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Biased Recommendations and Differentially Informed Consumers

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Abstract

We consider a monopolist selling an experience good to differentially informed consumers: some consumers are uncertain about their tastes, whereas other consumers are perfectly informed. The fully informed monopolist sets a uniform price and can make personalized product recommendations. We characterize conditions under which the monopolist *biases* its recommendations – that is, some consumers with match values lower than the marginal cost follow the recommendation to buy the product or some consumers with match values higher than the marginal cost follow the recommendation not to buy the product.

Keywords: information design, biased recommendations, recommender system

JEL-classification: L12, L15, D21, D42, M37

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1 Introduction

When consumers buy certain types of products irregularly, they may lack purchase-relevant information about how well the product matches their taste. Some consumers may have little information and instead base their purchase decision on whether the firm selling the product (or facilitating the sales as an intermediary) recommends it. With the advance of data collection and data analytics, firms are often able to infer match values with high precision and make personalized purchase recommendations. For example, e-commerce retailers (and platforms) collect a wealth of information about their customers and frequently make algorithmic purchase recommendations. In this article, we point to the importance that consumers are often heterogeneous with respect to the precision of this ex ante information. For instance, this holds in markets in which (i) consumers have heterogeneous skills that help them in assessing the expected match value prior to use, (ii) consumers have differential previous exposure to related products, or (iii) consumers arrive exogenously over time at a shop or website and late arrivals obtain match value information through word-of-mouth. This has important implications for the profit-maximizing recommendation strategy by the firm.

In this paper, we analyze information disclosure and price setting by a monopoly seller under uniform pricing.[1] There are two ex-ante consumer types: some consumers perfectly observe their match value while the others are uninformed and only know the prior distribution from which the match value is drawn. We assume that the monopoly seller knows the type of each consumer and their valuation. Because of ex-ante heterogeneity, the monopolist may want to recommend its product to consumers whose valuation is less than the

[1]There are several reasons for which a monopolist may decide not to engage in discriminatory pricing. First, discriminatory pricing opens the room for consumer arbitrage. Second, discriminatory pricing may trigger the intervention by a consumer protection agency, a sector regulator, or the legislator. Another reason could be that there is a consumer backlash and consumers stop buying from the firm if consumers uncover discriminatory pricing. It has been reported in the business press that “all the way back in 2000, when Amazon was mostly an online book and media store, it experimented with charging different prices to individual customers for the same DVDs. The customer response was so swift and negative that, nearly 20 years later, the e-tailer still avoids the practice.” Ben Unglesbee, ‘Why dynamic and personalized pricing strategies haven’t taken over retail – yet’, Retail Dive, 22 July 2019, available at: https://www.retaildive.com/news/why-dynamic-and-personalized-pricing-havent-taken-over-retail-yet/558975/ last accessed 17 April 2024.

marginal cost. We establish the conditions under which such inflated recommendations are part of the profit-maximizing monopoly strategy. We also provide alternative conditions under which recommendations are socially insufficient – that is, some consumers do not receive the recommendation to buy even though their valuation is strictly larger than the marginal cost.

We construct a numerical example to illustrate our point. Suppose that a monopoly firm sells at a constant marginal cost of production c. There is a unit mass of risk-neutral consumers and each consumer has a valuation in $\{1, 3, 5, 7, 9\}$, drawn independently across consumers. Each consumer independently receives a signal of its valuation, which is a fully informative signal with a probability of 1/2 and completely uninformative with the remaining probability of 1/2. A firm perfectly learns a consumer's valuation and the signal they have received. It commits to a recommendation policy and sets a uniform price for all consumers to maximize its profits.

To characterize the profit-maximizing solution for a range of c, we determine the demand for prices $p \in \{5, 7, 9\}$. At a price of $p = 5$, the firm can sell to at most 4/5 of consumers. It does so by recommending the product to all consumers with an uninformative signal who thus have an expected valuation of 5. In addition, it sells to all consumers with an informative signal with realization 5, 7, or 9.

At a price of $p = 7$, the firm can sell to at most 1/2 of consumers. It does so by recommending the product to all consumers with valuation $v \in \{5, 7, 9\}$ and an uninformative signal and, thus, is able to sell to them at the expected valuation conditional on receiving a recommendation, which is equal to 7 (i.e. 3/5 of all consumers with an uninformative signal). In addition, it sells to all consumers with an informative signal with realization 7 or 9 (i.e. 2/5 of all consumers with an informative signal).

At a price of $p = 9$, the firm can sell to at most 1/5 of consumers. It does so by recommending the product to all consumers with valuation $v = 9$ and an uninformative signal and, thus, is able to sell to them at expected valuation conditional on receiving a recommendation, which is equal to 9. In addition, it sells to all consumers with an informative signal with realization 9.

The firm maximizes its profit at $p = 7$ if $\frac{1}{2}(7-c) \geq \max\{\frac{4}{5}(5-c), \frac{1}{5}(9-c)\}$, which holds if and only if $c \in [\frac{5}{3}, \frac{17}{3}]$. For $c \in [\frac{5}{3}, 3)$, consumers with valuation 3 follow the recommendation not to buy the product, which is inefficient. For $c \in (5, \frac{17}{3}]$, consumers with valuation 5 follow the recommendation to buy the product even though their valuation is below marginal costs. In both instances, the firm gives biased recommendations from a welfare perspective. What is more, consumers with an uninformative signal and valuation 5 pay a price above their valuation. The insight of the numerical example carries over to continuous taste distributions, as we show in this paper.

Related literature. Our analysis of ex-ante heterogeneous information complements the analysis of ex-ante heterogeneous tastes in Peitz and Sobolev (forthcoming).[2] In this paper, the firm's choice of inflated versus insufficient recommendations depends on the shape of the virtual value function; our Proposition 2 adopts a result by Ivanov (2009), as we explain in the main text. More broadly, this paper belongs to the literature on Bayesian persuasion initiated by Kamenica and Gentzkow (2011) – for recent surveys, see Bergemann and Morris (2019) and Kamenica (2019). Lewis and Sappinton (1994) consider the edge case in which either all consumers receive a fully informative or a completely uninformative signal and shows that a monopolist does not have an incentive to bias its recommendations. Rayo and Segal (2010, Section VIII.C) consider a seller who discloses information and sets the product price in a setting in which a consumer draws their reservation value and show that the profit-maximizing disclosure rule is fully revealing. By contrast, we show that a monopoly seller maximizes its profit by partially informing consumers with an uninformative signal and providing biased recommendations.

In our analysis, the monopolist is restricted to set a uniform price. If the monopolist were able to segment consumers and price discriminate, it would not have an incentive to bias its recommendations. Thus, our work complements the work on information design under price discrimination (Bergemann et al., 2015).

Our model is presented in Section 2. In Section 3, we characterize sufficient conditions on

[2] As mentioned in the conclusion, we could restrict the firm to set its price and introduce an intermediary making recommendations as in Peitz and Sobolev (forthcoming). Other work on biased intermediaries includes Armstrong and Zhou (2011), Hagiu and Jullien (2011), and de Cornière and Taylor (2019).

the virtual value function, under which the seller inflates recommendations/provides insufficient recommendations. In section 4, we provide approximation results for sufficiently high and low marginal cost c. Section 5 concludes.

2 Model

Consider a seller offering an experience good. There is a unit mass of consumers with heterogeneous match values distributed according to the cumulative distribution function $F(\varepsilon) : [\underline{\varepsilon}, \overline{\varepsilon}] \subset \mathbb{R}_+$, which has a continuous positive density $f(\varepsilon)$. The marginal cost of production is $c \in (\underline{\varepsilon}, \overline{\varepsilon})$. We assume that $1 - F(\varepsilon)$ is log-concave on $[\underline{\varepsilon}, \overline{\varepsilon}]$. A fraction $\alpha \in [0, 1]$ of consumers are uninformed about their match values and believe that they are distributed according to $F(\varepsilon)$. They observe their match value after purchase. All other consumers are informed and know their match value before purchase.

The seller sets a uniform price and, in addition, can reveal information about the match values to uninformed consumers by providing personalized product recommendations. Uniform pricing may be due to a regulatory requirement or due to free arbitrage.[3]

The product recommendation policy μ represents a mapping from $[\underline{\varepsilon}, \overline{\varepsilon}] \times M \to [0, 1]$, where M is the set of messages.

We start by characterizing the profit-maximizing recommendation policy and price of the seller when all consumers are either fully informed ($\alpha = 0$) or uninformed ($\alpha = 1$).

Boundary cases. First, suppose that $\alpha = 0$ and, thus, all consumers are informed. The profit of the seller setting price p is $\pi^i(p) = (p - c)(1 - F(p))$. The first-order condition is

$$\frac{d\pi^i}{dp} = \left(\frac{1 - F(p)}{f(p)} - (p - c) \right) f(p) = 0.$$

By the log-concavity of $1 - F(p)$, the monopoly price p^i is uniquely defined and solves

$$p^i = c + \frac{1 - F(p^i)}{f(p^i)}.$$

[3]See also footnote 1. In the case of free arbitrage, we assume that ex-ante uninformed consumers learn their match value after consumption.

Second, suppose that $\alpha = 1$ and, thus, all consumers are uninformed about their match values. For any price $p \in (c, \bar{\varepsilon})$, the profit-maximizing recommendation policy is represented by a cutoff $\hat{\varepsilon} = \hat{\varepsilon}(p)$ (as shown in Lemma 1 in Appendix C of Peitz and Sobolev (2024)). That is, the seller recommends buying the good if and only if $\varepsilon \geq \hat{\varepsilon}$. The profit of the seller setting price p and recommendation policy $\hat{\varepsilon} \in [\underline{\varepsilon}, \bar{\varepsilon}]$, is given by

$$\pi^u(p) = (p - c)(1 - F(\hat{\varepsilon})), \text{ s.t. } p = \mathbb{E}[\varepsilon | \varepsilon \geq \hat{\varepsilon}].$$

The following proposition characterizes the profit-maximizing pricing and the recommendation policy when all consumers are uninformed about their match values.

Proposition 1. *Suppose that all consumers are uninformed ($\alpha = 1$). The seller sets price $p^u = \mathbb{E}[\varepsilon | \varepsilon \geq c]$ and recommends buying the product if and only if $\varepsilon \geq c$. Consumers follow the seller's recommendations. The equilibrium profit of the seller is given by*

$$(1 - F(c))\mathbb{E}[\varepsilon - c | \varepsilon \geq c].$$

Proof. The result directly follows from Lemma 2 in Appendix C of Peitz and Sobolev (2024), applied to the case in which $\underline{\theta} = \bar{\theta} = 0$. □

Since only consumers with $\varepsilon \geq c$ receive the recommendation to buy the product, the recommendation policy of the seller is efficient from the perspective of the social planner maximizing total welfare.

3 Biased Recommendations

In this section, we analyze the model in which both consumer groups are present; that is, $\alpha \in (0, 1)$. We explore how the presence of informed consumers changes the seller's profit-maximizing recommendation policy to characterize the conditions under which the seller *i)* inflates recommendations – some consumers with $\varepsilon < c$ receive buying recommendations; *ii)* provides insufficient recommendations – some consumers with $\varepsilon > c$ do not receive buying recommendations; *iii)* provides efficient recommendations – consumers receive buying recommendations if and only if $\varepsilon \geq c$.

We define $p^* = p^*(\alpha)$ as a profit-maximizing price of the seller for $\alpha \in (0, 1)$. In the following lemma, we show that $p^*(\alpha)$ is between the minimum and the maximum of p^u and p^i.

Lemma 1. *Suppose that $\alpha \in (0, 1)$ and $p^u \neq p^i$. Then, $p^* \in (\min\{p^u, p^i\}, \max\{p^u, p^i\})$.*

The proof of Lemma 1 is relegated to the Appendix. By Lemma 1, we have that if $p^i < p^u$, then the profit-maximizing price p^* satisfies $p^* < p^u$ and the seller's recommendation policy induces *inflated recommendations* – that is, some consumers with $\varepsilon < c$ are recommended to buy the good. If $p^i > p^u$, then $p^* > p^u$ and the seller provides insufficient recommendations. If $p^u = p^i$, then $p^* = p^u$ and the seller's recommendation policy is efficient. Hence, the problem of determining whether the seller inflates recommendations or provides insufficient recommendations for $\alpha \in (0, 1)$ boils down to comparing p^i with $p^u = \mathbb{E}[\varepsilon|\varepsilon \geq c]$.

We define the virtual value function as

$$\psi(\varepsilon) \equiv \varepsilon - \frac{1 - F(\varepsilon)}{f(\varepsilon)}.$$

The following proposition establishes that the shape of the virtual value function determines whether the monopolist provides inflated, efficient, or socially insufficient recommendations.

Proposition 2. *The seller's profit-maximizing strategy entails*

- *inflated recommendations, if $\psi(\cdot)$ is strictly concave;*
- *insufficient recommendations, if $\psi(\cdot)$ is strictly convex;*
- *efficient recommendations, if $\psi(\cdot)$ is linear.*

Proof. The derivative of π^i can be rewritten as

$$\frac{d\pi^i}{dp} = \left(\frac{1 - F(p)}{f(p)} - (p - c)\right) f(p) = (c - \psi(p))f(p).$$

The sign of the derivative of π^i at $p^u = \mathbb{E}[\varepsilon|\varepsilon \geq c]$ is determined by $c - \psi(p^u)$. Note that

$$\begin{aligned} c - \mathbb{E}[\psi(\varepsilon)|\varepsilon \geq c] &= \frac{1}{1 - F(c)} \int_c^{\bar{\varepsilon}} (c - \psi(\varepsilon))\, f(\varepsilon) d\varepsilon \\ &= \frac{1}{1 - F(c)} \int_c^{\bar{\varepsilon}} \frac{d\pi^i}{d\varepsilon} d\varepsilon \\ &= \frac{\pi^i(\bar{\varepsilon}) - \pi^i(c)}{1 - F(c)} = 0. \end{aligned}$$

Therefore, if ψ is strictly concave, by Jensen's inequality, we obtain that $c - \psi(p^u) < c - \mathbb{E}[\psi(\varepsilon)|\varepsilon \geq c] = 0$. This implies that the derivative of $\pi^i < 0$ at $p = p^u$ and $p^u > p^i$. By Lemma 1, the seller inflates recommendations. Similarly, if ψ is strictly convex, the seller induces insufficient recommendations. Finally, if $\psi(\cdot)$ is a linear function, then the seller provides efficient recommendations. □

Ivanov (2009) compares the monopoly price under full information p^i and the expected match value $\mathbb{E}[\varepsilon]$ for different distributions of match values. Theorem 1 of Ivanov (2009) establishes a sufficient condition for $p^i > (<)\mathbb{E}[\varepsilon]$ that depends on convexity vs. concavity of the virtual value function and on whether or not $\underline{\varepsilon} > c$. In Proposition 2, we adapt this result to our problem and show that only convexity vs. concavity of the virtual value function is sufficient to determine the relation between p^i and $\mathbb{E}[\varepsilon|\varepsilon \geq c]$.

As an example, consider the power distribution of match values, $F(\varepsilon) = \varepsilon^k$, $k > 0$. The virtual value function is given by $\psi(\varepsilon) = \varepsilon - \frac{1-\varepsilon^k}{k\varepsilon^{k-1}}$. The second derivative of ψ is $\psi'' = (1-k)/\varepsilon^{k+1}$. Therefore, if $k > 1$ and the distribution of match values is convex, then the virtual value function is concave and the seller inflates recommendations. Otherwise, if $k \in (0,1)$, then the seller provides insufficient recommendations. Figure 1 shows p^i and p^u as the functions of k. For $k > 1$, we have that $p^u > p^i$, and the seller inflates recommendations. Otherwise, if $k \in (0,1)$, we have that $p^i > p^u$ and the seller provides insufficient recommendations. In the borderline case $k = 1$, recommendations are efficient.

There exist many other classes of distribution functions, for which the virtual value function $\psi(\varepsilon)$ is either concave or convex. Among those, we have gamma, Weibull, log-logistic types of distribution functions.

As another example, we consider the class of the so-called ρ-linear demand functions (see Bulow and Pfleiderer, 1983). Suppose that

$$F(\varepsilon) = 1 - K\left(1 + \frac{1}{\rho}(a - b\varepsilon)\right)^{\rho},$$

which is defined on the support $[\underline{\varepsilon}, \overline{\varepsilon}]$ that depends on the parameters a, b, K and ρ such that $F(\overline{\varepsilon}) = 1$ and $F(\underline{\varepsilon}) = 0$. Note that the cost pass-through rate is constant (and equals to $\alpha = 1/(1+\rho)$) if and only if the demand function $1 - F(p)$ is ρ-linear. The density function

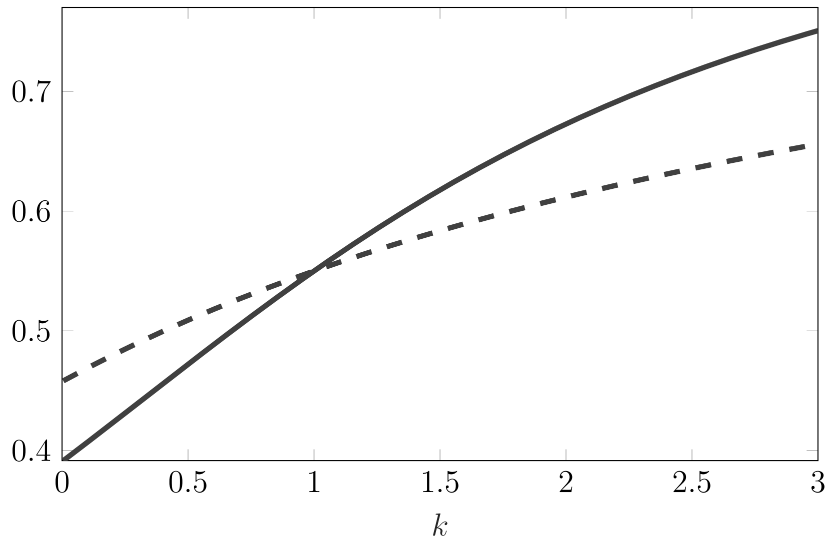

Figure 1: *Prices p^u (solid) and p^i (dashed) in k for $F(\varepsilon) = \varepsilon^k$, $k > 0$ and $c = 0.1$.*

is given by $f(\varepsilon) = \frac{b(1-F(\varepsilon))}{1+\frac{1}{\rho}(a-b\varepsilon)}$. Thus, it is straightforward to see that the virtual value function is linear and, according to Proposition 2, the seller provides efficient recommendations.

Remark 1. *Suppose that the demand function $1 - F(p)$ is ρ-linear. Then, the recommendation policy of the seller is efficient.*

We establish a connection between the concavity/convexity of the virtual value function and the behavior of the cost pass-through rate, $p'(c)$. By taking the derivative of the first-order condition of the seller's profit maximization problem when $\alpha = 0$, we obtain $\psi'_p p'_c = 1$. Therefore,

$$p''_{cc} = -\frac{\psi''_{pp} p'_c}{(\psi'(p))^2} = -\psi''_{pp}(p'_c)^3.$$

We observe that the cost pass-through rate strictly increases (strictly decreases) if and only if the virtual value function is strictly concave (convex). This implies that the sufficient conditions of Proposition 2 can be rewritten in terms of the behavior of the cost pass-through rate.

Remark 2. *Suppose that the cost pass-through rate $p'(c)$ strictly increases in marginal cost c. Then, the seller induces inflated recommendations. If the cost pass-through rate $p'(c)$ strictly decreases in cost c, then the seller provides insufficient recommendations.*

At the end of this section, we provide another sufficient condition for the outcome with inflated recommendations.

Proposition 3. *Suppose that* $\frac{dp^u}{d\hat{\varepsilon}} > \frac{dp^i}{dc}$ *for all* $p \in [\mathbb{E}[\varepsilon], \overline{\varepsilon}]$*. Then, the seller inflates recommendations for the uninformed type of consumers.*

4 Approximation results for high and low marginal costs

We study the conditions under which the seller inflates recommendations for high and low marginal costs. First, we explore the case of c sufficiently close to $\overline{\varepsilon}$. Second, we turn to the case of c sufficiently close to $\underline{\varepsilon}$. To determine whether or not the seller inflates recommendations, we derive Taylor approximation results for $p^u - p^i$.

The case of high c. We explore the sign of $p^u - p^i$ in a small neighborhood of $c = \overline{\varepsilon}$. Since for c sufficiently close to $\overline{\varepsilon}$, the price p^i is greater than $\mathbb{E}[\varepsilon]$, we have that, by Lemma 2, it is sufficient to explore the sign of $c - \hat{\varepsilon}^i$, where $\hat{\varepsilon}^i$ solves $p^i = \mathbb{E}[\varepsilon | \varepsilon| \geq \hat{\varepsilon}^i]$. In what follows, we require that the inverse hazard rate function $\frac{1-F(\varepsilon)}{f(\varepsilon)}$ to be twice differentiable at $\varepsilon = \overline{\varepsilon}$.

We start by seeking a second-order approximation of $\overline{\varepsilon} - c$ with respect to $\overline{\varepsilon} - p^i$ in the neighborhood of $c = \overline{\varepsilon}$. The first-order condition determining price p^i can be rewritten as

$$\overline{\varepsilon} - c = \overline{\varepsilon} - p + \frac{1 - F(p)}{f(p)}.$$

The first and the second derivatives of the inverse hazard rate are respectively given by

$$\left(\frac{1-F}{f}\right)' = -1 - \frac{f'(1-F)}{f^2},$$
$$\left(\frac{1-F}{f}\right)'' = -\frac{f'}{f}\left(\frac{1-F}{f}\right)' - \left(\frac{f'}{f}\right)'\frac{1-F}{f} = \frac{f'}{f} + \frac{1-F}{f}\left(\left(\frac{f'}{f}\right)^2 - \left(\frac{f'}{f}\right)'\right).$$

Therefore, the second-order Taylor approximation of the inverse hazard rate function at $p = \overline{\varepsilon}$ is given by

$$\frac{1 - F(p^i)}{f(p^i)} = -(p^i - \overline{\varepsilon}) + \frac{1}{2}\frac{f'(\overline{\varepsilon})}{f(\overline{\varepsilon})}(\overline{\varepsilon} - p^i)^2 + o((\overline{\varepsilon} - p^i)^2),$$

where $o(\cdot)$ is Landau's little-o.[4] By plugging this into the the first-order condition, which

[4] $o(\cdot)$ is Landau's little-o notation: $f(x) = o(g(x))$ in the neighborhood of $x = x_0$ if $f(x)/g(x) \underset{x \to x_0}{\longrightarrow} 0$.

determines price p^i, we obtain

$$\overline{\varepsilon} - c = \overline{\varepsilon} - p + \frac{1}{2}\frac{f'(\overline{\varepsilon})}{f(\overline{\varepsilon})}(\overline{\varepsilon} - p^i)^2 + o((\overline{\varepsilon} - p^i)^2).$$

Next, we derive a second-order approximation of $\overline{\varepsilon} - \hat{\varepsilon}^i$ with respect to $\overline{\varepsilon} - p^i$ in a small neighborhood of $c = \overline{\varepsilon}$. The equation determining $\hat{\varepsilon}^i$ can be rewritten as

$$\overline{\varepsilon} - p^i = \overline{\varepsilon} - \mathbb{E}[\varepsilon | \varepsilon \geq \hat{\varepsilon}^i] = \overline{\varepsilon} - \hat{\varepsilon}^i - \frac{\int_{\hat{\varepsilon}^i}^{\overline{\varepsilon}} (1 - F(\varepsilon)) d\varepsilon}{1 - F(\hat{\varepsilon}^i)}.$$

We derive the second-order approximation of the right-hand side with respect to $\overline{\varepsilon} - \hat{\varepsilon}^i$. Note that

$$\int_{\hat{\varepsilon}^i}^{\overline{\varepsilon}} (1 - F(\varepsilon)) d\varepsilon = \frac{1}{2} f(\overline{\varepsilon})(\overline{\varepsilon} - \hat{\varepsilon}^i)^2 - \frac{1}{6} f'(\overline{\varepsilon})(\overline{\varepsilon} - \hat{\varepsilon}^i)^3 + o((\overline{\varepsilon} - \hat{\varepsilon}^i)^3).$$

Moreover, we have that

$$\frac{\overline{\varepsilon} - \hat{\varepsilon}^i}{1 - F(\hat{\varepsilon}^i)} = \frac{1}{f(\overline{\varepsilon})} + \frac{1}{2}\frac{f'(\overline{\varepsilon})}{f^2(\overline{\varepsilon})}(\overline{\varepsilon} - \hat{\varepsilon}^i) + o(\overline{\varepsilon} - \hat{\varepsilon}^i).$$

Thus, we obtain that

$$\begin{aligned}
\frac{\int_{\hat{\varepsilon}^i}^{\overline{\varepsilon}} (1 - F(\varepsilon)) d\varepsilon}{1 - F(\hat{\varepsilon}^i)} &= \frac{\int_{\hat{\varepsilon}^i}^{\overline{\varepsilon}} (1 - F(\varepsilon)) d\varepsilon}{\overline{\varepsilon} - \hat{\varepsilon}^i} \frac{\overline{\varepsilon} - \hat{\varepsilon}^i}{1 - F(\hat{\varepsilon}^i)} \\
&= \left(\frac{1}{2} f(\overline{\varepsilon})(\overline{\varepsilon} - \hat{\varepsilon}^i) - \frac{1}{6} f'(\overline{\varepsilon})(\overline{\varepsilon} - \hat{\varepsilon}^i)^2 \right) \left(\frac{1}{f(\overline{\varepsilon})} + \frac{1}{2}\frac{f'(\overline{\varepsilon})}{f^2(\overline{\varepsilon})}(\overline{\varepsilon} - \hat{\varepsilon}^i) \right) + o((\overline{\varepsilon} - \hat{\varepsilon}^i)^2) \\
&= \frac{1}{2}(\overline{\varepsilon} - \hat{\varepsilon}^i) + \frac{1}{12}\frac{f'(\overline{\varepsilon})}{f(\overline{\varepsilon})}(\overline{\varepsilon} - \hat{\varepsilon}^i)^2 + o((\overline{\varepsilon} - \hat{\varepsilon}^i)^2).
\end{aligned}$$

By plugging this back into the equation determining $\hat{\varepsilon}^i$, we have that

$$\overline{\varepsilon} - p^i = \frac{1}{2}(\overline{\varepsilon} - \hat{\varepsilon}^i) - \frac{1}{12}\frac{f'(\overline{\varepsilon})}{f(\overline{\varepsilon})}(\overline{\varepsilon} - \hat{\varepsilon}^i)^2 + o((\overline{\varepsilon} - \hat{\varepsilon}^i)^2).$$

Note that $o((\overline{\varepsilon} - p^i)^n) = o((\overline{\varepsilon} - \hat{\varepsilon}^i)^n)$ for all $n \geq 0$ and $\overline{\varepsilon} - \hat{\varepsilon}^i = 2(\overline{\varepsilon} - p^i) + o(\overline{\varepsilon} - p^i)$. Thus, by plugging this into the approximation for $\overline{\varepsilon} - p^i$ and solving for $\overline{\varepsilon} - \hat{\varepsilon}^i$, we have that

$$\overline{\varepsilon} - \hat{\varepsilon}^i = 2(\overline{\varepsilon} - p^i) + \frac{2}{3}\frac{f'(\overline{\varepsilon})}{f(\overline{\varepsilon})}(\overline{\varepsilon} - p^i)^2 + o((\overline{\varepsilon} - p^i)^2).$$

After subtracting $\overline{\varepsilon} - c$ from $\overline{\varepsilon} - \hat{\varepsilon}^i$, we obtain the desired approximation,

$$c - \hat{\varepsilon}^i = \frac{1}{6}\frac{f'(\overline{\varepsilon})}{f(\overline{\varepsilon})}(\overline{\varepsilon} - p^i)^2 + o((\overline{\varepsilon} - p^i)^2).$$

We conclude that $c - \hat{\varepsilon}^i > 0$ if and only if $f'(\overline{\varepsilon}) > 0$. The following proposition summarizes the analysis above.

Proposition 4. *Suppose that the inverse hazard rate function* $\frac{1-F(\varepsilon)}{f(\varepsilon)}$ *is twice differentiable at* $\varepsilon = \overline{\varepsilon}$. *Then, in the neighborhood of* $c = \overline{\varepsilon}$, *the seller inflates recommendations if* $f'(\overline{\varepsilon}) > 0$ *and provides insufficient recommendation if* $f'(\overline{\varepsilon}) < 0$.

Note that for the power distribution $F(\varepsilon) = \varepsilon^k$, we have that $f'(\overline{\varepsilon}) = k(k-1)\overline{\varepsilon}^{k-2}$ is positive for $k > 1$, and the seller inflates recommendations. Conversely, $f'(\overline{\varepsilon})$ is negative, and the seller provides insufficient recommendations for $k \in (0,1)$.

The case of low c. We explore the sign of $p^u - p^i$ for $c = \underline{\varepsilon}$. The profit-maximizing price that the seller would set if all consumers were uninformed about their match values is equal to $p^u = \mathbb{E}[\varepsilon]$. The profit-maximizing price when all consumers are informed is given by p^i, which solves

$$p^i - \underline{\varepsilon} = \frac{1 - F(p^i)}{f(p^i)}.$$

By Proposition 2, if $\psi(\varepsilon)$ is strictly concave (strictly convex), then $\mathbb{E}[\varepsilon] > (<) p^i$. The following remark states the sufficient condition for inflated recommendations for $c = \varepsilon$.

Remark 3. *Suppose that* $c = \underline{\varepsilon}$. *If* $\psi(\varepsilon)$ *is strictly concave, the seller inflates recommendations. If* $\psi(\varepsilon)$ *is strictly convex, the seller provides insufficient recommendations.*

Finally, we provide another sufficient condition for inflated recommendations at $c = \underline{\varepsilon}$.

Remark 4. *Suppose that* f *is symmetric, i.e.,* $f\left(\frac{\overline{\varepsilon}+\underline{\varepsilon}}{2} - x\right) = f\left(\frac{\overline{\varepsilon}+\underline{\varepsilon}}{2} + x\right)$, *for all* $x \in \left[0, \frac{\overline{\varepsilon}-\underline{\varepsilon}}{2}\right]$. *Then, at* $c = \underline{\varepsilon}$, *the seller provides inflated recommendations if and only if* $f\left(\frac{\overline{\varepsilon}+\underline{\varepsilon}}{2}\right) < \frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}$.

Proof. The expected match value for the symmetric distribution is given by $p^u = \mathbb{E}[\varepsilon] = \frac{\overline{\varepsilon}+\underline{\varepsilon}}{2}$.

The first-order condition at price p^u, when all consumers are informed, is given by

$$\left.\frac{d\pi^i}{dp}\right|_{p=p^u} = -(p^u - \underline{\varepsilon}) f(p^u) + (1 - F(p^u)) = -\frac{\overline{\varepsilon} - \underline{\varepsilon}}{2} f\left(\frac{\overline{\varepsilon} + \underline{\varepsilon}}{2}\right) + \left(1 - F\left(\frac{\overline{\varepsilon} + \underline{\varepsilon}}{2}\right)\right).$$

Since the distribution is symmetric, we have that $F\left(\frac{\overline{\varepsilon}+\underline{\varepsilon}}{2}\right) = \frac{1}{2}$, implying that $p^u > p^i$ if and only if

$$f\left(\frac{\overline{\varepsilon} + \underline{\varepsilon}}{2}\right) > \frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}.$$

□

5 Conclusion

In this paper, we analyze a monopoly seller's price and recommendation policy where the monopolist can not price discriminate. Consumers draw their match value from the same distribution, but some consumers are perfectly informed about the realization while others are uninformed. We provide conditions such that the monopolist maximizes its profits by inflating recommendations – that is, some consumers receive a purchase recommendation and follow that recommendation even though the marginal cost is larger than the valuation.

Our finding appears to be robust to a more general setting in which consumers receive informative but noisy signals about their match value, as long as the level of noise is heterogeneous across consumers. Our finding also extends to a setting in which the firm setting the retail price is different from the firm making the purchase recommendation. For example, digital platforms may provide purchase recommendations and charge sellers for their intermediation service (that includes the recommendation service). In such a setting the intermediary biases its recommendations if it receives a fraction of the seller's profits (and, thus, the seller's and the intermediary's incentives are aligned).

Appendix

Proof of Lemma 1. Suppose that $p^u > p^i$. By Lemma 1 in Appendix C of Peitz and Sobolev (2024), the seller's recommendation policy is characterized by a cutoff $\hat{\varepsilon}^*$ that makes the incentive constraint of the uninformed consumers binding – that is, it solves $p^* = \mathbb{E}[\varepsilon|\varepsilon \geq \hat{\varepsilon}^*]$. Note that p^* can not be strictly higher than p^u, as then the seller could raise his profit from both groups of consumers by slightly decreasing the price and adjusting the recommendation strategy accordingly. Similarly, p^* cannot be strictly lower than p^i. This implies that $p^* \in [p^i, p^u]$.

Next, consider the profit of the seller setting price $p \in [p^i, p^u]$,

$$\pi(p) = \alpha\pi^i(p) + (1-\alpha)\pi^u(p).$$

The derivative of $\pi(p)$ with respect to p at $p = p^u$ equals to $\alpha\frac{d\pi^i}{dp}(p_u)$. It is easy to see that since $p^u > p^i$ and the fact that $1-F$ is strict log-concave, we have that $\alpha\frac{d\pi^i}{dp}(p^u) = \alpha\left(\frac{1-F(p^u)}{f(p^u)} - (p^u - c)\right) f(p^u) < \alpha\left(\frac{1-F(p^i)}{f(p^i)} - (p^i - c)\right) f(p^u) = 0$. This implies that $p^* < p^u$. Next, we evaluate the sign of the derivative of $\pi(p)$ at $p = p^i$ that is equal to $(1-\alpha)\frac{d\pi^u}{dp}(p^i)$. First, suppose that $p^i \geq \mathbb{E}[\varepsilon]$. Then, for any $p \in [p^i, p^u]$, there exists a corresponding $\hat{\varepsilon}$ that solves $p = \mathbb{E}[\varepsilon|\varepsilon \geq \hat{\varepsilon}]$. Thus, we have that the derivative of $\pi^u(p)$ at $p = p^i$ is given by

$$\begin{aligned}
\frac{d\pi^u}{dp}\Big|_{p=p^i} &= f(\hat{\varepsilon})\frac{d\hat{\varepsilon}}{dp}\left(\frac{1-F(\hat{\varepsilon})}{f(\hat{\varepsilon})\frac{d\hat{\varepsilon}}{dp}} - (p^i - c)\right) = f(\hat{\varepsilon})\frac{d\hat{\varepsilon}}{dp}\left(\frac{\int_{\hat{\varepsilon}}^{\overline{\varepsilon}}(\varepsilon - \hat{\varepsilon})dF(\varepsilon)}{1-F(\hat{\varepsilon})} - (p^i - c)\right) \\
&= f(\hat{\varepsilon})\frac{d\hat{\varepsilon}}{dp}\left(p^i - \hat{\varepsilon} - (p^i - c)\right) \\
&= f(\hat{\varepsilon})\frac{d\hat{\varepsilon}}{dp}(c - \hat{\varepsilon}).
\end{aligned}$$

Note that $\frac{d\hat{\varepsilon}}{dp} > 0$ and since $p^i < p^u$ we have that $\hat{\varepsilon} < c$. Thus, the profit function strictly increases at $p = p^i$, implying that $p^* > p^i$. Second, suppose that $p^i <\in (c, \mathbb{E}[\varepsilon])$, then the profit from an uninformed consumer equals $p^i - c$. The derivative of the profit function is $\frac{d\pi(p^i)}{dp} = (1-\alpha)\frac{d\pi^u(p^i)}{dp} = 1 - \alpha > 0$. We conclude that $p^* \in (p^i, p^u)$. The case of $p^u < p^i$ can be analyzed analogously.

□

Lemma 2. *The seller inflates recommendations for the uninformed consumers if and only if either $p^i \leq \mathbb{E}[\varepsilon]$ or $p^i > \mathbb{E}[\varepsilon]$ and $\hat{\varepsilon}^i < c$, where $\hat{\varepsilon}^i$ solves $p^i = \mathbb{E}[\varepsilon|\varepsilon \geq \hat{\varepsilon}^i]$. If $\hat{\varepsilon}^i > c$, then* *the seller induces insufficient recommendations. Otherwise, if $c = \hat{\varepsilon}^i$, recommendations are efficient.*

Proof. The derivative of $\pi^u(p)$ at $p > \mathbb{E}[\varepsilon]$ with respect to p is given by

$$\frac{d\pi^u}{dp} = f(\hat{\varepsilon})\frac{d\hat{\varepsilon}}{dp}(c - \hat{\varepsilon}).$$

It is straightforward to show that $\frac{d\hat{\varepsilon}}{dp} > 0$ for all $p \geq \mathbb{E}[\varepsilon]$. We obtain that if $p^i \leq \mathbb{E}[\varepsilon]$, then $p^u > p^i$. Otherwise, if $p^i > \mathbb{E}[\varepsilon]$, then we have that $p^u > p^i$ if and only if $\hat{\varepsilon}^i < c$, where $\hat{\varepsilon}^i$ solves $p^i = \mathbb{E}[\varepsilon | \varepsilon \geq \hat{\varepsilon}^i]$. □

Proof of Proposition 3. Consider an auxiliary function

$$\phi(p) = p - \frac{1 - F(p)}{f(p)} - \hat{\varepsilon}(p), \text{ where } p = \mathbb{E}[\varepsilon | \varepsilon \geq \hat{\varepsilon}(p)] \text{ and } p \in [\mathbb{E}[\varepsilon], \overline{\varepsilon}]\,.$$

The derivative of this function can be represented as

$$\frac{d\phi}{dp} = \frac{1}{dp^i/dc} - \frac{1}{dp^u/d\hat{\varepsilon}},$$

where $\frac{dp^i}{dc}$ is computed at c that solves solves $p - c = \frac{1-F(p)}{f(p)}$ and $\frac{dp^u}{d\hat{\varepsilon}}(\hat{\varepsilon})$ is computed at $\hat{\varepsilon} = \hat{\varepsilon}(p)$. Thus, $d\phi/dp > 0$ at $p \in [\mathbb{E}[\varepsilon], \overline{\varepsilon}]$ if and only if $\frac{dp^u}{d\hat{\varepsilon}} > \frac{dp^i}{dc}$.

If $p^i < \mathbb{E}[\varepsilon]$, then by Lemma 2, the seller provides inflated recommendations. Suppose tat $p^i > \mathbb{E}[\varepsilon]$. Recall that $\frac{dp^u}{d\hat{\varepsilon}} > \frac{dp^i}{dc}$ implies that $\frac{d\phi}{dp} > 0$ for all $p \in [\mathbb{E}[\varepsilon], \overline{\varepsilon}]$. Note that

$$\lim_{p \to \overline{\varepsilon}} \phi(p) = \overline{\varepsilon} - \frac{1 - F(\overline{\varepsilon})}{\overline{\varepsilon}} - \overline{\varepsilon} = 0.$$

Thus, $\phi(p)$ strictly increases on $[\mathbb{E}[\varepsilon], \overline{\varepsilon}]$ and tends to 0 when p goes to $\overline{\varepsilon}$. Therefore, we obtain that $\phi(p) < 0$ for all $p \in [\mathbb{E}[\varepsilon], \overline{\varepsilon}]$. Consequently, we have that

$$c - \hat{\varepsilon}^i = p^i - \frac{1 - F(p^i)}{f(p^i)} - \hat{\varepsilon}(p^i) = \phi(p^i) < 0.$$

We conclude that $\hat{\varepsilon}^i < c$. Therefore, by Lemma 2, which is stated in the Appendix above, the seller inflates recommendations for the uninformed type of consumers. □

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