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Dynamic Screening with Verifiable Bankruptcy

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Abstract

We consider a dynamic screening model, where the agent may go bankrupt due to, for example, cash constraints. We model bankruptcy as a verifiable event that occurs whenever the agent makes a per period loss. This leads to less stringent truth-telling constraints than those considered in the existing literature. We show that, for serially independent types, the weaker constraints do not affect optimal contracting, however. Moreover, we develop a novel method to study private values settings with continuous types and show that a regularity condition that has analogues in the literature on multi-dimensional screening ensures that the optimal contract is deterministic.

Keywords: Dynamic Screening, Bankruptcy, Verifiability, Mean Preserving Spread

JEL: D82, H57

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1 Introduction

A recent literature studies bankruptcy constraints in dynamic screening models where a procurer (the principal) procures goods or services over multiple periods from a supplier (the agent) whose costs evolve dynamically over time and are the supplier's private information (e.g. Krishna et al., 2010, Mirrokni et al., 2020, Krasikov and Lamba, 2021, Ashlagi et al., 2022). These bankruptcy constraints capture the fact that, in practice, suppliers are frequently unable to sustain short-term losses during the relationship, for example due to cash or credit constraints. The classical literature on dynamic screening/mechanism design (e.g., Baron and Besanko, 1984, Battaglini, 2005, Pavan et al., 2014, Esö and Szentes, 2017) neglects such concerns, effectively assuming that agents can sustain arbitrary losses after having accepted a contract.

We make two contributions to this literature. First, we explicitly impose cash constraints both on and off the equilibrium path. We do so by providing a more complete micro foundation of the contractual feasibility constraints, modeling bankruptcy as a *verifiable* event that occurs whenever the agent obtains a negative per-period utility.[1] In particular, bankruptcy may occur also off the equilibrium path. This differs from the approach of the existing literature, which focusses on direct revelation mechanisms and imposes cash constraints on but not off the equilibrium path.[2] As we show, our micro-foundation implies that an optimal contract has to satisfy only uni-directional incentive constraints that only prevent an agent from overstating her costs. The reason is that understating one's cost results in bankruptcy, thus revealing a lie. Even though our uni-directional incentive constraints are weaker than the feasibility constraints posited by the existing literature, a key insight of our paper is that optimal contracts do not, however, exploit the additional slack thus gained. In this sense, our approach validates the literature's approach to impose cash constraints only on but not off the equilibrium path.[3]

Second, we extend the existing literature's analysis of bankruptcy with two agent types and

[1]The verifiability of bankruptcy reflects existing institutional rules, since bankruptcy is a legal process formally verified and declared by the court system. For instance, bankruptcy is enshrined by the US Constitution in Article 1, Section 8, Clause 4, and in the case of business debtors further specified in its Bankruptcy Code under Chapter 11. In the UK, the first statute of law dealing with bankruptcy is the Statute of Bankrupts dating back to 1542.

[2]For instance, the optimal contracts as derived in Krishna et al. (2010) and Krasikov and Lamba (2021) impose losses on the agent when she deviates from truth-telling and thus violate cash constraints off the path. However, since the agent cannot be held liable for losses, some agent types could then do strictly better by selecting a contract that violates their cash constraints off path.

[3]Because our micro foundation implies that an optimal contract has to satisfy only uni-directional incentive constraints, our study of dynamic setting with bankruptcy constraints links to the literature that considers static settings in which such uni-directional incentive constraints exist for exogenous reasons (e.g., Moore, 1984, and Celik, 2006, Krähmer and Strausz, 2024). In line with our finding, this literature shows that, in static settings, these weaker incentive constraints do not give rise to different predictions in settings with private values or, more generally, when the aggregate surplus is monotone in the allocation.

private values to settings with continuous types. This extension is not straightforward, because with cash constraints the principal's ex ante payoff is a non-linear and non-monotone function of the agent's (future) information rents. Hence, contrary to dynamic screening without cash constraints, the problem cannot be reduced to maximizing a virtual surplus representation where allocations are additively separable by type, and is consequently difficult to solve when there are more than two types. To analyze the problem with more than two types, we therefore develop a novel solution method to identify an optimal contract.

The basic idea behind this method is based on the observation that every dynamic contract induces a continuation value for the agent which, from the principal's perspective, is a random variable, as it depends on the agent's privately known type. A standard argument from dynamic programming implies that the principal's continuation profit is concave in the agent's continuation value. This observation allows us to rank contracts in terms of second order stochastic dominance of the induced continuation value. As a result, we can identify an optimal contract as a contract that, among the set of feasible contracts, displays minimal dispersion in the second order sense. We show that, under a regularity condition, an optimal contract has a simple, deterministic cutoff structure where cost types below a cutoff produce the good and types above the cutoff do not. The regularity condition differs from the more familiar monotone virtual surplus kind of conditions, and also appears in the (static) multi-dimensional screening literature (e.g. Manelli and Vincent (2006)). The connection is that, as in this literature, we write the principal's optimization problem in terms of the agent's (continuation) value rather than the allocation rule.

Finally, we point out that cash constraints conceptually differ from withdrawal rights in the context of sequential screening problems with a single trading period (e.g. Krähmer and Strausz, 2015, Bergemann et al., 2020). This is so because with a withdrawal right the agent can voluntarily decide whether to sustain a loss ex post or not. Indeed, in contrast to verifiable bankruptcy, voluntary participation leads, if anything, to stricter incentive constraints, as it weakly increases an agent's utility from a misreport.[4]

2 The model

A principal (the buyer, she) and an agent (the seller, he) interact over two periods $\tau = 1, 2$.[5] In each period, the principal seeks to procure one good from the agent. In period τ, the terms

[4]On this point see also Compte and Jehiel (2009) who show that in mechanism design settings with ex post veto rights the potential to punish agents for misreports is limited because they can quit the mechanism ex post.

[5]At the end of Section 4, we show that our analysis and results extend to a setting with infinitely many periods.

of trade are the probability of trade x_τ and a transfer t_τ from the principal to the agent.[6] The principal's valuation for the good is v_τ, and the agent's cost to produce the good is θ_τ. While v_τ is commonly known, θ_τ, the agent's cost type in period τ, is privately known to the agent in period τ, and it is commonly known that θ_τ is distributed with cdf F_τ with support $\Theta_\tau \equiv [\underline{\theta}_\tau, \bar{\theta}_\tau]$ and differentiable pdf f_τ. We assume that θ_1 and θ_2 are stochastically independent. Moreover, we assume that production is only efficient when costs are low enough, i.e. $v_\tau \in (\underline{\theta}_\tau, \bar{\theta}_\tau)$.

The parties have time-separable quasi-linear utilities. That is, under the terms of trade x_τ, t_τ the principal's utility in period τ is $v_\tau x_\tau - t_\tau$, and the agent's utility is $t_\tau - \theta_\tau x_\tau$. A party's overall utility is the sum over the per-period utilities.

The novelty of our paper is to consider a situation in which the event that the agent makes a loss is verifiable. That is, if the agent were to make losses in period τ under the prevailing terms of trade, the agent would go bankrupt and this is publicly verifiable. In this case, both parties receive their reservation utility of zero.

The timing is as follows:

1. At the outset, the principal commits to a long-term contract which specifies the terms of trade over the two periods. If the agent rejects the contract, both parties receive their reservation utility of 0 and the game ends.

2. If the agent accepts, then in period 1, he privately learns θ_1. If $t_1 - \theta_1 x_1 \geq 0$, the terms of trade x_1, t_1 are implemented. If $t_1 - \theta_1 x_1 < 0$, bankruptcy occurs and both parties receive 0.[7,8]

3. In period 2, the agent privately learns θ_2. If $t_2 - \theta_2 x_2 \geq 0$, the terms of trade x_2, t_2 are implemented. If $t_2 - \theta_2 x_2 < 0$, bankruptcy occurs and both parties receive 0.

Example: To illustrate our analysis, we use the uniform example, where θ_1 and θ_2 are both uniformly distributed over the interval $[0, 1]$, and $v_1 = v_2 = \bar{\theta} = 1$. For this example, trade is efficient for all types and the per-period first-best surplus equals $S^{FB} = S_1^{FB} = S_2^{FB} = \int_0^1 1 - \theta \; d\theta = 1/2$, yielding an aggregate surplus of $S_1^{FB} + S_2^{FB} = 1$. In the static second best, the optimal mechanism

[6] As is standard, we interpret t_τ as the expected payment $t_\tau^{(0)}(1-x_\tau)+t_\tau^{(1)}x_\tau$, where $t^{(0)}$ (resp. $t^{(1)}$) is the payment when trade does not (resp. does) occur. Alternatively, for a divisible good, we may interpret x_τ as the share of the good traded.

[7] Related to footnote 6, if $x_1 \in (0, 1)$, bankruptcy occurs if $t^{(0)} < 0$ and the mechanism does not prescribe trade, and if $t^{(1)} - \theta_1 < 0$ and the mechanism does prescribe trade.

[8] Thus, we abstract from the possibility that there is any salvage value of a project that goes bankrupt and that there are any negative externalities of bankruptcy on third parties, such as workers or subcontractors whose labor or bills remain unpaid. These assumptions are without loss, as bankruptcy will not occur in equilibrium.

is a posted price of $1/2$, yielding the principal a per-period profit of $\Pi^{SB} \equiv (1-1/2)*1/2 = 1/4$ and the agent a per-period second best utility of $U^{SB} \equiv \int_0^{1/2} 1/2 - \theta \; d\theta = 1/8$. Implementing a posted price of $1/2$ for each of the two periods, yields an overall profit of $2\Pi^{SB} = 1/2$ to the principal and an overall utility of $2U^{SB} = 1/4$ to the agent, resulting in aggregate surplus of $3/4$.

In the benchmark case in which there is an interim participation constraint in period 1 but no bankruptcy constraint, the optimal mechanism implements a posted price of $p = 1/2$ for the first period, and extracts the whole surplus in the second period. This yields an overall profit of $\Pi^{SB} + S^{FB} = 3/4$ to the principal, an overall utility of $U^{SB} = 1/8$ to the agent, resulting in aggregate surplus of $7/8$. □

3 The principal's problem

The principal's objective is to design a contract to maximize her profits. Because the principal has full commitment, the revelation principle applies, implying that an optimal contract is in the class of direct mechanisms where, on the equilibrium path, the agent reports his type truthfully in every period (Myerson, 1986). Moreover, because bankruptcy in period 1 is verifiable, a mechanism can condition the terms of trade in period 2 on whether bankruptcy has occurred in period 1 or not. Without loss, we can therefore restrict attention to contracts of the form $(x_1, t_1, x_2^N, t_2^N, x_2^B, t_2^B)$, where

$$(x_1, t_1) : [\underline{\theta}_1, \bar{\theta}_1] \to [0,1] \times \mathbb{R}, \qquad (x_2^b, t_2^b) : [\underline{\theta}_1, \bar{\theta}_1] \times [\underline{\theta}_2, \bar{\theta}_2] \to [0,1] \times \mathbb{R}, \tag{1}$$

where $b \in \{B, N\}$ indicates whether bankruptcy has ($b = B$) or has not ($b = N$) occurred in period 1.

To state the incentive compatibility constraints, we denote for $b \in \{N, B\}$ the agent's expected period 2 utility from a report $\hat{\theta}_1$, conditional on truthfully reporting in period 2, by

$$U^b(\hat{\theta}_1) = \int_{\underline{\theta}_2}^{\bar{\theta}_2} \max\{0, t_2^b(\hat{\theta}_1, \theta_2) - \theta_2 x_2^b(\hat{\theta}_1, \theta_2)\} \, dF_2(\theta_2). \tag{2}$$

Moreover, let

$$\Theta_1^N = \{\theta_1 \mid t_1(\theta_1) - \theta_1 x_1(\theta_1) \geq 0\} \tag{3}$$

be the set of period 1 types where bankruptcy does not occur in period 1 under a given mechanism.

Definition 1 *A contract $(x_1, t_1, x_2^N, t_2^N, x_2^B, t_2^B)$ is feasible if:*
(i) It is incentive compatible in period 2, that is, for $b \in \{N, B\}$:[9]

$$\max\{0, t_2^b(\theta_1,\theta_2) - \theta_2 x_2^b(\theta_1,\theta_2)\} \geq \max\{0, t_2^b(\theta_1,\hat{\theta}_2) - \theta_2 x_2^b(\theta_1,\hat{\theta}_2)\} \quad \forall \theta_1, \theta_2, \hat{\theta}_2. \tag{4}$$

(ii) It is incentive compatible in period 1, that is:

◇ *For all $\theta_1 \in \Theta_1^N$, we have:*

$$\begin{aligned} t_1(\theta_1) - \theta_1 x_1(\theta_1) + U^N(\theta_1) &\geq t_1(\hat{\theta}_1) - \theta_1 x_1(\hat{\theta}_1) + U^N(\hat{\theta}_1) \quad \forall \hat{\theta}_1 : t_1(\hat{\theta}_1) - \theta_1 x_1(\hat{\theta}_1) \geq 0, && (5)\\ t_1(\theta_1) - \theta_1 x_1(\theta_1) + U^N(\theta_1) &\geq U^B(\hat{\theta}_1) \quad \forall \hat{\theta}_1 : t_1(\hat{\theta}_1) - \theta_1 x_1(\hat{\theta}_1) < 0, && (6) \end{aligned}$$

◇ *For all $\theta_1 \notin \Theta_1^N$, we have:*

$$\begin{aligned} U^B(\theta_1) &\geq t_1(\hat{\theta}_1) - \theta_1 x_1(\hat{\theta}_1) + U^N(\hat{\theta}_1) \quad \forall \hat{\theta}_1 : t_1(\hat{\theta}_1) - \theta_1 x_1(\hat{\theta}_1) \geq 0, && (7)\\ U^B(\theta_1) &\geq U^B(\hat{\theta}_1) \quad \forall \hat{\theta}_1 : t_1(\hat{\theta}_1) - \theta_1 x_1(\hat{\theta}_1) < 0. && (8) \end{aligned}$$

Part (i) of the definition captures the truth-telling constraints for the agent in period 2, taking into account that bankruptcy in period 2 occurs whenever the terms of trade would impose a loss on the agent. Similarly, part (ii) describes the truth-telling constraints for the agent in period 1. This requires a distinction between four cases, depending on whether truth-telling does or does not induce bankruptcy in period 1 and whether deviating from truth-telling does or does not induce bankruptcy in period 1.

The principal's problem is thus to select a feasible contract that maximizes her profits

$$\int_{\Theta_1^N} \left[v_1 x_1(\theta_1) - t_1(\theta_1) + \int_{\Theta_2^{N,N}(\theta_1)} v_2 x_2^N(\theta_1,\theta_2) - t_2^N(\theta_1,\theta_2)\, dF_2(\theta_2) \right] dF_1(\theta_1) \tag{9}$$

$$+ \int_{\Theta_1 \setminus \Theta_1^N} \left[0 + \int_{\Theta_2^{N,B}(\theta_1)} v_2 x_2^B(\theta_1,\theta_2) - t_2^B(\theta_1,\theta_2)\, dF_2(\theta_2) \right] dF_1(\theta_1), \tag{10}$$

[9]The revelation principle for dynamic games requires truthful reporting in period 2 only after a truthful report in period 1 (see Myerson, 1986). In our context, where types are independent, the support of period 2 types is "non-shifting", that is, is independent of the period 1 type. It then follows with standard arguments that if truth-telling in period 2 is optimal for the agent after telling the truth in period 1, then it is so after any report in period 1.

where

$$\Theta_2^{N,b}(\theta_1) \equiv \{\theta_2 \in \Theta_2 \mid t_2^b(\theta_1,\theta_2) - \theta_2 x_2^b(\theta_1,\theta_2) \geq 0\} \tag{11}$$

denotes the set of period 2 types θ_2 for whom bankruptcy does not occur in period 2 given the period 1 type θ_1 and given that bankruptcy has not ($b = N$) or has ($b = B$) occurred in period 1.

To solve the principal's problem, we first show that it is without loss to focus on contracts under which bankruptcy never occurs on the equilibrium path where the agent tells the truth.

The intuition is simply that the bankruptcy outcome is equivalent to not trading the good ($x = 0$) and making no payments ($t = 0$). Thus, the outcome of a mechanism γ in which bankruptcy occurs on the equilibrium path can be replicated by the mechanism which differs from γ only in that it specifies no trade and zero payments (and hence no bankruptcy) in case bankruptcy would occur under γ.

Second, it is without loss to focus on mechanisms in which the agent exactly breaks even in the first period, and backloads any potential profit for the agent in that it accrues only in the second period.[10] The reason is that if the agent were to make a profit in the first period, the principal could deduct it from the agent's first period 1 payments and pay it out in period 2 instead. This would not affect the principal's profit and would maintain truth-telling incentives for which only total payments matter.

We summarize these considerations in the next lemma.

Lemma 1 *For any feasible contract there is a payoff-equivalent feasible contract* $(x_1, t_1, x_2^N, t_2^N, x_2^B, t_2^B)$ *with the following properties:*

- *The agent exactly breaks even, and there is no bankruptcy in period 1 (on path):*

$$t_1(\theta_1) - \theta_1 x_1(\theta_1) = 0 \quad \textit{for all } \theta_1. \tag{12}$$

- *There is no bankruptcy in period 2 (on path):*

$$t_2^N(\theta_1,\theta_2) - \theta_2 x_2^N(\theta_1,\theta_2) \geq 0 \quad \textit{for all } \theta_1, \theta_2. \tag{13}$$

- *After the off-path event that there is bankruptcy in period 1, the relationship is terminated:*

$$x_2^B(\theta_1,\theta_2) = t_2^B(\theta_1,\theta_2) = 0 \quad \textit{for all } \theta_1, \theta_2. \tag{14}$$

[10]This argument also appears in Ashlagi et al. (2022).

Lemma 1 implies that we can find an optimal contract in the class of feasible contracts that satisfy (12)-(14). Since properties (12) and (14) pin down t_1, x_2^B, and t_2^B, we are actually left to determine only the triple (x_1, x_2^N, t_2^N). We therefore introduce the following definition.

Definition 2 *A triple* $(x_1, x_2, t_2) : [\underline{\theta}_1, \bar{\theta}_1] \to [0,1]^2 \times \mathbb{R}$ *is called a backloaded contract if*

$$NB_2 : \qquad t_2(\theta_1, \theta_2) - \theta_2 x_2(\theta_1, \theta_2) \geq 0 \quad \forall \theta_1, \theta_2. \tag{15}$$

A backloaded contract uniquely induces a contract $(x_1, t_1, x_2^N, t_2^N, x_2^B, t_2^B)$ with the properties (12)-(14) by setting $t_1 = \theta_1 x_1$, $x_2^N = x_2$, $t_2^N = t_2$, and $x_2^B = t_2^B = 0$. For a backloaded contract, we write

$$U(\theta_1) = \int_{\underline{\theta}_2}^{\bar{\theta}_2} t_2(\theta_1, \theta_2) - \theta_2 x_2(\theta_1, \theta_2) \, dF_2(\theta_2) \tag{16}$$

for the agent's expected period 2 utility. The next lemma characterizes when a backloaded contract is feasible.

Lemma 2 *A backloaded contract* (x_1, x_2, t_2) *induces a feasible contract* $(x_1, t_1, x_2^N, t_2^N, x_2^B, t_2^B)$ *if and only if*

$$IC_2 : \qquad t_2(\theta_1, \theta_2) - \theta_2 x_2(\theta_1, \theta_2) \geq t_2(\theta_1, \hat{\theta}_2) - \theta_2 x_2(\theta_1, \hat{\theta}_2) \quad \forall \theta_1, \theta_2, \hat{\theta}_2 \tag{17}$$

$$IC_1 : \qquad U(\theta_1) \geq (\hat{\theta}_1 - \theta_1) x_1(\hat{\theta}_1) + U(\hat{\theta}_1) \quad \forall \theta_1 < \hat{\theta}_1 \tag{18}$$

$$IC_1^0 : \qquad U(\theta_1) \geq U(\hat{\theta}_1) \quad \forall \hat{\theta}_1 \in \Theta_1^0, \forall \theta_1 \in \Theta_1. \tag{19}$$

Constraint IC_2 corresponds to the period 2 truth-telling constraints (4). The more interesting constraint is IC_1 which corresponds to the period 1 truth-telling constraints.[11]

The novelty is that IC_1 only requires that the agent does not report a higher type than his true type, but not that he does not report a lower type. The reason for this asymmetry is that if the agent reported lower costs in the first period, then, because any cost type breaks even in period 1, the agent would go bankrupt after such a lie. But, bankruptcy does not occur on path and is verifiable. Consequently, such a lie would be detected, and under a backloaded mechanism, the relationship would be terminated. This prospect is enough to dissuade the agent from understating his costs, and extra incentives are not needed to induce truth-telling.

[11] To see that IC_1 replaces the period 1 truth-telling constraints (5)-(8) note that because under a backloaded contract there is no bankruptcy in period 1 for any θ_1, constraints (6), (7) and (8) are all redundant, and the only relevant constraint is (5), which now has to hold for all θ_1, leading to IC_1.

A subtlety is however that bankruptcy can only occur when some production takes place. As a result, the previous reasoning applies only to types $\hat{\theta}_1$, who actually produce in period 1. The constraint IC_1^0 takes care of this by explicitly requiring that for types $\hat{\theta}_1$ with $x_1(\hat{\theta}_1) = 0$, the truthtelling constraints have to hold in both directions, as the verifiability of bankruptcy has no bite in this case.

We can now re-state the principal's problem as selecting an optimal backloaded contract. Under a backloaded contract, the principal's period 1 profit equals $v_1 x_1(\theta_1) - \theta_1 x_1(\theta_1)$ for all θ_1, because the agent breaks even in period 1. Moreover, her profit in period 2 is equal to $v_2 x_2(\theta_1, \theta_2) - t_2(\theta_1, \theta_2)$ for all (θ_1, θ_2), because no bankruptcy occurs in period 2. Thus, the principal's problem is

$$P: \qquad \sup_{x_1, x_2, t_2} \int_{\underline{\theta}_1}^{\bar{\theta}_1} \int_{\underline{\theta}_2}^{\bar{\theta}_2} v_1 x_1(\theta_1) - \theta_1 x_1(\theta_1) + v_2 x_2(\theta_1, \theta_2) - t_2(\theta_1, \theta_2)\, dF_2(\theta_2)\, dF_1(\theta_1)$$
$$s.t. \quad IC_2, NB_2, IC_1, IC_1^0$$

where IC_2, IC_1, and IC_1^0 are the feasibility constraints, and NB_2 ensures that the contract is a backloaded contract. Note that the constraint IC_1^0 leads to the technical complication that the feasibility set is not closed.[12]. For this reason, the principal's objective does not necessarily take on a maximum. We will address this issue explicitly.

Intuitively, for given θ_1, the principal faces the standard (intra-temporal) rent-efficiency trade-off in period 2, because she has to grant low cost types an information rent for truth-telling due to the presence of the no-bankruptcy constraint NB_2. Moreover, because a backloaded contract uses the *expected* period 2 information rent for incentivizing low cost types in period 1 to reveal the truth, the principal also faces an inter-temporal trade-off between maximizing profits in period 1 and 2.

Before solving problem P, we note that the no-bankruptcy constraint NB_2 is formally equivalent to a period 2 participation constraint. Hence, problem P corresponds to a two-period dynamic mechanism design problem with ex post participation constraints, but with the novelty that the first period incentive constraints IC_1 are asymmetric in that they only require higher types not to

[12]To see this, let $\Theta_1 = [0,1]$ and consider sequence of contracts with (x_1^n, U_1^n) given by

$$x_1^n(\theta_1) = \begin{cases} 1/n & if \quad \theta_1 \in [0, 1/2) \\ 1 & if \quad \theta_1 \in [1/2, 1] \end{cases}, \qquad U_1^n(\theta_1) = 1 - \theta_1. \tag{20}$$

Note that for every n, $\Theta_1^0 = \emptyset$ so that IC_1^0 is redundant, and it is easy to check that IC_1 is satisfied. Thus, every element in the sequence is feasible. However, the limit contract, as $n \to \infty$, is not feasible, because in the limit, $\Theta_1^0 = [0, 1/2)$ and so IC_1^0 is violated for any pair $(\theta_1, \hat{\theta}_1)$ with $\hat{\theta}_1 < \max\{\theta_1, 1/2\}$

mimic lower types, and the constraints IC_1^0 only apply to reports that imply no production.

4 Solution to the principal's problem

We solve the principal's problem by the well-known technique in dynamic programming to reduce the dynamic problem P to a sequence of static problems (Spear and Srivastava, 1987, Thomas and Worrall, 1990). Consequently, we proceed in two steps. In the first step, we solve for optimal period 2 terms of trade (x_2^U, t_2^U) that promise the agent a certain exogenously given expected period 2 utility U. In the second step, we then solve for an optimal period 1 allocation x_1 and an optimal continuation value U taking as given optimal period 2 terms of trade (x_2^U, t_2^U) from step 1 that supply the agent with U.

Step 1: Optimal period 2 terms of trade

Note first that the principal can promise any positive utility $U \geq 0$. Indeed, by NB_2, the principal cannot promise a negative utility while she can offer any utility $U \geq 0$ by, for example, offering the terms of trade $(x_2, t_2) = (0, U)$. Thus, the set of feasible promised utilities is $\{U \mid U \geq 0\}$.

For a given report θ_1, the principal's problem to optimally promise $U \geq 0$ is

$$P_2: \quad \Pi(U) \equiv \max_{x_2, t_2} \quad \int_{\underline{\theta}_2}^{\bar{\theta}_2} v_2 x_2(\theta_1, \theta_2) - t_2(\theta_1, \theta_2)\, dF_2(\theta_2) \quad s.t \tag{21}$$

$$IC_2: \quad t_2(\theta_1, \theta_2) - \theta_2 x_2(\theta_1, \theta_2) \geq t_2(\theta_1, \hat{\theta}_2) - \theta_2 x_2(\theta_1, \hat{\theta}_2) \quad \forall \theta_2, \hat{\theta}_2 \tag{22}$$

$$NB_2: \quad t_2(\theta_1, \theta_2) - \theta_2 x_2(\theta_1, \theta_2) \geq 0 \quad \forall \theta_2 \tag{23}$$

$$PK: \quad \int_{\underline{\theta}_2}^{\bar{\theta}_2} t_2(\theta_1, \theta_2) - \theta_2 x_2(\theta_1, \theta_2)\, dF_2(\theta_2) = U. \tag{24}$$

While the constraints IC_2 and NB_2 carry over from problem P, the constraint PK ensures that the agent receives his promised utility U.

Problem P_2 corresponds to a static monopoly problem where the agent has an interim outside option of 0 after learning θ_2 (as reflected by NB_2), and an ex-ante outside option of U before learning θ_2 (as reflected by PK). The solution is well-known from Samuelson (1984); for details see our Remark 1 below. We state the features that will be key for our purposes in the next lemma. In order to avoid uninteresting case distinctions, we impose the following mild condition.[13]

[13]It is well-known (e.g. Riley and Zeckhauser, 1983) that in the absence of PK, the solution to P_2 can be imple-

Assumption 1: The second best solution (x_2^{SB}, t_2^{SB}) to the relaxed version of P_2 where PK is missing is unique.

Given Assumption 1, the utilities in the second best solution for both the principal and the agent are unique and we denote them, respectively, by Π_2^{SB} and U_2^{SB}. Moreover, we denote the surplus associated with the period 2 first-best allocation $x_2^{FB}(\theta_1, \theta_2) \equiv \mathbf{1}_{[\underline{\theta}_2, \min\{v_2, \bar{\theta}_2\}]}(\theta_2)$ by[14]

$$S_2^{FB} = \int_{\underline{\theta}_2}^{\min\{v_2, \bar{\theta}_2\}} v_2 - \theta_2 \, dF_2. \tag{26}$$

Clearly, $U_2^{SB} \in (0, S_2^{FB})$.

Lemma 3 *The value of problem P_2 as a function of U, $\Pi(U)$, is concave in U with* $\Pi(0) = \Pi(S_2^{FB}) = 0$ *and attains a unique maximum* Π_2^{SB} *at* U_2^{SB}*, that is,* $\Pi(U_2^{SB}) = \Pi_2^{SB}$.

The concavity of the value follows from a standard mixing argument. Specifically, given two promises U' and U'', the principal can promise the agent the convex mixture $\bar{U} = \alpha U' + (1-\alpha)U''$ by appropriately randomizing between the optimal terms of trade for U' and U''. This would yield the principal a profit $\alpha\Pi(U_2') + (1-\alpha)\Pi(U_2'')$, but by re-optimizing, the principal can promise $\bar{U}$ at a higher profit, implying that Π is concave.

To see why $\Pi(0) = 0$, note that IC_2 and NB_2 imply that the only way to provide the agent with expected utility $U = 0$ is through no trade ($x_2 = t_2 = 0$), resulting in zero profits for the principal. To see that $\Pi(S_2^{FB}) = 0$, observe that the principal's profit Π is the total surplus generated minus the utility U supplied to the agent. Hence, if the principal promises the entire first-best surplus to the agent, $U = S_2^{FB}$, she cannot make a strictly positive profit. But she can at least guarantee herself zero profits by selecting terms of trade that generate the first-best surplus. Therefore, $\Pi(S_2^{FB}) = 0$. Finally, that the principal's profit is uniquely maximized at U^{SB} follows directly from the definition of the second best and Assumption 1.

Remark 1 (Period 2 implementation) We briefly discuss how the period 2 contract can be implemented. If the principal promises more than the second best surplus, $U \geq S_2^{FB}$, then it follows from

mented by a posted price p given by

$$p \in \arg\max_{\tilde{p}} \int_{\underline{\theta}_2}^{\tilde{p}} v_2 - \theta_2 - \frac{F_2(\theta_2)}{f_2(\theta_2)} \, dF_2(\theta_2). \tag{25}$$

Notice that the right hand side is generically a singleton in the sense that whenever it is not a singleton, a slight perturbation of $\frac{F_2(\theta_2)}{f_2(\theta_2)}$ would remove all but one solution. Hence, Assumption 1 is mild in that it holds generically.

[14]Given a set A, the indicator function $\mathbf{1}_A(a)$ is 1 if $a \in A$ and 0 otherwise.

the proof of Lemma 3 that the optimal contract displays the first-best allocation $x_2^{FB}(\theta_1, \theta_2)$. The intuition is that when guaranteeing the agent a utility exceeding the first-best level S_2^{FB}, the principal does not face the standard rent-efficiency trade-off anymore. Now, if the principal promises exactly $U = S_2^{FB}$, this can be indirectly implemented by an offer from the principal to procure the good at a price of v_2. If the principal promises strictly more, $U > S_2^{FB}$, then an optimal contract can be indirectly implemented by a two-part tariff. In particular, the principal makes an unconditional payment $U - S_2^{FB}$ and offers to procure the good at a price of v_2. This two-part tariff yields her an expected profit $\Pi(U) = S_2^{FB} - U$.

If, on the other hand, $U < S_2^{FB}$, it follows from Samuelson (1984) that an optimal trading probability is of the form

$$x_2(\theta_1, \theta_2) = \begin{cases} 1 & if \quad \theta_2 \in [0, \theta_2'] \\ \xi & if \quad \theta_2 \in (\theta_2', \theta_2''] \\ 0 & else \end{cases} \tag{27}$$

for some $\xi \in [0, 1]$, $\underline{\theta} \leq \theta_2' \leq \theta_2'' < v_2$ which all depend on U. If $\xi = 0$, the optimal contract can be implemented by an offer from the principal to procure the good at price θ_2'. If $\xi > 0$, the optimal contract can be implemented by a menu of three options for the agent: to not produce the good at a price of 0; to produce a "fraction" ξ of the good for a price of $\xi\theta_2'$,[15] or to produce the good at price θ_2''.

Whether ξ is strictly positive or not, depends on the distribution F_2 and on the size of U. For the special case that the hazard rate F_2/f_2 is increasing, we have that $\xi = 0$ for all U so that the optimal contract can be implemented with a posted price.

Example: For our uniform example, the hazard rate $F_2(\theta_2)/f_2(\theta_2) = \theta_2$ is increasing so that, as noted in Remark 1, a posted price in period 2 is optimal. In particular, , for $U \in [0, S_2^{FB}] = [0, 1/2]$, the optimal contract for $U \in [0, S_2^{FB}] = [0, 1/2]$ corresponds to a posted price p_2 which maximizes $p_2(v - p_2)$ subject to the promise keeping constraint $\int_0^{p_2} p_2 - \theta \, d\theta = U$. This constraint simplifies to $p_2^2/2 = U$ and therefore pins down $p_2(U) = \sqrt{2U}$. The resulting profit is $\Pi(U) = p_2(v - p_2) = \sqrt{2U}(1 - \sqrt{2U}) = \sqrt{2U} - 2U$. Moreover, for $U > 1/2$, we have $\Pi(U) = 1/2 - U$. Taken together,

[15] For an indivisible good the contract randomizes between trade and no trade and the agent is payed θ_2' if trade is the outcome.

we thus have:

$$\Pi(U)=\begin{cases}\sqrt{2U}-2U & if \quad U\leq 1/2;\\ 1/2-U & if \quad U>1/2.\end{cases} \tag{28}$$

Note that $\Pi(U)$ is not only continuous but also differentiable at $U=1/2$.

Step 2: Optimal period 1 terms of trade

Step 1 allow us to re-write the principal's problem P as a static maximization problem over the period 1 terms of trade x_1 and the agent's promised utility U. More specifically, any combination $(x_1(\theta_1),U(\theta_1))$ with $U(\theta_1)\geq 0$ corresponds to a backloaded contract $(x_1(\theta_1),x_2(\theta_1,\cdot),t_2(\theta_1,\cdot))$ where $(x_2(\theta_1,\cdot),t_2(\theta_1,\cdot))$ is the solution to P_2 with $U=U(\theta_1)$. We refer to (x_1,U) as a *reduced* backloaded contract (and simply as backloaded contract if there is no risk of confusion). Clearly, only contracts corresponding to reduced backloaded contracts can be optimal.

Recall that under a backloaded contract, the agent breaks even in period 1, that is, $t_1(\theta_1)=\theta_1 x_1(\theta_1)$. Consequently, the principal receives the profit $v_1x_1(\theta_1)-\theta_1x_1(\theta_1)$ in period 1 and the profit $\Pi(U(\theta_1))$ in period 2. Suppressing the time index for period 1, we can therefore rewrite the principal's problem as

$$P_1: \quad \sup_{x,U} \quad \int_{\underline{\theta}}^{\bar{\theta}} [v-\theta]x(\theta)+\Pi(U(\theta))\,dF(\theta) \quad s.t \tag{29}$$

$$IR: \quad U(\theta)\geq 0 \quad \forall\theta \tag{30}$$

$$IC: \quad U(\theta)\geq(\hat{\theta}-\theta)x(\hat{\theta})+U(\hat{\theta}) \quad \forall\theta<\hat{\theta} \tag{31}$$

$$IC^0: \quad U(\theta)\geq U(\hat{\theta}) \quad \forall\hat{\theta}\in\Theta^0,\forall\theta\in\Theta \tag{32}$$

$$UG: \quad x(\theta)\in[0,1] \quad \forall\theta. \tag{33}$$

The constraint IR is simply the feasibility constraint that the agent's expected utility cannot be negative, and the constraints IC and IC^0 are inherited from the original formulation of P. To deal with the problem that the feasibility set is not closed, we solve a relaxed version where we drop IC^0 leading to the following problem:

$$P_1': \qquad \max_{x,U} \quad \int_{\underline{\theta}}^{\bar{\theta}} [v-\theta]x(\theta)+\Pi(U(\theta))\,dF(\theta) \quad \text{s.t} \quad IR, IC, UG. \tag{34}$$

Problem P_1' looks similar to a standard monopoly problem where $U(\theta)$ is agent type θ's information rent, and constraint IR corresponds to a standard (interim) participation constraint. There are, however, two important differences.

First, unlike in the static monopoly problem with transferable utility, the principal's objective is not linear in the agent's information rent. This is due to the period 2 bankruptcy constraint which results in a rent-efficiency trade-off in period 2. In fact, if the principal did not face a bankruptcy constraint in period 2, her objective would be linear in the information rent because she would then optimally implement the first-best allocation in period 2 and could award the agent any (possibly negative) level of rent through an appropriate transfer.

To shed more light on the principal's costs of providing incentives, recall that Π is single-peaked with a maximum at U^{SB}. Therefore, if the period 1 type were publicly known, the principal would maximize the objective by picking an efficient $x(\theta)$ and setting $U(\theta)$ equal to the second best information rent U^{SB}. But since the period 1 type θ is private information, the principal has to create a spread in the information rents and award a higher information rent to low cost than to high cost types to induce the former to report truthfully. As a result, the cost of providing incentives through promising a certain information rent U in period 2 is not monotone in U. For example, "punishing" the agent with a zero information rent in period 2 is extremely costly, since $\Pi(0) = 0$ means that the principal has to sacrifice the entire surplus in period 2. Likewise, to reward the agent with a rent higher than U^{SB}, then because Π is maximal at U^{SB}, the principal has to give the agent a surplus share that exceeds the second best share of the surplus in period 2.

Second, the incentive compatibility constraints IC are uni-directional, only requiring that the agent does not report a less efficient type. In contrast to the setting with bi-directional incentive constraints, the uni-directional constraints prevent us from employing familiar solution techniques that are based on the characterization of incentive compatibility in terms of monotonicity of the trading probability and revenue equivalence. In fact, it is easy to see that IC does not even imply that $x(\theta)$ is monotone.[16] To address this issue, the next lemma provides necessary conditions for IC that allow us to relax problem P_1'.

Lemma 4 *If (x, U) satisfies IC, then it satisfies the two following conditions:*

M: *U is decreasing.*

IC_L: *$U'(\theta) \leq -x(\theta)$ for all θ where the derivative exists.*

[16] Analyzing a screening problem with uni-directional incentive constraints and discrete types, Celik (2006) makes the same observation. His techniques for solving the subsequent problem do not apply to our framework with continuous types.

Property M is straightforward and simply reflects that lower cost types can guarantee themselves at least the utility of a higher cost type by pretending to be that type. As to condition IC_L, note first that because U is decreasing, U is differentiable almost everywhere. Recall that in the standard case where IC is required for *all* reports $\hat{\theta}$, the derivative of the agent's utility is actually pinned down by the allocation x. In our case, where IC is required only for reports $\hat{\theta} > \theta$, it is only necessary that the derivative of the agent's utility is bounded by the allocation x.

The lemma implies that we obtain a relaxed version of P_1' if we replace IC with the monotonicity condition M and the "localized" condition IC_L:

$$R_1: \qquad \max_{x,U} \qquad \int_{\underline{\theta}}^{\bar{\theta}} [v-\theta]x(\theta) + \Pi(U(\theta))\, dF(\theta) \quad s.t \quad IR, M, IC_L, UG \tag{35}$$

We now solve R_1 and then show that its solution also solves P_1'. We proceed in two steps. We first show that at a solution to R_1, trade never happens if it is inefficient, and the constraint IC_L is binding. In the second step, we use these properties to establish a solution to R_1. To establish the first step, let Φ be the (non-empty) feasible set for problem R_1. We then obtain the following result.

Lemma 5 *Let $(\tilde{x}, \tilde{U}) \in \Phi$. Then there is $(x, U) \in \Phi$ which delivers the principal a (weakly) higher profit than $(\tilde{x}, \tilde{U})$ and has the following properties:*

(i) If $v < \bar{\theta}$, then $x(\theta) = 0$ for all $\theta > v$.

(ii) $U'(\theta) = -x(\theta)$ for all θ.

The first part makes the familiar point that an optimal contract induces a downward distortion. To understand the second part, recall that Π is concave with a maximum at U^{SB}. For a given trading probability x, the principal therefore seeks to choose U as closely as possible to U^{SB} while maintaining the incentive compatibility requirement that $U'(\theta) \leq -x(\theta)$. Thus, an optimal choice of U is maximally flat, implying that $U'(\theta) = -x(\theta)$.

We emphasize that although property (ii) corresponds to the revenue equivalence property from standard screening models where IC is required for *all* reports $\hat{\theta}$, in our setting, property (ii) expresses an optimality rather than a feasibility condition.

In standard screening models, property (ii) is useful, because it pins down the agent's utility U as an integral over the trading probability x. If, in addition, Π is linear, an integration by parts argument can be used to replace U in the objective function of (35), and the problem can then be solved by point-wise maximization over $x(\theta)$. In our case, because Π is concave, this approach does not work.

Our alternative approach is to instead use property (ii) to replace the trading probability x by the agent's utility function U in the objective function of (35) and then maximize over U. This allows us to show that an optimal contract is in the class of *cutoff-contracts* where the good is traded if and only if that agent's cost is below a cutoff $\theta_0 \in [\underline{\theta}.\bar{\theta}]$.

Definition 3 *A cutoff-contract (x,U) is characterized by two parameters: a cutoff $\theta_0 \in [\underline{\theta},\bar{\theta}]$ and an intercept $U_0 \geq \theta_0 - \underline{\theta}$ such that*

$$x(\theta) = \begin{cases} 1 \quad if \quad \theta \leq \theta_0 \\ 0 \quad else \end{cases}, \quad U(\theta) = \begin{cases} U_0 - (\theta - \underline{\theta}) \quad if \quad \theta \leq \theta_0 \\ U_0 - (\theta_0 - \underline{\theta}) \quad else. \end{cases} \tag{36}$$

We denote by Λ the set of cutoff contracts. Clearly, $\Lambda \subset \Phi$. We now state the main result of this section that, under a regularity condition, a cutoff-contract is a solution to the relaxed problem R_1.

Proposition 1 *Let $(v-\theta)\frac{f'(\theta)}{f(\theta)}$ be increasing on the range $[\underline{\theta}, \min\{v,\bar{\theta}\}]$. Consider $(\tilde{x},\tilde{U}) \in \Phi$. Then there is a cutoff-contract $(x,U) \in \Lambda$ which delivers a (weakly) higher profit than $(\tilde{x},\tilde{U})$.*

While we prove the proposition in the appendix, the underlying logic is best understood in the context of our uniform example. Note that the uniform example satisfies the regularity condition trivially, as $f'(\theta) = 0$.

Example: Consider some $(\tilde{x},\tilde{U}) \in \Phi$. As indicated earlier, we can use part (ii) of Lemma 5 to replace $\tilde{x}$ by $\tilde{U}'$ in the objective of (35). Using integration by parts and $v = 1$, the objective then rewrites as

$$\int_{\underline{\theta}}^{\bar{\theta}} [v-\theta]\tilde{x}(\theta) + \Pi(\tilde{U}(\theta))\, dF(\theta) = \int_0^1 -[v-\theta]\tilde{U}'(\theta) + \Pi(\tilde{U}(\theta))\, d\theta \tag{37}$$

$$= \tilde{U}(0) + \int_0^1 \tilde{U}(\theta)\, d\theta + \int_0^1 \Pi(\tilde{U}(\theta))\, d\theta. \tag{38}$$

We now construct a function U belonging to a cutoff-contract for which expression (38) is at least as large as for $\tilde{U}$. To do so, note that Lemma 5 implies that $\tilde{U}$ is a decreasing continuous function with a slope between -1 and 0. Therefore, because under a cutoff-contract, U has slope -1 up to the cutoff θ_0 and then slope 0, an intermediate value argument implies that we can find

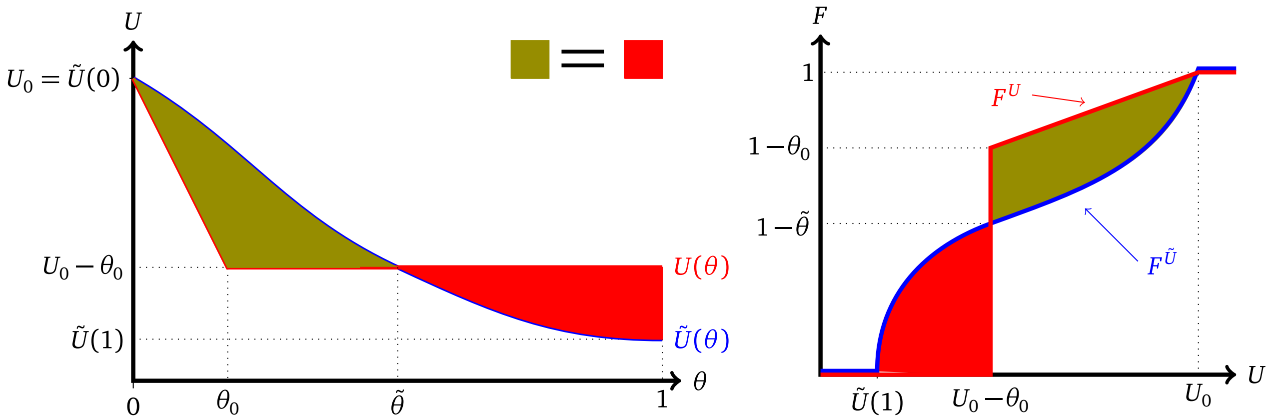

Figure 1: The left panel illustrates, given $\tilde{U}$ and that θ is uniformly distributed over $[0,1]$, the construction of the cutoff contract $U(.)$ such that $U_0 = \tilde{U}(0)$ and $\int_0^1 U(\theta)\,d\theta = \int_0^1 \tilde{U}(\theta)\,d\theta$. The right panel shows the associated probability distributions F^U and $F^{\tilde{U}}$ of U and $\tilde{U}$ in utility space. $F^{\tilde{U}}$ is a mean preserving spread of F^U.

U so that

$$U_0 = \tilde{U}(0), \qquad \int_0^1 U(\theta)\,d\theta = \int_0^1 \tilde{U}(\theta)\,d\theta. \tag{39}$$

In particular, there is a $\tilde{\theta} \in [0,1]$ so that

$$U(\theta) \leq \tilde{U}(\theta) \text{ for } \theta \leq \tilde{\theta} \text{ and } U(\theta) \geq \tilde{U}(\theta) \text{ for } \theta \geq \tilde{\theta}. \tag{40}$$

The first panel of Figure 1 illustrates the construction graphically.

By (39), the first two terms in (38) are the same for U and $\tilde{U}$. The key idea to analyze the third term in (38) is to interpret the agent's utility as a random variable which induces a probability distribution in utility space (the pushforward). Formally, and as illustrated in the second panel of Figure 1, the distributions induced by $\tilde{U}$ and U correspond to the cumulative distribution functions

$$F^{\tilde{U}}(u) = Pr(\theta \in [0,1] : \tilde{U}(\theta) \leq u) \quad and \quad F^U(u) = Pr(\theta \in [0,1] : U(\theta) \leq u). \tag{41}$$

The key observation is now that the second part of (39) and (40) imply that $F^{\tilde{U}}$ is a mean preserving spread of F^U. Therefore, because Π is concave, the third term in (38) is larger for U than for $\tilde{U}$. $\square$

For the general case without uniform distribution, the construction is analogous. The mean preserving spread argument carries over unchanged. The role of the regularity condition is to sign what corresponds to the first and second terms in (38), since these terms depend in general on the density f.

The regularity condition in Proposition 1 is not entirely new to the literature. In a context where the principal is a seller and the agent is a buyer, Manelli and Vincent (2006, Theorem 4) impose an equivalent regularity condition when characterizing the profit maximizing solution in a multi-dimensional screening problem. A sufficient condition for the regularity condition is that jointly $f' \leq 0$ and f is log-convex.[17] Examples include the family of power distributions $F(\theta) = \theta^\alpha$, $\theta \in [0,1]$, for $\alpha \leq 1$ or of exponential distributions $F(\theta) = 1 - e^{-\lambda\theta}$, $\theta \geq 0$, $\lambda \geq 0$.

Proposition 1 shows that a cutoff-contract is a solution to the relaxed problem R_1. It is straightforward to verify that any cutoff-contract satisfies the constraints IC of the original problem. Therefore, we have:

Proposition 2 *Let $(v - \theta)\frac{f'(\theta)}{f(\theta)}$ be increasing on the range $[\underline{\theta}, \min\{v, \bar{\theta}\}]$, then there is a cutoff-contract $(x, U) \in \Lambda$ which solves problem P_1'. Moreover, a cutoff contract also satisfies constraint IC^0. Thus, it is a solution to the original problem P.*

Since a cutoff-contract consists only of the two parameters θ_0, U_0, finding the optimal cutoff-contract comes down to solving an optimization problem in two variables. We illustrate this exercise in our running example.

Example: For our uniform example, the principal's objective is

$$W(\theta_0, U_0) \quad = \quad \int_0^{\theta_0} 1 - \theta \, d\theta + \int_0^{\theta_0} \Pi(U_0 - \theta) \, d\theta + \int_{\theta_0}^1 \Pi(U_0 - \theta_0) \, d\theta \tag{43}$$

with $U_0 \geq \theta_0$ and where Π is given by (28). To determine the maximizer, we first determine an optimal $\theta_0^*(U_0)$ for a given U_0. A tedious but otherwise straightforward analysis of the first and

[17] To see this, note

$$\frac{d}{d\theta}(v - \theta)\frac{f'(\theta)}{f(\theta)} = -\frac{f'(\theta)}{f(\theta)} + (v - \theta)\frac{d}{d\theta}\frac{f'(\theta)}{f(\theta)} = -\frac{f'(\theta)}{f(\theta)} + (v - \theta)\frac{d}{d\theta}\log(f(\theta)). \tag{42}$$

Because $v - \theta$ is positive on the range $[\underline{\theta}, \min\{v, \bar{\theta}\}]$, this expression is postive if $f' \leq 0$ and $\log f$ is increasing, that is, f is log-convex.

second order condition with respect to θ_0 yields[18]

$$\theta_0^*(U_0) = U_0 - 1/18. \tag{46}$$

Next, we maximize $W(\theta_0^*(U_0), U_0) = W(U_0 - 1/18, U_0)$ with respect to U_0. For $U_0 \leq 1/2$, this expression reduces to $109/648 + (5 + 4\sqrt{2U_0} - 9U_0)U_0/6$ which is strictly increasing for $U_0 \leq 1/2$ so that a maximum exhibits $U_0 \geq 1/2$. For $U_0 > 1/2$, the expression $W(U_0 - 1/18, U_0)$ reduces to the quadratic expression $41/324 + 4/3 \cdot U_0 - U_0^2$ which attains a maximum at $U_0 = 2/3$.

We therefore conclude that $(\theta_0^*, U_0^*) = (11/18, 2/3)$ maximizes $W(\theta_0, U_0)$ with a payoff of $185/324 \approx 0.571$, exceeding by 14% the principal's payoff of $2\Pi^{SB} = 1/2$, from charging twice the static optimal price $p = 1/2$.

Recall from above that in the uniform example, the period 2 terms of trade can be implemented by offering the agent a period 2 price $p_2 = \sqrt{2U}$. With this in mind, period 1 cost types θ above the cutoff $\theta_0^* = 11/18$ do not produce in period 1 and obtain expected period 2 utility of $U(\theta) = U_0^* = 2/3$, corresponding to a period 2 price offer $p_2 = 1/3$. All period 1 cost types θ below the cutoff $\theta_0^* = 11/18$ produce in the first period and obtain expected period 2 utility $U(\theta) = U_0^* - \theta = 2/3 - \theta$, corresponding to a period 2 price offer $p_2 = \min\{4/3 - 2\theta_1, 1\}$. Interestingly, period 1 cost types $\theta < 1/6$ obtain more than the utility 1/2. These types always produce in period 2, since they receive the offer to produce at a price of 1 in period 2.

The ex ante expected utility of the agent is 157/648 so that expected aggregate surplus is $185/324 + 157/648 = 527/648 \approx 0.813$, compared to the first best surplus of 1. Without bankruptcy constraints, aggregate surplus is 7/8=.875, while the twicely repeated static second best contract yields aggregate surplus of .75. Table 1 summarizes. □

Remark 2 (Implementation) We now briefly discuss how an optimal cutoff contract can be indirectly implemented by a menu of prices. For simplicity, suppose that the optimal period 2 terms

[18]The first order condition with respect to θ_0 is:

$$\frac{\partial W}{\partial \theta_0} = (1 - \theta_0)(1 - \Pi'(U_0 - \theta_0)) = 0 \quad \Leftrightarrow \quad \theta_0 = 1 \text{ or } \Pi'(U_0 - \theta_0) = 1. \tag{44}$$

It is easy to check that $\theta_0 = 1$ is not a maximizer of W. By (28), the unique solution to $\Pi'(U_0 - \theta_0) = 1$ is $\theta_0 = U_0 - 1/18$. This is indeed a maximizer of $W(\theta_0, U_0)$, because the second order condition is

$$\frac{\partial^2 W}{\partial \theta_0^2} = -1 + \Pi'(U_0 - \theta_0) + \Pi''(U_0 - \theta_0)(1 - \theta_0) < 0, \tag{45}$$

is satisfied for $\theta_0 = U_0 - 1/18$, since the first two terms cancel, while $\Pi''(U) = -\sqrt{2}U^{-3/2}/4 < 0$ for $U \leq 1/2$ and $U_0 - \theta_0 = 1/18 < 1/2$.

	(no A.I.) FB	2xSB	No bankruptcy	bankruptcy
P's payoff	1	0.5	0.75	0.571
U's ex ante payoff	0	0.25	.125	0.242
ex ante surplus	1	0.75	.875	0.813

Table 1: Payoff comparisons

of trade can be implemented by a posted price. Recall from Remark 1 that this is the case if, for example, F_2/f_2 is increasing.

An optimal contract can then be implemented by a menu $\{(r, p_2(r)) \mid r \in [\underline{\theta}, \theta_0]\}$ where the agent can choose to produce the good in period 1 for a price r and conditional on not going bankrupt in period 1, obtains the option to produce the good in period 2 for the price $p_2(r)$ where p_2 is decreasing in r. Moreover, if the agent goes bankrupt in period 1, the relationship is terminated.

To see this, recall that under a backloaded contract, the agent breaks even in period 1. Under a cutoff contract, the agent therefore receives in period 1 the transfer $\hat{\theta}_1$ and produces the good if he announces $\hat{\theta}_1 \in [\underline{\theta}, \theta_0]$ and does not go bankrupt. If he announces $\hat{\theta}_1 \in (\theta_0, \bar{\theta}]$ he receives the transfer 0 and does not produce the good. This corresponds to choosing a price $r = \hat{\theta}_1 \in [\underline{\theta}, \theta_0]$ at which to deliver the good in period 1. Moreover, after announcing $\hat{\theta}_1$, the agent obtains expected utility $U(\hat{\theta}_1)$ in period 2 which can be implemented by a posted price $p_2(\hat{\theta}_1)$ which is decreasing in $\hat{\theta}_1$ because $U(\hat{\theta}_1)$ is decreasing in $\hat{\theta}_1$. This corresponds to obtaining the option to produce the good at $p_2(r) = p_2(\hat{\theta}_1)$ in period 2 after choosing the price r in period 1.

Remark 3 (More than two periods) While we performed our analysis only for two periods, the extension to multiple periods is straightforward. To illustrate, suppose that there are infinitely many periods and that cost types θ_τ are i.i.d. with time-independent cdf F on the support $[\underline{\theta}, \bar{\theta}]$.[19] For the problem to be well-defined, assume that both parties discount future payoffs with a discount factor $\delta \in [0, 1)$. Under the dynamic programming formulation, the principal's choice variables are a probability of trade $x(\theta)$ for the current period and the expected continuation utility for the agent $U(\theta)$ that both depend on a report θ by the agent about his current type (as well as on the history of past reports which we suppress). The principal's value function $\Pi(V)$ is now defined recursively as a function of the agent's expected utility V (starting as of now)

[19]The extension to an arbitrary finite time horizon is analogous but all expressions are time-dependent.

according to the dynamic program:

$$
\begin{aligned}
P_\infty: \quad \Pi(V) = \max_{x,U} \quad & \int_{\underline{\theta}}^{\bar{\theta}} (v-\theta)x(\theta) + \delta\Pi(U(\theta))\, dF(\theta) \quad s.t. && (47)\\
IR: \quad & U(\theta) \geq 0 \quad \forall\theta && (48)\\
IC: \quad & U(\theta) \geq (\hat{\theta}-\theta)x(\hat{\theta}) + U(\hat{\theta}) \quad \forall\theta \leq \hat{\theta} && (49)\\
UG: \quad & x(\theta) \in [0,1] \quad \forall\theta && (50)\\
PK: \quad & \int_{\underline{\theta}}^{\bar{\theta}} \delta U(\theta) dF(\theta) = V. && (51)
\end{aligned}
$$

While problem P_∞ yields the principal's value function, the solution to the principal's overall problem starting in the initial period is obtained by maximizing Π with respect to V.

The essential difference between P_∞ and P_1' is the presence of the promise keeping constraint PK which ensures that the agent's expected utility from the contract is V. As above, we consider a relaxed problem where we localize IC and replace it with the constraints M and IC_L as stated in Lemma 4:

$$
R_\infty: \quad \tilde{\Pi}(V) = \max_{x,U} \int_{\underline{\theta}}^{\bar{\theta}} (v-\theta)x(\theta) + \delta\tilde{\Pi}(U(\theta))\, dF(\theta) \quad s.t \quad IR, M, IC_L, UG, PK. \tag{52}
$$

It follows from standard arguments (see Stockey and Lucas, 1989, or Krishna et al. 2013) that $\tilde{\Pi}$ exists. Crucially, as in the two-period case, $\tilde{\Pi}$ is concave. Recall that to establish the optimality of a cutoff contract for the two-period problem R_1, we exploited the concavity of $\tilde{\Pi}$ to construct for a every feasible contract $(\tilde{x}, \tilde{U})$ a feasible cutoff-contract (x, U) that is an improvement. Note that, in contrast to problem R_1, feasibility in problem R_∞ requires that a contract, in addition, satisfies PK. Therefore, to extend the argument from R_1 to R_∞, we have to ensure that the cutoff contract (x, U) that improves a given feasible contract does satisfy PK.

However, note that the cutoff contract (x, U) constructed in the two-period problem to improve upon $(\tilde{x}, \tilde{U})$ has the property that[20]

$$
\int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \tilde{U}(\theta) dF(\theta). \tag{53}
$$

Therefore, as $(\tilde{x}, \tilde{U})$ is an arbitrary feasible contract and thus satisfies PK by definition, so does

[20]This corresponds to the right part of (39) where we defined (x, U) in the uniform example.

(x, U). This shows that a cutoff contract is optimal also when there are more than two periods.

5 Conclusion

We study bankruptcy constraints in an otherwise standard dynamic screening model. We model bankruptcy as a verifiable event and show that it affects contractual feasibility constraints not only through participation but also incentive compatibility constraints. Thus, our analysis highlights the importance of spelling out explicitly the economic consequences of bankruptcy in terms of the underlying economic environment.

While our paper assumes that bankruptcy occurs whenever the agent makes short term losses, in practice the occurrence and consequences of bankruptcy may be more complicated than that, since bankruptcy may, for example, be partially discretionary or involve restructuring processes. It is an interesting avenue for future research to capture such richer forms of bankruptcy.

Our paper also makes a methodological contribution. In particular, we solve for an optimal contract using a new method that ranks contracts in terms of the spread of the distribution of the induced continuation values for the agent. An open question is to what extent our approach can be employed in a model with correlated cost types (as in Krasikov and Lamba, 2021). Such an extension is beyond the scope of the current paper because it implies that the agent's continuation value becomes type dependent, thus constraining the principal's choice of continuation values. Another interesting avenue is to apply our solution method to static mechanism design problems in which the principal's payoff is not linear in the agent's information rent.

Appendix

Proof of Lemma 1 Let $\tilde{\gamma} = (\tilde{x}_1, \tilde{t}_1, \tilde{x}_2^N, \tilde{t}_2^N, \tilde{x}_2^B, \tilde{t}_2^B)$ be a feasible contract. Our proof strategy is to first define an auxiliary contract $\hat{\gamma}$ that is feasible and payoff-equivalent to $\tilde{\gamma}$ but under which no bankruptcy occurs. In a second step, we modify $\hat{\gamma}$ to obtain the desired contract γ that has the properties stated in the lemma. In what follows, we indicate all variables pertaining to $\tilde{\gamma}$ and $\hat{\gamma}$ with a tilde and a hat.

Step 1: Define the auxiliary contract $\hat{\gamma} = (\hat{x}_1, \hat{t}_1, \hat{x}_2^N, \hat{t}_2^N, \hat{x}_2^B, \hat{t}_2^B)$ by

$$(\hat{x}_1(\theta_1), \hat{t}_1(\theta_1)) = \begin{cases} (\tilde{x}_1(\theta_1), \tilde{t}_1(\theta_1)) & \text{if } \theta_1 \in \tilde{\Theta}_1^N, \\ (0,0) & \text{otherwise} \end{cases} \tag{54}$$

$$(\hat{x}_2^N(\theta_1,\theta_2), \hat{t}_2^N(\theta_1,\theta_2)) = \begin{cases} (\tilde{x}_2^N(\theta_1,\theta_2), \tilde{t}_2^N(\theta_1,\theta_2)) & \text{if } \theta_1 \in \tilde{\Theta}_1^N, \theta_2 \in \tilde{\Theta}_2^{N,N}(\theta_1) \\ (\tilde{x}_2^B(\theta_1,\theta_2), \tilde{t}_2^B(\theta_1,\theta_2)) & \text{if } \theta_1 \notin \tilde{\Theta}_1^N, \theta_2 \in \tilde{\Theta}_2^{N,B}(\theta_1) \\ (0,0) & \text{otherwise,} \end{cases} \tag{55}$$

$$(\hat{x}_2^B(\theta_1,\theta_2), \hat{t}_2^B(\theta_1,\theta_2)) = (0,0) \quad \forall \theta_1, \theta_2. \tag{56}$$

We show that $\hat{\gamma}$ is feasible and payoff-equivalent to $\tilde{\gamma}$. To see this, note first that, by construction, we have $\hat{\Theta}_1^N = \Theta_1$ and $\hat{\Theta}_2^{N,N}(\theta_1) = \Theta_2$ for all θ_1. Furthermore,

$$\hat{U}^N(\theta_1) = \tilde{U}^N(\theta_1) \text{ for } \theta_1 \in \tilde{\Theta}_1^N \quad \textit{and} \quad \hat{U}^N(\theta_1) = \tilde{U}^B(\theta_1) \text{ for } \theta_1 \notin \tilde{\Theta}_1^N. \tag{57}$$

To see feasibility, observe that $\hat{\gamma}$ trivially satisfies (4) for $b = B$, and inherits (4) for $b = N$ by construction. To see (5), let $\hat{t}_1(\hat{\theta}_1) - \theta_1 \hat{x}_1(\hat{\theta}_1) \geq 0$. Consider first the case that $\theta_1 \in \tilde{\Theta}_1^N$ and $\hat{\theta}_1 \in \tilde{\Theta}_1^N$. Then, we have:

$$\begin{aligned} \hat{t}_1(\theta_1) - \theta_1 \hat{x}_1(\theta_1) + \hat{U}^N(\theta_1) &= \tilde{t}_1(\theta_1) - \theta_1 \tilde{x}_1(\theta_1) + \tilde{U}^N(\theta_1) & (58) \\ &\geq \tilde{t}_1(\hat{\theta}_1) - \theta_1 \tilde{x}_1(\hat{\theta}_1) + \tilde{U}^N(\hat{\theta}_1) & (59) \\ &= \hat{t}_1(\hat{\theta}_1) - \theta_1 \hat{x}_1(\hat{\theta}_1) + \hat{U}^N(\hat{\theta}_1), & (60) \end{aligned}$$

where the inequality follows, because $\tilde{\gamma}$ satisfies (5) and the two equalities follow from (57). The other cases can be shown analogously.

To see (6), note that the left hand side of (6) is non-negative by definition of $\hat{\gamma}$. Moreover, because $\hat{x}_2^B = \hat{t}_2^B = 0$, we have $\hat{U}^B(\hat{\theta}_1) = 0$ for all $\hat{\theta}_1$ so that the right hand side is zero. Therefore, (6) follows. To complete the proof of feasibility, note that (7) and (8) are void for $\hat{\gamma}$, because

$\hat{\Theta}_1^N = \Theta_1$.

Finally, $\hat{\gamma}$ and $\tilde{\gamma}$ are payoff-equivalent, because by construction, if bankruptcy does not occur under $\tilde{\gamma}$, then $\hat{\gamma}$ implements the same terms of trade as $\tilde{\gamma}$, and when bankruptcy occurs under $\tilde{\gamma}$, no trade occurs under $\hat{\gamma}$ so that under either contract both the principal and the agent get zero.

Step 2: We now construct a feasible contract $\gamma = (x_1, t_1, x_2^N, t_2^N, x_2^B, t_2^B)$ which is payoff-equivalent to $\hat{\gamma}$ and satisfies (12)-(14). To do so, note first that $\hat{\gamma}$ satisfies (13) and (14), but may violate (12) and display $\hat{t}_1(\theta_1) - \theta_1 \hat{x}_1(\theta_1) > 0$ for some θ_1.

Define γ as the contract that differs from $\hat{\gamma}$ only in that the period 1 profits for the agent are backloaded to period 2. Formally, γ displays $x_1 = \hat{x}_1, x_2^N = \hat{x}_2^N, x_2^B = \hat{x}_2^B, t_2^B = \hat{t}_2^B$ and payments

$$t_1(\theta) = \theta_1 x_1(\theta_1), \quad t_2^N(\theta_1, \theta_2) = \hat{t}_2^N(\theta_1, \theta_2) + \hat{t}_1(\theta_1) - t_1(\theta_1). \tag{61}$$

Note first that γ satisfies (12) by construction. Moreover, it inherits (14) from $\hat{\gamma}$ and also property (13) because

$$\begin{aligned} t_2^N(\theta_1, \theta_2) - \theta_2 x_2^N(\theta_1, \theta_2) &= \hat{t}_2^N(\theta_1, \theta_2) + \hat{t}_1(\theta_1) - t_1(\theta_1) - \theta_2 \hat{x}_2^N(\theta_1, \theta_2) && (62) \\ &= \hat{t}_2^N(\theta_1, \theta_2) - \theta_2 \hat{x}_2^N(\theta_1, \theta_2) + \hat{t}_1(\theta_1) - \theta_1 \hat{x}_1(\theta_1) \geq 0, && (63) \end{aligned}$$

where the inequality follows since under $\hat{\gamma}$ no bankruptcy occurs.

We next show that γ is feasible. Indeed, γ trivially satisfies (4) for $b = B$ because $x_2^B = t_2^B = 0$. For $b = N$, we have for all $\theta_1, \theta_2, \hat{\theta}_2$:

$$\begin{aligned} t_2^N(\theta_1, \theta_2) - \theta_2 x_2^N(\theta_1, \theta_2) &= \hat{t}_2^N(\theta_1, \theta_2) + \hat{t}_1(\theta_1) - t_1(\theta_1) - \theta_2 x_2^N(\theta_1, \theta_2) && (64) \\ &\geq \hat{t}_2^N(\theta_1, \hat{\theta}_2) + \hat{t}_1(\theta_1) - t_1(\theta_1) - \theta_2 x_2^N(\theta_1, \hat{\theta}_2) && (65) \\ &= t_2^N(\theta_1, \hat{\theta}_2) - \theta_2 x_2^N(\theta_1, \hat{\theta}_2), && (66) \end{aligned}$$

where the first and the third lines use the definition of t_2^N, and the second line follows because $\hat{\gamma}$ satisfies (4) for $b = N$ and since $x_2^N = \hat{x}_2^N$.

To see (5), consider $\theta_1, \hat{\theta}_1$ so that $t_1(\hat{\theta}_1) - \theta_1 x_1(\hat{\theta}_1) \geq 0$. Because $\hat{t}_1(\hat{\theta}_1) \geq t_1(\hat{\theta}_1)$ and $\hat{x}_1(\hat{\theta}) = x_1(\hat{\theta})$, this implies that also $\hat{t}_1(\hat{\theta}_1) - \theta_1 \hat{x}_1(\hat{\theta}_1) \geq 0$. Therefore, since $\hat{\gamma}$ satisfies (5), we have

$$\hat{t}_1(\theta) - \theta_1 \hat{x}_1(\theta_1) + \hat{U}^N(\theta_1) \geq \hat{t}_1(\hat{\theta}) - \theta_1 \hat{x}_1(\hat{\theta}_1) + \hat{U}^N(\hat{\theta}_1). \tag{67}$$

Moreover, by construction, we have that $t_1(\theta_1) + U^N(\theta_1) = \hat{t}_1(\theta_1) + \hat{U}^N(\theta_1)$. These two observations imply that

$$\begin{aligned}
t_1(\theta_1) - \theta_1 x_1(\theta_1) + U^N(\theta_1) &= \hat{t}_1(\theta) - \theta_1 \hat{x}_1(\theta_1) + \hat{U}^N(\theta_1) && (68)\\
&\geq \hat{t}_1(\hat{\theta}) - \theta_1 \hat{x}_1(\hat{\theta}_1) + \hat{U}^N(\hat{\theta}_1) && (69)\\
&= t_1(\hat{\theta}) - \theta_1 x_1(\hat{\theta}_1) + U^N(\hat{\theta}_1). && (70)
\end{aligned}$$

Furthermore, γ satisfies (6), because $U^B(\hat{\theta}_1) = 0$ for all $\hat{\theta}_1$ and the left hand side of (6) is non-negative. Finally, (7) and (8) are void for γ, because $\Theta_1^N = \hat{\Theta}_1^N = \Theta_1$.

It remains to show that γ and $\hat{\gamma}$ are payoff-equivalent. But this follows, because the only difference between the contracts is that the payments have been moved between periods, but the sum of payments over the two periods is the same. qed

Proof of Lemma 2 Let $\gamma = (x_1, t_1, x_2^N, t_2^N, x_2^B, t_2^B)$ be the contract induced by the backloaded contract (x_1, x_2, t_2). Hence, $t_1 = \theta_1 x_1$, $x_2^N = x_2$,$t_2^N = t_2$, $x_2^B = t_2^B = 0$. We have to show that γ is feasible if and only if IC_2 and IC_1 hold. To see this, observe first that γ trivially satisfies (4) for $b = B$ because $x_2^B = t_2^B = 0$. Moreover, for any backloaded-induced contract γ, the constraint (4) for $b = N$ rewrites as IC_2. Hence γ satisfies (4) if and only if it satisfies IC_2.

We next show that constraint (5) is equivalent to IC_1. Indeed, since $t_1(\hat{\theta}_1) = \hat{\theta}_1 x_1(\hat{\theta}_1)$ for all $\hat{\theta}_1$, we have

$$t_1(\hat{\theta}_1) - \theta_1 x_1(\hat{\theta}_1) \geq 0 \quad \Leftrightarrow \quad (\hat{\theta}_1 - \theta_1) x_1(\hat{\theta}_1) \geq 0 \quad \Leftrightarrow \quad \theta_1 \leq \hat{\theta}_1. \tag{71}$$

Hence, γ satisfies (5) if and only if for all $\theta_1 \leq \hat{\theta}_1$, we have $t_1(\theta_1) - \theta_1 x_1(\theta_1) + U(\theta_1) \geq t_1(\hat{\theta}_1) - \theta_1 x_1(\hat{\theta}_1) + U(\hat{\theta}_1)$. But because $t_1(\theta_1') - \theta_1' x_1(\theta_1') = 0$ for all θ_1' holds for any contract γ that is induced by some backloaded contract, this is equivalent to IC_1.

Moreover, γ always satisfies (6) because the right hand side of (6) is zero, and the left hand side is non-negative. Finally, (7) and (8) are void for γ because $\Theta_1^N = \Theta_1$. This completes the proof. qed

Proof of Lemma 3 To simplify notation, we omit θ_1 and suppress the time subindex. With standard screening arguments, we can write P_2 as a maximization problem that maximizes the virtual surplus with respect to the allocation $x(\cdot)$ and the rent of the most inefficient type $u(\bar{\theta})$ as follows:

$$
\begin{aligned}
P_2: \qquad \Pi(U) \equiv \max_{x,u(\bar{\theta})} \quad & \int_{\underline{\theta}}^{\bar{\theta}} \left(v-\theta-\frac{F(\theta)}{f(\theta)}\right) x(\theta)\, dF(\theta) - u(\bar{\theta}) \quad s.t && (72)\\
M: \quad & x(\theta) \text{ is decreasing in } \theta && (73)\\
NB_2: \quad & u(\bar{\theta}) \geq 0 && (74)\\
PK: \quad & u(\bar{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} x(\theta)\frac{F(\theta)}{f(\theta)}\, dF(\theta) = U && (75)
\end{aligned}
$$

That $\Pi(U)$ attains a maximum Π_2^{SB} at U_2^{SB} is explained in the main text.

To see the further claims of the Lemma note that PK pins down $u(\bar{\theta})$, and by substituting out $u(\bar{\theta})$ in the objective (73) and NB_2, the problem simplifies to

$$
\begin{aligned}
\hat{P}_2: \qquad \Pi(U) \equiv \max_{x} \quad & \int_{\underline{\theta}}^{\bar{\theta}} (v-\theta)x(\theta)\, dF(\theta) - U \quad s.t && (76)\\
M: \quad & x(\theta) \text{ is decreasing in } \theta && (77)\\
NB_2: \quad & \int_{\underline{\theta}}^{\bar{\theta}} x(\theta)\frac{F(\theta)}{f(\theta)}\, dF(\theta) \leq U && (78)
\end{aligned}
$$

To see that $\Pi(0)=0$, note that (78) implies that the only way to supply $U=0$ is to have $x(\theta)=0$ for all θ, resulting in zero profits, hence: $\Pi(0)=0$.

To see that Π is concave, let x' resp. x'' be solutions to $\hat{P}_2$ for U' resp. U''. Then the allocation $\bar{x} = \alpha x' + (1-\alpha)x''$ satisfies M and NB_2 for $U = \alpha U' + (1-\alpha)U''$. Moreover, $\bar{x}$ yields profit $\alpha\Pi(U') + (1-\alpha)\Pi(U'')$. The solution to $\hat{P}_2$ for $U = \alpha U' + (1-\alpha)U''$ must therefore yield at least $\bar{\Pi}$. Thus, we have $\Pi(\alpha U' + (1-\alpha)U'') \geq \alpha\Pi(U') + (1-\alpha)\Pi(U'')$, which establishes concavity of Π.

To see that $\Pi(S^{FB})=0$, note that, by definition, $\Pi + U \leq S^{FB}$ for any allocation $x(\cdot)$. Hence, we have $\Pi(S^{FB}) \leq 0$. To show $\Pi(S^{FB})=0$, it therefore suffices to show that, for $U = S^{FB}$, the first-best allocation $x^{FB}(\theta) = 1_{[\underline{\theta},\min\{v,\bar{\theta}\}]}(\theta)$ satisfies (77) and (78) and yields 0 for the objective (76). Indeed, $x^{FB}(\theta)$ clearly satisfies (77) and, together with $U = S^{FB}$, yields 0 for the objective (76). To see that the first-best allocation also satisfies (78) for $U = S^{FB} \int_{\underline{\theta}}^{\min\{v,\bar{\theta}\}} v-\theta\, dF(\theta)$, note that by integration by parts:

$$\int_{\underline{\theta}}^{\bar{\theta}} x^{FB}(\theta)\frac{F(\theta)}{f(\theta)}\,dF(\theta) = \int_{\underline{\theta}}^{\min\{v,\bar{\theta}\}} \frac{F(\theta)}{f(\theta)}\,dF(\theta) \tag{79}$$

$$= -(v-\theta)F(\theta)\Big|_{\underline{\theta}}^{\min\{v,\bar{\theta}\}} + \int_{\underline{\theta}}^{\min\{v,\bar{\theta}\}} v-\theta\,dF(\theta) \leq S^{FB}. \tag{80}$$

qed

Proof of Lemma 4 That U is decreasing is immediate from IC. Since U is decreasing, U has a derivative almost everywhere by Lebesque's Theorem. Now suppose that U' exists at θ. Note that for $h > 0$, we can write IC as $U(\theta-h)-U(\theta) \geq hx(\theta)$. Thus,

$$U'(\theta) = \lim_{h\to 0}\frac{U(\theta)-U(\theta-h)}{h} \leq -x(\theta), \tag{81}$$

as desired. qed

Proof of Lemma 5 Let $(\tilde{x},\tilde{U}) \in \Phi$ be such that it does not satisfy (i) or (ii). We construct an improvement $(x,U)\in\Phi$ that satisfies (i) and (ii).

Suppose that $v < \bar{\theta}$ and $(\tilde{x},\tilde{U})$ violates (i). Consider first the case that $\tilde{U}(v) \leq U^{SB}$, and define (x,U) as

$$x(\theta) = \begin{cases} \tilde{x}(\theta) & if \quad \theta \leq v \\ 0 & if \quad \theta > v \end{cases}, \quad U(\theta) = \begin{cases} \tilde{U}(\theta) & if \quad \theta \leq v \\ \tilde{U}(v) & if \quad \theta > v. \end{cases} \tag{82}$$

Clearly, $(x,U) \in \Phi$ and satisfies (i). We next argue that (x,U) is a (weak) improvement over $(\tilde{x},\tilde{U})$ by showing that

$$\int_{\underline{\theta}}^{\bar{\theta}} (v-\theta)x(\theta)\,dF(\theta) \geq \int_{\underline{\theta}}^{\bar{\theta}} (v-\theta)\tilde{x}(\theta)\,dF(\theta), \text{ and} \tag{83}$$

$$\int_{\underline{\theta}}^{\bar{\theta}} \Pi(U(\theta))\,dF(\theta) \geq \int_{\underline{\theta}}^{\bar{\theta}} \Pi(\tilde{U}(\theta))\,dF(\theta). \tag{84}$$

Inequality (83) is immediate from the definition of x. To see (84), note that because U is decreasing and $U(v) \leq U^{SB}$ by assumption, it follows by construction that for all $\theta > v$, we have $U^{SB} \geq U(\theta) \geq \tilde{U}(\theta)$. Thus, because Π is concave and uniquely maximized at U^{SB} by Lemma 3, this implies that $\Pi(U(\theta)) \geq \Pi(\tilde{U}(\theta))$ for all $\theta > v$. Since $U(\theta) = \tilde{U}(\theta)$ for all $\theta \leq v$, (84) follows.

Next consider the case that $\tilde{U}(v) > U^{SB}$. Define (x, U) as

$$x(\theta) = \begin{cases} \tilde{x}(\theta) & if \quad \theta \le v \\ 0 & if \quad \theta > v \end{cases}, \quad U(\theta) = \begin{cases} \tilde{U}(\theta) & if \quad \theta \le v \\ U^{SB} & if \quad \theta > v. \end{cases} \tag{85}$$

Clearly, $(x, U) \in \Phi$ and satisfies (i). It follows with similar arguments as in the previous paragraph that (x, U) is a (weak) improvement over $(\tilde{x}, \tilde{U})$.

Finally, suppose $(\tilde{x}, \tilde{U})$ violates (ii). Define

$$\tau = \sup\{\theta \mid \tilde{U}(\theta) \ge U^{SB}\}. \tag{86}$$

Because U is decreasing, we have that

$$\tilde{U}(\theta) \ge U^{SB} \text{ for all } \theta < \tau, \quad \text{and} \quad \tilde{U}(\theta) < U^{SB} \text{ for all } \theta > \tau. \tag{87}$$

Define (x, U) as $x(\theta) = \tilde{x}(\theta)$ for all θ, and

$$U(\theta) = U^{SB} - \int_{\tau}^{\theta} x(t)\, dt. \tag{88}$$

Clearly, $(x, U) \in \Phi$ and satisfies (ii). To show that (x, U) yields a higher profit than $(\tilde{x}, \tilde{U})$, observe that because $(\tilde{x}, \tilde{U})$ and (x, U) specify the same allocation x, it is sufficient to show that

$$\Pi(\tilde{U}(\theta)) \le \Pi(U(\theta)) \text{ for almost all } \theta. \tag{89}$$

To see this, consider first the case that $\theta < \tau$. It is well-known that the derivative of a decreasing function is (Lebesgue) integrable and that $\tilde{U}(\theta) - \tilde{U}(\tilde{\theta}) \ge \int_{\tilde{\theta}}^{\theta} \tilde{U}'(t)\, dt$ for all $\theta, \tilde{\theta}$. Hence, for all $\epsilon > 0$ with $\theta < \tau - \epsilon$:

$$\begin{aligned} \tilde{U}(\theta) &\ge \int_{\tau-\epsilon}^{\theta} \tilde{U}'(t)\, dt + \tilde{U}(\tau - \epsilon) && (90) \\ &= -\int_{\theta}^{\tau-\epsilon} \tilde{U}'(t)\, dt + \tilde{U}(\tau - \epsilon) && (91) \\ &\ge \int_{\theta}^{\tau-\epsilon} \tilde{x}(t)\, dt + \tilde{U}(\tau - \epsilon) && (92) \\ &= -\int_{\tau-\epsilon}^{\theta} x(t)\, dt + \tilde{U}(\tau - \epsilon), && (93) \end{aligned}$$

where the second inequality follows from IC_L, and the final equality from $x = \tilde{x}$ Because the inequality holds for all $\epsilon > 0$ and since $\tilde{U}(\tau - \epsilon) \geq U^{SB}$ by (87), we can infer that

$$\tilde{U}(\theta) \quad \geq \quad -\int_{\tau}^{\theta} x(t)\, dt + U^{SB} = U(\theta). \tag{94}$$

Moreover, since $\theta < \tau$, we have $U(\theta) \geq U^{SB}$, and accordingly, $\tilde{U}(\theta) \geq U(\theta) \geq U^{SB}$. Because Π is concave and uniquely maximized at U^{SB} by Lemma 3, these inequalities imply (89) for $\theta < \tau$. A symmetrical argument works to show (89) for $\theta > \tau$, and this completes the proof. qed

Proof of Proposition 1 To avoid case distinctions, we only consider the case $v < \bar{\theta}$. By Lemma 5, it is sufficient to prove the statement for $(\tilde{x}, \tilde{U}) \in \Phi$ which satisfies properties (i) and (ii) from Lemma 5. Consequently, we have:

(i') $\tilde{U}(\theta) = \tilde{U}(v)$ for all $\theta \geq v$.

We first construct a contract $(\hat{x}, \hat{U})$ which is *not* necessarily in Λ that delivers a (weakly) more profit than $(\tilde{x}, \tilde{U})$. In a second step, we then construct (x, U) which *is* in Λ that delivers a (weakly) higher profit than $(\hat{x}, \hat{U})$.

As to the first step, define for $\alpha \in [\tilde{U}(v), \tilde{U}(\underline{\theta})]$ the two functions

$$\hat{U}_\alpha(\theta) = \begin{cases} \tilde{U}(\underline{\theta}) - (\theta - \underline{\theta}) & if \quad \theta \in [\underline{\theta}, \hat{\theta}] \\ \alpha & if \quad \theta \in (\hat{\theta}, v) \\ \tilde{U}(\theta) & if \quad \theta \in [v, \bar{\theta}] \end{cases}, \qquad \Delta(\alpha) \equiv \int_{\underline{\theta}}^{\bar{\theta}} \hat{U}_\alpha(\theta) - \tilde{U}(\theta)\, dF(\theta),$$

where $\hat{\theta} \equiv \underline{\theta} + \tilde{U}(\underline{\theta}) - \alpha \in [\underline{\theta}, v]$.

In words, $\hat{U}_\alpha$ starts at $\tilde{U}(\underline{\theta})$, then decreases with slope -1 until it attains the value α at the point $\hat{\theta}$, then stays constant equal to α until it reaches the point $\theta = v$, at which it jumps downwards to $\tilde{U}(v)$ and stays constant from then on (since it coincides with $\tilde{U}$ which is constant on $[v, \bar{\theta}]$ by (i') above)

Next, we show that there is $\hat{\alpha} \in [\tilde{U}(v), \tilde{U}(\underline{\theta})]$ so that

$$\int_{\underline{\theta}}^{\bar{\theta}} \hat{U}_{\hat{\alpha}}(\theta)\, dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \tilde{U}(\theta)\, dF(\theta). \tag{95}$$

Indeed, by construction, for $\alpha = \tilde{U}(\underline{\theta})$, we have $\hat{U}_\alpha(\theta) - \tilde{U}(\theta) \geq 0$ for all θ, and for $\alpha = \tilde{U}(v)$, we have $\hat{U}_\alpha(\theta) - \tilde{U}(\theta) \leq 0$ for all θ. It follows that $\Delta(\tilde{U}(\underline{\theta})) \geq 0$ and $\Delta(\tilde{U}(v)) \leq 0$. Because $\Delta(\alpha)$ is continuous on $\alpha \in [\tilde{U}(v), \tilde{U}(\underline{\theta})]$, the intermediate value theorem applies, implying (95).

Moreover, because $\hat{U}_{\hat{\alpha}}$ and $\tilde{U}$ coincide on $[v, \bar{\theta}]$ by construction, the previous equality can equivalently be written as

$$\int_{\underline{\theta}}^{v} \hat{U}_{\hat{\alpha}}(\theta)\, dF(\theta) = \int_{\underline{\theta}}^{v} \tilde{U}(\theta)\, dF(\theta). \tag{96}$$

From now on, denote $\hat{U}_{\hat{\alpha}}$ simply by $\hat{U}$. Moreover, let

$$\hat{x}(\theta) = \begin{cases} 1 & if \quad \theta \in [\underline{\theta}, \hat{\theta}] \\ 0 & if \quad \theta > \hat{\theta}. \end{cases} \tag{97}$$

We now show that $(\hat{x}, \hat{U})$ yields a (weakly) higher profit than $(\tilde{x}, \tilde{U})$. This is trivially the case for $\hat{\alpha} = \tilde{U}(v)$, where we have $(\hat{x}, \hat{U}) = (\tilde{x}, \tilde{U})$. Hence, suppose $\hat{\alpha} > \tilde{U}(v)$. In this case, we have $\hat{U}(v) > \tilde{U}(v)$. Therefore, because $\hat{U}(\underline{\theta}) = \tilde{U}(\underline{\theta})$, $\tilde{U}'(\theta) \geq \hat{U}'(\theta) = -1$ for $\theta \in [\underline{\theta}, \hat{\theta}]$ and $\tilde{U}'(\theta) \leq \hat{U}'(\theta) = 0$ for $\theta \in [\hat{\theta}, v]$, there is a $\tilde{\theta} \in [\underline{\theta}, v]$ so that

$$\hat{U}(\theta) - \tilde{U}(\theta) \leq 0 \quad \forall \theta \leq \tilde{\theta} \quad \text{ and } \quad \hat{U}(\theta) - \tilde{U}(\theta) \geq 0 \quad \forall \theta \geq \tilde{\theta}. \tag{98}$$

Using the facts that $\hat{U}' = -\hat{x}$ and $\tilde{U}' = -\tilde{x}$, and $\hat{x}(\theta) = \tilde{x}(\theta) = 0$ for all $\theta > v$, we can write the difference in the principal's profits from $(\hat{x}, \hat{U})$ and $(\tilde{x}, \tilde{U})$ as

$$\begin{aligned} W(\hat{x}, \hat{U}) - W(\tilde{x}, \tilde{U}) &= \int_{\underline{\theta}}^{\bar{\theta}} (v - \theta)[\hat{x}(\theta) - \tilde{x}(\theta)] + \Pi(\hat{U}(\theta)) - \Pi(\tilde{U}(\theta))\, dF(\theta) \\ &= \int_{\underline{\theta}}^{v} (v - \theta)[\hat{x}(\theta) - \tilde{x}(\theta)]\, dF(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} \Pi(\hat{U}(\theta)) - \Pi(\tilde{U}(\theta))\, dF(\theta) \\ &= \int_{\underline{\theta}}^{v} (v - \theta)[\tilde{U}'(\theta) - \hat{U}'(\theta)]\, dF(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} \Pi(\hat{U}(\theta)) - \Pi(\tilde{U}(\theta))\, dF(\theta). \end{aligned}$$

Integrating the first integral by parts delivers

$$W(\hat{x}, \hat{U}) - W(\tilde{x}, \tilde{U}) = (v - \theta) f(\theta)[\tilde{U}(\theta) - \hat{U}(\theta)] \Big|_{\underline{\theta}}^{v} \tag{99}$$

$$- \int_{\underline{\theta}}^{v} [(v - \theta)\frac{f'(\theta)}{f(\theta)} - 1][\tilde{U}(\theta) - \hat{U}(\theta)]\, dF(\theta) \tag{100}$$

$$+ \int_{\underline{\theta}}^{\bar{\theta}} \Pi(\hat{U}(\theta)) - \Pi(\tilde{U}(\theta))\, dF(\theta). \tag{101}$$

We now argue that this expression is positive. Observe first that by construction, $\hat{U}(\underline{\theta}) = \tilde{U}(\underline{\theta})$, and thus the right hand side of (99) is equal to zero. Moreover, by (96), expression (100) can firstly be written as

$$-\int_{\underline{\theta}}^{v}[(v-\theta)\frac{f'(\theta)}{f(\theta)}][\tilde{U}(\theta)-\hat{U}(\theta)]\,dF(\theta) = Y, \tag{102}$$

and we can secondly add $\int_{\underline{\theta}}^{v}[(v-\tilde{\theta})\frac{f'(\tilde{\theta})}{f(\tilde{\theta})}][\tilde{U}(\theta)-\hat{U}(\theta)]\,dF(\theta) = 0$, with $\tilde{\theta}$ defined in (98), to obtain:

$$\begin{aligned} Y &= -\int_{\underline{\theta}}^{v}[(v-\theta)\frac{f'(\theta)}{f(\theta)} - (v-\tilde{\theta})\frac{f'(\tilde{\theta})}{f(\tilde{\theta})}][\tilde{U}(\theta)-\hat{U}(\theta)]\,dF(\theta) && (103)\\ &= -\int_{\underline{\theta}}^{\tilde{\theta}}[(v-\theta)\frac{f'(\theta)}{f(\theta)} - (v-\tilde{\theta})\frac{f'(\tilde{\theta})}{f(\tilde{\theta})}][\tilde{U}(\theta)-\hat{U}(\theta)]\,dF(\theta) && (104)\\ &\quad -\int_{\tilde{\theta}}^{v}[(v-\theta)\frac{f'(\theta)}{f(\theta)} - (v-\tilde{\theta})\frac{f'(\tilde{\theta})}{f(\tilde{\theta})}][\tilde{U}(\theta)-\hat{U}(\theta)]\,dF(\theta). && (105) \end{aligned}$$

Now, the assumption that $(v-\theta)\frac{f'(\theta)}{f(\theta)}$ is increasing implies that the first bracket under the integral (104) is negative for all $\theta \in [\underline{\theta}, \tilde{\theta}]$, and (98) implies that the second bracket under the integral (104) is positive for all $\theta \in [\underline{\theta}, \tilde{\theta}]$, so that, overall (104) is positive. Analogously, (105) is positive.

Finally, to see that (101) is positive, define for an arbitrary decreasing function U, the cdf F^U as the push-forward measure, that is, the utility distribution induced by U, given by

$$F^U(u) = Prob(\{\theta \mid U(\theta) \leq u\}). \tag{106}$$

By (95) and (98), $F^{\tilde{U}}$ is a mean preserving spread of $F^{\hat{U}}$. Thus, because Π is concave by Lemma 3, we have

$$\int_{\underline{\theta}}^{\tilde{\theta}} \Pi(\hat{U}(\theta)) - \Pi(\tilde{U}(\theta))\,dF(\theta) = \int_{\underline{\theta}}^{\tilde{\theta}} \Pi(u)\,dF^{\hat{U}}(u) - \int \Pi(u)\,dF^{\tilde{U}}(u) \geq 0. \tag{107}$$

This completes the first step of the proof.

As to the second step, let $(\hat{x}, \hat{U})$ from the first step be given. We construct $(x, U) \in \Lambda$ which delivers a (weakly) more profit than $(\hat{x}, \hat{U})$. Indeed, let (x, U) be a cutoff-contract with cutoff

$\theta_0 = \hat{\theta}$ and an intercept $U_0 \in [\hat{U}(v) + \hat{\theta} - \underline{\theta}, \hat{U}(\underline{\theta})]$ such that[21]

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta)\, dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \hat{U}(\theta)\, dF(\theta). \tag{108}$$

This also implies that

$$U(\theta) - \hat{U}(\theta) \leq 0 \quad \forall \theta \leq v \quad \text{and} \quad U(\theta) - \hat{U}(\theta) \geq 0 \quad \forall \theta \geq v. \tag{109}$$

Because $\theta_0 = \hat{\theta}$ implies $x = \hat{x}$, the difference in the principal's profit from (x, U) and $(\hat{x}, \hat{U})$ can be written as

$$\begin{aligned} W(x,U) - W(\hat{x}, \hat{U}) &= \int_{\underline{\theta}}^{\bar{\theta}} (v - \theta)[x(\theta) - \hat{x}(\theta)] + \Pi(U(\theta)) - \Pi(\hat{U}(\theta))\, dF(\theta) && (110) \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \Pi(U(\theta)) - \Pi(\hat{U}(\theta))\, dF(\theta). && (111) \end{aligned}$$

Similarly to the argument at the end of the first step, (108) and (109) imply that $F^{\hat{U}}$ is a mean preserving spread of F^U, and hence (111) is positive, and this completes the proof. qed

Proof of Proposition 2 Note first if there is a solution $(x, U) \in \Lambda$ to the relaxed problem R_1, then because $(x, U) \in \Lambda$ is obviously feasible for the problem P_1', it is also a solution to P_1'. Moreover, any contract $(x, U) \in \Lambda$ satisfies the constraint IC^0 and thus a solution $(x, U) \in \Lambda$ to P_1' is also a solution to P. To see this, observe that for $(x, U) \in \Lambda$ we have that $\Theta_1^0 = (\theta_0, \bar{\theta}]$ by (36). To show IC^0, we thus have to show that $U(\theta) \geq U(\hat{\theta})$ for all $\hat{\theta} \in (\theta_0, \bar{\theta}]$ and $\theta \in \Theta$. But this is immediate from the definition of U in (36).

It remains to show existence of a solution $(x, U) \in \Lambda$ to R_1. For this recall that a cutoff contract is characterized by cutoffs $\theta_0 \in [\underline{\theta}, \bar{\theta}]$ and $U_0 \geq \theta_0 - \underline{\theta}$. We first show the auxiliary claim that for any $(\tilde{x}, \tilde{U}) \in \Lambda$ there is a $(x, U) \in \Lambda$ which yields a (weakly) higher profit than $(\tilde{x}, \tilde{U})$ and has the property that

$$U_0 \leq U^{SB} + (\bar{\theta} - \underline{\theta}). \tag{112}$$

Indeed, consider a $(\tilde{x}, \tilde{U})$ with cutoffs $(\tilde{\theta}_0, \tilde{U}_0)$ that violates (112). Since $(\tilde{x}, \tilde{U})$ is a cutoff contract,

[21] Given $\theta_0 = \hat{\theta}$, the cutoff U_0 exists by the intermediate value theorem, because the integral on the left hand side of (108) is strictly larger than the right hand side for $U_0 = \hat{U}(\underline{\theta})$, strictly lower for $U_0 = \hat{U}(v) + \hat{\theta} - \underline{\theta}$, and changes continuously in U_0.

this implies that $\tilde{U}(\theta) > U^{SB}$ for all θ. Define $(x, U) \in \Lambda$ with cutoffs

$$\theta_0 = \tilde{\theta}_0, \quad U_0 = \tilde{U}_0 - (\tilde{U}(\bar{\theta}) - U^{SB}). \tag{113}$$

By construction, we have that $U^{SB} \leq U(\theta) \leq \tilde{U}(\theta)$ for all θ. Thus, because Π is concave and uniquely maximized at U^{SB} by Lemma 3, this implies that $\Pi(U(\theta)) \geq \Pi(\tilde{U}(\theta))$ for all θ. Therefore, and since $x = \tilde{x}$, we obtain the profit

$$W(x, U) = \int (v - \theta)\tilde{x}(\theta) + \Pi(U(\theta))\, dF(\theta) \geq \int (v - \theta)\tilde{x}(\theta) + \Pi(\tilde{U}(\theta))\, dF(\theta) = W(\tilde{x}, \tilde{U}) \tag{114}$$

and this proves the auxiliary claim.

Now, let $\bar{\Lambda}$ be the set of cutoff contracts that satisfy (112). That is, $(x, U) \in \bar{\Lambda}$ if we can express (x, U) as a cutoff contract with cutoff $\theta_0 \in [\underline{\theta}, \bar{\theta}]$ and intercept $U_0 \in [\theta_0 - \underline{\theta}, U^{SB} + (\bar{\theta} - \underline{\theta})]$.

The auxiliary claim and Proposition 1 then imply that there is a solution $(x, U) \in \Lambda$ to R_1 if there is a solution to the problem

$$Q: \qquad \max_{(x,U)} W(x, U) \quad s.t. \quad (x, U) \in \bar{\Lambda}. \tag{115}$$

Because the profit $W(x, U)$ of a cutoff contract is pinned down by (θ_0, U_0), problem Q boils down to the problem of choosing a two-dimensional variable (θ_0, U_0) from the compact set $[\underline{\theta}, \bar{\theta}] \times [\theta_0 - \underline{\theta}, U^{SB} + (\bar{\theta} - \underline{\theta})]$. Because profit is continuous in (θ_0, U_0), there is a solution to Q. Therefore, there is a solution $(x, U) \in \Lambda$ to R_1, and this completes the proof. qed

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