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Partisan Voting Under Uncertainty

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Abstract

We consider a common-value voting model in which voters are uncertain about the precision of the information they receive. With incomplete preference, party supporters adopt their own party as their status quo and vote for it whenever it is justifiable under some belief. Uncertainty is amplified by strategic consideration. As a result, voting becomes fully partisan and party supporters stick to their own party in large elections, even though all voters share the same preference. Additionally, voting is more partisan when voting is compulsory or when the population of party supporters is sufficiently large.

JEL classification: C72, D72, D81

Keywords: Common values, Elections, Information aggregation, Knightian uncertainty, Partisan voting

The most important voting cue is the party label. As Hershey (2017) states in *Party Politics in America*, "for tens of millions of Americans, the party label is a social identity, like that of an ethnic or religious group. It is the chief cue for their decisions about candidates or issues." In a widely cited paper, Bartels (2000) finds that "partisan loyalties had at least as much impact on voting behavior at the presidential level in the 1980s as in the 1950s, and even more in the 1990s than in the 1980s." An update on Bartels's study has confirmed that the level of partisan

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voting in U.S. presidential elections remains quite high (Weinschenk, 2013). Moreover, a sharp increase in party loyalty has been documented in Abramowitz and Webster (2016). In the latest presidential election, Trump received 92% support among Republicans and Republican leaners and Biden received 94% support among Democrats and Democratic leaners. The corresponding numbers for Trump and Clinton in 2016 were both 89% (Igielnik et al., 2021).

Despite the fact that a large proportion of American voters today are reluctant to openly acknowledge any affiliation with a political party, the new American voters are as partisan as ever, if not more so. According to the Gallup Poll, 43 percent of U.S. adults considered themselves independent in 2023 (Jones, 2024). When being pressed about their party preference, however, the picture changes completely. In the 2020 American National Election Studies (ANES) survey, 67 percent of voters identified with either the Democratic or Republican Party in response to the initial party identification question, but only 12 percent of voters fell into the "pure independent" category. Moreover, the population share of strong party identifiers in 2020 was 44 percent, the highest ever recorded in ANES surveys.[1] Scholars also find that leaners are attitudinally and behaviorally very similar to party identifiers (Keith et al., 1992; Magleby et al., 2011; Petrocik, 2009; Smith et al., 1995).

Political scientists have extensively discussed and investigated partisanship and its implication on American politics, especially partisan voting. "In fact, nearly every theory of voting in the American politics literature includes party identification as a critical—if not the only—factor explaining vote choice" (Schaffner and Streb, 2002). The classic conception, proposed in *The American Voter* (Campbell et al., 1960), views party identification as a "perceptual screen", which is formed early in life, remains stable throughout adulthood, and serves as an *unmoved mover* of more specific political attitudes and behavior. A newer conception, pioneered by Fiorina (1981), describes party identification as a *running tally* of retrospective evaluations of party promises and performances. Later scholars argue that partisanship is much more than a running tally and has more fundamental impact on shaping attitudes towards political objects (Bartels, 2002; Green et al., 2002; Iyengar et al., 2019; Levendusky, 2009). More recently, political scientists

[1] In the most recent ANES 2024 Pilot Study, 65 percent of voters are identified with either the Democratic or Republican Party, while 16 percent are identified as pure independents. Additionally, 40 percent of voters are classified as strong party identifiers.

have examined the rise of *negative partisanship* (Abramowitz and Webster, 2016; Bafumi and Shapiro, 2009; Huddy et al., 2015; Iyengar et al., 2012; Mason 2015), which is described as "the strongest potion in politics and media", as one CNN commentator (Stelter, 2020) put it during 2020 presidential election.

Despite being a major theme in the voting theory in political science for decades, economists have often paid far less attention to partisanship and its implication on voting. Party supporters are often simply assumed to be nonstrategic voters who consistently vote for one party (Feddersen and Pesendorfer, 1996; Myatt, 2007; Palfrey and Rosenthal, 1983). These papers assume partisan voting and examine its impact on voting behaviors of the ten percent pure independents, rather than focusing on the voting behaviors of the remaining ninety percent. Other models consider partisan voting as a result of difference in preference intensity (Aragones and Palfrey, 2002; Feddersen and Pesendorfer, 1999; Gul and Pesendorfer, 2009; Krishna and Morgan, 2011), as a result voters who prefer one party are more likely to vote for that party. These paper's perspectives thus align closely with Fiorina's concept of a running tally.

In this paper, we propose an alternative decision mechanism for the use of partisanship that is in line with the concept of an unmoved mover, and discuss how it affects voting and participation decisions. We consider a voting problem under Knightian uncertainty, in which partisanship assists voters in making voting decision when comparing candidates is challenging. We model party supporters as voters who take either party as a status quo choice, which is chosen when candidates are incomparable, while independents do not have such a status quo choice. The role of *status quo* is similar to that of the party cue to a party identifier or a partisan leaner: when voting decision is difficult to make, either because of lack of information (Bulllock, 2011; Downs, 1957) or information overloading (Lau and Redlawsk, 2006; Riggle, 1992), partisans vote according to the party label. This paper shows that the behavioral implication of such partisanship is so persistent that acts as an unmoved mover for party supporters and the persistence could be particularly strong in certain circumstances.[2]

Uncertainty in the voting problem can arise from various factors. In our main model, we

[2] In this paper, we do not attempt to answer why partisans use party cue for their decisions about candidates and issues. Instead, we proceed with the assumption that such behavior occurs and focus on examining how party supporters utilize party cues.

assume that voters are uncertain about the precision of the information they receives, as well as the information received by other voters. We believe that this type of uncertainty is particularly relevant in today's world due to the prevalence of misinformation, disinformation campaigns, party sorting and polarization.[3]

Rational voters are strategic, and they know their votes change the final outcome only when the election is close. A slight imbalance in the vote profile makes it far more likely for a single vote to matter in one state than in another. However, uncertainty about information quality causes almost any vote profile to become imbalanced under some beliefs. In large elections, this uncertainty is amplified through strategic voting, allowing party supporters to justify partisan voting with some belief that favors their own party, a justification often not feasible without uncertainty. Our main result on partisan voting is as follows.

> **Theorem** *In large elections, voting is partisan.*

This result does not depend on information quality. Unless voters have perfect information, private information cannot assure party supporters that the candidate of the other party is better than their own in all circumstances. This result suggests that partisan voting is not necessarily a result of lack of information in a low-information environment, or information overload in a high-information environment. Instead, partisan voting arises from amplified uncertainty due to strategic voting. Thus, we offer here a new perspective on how information environments interact with voters' use of partisanship.

We further consider voluntary participation in our setting. We show that voluntary participation makes voting more responsive, as party supporters may also vote for the other party or abstain. However, even though the possibility of abstention allows party supporters to respond to information, voting remains partisan when the support of the other party is strong enough.

> **Theorem** *In large elections with abstention, voting is partisan when the other party's supporters comprise at least one-third of the electorate.*

When considering both party identifiers and partisan leaners as partisans, the sufficient condition provided in the theorem is always satisfied in the United States in recent decades. Even

[3] For example, Fiorina (2016) states, "the natural consequence of party sorting is that each party gradually comes to have less contact with, knowledge of, and sympathy for the constituencies of the other."

when considering only party identifiers, this condition is often satisfied by Democratics and nearly so by Republicans. For details, see the following figures based on the ANES data.[4]

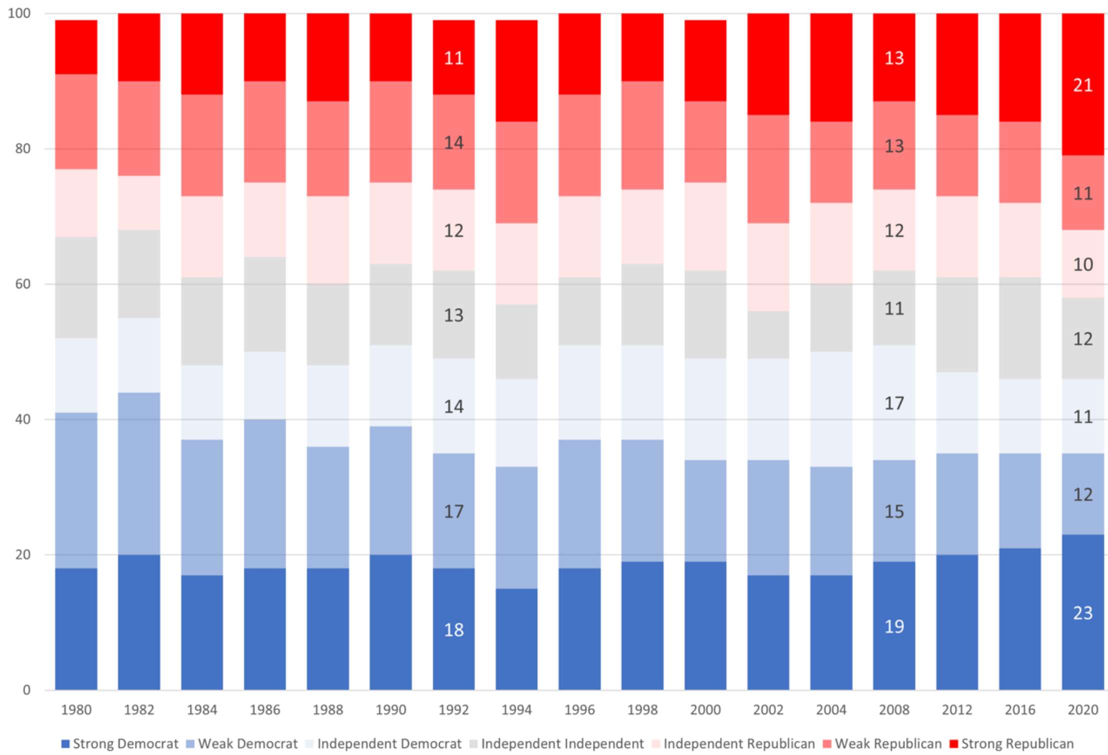

Figure 1: Partisanship, indepedents and partisan leaners, 1980-2020

1 Literature Review

This paper belongs to the common-value voting literature. According to the Condorcet Jury Theorem, elections can achieve the socially optimal choice by aggregating voters' information, regardless of how little an individual voter knows. However, the assumption of truthful voting in the classic Condorcet Jury Theorem has been challenged since Austen-Smith and Banks (1996). Strategic voting has been considered, and it has been shown that information is still aggregated in large elections, except under unanimity rule (Feddersen and Pesendorfer, 1998; McLennan, 1998; Myerson, 1998). Subsequent works have explored important extensions such as endogenous participation (Feddersen and Pesendorfer, 1996; Krishna and Morgan, 2012), costly information (Martinelli, 2006), pre-voting communication (Gerardi and Yariv, 2007).

[4]ANES Time Series studies: https://electionstudies.org/data-center/anes-time-series-cumulative-data-file/.

The expected utility analogue of our model has been studied by Krishna and Morgan (2012). We show that the introduction of uncertainty significantly alters the equilibria of the model. In large elections, party supporters must be partisan and independents are free to mix between the two candidates. In contrast, in Krishna and Morgan (2012), voters vote for one candidate with probability one after receiving one signal and mix between the two candidates after receiving the other.[5]

This paper is closely related to the voting literature considering partisan preference and partisan behavior. In some voting models, party supporters are simply assumed to be non-strategic and stick to a certain party (Feddersen and Pesendorfer, 1996; Myatt, 2007; Palfrey and Prosenthal, 1983). In these models, partisan population is merely a parameter. In some other models, partisan behavior results from difference in preference intensity and there is no behavioral difference between independents and party supporters (Aragones and Palfrey, 2002; Feddersen and Pesendorfer, 1999; Krishna and Morgan, 2011). In these models, partisanship is a descriptive term of voter's responsiveness to their information. In a Downsian setting, Gul and Pesendorfer (2009) consider a form of partisan preference, which they call personality preference. The personality preference they consider is rather weak, as voters only exhibit it when both candidates offer the same policy. However, even weak partisan preference significantly impacts on voting behaviors. In this paper, we also demonstrate that a weak partisan preference can greatly impact voting behaviors through strategic consideration.

This paper contributes to a strand of literature examining how uncertainty affects the voting behavior of voters with non-subjective expected utility (non-SEU) preference. Ghirardato and Katz (2006) discuss the possibility of a voter with maxmin preference choosing abstention to "hedge" against ambiguity but do not consider strategic voting. Ellis (2016) shows that ambiguity aversion can prevent information aggregation because strategic voters with maxmin preference have a strict preference for randomization. Ryan (2019) analyzes Ellis's model under the unanimity rule. In this paper, we consider a situation where voters with incomplete preference and partisanship takes the form of a status quo choice. With maxmin preference, as in Ellis (2016),

[5] Krishna and Morgan (2012) consider only an asymmetric setting in their paper. In the symmetric environment, there exists a truthful equilibrium, as demonstrated in Proposition 1.

expected payoffs are compared using all possible beliefs, leading voters to a strong tendency to randomize to minimize the chance of being pivotal. With incomplete preference, as in this paper, expected payoffs are compared for each possible belief one by one, with indeterminacy resolved by the status quo if present. Consequently, party supporters have a strong tendency to vote for their own party, while independents have no tendency towards any specific voting strategy. Our paper thus demonstrates that the modeling choice of how ambiguity affects behavior has significant consequences.

This paper is also related to the discussion on how small perturbation of the standard voting model could undermine information aggregation within the expected utility framework. The heterogeneity of voters' preferences and uncertainty about voters' preferences can block information aggregation (Bhattacharya, 2007; Feddersen and Presendorfer, 1997; Kim and Fey, 2007). More closely related to the current paper, Mandler (2012) introduces uncertainty about signal likelihoods into a common-value voting model but continues to assume that voters are expected utility maximizers and know the distributions of the signal likelihoods. He shows that information can fail to aggregate even when voters vote responsively to information. In contrast, we assume that voters face Knightian uncertainty. We show that in large elections, party supporters do not vote informatively but there is always an equilibrium in which independents vote informatively, allowing information to aggregate despite the additional uncertainty.

The rest of the paper is organized as follows. Section 2 reviews Knightian decision theory. Section 3 introduces the model. Section 4 discusses truthful voting. Section 5 presents the main results of the paper. Section 6 considers voluntary participation. Section 7 considers uncertainty about prior beliefs. Section 8 concludes. Appendix A contains omitted proofs from the main text. Appendix B relaxes one of the assumptions of the model, as detailed in Section 3.

2 Decision Theory Preliminaries

Before proceeding to the description of our model, we provide here a brief overview of Bewley's (2002) Knightian decision theory, which forms the basics of our model.

2.1 Incomplete Preference and Status Quo

Under uncertainty, completeness is not necessarily a reasonable axiom for individual decision problems. Bewley (2002) develops Knightian decision theory, which relaxes the axiom of completeness.

Under the completeness axiom, individual decision maker is able to rank any pair of alternatives. If preference is incomplete, some alternatives cannot be ranked. Bewley (2002) axiomatizes a model allowing for incompleteness with subjective probabilities.

Consider a finite state space N, the set of all probability distributions over N,

$$\triangle(N) := \left\{ \pi \in \mathbb{R}^N : \forall i = 1, ..., N,\ \pi_i \geq 0 \text{ and } \sum_{i=1}^{N} \pi_i = 1 \right\},$$

and two random monetary payoffs, $x, y \in X^N$, where $X \subseteq \mathbb{R}$ is finite. Bewley characterizes incomplete preference relation represented by a unique nonempty, closed, convex set of probability distributions $\Pi \subseteq \triangle(N)$ and a continuous, strictly increasing, concave function $u : X \to R$, unique up to a positive affine transformation, such that

$$x \succ y \text{ if and only if } \sum_{i=1}^{N} \pi_i u_i(x_i) > \sum_{i=1}^{N} \pi_i u_i(x_i) \text{ for all } \pi \in \Pi. \tag{1}$$

We say x *dominates* y and y is *dominated* by x if $x \succ y$. If neither x nor y is dominated, we say x and y are *incomparable.*

If the set of probabilities Π is a singleton, (1) is equivalent to an expected utility representation, so the ordering is complete. If Π is not a singleton, comparison between two alternatives are done "one probability distribution at a time". A strict preference is obtained only when one alternative is "strictly preferred" to the other unanimously according to all $\pi \in \Pi$.

Because preference is incomplete, a decision maker cannot make up her mind in some situations. By Bewley's maximality assumption, a decision maker does not choose a dominated alternative. Moreover, Bewley's inertia assumption helps to settle a choice among incomparable alternatives. If there is a *status quo*, a decision maker chooses the status quo whenever the status quo is not dominated. Therefore, when x and y are incomparable, decision maker with x (y) as status quo always chooses x (y), while decision maker without a status quo is free to choose x, y

or any distribution over them.

2.2 Party Identity as Status Quo

One difficulty in applying Bewley's inertia assumption is identifying a plausible candidate for the role of status quo. In the case of partisan voting, we find party identity, an "affective orientation" as Campbell et. al. (1960) put it in their classic *The American Voter*, a natural candidate for status quo. For all beliefs, voters compare the two candidates. Party supporters vote for their own party as long as it is not dominated. In other words, to motivate party supporters to vote against their own party, the other candidate must be clearly better than their own. When voting decision is difficult, voters resort to party cue to make their choice.

We also introduce *independents* into our model, who do not have a status quo. The absence of a status quo choice results in the indeterminacy of voting behaviors when the two candidates are incomparable. We do not make any assumption about the voting behaviors of independents under indeterminacy.

3 Model

There are two party candidates, $c \in \{A, B\}$, and an electorate of random size. Each voter of the electorate receives information about the state of the world $\theta \in \{\alpha, \beta\}$, and votes simultaneously for one of the two candidates. The election outcome $d \in \{A, B\}$ is determined by the simple majority rule, with ties decided by the toss of a fair coin. Let v_i denote voter i's vote. In the baseline model, no abstention is allowed, i.e., $v_i \in \{A, B\}$. In Section 6, we consider voluntary participation and abstention is allowed, i.e., $v_i \in \{A, B, \phi\}$, where ϕ denotes abstention. The common prior probability that the state is α is $p \in (0, 1)$.

Electorate. The size of the electorate is a random variable that follows the Poisson distribution with mean n. The probability that there are m voters is $e^{-n}\frac{n^m}{m!}$. After the size of electorate is drawn, voters' party identities are determined randomly. There are three types of voters, $t \in \{A, B, I\}$: party supporters of party A ($t = A$), party supporters of party B ($t = B$), and independents ($t = I$). Type A voters take candidate A as their status-quo choice, type B voters take candidate B as their status-quo choice, and independents have no status quo choice. A voter is type A, type B and type I with probability λ_A, λ_B, and λ_I, respectively, where $\lambda_A, \lambda_B, \lambda_I > 0$. We assume that the partisan population is not too large so that no party can secure a win only with its own base, i.e., $\lambda_A, \lambda_B < \frac{1}{2}$.

Payoffs. Voter payoffs are identical and depend on the election outcome d and the state θ. In state α (β), voters get a payoff of 1 if candidate A (B) is elected and -1 if candidate B (A) is elected.

Information. Before voting, each voter receives a private signal $s \in \{a, b\}$ regarding the state θ. Denote the probability of receiving signal a in state α by q_α, and the probability of receiving signal b in state β by q_β. Voters are uncertain about the information precision in each state, i.e., for each $\theta \in \{\alpha, \beta\}$, $q_\theta \in [\underline{q}_\theta, \overline{q}_\theta]$, where $\underline{q}_\theta, \overline{q}_\theta \in (\frac{1}{2}, 1)$.[6] Let $\Gamma := [\underline{q}_\alpha, \overline{q}_\alpha] \times [\underline{q}_\theta, \overline{q}_\theta]$. We further assume that there is some $(q_\alpha, q_\beta) \in \Gamma$ such that $q_\alpha = q_\beta$. The idea is that we mainly consider an information environment, in which signal asymmetry is not too large.[7] We say that there is *no uncertainty* when $\Gamma = \{(q, q)\}$ for some $q \in (\frac{1}{2}, 1)$. We also say that voters are *confident* if voting is truthful for all types in the hypothetical situation where a single voter's vote determines the outcome of the election.[8] Given $(q_\alpha, q_\beta) \in \Gamma$, let $l(s; q_\alpha, q_\beta)$ denote the posterior likelihood ratio of the state conditional on signal s, i.e.,

$$l(s; q_\alpha, q_\beta) = \begin{cases} \frac{p}{1-p} \frac{q_\alpha}{1-q_\beta} & \text{if } s = a, \\ \frac{p}{1-p} \frac{1-q_\alpha}{q_\beta} & \text{if } s = b. \end{cases}$$

Voters are confident if

$$l(b; q_\alpha, q_\beta) < 1 < l(a; q_\alpha, q_\beta) \tag{2}$$

for all $(q_\alpha, q_\beta) \in \Gamma$.

Strategy. Let $\gamma_c^t(s)$ be the probability that a type t voter votes for candidate c given signal s. Let $\gamma^t(s) := \left(\gamma_A^t(s), \gamma_B^t(s)\right)$. When abstention is allowed, we write $\gamma^t(s) := (\gamma_A^t(s), \gamma_\phi^t(s), \gamma_B^t(s))$,

[6] This means that a voter never confuses whether a signal is more indicative of state α or state β, and, in each state, a voter receives the "correct" signal with a probability strictly higher than 1/2, regardless of the precision level used for the calculation.

[7] This assumption immediately implies signal symmetry under no uncertainty. We discuss the effect of relaxing this assumption in Appendix B.

[8] The terminology is not standard and is inspired by Ellis (2016), who says that the voters *lack confidence* if, assuming the outcome of the election is determined by a single vote, the candidates are incomparable for all $s \in \{a, b\}$. Clearly, a voter who is confident according to our definition does not lack confidence in the sense of Ellis (2016).

where $\gamma_{\phi}^{t}(s)$ is the probability of abstention. A strategy for type t is given by the profile $\gamma^{t} := \left(\gamma^{t}(a), \gamma^{t}(b)\right)$. Voting is *truthful* for type t if $\gamma_{A}^{t}(a) = 1$ and $\gamma_{B}^{t}(b) = 1$. Voting is *responsive* for type t if $\gamma^{t}(a) \neq \gamma^{t}(b)$. Otherwise, voting is *nonresponsive*. Voting is *partisan* for type t if, for both $s \in \{a, b\}$, $\gamma_{c}^{t}(s) = 1$ if $t = c$.[9] We also say that type t is partisan if voting is partisan for type t.

Equilibrium. An equilibrium of our game consists of a strategy profile $\gamma := \left(\gamma^{A}, \gamma^{I}, \gamma^{B}\right)$, where, for each $s \in \{a, b\}$, all voters vote for the dominant candidate if there is one, and party supporters vote for their own party candidate if no candidate is dominant. An equilibrium is *truthful* if voting is truthful for *all* three types. An equilibrium is *responsive* if voting is responsive for *some* type. An equilibrium is *fully responsive* if voting is responsive for *all* types. An equilibrium is *fully partisan* if voting is partisan for both type A and type B. An equilibrium is *partially partisan*, if voting is partisan for either type A or B, but not both. A sequence of equilibria is an infinite sequence of strategy profiles $\{\gamma_n\}_{n \geq N}$, such that for each $n \geq N$, γ_n is an equilibrium given the electorate size n. Equilibria *in large elections* satisfy a property if there exists an $N > 0$, such that, if $\{\gamma_n\}_{n \geq N}$ is a sequence of equilibria, then for each $n \geq N$, γ_n satisfies that property.

3.1 Pivotal Events

An elementary event is a singleton consisting of a pair of vote totals (k, l), where k is the number of votes for party candidate A and l the votes for party candidate B. An event is an union of elementary events. An elementary event is *pivotal* if a single vote can affect the final outcome of the election. There are two types of elementary events where a vote for candidate A can have an effect on the final outcome: 1) there is a tie, and 2) party candidate A has one vote less or more than party candidate B. Let $T := \{(k, k) : k \geq 0\}$ denote the event that there is a tie, and let $T_{-1} := \{(k-1, k) : k \geq 1\}$ denote the event that A has one vote less than B, and let $T_{+1} := \{(k, k-1) : k \geq 1\}$ denote the event that A has one vote more than B. The event piv_A (pivotal if vote for A) is defined by $piv_A := T \cup T_{-1}$. The event piv_B is defined by $piv_B := T \cup T_{+1}$.

[9] An alternative definition for partisan voting is as follows: voting is partisan for type t if, for both $s \in \{a, b\}$, $\gamma_{c}^{t}(s) = 0$ if $c \in \{A, B\}$ and $c \neq t$. Without abstention, these two definitions are equivalent. With abstention, the alternative definition is less restrictive, so all results with regarding to partisan voting when abstention is allowed still hold.

For information precision $(q_\alpha, q_\beta) \in \Gamma$, let σ_A and σ_B be the probabilities that a random voter votes for A and B in state α, respectively. τ_A and τ_B are defined similarly for the probabilities in state β. Note that if abstention is not allowed, then $\sigma_A + \sigma_B = 1$ and $\tau_A + \tau_B = 1$; if abstention is allowed, then $\sigma_A + \sigma_B \leq 1$ and $\tau_A + \tau_B \leq 1$.

Consider the event that the size of the realized electorate is m and there are k votes in favor of party candidate A and l votes in favor of party candidate B. The number of abstentions is thus $m - k - l$. The probability of such an event in state α is

$$\Pr[\{(k,l)\}\,|\alpha] = e^{-n(\sigma_A+\sigma_B)} \frac{(n\sigma_A)^k}{k!} \frac{(n\sigma_B)^l}{l!}.$$

The probability of a tie in state α is

$$\Pr[T|\alpha] = e^{-n(\sigma_A+\sigma_B)} \sum_{k=0}^{\infty} \frac{(n\sigma_A)^k}{k!} \frac{(n\sigma_B)^k}{k!},$$

while the probability that candidate A has one vote less than candidate B in state α is

$$\Pr[T_{-1}|\alpha] = e^{-n(\sigma_A+\sigma_B)} \sum_{k=1}^{\infty} \frac{(n\sigma_A)^{k-1}}{(k-1)!} \frac{(n\sigma_B)^k}{k!},$$

and the probability that candidate B has one vote less than candidate A in state α is

$$\Pr[T_{+1}|\alpha] = e^{-n(\sigma_A+\sigma_B)} \sum_{k=1}^{\infty} \frac{(n\sigma_A)^k}{k!} \frac{(n\sigma_B)^{k-1}}{(k-1)!}.$$

Suppose $\lim_{n\to\infty} \sigma_A > 0$ and $\lim_{n\to\infty} \sigma_B > 0$, then, when n is large enough, these probabilities can be approximated by

$$\Pr[T|\alpha] \approx \frac{e^{-n\left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2}}{\sqrt{4\pi n\sqrt{\sigma_A\sigma_B}}}, \tag{3}$$

$$\Pr[T_{\pm m}|\alpha] \approx \left(\sqrt{\frac{\sigma_A}{\sigma_B}}\right)^{\pm m} \Pr[T|\alpha], \tag{4}$$

where m is an integer.[10] The corresponding probabilities in state β are obtained by substituting

[10]For details, see Myerson (2000) and Krishna and Morgan (2011, 2012).

σ for τ.

Define the pivotal ratio of the two states $\Omega_{piv}(q_\alpha, q_\beta, n, \gamma)$ by

$$\Omega_{piv}(q_\alpha, q_\beta, n, \gamma) := \frac{\Pr[piv_A|\alpha] + \Pr[piv_B|\alpha]}{\Pr[piv_A|\beta] + \Pr[piv_B|\beta]}.$$

Given $(q_\alpha, q_\beta) \in \Gamma$, let $U(c, s; q_\alpha, q_\beta)$ denote the expected payoff to a voter with an s signal from voting for candidate c. We have

$$U(A, s; q_\alpha, q_\beta) \gtreqless U(B, s; q_\alpha, q_\beta) \Longleftrightarrow l(s; q_\alpha, q_\beta)\, \Omega_{piv}(q_\alpha, q_\beta, n, \gamma) \gtreqless 1. \tag{5}$$

Using the approximations in (3) and (4), we conclude that

$$\Omega_{piv}(q_\alpha, q_\beta, n, \gamma) \approx \sqrt[4]{\frac{\tau_A \tau_B}{\sigma_A \sigma_B}} e^{n\left[\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2 - \left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2\right]} \left[\frac{2 + \left(\frac{\sigma_A}{\sigma_B}\right)^{\frac{1}{2}} + \left(\frac{\sigma_A}{\sigma_B}\right)^{-\frac{1}{2}}}{2 + \left(\frac{\tau_A}{\tau_B}\right)^{\frac{1}{2}} + \left(\frac{\tau_A}{\tau_B}\right)^{-\frac{1}{2}}}\right], \tag{6}$$

when n is large enough, provided that σ_A, σ_B, τ_A and τ_B are bounded away from 0.

4 Truthful Voting

Before presenting the main result, we discuss the existence of a truthful equilibrium in the absence of uncertainty. This illustrates the key difference in voting behaviors between voters with incomplete preference and those with SEU preference in large elections.

By (5) and Bewley's maximality and inertia assumptions, in order to have all three types vote truthfully in equilibrium, we must have

$$l(b; q_\alpha, q_\beta)\, \Omega_{piv}(q_\alpha, q_\beta, n, \gamma) < 1 < l(a; q_\alpha, q_\beta)\, \Omega_{piv}(q_\alpha, q_\beta, n, \gamma) \tag{7}$$

for all $(q_\alpha, q_\beta) \in \Gamma$.

When voting is truthful, the probabilities that a random draw results in a vote for candidate A and B in state α are q_α and $1 - q_\alpha$, respectively. The corresponding probabilities in state β

are $1-q_\beta$ and q_β, respectively. Therefore,

$$\begin{cases} \sigma_A = q_\alpha, \\ \sigma_B = 1-q_\alpha, \end{cases} \quad \text{and} \quad \begin{cases} \tau_A = 1-q_\beta, \\ \tau_B = q_\beta. \end{cases}$$

Consider $(q_\alpha, q_\beta) \in \Gamma$ such that $q_\alpha = q_\beta$, then $\sigma_A = \tau_B$ and $\sigma_B = \tau_A$. This means

$$\Omega_{piv}(q_\alpha, q_\beta, n, \gamma) = \frac{2e^{-n}\sum\limits_{k=0}^{\infty}\frac{(n\sigma_A)^k}{k!}\frac{(n\sigma_B)^k}{k!} + e^{-n}\sum\limits_{k=1}^{\infty}\frac{(n\sigma_A)^{k-1}}{(k-1)!}\frac{(n\sigma_B)^k}{k!} + e^{-n}\sum\limits_{k=1}^{\infty}\frac{(n\sigma_A)^k}{k!}\frac{(n\sigma_B)^{k-1}}{(k-1)!}}{2e^{-n}\sum\limits_{k=0}^{\infty}\frac{(n\tau_A)^k}{k!}\frac{(n\tau_B)^k}{k!} + e^{-n}\sum\limits_{k=1}^{\infty}\frac{(n\tau_A)^{k-1}}{(k-1)!}\frac{(n\tau_B)^k}{k!} + e^{-n}\sum\limits_{k=1}^{\infty}\frac{(n\tau_A)^k}{k!}\frac{(n\tau_B)^{k-1}}{(k-1)!}} = 1.$$

If there is no uncertainty, then, $\Omega_{piv}(q_\alpha, q_\beta, n, \gamma) = 1$ for all $(q_\alpha, q_\beta) \in \Gamma$. (7) reduces to (2). Thus, in this case, truthful voting is an equilibrium in large elections if and only if (2) holds.

Next, suppose there is uncertainty. Since $\sigma_A + \sigma_B = \tau_A + \tau_B = 1$, by (6), when n is large, we have

$$\Omega_{piv}(q_\alpha, q_\beta, n, \gamma) \approx e^{2n\left(\sqrt{\sigma_A\sigma_B} - \sqrt{\tau_A\tau_B}\right)} K(\sigma_A, \sigma_B, \tau_A, \tau_B),$$

where K is a function that is strictly positive and does not depend on n. Consider $q_\alpha > q_\beta$, then

$$\sqrt{\sigma_A\sigma_B} = \sqrt{q_\alpha(1-q_\alpha)} < \sqrt{q_\beta(1-q_\beta)} = \sqrt{\tau_A\tau_B},$$

which means that $\Omega_{piv}(q_\alpha, q_\beta, n, \gamma) \to 0$ as $n \to \infty$. It follows that, for n large enough, the payoff of voting for candidate A evaluated at such (q_α, q_β) is strictly lower than voting for candidate B. This means that type B voters would not vote for candidate A after receiving signal a. Similarly, if there exists some $(q_\alpha, q_\beta) \in \Gamma$ such that $q_\alpha < q_\beta$, type A voters would not vote for candidate B after receiving signal b. As a result, there could not be a truthful equilibrium in large elections. Summarizing our discussion, we conclude that

Proposition 1 (No Truthful Voting) *In large elections, truthful voting is an equilibrium if and only if there is no uncertainty and voters are confident.*

From the argument above, it is clear that even the smallest amount of uncertainty in precision would be amplified in large elections so rapidly that overwhelms any private information, no matter how precise it is. For any voting game where voters are confident, we can always find a large enough electorate such that truthful voting does not constitute an equilibrium.

When voting is strategic, voters decide how to vote based on their private information and conditional on being pivotal. Through the evaluation of pivotal events, uncertainty is amplified. There is some belief under which being pivotal is far more likely in state α, and another belief under which being pivotal is far more likely in state β. As a consequence, the two candidates become incomparable, leading party supporters to vote for their own party. This effectively rules out truthful voting. We focus on voting environments with uncertainty hereafter.

5 Partisan Voting

If voting is not truthful in large elections, how do voters vote? In this section, we first characterize some useful properties of equilibrium voting behaviors. Then, we show that voting is always partisan in large elections. Finally, we examine the voting behaviors of independents in large elections and discuss whether information aggregates.

Lemma 1 *In equilibrium,*

1. *Party supporters do not mix: for* $t \in \{A, B\}$, $c \in \{A, B\}$ *and* $s \in \{a, b\}$, $\gamma_c^t(s) \in \{0, 1\}$;
2. *Voting is monotone in types: (i)* $\gamma_A^A(s) = 0$ *implies* $\gamma_A^I(s) = \gamma_A^B(s) = 0$, *and* $\gamma_B^B(s) = 0$ *implies* $\gamma_B^I(s) = \gamma_B^B(s) = 0$; *(ii)* $\gamma_A^I(s) > 0$ *implies* $\gamma_A^A(s) = 1$ *and* $\gamma_B^I(s) > 0$ *implies* $\gamma_B^B(s) = 1$.
3. *Voting is monotone in signals for party supporters: for both* $t \in \{A, B\}$, $\gamma_A^t(a) \geq \gamma_A^t(b)$ *and* $\gamma_B^t(a) \leq \gamma_B^t(b)$.

The first part of Lemma 1 follows directly from Bewley's maximality and inertia assumptions: party supporters always vote for their own party unless it is dominated. Moreover, when one candidate is dominated, mixing between the two candidates leads to a lower payoff under all possible beliefs. This means that when abstention is not allowed, party supporters would only use pure strategies. This is not the case when abstention is allowed, see Section 6.

Secondly, if a party's supporters are willing to vote against their own party, then everybody else must be willing to do so as well. Similarly, if independents are willing to vote for a party, then supporters of that party must be willing to do so as well. Thus, voting is monotone in types.

Finally, for party supporters, voting is monotone in signals. Intuitively, if a party's supporters are willing to vote for a candidate given an unfavorable signal, they must be willing to do so given a favorable signal. However, this is not necessarily the case for independents. The two candidates could be incomparable under both signals. Since independents do not have a status quo, they are free to choose either candidate or mix between them. This indeterminacy allows for some counterintuitive behaviors, such as voting for a candidate given an unfavorable signal but voting against the same candidate given a favorable signal.

By Lemma 1, for party supporters, voting is either truthful or nonresponsive. By Proposition 1, truthful voting is not an equilibrium in large elections under uncertainty. The following theorem shows that voting is fully partisan in large elections.

Theorem 1 (Partisan Voting) *In large elections, any responsive equilibrium is fully partisan.*

Proof. By Proposition 1, there is no truthful equilibrium in large elections. By Lemma 1, this implies any responsive equilibrium must be at least partially partisan. Without loss of generality, suppose type A always votes for candidate A and type B votes truthfully. By Lemma 1, we have

$$\begin{cases} \gamma^A(a) = (1,0), \\ \gamma^I(a) = (1,0), \\ \gamma^B(a) = (1,0), \end{cases} \quad \text{and} \quad \begin{cases} \gamma^A(b) = (1,0), \\ \gamma^I(b) = \left(\gamma^I_A(b), 1-\gamma^I_A(b)\right), \\ \gamma^B(b) = (0,1), \end{cases}$$

where $\gamma^I_A(b)$ can take any value between 0 and 1. Given such a strategy profile, a random voter with signal a votes for candidate A with probability one, and a random voter with signal b votes for candidate A with probability $\lambda_A + \lambda_I \gamma^I_A(b)$. Therefore, we have

$$\begin{cases} \sigma_A = q_\alpha + (1-q_\alpha)\left(\lambda_A + \lambda_I \gamma^I_A(b)\right), \\ \sigma_B = (1-q_\alpha)\left(\lambda_I\left(1-\gamma^I_A(b)\right) + \lambda_B\right), \end{cases}$$

and

$$\begin{cases} \tau_A = 1 - q_\beta + q_\beta \left(\lambda_A + \lambda_I \gamma_A^I(b)\right), \\ \tau_B = q_\beta \left(\lambda_I \left(1 - \gamma_A^I(b)\right) + \lambda_B\right). \end{cases}$$

Consider some $(q_\alpha, q_\beta) \in \Gamma$ such that $q_\alpha = q_\beta = q$. We have

$$\begin{aligned} \sigma_A \sigma_B - \tau_A \tau_B &= \left(q + (1-q)\left(\lambda_A + \lambda_I \gamma_A^I(b)\right)\right)(1-q)\left(\lambda_I\left(1 - \gamma_A^I(b)\right) + \lambda_B\right) \\ &\quad - \left(1 - q + q\left(\lambda_A + \lambda_I \gamma_A^I(b)\right)\right) q \left(\lambda_I\left(1 - \gamma_A^I(b)\right) + \lambda_B\right) \\ &= (1-2q)\left(\lambda_A + \lambda_I \gamma_A^I(b)\right)\left(\lambda_I\left(1 - \gamma_A^I(b)\right) + \lambda_B\right) \\ &< 0. \end{aligned}$$

Since $\sigma_A + \sigma_B = \tau_A + \tau_B = 1$, by (6), when n is large, we have

$$\Omega_{piv}(q_\alpha, q_\beta, n, \gamma) \approx \sqrt[4]{\frac{\tau_A \tau_B}{\sigma_A \sigma_B}} e^{2n\left(\sqrt{\sigma_A \sigma_B} - \sqrt{\tau_A \tau_B}\right)} \left[\frac{2 + \left(\frac{\sigma_A}{\sigma_B}\right)^{\frac{1}{2}} + \left(\frac{\sigma_A}{\sigma_B}\right)^{-\frac{1}{2}}}{2 + \left(\frac{\tau_A}{\tau_B}\right)^{\frac{1}{2}} + \left(\frac{\tau_A}{\tau_B}\right)^{-\frac{1}{2}}}\right],$$

which goes to 0, as $n \to \infty$. Thus, for all $(q_\alpha, q_\beta) \in \Gamma$ such that $q_\alpha = q_\beta$, it is infinitely more likely to be pivotal in state β than in state α. As a result, given signal a, candidate A does not dominate candidate B. Since candidates A and B are incomparable, type B voters would not vote for candidate A after receiving signal a, which contradicts the assumed strategy profile. ■

This result on partisan voting is independent of signal precision. What drives partisan voting is not the amount of information available, but the uncertainty present in the evaluation of the pivotal event. This suggests that partisan voting could occur in both low-information and high-information elections. Moreover, even the smallest difference in signal precision across states is sufficient for partisan voting to arise when the electorate is large enough.

Note that it is not the uncertainty about one's own private information quality, but the uncertainty about others' information quality that drives this result. Thus, even a well-informed and experienced voter is not immune to partisan voting. Moreover, it is precisely the highly sophisticated voters who are prone to partisan voting. Naive voters, who do not realize that their votes would affect the outcome only when the election is very close, are unaffected by amplified uncertainty.

Next, we look at full partisan equilibrium. Does the election select the right candidate? Before we can answer this question, we need to define how information aggregation should be understood in our setting. The complication arises because, under certainty, different precision levels can lead to different winning probabilities.

Definition 1 (Information Aggregation) *A sequence of equilibria $\{\gamma_n\}_{n \geq N}$ aggregates information in large elections if for all $\varepsilon > 0$, there exists $N' \geq N$ such that for all $(q_\alpha, q_\beta) \in \Gamma$ and for all $n \geq N'$, $\Pr[d = B|\alpha] < \varepsilon$ and $\Pr[d = A|\beta] < \varepsilon$.*

Note that how voters vote depends on the signals they receive. Thus, the expected number of votes for a candidate in a given state depends on the probabilities of each signal being received, and whether information aggregates depends on the information precision (q_α, q_β) used for the evaluation. Definition 1 requires that the probabilities $\Pr[d = B|\alpha]$ and $\Pr[d = A|\beta]$ go to zero for all $(q_\alpha, q_\beta) \in \Gamma$.

A necessary condition for information aggregation is that equilibria must select the correct expected winners when the size of the electorate is large.

Definition 2 (Correct Expected Winners) *A strategy profile γ has the correct expected winners if*

$$\sigma_A > \sigma_B \text{ and } \tau_A < \tau_B$$

for all $(q_\alpha, q_\beta) \in \Gamma$.[11]

It is evident that having correct expected winners is necessary for information aggregation. Furthermore, if the two inequalities remain strict in the limit for a sequence of equilibria, this condition is also sufficient for information aggregation.

We show that information aggregates in large elections.

Proposition 2 *In large elections, there exists a sequence of full partisan equilibria that aggregates information.*

Proposition 2 suggests that the presence of a sizable partisan population does not preclude information aggregation. While party supporters do not contribute to information aggregation,

[11] The definition here follows Ellis (2016).

it is still achievable as long as the independent population is large enough such that some votes from independents are needed to win an election, which is guaranteed by our assumption.

With the possibility result for information aggregation established, we also want to examine how truthful voting performs in large elections, as truthful voting is particularly desirable due to its simplicity. When do independents vote truthfully, and does information aggregate when independents vote truthfully? We have

Proposition 3 *In large elections, there exists an equilibrium in which independents vote truthfully if* $\frac{\lambda_A-\lambda_B}{\lambda_I} \in (\underline{q}_\beta - \overline{q}_\alpha, \overline{q}_\beta - \underline{q}_\alpha)$.

Given the amount of uncertainty and the difference in partisan population, this condition suggests that independents are more likely to vote truthfully when their numbers are more substantial. When independents are a small minority, they might need to adjust their voting behavior to compensate for partisan voting. However, as the independent population increases, they can mitigate the impact of partisan voting and vote truthfully.

It is also worthwhile to mention that truthful voting of independents does not reply on the quality of information but on the independents' incomplete preference due to the presence of uncertainty. Even when voters do not receive information that is precise enough to counteract biased priors, i.e., voters are not confident, independents can still vote truthfully in equilibrium. This occurs because uncertainty renders the two candidates incomparable. In comparison to the amplified uncertainty caused by strategic consideration, the impact of biased prior is almost negligible. Independents are more likely to vote truthfully when there is a sufficient amount of uncertainty, rather than the other way around.[12]

However, information does not necessarily aggregate when independents vote truthfully. Our definition of information aggregation requires choosing the right candidates *under all beliefs*. Due to the presence of partisans, for some belief, truthful voting by independents results in excessive votes for the right candidate in one state and not enough votes in the other. Consider the following example.

[12] The condition we get on the relative partisan population, $\frac{\lambda_A-\lambda_B}{\lambda_I} \in (\underline{q}_\beta - \overline{q}_\alpha, \overline{q}_\beta - \underline{q}_\alpha)$, may not be very restrictive. According to the most recent Gallup poll of U.S. party affiliation, $\frac{\lambda_A-\lambda_B}{\lambda_I}$ is roughly around 0.044. This estimate is based on data from April 1 to April 22, 2024, which reports the following: Republicans at 27%, Independents at 45%, and Democrats at 25%. Note that Independents include partisan leaners in Gallup's data.

Example 1 (Truthful Voting and Failure of Information Aggregation) *Suppose* $p = \frac{1}{2}$, $\lambda_A = 0.25$, $\lambda_I = 0.4$, $\lambda_B = 0.35$ *and* $\Gamma = [0.6, 0.9]^2$. *There exists a full partisan equilibrium in which independents vote truthfully, because* $\frac{\lambda_A - \lambda_B}{\lambda_I} = -\frac{1}{4} \in (-0.3, 0.3)$. *The expected winner in state* α *is not correct evaluated at the precision level* $q_\alpha = 0.6$, *because* $\sigma_A = 0.25 + 0.4 \times 0.6 = 0.49 < 0.5$.

6 Voluntary Participation

In the previous section, we demonstrate that when abstention is not allowed, voting under certainty is fully partisan. While compulsory voting is practiced in some countries, it is far from universally adopted. Moreover, voluntary participation has been shown to help voters to select the correct candidate, as participation decisions factor in preference intensity (Feddersen and Pesendorfer, 1999; Krishna and Morgan, 2012) and information quality (Feddersen and Pesendorfer, 1996). A natural question, therefore, is how our results might change when abstention is allowed. In this section, we consider this extension and illustrate that while the added flexibility introduced by abstention leads to a new set of equilibria, voting remains fully partisan in large elections under a wide range of empirically plausible estimates of partisan populations.

We first establish some basic properties of equilibrium in the voting game with abstention in the form of Lemma 2, which is a generalization of Lemma 1.

Lemma 2 *Consider an election with abstention. In equilibrium,*

1. *Party supporters do not mix between voting for own party and the other options:* $\gamma_A^A(s) > 0$ *implies* $\gamma_A^A(s) = 1$, *and* $\gamma_B^B(s) > 0$ *implies* $\gamma_B^B(s) = 1$;

2. *Voting is monotone in types: (i)* $\gamma_A^A(s) = 0$ *implies* $\gamma_A^I(s) = \gamma_A^B(s) = 0$, *and* $\gamma_B^B(s) = 0$ *implies* $\gamma_B^I(s) = \gamma_B^B(s) = 0$; *(ii)* $\gamma_A^I(s) > 0$ *implies* $\gamma_A^A(s) = 1$ *and* $\gamma_B^I(s) > 0$ *implies* $\gamma_B^B(s) = 1$.

3. *Voting is monotone in signals for party supporters regarding own party: 1)* $\gamma_A^A(a) = 0$ *implies* $\gamma_A^A(b) = 0$; *2)* $\gamma_B^B(b) = 0$ *implies* $\gamma_B^B(a) = 0$.

The three characteristics outlined in Lemma 2 echo those in Lemma 1. Importantly, the introduction of abstention now allows for mixed strategies among party supporters. This significantly enlarges the set of potential equilibrium strategy profiles. How would voting behaviors change?

Our next two corollaries show that the basic insights of the previous sections remain valid. First, assuming voting is truthful, then no voter abstains. This means that if truthful voting cannot be supported as an equilibrium in the voting game without abstention, neither can it be an equilibrium in the voting game with abstention. Thus, by Proposition 1, we have

Corollary 1 *In large elections with abstention, voting is not truthful.*

The next corollary follows from Proposition 2 by verifying that the equilibrium constructed there remains an equilibrium in the voting game with abstention. This is because the equilibrium in Proposition 2 is constructed in a way such that, as the electorate size n goes to infinity, being pivotal in state α becomes infinitely more likely than being pivotal in state β under some belief, and vice visa under some other belief. This means that under these beliefs, there is a better choice than abstention. Therefore, abstention is not a dominant choice. As a result, the maximality assumption does not require any voter to abstain.

Corollary 2 *In large elections with abstention, there exists a sequence of full partisan equilibria that aggregates information.*

Given Corollaries 1 and 2, one may suspect whether abstention would change the equilibrium voting behaviors in any way in large elections. Our next example shows that the added flexibility offered by abstention indeed allows party supporters to behave more responsively under certain circumstances.

Example 2 *Suppose* $\lambda_A = 0.1$*,* $\lambda_I = 0.5$*,* $\lambda_B = 0.4$ *and* $\Gamma = \{(0.8)\} \times [0.6, 0.8]$*. For any* $p \in (0,1)$ *and* n *large enough, there exists a partial partisan equilibrium of the form*

$$\begin{cases} \gamma^A(a) = (1,0,0), \\ \gamma^I(a) = (\eta^n, 1-\eta^n, 0), \\ \gamma^B(a) = (\eta^n, 1-\eta^n, 0), \end{cases} \quad \text{and} \quad \begin{cases} \gamma^A(b) = (1,0,0), \\ \gamma^I(b) = (0,0,1), \\ \gamma^B(b) = (0,0,1), \end{cases} \tag{8}$$

where $\lim_{n\to\infty}\eta^n \approx 0.44569$ *is the solution of*

$$\sqrt{0.54} - \sqrt{0.1 + 0.36\eta} = \sqrt{0.1 + 0.72\eta} - \sqrt{0.18}.$$

Example 2 shows that the option to abstain enables party supporters to vote responsively, in contrast to the scenario under compulsory voting (Theorem 1). Remarkably, supporters of party B now not only may abstain with positive probability but also vote for party A with positive probability. While full partisan equilibria survive in large elections with abstention, abstention does introduce a new set of equilibria to the model. Nevertheless, we will show next that under a wide range of empirically plausible estimates for the partisan populations λ_A and λ_B, full partisan voting remains the only possible equilibrium outcome in large elections if the election is expected to select the correct candidates. We have

Theorem 2 (Partisan Voting with Abstention) *In large elections with abstention,*

1. *If both candidates receive positive votes with positive probability in equilibrium, voting is partisan for at least one partisan type;*[13]

2. *If the equilibrium has correct expected winners, for* $t, t' \in \{A, B\}$ *and* $t \neq t'$*, voting is partisan for type* t *if* $\lambda_{t'} \geq \frac{1}{3}$.

This result consists of two parts. The first part states that if both candidates receive some votes in equilibrium, then supporters of at least one of the two parties must be partisan, as illustrated in Example 2. The second part establishes a sufficient condition for partisan voting in large elections that select the correct candidates in expectation, namely, if a party's base is sufficiently large, supporters of the other party must be partisan. For instance, in Example 2, $\lambda_A = 0.1 < \frac{1}{3}$ and $\lambda_B = 0.4 \geq \frac{1}{3}$. Theorem 2 implies that in a large election with an equilibrium having correct expected winners, voting must be partisan for type A, while for type B, voting could be partisan (as in Corollary 2) or responsive (as in Example 2).[14] This suggests that

[13] Note that an equilibrium can be responsive and at the same time one candidate receives no vote in equilibrium. For example, this could be the case when the voters always vote for one candidate after receving one signal and abstain after receving the other.

[14] The bound provided in Theorem 2 holds for any Γ such that there is some $(q_\alpha, q_\beta) \in \Gamma$ such that $q_\alpha = q_\beta$. When the uncertainty is maximal, i.e., $\underline{q}_\theta \to \frac{1}{2}$ and $\overline{q}_\theta \to 1$ for both $\theta \in \{\alpha, \beta\}$, the bound is lower. In the limit, for $t, t' \in \{A, B\}$, voting is partisan for type t in equilibrium if $\lambda_{t'} \geq \frac{1}{9}$.

supporters of the weaker party are more likely to be partisan. This is because the smaller the supporter population of a party, the greater the need for its supporters to counterbalance the influence of the other party. As a result, the supporters of the weaker party are more inclined to vote strictly along party lines to ensure that their candidate has a winning chance.

We provide here a sketch of the proof. First, we consider an equilibrium in which all voters vote responsively. A truthful voting equilibrium is excluded by Corollary 1. Furthermore, if supporters of one party do not vote for their own candidate after receiving both signals, then this candidate is dominated after receiving both signals. Consequently, this candidate does not receive any votes in equilibrium.

It follows that if both candidates receive some votes and all party supporters vote responsively, party supporters must vote for their own party with probability one after receiving the signal favoring their own party, and mix between abstention and voting for the other party after receiving the signal favoring the other party. Such equilibria, however, can also be ruled out using logic similar to the proof of Proposition 1. This is because, in this case, voting for candidate A (B) must be dominated conditional on being pivotal and receiving signal b (a). Since the likelihood ratio of a single signal is finite, this means that the ratio of the pivotal probabilities in the two states evaluated *at all precision levels* must stay finite as the size of the electorate increases. But this is impossible. As a result, supporters of as least one of the two parties must be partisan.

Then, we consider the responsive equilibrium in which voting is partisan for either type A or B. Suppose voting is partisan for type A. We show that voting cannot be responsive for type B if the population size of type A is more than one-third of the electorate and the election selects the correct candidates in expectation. Intuitively, when the other party has a large supporter population, party supporters cannot afford to be responsive. Instead, they must support their own party to counterbalance the influence of the other party's supporters. Note that this result does not require the supporters of the other party to be partisan; the mere size of the other party's supporter population is enough to compel a party's supporters to stick to their own party. When the supporter population of neither party is smaller than one-third of the electorate, voting is fully partisan in large elections.

With SEU preference, voters evaluate all options according to a specific belief, and mixing is always between one party candidate and abstention (see Feddersen and Pesendorfer (1996) and Krishna and Morgan (2012) for instance). This is also true for voters with maxmin preference (see Lemma 6 in Ellis (2016)). However, with incomplete preference, mixing is more flexible for independents. For example, when the two candidates are incomparable and abstention is not dominated, it is possible for independents to fully mix among all three options in equilibrium. Incomplete preference imposes more restrictions on the voting behaviors of party supporters via status quo while imposing fewer restrictions on the voting behaviors of independents.

It is also worth mentioning that party supporters, due to partisan voting, fully participate. Their participation rate is always higher than that of independents. Party supporters do not abstain, not because they are particularly enthusiastic about voting or driven by partisan fervor, but because the option of abstention is not good enough for them to abandon their own party.

7 Uncertainty about Prior

In this section we consider an environment in which uncertainty is no longer about information precision, but about the prior belief. Assume the prior belief of the state being α is given by $p \in (\underline{p}, \overline{p})$, and signal precisions are given by $q_\alpha = q_\beta = q \in \left(\frac{1}{2}, 1\right)$. We say that voters are confident if voting is truthful for all voter types in the hypothetical situation where a single voter's vote determines the outcome of the election. That is, voters are confident if, for all $p \in [\underline{p}, \overline{p}]$,

$$l\,(b;p) < 1 < l\,(a;p)\,,$$

where $l\,(s;p)$ is the posterior likelihood ratio of the state conditional on signal s.

Following the proof of Proposition 1, given the truthful voting strategy profile γ, the pivotal ratio $\Omega_{piv}\,(n,\gamma) = 1$. Thus, if voters are confident, then

$$l\,(b;p)\,\Omega_{piv}\,(n,\gamma) < 1 < l\,(a;p)\,\Omega_{piv}\,(n,\gamma)$$

for all $p \in [\underline{p}, \overline{p}]$, which means that a truthful voting equilibrium exists.

Proposition 4 *Suppose uncertainty is about prior. Truthful voting is an equilibrium if voters* *are confident.*

Proposition 4 highlights a significant distinction between uncertainty about prior and uncertainty about signal precision. Contrasting this with Proposition 1, truthful voting becomes possible despite the presence of uncertainty. The difference can be explained by examining the posterior likelihood ratio $l(s)\,\Omega_{piv}$.

The term $l(s)$ represents the impact of voter's own information and prior belief and is independent of the population size n. On the other hand, the term Ω_{piv} represents the effect of strategic voting and is a function of the population size n. When uncertainty is about signal precision, it affects the posterior likelihood ratio through both $l(s)$ and Ω_{piv}. Consequently, uncertainty is amplified in large elections due to strategic consideration. In contrast, when uncertainty is about prior belief, it only affects strategic consideration through $l(s)$, thus not amplified in large elections to the same extent. When voters are confident, truthful voting is an equilibrium, regardless the size of the electorate.

The contrast between uncertainty about prior and uncertainty about signal precision also highlights the distinction between different forms of partisanship: one in terms of own party as status quo choice and the other in terms of additional utility gained from voting for one's own party. Consider a model with SEU preference in which party supporters gain some extra utility when voting for their own party. In large elections, voting remains partisan because the extra utility from voting for one's own party outweighs the potential utility loss from selecting the wrong candidate. This happens because the probability of a single vote affecting the outcome in large elections is negligible. This phenomenon is known as expressive voting.

Partisan voting driven by expressive incentive and partisan voting due to incomplete preference may look similar in some cases but differ significantly in others. The expressive incentive does not interact with other aspects of the voting problem. It is much less circumstantial and remains stable regardless of the information environment. In contrast, partisan voting due to incomplete preference is more context-dependent and sensitive to the information environment. This underscores the importance of understanding the underlying reasons for partisan voting.

8 Discussion

We discuss the role of partisanship in a common-value voting model with incomplete preference. Party cue helps voters to navigate the complex voting problem. In this paper, we propose a novel mechanism how voters use partisanship and how partisanship affects voting behaviors in different information environments. We show that its impact on partisan voting could be more overwhelming than a running tally, and that the effect of partisanship on voting behaviors is sensitive to the information environment.

In today's politics, there is a tremendous amount of uncertainty. Fake news and misinformation are prevalent, and the media landscape is increasingly fragmented and polarized. It is challenging for individuals to determine the trustworthiness of their own information sources, let alone to assess the reliability of others' sources. Our paper suggests that uncertainty regarding others' information quality can be amplified through strategic consideration, leading to increased partisan voting.

Great effort has been put into understanding how people view the world through their partisan lens and how this lens colors their perception. Our model highlights the importance of understanding how much people know about their own media exposure and others' media exposure, and how such knowledge affects their partisan behavior in voting. By examining these interactions, we can gain deeper insights into the mechanisms behind partisan voting and the broader implications for democratic processes.

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Appendix A

Proof of Proposition 1. In the main text. ■

Proof of Lemma 1. We consider each part separately.

1. We prove by contradiction. Suppose in equilibrium type A mixes between voting for candidates A and B. This means that voting for candidate A is dominated. Thus, there exists $\gamma^A(s) = \left(\gamma^A_A(s), \gamma^A_B(s)\right)$, such that for all $(q_\alpha, q_\beta) \in \Gamma$,

$$\gamma^A_A(s)\,U(A, s; q_\alpha, q_\beta) + \gamma^A_B(s)\,U(B, s; q_\alpha, q_\beta) > U(A, s; q_\alpha, q_\beta),$$

   which implies $U(B, s; q_\alpha, q_\beta) > U(A, s; q_\alpha, q_\beta)$ for all $(q_\alpha, q_\beta) \in \Gamma$. But this means that voting for candidate B dominates mixing between voting for candidates A and B, which is a contradiction. The case for type B is identical.

2. (i) Suppose in equilibrium, $\gamma^A_A(s) = 0$. This implies that voting for candidate A is dominated. Therefore, $\gamma^I_A(s) = \gamma^B_A(s) = 0$. The argument is the same for $\gamma^B_B(s) = 0$. (ii) Suppose $\gamma^I_c(s) > 0$. Then candidate c is not dominated. As a result, if $c = A$, we have $\gamma^A_A(s) = 1$. If $c = B$, we have $\gamma^B_B(s) = 1$.

3. Given a strategy profile γ. Since $l(a; q_\alpha, q_\beta) > l(b; q_\alpha, q_\beta)$ for all $(q_\alpha, q_\beta) \in \Gamma$, by (5), if $U(A, b; q_\alpha, q_\beta) \geq U(B, b; q_\alpha, q_\beta)$, then $U(A, a; q_\alpha, q_\beta) > U(B, a; q_\alpha, q_\beta)$. Therefore, if voting for candidate A is not dominated given signal b, it is not dominated given signal a. Thus, if $\gamma^A_A(b) = 1$, then $\gamma^A_A(a) = 1$. If $\gamma^A_A(b) = 0$, then trivially $\gamma^A_A(a) \geq \gamma^A_A(b)$. The argument is identical for type B. Therefore, voting is monotone in signals for both type A and type B.

■

Proof of Proposition 2. Consider the full partisan strategy profile γ given by

$$\begin{cases} \gamma^A(a) = (1,0), \\ \gamma^I(a) = \left(\frac{1-2\lambda_A}{2\lambda_I} + \varepsilon, \frac{1-2\lambda_B}{2\lambda_I} - \varepsilon\right), \\ \gamma^B(a) = (0,1), \end{cases} \quad \text{and} \quad \begin{cases} \gamma^A(b) = (1,0), \\ \gamma^I(b) = \left(\frac{1-2\lambda_A}{2\lambda_I} - \delta, \frac{1-2\lambda_B}{2\lambda_I} + \delta\right), \\ \gamma^B(b) = (0,1), \end{cases}$$

where $\varepsilon, \delta > 0$. Since $0 < \lambda_A, \lambda_B < 1/2$, γ is a valid strategy profile when ε and δ are small enough. For such a strategy profile to be an equilibrium, it must be the case that the two candidates are incomparable conditional on being pivotal after receiving either signal. We would like to show that we can choose ε and δ such that this requirement is satisfied in large elections and information aggregates. Under such a strategy profile, we have

$$\begin{cases} \sigma_A = \lambda_A + \lambda_I \left[q_\alpha \left(\frac{1-2\lambda_A}{2\lambda_I} + \varepsilon\right) + (1-q_\alpha)\left(\frac{1-2\lambda_A}{2\lambda_I} - \delta\right)\right], \\ \sigma_B = \lambda_B + \lambda_I \left[q_\alpha \left(\frac{1-2\lambda_B}{2\lambda_I} - \varepsilon\right) + (1-q_\alpha)\left(\frac{1-2\lambda_B}{2\lambda_I} + \delta\right)\right], \end{cases}$$

and

$$\begin{cases} \tau_A = \lambda_A + \lambda_I \left[(1-q_\beta) \left(\frac{1-2\lambda_A}{2\lambda_I} + \varepsilon\right) + q_\beta\left(\frac{1-2\lambda_A}{2\lambda_I} - \delta\right)\right], \\ \tau_B = \lambda_B + \lambda_I \left[(1-q_\beta) \left(\frac{1-2\lambda_B}{2\lambda_I} - \varepsilon\right) + q_\beta\left(\frac{1-2\lambda_B}{2\lambda_I} + \delta\right)\right]. \end{cases}$$

By (6), when n is large, we have

$$\Omega_{piv}(q_\alpha, q_\beta, n, \gamma) \approx e^{2n\left(\sqrt{\sigma_A\sigma_B} - \sqrt{\tau_A\tau_B}\right)} K(\sigma_A, \sigma_B, \tau_A, \tau_B),$$

where K is a function that is strictly positive and does not depend on n. Therefore, for the two candidates to be incomparable in large elections, there must exist some $(\hat{q}_\alpha, \hat{q}_\beta) \in \Gamma$ such that $\sigma_A\sigma_B = \tau_A\tau_B$ in the limit. Since $\gamma_A^I(a) > \gamma_A^I(b)$, we have $\sigma_A > \tau_A$. Thus, we need

$$\begin{aligned} \sigma_A &= \lambda_A + \lambda_I \left[\hat{q}_\alpha \left(\frac{1-2\lambda_A}{2\lambda_I} + \varepsilon\right) + (1-\hat{q}_\alpha)\left(\frac{1-2\lambda_A}{2\lambda_I} - \delta\right)\right] \\ &= \lambda_B + \lambda_I \left[(1-\hat{q}_\beta) \left(\frac{1-2\lambda_B}{2\lambda_I} - \varepsilon\right) + \hat{q}_\beta\left(\frac{1-2\lambda_B}{2\lambda_I} + \delta\right)\right] \\ &= \tau_B. \end{aligned}$$

Rewriting the above condition, we have

$$\varepsilon = \frac{1 - \hat{q}_\alpha + \hat{q}_\beta}{1 + \hat{q}_\alpha - \hat{q}_\beta}\delta. \tag{9}$$

Note that for any $(\hat{q}_\alpha, \hat{q}_\beta) \in \Gamma$, we can find small enough ε and δ that satisfy (9) so that the strategy profile γ is an equilibrium. To have correct expected winners, we must have for all $(q_\alpha, q_\beta) \in \Gamma$,

$$\sigma_A = \frac{1}{2} + \frac{\delta\lambda_I}{1 + \hat{q}_\alpha - \hat{q}_\beta}\left(2q_\alpha - 1 - \hat{q}_\alpha + \hat{q}_\beta\right) > \frac{1}{2}, \tag{10}$$

$$\tau_B = \frac{1}{2} + \frac{\delta\lambda_I}{1 + \hat{q}_\alpha - \hat{q}_\beta}\left(2q_\beta - 1 + \hat{q}_\alpha - \hat{q}_\beta\right) > \frac{1}{2}. \tag{11}$$

Since (10) and (11) are monotone in q_α and q_β, respectively, if they are satisfied for $(q_\alpha, q_\beta) = (\underline{q}_\alpha, \underline{q}_\beta)$, they are satisfied by for all $(q_\alpha, q_\beta) \in \Gamma$. Consider $(\hat{q}_\alpha, \hat{q}_\beta) \to (\underline{q}_\alpha, \underline{q}_\beta)$, we have

$$\sigma_A \to \frac{1}{2} + \frac{\delta\lambda_I}{1 + \underline{q}_\alpha - \underline{q}_\beta}\left(\underline{q}_\alpha + \underline{q}_\beta - 1\right) > \frac{1}{2},$$

$$\tau_B \to \frac{1}{2} + \frac{\delta\lambda_I}{1 + \underline{q}_\alpha - \underline{q}_\beta}\left(\underline{q}_\alpha + \underline{q}_\beta - 1\right) > \frac{1}{2}.$$

Thus, γ is an equilibrium in large elections and has correct expected winners.

Finally, we study the information aggregation property of large elections. In state α, it is optimal to elect A, so the probability of an incorrect decision is

$$\begin{aligned}\Pr\left[d = B|\alpha\right] &= \frac{1}{2}\Pr\left[T|\alpha\right] + \sum_{m=1}^{\infty}\Pr\left[T_{-m}|\alpha\right] \\ &< \sum_{m=0}^{\infty}\Pr\left[T_{-m}|\alpha\right],\end{aligned}$$

where $T_{-m} = \{(k - m, k) : k \geq m\}$ is the set of events in which B wins by exactly m votes. Using the approximation formulas (3) and (4), we have

$$\sum_{m=0}^{\infty} \Pr\left[T_{-m}|\alpha\right] \approx \frac{e^{-n\left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2}}{\sqrt{4\pi\sqrt{\sigma_A\sigma_B}}} \sum_{m=0}^{\infty}\left(\sqrt{\frac{\sigma_B}{\sigma_A}}\right)^m$$
$$= \frac{e^{-n\left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2}}{\sqrt{4\pi n\sqrt{\sigma_A\sigma_B}}} \frac{1}{1-\sqrt{\frac{\sigma_B}{\sigma_A}}}. \tag{12}$$

Since $\sigma_A > \sigma_B$, information must aggregate. ■

Proof of Proposition 3. Suppose $\frac{\lambda_A-\lambda_B}{\lambda_I} \in (\underline{q}_\beta - \overline{q}_\alpha, \overline{q}_\beta - \underline{q}_\alpha)$. Consider the strategy profile $\gamma = ((1,1),(1,0),(0,0))$. We have

$$\begin{cases} \sigma_A = \lambda_A + q_\alpha \lambda_I, \\ \sigma_B = \lambda_B + (1-q_\alpha)\lambda_I, \end{cases} \text{and} \begin{cases} \tau_A = \lambda_A + (1-q_\beta)\lambda_I, \\ \tau_B = \lambda_B + q_\beta \lambda_I. \end{cases}$$

Since $\frac{\lambda_A-\lambda_B}{\lambda_I} \in (\underline{q}_\beta - \overline{q}_\alpha, \overline{q}_\beta - \underline{q}_\alpha)$, there exists some $(q_\alpha, q_\beta) \in int(\Gamma)$ such that $\frac{\lambda_A-\lambda_B}{\lambda_I} = q_\beta - q_\alpha$, which implies that

$$\sigma_A - \tau_B = (\lambda_A + q_\alpha\lambda_I) - (\lambda_B + q_\beta\lambda_I) = 0.$$

Since $(q_\alpha, q_\beta) \in int(\Gamma)$, there always exists some $(q'_\alpha, q'_\beta) \in \Gamma$ such that $\sigma_A\sigma_B < \tau_A\tau_B$ and some $(q''_\alpha, q''_\beta) \in \Gamma$ such that $\sigma_A\sigma_B > \tau_A\tau_B$. By (6), in the first case, we have $\Omega_{piv}(q_\alpha, q_\beta, n, \gamma) \to 0$ as $n \to \infty$. In the second case, we have $\Omega_{piv}(q_\alpha, q_\beta, n, \gamma) \to \infty$ as $n \to \infty$. It follows that candidates A and B are incomparable conditional on being pivotal and receiving either signal. This means that γ is an equilibrium when n is large. ■

Proof of Lemma 2. We consider each part separately.

1. We prove by contradiction. Suppose in equilibrium type A mixes between voting for candidate A and the other two options. This means that voting for candidate A is dominated. Thus, there exists $\gamma^A(s) = (\gamma^A_A(s), \gamma^A_\phi(s), \gamma^A_B(s))$, such that for all $(q_\alpha, q_\beta) \in \Gamma$,

$$\gamma^A_A(s)\,U(A,s;q_\alpha,q_\beta) + \gamma^A_\phi(s)\,U(\phi,s;q_\alpha,q_\beta) + \gamma^A_B(s)\,U(B,s;q_\alpha,q_\beta) > U(A,s;q_\alpha,q_\beta),$$

which implies

$$\frac{\gamma_\phi^A(s)}{1-\gamma_A^A(s)}U(\phi,s;q_\alpha,q_\beta)+\frac{\gamma_B^A(s)}{1-\gamma_A^A(s)}U(B,s;q_\alpha,q_\beta)>U(A,s;q_\alpha,q_\beta)$$

for all $(q_\alpha, q_\beta) \in \Gamma$. But this means that mixing between voting for candidate B and abstention dominates any mixed strategy involving voting for candidate A, which is a contradiction. The case for type B is identical.

2. (i) Suppose in equilibrium, $\gamma_A^A(s) = 0$. This implies that voting for candidate A is dominated. Therefore, $\gamma_A^I(s) = \gamma_A^B(s) = 0$. The argument is the same for the case in which $\gamma_B^B(s) = 0$. (ii) $\gamma_c^I(s) > 0$ implies that candidate c is not dominated. As a result, if $c = A$, we have $\gamma_A^A(s) = 1$. If $c = B$, we have $\gamma_B^B(s) = 1$.

3. (i) Suppose in equilibrium, $\gamma_A^A(a) = 0$. This means voting for candidate A is dominated by the strategy $\gamma^A(a)$, where $\gamma^A(a) = (0, \gamma_\phi^A(a), \gamma_B^A(a))$.

$$\begin{aligned}
&U\left(\gamma^A(a), s; q_\alpha, q_\beta\right) - U(A, s; q_\alpha, q_\beta) \\
&= \gamma_\phi^A(a)\, U(\phi, s; q_\alpha, q_\beta) + \gamma_B^A(a)\, U(B, s; q_\alpha, q_\beta) - U(A, s; q_\alpha, q_\beta) \\
&= \gamma_\phi^A(a)\,(U(\phi, s; q_\alpha, q_\beta) - U(A, s; q_\alpha, q_\beta)) + \gamma_B^A(a)\,(U(B, s; q_\alpha, q_\beta) - U(A, s; q_\alpha, q_\beta))
\end{aligned}$$

Since $l(a; q_\alpha, q_\beta) > l(b; q_\alpha, q_\beta)$, if $U(\phi, s; q_\alpha, q_\beta) - U(A, s; q_\alpha, q_\beta)$ and $U(B, s; q_\alpha, q_\beta) - U(A, s; q_\alpha, q_\beta)$ are both monotone in s. Therefore, if $U(A, a; q_\alpha, q_\beta) < U(\gamma_A(a), a; q_\alpha, q_\beta)$ for all $(q_\alpha, q_\beta) \in \Gamma$, then $U(A, b; q_\alpha, q_\beta) < U(\gamma_A(a), b; q_\alpha, q_\beta)$ for all $(q_\alpha, q_\beta) \in \Gamma$. The argument is similar for the other cases.

■

Proof of Corollary 1. In the main text. ■

Proof of Corollary 2. In the main text. ■

Proof for Example 2. We would like to show that there exists an equilibrium of the form given by (8) in large elections. Consider the information precision $(0.8, 0.6) \in \Gamma$, we have

$$\begin{cases} \sigma_A = 0.1 + 0.72\eta^n, \\ \sigma_B = 0.18, \end{cases} \text{and} \begin{cases} \tau_A = 0.1 + 0.36\eta^n, \\ \tau_B = 0.54. \end{cases}.$$

Consider the pivotal ratios of voting for candidates A and B versus abstention. By (3) and (4), they are given by

$$\frac{\Pr[piv_A|\alpha]}{\Pr[piv_A|\beta]} \approx \sqrt[4]{\frac{\tau_A\tau_B}{\sigma_A\sigma_B}} e^{n\left[\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2-\left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2\right]} \left[\frac{1+\left(\frac{\sigma_A}{\sigma_B}\right)^{-\frac{1}{2}}}{1+\left(\frac{\tau_A}{\tau_B}\right)^{-\frac{1}{2}}}\right],$$

$$\frac{\Pr[piv_B|\alpha]}{\Pr[piv_B|\beta]} \approx \sqrt[4]{\frac{\tau_A\tau_B}{\sigma_A\sigma_B}} e^{n\left[\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2-\left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2\right]} \left[\frac{1+\left(\frac{\sigma_A}{\sigma_B}\right)^{\frac{1}{2}}}{1+\left(\frac{\tau_A}{\tau_B}\right)^{\frac{1}{2}}}\right],$$

respectively. Note that since $\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2 - \left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2 \to 0$ as $n \to \infty$, we can choose a sequence of η^n so that $e^{n\left[\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2-\left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2\right]}$ converges to any value between 0 and ∞. For large n, pick η^n so that

$$l\left(a; 0.8, 0.6\right)\frac{\Pr[piv_A|\alpha]}{\Pr[piv_A|\beta]} = 1.$$

This means that

$$l\left(a; 0.8, 0.6\right)\frac{\Pr[piv_B|\alpha]}{\Pr[piv_B|\beta]} \approx (2.2018)\, l\left(a; 0.8, 0.6\right)\frac{\Pr[piv_A|\alpha]}{\Pr[piv_A|\beta]} > 1.$$

Thus, we have

$$U\left(B, a; 0.8, 0.6\right) < U\left(A, a; 0.8, 0.6\right) = U\left(\phi, a; 0.8, 0.6\right). \tag{13}$$

On the other hand,

$$l\left(b; 0.8, 0.6\right)\frac{\Pr[piv_B|\alpha]}{\Pr[piv_B|\beta]} \approx \left(\frac{1-0.8}{0.6}\right)(1.1009) = 0.36697 < 1.$$

This means that

$$U\left(A, b; 0.8, 0.6\right) < U\left(\phi, b; 0.8, 0.6\right) < U\left(B, b; 0.8, 0.6\right). \tag{14}$$

Note that

$$\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2=\left(\sqrt{(0.9)\,q_\beta}-\sqrt{0.1+0.9\,(1-q_\beta)\,\eta^n}\right)^2$$

is strictly increasing in q_β. This means that for n large enough, we have, for all $q_\beta > 0.6$,

$$U\left(B,a;0.8,q_\beta\right)<U\left(A,a;0.8,q_\beta\right)\text{ and }U\left(B,a;0.8,q_\beta\right)<U\left(\phi,a;0.8,q_\beta\right). \tag{15}$$

Moreover, for n large enough, there exists $q_\beta \in (0.6, 0.8]$ such that

$$U\left(B,b;0.8,q_\beta\right)<U\left(\phi,a;0.8,q_\beta\right)<U\left(A,b;0.8,q_\beta\right). \tag{16}$$

By (13) and (15), conditional on signal a and being pivotal, voting for candidate B is dominated in large elections while voting for candidate A and abstention are not. By (14) and (16), conditional on signal b and being pivotal, none of the three options is dominated in large elections. Therefore, the strategy profile constitutes an equilibrium in large elections. ■

The proof of Theorem 2 makes use of three lemmas (Lemmas 3–5). The first lemma characterizes party supporters' strategies in an equilibrium where both candidates receive positive votes with positive probability. Note that Lemma 3 applies to small elections as well as large elections.

Lemma 3 *In an equilibrium of the voting game with abstention, if both candidates receive positive votes with positive probability, then $\gamma_A^A(a)=1$ and $\gamma_B^B(b)=1$. Moreover, if voting is responsive for type A (B), then $\gamma_A^A(b)=0$ ($\gamma_B^B(a)=0$).*

Proof. Suppose by way of contradiction that $\gamma_A^A(a)\neq 1$. By part 1 of Lemma 2, this means $\gamma_A^A(a)=0$. By part 3 of Lemma 2, voting is monotone in signals for type A, so $\gamma_A^A(b)=0$. By part 2 of Lemma 2, voting is monotone in types, so $\gamma_A^I(a)=\gamma_A^B(a)=0$ and $\gamma_A^I(b)=\gamma_A^B(b)=0$. Since no voter votes for candidate A with positive probability, candidate A does not receive any vote in equilibrium. This contradicts our assumption. We must have $\gamma_A^A(a)=1$. If voting is responsive for type A, then, $\gamma_A^A(b)\neq 1$. By part 1 of Lemma 2, this means $\gamma_A^A(b)=0$.

The proof for type B is similar. ■

The next lemma makes use of Lemma 3 and shows that in large elections with abstention, there is no equilibrium in which all voters vote responsively and both candidates receive positive votes with positive probability.

Lemma 4 *Under uncertainty, in large elections with abstention, if in equilibrium both candidates receive positive votes with positive probability, then at least one of types A and B is partisan.*

Proof. Suppose by way of contradiction that none of types A and B is partisan. By Lemma 2 and Lemma 3, the equilibrium strategy profile must satisfy

$$\begin{cases} \gamma^A(a) = (1,0,0), \\ \gamma^I(a) = \left(\gamma_A^I(a), \gamma_\phi^I(a), 0\right), \\ \gamma^B(a) = \left(\gamma_A^B(a), \gamma_\phi^B(a), 0\right), \end{cases} \quad \text{and} \quad \begin{cases} \gamma^A(b) = \left(0, \gamma_\phi^A(b), \gamma_B^A(b)\right), \\ \gamma^I(b) = \left(0, \gamma_\phi^I(b), \gamma_B^I(b)\right), \\ \gamma^B(b) = (0,0,1). \end{cases}$$

Therefore, we have

$$\begin{cases} \sigma_A = q_\alpha \left(\lambda_A + \lambda_I \gamma_A^I(a) + \lambda_B \gamma_A^B(a)\right), \\ \sigma_B = (1-q_\alpha)\left(\lambda_A \gamma_B^A(b) + \lambda_I \gamma_B^I(b) + \lambda_B\right), \\ \sigma_\phi = 1 - \sigma_A - \sigma_B, \end{cases}$$

and

$$\begin{cases} \tau_A = (1-q_\beta)\left(\lambda_A + \lambda_I \gamma_A^I(a) + \lambda_B \gamma_A^B(a)\right), \\ \tau_B = q_\beta \left(\lambda_A \gamma_B^A(b) + \lambda_I \gamma_B^I(b) + \lambda_B\right), \\ \tau_\phi = 1 - \tau_A - \tau_B. \end{cases}$$

Note that in order for the equilibrium strategy profile to be an equilibrium, it must be the case that conditional on being pivotal, candidate A (B) is the dominant choice after receiving signal a (b). By (3) and (4), the pivotal ratios of voting for candidates A and B are given by

$$\frac{\Pr[piv_A|\alpha]}{\Pr[piv_A|\beta]} \approx \sqrt[4]{\frac{\tau_A\tau_B}{\sigma_A\sigma_B}} e^{n\left[\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2 - \left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2\right]} \left[\frac{1+\left(\frac{\sigma_A}{\sigma_B}\right)^{-\frac{1}{2}}}{1+\left(\frac{\tau_A}{\tau_B}\right)^{-\frac{1}{2}}}\right],$$

$$\frac{\Pr[piv_B|\alpha]}{\Pr[piv_B|\beta]} \approx \sqrt[4]{\frac{\tau_A\tau_B}{\sigma_A\sigma_B}} e^{n\left[\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2 - \left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2\right]} \left[\frac{1+\left(\frac{\sigma_A}{\sigma_B}\right)^{\frac{1}{2}}}{1+\left(\frac{\tau_A}{\tau_B}\right)^{\frac{1}{2}}}\right],$$

respectively. We must have, for all $(q_\alpha, q_\beta) \in \Gamma$,

$$\lim_{n\to\infty}\left[\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2-\left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2\right]=0. \tag{17}$$

Otherwise, $\frac{\Pr[piv_A|\alpha]}{\Pr[piv_A|\beta]}$ and $\frac{\Pr[piv_B|\alpha]}{\Pr[piv_B|\beta]}$ will converge to zero or infinity for some $(q_\alpha, q_\beta) \in \Gamma$. This means that one of the two candidates will not be dominated after receiving either signal, which implies one of types A and B must be partisan, contradicting the equilibrium strategy profile γ. But it is also impossible to have (17) holds for all $(q_\alpha, q_\beta) \in \Gamma$. If (17) is satisfied in the interior of Γ, there must be some (q_α, q_β) in the neighborhood such that (17) fails. Therefore, γ cannot be an equilibrium. ■

The next lemma provides a sufficient condition for partisan voting in an equilibrium that has correct expected winners.

Lemma 5 *Under uncertainty, in large elections with abstention, if an equilibrium has correct expected winners, then voting is partisan for type $t \in \{A, B\}$ if $\lambda_{t'} \geq \frac{1}{3}$, where $t' \in \{A, B\}$ and $t' \neq t$.*

Proof. Suppose the equilibrium has correct expected winners and $\lambda_A \geq \frac{1}{3}$. By Lemma 4, supporters of one of the two parties must be partisan. If voting is partisan for type B, we are done. Suppose voting is partisan for type A. For $\gamma_A^A(a) = \gamma_A^A(b) = 1$, it must be the case that candidate A is not dominated conditional on being pivotal and receiving either signal. Meanwhile, type B votes responsively. By Lemma 3, $\gamma_B^B(a) = 0$ and $\gamma_B^B(b) = 1$. By Lemma 2, $\gamma_B^B(a) = 0$ implies that $\gamma_B^I(a) = 0$. Therefore, the equilibrium strategy profile is as follows:

$$\begin{cases} \gamma^A(a) = (1,0,0), \\ \gamma^I(a) = \left(\gamma_A^I(a), \gamma_\phi^I(a), 0\right), \\ \gamma^B(a) = \left(\gamma_A^B(a), \gamma_\phi^B(a), 0\right), \end{cases} \quad \text{and} \quad \begin{cases} \gamma^A(b) = (1,0,0), \\ \gamma^I(b) = \left(\gamma_A^I(b), \gamma_\phi^I(b), \gamma_B^I(b)\right), \\ \gamma^B(b) = (0,0,1). \end{cases}$$

Thus, we have

$$\begin{cases} \sigma_A = \lambda_A + q_\alpha \left(\lambda_I \gamma_A^I(a) + \lambda_B \gamma_A^B(a)\right) + (1-q_\alpha)\lambda_I \gamma_A^I(b), \\ \sigma_B = (1-q_\alpha)\left(\lambda_I \gamma_B^I(b) + \lambda_B\right), \\ \sigma_\phi = 1 - \sigma_A - \sigma_B, \end{cases}$$

and

$$\begin{cases} \tau_A = & \lambda_A + (1-q_\beta)\left(\lambda_I \gamma_A^I(a) + \lambda_B \gamma_A^B(a)\right) + q_\beta \lambda_I \gamma_A^I(b), \\ \tau_B = & q_\beta \left(\lambda_I \gamma_B^I(b) + \lambda_B\right), \\ \tau_\phi = & 1 - \tau_A - \tau_B. \end{cases}.$$

Consider the pivotal ratios of voting for candidates A and B, respectively:

$$\frac{\Pr[piv_A|\alpha]}{\Pr[piv_A|\beta]} \approx \sqrt[4]{\frac{\tau_A\tau_B}{\sigma_A\sigma_B}} e^{n\left[\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2 - \left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2\right]} \left[\frac{1+\left(\frac{\sigma_A}{\sigma_B}\right)^{-\frac{1}{2}}}{1+\left(\frac{\tau_A}{\tau_B}\right)^{-\frac{1}{2}}}\right]$$

$$\frac{\Pr[piv_B|\alpha]}{\Pr[piv_B|\beta]} \approx \sqrt[4]{\frac{\tau_A\tau_B}{\sigma_A\sigma_B}} e^{n\left[\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2 - \left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2\right]} \left[\frac{1+\left(\frac{\sigma_A}{\sigma_B}\right)^{\frac{1}{2}}}{1+\left(\frac{\tau_A}{\tau_B}\right)^{\frac{1}{2}}}\right]$$

If there exists some $(q_\alpha, q_\beta) \in \Gamma$ such that $\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2 < \left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2$, then

$$\lim_{n\to\infty} \frac{\Pr[piv_A|\alpha]}{\Pr[piv_A|\beta]} = \lim_{n\to\infty} \frac{\Pr[piv_B|\alpha]}{\Pr[piv_B|\beta]} = 0.$$

It follows that $U(A, s; q_\alpha, q_\beta) < U(\phi, s; q_\alpha, q_\beta) < U(B, s; q_\alpha, q_\beta)$ for both signals $s \in \{a, b\}$, which implies that candidate B is not dominated conditional on being pivotal and receiving either signal. Thus, $\gamma_B^B(a) = 1$, which contradicts the assumed strategy profile.

Therefore, we must have

$$\left(\sqrt{\tau_A}-\sqrt{\tau_B}\right)^2 \geq \left(\sqrt{\sigma_A}-\sqrt{\sigma_B}\right)^2, \tag{18}$$

for all $(q_\alpha, q_\beta) \in \Gamma$. By the assumption of correct expected winners, $\sigma_A > \sigma_B$ and $\tau_B > \tau_A$. It follows that (18) is equivalent to

$$\left(\sqrt{\sigma_B}+\sqrt{\tau_B}\right)-\left(\sqrt{\sigma_A}+\sqrt{\tau_A}\right)\geq 0 \tag{19}$$

for all $(q_\alpha, q_\beta)\in\Gamma$. We would like to show that if $\lambda_A \geq \frac{1}{3}$, then (19) is violated for all $(q_\alpha, q_\beta)\in\Gamma$ such that $q_\alpha = q_\beta$.

Define

$$F(\gamma) := \left(\sqrt{\sigma_B}+\sqrt{\tau_B}\right)-\left(\sqrt{\sigma_A}+\sqrt{\tau_A}\right).$$

Let $k_1 = \lambda_I\gamma_A^I(a)+\lambda_B\gamma_A^B(a)$, $k_2 = \lambda_I\gamma_A^I(b)$, and $k_3 = \lambda_I\gamma_B^I(b)+\lambda_B$. We have $k_1\in[0,\lambda_B+\lambda_I]$, $k_2\in[0,\lambda_I]$, $k_3\in[\lambda_B,\lambda_B+\lambda_I]$. Differentiate F, we have

$$\begin{aligned}
\tfrac{\partial F(\gamma)}{\partial k_1} &= -\tfrac{1}{2}\tau_A^{-\frac{1}{2}}(1-q_\beta)-\tfrac{1}{2}\sigma_A^{-\frac{1}{2}}q_\alpha < 0,\\
\tfrac{\partial F(\gamma)}{\partial k_2} &= -\tfrac{1}{2}\tau_A^{-\frac{1}{2}}q_\beta-\tfrac{1}{2}\sigma_A^{-\frac{1}{2}}(1-q_\alpha) < 0,\\
\tfrac{\partial F(\gamma)}{\partial k_3} &= \tfrac{1}{2}\tau_B^{-\frac{1}{2}}q_\beta+\tfrac{1}{2}\sigma_B^{-\frac{1}{2}}(1-q_\alpha) > 0.
\end{aligned}$$

Consider the following strategy $\tilde{\gamma}$ such that $k_1 = 0$, $k_2 = 0$, and $k_3 = \lambda_I+\lambda_B$. That is,

$$\begin{cases}\tilde{\gamma}^A(a) = (1,0,0),\\ \tilde{\gamma}^I(a) = (0,1,0),\\ \tilde{\gamma}^B(a) = (0,1,0),\end{cases}\quad\text{and}\quad\begin{cases}\tilde{\gamma}^A(b) = (1,0,0),\\ \tilde{\gamma}^I(b) = (0,0,1),\\ \tilde{\gamma}^B(b) = (0,0,1).\end{cases}$$

For all γ in which voting is partisan for type A and responsive for type B, $F(\tilde{\gamma})\geq F(\gamma)$. It follows that if

$$F(\tilde{\gamma}) = \left(\sqrt{(1-q_\alpha)(\lambda_I+\lambda_B)}+\sqrt{q_\beta(\lambda_I+\lambda_B)}\right)-2\sqrt{\lambda_A}<0,$$

then $F(\gamma)<0$. Consider $(q_\alpha,q_\beta)\in\Gamma$ such that $q_\alpha = q_\beta = q$,

$$\begin{aligned}
F(\tilde{\gamma}) &= \left(\sqrt{(1-q)}+\sqrt{q}\right)\sqrt{\lambda_I+\lambda_B}-2\sqrt{\lambda_A}\\
&< \sqrt{2(1-\lambda_A)}-2\sqrt{\lambda_A}\\
&\leq 0.
\end{aligned}$$

Thus, for all $(q_\alpha, q_\beta) \in \Gamma$ such that $q_\alpha = q_\beta$, $F(\gamma) < 0$ for all γ in which voting is partisan for type A and responsive for type B. This is a contradiction. Thus, voting must be partisan for type B as well. ■

Proof of Theorem 2. Theorem 2 simply combines Lemmas 4 and 5. ■

Proof of Proposition 4. In the main text. ■

Appendix B

In this appendix, we consider our main model when the assumption that there is some $(q_\alpha, q_\beta) \in \Gamma$ such that $q_\alpha = q_\beta$ is relaxed.

We begin by noting that the proof of Proposition 2 does not make use of this assumption. As a result, there is still a sequence of full partisan equilibria that aggregates information in large elections.

Proposition 5 *Under uncertainty, there exists a sequence of full partisan equilibria that aggregates information in large elections.*

Proof. Identical to the proof of Proposition 2. ■

Thus, full partisan equilibrium survives in large elections even if the assumption is dropped. But when is voting necessarily partisan? The following theorem is the counterpart of Theorem 1.

Theorem 3 *Under uncertainty, in any responsive equilibrium in large elections,*

1. *If $q_\beta \leq q_\alpha$ ($q_\beta \geq q_\alpha$) for some $(q_\alpha, q_\beta) \in \Gamma$,voting is partisan for type B (A).*

2. *If $\lambda_A \geq \frac{1}{3}$ ($\lambda_B \geq \frac{1}{3}$), voting is partisan for type B (A).*

Proof. Consider any responsive equilibrium γ. First, note that from the proof of Proposition 1 that if there exists some $(q_\alpha, q_\beta) \in \Gamma$, such that $q_\beta < q_\alpha$ or $q_\beta > q_\alpha$, then truthful voting cannot be an equilibrium. It follows from Lemma 1 that supporters of at least one of the two parties must be partisans. If both types A and B are partisans, then we are done. Since γ is an responsive equilibrium, one of these two types must vote truthfully. Without loss of generality, suppose type A always votes for candidate A and type B votes truthfully. We would like to show that voting is partisan for type B if one of the two stated conditions is satisfied. By Lemma 1, we have

$$\begin{cases} \gamma^{A}(a) = (1,0), \\ \gamma^{I}(a) = (1,0), \\ \gamma^{B}(a) = (1,0), \end{cases} \quad \text{and} \quad \begin{cases} \gamma^{A}(b) = (1,0), \\ \gamma^{I}(b) = \left(\gamma_{A}^{I}(b), 1-\gamma_{A}^{I}(b)\right), \\ \gamma^{B}(b) = (0,1), \end{cases}$$

where $\gamma_{A}^{I}(b)$ can take any value between 0 and 1. Given such a strategy profile, we have

$$\begin{cases} \sigma_{A} = q_{\alpha} + (1-q_{\alpha})\left(\lambda_{A} + \lambda_{I}\gamma_{A}^{I}(b)\right), \\ \sigma_{B} = (1-q_{\alpha})\left(\lambda_{I}\left(1-\gamma_{A}^{I}(b)\right) + \lambda_{B}\right), \end{cases}$$

and

$$\begin{cases} \tau_{A} = 1 - q_{\beta} + q_{\beta}\left(\lambda_{A} + \lambda_{I}\gamma_{A}^{I}(b)\right), \\ \tau_{B} = q_{\beta}\left(\lambda_{I}\left(1-\gamma_{A}^{I}(b)\right) + \lambda_{B}\right). \end{cases}$$

Note that for γ to be an equilibrium, there must be some $(q_{\alpha}, q_{\beta}) \in \Gamma$ such that $\sigma_{A}\sigma_{B} = \tau_{A}\tau_{B}$ in the limit, otherwise, by (6), $\Omega_{piv}(q_{\alpha}, q_{\beta}, n, \gamma)$ will converge to zero or infinity and either type A would not vote for candidate A or type B would not vote for candidate B. Since $q_{\alpha} > \frac{1}{2} > 1 - q_{\beta}$, it cannot be the case that $\sigma_{A} = \tau_{A}$, thus $\gamma_{A}^{I}(b)$ must solve $\sigma_{A} = \tau_{B}$ in the limit, which means that

$$\lambda_{A} + \lambda_{I}\gamma_{A}^{I}(b) = \frac{q_{\beta} - q_{\alpha}}{1 + q_{\beta} - q_{\alpha}}. \tag{20}$$

Because $\lambda_{A} + \lambda_{I}\gamma_{A}^{I}(b) > 0$ and $1 + q_{\beta} - q_{\alpha} > 0$, (20) is not satisfied for all $q_{\alpha} \geq q_{\beta}$. Thus, there must be some $(q'_{\alpha}, q'_{\beta}) \in \Gamma$ such that $q'_{\alpha} < q'_{\beta}$. Suppose there is also some $(q''_{\alpha}, q''_{\beta}) \in \Gamma$ such that $q''_{\alpha} \geq q''_{\beta}$. This means that there exists $(q'''_{\alpha}, q'''_{\beta}) \in \Gamma$ such that $q'''_{\alpha} = q'''_{\beta}$. The first part of Theorem 3 then follows from Theorem 1. Next, suppose $q_{\alpha} < q_{\beta}$ for all $(q_{\alpha}, q_{\beta}) \in \Gamma$. Because $q_{\alpha}, q_{\beta} \in \left(\frac{1}{2}, 1\right)$, $q_{\beta} - q_{\alpha} \in \left(0, \frac{1}{2}\right)$. Therefore, $\frac{q_{\beta} - q_{\alpha}}{1 + q_{\beta} - q_{\alpha}} < \frac{1}{3}$. If $\lambda_{A} \geq \frac{1}{3}$, (20) is not satisfied for any $(q_{\alpha}, q_{\beta}) \in \Gamma$. This proves the second part of Theorem 3. ■

Note that the first part of Theorem 3 implies Theorem 1. When there exists $(q_{\alpha}, q_{\beta}) \in \Gamma$ such that $q_\alpha = q_\beta$, voting must be partisan for both types A and B in any responsive equilibrium. The first part of Theorem 3 also implies that at least one of types A and B must be partisan in large elections. The second part of Theorem 3 further asserts that when the supporter population of the other party is large enough, the party supporters must be partisan. The intuition behind is similar to the second part of Theorem 2. Intuitively, when the other party has a large supporter population, then party supporters must support their own party to counter the influence of the other party's supporters.

Proposition 5 asserts that a full partisan equilibrium exists in large elections. The first part of Theorem 3 implies that at least one of types A and B must be partisan in large elections. But could there be any partial partisan equilibrium? The following example shows that a partial partisan equilibrium indeed exists under some parameter values.

Example 3 (Partial Partisan Equilibrium) *Suppose $\lambda_A = \lambda_B = 0.05$, $q_\alpha \in [0.6, 0.7]$ and $q_\beta \in [0.8, 0.9]$. For any $p \in (0,1)$ and N large enough, there exists a sequence of partial partisan equilibria $\{\gamma_n\}_{n>N}$ given by*

$$\begin{cases} \gamma^A(a) = (1,0), \\ \gamma^I(a) = (1,0), \\ \gamma^B(a) = (1,0), \end{cases} \quad \textit{and} \quad \begin{cases} \gamma^A(b) = (1,0), \\ \gamma^I(b) = (\eta^n, 1-\eta^n), \\ \gamma^B(b) = (0,1), \end{cases}$$

where $\eta^n \to \frac{1}{22}$ as $n \to \infty$.

Proof. Given the strategy profile γ_n, we have, for given $(q_\alpha, q_\beta) \in [0.6, 0.7] \times [0.8, 0.9]$,

$$\begin{cases} \sigma_A \to q_\alpha + \frac{1}{11}(1-q_\alpha), \\ \sigma_B \to \frac{10}{11}(1-q_\alpha), \end{cases}$$

and

$$\begin{cases} \tau_A \to 1 - q_\beta + \frac{1}{11} q_\beta, \\ \tau_B \to \frac{10}{11} q_\beta, \end{cases}$$

as $n \to \infty$. When $(q_\alpha, q_\beta) = (0.7, 0.8)$, we have $\lim_{n\to\infty} \sigma_A = \lim_{n\to\infty} \tau_B = \frac{8}{11}$. We can choose a sequence of η^n so that $e^{2n\left(\sqrt{\sigma_A\sigma_B} - \sqrt{\tau_A\tau_B}\right)}$ converges to any value between 0 and ∞. For large n, pick η^n so that

$$\frac{8}{3} > \Omega_{piv}\left(q_\alpha, q_\beta, n, \gamma\right) > \frac{2}{7}.$$

when $(q_\alpha, q_\beta) = (0.7, 0.8)$. This means that

$$U\left(B, a; 0.7, 0.8\right) < U\left(A, a; 0.7, 0.8\right),$$

and

$$U\left(B, b; 0.7, 0.8\right) > U\left(A, b; 0.7, 0.8\right).$$

Moreover, since $\frac{\partial}{\partial q_\alpha}\left(\sigma_A \sigma_B\right) < 0$ and $\frac{\partial}{\partial q_\beta}\left(\tau_A \tau_B\right) < 0$ for all $(q_\alpha, q_\beta) \in [0.6, 0.7] \times [0.8, 0.9]$ for n large enough, by (6), for all $(q_\alpha, q_\beta) \in [0.6, 0.7] \times [0.8, 0.9]$ such that $(q_\alpha, q_\beta) \neq (0.7, 0.8)$, $\Omega_{piv}\left(q_\alpha, q_\beta, n, \gamma\right)$ converges to infinity as n increases. This means that for n large enough, candidate A dominates candidate B conditional on being pivotal and receiving signal a, while the two candidates are incomparable conditional on being pivotal and receiving signal b. Thus, γ_n is a partial partisan equilibrium when n is large. ■