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Organizational Change and Reference-Dependent Preferences

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Organizational Change and Reference-Dependent Preferences†

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Abstract: Reference-dependent preferences can explain several puzzling observations about organizational change. We introduce a dynamic model in which a loss-neutral firm bargains with loss-averse workers over organizational change and wages. We show that change is often stagnant or slow for long periods followed by a sudden boost in productivity during a crisis. Moreover, it accounts for the fact that different firms in the same industry often have significant productivity differences. The model also demonstrates the importance of expectation management even if all parties have rational expectations. Social preferences explain why it may be optimal to divide a firm into separate entities.

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1 Introduction

In the early 1980s the iron ore and steel industry in the Great Lakes area was hit by a severe competitive shock. For more than 100 years it had been protected from foreign competition by its geographic location. Then it suddenly faced Brazilian competitors offering steel at substantially lower prices putting its own future into doubt. However, the local steel industry managed to survive and thrive by implementing cost reductions and boosting productivity. Within a few years, labor productivity doubled. This remarkable development was not due to the introduction of any new technologies, nor to major capital investments or the exit of low productivity firms. Instead, it was mainly caused by organizational improvements, in particular changes of inefficient work practices and the more efficient use of existing capital.[1] This raises the question of why these organizational improvements had not been implemented earlier. Why did it need a crisis to implement change?

This example is part of a larger puzzle: the existence of substantial and persistent heterogeneity in total factor productivity among firms within narrowly defined industries. Syverson (2004) reports for the US that a firm at 90th percentile of productivity has a TFP that is 1.9 times higher than the TFP of a firm at the 10th percentile (on average, in industries at the four-digit level). These differences cannot be accounted for by differences in observable inputs or heterogeneous prices. Increasing evidence suggests that differences in productivity are associated with organizational differences and differences in management practices.[2] However, if organizational and managerial practices are crucial for productivity, why don't all firms adopt best practices and use the most efficient organizational structures? What are the frictions that prevent profit-maximizing firms from producing at the efficiency frontier?

In this paper, we explore the implications of reference-dependent preferences, specifically loss aversion and social comparisons on the side of workers, for organizational change.[3] Technological changes create profitable opportunities for the firm that require organizational

[1]Schmitz (2007) offers a detailed and thought-provoking case study of this episode.

[2]See e.g., Bloom et al. (2014), Bloom et al. (2019).

[3]Loss aversion and social preferences are the most widely studied "biases" in behavioral economics. A recent meta study by Brown et al. (2023) reports that the average parameter of loss aversion across 607 empirical estimates is $1 + \lambda = 1.955$, i.e., losses weigh almost twice as much as gains. Fehr and Charness (2023) offer a recent survey on the vast literature on social preferences.

changes. In the basic version of the model we abstract away from any informational or contractual frictions and assume that organizational change is perfectly contractible. However, implementing change is costly, because workers have to be compensated for the psychological loss associated with the necessary change in their work practices. Without loss aversion the firm would implement the materially efficient change and compensate workers for the occuring adaptation cost. If workers are loss averse, there is a threshold such that for all technological changes below this threshold the firm will stick to the status quo. If the threshold is exceeded the firm will adjust, but it will adjust less than in the situation without loss aversion. Thus, as is well known, loss aversion creates inertia.

This changes in a crisis. A crisis reduces the outside options of workers as the firm is threatened by bankruptcy, which would lead to job loss and the need to accept lower-paying jobs in other industries. In order to keep their jobs, workers have to make concessions, which can take the form of wage cuts or changes in work practices. Both of these concessions are perceived as losses by workers. To induce workers to accept organizational changes the firm can now offer lower wage cuts (reduced losses), while it would have to offer higher wage increases (greater gains) in normal times. Thus, in a crisis the effect of loss aversion cancels out in the workers' marginal rate of substitution, and the firm will jump to the materially efficient organizational structure if there are sufficient worker rents to be appropriated by the firm. Thus, our model offers a microfoundation for why wages do not fall in a recession:[4] Workers will make concessions by working harder rather than by accepting wage cuts.

An important question for any model of loss aversion is how the reference point is determined. Kahneman and Tversky (1979) and much of the subsequent literature on loss aversion argue that in many situations a natural reference point is the status quo. In contrast, Köszegi and Rabin (2006) argue that the the reference point should be determined by rational expectations given the decision makers action. We take an agnostic position and assume that the reference point is some convex combination of the status quo and the rational expectation. Our qualitative results hold for any combination that puts positive weight on both arguments.

Our basic model captures the main intuition of why a crisis triggers organizational change, but it leaves several questions unanswered. The crisis has an effect only if workers enjoy a

[4]See, e.g., Bewley (1999).

rent. Where does this rent come from? What happens if the reference point adjusts to the new contract and the new expectations? How do rational players prepare for the possibility of crisis in the initial contract?

We develop a simple infinite horizon model that answers these questions. In this dynamic model there is technological change in every period. Loss aversion causes inertia, i.e., there is either no organizational adaptation to new technologies or less than material efficiency requires. If there is organizational change, workers suffer a one-period disutility due to adaptation cost and loss aversion associated to the adaptation. For these costs they are optimally compensated by a permanent wage increase. This gives rise to a quasi-rent that builds up over time. If the crisis hits, workers have to concede this quasi-rent by agreeing to substantial organizational change. If parties rationally anticipate the crisis, they will agree on higher wages to compensate for the expected utility loss in the crisis and they will delay change because change is cheaper to implement in the crisis. Thus, the model explains why there is often inertia or very slow change for extended periods of time, but then there is a sudden jump of productivity triggered by a crisis. Furthermore, the model explains why organizational change is history-dependent and may result in large productivity differences between firms that were founded at different points in time or faced idiosyncratic crises.

So far, we assume that change is both deterministic and perfectly contractible. In an extension of the model we consider a principal-agent problem between the owner of the firm and a manager who has to be incentivized to implement change stochastically. However, successful change always needs workers to go along with it. If the workers' reference point is (partially) determined by their expectations, an increase in the probability of successful change increases the reference point and makes it cheaper to pay workers to accept change. Thus, the owner will induce the manager to either implement the desired change with a high probability (or certainty), or to not implement it at all. This highlights the importance of expectations management, even if all parties form rational expectations. This is consistent with the emphasis on effective leadership, vision, and creating a sense of urgency for successful organizational change by practitioners and management consultants.[5]

Reference dependence is also a significant factor in social preferences, as individuals tend

[5]See e.g., Kotter (1995) and Burke (2017).

to assess their situation in comparison to others in their reference group. In a final version of the model we show that this phenomenon can explain effort compression within organizations. Effort is compressed in order to reduce wage inequality. It may be optimal to split a firm into separate entities in order to avoid social comparisons.

Our paper is related to three strands of the literature. First, there is an empirical literature on how competitive shocks affect productivity and productivity differences between firms. Bloom et al. (2014) report empirical evidence from the World Management Survey showing that there are large and persistent productivity differences across firms.[6] Higher total factor productivity is not only correlated with better management practices, but also with more intense competition. Performance increases in bad times, in particular, for low productivity firms. Bloom et al. (2019) show that the introduction of right-to-work laws in some states in the US is associated with improved managerial practices and efficiency increases. In Bloom et al. (2017) they also show that an increase of competition is associated with the introduction of better management techniques. Holmes and Schmitz (2010) survey case studies examining the behavior of firms that experienced dramatic changes in their competitive environment. They report that nearly all studies show that competitive shocks lead to increases in industry productivity. Plants that survive the competitive shock are typically found to have large productivity gains. Furthermore, these gains often account for most of the overall industry gains. Backus (2020) finds that an increase of competition in the ready-mix concrete industry has a direct positive effect on productivity (not driven by firm selection). All of this is consistent with our model. However, none of these papers explains, why it takes a competitive shock or a crisis to raise productivity.

Second, our paper contributes to the small but growing literature on reference-dependent preferences in dynamic models. Pagel (2017) shows that the incorporation of dynamic reference-dependent preferences into a macro model can account for empirically observed consumption-savings patterns.[7] Pagel (2016) uses reference-dependent preferences to explain the observed equity premium volatility and the equity premium puzzle. Eliaz and Spiegler (2014) develop a model of labor market dynamics in which workers have reference-dependent fairness pref-

[6]See Syverson (2011) and Gibbons and Henderson (2012) for surveys of the literature on productivity differences between firms in seemingly similar enterprises.

[7]Relatedly, Van Bilsen, Laeven, and Nijman (2020) find that with a backward-looking form of reference-dependence consumers delay painful consumption reductions.

erences which gives rise to wage stickiness in recessions, similar to the downward rigidity of wages in our model. Macera (2018) analyzes optimal incentive contracts in a dynamic moral hazard model with loss-averse agents. She shows that the principal backloads bonus compensation and pays a fixed wage in the present period if agents are sufficiently loss averse. Herweg, Karle, and Müller (2018) examine the role of loss aversion on renegotiation in a classical buyer-seller setting. They emphasize the role of expectations, showing that if buyers do not expect a renegotiation then the parties may indeed not be able to renegotiate, even if the outcome is ex-post inefficient. Karle and Schumacher (2017) analyze the incentives of a monopolist to release ex-ante product information. They show that good information gives rise to an attachment effect if consumers are loss averse and adjust their expectations. In the context of auctions von Wangenheim (2021) shows that with dynamic reference-dependent preferences the English auction yields lower revenues than a Vickrey auction due to a decrease of the attachment effect in the dynamic English auction. Rosato (2023) shows that a similar effect can explain empirically observed revenue declines in sequential auctions, since remaining bidders become less optimistic. Alesina and Passarelli (2019) study the effects of loss aversion in electoral competition. If there is an exogenous shock to voter preferences, the election outcome depends on the initial status quo. Furthermore, there are long-term cycles in policies with self-supporting movements to the right or to the left. Lockwood and Rockey (2020) apply loss aversion to electoral competition in a representative democracy. They show that an incumbent reacts less strongly to a shift in voter preferences than challengers.[8] None of these papers applies dynamic reference dependence to organizational change. Furthermore, the main effect driving our results is new and does not play a role in the previous literature.

Finally, there is a large literature in management science on organizational change (see e.g., Kotter (1995) and Burke (2017)). This literature emphasizes the role of expectation management, the importance of short-term wins, and the need for urgency (i.e., a crisis). This literature documents many interesting and important insights but lacks a rigorous microeconomic foundation.

The rest of the paper is organized as follows. The next section introduces the static version of the model. The model is "Coasean" in the sense that there are no informational or

[8]Lockwood, Le, and Rockey (2022) study the interaction of loss aversion and incomplete recall in dynamic electoral competition.

contractual frictions. The only friction is loss aversion. The model shows that loss aversion gives rise to inertia in normal times, but that parties will adjust toward material efficiency in a crisis. Section 3 sets up the dynamic, infinite-horizon model, in which the reference point adjusts over time. The model shows that the principal offers a permanent wage increase to compensate for change in normal times which gives rise to a quasi-rent that builds up over time. When a crisis hits, this quasi-rent is expropriated in order to implement drastic change. The model also shows how companies in the same industry using the same technology can have substantial productivity differences for extended periods of time. Section 4 introduces a principal-agent problem with a third party, the management, and shows how the principal can use expectation management to reduce the cost of implementing change. Section 5 looks at reference-dependent social preferences to explain wage and effort compression within a firm. Section 6 concludes.

2 A Coasean Model With Reference Dependence

There are two players, the owner of the firm (the principal, "she") and the workers, represented by a union (the agent, "he"), who negotiate on wages and the implementation of organizational change. We focus on the effect of reference-dependent preferences of workers on the negotiation outcome, so we abstract away from any informational or contractual frictions. The parties can implement any change via efficient ("Coasean") bargaining.

We start out with a simple one-period model. There is a state of the world, $\theta \in \Theta \subset \mathbb{R}$, that represents the current state of technology. Workers have to take a costly action $x \in \mathbb{R}^+$ that adapts the organization to the state of the world. This gives rise to a gross profit $v(x, \theta)$ that accrues to the owner.

The principal's profit function is given by

$$\Pi = v(x, \theta) - w - C \tag{1}$$

where $C \geq 0$ are all costs other than wages w. We assume that the gross profit function $v(x, \theta)$ is increasing and concave in x with $\frac{\partial v(x,\theta)}{\partial x} > 0$, $\frac{\partial^2 v(x,\theta)}{\partial x^2} < 0$, $\frac{\partial v(x,\theta)}{\partial \theta} > 0$, and $\frac{\partial v(x,\theta)}{\partial x \partial \theta} > 0$. Moreover, we assume $\lim_{x \to 0} \frac{\partial v(x,\theta)}{\partial x} = \infty$, $\lim_{x \to \infty} \frac{\partial v(x,\theta)}{\partial x} = 0$, and $\frac{\partial v(x,\theta)}{\partial x \partial \theta}$ strictly bounded away from zero in order to ensure interior solutions. The idea is that a higher state θ makes the change to more complex (i.e. higher) work practices x more profitable, but this change requires costly effort from workers. Without loss of generality we measure x by its cost, i.e., $c(x) = x$.

The utility function of the agent (the workers) is given by

$$U = w - x - \lambda[w^r - w]^+ - \lambda[x - x^r]^+, \tag{2}$$

where $[\cdot]^+ = \max\{\cdot, 0\}$. The agent's utility function is reference dependent. It consists of the material payoff, $w - x$, and the perceived experience of losses if the wage, w, is smaller than the reference wage, w^r, and/or if the action, x, is larger than its reference level, x^r.[9] The parameter $\lambda > 0$ measures the degree of loss aversion.[10] The reference point (w^r, x^r) is a convex combination of the status quo, (w_0, x_0) and the rational expectation (w^e, x^e), i.e., the values of w and x that workers expect to be realized at the end of the period:

$$w^r = \alpha w_0 + (1 - \alpha) w^e \ , \tag{3}$$

$$x^r = \alpha x_0 + (1 - \alpha) x^e \tag{4}$$

where $\alpha \in [0, 1]$ is the relative weight of the status quo.

For the baseline model we assume that organizational change is perfectly contractible. The principal makes a take-it-or-leave-it offer (w, x) to the agent (union/workers). If the agent rejects, the old contract (w_0, x_0) remains in place and the agent receives utility $U_0 \equiv w_0 - x_0 \geq 0$. Based on the offer and his rational expectation about his acceptance decision the agent forms his reference point.[11] We assume throughout that the agent expects to accept (w, x) if – given the expectation to accept – acceptance is optimal.[12] Then the agent decides whether to accept or reject the offer. After observing the agent's decision, the principal and

[9] An alternative interpretation of the model is in terms of concerns for fairness. In this interpretation w^r is the "fair wage" that workers expect to get, and x^r is the "fair effort level." Workers suffer from "inequity aversion" if the wage is below the fair wage or the requested effort above the fair effort. See Section 5.

[10] There could be different parameters of loss aversion for wages w and action x. As long as they are positive, our qualitative results hold.

[11] The idea that for $\alpha < 1$ the reference point (partly) adapts to the action chosen follows the logic of a choice-acclimating personal equilibrium in Köszegi and Rabin (2006).

[12] Without this assumption there are multiple equilibria, a common phenomenon in the literature on expectation-based reference points (Köszegi and Rabin (2006)). The other equilibria can be sustained by the workers' expectation to reject any wage offer below $\underline{w}$ with $x + \alpha\lambda(x - x_0) + U_0 < \underline{w} \leq x + \lambda(x - x_0) + U_0$. Hence, we assume that the firm can coordinate on the equilibrium in which the agent accepts change, if—given the reference point it induces—accepting change is optimal.

thereafter the agent have the option to terminate the relationship. If one of them does so, both parties get a utility of zero.

2.1 Inertia and Material Inefficiency in Normal Times

First, we consider the *normal case* where the old contract generates positive profits for the principal ($\Pi = v(x_0, \theta) - w_0 - C \geq 0$). This implies that the parties will continue their relationship even if the agent rejects the offer of the principal (which will be different in the case of a *crisis* to be discussed later). The agent correctly anticipates the negotiation outcome, so $(w^e, x^e) = (w, x)$ if he is going to accept the contract offer and $(w^e, x^e) = (w_0, x_0)$ otherwise. Because the agent anticipates that the relationship will be maintained even if he rejects the offer, his outside option utility in the normal case is given by $U_0 = w_0 - x_0 \geq 0$.

Thus, the principal's problem is:

$$\max_{w,x}\{v(x, \theta) - w - C\} \tag{5}$$

subject to

$$U \;=\; w - x - \lambda[w^r - w]^+ - \lambda[x - x^r]^+ \;\geq\; U_0 \tag{6}$$

As a benchmark consider the case where there is no loss aversion ($\lambda = 0$), so workers are only concerned about their material payoff. In this case the principal offers $w = x + U_0$ and chooses the materially efficient level of x such that

$$\frac{\partial v(x^{ME}, \theta)}{\partial x} = 1 \; . \tag{7}$$

Note that x^{ME} is an increasing function of θ (because $v_{x\theta} > 0$). Moreover, it does not depend on the status quo (w_0, x_0) or the firm's cost C.

Consider now the case with loss aversion. In order to implement x the principal has to pay

$$\begin{aligned} w &= x + \lambda[\alpha w_0 + (1-\alpha)w - w]^+ + \lambda[x - \alpha x_0 - (1-\alpha)x] + U_0 \\ &= x + \alpha\lambda[w_0 - w]^+ + \alpha\lambda[x - x_0]^+ + U_0 \; . \end{aligned} \tag{8}$$

We focus on the case where the principal wants to increase x (as compared to x_0) which implies that she also has to pay a higher wage to the workers. The case where the principal wants to decrease x is symmetric but less relevant, because θ, the state of technology can only go up. Thus, the principal maximizes

$$\Pi = v(x,\theta) - [x + \alpha\lambda(x - x_0) + U_0] - C \ .$$

The first order condition of this problem is

$$\frac{\partial v(x,\theta)}{\partial x} \leq 1 + \alpha\lambda \ . \tag{9}$$

with equality if $x > x_0$. Hence, the firm finds it optimal to increase x only if $\frac{\partial v(x_0,\theta)}{\partial x} > 1 + \alpha\lambda$.

The following proposition fully characterizes the optimal effort level x^* in normal times.

Proposition 1 (Optimal Contract, Normal Times)**.** *Suppose that the status quo contract* (w_0, x_0) *satisfies* $x_0 \leq x^{ME}(\theta)$ *and*

$$v(x_0,\theta) - w_0 - C > 0.$$

Define $\overline{x}(\theta)$ *implicitly by* $\frac{\partial v(\overline{x}(\theta),\theta)}{\partial x} = 1 + \alpha\lambda$*. The principal offers a contract* $(x^*(\theta), w^*(\theta))$ *to the agent that is given by*

$$x^*(\theta) \ = \ \begin{cases} x_0 & \text{if } x_0 \geq \overline{x}(\theta) \\ \overline{x}(\theta) & \text{if } x_0 < \overline{x}(\theta) \end{cases} \tag{10}$$

and

$$w^*(\theta) \ = \ \begin{cases} w_0 & \text{if } x_0 > \overline{x}(\theta) \\ w_0 + (1+\alpha\lambda)[\overline{x}(\theta) - x_0] & \text{if } x_0 < \overline{x}(\theta). \end{cases} \tag{11}$$

Proof. The proof follows directly from the arguments given in the text above. □

Proposition 1 shows that x^* differs from the materially efficient x^{ME}. Loss aversion drives a wedge between the marginal cost of the workers and the marginal benefit of the owner which induces the owner to stick to the status quo even if this is materially inefficient. Let $\overline{\theta}$ be implicitly defined by $\frac{\partial v(x_0,\overline{\theta})}{\partial x} = 1 + \alpha\lambda$. For $\theta \leq \overline{\theta}$ there is full inertia.[13] But even if $\theta > \overline{\theta}$, $x^*(\theta)$ is strictly smaller than the materially efficient $x^{ME}(\theta)$. The next proposition shows how the distortion depends on the degree of loss aversion, reference point formation, and the status quo.

[13] This implicitly assumes that $\theta \geq \underline{\theta}$ where $\underline{\theta}$ is defined by $x^{ME}(\underline{\theta}) = x_0$.

Proposition 2 (Comparative Statics)**.** *Suppose $x_0 < x^{ME}(\theta)$. The principal always implements less change than material efficiency requires, i.e., $x^*(\theta) - x_0 < x^{ME}(\theta) - x_0$.*

Furthermore, we have

(a) If λ (the degree of loss aversion) or α (the weight that workers put on the status quo in the formation of the reference point) increases, the amount of organizational change, $x^* - x_0$, decreases, i.e.

$$\frac{\partial x^*}{\partial \lambda}, \frac{\partial x^*}{\partial \alpha} < 0 \quad \text{if } \theta \geq \overline{\theta} \tag{12}$$

$$\frac{\partial x^*}{\partial \lambda}, \frac{\partial x^*}{\partial \alpha} = 0 \quad \text{if } \theta < \overline{\theta} \tag{13}$$

(b) An increase in λ or in α widens the range of inertia, i.e., $\frac{\partial \overline{\theta}}{\partial \lambda}, \frac{\partial \overline{\theta}}{\partial \alpha} > 0$, where $\overline{\theta}$ is implicitly defined by $\frac{\partial v(x_0, \overline{\theta})}{\partial x} = 1 + \alpha\lambda$.

(c) An increase of x_0 increases $\overline{\theta}$.

Proof. See Appendix.

Without loss aversion ($\lambda = 0$) or with a reference point that is fully determined by rational expectations ($\alpha = 0$) the principal will implement the materially efficient outcome x^{ME}. With $\lambda, \alpha > 0$ there is inertia. The larger the λ and α, the less change will be implemented and the larger the gap between the materially efficient effort level x^{ME} and the implemented effort level x^*. Furthermore, the larger the λ and α, the larger the range of inertia where the principal does not adjust x^* when θ increases. Finally, the range of inertia, i.e., $\overline{\theta}$, shifts upwards if the initial effort level x_0 increases.

2.2 A Parametric Example

Let $v(x, \theta) = \theta \ln x$. Then we have $\frac{\partial v(x,\theta)}{\partial x} = \frac{\theta}{x}$ and we get:

$$x^*(\theta) = \begin{cases} x_0 & \text{if } \theta \leq (1 + \alpha\lambda)x_0 \\ \frac{\theta}{1+\alpha\lambda} & \text{if } \theta > (1 + \alpha\lambda)x_0 \end{cases} \tag{14}$$

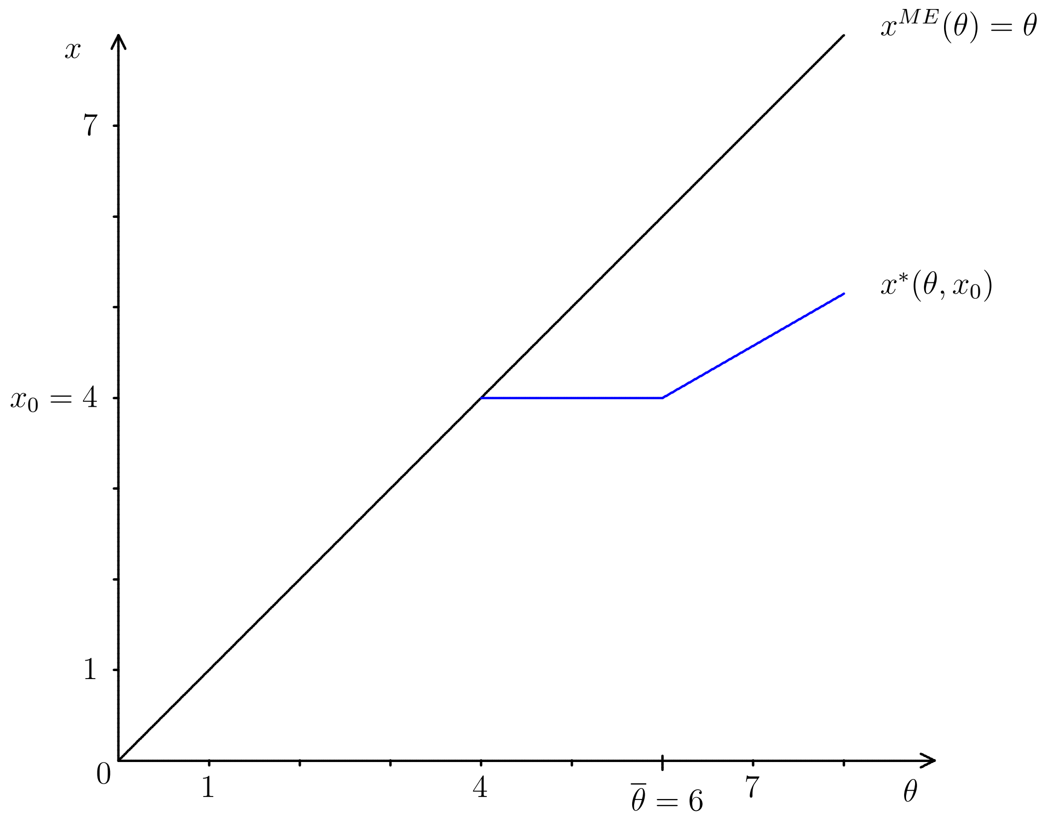

Figure 1: Organizational change as a function of θ with and without loss aversion.

Figure 1 illustrates the optimal choice of x^* by the principal for $\alpha = 0.5$, $\lambda = 1$, and $x_0 = 4$. These parameters imply $\overline{\theta} = (1 + \alpha\lambda)x_0 = 6$. The blue line depicts the optimal choice of $x^*(\theta)$ by the principal. If $\theta \leq \overline{\theta} = 6$, there is complete inertia. If $\theta > \overline{\theta}$, the principal adjusts x, but at a slope that is smaller than the slope of the materially efficient $x^{ME}(\theta)$. Thus, the larger the θ, the larger the gap between $x^{ME}(\theta)$ and $x^*(\theta)$.

2.3 The Effects of a Crisis

Suppose now that there is a crisis, i.e. a sudden shock to the firm's profits. The shock could be idiosyncratic (e.g. "Dieselgate" for Volkswagen) or it could affect the entire industry or economy (e.g. the Covid 19 pandemic). We model this exogenous decrease in profits as a shock to the firm's cost parameter C.[14] Consider a situation in which the cost parameter

[14]For tractability, we assume that the shock to profits does not affect the productivity of the worker.

C is such that the status quo contract (w_0, x_0) generates negative profits. Hence, the firm would rather terminate the relationship than continue with the old contract. This changes the workers' outside option which is now given by the utility level of unemployment that is normalized to 0. The next proposition shows how the firm uses the reduced outside option of the workers to improve the terms of the contract from its perspective.

Proposition 3 (Optimal Contract in a Crisis)**.** *Suppose that the status quo contract* (w_0, x_0) *satisfies* $x_0 \leq x^{ME}(\theta)$ *and*

$$v(x_0, \theta) - w_0 - C < 0.$$

Define $\hat{x}$ *implicitly by* $U(w_0, \hat{x}) = 0$ *and* $\overline{x}(\theta)$ *as in Proposition 1 by* $\frac{\partial v(\overline{x}(\theta), \theta)}{\partial x} = 1 + \alpha\lambda$.

1. *If* $\hat{x} \geq \overline{x}(\theta)$ *the firm offers a contract with* $x^* = \min\{\hat{x}, x^{ME}(\theta)\}$.

2. *If* $\hat{x} < \overline{x}(\theta)$ *the firm offers a contract with* $x^* = \overline{x}(\theta)$.

The offered wage w^* *always satisfies* $U(x^*, w^*) = 0$*, and the union accepts the offer.*

Proof. See the Appendix.

The workers know that they will lose their jobs if they reject the firm's offer. Therefore, any contract that offers them at least the utility level of unemployment will be accepted. The firm can use its improved bargaining position to either reduce wages w or to increase x. Thus, the firm pays for a higher x not with a higher wage (a gain), but with a smaller wage cut (a reduced loss). This changes the marginal rate of substitution between w and x. In a crisis, an increase in x comes at the same marginal cost to the worker as a wage cut of equal size. Because for $x < x^{ME}$ marginal productivity satisfies $\frac{\partial v(x_0, \theta)}{\partial x} > 1$. Hence, it is more efficient to increase x rather than decrease w. Additional wage cuts will only be implemented if the materially efficient level of x still generates positive utility for the workers at the status quo wage.

The second case in Proposition 3 considers the case where after increasing x to the workers' zero-utility threshold it is still smaller than the optimal level $\overline{x}(\theta)$ from Proposition 1. As in Proposition 1 the firm will then implement an additional increase of x up to the threshold $\overline{x}(\theta)$, which has to be compensated at a wage rate of $1 + \alpha\lambda$ to ensure the workers' consent.

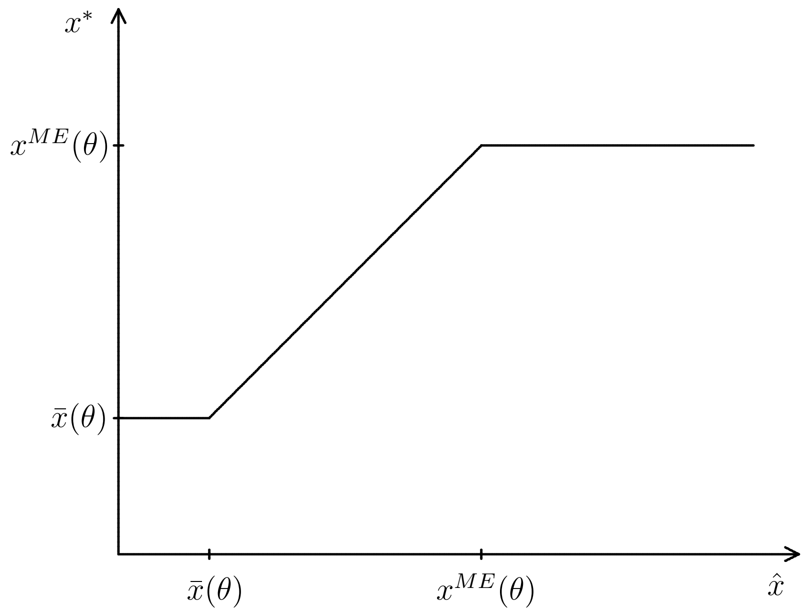

Figure 2: The Effect of a Crisis

Figure 2 illustrates Proposition 3. Consider a low status quo level of work practices, $x_0 < \overline{x}(\theta)$. Note that $\hat{x}$ linearly increases with the rent enjoyed by workers prior to the crisis. If this rent was small, the firm will increase x to the behaviorally efficient level $\bar{x}(\theta)$. If this rent was larger, the firm will increase x^* as much as possible, i.e. it sets $x^* = \hat{x}$. If the rent was so large that the firm could increase x even beyond the materially efficient level, it will stop at x^{ME} and reduce the workers' wages rather than increase x beyond x^{ME}.

The proposition shows that a crisis can induce a sudden jump in organizational efficiency. It also explains "why wages don't fall during a recession" (Bewley (1999)). In a recession, workers do make concessions, but firms and workers will not negotiate wage cuts but will rather implement more change.

The idea that a crisis has a positive effect on economic efficiency goes back to at least Schumpeter (1942). This effect is partly due to a composition effect if less efficient firms are driven out of the market while more efficient firms stay in. But, as shown in many empirical studies (Holmes and Schmitz, 2010), it is also caused by a sudden increase in the efficiency of existing firm. We give a new perspective for this observed pattern: The potential for the efficiency increase existed before the crisis already, but the firm could not exploit it because of resistance to change. The effect of the crisis is to weaken this resistance of workers (and other stakeholders). This reduces the relative cost of implementing organizational change and makes change possible.

It is worth noting that while concessions increase material efficiency and may be necessary to sustain the firm, they are not "behaviorally" efficient if we incorporate the behavioral adaptation cost. Indeed, if $x_0 \in (\overline{x}, x^{ME}]$, a marginal increase of x decreases workers' utility at a rate of $1 + \alpha\lambda$, whereas firm profits only increase at a rate of $\frac{\partial v(x_0,\theta)}{\partial x} \in [1, 1 + \alpha\lambda]$. The reason is that utility is not perfectly transferable. Yet, transferring utility via adapting organizational practices is still more efficient than doing so via wage cuts.

3 Rents and Organizational Change over Time

The one-period model captures the main intuition for why more change is implemented in a crisis than in normal times, but there are several open questions. First, the crisis has an effect only if workers enjoy a rent, i.e., $U_0 = w_0 - x_0 > 0$. Where does this rent come from? Second, it assumes that the reference point stays fixed after a change has been implemented. However, the reference point adjusts with some delay after the new contract becomes the new status quo. If parties anticipate this, how does it change the optimal contract? Finally, rational parties anticipate that a crisis may occur. How do they prepare for it in the initial contract?

In this section we consider a dynamic model in which the reference point adjusts over time. We do not attempt to build a fully general model that allows for arbitrary cost shocks. Instead, we focus on the most interesting case where the cost shock is such that the firm may survive it only if the workers make concessions. We show that the qualitative insights of Section 2 carry over to the dynamic model. Furthermore, we show that workers accumulate over time a (quasi-)rent as a compensation for organizational change in the past. Parties rationally anticipate reference point adjustments and the possibility of a crisis and adjust contracts optimally, which strengthens our results. We delineate the long-term dynamics, illustrate how productivity differences may persist over time, and how a crisis may increase productivity and reduce the productivity gap between comparable firms in the same industry.

We proceed as follows: In subsection 3.1 we set up the dynamic model. In subsection 3.2

we start out with the simpler case in which workers and firm do not anticipate that a crisis may occur. We show how x is adjusted in over time when there is no crisis, and how the optimal contract is adjusted if an unanticipated crisis occurs. In subsection 3.3 we show how productivity differences between firms facing the same technology may arise and how they are affected by an unanticipated crisis. Then, in subsection 3.4, we show that our results persist and are even stronger with agents who form rational expectations. Finally, subsection 3.5 considers the case where the cost of organizational change is not permanent but a temporary adjustment cost that vanishes after one period.

3.1 The Dynamic Model

Time $t = 1, 2, ...$ is discrete with an infinite horizon. We start in $t = 1$ with some state θ_1 and some status quo contract (w_0, x_0) that satisfies $U_0 = w_0 - x_0 \geq 0$ and $x_0 \leq x^{ME}(\theta_0)$. Any contractible action must satisfy $x_t \leq x^{ME}(\theta_t)$, where $x^{ME}(\theta_t)$ is the materially efficient effort level as defined in (7).[15]

In every period a new state of the world (θ_t, C_t) materializes. The state of technology θ_t increases deterministically over time, i.e., $\theta_{t+1} > \theta_t$.[16] For simplicity we assume that the firm's cost realization $C_t \in \{0, C_t^h\}$ is equal to 0 in "normal" periods and that there is at most one "crisis" with a high cost shock $C_t^h > 0$. Conditional on zero costs in all past periods, there is a crisis in the next period with probability $\mu > 0$. We are interested in the case where C_t^h is sufficiently large that the firm prefers to terminate the relationship if workers do not agree to make concessions (Assumption 1 below). A more general structure of cost shocks would give rise to many case distinctions which do not yield interesting additional insights.

Adapting more modern work practices, again, requires costly effort from workers. To fix ideas and ease the exposition we start with assuming that a "higher" work practice x requires permanently higher effort, and measure x without loss by its per period cost of effort. In Subsection 3.5, we allow for temporary adaptation cost and show that all our results are maintained or even strengthened.

[15]This assumption reflects the idea that future states of technology are unpredictable and the firm cannot introduce work practices for a technology that does not yet exist. It simplifies the analysis but does not affect the qualitative results.

[16]We assume implicitly that the growth in θ is bounded such that the present value of revenues $\sum_{t=0}^{\infty} \delta^t v(x_t, \theta_t)$ remains finite for all effort choices, such that present values are well defined.

The timing in each period is as follows. First, both parties observe the new state of the world. Then, the firm makes a take-it-or-leave-it offer (w_t, x_t) to the workers. The workers may accept or reject the offer. If they reject the current contract (w_{t-1}, x_{t-1}) remains in place. After observing the workers' decision both parties may or may not terminate the relationship.

Each party maximizes their expected utility, where future utility is discounted at a common discount factor $\delta < 1$. Hence, for an observed state (θ_t, C_t) and a contract (w_{t-1}, x_{t-1}) inherited from the previous period the firm's expected future profit evaluated at the beginning of period t is given by

$$\Pi((w_{t-1}, x_{t-1}), (\theta_t, C_t)) = \sum_{s=t}^{\infty} \delta^{s-t} \cdot \mathbb{E}_t[v(x_s, \theta_s) - w_s - C_s]. \tag{15}$$

The workers' reference point in period t is a convex combination of the status quo contract (w_{t-1}, x_{t-1}) and the correctly anticipated negotiation outcome (w_t, x_t). Again, workers receive reference-dependent utility by comparing the negotiation outcome in period t to the reference point in that period. Hence, the workers' continuation utility is

$$\begin{aligned} U_t(w_t, x_t | w_{t-1}, x_{t-1}) &= \sum_{s=t}^{\infty} \delta^{s-t} \cdot \mathbb{E}_t\big[w_s - x_s - \lambda[w_s^r - w_s]^+ - \lambda[x_s - x_s^r]^+\big] \\ &= \sum_{s=t}^{\infty} \delta^{s-t} \cdot \mathbb{E}_t\big[w_s - x_s - \lambda\alpha[w_{s-1} - w_s]^+ - \lambda\alpha[x_s - x_{s-1}]^+\big]. \end{aligned} \tag{16}$$

We restrict attention to Markov perfect equilibria in the sense that any offer (w_t, x_t) and each party's subgame-perfect decision to end or continue the relationship depends only on the current state, i.e., (θ_t, C_t), and the current reference point. In particular, both parties cannot commit to any future path of actions or transfers. Moreover, we assume that if one party is indifferent in its decision it breaks ties in favor of the other party.

3.2 An Unanticipated Crisis

In this section we analyze the more tractable case where the players anticipate that the reference point will change, but they do not anticipate that a crisis will occur with positive probability. The more general case in which the crisis is rationally anticipated is covered in Section 3.4.

Suppose that both parties believe that the cost realization will be zero in each period, i.e., $\mu = 0$. Ignoring participation constraints the (behaviorally) efficient contracts solve:

$$\max_{(w_t,x_t)_{t\geq 1}} W = \sum_{t=1}^{\infty} \delta^{t-1}\big(v(x_t,\theta_t) - x_t - \lambda\alpha[w_{t-1} - w_t]^+ - \lambda\alpha[x_t - x_{t-1}]^+\big).$$

Note that the wage affects efficiency only in the case of wage cuts. Falling wages create an inefficiency due to the behavioral cost. Hence, any behaviorally efficient effort choice that is accompanied by a weakly increasing wage schedule is a behaviorally efficient solution. The following lemma characterizes the behaviorally efficient sequence of effort levels in the dynamic model:

Lemma 1. *Define $\overline{x}(\theta)$ implicitly by the effort level that satisfies $\frac{\partial v(x,\theta)}{\partial x} = 1 + (1-\delta)\alpha\lambda$. For $\mu = 0$ the effort structure $(x_t^*)_{t\geq 1}$ of any behaviorally efficient contract satisfies $x_{t+1}^* \geq x_t^*$ with*

$$x_t^* = \begin{cases} x_{t-1} & \text{if } x_{t-1} \geq \overline{x}(\theta_t) \\ \overline{x}(\theta_t) & \text{if } x_{t-1} < \overline{x}(\theta_t). \end{cases}$$

Proof. See the Appendix.

The efficient effort is reminiscent of the effort schedule chosen by the firm in Proposition 1. Effort is weakly increasing, because θ_t is going up each period. Again, there is a region of inertia. Only if the mismatch between state θ and the associated effort is sufficiently strong is it optimal to increase the effort level. Even if effort increases, it stays below the materially efficient level of effort. However, the region of inertia and the material efficiency loss are strictly smaller than in the static benchmark. The reason is that a marginal effort increase induces a one-time marginal behavioral cost of $\lambda\alpha$ in the current period, but it reduces the behavioral cost in the next period by $\delta\lambda\alpha$ because the reference point has adjusted.

We are now ready to characterize the equilibrium in the benchmark case where a crisis is not anticipated ($\mu = 0$). The firm will simply implement the behaviorally efficient effort schedule and spread the necessary wage increase over all future periods.

Proposition 4. *If $\mu = 0$ the following is the unique equilibrium.*

1. In period $t \geq 1$ the firm offers the contract (w_t^*, x_t^*), where x^* is the efficient effort level characterized in Lemma 1 and $w_t^* = w_{t-1} + (1 + (1-\delta)\alpha\lambda)(x_t^* - x_{t-1})$.

2. The workers accept.

To see the intuition for this result, recall that the efficient effort is weakly increasing over time. In period t the contract (w_{t-1}, x_{t-1}) constitutes the workers' outside option, so the firm has to compensate the workers for any effort increase in order to guarantee the same utility as under contract (w_{t-1}, x_{t-1}). The compensation must cover the permanent higher cost of effort $x_t^* - x_{t-1}$ as well as the one-time behavioral adaptation cost of $\alpha\lambda(x_t^* - x_{t-1})$. The crucial observation is that the compensation for the behavioral cost must be spread out evenly over all future periods. Indeed, the present value of a permanent payment $(1-\delta)\alpha\lambda(x_t^* - x_{t-1})$ is

$$\sum_{s=t}^{\infty} \delta^{s-t}(1-\delta)\alpha\lambda(x_t^* - x_{t-1}) = \alpha\lambda(x_t^* - x_{t-1}).$$

Note that there is no other feasible compensation schedule to implement the behaviorally efficient effort. Since the firm has no commitment power it cannot backload the compensation to the future. It cannot frontload the compensation either, as this would imply that wages have to fall in some future periods. But the workers can block any future decline in wages in favor of the status quo. Thus, implementing the efficient effort in this way is the best the firm can do, because it generates the highest possible joint surplus, but leaves the workers with only the utility of their outside option.

Proof. Follows directly from the text. □

Note that the optimal contract in period t results in a utility loss for workers in period t, but a permanent utility increase in all future periods. Thus, starting in period $t+1$ workers enjoy a quasi-rent. Because the compensation is linear in the effort increase, the workers' quasi-rent that stems from the permanent compensations for past effort increases can easily be derived from the sum of past effort increases:

$$\begin{aligned} U_t^* = U_t^*(w_t, x_t) &= \sum_{s=t}^{\infty} \delta^{s-t}\big[w_0 - x_0 + (1-\delta)\alpha\lambda(x_{t-1} - x_0)\big] \\ &= U_1(w_0, x_0) + \alpha\lambda(x_{t-1} - x_0). \end{aligned} \tag{17}$$

The firm's equilibrium profit in period t consists of the discounted sum of future surpluses minus the utility left to the workers, i.e.,

$$\Pi_t^* = \sum_{s=t}^{\infty} \delta^{s-t}\big(v(x_s^*, \theta_s) - x_s^* - \lambda\alpha(x_s^* - x_{s-1}^*)\big) - \alpha\lambda(x_{t-1}^* - x_0) - U_1(w_0, x_0). \qquad (18)$$

Suppose that in some period t there is a cost shock that decreases the present value of the firm's expected future profits by C_t^h. If the firm's value remains positive then the shock has no impact on the firm's optimization problem. The interesting case is when the cost shock induces negative firm value for the status quo contract. Hence, the following assumption is made throughout the remainder of the paper.

Assumption 1. *In every period the potential cost shock satisfies $C_t^h > \Pi_t^*$, where Π_t^* is defined in(18).*

Assumption 1 implies that if there is a cost shock then the value of the firm will become negative if workers receive their status quo utility, even if the most efficient contract is implemented.[17] Hence, workers are willing to make concessions to prevent the firm from closing down. Therefore, the unemployment utility of zero constitutes the new threat-point in the negotiations. Note that we allow for the possibility of the cost shock being so high that the firm cannot be rescued even if workers give up their quasi-rent. In this case the firm closes down and workers become unemployed.

The following Proposition is a straightforward generalization of Proposition 3 to the dynamic case.

Proposition 5. *Suppose that Assumption 1 holds and there is a crisis in period t. Define $\hat{x}$ implicitly by $U_t(w_{t-1}, \hat{x}) = 0$, and $\overline{x}(\theta)$ as in Lemma 1 by $\frac{\partial v(\overline{x}(\theta),\theta)}{\partial x} = 1 + (1-\delta)\alpha\lambda$.*

1. If $\hat{x} \geq \overline{x}(\theta)$ the firm offers a contract with $x^* = \min\{\hat{x}, x^{ME}(\theta)\}$.

2. If $\hat{x} < \overline{x}(\theta)$ the firm offers a contract with $x^* = \overline{x}(\theta)$.

The offered wage w^* always satisfies $U(x^*, w^*) = 0$, and the union accepts the offer. The firm decides to continue the relationship if and only if its expected profit from the above contract is non-negative.

[17] Thus, a fortiori, the value of the firm becomes negative for any other feasible contract as well.

Proof. See the Appendix.

Again, the firm finds it more profitable to increase effort rather than decrease wages. Decreasing the wage or increasing the effort by one unit has identical effects on the workers' expected utility. For the firm, increasing the effort is even more appealing than in the static case: since θ is growing over time, a higher effort level today avoids costly adaptations in the future.

3.3 Long-Term Dynamics and Persistent Productivity Differences

We now illustrate the long-term dynamics and show how a crisis can help close the productivity gaps across firms. We continue with our previous example of $v(x, \theta) = \theta \ln(x)$. The following Figure 3 depicts the materially efficient line $x^{ME}(\theta) = \theta$ in black and the behaviorally efficient line $\overline{x}(\theta) = \frac{1}{1+(1-\delta)\alpha\lambda}\theta$ in red. We consider two firms that have access to the same production

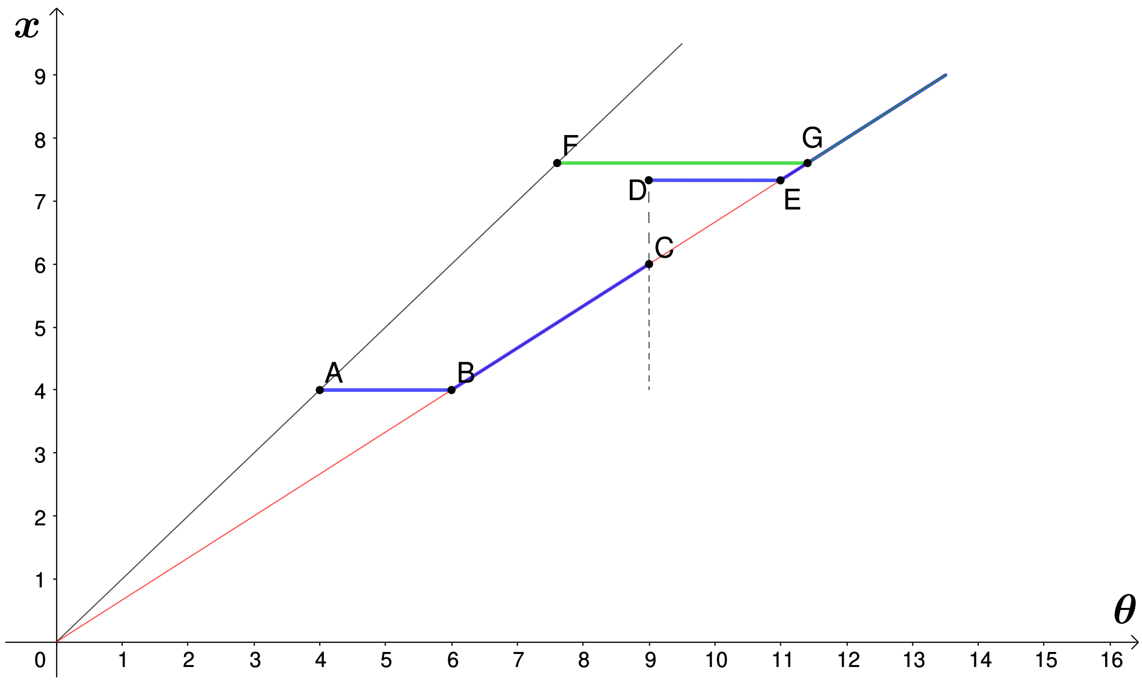

Figure 3: $v(x, \theta) = \theta \ln x$, $(1 - \delta)\alpha\lambda = 0.8$, firms created at $\theta = 4$ and $\theta = 7$, crisis at $\theta = 9$

technology. The productivity of the technology is summarized by θ, which grows over time. Firms are founded at different points in time. Firm 1 was founded when $\theta = 4$ and Firm 2 when $\theta = 7$, respectively. Thus, Firm 1 starts production at the materially efficient point A.

The blue line depicts the transition path as θ_t grows over time. Firm 1 is in the area of inertia until point B is reached. From point B onwards the contract follows the behaviorally efficient (red) line. The firm implements higher work practices each period, but these increases are smaller than the materially efficient increases. Because the compensation for the behavioral cost of adaptation is stretched out over time, workers build up a quasi-rent.

At $\theta = 7$ firm 2 is founded. Firm 2 also starts production at a materially efficient point (F) and follows the transition path along the green line. Note that there is a substantial productivity gap between the two firms. This gap gradually closes as Firm 1 slowly implements higher work practices, but it may persist for a long time.

When $\theta = 9$ is reached, the economy is hit by a crisis that threatens the profitability of Firm 1, but not of the more efficient Firm 2. By then Firm 1 is at point C, and the workers have accumulated a rent of $\alpha\lambda$ times the magnitude of total effort increases, illustrated by the length of the dashed black line. Firm 1 will use the threat of unemployment to expropriate this quasi-rent by increasing work practices from point C to point D.[18] If the firm is profitable at point D it will stay in business and continue producing, now again in the area of inertia until it hits point E.

Notice that the more efficient Firm 2 is not threatened by bankruptcy, even though it is hit by the same cost shock. Indeed, even after the effort adjustment of Firm 1, Firm 2 remains more profitable than Firm 1. But, the productivity gap between the two firms closes discontinuously in a crisis, as Firm 1 is able to implement more efficient work practices while Firm 2 is not.

It is worth noting that even if point D was on the green line, i.e., both firms use the same effort after the crisis, Firm 2 remains more profitable. The reason why is that it pays lower wages. Indeed, Firm 2 workers receive no rents up to point G. Firm 1, on the other hand, has to leave some future rents to its workers in order to compensate them for the effort adjustment in the crisis. Nevertheless, the profitability gap between the two firms narrows.

[18] Each additional unit of effort gives rise to an immediate behavioral cost of $\alpha\lambda$ and a permanent higher material effort cost of one, the discounted present value of which is $\frac{1}{1-\delta}$. The worker has accumulated a rent of $\alpha\lambda(x_t - x_0)$. Thus, this rent can be used to "pay for" an effort increase of $\alpha\lambda(x_t - x_0) \cdot \frac{1}{\alpha\lambda + \frac{1}{1-\delta}} = \frac{(1-\delta)\alpha\lambda}{1+(1-\delta)\alpha\lambda}(x_t - x_0)$.

The figure assumes equal levels of loss aversion of workers across firms. This does not necessarily have to be the case. Gächter, Johnson, and Herrmann (2022) find that loss aversion tends to increase in age, income, and wealth. This suggests that if younger firms employ a younger workforce, their workers suffer to a lesser degree from loss aversion than their older colleagues at older firms. In this case the inertia region of old firms is larger than that of young firms since workers from older firms are more reluctant to agree to organizational change. This could be another source of persistence productivity differences across firms.

3.4 The Equilibrium with Rational Expectations

Now we analyze the case in which both players correctly anticipate that the crisis may happen with probability μ each period. We show that all insights continue to hold and the range of inertia even widens.

We begin the analysis by noting that under Assumption 1 the occurrence of a crisis cannot be affected by the two parties. Hence, in the contracting game, both parties treat the probability of a crisis as exogenous.

Second, notice that the optimal reaction to a crisis for a given status quo contract is fully analyzed in Proposition 5. Indeed, due to the restriction to Markov strategies, the contract offer in a crisis depends only on the current reference point and the states θ_t and C_t^h. Hence, the problem reduces to finding the principal's optimal contract offer in normal periods, given that both parties correctly anticipate the effects of their contract on the adaptation in a potential crisis.

We start by calculating the necessary wage compensation to implement an effort increase from x_{t-1} to x_t in a normal period. The firm has to compensate the permanent effort cost and the one-time behavioral adaptation cost. Again, the compensation for the adaptation cost must be spread out equally over the current and all future normal periods. However, as compared to the case with an unanticipated crisis, both parties now anticipate that the crisis may occur in which case workers will lose their quasi-rent. Hence, the demanded per-period compensation for the adaptation cost is higher.

To simplify notation, let

$$\gamma \equiv \big[(1-\delta(1-\mu)\big]\alpha\lambda.$$

As illustrated in the proof of Lemma 2, γ is the permanent per-period compensation until the crisis hits that is necessary to compensate for the behavioral cost of a one unit effort increase.

Lemma 2. *In any equilibrium the effort is weakly increasing in every normal period. In order to implement $x_t > x_{t-1}$ in period t the firm offers the contract (w_t, x_t) with*

$$w_t = w_{t-1} + (1+\gamma)(x_t - x_{t-1}).$$

If there is no crisis until period t then

$$w_t = x_t + \gamma(x_t - x_0) + (w_0 - x_0), \tag{19}$$

and

$$U_t(w_{t-1}, x_{t-1}) = U_1(w_0, x_0) + \alpha\lambda(x_{t-1} - x_0).$$

Proof. See the Appendix. □

Again, over time, as no crisis happens, the workers build up rent that stems from the compensation for past effort increases. This is utility the firm can exploit to take away in a crisis, when the threat point of the workers is given by their unemployment utility of zero. Since workers anticipate that their quasi-rent will drop to zero in a crisis, the compensation they demand increases in the probability μ that a crisis will occur.

As established in Proposition 5, in a crisis the firm will use the workers' utility to implement higher effort. The following Lemma calculates by how much at most the firm can increase the effort in a crisis when keeping the wage constant.

Lemma 3. *For a given status quo contract (w_{t-1}, x_{t-1}) the effort increase Δx_t that satisfies $U_t(w_{t-1}, x_{t-1} + \Delta x_t) = 0$ is given by*

$$\Delta x_t = \frac{1}{1+(1-\delta)\alpha\lambda}\Big((w_0 - x_0) + \gamma(x_{t-1} - x_0)\Big). \tag{20}$$

Proof. See the Appendix. □

The next proposition characterizes the optimal effort schedule that the firm will implement in normal times in anticipation of how the contract will change if a crisis hits.

Proposition 6. *Suppose there is no crisis until period t. There is a threshold $\tilde{x}(\theta_t) \leq \overline{x}(\theta_t)$ such that the optimal effort x_t implemented in period t is given by*

$$x_t = \begin{cases} x_{t-1} & \text{if } x_{t-1} > \tilde{x}(\theta_t) \\ \tilde{x}(\theta_t) & \text{if } x_{t-1} \leq \tilde{x}(\theta_t) \end{cases}$$

Proof. See the Appendix. □

Comparing Proposition 6 to Proposition 5 shows that if parties rationally anticipate that a crisis may hit with probability $\mu > 0$, there will be even more inertia than if they do not anticipate a crisis, i.e., $\tilde{x}(\theta_t) \leq \overline{x}(\theta_t)$. Because the firm anticipates that it will become cheaper to adjust effort in a crisis, it delays the effort adaptation.

The proof of Proposition 6 requires a few case distinctions and is somewhat involved, but the intuition can be illustrated graphically with the logarithmic example that we used in Section 3.3.

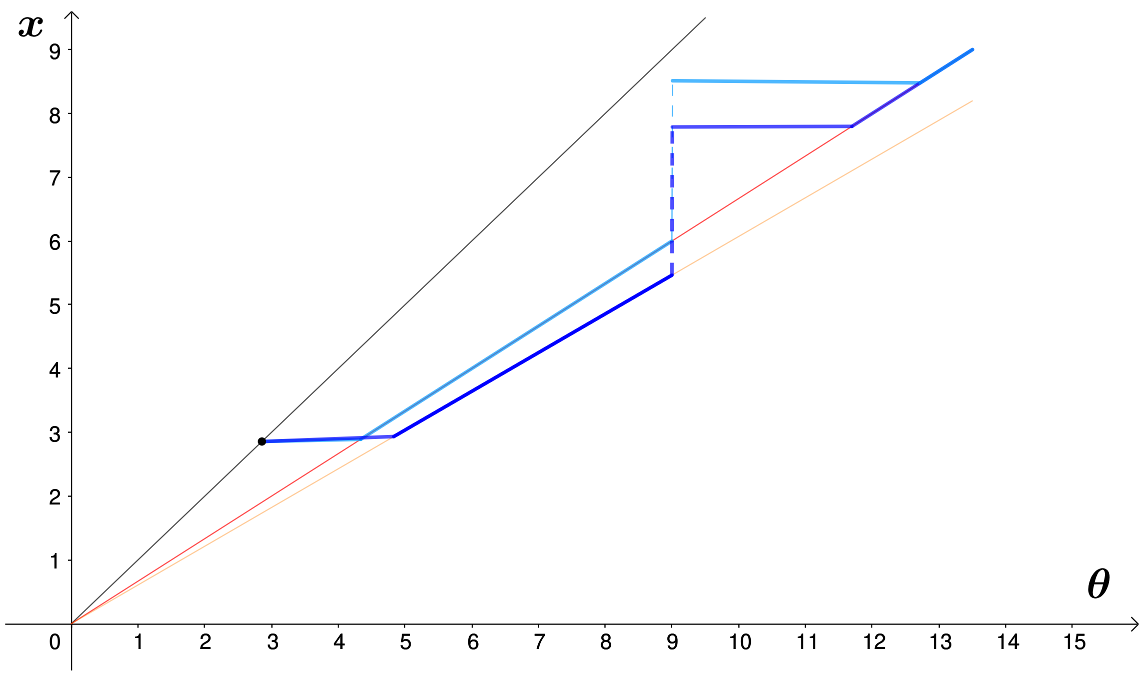

Figure 4: Logarithmic Example with Forward-Looking Players

Figure 4 reconsiders the example of Figure 3 with $\mu = 0.4$. The orange line depicts the boundary of the inertia region with players who form rational expectations. Note that it is further to the right than the behaviorally-efficient red line.

To see that the anticipation of the crisis increases inertia two cases have to be distinguished. First, if the cost shock C_t^h is sufficiently large, the workers' accumulated rent will not suffice to make the firm profitable and the principal will terminate the relationship. The anticipation of bankruptcy makes the implementation of higher effort levels less appealing, as the firm will not benefit from this effort after bankruptcy.

Second, and more interestingly, the inertia area also widens when workers' concessions suffice to keep the firm in business. There is more inertia than without anticipation of the crisis because the principal knows that effort in a crisis will increase anyway and therefore delays effort increases in normal times.[19] Intuitively, this behavior helps the principal to keep the effort level closer to the red behaviorally efficient line $\overline{x}(\theta)$. It optimally solves a tradeoff between "too low" effort in normal periods and "too high" effort as compared to the behaviorally efficient level in a crisis. The dark blue line depicts the optimal transition path of effort if a potential crisis is anticipated, as compared to the case of an unanticipated crisis (light blue line).

3.5 Temporary Adaptation Cost

So far we assumed that an increase in x requires permanently higher effort costs. While this is plausible in some applications, there are other applications where a change in x yields only a temporary higher effort cost. In these cases organizational change is costly for workers in the short-term because they have to adapt to new work practices, but once they got used to them, their jobs do not require higher effort costs than before the change.

In this extension we show that all insights of our model continue to hold if the adaptation cost to organizational change is only temporary. To keep the two versions of the model comparable we assume that the total material cost and the total behavioral cost of a change

[19]For the functional form of our logarithmic example the green inertia line of the bankruptcy case and the no-bankruptcy case coincide. For more general production functions this is not the case. The exact level of inertia then depends on whether the firm expects bankruptcy in a potential crisis.

in x remain of the same magnitude as before, but that the material cost is borne entirely in the period when x is changed. Recall that if the cost of the effort increase is permanent, a one unit increase in effort has a present value of material costs of $\sum_{t=0}^{\infty} \delta^t = \frac{1}{1-\delta}$, while the behavioral cost is $\alpha\lambda$. Thus, if the cost of the effort increase is temporary, the adaptation cost of a one unit effort increase in period t is $\frac{1}{1-\delta}$ (which is borne by workers in period t), while the behavioral cost (also borne in period t) is still $\alpha\lambda$.[20]

In this modified version of the model, the worker's baseline effort cost is constant and equal to x_0 in all but those periods in which x increases. In those periods the effort cost is $x_0 + \frac{x^t - x^{t-1}}{1-\delta}$. Thus, the utility function of workers is now given by:

$$U_t(w_t, x_t|w_{t-1}, x_{t-1}) = \sum_{s=t}^{\infty} \delta^{s-t} \cdot \mathbb{E}_t \left[w_s - x_0 - \frac{x_s - x_{s-1}}{1-\delta} - \lambda\alpha[w_{s-1} - w_s]^+ - \lambda\alpha[x_s - x_{s-1}]^+ \right]. \tag{21}$$

The following Lemma shows that with this utility function the total social surplus function remains unchanged.

Lemma 4. *If the material cost of effort adaptation is borne immediately when the change is implemented, the surplus function is given by*

$$W = \sum_{t=1}^{\infty} \delta^{t-1} \big(v(x_t, \theta_t) - x_t - \lambda\alpha[w_{t-1} - w_t]^+ - \lambda\alpha[x_t - x_{t-1}]^+ \big)$$

Since, the welfare function remains the same, Lemma 1 and Proposition 4 are unchanged under the new specification of worker utility. Again, the wage compensation for an effort increase is constant over time. However, the quasi-rent of the workers increases. They get the same permanent wage increase for a change in x in period t, but now not only the behavioral cost but also the entire adaptation cost is borne in period t. Thus the workers' quasi-rent in period $t+1$ as a function of past effort increases is given by

[20] This keeps the relative importance of the behavioral cost as compared to material cost constant. If the material costs would be reduced, the relative importance of the behavioral costs would increase which would further strengthen our results.

$$\begin{aligned}
U_t(w_t^*, x_t^*) = U_t(w_{t-1}^*, x_{t-1}^*) &= \sum_{s=t}^{\infty} \left(w_{t-1}^* - x_0\right) \\
&= \sum_{s=t}^{\infty} \Big(w_0 + (1 + (1-\delta)\alpha\lambda)(x_{t-1}^* - x_0) - x_0\Big) \\
&= \sum_{s=t}^{\infty} (w_0 - x_0) + \left[\frac{1}{1-\delta} + \alpha\lambda\right](x_{t-1}^* - x_0) \\
&= U_0(w_0, x_0) + \left[\frac{1}{1-\delta} + \alpha\lambda\right](x_{t-1}^* - x_0)
\end{aligned}$$

The new term $\frac{1}{1-\delta}(x_{t-1}^* - x_0)$ corresponds to the total value of compensations for pastly exerted effort cost.

In a crisis, this rent will, again, be used to increase effort. Since Proposition 5 and its proof do not rely on the size of these rents, they continue to hold unchanged. Because the quasi-rents are now higher than in the baseline model, the discontinuous jump in the crisis is even larger than in the baseline model.

Finally, we look at the case of forward-looking agents. The following Lemma parallels Lemma 2 :

Lemma 5. *Let* $\tilde{\gamma} \equiv \big[(1 - \delta(1 - \mu)\big]\left[\frac{1}{1-\delta} + \alpha\lambda\right]$. *In any equilibrium the effort is weakly increasing in every normal period. In order to implement* $x_t > x_{t-1}$ *in period* t *the firm offers the contract* (w_t, x_t) *with*

$$w_t = w_{t-1} + \tilde{\gamma}(x_t - x_{t-1}).$$

If there is no crisis until period t *we have*

$$w_t = \tilde{\gamma}(x_t - x_0) + w_0, \tag{22}$$

and

$$U_t(w_{t-1}, x_{t-1}) = U_0(w_0, x_0) + \left[\frac{1}{1-\delta} + \alpha\lambda\right].$$

Finally, the statement of Proposition 6 continues to hold with temporary adaptation cost:

Proposition 7. *Suppose there is no crisis until period* t. *If adaption costs are temporary there, again, is a threshold* $\tilde{x}(\theta_t) \leq \overline{x}(\theta_t)$ *such that the optimal effort* x_t *implemented in period* *t is given by*

$$x_t = \begin{cases} x_{t-1} & \textit{if } x_{t-1} > \tilde{x}(\theta_t) \\ \tilde{x}(\theta_t) & \textit{if } x_{t-1} \leq \tilde{x}(\theta_t) \end{cases}$$

4 Expectation Management

An important aspect of organizational change is expectation management. Management books on organizational change emphasize that it is of crucial importance to convince everyone concerned that change is inevitable and that it is better to get ready for it now. To model expectation management we have to endogenize the probability of change. To keep the model tractable, we return to the one-period model and take the amount of organizational change, Δx, as exogenously given. If there is a change of $\Delta x > 0$, the owner's gross profit increases by $\Delta v > \Delta x$.[21] However, change has to be implemented by a manager who has to spend effort to increase the probability that organizational change is successful. If he is unsuccessful there is no change. We assume that the manager chooses the probability of success, p, at cost $c(p)$. In order to derive a closed form solution, we consider a quadratic cost function $c(p) = \frac{c}{2}p^2$.

The owner of the firm, the principal, has to incentivize the manager to spend effort. The manager's chosen probability of success is unobservable to the principal and cannot be contracted upon. Change, however, is contractible, so the principal can offer a bonus payment b if the manager is successful in implementing it. The manager's outside option utility is normalized to 0 and he is wealth constrained, so he has to get a wage that is greater than or equal to 0 in both states of the world. We assume again that only workers suffer from loss aversion (as in Section 2) and that all other parties are loss and risk neutral and maximize the expected value of their payoff.

Note that in Section 2 everything is deterministic while in this section the outcome is stochastic. For this stochastic environment we assume that the workers' reference point is a convex combination of the status quo and the ex ante expected value of the outcome. [22]

[21] There is no problem in endogenizing the size of Δx, but it complicates the exposition significantly without adding any new insights.

[22] Köszegi and Rabin (2006) assume that the decision maker compares the realized outcome to each possible outcome in the ex-ante distribution rather than the expected value. In our simple model their specification boils down to our specification and yields exactly the same results.

Hence, if change Δx is implemented with probability p the workers' reference point in the effort dimension is

$$x^r = \alpha x_0 + (1-\alpha)\big((1-p)x_0 + p(x_0+\Delta x)\big) = \alpha x_0 + (1-\alpha)[x_0 + p\Delta x].$$

The time structure of the model is as follows: At stage 1 the principal makes a take-it-or-leave-it contract offer to the manager. In addition she pays workers to accept the change (if the manager is successful) by making a wage offer w.[23] At stage 2 the manager chooses the probability of success. At stage 3 nature determines whether change is successful and payments are made.

The principal's problem is a standard moral hazard problem with a risk-neutral but wealth constrained manager. It is straightforward to show that the owner will offer the manager a wage of 0 if he fails and a bonus $b \geq 0$ if he is successful.

Suppose that the principal offers a bonus b to the manager that induces him to choose a probability of success of p. Workers observe the bonus b and rationally anticipate that the manager chooses p. What wage does the owner have to pay to workers to make them accept the change? Note that $\Delta x > 0$ implies that $w > w_0$. Thus, the expected utility of workers is given by

$$\begin{aligned} U &= w - x_0 - p\Delta x - \alpha\lambda[w_0 - w]^+ - p\lambda[x_0 + \Delta x - (\alpha x_0 + (1-\alpha)(x_0 + p\Delta x))]^+ \\ &\quad -(1-p)\lambda[x_0 - (\alpha x_0 + (1-\alpha)(x_0 + p\Delta x))]^+ \\ &= w - x_0 - p\Delta x[1 + \lambda(1-(1-\alpha)p)] \geq U_0 \ . \end{aligned} \tag{23}$$

The following lemma characterizes the wage payment that workers have to be offered:

Lemma 6. *With probabilistic change the principal has to pay*

$$w = x_0 + p(1+\lambda)\Delta x - p^2(1-\alpha)\lambda\Delta x \ + U_0. \tag{24}$$

to workers. This wage function is concave in the probability of change p. It decreases in p if and only if

$$\frac{1+\lambda}{\lambda(1-\alpha)} < 2p. \tag{25}$$

[23]Note that the principal cannot do better by offering a wage payment conditional on the realization of change. Indeed, such a probabilistic wage would even create potential losses with respect to the reference point, if success is not realized.

Proof. See the Appendix. □

Equation 25 is more likely to hold if λ is large, α is small and p is close to 1.[24] If $\alpha = 1$, i.e., if the reference point is fully determined by the status quo, the wage increase is just a linear function of p. The more likely the change, the higher the wage increase that workers demand to accept it. However, if $\alpha < 1$, i.e., if the reference point is partly shaped by rational expectations, the wage increase is a concave and possibly decreasing function of p. The higher the probability of change, the higher the reference point x^r, and the less workers suffer from change. This has important implications for the probability of change that the principal wants to implement. If the weight on expectations for shaping the reference point is sufficiently large, i.e., if α is sufficiently small, the principal will induce change either with probability one or with probability zero. This is shown in the following proposition.

Proposition 8. *The probability of change that the principal wants to implement is characterized as follows:*

(a) If $c < (1-\alpha)\lambda\Delta x$ the principal's problem of inducing the manager to promote change is a convex problem. In this case the principal will implement a corner solution with $p = 1$ if $\Delta v \geq (1+\alpha\lambda)\Delta x + c$ and $p = 0$ otherwise.

(b) If $c > (1-\alpha)\lambda\Delta x$ the principal's problem is concave. In that case the principal implements $p > 0$ if and only if $\Delta v > (1+\lambda)\Delta x$, in which case p satisfies

$$p = \min\left\{\frac{\Delta v - (1+\lambda)\Delta x}{2[c-(1-\alpha)\lambda\Delta x]}, 1\right\} \tag{26}$$

Proof. See the Appendix □

The proposition shows that if α is small, i.e., if rational expectations have a large effect on the reference point, then the manager is induced to choose an extreme solution of either $p = 0$ or $p = 1$. Even if an interior solution is optimal, a decrease in α increases the probability of change. The reason is that if the reference point is at least partially determined by workers' expectations, then it becomes cheaper to implement change the more workers are convinced

[24] Note that (25) can only hold if $\lambda > 1$.

that change is going to take place. This resonates with the advice given in the literature on organizational change, that if you want to induce change you have to set the expectation that change is coming and that it is unavoidable.

5 Reference Dependence in Teams

So far we have assumed that people compare their current wage and effort level to the status quo and to their rational expectation of the future. In this section we consider a different reference point that is shaped by social comparisons. Each agent compares his situation to the situation of other agents in his reference group. He suffers a utility loss if his wage is lower than the reference wage and if his effort is higher than the reference effort.[25]

We restrict attention to the case of two workers, $i \in \{1, 2\}$ who may receive different wages and spend different amounts of effort. Furthermore, we assume that the reference point depends only on social comparisons and not on comparing the proposed wage and effort level to the status quo and the rational expectation of the future. It is straightforward to extend the analysis to the case of N workers and to have multi-dimensional reference points, but it does not add any interesting insights.

Each worker compares his own situation to that of his colleague. The parameter $\beta \in [0, 1]$ captures how much the reference point weighs the wage and effort of his colleague as compared to his own situation:

$$\begin{aligned} w_i^r &= (1-\beta)w_i + \beta w_j \\ x_i^r &= (1-\beta)x_i + \beta x_j \end{aligned}$$

If $\beta = 0$, a worker is only interested in his own situation and does not engage in social comparisons. In this case the model boils down to a model without reference dependence. For $\beta > 0$ the reference point is given by a weighted average of his own situation and the situation of his co-worker.[26]

[25]There are several recent empirical papers showing that workers are averse to pay inequality. See e.g., Card et al. (2012), Dube, Giuliano, and Leonard (2019), Cullen and Perez-Truglia (2022).

[26]The parameter β captures how much the worker compares his own situation with the situation of his

The utility of worker i is given by

$$U_i = w_i - x_i - \lambda[x_i - x_i^r]^+ - \lambda[w_i^r - w_i]^+, \tag{27}$$

and the worker accepts a contract only if he gets at least his outside option utility U_0. The following Lemma shows how social comparisons affect wages for given effort levels x_1 and x_2.

Lemma 7. *Suppose that $x_2 > x_1$. Then, wages are given by*

$$\begin{aligned} w_1 &= U_0 + x_1 + \lambda\beta[x_2 - x_1] \ , \\ w_2 &= U_0 + x_2 + \lambda\beta[x_2 - x_1] \ . \end{aligned}$$

Proof. See the Appendix. □

Lemma 7 shows that in order to induce $x_2 > x_1$ the wages of both workers have to exceed $U_0 + x_i$ by $\lambda\beta[x_2 - x_1]$. This term is increasing in λ, i.e., the weight of the reference point, and β, the weight put in the reference point on the other agent. However, the difference in wages is just equal to the difference in effort levels: $x_2 - x_1$. Thus, if there is wage compression, it must be due to the principal inducing smaller effort differences than without social comparisons.

Suppose that worker $i \in \{1, 2\}$ generates gross profits of $v_i(x_i, \theta)$. Suppose further without loss of generality that worker 2 is the more productive one, i.e., $\frac{\partial v_2(x,\theta)}{\partial x} > \frac{\partial v_1(x,\theta)}{\partial x}$ for all x and θ. The principal chooses x_1 and x_2 to maximize

$$\Pi = v_1(x_1, \theta) + v_2(x_2, \theta) - w_1 - w_2 - C$$

Without social comparisons the first best effort levels are given by

$$\frac{\partial v_1(x_1^{ME}, \theta)}{\partial x_1} = \frac{\partial v_2(x_2^{ME}, \theta)}{\partial x_2} = 1$$

The following proposition characterizes the optimal effort levels if workers engage social comparisons:

co-worker, while the parameter λ captures the magnitude of reference dependence on utility. However, in this very simple version of the model, only the product $\lambda\beta$ is going to matter. This is no longer the case if the reference point becomes multi-dimensional.

Proposition 9. *Let x^* be characterized by*

$$\frac{\partial v_1(x^*, \theta)}{\partial x} + \frac{\partial v_2(x^*, \theta)}{\partial x} = 2.$$

If $\frac{\partial v_2(x^*,\theta)}{\partial x} < 1+2\lambda\beta$ then the principal induces $x_1^* = x_2^* = x^*$. Otherwise, the principal induces the unique effort pair (x_1^*, x_2^*) that satisfies

$$\begin{aligned} \frac{\partial v_1(x_1^*, \theta)}{\partial x_1} &= 1 - 2\lambda\beta, \\ \frac{\partial v_2(x_2^*, \theta)}{\partial x_2} &= 1 + 2\lambda\beta. \end{aligned}$$

Proof. See the Appendix. □

The principal will induce the less productive worker to work more than his efficient effort level, while the more productive worker works less hard than required by efficiency. The principal compresses the difference in effort levels which also reduces the difference in wages. If the difference in productivity between the workers is sufficiently small, the principal finds it optimal to implement the same effort level for both workers. There is wage compression as compared to the material efficient solution because of this effort compression.

This is consistent with interesting empirical evidence provided by Hjort, Li, and Sarsons (2022) showing that multinationals use wages paid at their headquarters as a reference point for wages paid to employees in other countries, even if the establishment is located in a low-wage region.

Social comparisons are costly for the firm because they increase wages and distort effort levels. One possibility to reduce social comparisons is to organizationally separate workers. For example, if some tasks are contracted out to an independent company, the workers of this other company are less likely to compare their situation to that of the workers who are employed by a different company, as compared to a situation where all of them work for the same firm. This answers the question why it may be profitable to divide a company into two different entities (the so-called "Williamson puzzle").

6 Conclusions

We have shown that reference-dependent preferences can naturally account for several stylized facts about organizational change. Loss aversion explains why there is often no or slow change in normal times, but a sudden spur in productivity in a crisis. It explains why large productivity differences between firms can arise and persist for long periods of time if firms are founded at different points in time or face idiosyncratic shocks. Social preferences can explain why there is effort and wage compression within firms and why it may be optimal to split up a firm in order to avoid social comparisons.

Our model has several other interesting implications. For example, it implies that it is more difficult to implement change with older workers. Older workers have a shorter time-horizon until they retire. Furthermore, Gächter, Johnson, and Herrmann (2022) report that older people suffer more from loss aversion than younger people, that is, their λ is larger. For both reasons they need to receive a higher compensation to accept change, which makes change more costly to implement.

If the government protects workers with a generous social safety net and if it tries to prevent firm closures, it makes change more difficult to implement. For example, if firms can put their workers on short-term work rather than laying them off, there is less need for workers to make concessions. If there is a general unemployment insurance, workers lose less in the case of a crisis and are less willing to agree to changes. This resonates with the observation that European countries are often lagging behind Anglo-Saxon or Asian countries in the implementation of cutting-edge technologies.

In the model, we take the formation of the reference point as given. However, a company that wants to implement change could try to shape the reference point by reducing α, i.e., the weight that the reference point puts on the status quo. For example, it could focus attention on the future, by making it clear that change is unavoidable, that it will happen, and that it is better to embrace rather than resist. Alternatively, the company could hire a new manager who has a reputation for pushing change through, or a new owner who is committed to implementing change could take over the firm. All of this shifts attention to the rational expectation that change will happen.

The theoretical exploration of these effects and the empirical validation of the impact of reference-dependent preferences on organizational change are interesting directions for future research.

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A Online Appendix

Proof of Proposition 2. If $\theta \leq \overline{\theta}$ we have $x^*(\theta) = x_0$ and the first result is immediate. Suppose $\theta > \overline{\theta}$. The definition of $\overline{\theta}$ and $\frac{\partial^2 v(x,\theta)}{\partial x \partial \theta} > 0$ for all x and θ imply that $\frac{\partial v(x_0,\theta)}{\partial x} > \frac{\partial v(x_0,\overline{\theta})}{\partial x} = 1+\alpha\lambda$. For $\theta > \overline{\theta}$ the optimal $x^*(\theta)$ is characterized by

$$\phi(\cdot) = \frac{\partial v(x^*, \theta)}{\partial x} - 1 - \alpha\lambda = 0 \quad \Leftrightarrow \quad \frac{\partial v(x^*, \theta)}{\partial x} = 1 + \alpha\lambda. \tag{A1}$$

Recall that $x^{ME}(\theta)$ is characterized by $\frac{\partial v(x^{ME},\theta)}{\partial x} = 1$. Thus, because $v(x,\theta)$ is strictly concave in x, we must have $x_0 < x^* < x^{ME}$, which proves the first part of the proposition.

(a) For $\theta < \overline{\theta}$ we have $x^*(\theta) = x_0$ is constant, hence the result is immediate. For $\theta > \overline{\theta}$ the optimal $x^*(\theta)$ is characterized by (A1). Using the implicit function theorem, we get

$$\frac{\partial x^*}{\partial \lambda} = -\frac{\frac{\partial \phi}{\partial \lambda}}{\frac{\partial \phi}{\partial x}} = -\frac{-\alpha}{\frac{\partial^2 v}{\partial x^2}} = \frac{\alpha}{\frac{\partial^2 v}{\partial x^2}} < 0, \tag{A2}$$

$$\frac{\partial x^*}{\partial \alpha} = -\frac{\frac{\partial \phi}{\partial \alpha}}{\frac{\partial \phi}{\partial x}} = -\frac{-\lambda}{\frac{\partial^2 v}{\partial x^2}} = \frac{\lambda}{\frac{\partial^2 v}{\partial x^2}} < 0, \tag{A3}$$

while $x^{ME}(\theta)$ is independent of λ and α.

(b) The parameter $\overline{\theta}$ is implicitly defined by:

$$\frac{\partial v(x_0, \overline{\theta})}{\partial x} - 1 - \alpha\lambda = 0 \tag{A4}$$

By the implicit function theorem we get

$$\frac{\partial \overline{\theta}}{\partial \lambda} = -\frac{-\alpha}{\frac{\partial^2 v(x_0,\theta)}{\partial x \partial \theta}} > 0 \ . \tag{A5}$$

Thus, the range of inertia unambiguously widens when λ increases. An analog argument shows that $\frac{\partial \overline{\theta}}{\partial \alpha} > 0$, hence the same result holds for a change in α.

(c) Using the implicit function theorem again we get

$$\frac{\partial \overline{\theta}}{\partial x_0} = -\frac{\frac{\partial^2 v(x_0,\overline{\theta})}{\partial x^2}}{\frac{\partial^2 v(x_0,\overline{\theta})}{\partial x \partial \theta}} > 0 \ . \tag{A6}$$

Thus, an increase of x_0 increases $\overline{\theta}$.

$\square$

Proof of Proposition 3. Note first that if the union rejects the contract offer, the firm will terminate the contract because the status quo generates negative profits. In this case workers would receive a utility of zero. Therefore, it is optimal for the workers to accept any offer that yields (weakly) positive utility. Hence, the firm's maximization problem is given by

$$\max_{x,w} \quad \Pi(w,x) = v(x,\theta) - w - C \qquad s.t. \quad U(w,x) \geq 0. \tag{A7}$$

Since $x_0 \leq x^{ME}$ we have $\frac{\partial v(x_0,\theta)}{\partial x_0} \geq 1$. Therefore, it is not optimal to choose any effort level $x < x_0$. Moreover, the constraint must bind, since otherwise the firm could profitably decrease the wage. Hence, constraint (A7) implies

$$w = x + \alpha\lambda[w_0 - w]^+ + \alpha\lambda(x - x_0). \tag{A8}$$

Let $\hat{x}$ be defined by $U(w_0, \hat{x}) = 0$. Then, for any (w, x) with $U(w, x) = 0$ we have $w > w_0$ if and only if $x > \hat{x}$. If we solve A8 for w and plug it into the firm's objective function the maximization problem becomes

$$\max_{x} \quad \Pi(x) = \begin{cases} v(x,\theta) - (1+\alpha\lambda)x + \alpha\lambda x_0 - C & \text{if } x > \hat{x}, \\ v(x,\theta) - x + \frac{\alpha\lambda}{1+\alpha\lambda}(x_0 - w_0) - C & \text{if } x \leq \hat{x}. \end{cases}$$

Notice that

$$\frac{\partial \Pi(x)}{\partial x} = \begin{cases} \frac{\partial v(x,\theta)}{\partial x} - (1+\alpha\lambda) & \text{if } x > \hat{x}, \\ \frac{\partial v(x,\theta)}{\partial x} - 1 & \text{if } x < \hat{x}. \end{cases}$$

is decreasing in x.

We have to distinguish three cases.

1. If $\hat{x} \geq x^{ME}(\theta)$ then $\frac{\partial \Pi(x)}{\partial x} < 0$ for all $x > \hat{x}$, so the optimal solution $x^* \leq \hat{x}$. Since $\frac{\partial v(x^{ME}(\theta),\theta)}{\partial x} = 1$, we have $x^* = x^{ME}(\theta)$.

2. If $\hat{x} \in (\overline{x}(\theta), x^{ME}(\theta))$ then $\frac{\partial \Pi(x)}{\partial x} > 0$ for $x < \hat{x}$ and $\frac{\partial \Pi(x)}{\partial x} < 0$ for $x > \hat{x}$, and the optimal solution is given by $x^* = \hat{x}$.

3. If $\hat{x} \leq \overline{x}(\theta)$ then $\frac{\partial \Pi(x)}{\partial x} > 0$ for all $x < \hat{x}$. Since $\frac{\partial v(\overline{x}(\theta),\theta)}{\partial x} = 1 + \alpha\lambda$, the optimal solution is given by $x^* = \overline{x}(\theta)$.

□

Proof of Lemma 1. We first show that an optimal effort schedule cannot have decreasing effort. Consider an effort schedule that features $x_t < x_{t-1}$. Then,

$$\frac{\partial W}{\partial x_t} = \delta^{t-1}\left(\frac{\partial v(x_t,\theta)}{\partial x_t} - 1\right) + \delta^t(1 + \lambda\alpha\mathbb{1}_{x_{t+1}>x_t}).$$

Since $x_t < x_{t-1} < x^{ME}(\theta_t)$ we have $\frac{\partial v(x_t,\theta)}{\partial x_t} > 1$. This implies $\frac{\partial W}{\partial x_t} > 0$, which means that x_t cannot be optimal. Hence, optimal effort must be weakly increasing.

Suppose $(x_s)_{s\in\mathbb{N}}$ is an efficient effort schedule. We now fix some period $t \geq 1$ to prove the lemma. Since the efficient effort is weakly increasing there are three possible cases: Either the optimal effort remains constant from $t-1$ forever, or it remains constant for some periods and first increases in period $t+s$, $s \geq 1$, or it increases from period $t-1$ to t already. In the following, we show that the lemma holds in all three cases.

Case 1: The optimal effort is constant from x_{t-1} to infinity.

Denote with x the constant effort from period t to infinity. Then, an equal marginal increase of effort in all periods starting in t must be suboptimal. Hence, for $x = x_{t-1}$

$$0 \geq \frac{\partial W}{\partial x} = \delta^{t-1}\left(-\alpha\lambda + \sum_{s=0}^{\infty}\delta^s\left(\frac{\partial v(x,\theta_{s+t})}{\partial x} - 1\right)\right) > \delta^{t-1}\left(-\alpha\lambda + \sum_{s=0}^{\infty}\delta^s\left(\frac{\partial v(x,\theta_t)}{\partial x} - 1\right)\right),$$

where the last inequality exploits $\theta_t < \theta_{t+s}$ for $s > 0$. For $\delta > 0$ this implies

$$-\alpha\lambda + \frac{\frac{\partial v(x,\theta_t)}{\partial x} - 1}{1-\delta} \leq 0,$$

which is equivalent to

$$\frac{\partial v(x_t,\theta_t)}{\partial x_t} \leq 1 + (1-\delta)\alpha\lambda.$$

Since $x_{t-1} = x_t$ it follows that in Case 1 indeed $x_{t-1} = x_t \geq \overline{x}(\theta_t)$.

Case 2: The optimal effort is constant from x_{t-1} to x_s for some $s \geq t$, but $x_{s+1} > x_s$.

Because the optimal level is unchanged in period s but goes up in period $s+1$, the necessary condition for x_s implies

$$0 \geq \frac{\partial W}{\partial x_s} = \delta^{s-1} \left(\frac{\partial v(x_s, \theta_s)}{\partial x_s} - 1 - \alpha\lambda + \delta\lambda\alpha \right).$$

Since $x_{t-1} = x_t = x_s$ and $\theta_t \leq \theta_s$ this implies

$$\frac{\partial v(x_t, \theta_t)}{\partial x_t} \leq 1 + (1-\delta)\alpha\lambda.$$

Hence, we are again in the case $x_{t-1} = x_t \geq \overline{x}(\theta_t)$, and the lemma holds.

Case 3: The optimal effort increases in period t, i.e., $x_t > x_{t-1}$.

Notice first that in this case we must also have $x_{t+1} > x_t$. To see this suppose that $x_t = x_{t+1}$. If $\frac{\partial W}{\partial x_t} \geq 0$ then $\frac{\partial W}{\partial x_{t+1}} > 0$, contradicting the optimality of x_{t+1}. Thus, it must be that $\frac{\partial W}{\partial x_t} < 0$, but this contradicts the optimality of x_t. Hence, we are at an interior solution $x_t \in (x_{t-1}, x_{t+1})$. The necessary condition for the optimal x_t is then given by

$$0 = \frac{\partial W}{\partial x_t} = \delta^{t-1} \left(\frac{\partial v(x_t, \theta_t)}{\partial x_t} - 1 - \alpha\lambda \right) + \delta^t \lambda\alpha,$$

which solves to

$$\frac{\partial v(x_t, \theta_t)}{\partial x_t} = 1 + (1-\delta)\alpha\lambda.$$

Hence, in this case indeed $x_{t-1} < x_t = \overline{x}(\theta_t)$, and the lemma holds. □

Proof of Proposition 5. Because the unemployment utility of zero is the new threat point, workers accept an offer if and only if their continuation utility is weakly positive. Hence, the firm's maximization problem is given by

$$\max_{(w_t, x_t)} \quad \Pi_t(w_t, x_t) \qquad s.t. \quad U_t(w_t, x_t | w_{t-1}, x_{t-1}) \geq 0. \tag{A9}$$

Since $x_{t-1} \leq x^{ME}(\theta_s)$ for all $s \geq t$ we have $\frac{\partial v(x_{t-1}, \theta_s)}{\partial x} \geq 1$ for all $s \geq t$. Thus, decreasing effort in period t is suboptimal for the present and all future periods. This implies that the optimal effort level x_t satisfies $x_t \geq x_{t-1}$.

The workers anticipate that in all future offers they will be indifferent between the offer and their status quo. Hence, the expected utility of any contract (w_t, x_t) is the same as if working under this contract forever.

$$\begin{aligned} U_t(w_t, x_t | w_{t-1}, x_{t-1}) &= \sum_{s=t}^{\infty} \delta^{s-t}(w_t - x_t) - \alpha\lambda[w_{t-1} - w_t]^+ - \alpha\lambda[x_t - x_{t-1}]^+ \\ &= \frac{w_t - x_t}{1-\delta} - \alpha\lambda[w_{t-1} - w_t]^+ - \alpha\lambda[x_t - x_{t-1}]^+ \end{aligned}$$

Again, the constraint in (A9) must bind, since otherwise the firm could profitably decrease the wage. Therefore, the constraint implies

$$w_t = x_t + (1-\delta)\alpha\lambda[w_{t-1} - w_t]^+ + (1-\delta)\alpha\lambda(x_t - x_{t-1}). \tag{A10}$$

Let $\hat{x}$ be defined by $U_t(\hat{x}, w_{t-1}) = 0$. Then, for any (w_t, x_t) with $U_t(w_t, x_t) = 0$ we have $w_t > w_{t-1}$ if and only if $x_t > \hat{x}$. Hence, (A10) implies

$$w_t(x_t) = \begin{cases} (1 + (1-\delta)\alpha\lambda)x_t - (1-\delta)\alpha\lambda x_{t-1} & x_t \geq \hat{x} \\ x_t + \frac{(1-\delta)\alpha\lambda}{1+(1-\delta)\alpha\lambda}(w_{t-1} - x_{t-1}) & x_t < \hat{x}, \end{cases}$$

and therefore

$$w_t'(x_t) = \begin{cases} 1 + (1-\delta)\alpha\lambda & x_t > \hat{x} \\ 1 & x_t < \hat{x}. \end{cases} \tag{A11}$$

Recall that the firm maximizes

$$\max_{x_t} \quad \Pi_t(x_t) = v(x_t, \theta_t) - w(x_t) - C_t^h + \delta\Pi_{t+1}\big(x_{t+1}^*, w_{t+1}^* | x_t, w_t(x_t)\big),$$

where $\Pi_{t+1}\big(x_{t+1}^*, w_{t+1}^* | x_t, w_t(x_t)\big)$ is the continuation utility in $t+1$ for a given choice of $(x_t, w_t(x_t))$.

In $t+1$ the firm inherits a contract (w_t, x_t) from the former period and does not expect any further crisis. The optimal wage and effort choices for this case are established in Lemma 1 and Proposition 4: The firm chooses $(x_{t+s}^*, w_{t+s}^*) = (x_t, w_t)$ for some $s \leq n$, where $n \in \mathbb{N} \cup \{0\}$ is the duration for which effort stays in the inertia region.[27] For $s > n$ the firm chooses

$$(x_{t+s}^*, w_{t+s}^*) = (\overline{x}(\theta_{t+s}), w_t + (1 + (1-\delta)\alpha\lambda)(\overline{x}(\theta_{t+s}) - x_t).$$

[27] Note that n endogenously depends on the choice of x_t. For the remainder of the proof this is inconsequential.

Hence, the firm's profit function is[28]

$$\Pi(x_t) = \sum_{s=0}^{n} \delta^s (v(x_t, \theta_{t+s}) - w(x_t)) - C_t^h + \sum_{s=n+1}^{\infty} \delta^s \big(v(\overline{x}(\theta_{t+s}), \theta_{t+s}) - w(x_t) - (1 + (1-\delta)\alpha\lambda)(\overline{x}(\theta_{t+s}) - x_t) \big).$$

Using (A11) we obtain the derivative

$$\begin{aligned} \Pi'(x_t) &= \sum_{s=0}^{n} \delta^s \left(\frac{\partial v(x_t, \theta_{t+s})}{\partial x_t} - w'(x_t) \right) + \sum_{s=n+1}^{\infty} \delta^s \big(1 + (1-\delta)\alpha\lambda - w'(x_t) \big) \\ &= \begin{cases} \sum_{s=0}^{n} \delta^s \left(\frac{\partial v(x_t, \theta_t)}{\partial x_t} - 1 \right) + \sum_{s=n+1}^{\infty} \delta^s (1-\delta)\alpha\lambda & x < \hat{x} \\ \sum_{s=0}^{n} \delta^s \left(\frac{\partial v(x_t, \theta_t)}{\partial x_t} - 1 - (1-\delta)\alpha\lambda \right) & x > \hat{x}. \end{cases} \end{aligned}$$

Since $\frac{\partial v(x_t, \theta_{t+s})}{\partial x_t} \geq 1$ for all t and s this implies $\frac{\partial \Pi_t(x_t)}{\partial x_t} > 0$ for $x_t < \hat{x}$. Hence, as long as it is feasible, the firm will at least implement effort $\hat{x}$. Recall that, by assumption, the firm is unable to implement effort above $x^{ME}(\theta_t)$. This shows $x^* \geq \min\{\hat{x}, x^{ME}(\theta_t)\}$, with equality $x^* = x^{ME}(\theta_t)$ if $\hat{x} > x^{ME}(\theta_t)$.

Lastly, the firm implements $x > \hat{x}$ if and only if $\frac{\partial v(\hat{x}, \theta_t)}{\partial x_t} > 1 + (1-\delta)\alpha\lambda$, which is equivalent to $\hat{x} < \overline{x}(\theta_t)$. In this case the firm implements the inner solution $\overline{x}(\theta_t)$ which is defined by $\frac{\partial v(\overline{x}(\theta_t), \theta_t)}{\partial x_t} = 1 + (1-\delta)\alpha\lambda$. □

Proof of Lemma 2. Since $x_{t-1} \leq x^{ME}(\theta_{t-1}) < x^{ME}(\theta_t)$ choosing any $x_t < x_{t-1}$ decreases material efficiency. Hence, even disregarding behavioral costs from implementing change, it decreases efficiency in t and, since θ is increasing, a fortiori in all future periods. In any optimal offer the principal will make the workers indifferent to accepting, i.e., $U_t(x_{t-1}, w_{t-1}) = U_t(x_t, w_t)$, because otherwise the principal could decrease the wage slightly and the worker would still accept. Thus, a decrease in efficiency would decrease the firm's profits. This implies, effort must be weakly increasing in any equilibrium.

By Proposition 5 workers' utility drops to zero when a crisis occurs. Hence, workers

[28] If effort never leaves the inertia region the implicit convention is that $n = \infty$ and the second sum is empty.

discount future utility by a factor $\delta(1-\mu)$. Hence, for $w_t \geq w_{t-1}$ and $x_t \geq x_{t-1}$,

$$\begin{aligned}
& U_t(w_{t-1}, x_{t-1}) = U_t(w_t, x_t) \\
\Leftrightarrow \quad & \sum_{s=0}^{\infty}(\delta(1-\mu))^s(w_{t-1}-x_{t-1}) = -\alpha\lambda(x_t - x_{t-1}) + \sum_{s=0}^{\infty}(\delta(1-\mu)^s(w_t - x_t) \\
\Leftrightarrow \quad & \frac{(w_{t-1}-x_{t-1})}{1-\delta(1-\mu)} = -\alpha\lambda(x_t - x_{t-1}) + \frac{(w_t - x_t)}{1-\delta(1-\mu)} \\
\Leftrightarrow \quad & w_t = w_{t-1} + \Big[1 + \big(1-\delta(1-\mu)\big)\alpha\lambda\Big](x_t - x_{t-1}) = w_{t-1} + (1+\gamma)(x_t - x_{t-1}).
\end{aligned}$$

Iterating this formula yields

$$w_t = w_0 + (1+\gamma)(x_t - x_0) \quad \Leftrightarrow \quad w_t = x_t + \gamma(x_t - x_0) + (w_0 - x_0).$$

The last equation of Lemma 2 holds since

$$\begin{aligned}
U_t(w_{t-1}, x_{t-1}) &= \sum_{k=0}^{\infty}\big(\delta(1-\mu)\big)^k(w_{t-1}-x_{t-1}) \\
&= \sum_{k=0}^{\infty}(\delta(1-\mu))^k\gamma(x_{t-1}-x_0) + \sum_{k=0}^{\infty}(\delta(1-\mu))^k(x_0 - w_0) \\
&= \alpha\lambda(x_{t-1}-x_0) + U_0(x_0, w_0).
\end{aligned}$$

□

Proof of Lemma 3. Since there is no further threat of bankruptcy after the crisis, a contract (w_{t-1}, x_t) yields a utility of $w_{t-1} - x_t$ in the current and all periods but comes at a one-time behavioral cost of $\alpha\lambda\Delta x_t$. Hence, the effort level that generates zero utility satisfies

$$\alpha\lambda\Delta x_t = \sum_{i=0}^{\infty}\delta^i(w_t - x_t - \Delta x_t) = \frac{(w_t - x_t - \Delta x_t)}{1-\delta},$$

which is equivalent to

$$\Delta x_t = \frac{w_t - x_t}{1+(1-\delta)\alpha\lambda}.$$

Inserting $(w_t - x_t)$ as given in Equation 17 in Lemma 2 yields the result. □

Proof of Proposition 6. Consider some status quo contract (w_{t-1}, x_{t-1}) and no crisis in period t. Notice that by Lemma 2 the wage w_t is pinned down by the effort level x_t and the workers' initial utility level $w_0 - x_0$. Hence, we suppress the wage in the notation and denote with $W_t(x_t) \equiv W_t(x_t|x_{t-1})$ the expected welfare given that the contracted effort level at time t is x_t and future effort levels are chosen optimally given x_t.

Recall that in normal times for any implemented effort wages are set such that the workers only receive their status quo utility. Hence, the principal's expected profit at time t from implementing effort x_t is

$$\Pi_t(x_t) = W_t(x_t) - U_t(w_{t-1}, x_{t-1}).$$

Since the second term is a constant that does not depend on x_t, the principal aims to maximize $W_t(x_t)$ given x_{t-1}. To prove the proposition, we need to show that it is never welfare maximizing to set an effort level $x_t > \overline{x}(\theta_t)$.

Denote with $W^c_{t+1}(x_t)$ the expected welfare at time $t+1$ if the crisis occurs at $t+1$ and the effort level contracted in period t was x_t. Since, again, it cannot be optimal to lower effort in period t we have

$$W_t(x_t) = v(x_t, \theta_t) - x_t - \alpha\lambda(x_t - x_{t-1}) + \delta(1-\mu)W(x^*_{t+1}|x_t) + \delta\mu W^c_{t+1}(x_t).$$

Next, we establish an upper bound for $\frac{\partial W(x^*_{t+1}|x_t)}{\partial x_t}$. At $t+1$ the firm benefits most from a high x_t if $x^*_{t+1} > x_t$. But in this case we have

$$W(x^*_{t+1}|x_t) = W(x^*_{t+1}|x^*_{t+1}) - \alpha\lambda(x^*_{t+1} - x_t).$$

Hence, $\frac{\partial W(x^*_{t+1}|x_t)}{\partial x_t} \leq \alpha\lambda$, and we obtain the following upper bound for the effect of marginal effort increases on welfare:

$$\begin{aligned}\frac{\partial W_t(x_t)}{\partial x_t} &\leq \frac{\partial v_t(x_t, \theta_t)}{\partial x_t} - 1 - \alpha\lambda + \delta(1-\mu))\alpha\lambda + \delta\mu\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} \\ &= \frac{\partial v_t(x_t, \theta_t)}{\partial x_t} - \big(1 + (1-\delta)\alpha\lambda\big) + \delta\mu\left(\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} - \alpha\lambda\right).\end{aligned}$$

To show that it is never welfare maximizing to implement $x_t > \overline{x}(\theta_t)$ it suffices to show that the right-hand side of the above equation is negative for all $x_t > \overline{x}(\theta_t)$. Since for all $x_t > \overline{x}(\theta_t)$ we have $\frac{\partial v_t(x_t,\theta_t)}{\partial x_t} < 1 + (1-\delta)\alpha\lambda$ this amounts to showing that

$$\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} \leq \alpha\lambda \tag{A12}$$

for all $x_t > \overline{x}(\theta_t)$. The remainder of the proof is devoted to that task.

The firm will continue the relationship in the crisis if and only if her expected profits remain positive. Denote with $W^{cc}_{t+1}(x_t)$ the welfare without cost C^h_{t+1} if the principal continues the relationship. Since the principal continues the relationship if and only if it yields positive continuation payofff, we have

$$W^c_{t+1}(x_t) = \max\{0, W^{cc}_{t+1}(x_t) - C^h_{t+1}\}.$$

Notice that $W^c_{t+1}(x_t)$ is continuous everywhere and differentiable in all points except at the x_t that satisfies $W^{cc}_{t+1}(x_t) = C^h_{t+1}$. Depending on the implemented x_t the welfare $W^c_{t+1}(x_t)$ may be determined either by bankruptcy in $t+1$ or by the effort implemented in $t+1$, which follows one of the three cases laid out in the case distinctions in Proposition 5. We go through all case distinctions and show that Equation A12 holds in all of these cases.

- Case $W^{cc}_{t+1}(x_t) < C^h_{t+1}$.

The principal terminates the relationship and $W^c(x_t) = 0$. Hence,

$$\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} = 0 < \alpha\lambda.$$

In all other cases the firm continues the relationship. Recall that in the crisis the firm then uses the workers' quasi-rent to increase effort. For the remaining case distinctions outlined in Proposition 5, we denote with $\Delta(x_t) \equiv \Delta x_{t+1}$ the amount by which the firm can at most increase effort in a crisis at $t+1$ when holding wages fixed, as calculated in (20) of Lemma 3.

- Case $W^{cc}_{t+1}(x_t) > C^h_{t+1}$ and $\overline{x}(\theta_t) + \Delta(\overline{x}(\theta_t)) < \overline{x}(\theta_{t+1})$.

The second condition states that we are in Case 2. of Propposition 5: Even if the firm follows the behaviorally efficient path until t, the quasi-rent at time $t+1$ does not suffice to implement the behaviorally efficient level $\overline{x}(\theta_{t+1})$. This implies that the firm increases the wage for an additional effort increase to implement $x_{t+1} = \overline{x}(\theta_{t+1})$. This means the marginal effort choice at period t has no impact on the effort implemented in the crisis, and hence does not affect welfare in and after the crisis. The firm then implements the behavioral efficient effort level at time t, as outlaid in Proposition 4. Hence, the inequality in our proposition holds with equality, and inertia is the same as in the unanticipated case. We will see that in all other cases inertia is strictly larger than in the case of an unanticipated crisis.

If $\overline{x}(\theta_t) + \Delta(\overline{x}(\theta_{t+1})) > \overline{x}(\theta_{t+1})$, then a fortiori $x_t + \Delta(x_t) > \overline{x}(\theta_{t+1})$ for all $x_t > \overline{x}(\theta_t)$. In this case, following Proposition 5, the principal will implement effort $x_{t+1} = \min\{x^{ME}(\theta_{t+1}), x_t + \Delta x_t\}$, and the effort enters the region of inertia. This gives rise to the last two case distinctions

- Case $W^{cc}_{t+1}(x_t) > C^h_{t+1}$ and $x_t + \Delta x_t > x^{ME}(\theta_{t+1})$.

Then, in the crisis at $t + 1$ the principal implements the materially efficient effort level $x^{ME}(\theta_{t+1})$. The marginal choice of effort x_t has no impact on the effort implemented in a crisis. However, it has an impact on the associated wage cut in a crisis. A higher effort x_t leads to a stronger wage decrease in the crisis. Since a wage decrease comes with a behavioral cost this implies

$$\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} < 0 < \alpha\lambda.$$

- Case $W^{cc}_{t+1}(x_t) > C^h_{t+1}$ and $x_t + \Delta(x_t) \in (\overline{x}(\theta_{t+1}), x^{ME}(\theta_{t+1}))$.

This is the most interesting case of an interior solution, in which the marginal choice of effort x_t affects the effort path in and after the crisis. We start by deriving a closed form expression for the welfare $W^{cc}_{t+1}(x_t)$ in the crisis.

Let $T \in \{t + 1, t + 2, ...\} \cup \{\infty\}$ be the last period in which $x_{t+1} = x_t + \Delta(x_t)$ satisfies $x_{t+1} < \overline{x}(\theta_T)$, i.e., the period before the effort level x_{t+1} leaves the inertia region (if ever). Then,

$$\begin{aligned} W^{cc}_{t+1}(x_t) = -\alpha\lambda\Delta(x_t) + \sum_{s=t+1}^{T} \delta^{s-t-1}\big(v(x_{t+1}, \theta_s) - x_{t+1}\big) \\ -\delta^{(T-t)}\alpha\lambda(x_T - x_{t+1}) + \delta^{(T-t)} W_T(\overline{x}(\theta_T)) \end{aligned}$$

The first term corresponds to the behavioral cost from the effort adjustment in period $t + 1$. The sum is the welfare generated while effort is in the inertia region. The third term is the behavioral adjustment cost to the behaviorally efficient line after effort leaves the inertia region. The last term denotes the (discounted) welfare from the remaining game where effort follows the behaviorally efficient level and is independent of the past adjustment in the crisis.[29]

Then,

$$\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} = -\alpha\lambda\frac{\partial \Delta x_t}{\partial x_t} + \sum_{s=t+1}^{T} \delta^{s-t+1}\left(\frac{\partial v(x_{t+1},\theta_s)}{\partial x_t} - \frac{\partial x_{t+1}}{\partial x_t}\right) + \delta^{(T-t)}\alpha\lambda\frac{\partial x_{t+1}}{\partial x_t}.$$

Since $\frac{\partial x_{t+1}}{\partial x_t} = \frac{\partial(x_t+\Delta x_t)}{\partial x_t} = 1 + \frac{\partial \Delta x_t}{\partial x_t}$ we obtain

$$\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} = \alpha\lambda + \frac{\partial x_{t+1}}{\partial x_t}\left[-\alpha\lambda + \sum_{s=t+1}^{T} \delta^{s-t+1}\left(\frac{\partial v(x_{t+1},\theta_s)}{\partial x_{t+1}} - 1\right) + \delta^{(T-t)}\alpha\lambda\right].$$

For $s \in \{t+1, ..., T\}$ we are in the inertia region and have $\frac{\partial v(x_{t+1},\theta_s)}{\partial x_{t+1}} < 1 + (1-\delta)\alpha\lambda$, and hence

$$\begin{aligned}\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} &< \alpha\lambda + \frac{\partial x_{t+1}}{\partial x_t}\left[-\alpha\lambda + \sum_{s=t+1}^{T} \delta^{s-t+1}(1-\delta)\alpha\lambda + \delta^{(T-t)}\alpha\lambda\right] \\ &= \alpha\lambda + \alpha\lambda\frac{\partial x_{t+1}}{\partial x_t}\underbrace{\left[-1 + (1-\delta)\sum_{k=0}^{T-t-1}\delta^k + \delta^{T-t}\right]}_{=0} \\ &= \alpha\lambda.\end{aligned}$$

□

Proof of Lemma 4. The discounted sum of the firm's and the workers' utility is:

$$\begin{aligned}W &= \sum_{t=1}^{\infty}\delta^{t-1}\big(v(x_t,\theta_t) - x_0 - \frac{x_t - x_{t-1}}{1-\delta} - \lambda\alpha[w_{t-1}-w_t]^+ - \lambda\alpha[x_t - x_{t-1}]^+\big) \\ &= \sum_{t=1}^{\infty}\delta^{t-1}\big(v(x_t,\theta_t) - \frac{x_t}{1-\delta} - \lambda\alpha[w_{t-1}-w_t]^+ - \lambda\alpha[x_t - x_{t-1}]^+\big) - \frac{x_0}{1-\delta} + \sum_{t=1}^{\infty}\delta^{t-1}\frac{x_{t-1}}{1-\delta} \\ &= \sum_{t=1}^{\infty}\delta^{t-1}\big(v(x_t,\theta_t) - \frac{x_t}{1-\delta} - \lambda\alpha[w_{t-1}-w_t]^+ - \lambda\alpha[x_t - x_{t-1}]^+\big) + \sum_{t=1}^{\infty}\delta^{t}\frac{x_t}{1-\delta} \\ &= \sum_{t=1}^{\infty}\delta^{t-1}\big(v(x_t,\theta_t) - x_t - \lambda\alpha[w_{t-1}-w_t]^+ - \lambda\alpha[x_t - x_{t-1}]^+\big).\end{aligned}$$

□

[29] If $T = \infty$ the implicit convention is that the last two terms are zero.

Proof of Lemma 5. The proof is a simple adaptation of the proof of Lemma 2.

By Proposition 5 workers' utility drops to zero when a crisis occurs. Hence, workers discount future utility by a factor $\delta(1-\mu)$. Hence,

$$
\begin{aligned}
& U_t(w_{t-1}, x_{t-1}) = U_t(w_t, x_t) \\
\Leftrightarrow\quad & \sum_{s=0}^{\infty}(\delta(1-\mu))^s(w_{t-1}-x_0) = -\left[\frac{1}{1-\delta}+\alpha\lambda\right](x_t - x_{t-1}) + \sum_{s=0}^{\infty}(\delta(1-\mu)^s(w_t - x_0) \\
\Leftrightarrow\quad & \frac{w_{t-1}}{1-\delta(1-\mu)} = -\left[\frac{1}{1-\delta}+\alpha\lambda\right](x_t - x_{t-1}) + \frac{w_t}{1-\delta(1-\mu)} \\
\Leftrightarrow\quad & w_t = w_{t-1} + \tilde{\gamma}(x_t - x_{t-1}).
\end{aligned}
$$

Iterating this formula yields

$$w_t = w_0 + \tilde{\gamma}(x_t - x_0).$$

The last equation of Lemma 2 holds since

$$
\begin{aligned}
U_t(w_{t-1}, x_{t-1}) &= \sum_{k=0}^{\infty}\big(\delta(1-\mu)\big)^k(w_{t-1}-x_0) \\
&= \sum_{k=0}^{\infty}(\delta(1-\mu))^k\tilde{\gamma}(x_{t-1}-x_0) + \sum_{k=0}^{\infty}(\delta(1-\mu))^k(w_0 - x_0) \\
&= \left[\frac{1}{1-\delta}+\alpha\lambda\right](x_{t-1}-x_0) + U_0(x_0, w_0).
\end{aligned}
$$

□

Proof of Proposition 7. The proof is a simple adaptation of the proof of Proposition 6. We briefly sketch the main steps of the construction with a focus on the differences to the proof of Proposition 6. Since again

$$\Pi_t(x_t) = W_t(x_t) - U_t(w_{t-1}, x_{t-1}),$$

and $U_t(w_{t-1}, x_{t-1})$ does not depend on x_t, the firm maximizes welfare $W(x_t)$ at each normal period t. With temporary adaptation cost the welfare function reads

$$W_t(x_t) = v(x_t, \theta_t) - x_0 - \left(\frac{1}{1-\delta}+\alpha\lambda\right)(x_t - x_{t-1}) + \delta(1-\mu)W(x^*_{t+1}|x_t) + \delta\mu W^c_{t+1}(x_t),$$

where, again, $W^c_{t+1}(x_t)$ is the welfare at time $t+1$ if a crisis occurs.

If $x^*_{t+1} > x_t$ we have

$$W(x^*_{t+1}|x_t) = W(x^*_{t+1}|x^*_{t+1}) - \left(\frac{1}{1-\delta} + \alpha\lambda\right)(x^*_{t+1} - x_t).$$

Hence, $\frac{\partial W(x^*_{t+1}|x_t)}{\partial x_t} \leq \frac{1}{1-\delta} + \alpha\lambda$, and we obtain the following upper bound for the effect of marginal effort increases on welfare:

$$\begin{aligned}\frac{\partial W_t(x_t)}{\partial x_t} &\leq \frac{\partial v_t(x_t,\theta_t)}{\partial x_t} - \left(\frac{1}{1-\delta} + \alpha\lambda\right) + \delta(1-\mu))\left(\frac{1}{1-\delta} + \alpha\lambda\right) + \delta\mu\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} \\ &= \frac{\partial v_t(x_t,\theta_t)}{\partial x_t} - \big(1 + (1-\delta)\alpha\lambda\big) + \delta\mu\left(\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} - \alpha\lambda - \frac{1}{1-\delta}\right).\end{aligned}$$

To show that it is never welfare maximizing to implement $x_t > \overline{x}(\theta_t)$ it suffices to show that the right-hand side of the above equation is negative for all $x_t > \overline{x}(\theta_t)$. Since for all $x_t > \overline{x}(\theta_t)$ we have $\frac{\partial v_t(x_t,\theta_t)}{\partial x_t} < 1 + (1-\delta)\alpha\lambda$ it suffices to show that

$$\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} \leq \alpha\lambda + \frac{1}{1-\delta} \tag{A13}$$

for all $x_t > \overline{x}(\theta_t)$. The remainder of the proof is devoted to that task.

Let, again, $W^{cc}_{t+1}(x_t)$ be the welfare without cost C^h_{t+1} if the principal continues the relationship, such that

$$W^c_{t+1}(x_t) = \max\{0, W^{cc}_{t+1}(x_t) - C^h_{t+1}\}.$$

We go through all case distinctions to show that Equation A13 holds.

- Case $W^{cc}_{t+1}(x_t) < C^h_{t+1}$.

The principal terminates the relationship and $W^c(x_t) = 0$. Hence,

$$\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} = 0 < \alpha\lambda + \frac{1}{1-\delta}.$$

Denote, again, with $\Delta(x_t) \equiv \Delta x_{t+1}$ the amount by which the firm can at most increase effort in a crisis at $t+1$ when holding wages fixed, as calculated in (20) of Lemma 3.

- Case $W^{cc}_{t+1}(x_t) > C^h_{t+1}$ and $\overline{x}(\theta_t) + \Delta(\overline{x}(\theta_t)) < \overline{x}(\theta_{t+1})$.

We are in Case 2. of Propposition 5: Even if the firm follows the behaviorally efficient path until t, the quasi-rent at time $t+1$ does not suffice to implement the behaviorally efficient level $\overline{x}(\theta_{t+1})$. The firm implements an additional effort increase such that $x_{t+1} = \overline{x}(\theta_{t+1})$. Hence, the marginal effort choice at period t has no impact on the effort implemented in the crisis. The firm then implements the behavioral efficient effort level at time t. The inequality in our proposition holds with equality, and inertia is the same as in the unanticipated case.

If $\overline{x}(\theta_t) + \Delta(\overline{x}(\theta_{t+1})) > \overline{x}(\theta_{t+1})$, then a fortiori $x_t + \Delta(x_t) > \overline{x}(\theta_{t+1})$ for all $x_t > \overline{x}(\theta_t)$. In this case, following Proposition 5, the principal will implement effort $x_{t+1} = \min\{x^{ME}(\theta_{t+1}), x_t + \Delta x_t\}$, and the effort enters the region of inertia. This gives rise to the last two case distinctions

- Case $W^{cc}_{t+1}(x_t) > C^h_{t+1}$ and $x_t + \Delta x_t > x^{ME}(\theta_{t+1})$.

The principal implements the materially efficient effort level $x^{ME}(\theta_{t+1})$. The marginal choice of effort x_t has no impact on the effort implemented in a crisis. However, it has an impact on the associated wage cut in a crisis. A higher effort x_t leads to a stronger wage decrease in the crisis. Since a wage decrease comes with a behavioral cost this implies

$$\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} < 0 < \alpha\lambda + \frac{1}{1-\delta}.$$

- Case $W^{cc}_{t+1}(x_t) > C^h_{t+1}$ and $x_t + \Delta(x_t) \in (\overline{x}(\theta_{t+1}), x^{ME}(\theta_{t+1}))$.

Let, again, $T \in \{t+1, t+2, ...\} \cup \{\infty\}$ be the last period in which $x_{t+1} = x_t + \Delta(x_t)$ satisfies $x_{t+1} < \overline{x}(\theta_T)$, i.e., the period before the effort level x_{t+1} leaves the inertia region (if ever). Then,

$$\begin{aligned} W^{cc}_{t+1}(x_t) = &-\left(\alpha\lambda + \frac{1}{1-\delta}\right)\Delta(x_t) + \sum_{s=t+1}^{T} \delta^{s-t-1} v(x_{t+1}, \theta_s) \\ &-\delta^{(T-t)}\left(\alpha\lambda + \frac{1}{1-\delta}\right)(x_T - x_{t+1}) + \delta^{(T-t)} W_T(\overline{x}(\theta_T)) \end{aligned}$$

The first term corresponds to the total cost of effort adjustment in period $t+1$. The sum is the welfare generated while effort is in the inertia region. The third term is the adjustment cost to the behaviorally efficient line after effort leaves the inertia region. The last term denotes the (discounted) welfare from the remaining game where effort follows the behaviorally efficient level and is independent of the past adjustment in the crisis.[30]

Then,

$$\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} = -\left(\alpha\lambda + \frac{1}{1-\delta}\right)\frac{\partial \Delta x_t}{\partial x_t} + \sum_{s=t+1}^{T} \delta^{s-t+1}\frac{\partial v(x_{t+1},\theta_s)}{\partial x_t} + \delta^{(T-t)}\left(\alpha\lambda + \frac{1}{1-\delta}\right)\frac{\partial x_{t+1}}{\partial x_t}.$$

Since $\frac{\partial x_{t+1}}{\partial x_t} = \frac{\partial(x_t+\Delta x_t)}{\partial x_t} = 1 + \frac{\partial \Delta x_t}{\partial x_t}$ we obtain

$$\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} = \left(\alpha\lambda + \frac{1}{1-\delta}\right) + \frac{\partial x_{t+1}}{\partial x_t}\left(\alpha\lambda + \frac{1}{1-\delta}\right)\left[-1 + \delta^{(T-t)}\right] + \frac{\partial x_{t+1}}{\partial x_t}\sum_{s=t+1}^{T} \delta^{s-t+1}\frac{\partial v(x_{t+1},\theta_s)}{\partial x_{t+1}}.$$

For $s \in \{t+1,...,T\}$ we are in the inertia region and have $\frac{\partial v(x_{t+1},\theta_s)}{\partial x_{t+1}} < 1 + (1-\delta)\alpha\lambda$, and hence

$$\begin{aligned}
\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} &< \left(\alpha\lambda + \frac{1}{1-\delta}\right) + \frac{\partial x_{t+1}}{\partial x_t}\left(\alpha\lambda + \frac{1}{1-\delta}\right)\left[-1 + (1-\delta)\sum_{s=t+1}^{T}\delta^{s-t+1} + \delta^{(T-t)}\right] \\
&= \left(\alpha\lambda + \frac{1}{1-\delta}\right) + \left(\alpha\lambda + \frac{1}{1-\delta}\right)\frac{\partial x_{t+1}}{\partial x_t}\underbrace{\left[-1 + (1-\delta)\sum_{k=0}^{T-t-1}\delta^{k} + \delta^{T-t}\right]}_{=0} \\
&= \left(\alpha\lambda + \frac{1}{1-\delta}\right).
\end{aligned}$$

□

Proof of Lemma 6. The wage payment (24) follows directly from equation (23). Differentiating (24) twice with respect to p yields:

$$\begin{aligned}
\frac{\partial w}{\partial p} &= (1+\lambda)\Delta x - 2p(1-\alpha)\lambda\Delta x \\
\frac{\partial^2 w}{\partial p^2} &= -2(1-\alpha)\lambda\Delta x < 0
\end{aligned}$$

[30] If $T = \infty$ the implicit convention is that the last two terms are zero.

Note that

$$\frac{\partial w}{\partial p} < 0 \quad \text{if and only if} \quad \frac{1+\lambda}{\lambda(1-\alpha)} < 2p \ . \tag{A14}$$

□

Proof of Proposition 8. The principal maximizes

$$E\Pi = p(v + \Delta v - b) + (1-p)v - w \tag{A15}$$

subject to (24), $b \geq 0$, and

$$p \in \arg\max \left\{ pb - \frac{c}{2}p^2 \right\} . \tag{A16}$$

Since $b \geq 0$, constraint (A16) is equivalent to

$$p = \min \left\{ \frac{b}{c}, 1 \right\} . \tag{A17}$$

Evidently, any bonus $b > c$ is suboptimal as it induces the same probability of change but a higher manager compensation than $b = c$. By setting $p = \frac{b}{c}$ and plugging the wage from (24) into the objective, the principal's problem reduces to

$$\max_{b \in [0,c]} E\Pi = v - \left[x_0 + \frac{b}{c}\Delta x[1 + \lambda(1 - (1-\alpha)\frac{b}{c})] + U_0 \right] + \frac{b}{c}[\Delta v - b], \tag{A18}$$

which can be rewritten to

$$\max_{b \in [0,c]} v - x_0 - U_0 + \frac{b}{c}\left[\Delta v - (1+\lambda)\Delta x\right] - \frac{b^2}{c^2}\left[c - (1-\alpha)\lambda\Delta x\right] \ . \tag{A19}$$

This is a convex problem if and only if $c < (1-\alpha)\lambda\Delta x$. In the convex case we obtain a boundary solution, i.e., $b \in \{0, c\}$, or equivalently $p \in \{0, 1\}$. The boundary solution is $b = p = 0$ if and only if

$$\Delta v - (1+\lambda)\Delta x < c - (1-\alpha)\lambda\Delta x,$$

i.e., if and only if

$$\Delta v < (1+\alpha\lambda)\Delta x + c,$$

which proves (a) of the proposition.

For the concave case, the FOC for the problem is given by

$$\frac{\partial E\Pi}{\partial b} = \frac{\Delta v - (1+\lambda)\Delta x}{c} - \frac{2b[c-(1-\alpha)\lambda\Delta x]}{c^2} \leq 0 \ . \tag{A20}$$

Note that at $b=0$, $\frac{\partial E\Pi}{\partial b} = \frac{\Delta v-(1+\lambda)\Delta x}{c}$. Thus, the principal will choose the boundary solution $b=0$ if and only if $\Delta v \leq (1+\lambda)\Delta x$. Furthermore, if (A20) holds with equality we have

$$b = \frac{c}{2}\,\frac{\Delta v - (1+\lambda)\Delta x}{c-\lambda(1-\alpha)\Delta x} \quad \Leftrightarrow \quad p = \frac{\Delta v - (1+\lambda)\Delta x}{2[c-\lambda(1-\alpha)\Delta x]}. \tag{A21}$$

Hence, if $\Delta v > (1+\lambda)\Delta x$ the principal induces $p = \min\left\{\frac{\Delta v-(1+\lambda)\Delta x}{2[c-\lambda(1-\alpha)\Delta x]}, 1\right\}$.

□

Proof of Lemma 7. Because $x_2 > x_1$, we must have $w_2 > w_1$. Thus, the firm will offer wages that give both workers exactly their outside option utility U_0:

$$\begin{aligned} U_1 &= w_1 - x_1 - \lambda[w_1^r - w_1] = U_0, \\ U_2 &= w_2 - x_2 - \lambda[x_2 - x_2^r] = U_0. \end{aligned}$$

This implies

$$\begin{aligned} w_2 &= U_0 + x_2 + \lambda[x_2 - x_2^r] \\ &= U_0 + x_2 + \lambda\left[x_2 - (1-\beta)x_2 - \beta x_1\right] \\ &= U_0 + x_2 + \lambda\beta[x_2 - x_1]. \end{aligned}$$

This implies for w_1:

$$\begin{aligned} w_1 &= U_0 + x_1 + \lambda[w_1^r - w_1] \\ &= U_0 + x_1 + \lambda\left[(1-\beta)w_1 + \beta w_2 - w_1\right] \\ &= U_0 + x_1 + \lambda\beta[w_2 - w_1]. \end{aligned}$$

Collecting terms yields

$$\begin{aligned} [1+\lambda\beta]\, w_1 &= U_0 + x_1 + \lambda\beta w_2 \\ &= U_0 + x_1 + \lambda\beta\left[U_0 + x_2 + \lambda\beta(x_2 - x_1)\right] \\ &= (1+\lambda\beta)U_0 + \lambda\beta(1+\lambda\beta)x_2 + \left[1-(\lambda\beta)^2\right]x_1. \end{aligned}$$

Thus, we get:

$$\begin{aligned} w_1 &= U_0 + \lambda\beta x_2 + (1-\lambda\beta)x_1 \\ &= U_0 + x_1 + \lambda\beta[x_2 - x_1]. \end{aligned}$$

□

Proof of Proposition 9. Since worker 2 is the more productive one it is evidently suboptimal to implement $x_2 < x_1$. Denote in the following $\Delta \equiv x_2 - x_1 \geq 0$. Hence, the principal's problem can be written as

$$\max_{x_1, \Delta \geq 0} \Pi(x_1, \Delta) = v_1(x_1, \theta) + v_2(x_1 + \Delta, \theta) - [U_0 + x_1 + \lambda\beta\Delta] - [U_0 + x_1 + \Delta + \lambda\beta\Delta]$$

The first order conditions of a potential inner solution, $x_1 > 0, \Delta > 0$, are

$$\frac{\partial v_1(x_1, \theta)}{\partial x_1} + \frac{\partial v_2(x_2, \theta)}{\partial x_2} = 2,$$

$$\frac{\partial v_2(x_2, \theta)}{\partial x_2} = 1 + 2\beta\lambda.$$

Our regularity conditions imply that the first FOC is always at an inner solution, i.e., $x_1 > 0$. Hence, the first FOC binds with equality. The second FOC together with the constraint that $\Delta \geq 0$ implies the boundary solution $\Delta = 0$ if and only if the effort level $x_1 = x_2$ that satisfies the first FOC features $\frac{\partial v_2(x_2,\theta)}{\partial x_2} < 1 + 2\beta\lambda$. Hence, the result. □