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Optimal Disclosure of Private Information to Competitors

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Abstract

I study the incentives of an informed firm to share its private information with its competitor and the incentives of a regulator to constrain or enforce disclosure in order to benefit consumers. Firms offer differentiated goods, compete a là Bertrand and one firm has an information advantage about demand over its competitor. I show that full disclosure of information is optimal for the informed firm, because it increases price correlation and surplus extraction from consumers. A regulator can increase expected consumer surplus and welfare by restricting disclosure, but consumers can benefit from the regulator privately disclosing some information to the competitor. Disclosure increases the ability of firms to extract surplus from consumers, but private disclosure creates a coordination failure in firm pricing. The optimal disclosure policy is chosen to induce goods to be closer substitutes and intensify the competition across firms.

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Some firms can gather more information than their competitors about market features like demand, given their size or incumbency status. For instance, online platforms like Amazon engage in massive data analysis and demand estimation by gathering information generated through trade and consumer searches that other sellers on the platform can't replicate. As sellers themselves, they can use this information to guide their own pricing, control the information observed by other sellers, and potentially make price recommendations as in Amazon's Seller's coaching program. In those settings of information asymmetry, information disclosure between firms affects firm behavior and therefore also impacts consumers and welfare. The use of private information as a competitive advantage by online platforms and the role of price recommendations as a collusive device have attracted the attention of regulatory entities in the US and Europe.[1] This is because, when there is an uneven distribution of consumer data between firms, regulatory interventions to control information disclosure can redistribute surplus between firms and consumers, as well as impact welfare.

In this paper, I study the role of information disclosure as a pricing persuasion device through which a firm with an information advantage or a regulator can influence the pricing of a competing firm. I examine the informed firm's incentives to commit to share its private information with its competitor and the role of a regulator who commits to control information disclosure between firms to benefit consumers. Specifically, this paper analyzes a stylized duopoly model with information asymmetry about demand which can be low or high, in which an informed firm privately learns demand and an uninformed firm has no private information. Firms offer differentiated goods, such that consumer willingness to pay for a good depends on its substitutability with the competitor's. Firms compete by simultaneously and non-cooperatively setting prices to maximize their expected profits. In this context, I address the following questions: What is the informed firm's optimal disclosure policy as a competitor in the market? How can a regulator constrain or enforce information disclosure to benefit consumers?

I characterize the optimal disclosure for firms and consumers. The welfare implications of disclosure are determined by the degree of differentiation between goods, because it determines the extent to which disclosure affects firm pricing and relative demand in all markets. Regarding optimal disclosure for firms, I show that full disclosure is optimal for the informed firm and maximizes producer surplus. Intuitively, with substitutes, firm choices are strategic complements and the informed firm thus benefits from sharing its private information with the uninformed firm through increased price correlation. In particular, when prices

[1] See for example media coverage in Fung (2020), Green (2018) and Lardieri (2019).

are correlated, a firm's competitor will set a high price when it is optimal for the disclosing firm to set a high price, which raises the disclosing firm's profits because consumers can substitute between firms when prices are different. As a result, full disclosure is optimal for the informed firm. Full disclosure also maximizes producer surplus, because the uninformed firm also benefits from price correlation as well as from learning about the state. This result highlights that an informed firm may have incentives to share information even when it has no information to gain in return, because it can influence the pricing of its competitor.

Regarding optimal disclosure for consumers, my main result is that a regulator should restrict information disclosure, at least partially.[2] However, some information disclosure is not necessarily detrimental to consumers. First, the optimal disclosure is private, such that the informed firm doesn't observe the signal realization of the uninformed firm because this reduces the price correlation between firms which is to the detriment of consumers. Second, I show that optimal disclosure is determined by the degree of differentiation between goods. The optimal disclosure policy is chosen to induce goods to be closer substitutes and intensify the competition across firms.

To see the intuition, information disclosure creates a trade-off for consumers. On the one hand, it reduces the uninformed firm's uncertainty about the state, improving the ability of firms to extract surplus from consumers by increasing price correlation. On the other hand, private partial disclosure introduces uncertainty about the information observed by a firm's competitor. This expands the range of prices in each state, since firms price according to the expected price of its competitor and its own expected demand. This type of uncertainty implies that it becomes more likely that the two firms set different prices. Consumers benefit when this happens, ceteris paribus, because they can then choose from which firm to buy after observing prices. Overall, the regulator trades-off the opportunity to create this coordination failure in prices with allowing firms to better extract surplus from consumers. The net effect depends on the differentiation between goods, because it determines consumers' willingness to substitute between goods and therefore the extent to which disclosure affects relative demand across firms. In particular, note that when the goods are perfect substitutes, the latter (beneficial) effect of partial disclosure dominates. This is because consumers benefit greatly from any price differential, given that they just want to purchase the cheapest good when these are substitute.

To maximize expected welfare, the regulator trades off the effect of disclosure on con-

[2] Luco (2019) presents empirical evidence that full disclosure can be detrimental for consumers in the gasoline market in Chile.

sumers and firms. When firms offer sufficiently differentiated goods, no disclosure is optimal, since the expected loss from disclosure for consumers exceeds the expected gain for firms. Conversely, when firms offer sufficiently close substitutes, full disclosure is optimal. For intermediate levels of differentiation, partial disclosure maximizes expected welfare.

When partial disclosure is optimal, I also fully characterize the consumer and welfare optimal disclosure policies. I show that the optimal disclosure policy can be implemented as price recommendations, recommending a price to each firm conditional on the state subject to obedience constraints. Proposition 2 shows that the regulator recommends at most two prices. One of the prices is only recommended when the state is high, revealing the state to the uninformed firm. The other price is recommended in both states, obfuscating demand. The optimality of partial disclosure highlights that optimal disclosure is more nuanced when considering implications for consumers and welfare.

My results can inform current policy debates on the use of private information by firms who act as both a trading platform and a competitor in the market, as well as the debate about whether retail price recommendations act as a collusive device. As I show, it can be optimal for a regulator to intervene by completely preventing or forcing information disclosure, or by designing disclosure policies to partially inform the uninformed firm. Since disclosure policies can be interpreted as price recommendations, these recommendations can help consumers, and abstaining from regulation minimizes consumer surplus. Lastly, my results highlight that it is crucial to consider the strategic environment to understand the welfare consequences of information sharing.

Related literature. This paper contributes to the literature on strategic information sharing in oligopolies with commitment and the literature on information design in games.[3] Incentives for information sharing about demand among competing firms with symmetric private information and normally distributed linear demand were first studied in Novshek and Sonnenschein (1982), Clarke (1983) as well as Vives (1984), and later generalized in Raith (1996).[4] In these papers, firms commit to share their private information with an intermediary, which then discloses a common signal to all firms to maximize industry-wide

[3]Papers like Benoit and Dubra (2006) show that agents' ex-ante and ex-post incentives for information sharing can be disaligned, such that commitment is key.

[4]Other papers in this literature include Gal-Or (1985), Li (1985), Kirby (1988) and Vives (1990). Information sharing about costs are studied in papers like Fried (1984), Gal-Or (1986), Sakai (1986) and Shapiro (1986), in which incentives to share information are reversed for firms. Information sharing about costs with Bertrand competition is strategically equivalent to sharing about values in first price auctions (Engelbrecht-Wiggans et al. (1983), Fang and Morris (2006), and Bergemann et al. (2017)).

profits. These papers focus on the producer surplus optimal public disclosure and on the regulation of industry-wide information sharing by trading organizations. They show the optimality of full disclosure for firms when they compete by choosing prices and offer imperfect substitutes. Instead, I study the incentives of an individual firm to share information to influence its competitor's behavior in a setting of informational advantage, in which the distribution of the uninformed firm's signal is unrestricted.[5] My results show that it can be optimal for a firm to unilaterally disclose information about demand to a competitor even without receiving information in return, because disclosure influences competitor behavior and acts as a pricing persuasion device.[6] Further, full disclosure is not only optimal for the informed firm, but also for producer surplus. The intuition for this result relates to Angeletos and Pavan (2007), who study the social value of information with normally distributed signals and find that producer surplus increases with the precision of both public and private signals.

In contrast with this literature, I also analyze the effects of information disclosure on consumers. Vives (1984) and Calzolari and Pavan (2006) show that information disclosure is not necessarily harmful to consumers. Vives (1984) illustrates this by comparing the utility of a representative consumer across full and no disclosure when firms share symmetric private information. Calzolari and Pavan (2006) study a sequential setting in which the Stackelberg leader must provide incentives to consumers to reveal their private information to be able to share it with its follower. They focus on the leader's optimal disclosure policy, whereas I focus on the optimal disclosure for consumers. Regarding welfare, Vives (1984) also shows that full disclosure dominates no disclosure if and only if firms offer sufficiently close substitutes, yet I show that restricting to full and no disclosure is with loss of generality since partial disclosure can be consumer and welfare optimal. My results regarding welfare relate to Ui and Yoshizawa (2015), who study the social value of information restricted to symmetric normally distributed signals and symmetric equilibria. They show that welfare decreases in the precision of private information and increases in the precision of public information if goods are close substitutes, intuitively related to the optimality of either full or private partial disclosure as I fully characterize in this paper.

[5] Bergemann and Morris (2013), Bergemann et al. (2015b) and Eliaz and Forges (2015) analyze producer optimal disclosure in Cournot settings with perfect substitutes and information symmetry. They show that it is with loss of generality to restrict attention to a common and, hence, perfectly correlated disclosure.

[6] In sequential settings, the role of current choices as a costly persuasion device to influence the precision of future information has been studied in Mailath (1989), Mirman et al. (1993), Mirman et al. (1994), Harrington (1995), Keller and Rady (2003), Taylor (2004), Bernhardt and Taub (2015), Bonatti et al. (2017).

More broadly, the paper contributes to the literature on information design in games as studied in papers like Taneva (2019) and Mathevet et al. (2020). I characterize the optimal recommendation mechanism in a Bertrand setting with product differentiation and information asymmetry.[7] It is most closely related to the literature on consumer optimal information design, which analyzes the effect of information about buyers' valuation on pricing and welfare allocation. This literature has focused on buyer optimal learning, consumer optimal market segmentation and on the incentives of consumers to disclose their preferences to firms. Within the buyer optimal learning literature, Roesler and Szentes (2017) analyzes the effect of a buyer's information on monopoly pricing and characterizes optimal buyer learning. In a duopoly setting, Armstrong and Zhou (2019) studies competition between firms when consumers observe a private signal about their valuation and characterizes consumer optimal learning. Within the consumer optimal segmentation literature, Bergemann et al. (2015a) analyzes the welfare consequences of a monopolist having access to additional information about consumer preferences and characterize the feasible welfare outcomes achieved by segmentation. Li (2020) extends the insights from Bergemann et al. (2015a) to an oligopoly setting and characterizes the consumer-optimal market segmentation in competitive markets. Elliott et al. (2020) studies how information about consumer preferences should be distributed across firms which compete by offering personalized discounts to consumers and provides necessary and sufficient conditions under which perfect segmentation can be achieved. Lastly, Ichihashi (2020) studies the welfare effects of consumers disclosing information about their valuation with a monopolist, whereas Ali et al. (2020) analyzes the consumer optimal disclosure of information about their preferences in monopolistic and competitive markets. In contrast, I focus on the welfare consequences of an unequal distribution of consumer data across firms and the effect of information disclosure between firms. In particular, I characterize the consumer optimal disclosure policy between firms, which affects consumers indirectly by affecting prices.

The remainder of the paper is organized as follows: Section 1 presents the model, Section 2 derives the informed firm optimal disclosure, Section 3 derives the consumer optimal disclosure, Section 4 derives the producer and welfare optimal disclosures, Section 5 discusses extensions and robustness of results, and Section 6 concludes.

[7] In contrast, Bergemann et al. (2021) study a setting in which identical firms offer an homogeneous good, compete by setting prices and are uncertain about the number of price quotes a consumer receives. They identify how the equilibrium price dispersion depends on the distribution of the price count and the information firms have.

1 Model

Two symmetric firms offer horizontally differentiated substitutes and compete by simultaneously setting prices. Firm profits depend on the realization of a binary payoff-relevant state, $\theta \in \Theta = \{\theta_L, \theta_H\}$ with $\theta_H > \theta_L > 0$. Firms share a common prior about the state where the probability of $\theta \in \Theta$ is denoted by $\mu_\theta \in (0,1)$ and face symmetric demand. Firm i's demand at price vector $p = (p_i, p_{-i})$, $q(p;\theta)$, is continuous and differentiable and satisfies the following properties:

$$i)\ \frac{\partial q(p;\theta)}{\partial p_i} \leq 0,\ \frac{\partial q(p;\theta)}{\partial \theta} > 0 \text{ and } \frac{\partial q(p;\theta)}{\partial p_{-i}} > 0,$$

$$ii)\ |\frac{\partial q(p;\theta)}{\partial p_i}| > |\frac{\partial q(p;\theta)}{\partial p_{-i}}| \text{ for all } p \text{ and}$$

$$iii) \frac{\partial^2 q(p;\theta)}{\partial p_i \partial p_{-i}} \geq 0 \text{ for all } p \text{ and } \frac{\partial^2 q(p;\theta)}{\partial \theta \partial p_{-i}} \geq 0.$$

The first condition ensures that quantity demanded decreases as price increases, and that the state and the price of the competitor are positive demand shifters. The second condition implies that goods are differentiated and that a change of its own price has a bigger effect on the demand than a change of the price of a competitor.[8] Lastly, the third condition ensures that the elasticity of demand of firm i is a non-increasing function of the other firm' price and that the demand is supermodular in the state θ and the price of the other firm p_{-i}. I restrict attention to distributions of the payoff-relevant state that satisfy Assumption 1.

Assumption 1 *The difference between θ_L and θ_H is sufficiently small such that both firms are active in the market.*

Assumption 1 imposes an upper bound on the difference between the low and high state, ensuring that equilibrium prices and quantities are strictly positive for both firms, for any information they may have about the state. This assumption restricts attention to the effect of information disclosure on prices, isolating it from the potential effect of inducing a firm to be "priced out" of the market when it selects prices that are not competitive.

Firm ex-post profits. Assume that firms' costs are zero.[9] Hence, firm i's ex-post profits, $\Pi_i : \mathbb{R}^2_+ \times \Theta \to \mathbb{R}$, correspond to

$$\Pi_i(p;\theta) = p_i \cdot q(p;\theta).$$

Assume that firm's ex-post profits are strictly concave in p_i.

[8] This ensures that equilibrium prices are finite.

[9] Including linear or quadratic costs has no impact on the results.

Ex-post consumer surplus. Let $Q(\cdot;\theta) : \mathbb{R}^2_{++} \to \mathbb{R}^2_{+}$ with $Q_i(p,\theta) = q(p,\theta)$ denote the demand system. Under the stated assumptions, i) Q is continuously differentiable and

$$\begin{bmatrix} \dfrac{\partial q(p;\theta)}{\partial p_1} & \dfrac{\partial q(p;\theta)}{\partial p_2} \\ \dfrac{\partial q(p;\theta)}{\partial p_1} & \dfrac{\partial q(p;\theta)}{\partial p_2} \end{bmatrix}$$

is ii) symmetric and iii) negative semi-definite. Nocke and Schutz (2017) shows that quasi-linear integrability of demand is equivalent to these three properties of the demand system. In particular, under these conditions, the indirect utility $v : \mathbb{R}^2_{++} \to \mathbb{R}$ function is unique up to an additive constant, continuously differentiable and $\bigtriangledown v = -Q$. Hence, given the state θ and prices (p_i^*, p_{-i}^*), ex-post consumer surplus is characterized by $v(p^*;\theta)$.

Information environment. Firm 1 (the informed firm) learns the state, whereas firm 2 (the uninformed firm) initially has no information beyond the common prior. Assume that a designer can restrict (or require) information sharing between firms by choosing the information observed by the uninformed firm. In the spirit of information design, the designer selects and commits to an information structure before the realization of the state which discloses none, some, or all of the informed firm's private information to the uninformed firm. Let S_2 be the set of signal realizations observed by firm 2. An information structure consists of a set of signal realizations S_2 and a family of conditional distributions $\psi_2 : \Theta \to \Delta(S_2)$. The information structure is observed by both firms but signal realizations are private.

Specifically, the timing is as follows: (i) the designer selects and commits to an information structure (S_2, ψ_2) observed by both firms; (ii) the state θ is realized and privately observed by the informed firm; (iii) the signal realization is realized and privately observed by the uninformed firm according to (S_2, ψ_2); (iv) firms update their beliefs according to Bayes' rule and simultaneously choose prices; (v) payoffs are realized.

Two features on the information structure warrant further discussion. First, the commitment assumption is standard in the literature and can be interpreted as a reputation concern.[10] In practice, Amazon shares data with other firms through algorithmic price recommendations based on consumer searches and purchases. We should expect that Amazon to use automated recommendations, rather than designing a new algorithm each time demand for a given product is realized. Second, for any disclosure policy, consumers are better off when signals are private, while it has no effect on optimal disclosure for firms.[11] One

[10] See for example Mathevet et al. (2019), Vives (1984), Novshek and Sonnenschein (1982) or Bergemann et al. (2015b).

[11] Similarly, Bergemann et al. (2015b) shows in a Cournot setting that it is with loss of generality to restrict

interpretation of private disclosure is that the informed firm observes the uninformed firm's signal realization, but doesn't condition its pricing on it. For example, Amazon may observe the recommendations made to sellers, but any given recommendation typically does not feed back into its pricing.

Pricing game. *Fixing the information structure* (S_2, ψ_2), firms play a pricing game in which they condition their pricing on their information by selecting mappings

$$\beta_1 : \Theta \to \Delta(\mathbb{R}_+) \text{ and } \beta_2 : S_2 \to \Delta(\mathbb{R}_+)$$

to maximize their expected profits.[12] The solution concept is Bayes Nash equilibrium (BNE). A strategy profile (β_1, β_2) is a BNE if, for all $p_i \in \text{supp } \beta_i$, p_1 in state θ maximizes firm 1's expected profits given the signal distribution ψ_2 and the distribution of prices set by firm 2, β_2 whereas p_2 of signal s_2 maximizes firm 2's expected profits given the prior distribution of the state and the distribution of prices set by firm 1. Formally,

$$\int_{S_2}\int_{\mathbb{R}_+} \Pi_1((p_1, p_2); \theta) \mathrm{d}\beta_2(p_2|s_2) \mathrm{d}\psi_2(s_2|\theta) \geq \int_{S_2}\int_{\mathbb{R}_+} \Pi_1((p_1', p_2); \theta) \mathrm{d}\beta_2(p_2|s_2) \mathrm{d}\psi_2(s_2|\theta) \quad (1)$$

for all $p_1' \in \mathbb{R}_+$ and $\theta \in \Theta$ and

$$\sum_{\theta\in\Theta} \mu_\theta \int_{\mathbb{R}_+} \Pi_2((p_2, p_1); \theta) \mathrm{d}\beta_1(p_1|\theta) \geq \sum_{\theta\in\Theta} \mu_\theta \int_{\mathbb{R}_+} \Pi_2((p_2', p_1); \theta) \mathrm{d}\beta_1(p_1|\theta) \quad (2)$$

for all $p_2' \in \mathbb{R}_+$ and $s_2 \in S_2$.

For any information structure (S_2, ψ_2), the existence and uniqueness of the BNE is guaranteed by Ui (2016), which provides sufficient conditions for the existence and uniqueness of the BNE in Bayesian games with concave and continuously differentiable payoff functions. This result is formalized in Lemma 1. The proofs of this result and all subsequent others are in Appendix A.2.

Lemma 1 *For all information structures* (S_2, ψ_2)*, there is a unique BNE in the pricing game.*

Information disclosure. The choice of information structure (S_2, ψ_2) determines the equilibrium in the pricing game. The designer chooses an information structure to maximize its

attention to public information disclosure, since it comes at the cost of ex-ante welfare.

[12]Note that different information structures (S_2, ψ_2) induce different optimal strategies for both firms.

ex-ante expected payoff inducing $(\beta_1^*(p_1|\theta), \beta_2^*(p_2|s_2))$ as the BNE of the pricing game.[13] Formally, let $W(\psi_2;\gamma)$ denote the designer's expected payoff where $\gamma_i \in [0,1]$ with $\gamma_1+\gamma_2 \leq 1$ represents the welfare weight that the designer attaches to firm i. Then,

$$W(\psi_2;\gamma) = \sum_{i=1,2} \gamma_i \cdot \mathbb{E}_{(\mu,\psi_2)}\left[\Pi_i(p);\theta)\right] + (1-\gamma_1-\gamma_2)\mathbb{E}_{(\mu,\psi_2)}\left[v(p;\theta)\right],$$

where

$$\mathbb{E}_{(\mu,\psi_2)}\left[\Pi_i(p);\theta)\right] = \sum_{\theta\in\Theta} \mu_\theta \int_{S_2}\int_{\mathbb{R}_+}\int_{\mathbb{R}_+} \Pi_i((p_i,p_{-i});\theta)\mathrm{d}\beta_1^*(p_1|\theta)\mathrm{d}\beta_2^*(p_2|s_2)\mathrm{d}\psi_2(s_2|\theta)$$

is firm i's expected profits and

$$\mathbb{E}_{(\mu,\psi_2)}\left[v(p;\theta)\right] = \sum_{\theta\in\Theta} \mu_\theta \int_{S_2}\int_{\mathbb{R}_+}\int_{\mathbb{R}_+} v(p;\theta)\mathrm{d}\beta_1^*(p_1|\theta)\mathrm{d}\beta_2^*(p_2|s_2)\mathrm{d}\psi_2(s_2|\theta)$$

is consumer surplus.

The interpretation of the role of the designer varies depending on their objective function. If $\gamma_1 = 1$, the designer's objective is to maximize the informed firm's expected profits, then it is as if the informed firm is choosing how much information to disclose to its competitor. If $\gamma_1 = \gamma_2 = \frac{1}{2}$, the designer's objective is to maximize expected producer surplus, it is as if there is a collusive agreement between firms to determine optimal disclosure of information among them. If $\gamma_1 = \gamma_2 = 0$ or $\gamma_1+\gamma_2 \neq 1$, the designer's objective is to maximize expected consumer surplus or welfare respectively, then the interpretation of the designer is as a regulator.

Equivalence to recommendation mechanisms. The revelation principle for games of communication simplifies the information design problem by constraining the set of information structures. Taneva (2019) shows that it is without loss of generality to restrict attention to information structures where signals are equilibrium recommendations conditional on the state. I present an extension to compact action spaces and bounded, continuous real-valued payoff functions, restricting attention to $p_i \in [0,\overline{p}]$ for all $i \in \{1,2\}$.[14] In a recommendation

[13] I make the additional assumption that firms do not update their beliefs about the state after observing a deviant information structure. Since the designer chooses the information structure before the state is realized, strategic independence only requires this additional constraint and, therefore, this solution concept is equivalent to Perfect extended Bayesian Equilibrium. See Battigalli (1996) or Watson (2016) for details.

[14] The strict concavity of firm's ex-post profits in p_i imply that firms' profits are bounded and continuous functions. Moreover, there exists $\overline{p}$ such that it is without loss of generality to restrict attention to the compact action space $p_i \in [0,\overline{p}]$.

mechanism, the pricing rule $\sigma : \Theta \to \Delta\left([0,\overline{p}]^2\right)$ recommends a price for each firm such that the obedience constraints are satisfied, ensuring that firms are willing to follow the recommendation. Any pricing rule which satisfies the obedience constraints is a Bayes Correlated Equilibrium (BCE) as introduced by Bergemann and Morris (2013).[15] That is, a pricing rule $\sigma : \Theta \to \Delta\left([0,\overline{p}]^2\right)$ is a BCE if

$$\sum_{\theta\in\Theta} \mu_\theta \int_{p_{-i}\in[0,\overline{p}]} \Pi_i(p,\theta)\mathrm{d}\sigma(p|\theta) \geq \sum_{\theta\in\Theta} \mu_\theta \int_{p_{-i}\in[0,\overline{p}]} \Pi_i((p_i', p_{-i}),\theta)\mathrm{d}\sigma(p|\theta)$$

for all $p_i \in \operatorname{supp}\sigma$, $p_i' \in [0,\overline{p}]$, and $i \in \{1,2\}$, such that the distribution of the informed firm's price given the state is degenerated. Every possible BCE distribution can be replicated as a BNE by appropriately choosing the information structure. A detailed discussion of this equivalence is presented in Appendix A.1.

Existence of the optimal mechanism. The existence of the optimal recommendation mechanism stated in Lemma 2 is guaranteed by the Weierstrass extreme value theorem. First, the existence of correlated equilibria for games in which players receive private signals and simultaneously choose actions from compact sets is established in Stinchcombe (2011). Second, the set of BCE is compact in the weak* topology, since it is the set of all probability measures on a compact set.[16] Then, the designer's problem is to maximize a continuous function of σ over a non-empty compact set.

Lemma 2 *The optimal recommendation mechanism exists.*

Simplifications. The existence and uniqueness of BNE imply that it is sufficient to restrict attention to the distribution $\sigma(p_2|\theta)$ since for any obedient recommendation mechanism there exists a function $p_1(\theta, \sigma(p_2|\theta))$ which represents firm 1's best response when the state is θ and the price recommendations are given by σ where

$$p_1(\theta, \sigma(p_2|\theta)) = \arg\max_{p_1} \int_{p_2} \Pi_1(p;\theta)\mathrm{d}\sigma(p_2|\theta).$$

By Leibniz rule, $p_1(\theta, \sigma(p_2|\theta))$ is implicitly characterized by

$$\int_{p_2} q(p;\theta)\mathrm{d}\sigma(p_2|\theta) + p_1 \frac{\partial \int_{p_2} q(p;\theta)\mathrm{d}\sigma(p_2|\theta)}{\partial p_1} = 0.$$

[15] In my model, unlike in Bergemann and Morris (2013) in which both players are uninformed about the state, firm 1 learns the state before selecting prices. The definition of BCE is adapted to account for this.

[16] Firms have no incentives to set prices above the full disclosure price $p^F(\theta_H)$ or below $p^F(\theta_L)$, because such prices would never be part of a BNE of the pricing game. Hence, the support of any obedient recommendation mechanism must be a subset of $[p^F(\theta_L), p^F(\theta_H)]^2$.

Furthermore, for any information structure σ, the set of recommended equilibrium prices for firm 2 in the pricing game is a subset of the interval between the equilibrium prices with full disclosure. This is formalized in Lemma 3.

Lemma 3 *The support of any obedient distribution $\sigma(p_2|\theta)$ is a subset of $[p^F(\theta_L), p^F(\theta_H)]$ for all $\theta \in \Theta$ where $p^F(\theta)$ is the equilibrium price with full disclosure when the state θ is realized.*

2 Informed firm optimal disclosure

In this section, the informed firm directly determines its optimal information disclosure. That is, assume that the designer's objective is to maximize the informed firm's expected profits,

$$W(\sigma, (1,0)) = \mathbb{E}_{(\mu,\sigma)}[\Pi_1(p;\theta)] = \sum_{\theta\in\Theta} \mu_\theta \int \Pi_1(p;\theta)\mathrm{d}\sigma(p|\theta).$$

The informed firm chooses a feasible obedient recommendation mechanism σ to maximize its expected equilibrium profits in the pricing game. From its point of view, whether disclosure is private or public has no impact on the optimal disclosure policy. Proposition 1 states that it is optimal for the informed firm to share its information.

Proposition 1 (Informed firm optimal disclosure) *It is optimal for the informed firm to fully reveal its private information to the uninformed firm.*

The optimal disclosure policy is determined by the fact that pricing choices are strategic complements since the informed firm's expected equilibrium payoff conditional on the state, $\mathbb{E}_\sigma[\Pi_1^*(p;\theta)|\theta]$, is supermodular in the state and the price of the other firm given our assumptions on the demand system. This determines the effect of changes in the precision of the uninformed firm's signal on the informed firm's expected profits. Then, maximizing the informed firm's expected equilibrium profits is equivalent to maximizing the distance between the expected equilibrium prices set by the uninformed firm across states, $\mathbb{E}_\sigma[p_2|\theta_L]$ and $\mathbb{E}_\sigma[p_2|\theta_H]$. Increasing the precision of the signal observed by the uninformed firm increases the correlation between its expected price and the state and, therefore, variation in its expected price. As a result, full disclosure maximizes the informed firm's expected profits.[17]

[17]This result relies on Assumption 1. Otherwise, if the high state is sufficiently high and the uninformed

Intuitively, increasing the precision of the signal observed by the uninformed firm increases its certainty about the state, increasing (decreasing) expected demand when its posterior beliefs suggest that the high (low) state is more likely. Accordingly, the uninformed firm increases its expected equilibrium price in the high state and decreases it in the low state. As a result, with substitutes, more precise information disclosure increases (decreases) the informed firm's expected demand in the high (low) state. In the high state, a higher expected demand allows it to increase its price. The informed firm then increases its profits by raising the price on inframarginal consumers who were already buying its product and by gaining marginal consumers from the uninformed firm's market. The opposite is true in the low state since it charges a lower price and faces lower demand, but the expected profit gain in the high state exceeds the expected loss in the low state given the larger size of the market. Hence, the informed firm benefits from price correlation and its expected equilibrium profits increase in the precision of the uninformed firm's signal. Since this precision is maximized by full disclosure, it is optimal for the informed firm to fully disclose its private information. [18]

Information disclosure impacts the surplus allocation between firms. It is straightforward to show that it is also optimal for the uninformed firm to learn the state, because it increases the correlation between its pricing decisions and the state and, thus, full disclosure maximizes expected producer surplus.

This result is intuitively related to Kamenica and Gentzkow (2011), which finds that full disclosure is optimal when a sender's expected payoff is strictly convex. However, this result doesn't directly apply to my setting, since the informed firm (the sender) and the uninformed firm (the receiver) play a game after the uninformed firm privately observes its signal realization. As such, payoffs not only depend on the state and the action of the uninformed firm, but also on the action of the informed firm. My results highlight that their intuition holds more broadly, not only in decision problems, but also in games in which payoffs are supermodular in the state and the actions of others. Kolotilin and Wolitzky (2020) obtain a related result in a setting in which the sender and the receiver do not interact. They show that supermodularity of the sender's objective function with respect to the state and

firm observes a sufficiently uninformative signal about demand, its prices can be only competitive when the state is high. Then, the informed firm can be effectively a monopolist when demand is low if it shares sufficiently imprecise information about the state. Therefore, from the informed firm's perspective, sharing their private information could harm it, because it can induce more competition when demand is low.

[18] It is optimal for the informed firm to share none of its private information with the uninformed firm when firms offer complements. These results also extend to Cournot competition using usual equivalence arguments.

the receiver's action is a sufficient condition for the optimality of full disclosure in decision problems.

My results strengthen findings from previous work (Vives (1984), Vives (1990) and Raith (1996)), by showing the optimality of either full or no information disclosure in a setting of information asymmetry where the distribution of the uninformed firm's signal and the correlation with the informed firm's signal are unrestricted.[19] One takeaway is that it can be optimal for a firm to disclose information to a competitor even when it has no information to gain in return, because the firm can use disclosure to influence competitor prices.

3 Consumer optimal disclosure

In this section, I interpret the designer as a regulator whose objective is to maximize expected consumer surplus. We can interpret the regulator as a consumer protection agency who requires the informed firm to make its private information available to them. It can then privately share all or a subset of this information with the uninformed firm.[20]

In particular, assume that the designer's objective is to choose an obedient price recommendation mechanism σ that maximizes expected consumer surplus, given by

$$W(\sigma, (0,0)) = \mathbb{E}_{(\mu,\sigma)}[v(p;\theta)] = \sum_{\theta\in\Theta} \mu_\theta \int v(p;\theta)\mathrm{d}\sigma(p|\theta).$$

First, I show that the consumer optimal recommendation mechanism recommends at most two prices. If the optimal recommendation mechanism discloses some information, I show that it recommends a high price only recommended in the high state and a low price recommended in both states. The recommended prices maximize the uninformed firm's expected profits given its beliefs about the state. Then, the optimal price recommendation mechanism is characterized by the probability of recommending the low price in the high state, denoted by λ^*, where λ^* determines the recommended prices $\hat{p}_L$ and $\hat{p}_H$ and is chosen to maximize expected consumer surplus subject to firm optimal pricing.

Proposition 2 (Consumer optimal recommendation mechanism) *Any consumer optimal recommendation mechanism recommends at most two prices. If an optimal mechanism discloses information, then there exists an optimal mechanism that recommends one price* $\hat{p}_H$ *only when the state is high and another price* $\hat{p}_L$ *in both states.*

[19] They also strengthen results from Novshek and Sonnenschein (1982), Clarke (1983) and Gal-Or (1985), given that Cournot with substitutes (complements) is equivalent to Bertrand with complements (substitutes) from the point of view of firms, as discussed in Raith (1996).

[20] Alternatively, the regulator can require the informed firm to directly share a specific subset of its information with the uninformed firm, as long as the informed firm's pricing cannot be conditioned on it.

Intuitively, consumers gain from disclosure when there are differences in firm pricing. When the state is low and the informed firm sets a corresponding low price, recommending an intermediate rather than a high price to the uninformed firm would provide less benefit to consumers, implying that it is best for consumers for at most a low and a high price to be recommended. Given that no intermediate price would be recommended in the low state, an intermediate price recommendation would reveal to the uninformed firm that the state is high, but the uninformed firm would only be willing to set the high price in that case. Hence, an optimal price recommendation mechanism recommends at most two prices.

The sketch of the proof of Proposition 2 is as follows. First, I show that at most two prices are recommended in a state if only one price is recommended in the other state. If a unique price $\hat{p}$ is recommended in one state, observing any other recommendation $p_2 \neq \hat{p}$ reveals the state to the uninformed firm. When the uninformed firm knows demand, the obedience constraint implies that there is a unique price that it is willing to set. As a result, it is not possible to recommend more than two obedient prices across states. Second, I show that it is optimal for the regulator to recommend a unique price in one state. These results imply that the optimal information structure sends at most two price recommendations.

Some of the intuition behind the characterization of the optimal disclosure relates to Kamenica and Gentzkow (2011). They show that there exists an optimal mechanism which induces a distribution of posteriors whose support has no more than $|\Theta|$ elements, here corresponding to at most two prices. However, as they discuss themselves, their results do not apply to settings with multiple receivers whose payoffs depend on each others' actions and in which the designer (the regulator) can send private signals to each receiver. This is because, for a given set of beliefs that firms hold after observing their signals, their actions may vary as a function of the disclosure policy that produced those beliefs. Accordingly, I extend the intuition behind their results to a setting in which firms privately observe a signal about demand before engaging in Bertrand competition with differentiated goods.

The optimal disclosure is formalized in Proposition 3. Partial disclosure is optimal for consumers when firms offer sufficiently close substitutes. Otherwise, no disclosure is optimal.

Proposition 3 (Consumer optimal disclosure) *If the designer's objective is to maximize expected consumer surplus, the optimal disclosure policy is chosen to induce goods to be closer substitutes and intensify the competition across firms.*

Intuitively, the impact of disclosure on consumer surplus is determined through two channels. On the one hand, disclosure provides the uninformed firm with information about the state, which increases the correlation between its pricing and the state. Indirectly, this also increases pricing correlation across firms. Accordingly, in expectation, firms more accurately tailor their prices to the demand they face, allowing them to better extract surplus from consumers. On the other hand, it creates uncertainty in firms' pricing decisions, because both firms now have private information. Even if disclosure increases expected price correlation between firms, uncertainty about the signal realization observed by their competitor generates a pricing coordination failure with positive probability. That is, the uninformed firm may observe a signal realization that mismatches with the state, setting a price tailored to the incorrect state. In contrast, the informed firm sets a price tailored to the realized state. When the mismatch occurs and firms set different prices, consumers benefit by selecting from which firm to purchase after observing prices.

The relative impact of these effects is determined by the degree of differentiation between goods. When goods are close substitutes, a price differential between firms caused by partial private disclosure induces a large segment of the market to buy from the firm with a comparatively low price, creating large gains in consumer surplus with positive probability. In contrast, when goods are not close substitutes, the pricing coordination failure has little impact on the demand that firms face, yielding negligible benefits. Accordingly, when goods are sufficiently close substitutes, private partial disclosure creates a large enough expected benefit from a potential price coordination failure for the regulator to impose partial disclosure. Otherwise, no disclosure is optimal.

The sketch of the proof is as follows. First, I verify that no disclosure is always better for consumers than full disclosure. Second, I show that the difference between the expected consumer surplus with partial and no disclosure is a continuous and strictly increasing function of the degree of substitution. Third, I show that there exists a cutoff in the degree of differentiation above which partial disclosure is optimal.

The information environment I consider represents a lower bound on the potential benefits for consumers, because the benefits of private disclosure are minimized when there is complete information asymmetry between firms. This is because, when the informed firm is only partially informed, the regulator can induce a coordination failure in firm pricing with higher probability.

Furthermore, consumers can also benefit from disclosure when the uninformed firm's signal realization is noisily observed by the informed firm. As long as the informed firm is sufficiently uncertain about the information observed by the uninformed firm, it is possible for the regulator to create the coordination failure in prices with sufficiently high probability. Therefore, the optimality of partial disclosure doesn't rely on the fact that information disclosure is private, even though private disclosure maximizes the probability of inducing the coordination failure and is, as a result, optimal for consumers.

4 Welfare optimal disclosure

Assume that the designer, interpreted as a regulator, wants to maximize expected welfare, defined as the sum of expected consumer and producer surplus. Incentives for partial disclosure are driven by the effect of disclosure on consumer surplus. The qualitative features of the policy are shared with the consumer optimal one, that is, any optimal partially informative recommendation mechanism has binary support, recommends one price only when the state is low, and another price in both states.

In this context, the regulator trades off the effect of information disclosure on firms and consumers, given their conflicting preferences over disclosure policies. In particular, firms' expected profits are maximized by full disclosure, whereas expected consumer surplus is maximized by no or partial disclosure. However, the benefits from disclosure for both firms and consumers increase as firms offer closer substitutes. When firms offer sufficiently close substitutes, full disclosure is optimal since it is optimal for firms and their expected gains exceed expected losses for consumers. When firms offer sufficiently differentiated substitutes, no disclosure maximizes expected welfare since it is optimal for consumers and the expected gains for firms from disclosure are small. For intermediate levels of differentiation, partial disclosure is optimal. The levels of differentiation for which in each type of disclosure is optimal depend on the welfare weights, γ_1 and γ_1: for sufficiently high $1-\gamma_1-\gamma_2$, only no or partial disclosure can be optimal; for sufficiently low values of ω, full disclosure is always optimal.

Ui and Yoshizawa (2015) reach a similar conclusion, restricting attention to symmetric normally distributed private and public signals. When firms offer substitutes, they show that welfare decreases in the precision of private information and increases in the precision of public information, related to the optimality of either partial or full disclosure. These results suggest that a regulator whose objective is to maximize welfare faces a trade off between consumer and producer surplus, and must take into account the relationship between markets. They highlight that the task of a regulator can be more nuanced than simply banning or releasing information: the exact design of information matters.

5 Robustness of results

In this section, I discuss the robustness of the results and how they change as I enrich the model.

First, consumers are better off when signal realizations are private instead of public since they benefit from the induced uncertainty between firms, whereas firms' optimal disclosure remains the same. When signals are public, the gain from partial disclosure disappears and no disclosure is optimal for consumers. However, consumers can also benefit from disclosure when the uninformed firm's signal realization is noisily observed by the informed firm. As long as the informed firm is sufficiently uncertain about the information observed by the uninformed firm, it is possible for the regulator to create the coordination failure in prices with sufficiently high probability. Therefore, the optimality of partial disclosure doesn't rely on information disclosure being private.

Second, consumers benefit more from partial disclosure as the asymmetry in market size between firms increases, implying that their benefits from partial disclosure are minimized when firms face markets of the same size. While firm preferences for information remain unaltered, partial disclosure increases consumer surplus in the informed firm's market and reduces it in the uninformed firm's market. Then, when the informed firm faces a bigger market than the uninformed firm, incentives for partial disclosure are larger. Hence, partial disclosure is optimal for a bigger range of degrees of differentiation. In practice, this is specially relevant since firms with an information advantage may often be larger, like Amazon. Lastly, the features of the consumer and welfare optimal recommendation mechanism generalize to this case.

Third, the main intuition of the consumer optimal disclosure extends to the case in which N firms compete à la Bertrand and the designer selects an information structure from a constrained set. Suppose that the designer commits to an information structure with private signals to share all of the informed firm's private information with a subset of firms and no information with the rest. The informed firm's expected equilibrium profits are maximized by sharing its private information with all other firms, because it benefits from price correlation. However, I conjecture that if the designer's objective is to maximize expected consumer surplus, information disclosure between firms is at least partially restricted. The optimal information structure is determined by the degree of substitution and the number of firms in the market. It is optimal to share information with more firms as the number of firms increase in the market and as firms offer closer substitutes, but it is optimal to leave at least a fraction of firms uninformed. By leaving some firms uninformed, the designer is able to increase price heterogeneity, benefiting consumers.

Fourth, the informed firm's incentives to share information are amplified if it can charge a fee to the other firm to use its platform, but that consumer optimal and welfare optimal disclosures remain unaltered. Furthermore, the producer surplus optimal disclosure remains unchanged, since this represents a transfer between firms.

Fifth, the informed firm's incentives for information sharing are reversed when firms offer complements and the producer surplus optimal disclosure depends on the degree of complementary between goods. When goods are complements, disclosure increases the uninformed firm's profits, but reduces the informed firm's profits. In particular, if goods are sufficiently complementary, competitor prices have a significant impact on demand. Then, the negative effect of increased pricing correlation on the informed firm's profits exceeds the positive effect of learning about the state on the uninformed firm's profits. As a result, no disclosure is optimal. Otherwise, full disclosure is optimal.

Sixth, the proofs of Proposition 1 and Proposition 3 hold more generally for $[\theta_L, \theta_H]$, whereas the characterization of the optimal disclosure policy, Proposition 2, holds for $\{\theta_L, \theta_H\}$. In particular, considering more states would require increasing the number of price recommendations.

6 Conclusion

This paper studies information disclosure in a setting where two competing firms face ex-ante information asymmetry about demand. I examine the incentives of an informed firm to share its private information with a competitor in a market with product differentiation and price competition. I show that the informed firm can have incentives to fully disclose its private information even without receiving information in return, because it allows it to influence competitor pricing. When firms offer substitutes, they benefit from price correlation, which implies that it is optimal for the informed firm to fully reveal its private information to the uninformed firm.

Information disclosure also impacts consumers. Even though complete information disclosure can help firms, it hurts consumers. I find that a regulator with the objective of protecting consumers would either completely restrict information disclosure between firms or only allow private partial disclosure, determined by the degree of differentiation between goods. If goods are sufficiently close substitutes, partial disclosure is optimal, because it increases the benefit for consumers to reallocate across markets. The consumer optimal partial disclosure reveals low demand and obfuscates high levels to the uninformed firm.

Moreover, preferences for information disclosure between firms and consumers are not aligned. If firms offer sufficiently differentiated goods, no disclosure maximizes expected welfare. If firms offer sufficiently close substitutes, full disclosure is optimal. For intermediate levels of differentiation, partial disclosure is optimal. Since incentives for partial disclosure arise from consumers, the optimal partial disclosure also reveals low and obfuscates high demand to the uninformed firm. My results highlight the wide scope for potential intervention by regulators, depending on their objective function and product differentiation.

This paper speaks in a preliminary form about the competition issues that arise when there is an unequal distribution of consumer data among firms. An important aspect not considered in this paper is the effect of information disclosure on firm entry and exit decisions. In particular, the informed firm could reduce its information disclosure, reducing its current profits to increase its market share and future profits by inducing uninformed firms to exit the market. In this context, a regulator may have incentives to force information disclosure between firms to maintain competition, which could indirectly benefit consumers. Furthermore, if firms could choose their product offering and the state reflected consumer preferences over horizontally differentiated goods, an informed firm may not want to disclose information if it would lead their competitor to offer a similar product and intensify price competition.

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A Appendix

A.1 Equivalence to recommendation mechanisms

Consider an analogous information environment in which both firms observe a signal about the state with private signal realizations such that the informed firm's signal is perfectly informative. In what follows, I show that it is without loss of generality to interpret signals (s_1, s_2) as equilibrium recommendations in which each signal recommends a price to each firm. Define the information structure as the joint distribution of signals. Let S_i be the set of private signal realizations for firm i. An information structure consists of a set of signal realizations and a family of conditional distributions $\psi : \Theta \to \Delta(S)$, where $S = S_1 \times S_2 = \{s_L, s_H\} \times S_2$. Let $\psi_i : \Theta \to \Delta(S_i)$ be the marginal distribution of signal $s_i \in S_i$ given the information structure (S, π). The distribution ψ_1 is fully informative, which implies that the probability of observing signal s_k conditional on state θ_k is 1.

Given the information structure (S, ψ), firms play a pricing game in which they condition their pricing choices on their signal realization by selecting a mapping $\beta_i : S_i \to \Delta([0, \overline{p}])$ to maximize their expected profits. A strategy profile (β_1, β_2) is a BNE if, for all $p_i \in [0, \overline{p}]$ with $\beta_i(p_i|s_i) > 0$ for all i, we have

$$\sum_{\theta\in\Theta} \mu_\theta \int_{S_{-i}} \int_0^{\overline{p}} \Pi_i(p;\theta)\mathrm{d}\beta_{-i}(p_{-i}|s_{-i})\mathrm{d}\psi(s|\theta) \geq \sum_{\theta\in\Theta} \mu_\theta \int_{S_{-i}} \int_0^{\overline{p}} \Pi_i((p_i', p_{-i});\theta)\mathrm{d}\beta_{-i}(p_{-i}|s_{-i})\mathrm{d}\psi(s|\theta) \tag{3}$$

for all $p_i' \in [0, \overline{p}]$, $s \in S$ and $i \in \{1, 2\}$.Denote by $\mathcal{E}(S, \psi)$ the set of BNE.

Similarly, we can define a pricing rule $\sigma : \Theta \to \Delta\left([0, \overline{p}]^2\right)$ which is a BCE if

$$\sum_{\theta\in\Theta} \mu_\theta \int_{p_{-i}\in[0,\overline{p}]} \Pi_i(p, \theta)\mathrm{d}\sigma(p|\theta) \geq \sum_{\theta\in\Theta} \mu_\theta \int_{p_{-i}\in[0,\overline{p}]} \Pi_i((p_i', p_{-i}), \theta)\mathrm{d}\sigma(p|\theta) \tag{4}$$

for all $p_i \in \operatorname{supp} \sigma$, $p_i' \in [0, \overline{p}]$, and $i \in \{1, 2\}$, in which the distribution of the informed firm's price given the state is degenerated.

First, Lemma 4 is an equivalence result stating that every possible BCE distribution can be replicated as a BNE by appropriately choosing the information structure. Intuitively, any correlation between obedient pricing choices can be generated as a BCE. In a BNE, all the correlation between pricing choices is generated through the information structure (S, ψ).

Lemma 4 *The set of BCE coincides with* $\cup_{(S,\psi)}\mathcal{E}(S,\psi)$.

Second, Lemma 5 implies that it is without loss of generality to restrict attention to recommendation mechanisms. Formally, an information structure (S,ψ) is a recommendation mechanism if $S = \left[0, \frac{\theta_H}{a-b}\right]^2$. In a recommendation mechanism, signals act as pricing recommendations which firms are willing to follow as long as their competitor does as well.

Lemma 5 *For every* $\sigma \in \cup_{(S,\psi)}\mathcal{E}(S,\psi)$*, there exists a recommendation mechanism* $\left([0,\overline{p}]^2, \sigma\right)$ *such that* $\sigma \in \mathcal{E}\left([0,\overline{p}]^2, \sigma\right)$.

A.1.1 Proofs

Proof. Lemma 4. First, I show that the set of BCE is a subset of $\cup_{(S,\psi)}\mathcal{E}(S,\psi)$. Assume $\sigma \in BCE$. Then, σ satisfies

$$\sum_{\theta\in\Theta}\mu_\theta \int_{p_{-i}\in[0,\overline{p}]} \Pi_i(p,\theta)\mathrm{d}\sigma(p|\theta) \geq \sum_{\theta\in\Theta}\mu_\theta \int_{p_{-i}\in[0,\overline{p}]} \Pi_i((p_i', p_{-i}),\theta)\mathrm{d}\sigma(p|\theta) \tag{5}$$

for all $p_i \in \text{supp } \sigma$, $p_i' \in [0,\overline{p}]$ and $i \in \{1,2\}$.

Consider an information structure $\left([0,\overline{p}]^2, \psi^*\right)$ where $[0,\overline{p}]^2$ is the set of signal realizations and $\psi^* : \Theta \to \Delta([0,\overline{p}]^2)$ coincides with σ, i.e. $\sigma = \psi^*$. Let

$$\beta_i^*(p_i|p_i') = \begin{cases} 1 & if\ p_i = p_i' \\ 0 & otherwise \end{cases}$$

be the obedient strategy. Then, the right-hand side of (5) can be written as

$$\begin{aligned} &\sum_{\theta\in\Theta}\mu_\theta \int_{p_{-i}\in[0,\overline{p}]} \Pi_i((p_i', p_{-i}),\theta)\mathrm{d}\sigma(p|\theta) = \sum_{\theta\in\Theta}\mu_\theta \int_{p_{-i}\in[0,\overline{p}]} \Pi_i((p_i', p_{-i}),\theta)\mathrm{d}\psi^*(p|\theta) \\ &= \sum_{\theta\in\Theta}\mu_\theta \int_{s_{-i}\in[0,\overline{p}]}\int_{p_{-i}\in[0,\overline{p}]} \Pi_i((p_i', p_{-i});\theta)\mathrm{d}\beta_{-i}^*(p_{-i}|s_{-i})\mathrm{d}\psi^*(s|\theta) \end{aligned}$$

The first equality holds by definition of ψ^*. The second equality holds by definition of the obedient strategy and Fubini's theorem since, fixing θ, $\Pi_i(p;\theta)$ is σ-integrable because $\Pi_i|\theta : [0,\overline{p}]^2 \to \mathbb{R}_+$ is a bounded and continuous real-valued function on a compact set.[21] Hence, the BNE incentive-compatibility constraints are implied by the BCE obedience constraints.

[21]See theorem 11.27 from Aliprantis and Border (2013) where the condition of theorem are satisfied by Proposition 3.3 and Theorem 4.4 from from Royden (1968)

This, in turn, implies that if $\sigma \in BCE$, then σ is also a BNE of the game. Thus, the set of BCE is a subset of the set of BNE of the game.

Second, I show that $\cup_{(S,\psi)}\mathcal{E}(S,\psi)$ is a subset of BCE. Consider a BNE composed by an information structure $(\hat{S},\hat{\psi})$ with $\hat{\psi}:\Theta\to\Delta(S)$ and measurable behavioral strategies $(\hat{\beta}_i,\hat{\beta}_{-i})$.[22] Given the behavioral strategies $(\hat{\beta}_i,\hat{\beta}_{-i})$, define $\hat{\beta}:S\to\Delta\left([0,\overline{p}]^2\right)$ as the joint measure. Let $\hat{\sigma}:\Theta\to\Delta\left([0,\overline{p}]^2\right)$ be the composition of $\hat{\psi}$ and $\hat{\beta}$, defined as $\hat{\sigma}=\hat{\beta}\circ\hat{\psi}$. Then, by definition $\hat{\sigma}\in\cup_{(S,\psi)}\mathcal{E}(S,\psi)$. The definition of BNE implies that $(\hat{S},\hat{\psi})$ and $\hat{\beta}$ satisfy:

$$\begin{aligned}&\sum_{\theta\in\Theta}\mu_\theta\int_{\hat{S}_{-i}}\int_{p_{-i}\in[0,\overline{p}]}\Pi_i(p;\theta)\mathrm{d}\hat{\beta}_{-i}(p_{-i}|s_{-i})\mathrm{d}\hat{\psi}(s|\theta)\\ \geq&\sum_{\theta\in\Theta}\mu_\theta\int_{\hat{S}_{-i}}\int_{p_{-i}\in[0,\overline{p}]}\Pi_i((p_i',p_{-i});\theta)\mathrm{d}\hat{\beta}_{-i}(p_{-i}|s_{-i})\mathrm{d}\hat{\psi}(s|\theta)\end{aligned}\tag{6}$$

for all $p_i'\in[0,\overline{p}]$, $s\in S$ and $i\in\{1,2\}$. Integrating both sides of the BNE incentive-compatibility constraint, we have

$$\begin{aligned}&\sum_{\theta\in\Theta}\mu_\theta\int_{\hat{S}_i}\int_{\hat{S}_{-i}}\int_{p_{-i}\in[0,\overline{p}]}\Pi_i(p;\theta)\mathrm{d}\hat{\beta}_i(p_i|s_i)\mathrm{d}\hat{\beta}_{-i}(p_{-i}|s_{-i})\mathrm{d}\hat{\psi}(s|\theta)\\ \geq&\sum_{\theta\in\Theta}\mu_\theta\int_{\hat{S}_i}\int_{\hat{S}_{-i}}\int_{p_{-i}\in[0,\overline{p}]}\Pi_i((p_i',p_{-i});\theta)\mathrm{d}\hat{\beta}_i(p_i|s_i)\mathrm{d}\hat{\beta}_{-i}(p_{-i}|s_{-i})\mathrm{d}\hat{\psi}(s|\theta)\end{aligned}$$

Then, (6) implies that

$$\sum_{\theta\in\Theta}\mu_\theta\int_{p_{-i}\in[0,\overline{p}]}\Pi_i(p,\theta)\mathrm{d}\sigma(p|\theta)\geq\sum_{\theta\in\Theta}\mu_\theta\int_{p_{-i}\in[0,\overline{p}]}\Pi_i((p_i',p_{-i}),\theta)\mathrm{d}\sigma(p|\theta)$$

■

Proof. Lemma 5. Consider a distribution $\sigma\in\cup_{(S,\psi)}\mathcal{E}(S,\psi)$. Lemma 4 implies that $\sigma\in BCE$. Consider the recommendation mechanism $\left([0,\overline{p}]^2,\psi_\sigma\right)$ where $\psi_\sigma=\sigma$ for all $(p_1,p_2)\in[0,\overline{p}]^2$ and $\theta\in\Theta$ and the obedient behavioral strategy

$$\beta_i^*(p_i|p_i')=\begin{cases}1 & if\ p_i=p_i'\\ 0 & otherwise\end{cases}.$$

[22] Behavioral strategies $\beta_i:S_i\to\Delta\left(\left[0,\frac{\theta_H}{a-b}\right]\right)$ for all $i\in\{1,2\}$ are defined as a regular conditional probabilities as defined in Appendix C from Bass (2011).

The interim expected payoff of firm i when firm $-i$ follows β^*_{-i} is

$$\sum_{\theta\in\Theta}\mu_\theta\int_{S_{-i}}\int_0^{\overline{p}}\Pi_i((p_i', p_{-i});\theta)\mathrm{d}\beta^*_{-i}(p_{-i}|p'_{-i})\mathrm{d}\psi_\sigma((p_i, p'_{-i})|\theta)$$
$$=\sum_{\theta\in\Theta}\mu_\theta\int_0^{\overline{p}}\Pi_i((p_i', p_{-i});\theta)\mathrm{d}\psi_\sigma(p|\theta)$$
$$=\sum_{\theta\in\Theta}\mu_\theta\int_0^{\overline{p}}\Pi_i((p_i', p_{-i});\theta)\mathrm{d}\sigma(p|\theta) \tag{7}$$

for all i. Hence, the definition of BCE and (7) imply

$$\sum_{\theta\in\Theta}\mu_\theta\int_0^{\overline{p}}\Pi_i(p;\theta)\mathrm{d}\psi_\sigma(p|\theta)\geq\sum_{\theta\in\Theta}\mu_\theta\int_0^{\overline{p}}\Pi_i((p_i', p_{-i});\theta)\mathrm{d}\psi_\sigma(p|\theta)$$

for all $p_i'\in[0,\overline{p}]$ and i. The distribution of prices conditional on the state θ under β^* and $([0,\overline{p}]^2,\sigma)$ is $\psi_\sigma=\sigma$. Thus, $\sigma\in\mathcal{E}\left([0,\overline{p}]^2,\sigma\right)$. ■

A.2 Proofs

Proof. Lemma 1. The pricing game is a smooth concave game since $\Pi_i(\cdot, p_{-i};\theta):\mathbb{R}_+\to\mathbb{R}$ is concave and continuously differentiable for each $p_{-i}\in\mathbb{R}_+$. Define the payoff gradient as

$$\nabla\Pi(\mathbf{p},\theta):=\left(\frac{\partial\Pi_i(p;\theta)}{\partial p_i}\right)_{i\in\{1,2\}}.$$

The payoff gradient is continuously differentible and its Jacobian matrix, given by

$$F_{\nabla\Pi}(\mathbf{p},\theta):=\begin{pmatrix}\frac{\partial^2\Pi_1(p;\theta)}{\partial p_1^2} & \frac{\partial^2\Pi_1(p;\theta)}{\partial p_1\partial p_2}\\ \frac{\partial^2\Pi_2(p;\theta)}{\partial p_1\partial p_2} & \frac{\partial^2\Pi_2(p;\theta)}{\partial p_2^2}\end{pmatrix},$$

is negative definite because

$$\frac{\partial^2\Pi_i(p;\theta)}{\partial p_i^2}<0\text{ and }\frac{\partial^2\Pi_i(p;\theta)}{\partial p_i^2}\frac{\partial^2\Pi_{-i}(p;\theta)}{\partial p_{-i}^2}\geq\left(\frac{\partial^2\Pi_i(p;\theta)}{\partial p_i\partial p_{-i}}\right)^2.$$

This implies that the payoff gradient $\nabla\Pi(\mathbf{p},\theta)$ is strictly monotone by Lemma 4 from Ui (2016). Furthermore, since for all p, there exists $c>0$ such that $p^TF_{\nabla\Pi}(\mathbf{p},\theta)p<-cp^Tp$, the payoff gradient is also strongly monotone by the same lemma. Then, the uniqueness of the Bayesian Nash equilibrium of the pricing game follows from Proposition 1 from Ui (2016), which states that if the payoff gradient is strictly monotone, the Bayesian game as at most one Bayesian Nash equilibrium. The existence of a unique Bayesian Nash equilibrium follows from Proposition 2 from Ui (2016). ■

Proof. Lemma 2. The set of BCE is the collection of distributions $\sigma : \Theta \to \Delta([p^F(\theta_L), p^F(\theta_H)]^2)$ such that

i) $\sigma(p|\theta) \geq 0$ for all $p \in [p^F(\theta_L), p^F(\theta_H)]^2$ and $\theta \in \Theta$,

ii) $\displaystyle\int \mathrm{d}\sigma(p|\theta) = 1$ for all $\theta \in \Theta$ and

iii) $\displaystyle\sum_{\theta\in\Theta} \mu_\theta \int_{p_{-i}\in\mathbb{R}_+} \Pi_i(p,\theta)\mathrm{d}\sigma(p|\theta) \geq \sum_{\theta\in\Theta} \mu_\theta \int_{p_{-i}\in\mathbb{R}_+} \Pi_i((p_i', p_{-i}),\theta)\mathrm{d}\sigma(p|\theta)$ for all $p_i \in \text{supp } \sigma$, $p_i' \in \mathbb{R}_+$ and $i \in \{1, 2\}$.

First, Theorem A from Stinchcombe (2011) establishes the existence of Correlated equilibrium in games in which players receive private signals and then simultaneously choose actions from compact sets. Formally, consider a game in which the set of players I is finite and for each i, the type ω_i belongs to the measure space $(\Omega_i, \mathcal{F}_i)$. Each player i simultaneously chooses an action from a compact set A_i and denote by Δ_i the set of countably additive Borel probabilities in A_i, with the weak* topology. Let $\mathbb{B}_i(\mathcal{F}_i)$ be the set of i's behavioral strategies, defined as the $\mathcal{F}_i$-measurable functions from Ω_i to Δ_i. Given a vector $b \in \mathbb{B} := \times_i \mathbb{B}_i(\mathcal{F}_i)$, player i's expected utility if b is played is defined by

$$u_i^P(b) = \int_\Omega \langle u_i(\omega), \times_i b_i(\omega)\rangle P(\mathrm{d}\omega)$$

where $\langle f, \nu\rangle := \int_A f(a)\nu(\mathrm{d}a)$ for $f : A \to \mathbb{R}$ and Borel probabilities ν, and $\times_i b_i$ is the product probability on A having b_i as the marginal. $(\mathbb{B}_i(\mathcal{F}_i), u_i^P)_{i\in I})$ denotes the normal form game. Then, Theorem A shows that all games $(\mathbb{B}_i(\mathcal{F}_i), u_i^P)_{i\in I})$ have correlated equilibria. In the pricing game, two firms simultaneously choose a price to maximize their expected equilibrium profits from compact sets. Thus, this result implies that the set of BCE is non-empty.

Second, the set of BCE corresponds to the set of all probability measures on $[p^F(\theta_L), p^F(\theta_H)]^2$ for each $\theta \in \Theta$ where Θ is finite. Then, the set of BCE is compact since $[p^F(\theta_L), p^F(\theta_H)]^2$ is compact in the weak* topology, by Theorem 15.11 from Aliprantis and Border (2013). Third, the continuity of the objective function $W(\sigma; \gamma)$ in the weak* topology follows from Corollary 15.7 from Aliprantis and Border since because both $\Pi_i(p, \theta)$ and $q_i(p, \theta)$ are continuous and bounded functions. Hence, the integral $\int \Pi_i \mathrm{d}\sigma(p|\theta)$ and $\int v(p; \theta)\mathrm{d}\sigma(p|\theta)$ is continuous in σ. Thus, the designer's problem is to maximize a continuous objective function in a compact set. The existence of a solution is guaranteed by the Weierstrass extreme value theorem. ■

Proof. Lemma 3. The minimum and maximum price in any equilibrium is charged when both firms know that the state is low and that the state is high, respectively. That is, the highest and lowest equilibrium prices occur with full disclosure. Under full disclosure σ^F, there is no uncertainty about the state. Each firm chooses $p_i : \Theta \to \mathbb{R}_+$ to maximize $\Pi_i(p;\theta)$. That is, firm i's best response to p_{-i} is implicitly defined by

$$q(p;\theta) + p_i \frac{\partial q(p;\theta)}{\partial p_i} = 0$$

In equilibrium, both firms choose the same price, denoted by $p^F(\theta)$. Since $q(p;\theta_L) < q(p,\theta_H)$, the highest (lowest) equilibrium price the uninformed firm is willing to price is when both firms are certain that the state is high (low). Hence, the support of any obedient recommendation $\sigma(p_2|\theta)$ is a subset of $[p^F(\theta_L), p^F(\theta_H)]$. ■

Lemma 6 *When firms offer substitutes (complements),* $\mathbb{E}_\sigma[\Pi_1^*(p;\theta)|\theta]$ *is supermodular (submodular) in* θ *and* p_2*.*

Proof. Lemma 6. The informed firm expected equilibrium profits are

$$\int_{p_2} \Pi_1(p_1(\theta, \sigma(p_2|\theta)), p_2; \theta) \mathrm{d}\sigma(p_2|\theta),$$

where $p_1(\theta, \sigma(p_2|\theta))$ is firm 1's best response. When firms offer substitutes, for any obedient $\sigma(p_2|\theta)$ we have that

$$\int_{p_2} q_1(p;\theta)\mathrm{d}\sigma(p_2|\theta_H) \geq \int_{p_2} q_1(p;\theta)\mathrm{d}\sigma(p_2|\theta_L) \text{ and}$$
$$\frac{\partial \int_{p_2} q_1(p;\theta)\mathrm{d}\sigma(p_2|\theta_H)}{\partial p_1} \geq \frac{\partial \int_{p_2} q_1(p;\theta)\mathrm{d}\sigma(p_2|\theta_L)}{\partial p_1} \text{ for all } p_1 \text{ and } \theta \tag{8}$$

since $q_1(p;\theta)$ is strictly increasing in p_2, $\int_0^x \mathrm{d}\sigma(p_2|\theta_L) \geq \int_0^x \mathrm{d}\sigma(p_2|\theta_H)$ for all x and $\frac{\partial^2 q_1(p;\theta)}{\partial p_1 \partial p_2} > 0$. Then, since $p_1(\theta, \sigma(p_2|\theta))$ is implicitly defined by

$$\int_{p_2} q_1(p;\theta)\mathrm{d}\sigma(p_2|\theta) + p_1 \frac{\partial \int_{p_2} q_1(p;\theta)\mathrm{d}\sigma(p_2|\theta)}{\partial p_1} = 0,$$

(8) implies that $p_1(\theta, \sigma(p_2|\theta_H)) \geq p_1(\theta, \sigma(p_2|\theta_L))$ for all $\theta \in \Theta$. Furthermore, $\frac{\partial^2 q_1(p,\theta)}{\partial p_1 \partial p_2} \geq 0$ also implies that

$$\int_{p_2} q_1(p_1(\theta, \sigma(p_2|\theta_H)), p_2; \theta)\mathrm{d}\sigma(p_2|\theta_H) \geq \int_{p_2} q_1(p_1(\theta, \sigma(p_2|\theta_L)), p_2; \theta)\mathrm{d}\sigma(p_2|\theta_L).$$

Therefore, when firms offer substitutes,

$$\int_{p_2} \Pi_1(p_1(\theta, \sigma(p_2|\theta_H)), p_2; \theta) \mathrm{d}\sigma(p_2|\theta_H) - \int_{p_2} \Pi_1(p_1(\theta, \sigma(p_2|\theta_L)), p_2; \theta) \mathrm{d}\sigma(p_2|\theta_L) \geq 0 \tag{9}$$

for all $\theta \in \Theta$. Using Leibnitz rule, we have that for all θ',

$$\begin{aligned}
&\frac{\partial \int_{p_2} \Pi_1(p_1(\theta, \sigma(p_2|\theta')), p_2; \theta) \mathrm{d}\sigma(p_2|\theta')}{\partial \theta} \\
&= \frac{\partial p_1(\cdot)}{\partial \theta} + \left[\int_{p_2} q_1(p_1(\cdot)), p_2; \theta) \mathrm{d}\sigma(p_2|\theta') + p_1(\cdot) \frac{\partial \int_{p_2} q_1(p_1(\cdot), p_2; \theta) \mathrm{d}\sigma(p_2|\theta')}{\partial p_1} \right] \\
&+ p_1(\cdot) \frac{\partial \int_{p_2} q_1(p_1(\cdot), p_2; \theta) \mathrm{d}\sigma(p_2|\theta')}{\partial \theta} \\
&= p_1(\cdot) \frac{\partial \int_{p_2} q_1(p_1(\cdot), p_2; \theta) \mathrm{d}\sigma(p_2|\theta')}{\partial \theta}
\end{aligned}$$

where the last equality holds by the first order condition of the informed firm's pricing decision. Then, the left-hand side of (9) is non-decreasing in θ because $p_1(\theta, \sigma(p_2|\theta_H)) > p_1(\theta, \sigma(p_2|\theta_L))$ and $\frac{\partial^2 q_1(p;\theta)}{\partial \theta \partial p_2} > 0$. Thus, when firms offer substitutes,

$$\begin{aligned}
&\int_{p_2} \Pi_1(p_1(\theta_H, \sigma(p_2|\theta_H)), p_2; \theta_H) \mathrm{d}\sigma(p_2|\theta_H) - \int_{p_2} \Pi_1(p_1(\theta_H, \sigma(p_2|\theta_L)), p_2; \theta_H) \mathrm{d}\sigma(p_2|\theta_L) \\
\geq &\int_{p_2} \Pi_1(p_1(\theta_L, \sigma(p_2|\theta_H)), p_2; \theta_L) \mathrm{d}\sigma(p_2|\theta_H) - \int_{p_2} \Pi_1(p_1(\theta_L, \sigma(p_2|\theta_L)), p_2; \theta_L) \mathrm{d}\sigma(p_2|\theta_L)
\end{aligned}$$

which implies that $\mathbb{E}_\sigma[\Pi_1^*(p;\theta)|\theta]$ is supermodular in θ and p_2. The proof for the complement case is analogous. ■

Proof. Proposition 1. I show that if $\mathbb{E}_\sigma[\Pi_1^*(p;\theta)|\theta]$ is supermodular in p_2 and θ, full disclosure is optimal for the informed firm. Similarly, if $\mathbb{E}_\sigma[\Pi_1^*(p;\theta)|\theta]$ is submodular in p_2 and θ, no disclosure is optimal for the informed firm. Using Lemma 6, we obtain the desired result.

The fully disclosing information structure recommends prices $(p^F(\theta), p^F(\theta))$ with probability 1 for all $\theta \in \Theta$. Full disclosure is optimal for the informed firm if her expected equilibrium payoffs with full disclosure exceed her expected equilibrium payoffs induced by any other obedient recommendation mechanism. That is,

$$\sum_{\theta \in \Theta} \mu_\theta \Pi_1((p^F(\theta), p^F(\theta)); \theta) \geq \sum_{\theta \in \Theta} \mu_\theta \int \Pi_1(p; \theta) \mathrm{d}\sigma(p|\theta) \tag{10}$$

for all $\sigma : \Theta \to \Delta([p^F(\theta_L), p^F(\theta_H)]^2)$ that satisfy the obedience constraints. The obedience constraints requires that p_1 must be a best response for firm 1, denoted by $p_1(\theta, \sigma(p_2|\theta))$.

Consider first the case in which $\mathbb{E}_\sigma[\Pi_1^*(p;\theta)|\theta]$ is supermodular in p_2 and θ. Next, I show that for all σ and $p_2 \in [p^F(\theta_L), p^F(\theta_H)]$,

$$\mathbb{E}_{\sigma^F}[\Pi_1^*(p;\theta_L)|\theta_L] \leq \mathbb{E}_\sigma[\Pi_1^*(p;\theta_L)|\theta_L] \leq \mathbb{E}_\sigma[\Pi_1^*(p;\theta_H)|\theta_H] \leq \mathbb{E}_{\sigma^F}[\Pi_1^*(p;\theta_H)|\theta_H].$$

First,

$$\begin{aligned}\int_{p_2} \Pi_1(p_1(\theta_L, \sigma(p_2|\theta_L)), p_2; \theta_L)\mathrm{d}\sigma(p_2|\theta_L) &\geq \int_{p_2} \Pi_1(p_1(\theta_L, \sigma^F(p_2|\theta_L)), p_2; \theta_L)\mathrm{d}\sigma^F(p_2|\theta_L) \\ &= \Pi_1(p_1(\theta_L, \sigma^F(p_2|\theta_L)), p^F(\theta_L); \theta_L) \qquad (11)\end{aligned}$$

since $\sigma^F(p_2|\theta_L)$ recommends $p^F(\theta_L)$ with probability 1, the informed firm's demand is increasing in p_2 and $p_1(\theta_L, \sigma(p_2|\theta_L)) \geq p_1(\theta_L, \sigma^F(p_2|\theta_L))$.[23] Similarly,

$$\begin{aligned}\int_{p_2} \Pi_1(p_1(\theta_H, \sigma(p_2|\theta_H)), p_2; \theta_H)\mathrm{d}\sigma(p_2|\theta_H) &\leq \int_{p_2} \Pi_1(p_1(\theta_H, \sigma^F(p_2|\theta_H)), p_2; \theta_H)\mathrm{d}\sigma^F(p_2|\theta_H) \\ &= \Pi_1(p_1(\theta_H, \sigma^F(p_2|\theta_H)), p^F(\theta_H); \theta_H). \qquad (12)\end{aligned}$$

Second, supermodularity implies that

$$\int_{p_2} \Pi_1(p_1(\theta_L, \sigma(p_2|\theta_L)), p_2; \theta_L)\mathrm{d}\sigma(p_2|\theta_L) \leq \int_{p_2} \Pi_1(p_1(\theta_H, \sigma(p_2|\theta_H)), p_2; \theta_H)\mathrm{d}\sigma(p_2|\theta_H) \quad (13)$$

since $p_1(\theta_L, \sigma(p_2|\theta_L)) \leq p_1(\theta_H, \sigma(p_2|\theta_H))$, $\frac{\partial^2 q_1(p;\theta)}{\partial\theta\partial p_2} > 0$ and the state is a positive demand shifter, implying that $\sigma(p_2|\theta_H)$ recommends on average higher prices than $\sigma(p_2|\theta_L)$. Thus, (11), (12) and (13) imply that

$$\begin{aligned}\Pi_1(p_1(\theta_L, \sigma^F(p_2|\theta_L)), p^F(\theta_L); \theta_L) &\leq \int_{p_2} \Pi_1(p_1(\theta_L, \sigma(p_2|\theta_L)), p_2; \theta_L)\mathrm{d}\sigma(p_2|\theta_L) \\ \leq \int_{p_2} \Pi_1(p_1(\theta_H, \sigma(p_2|\theta_H)), p_2; \theta_H)\mathrm{d}\sigma(p_2|\theta_H) &\leq \Pi_1(p_1(\theta_H, \sigma^F(p_2|\theta_H)), p^F(\theta_H); \theta_H).\end{aligned}$$

Then,

$$\begin{aligned}\mathbb{E}_{\sigma^F,\mu}[\Pi_1^*(p;\theta)] &= \sum_{\theta\in\Theta} \mu_\theta \Pi_1(p_1(\theta, \sigma^F(p_2|\theta)), p^F(\theta); \theta) \\ &\geq \sum_{\theta\in\Theta} \mu_\theta \int_{p_2} \Pi_1(p_1(\theta, \sigma(p_2|\theta)), p_2; \theta)\mathrm{d}\sigma(p_2|\theta) \\ &= \mathbb{E}_{\sigma,\mu}[\Pi_1^*(p;\theta)]\end{aligned}$$

[23] The proof of $p_1(\theta_L, \sigma(p_2|\theta_L)) \geq p_1(\theta_L, \sigma^F(p_2|\theta_L))$ follows an analogous argument as in Lemma 6.

where the inequality holds by Jensen's inequality. Hence, full disclosure is optimal for the informed firm.

Consider now the case in which $\mathbb{E}_\sigma[\Pi_1^*(p;\theta)|\theta]$ is submodular in θ and $\sigma(p_2|\theta)$. Analogously as in the supermodular case, it is possible to show that

$$\Pi_1(p_1(\theta_L,\sigma^N(p_2|\theta_L)),p^N;\theta_L) \le \int_{p_2} \Pi_1(p_1(\theta_L,\sigma(p_2|\theta_L)),p_2;\theta_L)\mathrm{d}\sigma(p_2|\theta_L)$$
$$\le \int_{p_2} \Pi_1(p_1(\theta_H,\sigma(p_2|\theta_H)),p_2;\theta_H)\mathrm{d}\sigma(p_2|\theta_H) \le \Pi_1(p_1(\theta_H,\sigma^N(p_2|\theta_H)),p^N;\theta_H)$$

which in turn implies that

$$\begin{aligned}\mathbb{E}_{\sigma^N,\mu}[\Pi_1^*(p;\theta)] &= \sum_{\theta\in\Theta}\mu_\theta\Pi_1(p_1(\theta,p^N),p^N;\theta)\\ &\ge \sum_{\theta\in\Theta}\mu_\theta\int_{p_2}\Pi_1(p_1(\theta,\sigma(p_2|\theta)),p_2;\theta)\mathrm{d}\sigma(p_2|\theta_H)\\ &= \mathbb{E}_{\sigma,\mu}[\Pi_1^*(p;\theta)]\end{aligned}$$

where the inequality holds by Jensen's inequality. ■

Lemma 7 *Assume that σ is partially informative and $\sigma(p_2|\theta)$ is degenerate, placing all mass on $\hat{p}\in[p_L^F,p_H^F]$. For any obedient σ, supp $\sigma(p_2|\theta')=\{\hat{p},\hat{p}'\}$ for all $\theta\neq\theta'$.*

Proof. Lemma 7. The recommendation mechanism σ is not fully informative. First, I show that $\hat{p}\in$ supp $\sigma(p_2|\theta')$. Suppose not. Then, supp $\sigma|\theta\,\cap$ supp $\sigma|\theta'=\emptyset$ which implies that price recommendations fully reveal the state. However, this contradicts the assumption that σ is partially informative. Hence, $\hat{p}\in$ supp $\sigma(p_2|\theta')$.

Second, I show that the support of $\sigma(p_2|\theta')$ is binary. Firm i's obedience constraint is

$$\sum_{\theta\in\Theta}\mu_\theta\int_{p_{-i}\in\mathbb{R}_+}\Pi_i(p;\theta)\mathrm{d}\sigma(p|\theta)\ge\sum_{\theta\in\Theta}\mu_\theta\int_{p_{-i}\in\mathbb{R}_+}\Pi_i((p_i',p_{-i});\theta)\mathrm{d}\sigma(p|\theta)$$

for all i, $p_i\in$ supp σ and $p_i'\in[p_L^F,p_H^F]$. The uninformed firm obedience constraint of σ for $p_2\neq\hat{p}$ is

$$p_2\cdot q_2((p_1(\theta',p_2),p_2);\theta')\ge p_2'\cdot q_2((p_1(\theta',p_2'),p_2');\theta') \tag{14}$$

for all $p_2'\in[p_L^F,p_H^F]$. The uninformed firm's profits are strictly concave in p_2 which implies there exists a unique $\hat{p}'\in[p_L^F,p_H^F]$ that satisfies (14) and $\hat{p}'\neq\hat{p}$. Hence, the support of $\hat{\sigma}|\theta'$ is binary and given by $\{\hat{p},\hat{p}'\}$. ■

Lemma 8 *If the optimal recommendation mechanisms σ^* is partially informative, there exists a state in which a unique price is recommended.*

Proof. Lemma 8. Suppose not. Assume that the optimal recommendation mechanism σ^* is partially informative where both $\sigma^*(p_2|\theta)$ are not degenerate. Consider an alternative obedient partially informative recommendation mechanism $\hat{\sigma}$ as defined in Lemma 7. Let λ be the probability of recommending $\hat{p}$ in state θ', which fully characterizes $\hat{\sigma}$.

Next, I show that there exists $\lambda \in (0,1)$ such that consumer surplus under $\hat{\sigma}$ is greater or equal than under σ^*, where

$$\mathbb{E}_{(\mu,\sigma^*)}\left[v(p;\theta)\right] = \sum_{\theta\in\Theta}\mu_\theta \int v((p_1(p_2,\theta),p_2);\theta)\mathrm{d}\sigma^*(p_2|\theta) \text{ and}$$
$$\mathbb{E}_{(\mu,\hat{\sigma})}\left[v(p;\theta)\right] = \mu_\theta v((p_1(\hat{p},\theta),\hat{p});\theta) + (1-\mu_\theta)\left[\lambda v((p_1(\hat{p},\theta'),\hat{p});\theta') + (1-\lambda)v((\hat{p}',\hat{p}');\theta')\right].$$

Note that when $\hat{p}'$ is recommended is state θ', both firms are certain about the state and, thus, set the same price. Given the definition of v derived from integrating the demand system and since demand decreases in its own price, we have that there exists an obedient $\hat{p}$ such that

$$\mathbb{E}_{(\mu,\sigma^*)}\left[v(p;\theta)\right] \leq \mu_\theta v((p_1(\hat{p},\theta),\hat{p});\theta) + (1-\mu_\theta)\int v((p_1(p_2,\theta'),p_2);\theta')\mathrm{d}\sigma^*(p_2|\theta').$$

Since this is only compatible with recommending one additional price in state θ' given the obedience conditions, $\hat{p}$ is pinned down by λ. Therefore, there exists λ such that consumers are better off with $\hat{\sigma}$, contradicting the optimality of σ^*. ■

Lemma 9 *Consumers surplus is higher when two prices are recommended when the state is high and only one of those prices is recommended when the state is low.*

Proof. Lemma 9. Consider two recommendation mechanisms, σ^L and σ^H. Each mechanism only recommend two prices, $\hat{p}_L^k$ and $\hat{p}_H^k$ with $k \in \{L,H\}$. However, σ^ℓ recommends $\hat{p}_L^L$ is recommended both states whereas $\hat{p}_H^L$ is only recommended when the state is high. Analogously, σ^H recommends $\hat{p}_H^H$ is recommended both states whereas $\hat{p}_L^H$ is only recommended when the state is low. Denote by $\overline{p}_k'$ the unique obedient price in state θ_k' when σ^k reveals such state. Let $\lambda^k \in [0,1]$ be the probability of recommending $\hat{p}_k^k$ in θ_k' under the obedient recommendation mechanism σ^k for $k \in \{L,H\}$. Note that a given λ^k pins down the recommended price in state θ_k and that $\lambda^k = 0$ represents the fully informative recommendation mechanism whereas $\lambda^k = 1$, the uninformative one. Then, a recommendation mechanism σ^L

is characterized by $(\hat{p}_L^L(\lambda^L), \overline{p}_H, \lambda^L)$, where $\hat{p}_L^L$ increases in λ^L. Similarly, a recommendation mechanism σ^H is characterized by $(\overline{p}_L, \hat{p}_H^H(\lambda^H), \lambda^H)$, where $\hat{p}_H^H$ decreases in λ^H.

Under σ^L, the three possible equilibrium outcomes in terms of firm prices are: $(\tilde{p}_L^L, \hat{p}_L^L)$ where $\tilde{p}_L^L \leq \hat{p}_L^L$ with probability μ_L, $(\tilde{p}_I^L, \hat{p}_L^L)$ where $\tilde{p}_I^L \geq \hat{p}_L^L$ with probability $\mu_H \lambda^L$ and $(\overline{p}_H, \overline{p}_H)$ with the complementary probability. Similarly, under σ^H, the three possible equilibrium outcomes in terms of firm prices are: $(\overline{p}_L, \overline{p}_L)$ with probability $\mu_L(1-\lambda^H)$, $(\tilde{p}_I^H, \hat{p}_H^H)$ where $\tilde{p}_I^H \leq \hat{p}_H^H$ with probability $\mu_L \lambda^H$ and $(\tilde{p}_H^H, \hat{p}_H^H)$ where $\tilde{p}_H^H \geq \hat{p}_H^H$ with the complementary probability. Expected consumer surplus in each case is:

$$\mathbb{E}_{(\mu,\sigma^L)}\left[v(p;\theta)\right] = \mu_L v(((\tilde{p}_L^L, \hat{p}_L^L)); \theta_L) + \mu_H \left[\lambda^L v((\tilde{p}_I^L, \hat{p}_L^L); \theta_H) + (1-\lambda^L) v((\overline{p}_H, \overline{p}_H); \theta_H)\right] \text{ and}$$
$$\mathbb{E}_{(\mu,\sigma^H)}\left[v(p;\theta)\right] = \mu_L \left[(1-\lambda^H) v((\overline{p}_L, \overline{p}_L); \theta_L) + \lambda^H v((\tilde{p}_I^H, \hat{p}_H^H); \theta_L)\right] + \mu_H v((\tilde{p}_H^H, \hat{p}_H^H)); \theta_H)$$

Note that for any σ^L and σ^H, we have that

$$\overline{p}_L \leq \tilde{p}_L^L \leq \hat{p}_L^L \leq \tilde{p}_I^L \leq \tilde{p}_I^H \leq \hat{p}_H^H \leq \tilde{p}_H^H \leq \overline{p}_H$$

and that expected consumer surplus in non-linear in λ^k.

Define $\Delta\mathbb{E}_{(\mu,\sigma^k)}\left[v(p;\theta)\right]$ as the difference in consumer surplus between σ^k and the fully informative recommendation mechanism.

$$\Delta\mathbb{E}_{(\mu,\sigma^L)}\left[v(p;\theta)\right] = \mu_L[v((\tilde{p}_L^L, \hat{p}_L^L)); \theta_L) - v((\overline{p}_L, \overline{p}_L); \theta_L)] + \mu_H \lambda^L \left[v((\tilde{p}_I^L, \hat{p}_L^L); \theta_H) - v((\overline{p}_H, \overline{p}_H); \theta_H)\right] \text{ and}$$
$$\Delta\mathbb{E}_{(\mu,\sigma^H)}\left[v(p;\theta)\right] = \mu_L \lambda^H \left[v((\tilde{p}_I^H, \hat{p}_H^H); \theta_L) - v((\overline{p}_L, \overline{p}_L); \theta_L)\right] + \mu_H[v((\tilde{p}_H^H, \hat{p}_H^H)); \theta_H) - v((\overline{p}_H, \overline{p}_H); \theta_H)].$$

Note that the first term in square brackets is weakly negative in both cases, whereas the second one weakly positive. Moreover,

$$v((\tilde{p}_L^L, \hat{p}_L^L)); \theta_L) > v((\tilde{p}_I^H, \hat{p}_H^H); \theta_L) \text{ and } v((\tilde{p}_I^L, \hat{p}_L^L); \theta_H) > v((\tilde{p}_H^H, \hat{p}_H^H)); \theta_H).$$

Restricting attention to recommendation mechanisms such that $\Delta\mathbb{E}_{(\mu,\sigma^k)}\left[v(p;\theta)\right] \geq 0$, we have that $\Delta\mathbb{E}_{(\mu,\sigma^L)}\left[v(p;\theta)\right] \geq \Delta\mathbb{E}_{(\mu,\sigma^H)}\left[v(p;\theta)\right]$. ■

Proof. Proposition 2. Using Lemma 7, Lemma 8 and Lemma 9, we have that the consumer-optimal recommendation mechanism recommends a low price in both states and a high price only in the high state and it is fully characterized by λ^* where

$$\lambda^* \in \arg\max_{\lambda \in [0,1]} \mathbb{E}[v(p;\theta)](\lambda).$$

The existence of λ^* is guaranteed by Weierstrass theorem since the objective function is a continuous function of λ. ■

Proof. Proposition 3. Consider any partial disclosure policy σ and define $\sigma(s_2|\theta)$ the distribution of price recommendation p_2 conditional on the state θ. Expected consumer surplus is

$$\mathbb{E}_{(\mu,\sigma)}\left[v(p;\theta)\right] = \sum_{\theta\in\Theta}\mu_\theta \int v(p;\theta)\mathrm{d}\sigma((p_1,p_2)|\theta).$$

Proposition 2 implies that the optimal recommendation mechanism recommends at most two prices and it is fully characterized by a parameter λ which represents the probability of recommending a low price in the high state. Denote by $p_1^{k,\ell}$ firm 1's optimal price when firm 2 is recommended price $\hat{p}_k$ and the state is θ_ℓ, $p_1(\hat{p}_k,\theta_\ell)$. Hence, we can rewrite expected consumer surplus as follows:

$$\mathbb{E}_{(\mu,\sigma)}\left[v(p;\theta)\right] = \mu_L v((p_1^{L,L},\hat{p}_L);\theta_L) + \mu_H\left[\lambda v((p_1^{L,H},\hat{p}_L);\theta_H) + (1-\lambda)v((p_1^{H,H},\hat{p}_H);\theta_H)\right]$$

The regulator chooses λ to maximize the previous expression, taking into account that each λ induces a unique set of prices.

First, I show that full disclosure is never optimal for consumers since no disclosure induces higher expected consumer surplus when the likelihood of the high state is sufficiently low. Under full disclosure, both firms set prices $p^F(\theta)$ for each $\theta\in\Theta$, whereas with no disclosure, the informed firm sets $p_1^N(\theta)$ for $\theta\in\Theta$ and the uninformed firm sets $p_2^N\in[p_1^N(\theta_L),p_1^N(\theta_H)]\subset[p^F(\theta_L),p^F(\theta_H)]$. Expected consumer surplus with full and no disclosure is respectively given by

$$\mathbb{E}_{(\mu,\sigma^F)}\left[v(p;\theta)\right] = \sum_{\theta\in\Theta}\mu_\theta v((p^F(\theta),p^F(\theta));\theta) \text{ and}$$
$$\mathbb{E}_{(\mu,\sigma^N)}\left[v(p;\theta)\right] = \sum_{\theta\in\Theta}\mu_\theta v((p_1^N(\theta),p_2^N);\theta).$$

Define $\Delta\mathbb{E}[v(p;\theta)](\sigma)$ as the difference in expected consumer surplus with disclosure σ and no disclosure. Then,

$$\begin{aligned}\Delta\mathbb{E}[v(p;\theta)](\sigma^F) &= \mathbb{E}_{(\mu,\sigma^F)}\left[v(p;\theta)\right] - \mathbb{E}_{(\mu,\sigma^N)}\left[v(p;\theta)\right]\\ &= \mathbb{E}_\mu\left[v((p^F(\theta),p^F(\theta));\theta)\right] - \mathbb{E}_\mu\left[v((p_1^N(\theta),p_2^N);\theta)\right]\\ &\le \mathbb{E}_\mu\left[v((p^F(\theta),p^F(\theta));\theta)\right] - \mathbb{E}_\mu\left[v((p_1^N(\theta),p_1^N(\theta);\theta)\right]\\ &\le 0\end{aligned}$$

where the first inequality holds because v is convex in p_2 since q_2 is decreasing in p_2 and the second since v is concave in θ and $[p_1^N(\theta_L),p_1^N(\theta_H)]\subset[p^F(\theta_L),p^F(\theta_H)]$. Hence, either no or partial disclosure is optimal for consumers.

Second, I show that partial disclosure is optimal when information is chosen such that firms offer sufficiently close substitutes. Intuitively, given that v is increasing in both p_1 and p_2, $v(p;\theta_L) \leq v((p_1^N(\theta_L), p_2^N);\theta_L)$ and $v(p;\theta_H) \leq v((p^F(\theta_H), p^F(\theta_H);\theta_H)$ for all obedient recommendation mechanisms σ. That is, the designer would like to be as informative as possible in the high state while being as close to uninformative in the low state, hinting that disclosing some information may be beneficial for consumers. Note that the degree of differentiation determines how a change in the recommendation mechanism affects the equilibrium prices of both firms by determining how willing consumers are to substitute between goods offered by firms. Formally, the optimal partial disclosure is characterized by the FOC of the regulator's problem. Note that

$$\frac{\partial v}{\partial \lambda} = \frac{\partial p_2}{\partial \lambda}\left(\frac{\partial v}{\partial p_1}\frac{\partial p_1}{\partial p_2} + \frac{\partial v}{\partial p_2}\right) \text{ and } \frac{\partial v}{\partial p_i} = -q_i.$$

Then, the FOC of the regulator's problem is

$$\frac{\partial \mathbb{E}_{(\mu,\sigma)}\left[v(p;\theta)\right]}{\partial \lambda} = -r(\lambda) + l(\lambda) \text{ where}$$

$$r(\lambda) = \mu_L \frac{\partial \hat{p}_L}{\partial \lambda}\left(\frac{\partial p_1}{\partial p_2} q_1 + q_2\right)|_{at\ ((p_1^{L,L},\hat{p}_L);\theta_L)} + \mu_H \lambda \frac{\hat{p}_L}{\partial \lambda}\left(\frac{\partial p_1}{\partial p_2} q_1 + q_2\right)|_{at\ ((p_1^{L,H},\hat{p}_L);\theta_H)} \text{ and}$$

$$l(\lambda) = \mu_H \left[v(((p_1^{L,H}, \hat{p}_L);\theta_H)) - v((p_1^{H,H}, \hat{p}_H);\theta_H) - (1-\lambda)\frac{\partial \hat{p}_H}{\partial \lambda}\left(\frac{\partial p_1}{\partial p_2} q_1 + q_2\right)|_{at\ ((p_1^{H,H},\hat{p}_H);\theta_H)}\right]$$

Intuitively, $r(\lambda)$ represents the cost of recommending a lower price in the high state with higher probability while inducing a higher price in the low state. Similarly, $l(\lambda)$ represents the benefit of inducing a lower price in the high state when it is in fact recommended. Both $r(\lambda)$ and $l(\lambda)$ are continuous functions of λ and weakly positive for all $\lambda \in [0,1]$.[24] Moreover, $l(0) > l(1) = 0$ and $r(1) > r(0)$ and $l(0) > r(0)$ given that full disclosure is never optimal. Then, the intermediate value theorem implies that there exists $\lambda^* \in (0,1]$ such that $r(\lambda^*) = l(\lambda^*)$. Therefore, no disclosure is optimal for consumers when $r(1) = 0$ and partial disclosure is optimal otherwise.[25] In particular, the optimal disclosure policy λ^* is chosen to induce goods to be closer substitutes and intensify the competition across firms. ■

[24] Note that $q_1 \frac{\partial p_1}{\partial p_2} + q_2 > 0$, $\frac{\partial \hat{p}_L}{\partial \lambda} > 0$ and $\frac{\partial \hat{p}_H}{\partial \lambda} < 0$ since $\frac{\partial p_1}{\partial p_2} > 0$.

[25] Note that the SOC is satisfy given our assumptions on Q.