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The Role of Discounting in Bargaining with Private Information

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In this paper we analyze a continuous-time Coase setting with finite horizon, interdependent values, and different discount rates for the buyer and seller. We fully characterize the equilibrium behavior, which permits us to study how the agents' discount rates (i.e., patience levels) shape the bargaining outcome. We find that the seller's commitment problem persists even when she is fully patient, and that higher seller impatience may lead to higher equilibrium prices. Higher buyer impatience, on the other hand, incentivizes the buyer to trade earlier, which accelerates price decline since the seller's commitment problem is more severe at earlier times. Under appropriate conditions, we conclude that the buyer is better off when he is more impatient, *independently of his private valuation*; hence, higher bargaining costs may give negotiators with private information greater bargaining power.

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1 Introduction

Bargaining theory – the study of how bargaining outcomes depend on factors such as protocols, information, and costs – is a central tool in economics. It provides insights into areas such as price formation in decentralized markets (Osborne and Rubinstein, 1990), negotiations between unions and firms (Hart, 1989; Cramton and Tracy, 1992), cartel stability (Hnyilicza and Pindyck, 1976), and pretrial settlements (Spier, 1992). The seminal paper of Rubinstein (1982) showed that in a bargaining model with alternating offers and without private information, an agent's payoff is higher when she is more patient and lower when the other agent is more patient; furthermore, as she becomes fully patient, her payoff approaches the commitment payoff.

In this paper we consider the other canonical dynamic bargaining model, in which a seller (she) makes sequential offers to a buyer (he) with private information about his valuation. We fully characterize how discounting (or impatience) affects the outcome in this model, providing results which were largely unknown until now. Remarkably, we find that because of a feedback loop between the seller's commitment problem and rapid belief updating, the buyer may be better off when he is more impatient, as well as when the seller is more impatient. Also, in the presence of adverse selection, the seller sometimes offers higher prices when she becomes more impatient or when the deadline for negotiation is extended. Our analysis shows that when information is asymmetric, unlike in the Rubinstein model, lower impatience (i.e., a lower discount rate) does not necessarily lead to greater bargaining power.

Our setting is general, in that it encompasses several cases studied in the literature. First, we allow the discount (or interest) rates of the seller and the buyer – denoted by r_{s} and r_{b}, respectively – to be different. Second, we allow the seller's cost $c(v) \in [0, v]$ either to be independent of the buyer's valuation v (which corresponds to the private-values case, as in Gul et al., 1986, where $c(\cdot) = 0$), or to depend on v (which corresponds to the interdependent-values case, as in Deneckere and Liang, 2006, where $c(\cdot)$ is increasing). Third, while we focus on the "no-gap" case (where v is distributed on $[0, \overline{v}_0]$ and $c(0) = 0$), we require the time horizon T to be finite; hence, the game ends in finite time with probability one. This permits us to study the seller's commitment problem independently of her discount rate, which broadens the range of settings in which our results apply.[1]

[1] As Fuchs and Skrzypacz (2013a) point out, bargaining with a deadline is frequent in practice, for example, in pretrial negotiations, negotiations before international summits, sales of advertising or insurance before live events,

Our paper makes two main contributions. The first is that we provide a general, tractable continuous-time bargaining model with one-sided offers and asymmetric information, for which we obtain a full characterization of the equilibrium dynamics.[2] We characterize the unique reservation-price Markov perfect equilibrium, with the time t and the highest remaining buyer's valuation $\overline{v}_t$ as state variables. We find that in equilibrium, the seller screens the buyer slowly, no trade impasses occur, and the only trade burst occurs at the deadline.

Our second contribution is that, thanks to the tractability of our approach, we are able to document how the bargaining outcome is shaped by the agents' discount rates (impatience levels). We find that the seller's commitment problem is severe: independently of her own discount rate, that of the buyer, and the time horizon, her payoff is equal to the payoff she can obtain by waiting until the deadline and then selling at the monopolistic price. Furthermore, her commitment problem does not vanish in the limit where r_{s} approaches 0 while r_{b} remains fixed: in this limit, her equilibrium payoff converges to the static monopolistic payoff, while a seller with commitment can obtain a higher payoff by slowly lowering the price over time (see Fudenberg and Tirole, 1983).

We start by establishing that the equilibrium dynamics are fully determined by the following simple and easily interpreted equations:

$$p_t = c(\overline{v}_t) + e^{-r_{\mathrm{s}}(T-t)}\left(p^*(\overline{v}_t) - c(\overline{v}_t)\right), \text{ and} \tag{1}$$

$$\dot{p}_t = -r_{\mathrm{b}}\left(\overline{v}_t - p_t\right), \tag{2}$$

where $p^*(\overline{v}_t)$ is the static monopolistic price when the buyer's valuation is known to be lower than $\overline{v}_t$. Equation (1) shows that the seller's commitment problem is more severe at earlier times (when the price is close to the cost) and less severe at later times (when the price is close to the static monopolistic price). Equation (2) ensures that it is optimal for the buyer with valuation $\overline{v}_t$ to buy at time t at price p_t.

We then identify the key condition that determines how changes in the agents' discount rates affect the outcome: we say that *the no-lemons condition holds* if $p^*(\overline{v}) \geq c(\overline{v})$ for all $\overline{v}$ –

and negotiations to renew labor contracts before their expiration. Fuchs and Skrzypacz reference several empirical studies documenting "deadline effects" (last-minute deals) in labor contract negotiations and civil lawsuits.

[2] Although there are several recent studies of bargaining models in continuous time (see the literature review below), this paper is, to our knowledge, the first to analyze the standard one-sided-offers settings (Gul et al., 1986; Deneckere and Liang, 2006) directly in continuous time.

that is, if, in a setting where the distribution of buyer valuations is truncated above by $\overline{v}$, the static monopolist will not lose money when selling to a high-valuation buyer. For example, the no-lemons condition holds in the private-values case.

Now, if the buyer's discount rate r_{b} increases, then the equilibrium price must decrease more rapidly at each given state, in order to keep low-valuation buyers willing to delay trade (by equation (2)). When the no-lemons condition holds, increasing r_{b} has an additional effect: since $p^*(\overline{v}_t) > c(\overline{v}_t)$, the fact that every buyer type trades sooner means that the price he pays in equilibrium must be lower (by equation (1)). This creates a feedback loop: because the buyer is less willing to delay trade, the difference between his valuation and the purchase price increases (further decreasing the right-hand side of equation (2)), which makes him still less willing to delay and therefore further accelerates price decline. Consequently, the buyer is better off when he is more impatient, independently of his valuation: the reduction in equilibrium trade delay and purchase price more than compensates for the increase in the cost of delay.

We also find that the buyer is better off when the seller's discount rate r_{s} increases, provided the no-lemons condition holds. This is because increasing r_{s} lowers the price in each state (since it increases the second term of the right-hand side of (1)), which we show increases the buyer's payoff, independently of his valuation.

On the other hand, if the no-lemons condition fails, then the effect of increasing either r_{b} or r_{s} is ambiguous for the buyer. For example, if $p^*(\overline{v}_0) < c(\overline{v}_0)$, then in equilibrium, the seller will lose money if she sells early (because $p_0 < c(\overline{v}_0)$ by equation (1)), with the losses offset by higher profits if she sells later. An increase in r_{s} makes her less willing to incur these early losses, so she charges higher prices at the start to slow down trade. This benefits low-valuation buyers but hurts high-valuation buyers (who trade sooner). Similar intuition applies when r_{b} is increased.

Our results show that an agent with private information may actually benefit from needing to reach an agreement sooner (e.g., from facing a higher interest rate), as the fact of his impatience can lead to a faster decline in his opponent's optimism.[3] In other words, in bargaining settings with asymmetric information, it may not be appropriate to use patience as a measure of bargaining power.

[3]Such behavior by the seller is often interpreted as anxiety (see Brooks and Schweitzer, 2011). In our case, the seller becomes more anxious when she faces a weaker buyer (with higher bargaining costs).

To shed further light on our results, we compare our setting to one in which the seller has commitment power. Analyzing an example that admits a closed-form solution, we observe that when the seller is more patient than the buyer ($r_s < r_b$), giving her commitment power reduces the probability of trade; furthermore, unlike in the case where the seller and the buyer are equally patient, it increases trade delay.

Finally, motivated by the analysis in Hart (1989), we study a setting in which discount rates are time-dependent. We find that in the private-values case, the agreement rate is large during periods in which either r_s or r_b is large, but faster price decline occurs only when r_b is large. For example, if both r_s and r_b increase over time, then trade speeds up and the price declines more quickly near the deadline; thus, in periods with high impatience, there is a high probability of trade.

Literature review: Our paper contributes to the literature on bargaining with asymmetric information, as reviewed in Ausubel et al. (2002) and Fuchs and Skrzypacz (2020). To our knowledge, the role of discounting in one-sided bargaining with private information has been analyzed only by Sobel and Takahashi (1983) and Evans (1989). Sobel and Takahashi study two-period and infinite-horizon versions of a bargaining model with private values and find that each trader benefits from an increase in the other trader's impatience. Evans studies a two-type model with interdependent values and shows that a trade impasse may occur if the buyer is more patient than the seller. Our paper provides a complete analysis of the role of discounting for both the private-values case and the interdependent-values case. We obtain new results: for example, we show that in the interdependent-values case, a more impatient seller may charge higher prices, while in the private-values case, the buyer may benefit from being more impatient.

Several other papers have studied bargaining with a deadline. Most saliently, Fuchs and Skrzypacz (2013a) study how the efficiency of the bargaining outcome depends on the deadline and the disagreement payoff, in the case when the agents are equally patient and the buyer's valuation follows a power distribution. They find that a smaller disagreement payoff induces more trade before the deadline, while the length of the bargaining period may affect efficiency non-monotonically.[4]

[4]Other bargaining models with finite horizon are studied in Ma and Manove (1993), Fershtman and Seidmann (1993), Thépot (1998), Fanning (2016), Simsek and Yildiz (2016), and Berbeglia et al. (2019). See Ausubel and Deneckere (1989) and Fuchs and Skrzypacz (2013b) for analyses of the no-gap case with private values and inter-

We also contribute to the recent literature that models bargaining directly in continuous time. Examples include Ortner (2017) (bargaining in which the seller has time-varying private costs), Daley and Green (2020) and Lomys (2020) (bargaining with learning), and Chaves (2020) (bargaining with arrival of new traders). In this context, our paper provides a new approach to defining strategies and the corresponding outcomes. In particular, we place minimal assumptions on endogenous variables (for example, we do not impose the smoothness conditions required for the Bellman equation to hold, nor do we assume right-continuity or monotonicity of prices). Furthermore, we contribute to the study of non-stationary settings, given that our time horizon is finite.

The rest of the paper is organized as follows: Section 2 presents our model, Section 3 contains the equilibrium analysis, Section 4 provides the comparative statics results showing how discounting affects the bargaining outcome, and Section 5 concludes. The appendix contains the proofs of the results.

2 Model

We study a continuous-time bargaining model with a finite horizon, where time belongs to $[0, T]$. There is a seller of a durable good and a buyer. The buyer's private valuation for the good, v, is distributed according to some distribution F with a continuous and positive probability density function f and with support equal to $[0, \overline{v}_0]$, for some $\overline{v}_0 > 0$. The seller's valuation for the good is $c(v)$, where $c: [0, \overline{v}_0] \to \mathbb{R}_+$ is a continuously differentiable and non-decreasing function satisfying $c(v) < v$ for all $v \in (0, \overline{v}_0]$.[5] Hence, there is common knowledge of

dependent values, respectively.

[5]Note that we allow for the case of private values, where $c(v) = 0$ for all v. By allowing for non-constant $c(\cdot)$, our model is also applicable in many settings where it is plausible to assume that the informed agent has superior knowledge about the uninformed agent's valuation. For example, this may hold when the durable good is the procurement of a legal, medical, or repair service: the buyer may then know more than the seller about the cost of solving his particular problem.

In addition, our setting is equivalent to one where the seller sells to a unit mass of buyers, and the cost of producing $1 - F(\overline{v})$ units is $\int_{\overline{v}}^{\overline{v}_0} c(v)\, F(\mathrm{d}v)$. In this case, the decline in marginal cost can be attributed to learning-by-doing, for example. Finally, analogous results can be obtained if the roles of the seller and the buyer are reversed; in that case, the positively correlated valuations may correspond to the underlying quality of the good, which is known by the seller.

gains from trade, and there is no gap for lower buyer valuations.[6] At each instant $t \in [0, T]$, the seller makes an offer, and the buyer decides whether to accept it or not. The game ends when the buyer accepts an offer or when the deadline is reached. Both the seller and the buyer are risk-neutral, and their discount (or interest) rates are $r_s > 0$ and $r_b > 0$, respectively.

We make two assumptions throughout the paper:

Assumption 1. For any $\overline{v} \in (0, \overline{v}_0]$, the function $p \mapsto \int_p^{\overline{v}} (p - c(v))\, F(\mathrm{d}v)$ has a unique maximizer, henceforth denoted by $p^*(\overline{v})$, which is continuous and strictly increasing in $\overline{v}$.

Assumption 2. For any $\overline{v} \in [0, \overline{v}_0]$, $c(\overline{v}) \leq \frac{r_b}{r_s}\, \overline{v}$.

Assumption 1 is standard. In Section 3.3 we discuss and relax Assumption 2. Note that Assumption 2 is always satisfied in the private-values case (i.e., when $c(\overline{v}) = 0$ for all $\overline{v} \in (0, \overline{v}_0]$), and also when the seller is at least as patient as the buyer (i.e., when $r_s \leq r_b$).

We now formally define the continuous-time game.

Histories: A *history* is a measurable function from $[0, t]$ to $\mathbb{R}$, for some $t \in \{0^-\} \cup [0, T]$, generically denoted by $p^t \in \mathbb{R}^{[0,t]}$, where p^{0^-} is the empty history and $[0, 0^-] := \emptyset$. Note that p^t contains the prices offered up to time t, including the price at time t.

Seller's strategies: A *(pure) strategy for the seller* is a measurable function P specifying, for each history p^t, a continuation price path $P(p^t) \in \mathbb{R}^{(t,T]}$ (where $(0^-, T] := [0, T]$) such that

$$P_{t''}(p^t) = P_{t''}(p^t, P_{(t,t']}(p^t)) \quad \text{for all } t' > t \text{ and } t'' > t' \,, \tag{3}$$

where $P_{t''}(p^t)$ is the price $P(p^t)$ assigns to t'', and $(p^t, P_{(t,t']}(p^t))$ is the concatenation of the histories $p^t \in \mathbb{R}^{[0,t]}$ and $P_{(t,t']}(p^t) \in \mathbb{R}^{(t,t']}$. Intuitively, the consistency condition (3) requires that the seller does not deviate from her continuation strategy. More formally, note that when the seller follows strategy P on $(t, t']$ after history p^t, the history at time t' is $(p^t, P_{(t,t']}(p^t))$. Condition (3) requires that the price path specified by P after p^t and the one specified by P after $(p^t, P_{(t,t']}(p^t))$ both assign the same price to any time $t'' > t'$ (see Figure 1 for an illustration). We

[6] In finite-horizon Coase-conjecture models (e.g., Fuchs and Skrzypacz, 2013a), there is no significant difference between the equilibrium outcomes in the gap and no-gap cases, both of which typically feature Coasian dynamics (since backward induction from the last trading period can be used). The no-gap assumption is technically convenient in our model because it ensures that, in equilibrium, the game does not end for sure before the deadline.

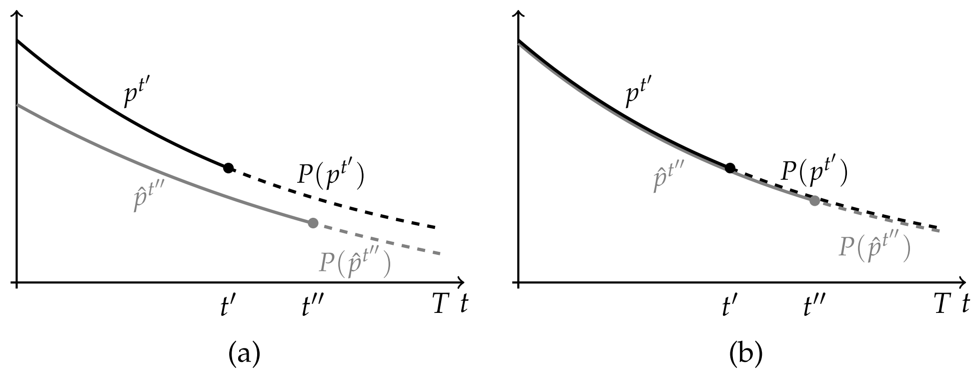


Figure 1: Subfigures (a) and (b) provide examples of two price histories, $p^{t'}$ and $\hat{p}^{t''}$, and the corresponding continuation price paths, $P(p^{t'})$ and $P(\hat{p}^{t''})$. Note that in (b), $\hat{p}^{t''}$ coincides with $P_{[0,t'']}(p^{t'})$, and hence the consistency condition (3) imposes that $P_{[0,T]}(p^{t'})=P_{[0,T]}(\hat{p}^{t''})$.

extend our notation by letting $P_{t'}(p^t)$ denote the price assigned by p^t to time $t'\leq t$. Condition (3) can then be written as follows: $P_{(t',T]}(p^t)=P_{(t',T]}(P_{[0,t']}(p^t))$ for all $t'\geq t$.

Our definition of a seller's strategy is a modeling innovation of this paper. Unlike reduced normal-form strategies (which, in our model, would specify only a single price path), our seller's strategies permit us to easily describe the continuation play after any on- or off-path history. Also, unlike behavior strategies (which, in our game, would specify a price for each information set of the seller), they avoid the usual continuous-time complications.[7] This is because, under our definition, a seller's strategy specifies a continuation price path after every on- or off-path history, which (as we will see), together with the buyer's strategy, pins down a unique continuation outcome. Note that we do not impose any monotonicity or regularity conditions on the choice of the price path. For example, the price path induced by strategy P for the entire bargaining period (until the buyer accepts) is $P(\emptyset)$, which can be any measurable function from $[0, T]$ to $\mathbb{R}$. The same applies to the continuation play after any history, $P(p^t)$.

Buyer's strategies: A *(pure) strategy for the buyer* specifies, for each history p^t and valuation v, an acceptance decision $a^v(p^t)\in\{0,1\}$, where $a^v(p^t)=1$ means "accept" and $a^v(p^t)=0$ means "reject". We assume that $\{v|a^v(p^t)=1\}$ is a measurable set for all p^t.

[7] It is known that when strategies are defined as maps from previous histories to current actions, they may generate non-unique outcomes even when there is only one agent taking actions. For example, if one specifies a strategy as a map from previous prices to the current price, multiple outcomes are consistent with the specification "$P_t=0$ if the price is 0 at all times in $[0,t)$ and $P_t=1$ otherwise".

Outcome: Fix some strategy profile (P,a), a history p^t, and a buyer's valuation v. Let

$$A^v(p^t;P,a) := \{\, t' > t \mid a^v(P_{[0,t']}(p^t)) = 1 \,\}$$

be the set of times at which the buyer with valuation v (the v-buyer) will accept after time t. The *transaction time* that (P,a) generates for the v-buyer after p^t is equal to

$$t^v(p^t;P,a) := \inf(A^v(p^t;P,a)) \in [t,T] \cup \{+\infty\} ,$$

where $t^v(p^t;P,a) = +\infty$ means that the v-buyer rejects all offers after time t. If $t^v(p^t;P,a) < +\infty$, the *transaction price* that (P,a) generates for the v-buyer after p^t is equal to

$$p^v(p^t;P,a) := \begin{cases} P_{t^v(p^t;P,a)}(p^t) & \text{if } t^v(p^t;P,a) \in A^v(p^t;P,a), \\ \lim_{t' \searrow t^v(p^t;P,a)} \inf \left(P_{(t^v(p^t;P,a),t'] \cap A^v(p^t;P,a)}(p^t) \right) & \text{otherwise.} \end{cases}$$

The use of the limit inferior guarantees that the transaction price is uniquely defined. The choice of the limit inferior (instead of the limit superior or a combination of the two) is innocuous for the equilibrium analysis.

Payoffs: Fix a strategy profile (P,a), a history p^t, and a buyer's valuation v. The *realized continuation payoff of the seller* is $e^{-r_s\,(t^v(p^t;P,a)-t)}\,(p^v(p^t;P,a) - c(v))$, and the *continuation payoff of the buyer* (with valuation v) is $e^{-r_b\,(t^v(p^t;P,a)-t)}\,(v - p^v(p^t;P,a))$.

2.1 Equilibrium concept

Perfect Bayesian equilibria

We now define perfect Bayesian equilibria in the usual way. A *belief process* is a function $F(\cdot|\cdot)$ assigning, to each history p^t, a posterior belief $F(\cdot|p^t) \in \Delta([0,\overline{v}_0])$.

Definition 2.1. A *(pure-strategy) perfect Bayesian equilibrium (PBE)* is given by a strategy profile (P,a) and a belief process $F(\cdot|\cdot)$ such that the following hold:

1. The seller's strategy P maximizes the seller's expected continuation payoff after each

history p^t, given the buyer's strategy and the belief $F(\cdot|p^t)$; that is, it maximizes

$$\int_0^{\overline{v}_0} e^{-r_{\mathrm{s}}\,(t^v(p^t;P^\dagger,a)-t)}\,\left(p^v(p^t;P^\dagger,a)-c(v)\right)F(\mathrm{d}v|p^t)$$

across all seller's strategies $P^\dagger$.

2. For any v, the buyer's strategy a maximizes the v-buyer's payoff after each history p^t; that is, it maximizes

$$e^{-r_{\mathrm{b}}\,(t^v(p^t;P,a^\dagger)-t)}\,(v-p^v(p^t;P,a^\dagger))$$

across all buyer's strategies $a^\dagger$.

3. Bayes' rule: $F(\cdot|\emptyset)=F(\cdot)$, and for all histories p^t and $t'<t$ (or $t'=0^-$), we have

$$F(v|p^t)=\frac{\int_0^v \mathbb{I}_{\{v''|a^{v''}(P_{[0,t'']}(p^t))=0\ \forall t''\in(t',t]\}}(v')\,\hat{F}(\mathrm{d}v'|P_{[0,t']}(p^t))}{\int_0^{\overline{v}_0} \mathbb{I}_{\{v''|a^{v''}(P_{[0,t'']}(p^t))=0\ \forall t''\in(t',t]\}}(v')\,F(\mathrm{d}v'|P_{[0,t']}(p^t))}$$

whenever the denominator is not 0.[8]

It is now convenient for us to state a standard property of the buyer's equilibrium behavior in bargaining models, which will facilitate the definition of Markov perfect equilibria:

Lemma 2.1 (skimming property)**.** *In any PBE $(P,a,F(\cdot|\cdot))$, the higher the buyer's valuation is, the earlier he trades and the more he pays; that is, for all p^t, $t^v(p^t;P,a)$ is decreasing in v and $p^v(p^t;P,a)$ is increasing in v.*

As usual, the skimming property permits us to focus, from now on, on a simple class of belief processes: namely, those that (on- or off-path) are upper truncations of F. The upper bound on the support of the belief distribution after some history for a fixed PBE and history p^t is then obtained as follows:

$$\overline{v}(p^t):=\sup(\operatorname{supp}(F(\cdot|p^t)))=\sup\{v|a^v(p^{t'})=0\ \forall t'\in[0,t]\}\ . \tag{4}$$

[8] As usual, for a set $A\subset\mathbb{R}$, $\mathbb{I}_A(\cdot)$ is the indicator function, defined by $\mathbb{I}_A(x)=1$ if $x\in A$ and $\mathbb{I}_A(x)=0$ otherwise. Hence, $\mathbb{I}_{\{v''|a^{v''}(P_{[0,t'']}(p^t))=0\ \forall t''\in(t',t]\}}(v')=1$ if and only if the v'-buyer does not trade in $(t',t]$.

When the history p^t is clear, we will use $\overline{v}_{t'}$ to denote $\overline{v}(P_{[0,t']}(p^t))$; that is, $\overline{v}_{t'}$ is the supremum of the support of the seller's belief at time t'. Note that by the skimming property, $\overline{v}_{t'}$ is decreasing in t'. Finally, note that optimality dictates that at time T, the seller offers the static monopolistic price for the residual demand (i.e., $p^*(\overline{v}_{T^-})$), and the buyer accepts if his valuation is above this price (i.e., $\overline{v}_T = p^*(\overline{v}_{T^-})$).

Reservation-price Markov perfect equilibria

Definition 2.2. A *reservation-price Markov perfect equilibrium* is a PBE $(P, a, F(\cdot|\cdot))$ with the following properties:

1. For all p^t and $\hat{p}^t$ such that $\overline{v}(p^t) = \overline{v}(\hat{p}^t)$, we have that $P_{t'}(p^t) = P_{t'}(\hat{p}^t)$ for all $t' > t$.
2. For every t and v there is some $p(t, v)$ such that, for any history p^t satisfying $\overline{v}(p^{t^-}) \geq v$ and $p_t \leq p(t, v)$, we have $a^v(p^t) = 1$.

The first property is standard: the price (and seller's continuation strategy) at time t depends on the *state variable* $(t, \overline{v}_t)$. The second requirement is analogous to the usual requirement that the v-buyer uses a *reservation-price strategy* (see Gul et al., 1986): at each history p^t where he has not traded before (i.e., such that $v \in [0, \overline{v}(p^{t^-})]$), the buyer accepts a price only if it is below his time-dependent reservation price. Note that such a requirement is natural for a Markov strategy: if the price at time t is the same for two t-histories in which the v-buyer has not traded before time t, and if this price is below $p(t, v)$, then the same buyer types remain after time t (by the skimming property), the continuation play is the same under both histories, and hence the incentive for the buyer to trade at time t is the same.[9] From now on, we refer to reservation-price Markov perfect equilibria simply as *equilibria*.

For a fixed equilibrium, we use $t^v(t, \overline{v}) \in [0, T] \cup \{+\infty\}$ and $p^v(t, \overline{v}) \in \mathbb{R}$ to denote the transaction time and price, respectively, for a buyer with valuation $v \in [0, \overline{v})$ at a history p^t with $\overline{v} = \overline{v}(p^t)$ (recall that $t^v(t, \overline{v}) = +\infty$ means that the v-buyer does not trade if the seller follows the equilibrium continuation strategy after $(t, \overline{v})$). It is convenient to use $\Pi(t, \overline{v})$ to denote the

[9]More formally, if p^t and $\hat{p}^t$ are such that $p_t = \hat{p}_t$, and if $\inf\{v | p(t, v) \geq p_t\} \geq \min\{\overline{v}(p^{t^-}), \overline{v}(p^{t^-})\}$ (i.e., if there is a type of buyer who has not traded before time t in either history, and who is willing to accept p_t), then $\overline{v}(p^t) = \overline{v}(\hat{p}^t) = \inf\{v | p(t, v) > p_t\}$; hence the continuation plays after histories p^t and $\hat{p}^t$ coincide. Note that, by the skimming property, $p(t, v)$ is (weakly) increasing in v.

normalized continuation payoff of the seller in state $(t, \overline{v})$,

$$\Pi(t, \overline{v}) := \int_0^{\overline{v}} e^{-r_s\,(t^v(t,\overline{v})-t)}\,(p^v(t, \overline{v}) - c(v))\,F(\mathrm{d}v)\ . \tag{5}$$

Note that the continuation payoff of the seller in state $(t, \overline{v})$ is $\Pi(t, \overline{v})/F(\overline{v})$.

Our definition of equilibrium has a small caveat: in principle, for a fixed equilibrium, there may be states which are never reached (not even after a seller's deviation); that is, there may be states $(t, \overline{v})$ for which there is no history p^t such that $(t, \overline{v}(p^t)) = (t, \overline{v})$. Although these states are irrelevant to the equilibrium analysis, it would clutter the exposition unnecessarily to specify conditions to exclude them. To circumvent this problem while keeping the argumentation simple, we make the following changes to our setting: from now on, we assume that there is some large $M > 0$ such that, if at time $t = 0$ (and only at this time) the seller sets a price that can be written as $-M + \overline{v}$ for some $\overline{v} \in [0, \overline{v}_0]$, then the buyer is "forced" to accept this price if his valuation is higher than $\overline{v}$, and to reject it otherwise. If she offers such a price, the seller receives an additional lump-sum payoff of $-M$. Of course, this is irrelevant for equilibrium behavior: setting a negative price at time 0 is strictly dominated. However, introducing this assumption has the following effect: now, for any equilibrium, $t \in (0, T]$, and $\overline{v} \in [0, \overline{v}_0]$, there is a history p^t such that $\overline{v}(p^t) = \overline{v}$, and the strategy is sequentially optimal afterward.[10]

Remark 2.1. While our focus is on reservation-price Markov perfect equilibria, we believe that our approach of first constructing (non-Markovian) strategies and PBE is beneficial, as it keeps the assumptions on endogenous variables simple. Indeed, in other models in the literature, only Markov strategies are defined, and technical conditions (such as continuity or differentiability) are then imposed, with unclear effects on equilibrium behavior, to guarantee that strategy profiles generate unique outcomes (e.g., to guarantee that a Bellman equation is necessary and sufficient for optimality). By contrast, our definition does not require additional regularity conditions, and we allow agents to deviate to non-Markovian strategies.

Also, we believe that focusing on reservation-price Markov perfect equilibria is natural in our setting. For example, for the gap case of the discrete-time model with infinite horizon, Gul et al. (1986) use backward induction from the last period with trade to show that all equilibria are reservation-price Markov perfect. We expect their result to generalize to a finite-

[10] An example of a history p^t such that $\overline{v}(p^t) = \overline{v}$ is the following: at time 0, the price is $-M + \overline{v}$, while at any other time $t' \in (0, t)$, the price offer is unacceptable (above $\overline{v}_0$).

horizon model even in the no-gap case, since backward induction can also be used there. In the continuous-time model, the restriction to reservation-price Markov perfect equilibria is necessary for tractability (and backward induction cannot be directly used); however, we do not rule out the possibility that all PBE in our model are reservation-price Markov perfect.

3 Equilibrium characterization

In this section, we fully characterize the unique equilibrium of our model, first presenting the main result and then explaining why it holds and how we obtain it. A reader interested in using our characterization of predicted trade dynamics under a price path can jump to Section 3.2. A reader interested in the implied comparative statics results can jump to Section 4. Section 3.3 provides a discussion of Assumption 2.

Our main result establishes that there is an essentially unique equilibrium (i.e., a unique outcome for every state, apart from a zero-measure set of times), which is fully characterized by the two simple equations presented in the Introduction (equations (1) and (2)).

Theorem 3.1. *There is an essentially unique equilibrium. For each state $(\hat{t}, \overline{v}_{\hat{t}})$, the on-path threshold type $\overline{v}_t$ and price p_t are continuously differentiable for all $t \in (\hat{t}, T)$, and are fully characterized by the following equations:*

$$p_t = (1 - e^{-r_\mathrm{s}\,(T-t)})\, c(\overline{v}_t) + e^{-r_\mathrm{s}\,(T-t)}\, p^*(\overline{v}_t)\ , \quad \text{and} \tag{6}$$

$$\dot{p}_t = -r_\mathrm{b}\,(\overline{v}_t - p_t)\ , \tag{7}$$

for all $t \in (\hat{t}, T)$, with $\lim_{t \searrow \hat{t}} \overline{v}_t = \overline{v}_{\hat{t}}$ and $\overline{v}_T = p_T = \lim_{t \nearrow T} p_t$. Furthermore, the seller's payoff is $e^{-r_\mathrm{s}\,T}\,\Pi^*(\overline{v}_0)$, where $\Pi^*(\overline{v}_0) := \int_{p^*(\overline{v}_0)}^{\overline{v}_0} (p^*(\overline{v}_0) - c(v))\, F(\mathrm{d}v)$ is the static monopolistic payoff.

3.1 Equilibrium properties

In this section, we derive some important properties that apply to any equilibrium of the game. (For brevity, we omit writing "In any equilibrium" before each statement.) Together, these properties imply that equations (6) and (7) are satisfied in any equilibrium. We then deduce the (essential) uniqueness of the equilibrium by showing that these equations have a unique solution.

No silent period

We begin with a property often referred to as "no silent period" (especially in discrete-time models) or "no trade gaps," which says that there are no intervals of time where the probability of trade is 0; equivalently, it says that all equilibrium offers are "serious" (i.e., not "losing") offers.[11] The no-silent-period property implies that if p^t is on path, then $t^{\overline{v}(p^t)}(t,\overline{v}(p^t))=t$ and $p^{\overline{v}(p^t)}(t,\overline{v}(p^t))=p_t=p(t,\overline{v}(p^t))$ (i.e., the $\overline{v}(p^t)$-buyer trades at time t and pays his reservation price).

Proposition 3.1 (no silent period)**.** *There is no time interval with no trade; that is, there is no history p^{t_1} and $t_2>t_1$ such that $\overline{v}_{t_1}(p^{t_1})=\overline{v}_{t_2}(p^{t_1})>0$.*

In some stationary settings, Proposition 3.1 is immediate, but in our setting it is not, for two reasons. The first is that our time horizon is finite; thus, although an interval without trade would delay revenue, it would also shorten the time left until the deadline, reducing the seller's commitment problem. (We show, however, that the price decline on the path of play is enough to guarantee that the seller's expected revenue increases if she sells earlier.) The second reason is that in the interdependent-values case, trading earlier increases the present value of the revenue but also the present value of the seller's cost. Thus, an early sale to the v-buyer at a price lower than the cost $c(v)$ may be detrimental for the seller. Such a sale may occur, for example, when the buyer is patient, since then the price that will induce him to trade earlier cannot be much higher than the equilibrium purchase price.

We now provide some intuition for Proposition 3.1, as illustrated in Figure 2. First, fix an equilibrium. We argue using contradiction: we assume that there is an interval of time without trade, (t_1,t_2), and then show that the seller has an incentive to speed up trade in this interval. Let $p_{t_2}\in[0,\overline{v}_{t_2}]$ be the equilibrium price at time t_2. Fix some small $\Delta>0$, and let $\hat{p}_{t_2-\Delta}$ be the price at which the $\overline{v}_{t_2}$-buyer is indifferent between buying at time $t_2-\Delta$ and buying at time t_2. We compare the seller's discounted payoffs from selling to the $\overline{v}_{t_2}$-buyer at t_2 at price p_{t_2} and from selling to the $\overline{v}_{t_2}$-buyer at $t_2-\Delta$ at price $\hat{p}_{t_2-\Delta}$; that is, we compare

$$e^{-r_\mathrm{s}\Delta}\left(p_{t_2}-c(\overline{v}_{t_2})\right) \quad \text{vs} \quad \underbrace{(1-e^{-r_\mathrm{b}\Delta})\,\overline{v}_{t_2}+e^{-r_\mathrm{b}\Delta}\,p_{t_2}}_{=\hat{p}_{t_2-\Delta}}-c(\overline{v}_{t_2})\,. \tag{8}$$

[11] Similar properties are found in models with arrival of buyers (Fuchs and Skrzypacz, 2010) and news arrival (Daley and Green, 2020).

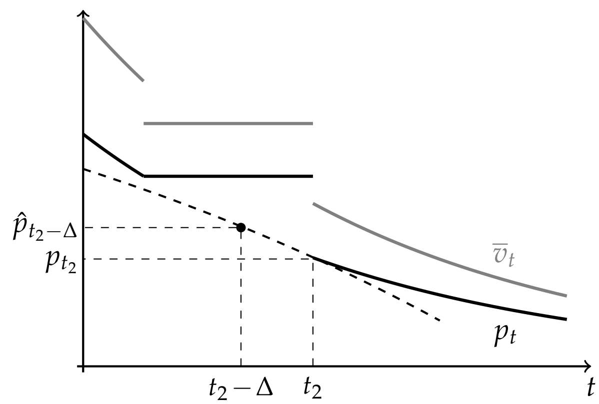


Figure 2: Illustration of the argument ruling out time intervals without trade. The dashed curve indicates the indifference curve for the $\overline{v}_{t_2}$-buyer. The seller can sell to the $\overline{v}_{t_2}$-buyer earlier (at time $t_2 - \Delta$ instead of t_2) and at a higher price ($\hat{p}_{t_2-\Delta}$ instead of p_{t_2}) but also has to pay the cost $c(\overline{v}_{t_2})$ earlier.

If $p_{t_2} = \overline{v}_{t_2}$, the gain from selling earlier is $(1 - e^{-r_\mathrm{b}\Delta})\,(\overline{v}_{t_2} - c(\overline{v}_{t_2}) > 0$. If $p_{t_2} = 0$, the gain from selling earlier is approximately $(r_\mathrm{b}\,\overline{v}_{t_2} - r_\mathrm{s}\,c(\overline{v}_{t_2}))\,\Delta$, which by Assumption 2 is non-negative. Hence, since the gain from selling earlier to the $\overline{v}_{t_2}$-buyer is linear in p_{t_2}, the earlier sale is profitable for the seller independently of the equilibrium price $p_{t_2} \in [0, \overline{v}_{t_2}]$. Now, if the seller chooses a price slightly below $\hat{p}_{t_2-\Delta}$ at $t_2 - \Delta$, then the buyer will purchase only when his valuation is close to $\overline{v}_{t_2}$, and hence this will be a profitable deviation for the seller. See Section 3.3 for further discussion.

Seller's equilibrium payoff

We now state, and provide heuristic arguments for, two results characterizing the seller's equilibrium payoff. These results can be interpreted in light of the seller's willingness to speed up or slow down her screening of the buyer. The intuition behind both results is shown in Figure 3.

Proposition 3.2. *For any $(t, \overline{v})$ with $t < T$,*

$$\Pi(t, \overline{v}) = \int_0^{\overline{v}} (p(t, v) - c(v))\, F(\mathrm{d}v)\ . \tag{9}$$

Proposition 3.2 can be interpreted as follows. Assume the seller deviates and decreases the price very quickly (but continuously) after time t. For example, as illustrated in Figure 3(b), she could set the price at $\hat{p}_{t'} = \frac{t+\varepsilon-t'}{\varepsilon}\, p(t, \overline{v})$ for all $t' \in (t, t+\varepsilon]$ for some small $\varepsilon > 0$; the price would then fall rapidly and continuously from $p(t, \overline{v})$ to 0, and so trade would occur for sure before

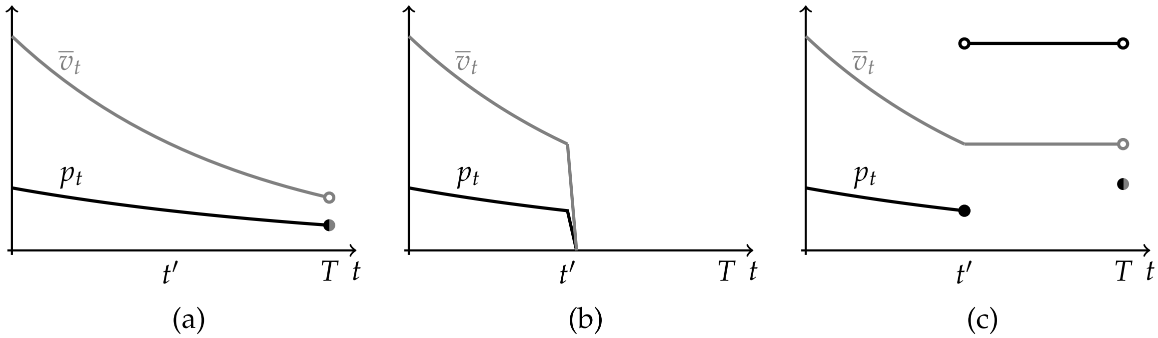


Figure 3: (a) An example of an equilibrium path for p and $\overline{v}$. (b) A deviation in which the seller lowers the price very quickly after time t', and thus screens the buyer very quickly. (c) A deviation in which the seller charges unacceptable prices during the interval (t', T), then charges $p^*(\overline{v}_T)$ at time T. Propositions 3.2 and 3.3 establish that all three strategies give the same payoff to the seller.

time $t+\varepsilon$. Under this deviation, each buyer type $v \in [0, \overline{v}]$ would buy at the first time $t' \in (t, t+\varepsilon]$ such that $\hat{p}_{t'} = p(t', v)$, and would therefore pay a price close to $p(t, v)$ (Lemma A.4 shows that $p(\cdot, v)$ is continuous). The seller's payoff from her deviation would then be approximately equal to the right-hand side of (9). Hence, Proposition 3.2 can be interpreted as establishing that in equilibrium, the seller is willing to screen the buyer "infinitely fast".

Proposition 3.3, by contrast, states that the seller is also willing not to screen the buyer at all. More formally, the seller's equilibrium payoff in state $(t, \overline{v})$ coincides with the payoff she would obtain if she made unacceptable offers (above $\overline{v}$, for example) until the deadline and then, at the deadline, charged the monopolistic price $p^*(\overline{v})$ (see Figure 3(c) for an illustration).[12]

Proposition 3.3. *For any $(t, \overline{v})$ with $t < T$, the seller's payoff equals the payoff she obtains from charging an unacceptable price until time T and then charging $p^*(\overline{v})$; that is,*

$$\Pi(t, \overline{v}) = e^{-(T-t)\, r_{\mathrm{s}}}\, \Pi^*(\overline{v}) \ , \tag{10}$$

where $\Pi^*(\overline{v}) := \int_{p^*(\overline{v})}^{\overline{v}} (p^*(\overline{v}) - c(v))\, F(\mathrm{d}v)$ equals the static normalized monopolistic payoff for the type distribution truncated at $\overline{v}$.

We now provide intuition for why Propositions 3.2 and 3.3 hold. Heuristically, the seller's continuation value is derived from the payoff from selling during the next small interval $\mathrm{d}t$

[12]Fuchs and Skrzypacz (2013a) obtain an analogous result in a model with private values, where the buyer's distribution follows a power distribution ($F(v) = v^a$ for $v \in [0, 1]$) and where the buyer and seller have the same impatience level ($r_{\mathrm{s}} = r_{\mathrm{b}}$).

plus the continuation value from not selling during this time, so

$$\Pi(t,\overline{v}) \simeq f(\overline{v})\,(p(t,\overline{v})-c(\overline{v}))\,\dot{\overline{v}}(t,\overline{v})\,\mathrm{d}t + (1-r_\mathrm{s}\,\mathrm{d}t)\,\Big(\Pi(t,\overline{v}) + \mathrm{d}t\,\big(\tfrac{\partial}{\partial t}\Pi(t,\overline{v}) + \tfrac{\partial}{\partial \overline{v}}\Pi(t,\overline{v})\,\dot{\overline{v}}(t,\overline{v})\big)\Big).$$

From the previous expression, we can heuristically obtain the standard Bellman equation:[13]

$$r_\mathrm{s}\,\Pi(t,\overline{v}) = \tfrac{\partial}{\partial t}\Pi(t,\overline{v}) + \Big(-f(\overline{v})\,(p(t,\overline{v})-c(\overline{v})) + \tfrac{\partial}{\partial \overline{v}}\Pi(t,\overline{v})\Big)\,\dot{\overline{v}}(t,\overline{v}), \tag{11}$$

where, by an abuse of notation, $\dot{\overline{v}}(t,\overline{v})$ denotes the speed at which the upper bound of the distribution of remaining types changes at state $(t,\overline{v})$. The interpretation of this equation is standard.

We can heuristically think of the seller's problem as that of choosing, at each instant, the screening speed $\dot{\overline{v}}(t,\overline{v})$ (by deciding how quickly to decrease the price). Faster screening (a more negative choice of $\dot{\overline{v}}(t,\overline{v})$) yields a higher flow payoff but also makes the continuation value decrease faster. From Proposition 3.1, we see that choosing $\dot{\overline{v}}(t,\overline{v})=0$ (i.e., not screening at all) cannot be strictly optimal. Similarly, if setting $\dot{\overline{v}}(t,\overline{v})=-\infty$ were strictly optimal for all $\overline{v}$, then the price would have to be 0 (since screening would be very fast, and the v-buyer never buys at a price above v). As a result, and because the right-hand side of equation (11) is linear in $\dot{\overline{v}}(t,\overline{v})$, it must be that $-f(\overline{v})\,(p(t,\overline{v})-c(\overline{v})) + \tfrac{\partial}{\partial \overline{v}}\Pi(t,\overline{v})=0$, which implies equation (9). Hence, the seller is indifferent between screening very quickly and screening very slowly (as well as screening at any intermediate rate).

Equilibrium price

Equations (9) and (10) provide two expressions for the normalized continuation payoff of the seller. Differentiating these expressions with respect to $\overline{v}$, we obtain

$$(p(t,\overline{v})-c(\overline{v}))\,f(\overline{v}) = \tfrac{\partial}{\partial \overline{v}}\Pi(t,\overline{v}) = e^{-r_\mathrm{s}\,(T-t)}\,\Pi^*(\overline{v}) = e^{-r_\mathrm{s}\,(T-t)}\,(p^*(\overline{v})-c(\overline{v}))\,f(\overline{v})\ ,$$

where the first equality follows from equation (9), the second from (10), and the third from using the envelope theorem on the static normalized payoff Π^*. Hence, equation (6) holds

[13] Unlike most of the literature modeling bargaining directly in continuous time, we do not make any regularity, continuity, or smoothness assumptions on strategies to guarantee that standard recursive analysis can be used. Nevertheless, as we will see, the equilibrium objects will be smooth enough for equation (11) to hold.

with p_t replaced by $p(t, \overline{v}_t)$ for any state $(t, \overline{v}_t) \in [0, T] \times (0, \overline{v}_0]$, on- or off-path (note that this expression is equivalent to the expression (1) in the introduction). The price at state $(t, \overline{v})$ is then a convex combination of the monopolistic price when the valuation is known to be below $\overline{v}$ (i.e., $p^*(\overline{v})$) and the seller's valuation of the good if the buyer's valuation is $\overline{v}$ (i.e., $c(\overline{v})$). Close to the deadline, the seller's commitment problem is reduced, so the weight on the monopolistic price increases, and $p(T, \overline{v}) = p^*(\overline{v})$.

Equation (6) provides a remarkably simple recipe for computing the price in a given state $(t, \overline{v})$. First, compute the surplus the seller obtains from the $\overline{v}$-buyer in the static monopolistic problem with valuations in $[0, \overline{v}]$ (which equals $p^*(\overline{v}) - c(\overline{v})$). Second, "discount" the surplus based on the time remaining until the deadline (at the seller's discount rate). The result is equal to the surplus the seller obtains from the $\overline{v}$-buyer in state $(t, \overline{v})$, that is, $p(t, \overline{v}) - c(\overline{v})$.

Note that Assumption 1 and the fact that F is differentiable imply that p^* is continuous and strictly increasing. Note also that $p(\cdot, \overline{v})$ is increasing if $p^*(\overline{v}) > c(\overline{v})$: intuitively, the seller's commitment problem becomes less severe as the deadline approaches, and she has more credibility in charging higher prices. Still, if $p^*(\overline{v}) < c(\overline{v})$, then $p(\cdot, \overline{v})$ is a decreasing function: as t approaches T, the seller is more willing to sell at a loss to the $\overline{v}$-buyer at time t in order to obtain the monopolistic payoff at the deadline. In both cases, the equilibrium price $p_t = p(t, \overline{v}_t)$ decreases over time, as the decrease in $\overline{v}_t$ more than compensates for the increase in t.

An important implication of equation (6) is that for a given history p^t, there are no "trade bursts" on $[t, T)$ (assuming the seller plays according to the equilibrium strategy on $[t, T]$). That is, there is no $t' \in [t, T)$ such that $\overline{v}_{t'}(p^t) > \overline{v}_{t'^+}(p^t)$. To see this, note that by the optimality of the buyer's strategy and Proposition 3.1, the price $p_{t'}(t, \overline{v})$ is continuous in t' on $[t, T]$. Furthermore, the right-hand side of equation (6) is continuous and strictly increasing in $\overline{v}$. The continuity of the on-path price $p_t = p(t, \overline{v}_t)$ implies that $\overline{v}_t$ is continuous too. This result is consistent with the finding in Fuchs and Skrzypacz (2013b) that the trade bursts obtained in the study of the interdependent-values case (Deneckere and Liang, 2006) disappear in the limit where the gap between the lowest seller's valuation and the lowest buyer's valuation vanishes.

Buyer optimality

We now use the optimality of the buyer's strategy to obtain the equilibrium price dynamics. For simplicity, we let p_t denote $P_t(\emptyset)$; that is, p_t is the price set by the seller at time t on the equilibrium path (the analysis of price dynamics after deviations is analogous). The "marginal buyer" at time $t \in (0, T)$ (i.e., the buyer with valuation $\overline{v}_t$) is willing to purchase at time t, and not before or after. Since, by the absence of trade bursts, $\overline{v}_t$ is continuous in t, equation (7) holds. The negative of the left-hand side of equation (7) is the $\overline{v}_t$-buyer's instantaneous gain from delaying trade by an instant. The negative of the right-hand side is the cost owed to the consequent delay in obtaining his surplus.

Equations (6) and (7) fully determine the equilibrium price dynamics. Indeed, they imply

$$\overbrace{\tfrac{\mathrm{d}}{\mathrm{d}t}\big(c(\overline{v}_t)+e^{-r_{\mathrm{s}}(T-t)}\,(p^*(\overline{v}_t)-c(\overline{v}_t))\big)}^{=\dot{p}_t}=-r_{\mathrm{b}}\,\Big(\overline{v}_t-\big(\overbrace{c(\overline{v}_t)+e^{-r_{\mathrm{s}}(T-t)}\,(p^*(\overline{v}_t)-c(\overline{v}_t))}^{=p_t}\big)\Big)\,. \tag{12}$$

Equation (12) gives an ordinary differential equation for the evolution of the upper threshold $\overline{v}_t$ and hence fully characterizes the equilibrium dynamics (note that the initial condition is that $\overline{v}_t$ at time 0 is equal to the parameter $\overline{v}_0$). The proof of Theorem 3.1 shows that the solution to equation (12) for $\overline{v}_t$ is, indeed, decreasing.

A final observation

From Theorem 3.1 we can make the following observations. As T converges towards 0, (i) the seller's payoff converges to the static monopolistic payoff $\Pi^*(\overline{v}_0)$, (ii) the initial price converges to $p^*(\overline{v}_0)$, and (iii) in equilibrium, the seller remains willing to screen infinitely fast. Hence, the theorem directly implies that the static monopolistic payoff equals the payoff that the seller would get in a market in which she could perfectly price-discriminate by charging, to each buyer type $\overline{v}$, the monopolistic price she would charge if it were known that the buyer's valuation was lower than $\overline{v}$ (i.e., $p^*(\overline{v})$). Thus, the static monopolistic payoff can be characterized as follows.

Corollary 3.1. *The static monopolistic payoff satisfies* $\Pi^*(\overline{v}_0)=\int_0^{\overline{v}_0}(p^*(v)-c(v))\,F(\mathrm{d}v)$.

Corollary 3.1 can be proven independently of Theorem 3.1 by applying the envelope theorem to $\Pi^*(\overline{v})$.

3.2 Equilibrium dynamics

In this section, we discuss the model dynamics implied by the equilibrium strategies both when the seller follows the prescribed price path and when she deviates.

We first explain how the state variable is obtained after a deviation by the seller. Assume that the seller has deviated on $[0,t)$ by setting a price history $p^t \in \mathbb{R}^{[0,t)}$. The state at time t is then $(t, \overline{v}(p^t))$, where $\overline{v}(p^t)$ is defined in equation (4). From the previous results, we can write

$$\overline{v}(p^t) = \inf\{\overline{v}(t', p_{t'}) | t' \in [0,t)\} \ ,$$

where $\overline{v}(t, p_t)$ is the (unique) solution to $p(t, \overline{v}(t, p_t)) = p_t$ in equation (6), when it exists, and $\overline{v}(t, p_t) = \overline{v}_0$ otherwise.[14] Intuitively, if the seller sets price p_t at time t, then $\overline{v}(t, p_t)$ is the valuation of the buyer who is indifferent between trading and not trading at time t. Therefore, for a price path p^t up to time t, a buyer with valuation $v > \overline{v}(t, p^t)$ must have traded before time t, because there was some time $t' \in [0,t)$ such that $p_{t'} \leq p(t', v)$. For example, when $c(v) = 0$, we have $\overline{v}(t, p_t) = e^{r_{\mathrm{s}}(T-t)}\, p^{*-1}(p_t)$.

We now illustrate the equilibrium dynamics through a specific example. Take F to be uniform on $[0, \overline{v}_0]$ and $c(\overline{v}) = 0$ for all $\overline{v} \in [0, \overline{v}_0]$ (the private-values case), so the static monopolistic price is $p^*(v) = v/2$. In this case, the on-path equilibrium dynamics (determined by (12)) are given by

$$\overline{v}_t = e^{-2\frac{r_{\mathrm{b}}}{r_{\mathrm{s}}}(e^{r_{\mathrm{s}} T} - e^{r_{\mathrm{s}}(T-t)}) - (r_{\mathrm{s}} - r_{\mathrm{b}})\, t}\, \overline{v}_0$$

and $p_t = e^{-r_{\mathrm{s}}(T-t)}\, \overline{v}_t / 2$. These are shown in Figure 4(a).

Next we describe the equilibrium dynamics if the seller deviates to a different price path p^T. First note that when F is uniform and c is 0, we have

$$\overline{v}(t, p_t) = e^{r_{\mathrm{s}}(T-t)}\, 2\, p_t \ .$$

Intuitively, close to the deadline, the v-buyer accepts a price close to the monopolistic price, that is, $v/2$, when the distribution of valuations is truncated at v. This is because, when the

[14] Note that p^* is continuous and strictly increasing, while c is non-decreasing, so the right-hand side is strictly increasing in $\overline{v}$.

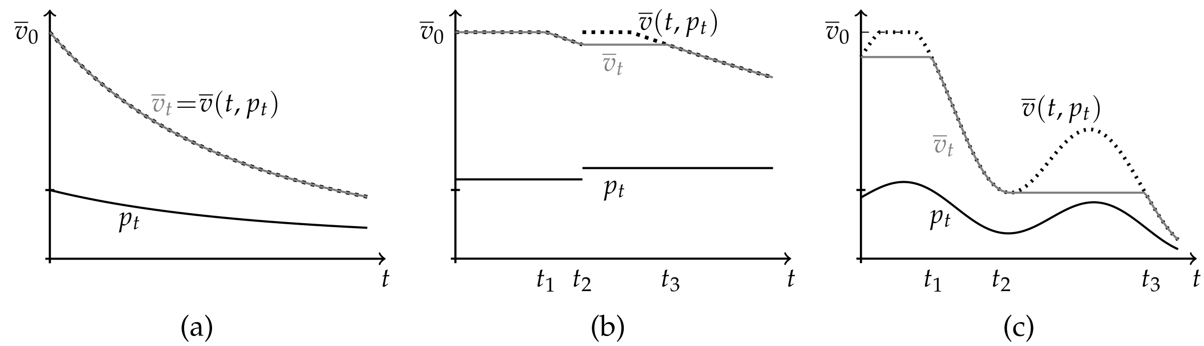


Figure 4: Example with $T{=}1$, $r_{\mathrm{b}}{=}r_{\mathrm{s}}{=}0.5$, F uniform in $[0,1]$, and $c(v){=}0$. Each subfigure depicts p_t (continuous black line), the implied $\overline{v}(t,p_t)$ (continuous gray line), and $\overline{v}_t$ (dotted black line). Subfigure (a) corresponds to the equilibrium price path, while subfigures (b) and (c) correspond to two deviations by the seller.

deadline is near, he does not expect much more screening, and the equilibrium price at the deadline coincides with the static monopolistic price for the residual demand. Far from the deadline, however, the expected price decline implies that the set of prices acceptable to the buyer shrinks.

Now suppose that the seller deviates by charging a price of $0.35\,\overline{v}_0$ until some time t_2 and $0.4\,\overline{v}_0$ afterward, as depicted in Figure 4(b). Under this deviation, at any given time t, the buyer trades if his valuation is $\min\{\overline{v}_0\,,\,2\,p_t\,e^{r_{\mathrm{s}}\,(T-t)}\}$ or above (provided he has not already traded). Note that there is no trade on $[0,t_1)$, as in this interval $p(t,\overline{v}_0){>}0.35\,\overline{v}_0$. Trade begins at t_1 satisfying $p(t_1,\overline{v}_0){=}0.35\,\overline{v}_0$, and then there is slow screening until t_2. The price increase at t_2 implies that $\overline{v}(t,p_t)$ also jumps; hence, there is no trade for some time. Trade resumes again between time t_3 and the deadline, when there is a burst (note that $\overline{v}_{T^-}{=}2\cdot 0.4$, so 0.4 is the static monopolistic price given the residual demand at the deadline).

Figure 4(c) illustrates another seller deviation. Here the price p_t rises and falls twice, and so does $\overline{v}(t,p_t)$. As explained above, the corresponding path for $\overline{v}_t$ is determined by the minimum value reached by $\overline{v}(t',p_{t'})$ before time t. In this case, since the initial price is below $p(0,\overline{v}_0)$, there is a trade burst at the game's outset. After that, there is no trade until time t_1. Trade resumes from t_1 to t_2, and again from t_3 until the deadline, where there is a trade burst.

Remarkably, the seller's payoff under the first deviation is the same as her equilibrium payoff. Indeed, note that conditionally on trading before the deadline, a buyer with valuation v pays $e^{-r_{\mathrm{s}}\,(T-t)}\,p^*(v)$ (from equation (6)), while a buyer who trades at the deadline pays $p^*(\overline{v}_T)$.

Hence, the seller's payoff is

$$\int_{\overline{v}_T}^{\overline{v}_0} e^{-r_{\mathrm{s}}\,t} e^{-r_{\mathrm{s}}\,(T-t)}\; p^*(v)\, F(\mathrm{d}v) + \int_{p^*(\overline{v}_T)}^{\overline{v}_T} e^{-r_{\mathrm{s}}\,T}\; p^*(\overline{v}_T)\, F(\mathrm{d}v)\ .$$

Using Corollary 3.1 (applied at $\overline{v}_T$ instead of $\overline{v}_0$), we can rewrite the second term of this expression so that the seller's payoff becomes

$$e^{-r_{\mathrm{s}}\,T} \int_0^{\overline{v}_0} p^*(v)\, F(\mathrm{d}v)\ ,$$

which again by Corollary 3.1 equals $e^{-r_{\mathrm{s}}\,T}\,\Pi^*(\overline{v}_0)$, the seller's equilibrium payoff.

By contrast, the seller's payoff under the second deviation is *lower* than her equilibrium payoff. The reason is that, under this deviation, there is a trade burst at time 0; hence, the seller obtains $e^{-r_{\mathrm{s}}\,T}\, p^*(\overline{v}_{0-}) < e^{-r_{\mathrm{s}}\,T}\, p^*(v)$ for buyer types $v \in (\overline{v}_{0+}, \overline{v}_0]$. In general, price paths p^t that induce a continuous $\overline{v}_t$ and such that $p_T = p^*(\overline{v}_T)$ (if $\overline{v}_T > 0$) give the seller the same payoff she obtains in equilibrium, while those not satisfying these conditions are suboptimal.[15]

3.3 Discussion of Assumption 2

Assumption 2 plays a critical role in the proof of Proposition 3.1. The intuitive argument outlined in Section 3.1 proceeds by contradiction, assuming there is no trade in some interval (t_1, t_2). It is shown that Assumption 2 guarantees that even if the price at t_2 is 0, the seller gains from deviating and selling earlier to a high-valuation buyer.

Proposition 3.1 is then used to prove Propositions 3.2 and 3.3. From these we conclude that the equilibrium price satisfies equation (6) when Assumption 2 holds. The cost–benefit argument used to prove Proposition 3.1 can be replicated using the equilibrium price instead of 0. Doing so yields the following condition, which is less restrictive than Assumption 2, yet still sufficient for the no-silent-period condition to hold.

Assumption 3. For any $\overline{v} \in (0, \overline{v}_0]$, we have $r_{\mathrm{b}}\,(\overline{v} - p^*(\overline{v})) \geq r_{\mathrm{s}}\,(c(\overline{v}) - p^*(\overline{v}))$.

It is easy to see that Assumption 3 is less restrictive than Assumption 2 (see the proof of Theorem 3.1). In fact, Assumption 2 is the least restrictive condition that guarantees that As-

[15]Note that, for the reasons given above, $p_T = p^*(\overline{v}_T)$ whenever there is trade towards the end of the game and p_t is continuous at T (as it is for the two deviations discussed).

sumption 3 holds *independently* of the distribution of buyer's valuations. That is, if Assumption 2 holds, then Assumption 3 holds as well (independently of F), and for any (c, r_s, r_b) not satisfying Assumption 2, there is some distribution F (and corresponding p^*) for which Assumption 3 does not hold. Additionally, the proof of Theorem 3.1 shows that Assumption 3 is necessary and sufficient for the solution of equation (12) for $\overline{v}_t$ to be decreasing. Hence, we have the following:

Corollary 3.2. *If Assumption 2 is not required to hold, the strategy profile described in Theorem 3.1 is an equilibrium if and only if Assumption 3 holds.*

When, then, does Assumption 3 *not* hold? That is, when is the strategy profile described in Theorem 3.1 *not* an equilibrium? For this to occur, it is necessary that (i) Assumption 2 does not hold,[16] and (ii) the seller's cost is above the monopolistic price for some of the buyer's valuations. Conditions (i) and (ii) hold if, for example, the seller is very impatient and has a very high cost for supplying high buyer types. In this case, the static monopolistic price $p^*(\overline{v})$ is low even for large values of $\overline{v}$, so $c(\overline{v}_0) > p^*(\overline{v}_0)$. Therefore, if the seller is impatient enough, she will not be willing to sell at time 0, since her sale price would be lower than $c(\overline{v}_0)$ by equation (6) (see Figure 5 below). Such trade delay, when the buyer is more patient than the seller, is consistent with the findings of Evans (1989).

4 Comparative statics and other results

The following measure of the degree of adverse selection will be an important factor in determining the qualitative features of the comparative statics results.

Definition 4.1. We say that *the no-lemons condition holds* if $p^*(\overline{v}) \geq c(\overline{v})$ for all $\overline{v} \in (0, \overline{v}_0]$ – that is, if for any $\overline{v}$ the static monopolistic payoff is non-negative valuation-by-valuation.

The no-lemons condition holds if, independently of the truncation of the type distribution, the static monopolist does not face any "lemons", that is, any buyer types to whom a sale at the static monopolistic price yields a negative payoff for the monopolist. This holds, for example, in the private-values case, where $c(\cdot) = 0$. Equation (1) implies that when the no-lemons condition

[16] Recall that, as explained above, Assumption 2 holds in two canonical cases: the private-values case and the case where the seller is (weakly) more patient than the buyer.

fails, the seller sells to some buyers at a loss, both in the static and in the dynamic model. If, for example, $p^*(\overline{v}_0) < c(\overline{v}_0)$, then the monopolist loses when selling to high buyer types. Such losses are offset by sales at $p^*(\overline{v}_0)$ to lower buyer types in the static setting, and by later sales at lower prices in the dynamic setting, making the overall profits positive in both cases. The following result establishes that when the no-lemons condition holds, the profile described in Theorem 3.1 is an equilibrium irrespective of the values of r_{s} and r_{b}.

Corollary 4.1. *If Assumption 2 is not required to hold, then the strategy profile described in Theorem 3.1 is an equilibrium for all values of r_{s} and r_{b} if and only if the no-lemons condition holds.*

4.1 Buyer's patience

We first investigate how changes in the buyer's level of impatience, r_{b}, affect the equilibrium outcome.

Clearly r_{b} does not affect either the seller's payoff (by Proposition 3.2) or the price at time 0 (by equation (6)). It does, however, affect the price path and the timing of trade. The following result establishes that if the no-lemons condition holds, then the buyer benefits ex-post from being more impatient. In other words, he is more willing to pay to enter the market when r_{b} is higher, independently of his valuation.

Proposition 4.1. *The seller's payoff is independent of r_{b}. If the no-lemons condition holds, then for any $v \in [0, \overline{v}_0]$, the v-buyer's payoff is increasing in r_{b}.*

An intuition for Proposition 4.1 is the following. Suppose the no-lemons condition holds. An increase in r_{b} does not affect the seller's commitment problem, and hence it does not change the equilibrium price in each state $(t, \overline{v})$ (see equation (6)) or the seller's payoff (see equation (10)). However, it increases the speed at which the price declines: if the buyer becomes more impatient, then for him to remain indifferent between buying now or an instant later, the price must decline faster (see equation (7)). In other words, when the buyer is more impatient, his rejection of a given price offer is a stronger signal of a low valuation, and thus forces the seller to lower the price faster. Therefore, the buyer is screened more rapidly. Importantly, in equilibrium, the faster price decline is reinforced by the fact that each given $\overline{v}$ is reached earlier, so that the $\overline{v}$-buyer pays a lower price (by the no-lemons condition and equation (6)). Hence, again by equation (7), the speed of price decline at the instant at which each $\overline{v}$-buyer trades increases more than proportionally to the increase in r_b. This reinforcement gives the result in the proposition: namely, that the decrease in the buyer's payoff due to an increase in his impatience is more than offset, in equilibrium, by the increase in his payoff due to the resulting increase in the speed of price decline.[17]

When the no-lemons condition fails, the effect of an increase in r_b is ambiguous. From the proof of Proposition 4.1, it is easily seen that if, for example, $p^*(\overline{v}_0)<c(\overline{v}_0)$, then high-valuation buyers are worse off when they are more impatient. However, increased impatience may benefit low-valuation buyers because it causes the seller to screen them more quickly. The example presented in the next section provides some intuition for this.

4.2 Seller's patience

We now present the comparative statics analysis with respect to the seller's level of impatience, r_s. From Proposition 3.3, the seller's payoff is decreasing in r_s. The effect of r_s on the buyer's payoff again depends on whether the no-lemons condition holds.

Proposition 4.2. *The seller's payoff is decreasing in r_s. If the no-lemons condition holds, then for any $v\in[0,\overline{v}_0]$, the v-buyer's payoff is increasing in r_s.*

To gather intuition for Proposition 4.2, assume the no-lemons condition holds. Then a more impatient seller faces a more severe commitment problem, and therefore charges a lower price given t and $\overline{v}_t$ (i.e., the right-hand side of equation (6) is decreasing in r_s). This implies that the price declines faster (by equation (6)), and therefore the highest buyer type willing to accept a given price p_t (i.e., $\overline{v}(t,p_t)$ as defined in Section 3.2) is higher as well. As a result, when r_s increases, p_0 decreases, and for any given time and price, the price decline speeds up. Therefore, the price is lower for all t, and all buyer types are better off.

Now assume that the no-lemons condition fails and, in particular, that $p^*(\overline{v}_0)<c(\overline{v}_0)$. In this case, a more impatient seller charges higher initial prices, because she is less willing to take losses on early trades in exchange for higher profits on later trades. This makes high-valuation buyers worse off. However, for values of $\overline{v}$ such that $p^*(\overline{v})>c(\overline{v})$, an increase in r_s decreases the

[17] Proposition 4.1 illustrates the advantages of the tractability of our approach: for a discrete-time model analogous to ours (with private values), Sobel and Takahashi (1983) say, "No general statements can be made about how the no-commitment equilibrium prices change when the buyer's discount factor changes" (p. 417).

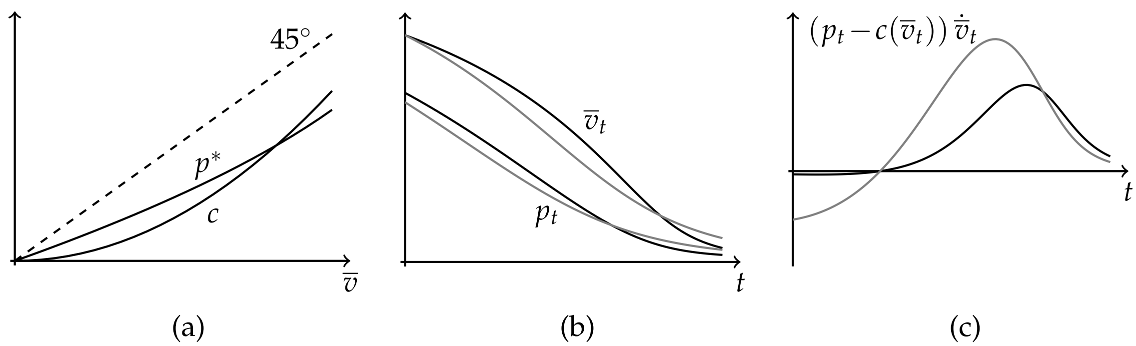


Figure 5: Example with $T=3$, $r_{\mathrm{b}}=1$, F uniform in $[0,1]$, and $c(v)=\frac{3}{4}v^2$, with $r_{\mathrm{s}}=0.1$ (gray) and $r_{\mathrm{s}}=1$ (black). (a) Plot of $p^*(\overline{v})$ and $c(\overline{v})$ with respect to $\overline{v}$. (b) Threshold types $\overline{v}_t$ (upper curves) and prices p_t (lower curves). (c) Seller's flow payoff, $(p_t - c(\overline{v}_t))\,\dot{\overline{v}}_t$.

price at each state $(t, \overline{v})$, and so accelerates the price decline. Hence, greater seller impatience may benefit low-valuation buyers.

Figure 5 depicts an example in which the no-lemons condition fails. Subfigure (a) shows that for high buyer valuations, the static monopolistic price is lower than the cost. Intuitively, this means the seller is willing to sell at a loss to high-valuation buyers so that she can sell at a higher profit to buyers with intermediate valuations. Subfigure (b) shows the price and threshold-type trajectories for small r_{s} (gray) and large r_{s} (black). Here we see that when the seller is more impatient, initial prices are higher while later prices are lower. Intuitively, a more impatient seller is less willing to accept early losses (by selling to high-valuation buyers), as her higher discount rate makes it more difficult to compensate for these with later sales to intermediate-valuation buyers. Finally, subfigure (c) shows that late in the bargaining period, when the cost is below the static monopolistic price, a higher r_{s} implies faster buyer screening: when the seller is more impatient, the initial flow payoff is less negative, but the later flow payoff is smaller. Overall, we see that when the seller is more impatient, she initially screens high-valuation buyers more slowly (by charging higher prices); later, she screens medium- and low-valuation buyers more quickly.

4.3 Time horizon

The effect of increasing the time horizon T is similar to that of increasing the seller's interest rate.

Proposition 4.3. *The seller's payoff is decreasing in T. If the no-lemons condition holds, then for any $v \in [0, \overline{v}_0]$, the v-buyer's payoff is increasing in T.*

Intuitively, increasing T lowers the seller's payoff because it gives her more time to screen the buyer, worsening her commitment problem. Furthermore, if the no-lemons condition holds, then changing the deadline from T to $T' > T$ reduces the equilibrium price at state $(T, \overline{v})$, leading to lower initial prices and therefore benefiting all buyer types. On the other hand, if the no-lemons condition fails, then the effect of increasing T is similar to that of increasing r_{s}: it worsens the seller's commitment problem, making sales at later times less profitable and inducing the seller to charge higher initial prices.

An interesting exercise is to consider simultaneous changes in r_{s} and T that keep $r_{\mathrm{s}}\,T$ constant. Such changes do not affect the equilibrium commitment problem of the seller: her payoff depends on r_{s} and T only through $r_{\mathrm{s}}\,T$. However, they do affect the buyer's payoff. An argument similar to that in the proof of Proposition 4.1 illustrates that, when the no-lemons condition fails, the buyer benefits from any such change provided that it makes the seller more patient (i.e., the buyer benefits when r_{s} decreases, T increases, and $r_{\mathrm{s}}\,T$ remains the same).

Infinite-horizon limit

We now analyze the limit where the time horizon becomes large. As T increases, our model approximates continuous-time versions of the discrete-time models studied in Gul et al. (1986) and Deneckere and Liang (2006).

If $c(\cdot) = 0$ (the private-values case), then by equation (6), $p(t, \overline{v}) \to 0$ as $T \to \infty$ for all $(t, \overline{v})$. In other words, we recover the conjecture of Coase (1972): even if the seller is more patient than the buyer, her inability to commit not to lower future prices dissipates all her rents from trade.

When $c(\cdot)$ is strictly increasing, we can combine equations (6) and (7) to obtain

$$\tfrac{\mathrm{d}}{\mathrm{d}t} c(\overline{v}_t) = -r_{\mathrm{b}} \left(\overline{v}_t - c(\overline{v}_t)\right) . \tag{13}$$

This is the same as the equation obtained in Fuchs and Skrzypacz (2013b) for the case $r_{\mathrm{s}} = r_{\mathrm{b}}$ in the double limit where both the gap between the lowest seller's and buyer's valuations and the length of the period vanish. It implies that the price dynamics are independent not only of the distribution of buyer's valuations in the infinite-horizon model (as observed by Fuchs and Skrzypacz), but also of the seller's patience level. Furthermore, for each value v, the payoff of the v-buyer is independent of r_s, r_b, and F.[18]

Note that the dynamics described in Theorem 3.1 in the limit as $T\to\infty$ coincide with the equilibrium dynamics in the limit as $r_s\to\infty$ when the no-lemons condition holds (and hence Assumption 3 holds for all r_s): in both cases, $p(t,\overline{v})=c(\overline{v})$ for all $(t,\overline{v})$, and the seller's payoff is zero. The limit outcome in the double limit as $r_s, T\to\infty$ can be reinterpreted as the outcome in a model with an infinite sequence of short-lived sellers, as in Hörner and Vieille (2009) (public-offers case). Our results imply that when the no-lemons condition holds and no gap exists between the lowest valuations of the seller and the buyer, the "trade impasse" disappears (and trade is smooth). In fact, unlike Hörner and Vieille, we find that there is no trade burst at time 0; trade is smooth, and it eventually occurs with probability one. This result can be seen as analogous to that of Fuchs and Skrzypacz (2013b): they show that the trade bursts predicted by Deneckere and Liang (2006) (for a long-lived seller having the same discount rate as the buyer) do not occur in the no-gap case.

4.4 Seller commitment

In our model, because the seller cannot commit not to lower the price in the future, her payoff is strictly lower than the one she would obtain if she could commit. This is clear when $r_s\geq r_b$: Stokey (1979) showed that in this case, when the seller can commit, trade occurs only at time 0 and at the monopolistic price $p^*(\overline{v}_0)$ (the result can be extended to the interdependent-values case).

When $r_s<r_b$, a seller with commitment power price-discriminates, taking advantage of the higher delay cost of the buyer (see Fudenberg and Tirole, 1983, and Landsberger and Meilijson, 1985).[19] The commitment solution is, in general, difficult to obtain. For intuition, we heuristically derive the optimal (non-stochastic) pricing strategy of a seller with commitment when the buyer's valuation is distributed uniformly on $[0,1]$ and the seller's cost is linear: $c(v)=k\,v$ for some $k\in[0,1)$. In this case, $p^*(\overline{v})=\frac{1}{2-k}\,\overline{v}$. We also focus, for simplicity, on the case where the seller is fully patient; that is, $r_s=0$.

[18]From equation (13), the payoff of the v-buyer is $\exp\left(-\int_v^{\overline{v}_0}\frac{c'(v')}{v'-c(v')}\,dv'\right)(v-c(v))$.

[19]Beccuti and Möller (2018) analyze a two-type, discrete-time, private-values model where the seller can offer a mechanism in each period and is more patient than the buyer. They obtain significant differences between selling and renting mechanisms.

To apply calculus of variations, we assume equation (7) holds in any interior time interval (t_1, t_2). Hence, the objective function of the seller in this interval is

$$\int_{t_1}^{t_2} (p_t - c(\overline{v}_t))\,\dot{\overline{v}}_t\,\mathrm{d}t = \int_{t_1}^{t_2} \underbrace{\left(p_t - k\left(p_t - r_{\mathrm{b}}^{-1}\,\dot{p}_t\right)\right)\left(\dot{p}_t - r_{\mathrm{b}}^{-1}\,\ddot{p}_t\right)}_{:=L(p_t,\dot{p}_t,\ddot{p}_t)}\,\mathrm{d}t\ .$$

Standard calculus of variations requires that the Euler–Lagrange equation holds:

$$0 = \frac{\partial L}{\partial p_t} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{p}_t} + \frac{\mathrm{d}^2}{\mathrm{d}t^2}\frac{\partial L}{\partial \ddot{p}_t} = -\frac{2\,\ddot{p}_t}{r_{\mathrm{b}}}\ .$$

That is, the price is a linear function of time. The seller's problem is then to find the optimal values for the prices at times 0 and T, denoted by p_0 and p_T, respectively. Equation (7) dictates that $\overline{v}(p) = p - \frac{p_T - p_0}{r_{\mathrm{b}}\,T}$, and hence the seller chooses p_0 and p_T to maximize

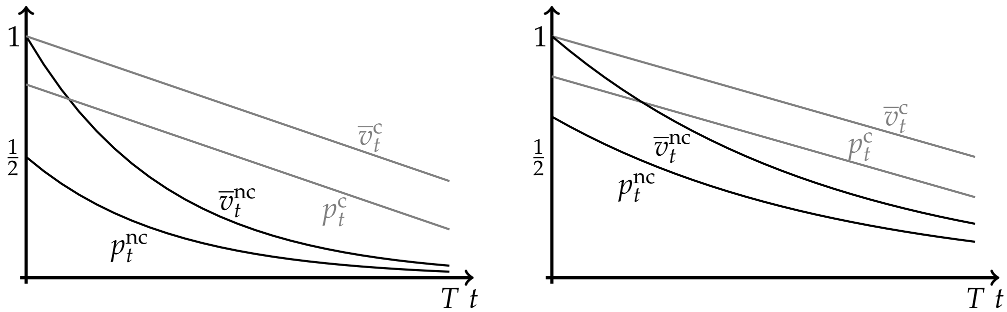
$$(1 - \overline{v}(p_0))\,p_0 + (\overline{v}(p_0) - \overline{v}(p_T))\,\tfrac{p_T + p_0}{2} + (\overline{v}(p_T) - p_T)\,p_T - \tfrac{k}{2}\,(1 - p_T^2)\ .$$

There is a unique pair $\{p_0^{\mathrm{c}}, p_T^{\mathrm{c}}\}$ maximizing this expression, from which we can obtain the price the committed seller charges:

$$p_t^{\mathrm{c}} := \frac{2 - k + (1-k)\,r_{\mathrm{b}}\,(T - t)}{2 - k + (1-k)\,r_{\mathrm{b}}\,T}\ .$$

From equations (6) and (7), we have that, in the absence of seller commitment, the equilibrium price is $p_t^{\mathrm{nc}} := \frac{1}{2-k}\,e^{-r_{\mathrm{b}}\,(1-k)\,t}$. Figure 6 depicts the price and upper valuation paths in the commitment and no-commitment cases for $k=0$ (the private-values case) and $k=\frac{1}{2}$ (the interdependent-values case).

Three observations are worth mentioning. First, since the seller is fully patient, her equilibrium payoff (with no commitment) is equal to the static monopolistic payoff $\Pi^*(\overline{v}_0)$. The equilibrium price, however, decreases over time. Hence, even though the seller price-discriminates regardless of whether she can commit, she fails to benefit from such price discrimination when she cannot commit. Second, in line with what occurs in the no-commitment case, the price at the deadline under commitment coincides with the static monopolistic price for the remaining buyer types. Hence, the seller's commitment problem arises not from her inability to commit to not lowering the price at the very end of the bargaining period (and the consequent backward induction argument), but from her inability to commit to not lowering the price quickly at an



(a) Private values ($k=0$). (b) Interdependent values ($k=\frac{1}{2}$.)

Figure 6: Price and upper valuation paths in the commitment solution (gray) and no-commitment solution (black) for F uniform on $[0,1]$, $r_{\mathrm{s}}=0$, $r_{\mathrm{b}}=1$, $T=3$, and $c(v)=k\,v$. For all cases, the only trade burst occurs at the deadline. As expected, the probability of trade is higher and the trade delay is lower in the no-commitment case than in the commitment case (i.e., $\overline{v}_t^{\mathrm{nc}}<\overline{v}_t^{\mathrm{c}}$ for all $t>0$). Also, there is a higher probability of trade and less trade delay in the private-values case.

earlier stage. Finally, in the commitment solution, every buyer type trades later than in the no-commitment solution (or does not trade at all). Thus, giving the seller commitment power not only decreases the probability of trade but also increases trade delay. This is the opposite of the result when the seller is more impatient than the buyer: in that case, when the seller has commitment power, trade occurs immediately.

4.5 Time-dependent discount rates

In this section, we consider a generalization of our main model in which the seller's and buyer's discount rates are time-dependent. In practice, changes in an agent's discount rate over time may correspond to increases in the probability of exogenous breakdown, a stochastic value decline (see Hart, 1989), or changes in the idiosyncratic interest rate.

Suppose that the seller's and buyer's discount rates are given by two bounded functions, $r_{\mathrm{s}}, r_{\mathrm{b}}\colon[0,T]\to\mathbb{R}_{++}$. We analyze the same model as in Section 2, the only difference being that if a transaction occurs at time t at price p_t, the respective payoffs of the seller and the buyer are

$$e^{-\int_0^t r_{\mathrm{s}}(t')\,\mathrm{d}t'}\left(p_t-c(v)\right) \text{ and } e^{-\int_0^t r_{\mathrm{b}}(t')\,\mathrm{d}t'}\left(v-p_t\right).$$

Assumption 2 is replaced by the requirement that for any $\overline{v}\in(0,\overline{v}_0]$ and t, $c(\overline{v})\leq\frac{r_{\mathrm{b}}(t)}{r_{\mathrm{s}}(t)}\,\overline{v}$.

It is not difficult to see that all the results in Sections 2.1 and 3.1 still hold (with appropriate changes to equation (10)). This can be best seen by normalizing the time unit so that the seller has a constant discount factor, for example, equal to 1. Most of the arguments for the previous results can then be extended immediately to this setting. In the normalized model, equation (6) holds with $r_{\mathrm{s}}{=}1$. The normalized, time-dependent discount rate for the buyer, $\tilde{r}_{\mathrm{b}}(t)$, modulates the speed at which the price changes through equation (7).

Now suppose that $r_{\mathrm{s}}(t)$ and $r_{\mathrm{b}}(t)$ increase over time, but $r_{\mathrm{b}}(t)/r_{\mathrm{s}}(t)$ is constant, equal to some κ, as in Hart (1989).[20] Let $\hat{r}_{\mathrm{s}}{:=}\int_0^T r_{\mathrm{s}}(t)\,\mathrm{d}t/T$, and consider a model with time horizon T in which the seller and the buyer have (constant) discount rates $\hat{r}_{\mathrm{s}}$ and $\kappa\,\hat{r}_{\mathrm{s}}$, respectively. Let $\hat{p}_t$ be the on-path price for this auxiliary model. The price in the model with time-dependent discount rates is then

$$p_t{=}\hat{p}_{\hat{t}(t)}\ , \quad \text{where } \ \hat{t}(t){:=}\hat{r}_{\mathrm{s}}^{-1}\int_0^t r_{\mathrm{s}}(t)\,\mathrm{d}t\ .$$

Because $r_{\mathrm{s}}(\cdot)$ is increasing, we have that $\hat{t}(\cdot)$ is convex. Hence, p_t is an "accelerated" version of the price in the auxiliary model: as delay becomes more costly, the probability of trade increases. The seller and buyer payoffs coincide with those in the normalized model.

For further intuition, consider the private-values case with general $r_{\mathrm{s}}(\cdot)$ and $r_{\mathrm{b}}(\cdot)$. Then, differentiating $p_t{=}p(t,\overline{v}_t)$ using equation (6), we obtain

$$\dot{p}_t{=}e^{-\int_t^T r_{\mathrm{s}}(t')\,\mathrm{d}t'}\,\big(\underbrace{r_{\mathrm{s}}(t)\,p^*(\overline{v}_t)}_{(*)}+\underbrace{p^{*\prime}(\overline{v}_t)\,\dot{\overline{v}}_t}_{(**)}\big)\ . \tag{14}$$

If $r_{\mathrm{s}}(\cdot)$ grows larger during a given interval of time while $r_{\mathrm{b}}(\cdot)$ remains roughly the same, then the price $p(t,\overline{v}_t)$ does not vary much in this interval (by equation (7)). Given that the term $(*)$ in equation (14) is larger, the term $(**)$ has to be more negative, meaning the buyer is screened more quickly in this interval. Conversely, if $r_{\mathrm{b}}(\cdot)$ grows larger during a given interval while $r_{\mathrm{s}}(\cdot)$ remains roughly the same, then the price decreases rapidly in this interval (by equation (7)). Since the term $(*)$ in equation (14) does not change much, the term $(**)$ must become more negative; hence, the buyer is again screened more quickly. Thus, faster buyer screening occurs when either the seller's or the buyer's relative discount rate is high; however, more rapid price decline occurs only when the buyer's relative discount rate is high.

[20] Hart (1989) studies a two-type model in discrete time where the probability of decline of the value of the good significantly increases in the period before a deadline. He finds that most trade occurs in the first and the last periods.

5 Conclusions

In real-life negotiations, it is common for the agents to reach an agreement only after a delay, or not at all. The outcome of a negotiation depends on numerous factors, including each party's bargaining costs, the gains from trade, and the deadlines that govern the bargaining process.

In this paper we have presented a tractable model of a negotiation with private information that enables us to characterize how all of these factors affect the trade dynamics. While some of our results validate standard intuitions, others are less straightforward. In particular, we identify the conditions on the gains from trade that determine how changes in buyer and seller impatience (discount rates) qualitatively affect the outcome. The crucial condition is the no-lemons condition, which says that the seller's cost for supplying high-valuation buyers is less than the static monopolistic price.

When the no-lemons condition holds (e.g., in the private-values case), the buyer is better off when his discount rate is higher. Intuitively, a higher buyer discount rate implies that the buyer is less willing to reject any given price offer; thus, a rejection is a stronger signal that his valuation is low. This induces the seller to decrease the price more quickly. There is also an additional effect: the seller's commitment problem is more severe at earlier times, which further accelerates the price decline. We find that the drop in prices is sufficient to offset the increase in the buyer's discount rate; thus, the buyer benefits independently of his valuation. Through similar arguments, we also find that the buyer is better off when the seller's discount rate is higher or the deadline is later.

On the other hand, when the no-lemons condition fails, the seller's payoff from early sales is negative. In equilibrium, these losses are offset by more profitable sales at later times. When the seller has a higher discount rate, this intertemporal trade-off of losses and gains is less attractive; hence, instead of speeding trade up by lowering prices, she delays trade by charging higher initial prices. Thus, an increase in the seller's discount rate makes high-valuation buyers worse off, although low-valuation buyers continue to benefit from rapid price decline at later times. A similar intuition applies when the deadline is extended: later sales are more heavily discounted and hence less profitable (from the perspective of the seller at time 0), so the seller chargers more initially to avoid losses.

In future research, it would be interesting to extend our results to settings not analyzed here. For example, as we argue in Section 3.3, other types of equilibria may arise when both the seller is very impatient and the no-lemons condition fails. In this case, there may be trade impasses, that is, intervals of time without trade. Similarly, the analysis could be extended to the gap case, where the buyer's valuation is bounded away from 0.[21] In this case, there may be equilibria in which trade occurs before the deadline with probability one. (Note, however, that because our results apply to a broad set of distributions, one could approximate the gap by assigning progressively lower probability to lower valuations.)

A Proofs of the results

A.1 Proofs of results in Section 2

Proof of Lemma 2.1

Proof. The proof follows the standard argument. Fix some PBE (P, a, F) and history p^t. Take two valuations $v > v'$ and assume, for the sake of contradiction, that $t^v > t^{v'}$ (we omit the explicit dependence on the strategy profile and the history). Note that the optimality of the buyer's strategy requires that $p^v < p^{v'} < v'$. Then, we have

$$\begin{aligned} \exp\left(-r_{\mathrm{b}}\, t^{v}\right)(v-v') &= \exp\left(-r_{\mathrm{b}}\, t^{v}\right)(v-p^{v}) - \exp\left(-r_{\mathrm{b}}\, t^{v}\right)(v'-p^{v}) \\ &\geq \exp\left(-r_{\mathrm{b}}\, t^{v'}\right)(v-p^{v'}) - \exp\left(-r_{\mathrm{b}}\, t^{v'}\right)(v'-p^{v'}) \\ &= \exp\left(-r_{\mathrm{b}}\, t^{v'}\right)(v-v')\ , \end{aligned}$$

which contradicts that $t^v > t^{v'}$. The inequality holds because the v-buyer is (weakly) worse off following the v'-buyer's equilibrium strategy and the v'-buyer is (weakly) better off following his equilibrium strategy. □

A.2 Proofs of results in Section 3

As explained in the main text, the proofs of Proposition 3.1–3.3 are used to prove Theorem 3.1. Hence, we prove them first, and then provide a proof of Theorem 3.1.

[21] In discrete time, the finite horizon plays a role similar to that of the gap. In particular, the folk theorem in Ausubel and Deneckere (1989) for the no-gap case fails when the horizon is finite, as one can use backward induction from the last period with trade.

Proof of Proposition 3.1

Proof. We begin the proof with an auxiliary result:

Lemma A.1. *Let p^t and $\hat{p}^t$ be such that $\overline{v}(p^t)<\overline{v}(\hat{p}^t)$. Then*

$$\Pi(t,\overline{v}(p^t))\geq\int_0^{\overline{v}(p^t)} e^{-r_\mathrm{s}\,(t^v(\hat{p}^t;P,a)-t)}\,\left(p^v(\hat{p}^t;P,a)-c(v)\right)F(\mathrm{d}v)\,. \tag{15}$$

Proof. Consider a deviation of the seller to the strategy $\hat{P}$ defined by $\hat{P}(p^{t'}):=P(P_{[0,t']}(\hat{p}^t))$ for all $p^{t'}$. Intuitively, for each t', the strategy $\hat{P}$ requires the seller to do what the strategy P specifies after history $P_{[0,t']}(\hat{p}^t)$. Then, from the second condition in Definition 2.2, all types of the buyer below $\overline{v}(p^t)$ buy at the same time and price when the seller uses $\hat{P}$ after p^t as they do when the seller uses strategy P after history $\hat{p}^t$. The seller's payoff from using strategy $\hat{P}$ after history p^t then coincides with the right-hand side of equation (15). Hence, the seller's payoff at state $(t,\overline{v}(p^t))$ can not be lower than the right-hand side of equation (15). □

(Proof of Proposition 3.1 continues.)

The proof of Lemma A.1 uses a mimicking argument. The seller at state $(t,\overline{v})$ can imitate the continuation strategy after state $(t,\overline{v}')$ with $\overline{v}'>\overline{v}$. By doing this, the seller's payoff from buyer types below $\overline{v}$ coincides with the one she obtains from these types after state $(t,\overline{v}')$.

Fix an equilibrium and history p^{t_1}, for some $t_1\in[0,T)$. Assume, for the sake of contradiction, that there is some $t_2>t_1$ satisfying that $\overline{v}_{t_1}=\overline{v}_{t_2^-}>0$ (as noted before, the explicit dependence on the history p^{t_1} is omitted).[22] We first observe that it must be that $\overline{v}_T<\overline{v}_{t_1}$, since $\overline{v}_{T^-}\leq\overline{v}_{t_2^-}$ (by the skimming property) and $\overline{v}_T=p^*(\overline{v}_{T^-})<\overline{v}_{T^-}$. It is convenient to pick t_2 to be the supremum among the times in $(t_1,T]$ satisfying that $\overline{v}_{t_2^-}=\overline{v}_{t_1}$. We consider two separate cases:

Case 1: There is a "burst" of trade at time t_2. Assume first $\overline{v}_{t_2}<\overline{v}_{t_2^-}$; that is, there is a positive probability of trade at time t_2 (at price $p_{t_2}:=P_{t_2}(p^{t_1})$). Fix some $t\in(t_1,t_2)$ and some $\varepsilon>0$, and consider the following deviation by the seller from time t on: offer price

$$\hat{p}_t:=(1-e^{-r_\mathrm{b}\,(t_2-t)})\,(\overline{v}_{t_1}-\varepsilon)+e^{-r_\mathrm{b}\,(t_2-t)}\,p_{t_2} \tag{16}$$

at date t, unacceptable prices on (t,t_2), and continue with the equilibrium strategy $P(p^{t_1})$

[22] Note that, for each t_1, there exists $t_2>t_1$ such that $\overline{v}_{t_1}=\overline{v}_{t_2^-}$ if and only if there exists $t_2'>t_1$ such that $\overline{v}_{t_1}=\overline{v}_{t_2'}$.

from date t_2 on (that is, "as if" she did not deviate at time t). Note that the $(\overline{v}_{t_1}-\varepsilon)$-buyer is indifferent between accepting $\hat{p}_t$ at time t and accepting p_{t_2} at time t_2. Hence, for all $v \in (\overline{v}_{t_1}-\varepsilon, \overline{v}_{t_1})$ the v-buyer obtains a strictly bigger payoff from accepting $\hat{p}_t$ at time t than from accepting p_{t_2} at time t_2. Let $\hat{v}_t$ denote $\overline{v}(P_{[0,t)}(p^{t_1}), \hat{p}_t)$.

There are two possibilities. The first is that there is no trade at time t when $\hat{p}_t$ is offered; that is, $\hat{v}_t = \overline{v}_{t_1}$.[23] Then, by the Markov property, the buyer believes that the seller's continuation strategy is such that there is no trade until t_2, where the price is p_{t_2}. This, nevertheless, leads to a contradiction, since as we observed before, there is a positive mass of buyer's valuations such that the buyer strictly prefers accepting $\hat{p}_t$ at t to accepting p_{t_2} at t_2. The second possibility is that there is a positive probability of trade at date t when $\hat{p}_t$ is offered; that is, $\hat{v}_t < \overline{v}_{t_1}$. By the argument in the proof of Lemma A.1, the seller obtains the same payoff from all types $v < \hat{v}_t$ under the deviation than under the equilibrium strategy (given that, under the deviation, her continuation strategy coincides with the continuation strategy if she did not deviate). The increase in the seller's payoff from the buyer when his valuation is in $[\hat{v}_t, \overline{v}_{t_1}]$ is given by

$$\overbrace{\hat{p}_t - \mathbb{E}[c(\tilde{v})|\tilde{v} \in [\hat{v}_t, \overline{v}_{t_1}]]}^{\text{payoff when } \hat{p}_t \text{ is offered}} - \overbrace{e^{-r_{\text{s}}(t_2-t)}\left(p_{t_2} - \mathbb{E}[c(\tilde{v})|\tilde{v} \in [\hat{v}_t, \overline{v}_{t_1}]]\right)}^{\text{equilibrium payoff}} .$$

Using equation (16) we have that, as $t \to t_2$, the previous expression can be written as:

$$\Big(\underbrace{r_{\text{b}}\left(\overline{v}_{t_1} - \varepsilon - p_{t_2}\right) - r_{\text{s}}\left(\mathbb{E}[c(\tilde{v})|\tilde{v} \in [\hat{v}_t, \overline{v}_{t_1}]] - p_{t_2}\right)}_{(*)}\Big)(t_2 - t) + O((t_2-t)^2) .$$

Note that, since $c(\cdot)$ is increasing, $\mathbb{E}[c(\tilde{v})|\tilde{v} \in [\hat{v}_t, \overline{v}_{t_1}]] \le c(\overline{v}_{t_1})$. Note also that the term $(*)$ is linear in p_{t_2}, larger than $r_{\text{s}}\left(\overline{v}_{t_1} - c(\overline{v}_{t_1})\right) - r_{\text{b}}\,\varepsilon$ when $p_{t_2} = \overline{v}_{t_1}$, and larger than $r_{\text{b}}\left(\overline{v}_{t_1} - \frac{r_{\text{s}}}{r_{\text{b}}}\,c(\overline{v}_{t_1}) - \varepsilon\right)$ when $p_{t_2} = 0$. Hence, using that $\overline{v}_{t_1} > c(\overline{v}_{t_1})$ and Assumption 2, we have that $(*)$ is postitive if ε is small enough. We then conclude that a profitable deviation for the seller exists, a contradiction.

Case 2: There is no "burst" of trade at time t_2. Assume now $\overline{v}_{t_2^-} = \overline{v}_{t_2}$. The logic for this case is similar to the logic for Case 1, but the argument is slightly more involving. First note that, by the observation above, it must be that $t_2 < T$ (because there is trade a burst at the deadline).

[23]Note that, as we have argued, the v-buyer obtains a strictly bigger payoff from accepting $\hat{p}_t$ at time t than from accepting p_{t_2} at time t_2 for all $v \in (\overline{v}_{t_1}-\varepsilon, \overline{v}_{t_1})$. Still, it could be that these none of these types accept $\hat{p}_t$: it depends on what the seller's continuation play after the deviation prescribed by the equilibrium.

Pick again some $t \in (t_1, t_2)$ and now let $\tilde{p}_t$ be such that the $(\overline{v}_{t_2} - \varepsilon)$-buyer is indifferent between accepting $\tilde{p}_t$ at time t or $p^{\overline{v}_{t_2} - \varepsilon}$ at time $t^{\overline{v}_{t_2} - \varepsilon}$. Noticing that, if ε is small enough, $t^{\overline{v}_{t_2} - \varepsilon}$ is close to t_2 (by the definition of t_2), the same argument as in Case 1 goes through. $\square$

Proof of Proposition 3.2

Proof. The proof is divided into three lemmas. Lemma A.2 sets an upper bound on the seller's payoff. Lemma A.3 establishes a lower and an upper bound on the seller's payoff in terms of the continuation payoffs at lower threshold valuations. Lemma A.4 establishes the continuity of $p(t, \cdot)$ for all t. We finally argue that these lemmas imply the result stated in Proposition 3.2.

We first present an auxiliary result. The result establishes that, after any t-history, the seller's payoff is no higher than that from selling to a range of higher buyer types at the price at time t, while continuing selling at the same time and prices to the lower buyer types. The result is intuitive, as the seller sells earlier and at a higher price to higher types. Nevertheless, the fact that the seller incurs the cost of selling to the higher types at an earlier time makes the result not trivial.

Lemma A.2. *Fix some state $(t, \overline{v})$. Then, for all $\overline{v}' < \overline{v}$ close enough to $\overline{v}$, we have*

$$\Pi(t, \overline{v}) \leq (F(\overline{v}) - F(\overline{v}')) \left(p(t, \overline{v}) - \mathbb{E}[c(\tilde{v}) | \tilde{v} \in [\overline{v}', \overline{v}]] \right) + \int_0^{\overline{v}'} e^{-r_\mathrm{s} \, (t^v(t,\overline{v}) - t)} \, \pi_{t^v(t,\overline{v})}(t, \overline{v}) \, F(\mathrm{d}v) \, , \quad (17)$$

where $\pi_{t'}(t, \overline{v}) := p_{t'}(t, \overline{v}) - c(\overline{v})$.

Proof. By the optimality of the buyer's strategy, it must be that, for each $v \in (\overline{v}', \overline{v})$,

$$v - p(t, \overline{v}) \leq e^{-r_\mathrm{b} \, (t^v(t,\overline{v}) - t)} \, (v - p_{t^v(t,\overline{v})}(t, \overline{v})) \, .$$

Rewriting the previous inequality, we have that the payoff the seller obtains from selling to the buyer with valuation v at time t at price $p(t, \overline{v})$ – which is equal to $p(t, \overline{v}) - c(v)$ – instead of at time $t^v(t, \overline{v})$ at price $p_{t^v(t,\overline{v})}(t, \overline{v})$ is no smaller than

$$v - c(v) - e^{-r_\mathrm{b} \, (t^v(t,\overline{v}) - t)} \, (v - p_{t^v(t,\overline{v})}(t, \overline{v})) \, .$$

By adding and subtracting the seller's payoff from selling to the buyer with valuation v at time $t^v(t, \overline{v})$ at price $p_{t^v(t,\overline{v})}(t, \overline{v})$ (as the continuation strategy prescribes), the previous expression is equal to

$$
\begin{aligned}
&e^{-r_{\mathrm{s}}\,(t^{v}(t,\overline{v})-t)}\,(p_{t^{v}(t,\overline{v})}(t,\overline{v})-c(v))\\
&\quad+\underbrace{(e^{-r_{\mathrm{b}}\,(t^{v}(t,\overline{v})-t)}-e^{-r_{\mathrm{s}}\,(t^{v}(t,\overline{v})-t)})\,p_{t^{v}(t,\overline{v})}(t,\overline{v})+(1-e^{-r_{\mathrm{b}}\,(t^{v}(t,\overline{v})-t)})\,v-(1-e^{-r_{\mathrm{s}}\,(t^{v}(t,\overline{v})-t)})\,c(v)}_{(*)}\ .
\end{aligned}
$$

Then, proving that the term $(*)$ is bigger than 0 when v is close enough to $\overline{v}$ shows the result. Note that $(*)$ is linear in $p_{t^{v}(t,\overline{v})}(t,\overline{v})$. By Proposition 3.1, we have that $\lim_{v\nearrow\overline{v}} t^{v}(t,\overline{v})=t$. Furthermore, note that if $p_{t^{v}(t,\overline{v})}(t,\overline{v})=0$ we have that $(*)$ is approximately equal to $(r_{\mathrm{b}}\,v-r_{\mathrm{b}}c(v))\,(p_{t^{v}(t,\overline{v})}(t,\overline{v})-t)$, which is non-negative by Assumption 2. When instead $p_{t^{v}(t,\overline{v})}(t,\overline{v})=v$, the term $(*)$ is equal to

$$
g(\hat{t}):=(1-e^{-r_{\mathrm{b}}\,\hat{t}})\,v-(1-e^{-r_{\mathrm{s}}\,\hat{t}})\,c(v)\ ,
$$

where $\hat{t}:=t^{v}(t,\overline{v})-t$. Note that $g(0)=0$ and $\lim_{\hat{t}\to\infty} g(\hat{t})=\overline{v}-c(\overline{v})>0$. Simple analysis shows that $g'(\cdot)$ is single peaked, $\lim_{\hat{t}\to\infty} g'(\hat{t})=0$, and, by Assumption 2, we have $g'(0)\geq 0$. Hence, the term $(*)$ is positive when $p_{t^{v}(t,\overline{v})}(t,\overline{v})=v$. Henceforth, the term $(*)$ is positive because $p_{t^{v}(t,\overline{v})}(t,\overline{v})\in(0,v]$. □

(Proof of Proposition 3.2 continues.)

We now establish bounds on the seller's payoff, both from below and from above:

Lemma A.3. *For any t, $\overline{v}$, and $\overline{v}'$, with $\overline{v}>\overline{v}'$,*

$$
\begin{aligned}
&(F(\overline{v})-F(\overline{v}'))\,\big(p(t,\overline{v}')-\mathbb{E}[c(\tilde{v})|\tilde{v}\in[\overline{v}',\overline{v}]]\big)+\Pi(t,\overline{v}')\\
&\qquad\leq\Pi(t,\overline{v})\leq(F(\overline{v})-F(\overline{v}'))\,\big(p(t,\overline{v})-\mathbb{E}[c(\tilde{v})|\tilde{v}\in[\overline{v}',\overline{v}]]\big)+\Pi(t,\overline{v}')\ .
\end{aligned}
\tag{18}
$$

Proof. The first inequality in equation (18) follows from the following observation. The seller has the option to "replicate" the continuation strategy she uses in state $(t,\overline{v}')$ when the state is, instead, $(t,\overline{v})$. By the same argument as in the proof of Lemma A.1, the seller obtains a payoff equal to the expression on the left-hand side of the first inequality in equation (18) by doing so.

To prove the second inequality, recall Lemma A.2. Using the optimality of the seller's continuation strategy at $(t^{\overline{v}'},\overline{v}')$, we have that the right-hand side of expression (17) is no larger than

$$(F(\overline{v})-F(\overline{v}'))\left(p(t,\overline{v})-\mathbb{E}[c(\tilde{v})|\tilde{v}\in[\overline{v}',\overline{v}]]\right)+e^{-r_s\,(t^{\overline{v}'}-t)}\,\Pi(t^{\overline{v}'},\overline{v}')\ , \tag{19}$$

and hence $\Pi(t,\overline{v})$ is smaller than expression (19). Note finally that, because the seller has the option of making unacceptable offers on $[t,t^{\overline{v}'})$, we have $e^{-r_s\,(t^{\overline{v}'}-t)}\,\Pi(t^{\overline{v}'},\overline{v}')\leq\Pi(t,\overline{v}')$, and hence the second inequality in expression (18) holds. □

(Proof of Proposition 3.2 continues.)

The following result establishes that $p(t,\cdot)$ is increasing and continuous:

Lemma A.4. *For all t, $p(t,\cdot)$ is increasing and continuous.*

Proof. **Proof that $p(t,\cdot)$ is increasing.** It follows directly from equation (18).

Proof that $p(t,\cdot)$ is continuous. The proof is similar to the proof of Proposition 3.1. We prove that $p(t,\cdot)$ is left-continuous (right-continuity is proven analogously). We do this by assuming, for the sake of contradiction, that $p(t,\cdot)$ is not left continuous at some $\overline{v}$; that is, there is a strictly increasing sequence $(\overline{v}_n)_n$ converging to $\overline{v}$ such that $p(t,\overline{v}_n)\to p_\infty\neq p(t,\overline{v})$. Since $p(t,\cdot)$ is increasing, it must be that $p_\infty<p(t,\overline{v})$. Let p^t be some history with $\overline{v}(p^t)=\overline{v}$, and we let $\overline{v}_{t'}$ denote $\overline{v}_{t'}(p^t)$. Also, for for each n, let p^t_n be a history with $\overline{v}(p^t_n)=\overline{v}_n$ (note they exist, see Footnote 10). Let finally t_n denote $t^{\overline{v}_n}(p^t)$, and note that $(t_n)_n$ (weakly) decreases toward t by Proposition 3.1.

We first prove that $t_n>t$ for all n. To see this, note that $t_n\geq t$ for all n, and so assume by contradiction that $t_{n-1}=t$ for some n (indicating that $\overline{v}_{t^+}\leq\overline{v}_{n-1}$). This implies that $\overline{v}_{t^+}<\overline{v}_n$. Consider the continuation price path $\hat{p}_{(t,T]}$ defined by

$$\hat{p}_{t'}:=\begin{cases}P_{t'}(p^t) & \text{if } t'\in(t,t+\varepsilon],\\ P_{t'}(p^t_{n+1}) & \text{if } t'\in(t+\varepsilon,T],\end{cases}$$

for some $\varepsilon>0$. As ε shrinks towards 0, the seller's payoff from the previous continuation play at history p_n^t converges to

$$\left(F(\overline{v}_n)-F(\overline{v}_{t^+}(p^t))\right)\left(p(t,\overline{v})-\mathbb{E}[c(\tilde{v})|\tilde{v}\in[\overline{v}_{t^+}(p^t),\overline{v}_n]]\right) \\ +\int_0^{\overline{v}_{t^+}(p^t)} e^{-r_{\mathrm{s}}\,(t^v(p_n^t;P,a)-t)}\left(p^v(p_n^t;P,a)-c(v)\right)F(\mathrm{d}v)\ .$$

Given that $p(t,\overline{v})>p(t,\overline{v}_n)$ and $F(\overline{v}_n)-F(\overline{v}_{t^+}(p^t))>0$, Lemma A.2 implies that the previous expression is strictly larger than $\Pi(t,\overline{v}_n)$. Since the seller has a profitable deviation, we reach a contradiction, and hence it must be that $t_n>t$ for all n.

Take some price $\hat{p}\in(p_\infty,p(t,\overline{v}))$. For each $t'>t$, let $\hat{v}_{t'}:=\overline{v}(p^t,P_{[0,t')}(p^t),\hat{p})$ be the upper valuation at time t' if the seller charges $\hat{p}$ at time t'. Consider a deviation of the seller after p^t, consisting in following the continuation path $P_{(t,t_n)}(p^t)$ on (t,t_n), then charging $\hat{p}$ at time t_n, and then continuing to follow the equilibrium strategy after t_n. Note that, since $p(t',\overline{v}_{t'})$ is continuous in t' on (t,t_n) (by the optimality of the buyer's strategy), we have that $p(t',\overline{v}_{t'})>\hat{p}$ if n is big enough. There are two cases:

1. In the first case, there is no trade at time t_n when the seller offers $\hat{p}$ at time t_n; that is, $\hat{v}_{t_n}=\overline{v}_{t_n}$. In this case, the continuation play after t_n is unchanged. Nevertheless, this implies that an interval of buyer's types $[\underline{v},\overline{v}_{t_n}]$, for some $\underline{v}<\overline{v}_{t_n}$, are willing to buy at prices larger than $\hat{p}$ at time t_n or later (recall that $p(t_n,\overline{v}_{t_n})>\hat{p}$), a contradiction.

2. In the second case, trade occurs with positive probability at time t_n when the seller offers $\hat{p}$; that is, $\hat{v}_{t_n}<\overline{v}_{t_n}$. Now, there are two possibilities:

   (a) The first possibility is that $\hat{v}_{t_n}\leq\overline{v}_{t_n}(p_n^t)$, but this implies that

   $$p(t_n,\hat{v}_{t_n})=\hat{p}>p_\infty>p(t,\overline{v}_n)>p_{t_n}(t,\overline{v}_n)=p(t_n,\overline{v}_{t_n}(p_n^t))\ ;$$

   contradicting that $p(t_n,\cdot)$ is increasing.

   (b) The second possibility is that $\hat{v}_{t_n}>\overline{v}_{t_n}(p_n^t)$. Now, consider the following deviation of the seller on the continuation strategy at history p_n^t for n is large enough: the seller charges unacceptable prices in (t,t_n), then charges $\hat{p}$ at time t_n, and then offers $P_{t''}(t,\overline{v}_n)$ for all $t''>t_n$. By the argument in the proof of Lemma A.1, using this strategy, the seller obtains the same payoff for all valuations in $[0,\hat{v}_{t_n}]$. Also, since the seller sells to the buyer when his valuation $[\hat{v}_{t_n},\overline{v}_n]$ at time t_n at price $\hat{p}$, the deviation is profitable (note that when n is large, t_n is close to t, but $p_{t'}(t,\overline{v}_n) < p_\infty < \hat{p}$ for all $t' \in (t,t_n)$), which is again a contradiction.

□

(Proof of Proposition 3.2 continues.)

From the first inequality in equation (A.4) and the continuity of $\pi(t,\overline{v}) := p(t,\overline{v}) - c(\overline{v})$ with respect to $\overline{v}$ (which follows from Lemma A.4 and the continuity of $c(\cdot)$) we have that

$$\liminf_{\overline{v}' \nearrow \overline{v}} \tfrac{\Pi(t,\overline{v}) - \Pi(t,\overline{v}')}{\overline{v} - \overline{v}'} \geq \pi(t,\overline{v})\, f(\overline{v}) \quad \text{and} \quad \liminf_{\overline{v} \searrow \overline{v}'} \tfrac{\Pi(t,\overline{v}) - \Pi(t,\overline{v}')}{\overline{v} - \overline{v}'} \geq \pi(t,\overline{v}')\, f(\overline{v}')\ . \tag{20}$$

Using the second inequality in equation (18) and, again, the continuity of $p(t,\cdot)$, we have that

$$\limsup_{\overline{v}' \nearrow \overline{v}} \tfrac{\Pi(t,\overline{v}) - \Pi(t,\overline{v}')}{\overline{v} - \overline{v}'} \leq \pi(t,\overline{v})\, f(\overline{v}) \quad \text{and} \quad \limsup_{\overline{v} \searrow \overline{v}'} \tfrac{\Pi(t,\overline{v}) - \Pi(t,\overline{v}')}{\overline{v} - \overline{v}'} \leq \pi(t,\overline{v}')\, f(\overline{v}')\ . \tag{21}$$

The four inequalities in expressions (20) and (21), together with the continuity of $p(t,\cdot)$ established in Lemma A.4, imply that $\Pi(t,\cdot)$ is differentiable, and the derivative is equal to

$$\tfrac{\mathrm{d}}{\mathrm{d}\overline{v}} \Pi(t,\overline{v}) = \pi(t,\overline{v})\, f(\overline{v})\ . \tag{22}$$

Integrating the previous expression gives equation (9). □

Proof of Proposition 3.3

Proof. Fix an equilibrium and history p^t. We write

$$\Pi(t,\overline{v}_t) = \int_{[\overline{v}_t,\overline{v}_{t+\varepsilon})} e^{-r_{\mathrm{s}}\,(t^v(t,\overline{v}_t)-t)}\ \pi_{t^v(t,\overline{v}_t)}(t,\overline{v}_t)\, F(\mathrm{d}v) + e^{-r_{\mathrm{s}}\,\varepsilon}\, \Pi(t+\varepsilon,\overline{v}_{t+\varepsilon})\ ,$$

where $\pi_{t^v(t,\overline{v}_t)}(t,\overline{v}_t) := p_{t^v(t,\overline{v}_t)}(t,\overline{v}_t) - c(v)$. Using Proposition 3.2, we have

$$0 = \int_{[\overline{v}_t,\overline{v}_{t+\varepsilon})} \left(\pi(t,v) - e^{-r_{\mathrm{s}}\,(t^v(t,\overline{v}_t)-t)}\ \pi_{t^v(t,\overline{v}_t)}(t,\overline{v}_t)\right) F(\mathrm{d}v) + e^{-r_{\mathrm{s}}\,\varepsilon}\, \Pi(t+\varepsilon,\overline{v}_{t+\varepsilon}) - \Pi(t,\overline{v}_{t+\varepsilon})\ .$$

Note that if $\overline{v}_{t^+} < \overline{v}_t$ then all types in $(\overline{v}_{t^+}, \overline{v}_t]$ trade at time t, hence they do not contribute to the previous expression. Hence, we can write the previous expression as

$$0 = \int_{[\overline{v}_{t^+}, \overline{v}_{t+\varepsilon})} \underbrace{\left(\pi(t,v) - e^{-r_s\,(t^v(t,\overline{v}_t)-t)}\, \pi_{t^v(t,\overline{v}_t)}(t,\overline{v}_t) \right)}_{(*)} F(\mathrm{d}v) + e^{-r_s\,\varepsilon}\, \Pi(t+\varepsilon, \overline{v}_{t+\varepsilon}) - \Pi(t, \overline{v}_{t+\varepsilon}) \ .$$

Note then that the term $(*)$ tends to 0 as $\varepsilon \to 0$. By Proposition 3.1, we have that $t^v(t,\overline{v}_t) \to t$ as $v \nearrow \overline{v}_t$. Furthermore, by the incentive compatibility of the buyer strategy (both when he has valuation v and $\overline{v}_t$), we have $p_{t^v(t,\overline{v}_t)}(t,\overline{v}_t) \to p(t,\overline{v}_t)$ as $v \nearrow \overline{v}_t$, hence $\pi_{t^v(t,\overline{v}_t)}(t,\overline{v}_t) \to p(t,\overline{v}_t)$. Since the term $(*)$ is integrated on $[\overline{v}_{t^+}, \overline{v}_{t+\varepsilon})$ and F has a continuous density, we have that the first term on the right hand side of the previous expression tends to 0 faster than ε (i.e., it is $o(\varepsilon)$). As a result, we have

$$\lim_{\varepsilon \searrow 0} \frac{e^{-r_s\,\varepsilon}\, \Pi(t+\varepsilon, \overline{v}_{t+\varepsilon}) - \Pi(t, \overline{v}_{t+\varepsilon})}{\varepsilon} = 0 \ .$$

A similar argument can be made when $\varepsilon \nearrow 0$. Hence, the function $t \mapsto e^{-r_s\,t}\, \Pi(t,\overline{v})$ (for a fixed $\overline{v}$) is continuous and differentiable (on $[0,T]$), with a derivative equal to 0. Since $\Pi(T,\overline{v}) = \Pi^*(\overline{v})$, we have that $\Pi(t,\overline{v}) = e^{-(T-t)\,r_s}\, \Pi^*(\overline{v})$, as desired. □

Proof of Theorem 3.1

Proof. We first argue that the strategies of the seller and the buyer are mutual best responses. Indeed, equation (7) guarantees that the buyer's strategy is optimal. To see that the seller does not have the incentive to deviate, fix some state $(t,\overline{v})$ with $t < T$ and $\overline{v} > 0$ and a history p^t such that $\overline{v}(p^t) = \overline{v}$. Assume that the seller deviates at history p^t to some strategy $\hat{P}$. The continuation payoff from the deviation is

$$\int_0^{\overline{v}} e^{-r_s\,(t^v(p^t;\hat{P},a)-t)}\, (p^v(p^t;\hat{P},a) - c(v))\, F(\mathrm{d}v) \ .$$

By equation (6), the price paid by the v-buyer is at most

$$\left(1 - e^{-r_s\,(T - t^v(p^t;\hat{P},a))}\right) c(\overline{v}) + e^{-r_s\,(T-t^v(p^t;\hat{P},a))}\, p^*(\overline{v}) \ .$$

Hence, the payoff the seller obtains from the deviation is no larger than

$$\int_0^{\overline{v}} e^{-r_s\,(T-t)}\,(p^*(\overline{v})-c(\overline{v}))\,F(\mathrm{d}v)\ .$$

By Corollary 3.1 (which is proven independently of Theorem 3.1), the payoff the seller obtains from the deviation is no larger than $\Pi(t,\overline{v})$.

We proceed by showing that the differential equation (12) (with the initial condition that $\overline{v}_t$ at time 0 is equal to the parameter $\overline{v}_0$), denoted $\overline{v}_t$, is decreasing. To verify this, we apply some algebra to equation (12) and obtain that

$$\dot{\overline{v}}_t=-\frac{r_b\,(\overline{v}_t-c(\overline{v}_t))+e^{-r_s\,(T-t)}\,(r_s-r_b)\,(p^*(\overline{v}_t)-c(\overline{v}_t))}{(1-e^{-r_s\,(T-t)})\,c'(\overline{v}_t)+e^{-r_s\,(T-t)}\,p^{*\prime}(\overline{v}_t)}\ . \tag{23}$$

From Assumption 1 we have that $p^*(\overline{v}_t)$ is increasing. Because $\overline{v}_t>c(\overline{v}_t)$ and $e^{-r_s\,(T-t)}\in(0,1]$, the right-hand side of (23) is non-negative for all t and $\overline{v}_t$ if and only if

$$0<r_b\,(\overline{v}_t-c(\overline{v}_t))+(r_s-r_b)\,(p^*(\overline{v}_t)-c(\overline{v}_t))\ . \tag{24}$$

This is equivalent to Assumption 3.

The fact that Assumption 2 implies Assumption 3 follows from the fact that $p^*(\overline{v}_t)\in(0,\overline{v}_t)$ and that the right-hand side of the expression (24) is linear in $p^*(\overline{v}_t)$, equal to $r_s\,(\overline{v}_t-c(\overline{v}_t))>0$ when $p^*(\overline{v}_t)=\overline{v}_t$, and equal to $r_b\,\left(\overline{v}_t-\frac{r_s}{r_b}\,c(\overline{v}_t)\right)$ (which is positive by Assumption 2) when $p^*(\overline{v}_t)=0$.

The uniqueness of the equilibrium follows from the arguments in Section 3.1, that clarify why the strategy profile described in statement of Theorem 3.1 is the unique candidate to be an equilibrium. □

Proof of Corollary 3.1

Proof. Using the envelope theorem, we have

$$\frac{\mathrm{d}}{\mathrm{d}\overline{v}_0}\Pi^*(\overline{v}_0)=\frac{\mathrm{d}}{\mathrm{d}\overline{v}_0}\left(\int_{p^*(\overline{v}_0)}^{\overline{v}_0}(p^*(\overline{v}_0)-c(v))\,F(\mathrm{d}v)\right)=f(\overline{v}_0)\,(p^*(\overline{v}_0)-c(\overline{v}_0))\ .$$

It is then clear that the statement of the Corollary holds. □

Proof of Corollary 3.2

Proof. That the strategy profile described in Theorem 3.1 is an equilibrium if Assumption 3 holds follows from the proof of Theorem 3.1. The converse follows from the fact that the right-hand (23) is negative for some t and $\overline{v}_t$ whenever Assumption 3 fails, but $\dot{\overline{v}}_t$ cannot be positive in equilibrium. □

A.3 Proofs of results in Section 4

Proof of Corollary 4.1

Proof. The result follows directly from Corollary 3.2 and the observation that, if $c(\overline{v}) > p^*(\overline{v})$ for some $\overline{v}$, then $r_\mathrm{b}\,(\overline{v} - p^*(\overline{v})) < r_\mathrm{s}\,(c(\overline{v}) - p^*(\overline{v}))$ for high enough r_s (hence Assumption 3 fails), while if $c(\overline{v}) \leq p^*(\overline{v})$ for all $\overline{v}$, then $r_\mathrm{b}\,(\overline{v} - p^*(\overline{v})) \geq r_\mathrm{s}\,(c(\overline{v}) - p^*(\overline{v}))$ for high enough r_s (because, $\overline{v} \geq p^*(\overline{v})$ for all $\overline{v}$). □

Proof of Proposition 4.1

Proof. We first note that a change in r_b can be reformulated as a change in the unit used to measure time. To see this, fix some $\lambda > 1$. We use *λ-model* to refer to the model where the discount rate of the buyer is $\lambda\, r_\mathrm{b} > r_\mathrm{b}$, while all other parameters are the same. The model where the discount rate of the seller is $r_\mathrm{s}^\lambda := r_\mathrm{s}/\lambda < r_\mathrm{s}$ and the time horizon is $T^\lambda := \lambda\, T > T$, while all other parameters are the same, is referred to as the *normalized λ-model*. Using $(p_t^\lambda, \overline{v}_t^\lambda)$ to denote the equilibrium outcome of the λ-model, it is easy to see that the normalized λ-model has a unique equilibrium outcome, denoted $(p_t^{*\lambda}, \overline{v}_t^{*\lambda})$, and that this equilibrium outcome satisfies $(p_t^{*\lambda}, \overline{v}_t^{*\lambda}) = (p_{t/\lambda}^\lambda, \overline{v}_{t/\lambda}^\lambda)$. As a result, both the seller and each valuation of the buyer obtain the same payoff in the λ-model and in the normalized λ-model.

Note that the product $r_\mathrm{s}^\lambda\, T^\lambda$ is independent of λ. From equations (6) and (7), we see that both $p_0^{*\lambda}$ and $\dot{p}_0^{*\lambda}$ are independent of λ as well. Furthermore, using equation (12), we have that

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\ddot{p}_0^{*\lambda} = -r_\mathrm{b}\,\frac{\mathrm{d}}{\mathrm{d}\lambda}\dot{\overline{v}}_0^{*\lambda} = -r_\mathrm{b}\,\frac{r_\mathrm{s}^\lambda\,(p^*(\overline{v}_0) - c(\overline{v}_0))}{\lambda^2\,((e^{r_\mathrm{s}^\lambda\, T^\lambda} - 1)\, c'(\overline{v}_0) - p^{*\prime}(\overline{v}_0))} < 0\,,$$

where we the last inequality holds because both c and p^* are increasing and because the no-lemons condition holds. Hence, since $\lambda > 1$, the price decreases faster around $t = 0$ in the normalized λ-model than in our base model. Finally, note the following. Assume that $p_t = p_t^{*\lambda}$ for some time $t \in (0, T)$. In this case, from equation (6), we have that

$$\left(1-e^{-r_{\mathrm{s}}(T-t)}\right) c(\overline{v}_t) + e^{-r_{\mathrm{s}}(T-t)}\, p^*(\overline{v}_t) = \left(1-e^{-r_{\mathrm{s}}^{\lambda}(T^{\lambda}-t)}\right) c(\overline{v}_t^{*\lambda}) + e^{-r_{\mathrm{s}}^{\lambda}(T^{\lambda}-t)}\, p^*(\overline{v}_t^{*\lambda}) \ . \tag{25}$$

Since (i) $e^{-r_{\mathrm{s}}(T-t)} > e^{-r_{\mathrm{s}}^{\lambda}(T^{\lambda}-t)}$, (ii) both c and p^* are increasing, and (iii) the no-lemons condition holds, we have that $\overline{v}_t < \overline{v}_t^{*\lambda}$.[24] Hence, from equation (7), it follows that $\dot{p}_t > \dot{p}_t^{*\lambda}$. Nevertheless, standard analysis of ordinary differential equations implies that p_t and $p_t^{*\lambda}$ can only cross once, and such crossing time is $t=0$.[25] As a result, $p_t > p_t^{*\lambda}$ for all $t \in (0, T]$. Since the buyer's discount rate is the same in the normalized λ-model and in our base model (equal to r_{b}), the buyer is better off in the normalized λ-model independently of his valuation, and therefore he is also better off in the λ-model than in our base model. In other words, the buyer is better off when he is more impatient. □

Proof of Proposition 4.2

Proof. The proof parallels the arguments in the proof of Proposition 4.1. Fix some $\lambda > 1$. We now define the *λ-model* as the model where the discount factor of the seller is $r_{\mathrm{s}}^{\lambda} := r_{\mathrm{s}}/\lambda < r_{\mathrm{s}}$, while the rest of the parameters remain the same. We use $(p_t^{\lambda}, \overline{v}_t^{\lambda})$ to denote the equilibrium outcome of the λ-model. Note that, because the no-lemons condition holds, $p_0 < p_0^{\lambda}$. Assume, by contradiction, there is some $t \in (0, T)$ such that $p_t^{\lambda} = p_t$. Equation (25) now holds with $T^{\lambda} = T$. Since $e^{-r_{\mathrm{s}}(T-t)} < e^{-r_{\mathrm{s}}^{\lambda}(T-t)}$ (note that this inequality is reversed in the proof of Proposition 4.1), an argument analogous to the one in the proof of Proposition 4.1 (see footnote 24) implies that now $\overline{v}_t > \overline{v}_t^{\lambda}$, and hence $\dot{p}_t < \dot{p}_t^{\lambda}$. As in the proof of Proposition 4.1, this leads a contradiction, and so $p_t < p_t^{\lambda}$ for all $t \in [0, T)$. Hence, all prices are higher in the λ-model than in our base model, and therefore the buyer is worse off when the seller is more patient. □

[24] Note that, since $e^{-r_{\mathrm{s}}(T-t)} > e^{-r_{\mathrm{s}}^{\lambda}(T^{\lambda}-t)}$, the weight on the static monopolistic price is larger on the left-hand side of equation (25) than on its right-hand side. Since the static monopolistic price is larger than the cost (because the no-lemons condition holds), and both are increasing functions, the threshold type of the left-hand side should be lower than the threshold type on the right-hand side; that is, $\overline{v}_t < \overline{v}_t^{*\lambda}$.

[25] Intuitively, if $p_t = p_t^{*\lambda}$ for some $t > 0$, we have that $\tilde{p}_t^{*\lambda}$ decreases faster (since $\overline{v}_t < \overline{v}_t^{*\lambda}$), and hence $p_t^{*\lambda}$ crosses p_t "from above." Nevertheless, we also showed that p_t^{λ} is smaller than $p_t^{*\lambda}$ for low values of t. This implies that p_t and p_t^{λ} cross (at most) once, that is, at $t=0$.

Proof of Proposition 4.3

Proof. The proof parallels the arguments in the proof of Propositions 4.1 and 4.2. Fix some $\lambda > 1$. We now define the *λ-model* as the model where the discount factor of the seller is $T^{\lambda} := \lambda\, T > T$, while the rest of the parameters remain the same. We use $(p_t^{\lambda}, \overline{v}_t^{\lambda})$ to denote the equilibrium outcome of the λ-model. Note that, because the no-lemons condition holds, $p_0 > p_0^{\lambda}$. Assume, by contradiction, there is some $t \in (0, T)$ such that $p_t^{\lambda} = p_t$. Equation (25) now holds with $r_s^{\lambda} = r_s$. Since $e^{-r_s\,(T-t)} > e^{-r_s\,(T^{\lambda}-t)}$ (note that this inequality is now the same as in the proof of Proposition 4.1), an argument analogous to the one in the proof of Proposition 4.1 (see Footnote 24) implies that $\overline{v}_t < \overline{v}_t^{\lambda}$, and hence $\dot{p}_t > \dot{p}_t^{\lambda}$. As in the proof of Proposition 4.1, this leads a contradiction, and hence $p_t > p_t^{\lambda}$ for all $t \in [0, T)$. Hence, all prices are lower in the λ-model than in our base model, and therefore the buyer is better off when the time horizon is longer. $\square$

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