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Incentives for Collective Innovation

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Gregorio Curello†

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Abstract

Agents exert hidden effort to produce randomly-sized innovations in a technology they share. Returns from using the technology grow as it develops, but so does the opportunity cost of effort, due to an 'exploration-exploitation' trade-off. As monitoring is imperfect, there exists a unique (strongly) symmetric equilibrium, and effort in any equilibrium ceases no later than in the single-agent problem. Small innovations may hurt all agents in the symmetric equilibrium, as they severely reduce effort. Allowing agents to discard innovations increases effort and payoffs, preserving uniqueness. Under natural conditions, payoffs rise above those of all equilibria with forced disclosure.

Keywords: dynamic games, imperfect monitoring, public goods, private information.

1 Introduction

Innovation often has the features of a collective-action problem, as innovators gradually improve a technology that they share, while simultaneously using it. For example, firms collaborate to refine their products,[1] non-profit organisations

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1. Knowledge exchange occurs within R&D partnerships but also among firms in the same 'network' (Pénin (2007)). The latter can generate a 'collective' kind of innovation, in which no single firm is the main driver of progress or its major beneficiary (Powell and Giannella (2010)). This occurred for biotechnologies (Powell, Koput, and Smith-Doerr (1996)), semiconductors (Chesbrough (2003) and Lim (2009)), and in the steel industry (von Hippel (1987)). See Bessen and Nuvolari (2016) for historical examples. 'User' innovation is collective in nature as well (Harhoff and Lakhani (2016)).

draw from a pool of shared knowledge to improve their programmes,[2] and workers in a team learn from each other how better to perform their tasks.[3]

However, innovation differs from traditional games of public-good provision in at least two respects. First, both the arrival time and the magnitude of innovations are typically uncertain. Since research effort is often unobserved,[4] it is then difficult to monitor the effort exerted by one's partners. Second, innovators face a resource trade-off between using and improving the shared technology. For example, firm that collaborate towards a technological improvement allocate funds between its achievement and private activities that use the technology being developed.[5] Non-profits split resources between developing new ideas and managing existing programmes,[6] and employees allocate their time and effort between creative and routine behaviour.[7] Thus, as the shared technology develops and using it becomes more profitable, the opportunity cost of improving it rises.[8]

In this paper, I analyse a game of public-good provision with these two key features. Long-lived identical agents exert hidden effort to induce randomly-sized increments in the stock of a good (their frequency being increasing with effort). Agents' flow payoffs are a general function of their effort and of the current stock, and the marginal cost of effort increases (weakly) with the stock. The 'stock' is the quality of the technology that the agents share, and 'effort' measures the quantity of resources that each agent invests in the advancement of the technology, the rest being devoted to its use.

2. Performance monitoring, which facilitates the identification and imitation of effective programmes, has recently become prominent in the social sector (Kroeger and Weber (2014)). Moreover, innovation is known to play a critical role for non-profits (Dover and Lawrence (2012)).

3. Research on workplace innovation has grown substantially in the recent years (Anderson, Potočnik, and Zhou (2014)), and knowledge exchange within teams is a known predictor (van Knippenberg (2017)). Knoweldge transfer among different *units* within a firm is also a key driver of innovation (Van Wijk, Jansen, and Lyles (2008)).

4. For example, Geroski (1995) notes that 'R&D ventures may be unable to overcome moral hazard problems that lead participants to invest less than promised in the joint venture, divert the energies of people nominally assigned to [it, and] assign less talented researchers to [it...]'

5. This trade-off is due to financial constraints, a known barrier to innovation for firms (Hottenrott and Peters (2012)).

6. Balancing this 'exploration-exploitation' trade-off is a major challenge for non-profits. See Dover and Lawrence (2012) and references therein.

7. This is Ford (1996)'s influential model of employee innovation. Unsworth and Clegg (2010) find confirming evidence for this model; they show that employees choose 'creative action' based on whether they judge it 'worthwhile', and argue that time pressure is pivotal to the decision.

8. In the example involving firms, 'using' the technology may also include running private R&D which does not benefit the other firms. Meyer (2003) highlights this incentive effect: 'With the establishment of a profitable industry, technological uncertainty is reduced and the collective invention process evaporates. Surviving firms run private R&D.' Similarly, Powell and Giannella (2010) note that 'as technological uncertainty recedes, firms develop private R&D and focus on their own specific applications. Reliance on collective invention accordingly wanes.'

I characterise the welfare-maximising benchmark (Proposition 1) and show that the game admits a unique strongly symmetric equilibrium (Theorem 1). As monitoring is imperfect, equilibrium effort is pinned down by the current stock; that is to say, no form of punishment is sustainable. Moreover, effort in *any* equilibrium ceases whenever it would cease in the single-agent setting (Proposition 2), and the Folk Theorem need not hold.

Continuation payoffs in the strongly symmetric equilibrium may fall after a small innovation (i.e. an increment in the stock), as all agents severely reduce their effort. Thus mediocre advances in a technology can hinder its *overall* progress, and decrease its expected profitability.[9] This phenomenon hinges on the model's main novel features: agents face a trade-off between using and improving the technology, *and* innovations are lumpy and have random size (Proposition 3). I obtain a necessary and sufficient condition for such 'detrimental' innovations to be produced (with positive probability) as long as effort is exerted (Proposition 4), and when payoffs are linear (Corollary 2).

Because innovations have adverse effects on incentives, agents who obtain them may wish to delay their disclosure and adoption. To explore this idea, I allow agents to discard innovations, after observing their size. Namely, the ensuing game admits a unique strongly symmetric equilibrium in which, after any history, effort and continuation payoffs are higher than in the symmetric equilibrium with forced disclosure (Theorem 2). Moreover, if the number of agents is sufficiently large and the size of innovations sufficiently dispersed, ex-ante payoffs in this equilibrium exceed those of any equilibrium with forced disclosure (Proposition 5).[10] Thus, even though discarding innovations is clearly inefficient, allowing the agents to do so enhances equilibrium welfare.[11]

The rest of the paper is organised as follows. I summarise the relevant literature in Section 2. I describe the model in Section 3, present the social-welfare benchmark in Section 4, and analyse the equilibria in Section 5. In Section 6, I examine when innovations are detrimental in the symmetric equilibrium, and I analyse the game with disposal in Section 7.

9. This phenomenon may explain Dover and Lawrence (2012)'s observation that among non-profits 'successful past innovation can act as brakes on new ideas. Past success [...] lead[s] to complacency [...], structural barriers to innovation [...], and a lack of questioning the status quo'.

10. See Azevedo et al. (2020) and the references therein for the importance of large and rare innovations.

11. All results would carry over if agents could conceal innovations, provided they cannot covertly refine the improvements that they hide. See the discussion at the end of Section 7.2. In Curello (2023), I allow agents refine concealed innovations, and I construct a symmetric perfect Bayesian equilibrium when payoffs are linear. If the hypotheses of Proposition 5 hold, this equilibrium improves on all equilibria with forced disclosure as well.

2 Literature review

This paper contributes to understanding the extent to which free-riding can be overcome in partnership games with frequent actions. Strongly Symmetric Equilibria (SSE) are known to sustain higher payoffs than symmetric Markov equilibria in dynamic games with perfect monitoring of the aggregate contribution, sometimes achieving efficiency.[12] In repeated games with imperfect (public) monitoring, only 'bad news' Poisson signals are helpful in SSE: there exist efficient SSE if perfectly-revealing bad news are available, but no SSE improves on the stage-game equilibrium payoffs if bad news are completely absent.[13] As noted by Georgiadis (2015), the impossibility of cooperation under Brownian noise extends to dynamic (non-repeated) games: no SSE induces higher payoffs than the symmetric Markov equilibrium. In the model I analyse, free-riding cannot be overcome in SSE: the unique SSE is a symmetric Markov equilibrium. As I note in Section 5, the result is not driven by the lack of 'bad news' signals.

Moreover, the Folk Theorem typically holds in partnership games with perfect monitoring, and an analogue for public prefect equilibria (PPE) applies in stochastic games with imperfect monitoring that satisfy irreducibility and identifiability conditions.[14] Yet Guéron (2015) shows that, in a partnership game with monitoring subject to 'smooth' noise, there exists no PPE sustaining investment beyond the individually-rational level. As I note in Section 5, the Folk Theorem can fail in my model as well, but this is not driven by the 'smoothness' of the noise.

The baseline model is closely related to the dynamic contribution games of Fershtman and Nitzan (1991), Marx and Matthews (2000), Lockwood and Thomas (2002), and Battaglini, Nunnari, and Palfrey (2014), as agents gradually add to the

12. Marx and Matthews (2000) analyse a dynamic game of private provision of a public good, and show that efficient SSE exist if payoffs jump upon reaching an exogenous goal, or if payoffs are 'kinked' at the goal and agents are sufficiently patient. Hörner, Klein, and Rady (2022) compute the payoffs attainable in SSE of games of experimentation, and show that efficiency is reached if payoffs have a diffusion component. See also Lockwood and Thomas (2002).

13. See Proposition 5 of Abreu, Milgrom, and Pearce (1991) for the former result and Sannikov and Skrzypacz (2010) for the latter. Other major contributions to the literature on repeated games with imperfect monitoring include Green and Porter (1984), Fudenberg, Levine, and Maskin (1994), Abreu, Pearce, and Stacchetti (1986), and Radner, Myerson, and Maskin (1986). Georgiadis (2015) and Cetemen, Hwang, and Kaya (2020) analyse (non-repeated) partnership games with imperfect monitoring.

14. The Folk Theorem holds in the partnership games of Marx and Matthews (2000), Lockwood and Thomas (2002), and Hörner, Klein, and Rady (2022), which feature perfect monitoring. Dutta (1995) derived a Folk Theorem for general stochastic games with perfect monitoring. Analogues for games with imperfect monitoring were obtained by Fudenberg and Yamamoto (2011), Hörner et al. (2011), and Pęski and Wiseman (2015).

stock of a public good, and the incentives to produce drop as the stock grows.[15] I contribute to this strand of the literature in two ways. First, I allow the production cost to depend on the current stock and show that, although a higher initial stock is beneficial absent its incentive effects, it may lead to lower equilibrium payoffs. Second, by allowing the stock to make discrete, randomly-sized jumps, I show that agents may have an incentive to discard increments in the stock, and I analyse the impact of allowing the agents to do so.

Games of strategic experimentation model social learning about the value of a *given* technology or project.[16] Flow payoffs and incentives to produce (information) move jointly in these games: 'good news' simultaneously makes agents better off and experimentation more attractive. This implies that good news always increases continuation payoffs in equilibrium, and agents have no incentive to conceal it.[17]

This paper is also related to the large theoretical literature on innovation.[18] Reinganum (1983) argued that industry leaders are likely to be overtaken by new entrants in the innovation race. Even though agents do not compete in my model, overtaking occurs as well. However, it is more severe in that the 'leader' may be ex-ante worse off than the 'follower'.[19] Cetemen, Urgun, and Yariv (Forthcoming) analyse a model of collective search where discoveries accumulate over time, building on past ones. Agents decide when to quit unilaterally and irreversibly, in order to exploit the best discovery to date. Search effort yields stochastic rewards and its marginal cost rises as progress is made, as in my model. Yet payoffs are guaranteed to increase with progress in equilibrium, as discoveries are arbitrarily small. In the endogenous-growth models of Lucas and Moll (2014) and Perla and

15. Important contributions to the theory of dynamic contribution games with a *fixed* goal include Admati and Perry (1991), Strausz (1999), and Georgiadis (2017).

16. Important contributions to this literature include Bolton and Harris (1999), Keller, Rady, and Cripps (2005), Keller and Rady (2010), Klein and Rady (2011), Bonatti and Hörner (2011), Heidhues, Rady, and Strack (2015), and Keller and Rady (2015, 2020).

17. Innovations *decrease* the incentive to exert effort in my model. If agents could learn about the productivity of their effort, the effect of innovations on incentives would be ambiguous, as they would raise the opportunity cost of effort but alter its (conjectured) productivity. However, sufficiently small innovations should naturally decrease productivity, thus exacerbating their effect on incentives. Therefore some innovations are likely to remain harmful in equilibrium.

18. Important contributions include Brander and Spencer (1983), Spence (1984), Katz (1986), d'Aspremont and Jacquemin (1988), Kamien, Muller, and Zang (1992), Suzumura (1992), and Leahy and Neary (1997).

19. In detail, consider two groups of agents (e.g. two distinct R&D partnerships) playing the equilibrium of the baseline game (Theorem 1). If the first innovation obtained across groups makes the group who obtained it (the 'leaders') worse off, the group of 'followers' (which therefore has a lower stock at the time of the innovation) is likely to have a higher stock in the near future. This is because the continuation payoffs of the followers are higher, even though their *flow* payoffs are lower as long as their stock lies below that of the leaders.

Tonetti (2014), agents face a resource trade-off between using an existing technology and improving it, exactly as in my model. However, innovations are always beneficial, as aggregate technological progress is deterministic.

3 Model

In this section, I describe the baseline model and discuss its main assumptions.

Time is continuous, indexed by $t \in \mathbb{R}_+$, and discounted at rate $r > 0$. There are $n \geq 2$ identical agents, indexed by $i \in \{1, \ldots, n\}$, and a public good. Its time-t stock is denoted x_t. At any time $t \geq 0$, agent i exerts effort $a_t^i \in [0, 1]$ and receives a flow payoff $r[b(x) - c(a_t^i, x_t)]$, where $b : \mathbb{R}_+ \to \mathbb{R}_+$ and $c : [0, 1] \times \mathbb{R}_+ \to \mathbb{R}_+$. Agents' effort is hidden.

The stock x_t takes some initial value $x_0 \geq 0$ and, for $t > 0$, is determined as follows. Agent i produces an increment in x_t at rate λa_t^i, where $\lambda > 0$. Each increment has random size $z \geq 0$, drawn from a CDF F with mean $\mu < \infty$ and such that $F(z) > 0$ for all $z > 0$. That is, arbitrarily small sizes are possible. [20] The production and the size of increments are independent across agents, time, and from each other. The arrival of increments, their size, as well as the identity of the agents inducing them, are public. Suppose without loss of generality that $r = 1$ (as usual, this is equivalent to rescaling λ by $1/r$).

Assume that b and c are twice continuously differentiable, and that $b(x) - c(a, x)$ is (weakly) increasing and (weakly) concave in x. That is, keeping the effort fixed, the payoff increases as the stock grows, but at a decreasing rate. Suppose that $c(a, x)$ is increasing and convex in a, and *strictly* increasing in a if $x > 0$. Suppose also that the first and second partial derivatives of c with respect to a (denoted c_1 and c_{11}, respectively) are increasing in x; that is, the cost of effort becomes steeper and more convex as the stock grows. Assume also that

$$\lim_{x \to \infty} \lambda \mu n b'(x) - c_1(a, x) < 0 \quad \text{for all } 0 < a \leq 1. \tag{1}$$

This means that any fixed amount of positive effort is inefficiently large if the stock is large enough. (Note that $\lambda \mu n b'(x)$ approximates the marginal social benefit of effort when the stock x is large, whereas $c_1(a, x)$ is its marginal cost.)

For some of the results,[21] we shall further assume that payoffs take the following

20. I (only) use this assumption to simplify the analysis of detrimental innovations (Proposition 4 and Corollary 2 in Section 6).

21. I impose (2) to derive closed-form expressions for the equilibrium I study (Corollary 1), and to fully characterise when innovations are detrimental (Corollary 2).

linear multiplicative form:

$$b(x) = x \quad \& \quad c(a, x) = ax. \tag{2}$$

In the main interpretation of the model, the stock x_t denotes the (quality of a shared) *technology*, and increments in x_t are *innovations*. These terms are used throughout the discussion. Given this interpretation, linear multiplicative payoffs may be understood as follows. In each period, (risk-neutral) agents face a binary decision between using and improving the technology. Improving the technology ($a_t^i = 1$) yields no payoff, and using it ($a_t^i = 0$) yields a payoff equal to its current quality, x_t. (We may interpret $0 < a_t^i < 1$ as improving the technology with probability a_t^i and using it with probability $1 - a_t^i$.)

Below is a brief description of histories, strategies and continuation payoffs. Note that (almost surely) only finitely many innovations are produced within any bounded period of time. Since effort is hidden, a (public) *history* is a finite sequence

$$h_m := (x_0, (t_1, z_1, i_1), \ldots, (t_m, z_m, i_m)) \tag{3}$$

such that agent i_1 obtains an innovation of size z_1 at time t_1, agent i_2 one of size z_2 at time $t_2 > t_1$, and so on. In particular, the stock after the mth innovation is

$$X(h_m) := x_0 + \sum_{l=1}^{m} z_l.$$

Agents simultaneously reach a new history whenever an innovation is produced. Since past exerted effort has no direct payoff relevance, we may without loss of generality restrict attention to public strategies (i.e., strategies that can be expressed as functions of public histories). Moreover:

Remark 1. *For any mixed perfect Bayesian equilibrium, there exists a public perfect equilibrium in pure strategies inducing the same distribution over public histories, and the same ex-ante payoffs.*

Remark 1 is proved in Appendix A. As a result, we may without loss restrict attention to pure strategies. A (public, pure) *strategy* σ^i specifies, for each history h_m (including $h_0 := x_0$), an effort schedule $\sigma^i(h_m) : (t_m, \infty) \to [0, 1]$ (where $t_0 := 0$).[22] Agent i exerts effort $[\sigma^i(h_m)](t)$ at any time $t > t_m$ such that no innovation was produced within the time interval $[t_m, t)$. If agents play a strategy

22. I restrict strategies to be measurable in a natural sense, adapting the arguments in Chapter IV of **warga1972<empty citation>**. **Explain measurability assumption?**

profile $\sigma := (\sigma^i)_{i=1}^n$, agent i's continuation payoff at a history h_m may be expressed as

$$v_\sigma^i(h_m) := \mathbb{E}\left(\sum_{l=m}^{\tilde{m}} \int_{\tilde{t}_l}^{\tilde{t}_{l+1}} e^{t_m - t} \Big[b\big(X(\tilde{h}_l)\big) - c\Big(\big[\sigma^i(\tilde{h}_l)\big](t), X(\tilde{h}_l)\Big)\Big] \mathrm{d}t \right) \tag{4}$$

where $\tilde{m} \in \{m, m+1, \ldots, \infty\}$ is the total number of innovations produced, $\tilde{h}_l$ is the history reached upon the lth innovation (at time $\tilde{t}_l$), $\tilde{h}_m := h_m$, $\tilde{t}_m := t_m$ and, if $\tilde{m} < \infty$, $\tilde{t}_{\tilde{m}+1} := \infty$.

Discussion of the assumptions. The map from effort to increments in the stock is stationary and ensures that (i) the stock is increasing and (ii) its trajectory does not perfectly reveal that of total effort. If effort were binary, these properties would uniquely pin down the map. Alternatively, effort could influence the distribution F; this would complicate the analysis but is likely to yield similar insights, provided effort not only increases the size of increments on average, but also its dispersion.

Innovations are lumpy in this model. This is crucial for the analysis (Remark 2 in Section 6), and may be viewed as reflecting the agents' inability to perfectly observe their opponents' progress in real time.

In the literature on dynamic public-good games, payoffs are typically assumed to be separable (formally, c is a function of a alone). The assumptions I impose on c are weaker and ensure that the efficient level of effort decreases as the stock grows, a central feature in the contribution games without a fixed goal. Moreover, (1) weakens the standard assumption of vanishing returns to production (i.e., $\lim_{x\to\infty} b'(x) = 0$).

4 Social-welfare benchmark

In this section, I describe how non-strategic agents should behave in order to maximise aggregate payoffs. The main features of this benchmark are common in dynamic public-good games. In particular, any innovation is beneficial.

A strategy is Markov if effort is pinned down by the current stock. Formally, a *Markov strategy* (for agent i) is a Borel measurable map $\alpha : \mathbb{R}_+ \to [0,1]$ such that $a_t^i = \alpha(x_t)$ for all t. If agents play a Markov profile $\alpha := (\alpha^i)_{i=1}^n$, the time-$t$ continuation payoff of each agent i is a function $v_\alpha^i(x_t)$ of the current stock x_t. Moreover,

$$v_\alpha^i(x) = b(x) - c\big(\alpha^i(x), x\big) + \lambda \sum_{j=1}^n \alpha^j(x) \big\{ \mathbb{E}_F\big[v_\alpha^i(x+\tilde{z})\big] - v_\alpha^i(x) \big\}. \tag{5}$$

That is to say, agent i's continuation payoff is the sum of the current payoff flow $b(x) - c(\alpha^i(x), x)$, and the net expected future benefit. This is given by the rate $\lambda \sum_{j=1}^{n} \alpha^j(x)$ at which innovations occur, multiplied by their expected social value; that is, the difference between the continuation payoff $v^i_\alpha(x+z)$ after an innovation of size z, and the current payoff $v^i_\alpha(x)$, weighted by the distribution F of z.

Welfare is the average of ex-ante payoffs across agents. Since agents are identical and the cost of effort $c(a, x)$ is convex in $a \in [0, 1]$, and the rate of arrival of innovations λa is linear in effort, it is efficient for all agents to exert the same amount of effort. Then, from (5), the Bellman equation for the maximal welfare achievable in the game is

$$v(x) = \max_{a\in[0,1]} \big\{ b(x) - c(a, x) + a\lambda n\{\mathbb{E}_F[v(x + \tilde{z})] - v(x)\} \big\}. \tag{6}$$

Standard dynamic-programming arguments imply that, if (6) admits a (well-behaved) solution v_* then, for any initial stock x_0, $v_*(x_0)$ is the maximal achievable welfare, across all (Markov and non-Markov) strategy profiles.[23] Moreover, any Markov strategy α solving

$$\alpha(x) \in \arg\max_{a\in[0,1]} \big\{ a\lambda n\{\mathbb{E}_F[v_*(x + \tilde{z})] - v_*(x)\} - c(a, x) \big\} \tag{7}$$

for all $x \geq 0$, induces welfare $v_*(x_0)$.

The following result describes how agents should behave to maximise welfare.

Proposition 1. *There exists a Markov strategy α_* that,*[24] *if played by all agents, maximises welfare for all initial stocks $x_0 \geq 0$. Effort $\alpha_*(x_t)$ is decreasing in the stock x_t, and $\alpha_*(\tilde{x}_t) \to 0$ a.s. as $t \to \infty$. Maximal welfare $v_*(x_0)$ is continuous and increasing in x_0, and $v_*(x_0) - b(x_0)$ is decreasing and vanishes as $x_0 \to \infty$.*

I prove Proposition 1 in Appendix B. Decreasing effort α_* is standard in dynamic public-good games without a fixed goal, and follows from the concavity of payoffs, as well as from the fact that the cost of effort becomes steeper and more convex as x_t grows. Maximal welfare v_* is increasing in the initial stock x_0 because higher x_0 yields a larger payoff flow without altering the productivity of effort. Thus, all innovations are beneficial. We will see that welfare need not increase with x_0 *in equilibrium*, and that innovations may be detrimental.

23. See e.g. Theorem 3.1.2 of Piunovskiy and Zhang (2020). It is sufficient that (6) admits a Borel solution $v : \mathbb{R}_+ \to \mathbb{R}$ such that $b(x) \leq v(x) \leq b(x) + n\lambda[b(\mu) - b(0)]$ for all $x \geq 0$.

24. Unless c is strictly convex, (7) may admit multiple solutions. α_* is the pointwise smallest.

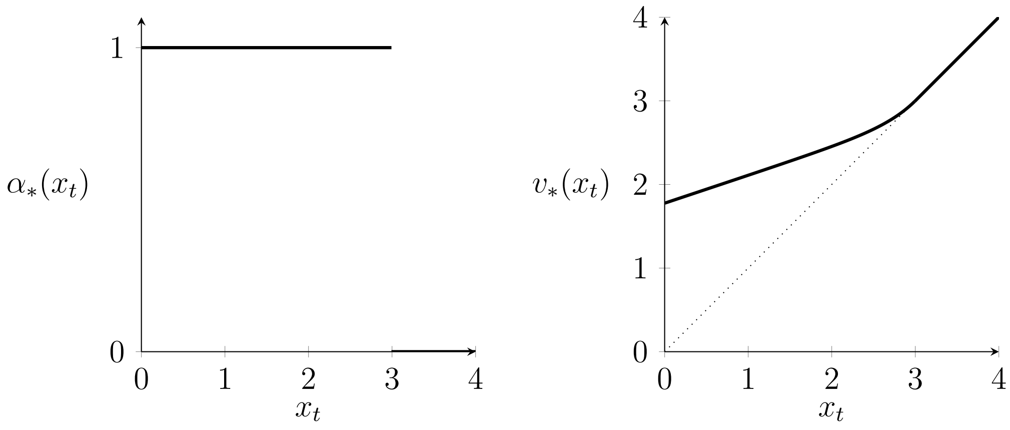

Figure 1: Effort (left) and welfare (right) in the social-welfare benchmark, as functions of the stock x_t. Parameter values are $b(x) = x$, $c(a, x) = ax$, F given by (**??**), $\lambda = 10$, $\rho = \epsilon = 0.01$, and $\zeta = n = 5$.

5 Equilibrium

In this section, I show that the game admits a unique strongly symmetric equilibrium (Theorem 1), and derive it in closed-form for linear and multiplicative payoffs (Corollary 1). I also show that effort in any public perfect equilibrium ceases when it would stop in the single agent problem (Proposition 2).

Given a history h featuring $m \in \{0, 1, \dots\}$ innovations, write $h \circ (t, z, i)$ for the history that features $m+1$ innovations and extends h, and in which the last innovation is produced by agent i at time t, and has size z. Recall that, if agents play a strategy profile $\sigma := (\sigma^i)_{i=1}^n$, $v_\sigma^i(h)$ is the payoff to agent i in the subgame after history h. Given h leading to some time $t_h \geq 0$, for all $t > t_h$, let $v_{\sigma,h}^i(t)$ be agent i's payoff at time t, given that no innovation was produced within the interval $[t_h, t)$ (and define $v_{\sigma,h}^i(t_h) := v_\sigma^i(h)$). Then, $v_{\sigma,h}^i : [t_h, \infty) \to \mathbb{R}$ is Lipschitz and, labelling x the stock that h leads to, for almost all $t > t_h$,[25]

$$\begin{aligned} v_{\sigma,h}^i(t) = \frac{\mathrm{d}v_{\sigma,h}^i}{\mathrm{d}t}(t) + b(x) - c\big(\big[\sigma^i(h)\big](t), x\big) \\ + \lambda \sum_{j=1}^n \big[\sigma^j(h)\big](t)\big\{\mathbb{E}_F\big[v_\sigma^i(h \circ (t, \tilde{z}, j))\big] - v_{\sigma,h}^i(t)\big\}. \end{aligned} \tag{8}$$

Compared to (5), effort depends on the current time (as the strategy profile played need not be Markov). As a consequence, agent i's payoff evolves at rate $\mathrm{d}v_{\sigma,h}^i/\mathrm{d}t$ even in the absence of innovations.

25. See Online Appendix Honline.pdf for details on how to derive (8), (9), (10), (11) and (12).

Let $\hat{v}^i_\sigma(h)$ be the largest payoff that agent i can achieve, across all strategies, in the subgame after history h. Assuming h leads to time $t_h \geq 0$, for all $t > t_h$, let $\hat{v}^i_{\sigma,h}(t)$ be the largest payoff achievable by agent i at time t, given that no innovation was produced within the interval $[t_h, t)$ (and define $\hat{v}^i_{\sigma,h}(t_h) := \hat{v}^i_\sigma(h)$). Then, the map $\hat{v}^i_{\sigma,h} : [t_h, \infty) \to \mathbb{R}$ is Lipschitz and, for almost all $t > t_h$,

$$\hat{v}^i_{\sigma,h}(t) = \frac{\mathrm{d}\hat{v}^i_{\sigma,h}}{\mathrm{d}t}(t) + b(x) + \max_{a\in[0,1]} \left\{a\lambda\left\{\mathbb{E}_F\left[\hat{v}^i_\sigma(h \circ (t, \tilde{z}, i))\right] - \hat{v}^i_{\sigma,h}(t)\right\} - c(a, x)\right\} + \lambda \sum_{j\neq i}^{n} \left[\sigma^j(h)\right](t)\left\{\mathbb{E}_F\left[\hat{v}^i_\sigma(h \circ (t, \tilde{z}, j))\right] - \hat{v}^i_{\sigma,h}(t)\right\}. \tag{9}$$

The strategy σ^i is a best response for agent i at all histories only if, given any history h leading to any time t_h, for almost all $t > t_h$,

$$[\sigma^i(h)](t) \in \arg\max_{a\in[0,1]} \left\{a\lambda\left\{\mathbb{E}_F\left[\hat{v}^i_\sigma(h \circ (t, \tilde{z}, i))\right] - \hat{v}^i_{\sigma,h}(t)\right\} - c(a, x)\right\}. \tag{10}$$

The profile σ is a *public perfect equilibrium* (PPE) if, for each i, σ^i is a best response for agent i at all histories. If moreover, $\sigma^i = \sigma^j$ for all i and j, then σ is a *strongly symmetric equilibrium* (SSE).

Recall the definition of Markov strategies from Section 4. If the opponents of a given agent play a Markov strategy α, the largest continuation payoff that she can achieve, across all strategies, is a (value) function $\hat{v}_\alpha$ of the current stock x_t, symmetric across agents. From (5), $\hat{v}_\alpha$ is the unique (well-behaved) solution to

$$v(x) = \max_{a\in[0,1]} \left\{b(x) - c(a, x) + \lambda[a + (n-1)\alpha(x)]\{\mathbb{E}_F[v(x + \tilde{z})] - v(x)\}\right\}. \tag{11}$$

Then, α is a best response, after any history, against opponents playing the same strategy if and only if

$$\alpha(x) \in \arg\max_{a\in[0,1]} \left\{a\lambda\{\mathbb{E}_F[\hat{v}_\alpha(x + \tilde{z})] - \hat{v}_\alpha(x)\} - c(a, x)\right\} \tag{12}$$

for all $x \geq x_0$.

The following result characterises the unique strongly symmetric equilibrium of the game. The proof is in Appendix C.

Theorem 1. *There exists a unique SSE, and it is induced by a Markov strategy α_f. Effort $\alpha_f(x_t)$ is continuous and decreasing in the stock x_t, and lies below the* *benchmark* $\alpha_*(x_t)$.[26] *Moreover,* $v_f(x_t) - b(x_t)$ *is decreasing in* x_t*, where* $v_f(x_t)$ *is the equilibrium continuation payoff given* x_t*.*

The analyses of Marx and Matthews (2000), Lockwood and Thomas (2002), and Hörner, Klein, and Rady (2022) suggests that the uniqueness of the SSE relies on the fact that monitoring is imperfect. If either aggregate effort were observable, or could be *perfectly* inferred from the trajectory of the stock, there would exist multiple SSE and some would induce higher payoffs than the (unique) symmetric Markov equilibrium. Uniqueness follows from a 'backward-induction' logic, as effort can be adjusted flexibly and must vanish in the long run by (1), and information is revealed only whenever the stock grows. The same logic would apply if past exerted effort were perfectly revealed after each increment in the stock, so that Theorem 1 would continue to hold.[27]

This game admits many asymmetric equilibria, some of which yield efficiency gains over the strongly symmetric one, due to the agents' ability to coordinate. However, as the next result shows, effort in any equilibrium ceases no later than in the single-agent problem. In the latter setting, effort is exerted as long as

$$\lambda\{\mathbb{E}_F[b(x_t + \tilde{z})] - b(x_t)\} \geq c_1(0, x_t), \tag{13}$$

where the left-hand side is decreasing in x_t and the right-hand side increasing.

Proposition 2. *In any PPE, no effort is exerted after any history leading to a stock* x_t *such that* (13) *fails.*[28]

Proposition 2 is proved in Appendix D. The logic behind it is similar to that explaining the analogous result for the Markov equilibria with 'finite switching' of Keller, Rady, and Cripps (2005). Specifically, effort cannot be sustained after the stock exceeds some cutoff $\hat{x}$. If the stock is below but close to $\hat{x}$, with high probability, no effort is exerted after an innovation. Then, the incentive to exert effort is essentially no higher than in the single-agent setting.

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26. Provided (**??**) holds, α_f is inefficient unless no effort is efficient. This is because v_* is continuous and hence, assuming that $\alpha_*(x_0) > 0$, some $a \in (0, 1)$ maximises the objective in (7) for some $x > x_0$. Then $\alpha_f < \alpha_*$ on $(\hat{x}, x)$ for some $\hat{x} < x$ by (12), as α_f is continuous and α_* decreasing. Thus $v_f(x_0) < v_*(x_0)$ by (**??**).

27. This logic fails in a repeated prisoners' dilemma. If actions played are revealed at a fixed rate as long as at least one player cooperates, there exists an SSE that is approximately efficient as the period length vanishes. See Proposition 5 of Abreu, Milgrom, and Pearce (1991).

28. Proposition 2 is similar to the main result of Guéron (2015), but holds for different reasons. In particular, it would continue to hold (with the same proof), if past exerted effort were perfectly revealed after each increment in the stock, even though this monitoring technology would not satisfy the 'smoothness' assumption that Guéron's result hinges on (Assumption 6).

Proposition 2 implies that, in the limit as agents become arbitrarily patient (i.e., as λ diverges), the largest achievable welfare across all equilibria need not approximate that of the welfare benchmark. That is, the 'Folk Theorem' need not hold. To see why, recall from Section 3 that payoffs are *linear and multiplicative* if $b(x) = x$ and $c(a, x) = ax$. In this case, (6) and (7) imply that welfare is maximised by the profile $\alpha_*(x) = \mathbb{1}_{x<\lambda\mu n}$. Moreover, $\lambda\mu$ is the unique solution to (13) and, using the appropriate analogues of (6) and (7), it is easy to show that among all strategy profiles such that no effort is exerted at stock levels exceeding $\lambda\mu$, welfare is maximised if all agents exert maximal effort until the stock exceeds $\lambda\mu$, and no effort thereafter. Moreover one can show that the welfare $\bar{v}(x_0)$ induced by this strategy profile satisfies $\limsup_{\lambda\to\infty} \bar{v}(x_0)/v_*(x_0) < 1$.[29]

In the SSE α_f, effort is exerted at any stock x such that (13) holds strictly, but not beyond it. Thus, if payoffs are linear and multiplicative and $x_0 < \lambda\mu$, effort in the SSE ceases whenever the stock reaches $\lambda\mu$ and the expected number of innovations produced in equilibrium is

$$M(x_0) := \mathbb{E}\left[\min\left\{m \in \mathbb{N} : x_0 + \sum_{l=1}^{m} \tilde{z}_l \geq \lambda\mu\right\}\right]$$

where $z_1, z_2, \ldots$ are i.i.d. draws from F. Note that there is a unique $y_f \in (0, \lambda\mu)$ such that

$$y_f(n-1) = \int_{y_f}^{\lambda\mu} \frac{M}{\lambda}.$$

The next result describes effort and payoffs in the SSE under the hypothesis that the payoff function is linear and multiplicative.

Corollary 1. *If payoffs are linear and multiplicative, effort α_f is maximal on $[0, y_f]$ and interior on $(y_f, \lambda\mu)$ and, for all $y_f \leq x \leq \lambda\mu$,*

$$\alpha_f(x) = \frac{1}{(n-1)x}\int_x^{\lambda\mu} \frac{M}{\lambda} \quad \& \quad v_f(x) = x + \int_x^{\lambda\mu} \frac{M}{\lambda},$$

and v_f is increasing globally if and only if it is increasing over $[y_f, \lambda\mu]$.

Corollary 1 is shown by verifying that the candidate α_f and v_f satisfy (11) and (12) (where $v_f(x) = \frac{\lambda n}{\lambda n+1}\mathbb{E}_F[v_f(x+\tilde{z})]$ on $[0, y_f]$, by (11)). The corollary implies that v_f is convex over $[y_f, \lambda\mu]$ and is increasing (is decreasing) over this

29. For example, if $F(z) = 1 - e^{-\mu z}$ then, for $x_0 \leq \lambda\mu$, $v_*(x) = \lambda\mu n e^{\frac{x_0/\mu - \lambda n}{1+\lambda n}}$ and $\bar{v}(x) = \frac{\lambda n\mu(1+\lambda)}{1+\lambda n} e^{\frac{x_0/\mu-\lambda}{1+\lambda n}}$ so that $\bar{v}(x_0)/v_*(x_0) \to e^{1-1/n}/n < 1$ as $\lambda \to \infty$, where the inequality holds since $n \geq 2$.

interval if $M(y_f) \leq \lambda$ (if $\lambda \leq 1$). Otherwise, assuming that (**??**) holds and that F is atomless, v_f is minimised over $[y_f, \lambda\mu]$ at the unique $x \in (y_f, \lambda\mu)$ such that $M(x) = \lambda$.

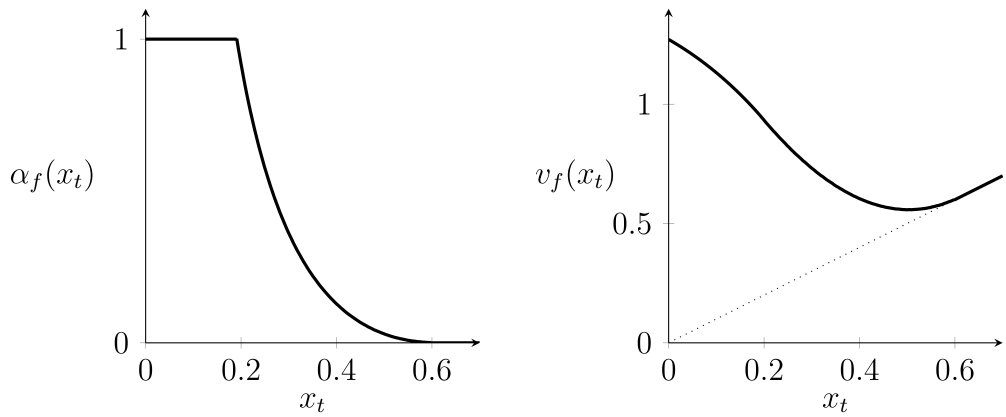

Figure 2: Effort (left) and payoffs (right) in the SSE, as functions of the stock x_t. Parameters are $b(x) = x$, $c(a, x) = ax$, F given by (**??**),, $\lambda = 10$, $\rho = \epsilon = 0.01$, and $\zeta = n = 5$.

6 Detrimental innovations

In this section, I argue that detrimental innovations (that is, increments that cause continuation payoffs to drop in the symmetric equilibrium) occur only if agents' marginal cost increases with the stock, and if innovations have random size (Proposition 3). I provide necessary and sufficient conditions for the risk of detrimental innovations to persist as long as effort is exerted (Proposition 4), and characterise its occurrence under linear payoffs (Corollary 1). We will see that the detrimental effects of innovations are driven by the size of the team, and by the dispersion of the distribution of innovation sizes.

Recall the SSE α_f (Theorem 1). Innovations are detrimental whenever continuation payoffs in α_f are not guaranteed to increase after each innovation:

Definition 1. *Innovations are detrimental* if $\Pr(t \mapsto v_f(\tilde{x}_t)$ is increasing$) < 1$, where $(\tilde{x}_t)_{t>0}$ describes the evolution of the stock when α_f is played.

The next result shows that the two main novel features of this model relative to the literature on dynamic public-good games are both necessary for innovations to be detrimental. In particular, assuming that increasing the stock has diminishing returns is insufficient.

Proposition 3. *If either innovations have fixed size, or the cost of effort does not increase with the stock, then innovations are not detrimental.*

Proposition 3 essentially follows from (12), and is proved in Appendix E. The fact that innovations are lumpy is necessary as well:

Remark 2. *In an alternative model where* $dx_t = \lambda \sum_{i=1}^{n} a_t^i + dz_t$ *for some diffusion process* z_t*, and payoffs* $u(a_t^i, x_t)$ *that are decreasing in* $a_t^i \in [0,1]$ *and increasing in* $x_t \in \mathbb{R}$*, ex-ante payoffs in any symmetric Markov equilibrium with decreasing effort are increasing in the initial stock* x_0*.*

To prove Remark 2 note that, conditional on any path $(z_t)_{t>0}$, and given any t, x_t is increasing in x_0. Then a_t^i is decreasing in x_0 and, hence, $u(a_t^i, x_t)$ is increasing.

The next result states a necessary and sufficient condition for the risk of detrimental innovations to persist as long as effort is exerted (provided it is not exerted forever). Recall from Proposition 2 that $\alpha_f(x) > 0$ only if (13) holds.

Proposition 4. *Suppose that* (**??**) *holds and* (13) *admits a largest solution* x_f*. Then, innovations are detrimental for any initial stock* $x_0 < x_f$ *if (and only if)*

$$b'(x_f)c_{11}(0, x_f) + (n-1)c_1(0, x_f)\{\lambda \mathbb{E}_F[b'(x_f + \tilde{z})] - c_{12}(0, x_f)\} < (\leq)\ 0. \quad (14)$$

Proposition 4 is proved in Appendix E. Condition (**??**) ensures that innovations are detrimental if (and only if) v_f is not increasing on $[x_0, \infty)$. Condition (14) describes the impact of a small-sized increment when the stock is close to, but below the cutoff x_f. Roughly speaking, the innovation increases the current payoff flow (first term), as well as future gross payoffs (first term inside braces), but increases the marginal cost of effort (second term inside braces). If the last force dominates, the drop in effort following the innovation is large enough to counter the aforementioned payoff increase.

From the perspective of Proposition 4, increasing the number of agents exacerbates the risk of detrimental innovations. Formally, if (14) holds (strictly), it continues to hold (strictly) after an increase in n, as x_f does not depend on n.

Under natural regularity conditions, detrimental innovations will persist in the long run, provided innovations are sufficiently large and rare, and the population is large. Specifically, suppose that b is unbounded and $b'(x)$ vanishes as x diverges, and that $c_1(0,0) = 0$ and c_{12} is strictly positive. Then, $0 < x_f < \infty$ and, for any $\lambda' \leq \lambda$ and $n' \geq n$, there is an FOSD-shift F' of F such that replacing λ, F and n by λ', F' and n', respectively, leaves x_f unchanged. Moreover, given sufficiently small λ' and large n', (14) holds after the substitution, no matter the F'. The next result characterises detrimental innovations for linear multiplicative payoffs. I state it without proof as it follows easily from Corollary 1. Recall the definitions of M and y_f from Section 5, and denote by '$\vee$' the 'max' operator.

Corollary 2. *Suppose that* (**??**) *holds and that payoffs are linear and multiplicative. Then, innovations are detrimental if and only if* $x_0 < \lambda\mu$ *and* $M(x_0 \vee y_f) > \lambda$. *In particular, payoffs drop with innovations raising the stock from any* $x \geq y_f$ *to any* $\hat{x} \in (x, \lambda\mu)$ *such that* $M(\hat{x}) > \lambda$.

Corollary 2 implies that, if innovations are sufficiently rare (that is, if $\lambda < 1$) and, given $x_0 \in (0, \lambda\mu)$, the population is sufficiently large so that $y_f \leq x_0$, *all* but the last innovation produced in equilibrium are harmful. It also implies that innovations are detrimental if, given any frequency λ of innovations and any distribution F of their size, the initial technology is sufficiently unproductive (i.e. x_0 is small enough), and the population is large enough. Indeed, in this case, $M(x_0 \vee y_f) \approx M(0) > \lambda$, as $\lim_{n\to\infty} y_f = 0$.

7 Disposal

In this section, I extend the model by allowing each agent to freely dispose of the innovations that she produces, after observing their size. This raises effort and payoffs in the strongly symmetric equilibrium. Under natural conditions, payoffs rise above those of all equilibria with forced disclosure.

7.1 Model

Enrich the model as follows. Whenever agent i obtains an innovation of size $z > 0$ at time $t \geq 0$, she (immediately) decides whether or not to reduce z to 0, after observing z. That is to say, agent i may either disclose the innovation (which results in its immediate adoption by all agents), or discard it. Assume that the arrival and size of innovations is private information, and that the disposal of innovations is unobserved. In particular, if an agent obtains and discards an innovation at some time t, her opponents will not be able to distinguish this event from the event that the agent does not obtain an innovation at time t. However, agents are immediately informed of any innovation that is disclosed, including its size and the identity of the agent disclosing it. As before, effort is hidden. I constrain agents to play pure strategies, and, for simplicity, I rule out strategies that condition on innovations discarded in the past.[30]

We recover the baseline model (Section 3) by restricting the agents' strategies so that all innovations produced are disclosed. I refer to the baseline model as the game *with forced disclosure,* and to this model as the game *with disposal*. As I argue at the end of Section 7.2, the analysis of strongly symmetric equilibria is unchanged if agents may conceal innovations instead of discarding them, but may not secretly refine the improvements that they hide.

Below is a brief description of histories, strategies and continuation payoffs. Formal definitions are in Online Appendix Ionline.pdf. Each agent reaches a new private history whenever she either produces or discards an innovation, or any agent discloses. Thus (almost surely), agents reach finitely many private histories within any bounded interval of time. Public histories are formally unchanged (see (3) in Section 3), but they now only record *disclosed* innovations. A strategy $\xi^i := (\sigma^i, \chi^i)$ specifies, for each public history h_m, an effort schedule $\sigma^i(h_m) : (t_m, \infty) \to [0,1]$ and a disclosure policy $\chi^i(h_m) : (t_m, \infty) \times (0, \infty) \to \{0, 1\}$. As before, agent i exerts effort $[\sigma^i(h_m)](t)$ at any time $t > t_m$ such that no innovation was disclosed within the time interval $[t_m, t)$. Moreover, if agent i produces an innovation of size z at such a time t, she discloses it if $[\chi(h_m)](t, z) = 1$, and discards it otherwise. Note that it is not necessary to keep track of discarded innovations, since strategies may not condition on them, by assumption. If agents play a strategy profile $\xi := (\sigma^i, \chi^i)_{i=1}^n$, the continuation payoff $v_\xi^i(h_m)$ to agent i at a public history h_m may be expressed as the right-hand side of (4), with the random path $(\tilde{h}_l)_{l=m}^{\tilde{m}}$ obviously having a different distribution.

7.2 Analysis

The social-welfare benchmark (characterised in Proposition 1) is unaffected by the introduction of disposal. This is because, in the non-strategic setting, the effects of disposal on incentives can be ignored. Moreover, disposal of innovations hinders the growth of the payoff flow, since $b(x) - c(a, x)$ is increasing in x.

Given a public history h featuring $m \in \{0, 1, \dots\}$ (disclosed) innovations, write $h \circ (t, z, i)$ for the public history that features $m + 1$ innovations and extends h, and in which the last innovation is disclosed by agent i at time t, and has size z. Given a profile $\xi := (\sigma^i, \chi^i)_{i=1}^n$, let $\hat{v}_\xi^i(h)$ be the largest payoff that agent i

30. Admitting the latter strategies would complicate the definition of the game without affecting the results. In particular, the unique strongly symmetric equilibrium (in which, by definition, strategies cannot condition on discarded innovations), would continue to exist.

can achieve, across all strategies, in the subgame after public history h, given that her opponents behave according to ξ. Given h leading to time $t_h \geq 0$, for all $t > t_h$, let $\hat{v}^i_{\xi,h}(t)$ be the largest payoff achievable by agent i at time t, assuming that no innovation was disclosed within the interval $[t_h, t)$, and that agent i does not produce an innovation at time t (and define $\hat{v}^i_{\xi,h}(t_h) := \hat{v}^i_\xi(h)$). Then, $\hat{v}^i_{\xi,h} : [t_h, \infty) \to \mathbb{R}$ is Lipschitz and, labelling x the stock that h leads to, for almost all $t > t_h$,[31]

$$\hat{v}^i_{\xi,h}(t) = \frac{\mathrm{d}\hat{v}^i_{\xi,h}}{\mathrm{d}t}(t) + b(x) + \max_{a\in[0,1]} \left\{a\lambda \mathbb{E}_F\left[\left(\hat{v}^i_\xi(h \circ (t, \tilde{z}, i)) - \hat{v}^i_{\xi,h}(t)\right) \vee 0\right] - c(a, x)\right\}$$
$$+ \lambda \sum_{j\neq i}^{n} \left[\sigma^j(h)\right](t)\mathbb{E}_F\left[\chi^j(t, \tilde{z})\left(\hat{v}^i_\xi(h \circ (t, \tilde{z}, j)) - \hat{v}^i_{\xi,h}(t)\right)\right]. \quad (15)$$

The strategy ξ^i is a best response for agent i at every private history only if, given any public history h leading to any time t_h, both of the following conditions hold: for almost all $t > t_h$,

$$\left[\sigma^i(h)\right](t) \in \arg\max_{a\in[0,1]} \left\{a\lambda \mathbb{E}_F\left[\left(\hat{v}^i_\xi(h \circ (t, \tilde{z}, i)) - \hat{v}^i_{\xi,h}(t)\right) \vee 0\right] - c(a, x)\right\} \quad (16)$$

and, furthermore, for all $t > t_h$ and $z > 0$,

$$\left[\chi^i(h)\right](t, z) \in \arg\max_{d\in\{0,1\}} \left\{d\left(\hat{v}^i_\xi(h \circ (t, z, i)) - \hat{v}^i_{\xi,h}(t)\right)\right\}. \quad (17)$$

Condition (17) states that agent i optimally adopts or discards an innovation of size z obtained at time t following history h. A strategy $\xi := (\sigma, \chi)$ induces a *strongly symmetric equilibrium* (SSE) if, given that all opponents play ξ, ξ is a best response at every private history.

A *Markov strategy* is a pair $\pi := (\alpha, \delta)$ of (Borel measurable) maps $\alpha : \mathbb{R}_+ \to [0, 1]$ and $\delta : \mathbb{R}_+ \times (0, \infty) \to \{0, 1\}$ such that $a^i_t = \alpha(x_t)$ for all $t \geq 0$ and, if agent i obtains an innovation of size $z > 0$ at time t, she discloses it if $\delta(x_t, z) = 1$ and discards it otherwise. If the opponents of any given agent play a Markov strategy $\pi := (\alpha, \delta)$, the largest continuation payoff that this agent can achieve, across all

31. See the end of Online Appendix Ionline.pdf for details on how to derive (15), (16), (18), (19) and (20). '$\vee$' denotes the 'max' operator.

strategies, is a (value) function $\hat{v}_\pi$ of the current stock x_t, solving

$$\hat{v}(x) = b(x) + \max_{a \in [0,1]} \left\{ a\lambda \mathbb{E}_F[(\hat{v}(x+\tilde{z}) - \hat{v}(x)) \vee 0] - c(a,x) \right\} \\ + \lambda(n-1)\alpha(x)\mathbb{E}_F[\delta(x,\tilde{z})(\hat{v}(x+\tilde{z}) - \hat{v}(x))]. \tag{18}$$

for all $x \geq 0$.[32] Then, π is a best response, after any private history, against opponents playing π if and only if, for all $x \geq 0$, both of the following hold:

$$\alpha(x) \in \arg\max_{a \in [0,1]} \left\{ a\lambda \mathbb{E}_F[(\hat{v}_\pi(x+\tilde{z}) - \hat{v}_\pi(x)) \vee 0] - c(a,x) \right\}, \tag{19}$$

$$\delta(x,z) \in \arg\max_{d \in \{0,1\}} \left\{ d(\hat{v}_\pi(x+z) - \hat{v}_\pi(x)) \right\} \quad \text{for all } z > 0. \tag{20}$$

The following result characterises the unique SSE of the game. Recall the SSE α_f of the game with forced disclosure, and the definition of detrimental innovations.

Theorem 2. *The game with disposal admits an (essentially) unique SSE, and it is induced by a Markov strategy (α_d, δ_d). In the absence of innovations at time t, and for any stock x_t, effort $\alpha_d(x_t)$ and continuation payoffs $v_d(x_t)$ are no lower than their analogues $\alpha_f(x_t)$ and $v_f(x_t)$ in the equilibrium with forced disclosure. Moreover, ex-ante payoffs $v_d(x_0)$ strictly exceed $v_f(x_0)$ if innovations are detrimental in α_f. Otherwise, no disposal occurs in (α_d, δ_d) and the equilibria coincide.*

I prove Theorem 2 in Appendix F. There I show that, although there may exist multiple equilibria, α_d and v_d are uniquely pinned down, and they inherit the properties of α_f and v_f described in Theorem 1.

Allowing agents to discard innovations increases equilibrium payoffs, and strictly so unless innovations are guaranteed to be beneficial in the SSE with forced disclosure. This is because individual and social incentives for the disposal of innovations are aligned, since the equilibrium is symmetric. Moreover, the fact that future detrimental innovations will be discarded increases continuation payoffs at all stock values exceeding the current one and, therefore, the current incentive to exert effort. As a result, equilibrium effort is higher in the game with disposal.

Theorem 2 remains valid if agents can conceal innovations instead of merely discarding them, provided they cannot covertly refine the improvements that they hide. (See Remark 6 in Appendix F for a precise statement.) This is because

32. This expression is valid if neither the agent nor one of her opponents produced an innovation at time t. If the agent obtains an innovation of size z at time t, the expression becomes $\hat{v}_\pi(x+z) \vee \hat{v}_\pi(x)$ (where $\hat{v}_\pi$ is pinned down by (18)). Assuming an opponent obtains the innovation instead, the expression is valid if $\delta(x,z) = 0$, and becomes $\hat{v}_\pi(x+z)$ if $\delta(x,z) = 1$.

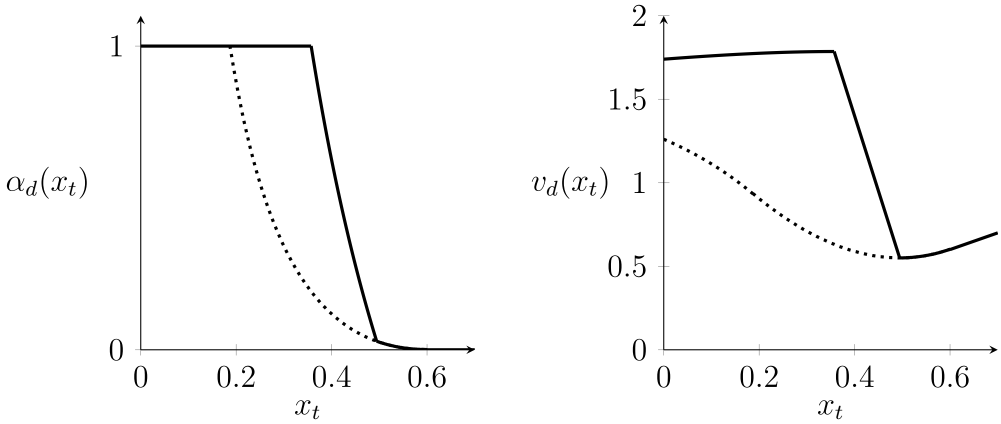

Figure 3: Effort (left) and payoffs (right) in the SSE of the game with disposal, as functions of the stock x_t. The dotted lines are effort α_f (left) and payoffs v_f (right) in the equilibrium of the game with forced disclosure. By (20), δ_d need not be a 'cutoff' strategy, since v_d is not quasi-concave. Parameter values are $b(x) = x$, $c(a, x) = ax$, $n = 5$, F given by (**??**), $\zeta = 5$, $\lambda = 0.1/\rho$, and $\epsilon = \rho/(1-\rho)$, where $\rho > 0$ is arbitrarily small.

SSE, by their public nature, rule out disclosing any innovation that was previously concealed. Thus (α_d, δ_d) is the only candidate equilibrium, and it *is* an equilibrium since it is stationary, so that no agent has an incentive to delay disclosure.

We end by deriving sufficient conditions under which the equilibrium with disposal induces higher payoffs than any PPE with forced disclosure. By Proposition 2, if (13) admits a largest solution $x \in (0, \infty)$, then in any such PPE the long-run stock $\lim_t \tilde{x}_t$ has the same distribution as $x_0 + \sum_{l=1}^{\tilde{m}} \tilde{z}_l$, where $z_1, z_2, \ldots$ are i.i.d. draws from F and $\tilde{m} = \min\{m \in \mathbb{N} : x_0 + \sum_{l=1}^{m} \tilde{z}_l \geq x\}$. Then, labelling this distribution G^x, individual ex-ante payoffs are at most $W(x) = \mathbb{E}_{G^x}(b(\tilde{x}))$.

Proposition 5. *Suppose that* (13) *admits a largest solution* $x \in (x_0, \infty)$ *and that*

$$\lambda \mathbb{E}_F[(b(x_0 + \tilde{z}) - W(x)) \vee 0] > c_1(0, x_0). \tag{21}$$

Then, if n *is sufficiently large, any agent is ex-ante better-off in* (α_d, δ_d) *than in any PPE of the game with forced disclosure.*

See Appendix F for the proof of Proposition 5. Assuming that payoffs are linear and multiplicative, (21) holds if x_0 is small or F is sufficiently 'dispersed'. For instance, if F is given by (**??**), (21) holds if substantial innovations are rare but drive progress in expectation (i.e., if ρ and ϵ are small and ζ large, keeping μ fixed).[33] If (21) holds, the ability to selectively discard innovations yields larger gains (in large populations) than the ability to coordinate.

33. To see why, note that $x = \lambda\mu$ if payoffs are linear and multiplicative, where $\mu = \rho\zeta + (1-\rho)\epsilon$.

References

Abreu, D., P. Milgrom, and D. Pearce. 1991. "Information and timing in repeated partnerships." *Econometrica* 59 (6): 1713–1733.

Abreu, D., D. Pearce, and E. Stacchetti. 1986. "Optimal cartel equilibria with imperfect monitoring." *Journal of Economic Theory* 39 (1): 251–269.

Admati, A., and M. Perry. 1991. "Joint projects without commitment." *The Review of Economic Studies* 58 (2): 259–276.

Anderson, N., K. Potočnik, and J. Zhou. 2014. "Innovation and creativity in organizations: A state-of-the-science review, prospective commentary, and guiding framework." *Journal of management* 40 (5): 1297–1333.

Azevedo, E. M., A. Deng, J. L. Montiel Olea, J. Rao, and E. G. Weyl. 2020. "A/b testing with fat tails." *Journal of Political Economy* 128 (12): 4614–4672.

Battaglini, M., S. Nunnari, and T. R. Palfrey. 2014. "Dynamic free riding with irreversible investments." *American Economic Review* 104 (9): 2858–2871.

Bessen, J., and A. Nuvolari. 2016. "Knowledge sharing among inventors: some historical perspectives." Chap. 7 in *Revolutionizing Innovation: Users, Communities and Open Innovation,* 135–157. MIT Press.

Bolton, P., and C. Harris. 1999. "Strategic experimentation." *Econometrica* 67 (2): 349–374.

Bonatti, A., and J. Hörner. 2011. "Collaborating." *American Economic Review* 101 (2): 632–63.

Brander, J. A., and B. J. Spencer. 1983. "Strategic commitment with R&D: the symmetric case." *The Bell Journal of Economics* 14 (1): 225–235.

Cetemen, D., I. Hwang, and A. Kaya. 2020. "Uncertainty-driven cooperation." *Theoretical Economics* 15 (3): 1023–1058.

Cetemen, D., C. Urgun, and L. Yariv. Forthcoming. "Collective Progress: Dynamics of Exit Waves." *Journal of Political Economy.*

Chesbrough, H. W. 2003. *Open innovation: The new imperative for creating and profiting from technology.* Harvard Business Press.

Curello, G. 2023. "Concealment in Collective Innovation." Working paper, May 2023. https://sites.google.com/view/gregoriocurello.

d'Aspremont, C., and A. Jacquemin. 1988. "Cooperative and noncooperative R&D in duopoly with spillovers." *The American Economic Review* 78 (5): 1133–1137.

Moreover, in the limit as ρ vanishes and ζ diverges while μ remains fixed, $W(\lambda\mu)$ is bounded and thus $\rho(x_0+\zeta-W(\lambda\mu))$ converges to $\mu-\epsilon$. Since $\mathbb{E}_F[(x_0+\tilde{z}-W(\lambda\mu))\vee 0] \geq \rho(x_0+\zeta-W(\lambda\mu))$, (21) holds for ρ sufficiently small, provided $\epsilon < \mu - x_0/\lambda$.

Dover, G., and T. B. Lawrence. 2012. “The role of power in nonprofit innovation.” *Nonprofit and Voluntary Sector Quarterly* 41 (6): 991–1013.

Dutta, P. K. 1995. “A folk theorem for stochastic games.” *Journal of Economic Theory* 66 (1): 1–32.

Fershtman, C., and S. Nitzan. 1991. “Dynamic voluntary provision of public goods.” *European Economic Review* 35 (5): 1057–1067.

Ford, C. M. 1996. “A theory of individual creative action in multiple social domains.” *Academy of Management review* 21 (4): 1112–1142.

Fudenberg, D., D. Levine, and E. Maskin. 1994. “The Folk Theorem with Imperfect Public Information.” *Econometrica* 62 (5): 997–1039.

Fudenberg, D., and Y. Yamamoto. 2011. “The folk theorem for irreducible stochastic games with imperfect public monitoring.” *Journal of Economic Theory* 146 (4): 1664–1683.

Georgiadis, G. 2017. “Deadlines and infrequent monitoring in the dynamic provision of public goods.” *Journal of Public Economics* 152:1–12.

———. 2015. “Projects and team dynamics.” *The Review of Economic Studies* 82 (1): 187–218.

Geroski, P. 1995. “Markets for technology: knowledge, innovation and appropriability.” Chap. 4 in *Handbook of the Economics Innovation and Technological Change,* 90–131. Blackwell.

Green, E. J., and R. H. Porter. 1984. “Noncooperative collusion under imperfect price information.” *Econometrica* 52 (1): 87–100.

Guéron, Y. 2015. “Failure of gradualism under imperfect monitoring.” *Journal of Economic Theory* 157:128–145.

Harhoff, D., and K. R. Lakhani. 2016. *Revolutionizing innovation: Users, communities, and open innovation.* MIT Press.

Heidhues, P., S. Rady, and P. Strack. 2015. “Strategic experimentation with private payoffs.” *Journal of Economic Theory* 159 (1): 531–551.

Hörner, J., N. Klein, and S. Rady. 2022. “Overcoming Free-Riding in Bandit Games.” *Review of Economic Studies* 89 (4): 1948—1992.

Hörner, J., T. Sugaya, S. Takahashi, and N. Vieille. 2011. “Recursive methods in discounted stochastic games: An algorithm for $\delta \to 1$ and a folk theorem.” *Econometrica* 79 (4): 1277–1318.

Hottenrott, H., and B. Peters. 2012. “Innovative capability and financing constraints for innovation: more money, more innovation?” *Review of Economics and Statistics* 94 (4): 1126–1142.

Kamien, M. I., E. Muller, and I. Zang. 1992. “Research joint ventures and R&D cartels.” *The American Economic Review* 82 (5): 1293–1306.

Katz, M. L. 1986. “An analysis of cooperative research and development.” *The RAND Journal of Economics* 17 (4): 527–543.

Keller, G., and S. Rady. 2015. “Breakdowns.” *Theoretical Economics* 10 (1): 175–202.

———. 2010. “Strategic experimentation with Poisson bandits.” *Theoretical Economics* 5 (2): 275–311.

———. 2020. “Undiscounted bandit games.” *Games and Economic Behavior* 124 (1): 43–61.

Keller, G., S. Rady, and M. Cripps. 2005. “Strategic experimentation with exponential bandits.” *Econometrica* 73 (1): 39–68.

Klein, N., and S. Rady. 2011. “Negatively correlated bandits.” *The Review of Economic Studies* 78 (2): 693–732.

Kroeger, A., and C. Weber. 2014. “Developing a conceptual framework for comparing social value creation.” *Academy of Management Review* 39 (4): 513–540.

Leahy, D., and J. P. Neary. 1997. “Public policy towards R&D in oligopolistic industries.” *The American Economic Review* 87 (4): 642–662.

Lim, K. 2009. “The many faces of absorptive capacity: spillovers of copper interconnect technology for semiconductor chips.” *Industrial and Corporate Change* 18 (6): 1249–1284.

Lockwood, B., and J. P. Thomas. 2002. “Gradualism and irreversibility.” *The Review of Economic Studies* 69 (2): 339–356.

Lucas, R. E., and B. Moll. 2014. “Knowledge growth and the allocation of time.” *Journal of Political Economy* 122 (1): 1–51.

Marx, L., and S. Matthews. 2000. “Dynamic voluntary contribution to a public project.” *The Review of Economic Studies* 67 (2): 327–358.

Meyer, P. B. 2003. *Episodes of collective invention.* Working Paper 368. U.S. Bureau of Labour Statistics.

Milgrom, Paul, and Ilya Segal. 2002. “Envelope theorems for arbitrary choice sets.” *Econometrica* 70 (2): 583–601.

Pénin, J. 2007. “Open knowledge disclosure: An overview of the evidence and economic motivations.” *Journal of Economic Surveys* 21 (2): 326–347.

Perla, J., and C. Tonetti. 2014. “Equilibrium imitation and growth.” *Journal of Political Economy* 122 (1): 52–76.

Pęski, M., and T. Wiseman. 2015. "A folk theorem for stochastic games with infrequent state changes." *Theoretical Economics* 10 (1): 131–173.

Piunovskiy, A., and Y. Zhang. 2020. *Continuous-Time Markov Decision Processes.* Springer.

Powell, W. W., and E. Giannella. 2010. "Collective invention and inventor networks." Chap. 13 in *Handbook of the Economics of Innovation,* 1:575–605. North Holland.

Powell, W. W., K. W. Koput, and L. Smith-Doerr. 1996. "Interorganizational collaboration and the locus of innovation: Networks of learning in biotechnology." *Administrative science quarterly* 41 (1): 116–145.

Radner, R., R. Myerson, and E. Maskin. 1986. "An example of a repeated partnership game with discounting and with uniformly inefficient equilibria." *The Review of Economic Studies* 53 (1): 59–69.

Reinganum, J. F. 1983. "Uncertain innovation and the persistence of monopoly." *The American Economic Review* 73 (4): 741–748.

Sannikov, Y., and A. Skrzypacz. 2010. "The role of information in repeated games with frequent actions." *Econometrica* 78 (3): 847–882.

Spence, M. 1984. "Cost reduction, competition, and industry performance." *Econometrica* 52 (1): 101–121.

Strausz, R. 1999. "Efficiency in sequential partnerships." *Journal of Economic Theory* 85 (1): 140–156.

Suzumura, K. 1992. "Cooperative and Noncooperative R&D in an Oligopoly with Spillovers." *The American Economic Review* 82 (5): 1307–1320.

Unsworth, K. L., and C. W. Clegg. 2010. "Why do employees undertake creative action?" *Journal of occupational and organizational psychology* 83 (1): 77–99.

Van Knippenberg, D. 2017. "Team innovation." *Annual Review of Organizational Psychology and Organizational Behavior* 4:211–233.

Van Wijk, R., J. J. P. Jansen, and M. A. Lyles. 2008. "Inter-and intra-organizational knowledge transfer: a meta-analytic review and assessment of its antecedents and consequences." *Journal of management studies* 45 (4): 830–853.

Von Hippel, E. 1987. "Cooperation between rivals: Informal know-how trading." *Research Policy* 16 (6): 291–302.

Appendices

A Proof of Remark 1

Fix a profile $\rho = (\rho^i)_{i=1}^n$ of mixed strategies and a profile β of beliefs making (ρ, β) a PBE. Assume for simplicity that each ρ^i has finite support and, given $h \in H$ satisfying (3) and i, let π_h^i be the distribution over effort schedules $(t_m, \infty) \to [0, 1]$ derived from ρ^i and $(\beta^j)_{j \neq i}$. Note that the distribution over public histories and the payoff of any strategy to any agent at any information set would not change if each agent i mixed according to π_h^i at each h.

Define the pure strategy $\sigma^i \in \Sigma$ by

$$[\sigma^i(h)](t) = \frac{\int a_s e^{-\lambda \int_{t_m}^t a} \mathrm{d}\pi_h^i(a)}{\int e^{-\lambda \int_{t_m}^t a} \mathrm{d}\pi_h^i(a)} \quad \text{for all } h \in H \text{ and } t > t_m$$

and note that $1 - e^{-\lambda \int_{t_m}^t \sigma^i(h)} = \int 1 - e^{-\lambda \int_{t_m}^t a} \mathrm{d}\pi_h^i(a)$ for all $t > t_m$. That is, the first time after t_m at which agent i produces an innovation is the same whether she plays σ^i or mixes according to π_h^i. Then $\sigma = (\sigma^i)_{i=1}^n$ and ρ induce the same distribution over public histories, and the value of agent i at any information set $(h, \beta^i(h))$ against opponents playing $(\rho^j)_{j \neq i}$ equals her value at h against opponents playing $(\sigma^j)_{j \neq i}$. Thus, it suffices to show that σ^i is a best response at any $h \in H$ against $(\sigma_j)_{j \neq i}$. That is, we have to show that, given h, (10) holds for almost all $t > t_m$. Since ρ is an equilibrium, given any a in the support of π_h^i, a_t maximises the objective in (10) for almost all $t \geq t_m$. Result follows since the objective is concave and, given any $t > t_m$, $a'_t \leq [\sigma^i(h)](t) \leq a''_t$ for some a', a'' in the support of π_h^i.

B Proof of Proposition 1

Given a Borel $v : \mathbb{R}_+ \to \mathbb{R}$ bounded below, define $L_f v, L_d v : \mathbb{R}_+ \to \mathbb{R} \cup \{\infty\}$ by

$$L_f v(x) := \mathbb{E}_F[v(x + \tilde{z})] \quad \& \quad L_d v(x) := \mathbb{E}_F[v(x) \vee v(x + \tilde{z})]. \tag{22}$$

Let $\bar{w} : \mathbb{R}_+ \to \mathbb{R}$ be given by $\bar{w}(x) = b(x) + \lambda n[b(\mu) - b(0)]$, and, given a map $w : \mathbb{R}_+ \to \mathbb{R}$ bounded below by $b(0)$, let B_w be the set of Borel $v : \mathbb{R}_+ \to \mathbb{R}$

bounded below by $b(0)$ such that $v \le w$. Define $\Gamma : [0,1] \times \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ by

$$\Gamma(a, x, l) := \frac{b(x) - c(a, x) + na\lambda l}{1 + na\lambda}. \tag{23}$$

Remark 3. *Let $v : \mathbb{R}_+ \to \mathbb{R}$ be Borel, bounded below, and such that $v - b$ is decreasing. Then $L_k v - b$ is decreasing for $k \in \{f, d\}$.*

Remark 4. *$\Gamma(a, x, L_f v(x)) \le \bar{w}(x)$ for all $a \in [0,1]$, $x \ge 0$, and $v \in B_{\bar{w}}$.*

Remarks 3 and 4 hold as b is increasing and concave.[34]

Proof of Proposition 1. Step 1. (6) *admits a solution $v_* \in B_{\bar{w}}$, v_* is increasing, and $v_* - b$ is decreasing (so that v_* is continuous).* Given $v \in B_{\bar{w}}$, let $P_* v : \mathbb{R}_+ \to \mathbb{R}$ be given by

$$P_* v(x) := \max_{a \in [0,1]} \Gamma(a, x, L_f v(x)). \tag{24}$$

Let V_* be the set of increasing $v \in B_{\bar{w}}$ such that $v - b$ is decreasing. Note that $v \in B_{\bar{w}}$ solves (6) if (and only if) it is a fixed point of P_*, that V_* (endowed with the point-wise order) is a complete lattice, and that P_* is increasing. Then it suffices to show that $P_* v \in V_*$ for all $v \in V_*$, as this implies that P_* has a fixed point in V_*, by Tarski's fixed-point theorem.

Fix $v \in V_*$ and note that $P_* v(x) \ge \Gamma(0, x, L_f v(x)) = b(x)$ and, from Remark 4, $Pv \le \bar{w}$. Moreover, v is continuous and $b'(x)$-Lipschitz over $[x, \infty)$ for any $x > 0$, as b is increasing and concave. Then, so is $L_f v$ and hence, given any $a \in [0,1]$, $\gamma : x \mapsto \Gamma(a, x, L_f v(x))$ is absolutely continuous on $\mathbb{R}_+$ and, for any $x > 0$ at which $L_f v$ is differentiable, γ is differentiable as well with

$$\gamma'(x) = \frac{b'(x) - c_2(a, x) + a\lambda n (L_f v)'(x)}{1 + a\lambda n} \le \frac{b'(x) - c_2(a, x) + a\lambda n b'(x)}{1 + a\lambda n} \le b'(x)$$

where the second inequality follows from Remark 3, and the third holds as $c_2 \ge 0$. Note that $L_f v$ is increasing as v is, so that $\gamma'(x) \ge 0$, as u is increasing in x. Then, Theorems 2 and 1 of Milgrom and Segal (2002) imply that $P_* v$ is absolutely continuous and that $0 \le (P_* v)' \le b'$ wherever $P_* v$ is differentiable, respectively. Thus $P_* v \in B_{\bar{w}}$, $P_* v$ is increasing and $P_* v - b$ is decreasing, so that $P_* v \in V_*$.

Step 2. v_* is induced by a decreasing Markov strategy α_*. Let α_* given by

$$\alpha_*(x) := \min \arg\max_{a \in [0,1]} a\lambda n\{\mathbb{E}_F[v_*(x + \tilde{z})] - v_*(x)\} - c(a, x). \tag{25}$$

34. To prove Remark 3, note that $L_k v(x_2) - L_k v(x_1) \le \mathbb{E}_F[\mathbb{1}_{k=d}\{(v(x_2 + \tilde{z}) - v(x_1 + \tilde{z})) \vee (v(x_2) - v(x_1))\} + \mathbb{1}_{k=f}(v(x_2 + \tilde{z}) - v(x_1 + \tilde{z}))] \le b(x_2) - b(x_1)$ for $0 \le x_1 \le x_2$. For Remark 4, note that $\Gamma(a, x, L_f v(x)) \le \Gamma(a, x, L_f \bar{w}(x)) \le \Gamma(1, x, L_f \bar{w}(x)) \le \bar{w}(x)$.

As $v_* \in B_{\overline{w}}$ solves (6), v_* is induced by α_*.[35] To show that α_* is decreasing, note:

$$\alpha_*(x) \in \min \arg \max_{a\in[0,1]} \Gamma(a, x, L_f v_*(x))$$

for all $x \geq 0$. Let $\gamma^* : [0,1] \times \mathbb{R}_+ \to \mathbb{R}$ be given by

$$\gamma^*(a,x) := L_f v_*(x) - [b(x) - c(a,x)] - \left(\frac{1}{n\lambda} + a\right) c_1(a,x). \tag{26}$$

and note that $a \mapsto \Gamma(a, x, L_f v_*(x))$ is differentiable on $(0,1)$ and its derivative has the same sign as $\gamma^*(a,x)$. Then, it suffices to show that γ^* is decreasing in a (so that $\Gamma(a, x, L_f v_*(x))$ is quasi-concave in a) as well as in x. Since $v_* \in V_*$ (from Step 1), γ^* is absolutely continuous with a.e. derivatives

$$\gamma_1^*(a,x) = -\left(\frac{1}{n\lambda} + a\right) c_{11}(a,x)$$
$$\gamma_2^*(a,x) = (L_f v_*)'(x) - [b'(x) - c_2(a,x)] - \left(\frac{1}{n\lambda} + a\right) c_{12}(a,x).$$

Then γ^* and γ_2^* are decreasing in a, as $c_{11} \geq 0$ and c_{12} is increasing in a. Hence $\gamma_2^*(a,x) \leq \gamma_2^*(0,x) \leq (L_f v_*)'(x) - b'(x) \leq 0$ where the second inequality holds as $c_2(0,x) = 0$ and $c_{12} \geq 0$, and the last follows from Remark 3, since $v_* \in V_*$.

Step 3. $\alpha_*(\tilde{x}_t) \to 0$ as $t \to \infty$, and $\lim_{x\to\infty} \alpha_*(x) = 0$. Note that $\lim_{t\to\infty} \tilde{x}_t$ exists as $\tilde{x}_t$ is increasing in t. Since α_* is decreasing, $\lim_{t\to\infty} \tilde{x}_t > x$ a.s. whenever $\alpha_*(x) > 0$. Then, $\alpha_*(\tilde{x}_t) \to 0$ as $t \to \infty$ if $\lim_{x\to\infty} \alpha_*(x) = 0$. Moreover, for $x > 0$,

$$\mathbb{E}_F[v_*(x + \tilde{z}) - v_*(x)] \leq \mathbb{E}_F[b(x + \tilde{z}) - b(x)] \leq b'(x)\mu$$

since $v_* - b$ is decreasing, and b concave, respectively. Then, for $a > 0$,

$$\lim_{x\to\infty} \lambda n\{\mathbb{E}_F[v_*(x + \tilde{z})] - v_*(x)\} - c_1(a,x) < 0$$

by (1), so that $\lim_{x\to\infty} \alpha_*(x) < a$. Hence $\lim_{x\to\infty} \alpha_*(x) = 0$.

Step 4. $\lim_{x\to\infty} v_*(x) - b(x) = 0$. Fix $x \geq 1$. As $v_* - b$ is decreasing, $v_*(x) \leq v_*(1) - b(1) + b(x)$ so that

$$L_f v_*(x) \leq v_*(1) - b(1) + L_f b(x) \leq v_*(1) - b(1) + b(x) + b'(1)\mu$$

35. This follows from e.g. Theorem 3.1.2 of Piunovskiy and Zhang (2020).

where the second inequality holds since b is concave. Hence

$$v_*(x) = \frac{b(x) - c(\alpha_*(x), x) + \alpha_*(x)\lambda n L_f v_*(x)}{1 + \alpha_*(x)\lambda n}$$
$$\leq b(x) + \frac{\alpha_*(x)\lambda n[v_*(1) - b(1) + b'(1)\mu] - c(\alpha_*(x), x)}{1 + \alpha_*(x)\lambda n}$$

Therefore, since $\lim_{x\to\infty} \alpha_*(x) = 0$ by Step 3,

$$\lim_{x\to\infty} v_*(x) - b(x) \leq - \lim_{x\to\infty} c(\alpha_*(x), x) \leq 0.$$

Then $\lim_{x\to\infty} v_*(x) - b(x) = 0$ since $v_* \geq b$. □

C Proof of Theorem 1

Theorem 1 follows immediately from Propositions 6 and 7, stated and proved below. Recall the definition of v_* from Section 4, and let V_f (V_d) be the set of pairs (α, v), where α is a Markov strategy of the game with forced disclosure and $v \in B_{v_*}$, such that (11) holds and (12) holds with $\hat{v}_\alpha = v$ (such that (18) holds for some δ satisfying (20) with $\hat{v}_\pi = v$ and (19) holds with $\hat{v}_\pi = v$), for all $x \geq 0$.

Proposition 6. *Given $k \in \{f, d\}$, $V_k = \{(\alpha_k, v_k)\}$. Moreover, α_k, v_k and $L_k v_k$ are continuous, α_k is decreasing and lies below α_*, and $v_k - b$ is decreasing.*

Proposition 7. *No strategy of the game with forced disclosure other than α_f induces a SSE. Moreover, a strategy $\xi := (\sigma, \chi)$ of the game with disposal induces a SSE only if $\sigma = \alpha_d$ and*

$$[\chi(h)](t, z) \in \arg\max_{d\in\{0,1\}} \{d[v_d(x + z) - v_d(x)]\} \quad \textit{for all } t > 0 \textit{ and } z > 0. \tag{27}$$

Claim 1. Let $\bar{v}, \underline{v} : \mathbb{R}_+ \to \mathbb{R}$ be Borel, bounded below, and such that $\bar{v} \geq \underline{v}$ and $\lim_{x\to\infty} \bar{v}(x) - \underline{v}(x) = 0$. Then $\bar{v} = \underline{v}$ if one of the following holds:

- $\bar{v} - \underline{v} \leq L_f\bar{v} - L_f\underline{v}$, or
- $\bar{v} - \underline{v} \leq L_d\bar{v} - L_d\underline{v}$ and $\lim_m \bar{v}(x_m) - \underline{v}(x_m) = 0$ for any bounded sequence $(x_m)_{m\in\mathbb{N}} \subset \mathbb{R}_+$ such that $\lim_m \Pr_F(\bar{v}(x_m + \tilde{z}) \leq \bar{v}(x_m)) = 1$.

Proof. Suppose that $\bar{v}(x) - \underline{v}(x) \geq \epsilon$ for some $\epsilon > 0$ and $x \geq 0$, and seek a contradiction. Let $\hat{x} := \inf\{x \geq 0 : \sup_{(x,\infty)} \bar{v} - \underline{v} < \epsilon\}$ and note that $\hat{x} < \infty$ since $\lim_{x\to\infty} \bar{v} - \underline{v} = 0$. Consider a sequence $(x_m)_{m\in\mathbb{N}} \subset [0, \hat{x}]$ with $\lim_m x_m = \hat{x}$

and such that $\lim_m \bar{v}(x_m) - \underline{v}(x_m) \geq \epsilon$. By hypothesis, $\bar{v} - \underline{v} \leq L_k\bar{v} - L_k\underline{v}$ for some $k \in \{d, e\}$. Then, it suffices to show that

$$\limsup_{m\to\infty} L_k\bar{v}(x_m) - L_k\underline{v}(x_m) < \epsilon. \tag{28}$$

By definition of $\hat{x}$, $\bar{v}(x) - \underline{v}(x) < \epsilon$ for all $x > \hat{x}$. Then, if $k = e$, (28) holds since $F(0) = 0$. Hence, assume that $k = d$. Let $\tilde{z} \sim F$ and, for all $m \in \mathbb{N}$, let E_m be the event '$\bar{v}(x_m + \tilde{z}) > \bar{v}(x_m)$'. By considering an appropriate subsequence, we may assume without loss that $\Pr(E_m) > 0$ for all $m \in \mathbb{N}$, and $\lim_m \Pr(E_m) > 0$.

Fix $m \in \mathbb{N}$ and note that

$$\begin{aligned} L_d\bar{v}(x_m) - L_d\underline{v}(x_m) &= \mathbb{E}[\bar{v}(x_m + \tilde{z}) \vee \bar{v}(x_m) - \underline{v}(x_m + \tilde{z}) \vee \underline{v}(x_m)] \\ &\leq \Pr(E_m)\mathbb{E}[\bar{v}(x_m + \tilde{z}) - \underline{v}(x_m + \tilde{z})|E_m] + [1 - \Pr(E_m)][\bar{v}(x_m) - \underline{v}(x_m)] \\ \Rightarrow \ L_d\bar{v}(x_m) - L_d\underline{v}(x_m) &\leq \mathbb{E}[\bar{v}(x_m + \tilde{z}) - \underline{v}(x_m + \tilde{z})|E_m] \end{aligned}$$

where the last step holds since $\bar{v} - \underline{v} \leq L_k\bar{v} - L_k\underline{v}$ and $\Pr(E_m) > 0$. Then, taking the limit $m \to \infty$ yields (28) since $\lim_m \Pr(E_m) > 0$, $F(0) = 0$ and, by definition of $\hat{x}$, $\sup_{(x,\infty)} \bar{v} - \underline{v} < \epsilon$ for all $x > \hat{x}$. □

Proof of Proposition 6. Step 1. There are $(\underline{\alpha}_k, \underline{v}_k)$, $(\bar{\alpha}_k, \bar{v}_k) \in V_k$ such that $\bar{\alpha}_k \leq \alpha_*$ and, for all $(\alpha, v) \in V_k$, $\underline{\alpha}_k \leq \alpha \leq \bar{\alpha}_k \leq \alpha_*$ and $\underline{v}_k \leq v \leq \bar{v}_k$. Note that, for any $x \geq (>)\ 0$ and $l > (\geq)\ b(x)$, there is a unique $p(x, l) \in [0, 1]$ such that

$$p(x, l) \in \arg\max_{a\in[0,1]} a\lambda \frac{l - [b(x) - c(p(x, l), x)]}{1 + \lambda n p(x, l)} - c(a, x). \tag{29}$$

Indeed, the objective is continuously differentiable and concave in a, and its derivative has the same sign as $\gamma(p(x, l), x, l)$ where

$$\gamma(a, x, l) := l - [b(x) - c(a, x)] - \left(\frac{1}{\lambda} + na\right)c_1(a, x) \tag{30}$$

is decreasing in a, as $c(a, x)$ is convex in a. Moreover, p is continuous. Recall (23) in Appendix B and, given $v \in B_{v_*}$, let $P_kv : \mathbb{R}_+ \to \mathbb{R}$ be given by

$$P_kv(x) := \Gamma(p(x, L_kv(x)), x, L_kv(x)),$$

Claim 2. $\Gamma(a, x, L_kv(x)) \leq P_kv(x)$ for all $x \geq 0$, $v \in B_{v_*}$, and $0 \leq a \leq p(x, L_kv(x))$.

Proof of Claim 2. Fix x, v, and a, and let $\hat{a} := p(x, L_kv(x))$ and $\phi : [0, 1] \to \mathbb{R}$ be given by

$$\phi(e) := \frac{b(x) - c(e,x) + \lambda[e + (n-1)\hat{a}]L_k v(x)}{1 + \lambda\lambda[e + (n-1)\hat{a}]}.$$

Note that ϕ is differentiable with

$$\phi'(e) = \lambda \frac{L_k v(x) - [b(x) - c(e,x)] - \left[\frac{1}{\lambda} + e + (n-1)\hat{a}\right] c_1(e,x)}{\{1 + \lambda[e + (n-1)\hat{a}]\}^2}. \tag{31}$$

Then, ϕ is quasi-concave since $c(e,x)$ is convex in e, and $\phi'(\hat{a})$ has the same sign as $\gamma(\hat{a}, x, L_k v(x))$, where γ was defined in (30). Hence, ϕ is maximised at $e = \hat{a}$. Moreover, $\phi(\hat{a}) = P_k v(x)$ so that $P_k v(x) \geq \phi(a)$, and thus it is enough to show that $\phi(a) \geq \Gamma(a, x, L_k v(x))$. Since ϕ is quasi-concave and maximised at $e = \hat{a}$, and $a \leq \hat{a}$, $\phi'(a) \geq 0$. Then, (31) implies that $b(x) - c(a,x) \leq L_k v(x)$, so that

$$\begin{aligned}
\phi(a) &= \frac{b(x) - c(a,x)}{1 + \lambda[a + (n-1)\hat{a}]} + \left[1 - \frac{1}{1 + \lambda[a + (n-1)\hat{a}]}\right] L_k v(x) \\
&\geq \frac{b(x) - c(a,x)}{1 + \lambda n a} + \left[1 - \frac{1}{1 + \lambda n a}\right] L_k v(x) = \Gamma(a, x, L_k v(x))
\end{aligned}$$

where the inequality holds since $a \leq \hat{a}$. □

I prove that P_k maps B_{v_*} to itself. Fix $v \in B_{v_*}$ and note that

$$L_k v \leq L_k v_* = L_f v_* \tag{32}$$

as $v \leq v_*$ and v_* is increasing (Proposition 1). Then, $\Gamma(a, x, l)$ is increasing in l,

$$P_k v(x) \leq \Gamma(p(x, L_k v(x)), x, L_f v_*(x)) \leq \max_{a \in [0,1]} \Gamma(a, x, L_f v_*(x)) = v_*(x).$$

Moreover, $L_k v$, p and, therefore, $P_k v$ are Borel, and $P_k v(x) \geq \Gamma(0, x, L_k v(x)) = b(x)$ from Claim 2. Hence $P_k v \in B_{v_*}$, as desired.

I show that P_k is increasing. Fix $v \leq w$ in B_{v_*} and note that $L_k w(x) \leq L_k v(x)$, so that $P_k w(x) \leq \Gamma(p(x, L_k w(x)), x, L_k v(x))$ as $\Gamma(a, x, l)$ is increasing in l, and $p(x, L_k w(x)) \leq p(x, L_k v(x))$ as $p(x, l)$ is increasing in l (since $\gamma(a, x, l)$ is). Then $\Gamma(p(x, L_k w(x)), x, L_k v(x)) \leq P_k v(x)$ by Claim 2, so that $P_k w(x) \leq P_k v(x)$.

Let $\bar{v}_k := \lim_m [(P_k)^m](v_*)$ and $\underline{v}_k := \lim_m [(P_k)^m](b(0))$, where we view $b(0)$ as a constant map. The limits are well-defined as P_k is increasing, and lie in B_{v_*} as P_k maps B_{v_*} to itself. Then $\bar{v}_k$ and $\underline{v}_k$ are fixed points of P_k, and they are the largest and smallest, respectively, as P_k is increasing. Define $\underline{\alpha}_k$ and $\bar{\alpha}_k$ by $\underline{\alpha}_k(x) := p(x, L_k \underline{v}_k(x))$ and $\bar{\alpha}_k := p(x, L_k \bar{v}_k(x))$ and note that, for any Markov strategy α and $v \in B_{v_*}$, $(\alpha, v) \in V_k$ if and only if $v = P_k v$ and $\alpha(x) = p(x, L_k v(x))$ for all x. Then $(\underline{\alpha}_k, \underline{v}_k), (\bar{\alpha}_k, \bar{v}_k) \in V_k$ and, for any $(\alpha, v) \in V_k$, $\underline{v}_k \leq v \leq \bar{v}_k$ so that $L_k \underline{v}_k \leq L_k v \leq L_k \bar{v}_k$ and therefore $\underline{\alpha}_k \leq \alpha \leq \bar{\alpha}_k$, as $p(x, l)$ is increasing in l.

It remains to prove that $\bar{\alpha}_k \leq \alpha_*$. Fix x and assume without loss that $\bar{\alpha}_k(x) > 0$ and $\alpha_*(x) < 1$. Then, from Step 2 of the proof of Proposition 1, it suffices to show that $\gamma^*(\bar{\alpha}_k(x), x) > 0$. Note that $\gamma^*(\bar{\alpha}_k(x), x) > \gamma(\bar{\alpha}_k(x), x, L_k \bar{v}_k(x)) \geq 0$, where the first inequality follows from (32) (as $\bar{v}_k \in B_{v_*}$) and the fact that $c_1(\bar{\alpha}_k(x), x) > 0$ (as $\bar{\alpha}_k(x) > 0$), and the second holds since $\bar{\alpha}_k(x) > 0$.

Step 2: V_k is a singleton. It suffices to show that $\underline{v}_k = \bar{v}_k$, as this implies that $\underline{\alpha}_k = \bar{\alpha}_k$. Note that $v_* \geq \bar{v}_k \geq \underline{v}_k \geq b$ since $\bar{v}_k, \underline{v}_k \in B_{v_*}$, so that $\lim_{x\to\infty} \bar{v}_k(x) - \underline{v}_k(x) = 0$ by Proposition 1. Moreover, if $k = d$, $\lim_m \bar{v}_d(x_m) - b(x_m) = 0$ for any bounded $(x_m)_{m\in\mathbb{N}} \subset \mathbb{R}_+$ such that $\lim_m \Pr_F(\bar{v}_d(x_m + \tilde{z}) \leq \bar{v}_d(x_m)) = 1$, by (18). Then, in light of Claim 1, it suffices to show that, for all $x \geq 0$,

$$\bar{v}_k(x) - \underline{v}_k(x) \leq L_k \bar{v}_k(x) - L_k \underline{v}_k(x) \tag{33}$$

Fix x and suppose first that $\bar{\alpha}_k(x) = 0$. Then $\underline{\alpha}_k(x) = 0$ as $\bar{\alpha}_k \geq \underline{\alpha}_k$, so that $\bar{v}_k(x) = b(x) = \underline{v}_k(x)$ and, therefore, (33) holds since $\bar{v}_k \geq \underline{v}_k$. If instead $\underline{\alpha}_k(x) = 1$, then $\bar{\alpha}_k(x) = 1$ so that, from (11) if $k = f$ and (18) if $k = d$,

$$[L_k \bar{v}_k(x) - \bar{v}_k(x)] - [L_k \underline{v}_k(x) - \underline{v}_k(x)] = \frac{\bar{v}_k(x) - \underline{v}_k(x)}{\lambda n} \geq 0$$

and thus (33) holds. Finally, if $\bar{\alpha}_k(x) > 0$ and $\underline{\alpha}_k(x) < 1$, by (12) for $k = f$ and (19) for $k = d$,

$$L_k \bar{v}_k(x) - \bar{v}_k(x) \geq \frac{c(\bar{\alpha}_k(x), x)}{\lambda} \geq \frac{c(\underline{\alpha}_k(x), x)}{\lambda} \geq L_k \underline{v}_k(x) - \underline{v}_k(x)$$

where the first inequality holds since $\bar{\alpha}_k(x) > 0$, the second since $\bar{\alpha}_k(x) \geq \underline{\alpha}_k(x)$, and the third since $\underline{\alpha}_k(x) < 1$. Then, (33) holds.

Step 3: α_k and $v_k - b$ are decreasing. Let $\widehat{V}$ be the set of $v \in B_{v_*}$ such that $v - b$ is decreasing. From Steps 1 and 2, it suffices to show that $p(x, L_k v(x))$ is decreasing in x for any $v \in B_{v_*}$, and to find a fixed point v of P_k in $\widehat{V}$. For the former, fix $v \in \widehat{V}$ and note that $L_k v - b$ is decreasing by Remark 3. Then, $\gamma(a, x, L_k v(x))$ is decreasing in x, as $c_1(a, x)$ and $c_{11}(a, x)$ are increasing in x. Hence $p(x, L_k v(x))$ is decreasing in x.

For the latter, note that $\widehat{V}$ is a complete lattice, and that P_k is increasing on $\widehat{V}$ from Step 1. Then, from Tarski's fixed-point theorem, it suffices to show that

P_k maps $\widehat{V}$ to itself. Fix $v \in \widehat{V}$ and note that $P_k v \in B_{v_*}$ from Step 1. Then it suffices to show that $P_k v(x_2) - P_k v(x_1) \leq b(x_2) - b(x_1)$ for all $x_1 \leq x_2$. Let $p_i = p(x_i, L_k v_i(x_i))$ for $i = 1, 2$, so that $p_1 \geq p_2$. Then

$$
\begin{aligned}
P_k v(x_2) - P_k v(x_1) &\leq \Gamma(p_2, x_2, L_k v(x_2)) - \Gamma(p_2, x_1, L_k v(x_1)) \\
&\leq \frac{b(x_2) - b(x_1) + \lambda n p_2 [L_k v(x_2) - L_k v(x_1)]}{1 + \lambda n p_2} \leq b(x_2) - b(x_1)
\end{aligned}
$$

where the first inequality follows from Claim 2 as $p_2 \leq p_1$, the second holds as $c(a, x)$ is increasing in x, and the third follows from Remark 3. Hence $P_k v \in \widehat{V}$.

Step 5. v_k, $L_k v_k$ and α_k are continuous. Note that v_k has bounded variation as $v_k \geq b(0)$ and $v_k - b$ is decreasing. Then we may define $\bar{v}, \underline{v} : \mathbb{R}_+ \to \mathbb{R}$ by

$$
\bar{v}(x) := \lim_{y \downarrow x} v_k(y) \quad \& \quad \underline{v}(x) := \begin{cases} v_k(0) & \text{if } x = 0 \\ \lim_{y \uparrow x} v_k(y) & \text{if } x > 0. \end{cases}
$$

It is easy to see that $\bar{v}$ and $\underline{v}$ are fixed points of P_k, so that $\bar{v} = v_k = \underline{v}$ by Step 2. Then v_k and, hence, $L v_k$ and α_k are continuous. □

Proof of Proposition 7. Let Ξ be the set of strategies $\xi := (\sigma, \xi)$ of the game with disposal, $H(x_0)$ be the set of public histories with initial stock $x_0 \geq 0$, $\Xi^*_d(x_0)$ be set set of $\xi \in \Xi$ inducing SSE in the game with disposal and initial stock x_0, and $\Xi^*_f(x_0)$ be the set of $\xi := (\sigma, \xi) \in \Xi$ where ξ is the 'full-disclosure' policy, and σ induces a SSE of the game with forced disclosure and initial stock x_0. Fix x_0 and k. We have to show that, for all $\xi := (\sigma, \xi) \in \Xi^*_k(x_0)$ and $h \in H(x_0)$, $\sigma(h) = \alpha_k(X(h))$ a.e. and, if $k = d$, then $\xi(h)$ satisfies (27).

I claim that it suffices to show that $v_{\xi,h}$ is constant with value $v_k(X(h))$ for all $h \in H(x_0)$ and $\xi \in \Xi^*_k(x_0)$. To see why note first that $L_k v_k(x) \geq v_k(x)$ where the inequality is strict unless $x > 0$. Indeed, if $L_k v_k(x) \leq v_k(x)$ then $v_k(x) = b(x)$ by (11) if $k = f$ and (18) if $k = d$, so that $L_k v_k(x) = b(x)$ and b must be constant above x since it is increasing and $v_k \geq b$, and thus $x > 0$. Now suppose that $v_{\xi,h}$ is constant with value $v_k(X(h))$ for all $h \in H(x_0)$ and $\xi \in \Xi^*_k(x_0)$. Fix $\xi := (\sigma, \chi) \in \Xi^*_k(x_0)$ and $h \in H(x_0)$, and note that (27) follows from (17) if $k = d$, so that it is enough to show that $\sigma(h) = \alpha_k(X(h))$ a.e. If $L_k v_k(X(h)) > v_k(X(h))$, then $\sigma(h) = \alpha_k(X(h))$ a.e. by (11) and (9) for $k = f$, and (18) and (15) for $k = d$. If instead $L_k v_k(X(h)) = v_k(X(h))$ then $X(h) > 0$, and thus $\sigma(h) \overset{\text{a.e.}}{=} 0 = \alpha_k(X(h))$ by (29) and (10) for $k = f$, (19) and (16) for $k = d$. Hence $\sigma(h) = \alpha_k(X(h))$ a.e.

Define $\bar{v}_k, \underline{v}_k : \mathbb{R}_+ \to \mathbb{R}$ by

$$\bar{v}_k(x) := \inf_{\epsilon>0} \sup\{v_\xi(y) : (x-\epsilon) \vee 0 \leq y \leq x+\epsilon, \xi \in \Sigma_k^*(y)\}$$
$$\underline{v}_k(x) := \sup_{\epsilon>0} \inf\{v_\xi(y) : (x-\epsilon) \vee 0 \leq y \leq x+\epsilon, \xi \in \Sigma_k^*(y)\}$$

and note that $\underline{v}_k(X(h)) \leq v_{\xi,h} \leq \bar{v}_k(X(h))$ for all $\xi \in \Xi_k^*(x_0)$ and $h \in H(x_0)$. Then, by the claim in the previous paragraph, it suffices to show that $\bar{v}_k = v_k = \underline{v}_k$.

I show that $\bar{v}_k = v_k$, relying on Claim 1. A similar reasoning yields that $v_k = \underline{v}_k$. Note first that $\bar{v}_k$ is upper-semicontinuous and, hence, Borel. Also, $b \leq v_k \leq \bar{v}_k \leq v_*$ (where the last inequality holds as v_* is continuous and $v_\xi(x) \leq v_*(x)$ for all x and $\xi \in \Xi_k^*(x)$), so that $\lim_{x\to\infty} \bar{v}_k - v_k = 0$ by Proposition 1.

I show that, if $k = d$, then $\lim_m \bar{v}_d(x^m) - b(x^m) = 0$ for any bounded sequence $(x^m)_{m\in\mathbb{N}} \subset \mathbb{R}_+$ such that $\lim_m \Pr_F(\bar{v}_d(x^m + \tilde{z}) \leq \bar{v}_d(x^m)) = 1$. Fix $(x^m)_{m\in\mathbb{N}} \subset \mathbb{R}_+$ and note that $\lim_m \mathbb{E}_F[0 \vee (\bar{v}_d(x^m + \tilde{z}) - \bar{v}_d(x^m))] = 0$ and that, for all $m \in \mathbb{N}$, $y \mapsto \mathbb{E}_F[0 \vee (\bar{v}_d(y + \tilde{z}) - \bar{v}_d(x_m))]$ is upper-semicontinuous at x_m, since $\bar{v}_d$ is upper-semicontinuous. Then, it is clear that there exist sequences $(y_m)_{m\in\mathbb{N}} \subset \mathbb{R}_+$ and $(\xi^m)_{m\in\mathbb{N}} := (\sigma^m, \chi^m)_{m\in\mathbb{N}} \in \prod_{m\in\mathbb{N}} \Xi_d^*(y_m)$ such that $(y_m)_{m\in\mathbb{N}}$ is bounded and $b(y^m) - b(x^m)$, $\bar{v}_d(x^m) - v_{\xi^m}(y^m)$, and $\mathbb{E}_F[0 \vee (\bar{v}_d(y^m + \tilde{z}) - v_{\xi^m}(y^m))]$ all vanish as $m \to \infty$. Hence, it suffices to show that $\lim_m v_{\xi^m}(y^m) - b(y^m) = 0$. Since $(y_m)_{m\in\mathbb{N}}$ is bounded, $(v_{\xi^m,y^m})_{m\in\mathbb{N}}$ is uniformly l-Lipschitz for some $l > 0$. Also, for all $\epsilon > 0$ and $m \in \mathbb{N}$, as v_{ξ^m,y^m} is bounded, there is $t_m \geq \epsilon$ such that v_{ξ^m,y^m} is increasing over $[0, t_m - \epsilon]$, and differentiable at t_m with derivative lower than ϵ. Then

$$\begin{aligned} v_{\xi^m}(y^m) &= v_{\xi^m,y^m}(0) \leq v_{\xi^m,y^m}(t_m - \epsilon) \leq l\epsilon + v_{\xi^m,y^m}(t_m) \\ &\leq (l+1)\epsilon + b(y^m) - c([\sigma^m(h)](t_m), y^m) \\ &\quad + \lambda[\sigma^m(h)](t_m) \textstyle\sum_{i=1}^n \mathbb{E}_F[0 \vee (v_{\xi^m}(y^m \circ (t_m, \tilde{z}, i))] - v_{\xi^m,y^m}(t_m))] \\ &\leq [l(1+\lambda n)+1]\epsilon + b(y^m) + \lambda n \mathbb{E}_F[0 \vee (\bar{v}_d(y^m + \tilde{z}) - v_{\xi^m}(y^m))] \end{aligned}$$

where the third inequality follows from (15), (16), and (17), and the last inequality holds since $v_{\xi^m}(y^m \circ (t_m, z, i)) \leq \bar{v}_d(y^m + z)$ for all i. Letting $m \to \infty$ yields $\lim_m v_{\xi^m}(y^m) - b(y^m) \leq [l(1+\lambda n)+1]\epsilon$, and result follows as $\epsilon > 0$ is arbitrary.

In light of Claim 1, it remains to show that $\bar{v}_k - v_k \leq L_k\bar{v}_k - L_k v_k$. Since v_k is continuous, it is enough to show that, for all $x \geq 0$ and $\xi \in \Xi_k^*(x)$,

$$v_\xi(x) - v_k(x) \leq L_k\bar{v}_k(x) - L_k v_k(x). \tag{34}$$

I show that $L\bar{v}_k \geq \bar{v}_k$. Suppose that $L\bar{v}_k(\hat{x}) < \bar{v}_k(\hat{x})$ for some $\hat{x} \geq 0$ and seek a contradiction. If so, then $k = f$ and, since $L_f\bar{v}_f$ is upper-semicontinuous (as $\bar{v}_f$ is), there exists $\epsilon > 0$ such that $L_f\bar{v}_f < \bar{v}_f(\hat{x})$ over $I := [(\hat{x}-\epsilon)\vee 0, \hat{x}+\epsilon]$. Then, there is $x \in I$ and $\sigma \in \Xi_f^*(x)$ such that $v_\sigma(x) > L_f\bar{v}_f(x)$. Let $t := \sup\{s \geq 0 : v_{\sigma,x} > L_f\bar{v}_f(x)$ over $[0,s]\}$ and note that $t > 0$ since $v_{\sigma,x}$ is continuous and $v_{\sigma,x}(0) = v_\sigma(x)$. But then $\mathbb{E}_F[v_\sigma(x\circ(s,\tilde{z},i)) - v_{\sigma,x}(s)] < 0$ for all i and $s \in [0,t)$, since $v_\sigma(x\circ(s,z,i)) \leq \bar{v}_f(x+z)$ for all $z > 0$, and thus $\sigma(x) = 0$ a.e. over $(0,t)$ by (10). Hence $v_\sigma(x) = b(x)$ if $t = \infty$ and $v_\sigma(x) = (1-e^{-t})b(x) + e^{-t}v_{\sigma,x}(t) = (1-e^{-t})b(x) + e^{-t}L_f\bar{v}_f(x)$ otherwise (as $v_{\sigma,x}$ is continuous at t). This contradicts the fact that $v_\sigma(x) > L_f\bar{v}_f(x)$, since $L_f\bar{v}_f(x) \geq L_f b(x) \geq b(x)$.

To prove (34), fix x and $\xi := (\sigma,\chi)$ and assume without loss that $v_\xi(x) > v_k(x)$. Let $t := \sup\{0\}\cup\{s > 0 : \sigma(x) \leq \alpha_k(x)$ a.e. on $(0,s)\}$ and define

$$\phi_* := \begin{cases} L_k\bar{v}_k(x) - L_k v_k(x) + v_k(x) & \text{if } t < \infty \\ 0 & \text{if } t = \infty \end{cases} \quad \& \quad \phi := \begin{cases} v_{\xi,x}(t) & \text{if } t < \infty \\ 0 & \text{if } t = \infty. \end{cases}$$

I claim that $\phi \leq \phi_*$. Indeed, if $t < \infty$ then $\alpha_k(x) < 1$ and, for a.e. $s \geq t$ such that $[\sigma(x)](s) > \alpha_k(x)$, and each i,

$$\begin{aligned} L_k\bar{v}_k(x) - v_{\xi,x}(s) &\geq \mathbb{E}_F\{[\chi(x)](s,\tilde{z})(v_\xi(x\circ(s,\tilde{z},i)) - v_{\xi,x}(s))\} \\ &\geq c_1([\sigma(x)](s),x)/\lambda \geq c_1(\alpha_k(x),x)/\lambda \geq L_k v_k(x) - v_k(x) \end{aligned}$$

where the first inequality holds since $\chi(x)$ is constant with value 1 if $k = f$, $L_k\bar{v}_k(x) \geq \bar{v}_k(x) \geq v_{\xi,x}(s)$ if $k = d$, and $\bar{v}_k(x+z) \geq v_\xi(x\circ(s,z,i))$ for all $z \geq 0$, the second follows from (10) if $k = f$, and from (16) and (17) if $k = d$, since $[\sigma(x)](s) > 0$, the third holds since $[\sigma(x)](s) \geq \alpha_k(x)$, and the last follows from (12) if $k = f$ and from (19) if $k = d$, since $\alpha_k(x) < 1$. Then, since $v_{\xi,x}$ is continuous at t, $\phi = v_{\xi,x}(t) \leq L_k\bar{v}_k(x) - L_k v_k(x) + v_k(x) = \phi_*$.

Let A_0 be the set of Borel $a : (0,\infty) \to [0,1]$. Given $a \in A_0^n$, let $N(\cdot,a)$ be the CDF of the random time τ of the first innovation, if agents exert effort according to a (with $\tau = \infty$ if no innovation is produced). Let

$$\Phi(a,v,w) := \int\int_0^{s\wedge t} b(x) - c(a^1(r),x)\mathrm{d}r + e^{-s\wedge t}(\mathbb{I}_{s\leq t}v + \mathbb{I}_{s>t}w)N(\mathrm{d}s,a),$$

for all $v, w \in \mathbb{R}$, and $\hat{v} := \max_{a\in A_0}\Phi((a,\alpha_k(x)^{n-1}), L_k\bar{v}_k(x), \phi_*)$. Note that, writing $a^n := (a, \ldots, a) \in A_0^n$ for any $a \in A_0$,

$$v_\xi(x) = \begin{cases} \Phi\big(\sigma(x)^n, \frac{1}{n}\sum_{i=1}^n \mathbb{E}_F[v_\xi(x \circ (s, \tilde{z}, i)], \phi\big) & \text{if } k = f \\ \Phi\big(\sigma(x)^n, \frac{1}{n}\sum_{i=1}^n \mathbb{E}_F[v_{\xi,x}(s) \vee v_\xi(x \circ (s, \tilde{z}, i))], \phi\big) & \text{if } k = d, \text{ by (17)} \end{cases}$$
$$\leq \Phi(\sigma(x)^n, L_k\bar{v}_k(x), \phi_*) \leq \hat{v}$$

where the first inequality holds since $v_{\xi,x}(s) \leq \bar{v}_k(x)$ and $v_\xi(x \circ (s, z, i)) \leq \bar{v}_k(x+z)$ for all $s, z \geq 0$ and i, and $\phi \leq \phi_*$, and the second since $\sigma(x) \leq \alpha_k(x)$ over $[0, t)$, $L_k\bar{v}_k(x) \geq b(x)$, and $L_k\bar{v}_k(x) \geq \phi_*$ (as $L_k v_k \geq v_k$).

Therefore, it suffices to show that $\hat{v} - v_k(x) \leq L_k\bar{v}_k(x) - L_k v_k(x)$. To this end, let $\hat{a} \in A_0$ achieve $\hat{v}$, and note that

$$v_k(x) = \max_{a \in A} \Phi\big((a, \alpha_k(x)^{n-1}), L_k v_k(x), v_k(x)\big) \geq \Phi\big((\hat{a}, \alpha_k(x)^{n-1}), L_k v_k(x), v_k(x)\big)$$

so that

$$\hat{v} - v_k(x) \leq [L_k\bar{v}_k(x) - L_k v_k(x)] \int e^{-s\wedge t} N(\mathrm{d}s, (\hat{a}, \alpha_k(x)^{n-1})) \leq L_k\bar{v}_k(x) - L_k v_k(x).$$

Thus (34) holds, and therefore $\bar{v}_k = v_k$, by Claim 1. □

D Proof of Proposition 2

Fix $\hat{x} \geq 0$ at which (13) fails. The map

$$\psi : x \mapsto \lambda\{\mathbb{E}_F[b(x + \tilde{z}) - b(x)\} - c_1(0, x)$$

is decreasing as b is concave and $c_1(0, x)$ is increasing. Then, there exist $\epsilon > 0$ and $\bar{x} > \hat{x}$ such that $\psi(x) + 2\lambda\epsilon < 0$ for all $x \geq \bar{x}$. By Proposition 1, we may choose $\bar{x}$ sufficiently large so that $v_*(x) - b(x) \leq \epsilon/n$ for all $x \geq \bar{x}$.

Fix a PPE $\sigma = (\sigma^i)_{i=1}^n$. Note that, for any i and history h such that $X(h) \geq \bar{x}$, $v_\sigma^i(h) \leq v_*(X(h)) + \epsilon$, for otherwise $nv_*(X(h)) \geq \sum_{j=1}^n v_\sigma^j(h) > v_*(X(h)) + \epsilon + (n-1)b(X(h))$, contradicting $v_*(X(h)) \leq b(X(h)) + \epsilon/n$. Then, for any history h such that $X(h) \geq \bar{x}$, any i and $s \geq T(h)$,

$$\begin{aligned} \lambda\{\mathbb{E}_F[v_\sigma^i(h \circ (s, \tilde{z}, i))] - v_{\sigma,h}(s)\} &\leq \lambda\{\mathbb{E}_F[v_*(X(h) + \tilde{z})] + \epsilon - b(X(h))\} \\ &\leq \lambda\{\mathbb{E}_F[b_*(X(h) + \tilde{z})] + (1 + 1/n)\epsilon - b(X(h))\} < c_1(0, X(h)) \end{aligned}$$

where the first inequality holds since $v_{\sigma,h}(s) \geq b(X(h))$ and, by the previous step, for all $z > 0$, $v_\sigma(h \circ (s, z, i)) \le v_*(X(h) + z) + \epsilon$. Then no effort is exerted in σ at history h, by (10).

Assume without loss that $\sigma^i(x_0) > 0$ is non-null, for some i (so that $x_0 < \bar{x}$), and define:

$$x_\sigma := \sup\{x \ge x_0 : \sigma^i(h) > 0 \text{ is non-null for some } i \text{ and } h \text{ with } X(h) \ge x\}.$$

Then, it suffices to show that $x_\sigma \le \hat{x}$. From above, $x_\sigma \le \bar{x} < \infty$. Fix $\hat{\epsilon} > 0$ and a history h such that $x_\sigma - \hat{\epsilon} \le X(h) \le x_\sigma$, and i such that $\sigma^i(h)$ is non-null. Then, there exists $t \ge T(h)$ such that $[\sigma^i(h)](t) > 0$ and (10) holds, and thus

$$\begin{aligned} c_1(0, X(h)) &\le c_1\big([\sigma^i(h)](t), X(h)\big) \\ &\le \lambda\{\mathbb{E}_F[v_\sigma(h \circ (t, \tilde{z}, i))] - v_{\sigma,h}(t)\} \\ &\le \lambda\{\mathbb{E}_F[\mathbb{1}_{\tilde{z} \le \hat{\epsilon}} \bar{w}(X(h) + \tilde{z}) + \mathbb{1}_{\tilde{z} > \hat{\epsilon}} b(X(h) + \tilde{z}))] - b(X(h))\} \end{aligned}$$

where $\bar{w}(x) := b(x) + n\lambda[b(\mu) - b(0)]$ is an upper bound on v_* (as established in the proof of Proposition 1), the second inequality follows from (10) since $[\sigma^i(h)](t) > 0$, and the last since $v_{\sigma,h}(t) \ge b(X(h))$ and, for all $z > 0$, $v_\sigma(h \circ (t, z, i)) \le \bar{w}(X(h) + z)$, and $v_\sigma(h \circ (t, z, i) = b(X(h) + z)$ for $z \ge x_\sigma - X(h)$. Since (13) holds with '$<$' for $x > \hat{x}$, letting $\hat{\epsilon}$ tend to 0 yields $X(h) \le \hat{x}$, so that $x_\sigma \le \hat{x}$, as desired.

E Proofs of Propositions 3 and 4

Proof of Proposition 3. For the first part note that $\mathbb{E}_F[v_f(x + \tilde{z})] \ge v_f(x)$ by (12), since if $\alpha_f(x) = 0$, then $v_f = b$ above x, as α_f is decreasing. For the second part, suppose that $c(a, x) = c(a)$. We shall show that v_f is increasing. recall from the proofs of Theorem 1 and proposition 6 that v_f is the unique fixed point of P_f in $\widehat{V}$, that P_f maps $\widehat{V}$ to itself and is increasing, and that $\widehat{V}$ is a complete lattice (with respect to the pointwise order). Let V' be the set of $v \in \widehat{V}$ that are increasing. It suffices to show that P_f admits a fixed point in V'. Note that V' is a complete lattice. Then, since P_f is increasing on $\widehat{V}$, by Tarski's fixed-point theorem, it suffices to show that P_f maps V' to itself. Fix $v \in V'$. Note that $P_f v \in \widehat{V}$, so that it remains to show that $P_f v$ is increasing.

Since v is increasing and $v - b$ is decreasing, v and, thus, $L_f v$ are continuous. Then, $p(L_f v(x), x)$ and, therefore, $P_f v$, are continuous. Moreover, from Step 3 of the proof of Proposition 6, $p(x, L_f v(x))$ is decreasing in x. Let $I_a := \{x \ge 0 : p(x, L_f v(x)) = a\}$ for $a \in \{0, 1\}$ and $I := \mathbb{R}_+ \backslash (I_0 \cup I_1)$. If $p(x, L_f v(x)) = 0$

for some $x \geq 0$, then I_0 is an interval, and $P_f v = b$ over I_0, so that it is increasing on I_0. If $p(x, L_f v(x)) \in (0,1)$ for some $x \geq 0$, then I is also an interval. Moreover, for any $x \in I$,

$$\lambda[L_f v(x) - P_f v(x)] = \lambda \frac{L_f v(x) - [b(x) - c(a,x)]}{1 + \lambda n a} = c'(a)$$

where $a := p(x, L_f v(x))$, the first equality follows by definition of P_f, and the second from (29). Then $P_f v$ is then increasing on I, as v is increasing and $p(x, L_f v(x))$ is decreasing in x. Finally, if $p(x, L_f v(x)) = 1$ for some $x \geq 0$, then I_1 is an interval, and

$$P_f v(x) = \frac{b(x) - c(1,x) + \lambda n L_f v(x)}{1 + \lambda n}$$

over I. Then, $P_f v$ is increasing on I_1 since $b(x) - c(1,x)$ and v are increasing. Therefore, $P_f v$ is decreasing since I_0, I_1 and I partition $\mathbb{R}_+$, and $P_f v$ is continuous. □

Remark 5. *Suppose that* (??) *holds. Then, innovations are detrimental if (and only if)* v_f *is not increasing on* $[x_0, \infty)$.

Remark 5 holds since v_f is continuous and increasing over the (possibly empty) interval $\{x \geq x_0 : \alpha_f(x) = 0\}$.

Proof. The 'only if' part is immediate, since $\tilde{x}_t$ increases over time. For the 'if' part, suppose that v_f is not increasing on $[x_0, \infty)$. Then, there are $x_0 \leq x' < x''$ such that $v_f(x') > v_f(x'')$.

We claim that we may choose x'' such that $\alpha_f(x'') > 0$. Without loss of generality, we may assume that $\alpha_f(x) = 0$ for some $x \geq 0$. Since α_f is continuous, we may define $x_f := \min\{x \geq 0 : \alpha_f(x) = 0\}$. As α_f is decreasing, $\alpha_f = 0$ over $[x_f, \infty)$, so that v_f matches b on $[x_f, \infty)$. In particular, v_f is increasing on $[x_f, \infty)$ and thus $x' < x_f$. Moreover, we may assume without loss of generality that $x'' \geq x_f$, so that $v_f(x_f) \leq v_f(x'')$. Since v_f is continuous and $v_f(x') > v_f(x'')$, $v_f(x') > v_f(\hat{x}'')$ for $\hat{x}'' \in (x', x_f)$ sufficiently close to x_f. Then, x' and $\hat{x}''$ satisfy the requirements of the claim.

Since v_f and α_f are continuous, there are neighbourhoods U' and U'' of x' and x'' such that $v_f(y') > v_f(y'')$ and $\alpha_f(y'') > 0$ for all $y' \in U'$ and $y'' \in U''$. Since α_f is decreasing, α_f is strictly positive on $[x_0, \sup U')$. Then, from (??), $\Pr\big(\exists t \geq 0, \tilde{x}_t^{\alpha_f} \in U' \big| x_0\big) > 0$, and $\Pr\big(\exists T > t, \tilde{x}_t^{\alpha_f} \in U'' \big| \exists t \geq 0, \tilde{x}_t^{\alpha_f} \in U'\big) > 0$. □

Proof of Proposition 4. We may assume without loss of generality that $c_1(0, x_f) > 0$, for otherwise $b'(x_f) = 0$ by (13), so that (14) holds with equality and there is nothing to prove. By (13), $\alpha_f(x) = (>)\ 0$ for all $x \geq (<)\ x_f$. Then $v_f(x) = x$ for all $x \geq x_f$, and so innovations are detrimental for any initial stock $x_0 < x_f$ if and only if v_f is non-monotone on (x, x_f) for all $x < x_f$, by Remark 5. Hence, it suffices to show that, for any $x < x_f$ sufficiently large and any $\hat{x} \in (x, x_f)$, $v_f(x) < (>)\ v_f(\hat{x})$ if (14) holds with '<' (with '>').

Since α_f is continuous (Theorem 1), $y_f := \max\{x \geq 0 : \alpha_f(x) = 1\} < x_f$. Then, as v_f is also continuous, for any $x \in [y_f, x_f]$,

$$\lambda\{\mathbb{E}_F[v_f(x + \tilde{z})] - v_f(x)\} = c_1(\alpha_f(x), x)$$

by (12). Hence, given $y_f > x > \hat{x} > x_f$, writing $\Delta v := v_f(x) - v_f(\hat{x})$,

$$\lambda(\Delta\mathbb{E} - \Delta v) = c_1(\alpha_f(x), x) - c_1(\alpha_f(\hat{x}), \hat{x}) = \Delta c_1 + c_1(\alpha_f(x), \hat{x}) - c_1(\alpha_f(\hat{x}), \hat{x}) \quad (35)$$

where $\Delta\mathbb{E} := \mathbb{E}_F[v_f(x + \tilde{z}) - b(\hat{x} + \tilde{z})]$ and $\Delta c_1 := c_1(\alpha_f(x), x) - c_1(\alpha_f(x), \hat{x})$. Note that $v_f(x) - b(x) + c(\alpha_f(x), x) = n\alpha_f(x)c_1(\alpha_f(x), x)$ for any $x \in [y_f, x_f]$, by (5). Then, given $y_f < x < \hat{x} < x_f$ such that $c_1(\alpha_f(x), \hat{x}) > c_1(\alpha_f(\hat{x}), \hat{x})$, there are $a, \hat{a} \in [\alpha_f(\hat{x}), \alpha_f(x)]$ such that $c(\alpha_f(x), \hat{x}) - c(\alpha_f(\hat{x}), \hat{x}) = [\alpha_f(x) - \alpha_f(\hat{x})]c_1(a, \hat{x})$ and $c_1(\alpha_f(x), \hat{x}) - c_1(\alpha_f(\hat{x}), \hat{x}) = [\alpha_f(x) - \alpha_f(\hat{x})]c_{11}(\hat{a}, \hat{x})$. Hence, setting $\Delta c := c(\alpha_f(x), x) - c(\alpha_f(x), \hat{x})$,

$$\begin{aligned}\Delta v - b(x) + b(\hat{x}) + \Delta c &= n\alpha_f(x)[c_1(\alpha_f(x), x) - c_1(\alpha_f(\hat{x}), \hat{x})] \\ &\qquad + [nc_1(\alpha_f(\hat{x}), \hat{x}) - c_1(a, \hat{x})][\alpha_f(x) - \alpha_f(\hat{x})] \\ &= n\alpha_f(x)\lambda(\Delta\mathbb{E} - \Delta v) + [nc_1(\alpha_f(\hat{x}), \hat{x}) - c_1(a, \hat{x})]\frac{\lambda(\Delta\mathbb{E} - \Delta v) - \Delta c_1}{c_{11}(\hat{a}, \hat{x})}\end{aligned}$$

where the second equality follows by (35). Then Δv has the sign of

$$\{b(x) - b(\hat{x}) - \Delta c + n\alpha_f(x)\lambda\Delta\mathbb{E}\}c_{11}(\hat{a}, \hat{x}) + [nc_1(\alpha_f(\hat{x}), \hat{x}) - c_1(a, \hat{x})](\lambda\Delta\mathbb{E} - \Delta c_1).$$

Note that, as $x \uparrow x_f$,

$$\frac{b(x) - b(\hat{x})}{x - \hat{x}} \to b'(x_f), \quad \frac{\Delta c}{x - \hat{x}} \to 0, \quad \frac{\Delta\mathbb{E}}{x - \hat{x}} \to \mathbb{E}_F[b'(x_f + \tilde{z})] \ \& \ \frac{\Delta c_1}{x - \hat{x}} \to c_{12}(0, x_f)$$

uniformly for all $\hat{x} \in (x, x_f)$, since $\alpha_f(x) \to 0$. Thus, for $x < x_f$ sufficiently large, $v_f(x) - v_f(\hat{x}) > (<)\ 0$ if (14) holds with '<' ('>').

Finally, suppose that, for all $x < x_f$, there exists $\hat{x} \in (x, x_f)$ such that $c_1(\alpha_f(x), \hat{x}) = c_1(\alpha_f(\hat{x}), \hat{x})$. Then $c_{11}(0, x_f) = 0$ and, by (35), for $x < x_f$ sufficiently large and any $\hat{x} \in (x, x_f)$ such that $c_1(\alpha_f(x), \hat{x}) = c_1(\alpha_f(\hat{x}), \hat{x})$, $v_f(x) >$ $(<)$ $v_f(\hat{x})$ if (14) holds with '$<$' (with '$>$'), as desired. □

F Appendix for Section 7

In this appendix I prove Theorem 2 and proposition 5, and argue that Theorem 2 extends to a setting in which agents can conceal their innovations, provided they cannot secretly refine them (Remark 6).

Proof of Theorem 2. For the first part note that, by Propositions 6 and 7, SSE are precisely the profiles induced by strategies (α_d, χ) such that χ solves (27), and that α_d and v_d inherit the properties of α_f and v_f described in Theorem 1.

I show that $v_d \geq v_f$. From Steps 1 and 2 of the proof of Proposition 6, it suffices to show that the map (P_f, P_d) admits a fixed point in the set $\widetilde{V}$ of pairs $(v, w) \in \widehat{V} \times \widehat{V}$ such that $v \leq w$. From Step 1, $\widetilde{V}$ is a complete lattice and (P_f, P_d) is increasing on $\widetilde{V}$. Then, from Tarski's fixed-point theorem, it suffices to show that (P_f, P_d) maps $\widetilde{V}$ to itself.

Fix $(v, w) \in \widetilde{V}$. From Step 1, $(P_f v, P_d w) \in \widehat{V} \times \widehat{V}$. Moreover, $L_f v \leq L_d w$ since $v \leq w$. Then, $p(x, L_f v(x)) \leq p(x, L_d w(x))$, since $p(x, l)$ is increasing in l (Step 1 of the proof of Proposition 6). Hence, for all $x \geq 0$,

$$\begin{aligned} P_f v(x) = \Gamma(p(x, L_f v(x)), x, L_f v(x)) &\leq \Gamma(p(x, L_f v(x)), x, L_d w(x)) \\ &\leq \Gamma(p(x, L_d w(x)), x, L_d w(x)) = P_d w(x) \end{aligned} \tag{36}$$

where the first inequality holds since $\Gamma(a, x, l)$ is increasing in l, and the second follows from Claim 2, as $p(x, L_f v(x)) \leq p(x, L_d w(x))$. Then $(P_f v, P_d w) \in \widetilde{V}$.

To show that $\alpha_d \geq \alpha_f$, note that $L_d v_d \geq L_f v_f$ since $v_d \geq v_f$. Then, $\alpha_d \geq \alpha_f$ as $\alpha_k(x) = p(x, L_k v_k(x))$ for all $x \geq 0$ and $k \in \{e, d\}$, and $p(x, l)$ is increasing in l.

For the last part, suppose first that innovations are not detrimental. Then, it is clear that the game with disposal admits a SSE that coincides on path with α_f. Then, (α_d, δ_d) coincides with α_f since it is the unique SSE of the game. Suppose now that innovations are detrimental. Let $X_0 = \{x \geq x_0 : \alpha_f(x) > 0$ and $\Pr_F(v_f(x + \tilde{z}) < v_f(x)) > 0\}$ and define $(X_m)_{m=1}^{\infty}$ recursively by $X_m := \{x \geq x_0 : \alpha_f(x) > 0$ and $\Pr_F(x + \tilde{z} \in X_{m-1}) > 0\}$ for all $m \geq 1$. By hypothesis, $x_0 \in X_m$ for some $m \geq 0$. Then, it suffices to show that $v_d > v_f$ over X_m for all $m \geq 0$. I proceed by backward induction on m. For the base case $m = 0$, fix $x \in X_0$ and note that $L_d v_d(x) \geq L_d v_f(x) > L_f v_f(x)$, where the first inequality holds since $v_d \geq v_f$. Then, $v_d(x) > v_f(x)$ by the argument used to derive (36), as $\alpha_f(x) > 0$. For the induction step, fix $m \geq 0$ and suppose that $v_d > v_f$ over X_m. Fix $x \in X_{m+1}$ and note that $L_d v_d \geq L_f v_d > L_f v_f$. Then, $v_d(x) > v_f(x)$ by the argument used to derive (36), as $\alpha_f(x) > 0$. □

Proof of Proposition 5. Write $\beta = \mathbb{E}_F[(b(x_0+\tilde{z}) - W(x)) \vee 0]$ and $\bar{u} = b(x_0) - c(1, x_0)$, fix $\hat{a} > 0$ such that $\lambda\beta > c_1(\hat{a}, x_0)$ and suppose that $n > (W(x) - \bar{u})/(\beta\hat{a})$. It suffices to show that $v_d(x_0) > W(x)$ or, equivalently, $\mathbb{E}_F[(b(x_0+\tilde{z}) - v_d(x_0)) \vee 0] < \beta$. If $\alpha_f(x_0) \leq \hat{a}$ then $\mathbb{E}_F[(b(x_0+\tilde{z}) - v_d(x_0)) \vee 0] \leq \mathbb{E}_F[(v_d(x_0+\tilde{z}) - v_d(x_0)) \vee 0] \leq c_1(\hat{a}, x_0)/\lambda < \beta$ where the second inequality follows from (19). If instead $\alpha_f(x_0) \geq \hat{a}$, supposing towards a contradiction that $W(x) \geq v_d(x)$,

$$\begin{aligned} W(x) \geq v_d(x_0) &= b(x_0) - c(\alpha_f(x_0), x_0) + \lambda n \alpha_f(x_0) \mathbb{E}_F[(v_d(x_0+\tilde{z}) - v_d(x_0)) \vee 0] \\ &\geq \bar{u} + n\hat{a}\beta, \end{aligned}$$

contradicting the hypothesis on n, where the equality follows from (18). □

Consider the following enrichment of the game with disposal. Each agent possesses a (time-varying) set of concealed increments $X_t^i \subset [0, \infty)$, initially empty. Whenever agent i obtains an innovation of size z at time t, the value $x_t + z$ is added to X_t^i (and the stock x_t does not grow). Moreover, each agent i picks a *disclosure* $d_t^i \in \{x \in X_t^i : x > x_t\} \cup \{x_t\}$ whenever she obtains an innovation,[36] and at any time $t \in \mathcal{T}$, where $\mathcal{T} \subset [0, \infty)$ is exogenous and $\mathcal{T} \cap [0, T]$ is finite for all $T > 0$. If a (non-empty) set of agents $I \subseteq \{1, \ldots, n\}$ pick disclosures $(d_t^i)_{i \in I}$ at time t, the public stock x_t rises (weakly) to $\max\{d_t^i : i \in I\}$. Say that agent i *discloses* at time t if $d_t^i > x_t$ and *conceals* if $d_t^i = x_t$. Assume that the arrival and size of innovations as well as the sets X_t^i are private information, and the concealment of innovations is unobserved. However, agents are immediately informed of any innovation that is disclosed, including its size and the identity of the agent disclosing it.

I refer to this model as the *extended game.* (Histories, strategies, information sets, and perfect Bayesian equilibria (PBE) are defined in the usual way.) We recover the game with disposal if we constrain agents to choose $d_t^i \in \{x_t, x_t + z\}$ whenever producing an innovation of size z at time t, and $d_t^i = x_t$ at any other time $t \in \mathcal{T}$. Note that public strategies of the extended game (i.e., strategies that may be expressed as a functions of public histories) coincide with strategies of the game with disposal; this is because they satisfy the aforementioned disclosure constraint and public histories of the extended game coincide with those of the game with

36. If agent i obtains an innovation of size z at time t then she can choose $d_t^i = x_t + z$.

disposal. Given a strategy profile $\xi := (\xi^i)_{i=1}^n$, a belief profile $\beta := (\beta^i)_{i=1}^n$, and a public strategy ξ' of the extended game, (ξ, β) is a *pseudo SSE induced by* ξ' if (ξ, β) is a symmetric PBE and each ξ^i (a) coincides with ξ' at any private history involving no past undisclosed innovations and (b) is a best response against all opponents playing ξ' at any of agent i's information sets induced by β^i.

Remark 6. *The extended game admits a pseudo SSE, and any pseudo SSE is induced by* (α_d, δ_d).

Proof. For the first part, let β be the symmetric belief profile such that, after any private history, each agent believes that opponents' private histories involve no past undisclosed innovations. Note that there exists a symmetric strategy profile ξ of the extended game such that (ξ, β) is a pseudo SSE induced by (α_d, δ_d). This is because (α_d, δ_d) is a best response for each agent i against all opponents playing (α_d, δ_d), at any information set induced by β^i and a private history involving no past undisclosed innovations.

For the second part, let ξ' induce a pseudo SSE. Note that we may view ξ' as a strategy of the game with disposal (as ξ' is public) and that ξ' induces an SSE. Then $\xi' = (\alpha_d, \delta_d)$ by Theorem 2. □