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Competition with Exclusive Contracts in Vertically Related Markets: An Equilibrium Non-Existence Result

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Abstract

I study a model in which two upstream firms compete to supply a homogeneous input to two downstream firms selling differentiated products. Upstream firms offer exclusive, discriminatory, public, two-part tariff contracts to the downstream firms. I show that, under very general conditions, this game does not have a pure-strategy subgame-perfect equilibrium. The intuition is that variable parts in such an equilibrium would have to be pairwise-stable; however, with pairwise-stable variable parts, downstream competitive externalities are not internalized, implying that upstream firms can profitably deviate. I contrast this non-existence result with earlier papers that found equilibria in related models.

Keywords: vertical relations, exclusive dealing, two-part tariffs, slotting fees.

1 Introduction

The competitive effects of exclusive dealing agreements are a hotly debated issue among economists and antitrust practitioners. Such contracts were seen with suspicion by antitrust authorities for much of the twentieth century. The main theory of harm was that exclusive dealing contracts allow an input manufacturer to exclude rival producers from the input market. Authors associated with the Chicago School challenged this view on the ground that

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a rational downstream buyer would need to be properly compensated to sign such a contract, which would dissipate the profitability of such agreements (Posner, 1976; Bork, 1978).

From the 1990s onward, a more strategic approach has been revisiting these issues using modern game-theoretical tools. Papers in that literature can by and large be organized into two groups. A first strand of literature, pioneered by Hart and Tirole (1990) and Bernheim and Whinston (1998), analyzes triangular market structures in which, by assumption, the upstream or the downstream market is supplied by a monopoly. A second strand of literature, initiated by Aghion and Bolton (1987), Rasmusen, Ramseyer, and Wiley (1991), and Segal and Whinston (2000), studies settings in which an upstream incumbent attempts to deter entry by signing exclusive contracts with downstream buyers before a potential upstream entrant makes its entry decision. In such models, the entrant cannot offer exclusive dealing contracts before entering, and there is therefore no competition for exclusives.

Yet, in most exclusive dealing cases, entrants or established competitors were already in the market when the incumbent was making exclusive offers, and multiple firms were present at both layers of the supply chain.[1] As Whinston (2006) pointed out in his chapter on exclusive dealing (p. 176), "Of course, in most actual markets there is more than one participant on both sides of the market. Thus, developing models that reflect this reality is a high priority." In this paper, I show that developing such models gives rise to non-trivial theoretical complications.

I study a model in which two identical upstream firms, U_1 and U_2, compete to supply a homogeneous input to two symmetrically differentiated downstream firms, D_1 and D_2. In the first stage of the game, upstream firms compete by offering exclusive, discriminatory, two-part tariff contracts to the downstream firms. In the second stage, downstream firms choose their upstream suppliers. In the third and last stage of the game, downstream firms simultaneously set their prices. All offers and acceptance decisions are publicly observable.

The extant literature suggests that the following outcome would be a natural equilibrium candidate: U_1 supplies D_1 and U_2 supplies D_2; due to upstream competition, the fixed parts of the tariffs redistribute upstream profits to the downstream firms; the variable parts of the tariffs are pairwise-stable as in Bonanno and Vickers (1988), Rey and Stiglitz (1988, 1995), and Shaffer (1991), in the sense that U_i's variable part maximizes the joint profits of U_i and D_i taking U_j's variable part as given, and vice versa. It is well known from the strategic delegation literature that such pairwise-stable tariffs entail variable parts above marginal cost, as this softens downstream competition.

The problem with this equilibrium candidate is that industry profit is not maximized, as competitive externalities between downstream firms are not internalized. In particular, upstream variable parts, and thus downstream prices, are too low from the viewpoint of

[1] Spector (2011) discusses this point in his introduction.

industry profit maximization. This opens the door to the following deviation: U_1 first becomes D_2's upstream supplier by slightly undercutting U_2's offer; next, it slightly increases the variable part and slightly decreases the fixed part of the tariff it is offering to D_1, in such a way that D_1 does not want to switch to U_2. As the channel profit of the structure $U_1 - D_1$ was maximized at the initial variable part, U_1 starts making losses on D_1 but these losses are second-order. On the other hand, as D_1 now has a higher marginal cost, it increases its downstream price in the continuation subgame, which results in D_2 selling more downstream, and thus buying more upstream. This latter effect gives rise to a first-order increase in the profits that U_1 earns from D_2, which makes the deviation profitable. I formalize this argument and show that, under very general conditions, the two-part tariff competition game with exclusive contracts does not have a subgame-perfect equilibrium in pure strategies.[2]

This non-existence problem seems surprising in light of the results reported in Shaffer (1991) and Chen and Riordan (2007). Shaffer (1991) solves a model similar to mine except that, in his model, a large number of identical upstream firms are competing in the input market. He argues that this game has an equilibrium, and that in any equilibrium, upstream firms make zero profit and variable parts are pairwise-stable. However, his analysis does not account for the deviation outlined above. In Section 4, I explain in greater detail how this deviation (and other potential issues) affects his equilibrium characterization, and conclude that the equilibrium set may be either empty or much larger than what Shaffer (1991) claimed.

The model developed in Chen and Riordan (2007) is also very close to mine except that downstream consumers are uniformly distributed on the Hotelling segment and downstream firms can perfectly price discriminate. In their model, all-out competition for each consumer drives (personalized) downstream prices down to the most efficient firm's marginal cost (net of transport cost). This mechanism nullifies the strategic delegation effect, and ensures that the only pairwise-stable variable parts are equal to upstream marginal costs. This also neutralizes the deviation outlined above, and ensures that Chen and Riordan (2007)'s equilibrium is indeed an equilibrium. In my model, under a very general class of demand functions and as long as downstream firms cannot price discriminate, pairwise-stable variable parts are always strictly larger than cost and an equilibrium therefore fails to exist. This issue makes it difficult to assess the robustness of Chen and Riordan (2007)'s results to alternative models of downstream competition.

Proposition 1, proven in the appendix, may be of independent interest. In this paper, I allow demand functions to be kinked at points where a firm's demand vanishes. The model therefore includes linear demand as a special case—in contrast to much of the industrial organization literature, which, when using general demand functions, typically assumes that

[2] Equilibrium non-existence problems are not infrequent in the vertical-relations literature (see, e.g., Rey and Vergé, 2004, 2010; Marx and Shaffer, 2006). The problem identified in the present paper is new.

demand is everywhere differentiable. With such kinks, the contraction-mapping theorem cannot be applied to establish the uniqueness of the Nash equilibrium in the downstream competition subgame because the best-response map is not necessarily a contraction.[3] Proposition 1 asserts that equilibrium existence and uniqueness still obtain provided the standard duopoly stability condition holds at every price vector at which both firms' demands are strictly positive.

Related literature. As mentioned above, a first strand of literature studies triangular market structures in which either the upstream or the downstream market is supplied by a monopoly. In a framework with one upstream firm, multiple downstream firms, and secret contracts, Hart and Tirole (1990) show that exclusive dealing can profitably be used to reduce downstream competition (see also O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Vergé, 2004; Rey and Tirole, 2007). Intuitively, without exclusive dealing, once the upstream monopolist has contracted with one downstream firm, it cannot refrain from offering contracts to the other downstream firms, even though those firms will end up competing with the first one. By signing an exclusive dealing contract with a single downstream firm, the upstream monopolist avoids this opportunism problem and thus restores its monopoly power.

In a model in which multiple upstream firms compete with non-linear contracts to supply a single downstream firm, O'Brien and Shaffer (1997) and Bernheim and Whinston (1998) show that foreclosure does not arise in (a Pareto-undominated) equilibrium. Foreclosure can, however, arise if upstream firms are restricted to offering linear contracts (Mathewson and Winter, 1987), the downstream firm must take non-verifiable actions that give rise to a moral-hazard problem (Bernheim and Whinston, 1998), or the downstream firm is privately informed about downstream demand conditions (Calzolari and Denicolo, 2013, 2015). Calzolari, Denicolo, and Zanchettin (2020) argue that the common feature of these three cases is that marginal prices are distorted due to the upstream firms' inability to perfectly extract rent from the downstream firm, and propose a framework that unifies these three approaches. Chambolle and Molina (2023) show that, even in the absence of such pricing distortions, inefficient exclusion may still arise due to buyer power.

A second strand of literature studies settings in which an upstream incumbent signs exclusive contracts with downstream buyers before a potential upstream entrant makes its entry decision. Aghion and Bolton (1987) show how an exclusive dealing contract along with a penalty escape clause can be used to deter entry and extract surplus from the entrant

[3] Cumbul and Virag (2018) show that a seemingly standard Bertrand oligopoly model with linear demand and cost heterogeneity can have a continuum of non-equivalent Nash equilibria if there are at least three firms. In those equilibria, some of the firms are inactive. The resulting kink in demand provides some leeway in the first-order conditions of active firms (which become inequalities instead of equalities), thus giving rise to equilibrium multiplicity. This suggests that my Proposition 1 cannot be extended to the case with three or more firms unless kinks are assumed away.

whenever entry takes place. The idea that a contract between an upstream incumbent and a downstream buyer can be used to extract rent from an upstream entrant is further explored in Marx and Shaffer (1999) in a setup with variable demand and in Choné and Linnemer (2015) under uncertainty about the surplus created by the entrant.

In a framework with multiple buyers, Rasmusen, Ramseyer, and Wiley (1991) and Segal and Whinston (2000) derive conditions under which an upstream incumbent finds it profitable to sign exclusive dealing contracts with a subset of the buyers, so as to prevent a potential entrant from reaching minimum viable size—that is, naked exclusion can be profitable. Spector (2011) shows that such inefficient exclusion may still arise (although it is less likely) if the potential entrant is already present in the market and can also offer exclusive dealing contracts. Miklós-Thal and Shaffer (2016) find that the divide-and-conquer exclusion strategy identified by Segal and Whinston (2000) fails if contract offers are not publicly observable, although exclusion may still arise due to a coordination failure among buyers. In a setup with uncertainty about the fixed cost of entry, Chen and Shaffer (2019) derive conditions under which the incumbent prefers using market-share contracts rather than exclusive dealing contracts.

The papers discussed in the previous paragraph all assume that downstream firms sell in independent markets. Fumagalli and Motta (2006) relax this assumption and find that inefficient exclusion does not arise if buyers compete in a downstream market, provided their products are sufficiently close substitutes. Abito and Wright (2008) show that this result is driven by the assumption that buyers must pay a small but positive fixed cost to operate in the downstream market. In the absence of such fixed costs, they find that exclusion is more likely to occur if downstream products are close substitutes. Simpson and Wickelgren (2007) and Gratz and Reisinger (2013) revisit the conclusions of the naked exclusion literature under the assumption that downstream buyers can breach exclusive contracts by paying expectation damages. Interestingly, Gratz and Reisinger (2013) find that exclusive dealing contracts can have pro-competitive effects if downstream products are imperfect substitutes, as the entrant has an incentive to set a very low input price to induce downstream firms to breach their contracts—this is reminiscent of the main mechanism in Aghion and Bolton (1987).

In addition to the articles by Shaffer (1991) and Chen and Riordan (2007) discussed above, a small set of papers investigate the competitive effects of exclusive dealings contracts in models with upstream and downstream competition in which all upstream firms can offer exclusives. In a model with linear wholesale contracts, two manufacturers of differentiated products, and multiple retailers, Besanko and Perry (1994) find that manufacturers tend to benefit from exclusive dealing but consumers are harmed. In their setup, exclusivity means that each manufacturer is able to sell to only half of the retailers.[4] Nocke and Rey

[4]See also Perry and Besanko (1991) and Li and Luo (2020) for models of exclusive dealing with linear

(2018) study a model with upstream product differentiation, secret contracting, and quantity competition in the downstream market. They find that upstream firms have an incentive to adopt exclusive distribution provisions to avoid the opportunism problem identified by Hart and Tirole (1990). Assuming also secret contracting, Rey and Vergé (2020) obtain similar insights in a model in which downstream competition is in prices, provided downstream firms are sufficiently close substitutes.

The remainder of the paper is organized as follows. I present the model in Section 2, solve its second and third stages in Section 3, and prove equilibrium non-existence in Section 4. In Section 5, I conclude and provide an overview of which modeling choices lead to equilibrium existence or non-existence. The appendix contains the proof of Proposition 1 and a derivation of the linear-demand example.

2 The Model

There are two identical upstream firms, U_1 and U_2, and two symmetric downstream firms, D_1 and D_2. Upstream firms produce an intermediate input at constant unit cost $m \geq 0$. Downstream firms purchase this input and transform it into the final product on a one-to-one basis. The transformation cost is normalized to zero.

The input is assumed to be homogeneous, whereas final products are differentiated. In the downstream market, firms D_1 and D_2 compete by simultaneously setting their prices p_1 and p_2. The demand for D_k's product is given by $q_k(p_1, p_2)$. As downstream firms are symmetric, D_k's demand can be written as $q_k = q(p_k, p_\ell)$, where the function $q(\cdot, \cdot)$ does not depend on k.

We make the following standard assumptions on the function q: (a) Individual demand is decreasing in own price; (b) final products are substitutes; and (c) total demand is non-increasing in prices. That is, for every p_k, p'_k, and p_ℓ such that $p_k > p'_k$, we have: (a) $q(p'_k, p_\ell) \geq q(p_k, p_\ell)$ with a strict inequality whenever $q(p'_k, p_\ell) > 0$; (b) $q(p_\ell, p_k) \geq q(p_\ell, p'_k)$ with a strict inequality whenever $q(p'_k, p_\ell) > 0$ and $q(p_\ell, p'_k) > 0$, and equality if instead $q(p'_k, p_\ell) = 0$; and (c) $q(p'_k, p_\ell) + q(p_\ell, p'_k) \geq q(p_k, p_\ell) + q(p_\ell, p_k)$. Moreover, q is continuous, and it is also twice continuously differentiable at every point (p_k, p_ℓ) such that $q(p_k, p_\ell) > 0$ and $q(p_k, p_\ell) > 0$.[5] I slightly strengthen the above monotonicity assumptions by requiring that $\partial_2 q(p_k, p_\ell) > 0$ whenever $q(p_k, p_\ell)q(p_\ell, p_k) \neq 0$. (Throughout the paper, I use the notation $\partial_k f$ for the partial derivative of the function f with respect to its k-th argument and $\partial^2_{ij} f$ for the second partial

wholesale prices.

[5]As discussed in the introduction, I do not assume that demand functions are everywhere twice continuously differentiable because standard linear demand à la Shubik and Levitan (1980) violates this assumption. (Linear demand functions are kinked at points where a firm's demand vanishes.)

derivative of f with respect to its i-th and j-th arguments.) I further assume that industry revenue tends to zero as both prices go to infinity, i.e., $\lim_{(p_k,p_\ell)\to(\infty,\infty)} p_k q(p_k, p_\ell) = 0$.[6]

Upstream firms offer discriminatory two-part tariff contracts. Let w_k^i (resp. T_k^i) denote the variable (resp. fixed) part of the contract that U_i offers to D_k. For technical reasons discussed in Section 4, I confine attention to contracts with variable parts no smaller than m. By contrast, I impose no restrictions on the sign of T_k^i, i.e., slotting fees are allowed. Upstream contracts are exclusive: If D_k signs a contract with U_i, then it cannot sign another contract with U_j. Contracts and acceptance decisions are publicly observed. If D_k does not sign an upstream contract, then it exits the industry and earns 0 profit; its rival receives a demand of $q(p_\ell, \infty) = \lim_{p_k\to\infty} q(p_\ell, p_k)$, which is assumed to be finite.

The game unfolds as follows. In stage 1, upstream firms simultaneously offer their contracts (w_k^i, T_k^i). In stage 2, D_1 and D_2 observe all upstream contracts, simultaneously decide which contract to accept (if any), and pay the corresponding fixed fees. In stage 3, acceptance decisions become common knowledge, and the downstream firms that accepted an upstream contract simultaneously set their downstream prices. I restrict D_k's strategy space at the pricing stage to $[w_k, \infty)$ ($k = 1, 2$) to refine away dominated equilibria in which D_k prices below cost and D_ℓ best-replies by setting a price such that D_k's demand vanishes.

I look for subgame-perfect equilibria in which: Firms use pure strategies in all stage-3 subgames; and firms use pure strategies on the equilibrium path in stages 1 and 2. The restriction to pure strategies is standard in the vertical relations literature. Note that I do allow downstream firms to mix in stage 2 off the equilibrium path, for reasons that will be made clear in Section 3.2.

3 Equilibrium Analysis: Supplier Choice and Downstream Competition

3.1 Downstream Competition

Consider the third stage of the game, and assume that D_k has signed a contract with variable part w_k, and let D_ℓ be D_k's downstream rival. I adopt the convention that, if D_k has signed no contract, then its variable part is $w_k = \infty$. D_k's profit in stage 3 (gross of the fixed fee) can be written as:

$$\pi_k(p_1, p_2, w_k) = (p_k - w_k)q(p_k, p_\ell) \equiv \pi(p_k, p_\ell, w_k),$$

[6]This assumption is automatically satisfied if demand has a finite choke price, i.e., if there exists a price $\bar{p}_k$ such that $q(p_k, p_\ell) = 0$ for every $p_k \geq \bar{p}_k$ and $p_\ell \geq 0$.

I make the following assumptions on this profit function. For every w_k and p_ℓ, the function $\pi(\cdot, p_\ell, w_k)$ is strictly quasi-concave on the set of prices p_k such that $q(p_k, p_\ell) > 0$. Moreover, prices are strategic complements and the duopoly stability condition holds: For every w_k, p_k, and p_ℓ such that $q(p_k, p_\ell) > 0$ and $q(p_\ell, p_k) > 0$, we have

$$\partial^2_{12}\pi(p_k, p_\ell, w_k) > 0 \quad \text{and} \quad \partial^2_{11}\pi(p_k, p_\ell, w_k) + \partial^2_{12}\pi(p_k, p_\ell, w_k) < 0.$$

Under these assumptions, I prove the following lemma:[7]

Proposition 1. *For all (w_1, w_2), the downstream competition subgame has a unique pure-strategy Nash equilibrium. At every point (w_1, w_2) such that the equilibrium is interior, equilibrium downstream prices are continuously differentiable in (w_1, w_2) with strictly positive partial derivatives.*

Proof. See Appendix A. □

In establishing equilibrium existence, I face the usual difficulty that action sets are unbounded above (see, e.g., Nocke and Schutz, 2018). I circumvent this difficulty by defining an auxiliary game with compact action sets, obtaining the existence of an equilibrium strategy profile in that game from the Debreu-Fan-Glicksberg theorem (see, e.g., Theorem 2.1 in Fudenberg and Tirole, 1991), and showing that the strategy profile remains an equilibrium in the original game with unbounded action sets.

Standard approaches to equilibrium uniqueness (based on the contraction-mapping theorem, the univalence theorem, or the Poincaré-Hopf theorem) have no bite, as they require the demand system to be differentiable everywhere, which I do not assume. In fact, due to the presence of kinks in the demand system, the best-response map may well fail to be a contraction. Consider for instance standard linear demand à la Shubik and Levitan (1980) (see Example 1 below for details), and let $r_2(p_1)$ denote D_2's best-response function. If p_1 is such that both firms receive strictly positive demand at $(p_1, r_2(p_1))$, then the slope of r_2 is locally given by $|\partial^2_{12}\pi/\partial^2_{11}\pi|$, which, under linear demand, is a constant strictly smaller than 1. By contrast, if p_1 is such that $q(p_1, r_2(p_1)) = 0$ and $q(p_1, r_2(p_1) + \varepsilon) > 0$ for all $\varepsilon > 0$ (i.e., D_2 best replies by playing its limit price), then the slope of r_2 is locally given by the reciprocal of the diversion ratio between the two firms, $|\partial_1 q/\partial_2 q|$.[8] Under linear demand, $|\partial_1 q/\partial_2 q|$ is a constant that is strictly greater than 1, implying that the best response map

[7] There is a slight abuse of terminology here. When downstream firms have similar marginal costs, only one pair of downstream prices can be sustained in a Nash equilibrium. If instead D_k's marginal cost is significantly lower than D_ℓ's, then there exists a continuum of Nash equilibria in which D_k sets its monopoly price and D_ℓ sets any price larger than or equal to its marginal cost. Note however that profits, consumer surplus, and demand are the same in all these equilibria. We select, without loss of economic substance, the equilibrium in which D_ℓ sets $p_\ell = w_\ell$.

[8] Here, $\partial_1 q$ and $\partial_2 q$ are one-sided partial derivatives.

is not a contraction. Using a different line of proof, I show in Appendix A that uniqueness of the Nash equilibrium nevertheless obtains in this framework.

Let $\hat{p}_k(w_1, w_2)$ denote the equilibrium downstream price set by D_k. By symmetry and equilibrium uniqueness, this function can be rewritten as $\hat{p}_k(w_1, w_2) = \hat{p}(w_k, w_\ell)$. I define downstream firms' equilibrium demands in stage 3 as

$$\hat{q}_k(w_1, w_2) \equiv \hat{q}(w_k, w_\ell) \equiv q(\hat{p}(w_k, w_\ell), \hat{p}(w_\ell, w_k)),$$

and downstream firms' equilibrium profits as

$$\hat{\pi}_k(w_1, w_2) \equiv \hat{\pi}(w_k, w_\ell) \equiv \pi(\hat{p}(w_k, w_\ell), \hat{p}(w_\ell, w_k), w_k).$$

I also denote the equilibrium upstream profits derived from selling the input to D_k by

$$\hat{u}_k(w_1, w_2) \equiv \hat{u}(w_k, w_\ell) \equiv (w_k - m)\hat{q}(w_k, w_\ell).$$

I make the following assumption

Assumption 1. $\partial_2 \hat{q}(w_k, w_\ell) > 0$ *whenever* $\hat{q}(w_k, w_\ell) > 0$ *and* $\hat{q}(w_\ell, w_k) > 0$.

An increase in D_ℓ's cost has a direct, positive impact on D_k's equilibrium demand (D_ℓ increases its price), and an indirect one (D_k changes its price as well). Assumption 1 means that direct effects dominate indirect ones.

I close this subsection with the following remark:

Example 1. *Consider the Shubik and Levitan (1980) demand system:*

$$q(p_k, p_\ell) = \begin{cases} \frac{1}{2}\left(1 - p_k - \gamma\left(p_k - \frac{p_k + p_\ell}{2}\right)\right) & \text{if } \frac{(2+\gamma)p_\ell - 2}{\gamma} \leq p_k \leq \frac{\gamma p_\ell + 2}{\gamma + 2}, \\ \frac{1+\gamma}{2+\gamma}(1 - p_k) & \text{if } p_k \leq \min\left(\frac{(2+\gamma)p_\ell - 2}{\gamma}, 1\right), \\ 0 & \text{otherwise.} \end{cases}$$

Then, all the assumptions made above are satisfied.

Proof. See Appendix B. □

3.2 Supplier Choice

As mentioned in Section 2, I am looking for subgame-perfect equilibria in pure strategies on path, but I allow downstream firms to mix in stage 2 off the equilibrium path. I now motivate this choice of equilibrium concept. I first prove the following result:

Lemma 1. *In Example 1, if γ is high enough, there exist profiles of upstream contracts such that the supplier-choice game has no pure-strategy equilibrium.*

Proof. Set $\gamma = \infty$, so that downstream products are homogeneous and normalize m to 0. Consider the following profile of upstream offers:

- $w_1^1 = w_2^2 = 0$ and $w_1^2 = w_2^1 = a < 1/2$.
- $T_1^1 = T_2^2 = a(1-a) - \varepsilon$ and $T_1^2 = T_2^1 = \eta$, where ε, η are small and strictly positive.

Let us compute the downstream firms' equilibrium profits for every possible profile of acceptance decisions:

- If D_i chooses the contract (a, η) and D_j does not accept any offer, then D_i sets its monopoly price, $p_i = (1+a)/2$, and earns a profit of $(1-a)^2/4 - \eta$, while D_j makes zero profit.
- If D_i accepts the contract $(0, a(1-a) - \varepsilon)$ and D_j accepts no offer, then D_i sets again its monopoly price, $p_i = 1/2$, and earns a profit of $1/4 + \varepsilon - a(1-a)$, whereas D_j still makes zero profit.
- If D_i chooses $(0, a(1-a) - \varepsilon)$ and D_j chooses (a, η), then asymmetric Bertrand competition drives both prices down to a. (Here we use the tie-breaking rule where consumers all purchase from the low-cost firm when prices are the same; the tie-breaking rule is irrelevant if γ is finite.) This results in D_i making a profit of ε and D_j earning $-\eta$.
- If both firms choose $(0, a(1-a) - \varepsilon)$, then Bertrand competition drives prices down to marginal cost and each firm earns $\varepsilon - a(1-a)$.
- If both firms choose (a, η), then Bertrand competition again drives prices down to marginal cost and each firm earns $-\eta$.

These equilibrium payoffs are summarized in Table 1.

Table 1: Payoff Matrix

	$(0, a(1-a) - \varepsilon)$	(a, η)	Exit
$(0, a(1-a) - \varepsilon)$	$(\varepsilon - a(1-a), \varepsilon - a(1-a))$	$(\varepsilon, -\eta)$	$(\frac{1}{4} + \varepsilon - a(1-a), 0)$
(a, η)	$(-\eta, \varepsilon)$	$(-\eta, -\eta)$	$(\frac{(1-a)^2}{4} - \eta, 0)$
Exit	$(0, \frac{1}{4} + \varepsilon - a(1-a))$	$(0, \frac{(1-a)^2}{4} - \eta)$	$(0, 0)$

Clearly, there is no equilibrium in which the firms accept the same contract, as both firms would make losses. The outcome in which both firms exit the industry is not an equilibrium either, as one firm would have an incentive to step in. Moreover, the strategy profile in which one firm accepts the contract with the low variable part while the other one chooses the contract with the high variable part is not an equilibrium, as the latter firm makes losses. Finally, the strategy profile in which one firm chooses the contract with the high variable part while its rival accepts no offer is not an equilibrium, as the latter firm can profitably deviate by accepting the contract with the low variable part. We are thus left with a unique equilibrium candidate (up to relabeling the firms), in which one firm chooses the contract $(0, a(1-a)-\varepsilon)$ while its rival exits. However, provided ε and η are small enough, there exists an open set of values $a < 1/2$ such that the difference

$$\left(\frac{(1-a)^2}{4} - \eta\right) - \left(\frac{1}{4} + \varepsilon - a(1-a)\right) = \frac{1}{4}a(2-3a) - \varepsilon - \eta$$

is strictly positive. If a satisfies this condition, there is no pure-strategy Nash equilibrium, as the firm that is supposed to accept the contract $(0, a(1-a)-\varepsilon)$ strictly prefers accepting the contract (a, η). By continuity, this result extends to high but finite values of γ. □

Thus, when demand is linear and γ sufficiently high, there exist subgames starting in stage 2 that do not have subgame-perfect equilibria in pure strategies. It follows that, for those values of γ, the entire game has no subgame-perfect equilibria in pure strategies. I sidestep this issue by allowing downstream firms to mix over their supplier choices in stage 2. As the supplier choice game is finite, it always has a mixed-strategy equilibrium.

In the following, I focus on subgame-perfect equilibria in which such mixing does not take place on the equilibrium path. Although I have not seen this stated explicitly, this seems to be the equilibrium concept that the existing literature studying competition in two-part tariffs contracts has been working with.

4 Equilibrium Analysis: Upstream Competition

I can now state and prove the main result of this paper:

Proposition 2. *The two-part tariff competition game with exclusive contracts does not have an equilibrium.*

The proof proceeds in several steps. I begin by ruling out equilibrium candidates in which one or two downstream firms are inactive (Lemma 2).[9] Next, I turn to equilibrium candidates in which both downstream firms are active. I show that upstream firms must make zero profit on the equilibrium path and that, for a downstream firm, accepting the contract it is meant

[9] A firm is active if it accepts a contract and its equilibrium quantity is strictly positive.

to choose on the equilibrium path strictly dominates exiting the industry (Lemma 3). Next, I prove that the variable parts at which downstream firms end up purchasing on path must be pairwise-stable in the sense of Bonanno and Vickers (1988), Rey and Stiglitz (1988, 1995), and Shaffer (1991) (Lemmas 4 and 5). I conclude the proof with Lemma 6, which shows that, even with pairwise-stable variable parts, there are profitable deviations for the upstream firms.

Lemma 2. *There is no equilibrium in which at least one downstream firm is inactive.*

Proof. We first show that there is no equilibrium in which both downstream firms are inactive on path, and then that there is no equilibrium in which exactly one downstream firm is inactive on path.

No equilibrium in which both downstream firms are inactive on path. Assume for a contradiction that such an equilibrium exists. Then, all firms make zero profit. As a first step, we show that for every $(k, i) \in \{1, 2\}^2$, the following property holds:

$$\text{For all } w \geq m, \ \hat{\pi}(w_k^i, w) - T_k^i \leq 0. \tag{1}$$

This condition is clearly satisfied if no firm accepts a contract on the equilibrium path because, in that case, $\pi(w_k^i, \infty) - T_k^i \leq 0$ for every k and i.

Next, suppose that only one firm accepts a contract on path: To fix ideas, suppose D_1 accepts U_1's contract. Then, $\hat{\pi}(w_1^2, \infty) - T_1^2 \leq \hat{\pi}(w_1^1, \infty) - T_1^1 = 0$, and condition (1) holds for $k = 1$ and $i = 1, 2$. Moreover, $\hat{q}(w_1^1, \infty) = 0$ (D_1 is inactive), and $T_1^1 = 0$ (no firm makes positive profits). Therefore, $q(w_1^1, \infty) = 0$. It follows that, for all $w \geq w_1^1$ and $w' \geq 0$, $q(w, w') = 0$ (as q is non-increasing in its first argument and non-decreasing in its second argument) and $q(w', w) = q(w', \infty)$ (as total demand is non-increasing in prices, q is non-decreasing in its second argument and $q(w, w') = 0$). As only D_1 accepts a contract on path, $\hat{\pi}(w_2^i, w_1^1) - T_2^i \leq 0$, $i = 1, 2$. However, as $q(w', w) = q(w', \infty)$ for all $w \geq w_1^1$ and $w' \geq 0$, it also follows that $\hat{\pi}(w_2^i, \infty) - T_2^i \leq 0$ for all i. This implies condition (1) for $k = 2$.

Finally, assume both downstream firms accept a contract on path: To fix ideas, suppose they both sign a contract with U_1. Then, $\hat{q}(w_1^1, w_2^1) = \hat{q}(w_2^1, w_1^1) = 0$. Therefore, $q(w_1^1, w_2^1) = q(w_2^1, w_1^1) = 0$. The argument in the previous paragraph implies that $q(w, w') = 0$ for every $w' \geq 0$ and $w \geq w_k^1$ ($k = 1, 2$), so that condition (1) holds for every k and i.

Now, consider the following deviation: U_1 offers (m, ε) to D_1 and (∞, ∞) to D_2, where $\varepsilon > 0$. As downstream products are differentiated, we have that $\hat{\pi}(m, w) - \varepsilon > 0$ for all $w \geq m$, provided ε is small enough. By condition (1) for $k = 1$ and $i = 2$, and as $w_2^j \geq m$, it is then a strictly dominant strategy for D_1 to accept U_1's contract. Hence, in any equilibrium of stage 2, D_1 accepts the deviation, U_1 earns ε, and the deviation is profitable.

No equilibrium in which only one downstream firm is inactive on path. Assume for a contradiction that there exists an equilibrium in which D_1 is active and purchases from

U_1, whereas D_2 is inactive.

Assume first that D_2 does not accept any offer on path. Then, $T_2^1 \geq 0$ and $\hat{\pi}(w_2^1, w_1^1) - T_2^1 \leq 0$. Moreover, as products are differentiated and $w_1^1 \geq m$, we also have that $T_2^1 > 0$ or $w_2^1 > m$ (otherwise, $\hat{\pi}(w_2^1, w_1^1) - T_2^1 \leq 0$ could not hold). I claim that U_2 can profitably deviate by offering (∞, ∞) to D_1 and (m, ε) to D_2, where $\varepsilon > 0$. Consider the acceptance choice subgame following this deviation. D_1 can either accept U_1's contract or exit the industry. If D_1 accepts U_1's contract, then D_2 strictly prefers accepting U_2's contract (provided ε is small enough), as $\hat{\pi}(m, w_1^1) - \varepsilon > 0 \geq \hat{\pi}(w_2^1, w_1^1) - T_2^1$, where the first inequality follows from the fact that products are differentiated and $w_1^1 \geq m$. If instead D_1 exits, then D_2 still strictly prefers U_2's contract, as $\hat{\pi}(m, \infty) - \varepsilon > \hat{\pi}(w_2^1, \infty) - T_2^1$ for ε small enough, where we have used the fact that $w_2^1 > m$ or $T_2^1 > 0$. Therefore, in any equilibrium of the supplier-choice subgame, D_2 accepts U_2's contract and U_2 earns ε.

Next, suppose D_2 accepts U_2's offer on the equilibrium path (but stays inactive). Then, D_2 makes zero profit on path (otherwise U_2 would be making losses and would have an incentive to withdraw its offers), and $\hat{\pi}(w_2^1, w_1^1) - T_2^1 \leq 0$. As before, this implies that $T_2^1 > 0$ or $w_2^1 > m$, and so U_2 can profitably deviate by offering (∞, ∞) to D_1 and (m, ε) to D_2, with $\varepsilon > 0$.

Finally, suppose D_2 accepts U_1's offer on the equilibrium path. Then, $T_2^1 \leq 0$, as D_2 is inactive. Assume for a contradiction that $T_2^1 < 0$. Then, $\hat{u}(w_1^1, w_2^1) + T_1^1 > 0$, for otherwise U_1 would be making strictly negative profits. U_2 can then profitably deviate by offering $(w_1^1, T_1^1 - \varepsilon)$ to D_1 and (∞, ∞) to D_2. In the subgame following this deviation, it is a dominant strategy for D_2 to accept U_1's offer, as D_1 already makes non-negative profits when accepting U_1's offer, and U_2's fixed fee is lower by ε. Therefore, at the unique Nash equilibrium of the acceptance stage, D_1 buys from U_2 and D_2 buys from U_1. U_2 earns $\hat{u}(w_1^1, w_2^1) + T_1^1 - \varepsilon$, which is strictly positive for ε small enough. Therefore, $T_2^1 = 0$ and $w_2^1 > m$. Then, as in the previous paragraph, U_2 can deviate by offering (∞, ∞) to D_1 and (m, ε) to D_2. □

The lengthy proof of Lemma 2 reveals an important issue that will have to be addressed repeatedly in this section: Starting from a given equilibrium candidate, and following a deviation by an upstream firm in stage 1, the continuation subgame starting in stage 2 may have multiple equilibria. To rule out the equilibrium candidate, I must ensure that the profit of the upstream deviator increases in any equilibrium of the continuation subgame. The restriction to variable parts no smaller than m proves very useful here, as it ensures that a downstream firm always strictly prefers accepting a contract (m, ε) to exiting the industry.

Next, let us turn to equilibrium candidates in which both downstream firms are active on path. I show that both upstream firms make zero profit, and exiting the industry is a strictly dominated strategy for the downstream firms:

Lemma 3. *Suppose that there exists an equilibrium in which both downstream firms are active, and denote by* (w_k^A, T_k^A) *(resp.* (w_k^R, T_k^R)*) the contract that is accepted (resp. rejected) by* D_k *on the equilibrium path. Then, upstream contracts satisfy the following properties:*

1. $\hat{\pi}(w_k^A, w_\ell^A) - T_k^A \geq \hat{\pi}(w_k^R, w_\ell^A) - T_k^R$ *for every* k.
2. $T_k^A = -\hat{u}(w_k^A, w_\ell^A)$, *for every* $k \neq \ell$.
3. $\hat{\pi}(w_k^A, w) - T_k^A > 0$ *for every* k *and* w.

Proof. The first assertion is obvious. To prove the second assertion, assume for a contradiction that U_1 supplies both downstream firms on the equilibrium path, and that $\hat{u}(w_1^A, w_2^A) + T_1^A > 0$. There are two cases to consider. Assume first that $\hat{\pi}(w_2^A, w_1^A) - T_2^A = 0$. Then, the profit that U_1 earns from selling the input to D_2 is equal to:

$$\hat{u}(w_2^A, w_1^A) + T_2^A = \underbrace{\hat{u}(w_2^A, w_1^A)}_{\geq 0 \text{ as } w_2^A \geq m} + \underbrace{\hat{\pi}(w_2^A, w_1^A)}_{>0 \text{ as } D_2 \text{ is active}} > 0.$$

Let us show that U_2 can profitably deviate by offering $(w_1^A, T_1^A - \varepsilon)$ to D_1 and $(w_2^A, T_2^A - \varepsilon)$ to D_2 with $\varepsilon > 0$. Accepting U_2's contract obviously strictly dominates accepting U_1's contract. Moreover, for $k \neq \ell$,

$$\hat{\pi}(w_k^A, \infty) - T_k^A + \varepsilon > \hat{\pi}(w_k^A, w_\ell^A) - T_k^A + \varepsilon > 0.$$

Hence, in this subgame, it is a strictly dominant strategy for both firms to accept U_2's deviating offer. The deviation is profitable for U_2 provided ε is small enough.

Suppose instead that $\hat{\pi}(w_2^A, w_1^A) - T_2^A > 0$. Then, U_2 can deviate by offering $(w_1^A, T_1^A - \varepsilon)$ to D_1 and (∞, ∞) to D_2. Then, it is a dominant strategy for D_1 to accept U_2's contract and for D_2 to stick to U_1's contract, and the deviation is profitable provided ε is small enough.

Next, suppose that U_1 supplies D_1 and U_2 supplies D_2 on the equilibrium path, and assume for a contradiction that $\hat{u}(w_1^A, w_2^A) + T_1^A > 0$. Clearly, $\hat{u}(w_2^A, w_2^A) + T_2^A \geq 0$.

Assume first that $w_1^A > m$. I claim that U_2 can profitably deviate by offering $\big(w_1^A, \min(0, T_1^A) - \varepsilon\big)$ to D_1 and $(w_2^A, T_2^A - \varepsilon)$ to D_2. It is a strictly dominant strategy for D_1 to accept U_2's contract. Moreover, given that D_1 accepts U_2's contract, D_2 strictly prefers accepting U_2's contract to exiting or accepting U_1's contract. Hence, the only equilibrium of the supplier choice subgame has both downstream firms accepting U_2's deviating offer. U_2's profit is either

$$\overbrace{\big(\hat{u}(w_1^A, w_2^A) + T_1^A\big)}^{>0} + \overbrace{\big(\hat{u}(w_2^A, w_1^A) + T_2^A\big)}^{\geq 0} - 2\varepsilon,$$

$$\text{or} \quad \underbrace{\big(\hat{u}(w_1^A, w_2^A)\big)}_{>0} + \underbrace{\big(\hat{u}(w_2^A, w_1^A) + T_2^A\big)}_{\geq 0} - 2\varepsilon.$$

Both expressions are strictly greater than $\hat{u}(w_2^A, w_1^A) + T_2^A$ when ε is small enough.

Assume instead that $w_1^A = m$. Then, $T_1^A > 0$, for otherwise U_1 would not be making positive profits. I claim that U_2 can profitably deviate by offering (m, ε) to D_1 and $(w_2^A, T_2^A - \frac{\varepsilon}{2})$ to D_2. If ε is small enough, D_1 strictly prefers accepting U_2's contract to accepting U_1's contract and to exiting (as products are differentiated and D_2's marginal cost cannot be lower than m). Moreover, given that D_1 accepts U_2's contract, D_2 is strictly better off accepting U_2's contract rather than exiting or accepting U_1's contract. The only equilibrium of the supplier choice subgame therefore has both downstream firms accepting U_2's deviating offer. U_2's profit is:

$$\left(\hat{u}(w_2^A, w_1^A) + T_2^A\right) + \frac{\varepsilon}{2} > \left(\hat{u}(w_2^A, w_1^A) + T_2^A\right),$$

and so the deviation is profitable.

Finally, I prove the third assertion of the lemma. I have shown that $T_k^A = -\hat{u}(w_k^A, w_\ell^A)$ for $k \neq \ell$. If $w_1^A > m$, then $T_1^A < 0$, and D_1 therefore makes a profit of at least $-T_1^A > 0$ if it accepts (w_1^A, T_1^A). If instead $w_1^A = m$, then $T_1^A = 0$. As D_2's marginal cost will always be at least m and products are differentiated, this implies that D_1 always makes strictly positive profits if it accepts (w_1^A, T_1^A). □

Intuitively, as upstream firms are competing in prices with homogeneous products, we cannot expect them to make positive profits in equilibrium. While this result seems obvious, the proof is tedious because of the potential equilibrium multiplicity in stage 2, discussed above. The third part of the lemma states that D_k strictly prefers accepting (w_k^A, T_k^A) to exiting, regardless of the variable part at which D_ℓ is purchasing. This result is useful, as it will allow me to ignore downstream firms' exit option when looking for equilibria of the supplier choice game, thereby turning this game into a two-by-two game.

The following concept is useful to study equilibria in which both downstream firms are active:

Definition 1. *A pair of linear upstream prices $(w_1^\star, w_2^\star)$ satisfies the Bonanno-Vickers-Rey-Stiglitz-Shaffer (BVRSS) conditions if $\hat{q}(w_1^\star, w_2^\star) > 0$, $\hat{q}(w_2^\star, w_1^\star) > 0$, and for every $k \neq \ell$ in $\{1, 2\}$,*

$$w_k^\star \in \arg\max_{w_k \geq m} \left(\hat{\pi}(w_k, w_\ell^\star) + \hat{u}(w_k, w_\ell^\star)\right). \tag{2}$$

That is, the BVRSS conditions hold if both downstream firms can be active and upstream prices are pairwise-stable. As is well-known (Bonanno and Vickers, 1988; Rey and Stiglitz, 1988, 1995; Shaffer, 1991), such upstream prices are strictly larger than marginal cost:

Lemma 4. *If $(w_1^\star, w_2^\star)$ satisfies the BVRSS conditions, then $w_1^\star > m$ and $w_2^\star > m$.*

Proof. Assume for a contradiction that $w_k^\star = m$. Differentiating $\hat{\pi}(w_k, w_\ell^\star) + \hat{u}(w_k, w_\ell^\star)$ with respect to w_k at $w_k = m$ and using the envelope theorem yields

$$\partial_1 \left(\hat{\pi}(m, w_\ell^\star) + \hat{u}(m, w_\ell^\star)\right) = \left[\hat{p}(m, w_\ell^\star) - m\right] \partial_2 q \left[\hat{p}(m, w_\ell^\star), \hat{p}(w_\ell^\star, m)\right] \partial_2 \hat{p}(w_\ell^\star, m) > 0,$$

which is a contradiction. □

The intuition is that a downstream firm with a high unit cost has a commitment to set a high downstream price, which is beneficial, as prices are strategic complements. In a model in which upstream firms do not compete by assumption, Bonanno and Vickers (1988) show that downstream firms purchase at upstream prices satisfying the BVRSS conditions. Shaffer (1991) argues that this outcome also emerges in a model with a large number of identical upstream firms. The following lemma confirms that, if there is an equilibrium in which both downstream firms are active, then the upstream variable parts at which they end up purchasing have to satisfy the BVRSS conditions:

Lemma 5. *Suppose that there exists an equilibrium in which both downstream firms are active, and denote by (w_k^A, T_k^A) (resp. (w_k^R, T_k^R)) the contract that is accepted (resp. rejected) by D_k on the equilibrium path. Then, (w_1^A, w_2^A) satisfies the BVRSS conditions.*

Proof. By Lemma 3, we must have that $T_k^A = -\hat{u}(w_k^A, w_\ell^A)$ for $k \neq \ell$, and D_k strictly prefers accepting (w_k^A, T_k^A) to exiting. Assume for a contradiction that (w_1^A, w_2^A) does not satisfy the BVRSS conditions. As both firms are active when they accept their equilibrium contracts, this means that condition (2) is not satisfied for some firm, say, D_1. Hence, there exists $\hat{w} \geq m$ such that

$$\hat{\pi}(\hat{w}, w_2^A) + \hat{u}(\hat{w}, w_2^A) > \hat{\pi}(w_1^A, w_2^A) + \hat{u}(w_1^A, w_2^A). \tag{3}$$

Assume first that U_1 supplies both D_1 and D_2. Then, I claim that U_2 can profitably deviate by offering (∞, ∞) to D_2, and a contract with variable part $\hat{w}$ and fixed part

$$\hat{T} = \hat{\pi}(\hat{w}, w_2^A) - \left(\hat{\pi}(w_1^A, w_2^A) + \hat{u}(w_1^A, w_2^A)\right) - \varepsilon \tag{4}$$

to D_1. It is a strictly dominant strategy for D_2 to stick to U_1's contract. Moreover, given that D_2 accepts U_1's contract, D_1 strictly prefers accepting U_2's contract, as

$$\hat{\pi}(\hat{w}, w_2^A) - \hat{T} = \hat{\pi}(w_1^A, w_2^A) + \hat{u}(w_1^A, w_2^A) + \varepsilon.$$

Therefore, at the only equilibrium of the supplier-choice subgame, D_1 accepts U_2's contract, and D_2 accepts U_1's contract. The profit of U_2 is equal to:

$$\hat{\pi}(\hat{w}, w_2^A) + \hat{u}(\hat{w}, w_2^A) - \left(\hat{\pi}(w_1^A, w_2^A) + \hat{u}(w_1^A, w_2^A)\right) - \varepsilon, \tag{5}$$

which is strictly positive for small enough ε: The deviation is profitable.

Next, assume that U_1 supplies D_1 and U_2 supplies D_2. Then, U_1 can profitably deviate by offering (∞, ∞) to D_2 and $(\hat{w}, \hat{T})$ to D_1, where $\hat{w}$ and $\hat{T}$ are defined in equations (3) and (4), respectively. It is a strictly dominant strategy for D_2 to keep accepting U_2's contract. Besides, as $\varepsilon > 0$, conditional on D_2 sticking to U_2's contract, D_1 strictly prefers accepting U_1's deviation to exiting or accepting (w_1^R, T_1^R). U_1's profit is equal to expression (5), which is strictly positive provided ε is small enough: The deviation is also profitable. □

However, even if downstream firms purchase at variable parts satisfying the BVRSS conditions, there still exist profitable deviations, which concludes the proof of Proposition 2:

Lemma 6. *There is no equilibrium in which downstream firms accept tariffs with variable parts satisfying the BVRSS conditions.*

Proof. Assume for a contradiction that such an equilibrium exists, and let (w_k^A, T_k^A) (resp. (w_k^R, T_k^R)) denote the contract that is accepted (resp. rejected) by D_k on the equilibrium path. By Lemma 3, $T_k^A = -\hat{u}(w_k^A, w_\ell^A)$ for $k \neq \ell$.

Assume first that U_1 supplies both downstream firms. Suppose U_2 deviates and offers $(w_1^A + \varepsilon, \hat{T}_1)$ to D_1 and $(w_2^A, T_2^A - \eta)$ to D_2, where ε and η are small and strictly positive, and $\hat{T}_1$ will be determined later. By Lemma 3 and as $\eta > 0$, accepting U_2's contract is a strictly dominant strategy for D_2. The equilibrium of the supplier-choice subgame is unique and such that D_1 also accepts U_2's contract if and only if

$$\hat{\pi}(w_1^A + \varepsilon, w_2^A) - \hat{T}_1 > \hat{\pi}(w_1^A, w_2^A) + \hat{u}(w_1^A, w_2^A), \tag{6}$$

where I have used the fact that $T_1^A = -\hat{u}(w_1^A, w_2^A)$. Adding $\hat{u}(w_1^A + \varepsilon, w_2^A)$ to both sides of inequality (6) and rearranging terms yields:

$$\hat{u}(w_1^A + \varepsilon, w_2^A) + \hat{T}_1 < \\ \left(\hat{\pi}(w_1^A + \varepsilon, w_2^A) + \hat{u}(w_1^A + \varepsilon, w_2^A)\right) - \left(\hat{\pi}(w_1^A, w_2^A) + \hat{u}(w_1^A, w_2^A)\right). \tag{7}$$

As the right-hand side is continuously differentiable, Taylor's theorem implies the existence of a function $h_1(\cdot)$ such that $\lim_{\varepsilon \to 0} h_1(\varepsilon) = 0$ and

$$\left(\hat{\pi}(w_1^A + \varepsilon, w_2^A) + \hat{u}(w_1^A + \varepsilon, w_2^A)\right) - \left(\hat{\pi}(w_1^A, w_2^A) + \hat{u}(w_1^A, w_2^A)\right) \\ = \partial_1 \left(\hat{\pi}(w_1^A, w_2^A) + \hat{u}(w_1^A, w_2^A)\right) \varepsilon + h_1(\varepsilon)\varepsilon = h_1(\varepsilon)\varepsilon,$$

where the second equality follows as (w_1^A, w_2^A) satisfies the BVRSS conditions. Set $\hat{T}_1$ so that $\hat{u}(w_1^A + \varepsilon, w_2^A) + \hat{T}_1 = h_1(\varepsilon)\varepsilon - \delta$, where $\delta > 0$ is a small number—note that inequality (7) holds as long as $\delta > 0$. Applying again Taylor's theorem, the profit that U_2 makes from selling to D_2 is equal to

$$\begin{aligned}\hat{u}(w_2^A, w_1^A + \varepsilon) + T_2^A - \eta &= \hat{u}(w_2^A, w_1^A) + T_2^A - \eta + \varepsilon\partial_2\hat{u}(w_2^A, w_1^A) + \varepsilon h_2(\varepsilon) \\ &= \varepsilon\partial_2\hat{u}(w_2^A, w_1^A) + \varepsilon h_2(\varepsilon) - \eta,\end{aligned}$$

where $\lim_{\varepsilon\to 0} h_2(\varepsilon) = 0$, and I have used the fact that $\hat{u}(w_2^A, w_1^A) + T_2^A = 0$ to obtain the second line. It follows that the total profit that U_2 earns when it deviates is equal to:

$$\Pi = \varepsilon(\partial_2\hat{u}(w_2^A, w_1^A) + h_1(\varepsilon) + h_2(\varepsilon)) - \eta - \delta. \tag{8}$$

As $h_1(\varepsilon)$ and $h_2(\varepsilon)$ both tend to zero as ε tends to zero and $\partial_2\hat{u}(w_2^A, w_1^A) > 0$ (by Assumption 1), there exists $\overline{\varepsilon} > 0$ such that $\partial_2\hat{u}(w_2^A, w_1^A) + h_1(\overline{\varepsilon}) + h_2(\overline{\varepsilon}) > 0$. Choose $\varepsilon = \overline{\varepsilon}$ and η and δ small enough so that the right-hand side of equation (8) is strictly positive. With these values of ε, η and δ, the deviation is strictly profitable. Hence, there is no equilibrium in which an upstream firm supplies both downstream firms with variable parts satisfying the BVRSS conditions.

Next, assume that U_1 supplies D_1 and U_2 supplies D_2 in equilibrium. I claim that

$$\hat{\pi}(w_k^A, w_\ell^A) - T_k^A = \hat{\pi}(w_k^R, w_\ell^A) - T_k^R,\ k \neq l \text{ in } \{1, 2\}. \tag{9}$$

Assume for a contradiction that

$$\hat{\pi}(w_1^A, w_2^A) - T_1^A > \hat{\pi}(w_1^R, w_2^A) - T_1^R.$$

Then, U_1 can profitably deviate by offering (∞, ∞) to D_2 and $(w_1^A, T_1^A + \varepsilon)$ to D_1: It is straightforward to check that, when ε is small enough, there exists a unique equilibrium of stage 2 in which D_1 accepts U_1's contract and D_2 accepts U_2's contract; therefore, U_1's profit increases from 0 to $\varepsilon > 0$, a contradiction. Hence, condition (9) holds.

Now, consider the following deviation: U_1 offers $(w_1^A + \varepsilon, \hat{T}_1)$ to D_1 and $(w_2^A, T_2^A - \eta)$ to D_2, where $\varepsilon,\ \eta > 0$ and $\hat{T}_1$ will be determined later. As before, it is a strictly dominant strategy for D_2 to accept U_1's contract. The equilibrium of the suppliers choice subgame is unique and such that D_1 accepts U_1's offer as well if

$$\hat{\pi}(w_1^A + \varepsilon, w_2^A) - \hat{T}_1 > \hat{\pi}(w_1^R, w_2^A) - T_1^R = \hat{\pi}(w_1^A, w_2^A) - T_1^A, \tag{10}$$

where the equality follows from condition (9). As inequality (10) is the same as inequality (6), the argument in the first part of the proof implies that we can find δ, ε, and $\hat{T}_1$ such that both downstream firms accept U_1's contracts at the unique equilibrium of the supplier choice subgame, and U_1 makes strictly positive profits. □

Intuitively, variable parts satisfying the BVRSS conditions are too low from the point of view of industry profit maximization, as they do not internalize competitive externalities between downstream firms. This opens the door to a deviation in which one of the upstream firms ends up supplying both downstream firms with variable parts higher than the initial BVRSS ones. Lemma 6 formalizes this intuition.

As mentioned in the introduction, Shaffer (1991) studies a model similar to mine except that a large number of identical upstream firms are competing in the upstream market. He claims that: The two-part tariff game with exclusive contracts has an equilibrium; in any equilibrium, downstream firms purchase at variable parts consistent with the BVRSS conditions, and upstream firms make zero profit. Unfortunately, he provides no information regarding the tariffs offered by the upstream firms whose contracts are not accepted on the equilibrium path.

To see why this matters, assume first that all upstream firms offer the same contracts. Then, just as in the proof of Lemma 6, an upstream firm can deviate by offering a slightly lower fixed part to one of the downstream firms, and a contract with a higher variable part and a lower fixed part to the other downstream firm. This deviation is profitable, as the deviating upstream firm makes a second-order loss on the latter firm and a first-order gain on the former.

This deviation might no longer be effective if some upstream firms offer contracts different from those that downstream firms are meant to accept on the equilibrium path. In this case, downstream firms might coordinate on another Nash equilibrium of the supplier choice subgame after an upstream deviation. This might make variable parts satisfying the BVRSS conditions sustainable in equilibrium. The problem is that this argument also applies to Lemma 5: variable parts that are not consistent with the BVRSS conditions might also be sustainable in equilibrium, because downstream firms might react to an upstream deviation by coordinating on another Nash equilibrium. By the same token, outcomes in which upstream firms make positive profits might also be sustainable, i.e., Lemma 3 might not extend to Shaffer (1991)'s framework. To sum up, the equilibrium set in Shaffer's model may be either empty or much larger than what Shaffer asserted in his paper.

5 Concluding Remarks

I have studied a model in which two upstream firms compete with exclusive, observable, two-part tariffs contracts to supply a homogeneous input to two downstream firms selling differentiated products. Under very general conditions, this model does not have a subgame-perfect equilibrium in pure strategies. Intuitively, in this framework, a natural equilibrium candidate would have both downstream firms purchasing input at variable parts that are pairwise-stable. However, such pairwise-stable variable parts fall short of maximizing industry profits, as they ignore competitive externalities between downstream firms. As a result, it is profitable and feasible for one of the upstream firms to step in and supply both downstream firms at variable parts above the original pairwise-stable ones.

I close the paper by providing an overview of modeling assumptions that lead to equilibrium existence or non-existence. A first observation is that different assumptions on the mode of downstream competition can restore equilibrium existence. In my model, downstream firms compete with uniform prices and differentiated products. Chen and Riordan (2007) study a variant of the model with spatial competition and perfect price discrimination. In equilibrium, every consumer ends up being supplied by the most efficient downstream firm (i.e., the firm with the lowest marginal cost net of transport costs) at a price equal to the marginal cost (net of transport costs) of the least efficient firm. From the point of view of a downstream firm, a variable part above marginal cost implies that (a) some downstream consumers will be lost to its rival, and (b) some consumers will receive inefficiently high prices. A high variable part only leads the rival to increase its prices for the consumers that it will eventually supply. This implies that, under downstream price discrimination, the only upstream prices consistent with the BVRSS conditions are equal to marginal cost. The argument in the proof of Lemma 6 then fails for the following reason: If an upstream firm increases the variable part it charges to a given downstream firm, it does not capture any of the additional profit that the other downstream firm earns, as that firm purchases the input at marginal cost.

A similar remark applies to the case where downstream firms compete in quantities. Under the standard assumption that quantities are strategic substitutes, upstream firms have an incentive to set their variable parts below, not above, marginal cost. Given my restriction to non-negative upstream margins, this implies that the only upstream prices consistent with the BVRSS conditions are again equal to marginal cost. It follows that the two-part tariff competition game has an equilibrium, in which the upstream market is supplied at cost.

I made the simplifying assumption that upstream products are homogeneous. I conjecture that an equilibrium could exist if upstream products were sufficiently horizontally differentiated. Intuitively, the reason why an equilibrium fails to exist in my model is that, starting from an outcome in which the downstream firms purchase from different upstream firms, one of the upstream firms has an incentive to step in and supply both downstream firms. Under sufficient upstream differentiation, this deviation may no longer be profitable, as it would reduce product variety and thus industry profit.

Another important modeling assumption is the type of tariffs that can be used in the upstream market. My model focuses on exclusive two-part tariffs. The simpler case of linear upstream tariffs was studied in Ordover, Saloner, and Salop (1990). Under the assumption that a downstream firm's equilibrium profit is decreasing in its marginal cost, it is straightforward to show that Bertrand competition in the upstream market drives the upstream price down to marginal cost (Ordover, Saloner, and Salop, 1990; Chen, 2001). A similar result obtains if upstream firms compete with *non-exclusive* two-part tariffs. In that case, as discussed in Chen (2001), negative fixed fees cannot arise in equilibrium, as, when faced with a contract with a negative fixed fee and another one with a lower variable part, a downstream firm would have an incentive to accept both contracts, pocket the negative fixed fee from the first contract and use the second contract for input purchases. Thus, upstream competition drives variable parts down to marginal cost and fixed fees down to zero.

A related question is whether equilibrium existence could be restored by allowing for a richer class of contracts. One possibility would be to allow the upstream firms to offer menus of two-part tariff contracts. The problem with this extension is that it runs into the difficulties mentioned in my discussion of Shaffer (1991)'s paper at the end of Section 4: With such menus of two-part tariffs, equilibrium multiplicity at the supplier-choice stage becomes a major concern. In principle, this potential multiplicity can be used to punish deviations away from many equilibrium candidates, including candidates in which on-path variable parts differ from the BVRSS ones, and candidates in which upstream firms make strictly positive profits. I have not been able to characterize the equilibrium set; no have I been able to determine whether it is non-empty.

Finally, I have maintained throughout the assumption that contracts are publicly observable. The private-contracting case was studied by Nocke and Rey (2018) under quantity competition and Rey and Vergé (2020) under price competition. In both setups, an equilibrium exists and equilibrium contracts are cost-based, i.e., the marginal input price is equal to the marginal cost of production. The intuition is that a contract in which the marginal input price would differ from marginal cost would not be bilaterally efficient and would bring in no strategic benefits under private contracting.

A Appendix: Proof of Proposition 1

This appendix is organized as follows. We begin by studying the properties of the firms' best-response functions. Leveraging those properties, we then establish existence and uniqueness of the equilibrium. Finally, we show that equilibrium downstream prices are differentiable and strictly increasing in upstream prices whenever the downstream equilibrium is interior.

Best responses. We begin by establishing basic properties of best-response functions:

Claim 1. *Firm D_k's best-response function, $r(p_\ell, w_k)$, is well defined. Moreover, the function $r(\cdot, w_k)$ is bounded on any closed interval.*

Proof. For every $w_k \geq m$ and $p_\ell \geq 0$, consider D_k's profit maximization problem:

$$\max_{p_k \geq w_k} \pi(p_k, p_\ell, w_k).$$

If $q(w_k, p_\ell) = 0$, then any $p_k \geq w_k$ solves this problem, and we set $r(p_\ell, w_k)$ equal to w_k without loss of generality. Suppose instead that $q(w_k, p_\ell) > 0$. Then, by continuity, we also have that $q(w_k+\eta, p_\ell) > 0$ for some $\eta > 0$, implying that D_k can earn at least $\eta q(w_k+\eta, p_\ell) > 0$. As industry revenue tends to 0 as prices tend to infinity, we have that $\pi(p_k, p'_\ell, w_k) \leq p_k q(p_k, p'_\ell) < \eta q(w_k+\eta, p_\ell)$ for p_k and p'_ℓ sufficiently high. As the function q is non-increasing in its second argument, this implies that $\pi(p_k, p_\ell, w_k) < \eta q(w_k+\eta, p_\ell)$ for p_k sufficiently large, say, above some threshold $\check{p}_k$. Hence, the above maximization problem has the same set of solutions as $\max_{p_k \in [w_k, \check{p}_k]} \pi(p_k, p_\ell, w_k)$. By continuity and compactness, the latter problem has a solution, and any solution p_k^* must be such that $q(p_k^*, p_\ell) > 0$. By strict quasi-concavity of $p_k \mapsto \pi(p_k, p_\ell, w_k)$ on the set of prices p_k such that $q(p_k, p_\ell) > 0$, the solution is unique, and we denote it by $r(p_\ell, w_k)$.

We now turn to the second part of the claim. Let $\hat{p}_\ell$ be equal to $\max\{p_\ell : \ q(w_k, p_\ell) = 0\}$ if $q(w_k, 0) = 0$; if $q(w_k, 0) > 0$, then $\hat{p}_\ell$ is left undefined. Clearly, $r(p_\ell, w_k) = w_k$ for every $p_\ell \leq \hat{p}_\ell$. Next, fix some $p_\ell > \hat{p}_\ell$ (resp., $p_\ell \geq 0$ if $\hat{p}_\ell$ is not defined), and let us show that $r(\cdot, w_k)$ is continuous at p_ℓ. Let $\bar{p}_\ell \in (\hat{p}_\ell, p_\ell)$ (resp., $\bar{p}_\ell = 0$ if $\hat{p}_\ell$ is not defined). Then, $\pi(\tilde{p}_k, \bar{p}_\ell, w_k) > 0$ for some $\tilde{p}_k > w_k$. As industry revenue tends to 0 as prices tend to infinity, and as q is non-increasing in its second argument, there exists a $\bar{p}_k > w_k$ such that $\pi(p_k, p'_\ell, w_k) < \pi(\tilde{p}_k, \bar{p}_\ell, w_k) \leq \pi(\tilde{p}_k, p'_\ell, w_k)$ for every $p_k \geq \bar{p}_k$ and $p'_\ell \geq \bar{p}_\ell$. Hence, for any $p'_\ell \geq \bar{p}_\ell$, maximization problems $\max_{p_k \geq w_k} \pi(p_k, p'_\ell, w_k)$ and $\max_{p_k \in [w_k, \bar{p}_k]} \pi(p_k, p'_\ell, w_k)$ have the same set of solutions. As π is continuous and $[w_k, \bar{p}_k]$ is compact, Berge's maximum theorem implies that $r(\cdot, w_k)$ is continuous at p_ℓ.

If $\hat{p}_\ell$ is not defined, we can already conclude: As $r(\cdot, w_k)$ is continuous on $[0, \infty)$, it is bounded on any compact set. Suppose instead that $\hat{p}_\ell$ is defined, and let us show that $r(\cdot, w_k)$ has a finite right-hand limit at $\hat{p}_\ell$ (the limit may or may not be equal to w_k). Assume for a contradiction that $q(\hat{p}_\ell, w_k) = 0$. Then, by definition of $\hat{p}_\ell$ and continuity of q, we also have that $q(w_k, \hat{p}_\ell) = 0$, which, by monotonicity of industry demand, implies that $q(p_\ell, w_k) + q(w_k, p_\ell) = 0$ for every $p_\ell > \hat{p}_\ell$. It follows that $q(w_k, p_\ell) = 0$ for any such p_ℓ, contradicting the definition of $\hat{p}_\ell$. Therefore, $q(\hat{p}_\ell, w_k) > 0$, and, by continuity, there exists $\varepsilon > 0$ such that $q(p_\ell, w_k) > 0$ for every $p_\ell \in (\hat{p}_\ell, \hat{p}_\ell + \varepsilon)$. Hence, for every $p_\ell \in (\hat{p}_\ell, \hat{p}_\ell + \varepsilon)$, we have that $q\left(r(p_\ell, w_k), p_\ell\right) > 0$ and $q\left(p_\ell, r(p_\ell, w_k)\right) > 0$. It follows that $\pi(\cdot, \cdot, w_k)$ is locally $\mathcal{C}^2$, and that $\partial_1 \pi(r(p_\ell, w_k), p_\ell, w_k) = 0$ for every $p_\ell \in (\hat{p}_\ell, \hat{p}_\ell + \varepsilon)$. We can thus apply the implicit function theorem to the first-order condition and use the fact that $\partial^2_{12}\pi > 0$ and $\partial^2_{11}\pi < 0$ to conclude that $r(\cdot, w_k)$ is locally increasing. It follows that $r(\cdot, w_k)$ has a finite right-hand limit at $\hat{p}_\ell$, as stated. We can conclude: The restriction of $r(\cdot, w_k)$ to any interval $[\hat{p}_\ell, p_\ell]$ is continuous and therefore bounded; $r(\cdot, w_k)$ is constant on any interval $[p_\ell, \hat{p}_\ell]$; hence, $r(\cdot, w_k)$ is bounded on any closed interval. □

The following property will be useful to prove equilibrium uniqueness. The thought experiment is the following. Suppose that D_k finds it optimal to drive D_ℓ out of the market when D_ℓ prices at some $\hat{p}_\ell$. Then, driving D_ℓ out of the market remains optimal for D_k whenever D_ℓ prices above $\hat{p}_\ell$.

Claim 2. *Suppose that $q(\hat{p}_\ell, r(\hat{p}_\ell, w_k)) = 0$. Then, $q(\tilde{p}_\ell, r(\tilde{p}_\ell, w_k)) = 0$ for every $\tilde{p}_\ell \geq \hat{p}_\ell$.*

Proof. Assume $q(\hat{p}_\ell, r(\hat{p}_\ell, w_k)) = 0$ and let $\tilde{p}_\ell > \hat{p}_\ell$. If $q(r(\hat{p}_\ell, w_k), \hat{p}_\ell) = 0$ and/or $q(\tilde{p}_\ell, p_k) = 0$ for all p_k, then the conclusion follows trivially.

Suppose instead that $q(r(\hat{p}_\ell, w_k), \hat{p}_\ell) > 0$ and there exists $\check{p}_k$ such that $q(\tilde{p}_\ell, \check{p}_k) > 0$. For $p_\ell \in [\hat{p}_\ell, \tilde{p}_\ell]$, let $\rho_0(p_\ell)$ be the highest p_k such that $q(p_\ell, p_k) = 0$ and $\bar{\rho}(p_\ell)$ the smallest p_k such that $q(p_k, p_\ell) = 0$ (or $+\infty$ if no such price exists). Clearly, $\rho_0(p_\ell) < \bar{\rho}(p_\ell)$; moreover, for every $p_\ell \in [\hat{p}_\ell, \tilde{p}_\ell]$ and $p_k \in (\rho_0(p_\ell), \bar{\rho}(p_\ell))$, we have that $q(p_k, p_\ell) > 0$ and $q(p_\ell, p_k) > 0$. Define the following function:

$$\psi_{p_\ell} : p_k \in (\rho_0(p_\ell), \bar{\rho}(p_\ell)) \mapsto \partial_1\pi\,(p_k, p_\ell, w_k)\,.$$

It follows from the stability condition that $\psi'_{p_\ell}(p_k) = \partial^2_{11}\pi(p_k, p_\ell, w_k) < 0$, i.e., $\psi_{p_\ell}(\cdot)$ is strictly decreasing on $(\rho_0(p_\ell), \bar{\rho}(p_\ell))$. Therefore, $\psi_{p_\ell}(\cdot)$ has a limit as p_k approaches $\rho_0(p_\ell)$ from the right, which we denote $\phi_0(p_\ell)$, and this limit is either finite or equal to $+\infty$. Note that $\phi_0(p_\ell) > \psi_{p_\ell}(p_k)$ for all $p_k > \rho_0(p_\ell)$. Moreover, as $\pi(\cdot, p_\ell, w_k)$ is strictly quasi-concave on the set of prices p_k such that $q(\cdot, p_\ell) > 0$, we immediately have that $q(p_\ell, r(p_\ell, w_k)) = 0$ if and only if $\phi_0(p_\ell) \leq 0$. Therefore, $\phi_0(\hat{p}_\ell) \leq 0$, and all I need to do is show that $\phi_0(\cdot)$ is non-increasing.

For all $\varepsilon \in (0, q(\tilde{p}_\ell, \infty))$, let $\rho_\varepsilon(p_\ell)(> \rho_0(p_\ell))$ be the unique solution in p_k of equation $q(p_\ell, p_k) = \varepsilon$, and put $\phi_\varepsilon(p_\ell) \equiv \psi_{p_\ell}(\rho_\varepsilon(p_\ell))$. Then, for all p_ℓ,

$$\phi_0(p_\ell) = \lim_{\varepsilon \downarrow 0} \phi_\varepsilon(p_\ell).$$

Differentiating ϕ_ε with respect to p_ℓ for $\varepsilon > 0$ yields:

$$\begin{aligned}
\phi'_\varepsilon(p_\ell) &= \rho'_\varepsilon(p_\ell)\partial^2_{11}\pi(\rho_\varepsilon(p_\ell), p_\ell, w_k) + \partial^2_{12}\pi(\rho_\varepsilon(p_\ell), p_\ell, w_k) \\
&= \frac{-\partial_1 q(p_\ell, \rho_\varepsilon(p_\ell))}{\partial_2 q(p_\ell, \rho_\varepsilon(p_\ell))}\partial^2_{11}\pi(\rho_\varepsilon(p_\ell), p_\ell, w_k) + \partial^2_{12}\pi(\rho_\varepsilon(p_\ell), p_\ell, w_k) \\
&\leq \partial^2_{11}\pi(\rho_\varepsilon(p_\ell), p_\ell, w_k) + \partial^2_{12}\pi(\rho_\varepsilon(p_\ell), p_\ell, w_k) \\
&< 0,
\end{aligned}$$

where the second line follows from the implicit function theorem, the third line from the local concavity of π and the fact that total demand is non-increasing in prices, and the last line from the stability condition. it follows that $\phi_\varepsilon(\cdot)$ is strictly decreasing for all $\varepsilon > 0$. Taking the limit as ε tends to 0, this implies that $\phi_0(\cdot)$ is non-increasing. □

Equilibrium existence. We would like to apply Theorem 1.2 in Fudenberg and Tirole (1991), but the firms' action sets are unbounded above and therefore not compact. The proof of the following claim circumvents this difficulty by establishing the existence of an equilibrium strategy profile in an auxiliary game with compact action sets and proving that the strategy profile remains an equilibrium in the original game:

Claim 3. *An equilibrium of the downstream competition subgame exists for every* (w_1, w_2).

Proof. Suppose $w_k \leq w_\ell$. If $q(w_k, w_\ell) = 0$, then the monotonicity properties of demand imply that $q(p_k, p_\ell) = q(p_\ell, p_k) = 0$ for every $p_k \geq w_k$ and $p_\ell \geq w_\ell$. Hence, there exists a (trivial) Nash equilibrium in which both firms price at marginal cost.

Suppose instead that $q(w_k, w_\ell) > 0$. By continuity, we also have that $q(w_k+\eta, w_\ell) > 0$ for some $\eta > 0$, implying that D_k can guarantee itself a profit of at least $\eta q(w_k+\eta, w_\ell)$ by pricing at $w_k + \eta$. As industry revenue tends to zero as prices go to infinity and q is non-increasing in its second argument, there exists a $\bar{p}_k$ such that $p_k q(p_k, p_\ell) < \eta q(w_k + \eta, w_\ell)$ for every (p_k, p_ℓ) such that $p_k \geq \bar{p}_k$ and $p_\ell \geq w_\ell$. Hence, for D_k, pricing above $\bar{p}_k$ is strictly dominated by pricing at $w_k + \eta$. After eliminating those strictly dominated strategies, we obtain the strategically equivalent game in which D_k chooses prices from $[w_k, \bar{p}_k]$ and D_ℓ chooses prices from $[w_\ell, +\infty)$. Let $\bar{p}_\ell < \infty$ be an upper bound for $r(\cdot, w_\ell)$ on $[w_k, \bar{p}_k]$; such a finite upper bound exists by the second part of Claim 1. The normal-form game in which D_k chooses its price from $[w_k, \bar{p}_k]$ and D_ℓ chooses its price from $[w_\ell, \bar{p}_\ell]$ has a Nash equilibrium by Theorem 1.2 in Fudenberg and Tirole (1991). Moreover, by definition of $\bar{p}_k$ and $\bar{p}_\ell$, that equilibrium is also an equilibrium of the original game. □

Equilibrium uniqueness. We prove a series of claims that jointly imply equilibrium uniqueness. We use the following terminology: An equilibrium is interior if both firms supply a strictly positive quantity; in a corner equilibrium, at least one firm receives zero demand. We have:

Claim 4. *There is at most one interior equilibrium.*

Proof. As products are differentiated, both firms must charge strictly positive markups in any interior equilibrium. Assume for a contradiction that $(\hat{p}_1, \hat{p}_2)$ and $(\tilde{p}_1, \tilde{p}_2)$ are distinct interior Nash equilibria. Assume without loss of generality that $\hat{p}_1 < \tilde{p}_1$. As $(\tilde{p}_1, \tilde{p}_2)$ is interior, $q(\tilde{p}_1, r(\tilde{p}_1, w_2)) = q(\tilde{p}_1, \tilde{p}_2) > 0$. It follows from Claim 2 that $q(p_1, r(p_1, w_2)) > 0$ for every $p \in [\hat{p}_1, \tilde{p}_1]$. Moreover, as $(\hat{p}_1, \hat{p}_2)$ is interior, we also have that $q(r(p_1, w_2), p) > 0$ for all $p_1 \in [\hat{p}_1, \tilde{p}_1]$. Therefore, firm 2's best response, $r(p_1, w_2)$, results in both firms having strictly positive demand for every $p_1 \in [\hat{p}_1, \tilde{p}_1]$. The implicit function theorem applied to D_2's first-order condition, the stability condition, and the strategic complementarity assumption imply that $r(\cdot, w_2)$ is continuously differentiable on $[\hat{p}_1, \tilde{p}_1]$, and that $\partial_1 r(p_1, w_2) \in (0, 1)$. This implies that $\hat{p}_2 < \tilde{p}_2$ and, using the mean value inequality, that

$$|\tilde{p}_2 - \hat{p}_2| = |r(\tilde{p}_1, w_2) - r(\hat{p}_1, w_2)| \leq \sup_{p_1 \in [\hat{p}_1, \tilde{p}_1]} |\partial_1 r(p_1, w_2)| |\tilde{p}_1 - \hat{p}_1| < |\tilde{p}_1 - \hat{p}_1|,$$

where the strict inequality follows as we are taking the supremum of a continuous function on a compact set. However, as $\hat{p}_2 < \tilde{p}_2$, the exact same argument can be used to show that $|\tilde{p}_1 - \hat{p}_1| < |\tilde{p}_2 - \hat{p}_2|$, a contradiction. □

Claim 5. *There is at most one corner equilibrium.*

Proof. Recall from footnote 7 that we view two corner equilibria as being one and the same if they result in the same level of demands, profits, and consumer surplus. Thus, assume for a contradiction that there exist two distinct corner equilibrium outcomes, $(\hat{p}_1, \hat{p}_2)$ and $(\tilde{p}_1, \tilde{p}_2)$. If $q(\hat{p}_1, \hat{p}_2) = q(\tilde{p}_2, \tilde{p}_1) = 0$, then we also have that $q(w_1, \hat{p}_2) = q(w_2, \tilde{p}_1) = 0$ and $q(w_1, w_2) = q(w_2, w_1) = 0$. Hence, both firms are receiving zero demand in both equilibria, which means that those equilibrium outcomes are one and the same, a contradiction.

Next, assume $q(\hat{p}_1, \hat{p}_2) = q(\tilde{p}_1, \tilde{p}_2) = 0$, and let $p_2^m (= r(\infty, w_2))$ be D_2's monopoly price. If $q(w_1, p_2^m) = 0$, then $\hat{p}_2 = \tilde{p}_2 = p_2^m$, $q(\tilde{p}_1, \tilde{p}_2) = q(\hat{p}_1, \hat{p}_2) = 0$, and $q(\tilde{p}_2, \tilde{p}_1) = q(\hat{p}_2, \hat{p}_1)$. Therefore, both equilibria lead to the same outcome, a contradiction. Assume instead that $q(w_1, p_2^m) > 0$. If $\hat{p}_1 > w_1$, then $\hat{p}_2$ is either equal to p_2^m or to the highest p_2 such that $q(\hat{p}_1, p_2) = 0$. In both cases, D_1 can profitably deviate by setting $p_1 = w_1 + \varepsilon$. It follows that $\hat{p}_1 = \tilde{p}_1 = w_1$ and $\hat{p}_2 = \tilde{p}_2 = r(w_1, w_2)$, which is a contradiction. □

Claim 6. *Corner and interior equilibria cannot coexist.*

Proof. Assume for a contradiction that there exists one interior equilibrium, $(\hat{p}_1, \hat{p}_2)$, and one corner equilibrium, $(\tilde{p}_1, \tilde{p}_2)$. To fix ideas, assume that, in the corner equilibrium, $q(\tilde{p}_2, \tilde{p}_1) > 0$

and $q(\tilde{p}_1, \tilde{p}_2) = 0$. As in the proof of Claim 5, if $q(w_1, p_2^m) > 0$, then $\tilde{p}_1 = w_1$ and $\tilde{p}_2$ is the highest p_2 such that $q(w_1, p_2) = 0$. Therefore, $q(w_1, r(w_1, w_2)) = 0$. As $q(\hat{p}_1, r(\hat{p}_1, w_2)) > 0$, it follows from Claim 2 that $\hat{p}_1 < w_1$, which is a contradiction. If instead $q(w_1, p_2^m) = 0$, then we also have that $q(w_1, r(w_1, w_2)) = 0$, and we obtain the same contradiction. □

Differentiability and monotonicity of equilibrium prices. We have:

Claim 7. *At every point (w_1, w_2) such that the equilibrium is interior, equilibrium downstream prices are continuously differentiable in (w_1, w_2) with strictly positive partial derivatives.*

Proof. Suppose there exists an interior equilibrium when upstream prices are $(\hat{w}_1, \hat{w}_2)$. Equilibrium downstream prices solve $\partial_1 \pi(p_1, p_2, \hat{w}_1) = 0$ and $\partial_1 \pi(p_2, p_1, \hat{w}_2) = 0$. As $\partial_1 \pi$ is (locally) continuously differentiable with non-singular Jacobian in (p_1, p_2), the implicit function theorem implies the existence of a neighborhood of $(\hat{w}_1, \hat{w}_2)$ such that, for all (w_1, w_2) in this neighborhood, the equilibrium is interior, and equilibrium downstream prices are continuously differentiable. The fact that downstream prices are increasing in upstream prices follows readily from a monotone comparative statics argument (see Vives, 1999, p.35), and from the fact that the downstream equilibrium is unique. □

B Appendix: Linear-Demand Example

Proof. As is well known (see Shubik and Levitan, 1980), the linear demand system can be derived from a representative consumer maximizing the quadratic, strictly concave net utility function

$$U(q_1, q_2) = q_1 + q_2 - \frac{1}{2}(q_1 + q_2)^2 - \frac{1}{1+\gamma}\left(q_1^2 + q_2^2 - \frac{1}{2}(q_1 + q_2)^2\right) - p_1 q_1 - p_2 q_2$$

over the compact set $[0, 1]^2$. The continuity of the demand system therefore follows from Berge's maximum theorem. The fact that the demand system is $\mathcal{C}^2$ at every price vector such that both firms' demands are strictly positive is immediate. The monotonicity properties stated in Section 2 are clearly satisfied. Moreover, as both firms' demands vanish when both prices exceed 1, the assumption that industry revenue goes to 0 as both prices tend to infinity holds. It is easily checked that, at any price vector at which both demands are strictly positive and regardless of the wholesale price, $\partial^2_{12}\pi(p_k, p_\ell, w_k) = \gamma/4$ and $\partial^2_{11}\pi(p_k, p_\ell, w_k) + \partial^2_{12}\pi(p_k, p_\ell, w_k) = -1 - \gamma/4$, so that the strategic complementarity assumption holds and the stability condition is satisfied.

Next, let us check that, for all $p_\ell > 0$, $\pi(\cdot, p_\ell, w_k)$ is strictly quasi-concave on the set of prices p_k such that $q(p_k, p_\ell) > 0$. If $p_\ell \geq 1$, then $\pi(p_k, p_\ell, w_k)$ is equal to $(p_k - w_k)\frac{1+\gamma}{2+\gamma}(1 - p_k)$

if $p_k \in (0,1)$, and to 0 if $p_k \geq 1$. Clearly, this function is strictly concave on $(0,1)$. Next, assume $p_\ell < 1$. Let $\underline{p}_k = \frac{(2+\gamma)p_\ell - 2}{\gamma}$ and $\overline{p}_k = \frac{\gamma p_\ell + 2}{\gamma+2}$. Then,

$$\pi(p_k, p_\ell, w_k) = \begin{cases} (p_k - w_k)\frac{1+\gamma}{2+\gamma}(1-p_k) & \text{if } p_k \in (0, \underline{p}_k], \\ (p_k - w_k)\frac{1}{2}\left(1 - p_k - \gamma\left(p_k - \frac{p_k+p_\ell}{2}\right)\right) & \text{if } p_k \in [\underline{p}_k, \overline{p}_k), \\ 0 & \text{if } p_k \geq \overline{p}_k. \end{cases}$$

Note that $\pi(\cdot p_\ell, w_k)$ is strictly concave on the intervals $[0, \underline{p}_k]$ and $[\underline{p}_k, \overline{p}_k)$. To establish its strict quasi-concavity on $(0, \overline{p}_k]$, all I need to do is show that $\lim_{p_k \uparrow \underline{p}_k} \partial_1 \pi(p_k, p_\ell, w_k) \geq \lim_{p_k \downarrow \underline{p}_k} \partial_1 \pi(p_k, p_\ell, w_k)$. This holds, as

$$\lim_{p_k \uparrow \underline{p}_k} \partial_1 \pi(p_k, p_\ell, w_k) = -(\underline{p}_k - w_k)\frac{1+\gamma}{2+\gamma} + q(\underline{p}_k, p_\ell),$$

$$\lim_{p_k \downarrow \underline{p}_k} \partial_1 \pi(p_k, p_\ell, w_k) = -(\underline{p}_k - w_k)\frac{1}{2}(1 + \frac{\gamma}{2}) + q(\underline{p}_k, p_\ell),$$

and $\frac{1+\gamma}{2+\gamma} < \frac{1+\gamma/2}{2}$.

Finally, let us check that Assumption 1 holds. For values of (w_1, w_2) such that the downstream equilibrium is interior, downstream equilibrium prices are given by the usual system of first-order conditions. The solution is available in closed form:

$$\hat{p}(w_k, w_\ell) = \frac{8 + 2w_k(2+\gamma)^2 + \gamma(6 + w_\ell(2+\gamma))}{(4+\gamma)(4+3\gamma)}.$$

Plugging these prices into the demand function yields:

$$\hat{q}(w_k, w_\ell) = \frac{(2+\gamma)(8 + \gamma(6 + w_\ell(2+\gamma)) - w_k(8 + \gamma(8+\gamma)))}{4(4+\gamma)(4+3\gamma)},$$

which is indeed strictly increasing in w_ℓ. □

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