Discussion Paper Series – CRC TR 224

Discussion Paper No. 612
Project C 03

International Policy Coordination in a Multisectoral Model of Trade and Health Policy

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November 2024

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Support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

International Policy Coordination in a Multisectoral Model of Trade and Health Policy*

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May 2024

Abstract

We analyze international trade and health policy coordination during a pandemic by developing a two-economy, two-sector trade model integrated with a micro-founded SIR model of infection dynamics. Disease transmission intensity can vary by goods (manufactured versus services, domestic versus foreign). Governments can implement domestic containment policies and impose import tariffs to prevent infections from abroad. Under the globally coordinated policy, the more-infected country aggressively contains the pandemic, raises tariffs, and shifts terms of trade in its favor, while the less-infected country lowers tariffs to share the economic burden. In contrast, in the uncoordinated Nash equilibrium, the more-infected country neglects global spread, lowering tariffs and its terms of trade, particularly in the contact-intensive services sector, while the less-infected country raises tariffs to counter the spread. Coordination is crucial: absent coordination, the health-cum-trade war results in reduced consumption, production, and smaller health gains due to inadequate global diversification of infection curves.

Keywords: Pandemic, SIR Model, COVID-19, Trade War, Terms of Trade

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*We thank Harald Fadinger, George Mailath, Volker Nocke, Raghu Sundaram, Michèle Tertilt, and Mathias Trabandt for comments. Steven Zheng, Cody Wan, Kunpeng Zhou, and Myeongrok Doh provided excellent research assistance. Support by the Deutsche Forschungsgemeinschaft (DFG) through CRC TR 224 (Project C03) is gratefully acknowledged.

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The Covid-19 pandemic has highlighted the potentially devastating global impact of infectious diseases. As the world becomes increasingly interconnected through economic linkages, local diseases can rapidly spread across borders, posing a significant threat to global health. The World Health Organization (WHO) anticipates future pandemics will be global in nature too, with the only uncertainty being the type of virus that will cause the next outbreak, and has actively advanced international health coordination in the last two years.[1] But given the economic nature of globalization, it is not enough to address the problem in terms of health policy alone. The problem is also a challenge for trade theory and policy.

The pandemic has generated significant debates over supply chain disruptions, border closures, and national protectionism. However, these observations reflect policy choices, often taken under domestic pressure and/or without sufficient scientific preparation, and may not provide an adequate account of the global economic turmoil caused by a pandemic. In fact, do the observed outcomes represent constrained-optimal policy decisions or indicate a suboptimal breakdown in international policy coordination? In particular, were the observed restrictions on imports and exports justified? Were the resulting distortions of the international terms of trade and their dynamics efficient from an international risk-sharing standpoint? Were the sectoral interventions appropriate, given the available economic gains from trade? More basically, how do health and trade policies affect each other during a pandemic, and in turn, how can one evaluate the resulting health and trade outcomes?

In this paper, we provide a theoretical framework to study these questions from first principles by embedding an epidemiological model of disease dynamics into an Armington-style international trade model, in which trade not only creates economic benefits by diversifying the choice of goods and services internationally, but also amplifies health risks by spreading diseases globally in its wake. Our model draws on the experience of the recent Covid-19 pandemic, but it is more general. It builds on the classic epidemiological model of SIR-dynamics along the lines of Kermack and McKendrick (1932), and integrates this into an economic context along the lines of the recent work by Eichenbaum, Rebelo and Trabandt (2021) and others. We generalize this latter class of models to include international trade in different goods and services and ask how domestic health and international trade policies interact. This interaction depends on the contact intensity, and thus the infectiousness of the associated consumption activity, which in the model can either be high ("services") or low ("manufactured goods").

The recent economic literature (Brotherhood et al., 2020; Garibaldi, Moen and Pissarides, 2020; Eichenbaum, Rebelo and Trabandt, 2021, and others) has emphasized that if a pandemic

[1] The recent WHO Pandemic Monitor at https://pandemichub.who.int/ is one of the WHO's monitoring and prevention initiatives in this respect. In March 2024, WHO member states agreed to finalize an international pandemic agreement to prepare for the threat of the next outbreak, declaring that "*[t]here is clear recognition from governments that the goal of a pandemic agreement is to prepare the world for preventing and responding to future pandemics*" (https://www.who.int/news/).

hits a single economy, local consumption and production adjust, but health externalities still justify domestic containment policies. In our model, governments impose domestic containment policies during the course of the domestic infection, which we model as distortionary "containment taxes" on domestic consumption similar to Eichenbaum, Rebelo and Trabandt (2021). This policy discourages potentially contagious consumption activity and can thus help internalize the health externalities. As a result, the levels of consumption and production in each country largely track the evolution of infected cases in each country.

In addition to the domestic containment policies, governments can levy import tariffs, differentially across sectors, as an instrument to address the international dimension of the problem. In the absence of pandemics, our model features a trade war, which as in the literature on international trade wars and negotiations (Brander and Spencer (1985), Bagwell and Staiger (1999) or Ossa (2014)), leads to high tariffs and to poor consumption choices between domestic and foreign goods. However, in a pandemic tariffs can potentially play a beneficial role, as they can alter the temporal structure of the terms of trade, inducing variation that is linked to the *relative* state of the pandemic in the two countries.

In this framework, we illustrate how national containment measures and trade policies impact disease transmission as well as economic welfare. We analyze the lack of international cooperation explicitly by modeling national policy-making as a non-cooperative game between governments and studying its Nash equilibria. We characterize and numerically simulate the resulting high-dimensional dynamic macroeconomic equilibrium, which involves a significant – and as far as we know hitherto unaddressed – degree of analytical and computational complexity. Using these simulations, we compare outcomes under different scenarios, and in particular study how and why non-cooperative policy-making differs from efficient international coordination.

One of the key insights from our model is that if the pandemic peaks at different times in different countries, international trade can potentially provide dynamic risk-sharing benefits in terms of both health and economic outcomes. Due to the cross-border nature of externalities, risk can be shared through trade policies that help pandemic-affected economies support their economic activity while minimizing the unavoidable health externalities that arise from it. Specifically, a globally efficient coordinated trade policy can soften the economic impact on an infected country by boosting its consumption of foreign goods and services, with adjustments made based on each sector's contribution to the country's overall health risk. This policy raises the terms of trade in favor of the infected country, which reduces the economic impact of the strong national containment policies needed to limit the spread of the disease. In our two-sector model, the more infected country imposes high import tariffs on less contact-intensive foreign goods to support its own industry, generate domestic income, and internalize the global spread of the pandemic. Simultaneously, it sets low tariffs for the sector with higher transmission intensity ("services") to avoid excessively discouraging imports. In parallel, the less-infected country subsidizes imports from the high-infection country for both manufacturing and service goods to support the infected country's economy. This mechanism requires the more-infected country to make a short-term economic sacrifice, while the less-infected country makes a short-term health sacrifice. However, these roles are reversed and there is reciprocation when the countries' pandemic situations change.

However, if the countries act non-cooperatively, the result is not only a trade war with lower economic welfare, but also a health war with worse disease outcomes. This latter feature is a novel result that complements the standard trade war logic on the economic front and addresses the often heard concern of a trade-off between health and economics. Specifically, the infected country seeks to boost domestic consumption by lowering import tariffs relative to the pure trade-war level, ignoring that this exacerbates the spread of infection in the foreign country. It lowers tariffs more, possibly even offering an import subsidy, i.e., a negative tariff, on the more contact-intensive services. However, this infect-thy-neighbor policy is only met by the less-infected country raising its tariff on services even more than in the pure trade-war case. The net result is that terms of trade do not favor the infected country in the services sector at the peak of its infection.

Figure 1 illustrates this key insight about equilibrium terms of trade in our model. The underlying model features pandemic waves that peak asynchronously in two countries, A and B. The dashed vertical lines signify the peak of the pandemic in each country (by assumption, country A peaks first, then country B).

Consider the uncoordinated (Nash) case first, which is depicted with the blue curve. When the pandemic first hits country A, it seeks to limit the spread of the disease domestically by imposing strong containment measures on domestic consumption, especially of services. To boost consumption, it provides import *subsidies* on foreign services. Altogether, the lower price level in country A incentivizes imports from A in B and leads to an increase in the risk of infection in B. In response, country B *raises* its import tariffs beyond the case without a pandemic. Other things equal, the infected country has to consume more of its own goods which generates more infections. In equilibrium, the infected country therefore *lowers* import tariffs drastically, in order to encourage its domestic households to consume more foreign services which are less conducive to infection than domestic services. As a result, uncoordinated policies modulate the tariff structure in a manner that skews the terms of trade *against* the infected country's production, aggravating economic risk-sharing possibilities in the midst of a pandemic. In contrast, the terms of trade for the manufacturing goods – which transmit the disease with lower intensity – are not much affected. In fact, they rise slightly in favor of the infected country because of reduced supply and labor shortage during the peak of the pandemic, highlighting the interaction of health and economic considerations.

Next, consider the case of optimal international coordination, depicted by the red curve in Figure 1. The structure of tariffs is also modulated in this case, but in a manner that is exactly the *opposite* of the uncoordinated case for services and more pronounced in case of manufactured goods. As domestic containment measures required to reduce domestic infections aggravate production and consumption in the infected country, the planner lowers the import tariffs in the foreign country and raises the import tariffs in the infected one, for both manufacturing and service goods. The structure of these tariffs might seem strange because it encourages both countries to consume more goods produced by the more infected country, raising the likelihood of infection. However, the terms of trade are now skewed in favor of the infected country's goods, not just for manufacturing but also for services, in order to ameliorate its economic situation. Furthermore, the effect is counteracted by an increase in production in the less infected country.

This intertemporal economic risk-sharing under coordinated policies also helps to share health risk. In particular, the less infected country imports a part of the infections by facilitating trade with the infected country and increasing production of its own good. This encourages the infected country to shift consumption towards foreign goods and thus reduce domestic labor at a time when work interactions are highly infectious (even as it reduces overall consumption in order to prevent its domestic infection rates from rising more strongly). In this sense, "*trade is essential to save both lives and livelihoods*" (OECD (2020)), i.e., there is no trade-off between economic and health performance in the international context. This normative conclusion of our model mirrors the argument by Antràs, Redding and Rossi-Hansberg (2023) who argue, using comparative statics around exogenous policy choices, that for countries with similar disease fundamentals, reducing trade frictions can increase the international spread of a pandemic, but that this effect is reversed if countries have sufficiently different health conditions. This latter situation arises endogenously in our model, as the disease spreads asymmetrically between countries. In fact, while the Nash equilibrium tariff policies reduce international disease transmission compared to laissez-faire policies, they still produce worse health outcomes in each country than socially optimal coordinated policies.

From a technical point of view, our analysis is, as far as we are aware, the first to study Nash equilibrium with fully dynamic economic and health policies. This is computationally demanding because strategies are high-dimensional and each iteration of the best-response algorithm requires solving a dynamic macroeconomic equilibrium model. For the sake of computational feasibility, we therefore model economic, health, and policy interactions as parsimoniously as possible. In particular, we restrict attention to open-loop Nash equilibria (see Fudenberg and Tirole (1991) or Dockner et al. (2000)) and thus assume that governments can commit to policy paths at the beginning of the interaction. Since there is no aggregate risk in our model and our extensive numerical checks indicate that the equilibrium is unique, we are confident (but cannot prove) that this constitutes no loss of generality. But certainly, our approach of solving for open-loop equilibria by using modifications of standard best-response algorithms can test the limits of large computing power.[2]

From a positive standpoint, our model helps to explain why, in the real-world scenario of uncoordinated decision-making by countries, terms of trade and economic outcomes may end up being excessively dire for the infected countries. As a consequence, an important normative insight of our model is that the purely epidemiological consideration of "closing the borders" for trade and travel as an extreme precautionary measure by less-infected countries to limit the spread of infections should be weighed against its implications for the economic and health risk-sharing between countries. In the extreme, we show that a policy of shutting down economic interaction massively by imposing drastic interventions ("Zero-Covid") yields not only less overall welfare, but also fares less well in health terms even than the non-cooperative Nash outcome. More generally, our model suggests that economic *and* health outcomes end up being superior with some coordination on trade, as countries that cooperate eventually reverse their roles (and the attendant sacrifices/gains) in being more or less affected by the pandemic.

Related Literature. Our paper contributes to a large literature that has emerged during the Covid-19 pandemic and studies the nexus between economics and disease. This literature is too large to be reviewed here extensively. In one of the few papers on the economics of disease dynamics before 2020, Greenwood et al. (2019) analyzed the dynamics of HIV in Africa and its economic consequences. Building on this work, Brotherhood et al. (2020) analyze a rich set of behavioral patterns and show the importance of heterogeneous lockdown policies for the Covid-19 environment. At a single country level, Eichenbaum, Rebelo and Trabandt (2021) (ERT) embed SIR disease dynamics into a macroeconomic model and study the trade-offs resulting from simple suppression policies quantitatively. Alvarez, Argente and Lippi (2021) study optimal lockdown policy as a planning problem in a single country. Health externalities arising from Covid-19 are analyzed in detail in Garibaldi, Moen and Pissarides (2020) and Assenza et al. (2020). Just like our paper, these early papers are mostly concerned with delaying or flattening the infection curve with a single infection peak. Modelling dynamics with several infection waves as observed in the global Covid-19 pandemic is more complex and requires additional model ingredients, as discussed by Atkeson (2021) or Krueger, Uhlig and Xie (2022).

Our paper naturally relates to that part of the literature that studies heterogeneity in macroeconomic SIR dynamics, such as Acemoglu et al. (2021) who develop an SIR model with

[2]Each government must choose a four-dimensional policy in each of the 260 weeks covered by our simulation, which generates a 2080-dimensional strategy space. Furthermore, the model features multiple agents, countries, and goods, which further increases the computational demands.

heterogeneous groups and lockdown policies, and Kaplan, Moll and Violante (2020) who integrate the SIR disease dynamics in a heterogeneous agent new-Keynesian model to analyze the distributional consequences of different containment strategies, with a focus similar to Glover et al. (2023). Fernandez-Villaverde and Jones (2022) estimate and simulate an SIR model by using disaggregated data from various locations, including international evolution of such data. In an international perspective, McKibbin and Roshen (2021) and Liu, Moon and Schorfheide (2021) estimate respectively a DSGE model and a Bayesian panel VAR in order to make global forecasts of different health-economics scenarios.

Similar to our paper, Antràs, Redding and Rossi-Hansberg (2023) study the economics of international trade and disease transmission conceptually. The authors develop a two-country model of household interaction in equilibrium with spatial frictions that jointly addresses the international spread of a disease and the gravity structure of international trade. While both our paper and theirs develop microfoundations of international SIR dynamics, the papers differ substantially otherwise. Our key focus is on governments, strategic national policies, and international coordination. In fact, unlike us, Antràs, Redding and Rossi-Hansberg (2023) treat the key policy frictions as exogenous parameters on which they perform comparative statics. Our paper models all individual behavioral responses as privately optimal throughout, which as Antràs, Redding and Rossi-Hansberg (2023) note is “challenging"; they mostly focus on the case where the disease either has no health or productivity effects or households are not aware of them in their decisions.

In terms of modeling international policy, our paper is closer to Beck and Wagner (2020) who also study cooperation across countries in containment policies in a simple two-stage model. However, their stylized model leaves aside the macroeconomic dynamics at the core of our model. In an early version of the present paper, Acharya et al. (2020), we discussed the role of terms of trade in a simpler trade model with a single sector. In parallel work, Engler et al. (2020) studied a simple generalization of ERT to two countries with no tariffs, in which a non-cooperative scenario is less computationally intensive, and also identified a positive impact of trade policies on the terms of trade and the spread of the disease. Their model is elegant, but both theoretically and computationally significantly simpler than the one developed in this paper. Later, Xie, Wang and Liu (2021) have used a two-country model similar to our earlier one in order to study tourism and travel restrictions. They restricted attention to partial equilibrium analysis, ignored the terms-of-trade effects that are central to our theory, and focus on specific policies such as border closures and travel bubbles. Also noteworthy is the early work generalizing ERT to multiple sectors by Krueger, Uhlig and Xie (2022). That paper also provides interesting discussions about consumption substitution patterns in theory and practice, but does not consider specific policy instruments, as we do, nor international trade.

In related, but more specialized work, Leibovici and Santacreu (2023) study the role of international trade in essential goods following global shocks in an open economy, multi-sector model of one country and analyze how the government can use industrial policy and trade policy in reaction to global shocks. Bonadio et al. (2021) and Yildirim et al. (2021) examine the role of global supply chains' impact on GDP growth across countries, while Meier and Pinto (2020) study the specific disruption of China-US supply chains and its impact on US production in March/April 2020 in detail.

Our paper owes much to the literature on trade wars and negotiations in international trade (Brander and Spencer, 1985; Perroni and Whalley, 2000; Broda, Limao and Weinstein, 2008; Ossa, 2011). Most closely related are Bagwell and Staiger (1999), which analyses a tractable static general equilibrium model with governments that non-cooperatively set tariffs to maximize different forms of national welfare in Nash equilibrium, and Ossa (2014), which quantitatively studies optimal tariffs that arise during a trade war and quantifies the costs of failures of coordination on trade policy. We add a temporal dimension to this work and study how international trade policy interacts with the global propagation of a pandemic. Our model generates many of the features present in these models of trade wars, while highlighting the novel interaction between trade wars, health outcomes, and international coordination of policies.[3]

1 The Model

We develop and study a two-country international trade model which embeds an epidemiological model of disease dynamics. The model has four key ingredients. First, households in each country have preferences for the consumption of manufactured goods and services produced in both countries. Second, the disease can be transmitted during the production of manufactured goods and services. Third, the consumption of services is contact-intensive and also facilitates disease transmission, both domestically and internationally across countries. The consumption of manufactured goods is not contact-intensive and avoids infections. Fourth, governments in each country can impose containment policies in the form of distortionary taxes on different forms of consumption and separately tariffs or subsidies on international consumption.

Specifically, we consider a global economy with two countries, $k = A, B$. Each country has households, competitive firms that either produce manufactured goods or services, and a government. Time is discrete ($t = 0, 1, 2, ...$). For all variables we use the following notational convention. Variables describing consumption, production, or government activity in country

[3] At a conceptual level, our paper connects to a recent and growing literature on the broader theme of international coordination in open economies. For example, Auray, Devereux and Eyquem (2022) consider protectionism and how its evolution under non-cooperative behavior depends on the exchange rate regime and price rigidity. Auray, Devereux and Eyquem (2024) and Egorov, Mukhin et al. (2019) study the strategic (non-) cooperation of governments in trade and monetary policy, when terms of trade and discretionary monetary policy interact in a world with potential trade wars and sticky prices.

$k \in \{A, B\}$ have the superscript k. When discussing a single country, the superscript $-k$ denotes the other country. The two goods in each country are labelled m (manufactured) and n (non-manufactured or services). For consumption, the subscript $(j, k) \in \{m, n\} \times \{A, B\}$ denotes good j from country k.

The households in each country are defined over a continuum of unit mass (we do not distinguish between individuals and households). They consume all four goods, but supply labor to firms that produce exactly one of them and never change their occupation, nor retire. Let μ_t^k denote the fraction of agents working in manufacturing in country k (with $1 - \mu_t^k$ working in services). Let S_t^{kj}, I_t^{kj}, R_t^{kj}, and D_t^{kj} denote the mass of susceptible, infected, recovered and deceased people in sector j in country k at time t. The total population of country k working in sector j at date t then is $N_t^{kj} = S_t^{kj} + I_t^{kj} + R_t^{kj}$. Individuals are infinitely lived except for deaths from the disease. Households within each of the three living categories are identical except for their occupation. S_t^{-kj}, I_t^{-kj}, R_t^{-kj}, and D_t^{-kj} are the masses of the respective groups in the other country, if we discuss activity in one country k. $h \in \{s, i, r\}$ indicates the three health types of living households.

1.1 Firms and Households

The two countries produce goods and services $j \in \{m, n\}$ with different specificity. Good (j, k) is produced by firms in sector j using country k labor only, according to the linear technology

$$y_t^{kj} = z^{kj} \left(\ell_t^{kjs} + \phi \ell_t^{kji} + \ell_t^{kjr} \right) \tag{1}$$

where ℓ_t^{kjh} is the amount of labor provided by employees of health status h in sector j, and z^{kj} is country k's productivity in sector j. In our baseline model, we assume constant and identical productivity across countries, i.e., $z_t^{kj} = z^j$. Infected individuals ($h = i$) have lower productivity, as given by ϕz_t^{kj} with $\phi < 1$. Firms are single-period lived, and act competitively, maximizing profits and taking prices as given.

Suppressing the time index for simplicity, the price of good j produced in country k is $p_{j,k}$, $k = A, B$ and $j = m, n$. There are no transport costs or other exogenous physical trade frictions between countries.

Households in each country provide labor and consume a basket of goods and services from countries A and B. Again suppressing the time index, denote the per household consumption of good j' from country κ by households of health status h employed in sector j in country k by $c_{j',\kappa}^{kjh}$. Households in each country consume domestic and foreign goods and services as a basket whose composition is given by the standard constant-elasticity-of-substitution (CES) aggregator

$$\bar{c}_{j'}^{kjh} = q(c_{j',k}^{kjh}, c_{j',-k}^{kjh}) = \left(\alpha(c_{j',k}^{kjh})^{\frac{\sigma_{j'}-1}{\sigma_{j'}}} + (1-\alpha)(c_{j',-k}^{kjh})^{\frac{\sigma_{j'}-1}{\sigma_{j'}}}\right)^{\frac{\sigma_{j'}}{\sigma_{j'}-1}} \tag{2}$$

for the consumption of manufactured goods (if $j' = m$) or services (if $j' = n$) by household type h working in sector j in country k. $\alpha \in (0.5, 1)$ is the home bias for domestic consumption goods or services, and $\sigma_j > 1$ the substitution elasticity between the domestic and the foreign good of type j. These parameters are identical in both countries in order to focus on the pure effects of disease transmission in international trade.[4]

At each time t, the households in any of the two countries have the following objective function, where we suppress notation for the household's home country, occupation, and health status to simplify the presentation:

$$U_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[v(x_\tau) - \frac{1}{2}\varphi \ell_\tau^2 \right], \tag{3}$$

where $0 < \beta < 1$ is the discount rate, $\varphi > 0$ the disutility of labor, ℓ_τ the labor supplied, and

$$x_\tau = (\bar{c}_{m,\tau})^\chi (\bar{c}_{n,\tau})^{1-\chi} \tag{4}$$

is a measure of the household's consumption-weighted basket, with $\chi \in (0, 1)$ denoting the relative preference for manufactured goods over services. The expectations operator in (3) refers to the individual uncertainty generated by infections and will be defined below.

The utility of consumption is of the constant-relative-risk-aversion type (with $\rho = 1$ corresponding to the log-utility case):

$$v'(x) = x^{-\rho}, \rho > 0. \tag{5}$$

Note that the households' optimal consumption decision in (4) will depend on its sector of employment, because its labor supply depends on the sector. We therefore have

$$\ell_\tau = \ell_\tau^{kjh}, \quad x_\tau = x_\tau^{kjh}.$$

In each country k, we denote aggregate consumption of the domestic good or service $j =$

[4]The symmetry assumption can be dispensed with. The most interesting feature of the asymmetric model is the possibility of multiple infection waves along the logic described by Antràs, Redding and Rossi-Hansberg (2023): if the wave in country A is naturally short and weak and that of country B strong, then this may lead to a second wave in country A.

m, n (“home consumption") by

$$H_{j,t}^{k} = S_t^{km} c_{j,k,t}^{kms} + I_t^{km} c_{j,k,t}^{kmi} + R_t^{km} c_{j,k,t}^{kmr} + S_t^{kn} c_{j,k,t}^{kns} + I_t^{kn} c_{j,k,t}^{kni} + R_t^{kn} c_{j,k,t}^{knr}, \tag{6}$$

and that of the foreign good j (“imports") by

$$F_{j,t}^{k} = S_t^{km} c_{j,-k,t}^{kms} + I_t^{km} c_{j,-k,t}^{kmi} + R_t^{km} c_{j,-k,t}^{kmr} + S_t^{kn} c_{j,-k,t}^{kns} + I_t^{kn} c_{j,-k,t}^{kni} + R_t^{kn} c_{j,-k,t}^{knr}. \tag{7}$$

Hence, in equilibrium the exports of good j by country k are $F_{j,t}^{-k}$.

1.2 Microfoundations of Disease Dynamics

Like Eichenbaum, Rebelo and Trabandt (2021), Brotherhood et al. (2020) and other recent contributions, we augment the classic SIR model by economic activity. Different from these contributions, we include not only domestic economic interactions, but also interactions due to international trade. In the basic SIR model following Kermack and McKendrick (1932), an infectious individual in any given area can spread the virus at the rate ηS_t (so-called “mass action incidence"), where S_t is the number of susceptibles in that area. Hence, the mass of newly infected people in that area at time t is given by $T_t = \eta S_t I_t$. Eichenbaum, Rebelo and Trabandt (2021) generalize this to transmission through consumption and work activities in a single country by splitting the individual's transmission rate ηS_t into three components:

$$T_t = \left[\pi_1 c_t(s) c_t(i) + \pi_2 \ell_t(s) \ell_t(i) + \pi_3\right] S_t I_t, \tag{8}$$

where $c_t(h)$ and $\ell_t(h)$ are respectively the representative consumer's consumption and labor. We differentiate between service goods and manufactured goods, and add an international trade channel to this transmission mechanism, taking into account that the consumption of imports leads to cross-border contacts that are potentially contagious. Typical examples of such contagious imports of country k would be the delivery and installation of manufactured goods and equipment in k by producers from country $-k$, or services, e.g., to tourists from country k in $-k$, or in consulting by firms from $-k$ to k. The trade of services is typically more contagious than that of manufactured goods. To make this point simply, we assume that transmission by consumption, nationally or internationally, only occurs through services.

Our transmission channel builds on the following generalization of the original SIR-type models, which we describe in more detail in Section A.5 in the Appendix. Dropping the time index for convenience, suppose individuals of country k employed in sector j and of health status h spend a fraction ℓ^{kjh} of their time at work, a fraction $\gamma c_{n,k}^{kjh}$ of their time consuming domestic services, a fraction $\gamma c_{n,-k}^{kjh}$ consuming foreign services, and a fraction

f out of their home for other reasons, neither consuming nor working. The assumption is that the time spent consuming services is proportional to the quantity consumed. Let η denote the probability of infection through contacts per unit of time spent on a given activity.[5] When consuming services, an individual is exposed to domestic residents and foreigners. Suppose there are I^{kj} infected domestic individuals and I^{-kj} infected foreigners in each sector j. Since the contact intensity for foreign and domestic consumption is likely to differ, let η^f and η^d denote the corresponding infection probabilities, respectively.[6] Then the probability of getting infected by domestic residents, per unit of time, from consuming domestic services is approximately $\eta^d\gamma\left(c_{n,k}^{kmi}I^{km}+c_{n,k}^{kni}I^{kn}\right)$ and that from consuming foreign services $\eta^d\gamma\left(c_{n,-k}^{kmi}I^{km}+c_{n,-k}^{kni}I^{kn}\right)$.[7] Similarly, the probability of getting infected by foreigners, per unit of time, from consuming domestic services is $\eta^f\gamma\left(c_{n,k}^{-kmi}I^{-km}+c_{n,k}^{-kni}I^{-kn}\right)$, and that from consuming foreign services $\eta^f\gamma\left(c_{n,-k}^{-kmi}I^{-km}+c_{n,-k}^{-kni}I^{-kn}\right)$. Hence, when consuming the services bundle $(c_{n,k}^{kjs}, c_{n,-k}^{kjs})$, the representative susceptible consumer in country k faces the probability of infection

$$\left[\left(c_{n,k}^{kjs}c_{n,k}^{kmi}+c_{n,-k}^{kjs}c_{n,-k}^{kmi}\right)I^{km}+\left(c_{n,k}^{kjs}c_{n,k}^{kni}+c_{n,-k}^{kjs}c_{n,-k}^{kni}\right)I^{kn}\right]\gamma^2\eta^d$$

from domestic residents, and

$$\left[\left(c_{n,k}^{kjs}c_{n,k}^{-kmi}+c_{n,-k}^{kjs}c_{n,-k}^{-kmi}\right)I^{-km}+\left(c_{n,k}^{kjs}c_{n,k}^{-kni}+c_{n,-k}^{kjs}c_{n,-k}^{-kni}\right)I^{-kn}\right]\gamma^2\eta^f$$

from foreigners.

We assume for simplicity that there are no international encounters in non-work-non-consumption situations, and we also ignore those at the workplace. In particular, the infection risk for susceptible individuals from working ℓ^{kjs} hours is $\eta^d\ell^{kjs}\ell^{kji}I^{kj}$ for $j=m,n$, and the background risk from non-work-non-consumption activity is $\eta^d f^2(I^{km}+I^{kn})$, both independent of foreign infections.

Hence, a susceptible individual occupied in sector j in country k who chooses ℓ^{kjs}, $c_{n,k}^{kjs}$,

[5] This is approximately equal to the contact rate (say φ) times the transmission probability per unit of time (say θ). Both these parameters depend on individual behavior and policy, but for tractability we take both as given. What matters for transmission is $\varphi\theta t_c$, where t_c is the duration of contacts. We model policy as influencing t_c.

[6] The difference between these two infection probabilities is related to, but different from, the difference between different types of services. An important component of services is tourism, which constitutes a large part of international trade in several countries (see, e.g., Culiuc (2014)). This type of import is particularly foreign contact intensive. On the other hand, imports of so-called *mode-3-services* (commercial presence) involve much less contact with foreigners, in particular in the age of Zoom.

[7] The true probability is a polynomial that can be well approximated around zero. See the Appendix for a more detailed derivation.

and $c^{kjs}_{n,-k}$ transits to the infectious state with (approximate) probability

$$\begin{aligned} & \tau(c^{kjs}_{n,k}, c^{kjs}_{n,-k}, \ell^{kjs}) \\ = \; & \pi_1 \left[\left(c^{kjs}_{n,k} c^{kmi}_{n,k} + c^{kjs}_{n,-k} c^{kmi}_{n,-k} \right) I^{km} + \left(c^{kjs}_{n,k} c^{kni}_{n,k} + c^{kjs}_{n,-k} c^{kni}_{n,-k} \right) I^{kn} \right] \\ + \; & \pi_2 \ell^{kjs} \ell^{kji} I^{kj} + \pi_3 (I^{km} + I^{kn}) \\ + \; & \pi_4 \left[\left(c^{kjs}_{n,k} c^{-kmi}_{n,k} + c^{kjs}_{n,-k} c^{-kmi}_{n,-k} \right) I^{-km} + \left(c^{kjs}_{n,k} c^{-kni}_{n,k} + c^{kjs}_{n,-k} c^{-kni}_{n,-k} \right) I^{-kn} \right] \end{aligned} \tag{9}$$

where $\pi_1 = \gamma^2 \eta^d$, $\pi_2 = \eta^d$, $\pi_3 = f^2 \eta^d$, and $\pi_4 = \gamma^2 \eta^f$.

In this equation, the π_1 term captures the infection risk from consuming service goods together with other domestic households, the π_2 term the risk from working, the π_3 term the background risk, and π_4 term the risk from consuming service goods together with foreigners, highlighting our assumption that infection probabilities do not depend on the consumption of manufactured goods. Therefore, as in (8), the first three terms on the right-hand side of (9) capture infections from domestic contacts arising during consumption, work, and all other local activity, respectively, and the fourth term describes infections arising from contacts with foreigners while importing or exporting. This is the international disease transmission mechanism at the heart of our analysis, of which the model of (8) is a special case, with only one consumption good per country and a single country.

As is standard in the literature, at the individual level, the remaining infection dynamics are simple. In each period, infected individuals recover with a constant probability p_r and die with probability p_d. As is commonly assumed in much of the applied epidemiological literature, we assume that recovered individuals remain in that category for sure (i.e., acquire at least temporary immunity).

By the Law of Large Numbers, these individual infection probabilities yield the following number of new infections in sector j in country k at date $t+1$:

$$T^{kj}_t = \tau(c^{kjs}_{n,k,t}, c^{kjs}_{n,-k,t}, \ell^{kjs}_t) S^{kj}_t \tag{10}$$

As in standard epidemiological models, the evolution of the transmission in any country is now given by

$$\begin{aligned} \text{susceptible:} \quad S^{kj}_{t+1} &= S^{kj}_t - T^{kj}_t, & (11) \\ \text{infected:} \quad I^{kj}_{t+1} &= I^{kj}_t + T^{kj}_t - (p_r + p_d(I^k_t)) I^{kj}_t, & (12) \\ \text{recovered:} \quad R^{kj}_{t+1} &= R^{kj}_t + p_r I^{kj}_t, & (13) \\ \text{deceased:} \quad D^{kj}_{t+1} &= D^{kj}_t + p_d(I^k_t) I^{kj}_t. & (14) \end{aligned}$$

To capture the potential crowding out of medical resources, we allow the transition probability p_d to be a function of the population currently infected $I_t^k = I_t^{km} + I_t^{kn}$.[8] In order to keep the computational complexity as low as possible, we assume that the death rate is a linear (affine) function of the infection rate: $p_d(I_t^k) = p_d(0) + \zeta I_t^k$, where $\zeta \geq 0$ measures the fragility of the national health system under intensive care pressure.

Note that the system (11)–(14) is deterministic, and the overall populations and sub-populations, $N_t^x = S_t^x + I_t^x + R_t^x$, decrease by $p_d(I_t^x)I_t^x$ each period. We normalize the total initial population in each country to $N_1^k = 1$.

We denote the current state of the disease by

$$\Theta_t = \left(S_t^{kj}, I_t^{kj}, R_t^{kj}\right)_{k=A,B,j=m,n} \tag{15}$$

and consider a situation in which initially, $S_1^A = 1 - \varepsilon$, $I_1^A = \varepsilon$, $R_1^A = 0$, and $S_1^B = 1$, $I_1^B = R_1^B = 0$, where $\varepsilon > 0$ is a small number. Hence, the pandemic begins with a small number of infections in country A and then spreads endogenously to country B.

1.3 The Role of Government

In each country, the government can impose measures to contain the spread of the pandemic. We follow the approach taken by Eichenbaum, Rebelo and Trabandt (2021) and assume that these measures act like ad valorem "containment taxes" $\xi_{j,t}^k \geq 0$. This means that households in country k have to pay an extra $\xi_{j,t}^k p_{j,\kappa,t}$ per unit of consumption of good $j = m, n$, $\kappa = k, -k$, regardless of its origin. This containment tax is collected by the government, and will be rebated back to the local households. We thus abstract from more complex containment policies, with quantity limits, non-linear price effects or other type-specific restrictions, in order to keep the model tractable.[9]

In addition, governments can intervene in the market for foreign goods and services. Formally, they do this by imposing proportional "import tariffs" $\nu_{j,t}^k \in \mathbb{R}$, incurred over and above the general domestic containment taxes $\xi_{j,t}^k$. If $\nu_{j,t}^k < 0$ this intervention involves an import subsidy. Again, these "import tariffs" are monetary and generate revenues for the government

[8] The role of such "congestion externalities" has been emphasized and modelled in the work on optimal containment policies, e.g. by Brotherhood et al. (2020), Kaplan, Moll and Violante (2020), Favero (2020), and Assenza et al. (2020).

[9] A number of papers have investigated different containment policies in less complex macro models, such as Berger, Herkenhoff and Mongey (2022) on the role of testing and case-dependent quarantine, Alon et al. (2020) on age-specific lockdown policies among sets of developing and advanced economies, and Jones, Philippon and Venkateswaran (2021) on work-from-home-policies. Early empirical work comparing pandemic policies internationally includes Ullah and Ajala (2020), who analyze effects of testing and lockdown in 69 countries, and Noy et al. (2020) who estimate measures of exposure, vulnerability and resilience to Covid-19 across countries. As we discuss in the conclusion, integrating such non-linear policies and disaggregated effects into our multi-sectoral trade model is an interesting and important avenue for future research.

(or require payments if negative).[10]

In any of the two countries $k = A, B$, households then have to pay $(1 + \xi^k_{j,t})p_{j,k,t}$ per unit of consumption of the domestic goods or services $j = m, n$, and $(1 + \xi^k_{j,t} + \nu^k_{j,t})p_{j,-k,t}$ per unit of consumption of the foreign goods or services. For each country k, we can thus simplify notation by defining the "consumer prices" as

$$\widehat{p}^k_{j,k,t} = (1 + \xi^k_{j,t})p_{j,k,t} \quad (16)$$

$$\widehat{p}^k_{j,-k,t} = (1 + \xi^k_{j,t} + \nu^k_{j,t})p_{j,-k,t} \quad (17)$$

for the domestic and foreign goods or services, respectively.

The government's budget constraint is therefore

$$(1 - D^k_t)g^k_t = G^k_t = \sum_{j=m,n} \left[\xi^k_{j,t}p_{j,k,t}H^k_{j,t} + (\xi^k_{j,t} + \nu^k_{j,t})p_{j,-k,t}F^k_{j,t}\right], \quad (18)$$

where g^k_t the per household government transfer to households, and $(1 - D^k_t)$ is the size of the population at time t, determined by the disease dynamics, and $H^k_{j,t}$ and $F^k_{j,t}$ denote respectively (as introduced before) aggregate consumption of domestic and foreign good of type j in country k.

Government policy therefore consists in setting the domestic containment policy ξ^k that controls overall consumption and the import policy ν^k that controls trade and the resulting import of infections. Once these policies are fixed, government spending g^k_t is given by the government budget constraint (18).

The Covid-19 pandemic has shown that import controls can be particularly important, even if contentious, especially with respect to international services, such as travel and tourism. Although import barriers can have a monetary benefit from tariffs, most of their strategic importance in a pandemic is broader, by balancing the consumption benefits of infectious goods or services against their health costs between countries and sectors. Next to direct monetary benefits, optimal import policies must achieve a number of partially conflicting goals. First, as usual, positive import barriers manipulate the terms of trade in favor of domestic goods and thus higher domestic labor income at the expense of lower domestic consumption choice. Second, import restrictions on services reduce infections resulting from foreign contacts, thus directly creating a beggar-thy-neighbor situation. Third, differential import frictions between

[10] Government policy is thus given by four linear distortionary instruments that are modelled as taxes or tariffs, capturing different aspects of non-pharmaceutical interventions. This is for reasons of tractability. Government interventions in national emergencies are neither necessarily monetary nor linear. However, given the complexity of the model, these instruments capture important dimensions of such interventions, making them comparable while keeping the theoretical and numerical analysis tractable. The four instruments should therefore not be taken as litteral descriptions of actual policy, but rather as first-order approximations of the shadow prices imposed by such policies.

manufactured goods and services from abroad influence the composition of domestic consumption and therefore also infections from domestic labor provision. And fourth, import frictions (or subsidies) can be used to influence the infection dynamics by shifting production internationally to where infection rates are lower.

Since the international infection dynamic (10) is deterministic, the interaction between the two governments is an infinite-horizon, deterministic, multi-stage game with observed actions (see Fudenberg and Tirole, 1991). In a single-agent framework, conditioning on the state of nature (here: the aggregate infection state) would therefore not be necessary, and every open-loop optimal path can be implemented by closed-loop strategies (i.e., strategies that depend on time t and the state) and vice versa. In a multi-agent framework, on the other hand, conditioning on the state of play (i.e., considering subgame-perfect equilibria, SPE) usually reduces the set of equilibria, by ruling out non-credible promises. Here, for computational reasons, we directly consider open-loop strategies, i.e., strategies that only depend on time t and not on the state. Hence, governments set their policy path initially once and for all.

While an important reason for this assumption is indeed the complexity of the computations and the theoretical model, there are at least two reasons why we believe that it is appropriate. First, it is not clear that countries do have a reason to deviate from the agreed course of equilibrium play (in the sense that Nash would not be credible). In fact, the country in our model that has the strongest incentives to do so for health reasons (country A after it has got over its infection wave) has little reason to do so for economic reasons: as we discuss in Section 4, during B's infection peak, in Nash equilibrium A already charges very high tariffs.

Second, since SPEs are Nash equilibria, the issue would be moot if our open-loop equilibrium were unique. Uniqueness of equilibrium is, of course, difficult to prove. But we have conducted extensive computational searches for other equilibria from different starting values, and always found the single Nash equilibrium reported in Section 4. Hence, there is computational evidence, even if it is not theoretically conclusive.

More interesting is the question how the cooperative outcome may actuallly be achieved, i.e. why it is of more than simple benchmarking interest. In general, there are two types of reasons why this may be the case. First, the participants may conclude an explicit agreement with a full description of policies and enforceable sanctions. Second, the pandemic episode modelled here may be viewed as one stage in a longer time horizon. In fact, while our work (as the whole literature) features only one pandemic – which means that any reciprocating agreement for a pandemic is only "one shot" in the longer horizon – it is increasingly likely that pandemics will occur again and that global policies are needed to address such recurring episodes. In this perspective, the Nash equilibrium discussed above can be a threat point in a tit-for-tat equilibrium that implements the cooperative outcome in each stage game. As we have noted in the introduction to this paper, the WTO is currently preparing a "pandemic agreement", which specifies policies deemed necessary for future international cooperation in pandemics. Combined with the stage-game Nash outcome in a tit-for-tat long-run equilibrium, such a pandemic agreement may offer a more effective mechanism to achieve cooperation than an explicit international treaty with sanctions and other dispute resolution provisions.[11]

To further simplify the computation, we assume that a vaccine or other cure is known to exist in a fixed, finite time T in the future. Hence, governments effectively work with a fixed time horizon, since after date T there are no more infections and the economies operate without any SIR-dynamics. As households are only concerned with current infections, the prospect of a vaccine in the future does not affect their current behavior.[12]

Returning to the households, in order to simplify the dynamics, we again follow Eichenbaum, Rebelo and Trabandt (2021), Brotherhood et al. (2020) and others, by assuming that households do not save or borrow. Hence, the only intertemporal link of household decisions is given by health concerns, and the budget constraint of a household of type h employed in sector j in country k at time t is static and given by

$$\sum_{j'=m,n} \left[\widehat{p}^{k}_{j',k,t} c^{kjh}_{j',k,t} + \widehat{p}^{k}_{j',-k,t} c^{kjh}_{j',-k,t}\right] = w^{kj}_t \ell^{kjh}_t + g^k_t + \sum_{j'} v^{kj'}_t \tag{19}$$

where w^{kj}_t is the domestic wage in sector j, and v^{kj}_t the per household profit of sector j in the country.[13] Furthermore, as discussed, households maximize their expected discounted utility, given government policy and the evolution of the disease. While the aggregate evolution is deterministic and known by assumption, the individual evolution is stochastic, and the expectation is formed with respect to the individual infection probability τ in (9) for susceptible households or the transition probabilities p_r, p_d for infected households. Let

$$V^{kjh}_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[v(x^{kjh_\tau}_\tau) - \frac{1}{2}\varphi(\ell^{kjh_\tau}_\tau)^2 \right] \tag{20}$$

denote the value function of households of health status h employed in sector j in country k at

[11] We are grateful to an anonymous referee for raising this issue.

[12] As our simulations of the infection dynamics in Section 4 show, this assumption does not influence aggregate variables and policy, either, which show the usual front-loading of policies by foreward-looking governments.

[13] Note that for simplicity we assume that neither government nor employers can condition their payments on the household's health/employment status. Determining optimal public transfer schemes is beyond the scope of the current paper and therefore simplifications are unavoidable – simply putting a health-occupation superscript on g^k_t, for example, may not be enough, because these schemes may depend on the aggregate state of the economy. But already making our lump sum transfer schemes type-dependent would increase the number of instruments of each government per period from 4 to 10. Hence, adding this high-dimensional optimization problem for the government to our already complex overall optimization problem would increase the dimension of the strategy space from the current 2 080 to 5 200 and risk making the computational problem intractable. For the same reason, we also do not consider direct transfer payments between governments. See also Alvarez, Argente and Lippi (2021) for a discussion of the difficulties of designing household-specific rules or transfers in an optimal planning model.

time t when making optimal consumption choices $x_\tau^{kjh_\tau}$ and labor choices $\ell_\tau^{kjh_\tau}$. Here h_τ is the household's health status at time τ along its optimal consumption-labor path. By symmetry, we assume that the government of country k maximizes the utilitarian welfare function

$$V^k = \mu_1^k \left[S_1^{km} V_1^{kms} + I_1^{km} V_1^{kmi}\right] + (1 - \mu_1^k) \left[S_1^{kn} V_1^{kns} + I_1^{kn} V_1^{kni}\right] \tag{21}$$

which sums up the expected utilities of all households living in country k at date 1. We assume that the sectoral share is $\mu_1^k = 1/2$. Then, the two sets of international health and trade policies we consider are determined as follows.

Uncoordinated Policy: Without coordination, the two governments play a non-cooperative game, where each country k chooses open-loop policy paths as described, so as to

$$\max_{\{\xi_t^k, \nu_t^k\}_t} V^k$$

taking the other government's policy path $\{\xi_t^{-k}, \nu_t^{-k}\}_t$ as given. A Nash equilibrium consists of two policy paths such that each is a best response to the other.

Coordinated Policy: Alternatively, we consider the benchmark of a single social planner who makes the policy decisions for both countries in order to maximize the sum of the two countries' welfare:

$$\max_{\{\xi_t^A, \nu_t^A, \xi_t^B, \nu_t^B\}_t} V^A + V^B \tag{22}$$

2 Equilibrium Analysis

We now turn to the characterization of equilibrium. Our focus is on understanding how international risk-sharing of health and economic well-being is affected by uncoordinated versus coordinated government policy.[14] Given government policy $\{\xi_t^k\}, \{\nu_t^k\}$, and $\{g_t^k\}$ in each country, firms maximize profits and households expected utility, taking prices and the economic and epidemiological constraints as given.

[14] It is worth repeating that risk-sharing in this context refers to individual risk. Once national policies are determined, the disease in our model runs its course deterministically, with aggregate transmissions determined by the Law of Large Numbers. Government policies, however, influence the laws of motion of the domestic transmissions and can shift aggregate infection rates internationally. This changes infection risks for the individuals in each country.

2.1 Firm behavior

Because of the constant-returns-to-scale structure (1), firms make zero profits in equilibrium and hire as much labor as is supplied by households. Hence, in equilibrium, aggregate output in sector j in country k is

$$Y_t^{kj} = z_t^j \left(S_t^{kj} \ell_t^{kjs} + \phi I_t^{kj} \ell_t^{kji} + R_t^{kj} \ell_t^{kjr} \right), \tag{23}$$

wages are $w_t^{kj} = p_{j,k,t} z_t^j$ for susceptible and recovered households, and firm profits $v_t = 0$.[15]

2.2 Household behavior

Households of each country k working in sector j at each date t maximize expected utility U_t given by (3) subject to the budget constraint (19), by choosing their domestic consumption bundle $(c_{m,k,t}^{kjh}, c_{n,k,t}^{kjh})$, the foreign consumption bundle $(c_{m,-k,t}^{kjh}, c_{n,-k,t}^{kjh})$, and labor ℓ_t^{kjh}, where $h = s, i, r$ is the household's health status. They know their own health status,[16] and the current state of the disease Θ_t, given by (15). Using (20) and dropping sub- and superscripts, in recursive terms, households choose current labor and consumption to maximize

$$v(x_t) - \frac{1}{2}\varphi \ell_t^2 + \beta \mathbb{E}_t[V_{t+1}(h_{t+1}; \Theta_{t+1})] \tag{24}$$

where the expectation operator is applied over the distribution of personal health status h_{t+1} next period.

Susceptible Households. For a susceptible individual there are two possible future health states - either she remains in s or she gets infected and transits to i. By (9), there are four possibilities to get infected. First, she may get infected while consuming local services (shopping, eating out, etc.). This probability is increasing with her own time spent on that activity and the total time that infected domestic or foreign individuals spend in that activity. This corresponds to the first and the third summand of the π_1-term and of the π_4-term in (9), respectively. Second, she may get infected at work with a similar logic, which corresponds to the π_2-term. Third, she may get infected in general encounters with infected people locally, not related to consumption or work, summarized by the π_3-term. Fourth, she may get infected during the consumption of foreign services (such as holidays abroad), which is summarized by the second and fourth summand of the π_1- and of the π_4-term. While the three types of interactions summarized by the π_1 to π_3 terms refer to infections from domestic households, the fourth,

[15] Remember that wages are sector-specific, but cannot be contingent of an individual's health status.

[16] Hence, we ignore the problem of asymptomatic or presymptomatic infections, which was of concern with Covid-19. See, for example, von Thadden (2020) for a detailed discussion.

summarized in the π_4 term, explicitly highlights the consumption risk from interaction with foreigners.

At time t, the value function of a susceptible individual working in sector j in country k therefore is

$$V_t^{kjs} = \max v(x_t^{kjs}) - \frac{1}{2}\varphi \left(\ell_t^{kjs}\right)^2 + \beta \left[\tau_t^{kjs} V_{t+1}^{kji} + (1 - \tau_t^{kjs}) V_{t+1}^{kjs}\right], \tag{25}$$

subject to the budget constraint (19), where the maximization is over $(c_{m,k,t}^{kjs}, c_{m,-k,t}^{kjs}, c_{n,k,t}^{kjs}, c_{n,-k,t}^{kjs}, \ell_t^{kjs}) \in \mathbb{R}_+^5$, x_t^{kjs} is given by (4), and τ_t^{kjs} is the household's infection probability discussed above, which, we assume, it takes into account. In doing so, the household takes as given the behavior of infected households, summarized in the 9 variables $c_{n,k}^{kmi}$, $c_{n,-k}^{kmi}$, $c_{n,k}^{kni}$, $c_{n,-k}^{kni}$, $c_{n,k}^{-kmi}$, $c_{n,-k}^{-kmi}$, $c_{n,k}^{-kni}$, $c_{n,-k}^{-kni}$, ℓ^{kji} that enter (9). We derive the solution to this problem in the appendix.

Infected Households. The behavior of infected households is simpler. Their behavior has no consequences for their future health, which is exogenously given by either recovery, with probability p_r, or death, with probability p_d. At time t an infected individual working in sector j in country k therefore chooses $(c_{m,k,t}^{kji}, c_{m,-k,t}^{kji}, c_{n,k,t}^{kji}, c_{n,-k,t}^{kji}, \ell_t^{kji}) \in \mathbb{R}_+^5$ so as to optimize the static decision problem

$$V_t^{kji} = \max v(x_t^{kji}) - \frac{1}{2}\varphi \left(\ell_t^{kji}\right)^2 + \beta \left[(1 - p_r - p_d) V_{t+1}^{kji} + p_r V_{t+1}^{kjr} + p_d V_{t+1}^{kjd}\right], \tag{26}$$

where the last term reflects the possibilities that the household remains infected, recovers, or dies. Via p_d, V_t^{kji} depends on the aggregate domestic pandemic state. This problem can be solved explicitly under our assumption of log preferences, as we show in the appendix.

Recovered Households. Similarly, when recovered, a type r household at time t chooses $(c_{m,k,t}^{kjr}, c_{m,-k,t}^{kjr}, c_{n,k,t}^{kjr}, c_{n,-k,t}^{kjr}, \ell_t^{kjr}) \in \mathbb{R}_+^5$ so as to optimize the static decision problem

$$V_t^{kjr} = \max v(x_t^{kjr}) - \frac{1}{2}\varphi \left(\ell_t^{kjr}\right)^2 + \beta V_{t+1}^{kjr}. \tag{27}$$

We also provide a solution to this problem in the appendix, which is the simplest case as recovered households no longer face any health risks (e.g., no “long COVID").

2.3 The macroeconomic synthesis

Each period, the following endogenous economic variables are determined in equilibrium:

- Households: 60 decisions $c_{j',\kappa,t}^{kjh}$, ℓ_t^{kjh}, for $k, \kappa \in \{A, B\}$, $j, j' \in \{m, n\}$, and $h = s, i, r$.

- Markets: 8 variables $p_{j,k,t}$, w_t^{kj} for $k \in \{A, B\}$, $j \in \{m, n\}$ where prices, consumer prices, and government policy are linked by (16)–(17).
- Government expenditures: 2 variables g_t^k, $k \in \{A, B\}$. In the absence of health dependent transfers g_t^h, fiscal policy is reduced to the balanced-budget rule (18).

As argued above, given the linear production technologies, the firm variables follow automatically from the household decisions.

Each period, the governments or the common social planner set the epidemiological and trade policy consisting of the 4 parameters $\xi_{j,t}^k$, $\nu_{j,t}^k$, $k \in \{A, B\}$, $j \in \{m, n\}$, for each country, which are exogenous from the point of view of market participants. These variables are implicit in the consumer prices $\widehat{p}_{j,k,t}^k, \widehat{p}_{j,-k,t}^k$.

Counting equations, we have for

- Households: in each country, there are 5 equations for each occupation-health (j, h) household type, for a total of 2 x 30 = 60 equations.
- Labor markets: 4 market clearing equations in 2 sectors and 2 countries.
- Goods markets: Each country produces two goods that are consumed domestically or exported, yielding 4 market clearing equations

$$Y_t^{kj} = H_{j,t}^k + F_{j,t}^{-k} \tag{28}$$

for $k \in \{A, B\}$ and $j \in \{m, n\}$, where output Y_t^{kj} is given by (23), domestic consumption $H_{j,t}^k$ by (6) and exports $F_{j,t}^{-k}$ by (7).

There are 12 value functions to be solved, V_t^{kjh}. As usual, we normalize the value functions $V_t^{kjd} = 0$, assuming that the cost of death is the lost utility of life.

In terms of aggregate variables, let us denote the total amount of labor provided in sector j of country k by

$$L_t^{kj} = S_t^{kj} \ell_t^{kjs} + I_t^{kj} \ell_t^{kji} + R_t^{kj} \ell_t^{kjr}. \tag{29}$$

As a simple measure of aggregate consumption for each country, we use the Cobb-Douglas aggregator (4) and define the aggregate consumption index for sector j of country k as

$$X_t^{kj} = S_t^{kj} x_t^{kjs} + I_t^{kj} x_t^{kji} + R_t^{kj} x_t^{kjr}. \tag{30}$$

The X_t^{kj} are preference-based aggregators of consumption quantities that are linearly homogenous (i.e., an increase of all consumption quantities by x percent increases X_t^{kj} by x percent). We therefore use the X_t^{kj} as indices of aggregate consumption. Note that L_t^{kj} and X_t^{kj} are sectoral variables, referring to the sector of employment.

In order to measure aggregate consumption of different goods, one can use pure quantity measures (which restricts the scope to one single good), or expenditures (which allows to aggregate several goods, but cannot distinguish between equilibrium quantity and price changes). A natural measure for consumption quantities, using (6) and (7), is the share of home consumption in total consumption of good j for country k,

$$C_{j,t}^{k} = \frac{H_{j,t}^{k}}{H_{j,t}^{k} + F_{j,t}^{-k}}\ , \tag{31}$$

where $C_{j,t}^{k}$ measures how much of the good $j \in \{m, n\}$ produced in country k is consumed in country k. The counterpart of this measure in terms of expenditures is the expenditure share for domestic goods and services in country k,

$$h_{t}^{k} = \frac{\widehat{p}_{m,k,t}^{k} H_{m,t}^{k} + \widehat{p}_{n,k,t}^{k} H_{n,t}^{k}}{\widehat{p}_{m,k,t}^{k} H_{m,t}^{k} + \widehat{p}_{m,-k,t}^{k} F_{m,t}^{k} + \widehat{p}_{n,k,t}^{k} H_{n,t}^{k} + \widehat{p}_{n,-k,t}^{k} F_{n,t}^{k}}. \tag{32}$$

The denominator of (32) describes aggregate consumption expenditure (on all four goods) in country k. Similarly, we can define the expenditure share for manufactured goods in country k as

$$m_{t}^{k} = \frac{\widehat{p}_{m,k,t}^{k} H_{m,t}^{k} + \widehat{p}_{m,-k,t}^{k} F_{m,t}^{k}}{\widehat{p}_{m,k,t}^{k} H_{m,t}^{k} + \widehat{p}_{m,-k,t}^{k} F_{m,t}^{k} + \widehat{p}_{n,k,t}^{k} H_{n,t}^{k} + \widehat{p}_{n,-k,t}^{k} F_{n,t}^{k}}. \tag{33}$$

Both, $C_{j,t}^{k}$ and h_{t}^{k}, are measures of the extent of "domestic consumption". But C_{j}^{k} refers to one single good ($(j, k) \in \{m, n\} \times \{A, B\}$) and can therefore be defined in terms of quantities, whereas h^{k} refers to consumption aggregates over several goods and therefore uses expenses, which confound prices and quantities. Both of these measures will be useful to disentangle the effects of government policies in our analysis of equilibrium behavior in Section 4.

Finally, as noted in the introduction, an important role for policy will be played by the terms of trade. We therefore define the terms of trade for sector $j \in \{m, n\}$ as the relative price of the output of country A to that of country B, before taxes and tariffs: $p_{j,A,t}/p_{j,B,t}$.

3 Parameterization

Our parameterization builds on Eichenbaum, Rebelo and Trabandt (2021). Table 1 provides a summary of our calibration choices. Each period in the model is a week. To make computation feasible in our high-dimensional environment, we assume log utility from consumption, i.e., we set $\rho = 1$, because this yields simple closed-form solutions to some expressions (see Appendix Section A.1).[17] We set $\beta = .96^{(1/52)}$ so that the value of life in autarky is approximately \$10 million.[18] Furthermore, for the sake of comparability we follow Eichenbaum, Rebelo and Trabandt (2021) and set $\phi = .8$, so that the average productivity loss for infected individuals is 20%.[19] We set productivity in both sectors and both countries to $z_t = \bar{z} = 39.835$ and $\phi = 0.001275$ so that in the pre-pandemic steady state each person works 28 hours per week and earns $58,000$ per year, which for the US is consistent with average data from the U.S. Bureau of Economic Analysis and the Bureau of Labor Statistics in 2018. Considering asymmetric productivities across sectors and countries, and thus international comparative advantages, would be an important direction for future research, but would distract from the current focus on the basic structure of disease transmission and trade. Initial populations are normalized to 1. In the pre-pandemic steady state the countries are symmetric.

For the elasticity of substitution between home and foreign goods, we follow Costinot and Rodríguez-Clare (2014) and set $\sigma = 6$. For the weights between the two sectors, we set $\chi = 1/2$ so that the two sectors are equally important. Finally, the home bias parameter $\alpha = 0.53$ is chosen such that the pre-pandemic steady-state consumption share towards domestic goods is roughly 67%.

To fix ideas, we assume that the infection originates in country A with an initial infected population of $I_1^A = \epsilon = 0.1\%$.[20] It then spreads to country B via international trade, at a speed that is endogenous to each country's policy. To parameterize our disease transmission, we again follow Eichenbaum, Rebelo and Trabandt (2021), who carefully derive values of π_1, π_2, and π_3 that make the model's pre-pandemic time-use and occupational predictions good matches for data from the U.S. Bureau of Labor Statistics 2018 American Time Use Survey and other U.S. based statistics. With these choices, in a closed economy 1/6 of transmission would occur through consumption, 1/6 of transmission through production, and the remaining 2/3 of transmission through other activities. This prominent role of exogenous, behavioral transmission, which cannot be influenced by the economic policies discussed in the present model, implies that infections indeed develop into pandemics in our model.[21] To adapt these

[17] Noting that ρ is also the inverse of the marginal rate of intertemporal substitution, Kaplan, Moll and Violante (2020) argue that also empirically $\rho = 1$ is a reasonable assumption.

[18] See, e.g., Hall, Jones and Klenow (2020) for a discussion.

[19] ϕ is difficult to calibrate and, of course, depends on the type of pandemic considered. The choice by Eichenbaum, Rebelo and Trabandt (2021), which is based on early data about asymptomatic Covid 19 infections from China, is likely too high for the Covid pandemic. See von Thadden (2020) for references and a detailed discussion. As discussed in our earlier paper Acharya et al. (2020), with lower values of ϕ the findings are even stronger.

[20] This is a relatively large number to start out with, but in the build-up phase for small early infection levels there is no noticeable behavioral reaction by agents and governments.

[21] A policy that makes sweeping use of curfews, quarantines, and other direct non-pharmaceutical interventions would provide a different and largely orthogonal channel to our analysis (with unmodeled dramatic economic consequences) and can potentially suppress early outbreaks by cutting these direct contacts. Given our interest in the transmission of infection waves of Covid-19 as observed in 2020/21, such radical alternatives are not very informative (except perhaps at the very early stage of the pandemic), and the current model seems more appropriate.

parameter values to our two-sector setting, we multiply the original π_1 value by 4 and the original π_2 value by 2. According to Eq. (9), consumption of one type of goods, which represents half of the economy, is matched (in a "contact" sense) with consumption of the same type of goods. So the contact intensity needs to be quadrupled to match the original setting. Similarly, agents only meet with half of the labor force in their own sector-specific production site, so the contact intensity needs to be doubled.

We then choose π_4 such that, without government intervention, the peak of the infection in country B occurs approximately 6 months after the peak of the infection in country A. Our theory of intertemporal risk-sharing through trade requires some degree of asynchronicity of infection dynamics, which is driven by π_4.

Moreover, we calibrate the benchmark transition probability p_r and $p_d(0)$ so that, when the infection rate approaches 0, the baseline mortality rate is 0.5% for the infected and it takes an average of 18 days to either recover or die from infection.[22] For the death rate $p_d(I_t) = p_d(0) + \zeta I_t$, we set $\zeta = 0.075$ in the benchmark case. This means that the mortality rate increases approximately 3.9-fold when the infection rate I_t is 10%. As noted earlier, for computational reasons we end the pandemic by assuming that a vaccine becomes available after T years. In calibrations, we use $T = 5$. These 260 weeks are well beyond the time frame for all processes that we report to settle at constant post-pandemic steady-state values. We provide technical details about our computation algorithm in Appendix Section A.6.

4 Simulations and Interpretation

We discuss below the key qualitative insights from the simulations of the laws of motion derived analytically in Sections 2.2 and A.1 in the appendix. First, we present the case with no policy, second the coordinated case, and third the Nash case. Then, we compare the three cases to highlight the structure of optimal health and trade coordination during a pandemic.

4.1 Laissez Faire

As a benchmark, Figure 2 illustrates the SIR dynamics and economic outcomes when there are no containment policies or tariffs (laissez-faire). The top row of plots shows tax and tariff policies which are all set at zero in the benchmark. The next eight plots in the second and the third row present the disease dynamics in both countries. As summarized in Table 3, starting with an initial infection rate of $I_0 = 0.001$ in country A, the pandemic takes off in country A

[22] Our calibration of the case fatality rate is at the lower end of the early estimates that we are aware of (see, for example, Fernandez-Villaverde and Jones (2022) or Verity et al. (2020)). These early estimates reflect high uncertainty, but also lack of experience with the treatment of severe cases.

and slowly spreads to country B, where it begins to take off after week 25. In each country, infection numbers are identical between the two sectors (m and n, represented by the blue and broken red lines, respectively), which is due to our assumption that work-induced infections spread identically in both sectors. The share of infected households in country A peaks at 4.0% in week 33 and declines thereafter. Around week 50, infections in country B overtake those in A and peak also at 4.0% in week 65. After week 104 the disease has run its course in country A, and after week 135 in country B, when almost 50% of the population in each country has become infected and around 0.5% of the population in each country has died.

The economic outcomes track local infection rates closely as shown in the fourth and the fifth row of Figure 2. When the infection wave hits country A, its labor declines by more than 20% in both sectors at the peak of the wave. In contrast, during most of the infection wave of country A the total amount of labor in country B stays more or less constant in both sectors. These patterns are reversed across countries when the pandemic eventually hits country B. As a consequence of the high infection rates at the work place, households in the country struck by the pandemic reduce their labor supply, which explains part of the decrease of equilibrium labor noted above. At first sight, it is surprising that this labor decrease is identical across sectors, because as the third plot in the 4th and the 5th row of Figure 2 shows, in both countries during their infection peaks, households decrease their local expenditure share h_t^k, defined in (32), in order to reduce the exposure to domestic infection (demand contraction). This adds a labor demand effect to the supply effect, and this effect is asymmetric across the sectors, given the greater contact-intensity of the services sector, as the plots of the expenditure share of manufactured goods m_t^k (defined in (33)) in the fourth plot in rows 4 and 5 show. In fact, in country A local service consumption as a share of total services produced in the country, $C_{n,t}^A$, falls, as well (shown in the first plot in row 4 of Figure 6). While the supply effect for labor is symmetric across sectors because infections are symmetric across sectors (plot 2 in rows 2 and 3), this asymmetry of labor demand should therefore lead to a differential fall in country A's labor L_t^{Aj}.

However, there is a general equilibrium price effect. While the terms of trade for country A in manufacturing, $p_{m,A}/p_{m,B}$, displayed in the two plots of row 6, move in a correlated manner with A's relative position in the disease waves, this is exactly the opposite for the terms of trade in services ($p_{n,A}/p_{n,B}$). Hence, country A's service goods become cheaper in both countries, which boosts their demand from households in both countries. This in turn increases the demand for labor in the service sector of country A, counteracting the original demand effect discussed above. As consequence, equilibrium labor in the two sectors co-moves perfectly, as shown in the second plot of rows 4 and 5.[23]

[23]The feature that these effects perfectly offset each other and that labor in both sectors evolves identically is a consequence of the special form of our logarithmic Cobb-Douglas utility function in (4) and (5).

Hence, the terms-of-trade effect implies that consumers of country B pick up some of the lost consumption of country A services from country A when the latter collapses during the peak of the crisis in A and their price falls. Interestingly, as seen in the third plot of row 5, this even yields a (small) decline of the local expenditure share of total consumption in country B, indicating that country B's households view the health risk from imports from country A as less important than the economic benefit from the improved exchange rate. Overall, these shifts in consumption shares have a perceptible impact on the terms of trade expressed by the relative prices of both goods (which, as shown in the rightmost plots of the bottom panel, change by at most 5%). But as noted above, taken together they do not impact aggregate production (as shown by the evolution of labor in the plots above).

However, these changes do have an impact on wages. As services become cheaper during the pandemic, wages in the service sector fall, too. This fall is significant (in our simulation, shown in the third plot of row 4 in Figure 6, more than 12% in country A from peak to trough) and implies that aggregate consumption by households employed in the service sector, X_t^{kn} as defined in (30), falls much more than that by households in the manufacturing sector, as evidenced by the first plot in rows 4 and 5. Hence, the pandemic also has distributional consequences in the labor market.

4.2 Fully Coordinated Government Policies

Next, we consider the optimal policy of a coordinated planner who maximizes the sum of the welfare of both countries' households as given by (22). At time 0, this planner determines both countries' domestic containment policies and tariffs from week 1 to 260 until the pandemic is over. Figure 3 reports these optimally coordinated policies (top row) and outcomes for variables of interest on health (second and third rows) and economic (fourth and fifth panels) dimensions. Terms of trade are shown in the sixth row.

The planner employs a dynamic state-contingent combination of domestic containment measures and international tariffs to achieve her desired health and economic outcomes. The severity of domestic containment measures in each country roughly tracks the level of infection rates in the country, with some front-running due to rational expectations and prevention. Given the higher transmission rate in services goods, containment measures are naturally more stringent for services consumption than manufacturing consumption. In contrast, tariffs have a pattern across time that is inversely symmetric between the two countries over time, and a pattern within each country between manufacturing and services sectors that is inversely symmetric to that for containment taxes for the two sectors. When the infection peaks in country A, the planner responds by *raising* tariffs in country A on B's goods, to close to 20% on services goods and over 35% for manufacturing goods, while imposing a *negative* tariff on A's goods in country B, i.e., an import subsidy, of close to 15% on services goods and 20% on manufacturing goods. The combined domestic containment tax and tariffs on foreign goods are higher for services than for manufactured goods in each country during its infection wave.

The health and economic consequences of these policy choices of the planner as well as the underlying rationales can be summarized as follows. As in the no-policy case, the second row shows that the pandemic quickly takes off in country A and slowly spreads to country B, where it begins to take off after around week 25. The infection in country A peaks around 40, over a month later than in the unfettered outbreak, and declines thereafter. But the peak infection rate at 1.8% is more than 50% lower and the disease lasts 65% (68 weeks) longer (see Table 3). The picture is almost identical in country B (the infection peak in B being slightly smaller than in country A). Less than 40% of the population becomes infected eventually in both countries, and total death rates are almost the same at 0.27% in both countries, around 45% lower than under laissez-faire. Hence, the planner's policies are effective at "flattening the curve" relative to laissez-faire.

Economic outcomes react to infection rates as well as to domestic containment and international tariff policies. When the wave of infection hits country A, its labor and aggregate consumption by households in both sectors exhibit a peak decline of over 25%, the decline being greater for the service sector. Both these declines are greater than under laissez-faire. Given that containment taxes in country A are higher than its tariffs, the local share of consumption by country A falls during its infection wave. This tilts the terms of trade in favor of A's goods, so that both $p_{m,A}/p_{m,B}$ and $p_{n,A}/p_{n,B}$ rise above one during A's infection wave. This differs from the laissez-faire both qualitatively and quantitatively. First, in the lassez-faire case, $p_{n,A}/p_{n,B}$ falls below, not rises above, one during A's infection wave. Second, $p_{m,A}/p_{m,B}$ rises above one in both laissez-faire and coordinated cases, but the increase in the coordinated case is an order of magnitude (5-10 times) larger. Notwithstanding the import subsidies in country B for A's goods, terms of trade shifting in A's favor cause the local share of consumption in country B to rise during A's infection wave. As A's infection wave subsides and B's picks up, it is now country B's turn to impose containment taxes and tariffs, while A provides import subsidies for B's goods, production and labor now shifting to country A from country B.

Key to understanding the planner's policy choices is to recognize that the infection curve is being flattened. The decline in both consumption and labor is greater than in the laissez-faire case, because the planner internalizes the infection externalities within and between the two countries. The inverse relationship of tariffs in the two countries is intriguing at a first pass because in the wave of infection for country A, it encourages (for given containment tax) both countries to consume more of country A's goods, which transmits the pandemic via consumption- and labor-induced interactions in country A and via imports to country B. However, these (marginal) health costs are dominated by the economic benefits, which ultimately make it possible to tighten (average) health standards in terms of raising domestic containment taxes significantly without losing too much on the consumption side. Specifically, tariffs raise the terms of trade for the infected country and its households earn higher wages. Given the higher wages, households can even reduce infectious labor contacts without sacrificing total income, and can thus enjoy a higher level of consumption, for a given level of containment measures. In this way, the planner achieves better international risk-sharing by alleviating the economic costs on the country that is hit harder by the pandemic at the moment.

Overall, the planner uses the asynchronous feature of the pandemic to design dynamic, state-contingent and country-specific policies that not only shift consumption, but also production, between sectors and countries, reducing infections and death rates significantly in the process while keeping economic costs contained.

4.3 Government Policy in Nash Equilibrium

We next consider the Nash equilibrium setting wherein each country's government determines its own domestic containment and tariff policies, taking those of the other country as given. Each country sets its policies in order to maximize the welfare of domestic households, defined as the weighted average of their lifetime utilities (21). We consider open-loop strategies, which is tantamount to assuming commitment and perfect foresight. This creates room for intertemporal trade-offs of the *do-ut-des* sort: governments can agree in advance on future actions to smooth health shocks. However, this also creates the potential to create the "Prisoners'-Dilemma" type blockades found in traditional theories of trade wars.

Figure 4 reports the policies as well as health and economic outcomes and Table 3 summarizes the basic health statistics. To interpret the policies (top row of the figure), it is helpful to begin at the end. Once the pandemic is over (around week 196 in country A, and 238 in country B), both governments impose a tariff of more than 25% on both manufacturing and services goods, due to the standard Nash logic that each country wants to boost its domestic employment and wages, given that the other country does so. However, this purely trade-war logic interferes with the objective of smoothing intertemporal shocks during the pandemic. During the pandemic, the infected country imposes substantial containment taxes to prevent domestic infections, but does not internalize the infection costs imposed on foreign country households. Hence, to compensate for the high containment tax on the more infectious domestic service sector, the infected country lowers tariffs on the less infectious foreign services goods – in fact, providing an import subsidy on these goods – and raises them on foreign manufacturing goods. This, however, is only met by the foreign country imposing tariffs at an even higher than the pure trade-war level on the infected country's services goods given its own private interest in limiting infections, while reducing the tariff slightly on manufacturing goods.

The evolution of the domestic expenditure share h_t^k in rows 4 and 5 makes it clear how the Nash outcome fails on both the economic and the health fronts. Even at the peak of the infection wave, when domestic services should be replaced as much as possible by foreign services, the Nash governments only reduce that share to around 74%, which is way above even the highest levels under optimal planning (or even Laissez-faire) at *any* point in time. This leads not only to bad economic diversification, but also to poor health diversification, i.e., to high contagion levels, which then must be (unsuccessfully) compensated by very high containment taxes on services (top row).

The resulting poor economic performance of the Nash outcome due to coordination failure is best seen in the last three rows of Figure 4. The local expenditure share h plots illustrate that during the infection waves, both countries reduce their domestic share of consumption relative to the level in the no-infection region, but that due to high tariffs the overall domestic share of consumption is high relative to the benchmark and the coordinated cases. The last row shows that terms of trade dynamics favor the infected country only in the manufacturing good but not in the services good. This pattern is similar to that in the benchmark case of no policy, but somewhat amplified. In effect, even though tariffs are reduced on the services good by the infected country below the pure trade-war level, its timing is inefficient as promoting trade of the high-transmitting services good spreads infection to the other country which retaliates by imposing an even higher tariff on services good of its own. The resulting weak terms of trade for the infected country in services then requires a stronger containment tax to restrict its consumption, worsening overall economic outcomes, as seen in the substantially lower aggregate consumption at infection peaks (first plot of the third and fourth row in Figure 4).

To summarize, trade war and beggar-thy-neighbor policies compound the economic and health problem in the Nash case and interact negatively. The trade-off between tariff predation and health protection in beggar-thy-neighbor policies of individual countries prevents trade-based risk-sharing seen in the coordinated case where terms of trade are modulated to favor the infected countries enabling them to implement less stringent containment measures.

4.4 Comparing Benchmark, Coordinated and Nash Policies

Figures 5, 6 and 7 compare between the Laissez-faire, coordinated, and Nash cases, respectively, (i) the health outcomes, (ii) the consumption and labor outcomes, and (iii) the government policies and terms of trade, displaying their evolution over the course of the pandemic in each case. As already seen above, both the Nash case and the Planner case feature similar paths of domestic containment policies with high values during the peak of the infection waves and no action outside this period. However, qualitatively the major difference between these cases is in the dynamics of the other policy instrument, viz., tariffs, and the induced terms of trade of manufacturing and services goods over the infection waves, summarized in the last row of Figure 7. This is most significant in the case of services, where the Nash governments' beggar-thy-neighbor policies actually depress the infected country's terms of trade, when optimal coordination requires to improve them.

In Table 2, we use our model to decompose the overall welfare effect of policy for each country. In Panel (a), we report these welfare effects in the unit of utils, which correspond to the value of the utility function (3). In Panel (b), we normalize these utils by the life-time utility under no policy and no pandemic. These numbers are presented in percentage points, so that -1 is equivalent to the welfare loss of losing 1% of the population.

Turning first to Panel (a), the first row reports, as a benchmark, the welfare loss of laissez-faire in a pandemic compared to the no-pandemic case. The second row compares the coordinated planner outcome to the no-pandemic case. Finally, the third and fourth rows provide a decomposition of the outcome of the Nash policies. We split the Nash policies into two parts. In the third row, we report the welfare loss from the trade war alone in the absence of the pandemic, again relative to the no-pandemic and no-policy equilibrium. The fourth row shows the *additional* loss in a pandemic due to the Nash policies. The total welfare loss in a pandemic with Nash policies, relative to no pandemic and no policy, is the sum of the last two rows. We decompose the households' utility loss (negative value) in each country relative to the no-pandemic case into two components: the welfare loss due to economic recession, and the welfare loss due to death. The former is the present value of the utility change in the consumption and labor of living households, from period 1 to the infinite future; the latter is the present value of the foregone utility due to death. Their sum is the total utility loss relative to laissez-faire.

The decomposition of Table 2, Panel (a) shows that in both countries the coordinated outcome alleviates death-related welfare losses relative to both the Nash equilibrium and the laissez-faire, whereas the laissez-faire by not undertaking any containment measures results in lower economic losses relative to the coordinated and Nash cases. Not surprisingly, Nash governments create an enormous economic loss in each country by using too strict domestic measures and ill-timed tariff policies. But Nash governments do better than Laissez-faire on the health front, because they punish imports indiscriminately, without sufficient regard to the gains from substitution in the service sector.

Next, Table 2, Panel (b) normalizes these welfare losses by life-time utility. Under the laissez-faire case, the welfare loss is equivalent to losing about 0.5% of the population in both countries. The coordinated policy under the planner's case reduces the welfare loss to about 0.35%. In contrast, the trade war alone in the Nash equilibrium is equivalent to losing 0.4% of the population, whereas the pandemic adds another about 0.35% to the toll.

Nash policies also fail economically on another important front, namely insurance in a situation of macroeconomic uncertainty. As discussed in Section 4.1, the collapse of demand for services during the pandemic is more pronounced than the collapse for manufactured goods (first plot in rows 4 and 5 of Figure 2). In the Laissez-faire case this leads to substantial income losses for households working in the domestic service sector, which imply substantial consumption and utility losses. Since these infections are random from an individual perspective and unavoidable in the aggregate, government policy must try to smooth consumption fluctuations in the pandemic as well as possible. Comparing the corresponding plots of X^{kj} for the Planner and the Nash cases in Figures 3 and 4 respectively shows that the consumption loss in both sectors of the economy from trough to peak of the pandemic is larger under Nash than under coordinated planning. In fact, under coordinated planning, in country A consumption by manufacturing workers falls by ca. 24% from trough to peak, while it falls by ca. 29% for service workers (28% compared to 34% in country B). Under Nash policies, in country A consumption by manufacturing workers falls by ca. 27%, while it falls by ca. 38% for service workers (28% compared to 42% in country B). This also shows that Nash policies fail relative to the coordinated case on the distributional front between sectors, as the service sector is hit worse than the manufacturing sector.

The economic intuition for the different welfare outcomes is worth repeating in the interest of emphasizing the key forces at work. In the benchmark case with no policy, the terms of trade is countercyclical (relative to the consumption cycle) for the manufacturing goods and procyclical for the services goods. This is because only the services goods transmit disease, so that households avoids its consumption in the infected country. Households also, however, withdraw labor from the manufacturing goods provision making them scarce and therefore expensive. In short, the services goods of the infected country are hit by a demand shock (fear of disease) and the manufacturing foods by a supply shock (lower production capacity).

The Nash case has similar cyclicality in the terms of trade for the two goods, but the procyclicality of services goods is amplified in magnitude. This is because the government in the country hit by the pandemic seeks to lower tariffs on foreign services goods which are less-transmitting than domestic services goods, and it does not internalize the spread of infections to the foreign households via this encouragement of imports. Of course, when not inducing this ill-timed procyclicality (from a risk-sharing standpoint), the usual trade-war incentive implies that Nash governments impose a positive tariff on average.

Finally, the planner implements a more stringent containment tax in infected countries and also shifts consumption away from services goods to manufacturing goods. This reverses the cyclicality of the terms of trade in the services goods. The planner uses tariff coordination to limit the economic damage from this stringency. She raises tariffs (but less than domestic containment taxes) on foreign goods in the infected country – with a smaller increase for the infectious service sector – and lowers tariffs into import subsidies for the infected country's goods in the other country – with a larger decrease for the less infectious manufacturing sector – reducing the overall share of domestic consumption in the infected country as well as increasing it in the foreign country, leading to improved risk-sharing. In other words, these tariff policies can be thought of as a wealth transfer that helps the more infected country to maintain consumption levels and reduce the economic impact of the pandemic.

4.5 "Zero-Covid" Policy

During the Covid-19 pandemic, one important policy response was a complete lockdown, as in the case of China, whose government imposed strong containment measures for consumption of both domestic goods and imports. To simulate such a policy in our framework, we assume that the country imposes 100% containment taxes on all consumption goods, and additionally 100% tariffs on all imports over the entire pandemic.

Figure B.1 in the Online Appendix plots the case in which country A imposes this strong containment policy, while country B does not impose any taxes or tariffs. In this case (which is the most favorable case for such a policy), the aggregate consumption and labor in country A are significantly depressed, even relative to the case of Nash policies, and ends up with a much lower welfare. Importantly, the death toll from this "infect-thy-neighbor" policy even in country A is only marginally lighter than under optimal international cooperation. This is because in our model, as in reality, this policy cannot actually achieve a literal "zero-Covid" outcome for two reasons. First, due to our assumption of unconditional transmission as captured by the π_3 term in the transmission dynamics, the disease spreads domestically even if all consumption and production were stopped. Life goes on, after all. Second, because of the CES consumption aggregator, households always want to consume some of each country's good. As a result, the pandemic also transmits internationally despite the stringent lockdown policy.

Figure B.2 in the Online Appendix plots the case in which both countries impose the strong containment policy. Now, the stringent lockdown policy significantly depresses aggregate consumption and labor in both countries, while the international transmission of the disease is only delayed. Interestingly, the zero-covid policy actually leads to greater death tolls in each country compared to the Nash outcome (and trivially, compared to the planner's policy). This is because the tariff component of the lockdown policy is an extreme trade war, which leads to a much higher home bias in consumption that promotes disease transmission. In essence, this policy eliminates the flexibility in fine-tuning the interaction between the domestic and trade-related containment measures and the sectoral reactions, which are at the heart of our paper.

5 Conclusion

This paper has developed a two-country two-sector model of epidemiology and international trade to study how international coordination, and the lack thereof, influence the impact of government policies on health and economic outcomes during a pandemic. A major insight from our work is that the interplay between domestic health policies and international trade policies makes it possible to share economic and health risks better across countries. This can be achieved by intertemporally modulating the terms of trade to shift consumption, production, and infection patterns internationally as a function of the global state of the pandemic. Crucially, the type of good, i.e., more contact-intensive services versus less contact-intensive manufactured goods, has a first-order bearing on such modulation. Coordinated policies influence the terms of trade in both types of goods such as to favor the country experiencing an infection wave when it needs it the most to counteract the adverse economic impact of containment measures. In contrast, uncoordinated policies aggravate overall outcomes by modulating manufactured goods' terms of trade far less in favor of the infected country, and modulating services goods' terms of trade *away* from the infected country. The trade-cum-health war arises due to countries not internalizing the externality from spreading infections globally, highlighting the importance of international cooperation in dealing with the health and economic fallout from a pandemic.

Our model, even if stylized, represents an advance over existing models, especially given its computational complexity. An ambitious and yet non-exhaustive list of directions to enrich it further includes (1) Labor choices leading to the reallocation of households across sectors as a function of the aggregate pandemic state; (2) Further heterogeneity among sectors, notably tradable versus nontradable sectors (which do not necessarily align with manufactured versus services sectors), and even further differentiated by contact intensity; (3) Factors that hinder trade and international risk-sharing, such as shipping costs or immigration delays, (4) Aggregate uncertainty around the evolution of the pandemic and the risk of multiple waves ("variants" due to pathogen evolution, temporary post-vaccine immunity, etc.) which create the risk of greater synchronicity of waves across countries and may add a new dimension to the state-contingent international reciprocation logic of our model; and, (5) More general intervention instruments: for instance, non-linear non-pharmaceutical interventions (NPIs) such as lockdowns that impact consumption as well as worker productivity, domestic transfers between household types based on their health, and international transfers in the form of direct payments between governments. While each of these extensions is likely to be challenging, our model can be a useful foundation and starting point.

Finally, some empirical implications and directions for model validation are also worth mentioning. For instance, the model implies that in case of cooperating countries in a pandemic, terms of trade in the services sector should favor the more-infected country, whereas in the case of non-cooperating countries, the pattern should reverse. A more specific implication would relate such terms of trade patterns to the observed data on tariffs and non-tariff barriers and link it to the model. Of course, an important assumption in testing such specific implication is that other endogenous outcomes – macroeconomic (GDP, consumption, labor), trade (imports, exports), and health (infection and death rates) – also line up with the model. This raises two challenges. One, it would be necessary to analyze macroeconomic and trade data at the level of individual sectors or at a minimum based on sectoral groups differentiated by contact intensity. Secondly, and perhaps more dauntingly, real world data are likely to have come from far richer heterogeneity across countries than in our model in terms of the timing of their first infections and subsequent waves, national policies and their relationship to those of trading partners, and evolutions in the quality of health data during pandemics. Further modeling advances such as the ones enumerated above can help better understand the empirical patterns of international terms of trade during pandemics. Clearly, the scope for further research in this area is immense, especially as the perceived risk of another pandemic is much greater than in the past.

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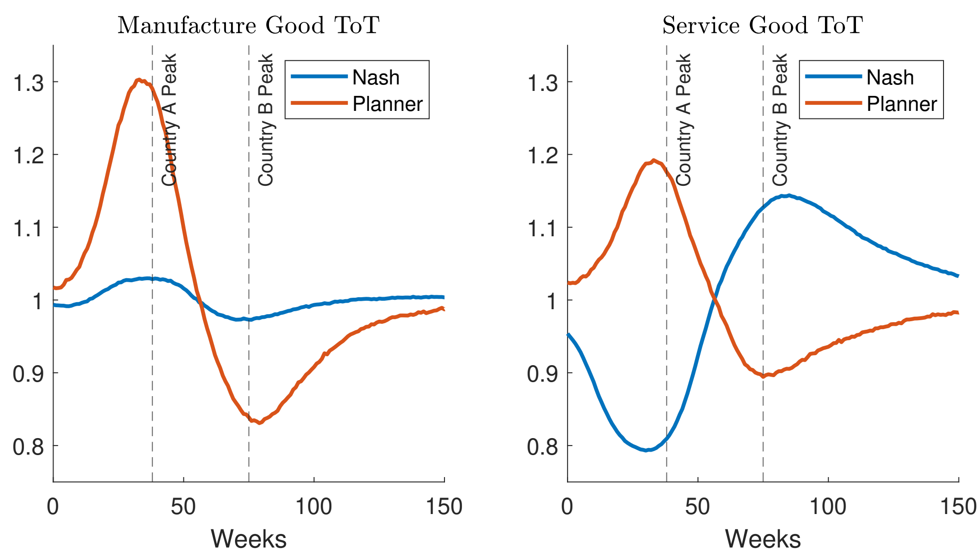
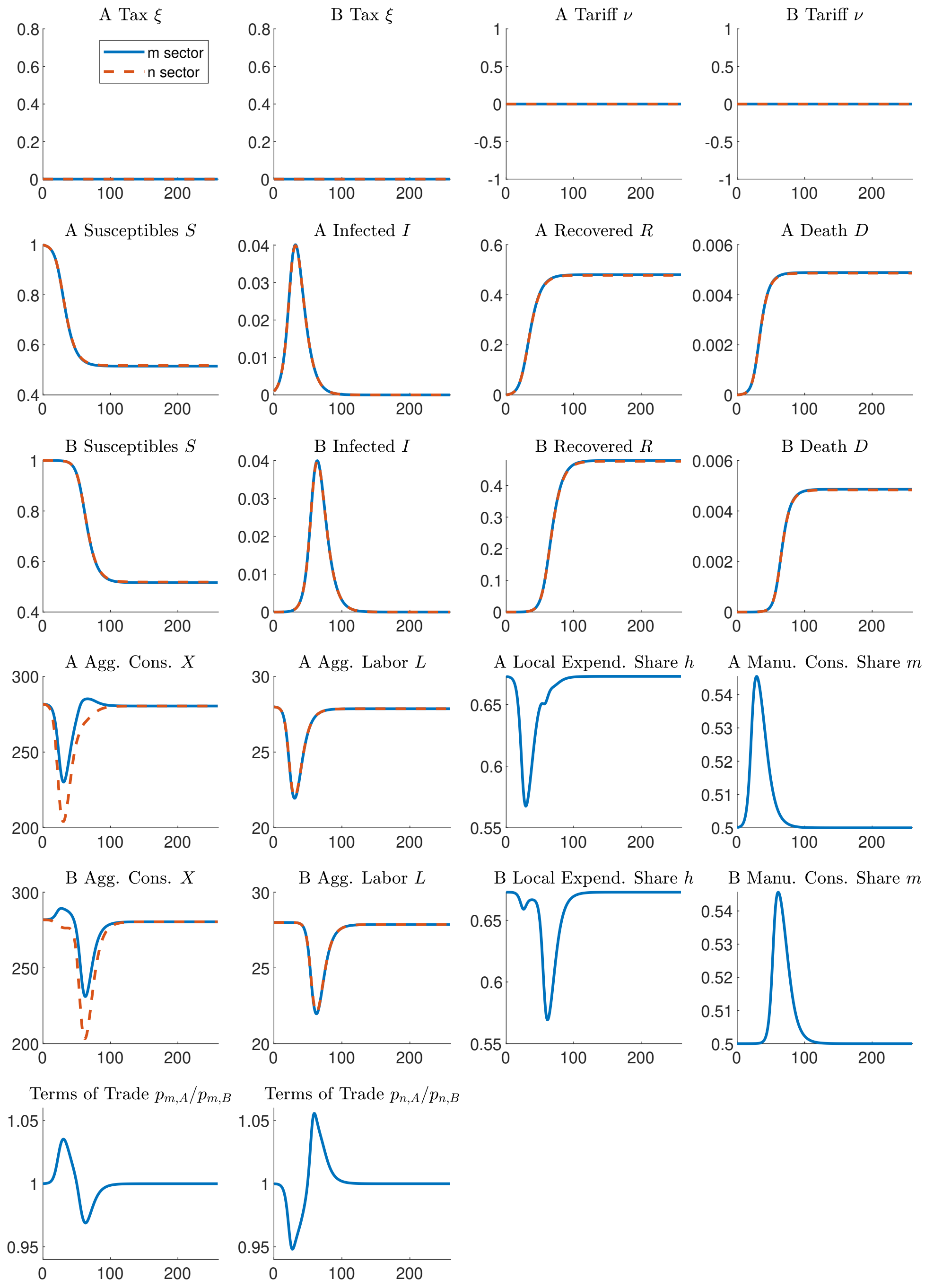
Figure 1: Terms of Trade With and Without Coordination



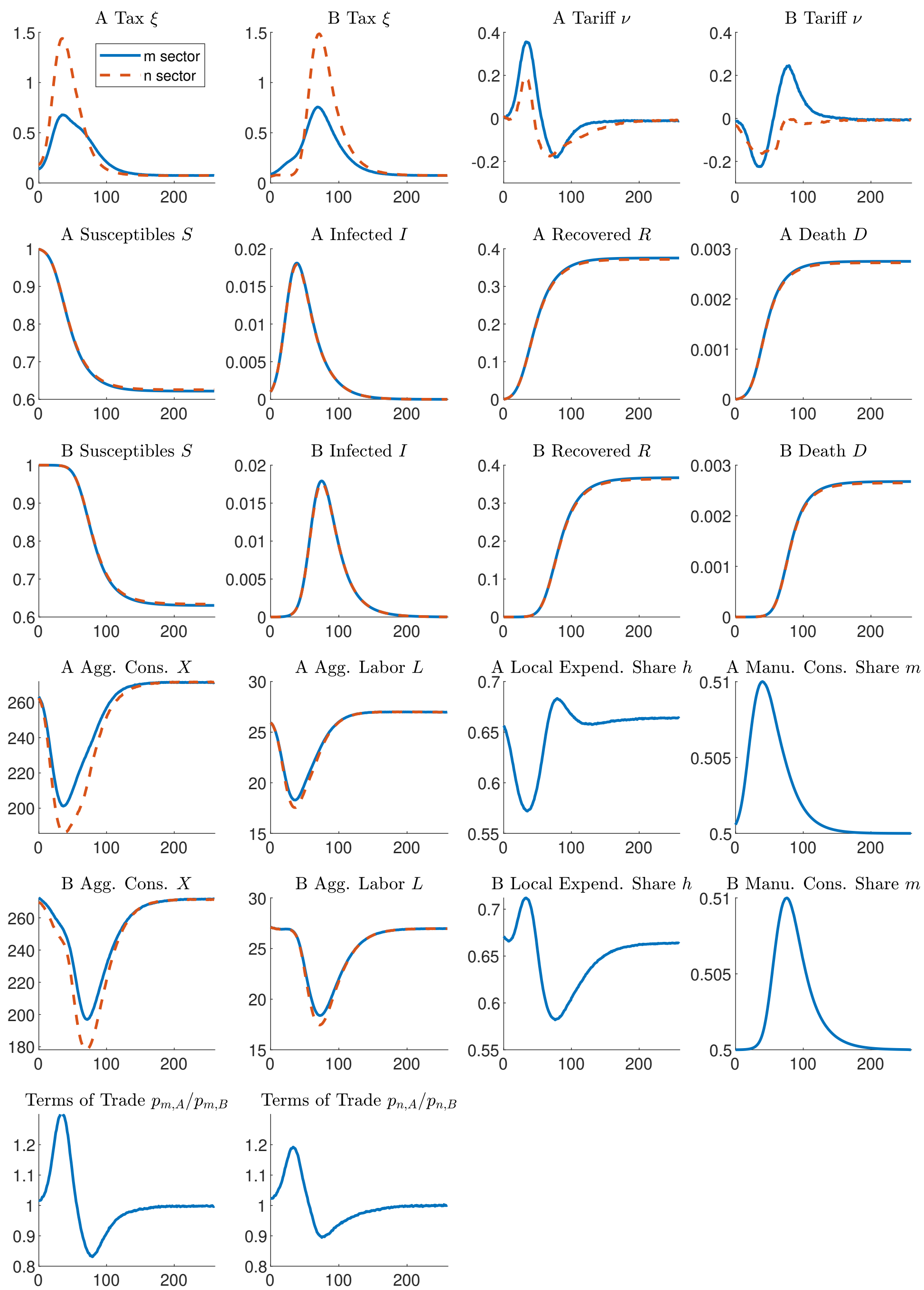
Note: Terms of Trade in uncoordinated (Nash) and coordinated (Planner) equilibrium. The dashed lines specify the approximate peak of maximum infections in country A and country B.

Figure 2: Benchmark SIR Dynamics



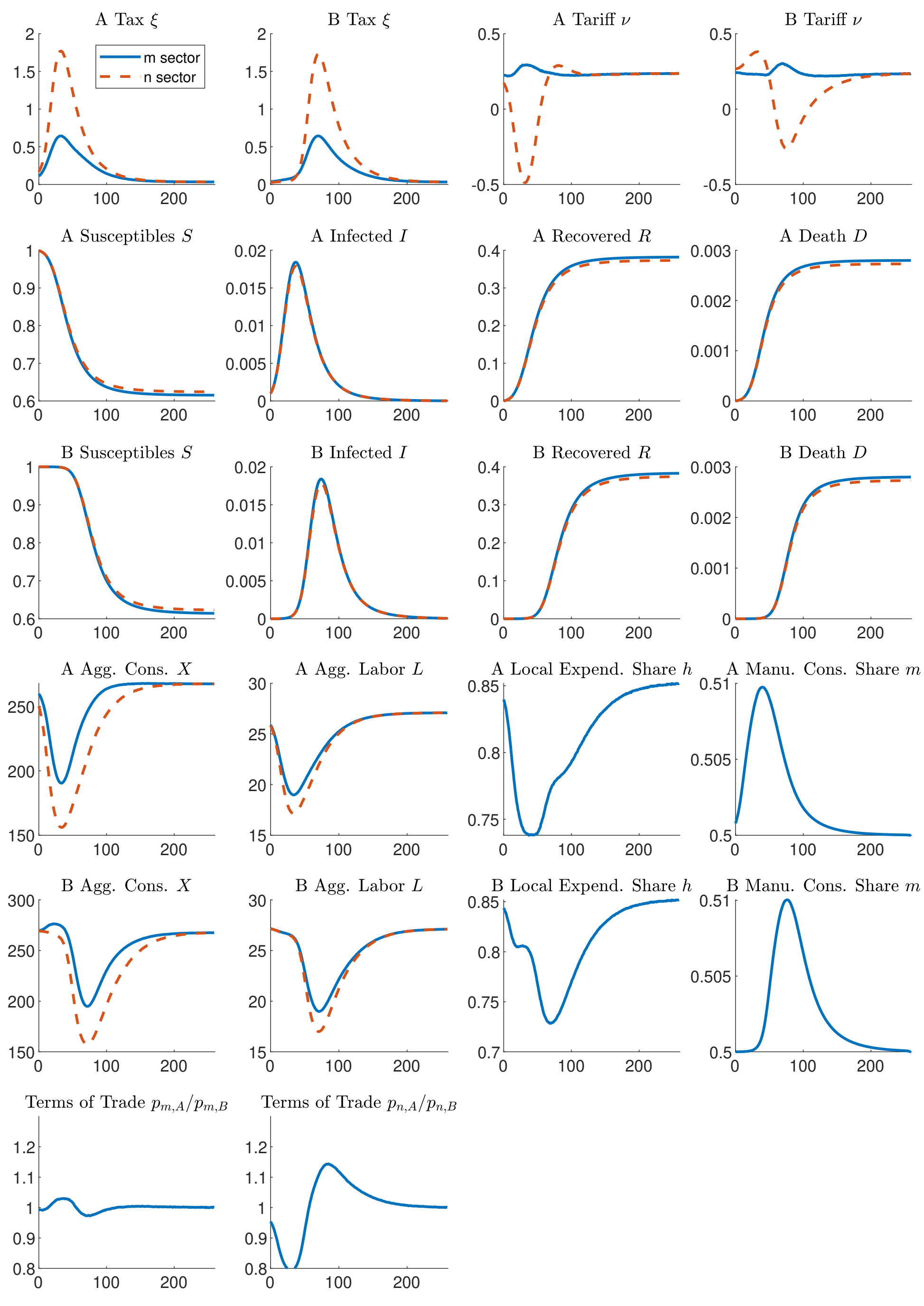
Note: Benchmark model with international transmission of pandemic. No government domestic containment policies or tariffs.

Figure 3: Coordinated Planning Equilibrium Outcomes



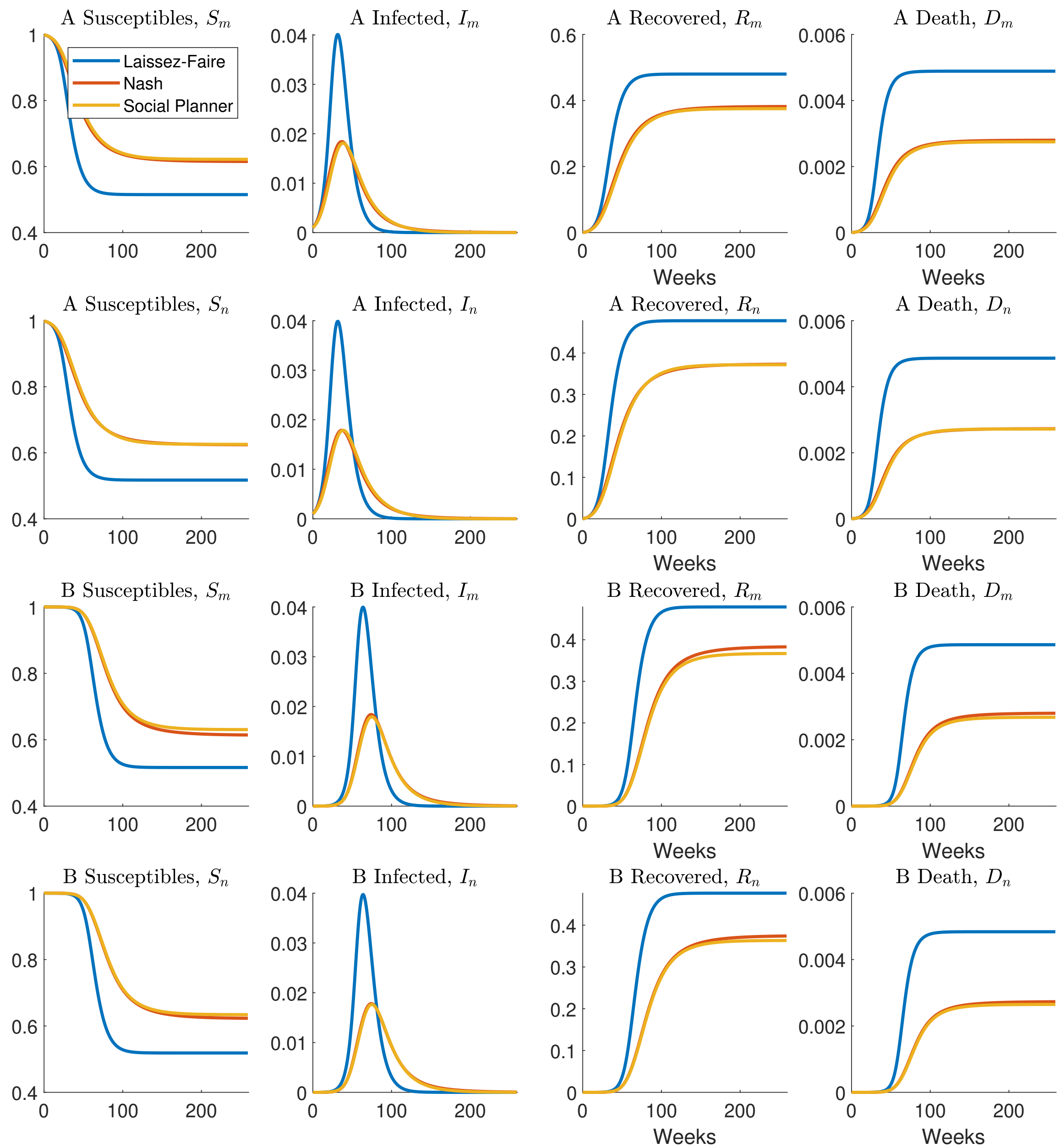
Note: Benchmark model with international transmission of pandemic. Equilibrium domestic containment policies and tariffs are determined by a global social planner that maximizes the sum of both countries' welfare.

Figure 4: Nash Equilibrium Outcomes



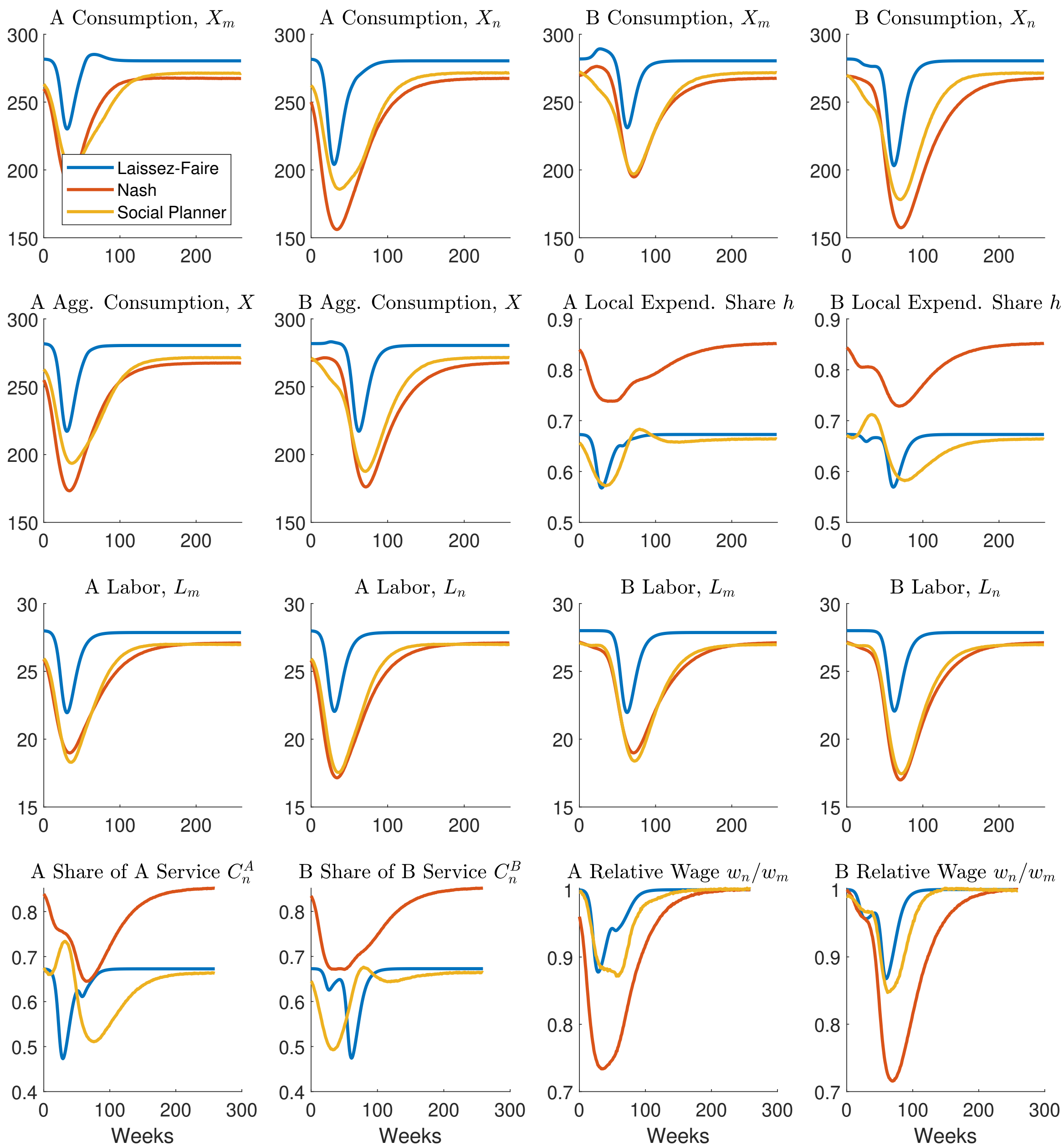
Note: Benchmark model with international transmission of pandemic. Equilibrium domestic containment policies and tariffs are the outcome of a Nash game between the two countries.

Figure 5: Comparing Equilibrium SIR Dynamics



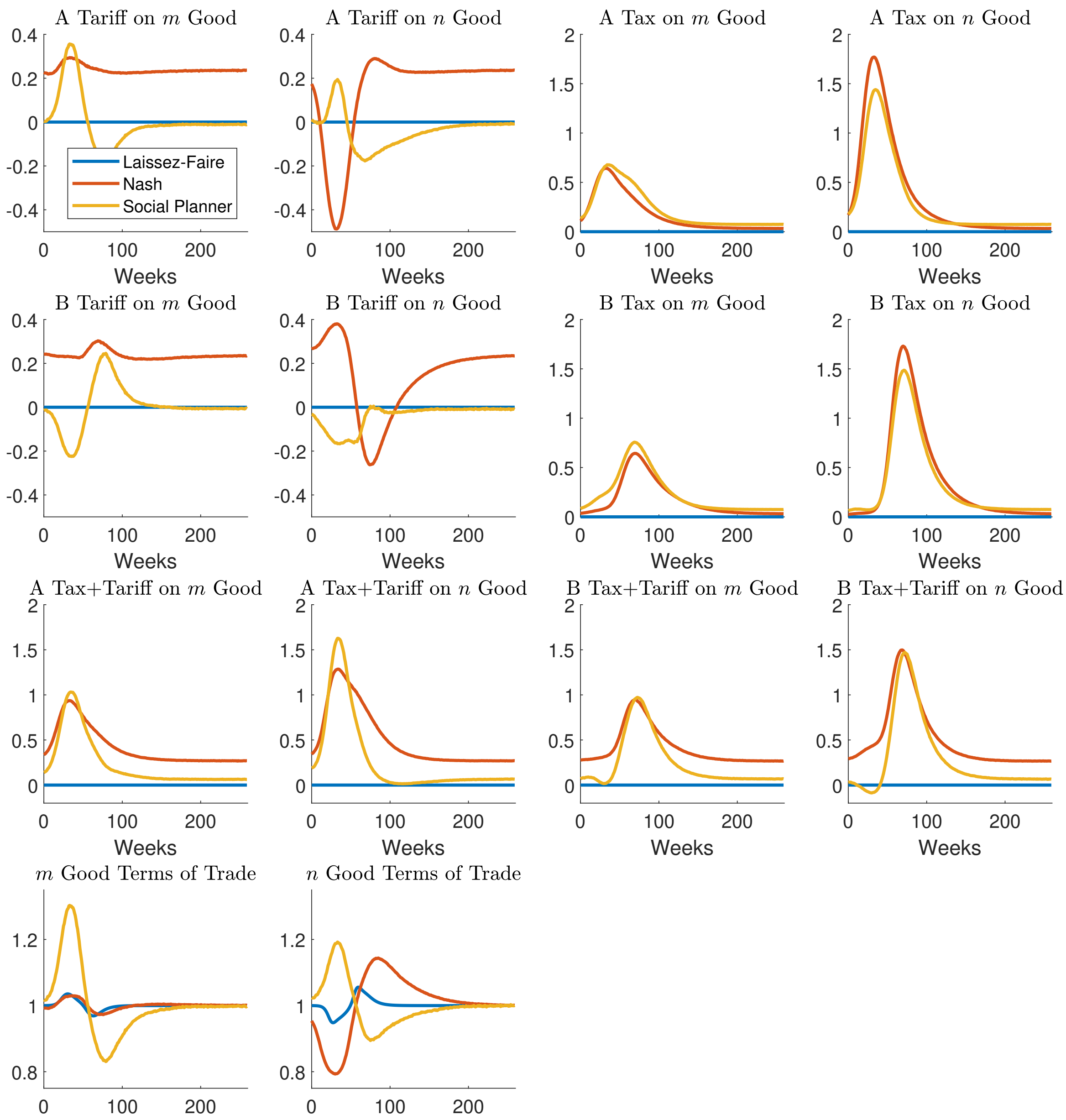
Note: Comparison of SIR dynamics in three cases: benchmark, Nash, and Planner. In the no policy case there are no domestic containment policies. In the Nash case, equilibrium domestic containment policies and tariffs are the outcome of a Nash game between the two countries. In the planner case, equilibrium domestic containment policies and tariffs are determined by a global social planner that maximizes the sum of both countries welfare.

Figure 6: Comparing Equilibrium Consumption and Labor Outcomes



Note: Comparison of economic outcomes in three cases: benchmark, Nash, and Planner. In the no policy case there are no domestic containment policies. In the Nash case, equilibrium domestic containment policies and tariffs are the outcome of a Nash game between the two countries. In the planner case, equilibrium domestic containment policies and tariffs are determined by a global social planner that maximizes the sum of both countries welfare.

Figure 7: Comparing Equilibrium Policies and Terms of Trade



Note: Comparison of government policies and the terms of trade in three cases: benchmark, Nash, and Planner. In the no policy case there are no domestic containment policies. In the Nash case, equilibrium domestic containment policies and tariffs are the outcome of a Nash game between the two countries. In the planner case, equilibrium domestic containment policies and tariffs are determined by a global social planner that maximizes the sum of both countries welfare.

Table 1: Parameter Choices

This table reports the values of calibrated parameters and our calibration targets. Our model is calibrated at the weekly frequency.

Symbol	Interpretation	Value	Calibration Target
β	discount factor	$0.96^{1/52}$	Value of life (Hall, Jones and Klenow, 2020)
p_d	probability of dying	$7 \times 0.5\%/18$	Fatality rate (Eichenbaum, Rebelo and Trabandt, 2021)
p_r	probability of recovering	$7 \times 1/18$	Recovery rate (Eichenbaum, Rebelo and Trabandt, 2021)
σ	Elasticity of substitution	6	CES Elasticity Costinot and Rodríguez-Clare (2014)
α	Home bias	0.53	Home consumption share (Costinot and Rodríguez-Clare, 2014)
χ	Sector share	0.5	Symmetry
κ	Labor disutility	0.13%	Bureau of Labor Statistics (Eichenbaum, Rebelo and Trabandt, 2021)
z	Productivity	39.84	Bureau of Labor Statistics (Eichenbaum, Rebelo and Trabandt, 2021)
ϕ	Productivity level of the infected	80%	Share of asymptomatic infected (Eichenbaum, Rebelo and Trabandt, 2021)
π_1	Consumption-based pandemic transmission	$1.4 \times 10^{-7} \times 4$	Share of infection through consumption (Eichenbaum, Rebelo and Trabandt, 2021)
π_2	Labor-based pandemic transmission	$1.2 \times 10^{-4} \times 2$	Share of infection through labor (Eichenbaum, Rebelo and Trabandt, 2021)
π_3	Unconditional pandemic transmission	0.39	Share of residual infection (Eichenbaum, Rebelo and Trabandt, 2021)
π_4	Interntional pandemic transmission	2.1×10^{-9}	6-month distance between wave peaks

Table 2: Welfare Decomposition

We report the welfare change relative to the steady-state level without pandemic and policy. We decompose the welfare change in each country into two components. The *economic* component is the present value of the utility change of living households due to changes in consumption and labor during the pandemic episode, and the *death* component is the present value of the foregone utility due to death. We also break down the welfare loss under the Nash case into a component due to the trade war in the absence of pandemic and a component due to the pandemic and the containment policy.

(a) In the Unit of Utils

	Country A			Country B		
	Total	Economic	Death	Total	Economic	Death
No Policy	-32.64	-1.57	-31.07	-31.69	-1.56	-30.13
Planner	-23.65	-6.40	-17.24	-22.70	-6.37	-16.32
Nash	-48.65	-31.29	-17.37	-47.71	-30.84	-16.87
- Trade War absent Pandemic	-25.23	-25.23	0.00	-25.23	-25.23	0.00
- Nash Containment Policy	-23.43	-6.06	-17.37	-22.48	-5.62	-16.87

(b) In the Unit of Lives (%)

	Country A			Country B		
	Total	Economic	Death	Total	Economic	Death
No Policy	-0.50	-0.02	-0.47	-0.48	-0.02	-0.46
Planner	-0.36	-0.10	-0.26	-0.35	-0.10	-0.25
Nash	-0.74	-0.48	-0.27	-0.73	-0.47	-0.26
- Trade War absent Pandemic	-0.39	-0.39	0.00	-0.39	-0.39	0.00
- Nash Containment Policy	-0.36	-0.09	-0.27	-0.34	-0.09	-0.26

Table 3: Health Dynamics

We report some statistics about infection and death rates in both countries.

	No Policy	Planner	Nash
Week of infection peak A	33.00	39.00	38.00
Week of infection peak B	65.00	76.00	75.00
Level of peak infection A (per 1000 households)	39.99	17.99	18.16
Level of peak infection B (per 1000 households)	39.85	17.79	18.11
Last week of pandemic A (over 0.01% infected)	104.00	172.00	196.00
Last week of pandemic B (over 0.01% infected)	135.00	210.00	238.00
Overall deaths A (per 1000 households)	4.88	2.73	2.76
Overall deaths B (per 1000 households)	4.85	2.66	2.76

A Model Appendix (For Online Publication)

A.1 The Static Model

Without pandemics, the model boils down to an essentially static two-country four-goods macro model. This is because, in order to focus on the epidemiological dynamics, in (19) we have ruled out economic dynamics. As a benchmark we now provide the basic properties of the static model. This analysis is also useful because it directly applies to the choice problems of the infected and the recovered households in the full model, who structurally solve the same static decision problems. The only truly dynamic decisions are made by susceptible households, whose choices influence their future health status.

To simplify notation, we drop country/occupation superscripts and time subscripts for the static analysis of households of country k in occupation j. Denote their wage by $w = w^{kj}$.

These consumers (who are not concerned with health) choose per-period consumption and labor $(c_{m,k}, c_{m,-k}, c_{n,k}, c_{n,-k}, \ell) \in \mathbb{R}^5_+$ in order to

$$\max v(x) - \frac{1}{2}\varphi\ell^2$$

$$\text{subject to} \qquad x = q(c_{m,k}, c_{m,-k})^{\chi} q(c_{n,k}, c_{n,-k})^{1-\chi} \tag{A.1}$$

$$\sum_{\substack{j=m,n \\ \kappa=k,-k}} \widehat{p}_{j,\kappa} c_{j,\kappa} = w\ell + g \tag{A.2}$$

where $\widehat{p}_{j,\kappa}$ are consumer prices and g is the public transfer. Let λ denote the Lagrange multiplier of the budget constraint. λ measures the pre-epidemic willingness to pay for utility, i.e. the "exchange rate between utils and dollars", which is needed to calibrate the model. As noted in Section 2.2, the solution is characterized by the following first-order conditions:

$$x^{-\rho} \frac{\partial x}{\partial c_{j,\kappa}} = \lambda \widehat{p}_{j,\kappa}, \qquad j = m, n, \kappa = k, -k \tag{A.3}$$

$$\varphi\ell = \lambda w \tag{A.4}$$

Evaluating the derivative and dividing the two equations for each index j of (A.3) by each other yields

$$\alpha^{\sigma_j} \widehat{p}_{j,-k}^{\sigma_j} c_{j,-k} = (1-\alpha)^{\sigma_j} \widehat{p}_{j,k}^{\sigma_j} c_{j,k} \tag{A.5}$$

for $j = m, n$. Hence, unsurprisingly, $c_{j,k}$ and $c_{j,-k}$ are linear functions of each other.

Inserting (A.5) into the CES aggregator (2) yields

$$\bar{c}_j = c_{j,k} \alpha^{-\sigma_j} \left(\alpha^{\sigma_j} + (1-\alpha)^{\sigma_j} \left(\frac{\widehat{p}_{j,k}}{\widehat{p}_{j,-k}} \right)^{\sigma_j - 1} \right)^{\frac{\sigma_j}{\sigma_j - 1}} \tag{A.6}$$

Letting

$$A_j = \alpha^{1-\sigma_j}\left(\alpha^{\sigma_j} + (1-\alpha)^{\sigma_j}\left(\frac{\widehat{p}_{j,k}}{\widehat{p}_{j,-k}}\right)^{\sigma_j - 1}\right) \tag{A.7}$$

the consumption aggregator x then is

$$x = c_{m,k}^{\chi} c_{n,k}^{1-\chi} A_m^{\frac{\sigma_m}{\sigma_m - 1}\chi} A_n^{\frac{\sigma_n}{\sigma_n - 1}(1-\chi)} \tag{A.8}$$

Evaluating the derivative and inserting (A.8) into (A.3) for $j = m, \kappa = k$, using (A.4), yields

$$\chi \alpha w x^{1-\rho} = \varphi \ell \widehat{p}_{m,k} c_{m,k} A_m \tag{A.9}$$

$$\Leftrightarrow \quad c_{m,k}^{(1-\rho)\chi - 1} c_{n,k}^{(1-\rho)(1-\chi)} = \frac{\varphi \widehat{p}_{m,k}}{\chi \alpha w} \ell A_m^{1-\frac{\sigma_m}{\sigma_m - 1}\chi(1-\rho)} A_n^{-\frac{\sigma_n}{\sigma_n - 1}(1-\chi)(1-\rho)} \tag{A.10}$$

Similarly, for $j = n, \kappa = k$,

$$(1-\chi)\alpha w x^{1-\rho} = \varphi \ell \widehat{p}_{n,k} c_{n,k} A_n \tag{A.11}$$

$$\Leftrightarrow \quad c_{m,k}^{(1-\rho)\chi} c_{n,k}^{(1-\rho)(1-\chi)-1} = \frac{\varphi \widehat{p}_{n,k}}{(1-\chi)\alpha w} \ell A_m^{-\frac{\sigma_m}{\sigma_m - 1}\chi(1-\rho)} A_n^{1-\frac{\sigma_n}{\sigma_n - 1}(1-\chi)(1-\rho)} \tag{A.12}$$

Inserting (A.5) into the budget constraint (A.2) to eliminate the $c_{j,-k}$ yields

$$\widehat{p}_{m,k} c_{m,k} A_m + \widehat{p}_{n,k} c_{n,k} A_n = \alpha(w\ell + g) \tag{A.13}$$

(A.10), (A.12), and (A.13) are a system of 3 equations in the 3 variables $c_{m,k}$, $c_{n,k}$, and ℓ.

By straightworward but lengthy computations we can eliminate, say, $c_{m,k}$ and ℓ and obtain the following characterization of $c_{n,k}$:

$$(c_{n,k})^{\rho+1} - E\,(c_{n,k})^{\rho} - F = 0 \tag{A.14}$$

where

$$E = \alpha(1-\chi)\frac{g}{\widehat{p}_{n,k} A_n} \tag{A.15}$$

$$F = \frac{\alpha^2 w^2}{\varphi}\left(\frac{\chi}{\widehat{p}_{m,k}}\right)^{\chi(1-\rho)}\left(\frac{1-\chi}{\widehat{p}_{n,k}}\right)^{(1-\chi)(1-\rho)+\rho+1} A_m^{\frac{1-\rho}{\sigma_m - 1}\chi} A_n^{\frac{1-\rho}{\sigma_n - 1}(1-\chi)-(\rho+1)} \tag{A.16}$$

are constants. (A.14) is a polynomial of degree $\rho + 1$ that has a unique positive root. Given this root, the other parts of the solution are

$$\ell = \frac{1}{\alpha(1-\chi)} A_n \frac{\widehat{p}_{n,k}}{w} c_{n,k} - \frac{g}{w} \tag{A.17}$$

$$c_{m,k} = \frac{\chi A_n \widehat{p}_{n,k}}{(1-\chi) A_m \widehat{p}_{m,k}} c_{n,k} \tag{A.18}$$

$c_{m,-k}$ and $c_{n,-k}$ are then obtained from (A.5). Hence, the household problem has a unique solution.

For the case $\rho = 1$, which we use in the numerical calibration, things are particular simple, as equation (A.14) is quadratic. In particular, we have

$$F = \frac{\alpha^2 w^2}{\varphi} \left(\frac{1-\chi}{\widehat{p}_{n,k} A_n} \right)^2 \tag{A.19}$$

(note that E is independent of ρ), and equation (A.14) becomes

$$(c_{n,k})^2 - E c_{n,k} - F = 0$$

which has the unique positive solution

$$c_{n,k} = \frac{1}{2} \alpha(1-\chi) \frac{g}{\widehat{p}_{n,k} A_n} \left[1 + \sqrt{1 + 4 \frac{w^2}{\varphi g^2}} \right]$$

for $g > 0$. If $g = 0$, $c_{n,k} = \frac{1}{\varphi} \alpha(1-\chi) \frac{w}{\widehat{p}_{n,k} A_n}$.

Substituting in from (A.17) and (A.18), this implies

$$c_{m,k} = \frac{1}{2} \alpha \chi \frac{g}{\widehat{p}_{m,k} A_m} \left[1 + \sqrt{1 + 4 \frac{w^2}{\varphi g^2}} \right]$$

$$\ell = \frac{1}{2} \left[\sqrt{\frac{g^2}{w^2} + \frac{4}{\varphi}} - \frac{g}{w} \right]$$

By (A.4), the multiplier λ, the "price of utility", is $\lambda = \frac{\varphi}{w} \ell$.

The remaining two optimal values are given by (A.5):

$$c_{n,-k} = \frac{1}{2} (A_n - \alpha)(1-\chi) \frac{g}{\widehat{p}_{n,-k} A_n} \left[1 + \sqrt{1 + 4 \frac{w^2}{\varphi g^2}} \right]$$

$$c_{m,-k} = \frac{1}{2} (A_m - \alpha) \chi \frac{g}{\widehat{p}_{m,-k} A_m} \left[1 + \sqrt{1 + 4 \frac{w^2}{\varphi g^2}} \right]$$

The above analysis describes the demand side of the static economy in the absence of health concerns and thus essentially the demand of infected and of recovered households in the full model in Section 2.2.

A.1.1 No-Pandemic Equilibria

We re-introduce country and occupation superscripts to describe market clearing in economies with no health concerns, be it pre-pandemic or after the arrival of a vaccine, but we omit the time subscripts. The market clearing conditions are

$$w^{kj} = p_{j,k} z^j \tag{A.20}$$

$$z^j \ell^{kj} = \sum_{\substack{\kappa=k,-k \\ j'=m,n}} c_{k,j}^{\kappa j'} \tag{A.21}$$

$k = A, B$, $j = m, n$, for the 4 labor markets and the 4 product markets, respectively.

Social Planner Under a benevolent social planner, government policy in each country will be $(\xi^k, \nu^k) = (0, 0)$: creating trading frictions for domestic or foreign goods is welfare reducing. Hence, the government collects no taxes, and by the budget constraint (18) transfers are $g^k = 0$. Consumer prices are undistorted,

$$\widehat{p}^k_{j,\kappa} = p_{j,\kappa},$$

and the 8 equations (A.20) and (A.21) together with the 4 budget constraints (A.2) are sufficient to determine the 12 values $w^{kj}, p_{j,k}, \ell^{kj},$, $k = A, B$, by using the solutions of (A.14), (A.17), and (A.18) obtained above. Of course, prices are determined only up to one degree of freedom, and by Walras' Law one of the above equilibrium relations is redundant.

Nash In Nash Equilibrium, $\xi^k_j = 0$ in each country. Yet, tariffs can be positive, for the standard economic reasons of trade wars discussed in the main text. Hence, consumer prices are

$$\begin{aligned} \widehat{p}^k_{j',k} &= p^k_{j',k} \\ \widehat{p}^k_{j,-k} &= (1 + \nu^k_j) p^k_{j,-k} \end{aligned}$$

and public transfers are given by the governments' receipts

$$g^k = \sum_{j=m,n} \nu^k_m p_{m,-k} c^{kj}_{m,-k} + \nu^k_n p_{n,-k} c^{kj}_{n,-k} \tag{A.22}$$

Now, for given government policies (ν^A, ν^B), we have the 14 equations (A.20), (A.21), (A.2), and (A.22) to determine the 14 endogenous variables $w^{kj}, \ell^{kj}, p_{j,k}, g^k$, $k = A, B, j = m, n$.

A.2 Optimization of s-indiviuals

The optimization problem of susceptible individuals is fully dynamic. If λ_t^{kjs} is the Lagrange multiplier of the budget constraint (19), the first-order conditions for the 4 consumption decisions of the domestic and foreign goods and services and that for labor are, respectively:

$$
\begin{aligned}
\left(x_t^{kjs}\right)^{-\rho} \frac{\partial x_t^{kjs}}{\partial c_{m,\kappa,t}^{kjs}} &= \lambda_t^{kjs} \widehat{p}_{m,\kappa,t}^k, \qquad \kappa = k, -k \\
\left(x_t^{kjs}\right)^{-\rho} \frac{\partial x_t^{kjs}}{\partial c_{n,\kappa,t}^{kjs}} + \beta \Big(\pi_1 \left[c_{n,\kappa,t}^{kmi} I_t^{km} + c_{n,\kappa,t}^{kni} I_t^{kn}\right] + & \\
\pi_4 \left[c_{n,\kappa,t}^{-kmi} I_t^{-km} + c_{n,\kappa,t}^{-kni} I_t^{-kn}\right]\Big) \left(V_{t+1}^{kji} - V_{t+1}^{kjs}\right) &= \lambda_t^{kjs} \widehat{p}_{n,\kappa,t}^k, \qquad \kappa = k, -k \\
\varphi \ell_t^{kjs} - \beta \pi_2 \ell_t^{kji} I_t^{kj} \left(V_{t+1}^{kji} - V_{t+1}^{kjs}\right) &= \lambda_t^{kjs} w_t^{kj}
\end{aligned}
$$

where the second terms in the third and fourth equations (those involving β) reflect the fact that consuming foreign services increases the chances of getting infected through contacts with domestic and foreign households. By assumption, the first two equations do not involve future utilities because the solitary consumption of home delivery pizza is not infectious.

Evaluating the derivatives, eliminating λ_t^{kjs}, and simplifying yields four first-order conditions for the optimal consumption choices $c_{j',\kappa,t}^{kjs}$, $j' = m, n$, $\kappa = k, -k$, and the labor supply ℓ_t^{kjs} of susceptible individuals of type kj. For this, we summarize the behavior of infected households (which the susceptible households take as given) by

$$
\mathcal{I}_\kappa^k = \pi_1 \left(c_{n,\kappa}^{kmi} I^{km} + c_{n,\kappa}^{kni} I^{kn}\right) + \pi_4 \left(c_{n,\kappa}^{-kmi} I^{-km} + c_{n,\kappa}^{-kni} I^{-kn}\right) \tag{A.23}
$$

$$
\mathcal{I}_\ell^{kj} = \pi_2 \ell^{kji} I^{kj} \tag{A.24}
$$

for $\kappa = k, -k$. The first-order conditions are then equivalent to

$$\begin{aligned}
& \chi\alpha w_t^{kj}\left(x_t^{kjs}\right)^{1-\rho} && \text{(A.25)} \\
= \; & \widehat{p}_{m,k,t}^{k}\left(c_{m,k,t}^{kjs}\right)^{1/\sigma_m}\left(\overline{c}_{m,t}^{kjs}\right)^{\frac{\sigma_m-1}{\sigma_m}}\left[\varphi\ell_t^{kjs}-\beta\mathcal{I}_{\ell,t}^{kj}\left(V_{t+1}^{kji}-V_{t+1}^{kjs}\right)\right] \\
& \alpha\widehat{p}_{m,-k,t}^{k}\left(c_{m,-k,t}^{kjs}\right)^{1/\sigma_m} = (1-\alpha)\widehat{p}_{m,k,t}^{k}\left(c_{m,k,t}^{kjs}\right)^{1/\sigma_m} && \text{(A.26)} \\
& (1-\chi)\alpha w_t^{kj}\left(x_t^{kjs}\right)^{1-\rho} && \text{(A.27)} \\
= \; & \left[\widehat{p}_{n,k,t}^{k}\varphi\ell_t^{kjs}-\beta\left(w_t^{kj}\mathcal{I}_{k,t}^{k}+\widehat{p}_{n,k,t}^{k}\mathcal{I}_{\ell,t}^{kj}\right)\left(V_{t+1}^{kji}-V_{t+1}^{kjs}\right)\right]\left(c_{n,k,t}^{kjs}\right)^{1/\sigma_n}\left(\overline{c}_{n,t}^{kjs}\right)^{\frac{\sigma_n-1}{\sigma_n}} \\
& (1-\chi)(1-\alpha)w_t^{kj}\left(x_t^{kjs}\right)^{1-\rho} && \text{(A.28)} \\
= \; & \left[\widehat{p}_{n,-k,t}^{k}\varphi\ell_t^{kjs}-\beta\left(w_t^{kj}\mathcal{I}_{-k,t}^{k}+\widehat{p}_{n,-k,t}^{k}\mathcal{I}_{\ell,t}^{kj}\right)\left(V_{t+1}^{kji}-V_{t+1}^{kjs}\right)\right]\left(c_{n,-k,t}^{kjs}\right)^{1/\sigma_n}\left(\overline{c}_{n,t}^{kjs}\right)^{\frac{\sigma_n-1}{\sigma_n}}
\end{aligned}$$

Together with the budget constraint, (A.25)–(A.28) determine the behavior of s-individuals as a function of current prices, the state of the pandemic, the current choices of infected agents, and the domestic policy parameters g_t^k and ξ_t^k, ν_t^k (which are inherent in the consumer prices $\widehat{p}_{k,t}^k, \widehat{p}_{-k,t}^k$).

The above system can be further simplified. Again, we simplify notation by dropping country/occupation and this time also health superscripts and time subscripts for households of country k in occupation j with health status s. Again, the wage is $w = w^{kj}$. Using the notation (A.23) and (A.24) that summarizes the behavior of infected individuals we let

$$\begin{aligned}
V_\Delta &= \beta\left(V_{t+1}^{kji}-V_{t+1}^{kjs}\right) \\
R_\kappa &= \frac{w}{p_{n,\kappa}}\mathcal{I}_\kappa+\mathcal{I}_\ell
\end{aligned}$$

for $\kappa = k, -k$. Then the first-order conditions (A.25)–(A.28) are

$$\chi\alpha w x^{1-\rho} = \widehat{p}_{m,k}\left[\varphi\ell-\mathcal{I}_\ell V_\Delta\right]c_{m,k}^{\frac{1}{\sigma_m}}\overline{c}_m^{\frac{\sigma_m-1}{\sigma_m}} \tag{A.29}$$

$$\alpha\widehat{p}_{m,-k}c_{m,-k}^{\frac{1}{\sigma_m}} = (1-\alpha)\widehat{p}_{m,k}c_{m,k}^{\frac{1}{\sigma_m}} \tag{A.30}$$

$$(1-\chi)\alpha w x^{1-\rho} = \widehat{p}_{n,k}\left[\varphi\ell-R_k V_\Delta\right]c_{n,k}^{\frac{1}{\sigma_n}}\overline{c}_n^{\frac{\sigma_n-1}{\sigma_n}} \tag{A.31}$$

$$(1-\chi)(1-\alpha)w x^{1-\rho} = \widehat{p}_{n,-k}\left[\varphi\ell-R_{-k}V_\Delta\right]c_{n,-k}^{\frac{1}{\sigma_n}}\overline{c}_n^{\frac{\sigma_n-1}{\sigma_n}} \tag{A.32}$$

Using (A.7) and (A.30) as in the static case, one can eliminate $c_{m,-k}$ in (A.29) to obtain

$$\chi\alpha w x^{1-\rho} = \widehat{p}_{m,k}\left[\varphi\ell-\mathcal{I}_\ell V_\Delta\right]c_{m,k}A_m \tag{A.33}$$

This is not possible for non-manufactured goods (n) because of the infection dynamics from the consumption of services. Dividing (A.31) by (A.32) and defining

$$B_n = \alpha^{1-\sigma_n}\left(\alpha^{\sigma_n} + (1-\alpha)^{\sigma_n}\left(\frac{\widehat{p}_{n,k}}{\widehat{p}_{n,-k}}\right)^{\sigma_n-1}\left(\frac{\varphi\ell - R_k V_\Delta}{\varphi\ell - R_{-k}V_\Delta}\right)^{\sigma_n-1}\right) \tag{A.34}$$

one obtains

$$(1-\chi)(1-\alpha)wx^{1-\rho} = \widehat{p}_{n,k}\left[\varphi\ell - R_k V_\Delta\right]c_{n,k}B_n \tag{A.35}$$

(A.35) is structurally different from (A.33) because the term B_n contains ℓ, i.e. is not constant (for the household's decision problem).

Proceding as in the static case for (A.13) yields the budget constraint

$$A_m p_{m,k}c_{m,k} + (B_n - D_n)p_{n,k}c_{n,k} = \alpha(w\ell + g) \tag{A.36}$$

where

$$D_n = \alpha\frac{(R_{-k}-R_k)V_\Delta}{\varphi\ell - R_{-k}V_\Delta} = \alpha w\frac{p_{n,k}\mathcal{I}_{-k} - p_{n,-k}\mathcal{I}_k}{p_{n,k}p_{n,-k}}\frac{V_\Delta}{\varphi\ell - R_{-k}V_\Delta}$$

Setting

$$f = \alpha\frac{p_{n,k}\mathcal{I}_{-k} - p_{n,-k}\mathcal{I}_k}{p_{n,k}p_{n,-k}}$$

this transforms (A.36) into

$$\begin{aligned} & \left[\varphi\ell - R_{-k}V_\Delta\right]A_m p_{m,k}c_{m,k} + \left[\varphi\ell - R_{-k}V_\Delta\right]B_n p_{n,k}c_{n,k} - fwV_\Delta p_{n,k}c_{n,k} \\ = \; & \alpha(w\ell+g)\left[\varphi\ell - R_{-k}V_\Delta\right] \end{aligned} \tag{A.37}$$

To conclude the characterization we now assume that $\rho = 1$.

Inserting (A.33) and (A.35) into (A.37) and using (A.36) yields

$$\begin{aligned} & B_n\left[\varphi\ell - R_{-k}V_\Delta\right]\left(\frac{\varphi\ell - R_k V_\Delta}{\varphi\ell - \mathcal{I}_\ell V_\Delta}\alpha\chi + (1-\alpha)(1-\chi) - \alpha(\ell + \frac{g}{w})\left[\varphi\ell - R_k V_\Delta\right]\right) \\ = \; & (1-\alpha)(1-\chi)wfV_\Delta \end{aligned} \tag{A.38}$$

(A.38) is an equation in ℓ only, in the sense that the other 4 of the 5 decision variables $(c_{m,k}, c_{m,-k}, c_{n,k}, c_{n,-k}, \ell) \in \mathbb{R}^5_+$ have been eliminated. If $\sigma_n \in \mathbb{N}$ then (A.38) is a polynomial of degree $\sigma_n + 2$. The solution of (A.38) determines B_n by (A.34). (A.33) and (A.35) then determine $c_{m,k}$ and $c_{n,k}$. $c_{m,-k}$ and $c_{n,-k}$ are then given by (A.30) and (A.32). Remember that these values are truly dynamic in the sense that next to current prices and wages, they depend on the future as given by V_Δ.

A.3 Optimization of i-indiviuals

Letting λ_t^{kji} denote the multiplier of the budget constraint, the problem yields the following 5 first-order conditions

$$\begin{aligned}\left(x_t^{kji}\right)^{-\rho}\frac{\partial x_t^{kji}}{\partial c_{j',\kappa,t}^{kji}} &= \lambda_t^{kji}\widehat{p}_{j',\kappa,t}^{k}, \qquad j'=m,n,\kappa=k,-k \\ \varphi\ell_t^{kji} &= \lambda_t^{kij}w_t^{kj}\end{aligned}$$

Simplifying and eliminating λ_t^{kji} as above yields the four conditions

$$\alpha\widehat{p}_{j',-k,t}^{k}\left(c_{j',-k,t}^{kji}\right)^{1/\sigma_{j'}} = (1-\alpha)\widehat{p}_{j',k,t}^{k}\left(c_{j',k,t}^{kji}\right)^{1/\sigma_{j'}}, \qquad j'=m,n \tag{A.39}$$

$$\chi\alpha w_t^{kj}\left(x_t^{kji}\right)^{1-\rho} = \varphi\ell_t^{kji}\widehat{p}_{m,k,t}^{k}\left(c_{m,k,t}^{kji}\right)^{1/\sigma_m}\widehat{c}_{m,t}^{kji} \tag{A.40}$$

$$(1-\chi)\alpha w_t^{kj}\left(x_t^{kji}\right)^{1-\rho} = \varphi\ell_t^{kji}\widehat{p}_{n,k,t}^{k}\left(c_{n,k,t}^{kji}\right)^{1/\sigma_n}\widehat{c}_{n,t}^{kji} \tag{A.41}$$

where (A.39) is (A.5). This system together with the budget constraint uniquely determines the 5 choice variables of infected households in t, as derived in Appendix A.1.

A.4 Optimization of r-individuals

As in the case of infected individuals, the unique solution of this problem is given by the 4 first-order conditions

$$\alpha\widehat{p}_{j',-k,t}^{k}\left(c_{j',-k,t}^{kjr}\right)^{1/\sigma_{j'}} = (1-\alpha)\widehat{p}_{j',k,t}^{k}\left(c_{j',k,t}^{kjr}\right)^{1/\sigma_{j'}}, \qquad j'=m,n \tag{A.42}$$

$$\chi\alpha w_t^{kj}\left(x_t^{kjr}\right)^{1-\rho} = \varphi\ell_t^{kjr}\widehat{p}_{m,k,t}^{k}\left(c_{m,k,t}^{kjr}\right)^{1/\sigma_m}\widehat{c}_{m,t}^{kjr} \tag{A.43}$$

$$(1-\chi)\alpha w_t^{kj}\left(x_t^{kjr}\right)^{1-\rho} = \varphi\ell_t^{kjr}\widehat{p}_{n,k,t}^{k}\left(c_{n,k,t}^{kjr}\right)^{1/\sigma_n}\widehat{c}_{n,t}^{kjr} \tag{A.44}$$

together with the budget constraint.

A.5 Disease Transmission

This subsection provides a more detailed microfoundation for the disease transmission dynamics (10) in Section 1.2.

In the basic SIR model (without economic choices) transmission occurs according to

$$T_t = \eta S_t I_t \tag{A.45}$$

This has the following logic. Let N be size of a given population. Let $N = S + I +$

R, where I is the number of infectious, and S that of susceptibles. Let φN be the rate of contacts of a single individual during which the disease can potentially be transmitted.[24] The assumption is that individuals spend a fixed proportion of their time (normalized to 1) outside the home, where they can transmit or contract the virus. Letting θ denote the probability that a contact leads to an infection, equation (A.45) can now be derived as follows.[25] One susceptible individual outside his home, per unit of time, on average has φN contacts. This leads to $\varphi N(I/N) = \varphi I$ contacts with infectious individuals. The probability of getting infected in these $k = \varphi I$ contacts is

$$\overline{\tau} = 1 - (1-\theta)^k = \theta \sum_{m=0}^{k-1} \binom{k}{m+1} (-\theta)^m \tag{A.46}$$

for $k > 0$, and the expected total number of transmissions per unit of time is $\overline{\tau} S$. $\overline{\tau}$ as a function of θ is a polynomial of degree k and strictly concave for $k > 1$. Hence, for small θ and large k, $\overline{\tau}$ is smaller than, but approximately equal to $k\theta$. In this case, letting $\eta = \theta\varphi$, the average rate of transmission is approximately equal to

$$\theta k S = \theta \varphi I S = \eta I S$$

as stated in (A.45).

A.5.1 The Macro-SIR Model

Eichenbaum et al. (2020) have incorporated economic activity into the above model, by distinguishing transmissions while consuming, at work, and during other activities outside the home. This model does not distinguish between foreign and domestic consumption goods, nor between different sectors.

To make that precise, dropping the time index for convenience, suppose that individuals spend a fixed fraction $f < 1$ of their time outside neither at work nor consuming. All durations are in terms of the unit of time chosen (which is scaled by φ).[26] To simplify, and different from Eichenbaum, Rebelo and Trabandt (2021); Brotherhood et al. (2020), we do not distinguish between utility from different types of leisure. Hence, individuals do not derive specific utility

[24]This is the so-called "mass incidence" model which is relevant for Covid-19 (differently from, say, HIV, as analyzed in Greenwood et al. (2019)): one infectious individual can infect a whole (sub-)group, no need for bilateral interaction.

[25]This is the perspective of susceptibles, which is most relevant for economic incentives. Usually, the derivation takes the perspective of infectious. See standard textbooks such as Brauer (2008).

[26]If this unit is a week and a day has 16 useful hours (e.g. McGrattan, Rogerson et al., 2004), then the individual has $112f$ hours of non-shopping leisure per week outside the home.

from leisure outside the home, and we therefore assume this fraction to be constant.[27] Suppose that individuals of health status h spend a fraction $\ell(h) < 1$ of their time at work, and a fraction $\gamma c(h) < 1$ consuming (shopping, dining, ...), the assumption being that the time spent on consumption is proportional to the quantity bought. We assume that $f + \ell(h) + \gamma c(h) < 1$, the remaining time being leisure alone at home.[28] Then, using the linear approximation of the infection probability $\overline{\tau}$, we have the following infection probabilities for susceptibles and aggregate average transmission rates:

1. During non-work-non-consumption time outside the home,
   - individual proba of becoming infected: $f^2\eta I$
   - expected total number of transmissions: $f^2\eta IS$
2. During work,
   - average rate of susceptible contacts with infected per unit of time: $\varphi\ell(i)I$
   - individual proba of becoming infected when working: $\ell(s)\eta\ell(i)I$
   - expected total number of transmissions at work: $\eta\ell(s)\ell(i)IS$
3. During consumption,
   - average rate of contacts with infected per unit of time: $\varphi\gamma c(i)I$
   - individual proba of becoming infected when consuming $c(s)$: $\gamma c(s)\eta\gamma c(i)I$
   - expected total number of transmissions from consumption: $\eta\gamma^2 c(s)c(i)IS$

Hence, an s individual faces the following transition probability to the infected state, if she chooses individual consumption $c(s)$ and labor supply $\ell(s)$:

$$\tau(c(s), \ell(s)) = f^2\eta I + \ell(s)\eta\ell(i)I + c(s)\eta\gamma^2 c(i)I \tag{A.47}$$

$$= \eta\left[\gamma^2 c(s)c(i) + \ell(s)\ell(i) + f\right] I \tag{A.48}$$

This yields the expected total number of transmissions from all activities, now with time indices:

$$T_t = \eta\left(\gamma^2 c_t(s)c_t(i) + \ell_t(s)\ell_t(i) + f\right) I_t S_t \tag{A.49}$$

A.5.2 International transmission

Again dropping the time index for convenience, suppose individuals of country k and health status h spend a fraction $\ell^k(h)$ of their time at work, a fraction $\gamma c_k^k(h)$ of their time consuming

[27] See Garibaldi, Moen and Pissarides (2020) for work that endogenizes f in a model of occupational choice, abstracting from the work-consumption choice considered here.

[28] We calibrate the parameter values such that the individual time constraints are satisfied in our simulations. Hence, we can ignore the time constraint in the household's optimization problem of (24).

the domestic good, a fraction $\gamma c_{-k}^{k}(h)$ consuming the foreign good, and a fraction f out of their home for other reasons.[29] When "shopping", an individual is directly exposed to home residents and foreigners. Since the contact intensity for foreign and domestic consumption is likely to differ we assume that the consumer has a contact rate $\varphi^{d}\gamma(C_{k}^{k}+C_{-k}^{k})$ with domestic residents and a contact rate $\varphi^{f}\gamma(C_{k}^{-k}+C_{-k}^{-k})$ with foreigners. In fact, when consuming the domestic good, an individual in country k meets foreign consumers who consume her domestic good, which leads to a number of contacts per unit of time of $\varphi^{f}\gamma C_{k}^{-k}$. And when consuming the foreign good, she meets foreign consumers who consume this good, i.e. their domestic good, which leads to a number of contacts per unit of time of $\varphi^{f}\gamma C_{-k}^{-k}$. Since the consumption of foreign goods is often intermediated by specialized import/export agents and thus likely to involve fewer direct contacts, we expect $\varphi^{f}<\varphi^{d}$.

We ignore international encounters at work and in non-work-non-consumption situations. Hence, the transmission dynamics is unchanged from the previous subsection as regards these two types of encounters. With respect to consumption related transmissions, a susceptible consuming the bundle $(c_{k}^{k}(s),c_{-k}^{k}(s))$ has an average rate of contacts with infected per unit of time of

$$\gamma\varphi^{d}(c_{k}^{k}(i)+c_{-k}^{k}(i))I^{k}+\gamma\varphi^{f}(c_{k}^{-k}(i)+c_{-k}^{-k}(i))I^{-k}$$

where $\gamma\varphi^{d}c_{x}^{k}(i)I^{k}$ are the contacts with domestic infected and $\gamma\varphi^{f}c_{x}^{-k}(i)I^{-k}$ those with foreign infected individuals.

Hence, her individual probability of becoming infected through consumption is approximately

$$c_{k}^{k}(s)\theta\gamma^{2}\left[\varphi^{d}c_{k}^{k}(i)I^{k}+\varphi^{f}c_{k}^{-k}(i)I^{-k}\right]+c_{-k}^{k}(s)\theta\gamma^{2}\left[\varphi^{d}c_{-k}^{k}(i)I^{k}+\varphi^{f}c_{-k}^{-k}(i)I^{-k}\right]$$

Adding the infection probabilities yields the formulas (9) and (10) in the main text (for the case of one good per country). These transmission dynamics are the simplest possible generalization of those of the single good case (A.49). The new terms reflect the transmissions through consumption interactions in exports ($c_{kt}^{-k}(i)$) and imports ($c_{-kt}^{k}(i)$) and therefore also involve foreign consumption abroad, $c_{-kt}^{-k}(i)$. More complicated interaction models (interactions at work or between consumption and leisure) do not change the results significantly.

A.6 Computation Details

The numerical algorithm for solving our model proceeds in a number of steps. We first detail the solution to the model for fixed containment policies and then detail the solution for the

[29]To simplify the exposition, we now consider the case of one single good per country. The generalization to two sectors is straightforward.

optimal coordinated and uncoordinated policies.

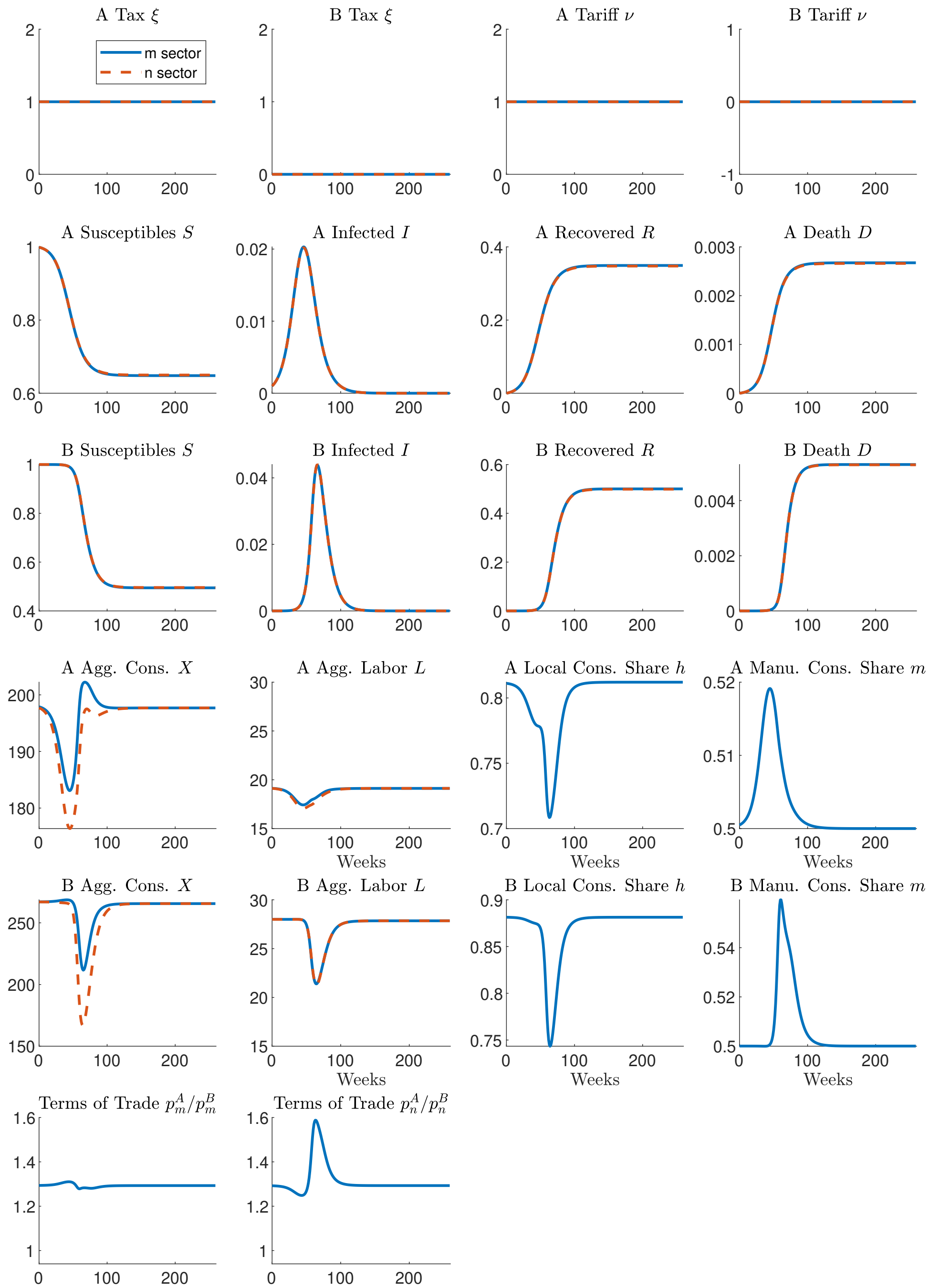
Solution for fixed policies. To solve the model for a fixed set of containment taxes, we begin with guesses for consumption, government transfers, and prices, normalizing the price of country A's manufactured good to 1 and also begin with guesses for the remaining 3 goods. With these guesses we compute labor supply in both countries and sectors. We then iterate forward on the SIR equations until the final period of the model, at which point consumption and labor return to their steady state values due to the vaccine's arrival. Next, we iterate backward to derive the present value of lifetime utility for each agent. We then use gradient-based methods to adjust our initial guesses until the all household types' first-order conditions, market clearing conditions, and government budget constraints hold. In this way, we confirm all equilibrium conditions are satisfied for a given set of containment policies.

Social planner solution. To solve for optimal containment policies from the perspective of a social planner, we nest the solution for fixed policies within another gradient-based optimizer. In this outer loop, we solve for containment policies and tariffs which maximize the present value of total time-0 utility, equally weighted across both countries.

Nash equilibrium solution. To solve for the Nash Equilibrium containment policies we begin with a guess for containment policies and tariffs across both countries. Given a fixed policy for a given country, we use a gradient-based optimizer to find the optimal policy response of the other country that maximizes the welfare of its own households. We then take this policy as fixed and find the optimal policy response of the other country. We iterate on this procedure until both countries' policies are the best responses to each other. We experiment with many different starting values but do not find any differences in the final result, which makes us believe that the identified Nash equilibrium is unique.

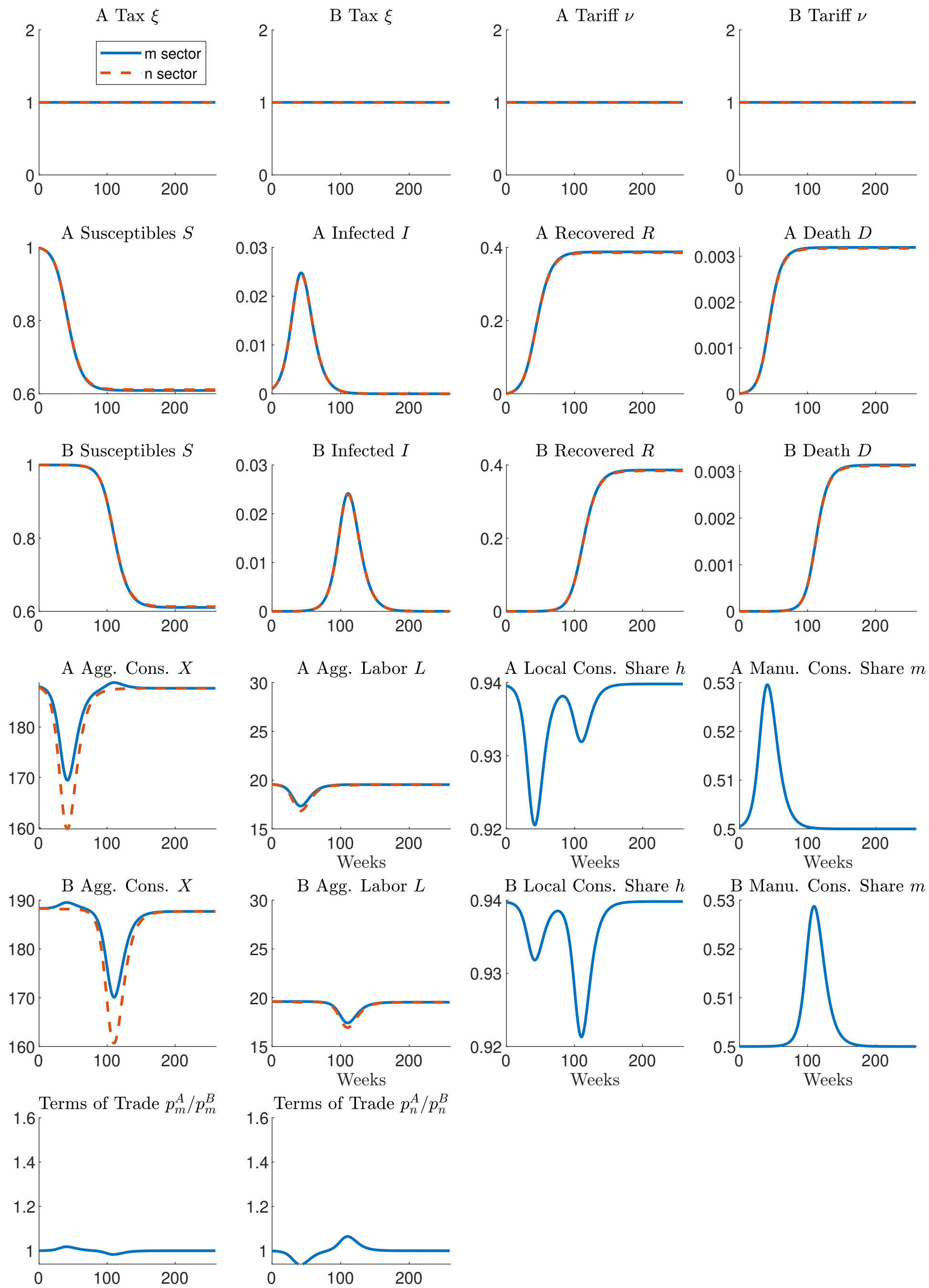
B Zero-Covid Policy (For Online Publication)

Figure B.1: SIR Dynamics with One-Country Zero-Covid Policy



Note: Model with international transmission of pandemic. Country A imposes 100% taxes and tariffs, while country B does not impose domestic containment policies or tariffs.

Figure B.2: SIR Dynamics with Two-Country Zero-Covid Policy



Note: Model with international transmission of pandemic. Country A and B impose 100% taxes and tariffs.