Discussion Paper Series – CRC TR 224

Discussion Paper No. 527
Project B 03

Pigou Meets Wolinsky: Search, Price Discrimination, and Consumer Sophistication

Carl-Christian Groh[1]
Jonas von Wangenheim[2]

April 2024

[1] University of Bonn, Email: cgroh@uni-bonn.de
[2] University of Bonn, Email: jwangenheim@uni-bonn.de

Support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Collaborative Research Center Transregio 224 - www.crctr224.de
Rheinische Friedrich-Wilhelms-Universität Bonn - Universität Mannheim

Pigou meets Wolinsky: Search, Price Discrimination, and Consumer Sophistication*

Carl-Christian Groh[†] Jonas von Wangenheim[‡]

April 5, 2024

Abstract

We study the competitive effects of personalized pricing in horizontally differentiated markets with search frictions. We integrate the possibility of first degree price discrimination into the classic Wolinsky (1986) framework of consumer search. If all consumers are rational, personalized pricing leads to higher consumer surplus if and only if there are no search frictions. If all consumers are unaware that firms price discriminate, i.e. are naive as in Eyster and Rabin (2005), this result is reversed: Personalized pricing improves consumer surplus unless search costs are prohibitive.

Keywords: search, price discrimination, welfare, bounded rationality, anonymity

JEL Codes: D21, D43, D83, D90

*We would like to thank Heiko Karle, Daniel Krähmer, Benny Moldovanu, Volker Nocke, Martin Peitz, and Marcel Preuss for valuable discussions and comments. Support by the Deutsche Forschungsgemeinschaft (DFG) through CRC TR 224 (Project B03) is gratefully acknowledged.

[†]Groh: Institute for Microeconomics, University of Bonn, Lennéstrasse 37, 53113 Bonn, Germany (email: cgroh@uni-bonn.de)

[‡]von Wangenheim: Institute for Microeconomics, University of Bonn, Adenauerallee 24-42, 53113 Bonn, Germany (email: jwangenheim@uni-bonn.de)

1 Introduction

This paper sheds new light on an old question: Do consumers benefit when firms can price discriminate against them? The relevance of this question is ever rising in the digital age, given the increasing availability of consumer data to firms operating in online markets and the mounting empirical evidence for price discrimination in these markets.[1] As a result, these business practices are being closely monitored by regulatory bodies around the world due to their potential negative effects on consumer surplus.[2]

In this article, we study how the competitive effects of price discrimination in horizontally differentiated markets are shaped by the level of search frictions and the degree to which consumers are aware that firms price discriminate. Accounting for search frictions is crucial in determining the optimal policy response to online price discrimination, given that search frictions in online markets are known to be substantial.[3] Similarly, it is important to factor in that consumers may not understand that the prices they receive are personalized, given that a majority of consumers are unaware of the exact ways in which their data is used.[4]

We establish the following results: First, when all consumers are rational, endowing firms with the ability to first-degree price discriminate increases consumer surplus if and only if there are no search frictions. This adds important context to the seminal finding of Thisse and Vives (1988), who showed that consumers benefit when firms can price discriminate in the classic Hotelling model without search frictions. Second, when all consumers are naive as in Eyster and Rabin (2005), i.e. would ignore the relationship between preferences and prices if firms price discriminate, endowing firms with the ability to price discriminate raises consumer surplus. The unifying intuition underlying these results is that price discrimination weakens the search incentives of rational consumers, while it strengthens the search incentives of naive consumers. We also study the optimal regulatory approach: Raising the share of consumers who are aware that firms price discriminate, which is viewed as desirable by regulatory authorities, reduces the welfare of all consumers.[5] By contrast, the establishment of a right to anonymity will (if it is exercised by some consumers) raise the welfare of all consumers, even though it is only utilized by rational consumers.

Formally, we study a duopoly version of the classic Wolinsky (1986) model of sequential search for horizontally differentiated products. Consumers have heterogeneous willingnesses-to-pay for the products of different firms. Importantly, consumers do not know their pref-

[1] See Hannak et al. (2014), Larson et al. (2015), Escobari et al. (2019), and Aryal et al. (2023).
[2] See OECD Secretariat (2016) and European Commission (2019).
[3] See, for example, Koulayev (2014), De los Santos (2018), and Jolivet and Turon (2019).
[4] See, for example, Computer Weekly (2019) and Internet Policy Review (2019).
[5] For details on policymakers' views, see OECD Secretariat (2016) and European Commission (2019).

erences ex ante and must engage in sequential search to discover their willingnesses-to-pay for the firms' goods and the prices offered by firms. We consider two search setups, namely frictionless and costly search. If search is frictionless, all consumers exogenously visit both firms without cost. Under costly search, consumers can costlessly visit one firm, but have to pay a search cost to visit a second firm after the first.

We extend this framework by integrating the possibility of first-degree price discrimination as defined in Pigou (1920): Whenever a consumer visits a firm, this firm finds out the consumer's willingness-to-pay for its product. If this type of information is available, firms offer a different price to every consumer. Importantly, firms do not know whether a given consumer has visited its rival before and have no information about any consumer's willingness-to-pay for the rival firm's product.

In our main analysis, we consider two versions of this framework: In the first version, all consumers are rational. This means that these consumers, when deciding whether or not to visit a second firm, take into account that firms price discriminate and form correct expectations about the distribution of the prices they will receive at the next firm. In the second version, all consumers are naive and do not take into account that prices are personalized when making their search decisions. Formally, these consumers are cursed as in Eyster and Rabin (2005) and expect that the price they receive at any firm is independent of their preferences.[6]

Within each version of the model, we derive the equilibrium outcomes in four different settings that vary in two dimensions, namely as to whether (i) firms can price discriminate or not and (ii) whether search is frictionless or consumers face positive search costs. Fixing a given level of search frictions, we then compare consumer surplus when firms can and cannot price discriminate. This allows us to establish how the welfare effects of price discrimination depend on the level of search frictions and the degree of consumer sophistication.

If consumers are rational, price discrimination raises consumer surplus if and only if search is frictionless. When firms cannot price discriminate, they offer a uniform price that is equal to the equilibrium price in Wolinsky (1986). When firms can price discriminate and search is frictionless, the standard result from Thisse and Vives (1988) is replicated: The possibility of price discrimination intensifies competition between firms, which implies that the average personalized price paid by consumers is lower than the aforementioned uniform price. Thus, when search is frictionless, consumer surplus is higher under personalized pricing.

This result is reversed when there are search frictions, no matter how small these are:

[6]The expectations of consumers are only correct ex ante: Before visiting a firm, naive consumers expect to receive a price that is drawn from the distribution of prices this firm offers to the entire mass of consumers.

If firms perfectly observe any consumer's willingness-to-pay for their product and there are search frictions, there is a unique perfect Bayesian equilibrium in which both firms charge any arriving consumer a price equal to the willingness-to-pay this consumer has for the firm's product. Thus, consumer surplus is zero. Said pricing strategy constitutes an equilibrium by the following logic: If both firms perfectly price discriminate, the surplus any consumer receives at either firm is always zero. Rational consumers anticipate this and thus never visit both firms if search is costly. Then, every firm has monopoly power over any arriving consumer and can appropriate the full surplus.

Put differently, endowing firms with the ability to price discriminate causes consumer harm by reducing the search incentives of rational consumers. The opposite holds true when all consumers are naive: In particular, naive consumers have stronger search incentives when firms price discriminate. Intuitively, this holds by the following logic: Endowing firms with the ability to price discriminate creates equilibrium price dispersion. Naive consumers do not understand the correlation between their willingness-to-pay for a firm's product and the price this firm charges them. Taken together, these arguments imply that the perceived spread of outcomes which naive consumers expect to attain by searching is higher when firms can price discriminate. This raises the search incentives of naive consumers, because any consumer's perceived gains of search are equal to the consumer's expectation of a function that is convex in the outcomes attainable through search.

When all consumers are naive, price discrimination raises consumer surplus unless search costs are prohibitively high. This follows from previous arguments: Endowing firms with the ability to price discriminate strengthens the search incentives of naive consumers. When all consumers are naive, the average number of firms consumers visit is thus significantly higher under price discrimination than when firms charge a uniform price. Hence, price discrimination by firms leads to more intense competition between them, which drives down their prices and benefits consumers.

Given that price discrimination has the potential of completely eroding consumer surplus, we also study the optimal policy response to these business practices. To make progress in this endeavour, we develop a framework in which there are both naive and rational consumers, which is an important feature of reality (Internet Policy Review, 2019). In this framework, we analyse the consequences of two policy measures, namely (i) an increase in the share of rational consumers, i.e. those who are aware that firms price discriminate, and (ii) the establishment of a right to anonymity, which enables any consumer to ensure that firms receive no information about her.

Raising the share of consumers who are aware that firms price discriminate, which is viewed as desirable by regulatory bodies (OECD Secretariat, 2016; European Commission, 2019), reduces the expected utility of naive and rational consumers. Intuitively, this follows from the fact that rational consumers impose a negative externality on naive consumers when firms price discriminate. This is because rational consumers visit less firms than naive consumers (in expectation), given that naive consumers overestimate their gains of search.

Endowing consumers with a right to anonymity will (if this right is utilized by some consumers) raise the expected utility of all consumers, including those who don't exercise this right. This right is only ever exercised by rational consumers, given that naive consumers don't understand that firms price discriminate and would thus not expect to attain any benefits from anonymity. When a right to anonymity is established and rational consumers exercise it, rational consumers will be strictly better off than in the absence of any regulation (otherwise, no rational consumer would exercise this right). Moreover, naive consumers also benefit. This is because the choice of anonymity of rational consumers reduces the share of rational consumers among those consumers for whom the firms have information. Since naive consumers search more than rational consumers, this raises the intensity of search for the group of consumers who do not anonymize, putting more competitive pressure on firms.

Literature: To the best of our knowledge, we are the first to integrate price discrimination based on consumers' willingnesses-to-pay into the classic sequential search model by Wolinsky (1986). Moreover, we are not aware of any paper which studies how consumer awareness of price discrimination affects competition in search markets. However, our work is related to three strands of the literature, namely (i) the recent body of research on price discrimination in search markets, (ii) the literature that studies the welfare effects of price discrimination in horizontally differentiated markets without search frictions, and (iii) the contributions on the role of biased consumer beliefs in search markets without price discrimination.

Naturally, our work relates to the recent literature that studies price discrimination in search markets. Armstrong and Zhou (2016), Preuss (2022), Groh and Preuss (2022), and Mauring and Williams (2023) consider models in which firms condition prices on a consumer's search history.[7] Fabra and Reguant (2020) study a simultaneous search setting where firms observe a consumer's desired quantity and price discriminate based on this information. Mauring (2022) and Atayev (2022) consider models in which firms receive imperfect information about the affiliation of a particular consumer to the groups of shoppers and non-shoppers as defined in Stahl (1989).[8] Bergemann et al. (2021) study a homogenous

[7]Garrett et al. (2019) consider a model of second-degree price discrimination in which consumers differ in their choice sets, but firms do not have information about consumers.

[8]Byrne et al. (2022) provide empirical evidence that, in bilateral negotiations, consumers with perceived

goods model with search frictions in which competing firms receive information about the number of price offers a consumer obtains. In contrast to our work, all these papers study models in which price discrimination is based on information about consumers' search costs or search histories, and not about their willingnesses-to-pay for firms' products. Moreover, all these papers only consider rational consumers.

There are some related papers that study search markets in which which firms price discriminate based on information about the willingnesses-to-pay of consumers. In Groh (2023a), firms price discriminate based on noisy information about consumer valuations in a homogenous goods market. In Groh (2023b), firms have differential access to information about consumers' valuations in a market in which consumers are either captive to some firm or consider the products of firms to be homogenous. Crucially, neither of these models resemble the Wolinsky (1986) framework. Marshall (2020) studies a model in which firms price discriminate based on information about consumer valuations. However, while Marshall (2020) sets up and empirically calibrates a structural model, he provides no analytical equilibrium characterization. Moreover, there are important differences in the model setup: In Marshall (2020), recall is impossible: When a consumer decides to search, she will never return to purchase at the firm she interacted with. Finally, all these three papers only consider rational consumers as well.

Bergemann and Bonatti (2022) study digital markets where a platform matches firms with consumers using targeted advertisements. Firms engage in second-degree price discrimination and consumers on the platform can also visit firms off the platform. Ke et al. (2023) study the information design problem of an intermediary in a market in which every consumer just has a match at one firm, i.e. has zero willingness-to-pay for the products of other firms. Firms and consumers do not know with which firm a consumer has a match. There is no price discrimination and no competition between firms in Ke et al. (2023).[9]

Our research is also related to previous work on the effect of price discrimination on consumer surplus in horizontally differentiated markets without search frictions. Thisse and Vives (1988) show that, in a Hotelling model, price discrimination leads to higher consumer surplus by reducing the price that any consumer pays. This seminal result also holds in papers that build on this model, namely Chen and Iyer (2002), Shaffer and Zhang (2002), and Montes et al. (2019). Anderson et al. (2022) study competitive personalized pricing in

lower switching costs receive more favorable offers from retail electricity providers in Australia. Gugler et al. (2023) document that German retail electricity providers differentiate their tariffs based on consumers' search intensity across different local markets.

[9]Wang and Watanabe (2021) and Teh et al. (2023) consider the incentives of a platform to facilitate access between buyers and sellers in settings where trade happens via first-price auctions.

a general discrete-choice model in which firms can send targeted discounts at a cost.[10]

Rhodes and Zhou (2022) consider a model of first-degree price discrimination and show that the classic result from Thisse and Vives (1988) rests on the assumption of full market coverage. Rhodes and Zhou (2022) demonstrate that price discrimination may reduce consumer surplus when the market is not fully covered and the market structure is exogenously given. While the work of Rhodes and Zhou (2022) is thus similar to our paper in spirit, the authors do not consider search frictions throughout their analysis.

Finally, our work relates to previous contributions on the role of (biased) beliefs in search markets. Janssen and Shelegia (2020) highlight that consumer beliefs about the identity of deviating parties play a crucial role in vertically related search markets. Mauring (2021) considers a model in which consumers engage in directed search based on signals about the price distributions which firms offer. Antler and Bachi (2022) consider a marriage market in which consumers do not take into account how an agent's attractiveness affects her propensity to accept or reject matches. Gamp and Krähmer (2022) study the equilibrium outcomes in markets where firms choose the quality of their goods and naive consumers are unable to correctly assess a product's quality.

Within this strand of the literature, the paper which is closest to our own is Gamp and Krähmer (2023). The authors consider markets in which naive consumers overestimate the spread of the potential outcomes that can be attained through search. Gamp and Krähmer (2023) demonstrate that the presence of consumers who are naive in that sense can break the Diamond paradox. The logic underlying this finding is similar to the intuition behind our result that consumer surplus is higher when consumers are unaware that firms price discriminate. Instead of analysing the surplus effects of price discrimination, Gamp and Krähmer (2023) establish how the presence of naive consumers can sustain equilibria with active search in a homogenous goods model. The authors provide an equilibrium analysis of a model without price discrimination and with no consumer or firm heterogeneity. Thus, our results regarding the welfare effects of price discrimination, and how these are shaped by the level of search frictions and consumer sophistication, are novel.

Outline: The rest of the paper proceeds as follows: Our modelling framework is introduced in section 2. Afterwards, we present the equilibrium analysis for the model variants with rational and naive consumers in sections 3 and 4, respectively. We discuss the aforementioned policy proposals in section 5 and conclude thereafter.

[10]Lu et al. (2022) consider a model in which consumers can purchase multiple products, while Jullien et al. (2023) solve a model in which there are multiple distribution channels.

2 Framework

We consider the following duopoly version of the Wolinsky (1986) model: There are two firms, indexed $j \in \{1, 2\}$, which produce a horizontally differentiated good at zero marginal cost. A unit mass of consumers wishes to buy at most one unit of the good. A consumer's willingness-to-pay for firm j's product, which is also referred to as the consumer's *match value* for firm j's product, is denoted by θ_j. These match values are independent random variables, where $\theta_j \overset{\text{iid}}{\sim} F$ and F is some continuously differentiable distribution with full support on $[0, 1]$. When buying the good of firm j at the price p_j, the consumer's utility, which we also refer to as the *surplus*, is:

$$S(\theta_j, p_j) = \theta_j - p_j \tag{1}$$

Ex ante, any consumer does not know her match values at the firms but must discover these by visiting firms sequentially. Visiting one firm is costless. Visiting a second firm after the first incurs a cost $c \geq 0$ for consumers. Consumers have free recall, i.e. they can costlessly access the offer of a previously visited firm. We say that search is *frictionless* if all consumers exogenously visit both firms at no cost.

We extend this standard version of the Wolinsky (1986) model by integrating the possibility of first-degree price discrimination: Whenever a firm j is visited by a consumer, this firm finds out the consumer's match value θ_j. Importantly, firms do not know a consumer's willingness-to-pay for their rival's product. Moreover, firms do not know a consumer's search history, i.e. do not know whether a given consumer has visited the rival before. Given the information available to them, firms can condition the price they offer to a consumer on the consumer's match value. Thus, a pure strategy of a firm j is a function $p_j(\theta_j)$.

The game's timing is as follows: Any consumer randomly visits either firm first with equal probability. Upon arriving at the first firm (say, firm j), nature draws the consumer's match value according to the distribution F and reveals this to the consumer and firm j. Firm j then chooses the price it offers to the consumer, based on the consumer's match value. Based on this price offer and her match value θ_j, the consumer decides whether or not to visit the other firm, namely firm $-j$, at cost $c \geq 0$. If the consumer visits firm $-j$, nature independently draws the match value θ_{-j} from the same distribution F and reveals this value to the consumer and to firm $-j$, which then offers the consumer a price, namely $p_{-j}(\theta_{-j})$. The consumer then decides from which firm to purchase the good or to exit the market, in which case she obtains zero utility.

We solve two versions of this model, one in which all consumers are rational (Section 4), and on in which consumers are naive (Section 5). Intuitively, rational consumers understand that firms price discriminate and take this into account when deciding whether or not to continue searching. By contrast, naive consumers are unaware that firms price discriminate and think that the price they receive at any firm is independent of their preferences.

Formally, rational consumers form correct expectations about the distribution of prices they receive at any firm. Thus, in this case the equilibrium concept we apply is perfect Bayesian equilibrium. Naive consumers are cursed in the sense of Eyster and Rabin (2005): They correctly anticipate the distribution of prices a firm offers to consumers in equilibrium, but neglect that firms condition the price on consumers' individual match value. Hence they expect that, independent of their match value at a firm j, this firm will offer them a price randomly drawn from the distribution of prices $p_j(\theta_j)$. All other specifications of what consists an equilibrium remain as in perfect Bayesian equilibrium.

Throughout the analysis, we define consumer surplus as the ex-ante utility of consumers.

3 Rational Consumers

In this section, we solve the model for the case of rational consumers. We compare the unique perfect Bayesian equilibrium for settings that vary in two dimensions, namely as to (i) whether firms can price discriminate or not and (ii) whether search is frictionless or consumers face strictly positive search costs c. The following analysis establishes that price discrimination raises consumer surplus if and only if there are no search frictions.

We begin by defining the equilibrium outcomes when firms cannot price discriminate. Then, firms offer a uniform price p^u, which was pinned down in Wolinsky (1986). We restate the classic result for a particular distribution of types, namely the uniform distribution:

Lemma 1 (Wolinsky, 1986)
Suppose firms cannot price discriminate and that $\theta_j \sim U[0,1]$. For any $c \geq 0$, there is a unique pure-strategy equilibrium in which firms offer the uniform price p^u, which solves:

$$p^u = \frac{1-(p^u)^2}{1+w^*} \quad ; \quad w^* = 1-\sqrt{2c}. \tag{2}$$

This result allows us to calculate the equilibria and the resulting consumer surplus when firms cannot price discriminate. The threshold w^* describes the search behaviour of consumers on the equilibrium path: they continue searching if and only if their initial match value is below w^*. Thus, $w^* = 1$ corresponds to the case of frictionless search, when the equilibrium price becomes $p^u = \sqrt{2}-1 \approx 0.414$. When $c \in [0, 1/8]$, the uniform price satisfies $p^u \in [\sqrt{2}-1, 0.5]$.

We continue by characterizing the equilibria when firms can price discriminate. Before doing so, it is useful to describe the optimal search behaviour of consumers. When a consumer receives the surplus S_{-j} at the first firm she visits, she will optimally continue searching if and only if:

$$\int_0^1 \max\{S_{-j}, \theta_j - p_j^*(\theta_j)\} dF(\theta_j) - c - S_{-j} > 0, \tag{3}$$

The optimal search rule of consumers is a cutoff rule, according to which consumers continue searching if and only if the surplus they receive at the first firm is below $\hat{S}$. This is because the match values at the two firms are drawn independently, which means that a consumers' incentives to continue searching are fully pinned down by the initially offered surplus.

Having established this, we characterize the equilibria under frictionless search, i.e. when all consumers exogenously visit both firms:

Proposition 1 (Frictionless search & price discrimination)
Suppose firms can price discriminate and that search is frictionless. There is a unique perfect Bayesian equilibrium in which firms price according to the following rule:

$$p^*(\theta_j) = \theta_j - \mathbb{E}[\theta_{-j} | \theta_{-j} < \theta_j] \tag{4}$$

If $\theta_j \sim U[0, 1]$, the equilibrium pricing rule is thus $p^*(\theta_j) = 0.5\theta_j$.

+The proof of the proposition is based on standard techniques from auction theory. Indeed, since buyers observe all prices, the firm's problem in this setting is equivalent to that of a bidder in a first-price auction.[11]

To understand the result, suppose that $\theta_j \sim U[0, 1]$ and that firms play the equilibrium strategy $p^*(\theta_j) = 0.5\theta_j$. Consider a firm j who is visited by a consumer with match value θ_j. Since this firm does not know the consumer's match value for the other firm's product and search is frictionless, firm j will maximize the following profit function through choice of p_j:

$$\Pi^0(p_j) = p_j \int_0^1 \mathbb{1}[\theta_j - p_j > 0.5\theta_{-j}] d\theta_{-j} = p_j \int_0^1 F\big(2(\theta_j - p_j)\big) d\theta_{-j} \tag{5}$$

It can be verified that the price $p_j = 0.5\theta_j$ locally maximizes this objective function, which

[11] To understand this, note that the individual match value θ_j is the value that a transaction with firm j generates. The surplus $S_j(\theta_j) = \theta_j - p_j(\theta_j)$ a firm offers to the buyer can be understood as the competitive bid a firm makes in order to guarantee a transaction.

is globally concave. Thus, said pricing strategy is mutually optimal for firms.

Next, we consider the case in which consumers have to incur a strictly positive cost $c > 0$ to visit a second firm. If firms can price discriminate, there is a unique equilibrium in which every consumer is charged their willingness-to-pay and consumer surplus is zero:

Proposition 2 (Search frictions & price discrimination)
Suppose that firms can price discriminate and consider any $c > 0$. There is a unique perfect Bayesian equilibrium in which firms price according to the following rule:

$$p^*(\theta_j) = \theta_j \tag{6}$$

Said pricing strategy is an equilibrium by the following logic: Suppose both firms price according to the rule $p^*(\theta_j) = \theta_j$. Then, consumers will receive zero surplus at any firm they visit. Rational consumers anticipate this and, because the costs of visiting a second firm are strictly positive, will never search beyond the first firm, even if they receive zero surplus there. Thus, it is optimal for firms to perfectly price discriminate.

To establish uniqueness, suppose that there exists an equilibrium (which may be asymmetric) in which some consumer type receives strictly positive surplus at either firm. Define $\hat{\theta}$ as the match value that receives the highest surplus in equilibrium and suppose that firm j offers a consumer the highest surplus. When visiting firm j first, a consumer who draws the match value $\theta_j = \hat{\theta}$ will strictly prefer to refrain from searching, because she anticipates that she cannot receive a higher surplus by visiting another firm and there are search costs. Because this preference is strict, said consumer would also not continue searching when receiving a surplus slightly below its equilibrium level. But this means that firm j has a profitable deviation — since the surplus it offers to any arriving consumer with $\theta_j = \hat{\theta}$ is strictly positive, it would be profitable for the firm to marginally increase the price it offers, as said consumer would still buy at this firm with certainty.

Now, we bring all the previous results together to pin down the effects of price discrimination on consumer surplus, which depends on whether there are search frictions or not:

Corollary 1 (Price discrimination & consumer surplus)
If search is frictionless and $\theta_j \sim U[0, 1]$, consumer surplus is higher under price discrimination. For any $c > 0$, consumer surplus is strictly lower when firms can price discriminate.

This result adds important context to the seminal finding of Thisse and Vives (1988), who show that price discrimination raises consumer surplus in a standard Hotelling model, in which search is frictionless. This result can also be replicated in our framework by considering a particular example, namely the case in which $\theta_j \sim U[0,1]$. However, when consumers face positive search costs, the result flips: The possibility of first-degree price discrimination allows firms to appropriate all surplus from consumers. Given that search frictions in online markets are known to be substantial (Koulayev, 2014; De los Santos, 2018; Jolivet and Turon, 2019), this suggests that the ability of firms to price discriminate in online markets may require policy interventions.

To understand the corollary, consider the case in which search is frictionless and $\theta_j \sim U[0,1]$. When firms cannot price discriminate, they offer a uniform price, which is $p^u = \sqrt{2} - 1 \approx 0.414$ by the results of lemma 1. When firms can price discriminate, they offer prices according to the rule $p^*(\theta_j) = 0.5\theta_j$. Because $\theta_j \sim U[0,1]$, prices are thus uniformly drawn from $[0, 0.5]$ and the average personalized price is 0.25. Hence, the average price consumers have to pay is lower under price discrimination, which implies that consumer surplus will be higher.

Now consider the case in which $c > 0$. When firms can price discriminate, they price according to the rule $p^*(\theta_j) = \theta_j$ and consumer surplus is zero. By contrast, consumer surplus will be strictly positive when firms cannot price discriminate. This is because firms will set a uniform price p^u that is strictly below 1 — else, they would make zero profits. Because a positive measure of consumers thus attains positive utility in equilibrium, consumer surplus must be strictly positive. This establishes that, when consumers are rational and there are search frictions, consumer surplus is strictly lower when firms can price discriminate.

4 Equilibrium analysis: Naive consumers

The previous section computed the competitive equilibria of our model when all consumers are rational. Now, we repeat this exercise with naive consumers. As discussed in section 3, we modify our equilibrium concept and impose that all consumers are cursed as in Eyster and Rabin (2005). Cursed consumers incorrectly expect that the price they are offered by a firm is independent of their preferences. Instead, cursed consumers have expectations that are merely correct ex ante: If firm j plays the strategy $p_j^*(\theta_j)$, the distribution of prices $H_j^*(y)$ this firm offers to consumers is given by

$$H_j^*(y) = \int_0^1 Pr(p_j^*(\theta_j) < y|\theta_j) f(\theta_j) d\theta_j. \tag{7}$$

Naive consumers expect that the price they receive at firm j is randomly drawn from this distribution. In particular they believe that the price the receive is independent of their match value at the respective firm. In an equilibrium in which a firm j prices according to the rule $p_j^*(\theta_j)$, any consumer thus continues searching after visiting firm $-j$ if and only if

$$\underbrace{\int_0^1 \int_0^1 \max\{S_{-j}, \theta_j - p_j^*(x)\} dF(x) dF(\theta_j) - c}_{\text{Expected utility after searching}} - S_{-j} > 0, \tag{8}$$

where S_{-j} is the surplus she received at the first firm she visited.

When consumers are naive and search costs are not prohibitively high, firms can no longer sustain perfect price discrimination as an equilibrium. We formalize this in the following proposition and characterize the equilibria that emerge instead:

Proposition 3 *Suppose firms can price discriminate, that all consumers are naive, and that $\theta_j \sim U[0,1]$. For every $c > 0$, there is a unique symmetric equilibrium.*

(1) If $c \leq \frac{1}{24}$, the firms' equilibrium pricing strategy is $p^*(\theta_j) = 0.5\theta_j$, and all consumers visit both firms.

(2) If $c \in \left(\frac{1}{24}, \frac{1}{6}\right)$ there exists a unique $\hat{S}^*(c)$ such that the firms' equilibrium strategy is

$$p^*(\theta_j) = \begin{cases} 0.5\theta_j & \theta_j < \frac{\hat{S}^*(c)}{1-\hat{S}^*(c)} \\ \theta_j - \hat{S}^*(c) & \theta_j \geq \frac{\hat{S}^*(c)}{1-\hat{S}^*(c)}, \end{cases} \tag{9}$$

and consumers continue searching if and only if they are offered a surplus below $\hat{S}^*(c)$.

(3) If $c \geq \frac{1}{6}$, the firms' equilibrium pricing strategy is $p^*(\theta_j) = \theta_j$ and no consumer visits both firms.

The proof of this proposition proceeds along two steps. First, we establish necessary conditions which characterize the structure of any symmetric equilibrium in this framework. Second, we show that there is a unique candidate for a symmetric equilibrium at any level of search costs and finally verify that these candidates are indeed equilibria.

Any equilibrium candidate can be fully characterized by the cutoff $\hat{S}$, where $\hat{S}$ describes the search strategy of consumers. To see this, note first that a consumer's incentives to search only depend on the surplus they initially obtain. Thus, they continue searching if and only if the surplus they initially receive is below a cutoff, which we call $\hat{S}$.

For a given $\hat{S}$, there is a unique pricing strategy that can be sustained in symmetric equilibrium. To see this, note first that optimality of the firm's pricing strategy requires that the surplus a consumer is offered is weakly increasing in her match value.[12] Together with the fact that consumers continue searching if and only if the offered surplus is below $\hat{S}$, this implies that there exists a cutoff type $\hat{\theta}$ such that consumers continue searching after visiting some firm if and only if their match value at this firm is below $\hat{\theta}$.

In a symmetric equilibrium and for a given $\hat{S}$, the prices firms can offer to consumers with types below and above $\hat{\theta}$ are uniquely pinned down. The pricing problem of a firm j facing a consumer with type $\theta_j < \hat{\theta}$ resembles the optimization problem of a bidder in a standard first-price auction, given that any such consumer will visit both firms. Thus, the price the firm offers to such a consumer in equilibrium must be given by $p^*(\theta_j) = \mathbb{E}[\theta_{-j}|\theta_{-j} < \theta_j] = 0.5\theta_j$. When facing a type $\theta_j > \hat{\theta}$, the only price that can be optimal and consistent with the characterization of equilibrium is the price $\theta_j - \hat{S}$, which makes the consumer indifferent between searching and not.[13] Finally, note that a firm must be indifferent between offering the prices $0.5\theta_j$ and $\theta_j - \hat{S}$ at $\theta_j = \hat{\theta}$, which implies that $\hat{\theta} = \frac{\hat{S}}{1-\hat{S}}$ must hold in equilibrium.[14] Summing up, we can thus note the following: Given a search strategy defined by $\hat{S}$, a mutually optimal pricing strategy of the firms must take the following form:

$$p^{BR}(\theta_j; \hat{S}) = \begin{cases} 0.5\theta_j & \theta_j < \frac{\hat{S}}{1-\hat{S}} \\ \theta_j - \hat{S} & \theta_j \geq \frac{\hat{S}}{1-\hat{S}}, \end{cases} \tag{10}$$

It remains to define when a given search strategy $\hat{S}$ constitutes an equilibrium. To do so, we must characterize the optimal search strategy of consumers for a given pricing function, which we call $\hat{S}^{BR}(p(\cdot))$. If firms price according to the rule $p^{BR}(\theta_j; \hat{S})$, a consumer whose initial surplus was $\hat{S}^{BR}(p(\cdot))$ must be indifferent between searching and not, so this object must solve:

$$\int_0^1 \int_0^1 \max\{\hat{S}^{BR}(\cdot), \theta_j - p^*(x)\} dx d\theta_j - c - \hat{S}^{BR}(\cdot) = 0 \tag{11}$$

An equilibrium search strategy (as defined by the cutoff $\hat{S}^*$) must be a fixed point of $\hat{S}^{BR}(p^{BR}(\theta_j; \hat{S}))$. One can show that this function always has a unique fixed point, which means that there exists an unique equilibrium. Roughly speaking, existence and uniqueness

[12] This is because the expected demand of any consumer only depends on the surplus that is offered to her, which implies that the derivative of profits with respect to the offered surplus is larger, the higher a consumer's type is.

[13] At any price below this price, the demand received by a firm is inelastic, implying the existence of a profitable deviation. When receiving a price above $\theta_j - \hat{S}$, consumers continue searching, which contradicts the postulated equilibrium structure.

[14] To see this, note that the profits of offering the price $p_j = \theta_j - \hat{S}$ are $(\theta_j - \hat{S})(1 + \hat{\theta})$ and the profits of offering the price $p_j = 0.5\theta_j$ are $(\theta_j)^2$

of such a solution can be verified using continuity and monotonicity of the outlined best response functions.

Unless $c \geq 1/6$, perfect price discrimination will not emerge in equilibrium. To understand this result, suppose that firms set prices according to the rule $p^*(\theta_j) = \theta_j$. From an ex ante perspective, the distribution of the price a consumer receives at any firm is the uniform distribution on $[0, 1]$. Naive consumers thus expect to receive a price drawn from this distribution at any firm they may visit, independent of their match value for this firm's product. When arriving at the first firm they randomly visit, all consumers will receive zero surplus by the firms' pricing rule. Because they are naive and search costs are small enough, all consumers will thus continue searching and visit both firms. But given this search behaviour, said pricing strategy is not optimal for firms: In the postulated equilibrium, both firms will sell to any consumer with probability 0.5. But then, any firm would always strictly prefer to set a price just below a consumer's match value to make the sale with certainty.

The intuition behind this result is reminiscent of Gamp and Krähmer (2023), who show that the Diamond paradox can fail to manifest in the presence of consumers who overestimate the spread of outcomes attainable via search. Just as the naive consumers in our framework, such consumers will search even when there are no gains to attain, which avoids an unfavorable outcome by ensuring competition between firms.

If search costs are sufficiently small, even consumers with a high willingness-to-pay for the initially inspected product continue searching in the hope for a similarly high match value but a lower price, as they don't understand the correlation between their match value for a firm's product and the price this firm will offer. When search costs are at intermediate levels, only consumers with low willingnesses-to-pay for the initial product they inspect will continue searching, whereas firms dissuade consumers with high types from continuing to search by offering them their cutoff surplus.[15] This form of search deterrence is only profitable at intermediate levels of search costs because the cutoff surplus needed to deter search by any consumer is falling in the level of search costs.

When considering the results of said proposition, it is instructive to visualize how the key equilibrium components, namely $\hat{\theta}^* := \frac{\hat{S}^*}{1-\hat{S}^*}$. and $\hat{S}^*$, depend on the level of search costs:

[15]When a consumer's match value is low, it is unprofitable for firms to deter search: For example, consumers with $\theta_j < \hat{S}$ would only buy directly if offered a negative price by the first firm they visit, which is clearly unprofitable for this firm.

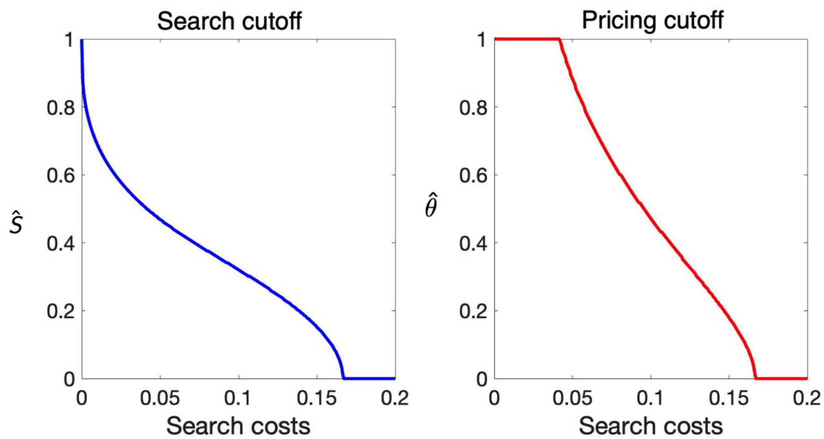

Figure 1: Equilibrium outcomes

Note that lower values of $\hat{S}^*$ mean that consumers have lower incentives to search. Similarly, lower values of $\hat{\theta}^*$ imply that less consumers search on the equilibrium path: If $\hat{\theta}^* = 1$, all consumers visit both firms. Conversely, if $\hat{\theta}^* = 0$, no consumer visits both firms.

Having characterized the equilibria under naivety, we are now ready to provide some more insights regarding the effects of price discrimination on consumer surplus. To do so, recall that we have defined consumer surplus as the ex-ante expected utility of consumers. In a symmetric equilibrium in which the firms' pricing strategy is $p^*(\theta_j)$ and consumers continue searching if and only if their initial surplus is below $\hat{S}^*$, consumer surplus is equal to:

$$C = \int_0^1 \int_0^1 u(\theta_j, \theta_{-j}) dF(\theta_j) dF(\theta_{-j}), \tag{12}$$

where $u(\theta_j, \theta_{-j})$ is given by:

$$\mathbb{1}[\theta_j - p^*(\theta_j) \geq \hat{S}^*](\theta_j - p^*(\theta_j)) + \mathbb{1}[\theta_j - p^*(\theta_j) < \hat{S}^*] \max\{\theta_j - p^*(\theta_j), \theta_{-j} - p^*(\theta_{-j}), 0\} \tag{13}$$

Corollary 2 *Suppose $\theta_j \sim U[0,1]$ and that all consumers are naive. For any $c \leq 1/8$, consumer surplus is strictly higher when firms can price discriminate.*

When firms cannot price discriminate, they will both offer a uniform price which replicates the equilibrium from the Wolinsky (1986). The resulting consumer surplus in the absence of price discrimination is plotted in red. When firms can price discriminate, the equilibrium from proposition (3) will be played. The associated consumer surplus is plotted in blue.

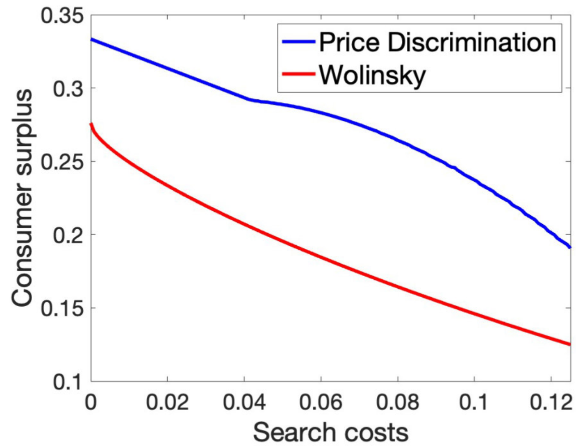

Figure 2: Consumer surplus under naivety

This figure visualizes that, when all consumers are naive, price discrimination raises consumer surplus unless search costs are prohibitively high (i.e. as long as $c \geq 1/8$, which guarantees an active market in Wolinsky (1986)). The reason for this revolves around one of the key insights of our paper: The welfare effects of price discrimination crucially depend on the strength of the search incentives of consumers. The search incentives of naive consumers are stronger under price discrimination than under uniform pricing — thus, there is more competitive pressure, which is beneficial for consumer surplus.

5 Policy implications

5.1 A framework with rational and naive consumers

In this subsection, we explore the effects of various regulatory approaches. To make progress in this endeavour, it is crucial to develop and solve a framework in which there are both rational and naive consumers, given that this is a feature of real-world markets (Internet Policy Review, 2019). We consider the following framework:

There is a unit mass of consumers, a share α of whom is sophisticated. A share $1-\alpha$ of consumers is naive. We assume that sophistication is uncorrelated with the distribution of valuations obtained at each firm: For any consumer, the match value at either firm is drawn independently and identically from the distribution F on $[\underline{\theta}, \overline{\theta}]$ with density f. For convenience, we restrict attention to the case in which $F = U[0,1]$ throughout the following analysis. Firms do not observe whether an arriving consumer is sophisticated or naive. In the following, we explicitly distinguish the two consumer *groups*, namely naive and sophisticated consumers. As before, we refer to a consumer's match value θ_j as her type.

A symmetric equilibrium in this framework consists of the objects $\left(p^*(\theta), \hat{S}^n, \hat{S}^r\right)$, where:

- The firms' pricing strategy $p^*(\theta)$ must be optimal for any given firm, given that its rival prices according to this strategy and rational (naive) consumers continue searching after visiting the first firm if and only if their initial surplus is below $\hat{S}^r$ ($\hat{S}^n$).

- Rational consumers continue searching if and only their initial surplus S_{-j} satisfies equation (3) for the given $p^*(\theta)$, which holds true if and only if $S_{-j} < \hat{S}^r$.

- Naive consumers continue searching if and only their initial surplus S_{-j} satisfies equation (8) for the given $p^*(\theta)$, which holds true if and only if $S_{-j} < \hat{S}^n$.

Throughout the following analysis, we focus on these equilibria. For convenience, we work with the equilibrium surplus function $S(\theta) = \theta - p^*(\theta)$ rather than the prices.

We start by establishing properties that any such equilibrium must satisfy. Recall that the decision to search for any given consumer (fixing the group the consumer belongs to) only depends on the surplus $S^*(\theta) = \theta - p(\theta)$ she receives at the first firm. Notice that, again, this surplus function must be weakly increasing in θ. This is because the realization of θ is uninformative about a consumer's group affiliation and her search history. Hence, a consumer's incentives to search and buy are, in expectation, identical for different types that are offered the same surplus. If a firm does not find it optimal to raise the surplus for some high type to increase the probability of a transaction, it cannot find this optimal for some lower type, for whom such an increase of surplus would be less profitable.

The fact that the surplus function $S^*(\theta)$ is increasing implies that the equilibrium search behaviour of consumers can be characterized by two cutoffs $\hat{\theta}^r$ and $\hat{\theta}^n$: Rational and naive consumers continue searching after visiting the first firm if and only if their match value at this firm is below $\hat{\theta}^r$ and $\hat{\theta}^n$, respectively. For simplicity, we restrict attention to equilibria in which all naive consumers visit both firms. This emerges as an equilibrium outcome if search costs are small enough. These equilibria are characterized by the following proposition:

Proposition 4 (Equilibria with rational & naive consumers)
Suppose $\theta_j \sim U[0,1]$ and consider an equilibrium in which all naive consumers search beyond the first firm and rational consumers search beyond the first firm if and only if their initial surplus is below $\hat{S}^r$. Then, firms offer a surplus function given by:

$$S^*(\theta) = \begin{cases} 0.5\theta & \theta < \hat{\theta}^r \\ \tilde{S}(\theta) & \theta \geq \hat{\theta}^r \end{cases} \tag{14}$$

The function $\tilde{S}(\theta)$ is given by:

$$\tilde{S}(\theta) = \frac{1}{\gamma(\hat{\theta}^r) + \theta}\left[0.5(\theta)^2 - 0.5(\hat{\theta}^r)^2 + \big(\gamma(\hat{\theta}^r) + \hat{\theta}^r\big)\hat{S}^r\right] \quad ; \quad \gamma(\hat{\theta}^r) = \frac{\alpha}{1-\alpha}\big(0.5 + 0.5\hat{\theta}^r\big) \tag{15}$$

The cutoff $\hat{\theta}^r$ must solve:

$$0.5(\hat{\theta}^r)^2 - (\hat{\theta}^r - \hat{S}^r)\big[\hat{\theta}^r + 0.5\alpha(1-\hat{\theta}^r)\big] = 0 \tag{16}$$

Having established the properties of any such equilibrium, we briefly comment on the way in which such an equilibrium is calculated: For a given candidate search rule $\hat{S}^r$, one calculates the firm-optimal surplus function $S^*(\theta)$ and the associated cutoff $\hat{\theta}^r$. If, given $S^*(\theta)$ and $\hat{\theta}^r$, all naive consumers optimally visit both firms and rational consumers optimally continue searching if and only if their initial surplus is below $\hat{S}^r$, one has found an equilibrium.

When studying the implications of various policy suggestions, one needs to examine the welfare of different groups. To that end, note that the ex-ante expected utility of rational consumers in an equilibrium of the aforementioned form (in which firms price discriminate), which we denote by $CS^{r,d}(\alpha)$, is given by:

$$\int_0^{\hat{\theta}^r(\alpha)}\int_0^1 \big[\max\{\theta_1 - p^*(\theta_1;\alpha), \theta_2 - p^*(\theta_2;\alpha)\} - c\big]d\theta_2 d\theta_1 + \int_{\hat{\theta}^r(\alpha)}^1 (\theta_1 - p^*(\theta_1;\alpha))d\theta_1 \tag{17}$$

We condition on the parameter α in the definition, given that this is a crucial parameter throughout the following analysis.

The ex-ante expected utility of naive consumers in an equilibrium of the aforementioned form (in which firms price discriminate) is given by:

$$CS^{n,d}(\alpha) = \int_0^1\int_0^1 \big[\max\{\theta_1 - p^*(\theta_1;\alpha), \theta_2 - p^*(\theta_2;\alpha)\} - c\big]d\theta_2 d\theta_1 \tag{18}$$

The ex-ante expected utility of consumers (naive or rational) when firms cannot price discriminate and set a uniform price $p^{nd,*}$ is given by:

$$CS^{nd} = \int_0^{\hat{\theta}^{nd}}\int_0^1 \big[\max\{\theta_1 - p^{nd,*}, \theta_2 - p^{nd,*}\} - c\big]d\theta_2 d\theta_1 + \int_{\hat{\theta}^{nd}}^1 (\theta_1 - p^{nd,*})d\theta_1 \tag{19}$$

where $\hat{\theta}^{nd}$ represents the equilibrium search behaviour of consumers in the sense that consumers continue searching after visiting the first firm if and only if their match value at this firm is below $\hat{\theta}^{nd}$. Having lined out the framework, we are now ready to study the impact of various regulatory proposals.

5.2 Raising awareness of price discrimination

Regulatory bodies such as the OECD and the European Commission have expressed their view that increasing consumer awareness of price discrimination is an important desideratum. In fact, recent regulation in the European Union mandates firms which engage in price discrimination to inform consumers about this fact (European Commission, 2019). Within our framework, this policy can be viewed as an increase of α, i.e. as an increase of the share of rational consumers.

In this section, we establish that such a policy will have undesirable effects. This was already foreshadowed by our results in sections 4 and 5: When firms price discriminate, consumer welfare is strictly higher when all consumers are naive, compared to the case in which all consumers are rational. Now, we demonstrate that these insights also hold on the margin: If firms can price discriminate, increases in the share of rational consumers (α) lead to reductions of consumer welfare. We visualize this result in the following graphs, where we plot the expected utility of rational consumers and naive consumers for different parameter combinations. Each graph corresponds to a fixed level of search costs $c \in \{0.02, 0.03, 0.04\}$, and different values of $\alpha \in (0, 1)$ are plotted on the x-axis.

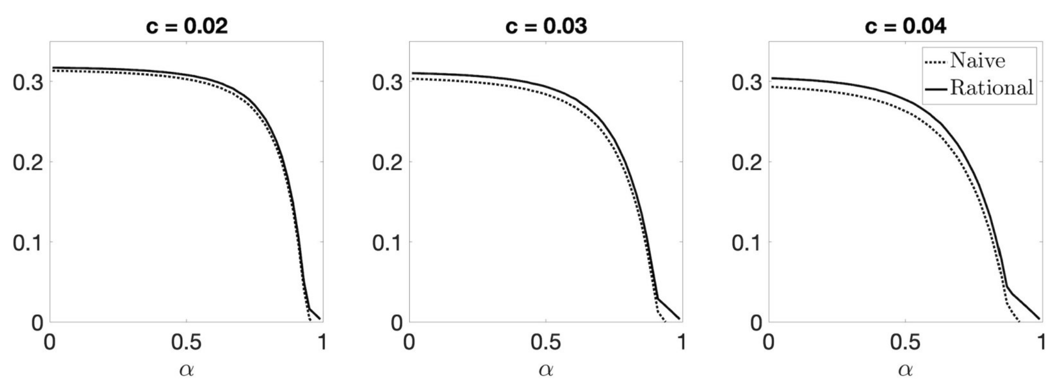

Figure 3: Consumer welfare & naivety

This figure confirms that total consumer surplus is falling in the share of rational consumers. The intuition underlying this result is as follows: Given that naive consumers overestimate their gains of search, they have stronger search incentives than rational consumers. This means that rational consumers visit (in expectation) less firms than naive consumers. As the share of rational consumers increases, the average number of firms consumers visit thus falls. This weakens competition between the firms and enables them to set higher prices, which is to the detriment of every consumer. In other words, rational consumers impose a negative externality on naive consumers, because the presence of rational consumers reduces the intensity of competition between firms.

5.3 A right to anonymity

An alternative regulatory approach would be to endow consumers with a right to anonymity, which enables every consumer to ensure that firms cannot price discriminate against them (e.g. by browsing using a VPN). Formally, we study the effects of such a regulatory approach by augmenting the framework laid out subsection 5.1 in the following way: A share $1 - \beta$ of all consumers are naive — these consumers cannot anonymize. The specification that naive consumers never anonymize is quite natural, given that naive consumers are not aware that firms price discriminate and would not expect to receive any benefits by anonymizing. A share β of consumers are rational and choose, before they begin the search process, whether or not to anonymize. As before, we assume that firms do not know whether an arriving consumer is rational or naive (ex ante) and that consumers' match values are uniformly distributed on the unit interval. Moreover, we impose that firms' beliefs are passive.

If a rational consumer chooses not to anonymize, every firm she visits will receive a perfect signal about the consumer's willingness to pay for its product. In other words, such a consumer is pooled with naive consumers and the game from subsection 5.1 is played after the consumer's initial decision. If the consumer chooses to anonymize, both firms never receive any information about the consumer. In that case, the standard game of Wolinsky (1986) will be played after the consumer anonymizes. Crucially, the consumer does not know her match values for the firms' products when making the decision whether to anonymize.

A rational consumer will choose to anonymize if and only if this is individually optimal for her. If she anonymizes, the expected utility she attains is equal to CS^{nd} as given in equation (19). This always holds true by our assumption that firms have passive beliefs. The expected utility a rational consumer attains by not anonymizing depends on how many rational consumers anonymize in equilibrium. To see this, define the probability with which a rational consumer anonymizes in equilibrium as ϕ. Consider an equilibrium in which all rational consumers anonymize (i.e. $\phi = 1$ holds). If a rational consumer does not anonymize, the expected utility she attains is equal to the expected utility defined in equation (17) when $\alpha = 0$ (i.e. if no consumers for whom firms observe information are rational). If rational consumers never anonymize in equilibrium, the expected utility a rational consumer attains by not anonymizing is given by expected utility defined in equation (17) when $\alpha = \beta$

(i.e. a share β of all consumers for whom firms have information are rational). Going forward, we thus define $CS^{r,d}(\phi)$ as the expected utility a rational consumer attains by not anonymizing, given that rational consumers anonymize with probability ϕ in equilibrium, which is equivalent to the expected utility defined in equation (17) if $\alpha = \frac{\beta(1-\phi)}{\beta(1-\phi)+(1-\beta)}$.

Before moving forward, note that $CS^{r,d}(\phi)$ is increasing in ϕ. As ϕ increases, a larger mass of rational consumers anonymize. This means that consumers for whom firms observe information are more likely to be naive. Given that naive consumers search more intensely, firms optimally offer lower prices when they can price discriminate, which translates into higher surplus for consumers that do not anonymize.

These arguments imply that the equilibrium share of rational consumers who anonymize can be determined using a case distinction: If $CS^{nd} \leq CS^{r,d}(0) < CS^{r,d}(1)$, all rational consumers will not anonymize in equilibrium. Alternatively, $CS^{nd} \in \big(CS^{r,d}(0), CS^{r,d}(1)\big)$, may hold, in which case rational consumers anonymize with some interior probability in equilibrium. In particular, the equilibrium share ϕ will then be such that $CS^{nd} = CS^{r,d}(\phi)$ holds, i.e. rational consumers are indifferent between anonymizing and not. Note that $CS^{nd} < CS^{r,d}(1)$ must hold by the insights of section 5: When consumers' match values are drawn from $U[0,1]$ and all consumers for whom firms have information are naive, naive consumers attain greater expected utility when price discrimination is possible than when it is not. Because rational consumers attain higher expected utility than naive consumers (holding the pricing strategy of firms fixed), a rational consumer would thus be better off under price discrimination when all consumers are naive, i.e. $CS^{nd} < CS^{r,d}(1)$.

We visualize these results in the following graphs, where we plot the share of rational consumers (ϕ) who anonymize in equilibrium. Every graph corresponds to a fixed level of search costs $c \in \{0.02, 0.03, 0.04\}$, and different values of β are plotted on the x-axis:

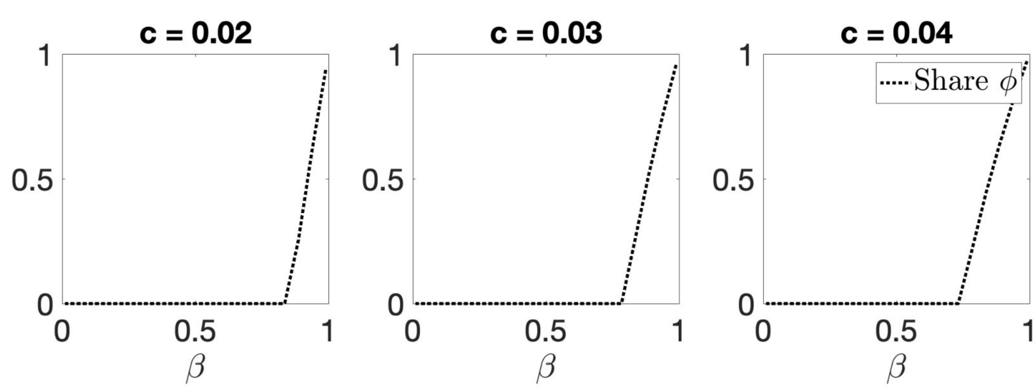

Figure 4: Share of anonymizing consumers

When β is small, the majority of consumers in the market are naive. In the absence of a right to anonymity, the large amount of naive consumers generates substantial competitive pressure, which means that the equilibrium prices under price discrimination would be comparatively low. Thus, rational consumers have no incentives to anonymize. This changes when β increases, i.e. when there are less naive consumers in the market and thus, price discriminating firms face less strong competitive pressure. Then, the equilibrium share ϕ will rise to an interior level at which rational consumers are exactly indifferent between anonymizing and not.

We now compare the effects of the establishment of a right to anonymity to two other regulatory approaches, namely the prohibition of price discrimination and a laissez-faire approach in which there is no regulatory intervention. When price discrimination is prohibited, both rational and naive consumers attain expected utility equal to CS^{nd} as defined in equation (19). In the absence of regulatory interventions, rational and naive consumers attain expected utility equal to $CS^{r,d}(\beta)$ as defined in equation (17) and $CS^{n,d}(\beta)$ as defined in equation (18), respectively.

When the right to anonymity is utilized (i.e. ϕ is interior), all rational consumers attain the same expected utility as when price discrimination is prohibited.[16] In the following graphs, we plot the expected utility of naive consumers under the various regulatory approaches. For expositional clarity, we focus on parameters for which some rational consumers utilize their right to anonymity, i.e. only consider $\beta \in (0.825, 1)$. If the right to anonymity is not utilized by any consumer, the establishment of this right yields the same expected utility for every consumer as a laissez-faire approach. Any graph corresponds to a fixed level of search costs $c \in \{0.02, 0.03, 0.04\}$, while varying levels of β are plotted on the x-axis. The expected utility that naive consumers attain under the laissez-faire approach, when firms cannot price discriminate, and when there is a right to anonymity, are labeled "Laissez-faire", "No data", and "Anonymity", respectively:

[16] This is because, in equilibrium, any rational consumer must be indifferent between anonymizing, which would yield expected utility equal to CS^{nd}, and not anonymizing, which yields expected utility equal to $CS^{r,d}(\phi)$. When price discrimination is prohibited, all consumers attain expected utility equal to CS^{nd}.

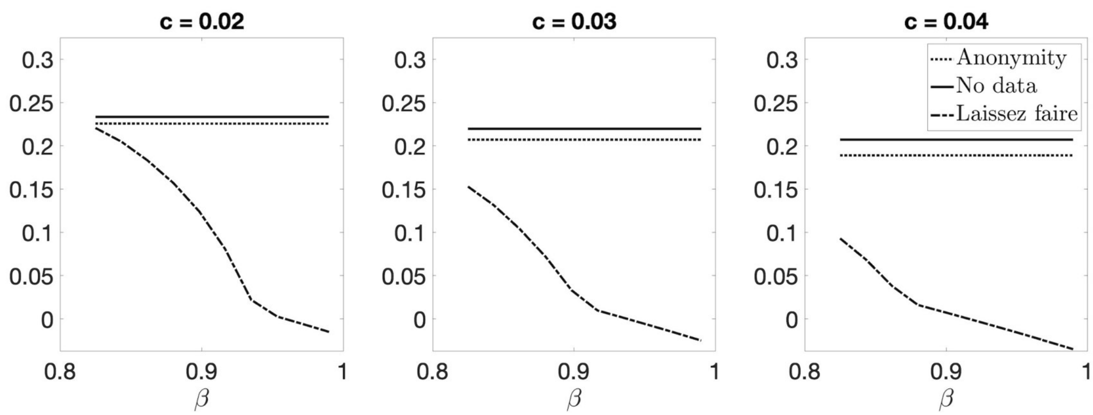

Figure 5: Consumer welfare & policy measures

These graphs establish two insights: First, the establishment of a right to anonymity will (if it is utilized) yield higher welfare for naive consumers than a laissez-faire approach, even though naive consumers never exercise this right. This is because rational consumers who exercise their right to anonymity reduce the mass of rational consumers for whom firms receive information, which means that the magnitude of the negative externality this consumer group imposes on naive consumers is reduced. Second, naive consumers attain lower expected utility than when price discrimination is prohibited. This holds true even though naive consumers would attain maximal expected utility if firms price discriminate against them and all rational consumers anonymize. However, this is not an equilibrium outcome — if all rational consumers anonymize, the prices that price discriminating firms set are so low that rational consumers would not optimally anonymize.

6 Conclusion

We have integrated the possibility of first-degree price discrimination as defined by Pigou (1920) into the classic Wolinsky (1986) framework of sequential search for horizontally differentiated products. In our analysis, we have considered different model variants, distinguishing (i) whether search is frictionless or not, (ii) whether firms can price discriminate or not, and (iii) whether consumers are aware that firms price discriminate or not. This approach has enabled us to study the effect of price discrimination on consumer surplus and how this depends on the level of search frictions and consumer sophistication.

Our work sheds new light on an old question, namely whether consumers benefit when firms can price discriminate against them. The relevance of this question is ever increasing, given that granular consumer data is becoming available to firms in online markets and there is mounting evidence of price discrimination in these markets. We show that, if firms can price discriminate and there are search frictions, there is a unique perfect Bayesian equilibrium in which consumer surplus is zero. This finding adds important context to the classic result of Thisse and Vives (1988), who find that price discrimination raises consumer surplus in the classic Hotelling framework, in which there are no search frictions. The relevance of our result is emphasized by the empirically documented fact that search frictions in online markets are substantial (Koulayev, 2014; De los Santos, 2018; Jolivet and Turon, 2019).

When studying these issues, establishing how the equilibrium outcomes are shaped by the extent to which consumers understand that firms price discriminate is important. This is because consumer awareness of the way in which their data is used is minimal and regulatory bodies such as the OECD and the European commission have stressed that improving consumer awareness of price discrimination is an important policy goal. We show that such policies may have unintended consequences: In our framework, consumer naivety with respect to the fact that firms price discriminate is beneficial for them. Intuitively, naive consumers overestimate their incentives to search, which induces them to visit multiple firms, thus enforcing favorable outcomes by promoting competition between firms.

A Proofs

A.1 Proof of Lemma 1

Suppose that search is frictionless. We derive the symmetric perfect Bayesian equilibrium in pure strategies, which builds on the arguments made in Wolinsky (1986).

Part 1: Characterization and existence of a symmetric equilibrium

Consider a symmetric equilibrium in which all firms set the price p^*. In general, the profit of a firm that sets an arbitrary price p_j is given by:

$$\Pi(p_j) = p_j \Big[\int_0^1 \int_0^1 \mathbb{1}[\theta_j - p_j > \theta_{-j} - p^*] \mathbb{1}[\theta_j > p_j] d\theta_{-j} d\theta_j \Big] = p_j \int_{p_j}^1 F(\theta_j - p_j + p^*) d\theta_j \tag{20}$$

By assumption, all consumers visit both firms. Thus, a consumer buys at firm j if and only if the match value at this firm is above the offered price <u>and</u> the surplus at firm j is highest. By independence of types across firms and given our information structure, profits thus take the aforementioned form.

For prices in an open ball around the equilibrium p^*, $F(\theta_j - p_j + p^*) \in [0,1]$ holds, so the profit function becomes:

$$\Pi(p_j) = p_j \Big[\int_{p_j}^1 (\theta_j - p_j + p^*) d\theta_j \Big] \tag{21}$$

The first-order condition reads:

$$\Big[\int_{p_j}^1 (\theta_j - p_j + p^*) d\theta_j \Big] + p_j \Big[\int_{p_j}^1 (-1) d\theta_j - (p^*) \Big] = 0 \tag{22}$$

Evaluating this at $p_j = p^*$ yields:

$$0.5(1 - p^* + p^*)^2 - 0.5(p^*)^2 + p^* \big[-(1-p^*) - p^* \big] = 0 \iff (p^*)^2 + 2p^* - 1 = 0 \tag{23}$$

Thus, an equilibrium price p^* must solve:

$$p^* = \frac{-2 + \sqrt{8}}{2} = 0.414 \tag{24}$$

There exists an equilibrium in which both firms set this price. Previous arguments establish that this price is locally optimal. Global optimality follows from strict concavity of $\Pi(p_j)$ in a symmetric equilibrium.

Part 2: Uniqueness

By the previous characterization, no other symmetric equilibrium can exist.

Moreover, there exists no asymmetric equilibrium. Suppose, for a contradiction, that there exists a pure-strategy equilibrium in which the two firms, call them $j \in \{1, 2\}$, set different prices $p_1^* \neq p_2^*$. Suppose further, without loss, that $p_1^* \leq p_2^*$.

We first show that the difference inbetween these two prices must be weakly smaller than 0.5. Suppose, for a contradiction, that $d = p_2^* - p_1^* > 0.5$. Given this setup, $p_2^* > 0.5$ must hold.

The monopoly price is equal to 0.5. The derivative of monopoly profits will be strictly negative for all $p_2 > 0.5$, i.e. we have:

$$\int_{p_2}^{1} 1 d\theta_j - p_2(1) < 0 \tag{25}$$

At $p_2 = p_2^*$, the derivative of our profit function will be strictly negative, because:

$$\int_{p_2}^{1} \underbrace{\frac{\theta_j - p_2 + p_1^*}{1 - p_2 + p_1^*}}_{<1} d\theta_j - p_2 < \int_{p_2}^{1} 1 d\theta_j - p_2 < 0 \tag{26}$$

Thus, there would be a profitable downward deviation from p_2^* in our supposed equilibrium, a contradiction.

Now consider arbitrary p_1^* and p_2^*, with $d = p_2^* - p_1^* \in [0, 0.5]$. The first-order conditions are:

$$D^1(p_1^*) + p_1^* \frac{\partial D^1(p_1^*)}{\partial p_1} = 0 \iff \int_{p_1^*}^{1} (\theta_j - p_1^* + p_2^*) d\theta_j - p_1^*(1 - p_1^* + p_2^*) = 0 \tag{27}$$

$$D^2(p_2^*) + p_2^* \frac{\partial D^2(p_2^*)}{\partial p_2} = 0 \iff \int_{p_2^*}^{1} (\theta_j - p_2^* + p_1^*) d\theta_j - p_2^*(1 - p_2^* + p_1^*) = 0 \tag{28}$$

We can apply the implicit function theorem to pin down best response functions $p_1^{br}(p_2)$ and $p_1^{br}(p_1)$ which pin down the unique optimal price of firm j as a function of the set price of firm $-j$. To see that these best response functions are always uniquely pinned down, note that the profit function of firm j is strictly concave in it's own price and that the appropriate boundary conditions are satisfied. Strict concavity holds because:

$$\frac{\partial \Pi^j}{\partial p_j} = -2(1 - p_j + p_{-j}^*) + p_j^* < -2(1 - 0.5) + p_1^* < 0 \tag{29}$$

This establishes that profit functions must be globally concave and the best response functions are uniquely pinned down.

In equilibrium, the price p_1^* must satisfy the following:

$$p_1^* - p_1^{br}\left(p_2^{br}(p_1^*)\right) = 0 \tag{30}$$

Let's investigate the derivative of the best function of a generic firm j, namely $p_j^{br}(p_{-j})$. This must satisfy the following equation:

$$T^j = \int_{p_j}^{1} (\theta_j - p_j + p_{-j}) d\theta_j - p_j(1 - p_j + p_{-j}) = 0 \tag{31}$$

This constitutes a second-order polynomial equation, the solution to which is given by:

$$p_j^{br}(p_{-j}) = \frac{2}{3}(1 + p_{-j}) - \frac{1}{3}\sqrt{(1 + p_{-j})^2 + 3(p_{-j})^2} \tag{32}$$

This derivative is below 1 because:

$$\frac{\partial p_j^{br}(p_{-j})}{\partial p_{-j}} = \frac{2}{3} - \frac{1}{3}\frac{1}{2}\frac{2(1 + p_{-j}) + 6p_{-j}}{\sqrt{(1 + p_{-j})^2 + 3(p_{-j})^2}} < \frac{2}{3} \tag{33}$$

Continuity of these best response functions follows from the implicit function theorem. Since the derivatives of the best response functions are below 1, there must be a unique fixed point p_1^* of the above expression. This completes the proof of uniqueness.

A.2 Proof of Proposition 1

Part 1: Equilibrium existence

We will show that, in the case of frictionless search, there exists an equilibrium in which firms set prices according to the rule $p^*(\theta) = 0.5\theta$.

Consider the pricing problem of a firm j who faces a consumer with match value θ_j. Given that the other firm prices according to $p^*(\theta_{-j}) = 0.5\theta_{-j}$, the profits of firm j for prices $p_j \leq \theta_j$ are:

$$\Pi^{0,*}(p_j) = p_j \int_0^1 \mathbb{1}[\theta_j - p_j > 0.5\theta_{-j}]d\theta_{-j} = p_j \int_0^{\min\{1,2(\theta_j - p_j)\}} 1 d\theta_{-j} \tag{34}$$

Suppose $\theta_j < 1$. For prices in an open ball around any equilibrium $p^*(\theta_j) < 1$, profits thus become:

$$\Pi^0(p_j) = 2p_j(\theta_j - p_j) \implies \frac{\partial \Pi^{0,*}(p_j)}{\partial p_j} = 0 \iff p_j^*(\theta_j) = 0.5\theta_j \tag{35}$$

The profit function $\Pi^{0,*}(p_j)$ is strictly concave and maximized by our price. For prices $p_j \in [\theta_j - 0.5, \theta_j]$, true profits are equal to $\Pi^{0,*}(p_j)$. For prices $p_j \in [0, \theta_j - 0.5]$, true profits are below $\Pi^{0,*}(p_j)$. Thus, there the postulated price is globally optimal and the price strategy we defined is mutually optimal.

Part 2: Uniqueness

There is a unique pricing equilibrium by the following logic: The problem of any firm is isomorphic to that of a bidder in a first-price auction, for which it is well-known that there exists a unique equilibrium.

To see why the problems are isomorphic, note that a firm who faces a consumer with type θ_j effectively chooses the surplus $S_j = \theta_j - p_j$ it offers to the consumer. This surplus can be understood as a "bid" in the language of first-price auctions. This is because, given the equilibrium surplus function $S^*(\theta)$, the profit of a firm in our pricing problem is given by:

$$\Pi^0(S_j) = \big(\theta_j - S_j\big) Pr\big(S_j > S^*(\theta_{-j})\big) \tag{36}$$

This profit function is exactly equal to the profit function of a bidder in a first-price auction.

A.3 Proof of Proposition 2

The proof that said equilibrium exists mirrors the discussion in the text.

To establish uniqueness, we define $\bar{S}_j$ as the supremum of $S_j^*(\theta_j)$ on $\theta_j \in [0,1]$, $\bar{S}_{-j}$ as the supremum of $S_{-j}^*(\theta_{-j})$ on $\theta_{-j} \in [0,1]$, and $\bar{S} = \max\{\bar{S}_j, \bar{S}_{-j}\}$.

Suppose, for a contradiction, that $\bar{S} > 0$. Suppose further, without loss, that $\bar{S} = \bar{S}_j$. There must exist a $\tilde{\theta}_j \in [0,1]$ that receives a surplus S_j in the interval $S_j \in (\bar{S} - s, \bar{S}]$. Consumers would never continue searching when receiving a surplus in this interval, since their gains of search are bounded from above by $\bar{S} - S_j - s < 0$. Thus, a consumer with type $\tilde{\theta}_j$ would strictly prefer to refrain from searching when receiving her equilibrium surplus at firm j. Any consumer with this type who arrives at firm j second must have received a surplus below $\bar{S} - s$. But then, firm j has a profitable deviation: It can marginally reduce the surplus offered to the type $\tilde{\theta}_j$, while still making the sale to all consumers with this type that arrive.

A.4 Proof of Corollary 1

Part 1: Frictionless search:

Suppose $\theta_j \sim U[0,1]$. Let's examine consumer surplus in the two different cases, beginning with the consumer surplus in the Wolinsky framework. There, consumer surplus is:

$$W^u = \int_0^1 \int_0^1 \max\{\theta_1 - p^*, \theta_2 - p^*, 0\} d\theta_2 d\theta_1$$

$$=$$

$$\int_0^{p^*} \left[0 + \left[0.5(\theta_2 - p^*)^2\right]_{p^*}^1\right] d\theta_1 + \int_{p^*}^1 \left[\theta_1(\theta_1 - p^*) + \left[0.5(\theta_2 - p^*)^2\right]_{\theta_1}^1\right] d\theta_1$$

$$=$$

$$\int_0^{p^*} \left[0.5(1-p^*)^2 - 0\right] d\theta_1 + \int_{p^*}^1 \theta_1(\theta_1 - p^*) d\theta_1 + \int_{p^*}^1 \left[0.5(1-p^*)^2 - 0.5(\theta_1 - p^*)^2\right] d\theta_1$$

$$=$$

$$\left[0.5(1-p^*)^2\theta_1\right]_0^{p^*} + \left[(1/3)\theta_1^3 - (1/2)p^*\theta_1^2\right]_{p^*}^1 + \left[0.5(1-p^*)^2\theta_1 - (1/6)(\theta_1 - p^*)^3\right]_{p^*}^1$$

$$=$$

$$\left[0.5(1-p^*)^2p^*\right]+\left[\frac{1}{3}-\frac{1}{2}p^*\right]-\left[\frac{1}{3}(p^*)^3-\frac{1}{2}(p^*)^3\right]+\left[\frac{1}{2}(1-p^*)^2-\frac{1}{6}(1-p^*)^3\right]-\left[0.5(1-p^*)^2p^*\right]$$
$$=$$
$$\left[\frac{1}{3}-\frac{1}{2}p^*\right]+\left[\frac{1}{6}(p^*)^3\right]+\left[\frac{1}{2}(1-p^*)^2-\frac{1}{6}(1-p^*)^3\right]$$
$$=$$
$$0.276$$

By contrast, consumer surplus in the price discrimination equilibrium is:

$$W^{pd} = \int_0^1\int_0^1 \max\{0.5\theta_1, 0.5\theta_2, 0\}d\theta_2 d\theta_1 = \int_0^1\left[\int_0^{\theta_1} 0.5\theta_1 d\theta_2 + \int_{\theta_1}^1 0.5\theta_2 d\theta_2\right]d\theta_1 =$$
$$\int_0^1\left[0.5(\theta_1)^2 + \left[(1/4)(\theta_2)^2\right]_{\theta_1}^1\right]d\theta_1 = \int_0^1\left[(1/4)+(1/4)(\theta_1)^2\right]d\theta_1 =$$
$$\left[(1/4)\theta_1 + (1/12)(\theta_1)^3\right]_0^1 = \frac{1}{4}\left[1+\frac{1}{3}\right]$$
$$=$$
$$0.333$$

Part 2: Search frictions:

When firms can price discriminate, consumer surplus is zero. When firms cannot price discriminate, the equilibrium from Wolinsky (1986) is played. In this equilibrium, firms will set a uniform price p^u strictly below 1. By our assumption that F has full support on $[0,1]$, a strictly positive measure of consumers will attain a match value $\theta_j \in [p^u, 1]$ at the initial firm. These consumers will receive strictly positive utility, so consumer surplus must be strictly positive.

A.5 Proof of Proposition 3

It is convenient to prove the proposition for the associated surplus function $S^*(\theta) = \theta - p^*(\theta)$. Hence, we need to show that in the unique symmetric equilibrium

$$S^*(\theta) = \begin{cases} 0.5\theta & \theta < \hat{\theta} \\ \frac{\hat{\theta}}{1+\hat{\theta}} & \theta \geq \hat{\theta}, \end{cases} \tag{37}$$

where

$$\hat{\theta} = \begin{cases} 1 & c \leq \frac{1}{24} \\ \in (0,1) & c \in \left(\frac{1}{24}, \frac{1}{6}\right) \\ 0 & c \geq \frac{1}{6}. \end{cases} \tag{38}$$

We start with the proof of two useful facts.

Part 1: In any equilibrium, the surplus function $S^*(\theta)$ is weakly increasing.

Notice that the consumer's purchase decision depends on the granted surplus at each firm, irrespective of the consumer's match values (θ_j, θ_{-j}). A higher granted surplus at firm i weakly decreases the incentives to search for first-arrivers and weakly increases the demand at firm i if the consumer visits both firms. Hence, the demand $D_i(S)$ at firm i is weakly increasing in the provided surplus S. Firm i's objective for a given visitor of type θ_i reads

$$\max_S \pi_i(\theta_i, S) = (\theta_i - S) D_i(S). \tag{39}$$

Because $D(S)$ is increasing in S, standard arguments concering the single-crossing property imply that $S^*(\theta_i)$ is weakly increasing in $\theta_i i$. To see this, consider two θ^1, θ^2, with $\theta^1 < \theta^2$ and define $S^1 := S(\theta^1)$ and $S^2 := S(\theta^2)$. Suppose, for a contradiction, that $S^1 > S^2$. The firm must not have a profitable deviations for either type, which requires:

$$\big(\theta^1 - S^1\big) D(S^1) > \big(\theta^1 - S^2\big) D(S^2) \iff D(S^1) - D(S^2) > \frac{S^1 D(S^1) - S^2 D(S^2)}{\theta^1}$$

$$\big(\theta^2 - S^2\big) D(S^2) > \big(\theta^2 - S^1\big) D(S^1) \iff \frac{S^1 D(S^1) - S^2 D(S^2)}{\theta^2} > D(S^1) - D(S^2)$$

Taking the two inequalities together yields:

$$\frac{S^1 D(S^1) - S^2 D(S^2)}{\theta^2} > \frac{S^1 D(S^1) - S^2 D(S^2)}{\theta^1} \iff \theta^1 > \theta^2 \tag{40}$$

The last implication follows because $S^1 > S^2$ and demand is increasing in the surplus, so $S^1 D(S^1) - S^2 D(S^2) > 0$. Thus, we have a contradiction.

Part 2: If in equilibrium some type $\hat{\theta}_i$ continues search after visiting firm i then so does any type $\theta_i < \hat{\theta}_i$.

Consider any equilibrium candidate $S^*(\theta)$. Consider a consumer with surplus $S = S^*(\theta_i)$ in hand after visiting firm i. When visiting firm j she expects a randomly drawn match value θ_j and a randomly and independently drawn price $p^*(x)$, where x is distributed according to the type distribution. Since the consumer buys at the firm that provides higher surplus, the consumer's additional surplus from search (excluding search costs) is $\max\{\theta_j - p^*(\theta_j) - S, 0\}$. Hence, with surplus S in hand a consumer refrains from search if and only if search costs exceed expected gains from search, i.e.

$$\int_{[\underline{\theta},\overline{\theta}]} \int_{[\underline{\theta},\overline{\theta}]} \max\{\theta_j - p^*(x) - S, 0\} f(\theta_j) d\theta_j f(x) dx \leq c. \tag{41}$$

Since the right hand side decreases in S and $S = S^*(\theta_i)$ increases in θ_i the claim follows.

Part 3: In equilibrium, there exists a $\hat{\theta} \in [0, 1]$ such that the firm's surplus function takes the following firm:

$$S^*(\theta; \hat{\theta}) = \begin{cases} 0.5\theta & \theta < \hat{\theta} \\ \hat{S} & \theta \geq \hat{\theta} \end{cases} \tag{42}$$

We continue with deriving necessary conditions for any equilibrium. Suppose in equilibrium some but not all consumers search. By Claim 2, search must follow a cutoff rule, by which consumers search after visiting firm i if and only if $\theta_i < \hat{\theta}$ with $\hat{\theta} \in (0, 1)$. We refer to such an equilibrium as a cutoff-equilibrium. Notice that equilibria in which no consumers or all consumers search can also be interpreted as cutoff-equilibrium in which the cutoff $\hat{\theta}$ is zero or larger one, respectively.

If consumers with $\theta_i \in [0, \hat{\theta})$ search, they compare prices and buy at the firm that provides the higher surplus. Hence, the firms' maximization problem, is

$$S(\theta_i) \in argmax_{s \in [0, S(\hat{\theta})]} \big\{ (\theta_i - s) Prob(s > S^*(\theta_j)) \big\}. \tag{43}$$

Notice that this problem is (locally) identical to the objective function in a standard first-price auction with independent private values $\theta_i \in [0, \hat{\theta}]$. Indeed, θ_i is firm i's gain from trade with the consumer, and S is the bid/surplus that the firm (i.e., the bidder) offers to the consumer (i.e., the auctioneer). One can show that the unique mutually optimal bidding strategy is:

$$\mathbb{E}[\theta_j|\theta_j \leq \theta] = 0.5\theta \qquad \text{for all} \;\; \theta < \hat{\theta}. \tag{44}$$

A mutually optimal surplus strategy $S^*(\theta)$ of the firms must be equal to the mutually optimal bidding strategy in a first-price auction. Suppose, for a contradiction, that it is not. Then, there exists some profitable deviation from the equilibrium bidding rule at some type. When offering the deviation surplus (bid), profits in our setting are always weakly above profits in the corresponding auction problem (since profits jump up when search is deterred). When offering the equilibrium surplus, profits are exactly equal to those attained when making the same bid because, by specification, search is not deterred when $\theta_i < \hat{\theta}$. This establishes the functional form of the equilibrium $S^*(\theta)$ for all $\theta < \hat{\theta}$: we must have $S^*(\theta) = 0.5\theta$.

Next, we establish that any equilibrium must provide constant surplus $\hat{S}$ to any consumer with $\theta_i \geq \hat{\theta}$. As argued above, a consumer with surplus S in hand refrains from search if and only if equation (41) holds.

Since the right-hand side in (41) strictly and continuously decreases in S and goes to zero for S sufficiently large, there is a unique $\hat{S}$, for which (41) holds with equality. In any equilibrium, firms can never offer a surplus above $\hat{S}$ to any consumer, since demand is constant for surpluses above this. In any equilibrium, Equation (41) must hold with equality since otherwise firms could profitably increase prices, and consumers would still refrain from search. Hence, we have established that any symmetric equilibrium must be of the form

$$S^*(\hat{\theta}, \theta) = \begin{cases} 0.5\theta & \theta < \hat{\theta} \\ \hat{S} & \theta \geq \hat{\theta} \end{cases} \tag{45}$$

Part 4: Suppose an equilibrium candidate satisfies (45) for some $\hat{\theta} \in (0, 1)$ and for this surplus function consumers indeed search if and only $\theta < \hat{\theta}$. Then, $S^*(\theta)$ is a best response for firm i if and only if $\hat{S} = \frac{\hat{\theta}}{1+\hat{\theta}}$.

We argue that firm behavior is optimal if and only if the firm is indifferent between setting prices $p(\hat{\theta}) = 0.5\hat{\theta}$ and $p(\hat{\theta}) = \hat{\theta} - \hat{S}$ when facing a consumer with type $\hat{\theta}$. Indifference is sufficient since because one can show that the incentives to deter weakly increase in θ. Indifference is necessary for $\hat{\theta} \in (0, 1)$ because for a given surplus S the firm's profits are continuous in θ. If the firm was not indifferent then it would either optimally induce some types $\theta_i < \hat{\theta}$ to search or it would optimally induce deterrence for some types $\theta_i < \hat{\theta}$.

Firm profit from search deterrence at $\hat{\theta}$ is

$$\pi_i(\hat{S}, \hat{\theta}) = \frac{1}{2}(\hat{\theta} - \hat{S}) + \frac{1}{2}Prob(\theta_j < \hat{\theta})(\hat{\theta} - \hat{S}) = 0.5(1 + \hat{\theta})(\hat{\theta} - \hat{S}). \tag{46}$$

The first summand represents profits from first-arrivers and the second summand is the profit from the share of consumers that first visit the rival, but continue searching since their match value for the rival's product is below $\hat{\theta}$. For the price $p_i(\hat{\theta}) = 0.5\hat{\theta}$ and assuming that the consumer continues searching, the firm's profit are

$$\pi_i(0.5\hat{\theta}, \hat{\theta}) = Prob(\theta_j < \hat{\theta})(\hat{\theta} - 0.5\hat{\theta}) = 0.5\hat{\theta}^2. \tag{47}$$

Hence, given consumers search according to the postulated rule, the pricing function in Proposition 3 is a firm's best response for $\hat{\theta} \in (0, 1)$ if and only if

$$0.5(1 + \hat{\theta})(\hat{\theta} - \hat{S}) = 0.5\hat{\theta}^2, \tag{48}$$

which is equivalent to $\hat{S}(\hat{\theta}) = \frac{\hat{\theta}}{1+\hat{\theta}}$.

We have established that any potential cutoff-equilibrium must be of the functional form described in (2) of Proposition 3, and that for such a surplus function firms indeed behave optimally given consumers search according to the aforementioned rule.

Part 5: Equilibrium structure when $c \in (1/24, 1/6)$.

To conclude the proof of (2) it remains to show that for $c \in (\frac{1}{24}, \frac{1}{6})$ there exists a unique $\hat{\theta}$ for which consumers indeed find it optimal to use the cutoff search if firms price according to $S^*(\hat{\theta}, \theta)$.

Suppose that $\hat{\theta} = 0$. Then, firms offer the surplus $\hat{S}$ to all consumers. Hence, $\hat{S} = 0$ must hold — else, any firm would make negative profits when facing consumers with a type below $\hat{S}$. However, since $c < 1/6$, one can show that $\hat{S} > 0$ would hold, given this pricing strategy, which is a contradiction.

Suppose that $\hat{\theta} = 1$. Then, all consumers search. However, one can show that it would be profitable for any firm to deter search by consumers with a type in an open ball below 1, a contradiction.

Thus, $\hat{\theta} \in (0,1)$ must hold in equilibrium, so the insights of part 4 apply.

Since consumer surplus $S(\hat{\theta},\theta) = \theta - p(\hat{\theta},\theta)$ is weakly increasing in θ, and the incentives to search decrease in the surplus in hand, consumers search according to the postulated rule if and only if type $\hat{\theta}$ is indifferent between search and no search, hence if and only if

$$c = \int_{[\underline{\theta},\overline{\theta}]} \int_{[\underline{\theta},\overline{\theta}]} \max\left\{\theta_j - p(\hat{\theta},x) - \frac{\hat{\theta}}{1+\hat{\theta}}, 0\right\} f(\theta_j)d\theta_j f(x)dx \tag{49}$$

It is easy to check that

$$-p(\hat{\theta},x) - \frac{\hat{\theta}}{1+\hat{\theta}} = \begin{cases} -0.5x - \frac{\hat{\theta}}{1+\hat{\theta}} & x < \hat{\theta} \\ -x & x \geq \hat{\theta} \end{cases} \tag{50}$$

is weakly decreasing in $\hat{\theta}$, and strictly so for $x < \hat{\theta}$.[17] This implies that the right-hand side in (49) is strictly decreasing in $\hat{\theta}$.

Part 6: Equilibrium characterization when $c \geq 1/6$ and $c \leq 1/24$.

To establish the cost bounds for search behavior, we examine incentives to search for surplus function $S(\hat{\theta},\theta)$ when $\hat{\theta} = 0$ and $\hat{\theta} = 1$. Recall that type $\theta = 0$ has the highest incentive to search. Since $S^*(0,0) = 0$, for $\hat{\theta} = 0$ type $\theta = 0$ finds it (weakly) optimal to search if and only if

$$\begin{aligned} c &\leq \int_{[\underline{\theta},\overline{\theta}]} \int_{[\underline{\theta},\overline{\theta}]} \max\{\theta_j - p(0,x), 0\} f(\theta_j)d\theta_j f(x)dx \\ &= \int_0^1 \int_x^1 (\theta_j - x) f(\theta_j)d\theta_j f(x)dx \\ &= \int_0^1 \frac{(1-x)^2}{2} f(x)dx \\ &= \frac{1}{6}. \end{aligned}$$

Hence, for $c > \frac{1}{6}$ in the unique equilibrium no consumer searches.

[17] Notice that the function is discontinuously increasing in x at $x = \hat{\theta}$. Hence, if the discontinuity point moves to the right, the function is decreasing at $x = \hat{\theta}$.

Conversely, type $\theta = 1$ has the lowest incentive to search. For $\hat{\theta} = 1$ we have $\hat{S} = 0.5$ for all types, and type $\theta = 1$ finds it (weakly) optimal not to search if and only if

$$\begin{aligned}
c &\geq \int_{[\underline{\theta},\overline{\theta}]} \int_{[\underline{\theta},\overline{\theta}]} \max\{\theta_j - 0.5x - 0.5, 0\} f(\theta_j) d\theta_j f(x) dx \\
&= \int_0^1 \left[\frac{(\theta_j - 0.5x - 0.5)^2}{2} \right]_{0.5+0.5x}^1 f(x) dx \\
&= \int_0^1 \frac{(0.5(1-x))^2}{2} f(x) dx \\
&= \frac{1}{24}.
\end{aligned}$$

Hence, for $c < \frac{1}{24}$ in the unique equilibrium all consumers search. Moreover, the right-hand side in (49) strictly decreases from $\frac{1}{6}$ for $\hat{\theta} = 0$ to $\frac{1}{24}$ for $\hat{\theta} = 1$, hence for each $c \in (\frac{1}{24}, \frac{1}{6})$ there is a unique $\hat{\theta} \in (0, 1)$ for which (45) is an equilibrium.

A.6 Proof of Corollary 2

Proof strategy:

Define $CS(c; k)$ as the consumer surplus at cost c, in the setup $k \in \{d, u\}$ (price discrimination, uniform pricing). As we verify later, the following holds:

- At $c' = 0.0651$, we have $\hat{S} = 0.42$ and:
  - The Wolinsky consumer surplus is 0.1792, i.e. $CS(0.0651; u) = 0.1792$.
  - The price discrimination consumer surplus is 0.2792, i.e. $CS(0.0651; d) = 0.2792$.
- At $c = 0$, we have:
  - The Wolinsky consumer surplus is 0.2761, i.e. $CS(0; u) = 0.2761$
- At $c = 0.127$, we have $\hat{S} = 0.24$ and
  - The price discrimination consumer surplus is 0.1865, i.e. $CS(0.127; p) = 0.1865$

One can show that consumer surplus falls in c in both setups. Given the above calculations, the desired result follows. Indeed, for all $c \in [0, 0.0651]$, consumer surplus under price discrimination must be higher, since:

$$CS(c;d) \geq CS(0.0651;d) > CS(0;u) \geq CS(c;u) \tag{51}$$

For all $c \in [0.0651, 0.127]$, we have:

$$CS(c;d) \geq CS(0.127;d) > CS(0.0651;u) \geq CS(c;u) \tag{52}$$

The proof strategy is visualized in the following graph:

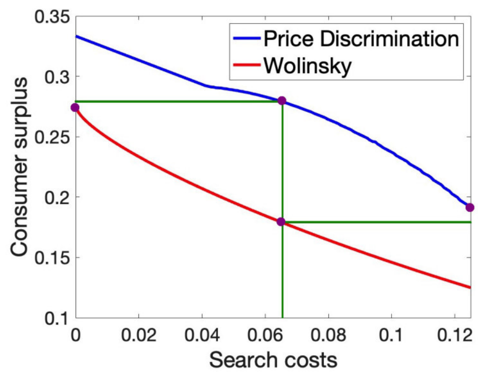

Figure 6: Corollary 2 — proof strategy

<u>Details: $c' = 0.0651$</u>

We begin by showing that, when the search costs are c', $\hat{S} = 0.42$ is the equilibrium search cutoff under naivety.

At $\hat{S} = 0.42$, we have $\hat{\theta} = \frac{0.42}{0.58} = \frac{42}{58}$. We evaluate:

$$\int_0^1 \left[\int_0^{\frac{42}{58}} \big[\max\{0.42, \theta_j - 0.5x\} - 0.42\big] dx + \int_{\frac{42}{58}}^1 \big[\max\{0.42, \theta_j - x + 0.42\} - 0.42\big] dx \right] d\theta_j = c'$$

$$\Longleftrightarrow$$

$$\int_{0.42}^1 \left[\int_0^{\frac{42}{58}} \big[\max\{0.42, \theta_j - 0.5x\} - 0.42\big] dx + \int_{\frac{42}{58}}^1 \big[\max\{0.42, \theta_j - x + 0.42\} - 0.42\big] dx \right] d\theta_j = c'$$

$$\Longleftrightarrow$$

$$\int_{0.42}^1 \int_0^{\frac{42}{58}} \big[\max\{0.42, \theta_j - 0.5x\} - 0.42\big] dx d\theta_j +$$

$$\int_{0.42}^{1}\int_{\frac{42}{58}}^{1}\big[\max\{0.42,\theta_j - x + 0.42\} - 0.42\big]dxd\theta_j = c' \tag{53}$$

To evaluate this, note that:

$$\int_{0}^{\frac{42}{58}}\big[\max\{0.42,\theta_j - 0.5x\} - 0.42\big]dx = \int_{0}^{\min\{2(\theta_j-0.42),\frac{42}{58}\}}\big[\theta_j - 0.5x - 0.42\big]dx \tag{54}$$

Note that:

$$2(\theta_j - 0.42) > \frac{42}{58} \iff \theta_j > \frac{21}{58} + \frac{42}{100} = \frac{2100}{5800} + \frac{2436}{5800} = \frac{4536}{5800} \tag{55}$$

For $\theta_j > \frac{4536}{5800}$, we have $\min\{2(\theta_j - 0.42), \frac{42}{58}\} = \frac{42}{58}$, so this integral becomes:

$$\int_{0}^{\frac{42}{58}}\big[\max\{0.42,\theta_j - 0.5x\} - 0.42\big]dx = \int_{0}^{\frac{42}{58}}\big[\theta_j - 0.5x - 0.42\big]dx =$$

$$\Big[\theta_j x - (1/4)x^2 - 0.42x\Big]_0^{\frac{42}{58}} = \Big[\frac{42}{58}\theta_j - \frac{1}{4}\frac{42}{58}\frac{42}{58} - \frac{42}{100}\frac{42}{58}\Big] = \Big[\frac{42}{58}\theta_j - \frac{441}{3364} - \frac{1764}{5800}\Big] \tag{56}$$

For $\theta_j \in [0.42, \frac{4536}{5800}]$, this integral becomes:

$$\int_{0}^{\min\{2(\theta_j-0.42),\frac{42}{58}\}}\big[\theta_j - 0.5x - 0.42\big]dx =$$

$$\Big[\big[\theta_j x - (1/4)x^2 - 0.42x\big]\Big]_0^{2(\theta_j-0.42)} = 2\theta_j\big(\theta_j - 0.42\big) - \big(\theta_j - 0.42\big)^2 - 0.42(2)(\theta_j - 0.42)$$

$$=$$

$$\big(2\theta_j - 2(0.42)\big)\big(\theta_j - 0.42\big) - \big(\theta_j - 0.42\big)\big(\theta_j - 0.42\big) = \big(\theta_j - 0.42\big)^2 \tag{57}$$

Taking previous results together implies that:

$$\int_{0.42}^{1}\int_{0}^{\frac{42}{58}}\big[\max\{0.42,\theta_j - 0.5x\} - 0.42\big]dxd\theta_j =$$

$$\int_{0.42}^{\frac{4536}{5800}}\int_{0}^{\frac{42}{58}}\big[\max\{0.42,\theta_j - 0.5x\} - 0.42\big]dxd\theta_j + \int_{\frac{4536}{5800}}^{1}\int_{0}^{\frac{42}{58}}\big[\max\{0.42,\theta_j - 0.5x\} - 0.42\big]dxd\theta_j =$$

$$\int_{0.42}^{\frac{4536}{5800}} \big[\theta_j - 0.42\big]^2 d\theta_j + \int_{\frac{4536}{5800}}^{1} \left[\frac{42}{58}\theta_j - \frac{441}{3364} - \frac{1764}{5800}\right] d\theta_j =$$

$$\left[\frac{1}{3}\big[\theta_j - 0.42\big]^3\right]_{0.42}^{\frac{4536}{5800}} + \left[\frac{21}{58}\theta_j^2 - \frac{441}{3364}\theta_j - \frac{1764}{5800}\theta_j\right]_{\frac{4536}{5800}}^{1}$$

$$=$$

$$\left[\frac{1}{3}\left(\frac{4536}{5800} - 0.42\right)^3\right] +$$

$$\left[\left(\frac{21}{58} - \frac{441}{3364} - \frac{1764}{5800}\right) - \left(\frac{21}{58}\left(\frac{4536}{5800}\right)^2 - \frac{441}{3364}\left(\frac{4536}{5800}\right) - \frac{1764}{5800}\left(\frac{4536}{5800}\right)\right)\right]$$

$$\Longrightarrow$$

$$\int_{0.42}^{1}\int_{0}^{\frac{42}{58}} \big[\max\{0.42, \theta_j - 0.5x\} - 0.42\big] dx d\theta_j = 0.0616 \tag{58}$$

Now let's evaluate the second integral, namely:

$$\int_{0.42}^{1}\int_{\frac{42}{58}}^{1} \big[\max\{0.42, \theta_j - x + 0.42\} - 0.42\big] dx d\theta_j \tag{59}$$

The argument of this integral is $\theta_j - x$ if $\theta_j > x$ and 0 otherwise. Note further that $0.42 < 42/58$. For any $\theta_j < \frac{42}{58}$, the argument of the integral must always be negative. For any $\theta_j \geq \frac{42}{58}$, the inner integral becomes:

$$\int_{\frac{42}{58}}^{1} \big[\max\{0.42, \theta_j - x + 0.42\} - 0.42\big] dx = \int_{42/58}^{\theta_j} \big[\theta_j - x\big] dx \tag{60}$$

This implies that our second integral becomes:

$$\int_{0.42}^{1}\int_{\frac{42}{58}}^{1} \big[\max\{0.42, \theta_j - x + 0.42\} - 0.42\big] dx d\theta_j =$$

$$\int_{0.42}^{42/58} (0) d\theta_j + \int_{42/58}^{1}\int_{42/58}^{\theta_j} \big[\theta_j - x\big] dx d\theta_j = \int_{42/58}^{1} \big[\theta_j x - 0.5x^2\big]_{42/58}^{\theta_j} d\theta_j =$$

$$\int_{42/58}^{1} \left[0.5\theta_j^2 - \frac{42}{58}\theta_j + 0.5\left(\frac{42}{58}\right)^2\right] d\theta_j = \left[\frac{1}{6}\theta_j^3 - \frac{21}{58}\theta_j^2 + 0.5\left(\frac{42}{58}\right)^2\theta_j\right]_{42/58}^{1} =$$

$$\left[\frac{1}{6} - \frac{21}{58} + 0.5\left(\frac{42}{58}\right)^2\right] - \left[-\frac{21}{58}\left(\frac{42}{58}\right)^2 + \frac{2}{3}\left(\frac{42}{58}\right)^3\right]$$

$$\Longrightarrow \int_{0.42}^{1}\int_{\frac{42}{58}}^{1}\big[\max\{0.42,\theta_j - x + 0.42\} - 0.42\big]dx d\theta_j = 0.0035 \tag{61}$$

Thus, we have:

$$0.0616 + 0.0035 = c' = 0.0651 \tag{62}$$

Hence, at c', our equilibrium tuple is $\hat{S} = 0.42$ and $\hat{\theta} = \frac{42}{58}$. Based on this, we can calculate consumer surplus:

$$CS = \int_0^{\hat{\theta}}\Big[\int_0^{\hat{\theta}}\big[\max\{0.5\theta_1, 0.5\theta_2\} - c\big]d\theta_2 + \int_{\hat{\theta}}^{1}\big[\hat{S} - c\big]d\theta_2\Big]d\theta_1 + \int_{\hat{\theta}}^{1}\hat{S}d\theta_1 =$$

$$\int_0^{\hat{\theta}}\int_0^{\hat{\theta}}\big[\max\{0.5\theta_1, 0.5\theta_2\} - c\big]d\theta_2 d\theta_1 + \hat{\theta}\big(1-\hat{\theta}\big)(\hat{S} - c) + \big(1-\hat{\theta}\big)\hat{S} =$$

$$\int_0^{\hat{\theta}}\int_0^{\theta_1}\big[0.5\theta_1\big]d\theta_2 d\theta_1 + \int_0^{\hat{\theta}}\int_{\theta_1}^{\hat{\theta}}\big[0.5\theta_2\big]d\theta_2 d\theta_1 - \hat{\theta}^2 c + \hat{\theta}\big(1-\hat{\theta}\big)\big(\hat{S} - c\big) + \big(1-\hat{\theta}\big)\hat{S} =$$

$$\int_0^{\hat{\theta}} 0.5\theta_1^2 d\theta_1 + \int_0^{\hat{\theta}}\big[(1/4)(\hat{\theta})^2 - (1/4)\theta_1^2\big]d\theta_1 + \hat{\theta}\big(1-\hat{\theta}\big)\big(\hat{S}\big) - \hat{\theta}c + \big(1-\hat{\theta}\big)\hat{S} =$$

$$\big[(1/6)\theta_1^3\big]_0^{\hat{\theta}} + \big[(1/4)(\hat{\theta})^2\theta_1 - (1/12)\theta_1^3\big]_0^{\hat{\theta}} + \hat{\theta}\big(1-\hat{\theta}\big)\big(\hat{S}\big) - \hat{\theta}c + \big(1-\hat{\theta}\big)\hat{S} =$$

$$(1/6)\hat{\theta}^3 + \big[(1/4)(\hat{\theta})^3 - (1/12)\hat{\theta}^3\big] + \hat{\theta}\big(1-\hat{\theta}\big)\big(\hat{S}\big) - \hat{\theta}c + \big(1-\hat{\theta}\big)\hat{S}$$

$$\Longrightarrow$$

$$CS = \big(1/3\big)(\hat{\theta})^3 + \hat{\theta}\big(1-\hat{\theta}\big)\big(\hat{S}\big) - \hat{\theta}c + \big(1-\hat{\theta}\big)\hat{S} \tag{63}$$

At $c' = 0.0651$, we have:

$$CS(c') = (1/3)\big(42/58\big)^3 + (42/58)\big(1-(42/58)\big)(0.42) - (42/58)c + \big(1-(42/58)\big)(0.42) = 0.2792 \tag{64}$$

<u>Details: $c'' = 0.127$</u>

We begin by showing that, when the search costs are c'', $\hat{S} = 0.24$ is the equilibrium search cutoff under naivety.

At $\hat{S} = 0.24$, we have $\hat{\theta} = \frac{0.24}{0.76} = \frac{24}{76}$. We try to evaluate:

$$\int_0^1 \left[\int_0^{\frac{24}{76}} \left[\max\{0.24, \theta_j - 0.5x\} - 0.24 \right] dx + \int_{\frac{24}{76}}^1 \left[\max\{0.24, \theta_j - x + 0.24\} - 0.24 \right] dx \right] d\theta_j = c''$$

$$\iff$$

$$\int_{0.24}^1 \left[\int_0^{\frac{24}{76}} \left[\max\{0.24, \theta_j - 0.5x\} - 0.24 \right] dx + \int_{\frac{24}{76}}^1 \left[\max\{0.24, \theta_j - x + 0.24\} - 0.24 \right] dx \right] d\theta_j = c''$$

$$\iff$$

$$\begin{aligned} &\int_{0.24}^1 \int_0^{\frac{24}{76}} \left[\max\{0.24, \theta_j - 0.5x\} - 0.24 \right] dx d\theta_j + \\ &\int_{0.24}^1 \int_{\frac{24}{76}}^1 \left[\max\{0.24, \theta_j - x + 0.24\} - 0.24 \right] dx d\theta_j = c' \end{aligned} \tag{65}$$

To evaluate this, note that:

$$\int_0^{\frac{24}{76}} \left[\max\{0.24, \theta_j - 0.5x\} - 0.24 \right] dx = \int_0^{\min\{2(\theta_j - 0.24), \frac{24}{76}\}} \left[\theta_j - 0.5x - 0.24 \right] dx \tag{66}$$

Note further that:

$$2(\theta_j - 0.24) > \frac{24}{76} \iff \theta_j > \frac{12}{76} + 0.24 \tag{67}$$

For $\theta_j > \frac{12}{76} + 0.24$, we have $\min\{2(\theta_j - 0.24), \frac{24}{76}\} = \frac{24}{76}$, so this integral becomes:

$$\begin{aligned} &\int_0^{\frac{24}{76}} \left[\max\{0.24, \theta_j - 0.5x\} - 0.24 \right] dx = \int_0^{\frac{24}{76}} \left[\theta_j - 0.5x - 0.24 \right] dx = \\ &\left[\theta_j x - (1/4)x^2 - 0.24x \right]_0^{\frac{24}{76}} = \left[\frac{24}{76}\theta_j - \frac{1}{4}\left(\frac{24}{76}\right)^2 - 0.24\frac{24}{76} \right] \end{aligned} \tag{68}$$

For $\theta_j \in [0.24, \frac{12}{76} + 0.24]$, this integral becomes:

$$\int_0^{\min\{2(\theta_j - 0.24), \frac{24}{76}\}} \left[\theta_j - 0.5x - 0.24 \right] dx =$$

$$\left[\left[\theta_j x - (1/4)x^2 - 0.24x \right] \right]_0^{2(\theta_j - 0.24)} = 2\theta_j \big(\theta_j - 0.24\big) - \big(\theta_j - 0.24\big)^2 - 0.24(2)(\theta_j - 0.24)$$

$$=$$

$$\big(2\theta_j - 2(0.24)\big)\big(\theta_j - 0.24\big) - \big(\theta_j - 0.24\big)\big(\theta_j - 0.24\big) = \big(\theta_j - 0.24\big)^2 \tag{69}$$

Taking previous results together implies that:

$$\int_{0.24}^{1}\int_{0}^{\frac{24}{76}} \big[\max\{0.24, \theta_j - 0.5x\} - 0.24\big] dx d\theta_j =$$

$$\int_{0.24}^{0.24+\frac{12}{76}}\int_{0}^{\frac{24}{76}} \big[\max\{0.24, \theta_j - 0.5x\} - 0.24\big] dx d\theta_j + \int_{0.24+\frac{12}{76}}^{1}\int_{0}^{\frac{24}{76}} \big[\max\{0.24, \theta_j - 0.5x\} - 0.24\big] dx d\theta_j =$$

$$\int_{0.24}^{0.24+\frac{12}{76}} \big[\theta_j - 0.24\big]^2 d\theta_j + \int_{0.24+\frac{12}{76}}^{1} \left[\frac{24}{76}\theta_j - \frac{1}{4}\left(\frac{24}{76}\right)^2 - 0.24\frac{24}{76}\right] d\theta_j =$$

$$\left[\frac{1}{3}\big[\theta_j - 0.24\big]^3\right]_{0.24}^{0.24+\frac{12}{76}} + \left[\frac{12}{76}\theta_j^2 - \frac{1}{4}\left(\frac{24}{76}\right)^2\theta_j - 0.24\frac{24}{76}\theta_j\right]_{0.24+\frac{12}{76}}^{1}$$

$$=$$

$$\left[\frac{1}{3}\frac{12}{76}\right] + \left[\frac{12}{76} - \frac{1}{4}\left(\frac{24}{76}\right)^2 - 0.24\frac{24}{76}\right] - \left[\left(\frac{12}{76}\right)\left(0.24 + \frac{12}{76}\right)^2 - \frac{1}{4}\left(\frac{24}{76}\right)^2\left(0.24 + \frac{12}{76}\right) - 0.24\frac{24}{76}\left(0.24 + \frac{12}{76}\right)\right]$$

$$\Longrightarrow$$

$$\int_{0.24}^{1}\int_{0}^{\frac{24}{76}} \big[\max\{0.24, \theta_j - 0.5x\} - 0.24\big] dx d\theta_j = 0.0736 \tag{70}$$

Now let's evaluate the second integral, namely:

$$\int_{0.24}^{1}\int_{\frac{24}{76}}^{1} \big[\max\{0.24, \theta_j - x + 0.24\} - 0.24\big] dx d\theta_j \tag{71}$$

The argument of this integral is $\theta_j - x$ if $\theta_j > x$ and 0 otherwise. Note further that $0.24 < 24/76$. For any $\theta_j < 24/76$, the argument of the integral must always be negative. For any $\theta_j \geq 24/76$, the inner integral becomes:

$$\int_{\frac{24}{76}}^{1} \big[\max\{0.24, \theta_j - x + 0.24\} - 0.24\big] dx = \int_{\frac{24}{76}}^{\theta_j} \big[\theta_j - x\big] dx \tag{72}$$

This implies that our second integral becomes:

$$\int_{0.24}^{1}\int_{\frac{24}{76}}^{1} \big[\max\{0.24, \theta_j - x + 0.24\} - 0.24\big] dx d\theta_j =$$

$$\int_{0.24}^{\frac{24}{76}}(0)d\theta_j+\int_{\frac{24}{76}}^{1}\int_{\frac{24}{76}}^{\theta_j}\left[\theta_j-x\right]dxd\theta_j=\int_{\frac{24}{76}}^{1}\left[\theta_j x-0.5x^2\right]_{\frac{24}{76}}^{\theta_j}d\theta_j=$$

$$\int_{\frac{24}{76}}^{1}\left[0.5\theta_j^2-\frac{24}{76}\theta_j+0.5\left(\frac{24}{76}\right)^2\right]d\theta_j=\left[\frac{1}{6}\theta_j^3-\frac{12}{76}\theta_j^2+0.5\left(\frac{24}{76}\right)^2\theta_j\right]_{\frac{24}{76}}^{1}=$$

$$\left[\frac{1}{6}-\frac{12}{76}+0.5\left(\frac{24}{76}\right)^2\right]-\left[\frac{2}{3}\left(\frac{24}{76}\right)^3-\frac{12}{76}\left(\frac{24}{76}\right)^2\right]$$

$$\Longrightarrow$$

$$\int_{0.24}^{1}\int_{\frac{24}{76}}^{1}\left[\max\{0.24,\theta_j-x+0.24\}-0.24\right]dxd\theta_j=0.0534 \tag{73}$$

Thus, we have:

$$0.0736+0.0534=c''=0.127 \tag{74}$$

Plugging $\hat{S}=0.24$ and $\hat{\theta}=24/76$ into our expression for consumer surplus yields:

$$CS(c'')=\big(1/3\big)(\hat{\theta})^3+\hat{\theta}\big(1-\hat{\theta}\big)\big(\hat{S}\big)-\hat{\theta}c+\big(1-\hat{\theta}\big)\hat{S}$$

$$=$$

$$(1/3)\big(24/76\big)^3+(24/76)\big(1-(24/76)\big)(0.24)-(24/76)c+\big(1-(24/76)\big)(0.24)=0.1865 \tag{75}$$

<u>Consumer surplus in the Wolinsky equilibrium</u>

We know the search cutoff is given by $w^*=1-\sqrt{2c}$. Given w^*, the equilibrium price solves:

$$p^*=(1-w^*)\frac{1-(p^*)^2}{1-(w^*)^2}\iff p^*=\frac{1-(p^*)^2}{1+w^*}\iff$$

$$p^*+w^*p^*=1-(p^*)^2\iff(p^*)^2+(1+w^*)p^*-1=0\iff$$

$$p^*=\frac{-(1+w^*)+\sqrt{(1+w^*)^2+4}}{2} \tag{76}$$

With these solutions, we can calculate consumer surplus in the Wolinsky equilibrium:

$$W^*=\int_{w^*}^{1}(\theta_1-p^*)d\theta_1+\int_{0}^{w^*}\left[\int_{0}^{\theta_1}\max\{\theta_1-p^*,0\}d\theta_2+\int_{\theta_1}^{1}\max\{\theta_2-p^*,0\}d\theta_2\right]d\theta_1-w^*c=$$

$$\int_{w^*}^{1}(\theta_1-p^*)d\theta_1+\int_0^{p^*}\int_{p^*}^{1}(\theta_2-p^*)d\theta_2 d\theta_1+\int_{p^*}^{w^*}\left[\int_0^{\theta_1}(\theta_1-p^*)d\theta_2+\int_{\theta_1}^{1}(\theta_2-p^*)d\theta_2\right]d\theta_1-w^*c$$

$$=$$

$$0.5\big[1-p^*\big]^2-0.5\big[w^*-p^*\big]^2+\int_0^{p^*}0.5\big[1-p^*\big]^2 d\theta_1+$$

$$\int_{p^*}^{w^*}\left[\theta_1(\theta_1-p^*)+0.5\big[1-p^*\big]^2-0.5\big[\theta_1-p^*\big]^2\right]d\theta_1-w^*c$$

$$=$$

$$0.5\big[1-p^*\big]^2-0.5\big[w^*-p^*\big]^2+0.5p^*\big[1-p^*\big]^2+$$

$$\left[(1/3)\theta_1^3-(1/2)\theta_1^2p^*+0.5\big[1-p^*\big]^2\theta_1-(1/6)\big[\theta_1-p^*\big]^3\right]_{p^*}^{w^*}-w^*c$$

$$=$$

$$0.5\big[1-p^*\big]^2-0.5\big[w^*-p^*\big]^2+0.5p^*[1-p^*]^2+\Big[(1/3)(w^*)^3-(1/2)(w^*)^2p^*+$$

$$0.5\big[1-p^*\big]^2(w^*)-(1/6)\big[w^*-p^*\big]^3\Big]-\Big[(1/3)(p^*)^3-(1/2)(p^*)^3+0.5\big[1-p^*\big]^2(p^*)\Big]-w^*c$$

$$=$$

$$0.5\big[1-p^*\big]^2-0.5\big[w^*-p^*\big]^2+\Big[(1/3)(w^*)^3-(1/2)(w^*)^2p^*+0.5\big[1-p^*\big]^2(w^*)-(1/6)\big[w^*-p^*\big]^3\Big]$$

$$+(1/6)(p^*)^3-w^*c \tag{77}$$

Plugging in the values of $c=0$ and $c=c'$ yield surplus levels of 0.2761 and 0.1792.

<u>Monotonicity of consumer surplus functions (in c)</u>

Consumer surplus in the Wolinsky equilibrium is falling in c (this can be shown using standard arguments from the literature).

Thus, it only remains to show that consumer surplus in the price discrimination equilibrium is monotonically falling in c. To see this, note firstly that $\hat{S}$ is weakly falling in c (apply the intermediate value theorem to the equation characterizing it). Now examine our expression for consumer surplus derived before:

$$CS=\big(1/3\big)\big(\hat{\theta}\big)^3+\big(1+\hat{\theta}\big)\big(1-\hat{\theta}\big)\hat{S}-\hat{\theta}c=$$

$$(1/3)\left(\frac{\hat{S}}{1-\hat{S}}\right)^3 + \left(1+\frac{\hat{S}}{1-\hat{S}}\right)\left(1-\frac{\hat{S}}{1-\hat{S}}\right)\hat{S} - \hat{\theta}c \tag{78}$$

The derivative of this object w.r.t c is thus:

$$\frac{\partial CS}{\partial c} = \frac{1}{3}(\hat{\theta})^2\frac{\partial\hat{\theta}}{\partial c} + \left((1-\hat{\theta})\frac{\partial\hat{\theta}}{\partial c} - (1+\hat{\theta})\frac{\partial\hat{\theta}}{\partial c}\right)\hat{S} + (1+\hat{\theta})(1-\hat{\theta})\frac{\partial\hat{S}}{\partial c} - \frac{\partial\hat{\theta}}{\partial c}c - \hat{\theta} \tag{79}$$

One can show that this is strictly negative.

A.7 Proof of Proposition 4

Part 1: Functional form of $S^*(\theta)$

We are considering an equilibrium in which all naive consumers search beyond the first firm and rational consumers continue searching if and only if the initial surplus they receive is below $\hat{S}^r$, or equivalently, their match is below $\hat{\theta}^r$.[18]

Notice that rational consumers with cutoff match $\hat{\theta}^r$ must be indifferent between searching and not searching, i.e. $S^*(\hat{\theta}^r) = \hat{S}^r$ must hold. If $S^*(\hat{\theta}^r) < \hat{S}^r$, these consumers would search, a contradiction. Thus, suppose for a contradiction, that $S^*(\hat{\theta}^r) > \hat{S}^r$. When offering $S^*(\hat{\theta}^r)$, firm j sells to all arriving rational consumers. First arrivers don't continue searching and second arrivers directly buy because they must have attained a surplus below $\hat{S}^r$. Moreover, firm j sells to all naive consumers with $\theta_{-j} < \hat{\theta}^r$. This implies that the firm can deviative by marginally reducing the surplus it offers without changing the demand from a consumer with type $\hat{\theta}^r$, a contradiction.

Next, we establish necessary conditions for the functional form of $S^*(\theta)$, beginning with $\theta < \hat{\theta}^r$. In equilibrium, all rational consumers search beyond the first firm when their initial type is in this interval. Consider any $\theta < \hat{\theta}^r$ and a surplus s in a small open ball around the equilibrium $S^*(\theta)$. If a consumer is rational, she will buy from this firm if any only if $\theta_j - p_j$ is above the surplus she attains at the other firm, given that first arrivers continue searching. Similarly, a naive consumer buys from this firm if and only if $\theta_j - p_j$ is above the surplus she attains at the other firm. Thus, the firm's profits are given by:

$$p_j \int_0^1 \mathbb{1}[\theta_j - p_j > \theta_{-j} - p^*(\theta_{-j})]d\theta_{-j} \tag{80}$$

[18] This formulation holds true because the surplus function must be increasing in equilibrium by the arguments laid out in the main text.

By previous arguments, $p^*(\theta) = 0.5\theta$ is a mutually optimal strategy for firms when $\theta \in [0, \hat{\theta}^r]$.

Now we pin down necessary conditions for $S^*(\theta)$ for $\theta > \hat{\theta}^r$. To that end, suppose firm j is visited by a consumer with an arbitrary $\theta_j > \hat{\theta}^r$. The firm knows that the consumer is one of the three following kinds. She can be (i) naive, (ii) sophisticated and a first arriver or (iii) sophisticated and a second arriver with type $\theta_{-j} < \hat{\theta}^r$. In equilibrium, the firm will make the sale to all consumers in categories (ii) and (iii). The firm makes the sale to consumers in category (i) if and only if the surplus this firm offers, which we call s, is above the surplus offered by the rival. When the firm offers the surplus s, it thus attains the following profits:

$$\Pi(s) = (\theta - s)\big[0.5\alpha + 0.5F(\hat{\theta}^r)\alpha + (1-\alpha)F(S^{-1}(s))\big] \tag{81}$$

$$= (\theta - s)\big[\alpha\big(1 - 0.5(1 - F(\hat{\theta}^r))\big) + (1-\alpha)F(S^{-1}(s))\big]. \tag{82}$$

To simplify notation, we define

$$\gamma \equiv \gamma(\hat{\theta}^r) \equiv \frac{\alpha\big(1 - 0.5(1 - F(\hat{\theta}^r))\big)}{1-\alpha} \tag{83}$$

as the relative share of sophisticated arriving consumers compared to naive ones.

By normalizing the firm's profit by the constant factor $(1-\alpha)$, the firm's objective is to maximize

$$\Pi(s) = (\theta - s)(\gamma + F(S^{-1}(s))) \tag{84}$$

This yields the following first-order condition:

$$(\theta - s)f(S^{-1}(s))(S^{-1})'(s) - \big[\gamma + F(S^{-1}(s))\big] = 0. \tag{85}$$

Exploiting that, in a symmetric equilibrium, the optimal s satisfies $s = S(\theta)$, we obtain the following differential equation that pins down $S^*(\theta)$ in symmetric equilibrium:

$$(\theta - S(\theta))\frac{f(\theta)}{S'(\theta)} - \gamma - F(\theta) = 0, \tag{86}$$

Plugging in the fact that consumer's valuations are uniformly distributed on $[0, 1]$ yields:

$$S'(\theta)\big[\gamma(\hat{\theta}^r) + \theta\big] + S(\theta) = \theta \tag{87}$$

Integrating up yields:

$$\int_{\hat{\theta}^r}^{\theta} \big(S'(x)[\gamma(\hat{\theta}^r)+x]+S(x)\big)dx = \int_{\hat{\theta}^r}^{\theta} x dx \iff \Big[S(x)[\gamma(\hat{\theta}^r)+x]\Big]_{\hat{\theta}^r}^{\theta} = 0.5\big[(\theta)^2-(\hat{\theta}^r)^2\big] \tag{88}$$

$$\iff$$

$$\Big[S(\theta)[\gamma(\hat{\theta}^r)+\theta]\Big] - \Big[S(\hat{\theta}^r)[\gamma(\hat{\theta}^r)+\hat{\theta}^r]\Big] = 0.5(\theta)^2 - 0.5(\hat{\theta}^r)^2 \tag{89}$$

$$\iff$$

$$S^*(\theta) = \frac{1}{\gamma(\hat{\theta}^r)+\theta}\Big[0.5(\theta)^2 - 0.5(\hat{\theta}^r)^2 + (\gamma(\hat{\theta}^r)+\hat{\theta}^r)\hat{S}^r\Big] \tag{90}$$

This establishes the functional form of $S^*(\theta)$ as described in the propostion.

Part 2: Calculating $\hat{\theta}^r$

When facing a consumer with type $\hat{\theta}^r$, the firm must be exactly indifferent between offering the surplus $\hat{S}^r$ and the optimal surplus it would set within the surpluses that induce rational consumers to search. If this is not the case, the firm would have a profitable deviation.

The profits a firm attains from consumers who do not continue searching are given by:

$$(\theta - S(\theta))\big[0.5\alpha + 0.5\hat{\theta}^r\alpha + (1-\alpha)\underbrace{S^{-1}(S(\theta))}_{=\theta}\big] \tag{91}$$

Plugging in $\theta = \hat{\theta}^r$ and $S^*(\hat{\theta}^r) = \hat{S}^r$ yields:

$$(\hat{\theta}^r - \hat{S}^r)\big[0.5\alpha + 0.5\hat{\theta}^r\alpha + (1-\alpha)\hat{\theta}^r\big] \tag{92}$$

The profits a firm attains from all consumers who continue searching are:

$$(\theta - S(\theta))\big[0.5\alpha\theta + 0.5\alpha\theta + (1-\alpha)\underbrace{S^{-1}(S(\theta))}_{=\theta}\big]$$

When the firm offers the surplus $S^*(\theta) = 0.5\theta$ to all these consumers, the profits are thus:

$$\big(0.5\theta\big)\big[\alpha\theta + (1-\alpha)\theta\big] = 0.5(\theta)^2 \tag{93}$$

Thus, the indifference condition that pins down $\hat{\theta}^r$ reads:

$$0.5(\hat{\theta}^r)^2 - (\hat{\theta}^r - \hat{S}^r)\big[\hat{\theta}^r + 0.5\alpha\big(1 - \hat{\theta}^r\big)\big] = 0 \tag{94}$$

Part 3: Equilibrium existence.

The key premise of this equilibrium was that naive consumers always search beyond the first firm. Note that any naive consumer continues searching after visiting the first firm if her initially obtained surplus S_{-j} satisfies:

$$\int_0^1 \int_0^1 \max\{S_{-j}, \theta_j - p_j^*(x)\} dF(x) dF(\theta_j) - c - S_{-j} > 0 \tag{95}$$

All naive consumers search even if consumers who attain the highest surplus continue searching. In other words, the necessary condition for equilibrium existence is the following:

$$\int_0^1 \int_0^1 \max\{S^*(1), \theta_j - p_j^*(x)\} dF(x) dF(\theta_j) - c - S^*(1) > 0 \tag{96}$$

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