left(T_{io}^{\frac{1}{\varepsilon_b}} \lambda'_{io} Z_{io} \right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}{\sum_{j\in\mathcal{I}_m} \left(T_{jo}^{\frac{1}{\varepsilon_b}} \lambda'_{jo} Z_{jo} \right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}.$$

Substituting the identified values for the revenue productivities $Z_{io} = \frac{P w_{io} L_{io}^{\delta}}{\beta_b \lambda_{io}}$ (see equation 25 in the main text), and amenities $\frac{s_{io|m}}{(P w_{io})^{\varepsilon_b}} \left(\frac{L_m}{\Gamma_b} \right)^{\varepsilon_b/\eta}$ (see section E.4 of this Online Appendix for the derivation) into the expression above and simplifying, we get:

$$s'_{io|m} = \frac{s_{io|m} \left(\lambda'_{io}/\lambda_{io} \right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}{\sum_{j\in\mathcal{I}_m} s_{jo|m} \left(\lambda'_{jo}/\lambda_{jo} \right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}.$$

Therefore, it is equivalent to compute the counterfactual employment shares within a local labor market using the observed employment shares and wedges in the baseline, or the identified amenities and revenue productivities. We can then use the revenue productivities, which are themselves a function of observed wages, employment levels and wedges to aggregate the counterfactual economy at the sector level. Following the same steps as in the baseline, the sector level system of equations in the counterfactual is analogous to (20) but with relative variables. Solving for relative sector prices we can compute the sector employment L'_b. Propositions 2 and 4 apply also in the 'hat' economy. Therefore, the solution for the counterfactuals exists and is unique.

Summing the counterfactual wage w'_{io} from (C2) to $\Phi'_m = \sum_{i\in\mathcal{I}_m} T_{io} w'^{\varepsilon_b}_{io}$ and factoring out the industry or economy wide constants we find the following relation,

$$\Phi'_m = \widetilde{\Phi}'^{\frac{1+\varepsilon_b\delta}{1+\eta\delta}}_m \frac{\widehat{P}_b^{\frac{\varepsilon_b}{(1-\alpha_b)(1+\eta\delta)}}}{P^{\frac{\varepsilon_b}{1+\eta\delta}}} \left(\frac{\Phi'}{L'} \right)^{\frac{\varepsilon_b\delta}{1+\eta\delta}}, \quad \widetilde{\Phi}'_m \equiv \sum_{io\in\mathcal{I}_m} T_{io} \omega_{io}^{\varepsilon_b}$$

Using the definition of $\Phi'_b \equiv \sum_{m\in\mathcal{M}_b} {\Phi'_m}^{\eta/\varepsilon_b}$ and $\Phi' \equiv \sum_{b\in\mathcal{B}} \Phi'_b \Gamma_b^\eta$, we have that:

$$
\begin{aligned}
\Phi'_b &= \widetilde{\Phi}'_b \frac{\widehat{P}_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}}}{P^{\frac{\eta}{1+\eta\delta}}} \left(\frac{\Phi'}{L'}\right)^{\frac{\eta\delta}{1+\eta\delta}}, \quad \widetilde{\Phi}'_b \equiv \sum_{m\in\mathcal{M}_b} \widetilde{\Phi'}_m^{\frac{(1+\varepsilon_b\delta)\eta}{(1+\eta\delta)\varepsilon_b}}, \\
\Phi' &= \widetilde{\Phi'}^{1+\eta\delta} P^{-\eta} L'^{-\eta\delta}, \quad \widetilde{\Phi}' \equiv \sum_{b\in\mathcal{B}} \widetilde{\Phi}'_b \widehat{P}_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \Gamma_b^\eta.
\end{aligned}
$$

Sector employment in the counterfactual is equal to:

$$
L'_b = \frac{\widehat{P}_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \widetilde{\Phi}'_b(\mathbf{s}'_b)\Gamma_b^\eta}{\sum_{b'\in\mathcal{B}} \widehat{P}_{b'}^{\frac{\eta}{(1-\alpha_{b'})(1+\eta\delta)}} \widetilde{\Phi}'_{b'}(\mathbf{s}'_{b'})\Gamma_{b'}^\eta} L',
$$

where counterfactual sector employment is a function of relative prices $\{\widehat{P}_b\}_{b\in\mathcal{B}}$ and counterfactual local labor market employment shares $\{\mathbf{s}'_b\}_{b\in\mathcal{B}}$. Establishment-occupation output in the counterfactual is:

$$
\begin{aligned}
y'_{io} &= {P'_b}^{\frac{\alpha_b}{1-\alpha_b}} A_{io} {L'_{io}}^{1-\delta} \\
&= P_b^{\frac{\alpha_b}{1-\alpha_b}} A_{io} \widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}} {L'_{io}}^{1-\delta} \\
&= \frac{\widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{PP_b} A_{io} P P_b^{\frac{1}{1-\alpha_b}} {L'_{io}}^{1-\delta} \\
&= \frac{\widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{PP_b} Z_{io} {L'_{io}}^{1-\delta}.
\end{aligned}
$$

The analogue expression for the baseline is: $y_{io} = \frac{1}{PP_b} Z_{io} L_{io}^{1-\delta}$. Aggregating up to industry b level, the counterfactual industry output Y'_b is ,

$$
Y'_b = \frac{\widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{PP_b} Z_b \Psi'_b {L'_b}^{1-\delta}, \quad \Psi'_b \equiv \sum_{io\in\mathcal{I}_b} \frac{Z_{io}}{Z_b} {s'_{io|m}}^{1-\delta} {s'_{m|b}}^{1-\delta},
$$

where Ψ'_b is a measure of misallocation based on revenue productivities and $Z_b \equiv \sum_{io\in\mathcal{I}_b} Z_{io} \tilde{s}_{io|m}^{1-\delta} \tilde{s}_{m|b}^{1-\delta}$ is a measure of sector revenue productivity that is the same in the baseline and in the counterfactuals. Note that because the revenue productivities are multiplied by a sector-level constant,

$$
\Psi'_b \equiv \sum_{io\in\mathcal{I}_b} \frac{Z_{io}}{Z_b} {s'_{io|m}}^{1-\delta} {s'_{m|b}}^{1-\delta} = \sum_{io\in\mathcal{I}_b} \frac{A_{io}}{A_b} {s'_{io|m}}^{1-\delta} {s'_{m|b}}^{1-\delta} \equiv \Omega'_b,
$$

where Ω'_b is a measure of misallocation in the counterfactual equilibrium. We keep the notational difference between Ψ'_b and Ω'_b to clarify that the former is computed using the revenue productivities, which are observed, instead of the physical productivities.

The baseline sector output is: $Y_b = \frac{1}{PP_b} Z_b \Psi_b L_b^{1-\delta}$ with Ψ_b analogue to the one defined for the counterfactual but with baseline employment shares, meaning $\Psi_b \equiv \sum_{io\in\mathcal{I}_b} \frac{Z_{io}}{Z_b} s_{io|m}^{1-\delta} s_{m|b}^{1-\delta}$. Taking the ratio, counterfactual sector output relative to the baseline is:

$$\widehat{Y}_b = \widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}} \widehat{\Psi}_b \widehat{L}_b^{1-\delta}, \tag{C3}$$

where $\widehat{\Psi}_b = \frac{\Psi'_b}{\Psi_b}$. Using L'_b and equation (3) we get a similar expression to (B8)

$$\widehat{P}_b^{\frac{1+\eta}{(1-\alpha_b)(1+\eta\delta)}} \widehat{\Psi}_b \left(\frac{\widetilde{\Phi}'_b \Gamma_b^\eta}{L_b} \right)^{1-\delta} = \prod_{b' \in \mathcal{B}} \left(\widehat{P}_{b'}^{\frac{\alpha_{b'}(1+\eta\delta)+\eta(1-\delta)}{(1-\alpha_{b'})(1+\eta\delta)}} \right)^{\theta_{b'}} \prod_{b' \in \mathcal{B}} \widehat{\Psi}_{b'}^{\theta_{b'}} \prod_{b' \in \mathcal{B}} \left(\frac{\widetilde{\Phi}'_{b'} \Gamma_{b'}^\eta}{L_{b'}} \right)^{(1-\delta)\theta_{b'}}. \tag{C4}$$

By taking the ratio, the elasticities θ_b and the economy wide constants cancel out on both sides. Rewriting, we get an expression very similar to equation (B10) in Proposition 4 but with hat variables:

$$\widehat{P}_b = \widehat{X}_b \widehat{X}^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}}, \tag{C5}$$

$$\widehat{X}_b = \left(\frac{L_b^{1-\delta}}{\widehat{\Psi}_b \left(\widetilde{\Phi'}_b \Gamma_b^\eta \right)^{1-\delta}} \right)^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}}, \quad \widehat{X} = \left(\prod_{b' \in \mathcal{B}} \widehat{X}_{b'}^{-\theta_{b'}} \right)^{\frac{1+\eta}{\sum_{b' \in \mathcal{B}} \theta_{b'}(1-\alpha_{b'})(1+\eta\delta)}}.$$

Fixed labor.
In the case where employment is fixed at the industry level b, the counterfactual wage (C2) becomes:

$$w'_{io} = \left(\beta_b \lambda_{io} \frac{Z_{io}}{T_{io}^\delta} \right)^{\frac{1}{1+\varepsilon_b \delta}} \frac{\widehat{P}_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b\delta)}}}{P^{\frac{1}{1+\varepsilon_b\delta}}} {\Phi'_m}^{(1-\eta/\varepsilon_b)\frac{\delta}{1+\varepsilon_b\delta}} \left(\frac{\Phi'_b}{L'_b} \right)^{\frac{\delta}{1+\varepsilon_b\delta}}.$$

Fixing lower levels than b would only change the last element. Keeping total employment at the local labor market fixed, the last term would become: $\left(\frac{\Phi'_m}{L'_m} \right)^{\frac{\delta}{1+\varepsilon_b\delta}}$. The constant Γ_b does not appear in this case as workers can't move across industries and the functional Γ_b is the same for all the local labor markets within an industry. Also, fixing lower levels than b clearly implies that L'_b is known and equal to the baseline labor in the industry L_b.

The counterfactuals where employment at b or lower level employment is fixed will give rise to a condition similar to (C4). Given that L'_b is known, we have that:

$$\widehat{P}_b^{\frac{1}{1-\alpha_b}} \widehat{\Psi}_b = \prod_{b' \in \mathcal{B}} \left(\widehat{P}_{b'}^{\frac{\alpha_{b'}}{1-\alpha_{b'}}} \widehat{\Psi}_{b'} \right)^{\theta_{b'}}.$$

Propositions 2 and 4 therefore also apply in the relative counterfactuals with fixed labor at the sector level b (or at a lower level).

C.2 Extension: Endogenous participation

We showed in the proof of Proposition 4 that the solution of sector prices $\mathbf{P}$ is homogeneous of degree zero with respect to total employment level which we denote here as L_e. We have that,

$$L_{io}(w_{io}) = \frac{T_{io}w_{io}^{\varepsilon_b}}{\Phi_m}\frac{\Phi_m^{\eta/\varepsilon_b}\Gamma_b^{\eta}}{\Phi}L = \frac{T_{io}w_{io}^{\varepsilon_b}}{\Phi_m}\frac{\Phi_m^{\eta/\varepsilon_b}\Gamma_b^{\eta}}{\Phi_e}L_e.$$

We have that $L_e = \frac{\Phi_e}{\Phi}L$ with $\Phi_e \equiv \sum_{m\in\mathcal{M}} \Phi_m^{\eta/\varepsilon_b}\Gamma_b^{\eta}$ is the part of Φ that comes from the employed and $\Phi_u \equiv \sum_{uo\in\mathcal{U}}(T_{uo}w_{uo}^{\varepsilon_b})^{\eta/\varepsilon_b}\Gamma_b^{\eta}$ is the part from the out of the labor force as in the main text.

The model aggregation steps are the same as in Section A with the exception that L_b now is $L_{b,e}$. We normalize all the reservation wages w_{uo} to 1. We recover the out-of-the-labor-force amenities T_{uo} to match the observed unemployment rate and we can compute Φ_u. There are no markdowns for the OTLF and we set the productivities of the fictitious OTLF establishments to zero such that they do not contribute to aggregate output.

Aggregating from (B5),

$$\Phi_{b,e} = \left(\frac{\Phi}{L}\right)^{\frac{\eta\delta}{1+\eta\delta}} \sum_{m\in\mathcal{M}_b} \widetilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b)}{\varepsilon_b(1+\eta\delta)}} P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} = \left(\frac{\Phi}{L}\right)^{\frac{\eta\delta}{1+\eta\delta}} \widetilde{\Phi}_{b,e}P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \tag{C6}$$

$$\widetilde{\Phi}_{b,e} \equiv \sum_{m\in\mathcal{M}_b} \widetilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b)}{\varepsilon_b(1+\eta\delta)}},$$

$$\Phi \equiv \Phi_e + \Phi_u,$$

and,

$$\Phi_e \equiv \left(\frac{\Phi}{L}\right)^{\frac{\eta\delta}{1+\eta\delta}} \sum_{b\in\mathcal{B}} \widetilde{\Phi}_{b,e}P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}}\Gamma_b^{\eta} = \left(\frac{\Phi}{L}\right)^{\frac{\eta\delta}{1+\eta\delta}} \widetilde{\Phi}_e \tag{C7}$$

$$\widetilde{\Phi}_e \equiv \sum_{b\in\mathcal{B}} \widetilde{\Phi}_{b,e}P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}}\Gamma_b^{\eta}.$$

Therefore,

$$L_{b,e} = \frac{\Phi_{b,e}\Gamma_b^{\eta}}{\Phi_e}L_e = \frac{\widetilde{\Phi}_{b,e}\Gamma_b^{\eta}P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}}}{\widetilde{\Phi}_e}L_e.$$

We can solve for the prices without knowing total employment level L_e. Total employment level,

$$L_e = \frac{\Phi_e}{\Phi}L,$$

where L is total labor supply (employed and out-of-the-labor-force), will determine the level of aggregate output. We can find it by solving for Φ_e in equation (C7),

$$\Phi_e^{\frac{1+\eta\delta}{\eta\delta}} L = (\Phi_e + \Phi_u)\widetilde{\Phi}_e^{\frac{1+\eta\delta}{\eta\delta}}.$$

The solution is obviously unique as the left hand side is convex and the right hand side linear. With the solution for Φ_e one can construct all the aggregates back.

C.3 Extension: Agglomeration

Plugging the labor supply into (29), the wage in the baseline economy is:

$$w_{io} = \left(\beta_b \lambda(\mu_{io}, \varphi_b) \frac{Z_{io}}{(T_{io}\Gamma_b^{\eta})^{\delta}}\right)^{\frac{1}{1+\varepsilon_b\delta}} \Phi_m^{\nu_b - \frac{\eta}{\varepsilon_b}\widetilde{\nu}_b} P^{-\frac{1}{1+\varepsilon_b\delta}} \left(\frac{\Phi}{L}\right)^{\widetilde{\nu}_b}, \quad \nu_b = \frac{\delta}{1+\varepsilon_b\delta}, \quad \widetilde{\nu}_b = \frac{\delta-\gamma}{1+\varepsilon_b\delta}.$$

The baseline wage can be written as: $w_{io} = \widetilde{w}_{io}\Phi_m^{\nu_b - \frac{\eta}{\varepsilon_b}\widetilde{\nu}_b} P^{-\frac{1}{1+\varepsilon_b\delta}} \left(\frac{\Phi}{L}\right)^{\widetilde{\nu}_b}$. Analogously, the counterfactual wage is: $w_{io} = \omega_{io}\widehat{P}_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b\delta)}} \Phi_m^{\nu_b - \frac{\eta}{\varepsilon_b}\widetilde{\nu}_b} P^{-\frac{1}{1+\varepsilon_b\delta}} \left(\frac{\Phi}{L}\right)^{\widetilde{\nu}_b}$. Aggregating to generate Φ_m,

$$\Phi_m = \widetilde{\Phi}_m^{\frac{1+\varepsilon_b\delta}{1+\eta(\delta-\gamma)}} P^{-\frac{\varepsilon_b}{1+\eta(\delta-\gamma)}} \left(\frac{\Phi}{L}\right)^{\frac{\varepsilon_b(\delta-\gamma)}{1+\eta(\delta-\gamma)}}. \tag{C8}$$

The counterfactual Φ'_m is analogously $\Phi'_m = \widetilde{\Phi'}_m^{\frac{1+\varepsilon_b\delta}{1+\eta(\delta-\gamma)}} \widehat{P}_b^{\frac{\varepsilon_b}{(1-\alpha_b)(1+\eta(\delta-\gamma))}} P^{-\frac{\varepsilon_b}{1+\eta(\delta-\gamma)}} \left(\frac{\Phi}{L}\right)^{\frac{\varepsilon_b(\delta-\gamma)}{1+\eta(\delta-\gamma)}}$.

In order to be able to find a solution to the model, we need that the exponents are bounded. This is equivalent to requiring $\gamma \neq \frac{1}{\eta} + \delta$. The parameter γ governs the strength of agglomeration forces within a local labor market, and δ and $\frac{1}{\eta}$ are related with dispersion forces. Those come from the decreasing returns to scale (δ) and from the variance of taste shocks ($\frac{1}{\eta}$). When the latter is high, the mass of workers having extreme taste shocks is higher. This implies that agglomeration forces will impact less as workers would be more inelastic to changes in wages. The standard condition for uniqueness of the equilibrium with agglomeration would be that it is sufficiently weak ($\gamma < \frac{1}{\eta} + \delta$). We instead find the weaker condition $\gamma \neq \frac{1}{\eta} + \delta$.

The counterfactual industry labor supply is:

$$L'_b = \frac{\widehat{P}_b^{\frac{\eta}{(1-\alpha_b)(1+\eta(\delta-\gamma))}} \widetilde{\Phi}'_b \Gamma_b^{\eta}}{\sum_{b\in\mathcal{B}} \widehat{P}_{b'}^{\frac{\eta}{(1-\alpha_{b'})(1+\eta(\delta-\gamma))}} \widetilde{\Phi}'_{b'} \Gamma_{b'}^{\eta}}, \quad \widetilde{\Phi}'_b \equiv \sum_{m\in\mathcal{M}_b} \widetilde{\Phi'}_m^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta(\delta-\gamma))}}$$

Turning to production, the establishment-occupation output y'_{io} and local labor market output Y_m in the counterfactual are:

$$y'_{io} = \frac{Z_{io}\widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{P_b P} {L'_{io}}^{1-\delta} {L'_m}^{\gamma}$$

$$Y'_m = \frac{Z_m(s')\widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{P_b P} {L'_m}^{1-\delta+\gamma}, \quad Z_m(s') = \sum_{i\in\mathcal{I}_m} Z_{io} {s'_{io|m}}^{1-\delta}.$$

The expressions for the baseline are analogous but setting $\widehat{P}_b = 1$. The counterfactual output of industry b, Y'_b, when there are agglomeration forces is:

$$Y'_b = \frac{Z_b(s')\widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{P_b P} {L'_b}^{1-\delta+\gamma}, \quad Z_b(s') = \sum_{m\in\mathcal{M}_b} Z_m {s'_{m|b}}^{1-\delta+\gamma},$$

where γ changed the returns to scale of the industry production function and the aggregation of productivities $Z_b(s')$. The intermediate good demand in the counterfactual relative to the baseline is:

$$\widehat{P}_b^{\frac{1}{1-\alpha_b}} \widehat{Z}_b \left(\frac{L'_b(\widehat{\mathbf{P}})}{L_b}\right)^{1-\delta+\gamma} = \prod_{b'\in\mathcal{B}} \widehat{P}_b^{\frac{\alpha_{b'}}{1-\alpha_{b'}}} \widehat{Z}_{b'} \left(\frac{L'_{b'}(\widehat{\mathbf{P}})}{L_{b'}}\right)^{1-\delta+\gamma}$$

$$\Leftrightarrow \widehat{P}_b^{\frac{1+\eta}{(1-\alpha_b)(1+\eta(\delta-\gamma))}} \widehat{Z}_b \left(\frac{\widetilde{\Phi}'_b \Gamma_b^\eta}{L_b}\right)^{1-\delta+\gamma} = \prod_{b'\in\mathcal{B}} \widehat{P}_{b'}^{\alpha_{b'}(1+\eta(\delta-\gamma))+\eta(1-\delta+\gamma)} \widehat{Z}_{b'} \left(\frac{\widetilde{\Phi}'_{b'} \Gamma_{b'}^\eta}{L_{b'}}\right)^{1-\delta+\gamma}.$$

Uniqueness of the solution to this system of equations is guaranteed by $\sum_{b\in\mathcal{B}} \alpha_b \theta_b < 1$. This condition being the same as for Proposition 4, uniqueness of the equilibrium with agglomeration forces only needs the additional requirement of $\gamma \neq \frac{1}{\eta} + \delta$.

C.4 Alternative production function

For completeness, in this section we lay out a model with an alternative Cobb-Douglas production function with generic capital and a labor composite that is at odds with the data.

Suppose that establishment i produces using some generic capital K_i and a labor composite H_i of different occupations:

$$y_i = \widetilde{A}_i K_i^{\alpha_b} H_i^{\beta_b} = \widetilde{A}_i K_i^{\alpha_b} \left(\prod_{o\in\mathcal{O}} L_{io}^{\gamma_o}\right)^{\beta_b}, \quad \sum_o \gamma_o = 1, \quad \alpha_b + \beta_b \leq 1. \tag{C9}$$

The first order conditions with respect to capital and the bargained wage are:

$$w_{io} = \beta_b \gamma_o \lambda(\mu_{io}, \varphi_b) P_b \frac{y_i}{L_{io}},$$
$$R_b = \alpha_b \widetilde{A}_i K_i^{\alpha_b - 1} H_i^{\beta_b}.$$

Substituting the first order condition for capital into the production function, the wage first order condition becomes:

$$w_{io} = \beta_b \gamma_o \lambda(\mu_{io}, \varphi_b) A_i H_i^{1-\delta} L_{io}^{-1} P_b^{\frac{1}{1-\alpha_b}},$$

where we plugged the labor supply and used the definition of $\delta = 1 - \frac{\beta_b}{1-\alpha_b}$ from the main text and $A_i = \widetilde{A}_i^{\frac{1}{1-\alpha_b}} \left(\frac{\alpha_b}{R_b}\right)^{\frac{\alpha_b}{1-\alpha_b}}$ as in the main text. Using those and solving for L_{io}, we can write the labor composite H_i as function of wages:

$$H_i^\delta = P_b^{\frac{1}{1-\alpha_b}} \prod_{o\in\mathcal{O}} \beta_b \gamma_o \lambda(\mu_{io}, \varphi_b) w_{io}^{-1}$$

Substituting the wage equation with the labor supply (12) into the expression above, we get:

$$H_i^{1+\varepsilon_b \delta} = P_b^{\frac{\varepsilon_b}{1-\alpha_b}} \prod_{o\in\mathcal{O}} \left(\beta_b \gamma_o \lambda(\mu_{io}, \varphi_b) A_i (T_{io} \Gamma_b^\eta)^{1/\varepsilon_b}\right)^{\varepsilon_b \gamma_o} \prod_{o\in\mathcal{O}} \left(\Phi_m^{1-\eta/\varepsilon_b} \frac{\Phi}{L}\right)^{-\gamma_o}$$
$$= P_b^{\frac{\varepsilon_b}{1-\alpha_b}} (\beta_b \Upsilon A_i)^{\varepsilon_b} T_i \Gamma \prod_{o\in\mathcal{O}} \lambda(\mu_{io}, \varphi_b)^{\varepsilon_b \gamma_o} \prod_{o\in\mathcal{O}} \left(\Phi_m^{1-\eta/\varepsilon_b} \frac{\Phi}{L}\right)^{-\gamma_o},$$

where $\Upsilon \equiv \prod_{o \in \mathcal{O}} \gamma_o$, $\Gamma \equiv \prod_{o \in \mathcal{O}} \Gamma_b^{\eta}$ and $T_i \equiv \prod_{o \in \mathcal{O}} T_{io}$. Plugging back into the wage equation and rearranging, we get:

$$w_{io} = \left[\lambda(\mu_{io}, \varphi_b) \frac{\gamma_o}{T_{io}\Gamma_b^{\eta}} (\beta_b A_i)^{\frac{1+\varepsilon_b}{1+\varepsilon_b\delta}} (\Upsilon (T_i \Gamma)^{1/\varepsilon_b})^{\frac{\varepsilon_b(1-\delta)}{1+\varepsilon_b\delta}} \right.$$
$$\left. \times \left(\prod_{o' \in \mathcal{O}} \lambda(\mu_{io'}, \varphi_b)^{\varepsilon_b \gamma'_o} \right)^{\frac{1-\delta}{1+\varepsilon_b\delta}} \left(\prod_{o' \in \mathcal{O}} \Phi_{m'}^{(\eta/\varepsilon_b - 1)\gamma'_o} \right)^{\frac{1-\delta}{1+\varepsilon_b\delta}} \Phi_m^{1-\eta/\varepsilon_b} \right]^{\frac{1}{1+\varepsilon_b}} \left(\frac{\Phi}{L} \right)^{\frac{1}{1+\varepsilon_b}} P_b^{1/\chi_b}, \quad \text{(C10)}$$

with $\chi_b = (1-\alpha_b)(1+\varepsilon_b\delta)$. Define the following:

$$c_{io} \equiv \frac{\gamma_o}{T_{io}\Gamma_b^{\eta}} (\beta_b A_i)^{\frac{1+\varepsilon_b}{1+\varepsilon_b\delta}} (\Upsilon (T_i \Gamma)^{1/\varepsilon_b})^{\frac{\varepsilon_b(1-\delta)}{1+\varepsilon_b\delta}},$$
$$C_l \equiv \prod_{o' \in \mathcal{O}} \left(\Phi_{m'}^{(\eta/\varepsilon_b - 1)\gamma_o} \right)^{\frac{\delta}{1+\varepsilon_b\delta}} \left(\frac{\Phi}{L} \right)^{\frac{1}{1+\varepsilon_b}},$$
$$F_b \equiv P_b^{1/\chi_b},$$

where C_l is a location constant with $l = n \times h$. Rearranging we have that:

$$w_{io} = \left[\lambda(\mu_{io}, \varphi_b) c_{io} \left(\prod_{o' \in \mathcal{O}} \lambda(\mu_{io'}, \varphi_b)^{\varepsilon_b \gamma'_o} \right)^{\frac{1-\delta}{1+\varepsilon_b\delta}} \frac{\Phi_m^{1-\eta/\varepsilon_b}}{\prod_{o' \in \mathcal{O}} \Phi_{m'}^{(1-\eta/\varepsilon_b)\gamma'_o}} \right]^{\frac{1}{1+\varepsilon_b}} C_l F_b. \quad \text{(C11)}$$

The last system is equivalent to the one in (C10) and has the benefit to being able to write the wages as $w_{io} = \widetilde{w}_{io} C_m F_b$, where we want $\widetilde{w}_{io}$ to be homogeneous of degree zero with respect constants to m level. Note that the last term inside the brackets is homogeneous of degree zero with respect to location l constants shared by all the occupations of a establishments. Then, defining $\widetilde{\Phi}_m \equiv \sum_{i \in \mathcal{I}_m} T_{io} w_{io}^{\varepsilon_b}$, the establishment-occupation or normalized wage is:

$$\widetilde{w}_{io} \equiv \left[\lambda(\mu_{io}, \varphi_b) c_{io} \left(\prod_{o' \in \mathcal{O}} \lambda(\mu_{io'}, \varphi_b)^{\varepsilon_b \gamma'_o} \right)^{\frac{1-\delta}{1+\varepsilon_b\delta}} \frac{\widetilde{\Phi}_m^{1-\eta/\varepsilon_b}}{\prod_{o' \in \mathcal{O}} \widetilde{\Phi}_{m'}^{(1-\eta/\varepsilon_b)\gamma'_o}} \right]^{\frac{1}{1+\varepsilon_b}}. \quad \text{(C12)}$$

$\widetilde{w}_{io}$ is homogeneous of degree zero with respect to location l constants shared by all occupations. This property, makes the model with the alternative production function also block recursive. That is, it allows solving for the normalized wages of every location l (combinations of commuting zone n and sub-industry h combinations) independently and then recover the aggregate constants. Aggregating (C12) and solving for $\widetilde{\Phi}_m$, we have:

$$\widetilde{\Phi}_m = \left[\frac{\sum_{i \in I_m} \left(\lambda(\mu_{io}, \varphi_b) c_{io} T_{io}^{\frac{1+\varepsilon_b}{\varepsilon_b}} \prod_{o' \in \mathcal{O}} \lambda(\mu_{io'}, \varphi_b)^{\varepsilon_b \gamma'_o} \right)^{\frac{1-\delta}{1+\varepsilon_b\delta}}}{\prod_{o' \in \mathcal{O}} \widetilde{\Phi}_{m'}^{(1-\eta/\varepsilon_b)\gamma'_o}} \right]^{\frac{\varepsilon_b}{1+\eta}}.$$

Taking everything to the power $(1-\eta/\varepsilon_b)\gamma_o$ and taking the product,

$$\mathcal{L}_l \equiv \prod_{o'\in\mathcal{O}} \widetilde{\Phi}_{m'}^{(1-\eta/\varepsilon_b)\gamma'_o} = \prod_{o'\in\mathcal{O}} \left[\sum_{i\in I_m} \left(\lambda(\mu_{io},\varphi_b) c_{io} T_{io}^{\frac{1+\varepsilon_b}{\varepsilon_b}} \prod_{o'\in\mathcal{O}} \lambda(\mu_{io'},\varphi_b)^{\varepsilon_b\gamma'_o} \right)^{\frac{1-\delta}{1+\varepsilon_b\delta}} \right]^{\gamma_{o'}\frac{\varepsilon_b-\eta}{1+\varepsilon_b-\eta}},$$

which recovers all the local labor market constants inside $\widetilde{w}_{io}$.

In order to prove the existence and uniqueness of the solution of the system (C12), define $\widehat{w}_{io}$ as:

$$\begin{aligned} \widehat{w}_{io} &= \left[\lambda(\mu_{io},\varphi_b) \left(\prod_{o'\in\mathcal{O}} \lambda(\mu_{io'},\varphi_b)^{\varepsilon_b\gamma'_o} \right)^{\frac{1-\delta}{1+\varepsilon_b\delta}} \right]^{\frac{1}{1+\varepsilon_b}} c_{io}^{\frac{1}{1+\varepsilon_b}} \\ w_{io} &= \widehat{w}_{io} \left[\frac{\widetilde{\Phi}_m^{1-\eta/\varepsilon_b}}{\mathcal{L}_l} \right]^{\frac{1}{1+\varepsilon_b}} C_l F_b = \widehat{w}_{io} z_l = \widetilde{w}_{io} C_l F_b. \end{aligned} \tag{C13}$$

We can show that the system formed by (C13) has a solution and is unique.

Proposition 3. *For given parameters $0 \leq \alpha_b, \beta_b < 1$, $1 < \eta < \varepsilon_b$, $0 \leq \delta \leq 1$, transformed price F_b, constants C_l, $\widetilde{\Phi}_m$, $\mathcal{L}_l$ and non-negative vectors of productivities $\{A_i\}_{i\in m}$ and amenities $\{T_{io}\}_{io\in m}$, there exists a unique vector of wages $\{w_{io}\}_{io\in I_m}$ for every location l (combination of commuting zone n and sub-industry h) that solves the system formed by* (C13).

Proof. For existence, first note that $\lambda(\mu_{io},\varphi_b) \in \left[(1-\varphi_b)\frac{\eta}{1+\eta} + \varphi_b\frac{1}{1-\delta}, (1-\varphi_b)\frac{\varepsilon_b}{1+\varepsilon_b} + \varphi_b\frac{1}{1-\delta}\right], \forall i, o$. Define a vector $\mathbf{w}$ with wage of all the establishment-occupations at location l, $\mathbf{w} \equiv \{w_{11}, w_{12}, ..., w_{1O}, ..., w_{I1}, ..., w_{IO}\}$. Taking for now the elements of z_l as constants. The system to solve is: $f_{io}(\mathbf{w}) = \widehat{w}_{io} z_l$. We have that

$$\begin{aligned} \mathbf{w} \in \mathcal{C} \equiv & \left[\left((1-\varphi_b)\frac{\eta}{1+\eta} + \varphi_b\frac{1}{1-\delta} \right)^{\frac{1}{1+\eta\delta}} c_{11}^{\frac{1}{1+\varepsilon_b}} z_{l1}, \left((1-\varphi_b)\frac{\varepsilon_b}{1+\varepsilon_b} + \varphi_b\frac{1}{1-\delta} \right)^{\frac{1}{1+\eta\delta}} c_{11}^{\frac{1}{1+\varepsilon_b}} z_{l1} \right] \\ & \times ... \times \left[\left((1-\varphi_b)\frac{\eta}{1+\eta} + \varphi_b\frac{1}{1-\delta} \right)^{\frac{1}{1+\eta\delta}} c_{IO}^{\frac{1}{1+\varepsilon_b}} z_{lO}, \left((1-\varphi_b)\frac{\varepsilon_b}{1+\varepsilon_b} + \varphi_b\frac{1}{1-\delta} \right)^{\frac{1}{1+\eta\delta}} c_{IO}^{\frac{1}{1+\varepsilon_b}} z_{lO} \right]. \end{aligned}$$

The system f_{io} is continuous on wages and maps into itself on $\mathcal{C}$. The last set being a compact set we can apply Brower's fixed point theorem.

For uniqueness, once the product of the wedges is substituted, $\widehat{w}_{io}$ is:

$$\widehat{w}_{io} = \left[\lambda(\mu_{io},\varphi_b) c_{io} \prod_{o'\in\mathcal{O}} (w_{io'} c_{io}^{-\frac{1}{1+\varepsilon_b}})^{\gamma'_o\varepsilon_b(1-\delta)} \right]^{\frac{1}{1+\varepsilon_b}}$$

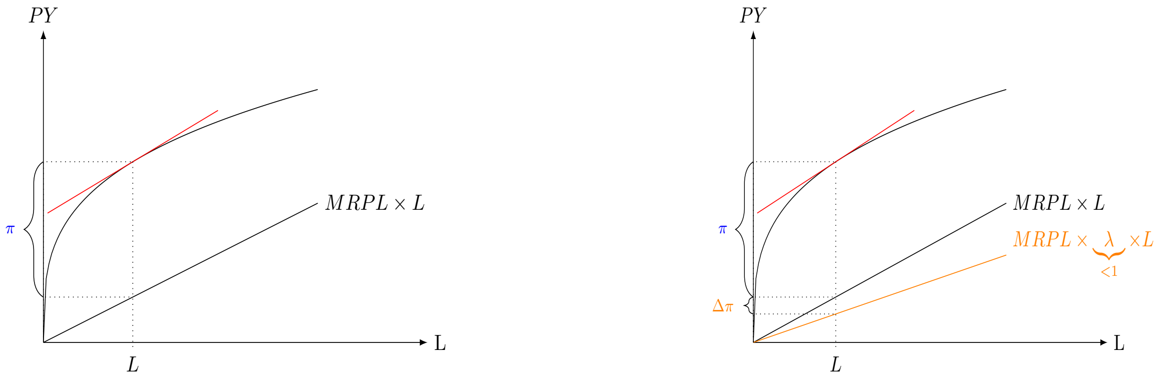
Define the function $g_{io}(\mathbf{w}) = f_{io}(\mathbf{w}) - w_{io}$. Gross substitution is fulfilled if $\frac{\partial g_{io}(\mathbf{w})}{\partial w_{jo}} > 0, \forall j \neq i$ with $j \in \mathcal{I}_l$ and $\frac{\partial g_{io}(\mathbf{w})}{\partial w_{io'}}, \forall o'$. Gross substitution resumes to taking the partial derivatives of $\widehat{w}_{io}$ which are positive by similar reasoning as in the main proof. Finally, $\widehat{w}_{io}$ is homogeneous of degree $\frac{\varepsilon_b}{1+\varepsilon_b}(1-\delta) < 1$. Therefore the solution to the system (C13) exists and is unique. □

Finally, the model can be aggregated up to the industry level following similar steps as in Proposition 3.

D Distributional and productivity consequences

Figure D1: Distributional Consequences

(a) Perfect Competition (b) Labor Market Power



Here we illustrate the distributional and productivity effects when the labor wedge λ is below one. Figure D1 illustrates the effect of labor market power on the distribution of value added into profits and wage payments. For simplicity, we illustrate with the case of a production function using only labor with a decreasing returns to scale technology. On the left panel, we have the case of perfect competition in the labor market where wages are equal to the marginal revenue product of labor and the firm earns quasi-rents generated from having decreasing returns. On the right panel, we illustrate the case with labor market power where employer monopsony power dominates. Wages are below the marginal revenue product because the wedge λ is below one. This generates additional profits for the firm, reducing wage bill payments and therefore the labor share.

Figure D2: Productivity Consequences

(a) Homogeneous Wedges (b) Heterogeneous Wedges

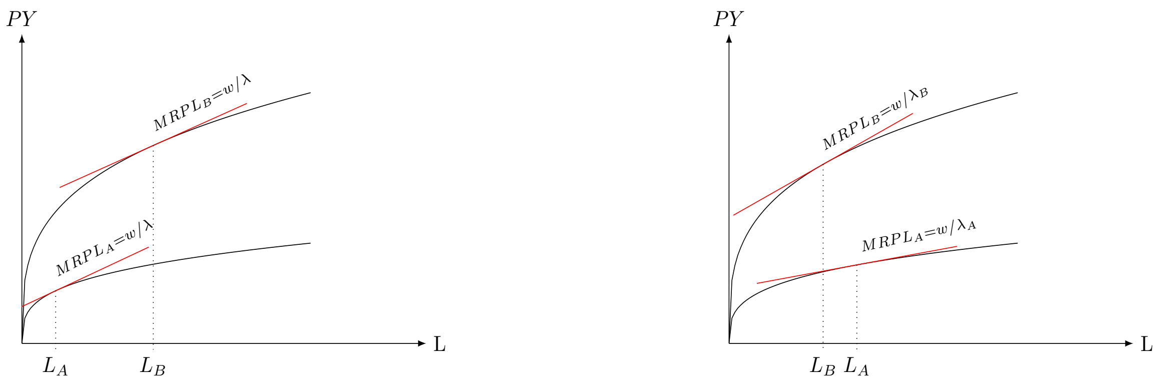


Figure D2 shows the productivity consequences due to the misallocation of resources. The left panel shows two firms with the same labor wedge. For simplicity we assume that all firms and local labor markets have the same amenities and workers are indifferent across establishments and local labor markets so all establishments will have the same wage in equilibrium. With homogeneous wedges, the marginal revenue products are equalized across establishments. In particular, if firm B is more productive we have in equilibrium $L_B > L_A$. On the right panel we show an example with heterogeneous wedges. Firm B being more productive is more likely to have a higher employment share at the local labor market and therefore a more important markdown. That is, $\mu_B < \mu_A$

and therefore $\lambda_B < \lambda_A$. Wages being equalized for all the establishments $MRPL_B > MRPL_A$. We illustrate the extreme case where the distortion generated by labor market power flips the employment size of both firms and we have $L_A > L_B$. Shifting employment from A to B, from low to high marginal revenue product firms, there could be productivity gains.

E Identification and estimation

E.1 Identification of common parameters η and δ

In order to identify the across markets labor supply elasticity η and the labor demand elasticity δ we exploit the fact that in local labor markets where there is only one establishment, the wedge $\lambda(\mu, \phi_b)$ is constant within industries b. We denominate this type of establishments as *full monopsonists.* Additionally, the effect of wages on the labor supply of full monopsonists is only affected by the parameter η as the within market labor supply elasticity ε_b is irrelevant in local labor markets with only one establishment. Taking the logarithm for the labor supply that full monopsonists face (12), we get:

$$\ln(L_{io,s=1}) = \eta \ln(w_{io}) + \ln(\tilde{T}_{io}) + \ln(\Gamma_b^{\eta} L/\Phi),$$

where $\tilde{T}_{io} = T_{io}^{\eta/\varepsilon_b}$. As mentioned in the main text, full monopsonists apply a constant markdown equal to $\mu(s=1) = \frac{\eta}{\eta+1}$ that in turn will imply a constant wedge $\lambda(\mu, \varphi_b)$ within industry b. Their labor demand (15) in logs is:

$$\ln(w_{io,s=1}) = \ln(\beta_b) + \ln\left((1-\varphi_b)\frac{\eta}{\eta+1} + \varphi_b \frac{1}{1-\delta}\right) + \ln(A_{io}) - \delta \ln(L_{io}) + \frac{1}{1-\alpha_b}\ln(P_b).$$

In order to get rid of industry and economy wide constants, we demean $\ln(L_{io,s=1})$ and $\ln(w_{io,s=1})$ by removing the industry b averages per year. Denoting with $\overline{\ln(X)}$ the demeaned variables, we rewrite the labor supply and demand equations as:

$$\begin{aligned}
\overline{\ln(L_{io})} &= \eta \overline{\ln(w_{io})} + \overline{\ln(\tilde{T}_{io})}, \\
\overline{\ln(w_{io})} &= -\delta \overline{\ln(L_{io})} + \overline{\ln(A_{io})}.
\end{aligned} \tag{E1}$$

The above system is a traditional demand and supply setting and as it is well known, is under-identified. It is the classic textbook example of simultaneity bias. The reason for this under-identification is the following: while the variance-covariance matrix of $\left(\overline{\ln(L_{io})}, \overline{\ln(w_{io})}\right)$ gives us three moments from the data, the system above has five unknowns, which are the elasticities, η and δ, plus the three components of the variance-covariance matrix of the structural errors $\overline{\ln(\tilde{T}_{io})}$ and $\overline{\ln(A_{io})}$. Therefore, in absence of valid instruments that would exogenously vary either the supply or demand equations in (E1) we can not identify the elasticities through exclusion restrictions.

In order to identify the elasticities using the labor supply and demand equations in (E1), we impose restrictions on the variance-covariance matrix of the structural errors while exploiting the differences in the variance-covariance matrix of the employment and wages across occupations. This way of achieving identification is known in the literature as *identification through heteroskedasticity* (see Rigobon (2003)). We classify our four occupations into two broader categories $S \in \{1, 2\}$ which we denote as blue collar and white collar. Our identification assumption is that the covariance between the transformed productivity $\overline{\ln(A_{io})}$ and amenities $\overline{\ln(\tilde{T}_{io})}$, that we denote σ_{TA}, is constant within each category S. The fact that the elasticities are the same across occupational groups within the categories, in addition to the assumption of common covariance of the structural errors within broad categories, are the reason we can achieve identification. While the four occupational categories give us $3 \times 4 = 12$ moments, the unknowns to be identified are also 12: 2, δ and η, plus 2, the broad category covariances, plus 8, the variances of the transformed productivities and amenities for each of the four occupational categories.[7]

We can rewrite the system (E1) in the following way:

$$\begin{aligned}\overline{\ln(\tilde{T}_{io})} &= \overline{\ln(L_{io})} - \eta\,\overline{\ln(w_{io})},\\ \overline{\ln(A_{io})} &= \delta\,\overline{\ln(L_{io})} + \overline{\ln(w_{io})}.\end{aligned} \tag{E2}$$

Denote the covariance matrix of the structural errors for occupation o in category S (meaning the left hand side of system (E2)) by E_{oS}. Denote the covariance matrix between employment and wages of the full monosponists by Ω_{oS}. The covariance of system (E2) writes as:

$$E_{oS} = D\Omega_{oS}D^T, \quad D = \begin{pmatrix} 1 & -\eta \\ \delta & 1 \end{pmatrix}, \quad \Omega_{oS} = \begin{pmatrix} \sigma^2_{L,oS} & \sigma_{LW,oS} \\ \sigma_{LW,oS} & \sigma^2_{W,oS} \end{pmatrix},$$

where D^T denotes the transpose of matrix D. Defining an auxiliary parameter $\widetilde{\delta} = -\delta$ and using our identifying assumption that $\sigma_{AT,oS} = \sigma_{AT,o'S} = \sigma_{AT,S}$ for occupations that belong to the same category S, the system writes as:

$$\begin{pmatrix} \sigma^2_{T,oS} & \sigma_{TA,S} \\ \sigma_{TA,S} & \sigma^2_{A,oS} \end{pmatrix} = \begin{pmatrix} 1 & -\eta \\ -\widetilde{\delta} & 1 \end{pmatrix} \begin{pmatrix} \sigma^2_{L,oS} & \sigma_{LW,oS} \\ \sigma_{LW,oS} & \sigma^2_{W,oS} \end{pmatrix} \begin{pmatrix} 1 & -\widetilde{\delta} \\ -\eta & 1 \end{pmatrix}$$

This system only allows us to identify η and δ. Denote by $\Omega_S \equiv \Omega_{oS} - \Omega_{o'S}$ the difference between the variance covariance matrix within category S, $\Delta_S \equiv E_{oS} - E_{o'S}$, and $\Omega_{S,[i,j]} = \omega_{ij,S}$ the element on ith row and jth column of Ω_S. The system of differences is:

$$\Delta_S = D\Omega_S D^T, \quad \forall S \in \{1,2\}$$

With the identification assumption of equal covariance within category, we have that:

$$\Delta_{S,[1,2]} = 0 = -\eta\omega_{22,S} + (1 + \eta\widetilde{\delta})\omega_{12,S} - \widetilde{\delta}\omega_{11,S}.$$

Solving for η,

$$\eta = \frac{\omega_{12,S} - \widetilde{\delta}\omega_{11,S}}{\omega_{22,S} - \widetilde{\delta}\omega_{12,S}}, \quad \forall S \in \{1,2\}$$

Equalizing the above across both occupation categories we get a quadratic equation in $\widetilde{\delta}$ that solves:

$$\widetilde{\delta}^2[\omega_{11,1}\omega_{12,2} - \omega_{11,2}\omega_{12,1}] - \widetilde{\delta}[\omega_{11,1}\omega_{22,2} - \omega_{11,2}\omega_{22,1}] + \omega_{12,1}\omega_{22,2} - \omega_{12,2}\omega_{22,1} = 0. \tag{E3}$$

This is the same system as the simple case with zero covariance between the fundamental shocks

[7] Of course we could have a more stringent identification assumption that would leave us with an overidentified system, for example, that all covariances are equal to zero. As an additional exercise we also estimated the parameters following a different identification strategy: we assume that the covariances of the structural errors were the same among all the occupational groups. This gives us a system with one overidentification restriction. The point estimates using this assumption and the one we mentioned above are pretty similar.

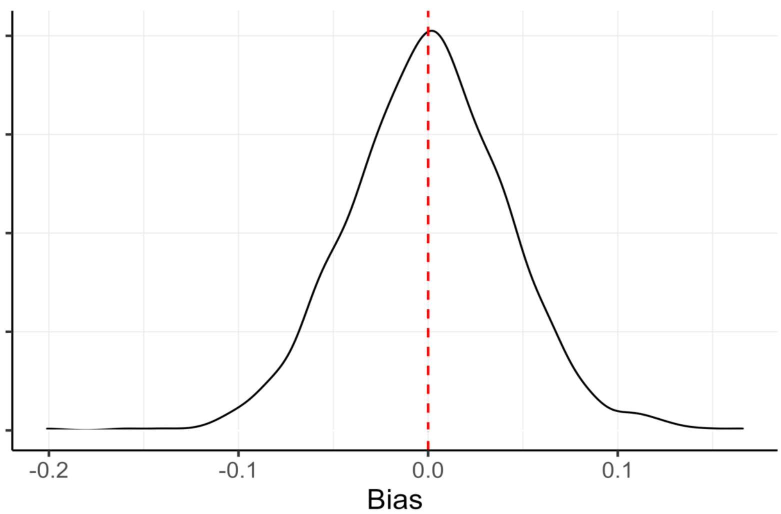
in Rigobon (2003). Different to him, Ω_S is not directly the estimated variance-covariance matrix of each of the 4 occupations but rather the matrix of covariance differences within category or state S. As mentioned by Rigobon (2003) there are two solutions to the previous equation. One can show that if $\widetilde{\delta}^*$ and η^* are a solution then the other solution is equal to $\widetilde{\delta} = 1/\eta^*$ and $\eta = 1/\widetilde{\delta}^*$. This means that the solutions are actually the two possible ways the original structural system (E1) can be written. We have that by assumption η is positive while $\widetilde{\delta}$ is negative. Therefore as long as the two possible solutions for $\widetilde{\delta}$ have different signs, we just need to pick the negative one.

E.2 Validation of the identification of ε_b

In this section, we validate our identification strategy of the within-labor market labor supply elasticities via simulations. We perform 1000 simulations of an economy populated with 200 local labor markets. For each simulation we have 14 years as in the application. The number of competitors in the local labor market follows an exponential distribution with mean 4 and standard deviation of 1, and the logarithm of productivities and amenities are normally distributed with means of 1 for both and standard deviations of 0.8 and 0.1 respectively. Population is assumed to be symmetrically distributed across local labor markets. We simulate productivities, amenities and number of competitors in local labor markets of the *Food* sector. We solve for each local labor market independently of aggregates and therefore characterize $w_{io} = \left(T_{io}^{\frac{1}{\varepsilon_b}} \lambda_{io} A_{io}\right)^{\frac{1}{1+\varepsilon_b \delta}}$ for each establishment.

We estimate equation (23) in the simulated equilibrium by regressing the logarithm of establishment employment on the logarithm of wages. We control for the strategic interactions within the local labor market by introducing local labor market fixed effects and therefore only use within-local labor market variation to identify the local elasticity of substitution. Figure E1 presents the bias of the IV estimates when we instrument for contemporaneous log wages by a proxy of establishment revenue productivity: $\widehat{A}_{iot} = \frac{P_{bt} Y_{jt}}{L_{iot}^{1-\delta}}$. The figure shows that even in the presence of amenities, which are labor supply shifters that correlate with wages, our identification strategy recovers the local elasticities of substitution as the density is centered around 0.

Figure E1: Bias of estimated ε_b



Note: The figure presents the estimation results from simulating local labor markets of sector 15 *Food*. It shows the density of the difference between the estimated local elasticity of substitution and the true parameter when simulating the model.

E.3 Identification of φ_b

In order to identify the sector specific workers bargaining power, we need to construct the model counterparts of the industry labor share at every period t:

$$LS_{bt}^{M}(\varphi_b) = \frac{\beta_b \sum_{io\in\mathcal{I}_b} w_{iot}L_{iot}}{\sum_{io\in\mathcal{I}_b} w_{iot}L_{iot}/\lambda(\mu_{io},\varphi_b)},$$

$\mathcal{I}_b$ being the set of all establishment-occupations that belong to sector b. We target the average across time industry labor share. That is, we pick ϕ_b such that:

$$\mathbb{E}_t\left[LS_{bt}^{M}(\varphi_b) - LS_{bt}^{D}\right] = 0, \tag{E4}$$

where LS_{bt}^{D} is the labor share of sector b at time t that we observe in the data. Given that the wedge $\lambda(\mu_{io},\varphi_b)$ is increasing in φ_b, then $LS_{bt}^{M}(\varphi_b)$ is increasing in φ_b as well. Therefore, if a solution exists for (E4) with $\varphi_b \in [0,1]$ this has to be unique.[8]

E.4 Amenities

In order to perform counterfactuals we still need to compute other policy invariant parameters, or fundamentals, from the data. In particular we need to recover establishment-occupation amenities and TFPRs, while ensuring that in equilibrium the wages and labor allocations are the same as in the data.

Using the establishments labor supply (12), we can back out amenities, up to a constant:

$$T_{io} = \frac{s_{io|m}}{(Pw_{io})^{\varepsilon_b}}\Phi_m.$$

The sub-market level Φ_m is a function of the amenities of all plants in m. We proceed by normalizing one particular local labor market. Note that the allocation of resources is independent from this normalization. We denote the local labor market that we normalize as 1. The relative employment share of market m with respect to the normalized one is: $\frac{L_m}{L_1} = \frac{\Phi_m^{\eta/\varepsilon_b}}{\Phi_1^{\eta/\varepsilon_{b'}}}\frac{\Gamma_b}{\Gamma_1}$. The local labor market aggregate is then:

$$\Phi_m = \left(\frac{L_m}{L_1}\frac{\Gamma_1}{\Gamma_b}\Phi_1^{\frac{\eta}{\varepsilon_{b'}}}\right)^{\frac{\varepsilon_b}{\eta}}$$

Substituting into the above we have that:

$$T_{io} \propto \frac{s_{io|m}}{(Pw_{io})^{\varepsilon_b}}\left(\frac{L_m}{\Gamma_b}\right)^{\varepsilon_b/\eta}.$$

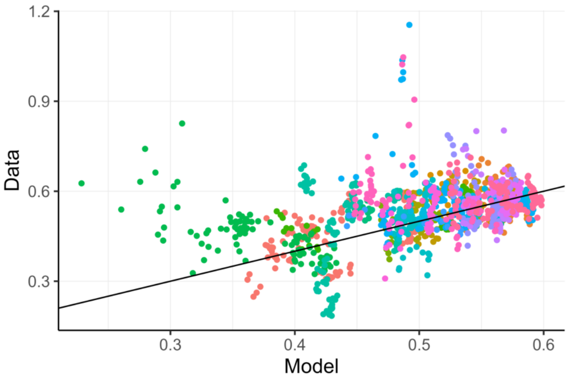
[8] It can be the case that the solution does not exist. For example, if given values of β_b, ε_b and η, even with $\varphi_b = 1$ the labor share generated by the model is too small to the one in the data.

E.5 Non targeted moments

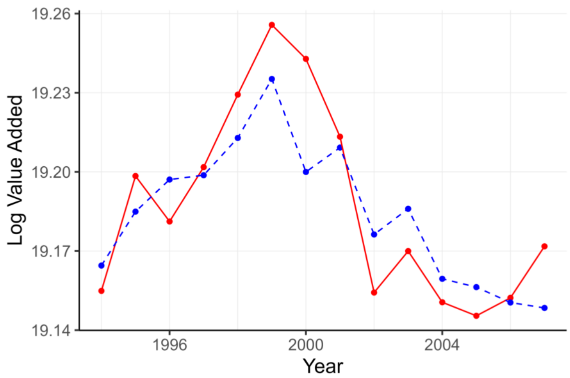
In panel (a) of Figure E2 we have 3-digit industry labor shares per year. On the horizontal axis, we have the model generated moments, while on the vertical axis, we have the corresponding observed moment in the data. If the fit was perfect, each dot would be on the 45 degree line. Each color represents a 2-digit industry. We see that most of the dots are aligned around the 45 degree line.

Panel (b) shows the model matches the evolution of aggregate value added. This in fact might not be surprising as there is a very strong relationship between establishment's production and wage bill in the model and in the data. Since the model matches the establishment's wages and labor allocations exactly, it also has a good fit of the value added.

Figure E2: Model Fit Non Targeted Moments



(a) Sub-industry Labor Share



(b) Aggregate Value Added. Model in dashed blue, data in red.

E.6 Additional estimation results

Table E1 presents the estimated output elasticities with respect to labor, within industry elasticities and the workers' bargaining power for every 2-digit industry.

We calibrate the elasticities of the final good production function $\{\theta_b\}_{b\in\mathcal{B}}$ for every year of the sample such that the industry expenditure shares are equal to the shares of industry value added in the data. Table E2 has the calibrated final good production function elasticities of the intermediate the $\{\theta_b\}_{b=1}^{\mathcal{B}}$ and the rental rate of capital $\{R_b\}_{b=1}^{\mathcal{B}}$ for the year 2007. Table E3 presents a comparison of the estimated within local labor market labor supply elasticities one period lagged instrument to the ones instrumented with a two period lagged revenue productivity. We take the estimates with a one period lagged instrument as our baseline estimation.

Table E1: Sector Estimates

Sector Code	Industry Name	$\widehat{\beta}_b$	$\widehat{\varepsilon}_b$	$\widehat{\varphi}_b$
15	Food	0.74	1.69	0.22
17	Textile	0.74	1.49	0.51
18	Clothing	0.84	1.41	0.31
19	Leather	0.85	2.09	0.26
20	Wood	0.77	1.51	0.42
21	Paper	0.61	3.06	0.55
22	Printing	0.84	1.52	0.18
24	Chemical	0.67	3.25	0.06
25	Plastic	0.73	2.51	0.35
26	Other Minerals	0.65	1.62	0.43
27	Metallurgy	0.61	3.77	0.59
28	Metals	0.81	1.22	0.38
29	Machines and Equipments	0.79	2.18	0.32
30	Office Machinery	0.81	3.33	0.20
31	Electrical Equipment	0.65	3.02	0.67
32	Telecommunications	0.62	3.54	0.73
33	Optical Equipment	0.75	1.91	0.45
34	Transport	0.57	4.05	0.69
35	Other Transport	0.72	3.49	0.44
36	Furniture	0.81	1.57	0.43

Notes: All the estimated parameters are 2-digit industry specific. $\widehat{\beta}_b$ are the estimated output elasticities with respect of labor, $\widehat{\varepsilon}_b$ are the within local labor market elasticities and $\widehat{\varphi}_b$ are union bargaining powers.

Table E2: Calibrated $\{\theta_b\}$ and $\{R_b\}$

Industry Code	Industry Name	θ_b	R_b
15	Food	0.13	0.11
17	Textile	0.02	0.14
18	Clothing	0.01	0.14
19	Leather	0.01	0.14
20	Wood	0.02	0.13
21	Paper	0.02	0.13
22	Printing	0.06	0.13
24	Chemical	0.14	0.16
25	Plastic	0.06	0.15
26	Other Minerals	0.05	0.15
27	Metallurgy	0.03	0.14
28	Metals	0.10	0.14
29	Machines and Equipments	0.09	0.17
30	Office Machinery	0.00	0.17
31	Electrical Equipment	0.04	0.23
32	Telecommunications	0.04	0.23
33	Optical Equipment	0.04	0.23
34	Transport	0.04	0.19
35	Other Transport	0.06	0.19
36	Furniture	0.03	0.14

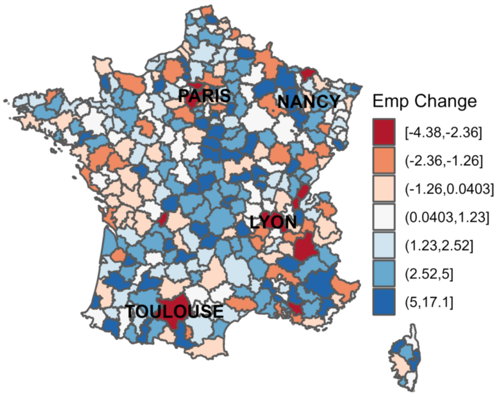
Notes: All the calibrated parameters are 2-digit industry specific for the year 2007. θ_b are the intermediate good elasticities in the final good production function and R_b are the capital rental rates for 2007. We construct the rental rates following Barkai (2020).

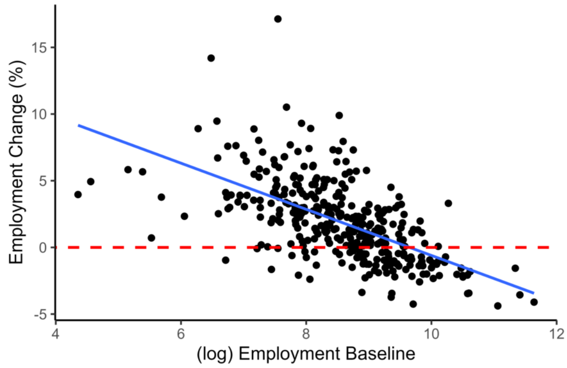
Table E3: Estimated Within Elasticities for Different Lags

Industry Code	Industry Name	1 Lag $\widehat{\varepsilon}_b$	2 Lags $\widehat{\varepsilon}_b$
15	Food	1.69	1.99
17	Textile	1.49	1.83
18	Clothing	1.41	1.69
19	Leather	2.09	2.50
20	Wood	1.51	1.77
21	Paper	3.06	3.39
22	Printing	1.52	1.79
24	Chemical	3.25	3.56
25	Plastic	2.51	3.04
26	Other Minerals	1.62	1.77
27	Metallurgy	3.77	4.35
28	Metals	1.22	1.48
29	Machines and Equipments	2.18	2.63
30	Office Machinery	3.33	3.72
31	Electrical Equipment	3.02	3.61
32	Telecommunications	3.54	4.08
33	Optical Equipment	1.91	2.36
34	Transport	4.05	4.56
35	Other Transport	3.49	4.05
36	Furniture	1.57	1.90

Notes: All the estimated parameters are 2-digit industry specific. *1 Lag* $\widehat{\varepsilon}_b$ are the estimated within local labor market elasticities when we instrument for the wages with one lag and *2 Lags* $\widehat{\varepsilon}_b$ present the analogous when we instrument with two lags.

Figure F1: Employment Change (%) in the Counterfactual: Perfect Competition





Notes: The map presents employment changes with respect to the baseline economy in percentages within commuting zones. The counterfactual without labor wedges is performed for the year 2007. The figure in the right plots the employment change in the counterfactual versus the log of employment in the baseline. The blue line is a fitted line from an OLS regression.

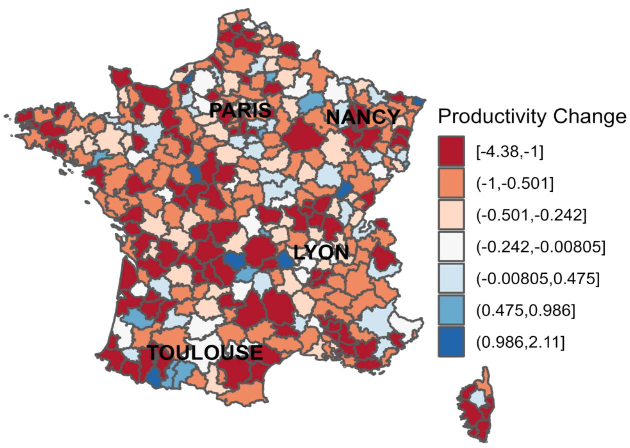
F Counterfactuals

We present additional results of the main counterfactual and other implications of labor market power on urban-rural differences.

F.1 Main counterfactuals

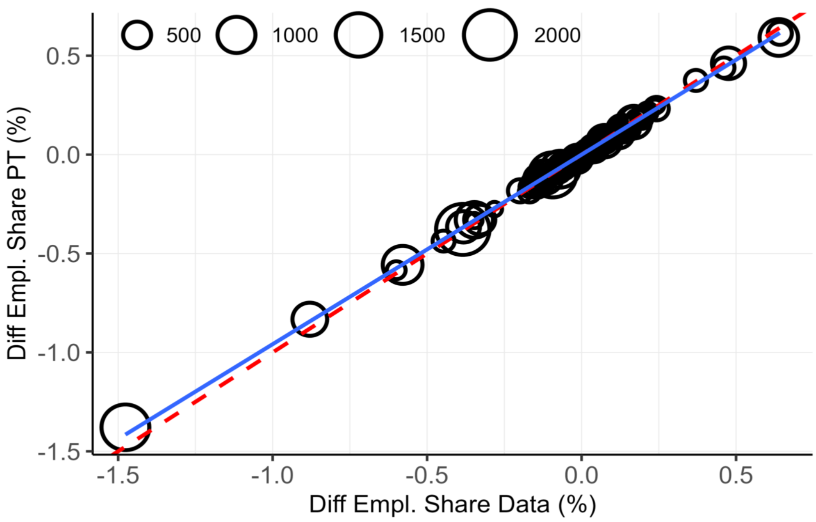
Figure F2 shows productivity changes in the counterfactual with oligopsonistic competition relative to the baseline. The map shows that the biggest productivity losses happen outside big cities and some commuting zones increase overall productivity due to labor mobility across sectors.

Figure F2: Productivity Change (%) in the Counterfactual: Oligopsonistic Competition



Notes: The map presents productivity changes with respect to the baseline economy in percentages. Each block constitutes a commuting zone. Local labor markets are aggregated up to the commuting zone. Commuting zone productivity is an employment weighted average of individual productivities. Following the discussion in Section C.1, keeping fixed the baseline revenue productivities, any change in the counterfactual comes from changes in aggregate productivities from the reallocation of workers. Counterfactuals are performed for the year 2007.

Figure F3: De-industrialization differences



Notes: The x-axis shows the percentage differences of commuting zone employment shares out of manufacturing over time in the data ($\Delta^D = S^D_{07} - S^D_{94}$). The y-axis presents the analogous for the counterfactual without wedges ($\Delta^M = S^{PT}_{07} - S^{PT}_{94}$). The first year is 1994 and the last one is 2007. The bubble size represents the level of employment in thousands at the commuting zone for the first year. The blue line represents a fitted line from an OLS regression. A weighted least squares regression using initial employment as weights gives a vert similar result.

F.2 The effect of labor market power on urban-rural differences

Figure F1 suggests an important labor reallocation from cities to rural areas in the counterfactual without labor wedges. This section explores the impact of employer and union labor market power on the urban-rural mobility over time and the urban-rural wage gap.

Employment changes

We compare the urban-rural manufacturing employment changes over time observed in the data to the ones from yearly counterfactuals without union and firm labor market power. In the data, the de-industrialization or the reduction of manufacturing employment occurred primarily in cities leading to the gain in relative importance of rural areas within manufacturing. Figure F3 compares the relative employment shares observed in the data to the one in a counterfactual without labor wedges for each commuting zone.

First, we performed the main counterfactual where there are no labor wedges because establishments and unions act as price takers (PT) for the initial year 1994. Then we compute the commuting zone employment share out of total manufacturing employment for the initial and final years (1994 and 2007 respectively) and for the different scenarios. To compare mobility over time, we compute the differences over time of the commuting zone employment shares in the data ($\Delta^D = S^D_{07} - S^D_{94}$) and in the counterfactual ($\Delta^M = S^{PT}_{07} - S^{PT}_{94}$). Figure F3 in presents this comparison. The x axis shows the time difference in the data Δ^D and the y axis shows the time difference in the model counterfactual without labor wedges Δ^M. The size of the dots is the initial level of manufacturing employment of the commuting zone. The counterfactual urban-rural mobility is very similar to the process observed in the data which is mostly guided by exogenous productivity and firm location decisions and not by labor market distortions.

The line generated by the largest population commuting zones in Figure F3 is slightly flatter than

Table F2: Counterfactuals: Agglomeration. Perfect Competition

	ΔY (%)	Δ Prod (%)	Contribution ΔY (%)		
			GE	Productivity	Labor
No Agglomeration	1.62	1.33	9	83	8
Agglomeration					
$\gamma = 0.05$	1.73	1.40	8	82	10
$\gamma = 0.1$	1.84	1.48	7	81	12
$\gamma = 0.2$	2.08	1.66	5	80	15
$\gamma = 0.25$	2.22	1.75	3	80	17
$\gamma = 0.3$	2.36	1.86	2	80	18

Notes: Results are in percentages. First column ΔY is the change of aggregate output with respect to the baseline, Δ *Prod* is the change in aggregate productivity from decomposition (28). Last three columns present the contribution of each of the elements of the decomposition (28) to output gains. *No Agglomeration* is the main counterfactual without wedges, under free mobility of labor, fixed total labor supply and no agglomeration forces. All the other counterfactuals in this table allow for agglomeration within the local labor market. Similarly to the main counterfactual, workers are freely mobile and total employment is fixed. We present different counterfactuals depending on the agglomeration parameter γ.

the 45 degree line. Cities would loose their relative importance a bit slower in the counterfactual. A potential reason is the closure of manufacturing firms in the largest cities, which became more concentrated over time leading to distortions closer to the ones present in rural areas.

Wage gap

Table F1: Wage Gap

	Rural Wage	Urban Wage	Gap (%)
Baseline	33.319	45.210	36
Counterfactual. Oligopsony	24.592	36.861	50
Counterfactual. No wedges	49.486	60.675	23

Note: Wages in constant 2015 euros. We classify as *Urban* the 10 biggest commuting zones: Paris, Marseille, Lyon, Toulouse, Nantes, and the Paris surrounding areas of Boulogne-Billancourt, Creteil, Montreuil, Saint-Denis and Argenteuil. The rest are considered as *Rural*. Wages are employment weighted averages per urban/rural location for the year 2007.

Table F1 presents wage levels and the urban-rural wage gap.[9] Both urban and rural areas experience important wage gains in the counterfactual. Gains being bigger outside cities the wage gap is reduced from 36% to 23% in the counterfactual. This reveals that labor market distortions account for more than a third of the urban-rural wage gap.

F.3 Extensions

Table F2 presents counterfactuals with agglomeration externalities under perfect competition where wages are equal to the marginal revenue product of labor. Baseline gains (1.62%) are amplified with agglomeration due to the productivity gains.

[9] We consider urban the 10 biggest commuting zones: Paris, Marseille, Lyon, Toulouse, Nantes, and the Paris surrounding areas of Boulogne-Billancourt, Creteil, Montreuil, Saint-Denis and Argenteuil. Rural is the rest of the commuting zones.

G Data details

We provide details about sample selection and variable construction.

G.1 Sample selection

Ficus/Fare. This data source comes from tax records therefore we observe yearly firm information. We exclude the source tables belonging to public firms.[10] Before 2000 we take table sources in euros and from 2001 onward we use data from consolidated economic units.[11] After excluding firms without a firm identifier, the raw data sample contains about 29 million firms, of which about 2.8 million are manufacturing firms.[12] Manufacturing sector (sector code equal to D) constitutes on average 10% of the observations, 19.2% of value added and 27.2% of employment.

Postes. *DADS Postes* covers all the employment spells of a salaried employee per year. If a worker has several spells in a year we would have multiple observations. The main benefit of this employer-employee data source is that we can know the establishment and employment location of the workers. We exclude workers in establishments with fictitious identifiers (SIREN starting by F) and in public firms. For every establishment identifier (SIRET) we sum the wage bill and the full time equivalent number of employees.

Merged data. After merging both data sources, we end up with data that include yearly establishment observations. After the filters and merging the sample consists of 1.3 million firms and 1.6 million establishment observations. In the process of filtering and merging, about half of the original firms are lost. Wages and value added are deflated using the Consumer Price Index.[13]

Labor and wage data, coming from the balance sheets (at the firm level) and the one from employee records, needs to be consolidated. In order to be consistent with balance sheet information we assign labor and employment coming from *FICUS* to the establishments according to their respective shares. We proceed in several steps. First, we filter out observations with no wage or employment information from *Postes* from firms present at different commuting zones. Second, we get rid of observations with no labor, capital and wage bill information coming from *FICUS* and also observations with non existing or missing commuting zone. Third, we aggregate employee data to the firm times commuting zone level.[14] What we call establishment throughout the text is the entity aggregated at the commuting zone level. Then we compute the labor and wage shares of these entities out of the firm's aggregates. Finally, we split firm data from the balance sheet according to those shares. This procedure leaves the firms in a unique commuting zone with their balance sheet data but allows to split wage bill and employment data coming from the balance

[10] We only use the Financial units (*FIN*) and Other units (*TAB*) tables and exclude Public administration (*APU*).

[11] The profiling of big groups consolidates legal units into economic units. In 2001 the Peugeot-Citroën PSA was treated, Renault in 2003 and the group Accor in 2005. This implies the definition of new economic entities and would therefore lead to the creation of new firm identifiers. Given the potential impact of big establishments in local labor markets we opted to maintain them.

[12] We consider a missing firm identifier (SIREN) also if the identifier equals to zero for all the 9 digits.

[13] Nominal variables are expressed in constant 2015 euros.

[14] Data from 1994 and 1995 do not have commuting zone information. We therefore impute it using correspondence tables between city code and commuting zone. A city code has 5 digits coming from the department and city. Some commuting zone codes beyond the 2 missing years were modified or cleaned. City codes (*commune* codes) of Paris, Marseille and Lyon were divided into different *arrondissements*. We assign them codes 75056, 13055 and 69123 respectively. Then we proceed to the cleaning of commuting zones by assigning to the non existing codes the one corresponding to the city where the establishment is located. We get rid of non matched or missing commuting zone codes. We aggregate the data coming from *Postes* at the commuting zone level after this cleaning.

sheet for multi-location firms. Establishment wage is simply the average wage. That is, wage bill over total full time equivalent employees.

We further exclude Tobacco (2-digits industry code 16), Refineries & Nuclear industry (code 23) and Recycling (code 37). We finally get rid of the outliers reducing the sample 1.5% and finish with 4,156,754 establishment-occupation-year observations that belong to 1.25 million firms.[15]

G.2 Variable construction

Ficus:

- Value added: value added net of taxes (*VACBF*). We restrict to firms with strictly positive value added.[16]
- Capital: tangible and intangible capital without counting depreciation. It is the sum of the variables *IMMOCOR* and *IMMOINC*.
- Employment: full time equivalent employment at the firm (*EFFSALM*).
- Wage bill: gross total wage bills. Is the sum of wages (*SALTRAI*) and firm taxed (*CHARSOC*).[17]
- Industry: industry classification comes from *APE*. The sub-industries h are 3 digit industries and industries b are at two digits.

Postes:

- Occupation: original occupation categories come from the two digit occupations (*CS2*). We group occupations with first digits 2 and 3 into a unique occupation group.[18] This regrouping is done to avoid substantial changes in occupation groups between 1994 and 2007. Observations with missing occupation information are excluded.
- Employment: full time equivalent at the establishment-occupation level (*etp*).
- Wage: is the gross wage (per year) of individual worker (*sbrut*). If the spell is less than a year is the gross wage corresponding to the spell.
- Commuting zone: depending on the year, the commuting zone classification is taken from the variable *zemp* or *zempt*. Commuting zone information is missing for the years 1994 and 1995 and is imputed using the city codes.[19]

G.3 Construction of required rates

In order to construct the required rates for the different sectors we follow the methodology proposed by Barkai (2020) using the Capital Input Data from the EU KLEMS database, December 2016 revision. In this dataset one can find, for a given industry, different depreciation rates and price indices for different types of capital. The types of capital that are present in the manufacturing sector are: Computing Equipment, Communications Equipment, Computer Software and Databases,

[15] We get rid of wage per capita outliers by truncating the sample at the 0.5% below and 99.5%.
[16] We follow the advise of the French statistical instiute (INSEEE) in using net value added to perform comparisons across industries.
[17] For firms declaring at the BIC-BRN regime (*TYPIMPO*= 1) we only take *SALTRAI* .
[18] Occupations with first digit 1 and 7 are excluded. They constituted less than 0.05% of the matched sample.
[19] City codes are the concatenation of department (*DEP*) and city (*COM*).

Transport Equipment, Buildings and structures (non-residential), and Research and Development. We construct a required rate for each of the industries at the 2 digit level defined in the NAF classification. However, unlike the NAF classification, that we have up to 20 different industries, there are only 11 industries classified within manufacturing within the EU KLEMS database. Any industry classification in EU KLEMS is just an aggregation of the 2 digit industry classification in NAF. Therefore, there are industries within the NAF classification that will have the same required rate of return on capital.

For a type of capital s and sector b, we define the the required rate of return R_{sb} as:

$$R_{sb} = \left(i^D - \mathbb{E}\left[\pi_{sb}\right] + \delta_{sb}\right),$$

where i^D is a the cost fo debt borrowing in financial markets, and π_{sb} and δ_{sb} are, respectively, the inflation and depreciation rates of capital type s in sector b.

Then we define the total expenditures on capital type s in sector b as:

$$E_{sb} = R_{sb} P^K_{sb} K_{sb},$$

where $P^K_{sb} K_{sb}$ is the nominal value of capital stock of type s. Summing over all types of capital within a sector we can obtain the total expenditures of capital of such sector:

$$E_b = \sum_{sb} R_{sb} P^K_{sb} K_{sb}.$$

Multiplying and dividing by the total nominal value of capital stock we obtain the following decomposition:

$$\sum_s R_{sb} P^K_{sb} K_{sb} = \underbrace{\sum_s \frac{P^K_{sb} K_{sb}}{\sum_{s'} P^K_{s'b} K_{s'b}} R_{sb}}_{R_b} \underbrace{\sum_s P^K_{sb} K_{sb}}_{P^{Kb} K_b},$$

where the first term R_b is the interest rate that we use in the model.

H Summary statistics

Tables H1, H2 and H3 contain summary statistics of sub-industries, local labor markets and commuting zones for the year 2007, which is the year we use for our counterfactuals. Table H4 presents worker transition probabilities across occupations, industries and commuting zones.

Table H1: Sub-industry Summary Statistics. Baseline Year

Variable	Mean	Pctl(25)	Median	Pctl(75)	St. Dev.
N_h	2,840	493	1,261	2,639	4,530.5
L_h	30,466	7,559	15,070	50,036	33,899.3
$\overline{w}_h$	34.6	29.6	33.0	37.531	6.9
LS_h	0.52	0.48	0.53	0.58	0.10
KS_h	0.26	0.17	0.23	0.32	0.13

Notes: There are 97 3-digit industries, or sub-industries, in the sample. N_h is the number of establishments per 3-digit industry h, L_h is total employment of h, $\overline{w}_h$ is the average establishment wage of h, LS_h is the labor share and KS_h is the capital share. We get the capital shares following Barkai (2020). All the nominal variables are in thousands of constant 2015 euros.

Table H2: Local Labor Market Summary Statistics. Baseline Year

Variable	Mean	Pctl(25)	Median	Pctl(75)	St. Dev.
N_m	4.76	1	2	4	14.4
L_m	51.0	2.8	9.4	34.9	196.2
$\overline{w}_m$	36.6	24.3	30.2	42.5	36.1
$\widehat{w}_m$	36.2	24.1	30.0	42.2	25.6
HHI($s_{io\|m}$)	0.67	0.38	0.68	1.00	0.32
HHI($s^w_{io\|m}$)	0.68	0.39	0.70	1.00	0.32

Notes: There are 57,940 local labor markets in the year 2007. N_m is the number of competitors in the local labor market m, L_m is total employment in m, $\overline{w}_m$ is the mean w_{iot} of the establishment-occupations in m, $\widehat{w}_m$ is the weighted average wage at m with weights equal to employment shares, HHI($s_{io|m}$) and HHI($s^w_{io|m}$) are respectively the Herfindahls with employment and wage shares. All the nominal variables are in thousands of constant 2015 euros.

Table H3: Commuting Zones Summary Statistics. Baseline Year

Variable	Mean	Pctl(25)	Median	Pctl(75)	St. Dev.
N_n	773.798	266.8	461	861.2	1,168.407
L_n	8,300.567	2,567.403	5,244.300	10,086.210	11,322.000
$\overline{L}_n$	11.389	8.148	10.878	13.547	6.043
$\overline{w}_n$	34.399	32.707	34.161	35.593	3.242

Notes: There are 356 commuting zones in the sample. N_n is the number of establishments at the CZ, L_n is full time equivalent employment at CZ, $\overline{L}_n$ is the average L_{iot} of establishment-occupations at n, $\overline{w}_n$ is the mean w_{iot} of the establishment-occupations at n in thousands of constant 2015 euros.

Table H4: Transition Probabilities

Occupation	Commuting Zone	Industry	Trans. Prob. FTE	Trans. Prob.
0	0	0	91.39	91.01
0	0	1	2.37	2.36
0	1	0	0.02	0.02
1	0	0	6.03	6.40
1	0	1	0.20	0.21
1	1	0	0.00	0.00
1	1	1	0.00	0.00

Notes: The transition rates are computed over the whole sample period 1994-2007. *Occupation* is an indicator function of occupational change, *Commuting Zone* is an indicator function of commuting zone change, *Industry* is an indicator function of 3-digit industry change, *Trans. Prob. FTE* are the unconditional transition probabilities based on full time equivalent units and *Trans. Prob.* are the unconditional transition probabilities based on counts of working spells independently of duration and part-time status.

I Empirical evidence

In this section, we provide the link between the reduced form relating labor market power to wages and our structural framework. We also present additional results, robustness checks and results on rent sharing elasticities.

I.1 Labor market power and wages

I.1.1 Instrument: Mass layoff shock

The mass layoff shock instrument we use intends to capture the effect of a negative idiosyncratic productivity shock on close competitors. To provide some intuition on how the instrument works, it will be helpful to focus on a local labor market with only 2 competitors. Using Proposition 1 and getting rid of the occupational subscript o and assuming constant amenities for simplicity, the employment share of establishment 1 is:

$$s_{1|m} = \frac{\left(\lambda(s_{1|m})A_1\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}{\left(\lambda(s_{1|m})A_1\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} + \left(\lambda(1-s_{1|m})A_2\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}},$$

where the numerator is the aggregator Φ_m. This equation completely characterizes the equilibrium in the local labor market as the employment share of the other establishment in the market is equal to $1 - s_{1|m}$.

The above equation implicitly defines $s_{1|m}$ as a function of A_2 and $\lambda(g(s_{1|m}))$, where $g(s_{1|m}) = s_{1|m}$ or $g(s_{1|m}) = 1 - s_{1|m}$. We can represent the above system as: $F\left(s_{1|m}, A_2, \lambda(g(s_{1|m}))\right)$. Using the implicit function theorem we have that: $\frac{ds_{1|m}}{dA_2} = -\frac{\frac{\partial F(\cdot)}{\partial A_2}}{\frac{\partial F(\cdot)}{\partial s_{1|m}}}$. Developing the partial derivatives, we get:

$$\frac{\partial F(\cdot)}{\partial A_2} = -\Phi_m^2 \frac{\varepsilon_b}{1+\varepsilon_b\delta}\lambda(1-s_{1|m})^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} A_2^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}-1} < 0,$$

and,

$$\begin{aligned}
\frac{\partial F(\cdot)}{\partial s_{1|m}} &= \Phi_m^{-2}\frac{\varepsilon_b}{1+\varepsilon_b\delta}\frac{\partial\lambda(s_{1|m})}{\partial s_{1|m}}\left(\lambda(s_{1|m})A_1\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}\lambda(s_{1|m})^{-1}\Phi_m - \left(\lambda(s_{1|m})A_1\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}\frac{\varepsilon_b}{1+\varepsilon_b\delta}\Phi_m^{-2} \\
&\left[\left(\lambda(s_{1|m})A_1\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}\lambda(s_{1|m})^{-1}\frac{\partial\lambda(s_{1|m})}{\partial s_{1|m}} - \left(\lambda(1-s_{1|m})A_2\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}\lambda(1-s_{1|m})^{-1}\frac{\partial\lambda(1-s_{1|m})}{\partial(1-s_{1|m})}\right] - 1 \\
&= \frac{\varepsilon_b}{1+\varepsilon_b\delta}s_{1|m}\left\{\frac{\partial\lambda(s_{1|m})}{\partial s_{1|m}}\lambda(s_{1|m})^{-1} - \left[s_{1|m}\frac{\partial\lambda(s_{1|m})}{\partial s_{1|m}}\lambda(s_{1|m})^{-1} - (1-s_{1|m})\lambda(1-s_{1|m})^{-1}\frac{\partial\lambda(1-s_{1|m})}{\partial(1-s_{1|m})}\right]\right\} - 1 \\
&= \frac{\varepsilon_b}{1+\varepsilon_b\delta}s_{1|m}(1-s_{1|m})\left\{\frac{\partial\lambda(s_{1|m})}{\partial s_{1|m}}\lambda(s_{1|m})^{-1} + \lambda(1-s_{1|m})^{-1}\frac{\partial\lambda(1-s_{1|m})}{\partial(1-s_{1|m})}\right\} - 1 < 0,
\end{aligned}$$

where we used the expression of the employment share above and the fact that $\frac{\partial\lambda(s_{1|m})}{\partial s_{1|m}} < 0$ and $\frac{\partial\lambda(1-s_{1|m})}{\partial(1-s_{1|m})} < 0$. We therefore have that: $\frac{ds_{1|m}}{dA_2} < 0$. In turn, abstracting from market level constants,

$\log(w_1) = \left(\lambda(s_{1|m})A_1\right)^{\frac{1}{1+\varepsilon_b\delta}}$. The effect of a change in A_2 on $\log(w_1)$ is:

$$\begin{aligned}\frac{d\log(w_1)}{dA_2} &= \frac{\partial\log(w_1)}{\partial A_2} + \frac{\partial\log(w_1)}{\partial s_{1|m}}\frac{ds_{1|m}}{dA_2} \\ &= \quad 0 \quad + \frac{\partial\log(w_1)}{\partial s_{1|m}}\frac{ds_{1|m}}{dA_2} \\ &= \underbrace{\frac{\partial\log(w_1)}{\partial\log(\lambda(s_{1|m}))}}_{>0}\underbrace{\frac{\partial\log(\lambda(s_{1|m}))}{\partial s_{1|m}}}_{<0}\underbrace{\frac{ds_{1|m}}{dA_2}}_{<0} > 0.\end{aligned}$$

Therefore, when a shock occurs to a competitor's productivity, the covariance between employment shares and log wages becomes negative. If we use an IV regression, we can identify the reduced-form effect. However, the reduced-form effect would be different from the structural estimate obtained when there is a change in the employment share—triggered by a change in a competitor's productivity—while holding everything else constant. This is because, as explained in section 5.2 of the main text, strategic interactions can trigger responses from other market participants, which changes the underlying environment. However, as explained by Berger et al. (2022), the reduced-form estimate is still informative of the structural response. Reassuringly, our reduced-form estimate provides the same qualitative result as the structural one: a negative relation between employment share and log wages after a competitor's shock.

Definition of a mass layoff. The definition of a mass layoff is firm-occupation specific. Denote by ML the set of firm-occupations with a *national* mass layoff. That is, firm-occupations with all the establishments suffering a mass layoff. We instrument the employment share of the establishments of firm-occupations not suffering the national mass layoff $j \notin ML$ by the exogenous event of a firm present at the local labor market having a negative shock. We restrict the analysis to non-shocked multi-location firm-occupations with at least one establishment in a sub-market where a competitor has suffered a mass layoff and another establishment whose competitors do not belong to firms in ML.

Defining a cut-off value κ, we identify a firm-occupation $j \in ML$ if employment at t is less than κ% employment last year for all the establishment-occupations. That is, a firm j at occupation o has a mass layoff shock if $L_{io,t}/L_{io,t-1} < \kappa \, \forall i$ belonging to firm j. A local labor market is identified as shocked $D_{m,t} = 1$ if at least one establishment at the local market belongs to a firm in ML.

The first stage is:

$$s_{io|m,t} = \psi_{\mathbf{J}(i),o,t} + \delta_{\mathbf{N}(i),t} + \gamma\, D_{m,t} + \epsilon_{io,t}$$

where as before, $\psi_{\mathbf{J}(i),o,t}$ is a firm-occupation-year fixed effect and $\delta_{\mathbf{N}(i),t}$ is a commuting zone times year fixed effect. Using the fitted values we consider the following model for the second stage:

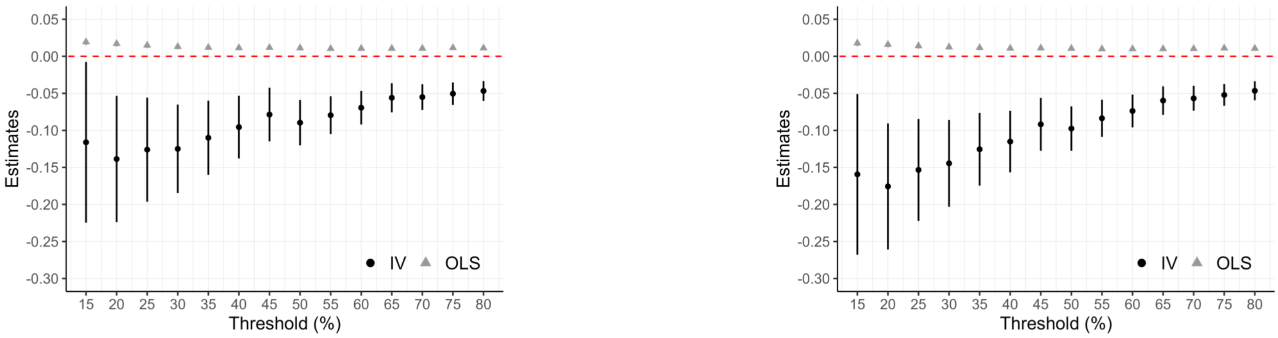
$$\log(w_{io,t}) = \psi_{\mathbf{J}(i),o,t} + \delta_{\mathbf{N}(i),t} + \alpha\, \widehat{s_{io|m,t}} + u_{io,t} \tag{I1}$$

I.1.2 Robustness checks

This section presents robustness checks of the reduced form evidence. First, we consider a different instrument for the employment shares and we change the main specification by taking commuting zone fixed effects. The results in the main text are with commuting zone-year fixed effects. Second, we present a robustness check to a different definition of local labor markets.

Instrument. Panel (a) of Figure I1 shows a robustness check where the new instrument is not binary anymore and takes into account the original employment share of the mass layoff establishments. Panel (b) of the same figure shows the results using the main text speciication but with commuting zone fixed effects. Results are qualitatively unchanged from the baseline in both cases.

Figure I1: Robustness



(a) Instrument: Intensive Share

(b) CZ fixed effects

Notes: This figures present the point estimates and 95% confidence bands of the OLS and IV exercises on the y-axis. The x-axis presents different thresholds κ that define a mass layoff shock. In both cases we focus on non-affected competitors (not suffering a mass layoff shock). The instrument in Panel (a) is the presence of a mass layoff shock firm in the local labor market interacted with the employment share of the affected firm. Panel (b) presents the results with commuting zone fixed effects.

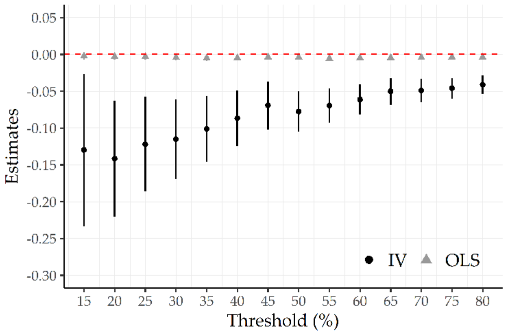
Controlling for labor demand. When there are decreasing returns to scale, establishments would have a demand with a negative slope. Thus, an increase in the employment level could lead to wage reductions if there is movement along the labor demand curve. To take into account the potential effects of changes along the labor demand curve after the mass-layoff shock, we control for the logarithm of establishment-occupation employment level as in the following model:

$$\log(w_{io,t}) = \beta s_{io|m,t} + \gamma \log(L_{io,t}) + \psi_{\mathbf{J}(i),o,t} + \delta_{\mathbf{N}(i),t} + \epsilon_{io,t}\,, \tag{I2}$$

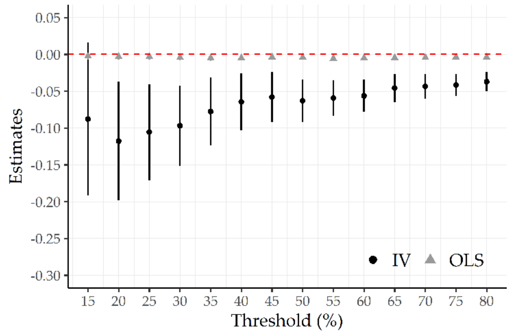
where $\log(w_{io,t})$ is the log average wage at plant i of firm j and occupation o at local labor market m in year t, $s_{io|m,t}$ is the employment share of the plant out of the market m, $\log(L_{io,t})$ is the logarithm of the establishment-occupation employment, $\psi_{\mathbf{J}(i),o,t}$ is a firm-occupation-year fixed effect, $\delta_{\mathbf{N}(i),t}$ is a commuting zone-year fixed effect and $\epsilon_{io,t}$ is an error term. Our parameter of interest is β.

There are two potentially endogenous variables, $s_{io|m,t}$ and $\log(L_{io,t})$, so we follow two approaches. First, we instrument $s_{io|m,t}$ with the presence of mass-layoff shocks in the local labor market and add the contemporaneous logarithm of employment as a control. Even if this last instrument would not satisfy the standard exclusion restriction, we can still get a consistent estimate of β with a different conditional mean independence assumption. To see this, let Z be the

Figure I2: Additional Robustness



(a) Control $\log(L_{io,t})$



(b) Control $\log(L_{io,t})$. Instrument $\log(L_{io,t-1})$

Notes: This figures present the point estimates and 95% confidence bands of the OLS and IV exercises on the y-axis. The x-axis presents different thresholds κ that define a mass layoff shock. The instrument is the presence of a firm with a mass layoff shock in the local labor market. We focus on non-affected competitors (not suffering a mass layoff shock). The specification is equation (I2). The left figure controls directly for $\log(L_{io})$ and the right figure instruments the logarithm of employment with its lagged value.

mass-layoff shock instrument, and W is the vector of controls, which includes the logarithm of employment and the fixed effects. We have abstracted from subscripts to ease on notation. Then, if $\mathbb{E}(\epsilon|Z,W) = \mathbb{E}(\epsilon|W) = W\xi$ we can still obtain a consistent estimate of β using instrumental variables.[20] In the second approach, we use lagged values of the employment logarithm as an instrument instead of its contemporaneous value. The left panel of Figure I2 we present the estimates for β estimating the model (I2) using the first approach. In the right panel, we do the same but using the second approach.

Local labor market. Figure I3 does the same exercise as in the main empirical strategy but changing the definition of local labor market. Local labor markets are here defined with 2-digit industries instead of 3-digit industries.[21] The specification includes commuting zone fixed effects as in Figure I1 Panel (b).

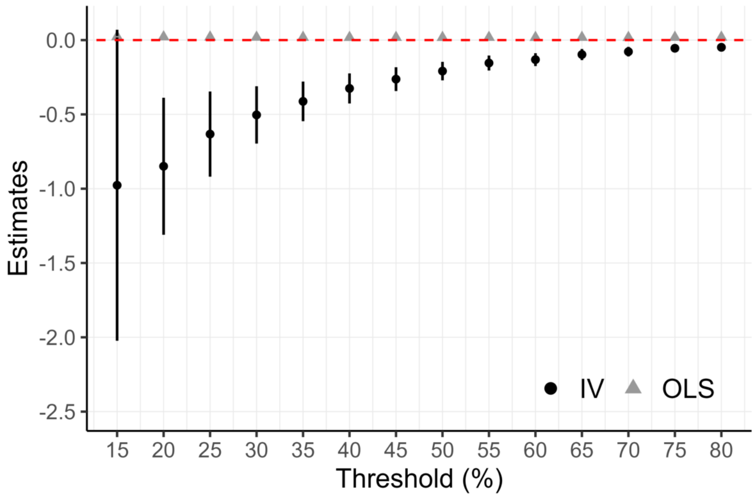
Alternative instrument. We build an additional instrument for the employment share by lagged concentration measures. More specifically, we instrument the employment share $s_{io|m,t}$ by the lagged inverse of the number of competitors in the local labor market $1/N_{m,t-1}$. Lagged concentration measures exclude potential endogeneity of the market structure to current period shocks. The correlation between employment shares and lagged concentration measures is 0.77.

Table I1 shows the results. The first two columns recover estimates of the specification (1) with commuting zone (CZ) fixed effects and the last two columns with commuting zone-year fixed effects. Columns 1 and 3 present the Ordinary Least Squares (OLS) estimates. The model reflects both labor demand and supply therefore a direct estimation by OLS is problematic and expected to be biased towards zero. We indeed find that both OLS estimates are very close to zero and positive. Columns 2 and 4 present the results once we instrument for the employment share. Both specifications (with CZ and CZ-year fixed effects) give the same point estimates. These estimates

[20] Proof: Let the original regression be $y = \beta s + W\tilde{\gamma} + \epsilon$. Then, assume that $\mathbb{E}(\epsilon|Z,W) = \mathbb{E}(\epsilon|W) = W\xi$. This implies that $y = \beta s + W\tilde{\gamma} + \epsilon - \mathbb{E}(\epsilon|W) + \mathbb{E}(\epsilon|W) = \beta s + W(\tilde{\gamma} + \xi) + \tilde{\epsilon}$, where $\tilde{\epsilon} = \epsilon - \mathbb{E}(\epsilon|W)$. Then $\mathbb{E}(\tilde{\epsilon}|Z,W) = \mathbb{E}(\epsilon|Z,W) - \mathbb{E}(\epsilon|W) = \mathbb{E}(\epsilon|Z,W) - \mathbb{E}(\epsilon|Z,W) = 0$. Thus, an IV regression can obtain consistent estimates of β and $(\tilde{\gamma} + \xi)$.

[21] That is, a local labor market is defined as a combination between commuting zone, 2-digit industry and occupation.

Figure I3: Robustness. Local Labor Market at 2-digit Industry



Notes: This figure presents the point estimates and 95% confidence bands of the OLS and IV exercises on the y-axis. The x-axis presents different thresholds κ that define a mass layoff shock. We focus on non-affected competitors (not suffering a mass layoff shock). The instrument is the presence of a mass layoff shock firm in the local labor market. The definition of local labor market is a combination of commuting zone, 2-digit industry and occupation. The difference with respect to the figure in the main text is that the local labor market is at 2-digit rather than 3-digit industry.

Table I1: Wage Regression. Multilocation firm-occupations

	Dependent variable: $log(w_{io,t})$			
	OLS	IV	OLS	IV
$s_{io\|m,t}$	0.010***	−0.030***	0.007***	−0.030***
	(0.001)	(0.002)	(0.001)	(0.002)
Firm-occ-year FE	Y	Y	Y	Y
CZ FE	Y	Y	N	N
CZ-year FE	N	N	Y	Y
Observations	792,656	733,576	792,656	733,576
R^2	0.833	0.861	0.853	0.862

Notes: The instruments in this table are lagged concentration measures $1/N_{m,t-1}$. Columns 1 and 2 present estimates with commuting zone (CZ) fixed effects for the ordinary least squares (OLS) and instrumental variable (IV) exercises. Columns 3 and 4 present the analogous with commuting zone-year fixed effects. The dependent variable $log(w_{io,t})$ is the logarithm of establishment-occupation wage at time t. $s_{io|m,t}$ is the establishment-occupation employment share at time t. *p<0.1; **p<0.05; ***p<0.01

imply that an increase of one percentage point (p.p. henceforth) of the local labor market share is associated with a decrease of 0.03% of the plant wage. This implies that the same establishment passing from the first to the third quartile of the employment share distribution reduces wages by 0.68%. This elasticity translates into a reduction of roughly 190 euros of the median yearly establishment-occupation wage.

I.2 Labor market concentration and the labor share

We follow similarly to the literature by establishing the relationship between aggregate concentration measures and the labor share. A standard measure of concentration is the Herfindahl-Hirschman Index (HHI). From our definition of local labor market m, the HHI of market m at time t, HHI_{mt}, is the sum of the squared employment shares of the plants present in m at a given year. The labor share at the 3-digit industry level, LS_{ht}, is the ratio of the wage bill over value added at time t. Due to data restrictions of observing value added only at the firm level, we cannot compute

Table I2: Concentration and Labor Share

	Dependent variable: $\log(LS_{h,t})$		
	(1)	(2)	(3)
$\log(\overline{HHI}_{h,t})$	-0.064^{***}	-0.054^{***}	-0.056^{***}
	(0.013)	(0.013)	(0.014)
Sector FE	N	Y	N
Sector-year FE	N	N	Y
R^2	0.017	0.290	0.343

Notes: The number of observations is 1,357. This table presents estimates of equation (I3). Column 1 presents the estimate without any fixed effect. Column 2 shows results with sector fixed effects and column 3 has sector-year fixed effects. The dependent variable is the logarithm of 3-digit industry h labor share $\log(LS_{h,t})$ at time t. $\log(\overline{HHI}_{h,t})$ is the logarithm of the employment weighted average of the local labor market Herfindahl Index. *p<0.1; **p<0.05; ***p<0.01

labor shares at the local labor market level. We therefore build a sub-industry concentration index $\overline{HHI}_{ht}$ by taking the employment weighted mean of HHI_{mt} across different local labor markets.[22]

We run the following linear regression:

$$\log(LS_{h,t}) = \delta_{b,t} + \beta \log(\overline{HHI}_{h,t}) + \varepsilon_{h,t}. \tag{I3}$$

Table I2 presents the results which indicate that more concentrated sub-industries have a lower labor share. Sector fixed effects capture differences in the usage of capital. The focus of the paper being the cross sectional allocation of resources we also take sector-year fixed effects to use only cross sectional variation.[23] Column 3 shows that the negative relation between employment concentration and the labor share is robust to controlling for sector and sector-year fixed effects.

This regression gives a sense of the importance of the labor wedge heterogeneity to generate output and labor share losses. At face value, the estimate with sector fixed effects (column 2) implies a reduction of 1 percentage point of the labor share when passing from the first to the third quartile of concentration.[24] Estimates in column 3 with sector-year fixed effects are very similar. The low estimated effects imply that wages, and therefore labor shares, are not very responsive to differentiated levels of concentration. Nevertheless, one cannot interpret that they rule out employer labor market power because in a setting where all the firms acted as pure monopsonists facing an equal labor supply elasticity, wages (and the labor share) would be insensitive to concentration as all establishments would have the same markdown.

The small estimated coefficient is most likely a result of level effects as the regression does not take into account the effect of concentration on the average level of the labor share as this is absorbed by the fixed effects.

[22]The HHI index at market m and year t is: $HHI_{mt} = \sum_{i \in \mathcal{I}_{m,t}} s^2_{io|m,t}$ where shares at the market are accounted as shares of full time equivalent employees and $\mathcal{I}_{m,t}$ is the set of all firms in the sub-market m at year t. The sub-industry concentration index $\overline{HHI}_{ht}$ is:

$$\overline{HHI}_{ht} = \frac{1}{|\mathcal{M}_{ht}|} \sum_{m \in \mathcal{M}_{ht}} HHI_{mt} \frac{L_{mt}}{L_{ht}},$$

where $|\mathcal{M}_{ht}|$ is the number of local labor markets that belong to h in t, L_{mt} is the local labor market employment and L_{ht} is the 3-digit industry employment.

[23]The inclusion of fixed effects absorbs changes in the HHI that stem from the entry of more establishments in the economy.

[24]Local labor market summary statistics including quartiles of $HHI(s_{io|m})$ are in Table H2 in Appendix H.

Table I3: Rent Sharing: Industry

Industry Code	Industry Name	Rent Sharing	Std Err ($\times 10^2$)
15	Food	0.40	0.09
17	Textile	0.22	0.23
18	Clothing	0.31	0.18
19	Leather	0.31	0.39
20	Wood	0.32	0.24
21	Paper	0.22	0.37
22	Printing	0.34	0.11
24	Chemical	0.17	0.17
25	Plastic	0.23	0.21
26	Other Minerals	0.25	0.18
27	Metallurgy	0.14	0.40
28	Metals	0.37	0.12
29	Machines and Equipments	0.30	0.14
30	Office Machinery	0.33	0.56
31	Electrical Equipment	0.25	0.23
32	Telecommunications	0.23	0.27
33	Optical Equipment	0.32	0.18
34	Transport	0.22	0.33
35	Other Transport	0.31	0.32
36	Furniture	0.37	0.17

Table I4: Rent Sharing: Occupation

Occupation	Rent Sharing	Std Err ($\times 10^2$)
Top management	0.38	0.08
Supervisor	0.27	0.06
Clerical	0.29	0.06
Blue collar	0.30	0.05

I.3 Unions

Tables I3 and I4 present respectively the rent sharing elasticities for industries and occupations. As it is clear from comparing the tables, there is more heterogeneity in the rent sharing elasticities across industries than across occupations. This is one reason why we choose the bargaining powers to vary across industries instead of across occupations.

References

Allen, T., C. Arkolakis, and X. Li (2016): "On the Existence and Uniqueness of Trade Equilibria," *Unpublished manuscript*.

——— (2021): "On the Equilibrium Properties of Network Models with Heterogeneous Agents," Tech. rep., National Bureau of Economic Research.

Barkai, S. (2020): "Declining Labor and Capital Shares," *The Journal of Finance*, 75, 2421–2463, _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/jofi.12909.

Berger, D., K. Herkenhoff, and S. Mongey (2022): "Labor Market Power," *American Economic Review*, 112, 1147–93.

Centre d'accès sécurisé aux données (CASD) (2020a): "BTS-Postes 1994-2007 (previously DADS Postes)," Dataset.

——— (2020b): "FICUS 1994-2007," Dataset.

Costinot, A. and A. Rodríguez-Clare (2014): "Trade Theory with Numbers: Quantifying the Consequences of Globalization," in *Handbook of International Economics*, Elsevier, vol. 4, 197–261.

Hsieh, C.-T. and P. J. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *The Quarterly journal of economics*, 124, 1403–1448.

Kucheryavyy, K. (2012): "Continuity of a Model with a Nested CES Utility Function and Bertrand Competition," *Economics Letters*, 117, 473–476.

Rigobon, R. (2003): "Identification through Heteroskedasticity," *Review of Economics and Statistics*, 85, 777–792.